A Course on DSGE Models with Financial Frictions
Part 2: Simple RBC & Dynare Introduction

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March 24, 2021

A simple Business Cycle Model

- There is a representative household
- Chooses optimally consumption and labour subject to its budget constraint
- Firms produce output according to a production technology and choose labour and capital inputs to minimize cost
- Labour, capital and output markets clear

Summary

- Households choose hours worked (H_t) and consumption (C_t) to maximize their utility

- Their utility is:

$$U = U(C_t, L_t)$$

where C_t is consumption and all variables are expressed in real terms relative to the price of retail output. We assume that

$$U_C > 0,\ U_L > 0\ U_{CC} \leq 0,\ U_{LL} \leq 0 \tag{1}$$

- In a stochastic environment, the **value function** of the representative household at time t is given by

$$V_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}, L_{t+s}) \right]\ ;\ \beta \in (0,1) \tag{2}$$

The Household Optimization Problem

- Household chooses $\{C_t\}$, leisure, $\{L_t\}$, labour supply $\{H_t = 1 - L_t\}$, capital stock $\{K_t\}$ and investment $\{I_t\}$ to maximize V_t given by (2) given the budget constraint:

$$B_t = R_{t-1}B_{t-1} + r_t^K K_{t-1} + W_t H_t - C_t - I_t - T_t \tag{3}$$

- B_t is the value of the stock of one-period bonds (price$\times$number of bonds) at the end of period t.
- r_t^K is the rental rate for capital, W_t is the wage rate and R_{t-1} is the interest rate set in period $t-1$ paid in period t on bonds held at the end of period $t-1$. Note K_t is *end-of-period* capital stock.
- Capital stock accumulates according to

$$K_t = (1-\delta)K_{t-1} + I_t$$

Solution to the Household Optimization Problem

First order conditions are

$$\begin{aligned} \text{Euler Consumption} \quad &: \quad R_t \mathbb{E}_t \left[\Lambda_{t,t+1} \right] = 1 \\ \text{Labour Supply} \quad &: \quad \frac{U_{H,t}}{U_{C,t}} = -W_t \\ \text{Capital Supply} \quad &: \quad \mathbb{E}_t \left[\Lambda_{t,t+1} R^K_{t+1} \right] = 1 \end{aligned}$$

where the gross return on capital is given by

$$R^K_t = \left[r^K_t + (1 - \delta) \right]$$

and $\Lambda_{t,t+1} \equiv \beta \frac{U_{C,t+1}}{U_{C,t}}$ is the *real stochastic discount factor* $[t, t+1]$.

Solution to the Household Optimization Problem

- Then we have the arbitrage condition

$$1 = R_t \mathbb{E}_t \left[\Lambda_{t,t+1} \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} R_{t+1}^K \right]$$

- In our financial frictions models $R_t \mathbb{E}_t \left[\Lambda_{t,t+1} \right] \neq \mathbb{E}_t \left[\Lambda_{t,t+1} R_{t+1}^K \right]$

Production Side and Closing the Model

- Output and the firm's behaviour is summarized by:

$$\begin{aligned} \text{Output} &: \; Y_t = A_t K_t^{\alpha} H_t^{1-\alpha} \\ \text{Labour Demand} &: \; \frac{\alpha Y_t}{H_t} = W_t \\ \text{Capital Demand} &: \; \frac{(1-\alpha) Y_t}{K_{t-1}} = r_t^K \end{aligned}$$

- The model is completed with an output equilibrium and a balanced budget constraint with lump-sum taxes (T_t).

$$\begin{aligned} Y_t &= C_t + G_t + I_t \\ G_t &= T_t \end{aligned}$$

where G_t is government spending and A_t follows an AR(1) process:
$\ln A_t - \ln \bar{A}_t = \rho_A (\ln A_{t-1} - \ln \bar{A}_{t-1}) + \epsilon_{A,t}$

The Steady State

- We assume a **zero-growth** steady state and a CRRA utility $U = \ln C_t - \omega \frac{H_t^{1+\phi}}{1+\phi}$ where $\omega > 0$ indicates how leisure is valued relative to consumption, and $\phi > 0$ is the inverse of the labour supply elasticity
- $\bar{A}_t = \bar{A}_{t-1} = A$, say and $\bar{G}_t = \bar{G}_{t-1} = G$. $K_t = K_{t-1} = K$ etc
- The zero-growth steady state in recursive form is given by:

$$
\begin{aligned}
R &= \frac{1}{\beta} \\
\frac{K}{Y} &= \frac{(1-\alpha)}{R-1+\delta} \\
\frac{I}{Y} &= \frac{\delta K}{Y} = \frac{(1-\alpha)\delta}{R-1+\delta} = \frac{(1-\alpha)\delta}{R-1+\delta} \\
\frac{C}{Y} &= 1 - \frac{I}{Y} - \frac{G}{Y} = 1 - \frac{I}{Y} - g_y
\end{aligned}
$$

Zero Growth Steady State in Recursive Form (cont)

$$
\begin{aligned}
H &= \frac{\alpha}{C/Y}\frac{1}{\omega} \\
Y &= (AH)^{\alpha}K^{1-\alpha} = (AH)^{\alpha}\left(\frac{K}{Y}\right)^{1-\alpha}(Y)^{1-\alpha} \\
&\Rightarrow Y = (AH)(K/Y)^{\frac{1-\alpha}{\alpha}} \\
G &= g_y Y = T \\
W &= \alpha\frac{Y}{H} \\
I &= \frac{I}{Y}Y\,;\; C = \frac{C}{Y}Y\,;\; K = \frac{K}{Y}Y \\
A &= 1
\end{aligned}
$$

Solve the Model with Dynare

- Dynare uses a technique called **perturbation**. For more : Judd (1998)

- Computes first, second and third order Taylor series approximation of the policy rules *around the steady state*

It also:

- Computes the steady state (numerically) of the model

- Computes the solution of deterministic models

- Estimates (either by maximum-likelihood(ML) or Bayesian approach) parameters of DSGE models and their distribution

Dynare Starters

- It is a big collection of Matlab functions that use Matlab in order to solve the model with perturbation

- We just have to set the external path of Matlab to the Dynare folder

- Download it at `https://www.dynare.org`

- Install it. Go to Matlab on menu File/Set Path to add the path to the Dynare subdirectory (to store all the subroutines), e.g. the path would be set to $c : \backslash dynare \backslash 4.4.y \backslash matlab$

Dynare Model [`.mod`] file

- The .mod file is the file where you write down your DSGE model

- It includes several blocks
  - Variable block
  - Parameter block
  - Parameter values block
  - Model block
  - Steady state block
  - Shocks block
  - Solution (or estimation) block

  Super useful reading: Adjemian *et al.* (2011)

Variables and Parameters Block

- `var` block: Names of the endogenous variables
  example:
  `var K C G A;`

- `varexo` block: Names of the shocks
  example:
  `varexo epsA epsG;`

- `parameters` block: Names of the parameters ; Values of the parameters
  example:

```
parameters alpha beta delta ;
alpha=0.3;
beta=0.99;
delta=0.025;
```

Model Block

- Starts with `model;` and ends with `end;`

- Type equations ending with ;
  - `x(-1)` for predetermined variables. The variable is decided in $t-1$ (predetermined), e.g. the capital stock, write it as `x(-1)` instead of`x`
  - `x(+1)` for expectations

  example:

  ```
  K = (1-delta)*K(-1)+I;
  ```

Shocks Block

- Starts with `shocks;` and ends with `end;`

- In between declare shock standard deviations
  example:
```
shocks;
var epsA;
stderr 0.02;
end;
```

- The variances (and covariances) of the shocks are defined within these commands

- Sets the std. error of this exogenous variable = 0.02

Some info

- **Note** that each instruction of the .mod file must be terminated by a semicolon

- Also Dynare uses 2 forward slashes (//) to comment out any line (whereas MATLAB uses %). (Note: for Dynare the two are equivalent!)

- There need to be as many equations as your endogenous variables declared (except for optimal policy)

- Names are case sensitive

- The stability "Blanchard-Kahn" conditions are met only if the number of jumpers equals the number of eigenvalues greater than one. (See Topic 2).

The Steady State Block

- It's the most difficult and time consuming part

- There are two options
  - Let Dynare calculate the steady state (sounds good, does not always work)
  - Calculate it ourselves and then add this as a Matlab function

The Steady State Block: Option 1

- Dynare solves for the steady state of the model, it just need (good!) initial values
- Starts with `initval;` and ends with `end;`
- In between, add initial values for all variables
- Initial values can be exact numbers or functions that depend on parameters or steady state variables
- Then, *steady* command computes the steady state
- If the model is quite complicated and the initial values not close to the truth there will be problems $\rightarrow$ **Option 2**

The Steady State Block: Option 2

- Find the analytical solution for the steady state
- Import it to a Matlab function doing the computation externally with a Matlab program FILENAME_steadystate.m
- Dynare understands that this function gives the steady state of the model
- Needs a specific preamble and ending that is provided in these files

Solution Block

- `stoch_simul` starts the solution routine for stochastic models and `simul` for deterministic simulations
  example
  `stoch simul(order=1,IRF=20, periods =10000) ;`

- There are many options for the stochastic simulation (see Dynare manual for more)

- *periods* - specifies the number of simulation periods

- *irf* sets the number of periods for which to compute impulse responses

- $order = 1$ sets the order of the Taylor approximation (default is two)

Solve your Model

- Just type in Matlab `dynare modfilename.mod`
- Dynare output is (among many others):
  - Policy rules
  - Moments
  - Impulse response functions
  - Almost everything is in the folder `oo_`. You will find it in Matlab's workspace right after the solution takes place

Adjemian, S., Bastani, H., Juillard, M., Karamé, F., Maih, J., Mihoubi, F., Mutschler, W., Perendia, G., Pfeifer, J., Ratto, M., and Villemot, S. (2011). Dynare: Reference manual version 4. Dynare Working Papers 1, CEPREMAP.

Judd, K. L. (1998). *Numerical methods in economics*. MIT press.