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Problem Set 7

Problem 1

a

The budget constraint in period one and two satisfy that

$$P_1C_1 + S = P_1Y_1$$
$$P_2C_2 = P_2Y_2 + (1+i)S$$

The intertemporal budget constraint is the same as the model with sticky prices with $P_1 = P_2 = 1$:

$$C_1 + \frac{C_2}{1+i} = Y_1 + \frac{Y_2}{1+i} = y$$

b

$$\mathcal{L}(C_1, C_2, \lambda) = \log(C_1) + \beta\log(C_2) - \lambda(C_1 + \frac{C_2}{1+i} - y)$$
$$\frac{\partial\mathcal{L}}{\partial C_1} = \frac{1}{C_1} - \lambda = 0$$
$$\frac{\partial\mathcal{L}}{\partial C_2} = \frac{\beta}{C_2} - \frac{\lambda}{1+i} = 0$$
$$\frac{\partial\mathcal{L}}{\partial \lambda} = C_1 + \frac{C_2}{1+i} - y = 0$$

Taking that $\lambda = C_1$,

$$\frac{C_2}{C_1} = \beta(i+i)$$

Then, we have that if the central bank wishes to ensure $C_1 = Y_1, C_2 = Y_2$, then $\beta(1+i) = \implies i = 0.1$. In equilibrium, we have that $C_1 = Y_1 = 10$, and that savings is 0.

c

$$E_1(\overline{Y_2}) = \frac{1}{2}(6) + \frac{1}{2}(12) = 9$$

d

We are subject to the following constraints, assuming that nominal prices are fixed at 1:

$$C_1 + S = Y_1 \implies C_1 = Y_1 - S$$
$$C_2^g = Y_2^g + (1+i)S$$
$$C_2^b = Y_2^b + (1+i)S$$

We now have expected utility

$$V(S) = \log(Y_1 - S) + \frac{\beta}{2}\log(Y_2^g + (1+i)S) + \frac{\beta}{2}\log(Y_2^b + (1+i)S)$$

Maximizing with respect to savings,

$$\frac{\partial V}{\partial S} = 0 = \frac{1}{Y_1 - S} + \frac{\beta(1+i)}{2(Y_2^g + (1+i)S)} + \frac{\beta(1+i)}{2(Y_2^b + (1+i)S)}$$
$$\frac{1}{Y_1 - S} = \frac{\beta(1+i)}{2(Y_2^g + (1+i)S)} + \frac{\beta(1+i)}{2(Y_2^b + (1+i)S)}$$

If the central bank does not adjust monetary policy (i.e. keeps rates at $i = 0.1$), then we have that

$$\frac{1}{10 - S} = \frac{1}{2(6 + 1.1S)} + \frac{1}{2(12 + 1.1S)}$$

This results in $S = 0.905$, and so $C_1 = 9.095$.

e

In this class, we have worked with the assumption that the market clears, such that $S = 0$. Then, we have that

$$\frac{1}{1+i} = \frac{\beta}{Y_1}\left(\frac{1}{2Y_2^g} + \frac{1}{2Y_2^b}\right) = 0.011$$

Thus, we have that we are in a liquidity trap, and the optimal policy is $i = 0$.

At the zero lower bound, we have that

$$\frac{1}{10 - S} = \frac{\beta}{2(6 + 1.1S)} + \frac{\beta}{2(12 + 1.1S)}$$

such that $S = 0.514$, and thus $C_1 = 9.486$.

Problem 2

a

This is identical to before; $i^* = 0.1$.

b

We still have that, since the market clears and is in full employment in the long run, $C_1 = Y_1, C_2 = \overline{Y}$

$$\frac{C_2}{C_1} = \beta(1+i) \implies Y_1 = \frac{\overline{Y}}{\beta(1+i)}$$

Thus, we have that $Y_1 = 8.18$, and that the output gap is

$$\left(\frac{\overline{Y}}{Y_1} - 1\right) 100 = 22.2$$

c

If the bank acts quickly, then we have that $\beta(1+i) = 1 \implies i < 0$, such that we are in a liquidity trap. At $i = 0$, we have that $Y_1 = 0.9\overline{Y}$, and so the output gap is 11.1.

d

We wish to spend G^* such that at $i = 0$, $Y_1 = \overline{Y}$. In particular, since we still have that

$$\frac{C_2}{C_1} = \beta(1+i)$$

we now arrive at

$$Y_1 = \frac{\overline{Y}}{\beta(1+i)} + G^*$$

Thus, if we want that $Y_1 = \overline{Y}$, it must be that $G^* = 0.1\overline{Y} = 1$.

e

If instead $\tilde{G} = 0.11\overline{Y}$ is spent, then the central bank will raise interest rates to compensate, such that

$$1 = \frac{1}{\beta(1+i)} + 0.11 \implies i = 0.123$$

Private consumption is now $\overline{Y} - G = 0.89\overline{Y} = 8.9$.

We see that government fiscal spending has crowded out private consumption due to its miscalculation, such that the government spending multiplier is now < 1.