# Lista 2

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1) $x[n] = \left(\frac{1}{4}\right)^n \cdot u[n-4]$ , $h[n] = 2^n\, u[2-n]$

$y[n] = x[n] * h[n] = \sum_{K=-\infty}^{\infty} x[K] \cdot h[n-K]$

Substituindo $x[n] \Rightarrow x[K]$ e $h[n] \Rightarrow h[n-K]$

$y[n] = \sum_{K=-\infty}^{\infty} x[K] \cdot \left(2^{n-K} \cdot u[2-n+K]\right)$

$u[2-n+K] \begin{cases} 0, & K < n-2 \\ 1, & K \geq n-2 \end{cases}$ $\Rightarrow$ portanto pode considerar todos acima de $n-2$

$y[n] = \sum_{K=n-2}^{\infty} \left(\frac{1}{4}\right)^K \cdot u[K-4] \cdot 2^{n-K} \cdot [1] = \sum_{K=n-2}^{\infty} \frac{2^n}{8^K} \cdot u[K-4]$

$u[K-4] \begin{cases} 0, & K < 4 \\ 1, & K \geq 4 \end{cases}$ $\Rightarrow$

Se $n \geq 6 \Rightarrow y[n] = \sum_{K=n-2}^{\infty} \frac{2^n}{8^K} \cdot [1] = 2^n \sum_{K=n-2}^{\infty} \left(\frac{1}{8}\right)^K = 2^n \cdot \left(\frac{1}{8}\right)^{n-2} \cdot \frac{1}{1-(1/8)}$

$\Rightarrow y[n] = \frac{2^n \cdot 8^2}{8^n} \cdot \frac{1}{7/8} = \frac{2^n \cdot 8^2}{8^n} \cdot \frac{8}{7} = \frac{2^n \cdot 8^{(3-n)}}{7}$

Se $n < 6 \Rightarrow y[n] = \sum_{K=4}^{\infty} \frac{2^n}{8^K} \cdot [1] = 2^n \sum_{K=4}^{\infty} \left(\frac{1}{8}\right)^K = 2^n \cdot \left(\frac{1}{8}\right)^4 \cdot \frac{1}{1-(1/8)}$

$y[n] = \frac{2^n}{8^4} \cdot \frac{1}{7/8} = \frac{2^n \cdot 8}{8^4 \cdot 7} = \frac{2^n}{8^3 \cdot 7}$

$\therefore$

$$\boxed{\begin{array}{l} \text{Se } n \geq 6 \Rightarrow y[n] = \dfrac{2^n \cdot 8^{(3-n)}}{7} \\ \text{Se } n < 6 \Rightarrow y[n] = \dfrac{2^n}{8^3 \cdot 7} \end{array}}$$

2) $S_1 \Rightarrow \quad w[n] = \frac{1}{4} w[n-1] + x[n]$

$S_2 \Rightarrow \quad y[n] = \alpha . y[n-1] + \beta w[n]$

$S_3 \Rightarrow \quad y[n] = -\frac{1}{4} y[n-2] + \frac{5}{4} y[n-1] + x[n]$

a) Reescrevendo $S_2$ isolando $w[n]$:

$$w[n] = \frac{y[n]}{\beta} - \frac{\alpha . y[n-1]}{\beta} \quad (1)$$

Substituindo $n$ por $n-1$ e multiplicando a eq. por $\frac{1}{4}$:

$$\frac{w[n-1]}{4} = \frac{y[n-1]}{4 . \beta} - \frac{\alpha . y[n-2]}{4 . \beta} \quad (2)$$

fazendo (1) - (2):

$$w[n] - \frac{w[n-1]}{4} = \frac{y[n]}{\beta} - \frac{y[n-1]}{4 . \beta} - \frac{\alpha . y[n-1]}{\beta} + \frac{\alpha \, y[n-2]}{4 . \beta}$$

$S_1$ pode ser escrito como:

$$x[n] = w[n] - \frac{w[n-1]}{4} = \frac{y[n]}{\beta} - \frac{y[n-1]}{4 . \beta} - \frac{\alpha y[n-1]}{\beta} + \frac{\alpha \, y[n-2]}{4 . \beta}$$

isolando $y[n]$ para comparar com $S_3$:

$$y[n] = \frac{\cancel{\beta} . y[n-1]}{4 \cancel{\beta}} + \frac{\alpha . \cancel{\beta} . y[n-1]}{\cancel{\beta}} - \frac{\alpha . \cancel{\beta} y[n-2]}{4 . \cancel{\beta}} + \beta . x[n] \Rightarrow$$

$$y[n] = -\frac{\alpha}{4} . y[n-2] + \left(\frac{1}{4} + \alpha\right) y[n-1] + \beta x[n] \quad (3)$$

Comparando (3) com $S_3$:

$-\frac{\alpha}{4} = -\frac{1}{4}$ ; $\frac{1}{4} + \alpha = \frac{5}{4}$ ; $\beta = 1$ $\Rightarrow \boxed{\alpha = 1 \quad \beta = 1}$ //

$\Downarrow$ $\qquad$ $\Downarrow$

$\alpha = 1$ $\qquad$ $\alpha = 1$

b) Os sistemas $S_1$ e $S_2$

$S_1 \Rightarrow w[n] = \frac{1}{4} w[n-1] + x[n]$

$S_2 \Rightarrow y[n] = y[n-1] + W[n]$

Considerando uma entrada $x[n] = K\,\delta[n]$

$x[n] = 0$ para $n < 0$

para $n \geq 0$:

$$w[0] = \frac{1}{4} \cancelto{0}{w[-1]} + x[0] = K$$

$$w[1] = \frac{1}{4} \cancelto{K}{w[0]} + \cancelto{0}{x[1]} = \frac{1}{4} K$$

$$w[2] = \frac{1}{4} \cancelto{\frac{1}{4}K}{w[1]} + \cancelto{0}{x[2]} = \left(\frac{1}{4}\right)^2 K$$

$$\vdots$$

$$w[n] = \frac{1}{4} w[n-1] + x[n] = \left(\frac{1}{4}\right)^n K$$

Como a condição de repouso inicial é LIT, seu comportamento entrada-saída é caracterizado pela resposta impulso.

Estabelecendo $K = 1$:

$$h_1[n] = \left(\frac{1}{4}\right)^n \cdot u[n]$$

fazendo o mesmo raciocinio para $h_2$:

$$h_2[n] = u[n]$$

$$h[n] = h_1[n] * h_2[n] = \sum_{K=-\infty}^{\infty} h_1[K]\, h_2[n-K]$$

$$= \sum_{K=-\infty}^{\infty} \left(\frac{1}{4}\right)^K \cdot u[K] \cdot u[n-K]$$

$$u[n-K] \begin{cases} 0, & K > n \\ 1, & K \leq n \end{cases} \qquad u[K] \begin{cases} 0, & K < 0 \\ 1, & K \geq 0 \end{cases}$$

$$h[n] = \sum_{K=0}^{n} \left(\frac{1}{4}\right)^K \cdot 1 \cdot 1 \Rightarrow \boxed{h[n] = \left(\frac{1 - (1/4)^{n+1}}{1 - (1/4)}\right) \cdot u[n]}$$