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Problem Set 2

Part a

If we have any positive income to the bottom 90%, then $Y_t^{90} > 0$; and since we know that $Y_t > 0$, we have that $\alpha_t > 0$.

Since we have that $i < j \implies y_t^i \leq y_t^j$, we have that $\sum_{i=k}^{k+9} y_t^i \leq \sum_{i=91}^{100} y_t^i$ for any $k < 91$. Taking $k = 1, 11, ..., 81$ and summing, we have that $\sum_{i=1}^{90} y_t^i \leq 9 \sum_{i=91}^{100}$.

$$\begin{aligned}
\alpha_t &= \frac{Y_t^{90}}{Y_t} \\
&= \frac{Y_t^{90}}{Y_t^{90} + Y_t^{10}} \\
&= 1 - \frac{Y_t^{10}}{Y_t^{90} + Y_t^{10}} \\
&= 1 - \frac{\sum_{i=91}^{100} y_t^i}{\sum_{i=1}^{90} y_t^i + \sum_{i=91}^{100} y_t^i} \\
&\leq 1 - \frac{\sum_{i=91}^{100} y_t^i}{10 \sum_{i=91}^{100} y_t^i} \\
&= 1 - \frac{1}{10} = 0.9
\end{aligned}$$

Part b

If there is no income inequailty, for any i, j we have that $y_t^i = y_t^j$.

$$\begin{aligned}
\alpha_t &= \frac{Y_t^{90}}{Y_t} \\
&= \frac{\sum_{i=91}^{100} y_t^i}{\sum_{i=1}^{100} y_t^i} \\
&= \frac{90 y_t^1}{100 y_t^1} \\
&= 0.9
\end{aligned}$$

Part c

$$\frac{Y_{2018}^{90}}{Y_{1980}^{90}} = \frac{3Y_{1947}^{90}}{2Y_{1947}^{90}} = \frac{3}{2} = 1.5$$

This implies that the total amount of income accrued to the bottom 90% increased by a magnitude of 50% over the 38 years inbetween 1980 and 2018.

Part d

$$\frac{Y_{2018}}{Y_{1980}} = \frac{4Y_{1947}}{2Y_{1947}} = 2$$

This implies that the total amount of income to the entire population doubled over the 38 years inbetween 1980 and 2018; since this is significantly higher than the previous $Y^{90}_{2018}/Y^{90}_{1980}$, it implies that income grew much faster for the top decile than the rest.

Part e

$$\begin{aligned}
Y^{10}_{1980} &= \alpha_{1980} Y_{1980} \\
\frac{Y^{10}_{2018}}{Y^{10}_{1980}} &= \frac{Y_{2018} - Y^{10}_{2018}}{Y_{1980} - Y^{90}_{1980}} \\
&= \frac{2Y_{1980} - \frac{3}{2}Y^{90}_{1980}}{Y_{1980} - Y^{90}_{1980}} \\
&= \frac{2Y_{1980} - \frac{3\alpha_{1980}}{2}Y_{1980}}{Y_{1980} - \alpha_{1980}Y_{1980}} \\
&= \frac{4 - 3\alpha_{1980}}{2 - 2\alpha_{1980}}
\end{aligned}$$

Part f

$$\frac{Y^{10}_{2018}}{Y^{10}_{1980}} = \frac{4 - 3\alpha_{1980}}{2 - 2\alpha_{1980}} = \frac{4-2}{2 - \frac{4}{3}} = 3$$

This means that the amount of income accrued to the top decile in 2018 is triple of what it was in 1980.

Part g

$$\alpha_{2018} = \frac{Y^{90}_{2018}}{Y_{2018}} = \frac{\frac{3}{2}Y^{90}_{1980}}{2Y_{1980}} = \frac{3}{4}\alpha_{1980}$$

Part h

We have that

$$Y^{90}_t = \alpha_t Y_t = \alpha_t (Y^{90}_t + Y^{10}_t) \implies \frac{Y^{10}_t}{Y^{90}_t} = \frac{1 - \alpha_t}{\alpha_t}$$

$$\frac{x_t^{10}}{x_t^{90}} = \frac{9Y_t^{10}}{Y_t^{90}}$$
$$= \frac{9-9\alpha_t}{\alpha_t}$$

For the respective years, we have that

$$\frac{x_{1980}^{10}}{x_{1980}^{90}} = \frac{3}{\frac{2}{3}} = \frac{9}{2},$$

$$\frac{x_t^{10}}{x_t^{90}} = \frac{\frac{9}{2}}{\frac{1}{2}} = 9,$$

We have that inequailty explodes from $1980 - 2018$, as we see that the average real income per capita doubles over this period.