New SRT: Understanding SRT Sensitivity

1 Objective

- Use simulated radiometer data to plot the relationship between S(N) and N, where N is the number of samples included in an average and S(N) is the standard deviation of those averages.

2 Introduction

The spectral power density P collected by the SRT is given by

$$P = \frac{1}{2}\eta AF * 10^{-26} wHz^{-1} \tag{1}$$

where
A = geometrical area of the dish, $7.3m^2$ for a 10 foot dish
η = efficiency of dish, » 50%
F = Flux density of radio source, in Janskys (1 Jy = $10^{-26}wm^{-2}Hz^{-1}$)

The factor of $\frac{1}{2}$ arises because the antenna receives only one polarization and by convention the flux density that in both polarizations. Using the knowledge that a resistor at temperature T produces a spectral power density of kT, we can express the power density into units of temperature T_A such that:

$$T_A = \frac{\eta AF}{2k} \tag{2}$$

where k = Boltzmann's constant ($1.38 * 10^{-23}wHz^{-1}$. For example, a 1000 Jy source will produce a 1.3 K signal in a 10 foot antenna with 50% aperture efficiency. If a radio source radiates thermal radiation as a "black body" of temperature T_B, the flux density in the radio wavelength is given by the Rayleigh-Jeans law:

$$F = \frac{2kT_B\Omega_s}{\lambda^2} \tag{3}$$

where
T_B = blackbody temperature in K
Ω_s = solid angle subtended by source in rad^2
λ = wavelength in m

For example, the moon radiates as an approximately 190 K black body at 21 cm wavelength and since it subtends about 0.5 degrees at the Earth, the flux is 710 Jy. This flux will produce an output on the SRT of about 1K. From another viewpoint, the moon only covers about 1% of the beam area and so the 190 K is diluted down to 1 K (accounting for the efficiency). The 1 K signal from the moon is not a strong signal for the SRT since the system temperature is over 100 K, it is less than 1% of the receiver noise power.

Signals which are only a small fraction of the receiver noise can be detected through the use of averaging. The SRT has 50 kHz of bandwidth, which is tunable and can be scanned over a wide frequency range. If the signal power is averaged for an integration time τ with an instantaneous bandwidth of B, and system temperature T_s the 1 sigma noise DT_A in the average is given by:

$$\Delta T_A = T_s(B\tau)^{-\frac{1}{2}} \tag{4}$$

For example, the SRT hardware averages the power for each frequency in a scan for 0.1 sec, so that with a 200K system temperature, the noise is about 3K. Further reduction in noise can be obtained by averaging the averages so that:

$$\Delta T_A = T_s(B\tau N)^{-\frac{1}{2}} \tag{5}$$

where N is the number of data averages. Thus, if the receiver is limited only by its own noise, we will have to average 800 0.1 second data points to lower the noise to 0.1 K. From this analysis, it looks like the sensitivity can be increased by observing longer and longer. This is generally true, but there is often a limit reached when the fluctuations are no longer random but become systematic. For example, in order to observe the signal from the moon we need to move the dish so that it points at the moon and then points off the moon at some comparison region. This is known as "beam switching". When we move the dish on and off the moon, other things change, like the surroundings in the spillover from the feed. We need to switch back and forth from "signal" to "comparison". In this case, the effective noise will be doubled for a given total averaging time because we have to difference the "on" and "off" averages and we can only spend half the time observing the signal. This factor of 2 arises because:

1. The standard deviation of the difference of 2 independent random variable each with standard deviation s is $\sqrt{2}s$
2. The total time is cut in half so that the standard deviation increases by another factor of $\sqrt{2}$

For continuum observations, the SRT could benefit from the use of a wider bandwidth – but ultimately will be limited by systematic errors. For spectral line observations we need the narrow bandwidth to obtain sufficient spectral resolution. Also in spectral line observations we don't require a comparison as the wings of the spectrum itself provide a comparison. We may, however, need to take out a "baseline" slope as a first order correction to the shape of the overall receiver bandpass. [Haystack-Observatory,]

3 Procedure

3.1 Old SRT

Take a copy of the SRT software and load it on your computer. [You don't need any SRT hardware for this experiment – you can try it on the real hardware later.] Run the SRT in simulate mode (java srt 1 1). Select 40 frequency bins and calibrate. Open an output file and record the simulated radiometer data. Let the program run for 10 minutes to acquire approximately 5000 radiometer samples of 0.1 seconds each.[Haystack-Observatory,]

3.2 New SRT

1. Open the SRT Dash interface in simulate mode, select 40 frequency bins, and calibrate.
2. Start recording data, and let the program run for 10 minutes to acquire approximately 5000 radiometer samples of 0.1 seconds each.
3. Open the jupyter notebook corresponding to this lab and indicate the directory in which you saved your data.
4. Conduct analysis on the data using the code and functions provided in the notebook.

4 Analysis

The data can be analyzed by forming averages of N points at a time, then computing the standard deviation of the averages. For example, for N=10 take the first 10 points and form an average and then the next 10 points etc. The standard deviation s is then derived as follows: [Haystack-Observatory,]

$$\sigma(N) = (\sum_k \frac{(A_k(N) - \bar{A})^2}{M-1})^{\frac{1}{2}} \tag{6}$$

where
A_k = the k^{th} average
$\bar{A}$ = overall average of all points
M = number of averages in sum
Using the jupyter notebook corresponding to this lab, make a plot of s(N) vs N for N = 1,3,10,30,100,300

and see if it follows the $N^{-\frac{1}{2}}$ dependence. One way to do this is to plot $10\log_{10} s(N)$ vs $10\log_{10} N$ and check that the slope is $-\frac{1}{2}$. Sample results are shown in figs. 1 2.

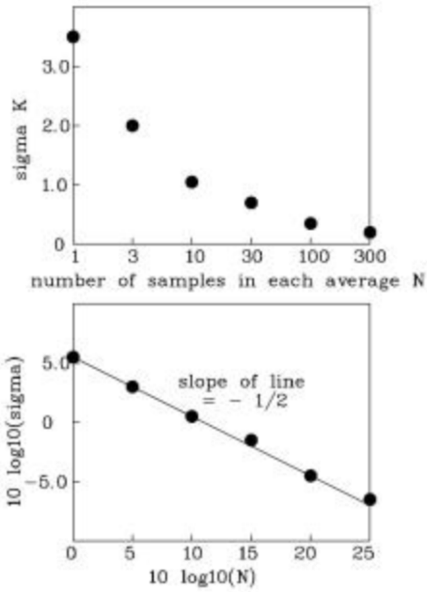

Figure 1: Old SRT: Sample results illustrating the $N^{-\frac{1}{2}}$ dependence.[Haystack-Observatory,]

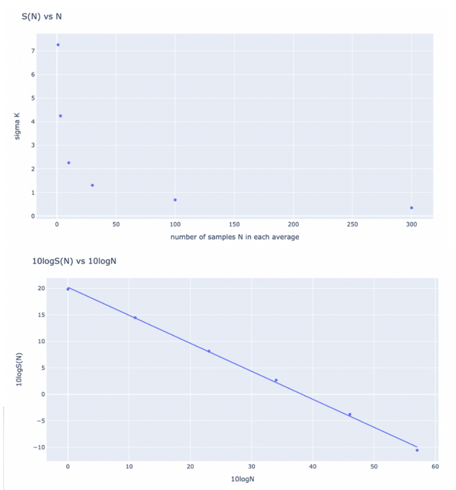

Figure 2: New SRT: Sample results illustrating the $N^{-\frac{1}{2}}$ dependence.

5 List of Radio Sources

With the current SRT design, the following sources are observable:

Source	Expected $T_A K$	Comments
Sun	250-3000	Strong source, can use 25 point map
Moon	1	Requires beamswitching
Cass A	3	Requires beamswitching
Cygnus X	7	Requires beamswitching
Galaxy	1-50	Strong signals, only a few minutes needed to obtain good spectra
Andromeda	0.5	Very weak- difficult experiment, requires days of observing

6 Discussion Questions

1. Explain why the standard deviation can be less than one even though the data in the file is written out as whole numbers
2. Perform the same analysis on real data from the SRT radiometer.

References

[Haystack-Observatory,] Haystack-Observatory. Understanding srt sensitivity. https://www.haystack.mit.edu/edu/undergrad/srt/SRT%20Projects/ActivitySRTsensitivity.html.