A Study of Khintchine Type Inequalities for Random Variables

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1 An Overview of Sharp Khintchine Type Inequalities

2 Khintchine Type Inequalities for Classes of Random Variables

- General Form

2.1 Random Signs

- Definition
- Haagerup's Approach
- Variations: Dependent random signs.

file:///C:/Users/Alex/Downloads/Haagerup

2.2 The Classical Vector Setting

- Definitions
- Khintchine Kahane

2.3 Ultra Sub Gaussian Random Variables

- Definitions
- Log Concavity
- Khintchine Results
- Examples
- Connection to Strong Concavity

2.4 Type L Random Variables

- Definitions
- Khintchine Results
- Examples: Polya, Newman, Renyi etc
- Connection to Strong Concavity and Ultra Sub Gaussianity

2.5 Generalized Random Signs

- Definitions
- Khintchine Results
- Connection to Type L

https://www.math.cmu.edu/ ttkocz/mypapers/mathematics/khinchin-disc-unif.pdf

2.6 Vectors on Spheres

- Results in the Uniform Case
- Majorization
- Bisubharmonicity
- Uniform Result on Sphere for Bisubharmonic Functions
- Results with Rotational Invariance

https://www.researchgate.net/publication/226779758$_{B}est_{K}hintchine_{T}ype_{I}nequalities_{f}or_{S}ums_{o}f_{I}ndependent_{R}$
https://www.mimuw.edu.pl/ rlatala/papers/cm6826.pdf
https://pdfs.semanticscholar.org/30af/d8b3a15c3b180e94f8d39c3cdfd90540bba5.pdf

2.7 Gaussian Mixtures

- Mixtures
- Results for Gaussian Mixtures

https://arxiv.org/pdf/1611.04921.pdf

2.8 The Exponential Family

- Definitions
- Results for Exponential Family

https://arxiv.org/abs/1801.07597

3 Applications

- Littlewood-Payley Decomposition
- Banach Space Type
- Grothendieck Inequality
- Projections of Cross-Polytopes

Banach space: absolute convergence of series. Reimann's theorem: series is unconditinoal if absolute. But if not absolutely converging then not unconditionally converging

Banach: Permutation invariant, iff convering absolutely: If + and - is convering: Classical absolute convergene is too strong

4 Notes

4.1 Ultra Sub Gaussanity

`https://link.springer.com/content/pdf/10.1007/s11117-011-0130-z.pdf`

Remark 1. Haagerup finding best constants for $p \in (2,3)$ and q in $(0,2)$ resolved parts important in applictaions since 2 norm easily computable

Remark 2. Problem of finding optimal constants for general $C_{p,q}$ still open. Note $C_{p,q}$ is for $(\mathbb{E}|S|^p)^{1/p} \leq C_{p,q}(\mathbb{E}|S|^q)^{1/q}$

References

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