Plant Tillering - Fibonacci Numbers

Gunther Krauss

Crop Science Research Group

Institute of Crop Science and Resource Conservation (INRES)

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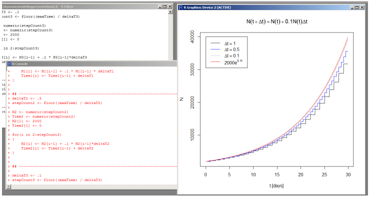


Plant tillering

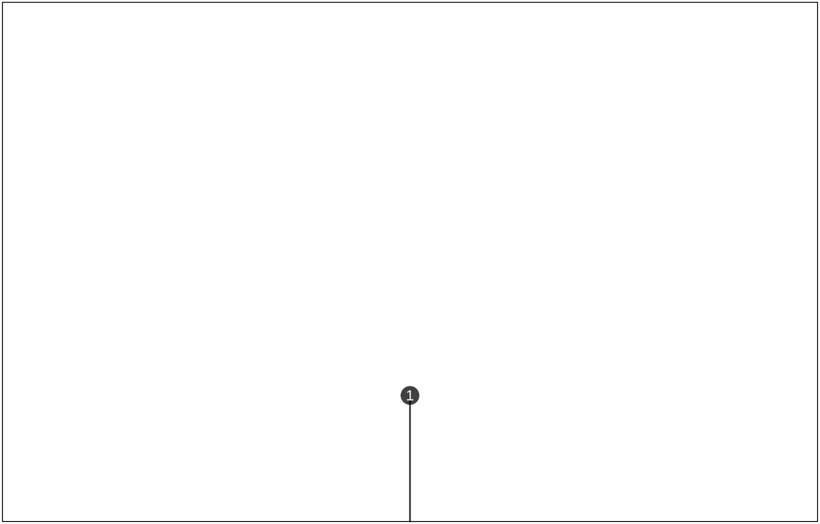
Under optimal conditions the number of new tillers at each stage follow a mathematic pattern:

$$t_1 = 1 \tag{1}$$
$$t_2 = 1 \tag{2}$$
$$t_n = t_{n-1} + t_{n-2} \quad \text{if} \quad n \geq 3 \tag{3}$$

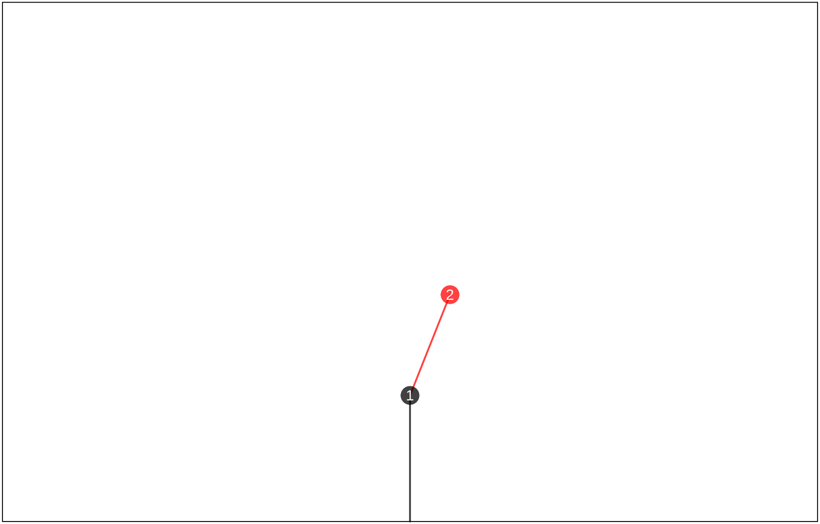
where t_n is the number of tillers at stage n.

These numbers are well known under the name of **Fibonacci Numbers**

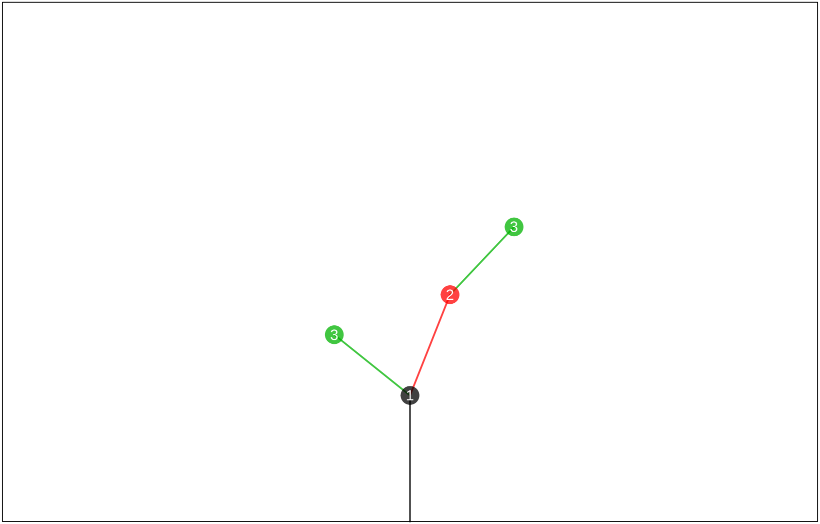
Visualization of tillering stage 1



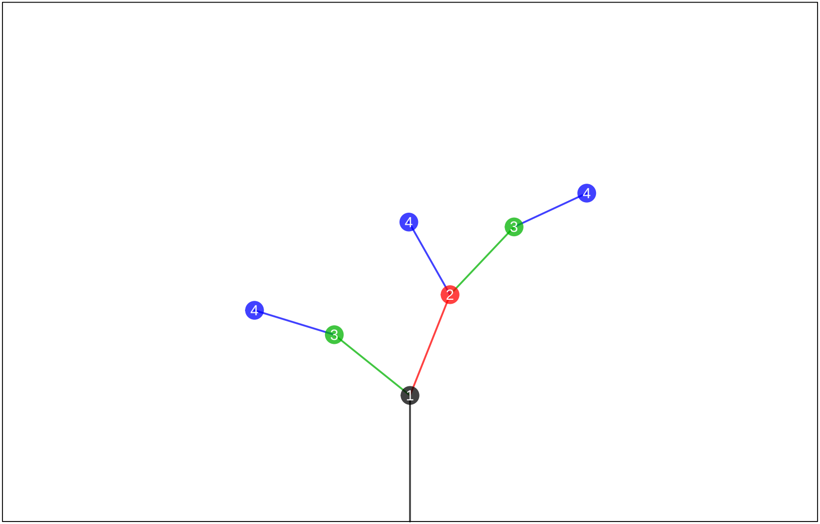
Visualization of tillering stage 2



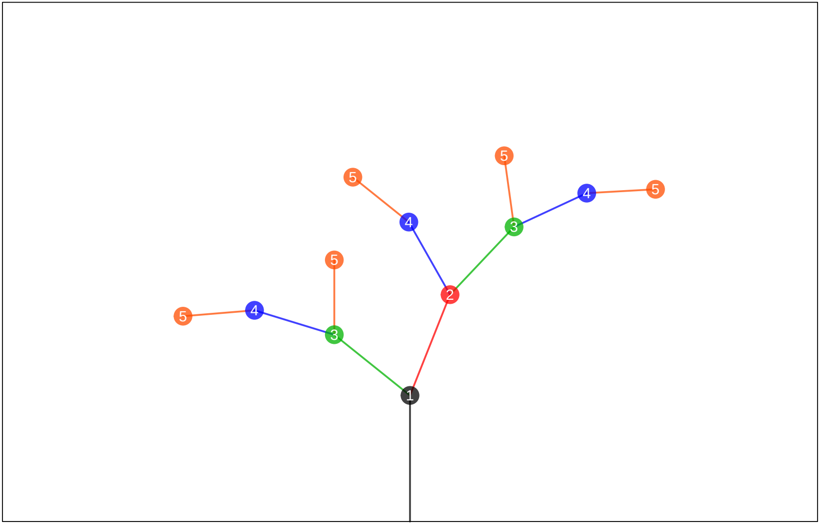
Visualization of tillering stage 3



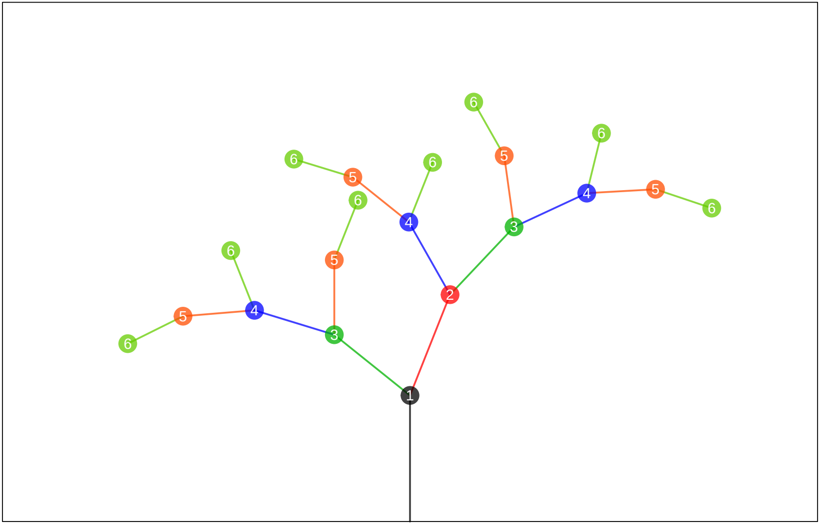
Visualization of tillering stage 4



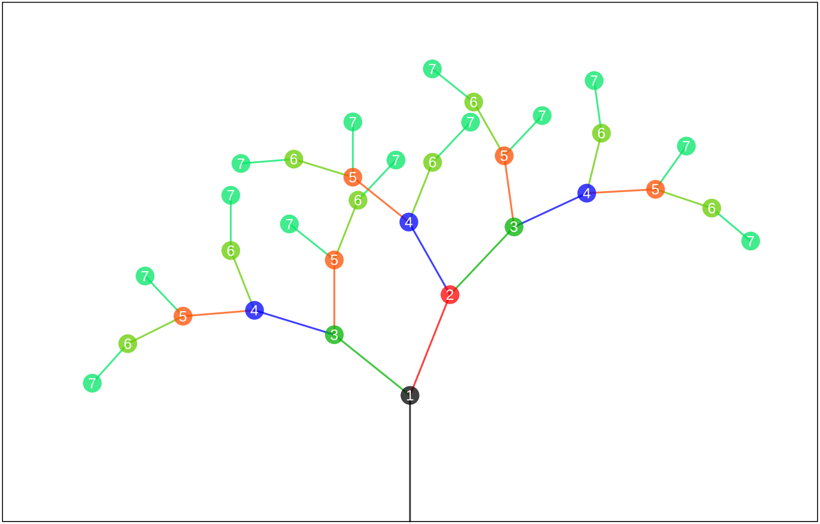
Visualization of tillering stage 5



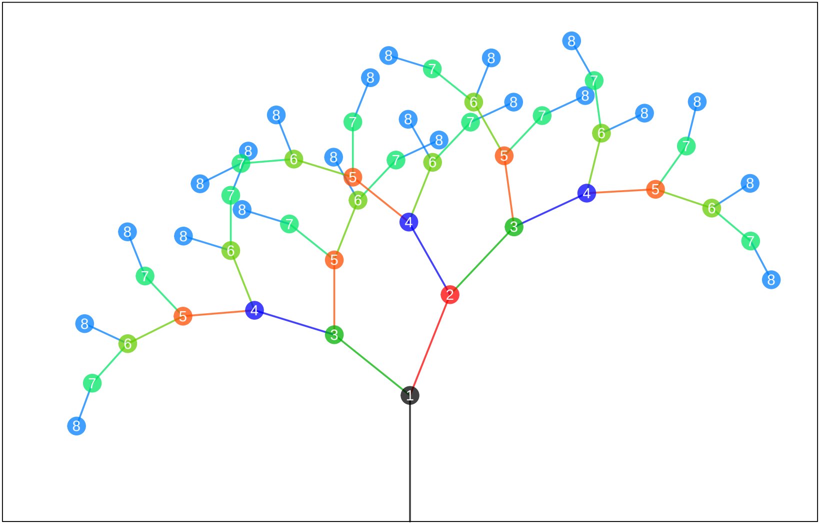
Visualization of tillering stage 6

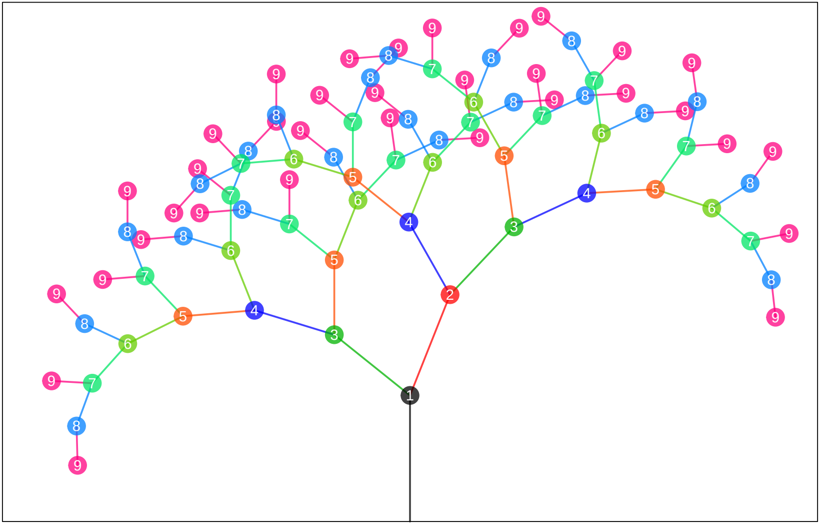


Visualization of tillering stage 7



Visualization of tillering stage 8





Questions

1. How can we calculate the number of new tillers at each stage?
2. How can we calculate the number of total tillers at each stage?
3. Given phyllochron and daily mean temperature, how many tillers are grown after a specific time?
4. How is the relationship between daily mean temperature and number of tillers?

We use R to answer these questions.

Calculation - first approach

Remind the formula

$$t_1 = 1, \quad t_2 = 1, \quad t_n = t_{n-1} + t_{n-2}$$

```
t1 <- 1
t2 <- 1
t3 <- t2 + t1
t3
```

```
# [1] 2
```

```
t4 <- t3 + t2
t4
```

```
# [1] 3
```

```
t5 <- t4 + t3
t6 <- t5 + t4
t7 <- t6 + t5
t8 <- t7 + t6
t8
```

```
# [1] 21
```

What is the value of t12? And t50???

Using the for-loop

We use the for-loop to calculate the numbers of new tillers up to stage 50.

```
m <- 50
t <- numeric(m)
t[1] <- 1
t[2] <- 1
for(n in 3:m)
{
  t[n] <- t[n-1] + t[n-2]
}
```

Displaying the number of new tillers

1	89	10946	1346269	165580141
1	144	17711	2178309	267914296
2	233	28657	3524578	433494437
3	377	46368	5702887	701408733
5	610	75025	9227465	1134903170
8	987	121393	14930352	1836311903
13	1597	196418	24157817	2971215073
21	2584	317811	39088169	4807526976
34	4181	514229	63245986	7778742049
55	6765	832040	102334155	12586269025

Total numbers of tillers

Total number of tillers of a stage is the sum of the actual stage tillers and previous stages tillers.

```
tt2 <- t[1] + t[2]
tt3 <- t[1] + t[2] +t[3]
```

Better to use the sum function

```
tt4 <- sum(t[1:4])
tt5 <- sum(t[1:5])
```

Total numbers of tiller

How can we do this more efficient? For-loop?

```
tt<- numeric(m)
for(n in 1:m)
{
  tt[n] <- sum(t[1:n])
}
```

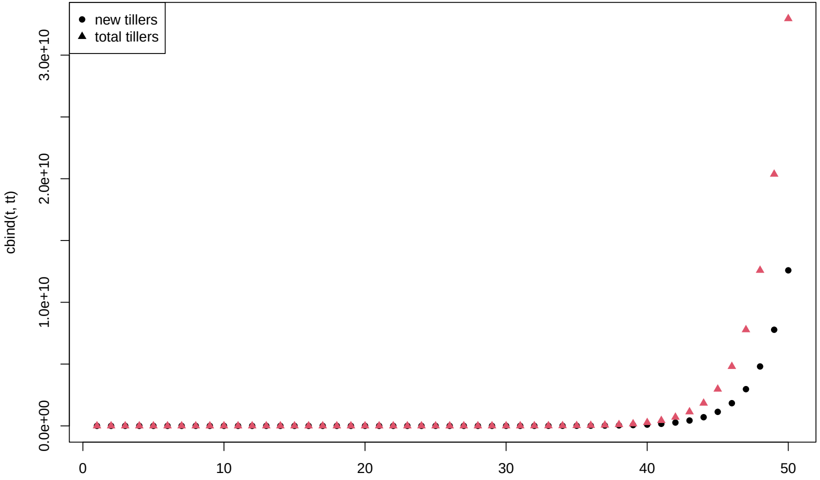
We can do it even simpler, as R already has the `cumsum` function:

```
tt <- cumsum(t)
```

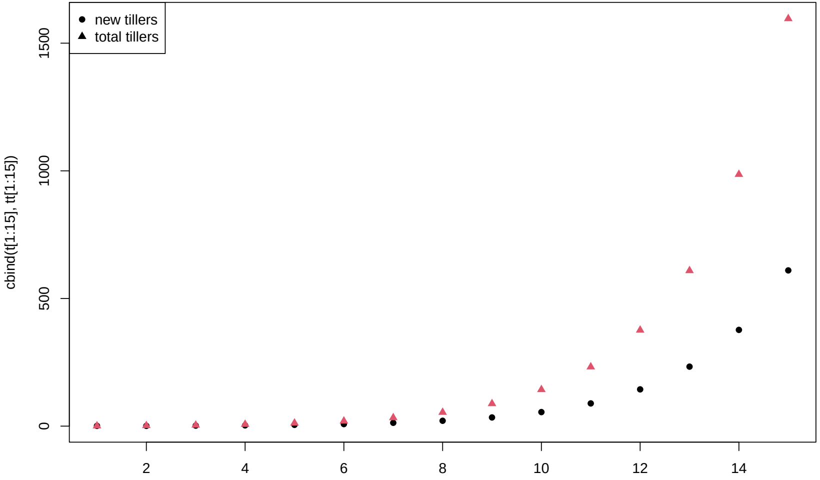
Displaying the number of total tillers

1	232	28656	3524577	433494436
2	376	46367	5702886	701408732
4	609	75024	9227464	1134903169
7	986	121392	14930351	1836311902
12	1596	196417	24157816	2971215072
20	2583	317810	39088168	4807526975
33	4180	514228	63245985	7778742048
54	6764	832039	102334154	12586269024
88	10945	1346268	165580140	20365011073
143	17710	2178308	267914295	32951280098

Plot new and total tillers



Zoom in



new tillers
total tillers
cbind(t[1:15], tt[1:15])
0
500
1000
1500
2
4
6
8
10
12
14

Phyllochron

Phyllochron is the accumulated temperature sum between two stages. Let's take a value of *120 °C day* for the phyllochron.

How many days does it take between two stages, if the daily mean temperature is *8 °C*? And when it is *10 °C*? *11 °C*?

```
Tphyl <- 120
Tphyl / 8
```

```
# [1] 15
```

```
Tphyl / 10
```

```
# [1] 12
```

```
Tphyl / 11
```

```
# [1] 10.90909
```

As R can do arithmetic calculations with vectors, we can perform it in one step:

```
Temp <- c(8,10,11)
Tphyl/Temp
```

```
# [1] 15.00000 12.00000 10.90909
```

Temperature, Phyllochron => Number of Tillers

If we have phyllochron and temperature, we want to calculate the number of tillers after a specific time period:

Temperature, Phyllochron => Number of Tillers

We already have calculated the number of tillers for different stages. So we need only to calculate the stages from temperature and phyllochron:

Temperature, Phyllochron => Stages

Stages => Number of Tillers

Environment and Crop properties

- Temperature is an environmental variable, phyllochron is a crop property.
- In this exercise we will analyze the relationship between environment and number of tillers:
  Temperature, Phyllochron => Number of Tillers
- In the assignment you will analyze the relationship between different crop varieties and number of tillers:
  Temperature, **Phyllochron** => Number of Tillers

Calculation of stages

We consider a crop with a phyllochron `Tphyl` of 120 °C day.

For simplicity, we assume that we have the same daily mean temperature `Tdmean` on each day.

How many stages can occur within 100 days when Tphyl is 120 °C day and Tdmean is 8 °C?

Calculation of stages - Result

```
Tdmean <- 8
Tphyl <- 120
time <- 100
Tsum <- Tdmean * time
stages <- Tsum/Tphyl
stages
```

```
# [1] 6.666667
```

6.6666667 means that 6 stages are completed. That means that we have to round down the calculated number. R has three functions ***floor*** (round down), ***ceiling*** (round up) and ***round***.

Rounding

```
nm <- c(11.7, 14.3)
round(nm)
```

```
# [1] 12 14
```

```
floor(nm)
```

```
# [1] 11 14
```

```
ceiling(nm)
```

```
# [1] 12 15
```

Calculating number of completed stages

Calculate the stages for all Tdmeans from *6 °C* to *18 °C*.

Calculating number of completed stages - Result

```
Tphyl <- 120
Tdmean <- 6:18
time <- 100
Tsum  <- Tdmean * time
stages <- floor(Tsum/Tphyl)
```

Td	6	7	8	9	10	11	12	13	14	15	16	17	18
stages	5	5	6	7	8	9	10	10	11	12	13	14	15

Finally the number of tillers.

Assume the daily mean temperature is *8 °C* and a time period of *100 days*, then with a phyllochron of *120 °C* day, *6* stages are completed. To get the number of new tillers, we take t[6], and the total number of tillers is tt[6].

```
t[6]
```

```
# [1] 8
```

```
tt[6]
```

```
# [1] 20
```

Number of tillers for all Tdmean

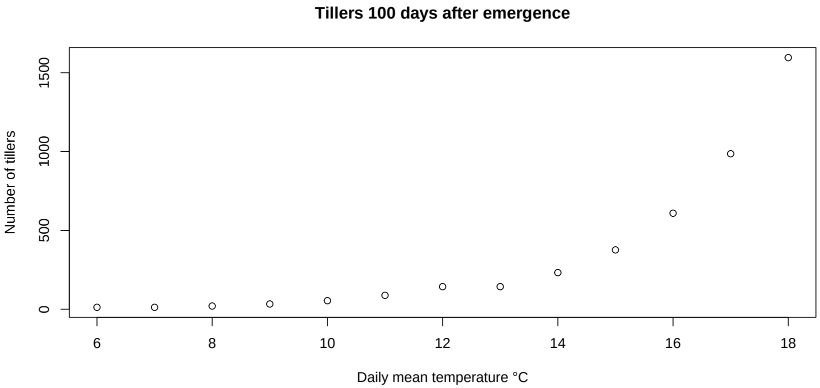
```
Tphyl <- 120
Tdmean <- 6:18
time <- 100
Tsum <- Tdmean * time

stages <- floor(Tsum/Tphyl)

newtil <- t[stages]
tottil <- tt[stages]
```

Plot total tillers vs daily means

```
plot(Tdmean, tottil,
     xlab="Daily mean temperature °C",
     ylab="Number of tillers",
     main="Tillers 100 days after emergence")
```



To loop or not to loop?

- We used a loop to calculate the number of tillers for different stages
- We calculated the stages for different temperatures without an explicit loop

Avoiding explicit loops make the code often simpler and better understandable.

```
x <- c(1,5,7,2,1)
y <- c(3,4,3,2,1)
z <- x + y
```

```
x <- c(1,5,7,2,1)
y <- c(3,4,3,2,1)
z <- numeric(length(x))
for(i in seq_along(x))
{
 z[i] <- x[i] + y[i]
}
```

Why did we then use a loop for calculating the tiller numbers?

We can't avoid explicit loops if one element in the vector depends from the previous elements, i.e. when we have to calculate the values iteratively.

```
stages[i] <- floor(Tdmean[i]*time/Tphyl)
```

```
t[i] <- t[i-1] + t[i-2]
```