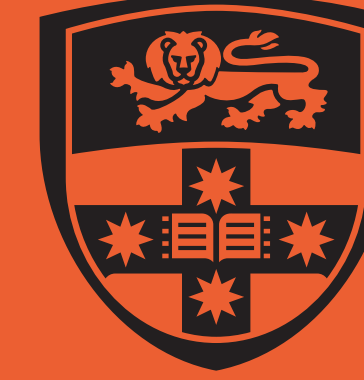


# Optimal Look-back Horizon for Time Series Forecasting in Federated Learning

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## Abstract

Selecting an appropriate look-back horizon remains a fundamental challenge in time series forecasting, particularly in federated learning where data is decentralized and heterogeneous. We present a principled framework for adaptive horizon selection via an intrinsic-space formulation grounded in a structured synthetic data generator.

## Introduction

Existing scaling-law analyses typically assume centralized IID data and fail to account for client heterogeneity. In federated environments, clients differ in dynamics, noise, and temporal structure, making fixed horizons suboptimal.

- Client-aware intrinsic representation space
- Explicit Bayesian + approximation error decomposition
- Provably optimal, adaptive horizon criterion

## Synthetic Data Generator

For client  $k$ , feature  $f$ , and time  $t$ :

$$\hat{\mathbf{x}}_{f,t,k} = \sum_i A_{f,j,k} \sin\left(\frac{2\pi t}{T_{f,j,k}} + \theta_{f,j,k}\right) + \sum_i \phi_{k,i} \mathbf{x}_{f,t-i,k} + \beta_{f,k} t + \epsilon_{f,t,k}.$$

## Loss Decomposition

The global prediction loss decomposes as:

$$L(H) = L_{\text{Bayes}}(H) + L_{\text{approx}}(H),$$

separating irreducible client uncertainty from approximation error induced by finite samples and model capacity.

## Smallest Sufficient Horizon

For tolerance  $\delta > 0$ , define:

$$H_k^*(\delta) = \min\{H : |\Delta L_{\text{Bayes}}^{(k)}(H)| \leq \delta\}.$$

This is the smallest horizon at which additional history yields negligible irreducible error reduction.

## Conclusion

The total forecasting loss is unimodal in horizon length. The smallest sufficient horizon minimizes loss by balancing information identifiability against approximation error growth, yielding a principled, client-adaptive horizon selection rule.