80-100 Final Rough Draft 1

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1 Introduction

In “The Mathematical Universe” physicist Max Tegmark theorizes that the observable universe is “isomorphic to a mathematical structure”. In this sense, the universe is not described by math but physically represents some mathematical structure. This *TOE* - Theory of everything, is an attempt to answer the question “Why does mathematics, a man-made system, so effectively describe the universe around us?” most famously raised by physicist Eugene Wigner in his paper “The Unreasonable Effectiveness of Mathematics in the Natural Sciences”. I aim to comment on the substance behind Tegmark's *MUH* - Mathematical Universe Hypothesis and explore thoughts about the related question "Is the universe perfectly modelable by mathematics?". Assuming the ERH - external reality hypothesis to be true, I will explore the related questions "Can we perfectly model the universe mathematically?" and "Is the universe mathematical in nature?". I posit that universe is probably not mathematical in nature. Furthermore if it is mathematical, it is so in a way that is less meaningful than Tegmark asserts, and that it is not perfectly modelable.

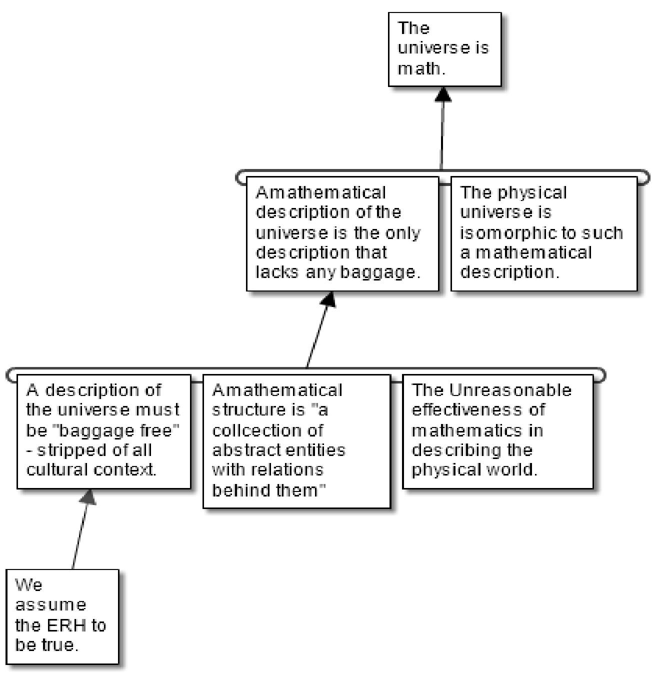
2 Background

Eugene Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" comments on the astounding success mathematics has in describing physics. Wigner loosely defines this effectiveness as a mathematical model's ability to predict future events. Furthermore he echoes Einstein in that it is amazing such relatively simple man made systems can model such vast arrays of events in the universe. He gives many examples, his seemingly favorite being the complex numbers as central to quantum mechanics. Possibly math describes physics so well because we restrict physics to what can be described by mathematics. Yet even this seems unlikely as math allows for a prediction that far exceeds our senses' abilities to perceive and thus screen out non-mathematical material. Yet surprisingly, we somehow still have very little contradiction in this physics based system explaining such a wide variety of phenomenon, leading us to believe that math is the "right language"(Wigner). Wigner's best example of this is the Lamb shift theory in quantum electrodynamics. This serves a purely mathematical theory entirely independent of human experience. It is predicted entirely by mathematics. And yet, it is corroborated extensively by empirical experiments. Newton's laws of gravitation serve as the best known example of relatively simple mathematical laws that were amazingly accurate. It appears that our array of theories is contradiction free. Yet again as we learned with Duhem and Quine, is this because we selectively choose theories that predict the most while simultaneously conflicting with current theory the least? Wigner asks what this says about the validity of such theories. He wonders, as mathematics progresses through the sciences, will we inevitably reach an impassable disagreement between theories of consciousness or other biological phenomena which fundamentally disagree with the developed laws of physics. This "would give us a deep sense of frustration in our search for what I called "the ultimate truth." The reason that such a situation is conceivable is that, fundamentally, we do not know why our theories work so well. Hence, their accuracy may not prove their truth and consistency."(Wigner)

20 years later American Mathematician R.W. Hamming published "The Unreasonable Effectiveness of Mathematics"; it was inspired by Wigner's now-famous piece in an effort to more thoroughly answer the questions raised by Wigner. Hamming underscores the importance of invariance, captured by mathematical equations, that allows the development of physics to occur. Mathematical descriptions assume invariance of the phenomena under a near infinite number of variables. If we did not assume such invariance to exist, it would presumably render mathematical equations too cumbersome and impractical, decreasing the effectiveness of the model. As Einstein has said, it is no surprise that the simplest yet most versatile models are those we deem the most beautiful. While Wigner simply characterizes mathematics as "chosen for their amenability to clever manipulations and striking, brilliant arguments" but goes into little more detail, Hamming seeks to give a clearer picture of mathematics through examples. He breaks the discipline into four categories: logic, geometry and its subsequent protege, number theory and arithmetic and algebra etc, and "artistic taste". He then proceeds to give a brief overview of math as being developed through necessity and convenience to support the activities of humans. Such examples include numbers being developed for counting and geometry being developed for design. But this continues to raise the question: How can a system developed exclusively on human experience and reason possibly grow to explain vast ranges of phenomena outside of our immediate experience? What is more, standards of mathematical rigor are continually increasing. Hamming uses Euclid's proofs as an example of this. While his proofs have been found to be deficient in rigor, every theorem he introduced has been re-proven true using additional axioms, suggesting that at the very least his system of proofs is contradiction free. This further pushes us to believe that math is "fundamentally true". Hamming explains these observations in four arguments. First, "we see what we look for"(Hamming). In other words we make a fundamental set of assumptions about the physical world around us, and then we deduce what we can from those assumptions with the aide of experiments. But this is very like what mathematicians do in constructing/discovering new mathematics. Second,

"we select the mathematics to fit the situation". It is certainly true that much math has been developed that does not appear to be applicable to the universe. Third, "science in fact answers comparatively few problems". Lastly, "The evolution of man has provided the model." This is the theory that evolution selects for species with the best mental model of the external reality.

The last paper we will be considering in the context of this piece is Max Tegmark's aforementioned "The Mathematical Universe". His thesis argues that if the external reality hypothesis is true, then the universe is precisely mathematical in nature. He first states the *ERH* requires that a description of the universe must be "well-defined according to non-human sentient beings that lack common understanding of concepts we humans have evolved"(Tegmark). So then such a description must be "baggage free"(Tegmark)-ie. stripped of cultural context. Tegmark claims that a mathematical description is **the only** theory that fulfills such a requirement. He defines a mathematical structure as "*a collection of abstract entities with relations behind them*". Then, because such a mathematical structure can be said to be isomorphic to the physical universe, Max Tegmark concludes the universe ***is*** a mathematical structure. As circumstantial evidence Tegmark cites the overwhelming effectiveness of math as touched upon by Wigner and many others at describing our universe. This reduces physics to mathematically describing a physical universe isomorphic to a mathematical structure. In effect math approximates math. This argument diagram provides a rough outline:



3 Arguments

Now we will consider these works in the context of the two questions: Is the universe mathematical in nature? Can we completely model the universe mathematically?

3.1 Is the universe mathematical in nature?

Both Wigner and Hamming alluded to this question but failed to address it directly. Max Tegmark, influenced by Wigner, suggests that the universe is a mathematical structure and therefore mathematical in nature.

Tegmark asserts that for a reality independent of humans to exist, it must be a mathematical structure through a chain of logic highlighted above. First note how Tegmark defines a mathematical structure. Informally he defines a mathematical object as any collection of objects with relationships between them. This is an incredibly broad definition, which seems to reduce its utility. Admitting this has far ranging implications. Immediately it seems to classify everything as mathematical. By the finite nature of human existence, it seems that every form of experience can described as a 'set of objects' with 'relationships'. Intuitively it seems as if such a definition is missing something crucial. Mathematics demands a certain level of precision. It is precisely because of this lack of rigor that Max Tegmark's definition of a mathematical structure is so encompassing. Yet because of it the definition seems in itself not rigorous enough to be mathematical. As Wigner aptly stated, math is the art of useful definitions, and I contend Tegmark's definition is not useful.

However let us assume that Tegmark's definition is useful in order to examine his other premises and conclusions. The conclusion that a mathematical description of the universe is the only description lacking baggage is too questionable. Is it fair to say that mathematics is the only viable descriptive system? Is it even fair to say that mathematics is completely culture free? While I do not know whether or not mathematics is the only system, claiming it is requires proof. And Tegmark gives none. While if we accept that math is completely free of baggage, this does not mean that no other systems are similarly free of baggage. I suspect an argument can be made that any system that contains no baggage is isomorphic to a mathematical system. However such an argument likely runs into problems similar to those encountered when informally defining a mathematical structure as Tegmark does.

*Start of major new section***

Yet, is it even fair to say that mathematics is a completely baggage free, acultural system? I don't think this is necessarily true either. Hamming alludes to this in his fourth argument. He says "The evolution of man has provided the model". Man's creation, or discovery of mathematics, has evolved with us. While it is a highly abstract system, it still depends on fundamental assumptions made by the human race. Consider a simple alien race thought experiment. It is not inconceivable that another race could have evolved with an entirely separate system of logic and thus mathematics, that is still equally powerful in explaining the external world around them. As Hamming suggests, it is not inconceivable that there are thoughts we cannot think, just as there are experiences we cannot experience. Therefore I contend that Max Tegmark also assumes that any sentient race shares the same foundations of logic that we do in order to interpret the structures he defines as mathematical.

It is for these reasons that I disagree strongly with Tegmark's chain of logic. What's more, I also disagree with Hamming's suggestion that natural selection selects for the individual with the best "mental model" of the exterior world. Natural selection selects for reproducability, and it is a logical leap to say that a better understanding of the world increases the chances of reproducing. Certainly "mental models" are a factor. Yet they are far from the only factor, and I suspect much less important than believed.

Many physicists, including notably Max Tegmark, have said "We should expect our common intuitions to be violated" when discussing theories of nature such as relativity and quantum theory. Ironically, I argue that these "common intuitions" include the cognitively evolved foundations of logic and reason that we use to construct mathematics. As I stated above, we have no reason to believe that these cognitive faculties allow us to capture the intricacies of our universe so perfectly. In fact to assume otherwise seems somewhat arrogant and xenocentric. We should expect science's quest to discover the foundations of the universe to violate not only our common intuitions, but also our senses of reason, logic, and mathematics.

This elevates the challenge of science to an even greater level that demands us either create or discover cognitive faculties that can augment our current mathematics and logics in their description of the universe.

End of major new section

However, let us assume for further discussion that our observable universe is formalizable through some system, perhaps even mathematics. I do not believe that math is the only system uniquely capable of completely representing the physical universe. Even assuming this, the question of whether the universe is mathematical is not well-formed. We lack both a clear idea of what the bounds of the universe are, and what constitutes mathematics. Therefore the answer to the question depends highly on the scope of both terms, as we have discovered. All we can say with certainty is the following: Assuming the ERH allows tells us that objects exist independently of humans. There exist some relationships between these objects, even if that relationship is no relationship. Based on this we can say some kind of structure is present in the universe around us. Because that structure is independent of us, it is externally observable. Then, if we extend our definition of mathematics to include construction of such a structure, then we can say the universe is mathematical in nature. The problem comes in this last step.

This parallels the question of whether mathematics is created or discovered. Yet this leads us to implications in the mind-body problem. For example, if we accept the ERH, and simultaneously some non cartesian-dualist point of view that the mind really is nothing more than the result of physical interactions with the brain, then this suggests that we must accept mathematics, constructed out of chains of logical thought, as discovered properties of a physical universe. So then all that can be mathematically discovered is contained within physical arrangements of universe. 20th century logician Kurt Godel tended toward this view of mathematics in the later years of his life. This is significant since his incompleteness theorems will be central to our analysis of the modelability of the universe. This would again imply that the universe is ultimately mathematical in nature and exists as isomorphic to a mathematial structure.

3.2 Is the Universe Perfectly Modelable with Mathematics?

First note that, if the universe is not inherently mathematical in nature then we cannot completely model it mathematically.

Now we assume for the sake of discussion that the universe is inherently mathematical. I contend that we cannot mathematically model such a universe completely. We should briefly introduce the results of Godel's incompleteness theorems in order to explain the argument. Godel's first incompleteness theorem informally states that no consistent axiomatic system of arithmetic can be complete. A system whose axioms do not lead to logical contradictions contains theorems that are true but cannot be proven true. The universe is clearly a consistent system, as saying the universe is inconsistent seems nonsense. But then this implies that any mathematical system we construct to model the mathematical universe is doomed to either inconsistency, in which case the model is useless, or incompleteness.

This seems to imply we will never develop a perfect model of the universe. Or rather we cannot prove that we have developed a perfect model of the universe. One natural question is whether or not there is a difference between these questions. I claim there is. If we could never model the universe perfectly, given a mathematical structure, then given sufficient time discrepancies would occur between our best models and the universe. Yet if we do model the universe perfectly, but cannot prove it, no discrepancies will ever occur. The difference between these is massive philosophically, since if the former were true, this would have massive implications for the greater mission of science at larger.

Therefore I conclude it is possible to perfectly model the universe if it is represented by a mathematical structure. The difficulty is in finding the appropriate structure/model. Then the question arises, will humans ever be able to find this structure? I do not think so. While Godel's theorem does not prevent us from finding such a model theoretically, it does prevent us from finding such a model practically. The reason is a statistical one. There exists only one model that perfectly describes the universe: the mathematical structure to which the universe is isomorphic. Yet there exists perhaps a humongously finite, countably infinite, or uncountably infinite number of mathematical structures that could be used to describe it. Godel's theorem does tell us that we have no way to prove that any one model is the correct one. All we can do is disprove incorrect models through discrepancies. Because we humans only have a finite amount of time to detect such discrepancies, the likelihood we come across the correct model is effectively 0. What is more, the closer we approach the correct model, the longer it will take to detect disqualifying discrepancies!

4 Counterarguments

My thoughts on the matter are themselves likely neither complete nor consistent. Consider the following.

4.1 Mathematics is Created, Not Discovered

Many mathematicians strongly believe that the mathematics we have is created purely by humans, as opposed to discovered. In this case the argument would go that mathematics is in fact a more abstract form of knowledge than knowledge about the universe, and finding a mathematical structure that represents the universe is a concrete application of the abstract. However, consider again my argument surrounding the mind-body problem in conjunction with a mathematically structured universe. If one admits that the mind is nothing more than the result of physical arrangements of physical structures within a physical universe, then it seems that mathematics must in fact be discovered. Hamming's thoughts on the limits of thought come prove particularly relevant here. If we assume the idea of a mathematical thought in humans to be some physical arrangement of neurons, which is in turn some physical arrangement of the universe, is this not included in the mathematical structure that is said to describe the universe? This produces a contradiction. Excluding the existence of a Cartesian Dualist existence of the brain, which I choose to exclude for sake of argument, is there some way to reconcile holding the belief that mathematics is created by humans with the belief that the universe is mathematical in nature?

4.2 Does our Abstract and Non-rigorous Definition of Math Guarantee That it Describes the Whole Universe?

Next, how do we know that even with as vague a definition as a "mathematical structure is *A collection of objects with relationships*", we can describe the entire universe. Once again, consider the words of Hamming: "Is it inconceivable that there are certain thoughts we cannot think?". Hamming conjectured that evolution has selected towards organisms with the best mental models of the external universe. But clearly even organisms such as humans have severe limitations on what they are able to perceive in the universe. We classify light according to whether or not it is observable to our eye, and sound according to whether or not we can hear it. Perhaps there are types of thoughts that we do not have access to? Perhaps within these classes of thought is contained entirely foreign pieces of logic, crucial to completely defining the universe. Ultimately, is it possible to describe that which we do not know exists? Intuitively the answer is no. Then how can we define the entire universe mathematically without even knowing if mathematics is the most ”abstract” form of thought?

5 Counter-Counterarguments

5.1 On 4.1

If we assume that a mathematical structure describes the physical universe, then every physical arrangement of the universe is described by the structure. Then, under the assumption that our mind is purely physical, our mathematical thoughts can be analogues of certain arrangements of physical structures, which are in turn described by the mathematical structure of the universe. Given the assumptions, this claim seems irrefutable. This suggests that the belief in creating math is incompatible with the belief that 1) the universe is mathematical in nature and 2) our thoughts are nothing more than physical arrangements. These two beliefs are logically incompatible. I therefore claim that, to simultaneously believe in a mathematical universe and creation of mathematics, one must be a cartesian dualist. Otherwise our very thoughts are governed by the mathematical structure we are supposedly creating. What does this say about mathematics that seems impossible in the universe? Remember, the mathematical structure that describes the universe is an abstract structure that does not actually exist, but simply defines the physical nature of the universe mathematically. My conclusion is not, for example, that numbers actually physically exist; I only conclude that the concept of numbers is included within the abstract mathematical description of the physical universe. This then asks the further question ”What about mathematics that does not seem to describe anything in the universe?” My inadequate response is that science has simply not yet discovered the physical processes requiring such mathematical explanation. Much like Wigner, I hold the suspicion that the more we attempt to quantify the extremely complicated processes of biology, the more complicated models we will need, requiring more of the math we currently see no physical application for. A corollary to this is the distinction between theoretical and applied mathematics is artificial.

5.2 On 4.2

This seems to me a much more problematic counterargument, which because it is a purely skeptical objection seems nearly impossible to counter. Because by nature we do not know the limits of the universe, it is impossible to say that we know it is mathematically describable. As the famous linguist Ludwig Witgenstein said: ”The limits of my language are the limits of my world”. We cannot conclude anything about that which we do not know. This problem of lack of knowledge was solved by the skeptic Descartes with his belief in an almighty God. without such an assumption, the conclusion that human reason is supreme seems impossible. I cannot Wstify saying reason is certainly the most abstract form of knowledge in physical existence. For sake of argument, define the physical universe in which reason is the highest form of abstract thought. Then a mathematical structure can be found to describe it, and my previous analysis holds. Let us call this universe "our physical universe" and the whole physical universe "the physical universe". Do the limitations of our thoughts prevent us from ever interacting with anything outside our physical universe? I think so. So then for all purposes the only part of the physical universe we can come to know is our physical universe, where our analysis holds. So for all purposes we can say the physical universe is our physical universe.

The ultimate problem is that by the unknown's very nature, we can never rule out the possibility of the unknown.

6 Concluding Thoughts

Assuming the external reality hypothesis, my thesis asked the questions "Is the universe mathematical in nature?" and "Can we ever perfectly model the universe mathematically?". I concluded that the universe, specifically our universe, is mathematical in nature given a sufficiently broad definition of math, supplied by Tegmark. Secondly I concluded it is impossible for us to perfectly model even a mathematical universe mathematically. My analysis of these questions lead me to the additional claims:

1. Tegmark's definition of a mathematical structure is too non-rigorous to constitute an rigorously defined mathematical structure.
2. Tegmark's claim that the universe "is" math, in the most literal sense, is false.
3. Believing in a mathematical universe and rejecting Cartesian dualism implies mathematics is discovered rather than created.
4. Godel's Theorem does not eliminate the possibilty of finding the perfect mathematical model; statistical improbability does.

This concludes my analysis of a mathematical universe.

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