

EXISTENCE AND SIMULATION OF GIBBS-LAGUERRE-DELAUNAY TETRAHEDRIZATIONS

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We present revolutionary unprecedented results using newly developed methods. Songs will be sung about this paper.

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Classification: 60K35, 60G55

1. INTRODUCTION

This is an interesting field. We did some things in this interesting field. Here they are.

2. PRELIMINARIES

See [1].

- Laguerre geometry
- Gibbs point process - already tie in [2].

3. EXISTENCE

Basically [2].

- Specification of the model
- Proof of existence

Smooth interaction: For $\eta \in \mathcal{LD}(\mathbf{x})$ define the potential φ_S as a unary potential such that

$$\varphi_S(\eta, \mathbf{x}) \leq K_0 + K_1 \chi(\eta)^\beta$$

for some $K_0, K_1 \geq 0, \beta > 0$

Hard-core interaction: For $\eta \in \mathcal{LD}(\mathbf{x})$ define the potential φ_{HC} as a unary potential such that

$$\sup_{\eta: d_0 \leq \chi(\eta) \leq d_1} \varphi_{HC}(\eta, \mathbf{x}) < \infty \text{ and } \varphi_{HC}(\eta, \mathbf{x}) = \infty \text{ if } \chi(\eta) > \alpha$$

How exactly
does this
look? Why?

for some $0 \leq d_0 < d_1 \leq \alpha$.

- (S) *Stability.* The energy function H is called *stable* if there exists a constant $c_S \geq 0$ such that

$$H_{\Lambda, \mathbf{x}}(\zeta) \geq -c_S \cdot \text{card}(\zeta \cup \partial_{\Lambda} \mathbf{x})$$

for all $\Lambda \in \mathcal{B}_0, \zeta \in \mathbf{N}_{\Lambda}, \mathbf{x} \in \mathbf{N}_{\text{cr}}^{\Lambda}$.

- (R) *Range condition.* There exist constants $\ell_R, n_R \in \mathbb{N}$ and $\chi_R < \infty$ such that for all $(\eta, \mathbf{x}) \in \mathcal{E}$ there exists a finite horizon Δ satisfying: For every $x, y \in \Delta$ there exist ℓ open balls B_1, \dots, B_{ℓ} (with $\ell \leq \ell_R$) such that

- the set $\cup_{i=1}^{\ell} \bar{B}_i$ is connected and contains x and y , and
- for each i , either $\text{diam} B_i \leq \chi_R$ or $N_{B_i}(\mathbf{x}) \leq n_R$.

- (U) *Upper regularity.* M and Γ can be chosen so that the following holds.

(U1) *Uniform confinement:* $\bar{\Gamma} \subset \mathbf{N}_{\text{cr}}^{\Lambda}$ for all $\Lambda \in \mathcal{B}_0$ and

$$r_{\Gamma} := \sup_{\Lambda \in \mathcal{B}_0} \sup_{\mathbf{x} \in \bar{\Gamma}} r_{\Lambda, \mathbf{x}} < \infty. \quad (1)$$

(U2) *Uniform summability:*

$$c_{\Gamma} := \sup_{\mathbf{x} \in \bar{\Gamma}} \sum_{\eta \in \mathcal{E}(\mathbf{x}) : \eta' \cap C \neq \emptyset} \frac{\varphi(\eta, \mathbf{x})}{\#(\hat{\eta})} < \infty,$$

where $\hat{\eta} := \{k \in \mathbb{Z}^3 : \eta \cap C(k) \neq \emptyset\}$.

(U3) *Strong non-rigidity:* $e^{z|C|} \Pi_C^z(\Gamma) > e^{c_{\Gamma}}$.

For some models it is possible to replace the upper regularity assumptions by their alternative and prove the existence for all $z > 0$.

- ($\hat{\text{U}}$) *Alternative upper regularity.* M and Γ can be chosen so that the following holds.

($\hat{\text{U}}$ 1) *Lower density bound:* There exist constants $c, d > 0$ such that

$$\text{card}(\zeta) \geq c|\Lambda| - d$$

whenever $\zeta \in \mathbf{N}_f \cap \mathbf{N}_{\Lambda}$ is such that $H_{\Lambda, \mathbf{x}}(\zeta) < \infty$ for some $\Lambda \in \mathcal{B}_0$ and some $\mathbf{x} \in \bar{\Gamma}$.

($\hat{\text{U}}$ 2) = (U2) *Uniform summability.*

($\hat{\text{U}}$ 3) *Weak non-rigidity:* $\Pi_C^z(\Gamma) > 0$.

4. SIMULATION

Basically [3].

- Specification of the simulated model.
- Results

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