

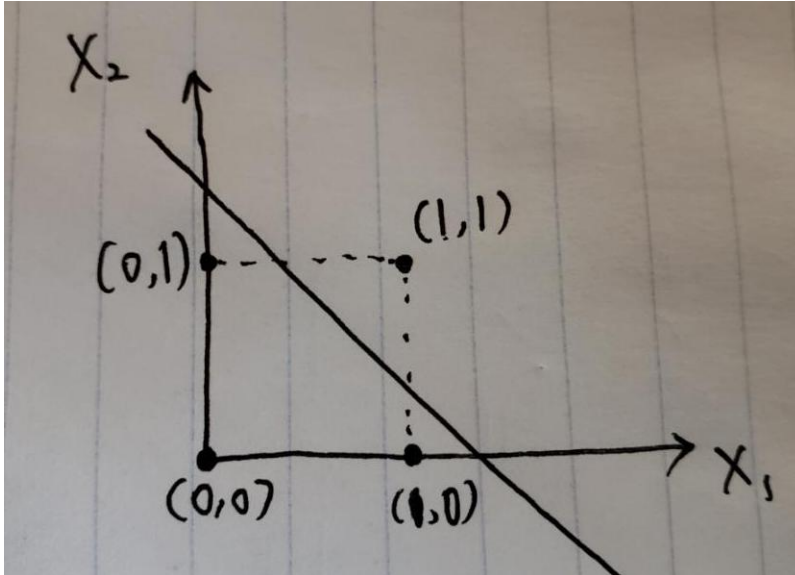
## JichenDai\_HW#2

10448922

1.

(1) Yes, It is linearly separable.

As we can see in the graph below, there is a line separate positive and negative.



(2) Yes, it is possible.

Set Learning Rate = 0.2.

Since  $x_1 + x_2 - 1/2 = 0$ ,

$w_0 = -0.5, w_1 = 1, w_2 = 1$

Input P1:

$$\begin{aligned} y &= 0*1 + 1*1 - 0.5 \\ &= 0.5 > 0 \end{aligned}$$

Prediction is **wrong**, update the weights:

$$w_0 = w_0*(1+0.2) = 0.6$$

$$w_1 = w_1*(1-0.2) = 0.8$$

$$w_2 = w_2*(1-0.2) = 0.8$$

Input P2:

$$\begin{aligned} y &= 1*0.8 + 1*0.8 - 0.6 \\ &= 1 > 0 \end{aligned}$$

Prediction is **correct**

Input P3:

$$\begin{aligned} y &= 1*0.8 + 0*0.8 - 0.6 \\ &= 0.2 > 0 \end{aligned}$$

Prediction is **wrong**, update the weights:

$$w0 = w0*(1+0.2) = 0.72$$

$$w1 = w1*(1-0.2) = 0.64$$

$$w2 = w2*(1-0.2) = 0.64$$

**Input P4:**

$$y = 0*0.64 + 0*0.64 - 0.72$$

$$= -0.72 < 0$$

Prediction is **correct**.

**Input P1:**

$$y = 0*0.64 + 1*0.64 - 0.72$$

$$= -0.08 < 0$$

Prediction is **correct**.

**Input P2:**

$$y = 1*0.64 + 1*0.64 - 0.72$$

$$= 0.56 > 0$$

Prediction is **correct**.

**Input P3:**

$$y = 1*0.64 + 0*0.64 - 0.72$$

$$= -0.08 < 0$$

Prediction is **correct**.

**Input P4:**

$$y = 0*0.64 + 0*0.64 - 0.72$$

$$= -0.72 < 0$$

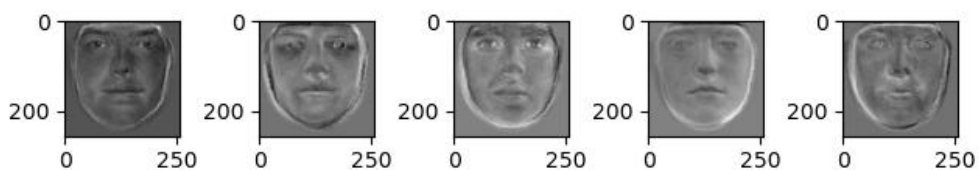
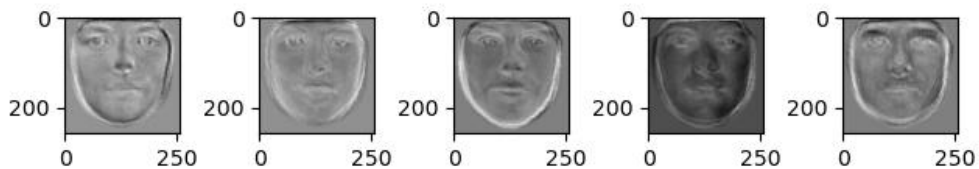
Prediction is **correct**.

So, now the weight can correctly classify all 4 points, final decision boundary is:

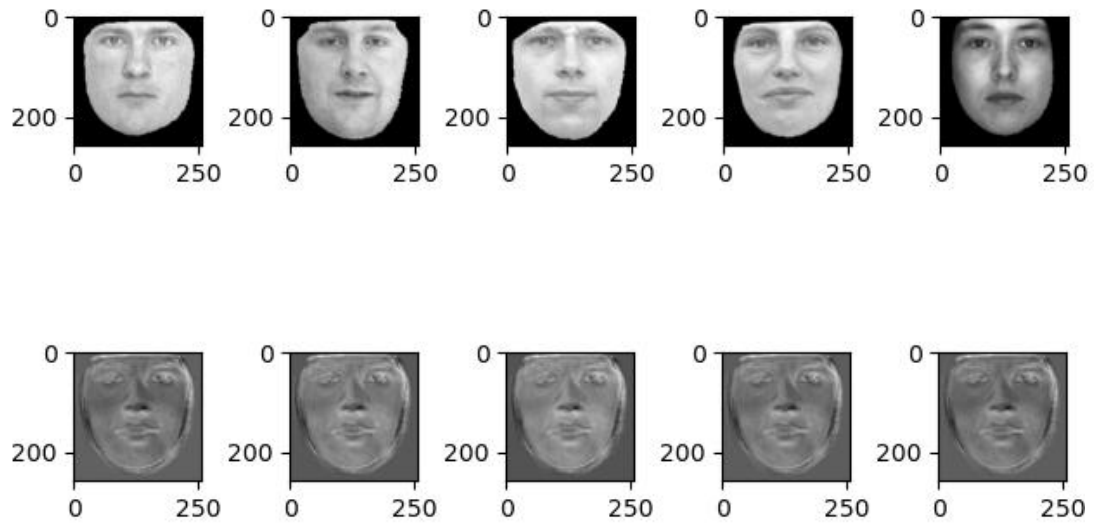
$$0.64 * X1 + 0.64 * X2 - 0.72 = 0$$

**2**

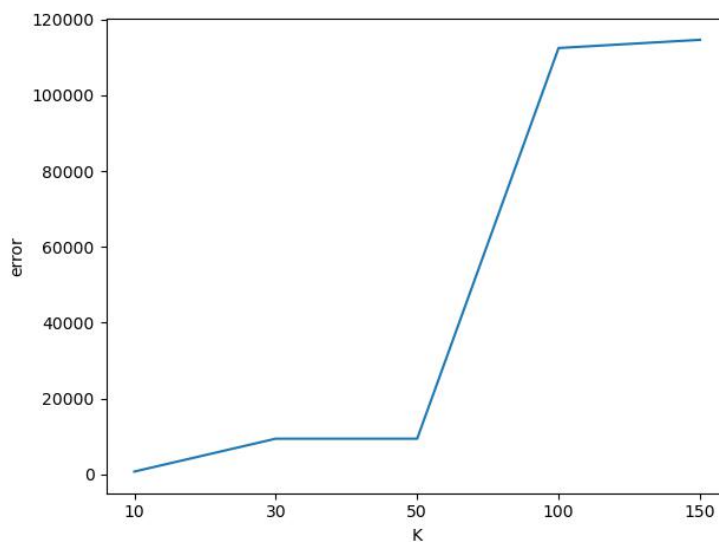
**(1)**



**(2) error is 9441**



**(3)**



**Source code:**

```
import numpy as np
from PIL import Image
from os import listdir
import matplotlib.pyplot as plt

# load data with shape 177*65536
def load_image(path):
    vectors = []
    for bmp in listdir(path):
        vector = np.array(Image.open("face_data/" + bmp)).flatten()
```

```

        vectors.append(vector)
    return np.array(vectors, dtype=np.float64)

def plot(images, row, col):
    plt.figure()
    for i in range(col * row):
        plt.subplot(row, col, i + 1)
        image = images[i].reshape(256,256)
        plt.imshow(image, cmap = 'gray')
    plt.show()

def cal_error(origin, recon):
    row, col = origin.shape
    sum = 0
    for i in range(origin.shape[0]):
        for j in range(col):
            sum += (origin[i,j] - recon[i,j])**2
    return sum / (row*col)

def norm(faces):
    norm = []
    for face in faces:
        if(face.sum() != 0):
            norm.append(face / face.sum())
        else:
            norm.append(face)
    return np.array(norm)

#####
# load data
data = load_image("face_data")
training, test = data[:156], data[157:]

# calculate mean (Y = X - avg(X))=>(156*65536)
[r, c] = training.shape
mean = np.mean(training, 0)
Y = training - mean

# compute covariance (Y * Y_t / size_train)=>(156*156)
cov = np.dot(Y, Y.T) / c

```

```

# get eigen vetor
values, vectors = np.linalg.eig(cov)

# sort vectors according to values
ascending_order = np.argsort(values)
descending_order = ascending_order[::-1]
vectors = vectors[:, descending_order]

# select top (K = 30)=>(30*156)
selected_V = vectors[:30]

# do projection (Y_t * eigenvectors) =>(30*65536)
eigenfaces = np.dot(selected_V, Y)
eigenfaces = norm(eigenfaces)

# display top 10 eigenfaces
plot(eigenfaces[:10], 2, 5)

##### Question 2 #####
# show 5 original testing images
plot(test[:5], 1, 5)

# show 5 reconstructed ones.
Y_test = np.dot(test[:5] - mean, eigenfaces.T)
Y_test = np.dot(Y_test, eigenfaces) + mean
plot(Y_test, 1, 5)

# compute error value
error = cal_error(test[:5], Y_test)
print(error)

##### Question 3 #####
def given_K_return_error(K):
    eigenfaces = np.dot(vectors[:K], Y)
    eigenfaces = norm(eigenfaces)

    Y_test = np.dot(test[:5] - mean, eigenfaces.T)
    Y_test = np.dot(Y_test, eigenfaces) + mean

    error = cal_error(test[:5], Y_test)
    return error

```

```
Ks = [10, 20, 50, 100, 150]
errors = []

for K in Ks:
    errors.append(given_K_return_error(K))

plt.plot(['10', '30', '50', '100', '150'], errors)
plt.xlabel("K")
plt.ylabel("error")
plt.show()
```