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Problem Set 3

Problem 1

a

We have that the final overall steady state criterion is that

$$\begin{aligned}
K^* &= (1-\delta)K^* + \sigma I^* \\
\implies \quad k^* &= \frac{1-\delta}{(1+g)(1+n)}k^* + \frac{\sigma\sqrt{k^*}}{(1+g)(1+n)} \\
\implies \sigma\sqrt{k^*} &= ((1+g)(1+n) - (1-\delta))k^* \\
\implies \quad k^* &= \left(\frac{\sigma}{(1+n)(1+g) - (1-\delta)}\right)^2
\end{aligned}$$

b

From above, we have that

$$\begin{aligned}
k^* &= \left(\frac{\sigma}{(1+n)(1+g) - (1-\delta)}\right)^2 \\
&= \left(\frac{0.25}{(1+0.015)(1+0.02) - (1-0.10)}\right)^2 \\
&= 3.414
\end{aligned}$$

Since we have that in steady state $y^* = \sqrt{k^*}$,

$$y^* = \sqrt{3.414} = 1.847$$

c

$$\begin{aligned}
\frac{Y_{t+1}}{L_{t+1}E_{t+1}} &= \frac{Y_t}{L_tE_t} \\
\implies \quad Y_{t+1} &= (1+g)(1+n)Y_t \\
&= 1.0353Y_t
\end{aligned}$$

Thus, it takes $\frac{\log(2)}{\log(1.0353)} = 20$ years for overall output to double.

d

$$\begin{aligned} \frac{Y_{t+1}}{L_{t+1}E_{t+1}} &= \frac{Y_t}{L_tE_t} \\ \implies \qquad \frac{Y_{t+1}}{L_{t+1}} &= (1+n)\frac{Y_t}{L_t} \\ \implies \qquad \frac{Y_{t+1}}{L_{t+1}} &= 1.015\frac{Y_t}{L_t} \end{aligned}$$

Thus, it takes $\frac{\log(2)}{\log(1.015)} = 46.5$ years for output per capita to double.

e

$$\begin{aligned} \frac{K_{t+1}}{L_{t+1}E_{t+1}} &= \frac{K_t}{L_tE_T} \\ \implies \qquad K_{t+1} &= (1+g)(1+n)K_t \end{aligned}$$

From above, it takes also 20 years for the capital stock to double.

Problem 2

a

We have from SGU that as growth in GDP per capita is computed to be near constant over 1870 to 2016, and is equal to 1.97%. Thus, GDP per capital grew about 18 fold over the mentioned 148 years.

b

No, there is not; in fact, the plot has the lowest values for GDP per capital are about 500 in 1990 $, and the highest at about 3000. This is then a maximum of a 6-fold increase in any time period shown, so no country sees growth like the US from 1870 to 2018.

c

Keynes is mostly correct: there are varations, but all countries on the plotted timespan have maximum value about 200% of their minimum value, and so there is no such explosive, continual growth.

d

Long flat growth followed by nonzero growth looks like a hockey stick on its side when output is plotted logarithmically.