Data Mining

Assignment 2

Data Exploration and Preprocessing

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Dataset investigation

The dataset used is the combination of the files :segmentation.data and segmentation.test

- **number of readings and attributes**
  - the data file consists of 210 instances (rows) and 19 features/attributes (columns)
  - the test file consists of 2100 instances (rows) with 19 features/attributes (columns)
  - the merged data consists of 2310 instances (rows) with 20 features (columns)
- **classes**
  - The class column has no header. We add a name to it and reset the index to be a zero based index
  - The different class values are (['BRICKFACE', 'SKY', 'FOLIAGE', 'CEMENT', 'WINDOW', 'PATH', 'GRASS'])
  - each unique value in class has 330 instances

Index	class	ION-CENTROID-(	ION-CENTROID-R	GION-PIXEL-COU	ORT-LINE-DENSIT	ORT-LINE-DENSIT	VEDGE-MEAN	VEDGE-SD	HEDGE-MEAN	HEDGE-SD	NTENSITY-MEAN	RAWRED-MEAN	RA
0	BRICKFACE	140	125	9	0	0	0.277778	0.062963	0.666667	0.311111	6.18518	7.33333	7.
1	BRICKFACE	27	68	9	0	0	1.38889	1.48519	1.77778	5.0963	21.5926	20.4444	28.
2	BRICKFACE	29	100	9	0	0	2.22222	1.36296	2.66667	3.64444	20.6296	20.8889	25.
3	BRICKFACE	23	113	9	0	0	0.722222	0.996296	2.77778	2.78518	14.7037	17.1111	16.
4	BRICKFACE	40	85	9	0.111111	0	1.11111	0.518519	2.5	0.655555	21.1111	20.7778	27.
5	BRICKFACE	77	78	9	0	0	2.22222	2.34074	2	5.86667	24.5926	24	31.
6	BRICKFACE	96	92	9	0.111111	0	3.33333	2.97778	1.77778	0.651851	22.8889	21.7778	29.
7	BRICKFACE	147	92	9	0	0	1.11111	0.429631	1.38889	2.24074	23.4444	22.7778	30
8	BRICKFACE	42	59	9	0	0	1.83333	3.5	2.05556	0.862963	21.8148	20.7778	28.
9	BRICKFACE	2	63	9	0	0	1.22222	0.562963	1.5	0.700001	18.0741	17.2222	23.
10	BRICKFACE	31	106	9	0	0	1.61111	0.72963	0.722222	1.17407	19.1111	19.5556	23.
11	BRICKFACE	33	104	9	0.111111	0	1.55556	0.962963	3.05556	5.04074	20.1852	20.2222	24.
12	BRICKFACE	81	98	9	0.111111	0	1.22222	0.42963	1.88889	3.22963	20.8889	21	25.
13	BRICKFACE	6	90	9	0	0	1.94444	0.596296	1.33333	1.82222	18.1111	18.7778	22.
14	BRICKFACE	76	81	9	0.111111	0	1.88889	1.67407	1.33333	2.4	22.7037	22.3333	28.
15	BRICKFACE	145	90	9	0	0	1	0.577778	2	0.399999	24.8519	23.6667	32.
16	BRICKFACE	133	67	9	0	0	1.38889	1.3963	2.11111	1.22963	27.1481	25.1111	35.
17	BRICKFACE	80	95	9	0	0	1.22222	1.00741	0.944444	0.551851	21.4074	21.3333	26.
18	BRICKFACE	91	115	9	0.111111	0	1.72222	1.44074	3.88889	2.82963	19.9259	20.5556	24.
19	BRICKFACE	75	107	9	0.111111	0	1.16667	0.344445	1.77778	1.54074	16.7037	18.7778	19.
20	BRICKFACE	121	60	9	0	0	2.27778	2.32963	2.88889	2.87407	26.7407	24.6667	35.

- **data description**

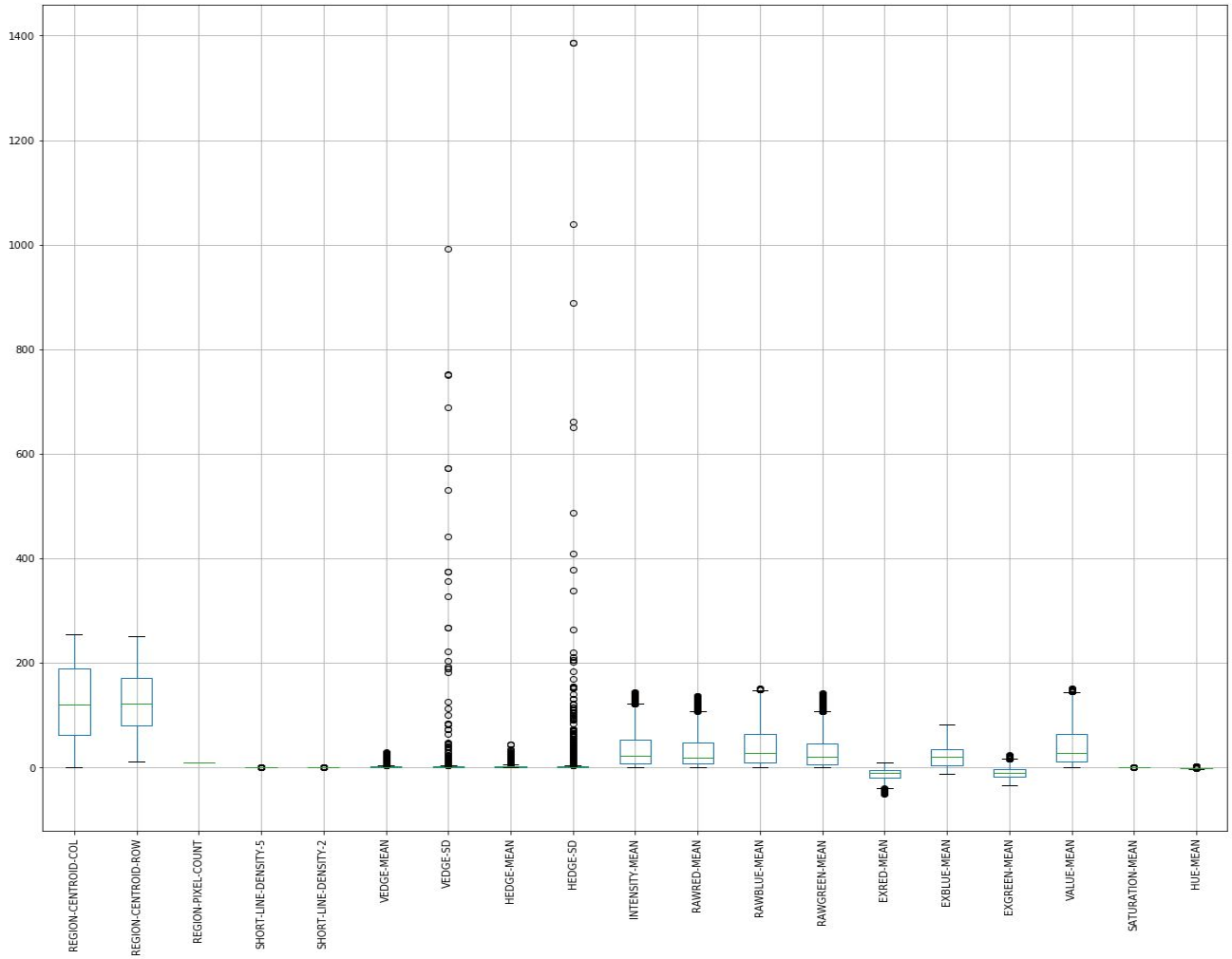
x - DataFrame

Index	ION-CENTROID-(	ION-CENTROID-R	GION-PIXEL-COU	ORT-LINE-DENSIT	ORT-LINE-DENSIT	VEDGE-MEAN	VEDGE-SD	HEDGE-MEAN	HEDGE-SD	NTENSITY-MEAN	RAWRED-MEAN	RAWBLUE-MEAN	RAWGR
count	2310	2310	2310	2310	2310	2310	2310	2310	2310	2310	2310	2310	2310
mean	124.914	123.417	9	0.0143338	0.0047138	1.89394	5.70932	2.42472	8.24369	37.0516	32.8213	44.1879	34.14
std	72.9565	57.4839	0	0.0401541	0.0242343	2.69891	44.8465	3.61008	58.8115	38.1764	35.0368	43.5275	36.36
min	1	11	9	0	0	0	0	0	-1.58946e-08	0	0	0	0
25%	62	81	9	0	0	0.722222	0.355555	0.77778	0.421637	7.2963	7	9.55556	6.027
50%	121	122	9	0	0	1.22222	0.833333	1.44444	0.962963	21.5926	19.5556	27.6667	20.33
75%	189	172	9	0	0	2.16667	1.80637	2.55556	2.18327	53.213	47.3333	64.8889	46.5
max	254	251	9	0.333333	0.222222	29.2222	991.718	44.7222	1386.33	143.444	137.111	150.889	142.5

- **dataset information**

```
In [52]: display (dataset.info())
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 2310 entries, 0 to 2309
Data columns (total 20 columns):
class                  2310 non-null object
REGION-CENTROID-COL    2310 non-null float64
REGION-CENTROID-ROW    2310 non-null float64
REGION-PIXEL-COUNT     2310 non-null int64
SHORT-LINE-DENSITY-5   2310 non-null float64
SHORT-LINE-DENSITY-2   2310 non-null float64
VEDGE-MEAN             2310 non-null float64
VEDGE-SD               2310 non-null float64
HEDGE-MEAN             2310 non-null float64
HEDGE-SD               2310 non-null float64
INTENSITY-MEAN         2310 non-null float64
RAWRED-MEAN            2310 non-null float64
RAWBLUE-MEAN           2310 non-null float64
RAWGREEN-MEAN          2310 non-null float64
EXRED-MEAN             2310 non-null float64
EXBLUE-MEAN            2310 non-null float64
EXGREEN-MEAN           2310 non-null float64
VALUE-MEAN             2310 non-null float64
SATURATION-MEAN        2310 non-null float64
HUE-MEAN               2310 non-null float64
dtypes: float64(18), int64(1), object(1)
memory usage: 361.0+ KB
None
```

- ## Dataset visualisation
  - ### Boxplot



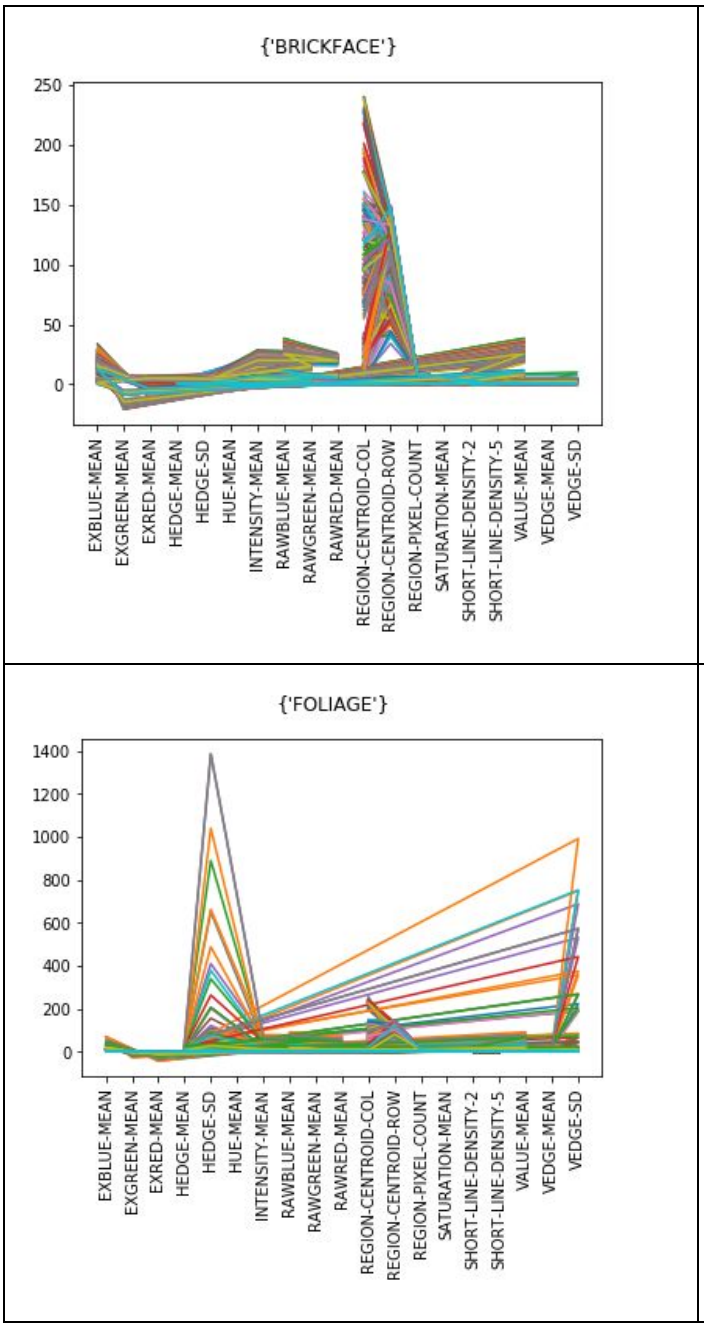
Observations:

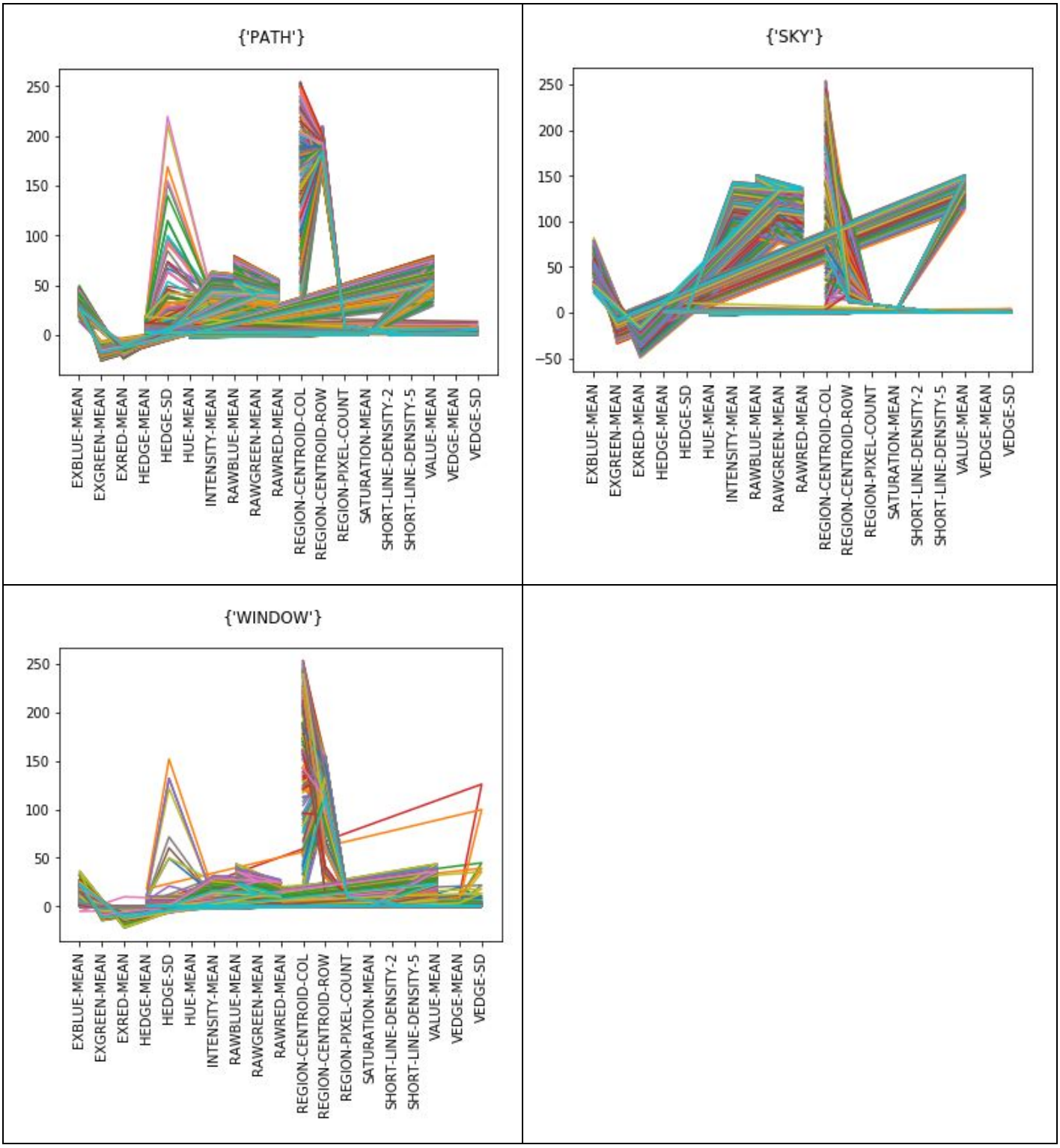
- The dataset attributes are different in ranges
- *REGION-CENTROID-COL* have a normal distribution and no outliers
- *REGION-PIXEL-COUNT* has a zero standard deviation because it has fixed value = 9. (redundant dimension to be removed).
- *SHORT-LINE-DENSITY-5* and SHORT-LINE-DENSITY-2 has a small standard deviation
- *VEDGE-MEAN* and *HEDGE-MEAN* have outliers that affects mean and standard deviation.
- *HEDGE-SD* and *VEDGE-SD* have very large outliers

Conclusion

- The data needs to be normalized
- Outliers need to be removed
- Redundant attributes need to be removed

  - **plots for each class**

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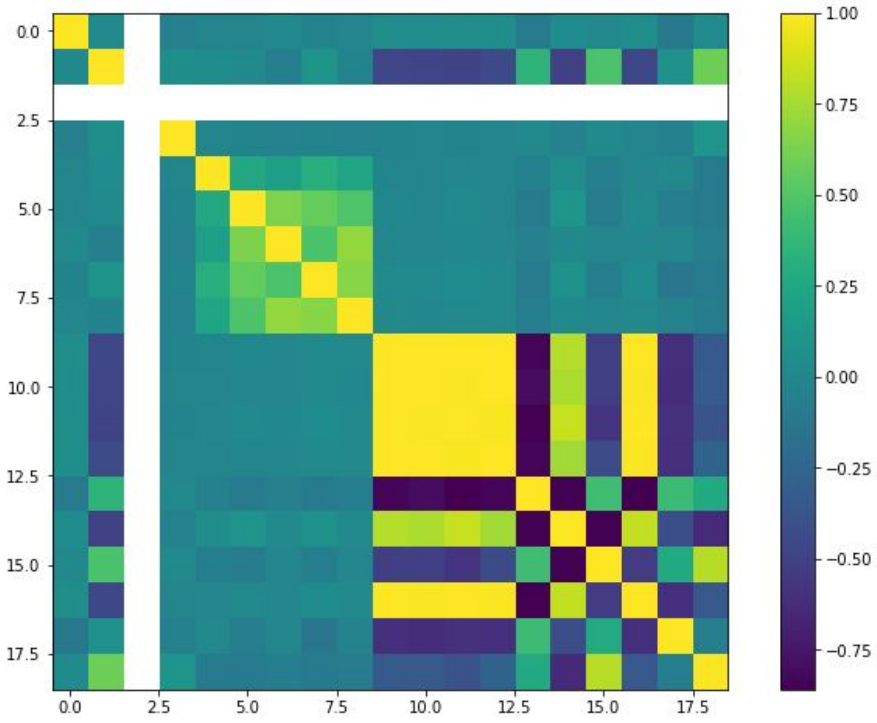
EXBLUE-MEAN
EXGREEN-MEAN
EXRED-MEAN
HEDGE-MEAN
HEDGE-SD
HUE-MEAN
INTENSITY-MEAN
RAWBLUE-MEAN
RAWGREEN-MEAN
RAWRED-MEAN
REGION-CENTROID-COL
REGION-CENTROID-ROW
REGION-PIXEL-COUNT
SATURATION-MEAN
SHORT-LINE-DENSITY-2
SHORT-LINE-DENSITY-5
VALUE-MEAN
VEDGE-MEAN
VEDGE-SD

Data exploration

- Pearson’s correlation
  - matrix

Index	ION-CENTROID-(	ION-CENTROID-F	GION-PIXEL-COU	ORT-LINE-DENSIT	ORT-LINE-DENSIT	VEDGE-MEAN	VEDGE-SD	HEDGE-MEAN	HEDGE-SD	NTENSITY-MEAN	RAWRED-MEAN	RAWBLUE-MEAN	RAWGR
REGION-CENTROID-COL	1	0.0267683	nan	-0.0519617	-0.0159643	-0.0113042	0.0219603	-0.0189142	-0.00193879	0.0589574	0.054673	0.0581691	0.063
REGION-CENTROID-ROW	0.0267683	1	nan	0.0648913	0.0418694	0.0261463	-0.053578	0.105223	-0.0210774	-0.46524	-0.468009	-0.481521	-0.43
REGION-PIXEL-COUNT	nan	nan	nan	nan	nan	nan	nan	nan	nan	nan	nan	nan	nan
SHORT-LINE-DENSITY-5	-0.0519617	0.0648913	nan	1	-0.00902435	-0.0202057	-0.0327814	-0.0212863	-0.0379961	-0.0182106	-0.0167553	-0.0213921	-0.01
SHORT-LINE-DENSITY-2	-0.0159643	0.0418694	nan	-0.00902435	1	0.262575	0.193728	0.303182	0.243155	-0.00691096	-0.0124706	0.00307818	-0.01
VEDGE-MEAN	-0.0113042	0.0261463	nan	-0.0202057	0.262575	1	0.637452	0.559491	0.488347	0.0051292	-0.00548196	0.0204975	-0.00
VEDGE-SD	0.0219603	-0.053578	nan	-0.0327814	0.193728	0.637452	1	0.471016	0.703049	0.00300641	-0.00213776	0.00678241	0.003
HEDGE-MEAN	-0.0189142	0.105223	nan	-0.0212863	0.303182	0.559491	0.471016	1	0.668179	0.0339725	0.0260589	0.0438457	0.029
HEDGE-SD	-0.00193879	-0.0210774	nan	-0.0379961	0.243155	0.488347	0.703049	0.668179	1	0.013518	0.00853753	0.0168992	0.014
INTENSITY-MEAN	0.0589574	-0.46524	nan	-0.0182106	-0.00691096	0.0051292	0.00300641	0.0339725	0.013518	1	0.998112	0.995809	0.995
RAWRED-MEAN	0.054673	-0.468009	nan	-0.0167553	-0.0124706	-0.00548196	-0.00213776	0.0260589	0.00853753	0.998112	1	0.990813	0.994
RAWBLUE-MEAN	0.0581691	-0.481521	nan	-0.0213921	0.00307818	0.0204975	0.00678241	0.0438457	0.0168992	0.995809	0.990813	1	0.984
RAWGREEN-MEAN	0.0633807	-0.437971	nan	-0.0156042	-0.013435	-0.00309891	0.00340993	0.0294059	0.014121	0.995842	0.994056	0.984659	1
EXRED-MEAN	-0.0868165	0.353175	nan	0.0280127	-0.0448293	-0.100457	-0.0491233	-0.0994335	-0.0561856	-0.830261	-0.794457	-0.855058	-0.82
EXBLUE-MEAN	0.0430985	-0.490219	nan	-0.036164	0.0609787	0.106744	0.0276592	0.0937381	0.0336465	0.792257	0.76997	0.844741	0.742
EXGREEN-MEAN	0.0140351	0.476421	nan	0.0331823	-0.0583623	-0.0801201	0.00239638	-0.0591112	-0.000666109	-0.509756	-0.507899	-0.573816	-0.42
VALUE-MEAN	0.0601893	-0.458388	nan	-0.0158859	-0.000145206	0.0181477	0.00480412	0.0422324	0.0148579	0.997385	0.992062	0.998644	0.990
SATURATION-MEAN	-0.108214	0.0815563	nan	-0.0432207	0.0162084	-0.0648269	0.0023061	-0.125955	-0.0241491	-0.60829	-0.616928	-0.595166	-0.60
HUE-MEAN	0.0392985	0.59293	nan	0.112989	-0.0829386	-0.0979591	-0.0615915	-0.0938031	-0.0699882	-0.329845	-0.328574	-0.384925	-0.26

- visualisation



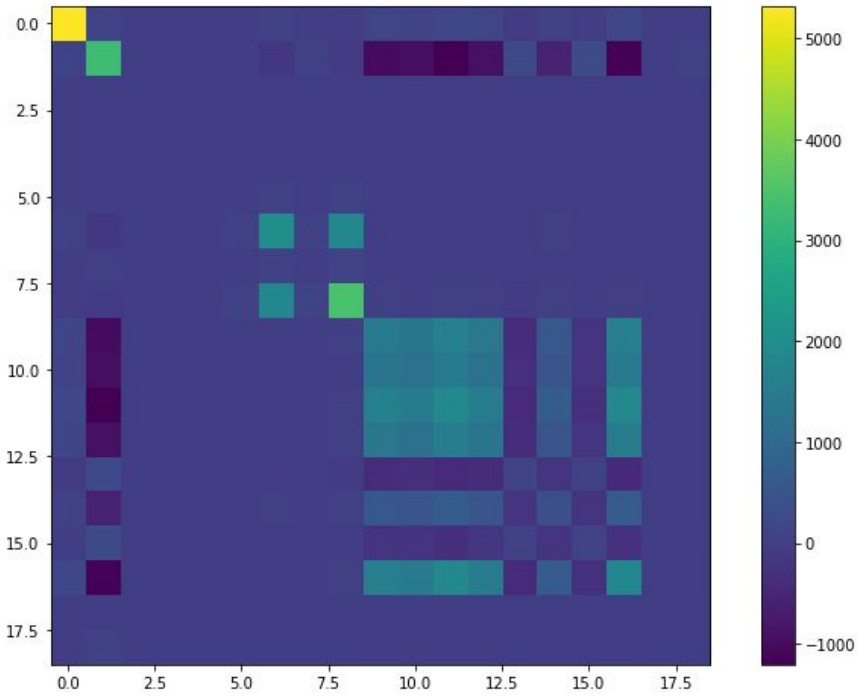
● covariance matrix

- matrix

covariance - DataFrame

Index	ION-CENTROID-(	ION-CENTROID-F	GION-PIXEL-COU	ORT-LINE-DENSIT	ORT-LINE-DENSIT	VEDGE-MEAN	VEDGE-SD	HEDGE-MEAN	HEDGE-SD	NTENSITY-MEAN	RAWRED-MEAN	RAWBLUE-MEAN	AWGR
REGION-CENTROID-COL	5322.66	112.262	0	-0.152222	-0.0282257	-2.22583	71.8508	-4.98161	-8.31874	164.209	139.753	184.723	168.1
REGION-CENTROID-ROW	112.262	3304.39	0	0.149783	0.0583275	4.05643	-138.121	21.8359	-71.2565	-1020.98	-942.593	-1204.83	-915.
REGION-PIXEL-COUNT	0	0	0	0	0	0	0	0	0	0	0	0	0
SHORT-LINE-DENSITY-5	-0.152222	0.149783	0	0.00161235	-8.78166e-06	-0.00218974	-0.0590317	-0.00308566	-0.0897287	-0.0279157	-0.0235725	-0.0373893	-0.02
SHORT-LINE-DENSITY-2	-0.0282257	0.0583275	0	-8.78166e-06	0.000587302	0.0171741	0.210548	0.0265247	0.346558	-0.00639388	-0.0105887	0.00324704	-0.01
VEDGE-MEAN	-2.22583	4.05643	0	-0.00218974	0.0171741	7.2841	77.1549	5.45128	77.5139	0.528485	-0.51838	2.40798	-0.30
VEDGE-SD	71.8508	-138.121	0	-0.0590317	0.210548	77.1549	2011.2	76.2572	1854.28	5.14721	-3.35901	13.2396	5.561
HEDGE-MEAN	-4.98161	21.8359	0	-0.00308566	0.0265247	5.45128	76.2572	13.0327	141.864	4.6821	3.29607	6.88981	3.860
HEDGE-SD	-8.31874	-71.2565	0	-0.0897287	0.346558	77.5139	1854.28	141.864	3458.79	30.3509	17.5921	43.2605	30.20
INTENSITY-MEAN	164.209	-1020.98	0	-0.0279157	-0.00639388	0.528485	5.14721	4.6821	30.3509	1457.44	1335.05	1654.76	1382.
RAWRED-MEAN	139.753	-942.593	0	-0.0235725	-0.0105887	-0.51838	-3.35901	3.29607	17.5921	1335.05	1227.58	1511.05	1266.
RAWBLUE-MEAN	184.723	-1204.83	0	-0.0373893	0.00324704	2.40798	13.2396	6.88981	43.2605	1654.76	1511.05	1894.64	1558.
RAWGREEN-MEAN	168.152	-915.53	0	-0.0227852	-0.01184	-0.304143	5.56103	3.86041	30.2001	1382.5	1266.53	1558.58	1322.
EXRED-MEAN	-73.3683	235.168	0	0.0130295	-0.0125845	-3.1406	-25.5187	-4.15807	-38.2763	-367.157	-322.431	-431.123	-347.
EXBLUE-MEAN	61.54	-551.527	0	-0.0284209	0.0289228	5.63848	24.2772	6.62315	38.7288	591.96	527.994	719.645	528.2
EXGREEN-MEAN	11.8283	316.359	0	0.0153915	-0.0163383	-2.49789	1.24144	-2.46507	-0.452533	-224.802	-205.563	-288.522	-180.
VALUE-MEAN	188.478	-1130.98	0	-0.0273791	-0.00015104	2.10227	9.2474	6.54395	37.5056	1634.31	1491.9	1865.74	1545.
SATURATION-MEAN	-1.80249	1.07035	0	-0.000396228	8.96796e-05	-0.0399454	0.0236118	-0.103814	-0.324256	-5.30187	-4.93495	-5.9146	-5.05
HUE-MEAN	4.43061	52.671	0	0.00701116	-0.00310606	-0.40856	-4.26846	-0.523308	-6.36077	-19.4593	-17.7901	-25.8918	-14.6

- visualisation

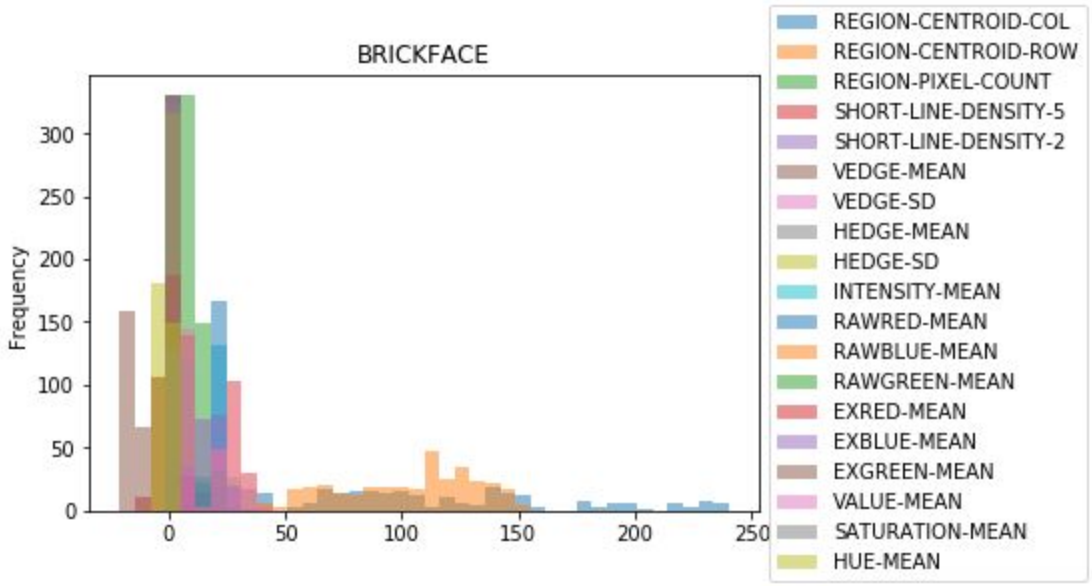


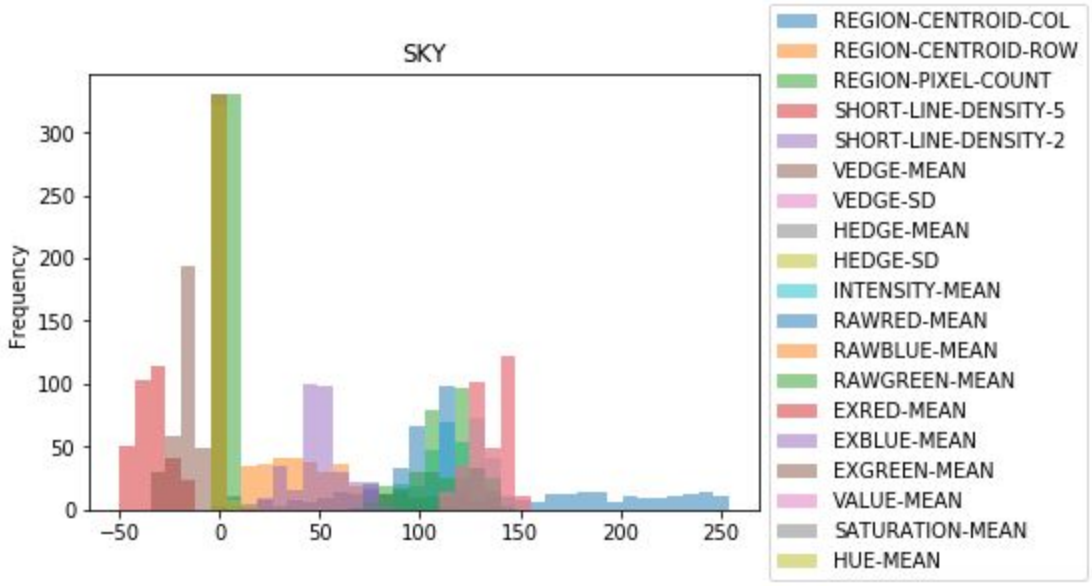
- **What is the relation between the covariance matrix of the dataset and the Pearson's correlation matrix of it?**

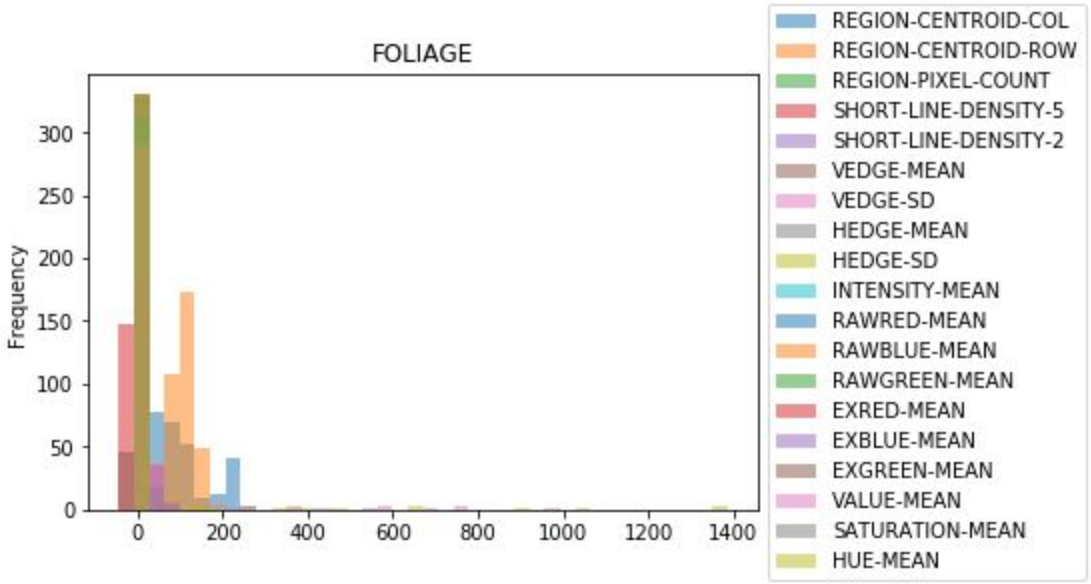
  - Correlation is a special case of covariance where the matrix is standardized
  - Both measures only linear relationship between two variables, i.e. when the correlation coefficient is zero, covariance is also zero.
  - A measure used to indicate the extent to which two random variables change in tandem is known as covariance. A measure used to represent how strongly two random variables are related known as correlation.
  - Covariance is a measure of correlation. On the contrary, correlation refers to the scaled form of covariance.
  - The value of correlation takes place between -1 and +1. Conversely, the value of covariance lies between -∞ and +∞
  - Correlation is dimensionless, i.e. it is a unit-free measure of the relationship between variables. Unlike covariance, where the value is obtained by the product of the units of the two variables.

$$r_{A,B} = \frac{Cov(A,B)}{\sigma_A \sigma_B}$$

- Histograms
  - Histograms for each class



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1200
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REGION-CENTROID-ROW
REGION-PIXEL-COUNT
SHORT-LINE-DENSITY-5
SHORT-LINE-DENSITY-2
VEDGE-MEAN
VEDGE-SD
HEDGE-MEAN
HEDGE-SD
INTENSITY-MEAN
RAWRED-MEAN
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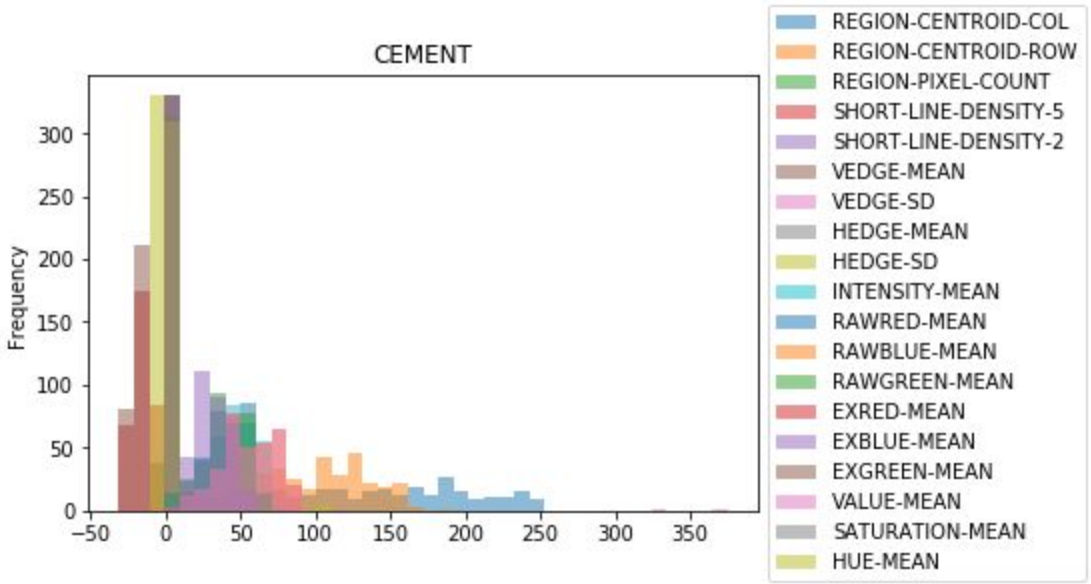
EXGREEN-MEAN
VALUE-MEAN
SATURATION-MEAN
HUE-MEAN


CEMENT
Frequency
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200
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150
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250
300
350
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REGION-CENTROID-ROW
REGION-PIXEL-COUNT
SHORT-LINE-DENSITY-5
SHORT-LINE-DENSITY-2
VEDGE-MEAN
VEDGE-SD
HEDGE-MEAN
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INTENSITY-MEAN
RAWRED-MEAN
RAWBLUE-MEAN
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EXRED-MEAN
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VALUE-MEAN
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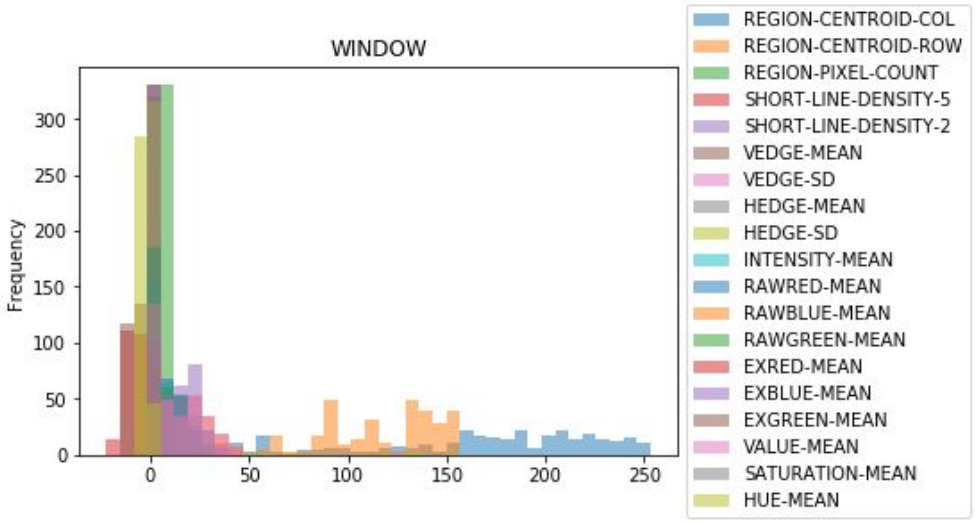
HUE-MEAN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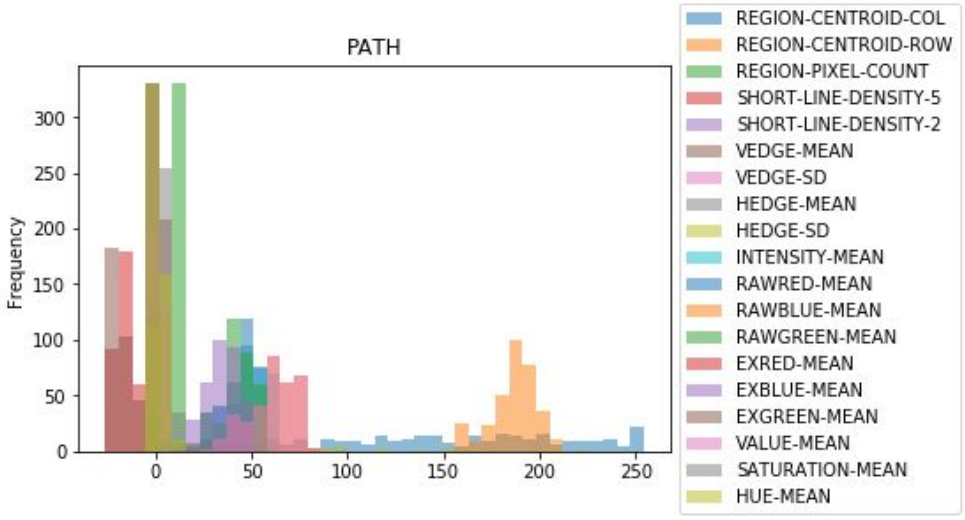
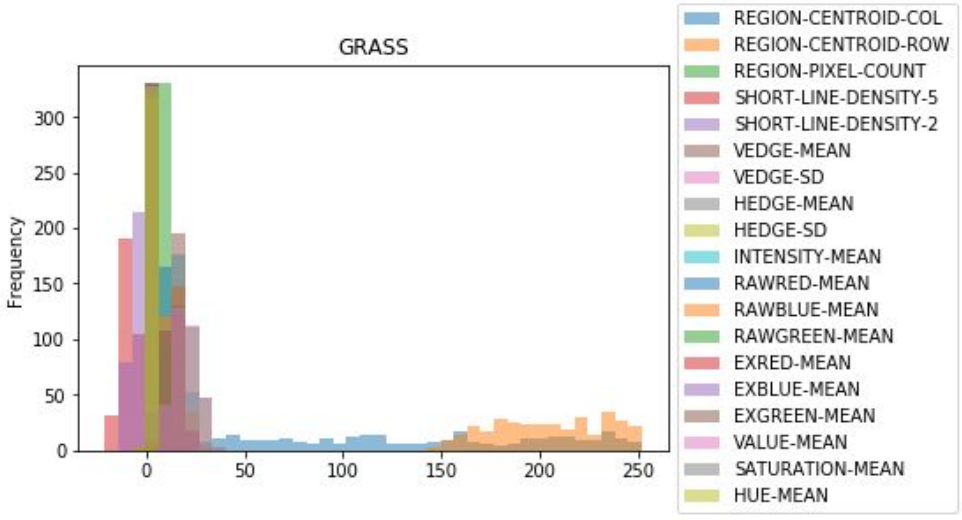
250
300
REGION-CENTROID-COL
REGION-CENTROID-ROW
REGION-PIXEL-COUNT
SHORT-LINE-DENSITY-5
SHORT-LINE-DENSITY-2
VEDGE-MEAN
VEDGE-SD
HEDGE-MEAN
HEDGE-SD
INTENSITY-MEAN
RAWRED-MEAN
RAWBLUE-MEAN
RAWGREEN-MEAN
EXRED-MEAN
EXBLUE-MEAN
EXGREEN-MEAN
VALUE-MEAN
SATURATION-MEAN
HUE-MEAN


PATH
Frequency
0
50
100
150
200
250
300
REGION-CENTROID-COL
REGION-CENTROID-ROW
REGION-PIXEL-COUNT
SHORT-LINE-DENSITY-5
SHORT-LINE-DENSITY-2
VEDGE-MEAN
VEDGE-SD
HEDGE-MEAN
HEDGE-SD
INTENSITY-MEAN
RAWRED-MEAN
RAWBLUE-MEAN
RAWGREEN-MEAN
EXRED-MEAN
EXBLUE-MEAN
EXGREEN-MEAN
VALUE-MEAN
SATURATION-MEAN
HUE-MEAN


GRASS
Frequency
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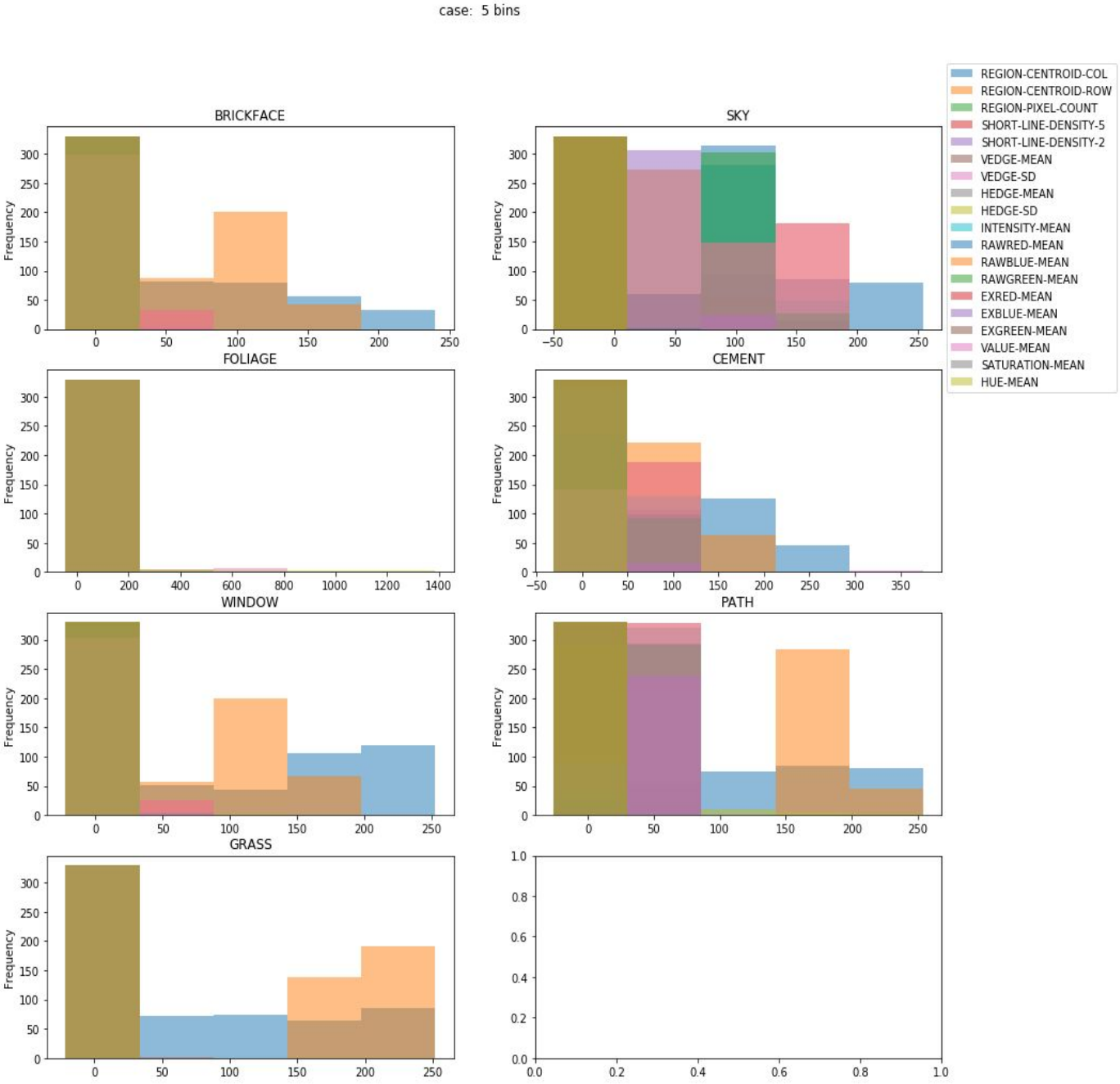
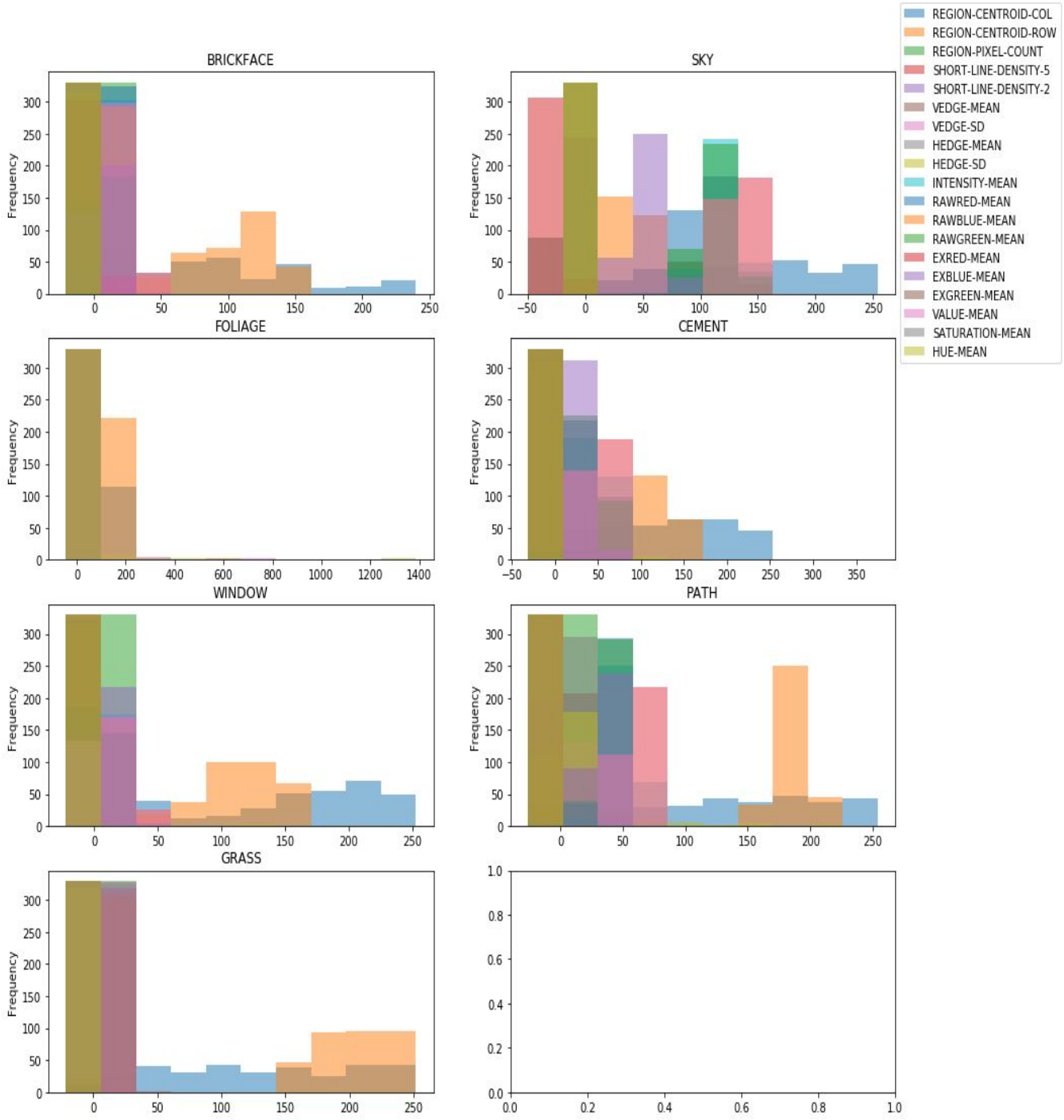
50
100
150
200
250
300
REGION-CENTROID-COL
REGION-CENTROID-ROW
REGION-PIXEL-COUNT
SHORT-LINE-DENSITY-5
SHORT-LINE-DENSITY-2
VEDGE-MEAN
VEDGE-SD
HEDGE-MEAN
HEDGE-SD
INTENSITY-MEAN
RAWRED-MEAN
RAWBLUE-MEAN
RAWGREEN-MEAN
EXRED-MEAN
EXBLUE-MEAN
EXGREEN-MEAN
VALUE-MEAN
SATURATION-MEAN
HUE-MEAN

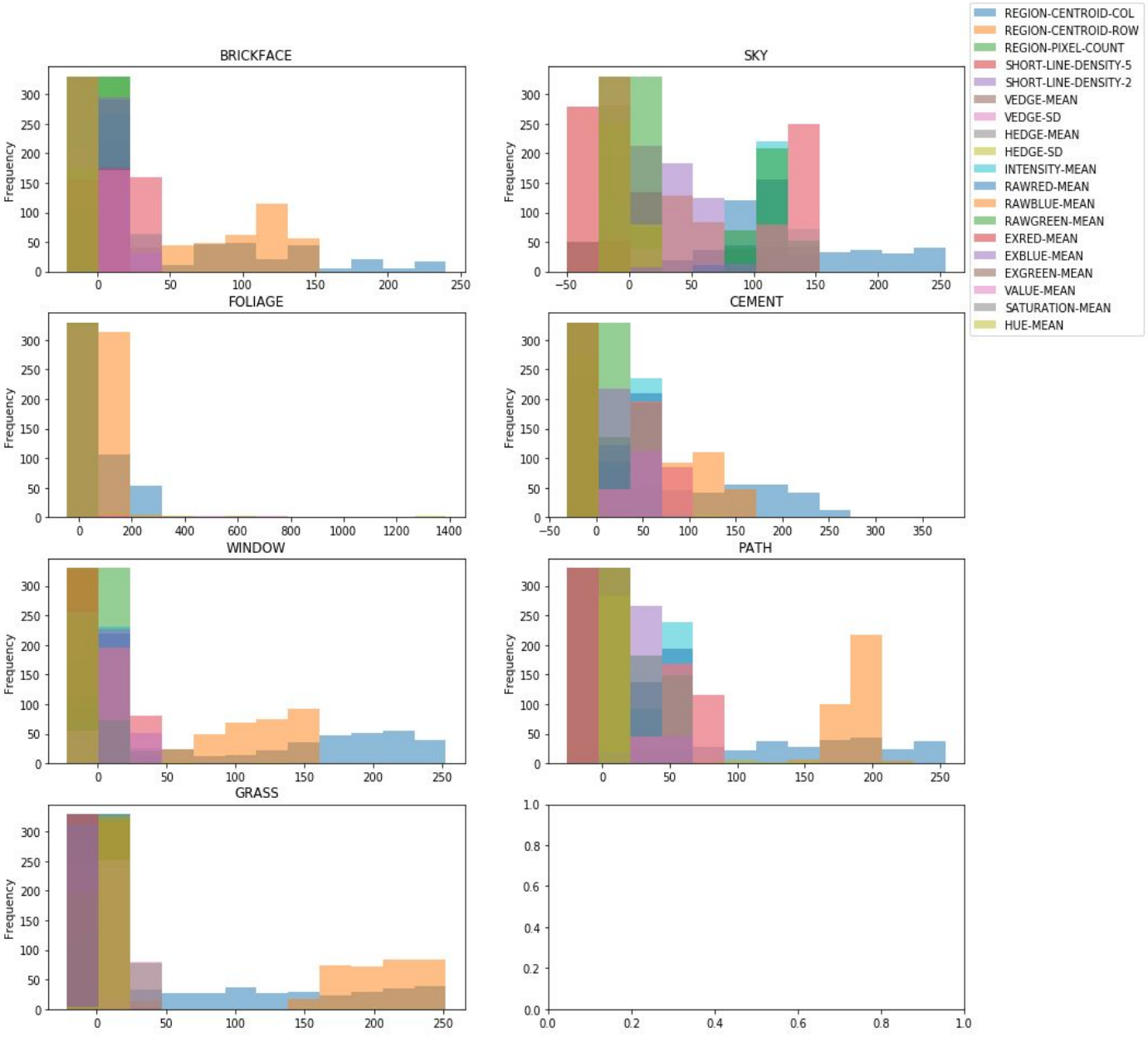
- bins 5, 10, 12


case: 5 bins
BRICKFACE
SKY
FOLIAGE
CEMENT
WINDOW
PATH
GRASS
Frequency
REGION-CENTROID-COL
REGION-CENTROID-ROW
REGION-PIXEL-COUNT
SHORT-LINE-DENSITY-5
SHORT-LINE-DENSITY-2
VEDGE-MEAN
VEDGE-SD
HEDGE-MEAN
HEDGE-SD
INTENSITY-MEAN
RAWRED-MEAN
RAWBLUE-MEAN
RAWGREEN-MEAN
EXRED-MEAN
EXBLUE-MEAN
EXGREEN-MEAN
VALUE-MEAN
SATURATION-MEAN
HUE-MEAN

case: 10 bins


BRICKFACE
SKY
FOLIAGE
CEMENT
WINDOW
PATH
GRASS
Frequency
REGION-CENTROID-COL
REGION-CENTROID-ROW
REGION-PIXEL-COUNT
SHORT-LINE-DENSITY-5
SHORT-LINE-DENSITY-2
VEDGE-MEAN
VEDGE-SD
HEDGE-MEAN
HEDGE-SD
INTENSITY-MEAN
RAWRED-MEAN
RAWBLUE-MEAN
RAWGREEN-MEAN
EXRED-MEAN
EXBLUE-MEAN
EXGREEN-MEAN
VALUE-MEAN
SATURATION-MEAN
HUE-MEAN

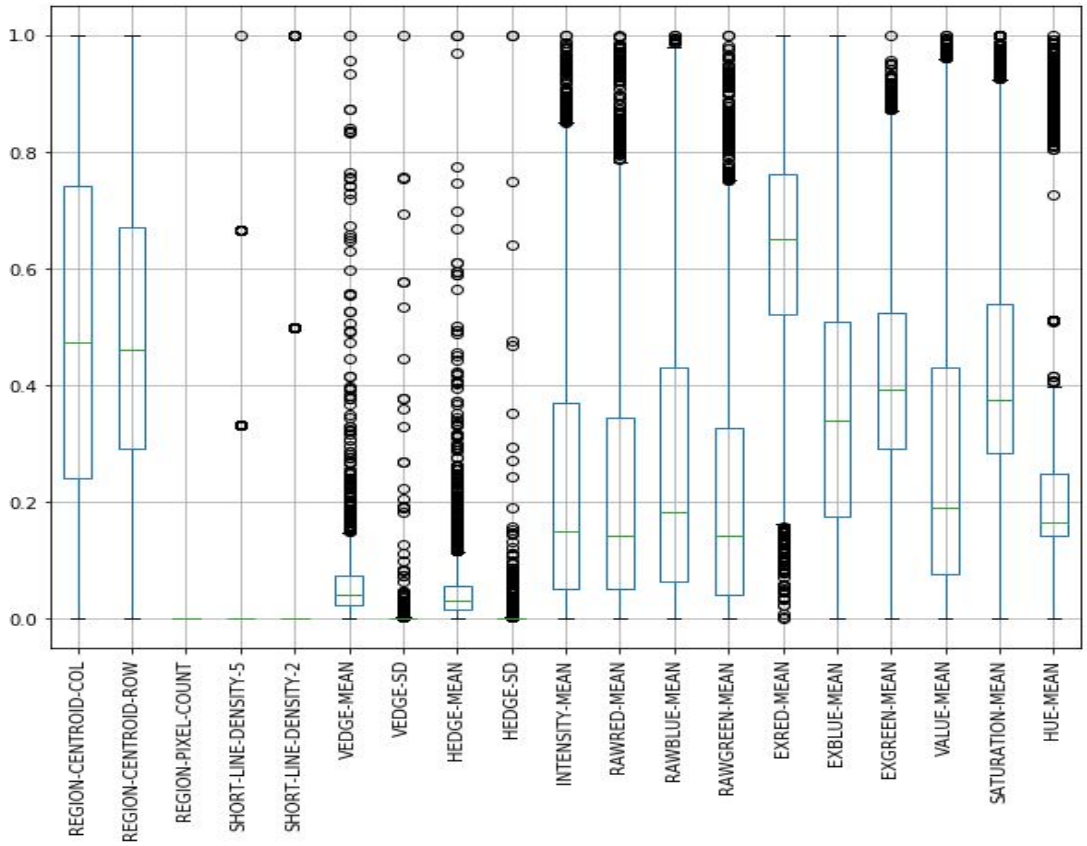
case: 12 bins


BRICKFACE
SKY
FOLIAGE
CEMENT
WINDOW
PATH
GRASS
Frequency
REGION-CENTROID-COL
REGION-CENTROID-ROW
REGION-PIXEL-COUNT
SHORT-LINE-DENSITY-5
SHORT-LINE-DENSITY-2
VEDGE-MEAN
VEDGE-SD
HEDGE-MEAN
HEDGE-SD
INTENSITY-MEAN
RAWRED-MEAN
RAWBLUE-MEAN
RAWGREEN-MEAN
EXRED-MEAN
EXBLUE-MEAN
EXGREEN-MEAN
VALUE-MEAN
SATURATION-MEAN
HUE-MEAN

Preprocessing

- Normalization
    1. **Min-max scaler**

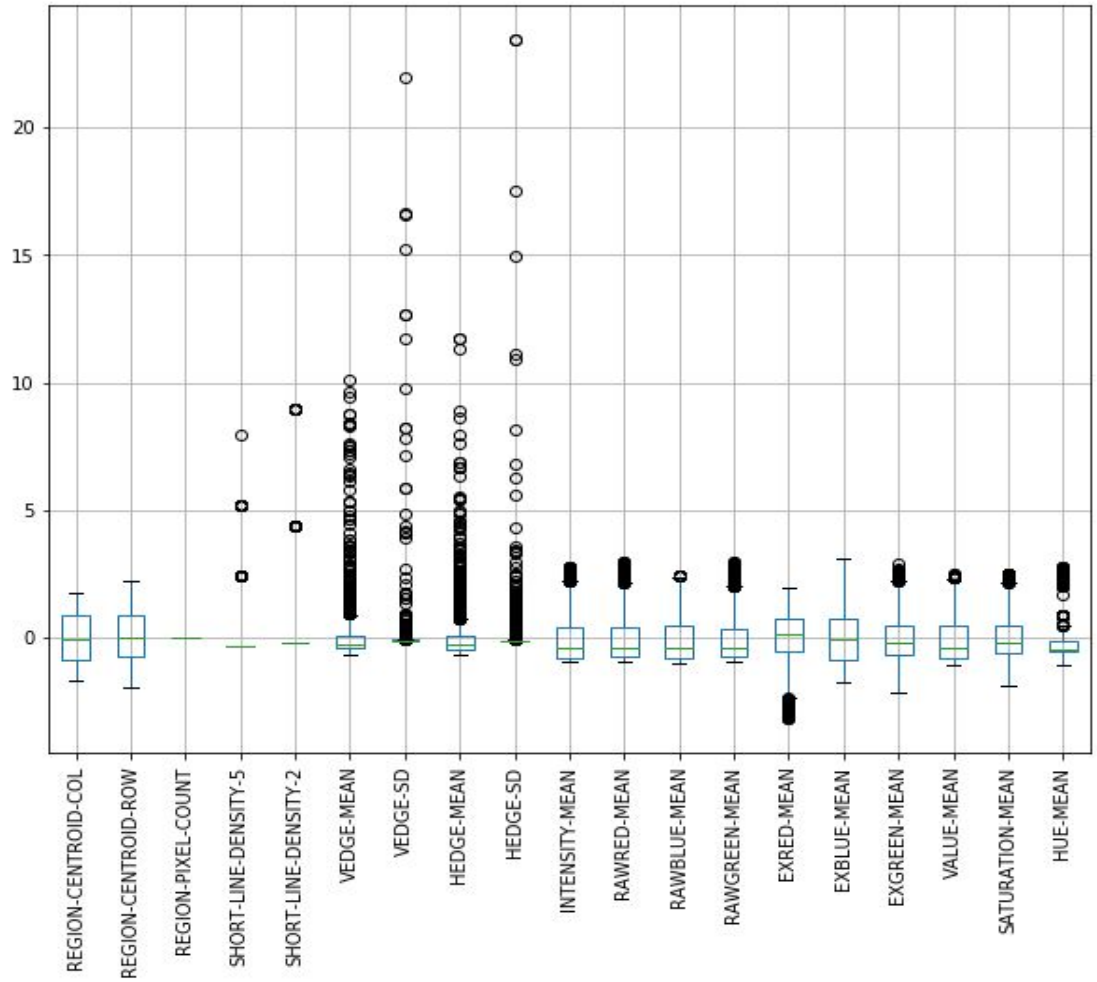
After the max-min normalization, all data are scaled to be in range from 0 to 1



Index	ION-CENTROID-(	ION-CENTROID-F	GION-PIXEL-COU	ORT-LINE-DENSIT	ORT-LINE-DENSIT	VEDGE-MEAN	VEDGE-SD	HEDGE-MEAN	HEDGE-SD	NTENSITY-MEAN	RAWRED-MEAN	RAWBLUE-MEAN	RAWGR
count	2310	2310	2310	2310	2310	2310	2310	2310	2310	2310	2310	2310	2310
mean	0.489778	0.468405	0	0.0430014	0.0212121	0.0648116	0.005757	0.0542174	0.00594642	0.258299	0.239377	0.29285	0.239
std	0.288366	0.239516	0	0.120462	0.109054	0.0923581	0.045221	0.0807224	0.0424225	0.266141	0.255536	0.288474	0.255
min	0	0	0	0	0	0	0	0	0	0	0	0	0
25%	0.241107	0.291667	0	0	0	0.0247148	0.000358524	0.0173914	0.000304139	0.050865	0.0510535	0.0633284	0.042
50%	0.474308	0.4625	0	0	0	0.0418252	0.000840292	0.0322981	0.000694614	0.150529	0.142626	0.183358	0.142
75%	0.743083	0.670833	0	0	0	0.0741445	0.00182145	0.0571429	0.00157486	0.370966	0.345219	0.430044	0.326
max	1	1	0	1	1	1	1	1	1	1	1	1	1

2. Z-score normalization

All values almost have mean = 0, sd = 1. This is good for the normally distributed features.



The difference is that: the feature with high range will not dominate after normalization.

Z-score method preserves range (maximum and minimum) and introduces the dispersion of the serie (standard deviation / variance). If data follow a gaussian distribution, they are converted into a N(0,1) distribution and the comparison between series (probabilities calculation) will be easier.

- ## Dimensionality reduction
    1. ### Feature Projection

        chosen components_num = [1, 2, 4, 6, 8, 10, 13, 16, 19]

        - [0.42341135]
        - [0.42341135 0.16203649]
        - [0.42341135 0.16203649 0.09959451 0.05857283]
        - [0.42341135 0.16203649 0.09959451 0.05857283 0.05197997 0.05050372]
        - [0.42341135 0.16203649 0.09959451 0.05857283 0.05197997 0.05050372 0.04041415 0.03120143]
        - [0.42341135 0.16203649 0.09959451 0.05857283 0.05197997 0.05050372 0.04041415 0.03120143 0.02999802 0.02195028]
        - [0.42341135 0.16203649 0.09959451 0.05857283 0.05197997 0.05050372 0.04041415 0.03120143 0.02999802 0.02195028 0.0142209   0.00993527 0.00616366]
        - [4.23411347e-01   1.62036489e-01   9.95945088e-02 5.85728321e-02 5.19799667e-02   5.05037229e-02   4.04141498e-02 3.12014310e-02 2.99980217e-02   2.19502844e-02   1.42209011e-02 9.93526978e-03 6.16366453e-03   1.74116798e-05   1.58780274e-16 1.30219428e-16]
        - [4.23411347e-01   1.62036489e-01   9.95945088e-02   5.85728321e-02   5.19799667e-02   5.05037229e-02   4.04141498e-02   3.12014310e-02   2.99980217e-02   2.19502844e-02   1.42209011e-02   9.93526978e-03   6.16366453e-03   1.74116798e-05   1.58780274e-16   1.30219428e-16   1.03964749e-16   9.55203032e-17   1.39078630e-34]

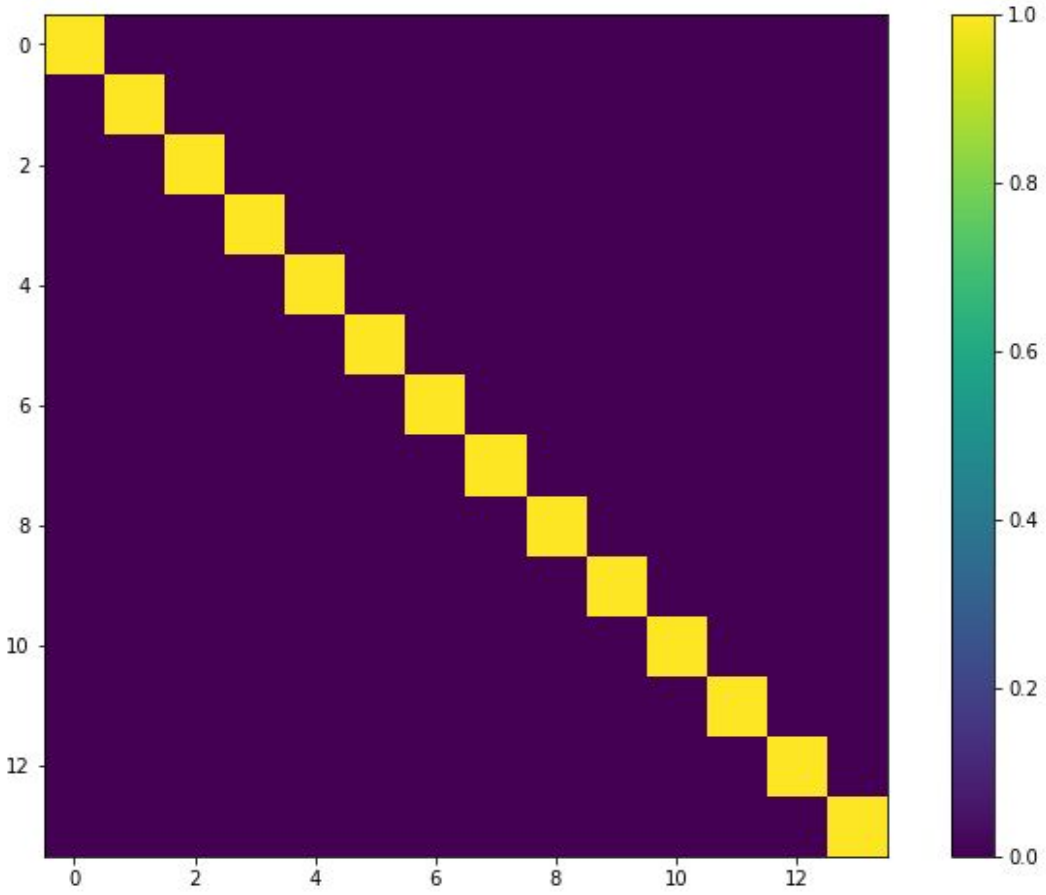
index	captured variance sum	component number
0	0.423411	1
1	0.585448	2
2	0.743615	4
3	0.846099	6
4	0.917714	8
5	0.969663	10
6	0.999983	13
7	1.000000	16
8	1.000000	19

We notice that the last 3 components are nearly 1 so, we can take the first 14 components

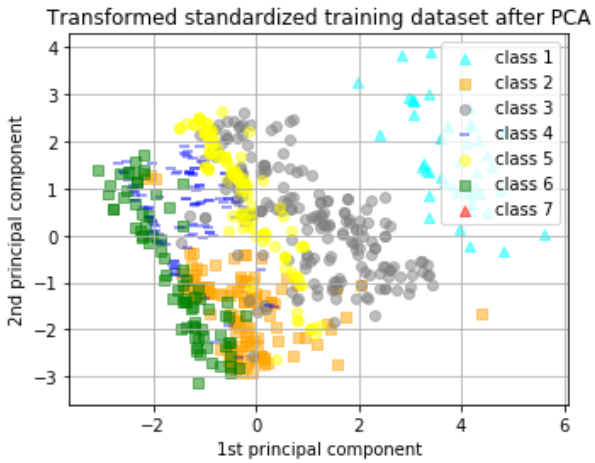
showing the matrix resultant after the PCA

Index	0	1	2	3	4	5	6	7	8	9	10	11	
0	-2.34101	-0.568638	-0.650548	-0.477064	-0.0547828	-0.136842	0.012842	-0.62045	0.276409	0.263032	-0.284534	-0.0399267	-0.
1	-0.612479	-0.191599	-1.42468	0.624449	-0.971095	0.178148	-0.291384	-0.656609	0.596323	-0.00602486	0.327241	-0.00252681	-0.
2	-0.986023	0.04876	-0.985904	0.647385	-0.896088	0.293113	-0.68396	-0.65327	0.726483	-0.0861032	-0.0956005	-0.060829	-0.
3	-1.78441	-0.21824	-0.815338	0.692126	-1.0088	0.378406	-0.564007	-1.03559	0.533698	0.11252	-0.387638	0.0935647	-0.
4	-0.840568	-0.284457	-0.870496	2.50717	0.477375	-1.11299	-0.0888781	-0.778695	0.351134	-0.0601369	0.0171944	0.107554	-0.
5	-0.497166	-0.0268167	-1.14837	0.190721	-0.512381	-0.0191702	-0.440232	-0.619748	0.764301	-0.0907055	0.101014	-0.135551	-0.
6	-0.616498	0.0286702	-0.721122	1.99571	0.987315	-1.40613	-0.139021	-0.186877	0.674429	-0.231794	-0.0196083	-0.251919	-0.
7	-0.606379	-0.345504	-0.985121	-0.466912	0.139852	-0.240579	-0.331362	-0.687787	0.523784	0.144244	0.0124836	-0.0631898	-0.
8	-0.556099	-0.0993556	-1.4741	0.489699	-0.853301	0.0663909	-0.261186	-0.618027	0.706274	-0.171118	0.324907	0.0338353	-0.
9	-0.920873	-0.297635	-1.42826	0.831645	-1.24293	0.268776	-0.190548	-0.732222	0.706257	-0.0779116	0.368993	0.0473832	-0.
10	-1.15722	-0.333159	-0.996147	0.598124	-0.889067	0.279689	-0.521061	-0.59979	0.835949	0.274237	-0.101258	-0.188172	-0.
11	-1.06347	-0.0946611	-0.529351	2.58179	0.414788	-1.01566	-0.28929	-0.806169	0.39359	-0.108707	-0.13508	0.06411	-0.
12	-0.985916	-0.336779	-0.550753	2.12227	0.81748	-1.26563	-0.0918333	-0.818186	0.47142	0.0291417	-0.12493	-0.0276746	-0.
13	-1.21033	-0.198951	-1.05523	0.825672	-1.16814	0.326482	-0.487676	-0.727645	0.963697	0.0178624	-0.00487703	-0.148549	-0.
14	-0.717101	-0.290457	-0.793273	2.16101	0.751873	-1.34338	0.023042	-0.628421	0.643131	-0.046785	0.00645112	-0.142351	-0.
15	-0.425876	-0.296811	-1.03836	-0.436665	0.151791	-0.208142	-0.377001	-0.662763	0.387249	0.0892526	0.0760996	0.0520209	-0.
16	-0.0626347	-0.205131	-1.27661	-0.330228	0.0147155	-0.248289	-0.265724	-0.58549	0.445431	-0.069919	0.325137	0.0492806	-0.
17	-0.886784	-0.387908	-0.969632	0.141301	-0.480332	0.0452911	-0.390777	-0.720113	0.745749	0.201338	-0.0124774	-0.0916619	-0.
18	-1.16836	0.0186746	-0.288599	2.069	0.952813	-1.21371	-0.388945	-0.859736	0.327125	-0.226374	-0.341839	0.12225	-0.
19	-1.5681	-0.386736	-0.398264	2.18156	0.73347	-1.23042	-0.124791	-0.995216	0.638688	0.0564065	-0.394325	-0.0724101	-0.
20	-0.0685063	0.0718931	-1.27563	-0.214001	-0.122107	-0.237934	-0.312053	-0.526885	0.548046	-0.353922	0.351167	0.0272227	-0.

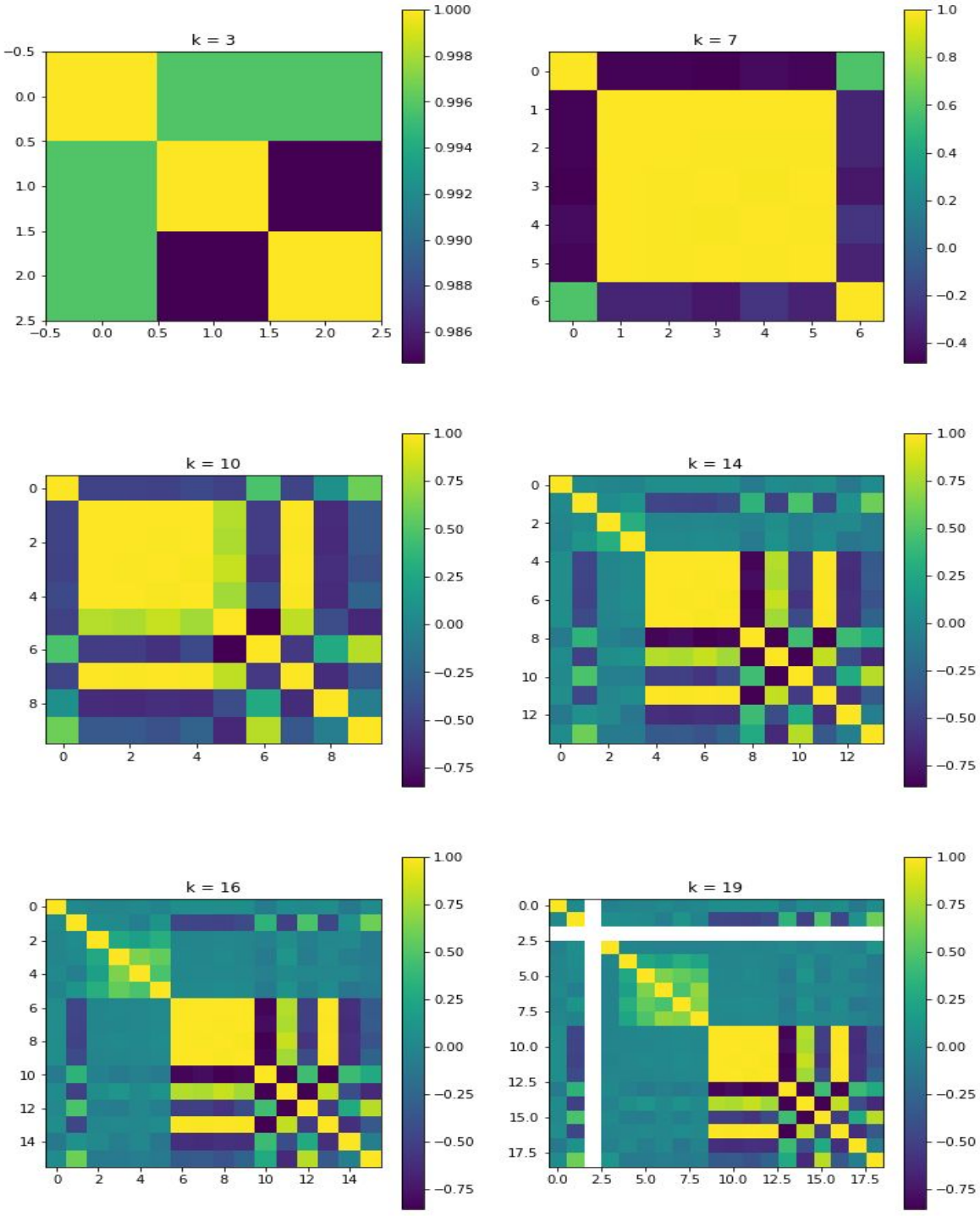
Visualization:



The visualisations shows that the correlation between each 2 attributes is 0.
So we have chosen the best uncorrelated features to avoid redundancy

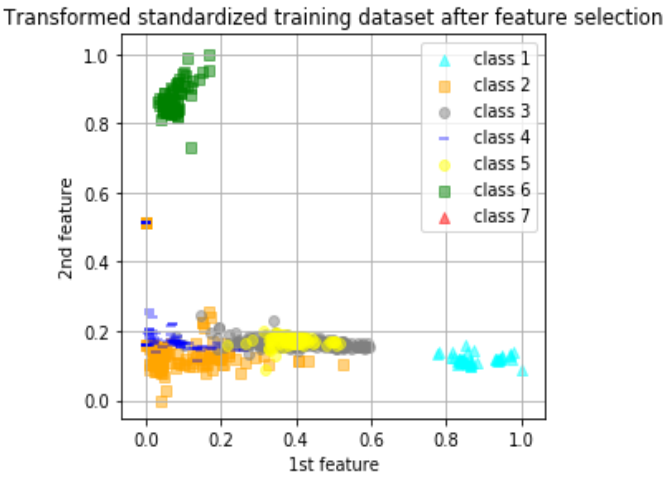


2. Feature selection



The first 5 attributes are very correlated as shown at k=4 and k=7 starting from index 1. Also the 3rd attribute appears only when k=19 which means that it is the worst attribute as we expected, due to the zero standard deviation . It gives no information.

K = 2



Correlation matrix

- **K =5**

[[0.04311903 0.05357143 0.05081001 0.05154639 0.32532112]
[0.0464756 0.06087663 0.05154639 0.05596466 0.35865966]
[0.04260263 0.0551948 0.04786451 0.05007364 0.35169946]
...
[0.78156473 0.71022725 0.86450659 0.86450659 0.09970969]
[0.76658921 0.71266236 0.84462441 0.84462441 0.11727551]
[0.7384457 0.6712662 0.82768774 0.82768774 0.1120159]]

- **k=9**

[[0.475 0.04311903 0.05357143 ... 0.43220337 0.05154639 0.32532112]
[0.50833333 0.0464756 0.06087663 ... 0.4237288 0.05596466 0.35865966]
[0.53333333 0.04260263 0.0551948 ... 0.43644067 0.05007364 0.35169946]
...
[0.02916667 0.78156473 0.71022725 ... 0.38559321 0.86450659 0.09970969]
[0.17916667 0.76658921 0.71266236 ... 0.29237287 0.84462441 0.11727551]
[0.29166667 0.7384457 0.6712662 ... 0.30084744 0.82768774 0.1120159]]

- **k=19**

[[0.5515873 0.475 0. ... 0.05154639 0.5456349 0.32532112]
[0.74206349 0.50833333 0. ... 0.05596466 0.53858024 0.35865966]
[0.41269841 0.53333333 0. ... 0.05007364 0.5326279 0.35169946]
...
[0.59920635 0.02916667 0. ... 0.86450659 0.2546684 0.09970969]
[0.32539683 0.17916667 0. ... 0.84462441 0.23450433 0.11727551]
[0.45238095 0.29166667 0. ... 0.82768774 0.26421982 0.1120159]]

High variance for larger k so the features.