Optimal Look-back Horizon for Time Series Forecasting in Federated Learning

Dahao Tang[1], Nan Yang[1], Yanli Li[1], Zhiyu Zhu[2], Zhibo Jin[2], Dong Yuan[1]
[1]The University of Sydney [2]University of Technology Sydney
dahao.tang@sydney.edu.au, n.yang@sydney.edu.au, yanli.li@sydney.edu.au, zhiyu.zhu@student.uts.edu.au, zhibo.jin@student.uts.edu.au, dong.yang@sydney.edu.au

Abstract

Selecting an appropriate look-back horizon remains a fundamental challenge in time series forecasting, particularly in federated learning where data is decentralized and heterogeneous. We present a principled framework for adaptive horizon selection via an intrinsic-space formulation grounded in a structured synthetic data generator.

Introduction

Existing scaling-law analyses typically assume centralized IID data and fail to account for client heterogeneity. In federated environments, clients differ in dynamics, noise, and temporal structure, making fixed horizons suboptimal.

- Client-aware intrinsic representation space
- Explicit Bayesian + approximation error decomposition
- Provably optimal, adaptive horizon criterion

Synthetic Data Generator

For client k, feature f, and time t:

$$\hat{x}_{f,t,k} = \sum_j A_{f,j,k} \sin\left(\frac{2\pi t}{T_{f,j,k}} + \theta_{f,j,k}\right) + \sum_i \phi_{k,i} x_{f,t-i,k} + \beta_{f,k} t + \epsilon_{f,t,k}.$$

Loss Decomposition

The global prediction loss decomposes as:

$$L(H) = L_{\mathrm{Bayes}}(H) + L_{\mathrm{approx}}(H),$$

separating irreducible client uncertainty from approximation error induced by finite samples and model capacity.

Smallest Sufficient Horizon

For tolerance $\delta > 0$, define:

$$H_k^*(\delta) = \min\{H : |\Delta L_{\mathrm{Bayes}}^{(k)}(H)| \le \delta\}.$$

This is the smallest horizon at which additional history yields negligible irreducible error reduction.

Conclusion

The total forecasting loss is unimodal in horizon length. The smallest sufficient horizon minimizes loss by balancing information identifiability against approximation error growth, yielding a principled, client-adaptive horizon selection rule.