JichenDai_CS600_HW#11

R-26.5

Slack form:

maximize: $z = x_1 + x_2$

subject to: $x_3 = 77 - 3x_1 - 5x_2$

$x_4 = 56 - 7x_1 - 2x_2$

$x_1, x_2, x_3, x_4 \geq 0$

Pivot x1 and x4 c_* **= 8**

maximize: $z = 8 + \frac{5}{7}x_2 - \frac{1}{7}x_4$

subject to: $x_3 = 53 - \frac{29}{7}x_2 + \frac{3}{7}x_4$

$x_1 = 8 - \frac{2}{7}x_2 - \frac{1}{7}x_4$

$x_1, x_2, x_3, x_4 \geq 0$

Pivot x2 and x3 c_* **= 497 / 29**

maximize: $z = \frac{497}{29} - \frac{7}{29}x_3 - \frac{2}{29}x_4$

subject to: $x_2 = \frac{371}{29} + \frac{3}{29}x_4 - \frac{7}{29}x_3$

$x_1 = \frac{126}{29} - \frac{5}{29}x_4 + \frac{2}{29}x_3$

$x_1, x_2, x_3, x_4 \geq 0$

Now, both coefficients is negative, so the optimal value is **497 / 29**, with x1 = 126/29, x2 = 371/29.

R-26.7

minimize: $z = 3y_1 + 2y_2 + y_3$

subject to: $-3y_1 + y_2 + y_3 \geq 1$

$2y_1 + y_2 - y_3 \geq 2$

$y_1, y_2, y_3 \geq 0$

Since that a standard linear program has the following form:

$$\text{maximize:} \quad z = c_* + \sum_{j \in F} c_j x_j$$

$$\text{subject to:} \quad x_i = b_i - \sum_{j \in F} a_{ij} x_j, \quad \text{for } i \in B$$

$$x_i \geq 0 \text{ for } 1 \leq i \leq m + n.$$

Standard form is:

Maximize: $z = -3y_1 - 2y_2 - y_3$

Subject to: $3y_1 - y_2 - y_3 \leq -1$

$-2y_1 - y_2 + y_3 \leq -2$

$y_1, y_2, y_3 \geq 0$

A-26.3

Let the maximum budget be ***m***, then we transfer this question into a slack form:

Slack form:

maximize: $z = ax_1 + bx_2 + cx_3$

subject to : $10000x_1 + 70000x_2 + 110000x_3 + x_4 = m$

$x_1 + x_5 = 25$

$x_2 + x_6 = 7$

$x_3 + x_7 = 15$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$

Let $\boldsymbol{A}$ =

$$\begin{bmatrix} 10000 & 70000 & 110000 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let $\boldsymbol{B}$ = {4, 5, 6, 7}, $\boldsymbol{F}$ = {1, 2, 3}, $\boldsymbol{c}$ = (a, b, c), $\boldsymbol{b}$ = (m, 25, 7, 15), c_* **= 0.**

Then, we employ algorithm 26.5 ***SimplexMethod(A, b, c,*** c_* ***, F, B)*** (textbook page 744) to solve this problem.

This algorithm's return contain c = (a', b', c') and c_* .

Which means this candidate should buy a' radio advertise, b' print advertise, c' TV advertise. The maximum impact is c_***.**