1 10/23

Notes

2 10/29

Questions:

How do we extend to khintchine inequality?

3 11/19

Goals:

Show distribution inequality. Involving lambert w functions.

Plan:

Break into cases:

Note first over $t \in (t_{lm}, 1]$ we know $G(t) \geq F(t)$ since $t_{lm} = 1/2e^{-3/2}$ is a local optimium for G and above this threshold is it strictly dominated. (For some reason graph of $p = 2$ not reflective of this? Should be dominated regardless of distribution of mass). So dif $D = F - G$ negative.

Then note from (t_+, t_{lm}) we seek to show the derivative D' negative.

From $(0, t_+)$ seek to show D postive.

This will show we have exactly one crossing from + to -.

Issue: t_+ seems to depend on p.

Confusions:

The plots don't seem to be reflecting a switch from positive to negative given a fixed variance? But it must be there?

Upon closer inspection just seems to go negative very quickly for large p. So G very quickly dominates F as p increases. Increasing p concentrates mass at 0? Makes sense cause we know gaussian F dominates G near 0(so G larger since measuring under mass).

Also all plots regardless of p should be negative at t_{lm} but this doesn't seem to be the case. Makes me worry the formulation is incorrect.

Why does it suffice to compute measure for $x > 0$ in distribution argument?

3.1 The Lambert Function

$W(x)e^{W(x)} = x$. But multiple solutions to $W(xe^x)$ (if negative).

Implcitly defined. But similar to logs and exponentials:

$$\lim_{x \to \infty} \frac{W_0(x)}{ln(x)} = 1$$
$$\lim_{x \to 0^-} \frac{W_{-1}(x)}{ln(-x)} = 1$$

References

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