SWJTU-Leeds Joint School

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西南交通大学－利兹学院
SWJTU-LEEDS JOINT SCHOOL

Coursework Submission – Cover Sheet

Please complete ALL information

Leeds Student ID Number: 201487695

SWJTU Student ID Number: 2020110001

Student Name: Yizhong Ding

Module Code & Name: XJCO1511 Introduction to Discrete Mathematics

Title of Coursework Item: Coursework

For the Attention of: Prof Tianrui Li, Dr Yin Tong

Deadline Time: 5 p.m

Deadline Date: 8/4/2022

Student Signature: 丁奕中

For office use:

date stamp here

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1. In how many ways can 30 identical balls be distributed into 6 distinct boxes(numbered box 1, ... , box6) subject to the following conditions.

(a) [4 marks] Each of the odd numbered boxes gets exactly one ball and each of the even numbered boxes gets at least one ball.

<1> We put three balls into the three odd numbered boxes, because the 30 balls are identical, we just consider the remaining 27 balls.

<2> Again, we put three balls into the three even numbered boxes. Then we have 24 balls remained. Here every distribution of 24 balls into 3 distinct boxes can be modeled with a sequence of 24 0's and 2|'s.

So there is a one-to-one correspondence between the distributions of 24 identical balls into 3 distinct boxes and the sequence of 24 0's and 2|'s.

Consider the following procedure for counting the number of sequence of 24 0's and 2|'s:

(1) Choose 24 positions in the sequence for the 0's.(C (26,7) $ways$)

(2) Choose the remaining 2 positions in the sequence for the |'s.(1 ways)

--> There are $C\,(26,7) = 325$

(b) [4 marks] Each box gets an odd number of balls.

<1> Put one ball in 3 boxes, then we just consider the remaining 24 balls.

<2> Put 24 balls to 6 boxes. According to the statement, we could regard the 2 balls as a whole. Thus the question could be changed to putting 12 balls(whole) in 6 distinct boxes. We could get that there are $C\,(12+6-1, 6-1) = C\,(17,5) = 6188$

2. [4 marks] What is the coefficient of term x^4y^6 when the expression $(x+y+3)^{15}$ is expanded.

(Hint: do not use the Binomial Theorem, but use its proof as an inspiration.)

$(x+y+3)^{15} = (x+y+3)\ldots(x+y+3)$ (15 factors)

In the expansion of $(x+y+3)^{15}$, a term of the form x^4y^6 arises from choosing y from 6 factors and x from 4 factors. This can be done in $C_{15}^4 C_{11}^6$ ways.

Thus x^4y^6 appears $C_{15}^4 C_{11}^6$ times in the expansion. We could get that the coefficient of term x^4y^6 when the expression $(x+y+3)^{15}$ is expanded is $3^5 C_{15}^4 C_{11}^6 = 153243090$

3. [4 marks] Give a combinatorial argument to show that if n is a positive integer, then$\binom{2n}{2}$=2 $\binom{n}{2}$+n^2.

(Hint: Let X and Y be disjoint sets such that |X|=|Y|=n, and let Z=X∪Y. Each side counts a particular selection from Z; state which selection, and then count this selection in two different ways.)

We could set an example here to prove this equation.

Let X and Y be disjoint sets such that |X|=|Y|=n, let Z=X∪Y and |Z|=2n

According to the statement of the title we could suppose that there are n identical balls in both box X and Y. Here we mix the balls in X and Y together and we could get 2n balls in Z.

<1> Select two balls from Z $\binom{2n}{2}$

<2> Select two balls from X and Y. This action contains three options:

(1) 2 balls come from X
(2) 2 balls come from Y
(3) 1 ball comes from X and 1 ball comes from Y

Thus, there exists $\binom{2n}{2}$=2 $\binom{n}{2}$+n^2 kinds of methods in total. The equation of $\binom{2n}{2}$=2 $\binom{n}{2}$+n^2 could be proven.

4. [4 marks] A standard deck of cards contains 52 cards and is comprised of 13 ranks (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King) in each of the four suits (clubs, diamond, hearts, spades). How many cards must be selected from a standard deck of cards to guarantee that at least three cards are of the same rank?

In order to guarantee that at least three cards are of the same rank, we could just make sure that there exists one combination of three cards in the same rank. Then we could consider that:

<1> First we pick two cards from each of 13 ranks(13*2 = 26)

<2> Pick another one card at random from these 13 ranks then we could make sure that at least three cards are of the same rank(13*2 + 1 = 27)

Thus 27 cards must be selected from a standard deck of cards to guarantee that at least three cards are of the same rank.