Dynamic Models with R

Gunther Krauss

January 18, 2022

This handout is a supplement to the presentation of the second modeling exercise. You can use it if you want to type and run the R code that was presented during the exercise.

1 General scheme to calculate a dynamic model

If we have a dynamic model where we know the initial state and know the rate calculation function, we can calculate all the states step by step.[1]

$$
\begin{aligned}
S_1 &= a && \text{start value} \\
&\dots \\
\Delta S_{i-1} &= \dots && \text{rate calculation formula} \\
S_i &= S_{i-1} + \Delta S_{i-1} && \text{calculate new state} \\
&\dots
\end{aligned}
$$

This scheme is known as **Euler Method**.

[1] For dynamic models, that are not **difference** equations $\Delta S = \dots$ but **differential** equations $\frac{dS}{dt} = ..$ one has to use possibly more advanced calculation methods.

2 Unrestricted growth

We use an unrestricted growth model with have the rate formula

$$\Delta N_i = 0.1 N_i \quad \text{for all } i$$

.

and an initial value $N_1 = 2000$

2.1 Manual calculation

With *R* we can calculate the number:

```
N1 <- 2000

deltaN1 <- 0.1 * N1
N2 <- N1 + deltaN1

deltaN2 <- 0.1 * N2
N3 <- N2 + deltaN2

deltaN3 <- 0.1 * N3
N4 <- N3 + deltaN3

N4
```

```
# [1] 2662
```

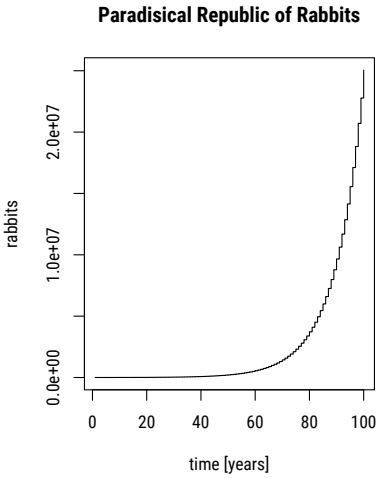
2.2 Using a loop

```
EndTime <- 100
N <- numeric(EndTime)
deltaN <- numeric(EndTime)
N[1] <- 2000
for(n in 2:EndTime)
{
  deltaN[n-1] <- 0.1 * N[n-1]
  N[n] <- N[n-1] + deltaN[n-1]
}
N[EndTime]
```

```
# [1] 25055659
```

2.3 Plot the result

```
plot(N, type="s", main="Paradisical Republic of Rabbits",
  xlab="time [years]", ylab="rabbits")
```



3 Restricted growth

We now use following formula for a restricted growth model

$$\Delta NR_i = \begin{cases} 0.1 NR_i & \text{if} \quad NR_i \leq 1000000 \\ 100000 & \text{else} \end{cases}$$

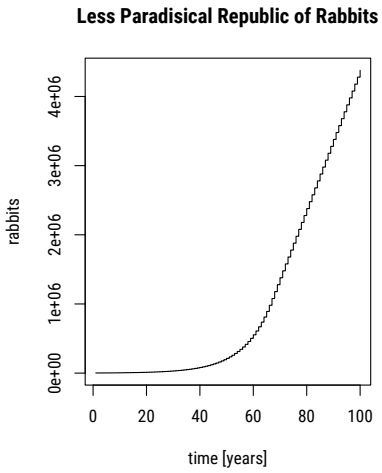
3.1 Restricted growth - calculation

```
EndTime <- 100
NR <- numeric(EndTime)
deltaNR <- numeric(EndTime)
NR[1] <- 2000
for(n in 2:EndTime)
{
  if(NR[n-1] <= 1000000)
  {
    deltaNR[n-1] <- 0.1 * NR[n-1]
  }
  else
  {
    deltaNR[n-1] <- 100000
  }
  NR[n] <- NR[n-1] + deltaNR[n-1]
}
NR[EndTime]
```

```
# [1] 4378816
```

3.2 Restricted growth - plot

```
plot(NR, type="s",
         main="Less Paradisical Republic of Rabbits",
         xlab="time [years]", ylab="rabbits")
```



3.3 Comparison - plot

```
matplot(cbind(N,NR), type=c("s","s"),lty=c(1,1),
         main="Unrestricted and restricted growth",
         xlab="time [years]", ylab="rabbits",
         col=c("red","blue"))
legend(x=0,y=2.5E7,
       legend=c("Unrestricted","Restricted"),
       col=c("red","blue"),lty=c(1,1)
       )
```