Point Spread Function Modeling of Cometary Comae

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Background

Comae are the nebulous structures surrounding comets, consisting of ice and dust that form due to sublimation as comets pass close to the sun. Cometary nuclei are often obscured by these comae, making it difficult to determine where exactly the nucleus is. Through the process of astrometry, it is possible to measure the position of an astronomical object in the sky and calculate its ephemeris, which predicts the object's location at future points in time, by transforming the cartesian coordinates of the object in an image to the spherical coordinates (typically right ascension, declination) of the object in the sky. One of the first steps in producing an astrometric fit is using a centroiding method to approximate the center of the object a given image. In the case of comets, this process is often complicated both by the dispersion of brightness caused by the coma and the significant variance in the shape and opacity of the coma over time, as illustrated in fig. 1. Because comets and comet-like objects such as Centaurs are so small and far away, astrometric accuracy is needed on the scale of milli-arcseconds (less than 1 pixel in a typical image) to generate useful ephemerides, meaning even the smallest differences in centroided location of the cometary nucleus can have significant consequences.

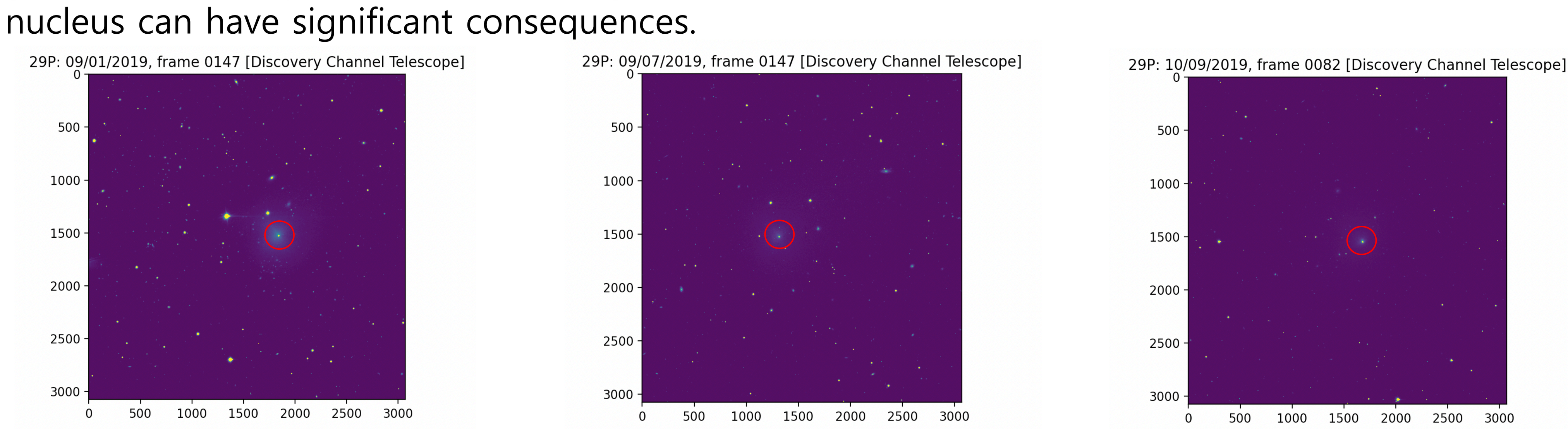

Fig. 1: Images of comet 29P from 3 different nights illustrating the variability of its coma over time

Methodology

The process of deriving the position of the cometary nucleus in an image involves fitting a circular gaussian point spread function to the image signal in order to obtain sub-pixel precision in centroiding. However, this process does not take into account the asymmetric nature of cometary comae, and literature suggests that signal from the coma drops off at a rate proportional to 1/r from the nucleus. With the goal of enabling the more accurate and consistent determination the true location of the comet nucleus, that finding was used in this project to create a new point spread function for use in the centroiding of cometary nuclei by modifying the generalized circular gaussian to include this 1/r relationship. Shown below are $G(x,y)$, the original Gaussian model (eq. 1), and $G'^{(x,y)}$, the new model (eq.2) adjusted to include a 1/r "skirt".

$$G(x,y) = \beta + \alpha * \exp\left(\frac{(x-x_0)^2+(y-y_0)^2}{2\sigma}\right) \quad [1]$$

$$G'^{(x,y)} = \beta + \frac{k}{\sqrt{(x-x_0)^2+(y-y_0)^2}} + \alpha * \exp\left(\frac{(x-x_0)^2+(y-y_0)^2}{2\sigma}\right) \quad [2]$$

The parameters for the new model were fit in Mathematica, using multivariable least squares optimization to determine peak (the maximum value of the function), row center (y_0), column center (x_0), diameter (full width half max), shape index (σ), k, and background (β). Because $\lim_{r\to 0} 1/r = \infty$, the new point spread function was implemented as a piecewise where the 1/r "skirt" was applied only outside a cutoff distance from the center whose value was determined as a fraction of the Full Width Half Max (FWHM) manually set through testing different values to see which resulted in a smaller chi square value (in order to avoid undifferentiable discontinuities in optimization) (eq. 3).

$$F(x,y) = \begin{cases} G(x,y) & \sqrt{(x-x_0)^2+(y-y_0)^2} < cutoff \\ G'(x,y) & \sqrt{(x-x_0)^2+(y-y_0)^2} > cutoff \end{cases} \quad [3]$$

The optimized parameter values were adjusted over a multiple-step process that involved first determining preliminary estimates using 3 calibration stars, then doing a "core fit" of the gaussian only on pixels within the cutoff distance to best characterize the comet nucleus, then doing a "skirt fit" of the k and background parameters only on pixels outside the cutoff distance to best characterize the coma, and finally refitting the gaussian parameters to let the model settle on a final configuration with a minimized sum of squared residuals. Fig. 2 shows the slightly broader wings of the model (green) resulting from the 1/r skirt compared with the original circular gaussian (blue).

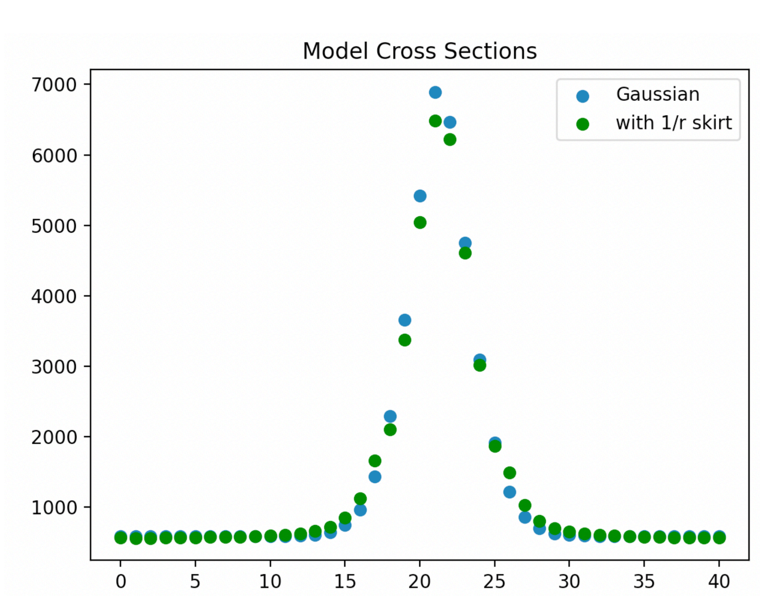

Fig. 2: Cross sections of $G(x,y)$ (blue) and $G'(x,y)$ (green)

Results

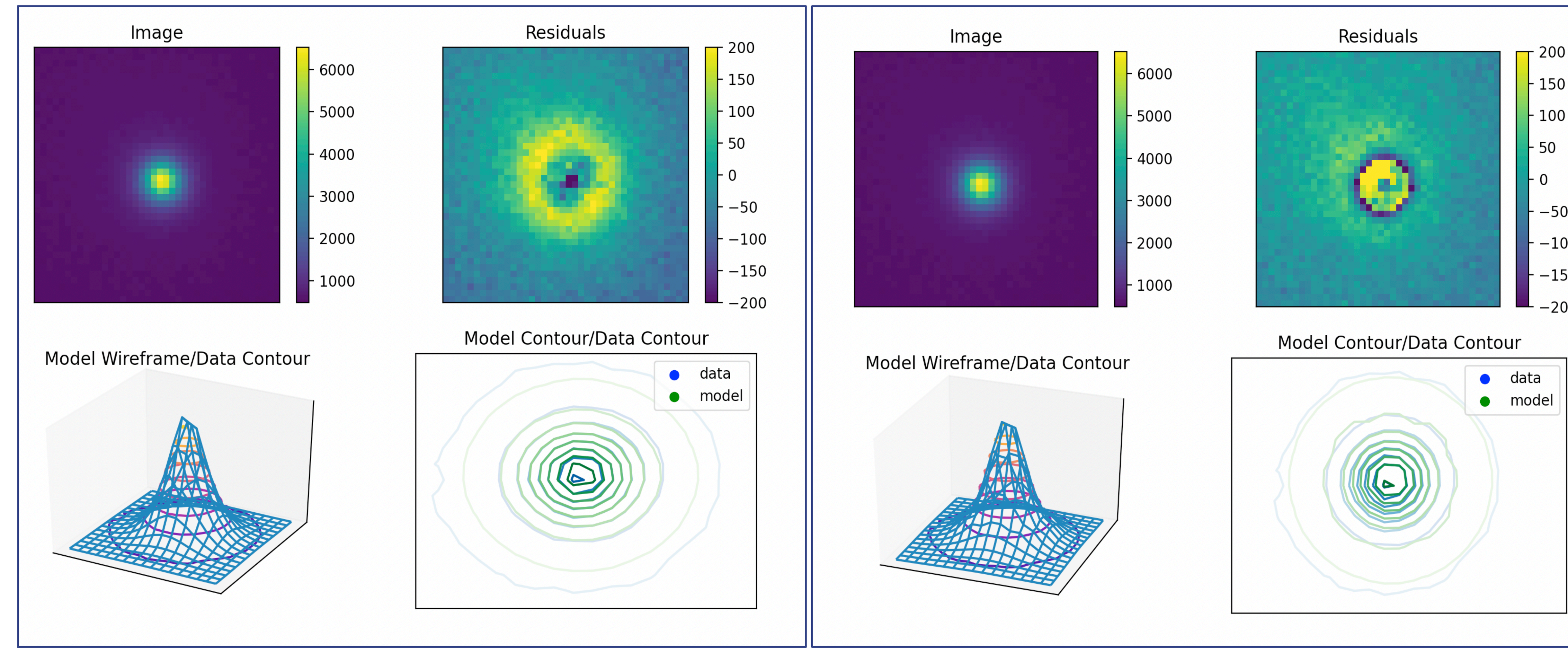

Fig. 3: Various representations of the fits for the original circular gaussian (left) and the new gaussian modified with a 1/r "skirt" (right). Fits shown applied to frame 147 of 29P (leftmost in fig.1) from the Discovery Channel Telescope on 9/01/2019

Images of 29P from the Discovery Channel Telescope on the night of 9/01/2019 were used to develop and test the model because, as shown in fig.1, this night was the one in which 29P was the most active among all nights in the dataset. On average, the χ^2 value decreased by a factor of 15 between the original and modified gaussian models (from $3.2 * 10^7$ to $2.6 * 10^6$ in the specific fits shown), suggesting that the modified model does indeed provide a better fit to the comet signal than the original. This is demonstrated in the above residual plots in fig.3 as well, as it is evident that the original gaussian (left) does a poor job of fitting the outer coma while in the modified gaussian (right), the largest residuals are much more constrained to the nucleus and the peak signal value also more closely matches the data. This same effect can be seen in the contour plots, where there is a much larger divergence between the data and the original model in the outer coma region as compared to the modified model.

Next Steps

In order to better validate the effectiveness of this model as compared to the standard circular gaussian, the immediate next step is to continue testing on additional nights of data on 29p and other cometary bodies. In addition, further work needs to be done to evaluate whether this modified model is significantly more consistent and accurate in the astrometric calculations it produces. The plan is to do this by using this new model and the standard gaussian to centroid multiple images of 29P on multiple nights, then correct for field distortion and convert the produced cartesian (x,y) centroids to spherical (right ascension, declination) coordinates and perform an additional topocentric to geocentric conversion to get analogous (right ascension, declination) coordinates to those calculated in JPL's ephemerides for 29P. Through analysis of the variance of the coordinates produced to evaluate consistency and comparison to JPL values as an imperfect but potentially useful assessment of accuracy, the goal is to reach a more robust conclusion in terms of whether this new model is a viable substitute for the standard that can significantly improve the accuracy of ephemerides for comets and comet-like bodies. Ultimately, this can result in better prediction of stellar occultations and other astronomical events involving these objects, which would allow us to gain deeper insight into their composition, history, and structure.

References

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