Comments for *Algebraic Geometry* with EGA & SGA

LOCAL PROPERTIES OF SCHEMES

The general philosophy here is read as: any local property of rings naturally corresponds to a local property of schemes. In particular, we also concern about those pointwise properties of rings.

Definition 1. Let X be a scheme.

(1) X is *reduced* if, for any affine open subset $\operatorname{Spec} A \subseteq X$, the ring A is reduced; or equivalently, for each $x \in X$, the stalk $\mathcal{O}_{X,x}$ is a reduced ring.
(2) X is *normal* if those A's in (1) are all normal.
(3) X is *locally noetherian* if those A's in (1) are all noetherian rings.
Caveat. The noetherian condition is not pointwise.
(4) X is *regular* if those A's in (1) are all regular rings. That is, X is locally noetherian and, for each $x \in X$, $\mathcal{O}_{X,x}$ is a regular ring.[1]
(5) X is *Dedekind* if it is regular of $\dim X \leqslant 1$.[2] Or alternatively, X is locally noetherian, and each $\mathcal{O}_{X,x}$ is either a field or a discrete valuation ring.
(6) X is *Krull* if, for any $x \in X$, there exists an affine open neighborhood $\operatorname{Spec} A \subseteq X$, such that A is a Krull integral domain.
(7) X is *Artinian* if, X is noetherian, and all $\mathcal{O}_{X,x}$ are Artinian local rings. Equivalently, $X \simeq \operatorname{Spec} A$ for some Artinian ring A. (c.f. [EGA I, §6.2].)

We give the following summary on relations between schemes with various local properties induced by those of rings.

Proposition 2. *We obtain the following inclusions:*

$$\{\textit{Dedekind schemes}\} \subsetneq \{\textit{regular schemes}\} \subsetneq \{\textit{normal locally noetherian schemes}\}$$
$$\subsetneq \{\textit{Krull schemes}\} \subsetneq \{\textit{normal schemes}\}.$$

School of Mathematical Sciences, Peking University, 100871, Beijing, China
Email address: daiwenhan@pku.edu.cn

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[1] In fact one can define similarly the *Gorenstein* schemes and the *Cohen–Macaulay* schemes.

[2] Some other materials may require $\dim X = 1$.