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Problem Set 6

ECON 3213
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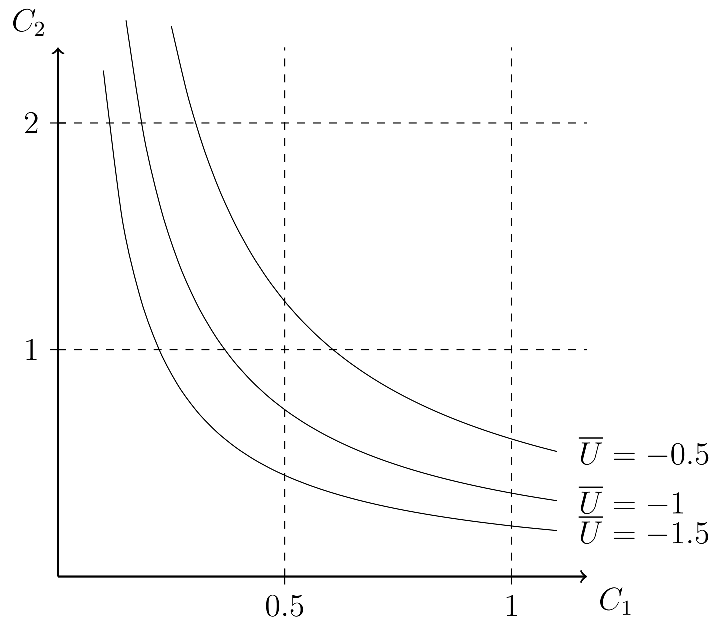
Problem 1

a



b

$$
\begin{aligned}
U &= \ln(C_1) + \ln(C_2) \\
0 &= \frac{1}{C_1} + \frac{dC_2}{dC_1}\frac{1}{C_2} \\
\frac{dC_2}{dC_1} &= -\frac{C_2}{C_1}
\end{aligned}
$$

Computing the marginal rate of substitution,

$$
\begin{aligned}
\frac{MU_1}{MU_2} &= \frac{1/C_1}{1/C_2} \\
&= \frac{C_2}{C_1} \\
&= -\frac{dC_2}{dC_1}
\end{aligned}
$$

c

$$\frac{d^2C_2}{dC_1^2} = -\frac{C_1\frac{dC_2}{dC_1} - C_2}{C_1^2} = -\frac{-C_2 - C_2}{C_1^2} = \frac{2C_2}{C_1^2}$$

Since we have that $C_2 > 0$, as we have a positive amount of consumption in both periods to have real utility, and $C_1^2 > 0$ as well, $\frac{d^2C_2}{dC_1^2} > 0$.

d

Note that we have $C_2 = \frac{e^U}{C_1}$. Then, $\frac{dC_2}{dC_1} = -\frac{e^U}{C_1^2} = -e^U$ when $C_1 = 1$.
For $U = -1.5$, $\frac{dC_2}{dC_1} = -e^{-1.5}$.
For $U = -1.0$, $\frac{dC_2}{dC_1} = -e^{-1.0}$.
For $U = -0.5$, $\frac{dC_2}{dC_1} = -e^{-0.5}$.
The reason that we see as utility diminishes the slope becomes flatter is that a flatter slope represents a lower marginal rate of substitution, which in this case is a lower willingness to trade C_2 for C_1. Since at lower utilities with C_1 held constant we must have C_2 lower (since utility must be increasing), and since preferences are convex, we would expect to see at lower utilities a diminished willingness to trade C_2 for C_1, and thus a flatter slope.

Problem 2

a

Period 1:

$$C_1 + S_1 = Y_1$$

where $S_1 = \sigma Y_1$ is the savings (or borrowing if $\sigma < 0$).
Period 2:

$$C_2 = Y_2 + S_1(1 + r)$$

b

$$C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}$$

c

$$\max_{C_1,C_2}\{\sqrt{C_1}+\sqrt{C_2}\} \text{ such that } C_1+\frac{C_2}{1+r}=Y_1+\frac{Y_2}{1+r}$$

d

Put $y = Y_1 + \frac{Y_2}{1+r}$.

$$\begin{aligned}
\mathcal{L}(C_1,C_2,\lambda) &= \sqrt{C_1}+\sqrt{C_2}-\lambda(C_1+\frac{C_2}{1+r}-y)\\
\frac{\partial\mathcal{L}}{\partial C_1} &= \frac{1}{2\sqrt{C_1}}-\lambda=0\\
\frac{\partial\mathcal{L}}{\partial C_2} &= \frac{1}{2\sqrt{C_2}}-\frac{\lambda}{1+r}=0\\
\frac{\partial\mathcal{L}}{\partial \lambda} &= C_1+\frac{C_2}{1+r}-y=0\\
\lambda &= \frac{1}{2\sqrt{C_1}}=\frac{1+r}{2\sqrt{C_2}}\\
C_1 &= \frac{C_2}{(1+r)^2}\\
y &= C_1+(1+r)C_1\\
C_1 &= \frac{y}{2+r}=\frac{Y_1}{2+r}+\frac{Y_2}{(1+r)(2+r)}=\frac{1}{2+r}(Y_1+\frac{Y_2}{1+r})\\
C_2 &= \frac{y(1+r)^2}{2+r}=(1+r)^2(\frac{Y_1}{2+r}+\frac{Y_2}{(1+r)(2+r)})\\
&= \frac{Y_1(1+r)^2}{2+r}+\frac{Y_2(1+r)}{2+r}=\frac{1}{2+r}(Y_1(1+r)^2+Y_2(1+r))\\
S_1 &= Y_1-C_1\\
&= \frac{1}{2+r}((1-r)Y_1-\frac{Y_2}{1+r})
\end{aligned}$$

e

For $r=0, Y_1=Y_2=10$, one can note that this makes the problem symmetric in C_1, C_2. Then, $C_1+C_2=Y_1+Y_2 \implies C_1=C_2=\frac{1}{2}(Y_1+Y_2)=10, S_1=0$.
Anyway, substituting from above, we have that

$$C_1=\frac{1}{2}(10+10)=10, C_2=\frac{1}{2}(10+10)=10, S_1=10-10=0$$

For $r = 0.1$,

$$C_1 = \frac{1}{2.1}(10 + \frac{10}{2.1}) = 9.09, C_2 = 9.1(1.1)^2 = 11.01, S_1 = 10 - 9.09 = 0.91$$