Homework problem of Stochastic Calculus

(1) Le Gall's book, Exercise 1.15

(2) 1.16

(3) 1.18

(4) Let B be a standard Brownian motion, i.e., a centered Gaussian process on $\mathbb{R}_+$ with covariance function $\Gamma(s,t) = \min(s,t)$. Fix $t > 0$, and consider the partition $0 = t_0 < t_1 \ldots < t_n = t$ and the summation depending on $p \in [0,1]$ and smooth function $f \in C_c^\infty(\mathbb{R})$

$$S_f(p) = \sum_{j=0}^{n-1} f(B_{(1-p)t_j + pt_{j+1}})(B_{t_{j+1}} - B_{t_j})$$

(i) in the case of $f(x) = x$, show $S_f(p) - S_f(0)$ converges in probability as the size of the partition goes to zero; (ii) redo (i) for general smooth f.