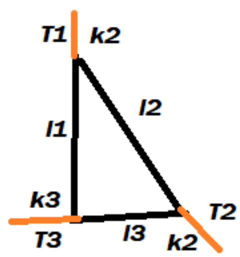
Note: After writing this up I remembered your comment last time about indexing edges with doubled indices... Will use that notation next time

# 1 Computing Constraint Spaces

Take this to be our prototypical curve(clockwise oriented):




We aim to develop a flow in reduced coordinates(in terms a lengths and angles) satisfying the following constraints:

$$\sum_{i=1}^{n} l_i T_i = 0 \tag{1}$$

$$\sum_{I=1}^{n} \kappa_i = 2\pi k \tag{2}$$

where $T_i$ are unit tangents. In terms of the above this is

$$l_1T_1 + l_2T_2 + l_3T_3 = 0$$
$$\kappa_1 + \kappa_2 + \kappa_3 = 0$$

Write this as $g : \mathbb{R}^6 \to R^3$. Examining the jacobian we find

$$\begin{bmatrix} T_1^x & T_2^x & T_3^x & (-sin\kappa_1, cos\kappa_1)\cdot l_1T_3 & (-sin\kappa_2, cos\kappa_2)\cdot l_2T_1 & (-sin\kappa_3, cos\kappa_3)\cdot l_3T_2 \\ T_1^y & T_2^y & T_3^y & (-cos\kappa_1, -sin\kappa_1)\cdot l_1T_3 & (-cos\kappa_2, -sin\kappa_2)\cdot l_2T_1 & (-cos\kappa_3, -sin\kappa_3)\cdot l_3T_2 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

where we know($Q_c$ is the clockwise rotation by c)

$$T_{i+1} = Q_{\kappa_{i+1}} T_i = (cos(\kappa_{i+1})T_i^x + sin(\kappa_{i+1})T_i^y, -sin(\kappa_{i+1})T_i^x + cos(\kappa_{i+1})T_i^y)$$

so $\frac{dT_i}{d\kappa_i} = (-sin(\kappa_i)T^x_{i-1} + cos(\kappa_i)T^y_{i-1}, -cos(\kappa_i)T^x_{i-1} - sin(\kappa_i)T^y_{i-1})$

So for a point $(l_1, l_2, l_3, \kappa_1, \kappa_2, \kappa_3)$ we have vectors spanning the normal space.

Our energy to minimize is defined by

$$E(l, \kappa) = \frac{2\kappa_1^2}{l_3 + l_1} + \frac{2\kappa_2^2}{l_1 + l_2} + \frac{2\kappa_3^2}{l_2 + l_3}$$

Then

$$E_{\kappa_i}(l, \kappa) = \frac{4\kappa_i}{l_{i-1} + l_i}$$

$$E_{l_i}(l, \kappa) = \frac{-2\kappa_i^2}{(l_i + l_{i-1})^2} + \frac{-2\kappa_{i+1}^2}{(l_{i+1} + l_i)^2}$$