Lista 4

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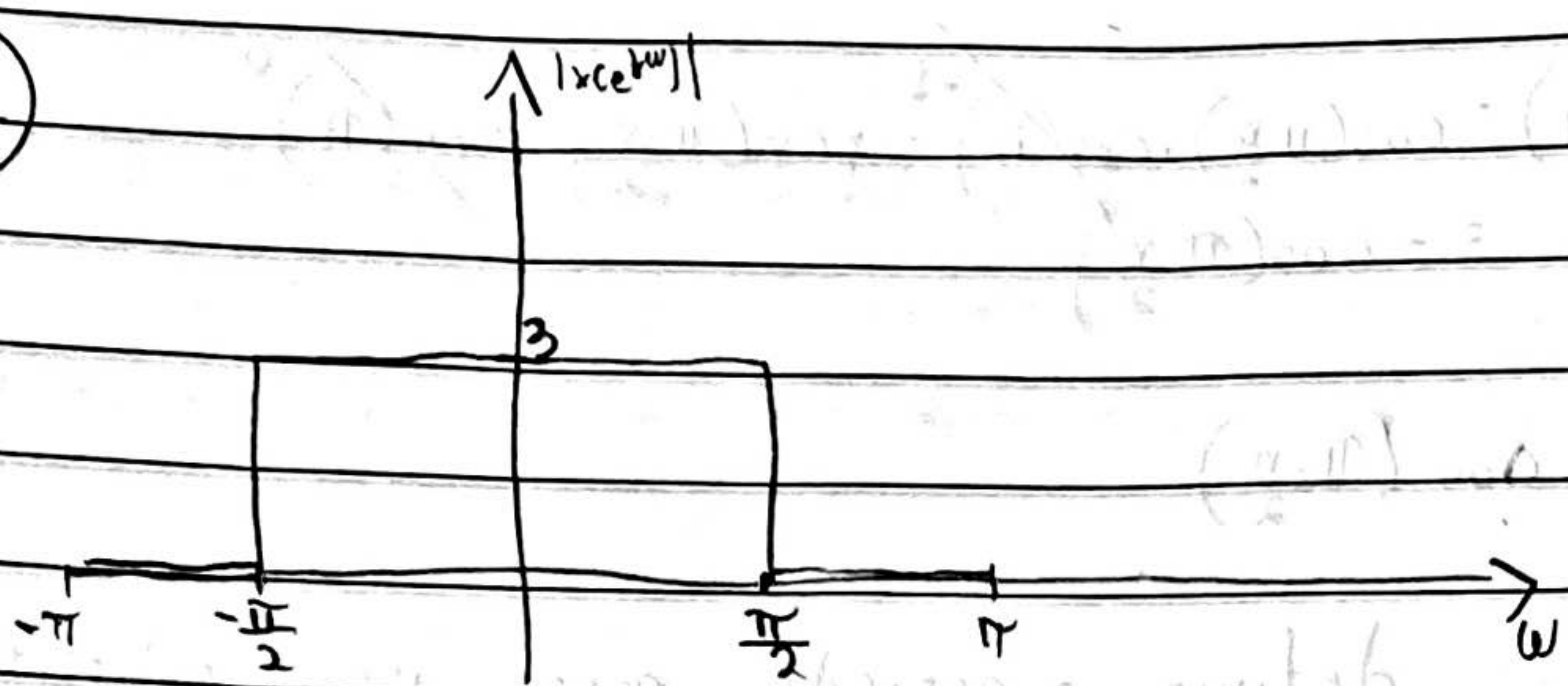
1) $|X(e^{jw})| = 3\{u(w+\pi/2) - u(w-\pi/2)\}$

a)



b) $x[n] = \frac{1}{2\pi}\int_0^{2\pi} X(e^{jw}) \cdot e^{jwn}\, dw \Rightarrow$

$$x[n] = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\pi/2} 3 \cdot e^{jwn}\, dw$$

$$= \frac{3}{2\pi} \cdot \int_{-\pi/2}^{\pi/2} e^{jwn}\, dw$$

$$= \frac{3}{2\pi} \cdot \left[\frac{e^{jwn}}{jn}\right]_{-\pi/2}^{\pi/2} = \frac{3}{2\pi} \cdot \left(\frac{e^{\frac{\pi}{2}\cdot j \cdot n}}{jn} - \frac{e^{-\frac{\pi}{2}\cdot j\cdot n}}{jn}\right)$$

$$= \frac{3}{\pi \cdot n} \cdot \left(\frac{e^{\frac{\pi}{2}\cdot j\cdot n} - e^{-\frac{\pi}{2}\cdot j\cdot n}}{2j}\right)$$

$$\boxed{x[n] = \frac{3}{\pi \cdot n} \cdot \operatorname{sen}\left(\frac{\pi}{2}\cdot n\right)}$$

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2) $x[n] = 2 + \cos(\pi n/2 + \pi)$

Como $\cos(a+b) = \cos a \cdot \cos b - \text{sen}\, a \cdot \text{sen}\, b$

$\cos(\pi n/2 + \pi) = \cos\left(\frac{\pi n}{2}\right) \cdot \cancel{\cos(\pi)}^{-1} - \text{sen}\left(\frac{\pi . n}{2}\right) \cdot \cancel{\text{sen}(\pi)}^{0}$
$= -\cos\left(\frac{\pi . n}{2}\right)$

$\Rightarrow x[n] = 2 - \cos\left(\frac{\pi . n}{2}\right)$

a) Primeiro é definir o periodo para isso substituir n por n + N

$x[n+N] = 2 - \cos\left(\frac{\pi . (n+N)}{2}\right) = 2 - \cos\left(\frac{\pi n}{2} + \frac{\pi . N}{2}\right)$

Para definir o periodo, $\frac{\pi . N}{2}$ deve ser multiplo de 2π

$\frac{N . \pi}{2} = m . 2\pi \Rightarrow N = \frac{m . 2 . \cancel{\pi} . 2}{\cancel{\pi}} \Rightarrow N = m . 4$

Agora para definir os coeficientes:

$x[n] = \sum_{K=\langle N \rangle} a_K . e^{jK.\frac{\pi}{2}.n}$

$\Rightarrow x[n] = \sum_{K=0}^{4} a_K . e^{jK.\frac{\pi}{2}.n} = a_0 + a_1 . e^{\frac{j\pi}{2}.n} + a_2 . e^{j.\pi.n} + a_3 . e^{j.\frac{3.\pi}{2}.n}$

$x[n] = 2 - \cos\left(\frac{\pi . n}{2}\right) = 2 - \frac{e^{\frac{j\pi . n}{2}}}{2} - \frac{e^{-\frac{j\pi . n}{2}}}{2}$

como $-\frac{\pi}{2}$ não esta na equação, vai ser somado 2π

$-\frac{\pi}{2} + 2\pi = -\frac{\pi}{2} + \frac{4\pi}{2} = \frac{-\pi + 4\pi}{2} = \frac{3\pi}{2} \Rightarrow$ este tem na eq.

$\Rightarrow x[n] = 2 - \frac{e^{\frac{j\pi n}{2}}}{2} - \frac{e^{\frac{j.3\pi . n}{2}}}{2} = a_0 + a_1 . e^{\frac{j\pi n}{2}} + a_2 . e^{j.\pi.n} + a_3 . e^{j.\frac{3.\pi}{2}.n}$

$\boxed{a_0 = 2}$ $\boxed{a_1 = -\frac{1}{2}}$ $\boxed{a_2 = 0}$ $\boxed{a_3 = -\frac{1}{2}}$

(b) $x[n] = 2 - \cos\left(\frac{\pi . n}{2}\right)$

Utilizando as fórmulas da Transformada de Fourier

$2 = 2\pi . 2 . \delta(\omega) = 4\pi . \delta(\omega)$

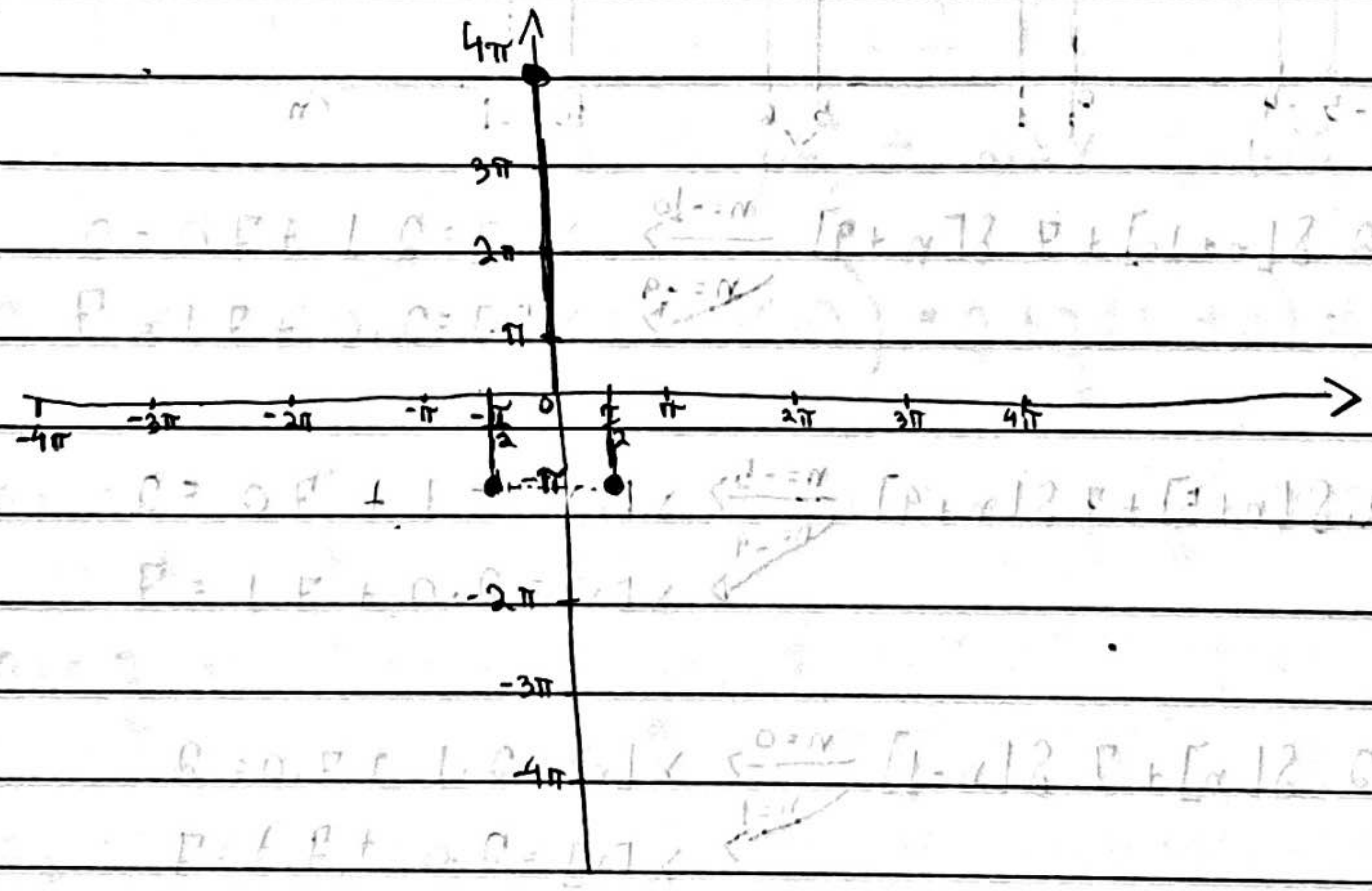
$\cos\left(\frac{\pi . n}{2}\right) = \pi . \left[\delta\left(\omega - \frac{\pi}{2}\right) + \delta\left(\omega + \frac{\pi}{2}\right)\right]$

Portanto:

$x[n] = 2 - \cos\left(\frac{\pi . n}{2}\right) = \boxed{4\pi . \delta(\omega) - \pi . \delta\left(\omega - \frac{\pi}{2}\right) - \pi\delta\left(\omega + \frac{\pi}{2}\right)}$

∴

(c)



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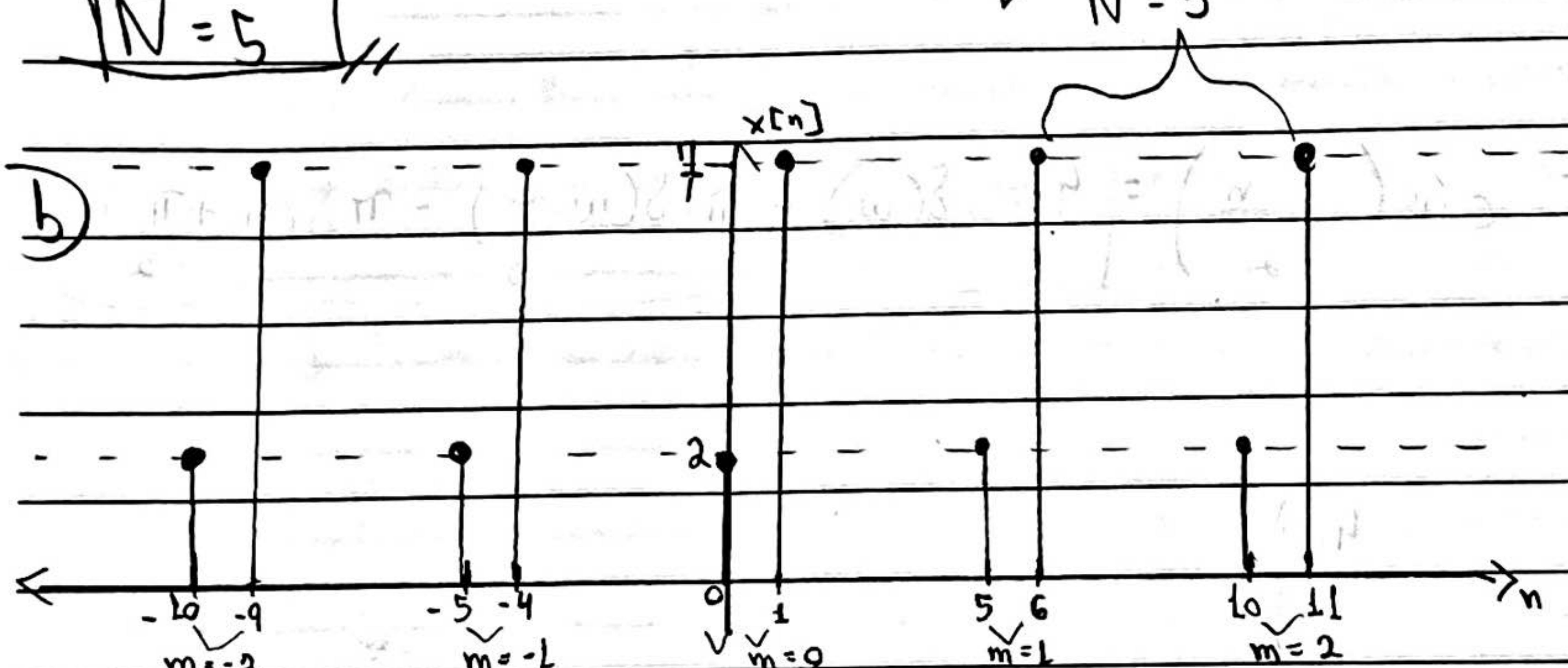
3) $x[n] = \sum_{m=-\infty}^{\infty} \{2\delta[n-5m] + 7\delta[n-1-5m]\}$

a) $x[n+N] = \sum_{m=-\infty}^{\infty} \{2\delta[n+N-5m] + 7\delta[n+N-1-5m]\}$

Para definir o periodo N-5m deve ser periodico, portanto N = 5m, então o periodo é 5, pois como é uma função impulso, a cada 5 unidades o valor se repete → N = 5

$\boxed{N=5}$ //

b)



$m=-2$: $x[n] = 2.\delta[n+10] + 7.\delta[n+9] \xrightarrow{n=-10} x[n] = 2.1 + 7.0 = 2$
$\xrightarrow{n=-9} x[n] = 2.0 + 7.1 = 7$

$m=-1$: $x[n] = 2.\delta[n+5] + 7.\delta[n+4] \xrightarrow{n=-5} x[n] = 2.1 + 7.0 = 2$
$\xrightarrow{n=-4} x[n] = 2.0 + 7.1 = 7$

$m=0$: $x[n] = 2.\delta[n] + 7.\delta[n-1] \xrightarrow{n=0} x[n] = 2.1 + 7.0 = 2$
$\xrightarrow{n=1} x[n] = 2.0 + 7.1 = 7$

$m=1$: $x[n] = 2.\delta[n-5] + 7.\delta[n-6] \xrightarrow{n=5} x[n] = 2.1 + 7.0 = 2$
$\xrightarrow{n=6} x[n] = 2.0 + 7.1 = 7$

$m=2$: $x[n] = 2.\delta[n-10] + 7.\delta[n-11] \xrightarrow{n=10} x[n] = 2.1 + 7.0 = 2$
$\xrightarrow{n=11} x[n] = 2.0 + 7.1 = 7$

3) c)

$$a_K = \frac{1}{N} \sum_{n=-N_1}^{N_1} x[n] \cdot e^{-jK\left(\frac{2\pi}{N}\right)n}$$

como $N = 5$

$$a_K = \frac{1}{5} \sum_{n=0}^{4} x[n] \cdot e^{-jK\left(\frac{2\pi}{5}\right) \cdot n}$$

Resolvendo o somatório com $x[n]$:

$$\sum_{n=0}^{4} x[n] \cdot e^{-jK\left(\frac{2\pi}{5}\right) \cdot n} = \left(2 + 7 \cdot e^{-j \cdot K \cdot \left(\frac{2\pi}{5}\right)}\right)$$

$$\Rightarrow a_K = \frac{1}{5} \cdot \left(2 + 7 \cdot e^{-j \cdot K \cdot \left(\frac{2\pi}{5}\right)}\right) = \frac{2}{5} + \frac{7 \cdot e^{-j \cdot K \cdot \left(\frac{2\pi}{5}\right)}}{5}$$

$$a_K = \frac{2}{5} + \frac{7}{5} \cdot \left(\cos\left(\frac{-K \cdot 2 \cdot \pi}{5}\right) + j \cdot \text{sen}\left(\frac{-K \cdot 2 \cdot \pi}{5}\right)\right)$$

$$a_0 = \frac{2}{5} + \frac{7}{5} \cdot \left(\cos(0) + j \cdot \text{sen}(0)\right) = \frac{2}{5} + \frac{7}{5}(1 + 0) = \boxed{\frac{9}{5} = a_0}$$

$$a_1 = \frac{2}{5} + \frac{7}{5} \cdot \left(\cos\left(\frac{-2\pi}{5}\right) + j \cdot \text{sen}\left(\frac{-2\pi}{5}\right)\right) = \frac{2}{5} + \frac{7}{5}(0{,}309 - j \cdot 0{,}951) \Rightarrow$$

$$a_1 = \frac{2}{5} + 0{,}4326 - j \cdot 1{,}3314 = \boxed{0{,}83 - j \cdot 1{,}33 = a_1}$$

$$a_2 = \frac{2}{5} + \frac{7}{5} \cdot \left(\cos\left(\frac{-4\pi}{5}\right) + j \cdot \text{sen}\left(\frac{-4\pi}{5}\right)\right) = \frac{2}{5} + \frac{7}{5}\left(-0{,}81 + j \cdot (-0{,}588)\right) \Rightarrow$$

$$a_2 = \frac{2}{5} + (-1{,}134) - j \cdot 0{,}8232 = \boxed{-0{,}734 - j \cdot 0{,}8232 = a_2}$$

$$a_3 = \frac{2}{5} + \frac{7}{5}\left(\cos\left(\frac{-6\pi}{5}\right) + j \cdot \text{sen}\left(\frac{-6\pi}{5}\right)\right) = \frac{2}{5} + \frac{7}{5}(-0{,}81 + j \cdot 0{,}588) = \boxed{-0{,}734 + j \cdot 0{,}8232 = a_3}$$

$$a_4 = \frac{2}{5} + \frac{7}{5} \cdot \left(\cos\left(\frac{-8\pi}{5}\right) + j \cdot \text{sen}\left(\frac{-8\pi}{5}\right)\right) = \frac{2}{5} + \frac{7}{5}(0{,}309 + j \cdot 0{,}951) = \boxed{0{,}83 + j \cdot 1{,}33 = a_4}$$