# Problem 1

## a)

The consumer would never consume at a corner of their budget constraint. This is true in general of Cobb-Douglass utility functions, as taking either quantity as 0 would vanish utility. Thus, in any case, they consume at least some of either good, as we take the goods to be perfectly divisible.

## b)

We simply can note that the problem is symmetric across $q_x, q_y$ and conclude that equal quantities must be bought, and thus $q_x^* = q_y^* = 4$.

However, I suspect that this may not be satisfactory for the TA's. The problem then is $\max[q_x^{0.5} q_y^{0.5}]$ over $q_x, q_y$, such that $2q_x + 2q_y = 16$.

$$
\begin{aligned}
\mathcal{L}(q_x, q_y, \lambda) &= q_x^{\frac{1}{2}} q_y^{\frac{1}{2}} - \lambda(p_x q_x + p_y q_y - M) \\
\frac{\delta \mathcal{L}}{\delta q_x} &= \frac{1}{2}(\frac{q_y}{q_x})^{\frac{1}{2}} - p_x \lambda = 0 \\
\frac{\delta \mathcal{L}}{\delta q_y} &= \frac{1}{2}(\frac{q_x}{q_y})^{\frac{1}{2}} - p_y \lambda = 0 \\
\frac{\delta \mathcal{L}}{\delta \lambda} &= -p_x q_x - p_y q_y + M = 0 \\
\implies \qquad \frac{q_y^*}{q_x^*} &= \frac{p_x}{p_y} \\
\implies \qquad q_y^* &= \frac{p_x q_x^*}{p_y} \\
\implies \qquad 2p_x q_x^* &= M \\
\implies \qquad q_x^* &= \frac{M}{2p_x} = 4 \\
\implies \qquad q_y^* &= \frac{M}{2p_y} = 4
\end{aligned}
$$

The consumer has utility $\overline{U} = (4)^{0.5}(4)^{0.5} = 4$.

## c)

### c.1

The cost of the original bundle would be $8(4) + 2(4) = 40$.

### c.2

Her utility function remains the same, so she would purchase then $q_x = \frac{40}{2(8)} = 2.5, q_y = \frac{40}{2(2)} = 10$.

### c.3

The difference in good $x$ is $2.5 - 4 = -1.5$, as the Slutsky bundle has 1.5 units of $x$ less (own substitution).

### c.4

The difference in good $y$ is $10 - 4 = 6$, as the Slutsky bundle has 6 units of $y$ more (cross substitution).

### d.1

The new optimal bundle is at $q_x = \frac{16}{2(8)} = 1, q_y = \frac{16}{2(2)} = 4$.

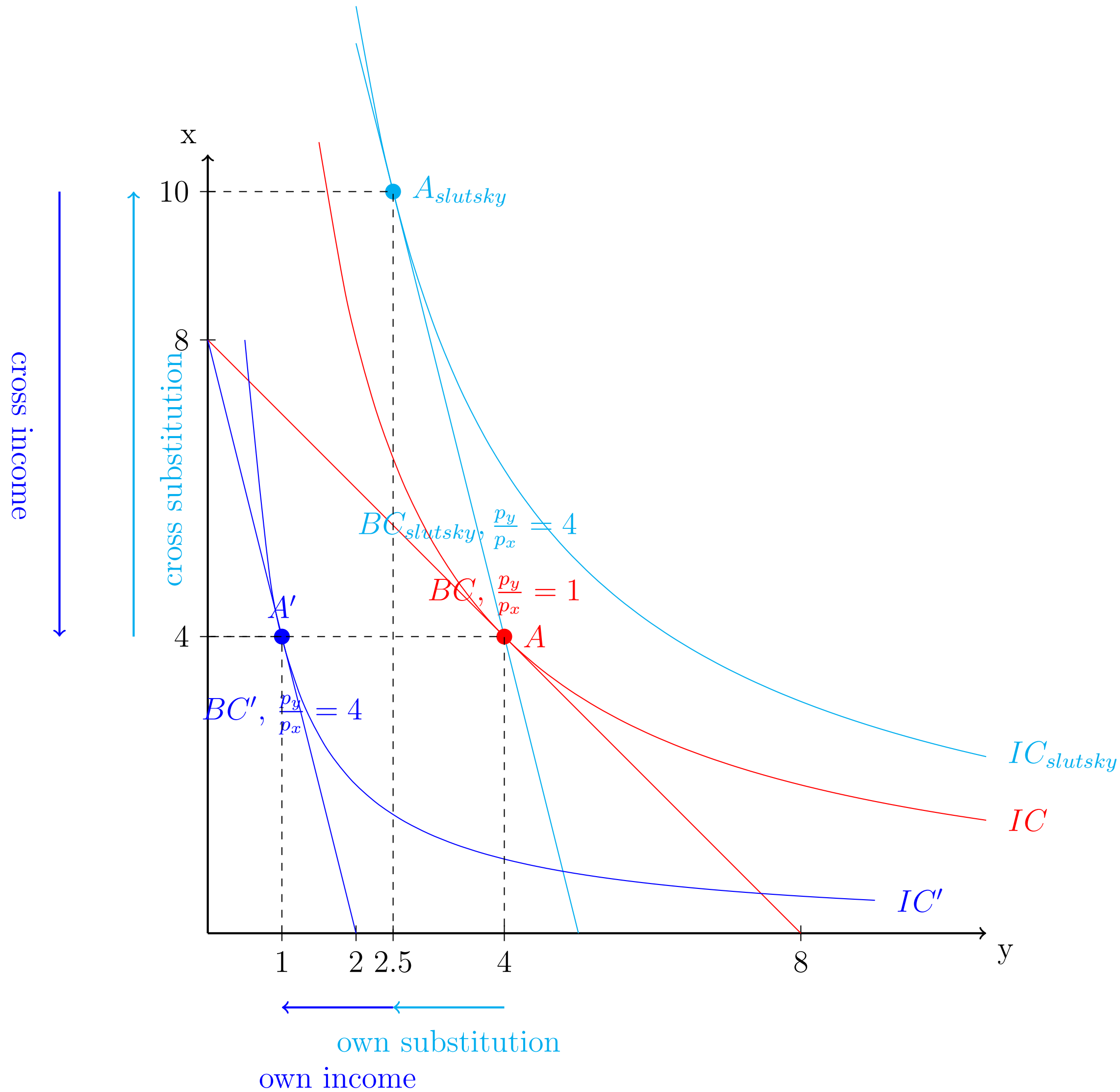
### d.2

The difference in good $x$ is $1 - 2.5 = -1.5$, such that the new optimal bundle has 1.5 units of $x$ less (own income).

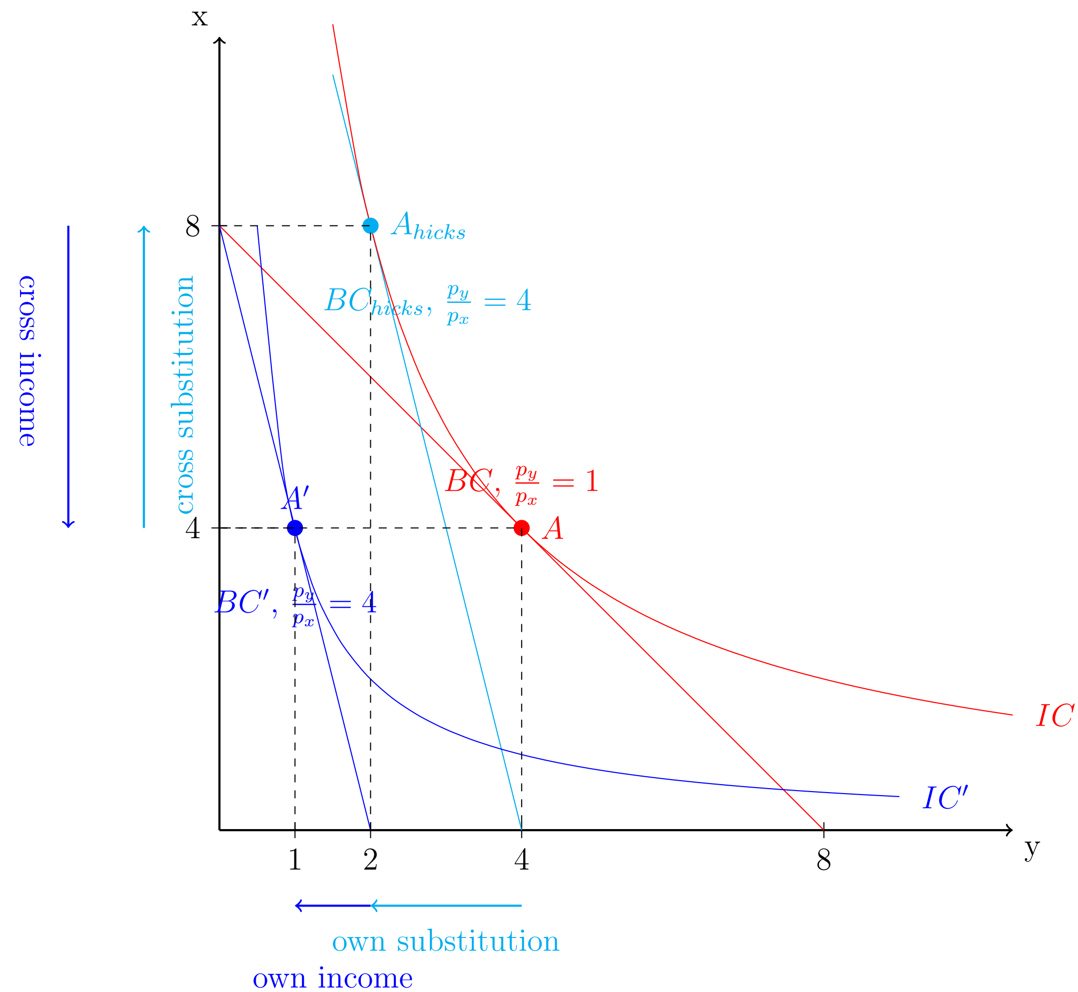
### d.3

The difference in good $y$ is $4 - 10 = -6$, such that the new optimal bundle has 6 units of $x$ less (cross income).

## e)


x
10
8
4
$A_{slutsky}$
cross income
cross substitution
$BC_{slutsky}$, $\frac{p_y}{p_x} = 4$
$BC$, $\frac{p_y}{p_x} = 1$
$A'$
$A$
$BC'$, $\frac{p_y}{p_x} = 4$
$IC_{slutsky}$
$IC$
$IC'$
1
2
2.5
4
8
y
own substitution
own income

## f)




## g)

### g.1

To find the Hicksian bundle, we have that $U(q_x, q_y) = 4$, and that the Hicks line has slope $-\frac{p_q}{p_x} = -4$. Then $\frac{dq_x}{dq_y} = -\frac{(\frac{q_y}{q_x})^{\frac{1}{2}}}{(\frac{q_x}{q_y})^{\frac{1}{2}}} = -\frac{q_y}{q_x}$ along $U(q_x, q_y) = 4$. Thus, we have that $\frac{q_y}{q_x} = 4 \implies q_y = 4q_x$. In order to have $U(q_x, 4q_x) = 2q_x = 4$, we must have $q_x^* = 2, q_y^* = 8$.

#### g.2

The own substitution effect is $2 - 4 = -2$, as there are 2 less units of $x$ in the Hicksian bundle. The cross substitution effect is $8 - 4 = 4$, as there are 4 more units of good $y$ in the Hicksian bundle.

#### g.3

The own income effect is $1 - 2 = -1$, as there is 1 less unit of $x$ in the new optimal bundle from the Hicksian bundle. Similarly, since there are 4 less units of $y$, we have that the cross income effect is $4 - 8 = -4$.

# Problem 2

## a)

The optimization problem faced is to $\max[q_R^{\frac{1}{2}} q_B^{\frac{1}{2}}]$ over $q_R, q_B$, such that $P_R q_R + P_B q_B = M$.

## b)

This is identical to the system in problem 1, given a few name changes.

#### b.1

$$\mathcal{L}(q_R, q_B, \lambda) = q_R^{\frac{1}{2}} q_B^{\frac{1}{2}} - \lambda(P_R q_R + P_B q_B - M)$$

#### b.2

$$\begin{aligned}
\frac{\delta \mathcal{L}}{\delta q_R} &= \frac{1}{2}(\frac{q_B}{q_R})^{\frac{1}{2}} - P_R \lambda = 0 \\
\frac{\delta \mathcal{L}}{\delta q_B} &= \frac{1}{2}(\frac{q_R}{q_B})^{\frac{1}{2}} - P_B \lambda = 0 \\
\frac{\delta \mathcal{L}}{\delta \lambda} &= -P_R q_R - P_B q_B + M = 0
\end{aligned}$$

**b.3**

$$
\begin{aligned}
\implies \quad \frac{q_B^*}{q_R^*} &= \frac{P_R}{P_B} \\
\implies \quad q_B^* &= \frac{P_R q_R^*}{P_B} \\
\implies 2P_R q_R^* &= M \\
\implies \quad q_R^* &= \frac{M}{2P_R} \\
\implies \quad q_B^* &= \frac{M}{2P_B} \\
\implies \quad \lambda &= \frac{1}{2(P_R P_B)^{\frac{1}{2}}}
\end{aligned}
$$

**b.4**

$$
\begin{aligned}
V(P_B, P_R, M) &= (\frac{M}{2P_R})^{\frac{1}{2}}(\frac{M}{2P_B})^{\frac{1}{2}} \\
&= \frac{M}{2(P_R P_B)^{\frac{1}{2}}}
\end{aligned}
$$

## c)

His optimal bundle is $q_R^* = \frac{M}{2P_R} = \frac{2400}{2(200)} = 6, q_B^* = \frac{M}{2P_B} = \frac{2400}{2(50)} = 24$.
His indirect utility is $V_0 = \frac{2400}{2\sqrt{50(200)}} = 12$

## d)

The current cost of his optimal bundle is $6(120) + 24(80) = \$2640$, which is not affordable. This however is not the optimal bundle at the other town, which is $q_R^* = \frac{2400}{2(120)} = 10, q_B^* = \frac{2400}{2(80)} = 15$ at a cost equal to $M = 2400$.

## e)

$V(120, 80, 2400) = \frac{2400}{2(120(80))^{\frac{1}{2}}} = \frac{1200}{40\sqrt{6}} = \frac{30}{\sqrt{6}} = 5\sqrt{6} \approx 12.25$.

## f)

Roger's utility is higher when shopping in the other town rather than the local town.

## g)

### g.1

$V(200, 50, 2400) = 12 = V(120, 80, 2400 + CV)$.

$$\begin{aligned} V(120, 80, 2400 + CV) &= \frac{2400 + CV}{2(120(80))^{\frac{1}{2}}} \\ \implies \quad 12 &= \frac{2400 + CV}{2(40)\sqrt{6}} \\ \implies \quad 960\sqrt{6} &= 2400 + CV \\ \implies \quad CV &= 960\sqrt{6} - 2400 = -\$48.49 \end{aligned}$$

It is negative to indicate willingness to pay to get access to the new bundle.
(Although this is taken to be positive in recitation for some reason?)

### g.2i

The problem is to $\min[120q_R + 80q_B]$ over $q_R, q_B$, such that $q_R^{\frac{1}{2}} q_B^{\frac{1}{2}} = 12$.

### g.2ii

$$\mathcal{L}(q_R, q_B, \lambda) = 120q_R + 80q_B - \lambda(q_R^{\frac{1}{2}} q_B^{\frac{1}{2}} - 12)$$

### g.2iii

$$\begin{aligned} \frac{\delta \mathcal{L}}{\delta q_R} &= 120 - \frac{\lambda}{2}\left(\frac{q_B}{q_R}\right)^{\frac{1}{2}} = 0 \\ \frac{\delta \mathcal{L}}{\delta q_B} &= 80 - \frac{\lambda}{2}\left(\frac{q_R}{q_B}\right)^{\frac{1}{2}} = 0 \\ \frac{\delta \mathcal{L}}{\delta \lambda} &= q_R^{\frac{1}{2}} q_B^{\frac{1}{2}} - 12 = 0 \end{aligned}$$

**g.2iv**

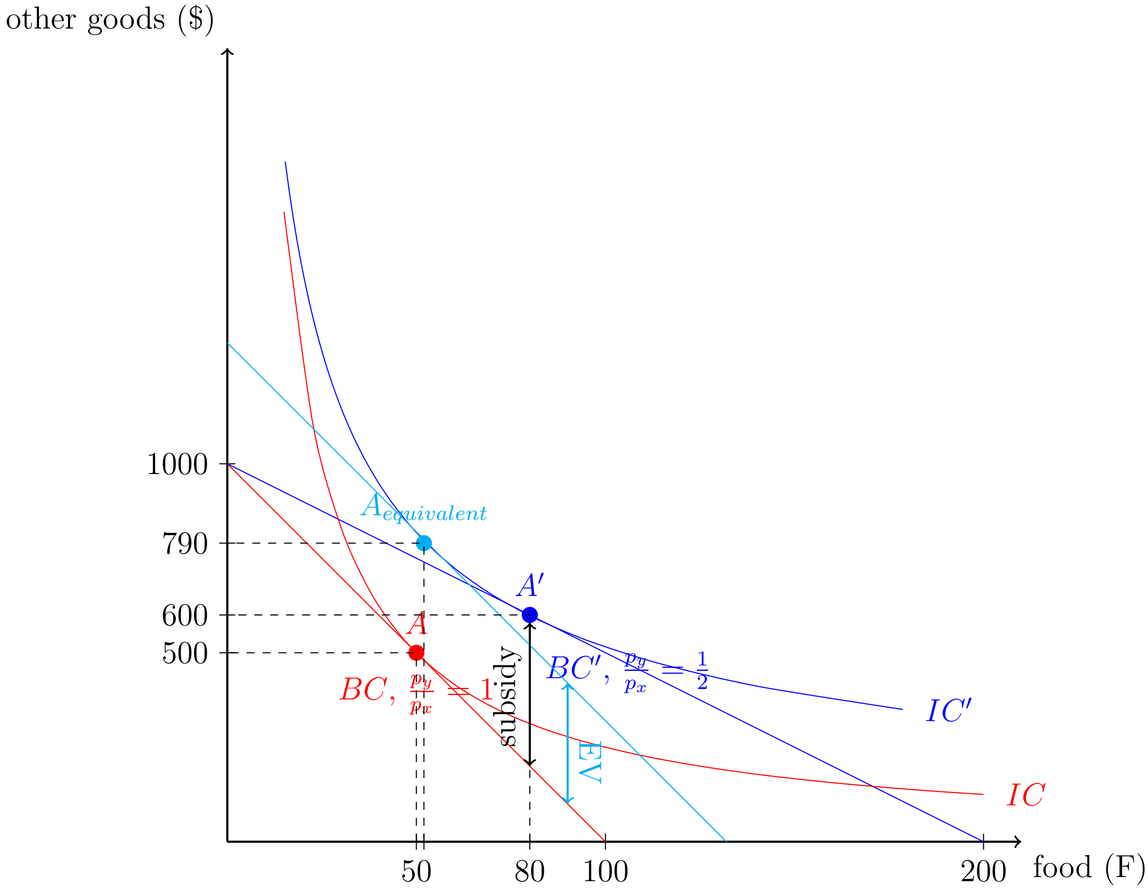
$$
\begin{aligned}
\frac{q_B}{q_R} &= \frac{120}{80} \\
\implies \quad q_B &= \frac{3}{2}q_R \\
\implies (\frac{3}{2})^{\frac{1}{2}}q_R &= 12 \\
\implies \quad q_R &= 12(\frac{2}{3})^{\frac{1}{2}} = 4\sqrt{6} \\
\implies \quad q_B &= 12(\frac{3}{2})^{\frac{1}{2}} = 6\sqrt{6} \\
\implies \quad \lambda &= 80(2(\frac{3}{2})^{\frac{1}{2}}) = 80\sqrt{6}
\end{aligned}
$$

**g.2v**

The cheapest bundle costs $120(4\sqrt{6}) + 80(6\sqrt{6}) = 960\sqrt{6}$. Roger's compensating variation is then $960\sqrt{6} - 2400 = -\$48.49$.

# Problem 3

## a), b), c), d), e)




We can note that we have $EV$ less than the subsidy.

## f)

The fact that $EV$ is less than the subsidy means that a straight cash transfer equal to $EV$ would've pushed utility to the same level as the subsidy for less spending.

A benefit, however, to specific subsidies like the one on food here is that it directly increases consumption of food even more than a straight cash transfer. Thus, if the main goal is to reduce hunger drastically, it might wind up being advantageous to directly subsidize food.

Also, people won't vote for direct cash transfers, so...

# Problem 4

## a)

The consumer's time constraint is $T = H + N$, where $H$ is the amount of time spent working and $N$ is the amount of time not working.

## b)

The consumer's budget constraint is $pC = wH$, as they have no nonwage sources of income.

## c)

Putting these together, we arrive at $pC = wT - wN$, or $pC + wN = wT$.

## d)

### d.1

$$\mathcal{L}(C, N, \lambda) = CN^2 - \lambda(pC + wN - wT)$$

### d.2

$$\frac{\delta\mathcal{L}}{\delta C} = N^{*2} - p\lambda = 0$$
$$\frac{\delta\mathcal{L}}{\delta N} = 2C^*N^* - w\lambda = 0$$
$$\frac{\delta\mathcal{L}}{\delta \lambda} = pC^* + wN^* - wT = 0$$

### d.3

$$
\begin{aligned}
p\lambda &= N^{*2} \\
w\lambda &= 2C^*N^* \\
\implies N^{*2} &= 2C^*N^*\frac{p}{w} \\
\implies N^* &= \frac{2p}{w}C^* \\
\implies 3pC^* &= wT \\
\implies C^* &= \frac{wT}{3p} \\
\implies N^* &= \frac{2T}{3} \\
\implies \lambda &= \frac{4T^2}{9p}
\end{aligned}
$$

## e)

The labor supply is simply $H = T - N^* = \frac{T}{3}$.

## f)

Since $\frac{dH}{dw} = 0$, a wage increase has no effect on the amount worked.

## g)

Similarly, $\frac{dH}{dp} = 0$, so they still would not work longer hours.

## h)

The time constraint remains the same, but the budget constraint becomes $pC = wH - A = wT - wN - A \implies pC + wN = wT - A$.

$$\begin{aligned}
\mathcal{L}(C,N,\lambda) &= CN^2 - \lambda(pC + wN - wT + A) \\
\frac{\delta\mathcal{L}}{\delta C} &= N^{*2} - p\lambda = 0 \\
\frac{\delta\mathcal{L}}{\delta N} &= 2C^*N^* - w\lambda = 0 \\
\frac{\delta\mathcal{L}}{\delta \lambda} &= pC^* + wN^* - wT + A \\
\implies \qquad p\lambda &= N^{*2} \\
\implies \qquad w\lambda &= 2C^*N^* \\
\implies \qquad N^{*2} &= 2C^*N^*\frac{p}{w} \\
\implies \qquad N^* &= \frac{2p}{w}C^* \\
\implies \qquad 3pC^* &= wT - A \\
\implies \qquad C^* &= \frac{wT - A}{3p} \\
\implies \qquad N^* &= \frac{2T}{3} - \frac{2A}{3w}
\end{aligned}$$

## i)

In this case, we have that $H = T - N^* = \frac{T}{3} + \frac{2A}{3w}$. Thus, $\frac{dH}{dw} = -\frac{2A}{3w^2} < 0$, so an increase in wages would lead to a decrease in the time worked, approaching just $\frac{T}{3}$ hours worked as wages grow large.

## j)

We still however have that $\frac{dH}{dp} = 0$, so inflation does not cause them to work more or less.