

# 3DCV Hw1

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tags: 3DCV Python Report

## Problem 1: Homography estimation

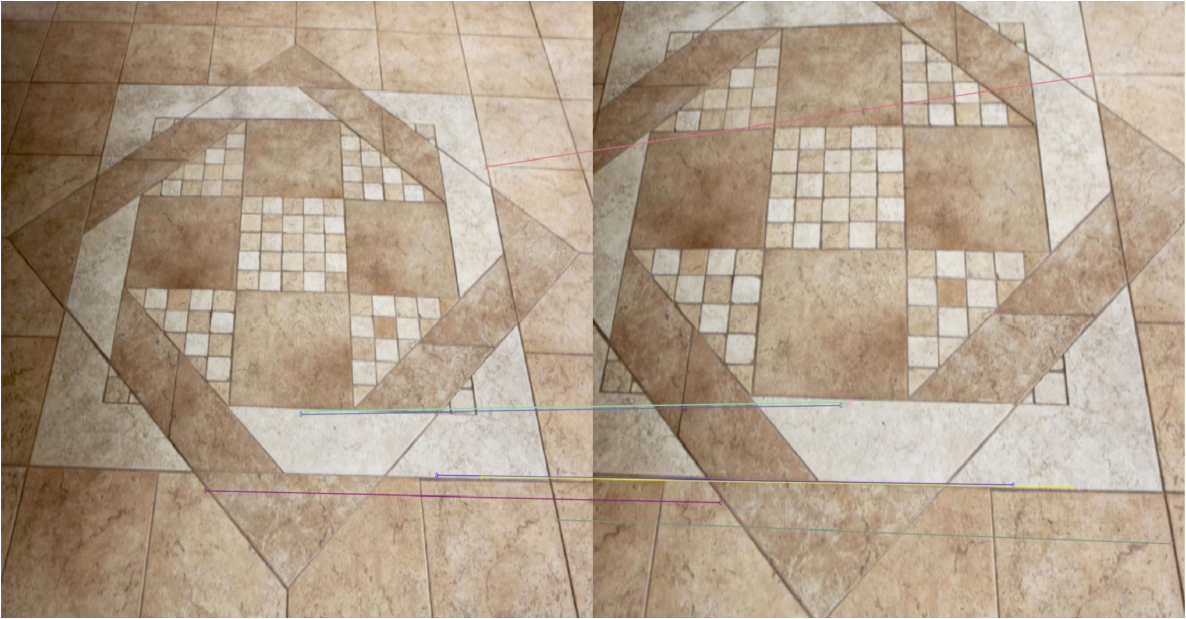
- Settings:
  - Execute codes:

```
python .\1.py .\images\1-0.png .\images\1-[1or2].png
.\groundtruth_correspondences\correspondence_0[1or2].npy [k_toppairs]
```

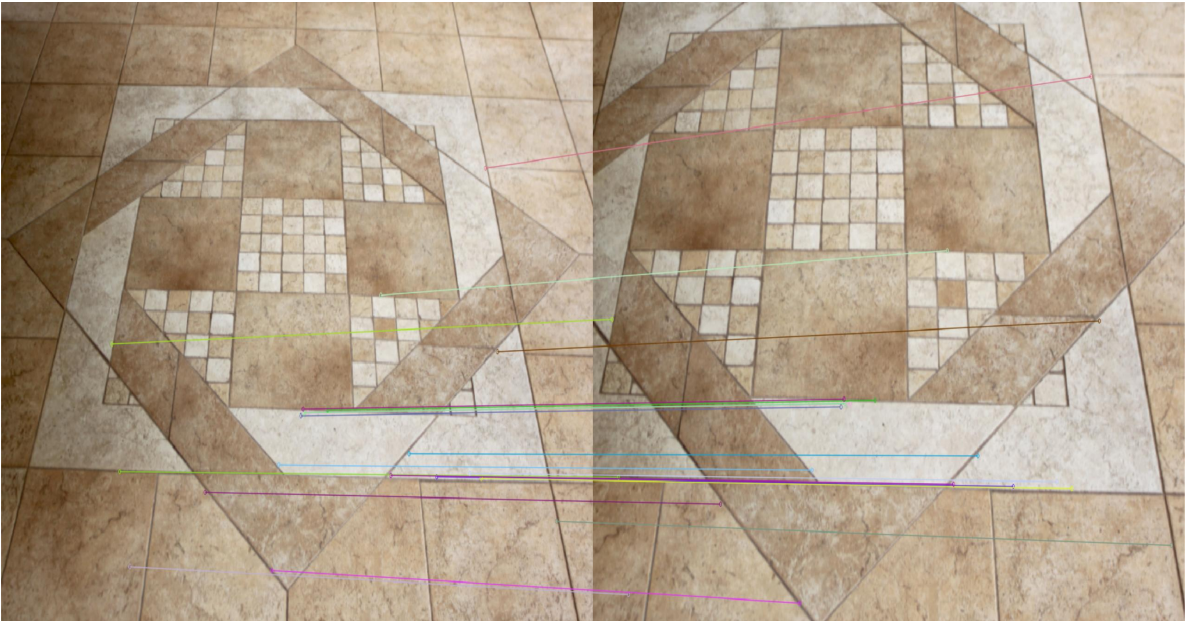
    - ex: `python .\1.py .\images\1-0.png .\images\1-2.png`  
`.\groundtruth_correspondences\correspondence_02.npy 20`
  - Package:
    - sys, numpy, cv2, math
  - Environment:
    - python: 3.7.9
    - opencv-python == 4.5.1.48
- Screenshots of sampling k correspondences
  - img 1-0 and 1-1
    - k = 4



■  $k = 8$



■  $k = 20$



○ img 1-0 and 1-2

■  $k = 4$





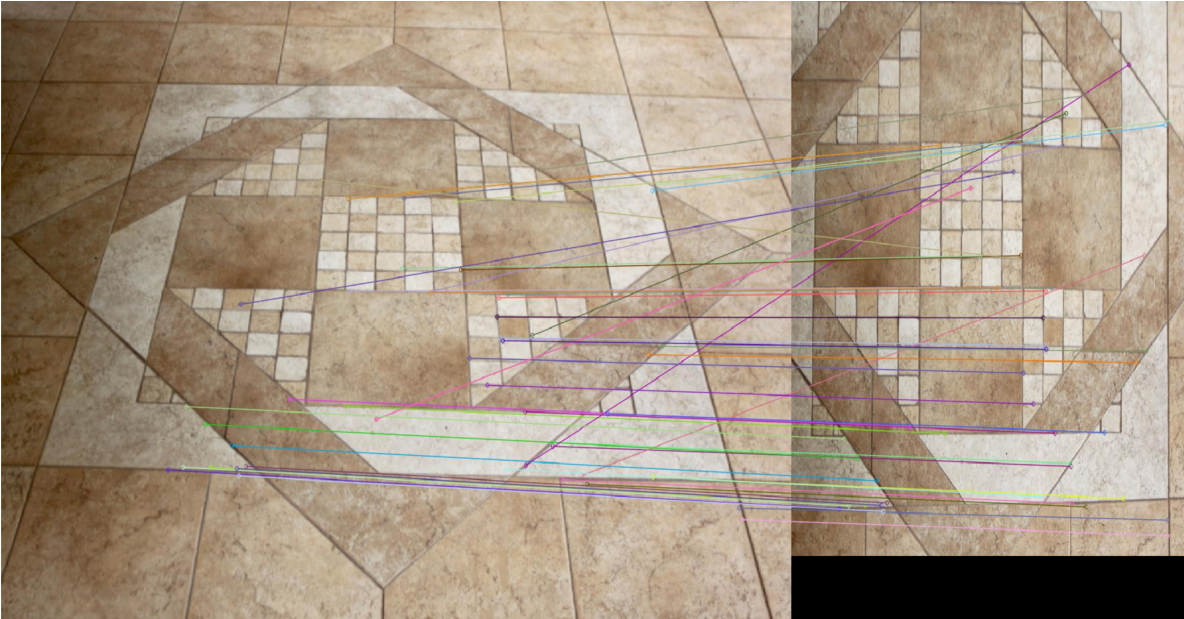
■  $k = 8$



■  $k = 20$



■  $k = 50$



- Briefly explain code:
  - First of all, the function `get_sift_correspondences` acquires matched points of two images. In addition, with Lowe's ratio test, it rejects some outliers whose two best matches of a point is not sufficiently different. In addition, choose only the top  $k$  pairs of the correspondences as the matching points.

```

7  def get_sift_correspondences(img1, img2, k_toppairs):
8      ...
9      Input:
10         img1: numpy array of the first image
11         img2: numpy array of the second image
12
13      Return:
14         points1: numpy array [N, 2], N is the number of correspondences
15         points2: numpy array [N, 2], N is the number of correspondences
16         ...
17
18         # sift = cv.xfeatures2d.SIFT_create() # opencv-python and opencv-contrib-python version == 3.4.2.16 or enable nonfree
19         sift = cv.SIFT_create()
20         # find keypoints and descriptors
21         kp1, des1 = sift.detectAndCompute(img1, None)
22         kp2, des2 = sift.detectAndCompute(img2, None)
23
24         matcher = cv.BFMatcher() # BFMatcher: Brute-Force Matcher
25         matches = matcher.knnMatch(des1, des2, k=2) # Match descriptors
26
27         # Ratio test
28         good_matches = []
29         for m, n in matches:
30             if m.distance < 0.75 * n.distance:
31                 good_matches.append(m)
32
33         good_matches = sorted(good_matches, key=lambda x: x.distance)
34
35         # Sample k pairs of correspondences
36         good_matches = good_matches[:k_toppairs]
37
38         points1 = np.array([kp1[m.queryIdx].pt for m in good_matches])
39         points2 = np.array([kp2[m.trainIdx].pt for m in good_matches])
40
41         img_draw_match = cv.drawMatches(img1, kp1, img2, kp2, good_matches, None, flags=cv.DrawMatchesFlags_NOT_DRAW_SINGLE_POINTS)
42         cv.namedWindow('match', cv.WINDOW_NORMAL)
43         cv.imshow('match', img_draw_match)
44         cv.waitKey(0)
45
46         return points1, points2

```

- Calculate homography matrix by Direct Linear Transform.
  - Build the matrix  $A$  by the  $k$  pairs of correspondences.
  - Acquire  $V$  from SVD of  $A$  and homography matrix is the last row of  $V$  after reshaped.

```

48 # Direct Linear Transform
49 def DLT(k_points1, k_points2):
50     """
51     Input:
52         points1: numpy array [k, 2], k is the number of top k pairs of correspondences
53         points2: numpy array [k, 2], k is the number of top k pairs of correspondences
54
55     Return:
56         Homography matrix: 3 x 3 matrix
57     """
58
59     # Construct A matrix from point correspondences
60     k_toppairs = len(k_points1)
61     A = np.zeros([k_toppairs * 2, 9], dtype=float)
62     for i in range(k_toppairs):
63         A[i * 2, 3] = -k_points1[i, 0]
64         A[i * 2, 4] = -k_points1[i, 1]
65         A[i * 2, 5] = -1
66         A[i * 2, 6] = k_points2[i, 1] * k_points1[i, 0]
67         A[i * 2, 7] = k_points2[i, 1] * k_points1[i, 1]
68         A[i * 2, 8] = k_points2[i, 1]
69
70         A[i * 2 + 1, 0] = k_points1[i, 0]
71         A[i * 2 + 1, 1] = k_points1[i, 1]
72         A[i * 2 + 1, 2] = 1
73         A[i * 2 + 1, 6] = -k_points2[i, 0] * k_points1[i, 0]
74         A[i * 2 + 1, 7] = -k_points2[i, 0] * k_points1[i, 1]
75         A[i * 2 + 1, 8] = -k_points2[i, 0]
76
77     # SVD
78     U, S, V = np.linalg.svd(A)
79     h = V[-1]
80     H = h.reshape(3, 3)
81     H /= H[2, 2]
82
83     return H

```

- Calculate normalized homography matrix by normalized DLT.
  - First compute similarity transform for both matching points that translates the centroid to the origin and scales the average distance from the origin is the root of 2.
  - Apply DLT on the normalized points to obtain a homography matrix.
  - Last, denormalize and get the true homography.

```

85 # Normalized Direct Linear Transform
86 def norm_DLT(k_points1, k_points2):
87     """
88     Input:
89         points1: numpy array [k, 2], k is the number of top k pairs of correspondences
90         points2: numpy array [k, 2], k is the number of top k pairs of correspondences
91         k_toppairs: k_toppairs
92
93     Return:
94         Homography matrix: 3 x 3 matrix
95     """
96
97     k_toppairs = len(k_points1)
98
99     # Calculate mean of u, v and u', v'
100     mu_u1 = k_points1[:, 0].sum() / k_toppairs
101     mu_v1 = k_points1[:, 1].sum() / k_toppairs
102     mu_u2 = k_points2[:, 0].sum() / k_toppairs
103     mu_v2 = k_points2[:, 1].sum() / k_toppairs
104
105     # Scalar quantities
106     s1 = 0
107     for pnt in k_points1:
108         s1 += np.sqrt(np.square(pnt[0] - mu_u1) + np.square(pnt[1] - mu_v1))
109     s1 /= math.sqrt(2) * k_toppairs
110     s1 = np.reciprocal(s1)
111
112     s2 = 0
113     for pnt in k_points2:
114         s2 += np.sqrt(np.square(pnt[0] - mu_u2) + np.square(pnt[1] - mu_v2))
115     s2 /= math.sqrt(2) * k_toppairs
116     s2 = np.reciprocal(s2)

```



```

118 # Calculate similarity transform T, T'
119 T1 = np.zeros([3, 3], dtype=float)
120 T1[0, 0] = s1
121 T1[0, 2] = -s1 * mu_u1
122 T1[1, 1] = s1
123 T1[1, 2] = -s1 * mu_v1
124 T1[2, 2] = 1
125
126 T2 = np.zeros([3, 3], dtype=float)
127 T2[0, 0] = s2
128 T2[0, 2] = -s2 * mu_u2
129 T2[1, 1] = s2
130 T2[1, 2] = -s2 * mu_v2
131 T2[2, 2] = 1
132
133 # p1 = [ui, vi, 1], norm_p = T dot p
134 col_ones = np.ones(k_toppairs)
135 p1 = np.column_stack((k_points1, col_ones))
136 p2 = np.column_stack((k_points2, col_ones))
137
138 norm_p1 = np.matmul(p1, T1.T)
139 norm_p2 = np.matmul(p2, T2.T)
140
141 # DLT of normalized points
142 H = DLT(norm_p1, norm_p2)
143
144 # Denormalize to get the normalized Homography matrix
145 norm_H = np.matmul(np.linalg.inv(T2), np.matmul(H, T1))
146 norm_H /= norm_H[2, 2]
147
148 return norm_H

```

- Compute error by the L2 norm of the estimated target points and ground truth matching pairs.

```

142 # Compute the reprojection error with the ground truth matching pairs
143 def compute_error(gt_correspondences, homography_mat):
144     # p_s: 100 source points, p_t: 100 target points
145     num_pts = len(gt_correspondences[0]) # 100
146     p_s = gt_correspondences[0]
147     p_t = gt_correspondences[1]
148
149     col_ones = np.ones(num_pts)
150     p_s = np.column_stack((p_s, col_ones))
151     p_t = np.column_stack((p_t, col_ones))
152     p_t_est = np.matmul(p_s, homography_mat.T)
153
154     error = np.sum(np.linalg.norm((p_t[:, :2] - p_t_est[:, :2]), ord=2, axis=0)) / num_pts
155
156     return error

```

- Compare the errors:

- Error of img 1-1 with Lowe's ratio = 0.75

img 1-1	k = 4	k = 8	k = 20	k = 1467
DLT	74.5550	7.3490	7.7314	791.5816
Norm DLT	74.5550	7.3710	7.7348	11.6377

- Error of img 1-2 with Lowe's ratio = 0.75

img 1-2	k = 4	k = 8	k = 20	k = 50	k = 88
DLT	75.2951	102.7948	102.8110	112.1455	122.6268
Norm DLT	75.2951	101.5104	94.4720	65.2852	36.0418

- Discussion

- The error between DLT and normalized DLT is not obvious in img 1-1, but it is significant in img 1-2. It is because that the viewpoint of 1-2 is much different than 1-1. It might be harder to get the correct SIFT correspondences in img 1-2.

- When setting  $k$  top points to all correspondences, the error is significantly different between DLT and normalized DLT. In addition, the errors of normalized DLT in img 1-2 are all smaller with various  $k$ . It might be caused by the advantage of normalization, which reduces the problem of ill-conditioning number and finds a better result of the homography matrix by solving SVD.

## Problem 2: Document rectification

- Settings:
  - Execute codes:

```
python .\2.py .\images\2-1 [path to save the warped result]
```

    - if prefer to save the warped image, add saving path at the end`
    - ex: `python .\2.py .\images\2-1 .\images\warped_img.png`
  - Package:
    - sys, numpy, cv2
  - Environment:
    - python: 3.7.9
    - opencv-python == 4.5.1.48
- The input document image (must be captured by yourself)



- Rectified results



- Briefly explain your method (how you choose the corners, warping efficiency)
  - I make use of the function in mouse\_click\_example.py to manually get four corners by the user (order: top-left, top-right, bottom-left, bottom-right). Also, it will automatically end the window after selecting four points.

```

5  def on_mouse(event, x, y, flags, param):
6      if event == cv.EVENT_LBUTTONDOWN:
7          param[0].append([x, y])
8
9      # Use mouse to get 4 corner points of the image
10     def get_corners(img):
11         points_add = []
12         cv.namedWindow('get_corners', cv.WINDOW_NORMAL)
13         cv.setMouseCallback('get_corners', on_mouse, [points_add])
14
15         # Break when it gets 4 points
16         while len(points_add) < 4:
17             img_ = img.copy()
18             for i, p in enumerate(points_add):
19                 # draw points on img_
20                 cv.circle(img_, tuple(p), 2, (0, 255, 0), -1)
21                 cv.imshow('get_corners', img_)
22
23             key = cv.waitKey(20) % 0xFF
24             if key == 27: break # exist when pressing ESC
25
26         cv.destroyAllWindows()
27
28         print('{} corner points added'.format(len(points_add)))
29         print(points_add)
30
31     return np.array(points_add)

```

- Set the projected image size half of the original image. Then, calculate the homography matrix by Direct Linear Transform (the result is as good as the one of normalized DLT).

```

131     # left-top, right-top, left-bottom, right-bottom
132     corners = get_corners(img)
133     #corners = np.array([[696, 1270], [2951, 294], [1395, 2747], [3556, 1841]], dtype=float)
134
135     proj_corners = np.array([[0, 0],
136                             [img_w - 1, 0],
137                             [0, img_h - 1],
138                             [img_w - 1, img_h - 1]])
139
140     H = DLT(corners, proj_corners)

```

- Warp the image by the function, back\_warping with the parameters of original image and homography matrix.



- At first, I take the inverse of the homography matrix for inverse mapping from destination pixels to original pixel positions.

```

69 # Inverse/Backward warping
70 def backward_warping(ori_img, homography_mat):
71     """
72     Input:
73         ori_img: original image
74         homography_mat: 3 x 3 matrix
75
76     Return:
77         warp_img: the warped image after doing backward warping
78     """
79
80     inv_homography_mat = np.linalg.inv(homography_mat)
81     img_h, img_w, _ = ori_img.shape
82     img_h, img_w = int(img_h/2), int(img_w/2)
83     warp_img = np.empty((img_h, img_w), dtype=np.uint8)

```

- Then, construct matrix dst\_coordinates (wi, hi, 1), which size is # of pixels x 3. And warp all of those coordinates back to original coordinates.

```

85 # Construct matrix dst_coordinates [wi hi 1], size = (img_w * img_h, 3)
86 # ex: image size = 2 x 3
87 # | 0 0 1 |
88 # | 1 0 1 |
89 # | 2 0 1 |
90 # | 0 1 1 |
91 # | 1 1 1 |
92 # | 2 1 1 |
93
94 w_arr = np.arange(img_w).reshape((img_w, 1))
95 h_arr = np.zeros((img_w, 1))
96 dst_coordinates = np.hstack((w_arr, h_arr))
97
98 for i in range(1, img_h):
99     h_arr = np.ones((img_w, 1)) * i
100     dst_coordinates = np.vstack((dst_coordinates, np.hstack((w_arr, h_arr))))
101
102 one_arr = np.ones((len(dst_coordinates), 1))
103 dst_coordinates = np.hstack((dst_coordinates, one_arr))

```

- Calculate RGB values by the four nearest points of the original image.

```

105 # Inverse warping to source coordinates
106 src_coordinates = np.matmul(dst_coordinates, inv_homography_mat.T)
107 src_coordinates = src_coordinates.T / src_coordinates[:, 2]
108 src_w_arr, src_h_arr = src_coordinates[0], src_coordinates[1]
109
110 # Calculate values of each pixel
111 h_l_arr, h_r_arr = np.floor(src_h_arr).astype(int), np.ceil(src_h_arr).astype(int)
112 w_l_arr, w_r_arr = np.floor(src_w_arr).astype(int), np.ceil(src_w_arr).astype(int)
113
114 warp_img = (ori_img[h_l_arr, w_l_arr, :] * (src_h_arr - h_l_arr).reshape(-1, 1) * (src_w_arr - w_l_arr).reshape(-1, 1)
115             + ori_img[h_l_arr, w_r_arr, :] * (src_h_arr - h_l_arr).reshape(-1, 1) * (w_r_arr - src_w_arr).reshape(-1, 1)
116             + ori_img[h_r_arr, w_l_arr, :] * (h_r_arr - src_h_arr).reshape(-1, 1) * (src_w_arr - w_l_arr).reshape(-1, 1)
117             + ori_img[h_r_arr, w_r_arr, :] * (h_r_arr - src_h_arr).reshape(-1, 1) * (w_r_arr - src_w_arr).reshape(-1, 1)).reshape((img_h, img_w, 3)).astype(np.uint8)
118
119 return warp_img

```

## • Discussion

- My warped result was nearly blanked at first, and I figured out I should set the type to uint32.
- Besides, I used two for loops to calculate values of each pixel which was super slow. Therefore, I improved the computational time by constructing a matrix with destination pixel values and multiplying it with inverse homography matrix.