EXISTENCE AND SIMULATION OF GIBBS-LAGUERRE-DELAUNAY TETRAHEDRIZATIONS

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We present revolutionary unprecedented results using newly developed methods. Songs will be sung about this paper.

Keywords: Gibbs point process, Laguerre-Delauay triangulation, tetrahedralization, MCMC simulation

Classification: 60K35, 60G55

1. INTRODUCTION

This is an interesting field. We did some things in this interesting field. Here they are.

2. PRELIMINARIES

See [1].

- Laguerre geometry
- Gibbs point process - already tie in [2].

3. EXISTENCE

Basically [2].

- Specification of the model
- Proof of existence

Smooth interaction: For $\eta \in \mathcal{LD}(\mathbf{x})$ define the potential φ_S as a unary potential such that

$$\varphi_S(\eta, \mathbf{x}) \leq K_0 + K_1 \chi(\eta)^\beta$$

for some $K_0, K_1 \geq 0, \beta > 0$

Hard-core interaction: For $\eta \in \mathcal{LD}(\mathbf{x})$ define the potential φ_{HC} as a unary potential such that

$$\sup_{\eta : d_0 \leq \chi(\eta) \leq d_1} \varphi_{HC}(\eta, \mathbf{x}) < \infty \text{ and } \varphi_{HC}(\eta, \mathbf{x}) = \infty \text{ if } \chi(\eta) > \alpha$$

DOI:

How exactly does this look? Why?

for some $0 \leq d_0 < d_1 \leq \alpha$.

3.1. Existence theorems

Theorem 3.1. For every hypergraph structure $\mathcal{E}$, hyperedge potential φ and activity $z > 0$ satisfying **(S)**, **(R)** and **(U)** there exists at least one Gibbs measure.

Theorem 3.2. For every hypergraph structure $\mathcal{E}$, hyperedge potential φ and activity $z > 0$ satisfying **(S)**, **(R)** and **($\hat{\mathbf{U}}$)** there exists at least one Gibbs measure.

Proofs of both theorems can be found in [2], see also Remark 3.7. in the same paper about the marked case.

(S) *Stability.* The energy function H is called *stable* if there exists a constant $c_S \geq 0$ such that

$$H_{\Lambda,\mathbf{x}}(\zeta) \geq -c_S \cdot \mathrm{card}(\zeta \cup \partial_\Lambda \mathbf{x})$$

for all $\Lambda \in \mathcal{B}_0, \zeta \in \mathbf{N}_\Lambda, \mathbf{x} \in \mathbf{N}_{\mathrm{cr}}^\Lambda$.

(R) *Range condition.* There exist constants $\ell_R, n_R \in \mathbb{N}$ and $\chi_R < \infty$ such that for all $(\eta, \mathbf{x}) \in \mathcal{E}$ there exists a finite horizon Δ satisfying: For every $x, y \in \Delta$ there exist ℓ open balls $B_1, \ldots, B_\ell$ (with $\ell \leq \ell_R$) such that

- the set $\cup_{i=1}^\ell \bar{B}_i$ is connected and contains x and y, and
- for each i, either $\mathrm{diam} B_i \leq \chi_R$ or $N_{B_i}(\mathbf{x}) \leq n_R$.

(U) *Upper regularity.* M and Γ can be chosen so that the following holds.

(U1) *Uniform confinement*: $\bar{\Gamma} \subset \mathbf{N}_{\mathrm{cr}}^\Lambda$ for all $\Lambda \in \mathcal{B}_0$ and

$$r_\Gamma := \sup_{\Lambda \in \mathcal{B}_0} \sup_{\mathbf{x} \in \bar{\Gamma}} r_{\Lambda,\mathbf{x}} < \infty. \tag{1}$$

(U2) *Uniform summability*:

$$c_\Gamma := \sup_{\mathbf{x} \in \bar{\Gamma}} \sum_{\eta \in \mathcal{E}(\mathbf{x}) : \eta' \cap C \neq \emptyset} \frac{\varphi(\eta, \mathbf{x})}{\#(\hat{\eta})} < \infty,$$

where $\hat{\eta} := \{k \in \mathbb{Z}^3 : \eta \cap C(k) \neq \emptyset\}$.

(U3) *Strong non-rigidity*: $e^{z|C|} \Pi_C^z(\Gamma) > e^{c_\Gamma}$.

For some models it is possible to replace the upper regularity assumptions by their alternative and prove the existence for all $z > 0$.

($\hat{\mathbf{U}}$) *Alternative upper regularity.* M and Γ can be chosen so that the following holds.

($\hat{\mathrm{U}}$1) *Lower density bound*: There exist constants $c, d > 0$ such that

$$\mathrm{card}(\zeta) \geq c|\Lambda| - d$$

whenever $\zeta \in \mathbf{N}_f \cap \mathbf{N}_\Lambda$ is such that $H_{\Lambda,\mathbf{x}}(\zeta) < \infty$ for some $\Lambda \in \mathcal{B}_0$ and some $\mathbf{x} \in \bar{\Gamma}$.

($\hat{\mathrm{U}}$2) = (U2) *Uniform summability.*

($\hat{\mathrm{U}}$3) *Weak non-rigidity*: $\Pi_C^z(\Gamma) > 0$.

3.2. Verification

Fix some $A \subset C \times S$ and define

$$\Gamma^b = \{\zeta \in \mathbf{N}_C : \zeta = \{p\}, p \in B(0,b) \times \left[0, \sqrt{\frac{a}{2}(1-2\rho)}\right]\},$$

the set of configurations consisting of exactly one point in the set A. The set of pseudo-periodic configurations $\bar{\Gamma}$ thus contains only one point in each $C(k), k \in \mathbb{Z}^3$.

Let M be such that $|M_i| = a > 0$ for $i = 1, 2, 3$ and $\angle(M_i, M_j) = \pi/3$ for $i \neq j$. We further assume $\rho < 1/4$, see Appendix **??**.

4. SIMULATION

Basically [3].

- Specification of the simulated model.
- Results

ACKNOWLEDGEMENT

This work was partially supported by GACR 17-00393J (investigator V. Beneš). This work was was fully supported by Panda.

(Received ????)

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