Lista 5

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1) $\tau(w) = -\frac{d}{dw} \angle H(e^{jw})$

$|H(e^{j\pi/3})| = 4, \quad \angle H(e^{j0}) = 0, \quad \tau\left(\frac{\pi}{3}\right) = 4$

a) $\cos(\pi \cdot n/3) = \frac{e^{j\frac{\pi \cdot n}{3}} + e^{-j\frac{\pi \cdot n}{3}}}{2}$

$\Rightarrow x_1[n] = \frac{e^{j\frac{\pi n}{3}}}{2} \qquad x_2[n] = \frac{e^{-j\frac{\pi n}{3}}}{2}$

Passando pelo sistema LiT, o atraso é de 4, então:

$y[n] = 4x_1[n-4] + 4x_2[n-4]$

$\Rightarrow y[n] = 4\left(\frac{e^{j\pi\frac{(n-4)}{3}} + e^{-jw\frac{(n-4)}{3}}}{2}\right) = \boxed{4 \cdot \cos\left(\frac{\pi}{3} \cdot n - \pi\right)}$

b) $\operatorname{sen}\left(\frac{11\pi \cdot n}{3} + \frac{\pi}{6}\right) = \frac{e^{j\cdot\left(\frac{11}{3}\cdot\pi\cdot n + \frac{\pi}{6}\right)} - e^{-j\left(\frac{11}{3}\cdot\pi\cdot n + \frac{\pi}{6}\right)}}{2j}$

Como o atraso é de 4 passando pelo sistema LiT:

$\Rightarrow y[n] = 4x[n-4] = 4 \cdot \operatorname{sen}\left(\frac{11\pi}{3}\cdot(n-4) + \frac{\pi}{6}\right) =$

$= 4 \cdot \operatorname{sen}\left(\frac{11\pi \cdot n}{3} - \frac{44\pi}{3} + \frac{\pi}{6}\right) = 4 \cdot \operatorname{sen}\left(\frac{11 \cdot \pi n}{3} + \frac{\pi(3-264)}{18}\right)$

$y[n] = 4 \cdot \operatorname{sen}\left(\frac{11 \cdot \pi \cdot n}{3} - \frac{\pi \cdot 261}{18}\right) = \boxed{4 \cdot \operatorname{sen}\left(\frac{11\pi \cdot n}{3} - \frac{\pi \cdot 87}{6}\right)}$

2) $y[n] = c\{b\,x[n-2] + a\,x[n-1] + x[n] + a\,x[n+1] + b\,x[n+2]\}$

a) Fazendo a Transformada de Fourier

$Y(\Omega) = c\{b.e^{-2.j.\Omega}.X(\Omega) + a.e^{-j\Omega}.X(\Omega) + X(\Omega) + a.e^{j\Omega}.X(\Omega) + b.e^{2j.\Omega}.X(\Omega)\}$

$Y(\Omega) = c.\{X(\Omega).(b.e^{-2.j.\Omega} + b.e^{2.j\Omega} + a.e^{-j\Omega} + a.e^{j\Omega} + 1)\}$

$Y(\Omega) = c.\{X(\Omega).(b.(e^{-2.j.\Omega} + e^{2.j.\Omega}) + a.(e^{-j\Omega} + e^{j\Omega}) + 1)\}$

$\frac{Y(\Omega)}{X(\Omega)} = c.\left\{2.b.\left(\frac{e^{-2j\Omega} + e^{2j\Omega}}{2}\right) + 2.a\left(\frac{e^{-j\Omega} + e^{j\Omega}}{2}\right) + 1\right\}$

$$\boxed{H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = c.\{2.b.\cos(2\Omega) + 2.a.\cos(\Omega) + 1\}}$$

b) frequencia zero e ganho unitario

$c.\{2.b.\cos(2.0) + 2a.\cos(0) + 1\} = 1 \Rightarrow c.\{2b + 2a + 1\} = 1$

$$\Rightarrow \boxed{c = \frac{1}{2a + 2b + 1}}$$

c) $a = b = 0,5$; $\quad c = \frac{1}{\frac{2.1}{2} + \frac{2.1}{2} + 1} = \frac{1}{3}$

$H(e^{jw}) = \frac{1}{3}.\left[\frac{2}{2}.\cos(2.w) + \frac{2}{2}.\cos(w) + 1\right] = \frac{1}{3}.[\cos(2w) + \cos(w) + 1]$

$$\boxed{H(e^{jw}) = \frac{1}{3}.[\cos(2w) + \cos(w) + 1]}$$

3) $y[n] + \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = x[n]$

a) $Y(\Omega) + \frac{3}{4} e^{-j\cdot\Omega} \cdot Y(\Omega) + \frac{1}{8} \cdot e^{-2j\cdot\Omega} \cdot Y(\Omega) = X(\Omega)$

$Y(\Omega) \cdot \left(1 + \frac{3}{4} \cdot e^{-j\Omega} + \frac{1}{8} \cdot e^{-2j\cdot\Omega} \right) = X(\Omega)$

$Y(\Omega) \cdot \left(\frac{8 + 6 \cdot e^{-j\Omega} + e^{-2j\cdot\Omega}}{8} \right) = X(\Omega)$

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{8}{8 + 6 \cdot e^{-j\cdot\Omega} + e^{-2j\cdot\Omega}} //$$