

TRAIL *OF* **BITS**

On the Optimization of Equivalent Concurrent Computations*

Henrich Lauko, Lukáš Korenčík & Peter Goodman

*Research funded by DARPA SIEVE program.

On the Optimization of ~~Equivalent Concurrent Computations~~ Common Subexpressions*

Henrich Lauko, Lukáš Korenčík & Peter Goodman

*Research funded by DARPA SIEVE program.

Problem Statement

- Find **common subexpressions** independent of their arguments
- Factor out **all of them or none** to **a separate function**

$$f(e(a_1, a_2)) \wedge h(e(b_1, b_2))$$

↓

$$f(g(a_1, a_2)) \wedge h(g(b_1, b_2)) \wedge g(c_1, c_2) = e(c_1, c_2)$$

Pattern Extraction Example

$$v_1 = a_1 \times (a_2 \times 2) \wedge v_2 = b_1 \times (b_2 \ll 1)$$



extract pattern of form $c_1 \times (c_2 \times 2)$



$$v_1 = g(a_1, a_2) \wedge v_2 = g(b_1, b_2) \wedge g(c_1, c_2) = c_1 \times (c_2 \times 2)$$

Applications

- Program refactoring
- Term simplification

Optimization of arithmetic circuits for ZK proofs

- Application targets optimization of circuits for the ZK proofs
- Extraction of common arithmetic logic units from generated circuit

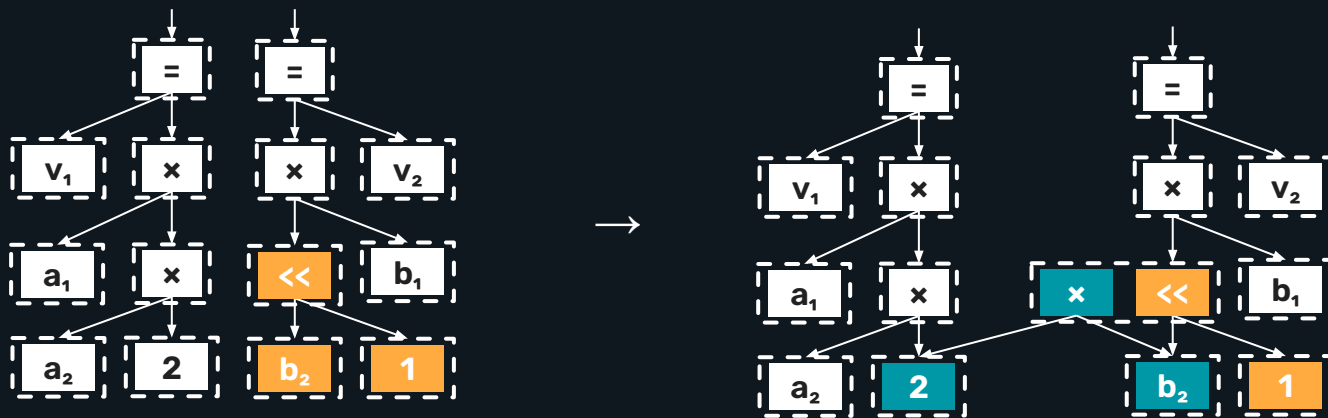
Problems to Solve

1. Keep the equality saturation algorithm
2. Extend the ematch to find **common subexpressions**
3. Represent **subexpression relation** in the e-graph
4. Extract refactored structure

1. Apply rewrite rules as usual

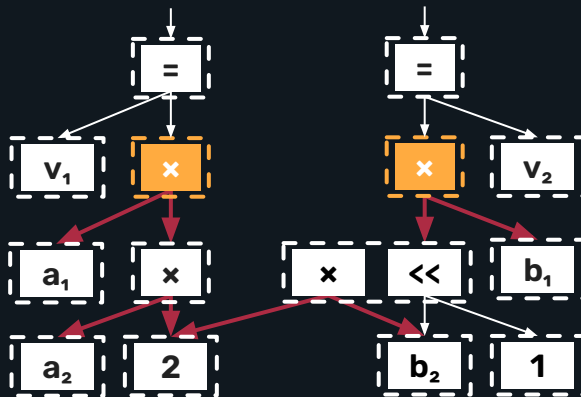
Original formula: $v_1 = a_1 \times (a_2 \times 2) \wedge v_2 = b_1 \times (b_2 \ll 1)$

Rewrite rule: $? \ll 1 \rightarrow ? \times 2$



2. Find common subexpression

- **New ematch rule:** $(\text{let } E \text{ } (?_1 \times (?_2 \times 2))) \text{ (match } E \dots) \rightarrow$
- **Eagerly match all patterns of a given form**

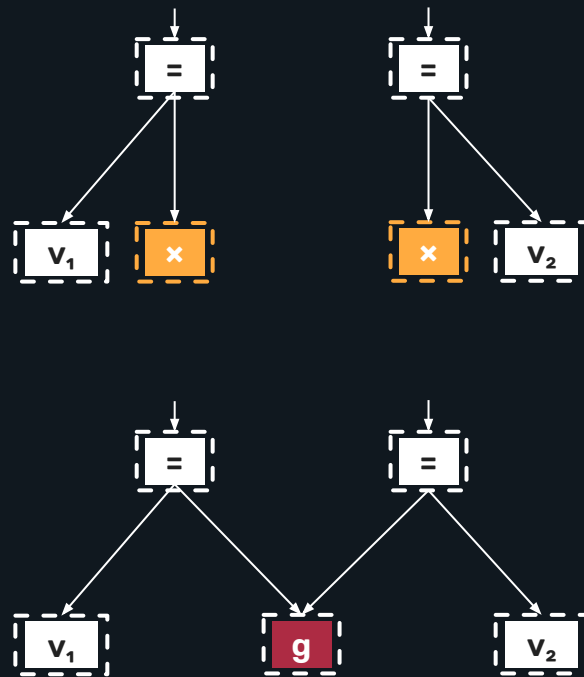
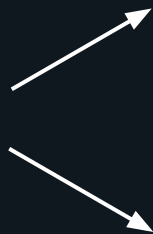
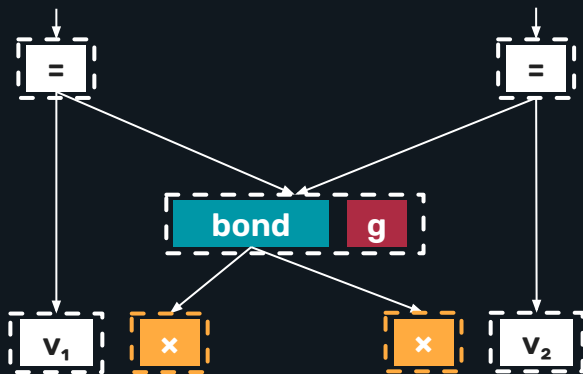


3. Relate common subexpressions

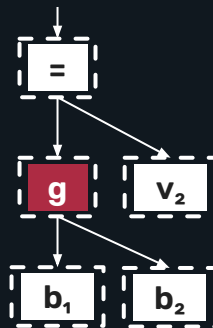
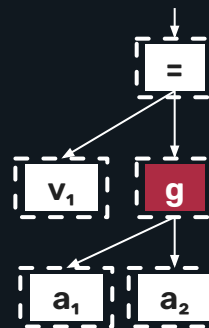
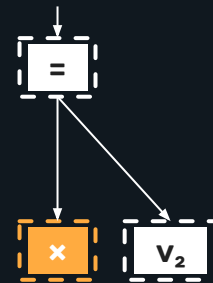
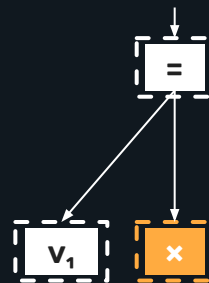
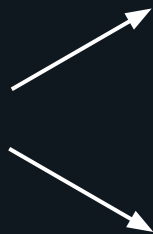
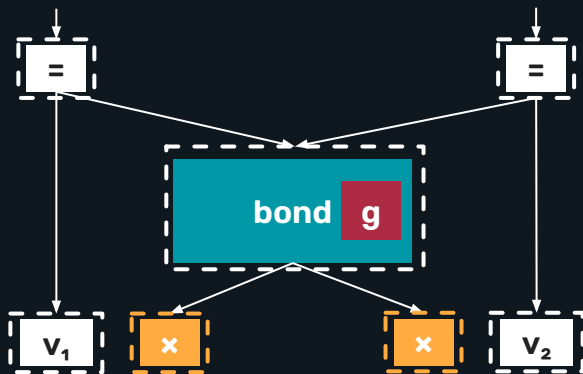
- Extend E-GRAPH by a new type of node – **b-node** (bond node)
- Represents relation between multiple e-classes
- Keeps relation between parent and **children** e-classes
- **New rule action:** $(\text{let } E (?_1 \times (?_2 \times 2))) (\text{match } E\ldots) \rightarrow (\text{bond } E\ldots)$



Bond lowering

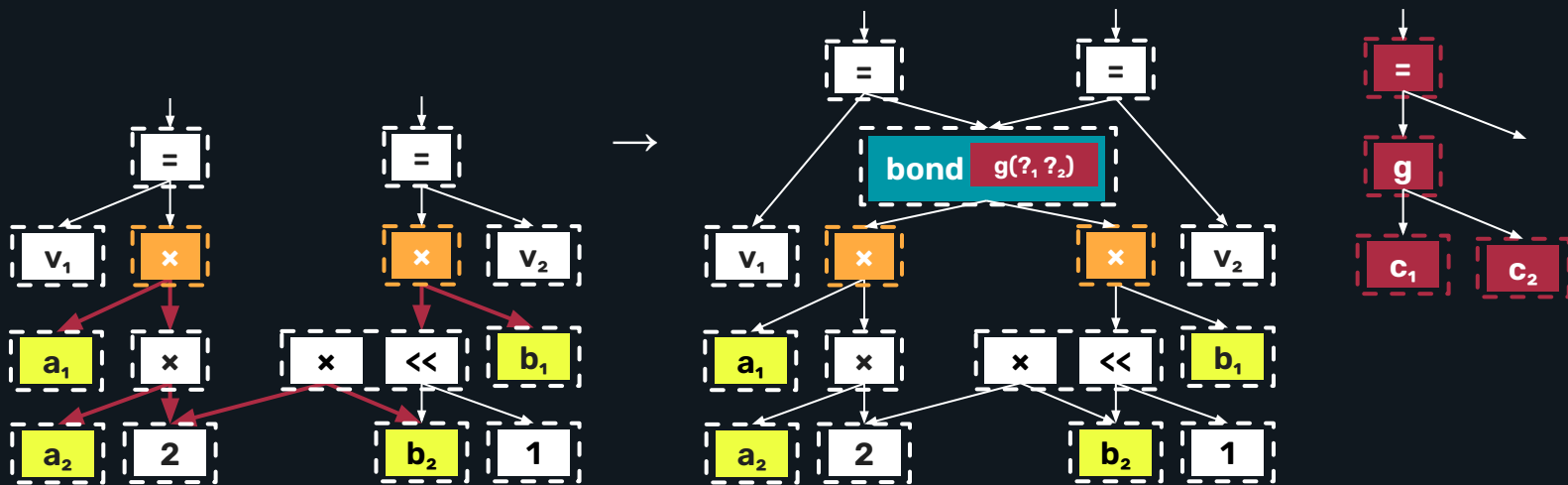


Bond lowering



The final bonding rule application

$(\text{let } E \text{ } (?_1 \times (?_2 \times 2))) (\text{match } E... \text{ with vars } ?_1 ?_2) \rightarrow$
 $((\text{let } G \text{ } (g(c_1, c_2) = c_1 \times (c_2 \times 2))) (\text{bond } E... \text{ with } g(?_1, ?_2)))$



Experiments

Benchmark	Circuit Size		AND Gates		Multiplications	
	UNOPTIMIZED	EQSAT	UNOPTIMIZED	EQSAT	UNOPTIMIZED	EQSAT
x86 mul-forms	94,286	71,102	42,591	31,043	25	5
3D toolkit	124,795	97,881	61,783	49,590	12	2
Router sim.	109,596	86,585	55,242	40,757	13	2
LAN simulator	126,657	104,184	63,430	49,232	19	3

Implementation

- Inspired by egg
- Soon to be released as C++ library:
<https://github.com/lifting-bits/eqsat>
- Extends the language of patterns and e-graph modifiers
- Used as optimization ZK circuit compiler in the tool circuitous

Implementation

- Inspired by egg
- Soon to be released as C++ library:
<https://github.com/lifting-bits/eqsat>
- Extends the language of patterns and e-graph modifiers
- Used as optimization ZK circuit compiler in the tool circuitous

Thank You