Coursework Submission – Cover Sheet

Please complete ALL information

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Module Code & Name: XJCO1511 Introduction to Discrete Mathematics

Title of Coursework Item: Coursework

For the Attention of: Prof Tianrui Li, Dr Yin Tong

Deadline Time: 5 p.m.

Deadline Date: 18/3/2022

Student Signature: 丁奕中

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1. In a standard deck of cards there are 4 suits: clubs, diamonds, hearts and spades. Each suit contains 13 cards: 2, 3,..., 10, Jack, Queen, King, Ace. So there are 4 · 13 = 52 cards in the deck.

A sequence of 5 cards is drawn from a standard deck of cards, with replacement (meaning that when a card is drawn it is placed back into the deck before the next card is chosen).

(a) [4 marks] How many of such sequences begin with a King and end with a Queen?

<1>Choose a King for beginning (4 choices)

<2>Choose one card for the second place (52 choices)

<3>Choose one card for the third place (52 choices)

<4>Choose one card for the forth place (52 choices)

<5>Choose a Queen for ending (4 choices)

Thus, the total sequences begin with a King and end with a Queen have 4·52·52·52·4 = 2249728 kinds.

(b) [4 marks] How many of such sequences begin with a King or end with a Queen?

<1>Choose a sequence of 5 cards beginning with a King (4·52·52·52·52 choices)

<2>Choose a sequence of 5 cards ending with a Queen (52·52·52·52·4 choices)

<3>Choose a sequence of 5 cards beginning with a King and ending with a Queen (4·52·52·52·4 choices)

The sequence of <3> is a repetitive part. Thus the sequences begin with a King or end with a queen should have <1> + <2> − <3> =

4·52·52·52·52 + 52·52·52·52·4 - 4·52·52·52·4 = 56243200 kinds.

(c) [4 marks] How many of such sequences will contain at least one King or one Queen (or both)

<1>Choose a sequence of 5 cards (52·52·52·52·52 choices)

<2>Choose a sequence without containing King or Queen {(52-8)·(52-8)·(52-8)·(52-8)·(52-8) choices}

The sequence of <2> is what we do not want, so the sequences contain at least one King or one Queen(or both) should have <1> − <2> =

52·52·52·52·52 −(52-8)·(52-8)·(52-8)·(52-8)·(52-8) = 215287808 kinds.

2. [4 marks] In the game of poker a player receives a subset of 5 cards, called a poker hand, from the standard deck of 52 cards. The order in which the cards are received is not important, just the actual cards themselves. How many poker hands contain exactly two suits?

<1>Divide the subset of 5 cards(poker hand) into two parts, one part is 4 cards and the other part is 1 card. Then these two parts have exactly two suits. One suit has 13 cards and here we choose 4 cards (one part) from one suit and the other 1 card(one part) from another suit. Then we could get $C_{13}^{4}C_{13}^{1}$. The following we choose one suit from the total 4 suits and the other suit from the rest of 3 suits. Here we could get $C_{4}^{1}C_{3}^{1}$.

<2>In the same way, divide the subset of 5 cards(poker hand) into two parts, one part is 3 cards and the other part is 2 cards. Here we choose 3 cards (one part) from one suit and the other 2 cards(one part) from another suit. Then we could get $C_{13}^{3}C_{13}^{2}$. The following we choose one suit from the total 4 suits and the other suit from the rest of 3 suits. Here we could get $C_{4}^{1}C_{3}^{1}$. Because the order in which the cards are received is not important, just the actual cards themselves. There are $C_{13}^{4}C_{13}^{1}\,C_{4}^{1}C_{3}^{1} + C_{13}^{3}C_{13}^{2}\,C_{4}^{1}C_{3}^{1}$ = 379236 poker hands.

3. [4 marks] On the menu of a Chinese restaurant there are 7 chicken dishes, 5 beef dishes, 4 seafood dishes and 10 vegetable dishes. In how many ways can they order if at most one dish of each kind is chosen?

<1>Chicken dishes: there are 8 choices for chicken dishes, including 7 kinds of chicken dishes and 1 kind of choice to order nothing.

<2>Beef dishes: there are 6 choices for chicken dishes, including 5 kinds of chicken dishes and 1 kind of choice to order nothing.

<3>Seafood dishes: there are 5 choices for chicken dishes, including 4 kinds of chicken dishes and 1 kind of choice to order nothing.

<4>Vegetable dishes: there are 11 choices for chicken dishes, including 10 kinds of chicken dishes and 1 kind of choice to order nothing.

Thus, if at most one dish of each kind is chosen, there are $8\cdot6\cdot5\cdot11 = 2640$ ways the customers can order.