dc3451
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Problem Set 2

ECON 3211
September 19, 2019

Math Review

a)

$$\begin{aligned} dU = \frac{\delta U}{\delta x}dx + \frac{\delta U}{\delta y}dy = 0 \\ \implies \quad -\frac{\delta U}{\delta y}dy = \frac{\delta U}{\delta x}dx \\ \implies \quad \frac{dy}{dx} = -\frac{\frac{\delta U}{\delta x}}{\frac{\delta U}{\delta y}} \end{aligned}$$

b)

$$\begin{aligned} & R(x,y) = \$10x + \$5y - \$100 = 0 \\ \implies \quad & dR = \frac{\delta R}{\delta x}dx + \frac{\delta R}{\delta y}dy = \$10dx + \$5dy = 0 \\ \implies \quad & \frac{dy}{dx} = -2 \end{aligned}$$

c)

This does not exist for some reason.

d)

$$\mathcal{L}(x, y, \lambda) = 2x^{0.5} + 2y^{0.5} - \lambda(x + y - 16)$$

e)

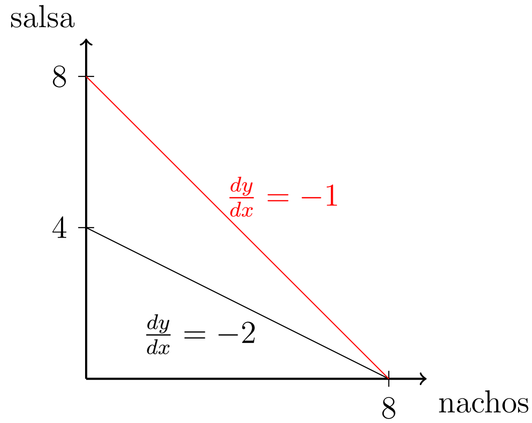
$$\mathcal{L}(x, y, \lambda) = x + y - \lambda(x^2 + y^2 - 16)$$

Problem 1

(What a terrible world to live in.)

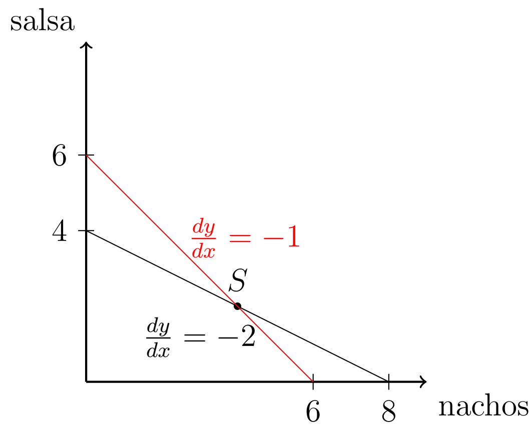
a)

a.1, a.2



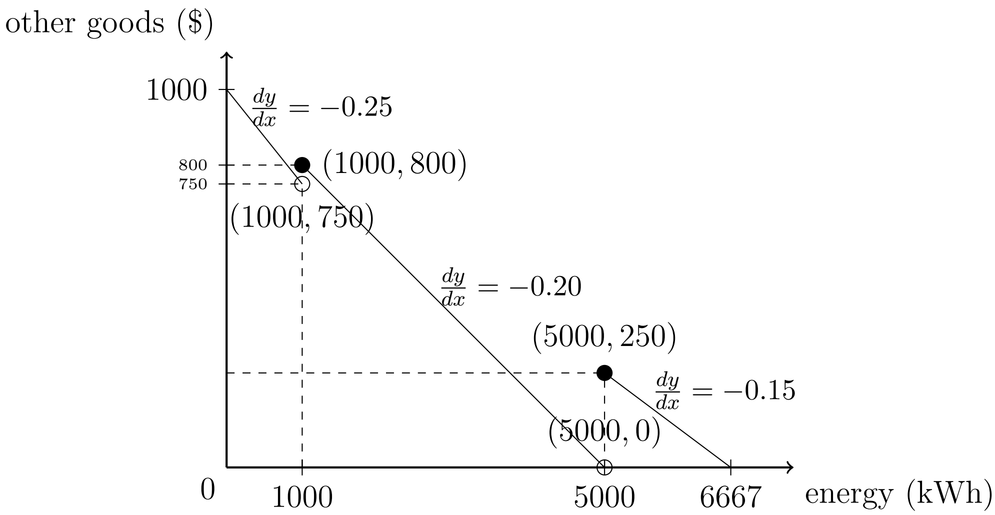
a.3, a.4

Note that this is the same exact situation as above before the price shift. S is the inital bundle of goods.



b)

b.1

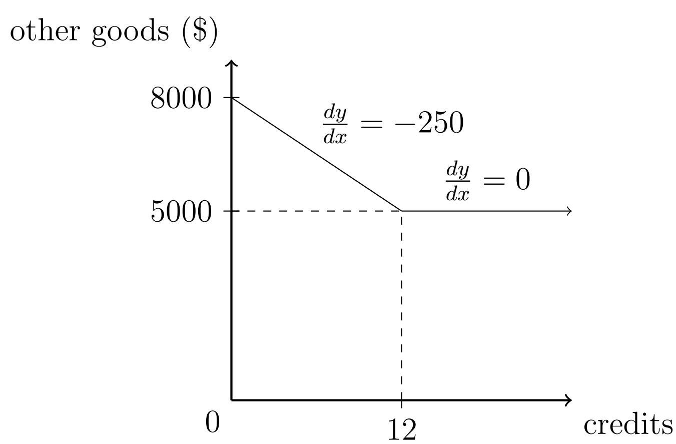


b.2

$$y = \begin{cases} 1000 - 0.25x & 0 \leq x < 1000 \\ 1000 - 0.20x & 1000 \leq x < 5000 \\ 1000 - 0.15x & 5000 \leq x \end{cases}$$

c)

c.1



c.2

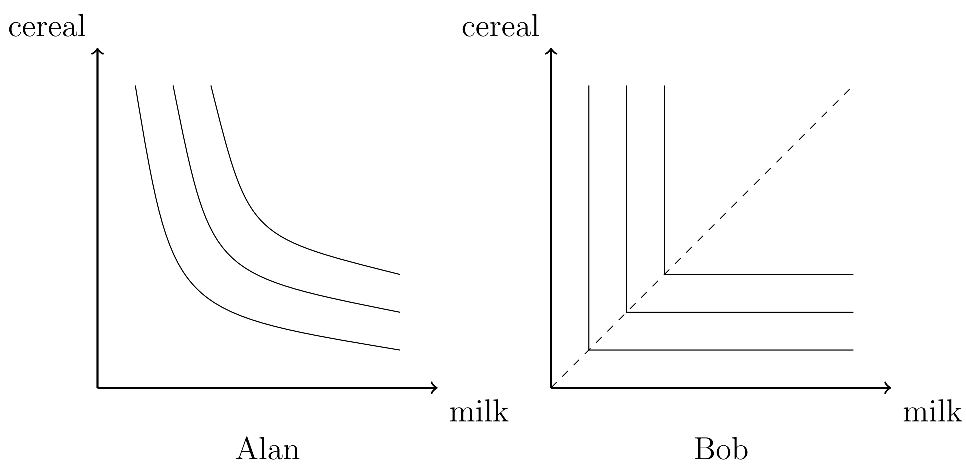
At 10 credits, the student has an opportunity cost of \$250 that could be spent on other goods for taking an extra credit.

However, at 13 credits, the student would just be paying the bulk payment of \$3000 for the semester, so there is no opportunity cost for another credit other than the time that she must invest into the credit.

Problem 2

a)

a.1



a.2

An example utility function for Alan can be the Cobbs-Douglas utility function, $U(q_c, q_m) = q_c^{0.5} q_m^{0.5}$.

An example utility function for Bob can be $U(q_c, q_m) = \min(q_c, q_m)$. The dashed line marks the kinks of the contours of the utility function.

b)

b.1

We have that $MRS_{Y,X} = -\frac{MU_X}{MU_Y}$.

$$MU_X = \frac{\delta U}{\delta x} = (y+z)$$
$$MU_Y = \frac{\delta U}{\delta y} = x$$
$$\implies MRS_{Y,X} = -\frac{y+z}{x}$$

b.2

$|MRS_{Y,X}|$ would fall from $\frac{y+100}{x}$ to $\frac{y+80}{x}$, a fall in magnitude of $\frac{20}{x}$. This means that consumers would be less willing to substitute air travel for other goods (specifically they would be willing to give up $\frac{20}{x}$ less for an extra mile traveled by air).

b.3

This makes sense, because after 9/11 airline safety would go down, so that consumers are less willing to sacrifice other spending for air travel, decreasing the amount of people traveling by air. In the context of 9/11, this would suggest not particularly that people's tastes have changed, but their information has changed (9/11 makes consumers think that airline travel is unsafe), leading them to stay away from air travel.

b.4

We now have that $MRS_{Y,X} = \frac{yz}{xz} = \frac{y}{x}$. This means that $MRS_{Y,Z}$ is independent of Z, which doesn't make that much sense, as you would expect a change in airline safety to either increase or decrease travel.

In another sense this is strange also because $MU_Y = xz$. Why would the marginal utility of other spending depend specifically on the safety of air travel?

Problem 3

a)

Carla generally likes more even bundles (though skewed torwards clothes), as they appear to be imperfect substitutes to each other as shown by a Cobbs-Douglas utility function. However, since the exponent of clothing is higher in her utility function, we can conclude that she enjoys clothes more than food per each respective unit.

b)

$$MU_X = \frac{\delta U}{\delta x} = \frac{1}{3}x^{-2/3}y^{2/3} = \frac{1}{3}(\frac{y}{x})^{2/3}.$$

This diminishes monotonically as x increases, meaning that her marginal utility for food decreases as she consumes more food.

c)

$$5x + 25y = 900$$

d)

We can formally describe Carla's utility maximization problem as the following: find the maximum value of $U(x, y) = x^{1/3}y^{2/3}$ with $x, y \geq 0$ such that $5x + 25y = 900$.

e)

e.1

$$\mathcal{L}(x, y, \lambda) = x^{1/3}y^{2/3} - \lambda(5x + 25y - 900)$$

e.2

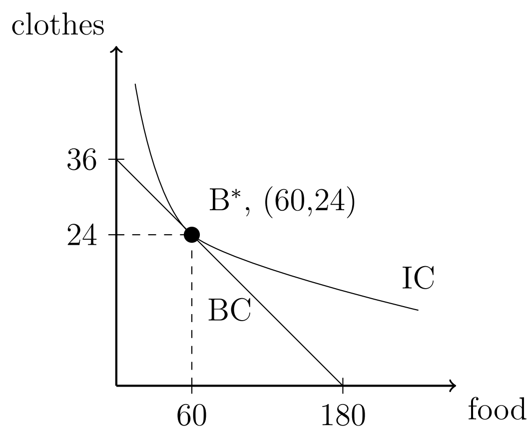
$$\begin{aligned}
\frac{\delta\mathcal{L}}{\delta x} &= \frac{1}{3}(\frac{y}{x})^{2/3} - 5\lambda \\
\frac{\delta\mathcal{L}}{\delta y} &= \frac{2}{3}(\frac{y}{x})^{-1/3} - 25\lambda \\
\frac{\delta\mathcal{L}}{\delta \lambda} &= 5x + 25y - 900
\end{aligned}$$

e.3

Setting them all equal to zero, we have the following:

$$
\begin{aligned}
\frac{1}{3}(\frac{y}{x})^{2/3} &= 5\lambda \\
\frac{2}{3}(\frac{y}{x})^{-1/3} &= 25\lambda \\
\implies \quad \frac{1}{2}(\frac{y}{x}) &= \frac{1}{5} \\
\implies \quad y &= \frac{2}{5}x \\
\implies 5x + 10x &= 900 \\
\implies \quad x &= 60 \\
\implies \quad y &= \frac{2}{5}(60) = 24 \\
\implies \quad \lambda &= \frac{1}{15}(\frac{24}{60})^{2/3} = 0.036
\end{aligned}
$$

f)



The curve labeled IC is the indifference curve tangent to the budget constraint BC. They meet at the optimal bundle B*, which contains 60 food and 24 clothes.

g)

If she purchases her optimal bundle, then she spends $60(5) = 300$, or $1/3$ of her total budget on food. Similarly, she will spend $24(25) = 600$, or $2/3$ of her total budget on clothing.

h)

We notice quickly that the share of her budget spent on a good is exactly the exponent assigned to the good in the utility function.

i)

Instead of 900, we can solve the system for an arbitrary budget M. In that case, we arrive at $5x+10x=M$ from part **e.3** instead of $5x+10x=900$ (as this is the first step where M does not vanish in the manipulated equations after taking the partial derivative), meaning that $x=\frac{M}{15}$ and that $y=\frac{2M}{75}$ for the optimal bundle. This yields shares of $\frac{M}{15}(5)=\frac{1}{3}M$ spent on food and $\frac{2M}{75}(25)=\frac{2}{3}M$ spent on clothing. This implies that the shares spent on the goods are fixed independently of budget.

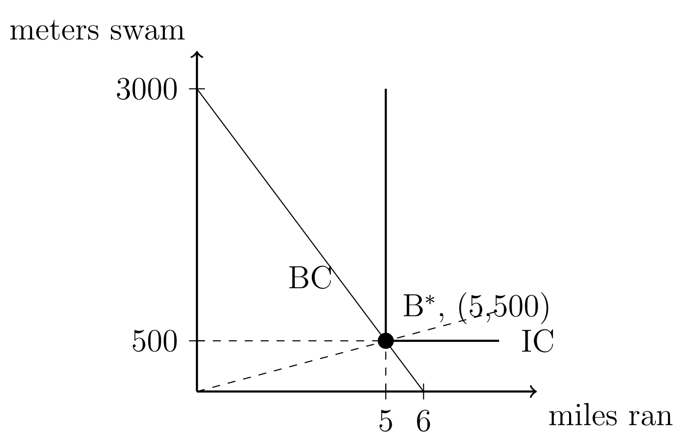
Problem 4

a)

She needs to complete exercise such that for every 10 minutes of running (or mile ran) she swims 2 minutes. This means that if x represents the miles run, then $10x+2x=60$, so she ought to run 5 miles and 500m (50 minutes running, 10 minutes swimming).

As an aside, Vera is very very fit. That's a lot of cardio!

b)



The line marked BC is the budget constraint. The indifference curve, labeled IC, is shaped as an L as the two are perfect compliments. The optimal bundle is marked B*, at 5 miles ran, 500m swam. The slanted dotted line marks the ratio at which Vera wants to split running and swimming.