# Învățare automată - Temă S10

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## Problema 0.164A / pag. 254

Cunoaștem $f$ funcție derivabilă, $f : \mathbb{R}^d \to \mathbb{R}$.

Regula de actualizare este: $x_i^{(k+1)} \leftarrow x_i^{(k)} - learningRate \cdot \frac{\partial}{\partial x_i} f(x^{(k)})$

**a.**

$f(x) = 4x^2 - 2x + 1$

$learningRate = 0.1$

$\frac{\partial}{\partial x} f(x) = \frac{\partial}{\partial x}(4x^2 - 2x + 1) = 8x - 2$

$x^{(1)} = 1$

Pentru $k = 0$:

$\frac{\partial}{\partial x} f(x^{(0)}) = 6$

$x^{(1)} = x^{(0)} - learningRate \cdot 6 = 1 - 0.1 \cdot 6 = 0.4$

$f(x^{(1)}) = f(0.4) = 0.84$

Pentru $k = 1$:

$\frac{\partial}{\partial x} f(x^{(1)}) = 1.2$

$x^{(2)} = x^{(1)} - learningRate \cdot 1.2 = 0.4 - 0.1 \cdot 1.2 = 0.28$

$f(x^{(2)}) = f(0.28) = 0.7536$

Pentru $k = 2$:

$\frac{\partial}{\partial x} f(x^{(2)}) = 0.24$

$x^{(3)} = x^{(2)} - learningRate \cdot 0.24 = 0.28 - 0.1 \cdot 0.24 = 0.256$

$f(x^{(3)}) = f(0.256) = 0.7501$

**b.**

$f(x_1, x_2) = x_1^2 + sin(x_1 + x_2) + x_2^2$

$x^{(k)} = (x_1^{(k)}, x_2^{(k)})$

$x_1^{(k+1)} \leftarrow x_1^{(k)} - learningRate \cdot \frac{\partial}{\partial x_1} f(x_1^{(k)}, x_2^{(k)})$
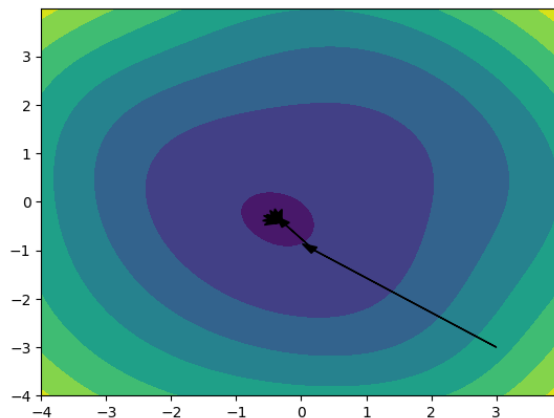
$x_2^{(k+1)} \leftarrow x_2^{(k)} - learningRate \cdot \frac{\partial}{\partial x_2} f(x_1^{(k)}, x_2^{(k)})$

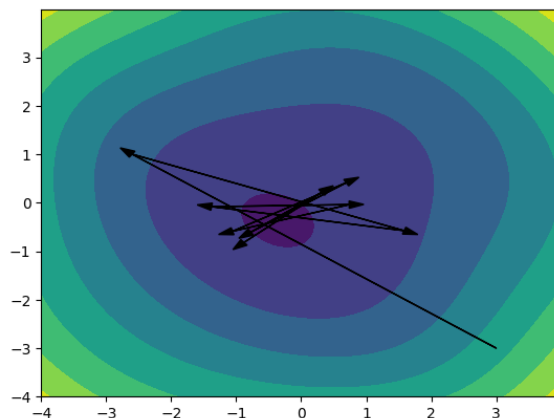$\frac{\partial}{\partial x_1} f(x_1^{(k)}, x_2^{(k)}) = 2x_1 + cos(x_1 + x_2)$

$\frac{\partial}{\partial x_2} f(x_1^{(k)}, x_2^{(k)}) = 2x_2 + cos(x_1 + x_2)$

$f : [-4, 4] \to [-4, 4]$

**i.** Primii 10 pași făcuți de **GD** începând cu $(x_1^{(0)}, x_2^{(0)}) = (3, -3)$ și $learningRate = 0.4$:

**ii**. Primii 10 pași făcuți de **GD** începând cu $(x_1^{(0)}, x_2^{(0)}) = (3, -3)$ și *learningRate* $= 0.8$:



Se observă faptul că punctele converg către o valoare de minim. Mai mult, *learningRate*-ul configurează dimensiunea "pașilor" cu care covergența are loc.

## Problema 1.34 / pag. 332

Vom schimba setul de date inițial cu cel de la **Problema 4.6 / pag. 491**.
Astfel, vom face următoarele translări pentru a ne ajuta în rezolvarea problemei curent:

- Clasa: $X_1 \in \{0, 1\}$, unde I = 0, Inferioară = 1;

- Sexul: $X_2 \in \{0, 1\}$, unde Masculin = 0, Feminin = 1;

- Vârsta: $X_3 \in \{0, 1\}$, unde Copil = 0, Adult = 1;

- Supraviețuitor: $Y \in \{0, 1\}$, unde Nu = 0, Da = 1.

Setul de date devine:

| Indecși | Număr | $X_1$ | $X_2$ | $X_3$ | $Y$ |
|---|---|---|---|---|---|
| $[1,5]$ | 5 | 0 | 0 | 0 | 1 |
| $[6,123]$ | 118 | 0 | 0 | 1 | 0 |
| $[124,180]$ | 57 | 0 | 0 | 1 | 1 |
| $[181,181]$ | 1 | 0 | 1 | 0 | 1 |
| $[182,185]$ | 4 | 0 | 1 | 1 | 0 |
| $[186,325]$ | 140 | 0 | 1 | 1 | 1 |
| $[326,360]$ | 35 | 1 | 0 | 0 | 0 |
| $[361,384]$ | 24 | 1 | 0 | 0 | 1 |
| $[385,1595]$ | 1211 | 1 | 0 | 1 | 0 |
| $[1596,1876]$ | 281 | 1 | 0 | 1 | 1 |
| $[1877,1893]$ | 17 | 1 | 1 | 0 | 0 |
| $[1894,1920]$ | 27 | 1 | 1 | 0 | 1 |
| $[1921,2025]$ | 105 | 1 | 1 | 1 | 0 |
| $[2026,2201]$ | 176 | 1 | 1 | 1 | 1 |

**a.**

$l(w) = \sum_{i=1}^{2201}(y^{(i)} \ln \sigma(w \cdot x^{(i)}) + (1-y^{(i)}) \ln(1 - \sigma(w \cdot x^{(i)}))) = 5 \cdot 1 \cdot \ln \sigma(w \cdot (1,0,0,0)^T) + 118 \cdot (1-0) \cdot \ln(1 - \sigma(w \cdot (1,0,0,1)^T)) + 57 \cdot 1 \cdot \ln \sigma(w \cdot (1,0,0,1)^T) + 1 \cdot 1 \cdot \ln \sigma(w \cdot (1,0,1,0)^T) + 4 \cdot (1-0) \cdot \ln(1 - \sigma(w \cdot (1,0,1,1)^T)) + 140 \cdot 1 \cdot \ln \sigma(w \cdot (1,0,1,1)^T) + 35 \cdot (1-0) \cdot \ln(1 - \sigma(w \cdot (1,1,0,0)^T)) + 24 \cdot 1 \cdot \ln \sigma(w \cdot (1,1,0,0)^T) + 1211 \cdot (1-0) \cdot \ln(1 - \sigma(w \cdot (1,1,0,1)^T)) + 281 \cdot 1 \cdot \ln \sigma(w \cdot (1,1,0,1)^T) + 17 \cdot (1-0) \cdot \ln(1 - \sigma(w \cdot (1,1,1,0)^T)) + 27 \cdot 1 \cdot \ln \sigma(w \cdot (1,1,1,0)^T) + 105 \cdot (1-0) \cdot \ln(1 - \sigma(w \cdot (1,1,1,1)^T)) + 176 \cdot 1 \cdot \ln \sigma(w \cdot (1,1,1,1)^T) = 5 \ln \sigma(w_0) + 118 \ln(1 - \sigma(w_0 + w_3)) + 57 \ln \sigma(w_0 + w_3) + \ln \sigma(w_0 + w_2) + 4 \ln(1 - \sigma(w_0 + w_2 + w_3)) + 140 \ln \sigma(w_0 + w_2 + w_3) + 35 \ln(1 - \sigma(w_0 + w_1)) + 24 \ln \sigma(w_0 + w_1) + 1211 \ln(1 - \sigma(w_0 + w_1 + w_3)) + 281 \ln \sigma(w_0 + w_1 + w_3) + 17 \ln(1 - \sigma(w_0 + w_1 + w_2)) + 27 \ln \sigma(w_0 + w_1 + w_2) + 105 \ln(1 - \sigma(w_0 + w_1 + w_2 + w_3)) + 176 \ln \sigma(w_0 + w_1 + w_2 + w_3)$

**b.**

**i.**

$\nabla_w l(w) = \sum_{i=1}^{2201}[y^{(i)} - \sigma(w \cdot x^{(i)})]x^{(i)} = 5[1 - \sigma(w_0)](1,0,0,0)^T - 118\sigma(w_0 + w_3)(1,0,0,1)^T + 57[1 - \sigma(w_0 + w_3)](1,0,0,1)^T + [1 - \sigma(w_0 + w_2)](1,0,1,0)^T - 4\sigma(w_0 + w_2 + w_3)(1,0,1,1)^T + 140[1 - \sigma(w_0 + w_2 + w_3)](1,0,1,1)^T - 35\sigma(w_0 + w_1)(1,1,0,0)^T + 24[1 - \sigma(w_0 + w_1)](1,1,0,0)^T - 1211\sigma(w_0 + w_1 + w_3)(1,1,0,1)^T + 281[1 - \sigma(w_0 + w_1 + w_3)](1,1,0,1)^T - 17\sigma(w_0 + w_1 + w_2)(1,1,1,0)^T + 27[1 - \sigma(w_0 + w_1 + w_2)](1,1,1,0)^T - 105\sigma(w_0 + w_1 + w_2 + w_3)(1,1,1,1)^T + 176[1 - \sigma(w_0 + w_1 + w_2 + w_3)](1,1,1,1)^T = (711 - 5\sigma(w_0) - 175\sigma(w_0 + w_3) - \sigma(w_0 + w_2) - 144\sigma(w_0 + w_2 + w_3) - 59\sigma(w_0 + w_1) - 1492\sigma(w_0 + w_1 + w_3) - 44\sigma(w_0 + w_1 + w_2) - 281\sigma(w_0 + w_1 + w_2 + w_3), 508 - 59\sigma(w_0 + w_1) - 1492\sigma(w_0 + w_1 + w_3) - 44\sigma(w_0 + w_1 + w_2) - 281\sigma(w_0 + w_1 + w_2 + w_3), 344 - \sigma(w_0 + w_2) - 144\sigma(w_0 + w_2 + w_3) - 44\sigma(w_0 + w_1 + w_2) - 281\sigma(w_0 + w_1 + w_2 + w_3), 654 - 175\sigma(w_0 + w_3) - 144\sigma(w_0 + w_2 + w_3) - 1492\sigma(w_0 + w_1 + w_3) - 281\sigma(w_0 + w_1 + w_2 + w_3))^T$

**ii.**

Alegem $j = 2$:

$\frac{\partial}{\partial w_2} l(w) = \frac{\partial}{\partial x_2} \ln \sigma(w_0 + w_2) + \frac{\partial}{\partial x_2} 4 \ln(1 - \sigma(w_0 + w_2 + w_3)) + \frac{\partial}{\partial x_2} 140 \ln \sigma(w_0 + w_2 + w_3) + \frac{\partial}{\partial x_2} 17 \ln(1 - \sigma(w_0 + w_1 + w_2)) + \frac{\partial}{\partial x_2} 27 \ln \sigma(w_0 + w_1 + w_2) + \frac{\partial}{\partial x_2} 105 \ln(1 - \sigma(w_0 + w_1 + w_2 + w_3)) + \frac{\partial}{\partial x_2} 176 \ln \sigma(w_0 + w_1 + w_2 + w_3)$