

To express Constraint 2, note that

$$\begin{aligned} \text{Total lab time used annually} &= \text{time used annually to process raw material} \\ &\quad + \text{time used annually to process Luxury Brute} \\ &\quad + \text{time used annually to process Luxury Chanelle} \\ &= x_5 + 3x_2 + 2x_4 \end{aligned}$$

Then Constraint 2 becomes

$$3x_2 + 2x_4 + x_5 \leq 6,000 \quad (56)$$

After adding the sign restrictions $x_i \geq 0$ ($i = 1, 2, 3, 4, 5$), many students claim that Rylon should solve the following LP:

$$\begin{aligned} \max z &= 7x_1 + 14x_2 + 6x_3 + 10x_4 - 3x_5 \\ \text{s.t.} \quad &x_5 \leq 4,000 \\ &3x_2 + 2x_4 + x_5 \leq 6,000 \\ &x_i \geq 0 \quad (i = 1, 2, 3, 4, 5) \end{aligned}$$

This formulation is incorrect. Observe that the variables x_1 and x_3 do not appear in any of the constraints. This means that any point with $x_2 = x_4 = x_5 = 0$ and x_1 and x_3 large can yield arbitrarily large profits. Thus, this LP is unbounded. Our mistake is that the current formulation does not indicate that the amount of raw material purchased determines the amount of Brute and Chanelle that is available for sale or further processing. More specifically, from Figure 10 (and the fact that 1 oz of processed Brute yields exactly 1 oz of Luxury Brute), it follows that

Ounces of Regular Brute Sold

$$\begin{aligned} + \text{ ounces of Luxury Brute sold} &= \left(\frac{\text{ounces of Brute produced}}{\text{pounds of raw material purchased}} \right) = \\ &= 3x_5 \end{aligned}$$

This relation is reflected in the constraint

$$x_1 + x_2 = 3x_5 \quad \text{or} \quad x_1 + x_2 - 3x_5 = 0 \quad (57)$$

Similarly, from Figure 10 it is clear that

Ounces of Regular Chanelle sold + ounces of Luxury Chanelle sold = $4x_5$ This relation yields the constraint

$$x_3 + x_4 = 4x_5 \quad \text{or} \quad x_3 + x_4 - 4x_5 = 0 \quad (58)$$

Constraints (57) and (58) relate several decision variables. Students often omit constraints of this type. As this problem shows, leaving out even one constraint may very well

$3x_5$ oz Brute

x_5 lb Raw material

x_2 oz Reg. Brute processed into Lux. Brute

$4x_5$ oz Chanelle

FIGURE 10