

Theory of Information

that the sailor has to represent. For example, to compose any number from 0 up to 2100, the sailor would need twenty-four flags in the binary system and only twenty-one flags in the ternary system. To do the same thing for all numbers from 0 to 17000, the sailor would need twenty-eight flags in the binary system and only twenty-four flags in the ternary system.

In spite of all its success, Shannon's theory of the amount of information was criticized by many later authors as merely a theory of syntactical information, because it excluded the semantic and pragmatic levels. The price to pay the ability to build such an elegant and advanced mathematical theory is that the Hartley-Shannon approach does not deal at all with many important aspects of information. That is why statistical information theory is rightly seen as lacking indications for a conceptual clarification of information. Besides, any theory of information would necessarily reach an impasse. To this day, though, there is no measure in information theory which is as well-supported and as generally accepted as Shannon's quantity of information.

There were many generalizations of the Shannon's approach to information measurement aimed at improving some capabilities of this theory. Here we consider some of them.

Rényi (1960) introduced a new kind of entropy by the following formula

$$H_{\alpha}(p_1, p_2, \dots, p_n) = (1/(1 - \alpha)) \ln \left(\sum_{i=1}^n p_i^{\alpha} \right) \quad (3.2.8)$$

Now this information measure is called the Rényi entropy.

In the limit, when $\alpha \rightarrow 1$, the Rényi entropy is equal to a version of the Shannon entropy with the natural logarithm \ln instead of \log_2 :

$$\lim_{\alpha \rightarrow 1} H_{\alpha}(p_1, p_2, \dots, p_n) = - \sum_{i=1}^n p_i \ln p_i$$

The Rényi entropy was used to express quantum mechanical uncertainty relations, such as the Heisenberg uncertainty principle (Bialynicki-Birula, 2006).

Klir and his collaborators (Higashi and Klir, 1983; Klir and Mariano, 1987; Klir, 1991) developed a generalized information theory. The main idea of this generalization is based on the following consideration.