## Short wave propagation

### Wave action balance

The wave forcing in the shallow water momentum equation is obtained from a time dependent version of the wave action balance equation. Similar to Delft University’s (stationary) HISWA model (Holthuijsen et al., 1989) the directional distribution of the action density is taken into account whereas the frequency spectrum is represented by a frequency, best represented by the spectral parameter *fm-1,0*.The wave action balance is then given by:



In which the wave action *A* is calculated as:



In *θ* represents the angle of incidence with respect to the x-axis, *Sw* represents the wave energy density in each directional bin and *σ* the intrinsic wave frequency. The wave action propagation speeds in x- and y-direction are given by:



With *uL* and *vL* the cross-shore and alongshore depth-averaged Lagrangian velocities respectively (defined below), and the group velocity cg obtained from linear theory. If wave-current interaction is turned off (*wci=0*) then the last term in either equation is not taken into account. The propagation speed in θ-space is obtained from:



In *h* represents the total water depth and in this formulation bottom refraction (first term) and wave-current interaction (last two terms) are taken into account. If wave-current interaction is turned off (*wci=0*) then the last two terms are neglected.

The wave number *k* is obtained from the eikonal equations that is described in . In this formulation the subscripts refer to the direction of the wave vector components and *ω* represents the absolute radial frequency.



The wave number is then obtained from .



The absolute radial frequency *ω* is given by . The intrinsic frequency *σ* is obtained from the linear dispersion relation. If wave-current interaction is turned off (*wci=0*) then the last two terms are not taken into account.



### Dissipation

The set of equations of the wave action balance closes with dissipation terms. Given the spatial distribution of the wave action (and therefore wave energy) the radiation stresses can be evaluated by using linear wave theory as described in:



#### Breaking

There are in four different wave breaking formulations implemented in XBeach. The formulations are coded with the keyword *break*.

1. Non-stationary waves: formulation of Roelvink (1993a)
2. Stationary waves: formulation of Baldock et al. (1998)
3. Non-stationary waves: adaptation of break=1
4. Non-stationary waves: adaptation of break=1 (Daly et al. ,2010)

For the non-stationary (surf beat) approach the total wave energy dissipation, i.e. directionally integrated, due to wave breaking is modelled according to Roelvink (1993a). This is coded as *break=1*. In *α* is applied as wave dissipation coefficient, *Qb* is the fraction breaking waves, *p* stands for the water density and *γ* is the breaker index. The total wave energy *Ew* is calculated by integrating over the wave direction per directional bin.



In a variation of , one could also use the third wave breaking formulation, presented in . This formulation is somewhat different than the formulation of Roelvink (1993a). This is coded as *break=3.*



On top of that, Daly et al. (2010) developed a formulation presented in , which states that waves are fully breaking if the wave height exceeds a threshold (*γ*) and stop breaking if the wave height fall below another threshold (*γ2*). This is coded as *break=4*.



In the stationary case Baldock et al. (1998) is applied, which is presented in . In this breaking formulation the fraction breaking waves *Qb* and breaking wave height *Hb* is calculated differently compared to the breaking formulations used for the non-stationary situation. In *α* is applied as wave dissipation coefficient, *frep* represents a representative intrinsic frequency and *y* is a calibration factor. The stationary wave breaking formulation is coded with *break=4*.



In either the non-stationary or stationary case the total wave dissipation is distributed proportionally over the wave directions with the formulation in .



#### Bottom friction

The bottom friction dissipation is modelled as



#### Vegetation

Arnold