Alex Havrilla

alumhavr

Hw 5

Question 1:

Let $\mathcal{M}$ be a Riemann manifold with affien connection ∇. Let $\gamma : I \to \mathcal{M}$ be a curve. Let $P_{\gamma,t_0,t} : T_{\gamma(t_0)}\mathcal{M} \to T_{\gamma(t)}\mathcal{M}$ be the mapping taking tangent vector V_0 at $\gamma(t_0)$ to $V(t)$ where V is the parallel transport of V_0 along γ.

Let X and Y be vector fields on $\mathcal{M}$. Consider curve γ as an integral curve for X. Then $\frac{d\gamma}{dt} = X|_{\gamma(t)}$.

Prop 1. *Where ∇ is the Riemann connection then*

$$\nabla_X Y|_{\gamma(t_0)} = \frac{d}{dt}(P^{-1}_{\gamma,t_0,t} Y|_{\gamma(t)})|_{t=t_0}$$

Proof. □

Question 2:

Let $\mathcal{M}$, $\overline{\mathcal{M}}$

Question 3:

Set $\mathbb{R}^2_+ = \{(x, y) \in \mathbb{R}^2, y > 0\}$

with metric coefficients $g_{11} = g_{22} = \frac{1}{y^2}$ and $g_{12} = 0$.

Prop 2. *The christoffel symbols of the Riemannian connection are* $\Gamma^1_{11} = \Gamma^2_{12} = \Gamma^1_{22} = 0$ *and* $\Gamma^2_{11} = \frac{1}{y}, \Gamma^1_{12} = \Gamma^2_{22} = \frac{-1}{y}$

Proof. test □

Question 4:
