Department of Probability and Mathematical Statistics

FACULTY
OF MATHEMATICS
AND PHYSICS
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Existence of Gibbs-Laguerre-Delaunay Tetrahedrizations

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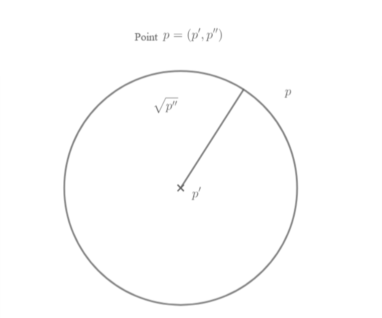
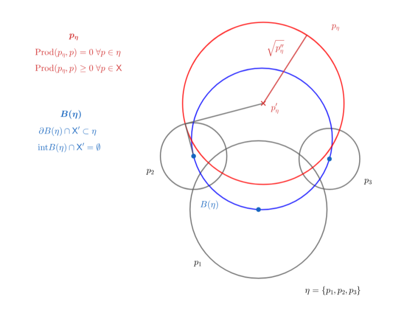
Point and empty sphere property

Geometrical interpretation

p





where for $p, q \in \mathbb{R}^3 \times S$

$$\mathrm{Prod}(p, q) = \|p' - q'\|^2 - p'' - q''.$$

Range condition

- (R) *Range condition*. There exist constants $\ell_R, n_R \in \mathbb{N}$ and $\delta_R < \infty$ such that for all $(\eta, \mathbb{x}) \in \mathcal{E}$ there exists a finite horizon Δ satisfying: For every $x, y \in \Delta$ there exist ℓ open balls $B_1, \dots, B_\ell$ (with $\ell \leq \ell_R$) such that
  - the set $\cup_{i=1}^{\ell} \bar{B}_i$ is connected and contains x and y, and
  - for each i, either $\operatorname{diam} B_i \leq \delta_R$ or $\#(\mathbb{x} \cap (B_i \times S)) \leq n_R$.

Stability

- *Stability*. The hyperedge potential φ is called *stable* if there exists a constant $c_S \geq 0$ such that

$$H_{\Lambda,\mathbb{x}}(\zeta) \geq -c_S \#(\zeta \cup \partial_\Lambda \mathbb{x})$$

for all $\Lambda \in \mathcal{B}_0, \zeta \in N_\Lambda, \mathbb{x} \in N_{\mathrm{cr}}^\Lambda$.

Upper regularity

- **(U)** *Upper regularity*. M and Γ can be chosen so that the following holds.
  - **(U1)** *Uniform confinement*: $\bar{\Gamma} \subset N_{\mathrm{cr}}^{\Lambda}$ for all $\Lambda \in \mathcal{B}_0$ and

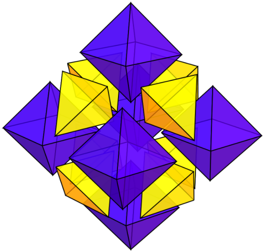
$$r_\Gamma := \sup_{\Lambda \in \mathcal{B}_0} \sup_{\mathbb{x} \in \bar{\Gamma}} r_{\Lambda, \mathbb{x}} < \infty$$

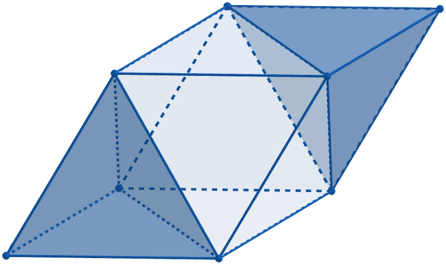
  - **(U2)** *Uniform summability*:

$$c_\Gamma^+ := \sup_{\mathbb{x} \in \bar{\Gamma}} \sum_{\eta \in \mathcal{E}(\mathbb{x}) : \eta \cap C \neq \emptyset} \frac{\varphi^+(\eta, \mathbb{x})}{\#(\hat{\eta})} < \infty,$$

    where $\hat{\eta} := \{k \in \mathbb{Z}^3 : \eta \cap C(k) \neq \emptyset\}$ and $\varphi^+ = \max(\varphi, 0)$ is the positive part of φ.
  - **(U3)** *Strong non-rigidity*: $e^{z|C|} \Pi_C^z(\Gamma) > e^{c_\Gamma}$, where c_Γ is defined as in (U2) with φ in place of φ^+.

Creates a octahedral-tetrahedral honeycomb tessellation





Alternative upper regularity

- (Û) *Alternative upper regularity*. M and Γ can be chosen so that the following holds.
  - (Û1) *Lower density bound*: There exist constants $c, d > 0$ such that $\#(\zeta) \geq c|\Lambda| - d$ whenever $\zeta \in N_f \cap N_\Lambda$ is such that $H_{\Lambda,\mathbb{x}}(\zeta) < \infty$ for some $\Lambda \in \mathcal{B}_0$ and some $\mathbb{x} \in \bar{\Gamma}$.
  - (Û2) = (U2) *Uniform summability*.
  - (Û3) *Weak non-rigidity*: $\Pi^z_C(\Gamma) > 0$.