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Problem Set 9

ECON 3211
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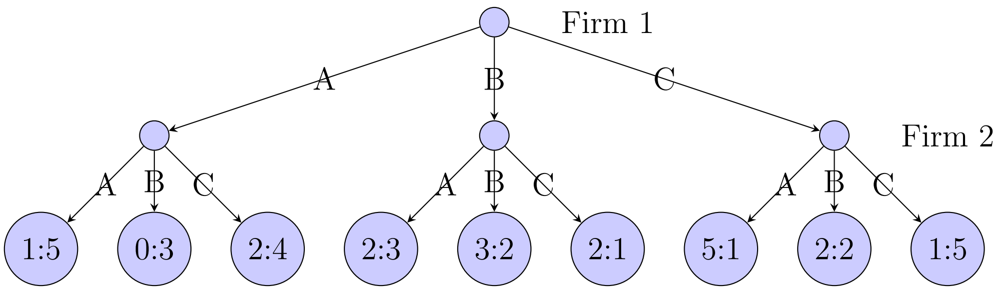
Problem 1

a)

The payoffs are written on the bottom in the format Firm 1:Firm 2 in millions, where Firm 1 is on the side of the matrix and Firm 2 on the top (as in the notes).



b)

$$BR_1(s_2) = \begin{cases} C & s_2 = A \\ B & s_2 = B \\ A \text{ or } B & s_2 = C \end{cases}$$

c)

Firm 1 does not have a dominant or weakly dominant strategy; however, Version A is weakly dominated by Version B.

d)

$$BR_2(s_1) = \begin{cases} A & s_1 = A \\ A & s_1 = B \\ C & s_1 = C \end{cases}$$

e)

There are no pure strategy Nash equilibria.

Put probability θ_X for the probability that Firm 1 chooses Version X, and ρ_X the probability that Firm 2 chooses Version X.

Now, consider that we see that Version B is strongly dominated by a mixture of Version A and Version C (for example, $\theta_A = 0.5, \theta_C = 0.5$) for Firm 2. Eliminating this, we will check the resulting 2×3 game for mixed Nash equilibria. Seeing that Version A is weakly dominated by Version B for Firm 1, we easily check that no Nash equilibrium occurs when mixing over $\{A, B, C\}$ for Firm 2. We arrive at the game

	A	C
B	2,3	2,1
C	5,1	1,5

Computing the Nash equilibrium, we see that it ought to be $3\theta_B + 1\theta_C = \theta_B + 5\theta_C \implies \theta_B = \frac{2}{3}, \theta_C = \frac{1}{3}$ and $2\rho_A + 2\rho_B = 5\rho_A + \rho_B \implies \rho_A = \frac{1}{4}, \rho_C = \frac{3}{4}$.
Therefore, the Nash equilibrium is then for Firm 1 $(0, \frac{2}{3}, \frac{1}{3})$ and Firm 2 $(\frac{1}{4}, 0, \frac{3}{4})$.

Problem 2

a),b)

The pure cooperation game is an example of what we want. Take payoffs $P_i : \{A, B\} \times \{A, B\} \to \mathbb{R}$:

$$P_1(s_1, s_2) = \begin{cases} 1 & s_1 = s_2 \\ 0 & s_1 \neq s_2 \end{cases}$$

$$P_2(s_1, s_2) = \begin{cases} 1 & s_1 = s_2 \\ 0 & s_1 \neq s_2 \end{cases}$$

	A	B
A	1,1	0,0
B	0,0	1,1

There are two pure Nash equilibria: (A, A) and (B, B).

c)

Let θ, ρ be the probabilities of player 1 (row player) and player 2 (column) player choosing strategy A, respectively. The mixed strategy Nash equilibrium then satisfies $\theta = 1 - \theta, \rho = 1 - \rho \implies \theta = \frac{1}{2}, \rho = \frac{1}{2}$.

Problem 3

a)

The profit maximizing quantity satisfies that $MR = 70-2Q = 10 = MC \implies Q = 30*, P = 40*$.

b)

Consumer surplus in this market can be computed to be $\frac{1}{2}(70-40)(30) = 450$. The monopolist's surplus is then $(40-10)(30) = 900$.

c)

The game in normal form would have the set of players $S = \{1, 2\}$, picking actions $q_1, q_2 \in \mathbb{R}_{\geq 0}$ representing their quantity produced. We define payoff functions then as follows: $P_i : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ is equal to $\pi_1(q_1, q_2) = (70-10(q_1+q_2))q_1 - 10q_1$, $\pi_2(q_1, q_2) = (70-10(q_1+q_2))q_2 - 10q_2$.

d),f)

We have that

$$
\begin{aligned}
BR_2(\bar{q_2}) &= \max_{q_1}(70 - (q_1 + \bar{q_2})q_1 - 10q_1 - 450) \\
\frac{\partial BR_1}{\partial q_1} &= 70 - 2q_1 - \bar{q_2} - 10 = 0 \\
q_1^* &= \frac{60 - \bar{q_2}}{2} \\
BR_2(\bar{q_1}) &= \max_{q_2}(70 - (q_2 + \bar{q_1})q_2 - 10q_2 - 450) \\
\frac{\partial BR_2}{\partial q_2} &= 70 - 2q_2 - \bar{q_1} - 10 = 0 \\
q_2^* &= \frac{60 - \bar{q_1}}{2} \\
&= \frac{60 - \frac{60-\bar{q_2}}{2}}{2} \\
&= \frac{60 + q_2^*}{4} \\
&= \frac{q_2^*}{4} + 15
\end{aligned}
$$

$$q_2^* = \frac{4}{3}15 = 20$$
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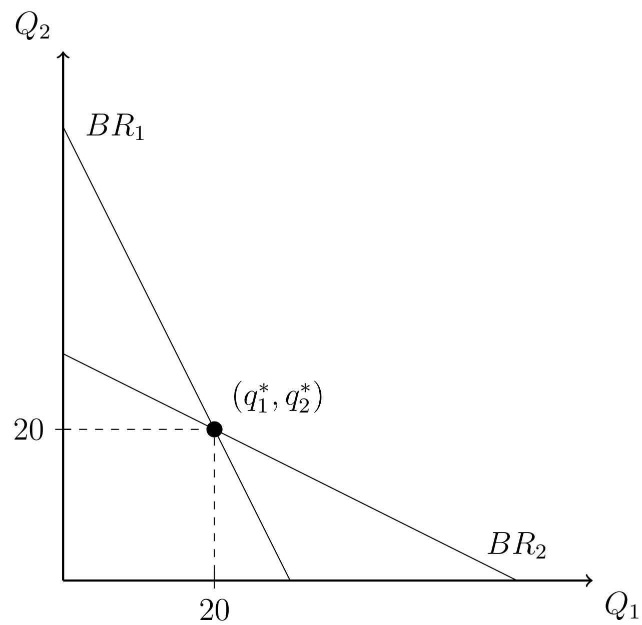
e),f)



g)

The total quantity is 40, at a price of 30. This is more output for a lower price than the sole monopolist, and leads to a closer to socially optimal outcome (although not actually optimal).

h)

There is now a consumer surplus of $\frac{1}{2}(70 - 30)(40) = 800$ and a producer surplus of $(30 - 10)(20) = 400$ each, for a total producer surplus of 800 and a total surplus of 1600. Producer surplus fell, consumer surplus rose, the total surplus rose ($1350 \to 1600$).

i)

The firms make in profit $\pi = 30(20) - 10(20) - 450 = -50$ each, so they are losing money; previously, a monopoly would make $\pi = 40(30) - 10(30) - 450 = 450$. A regulator should not push for a duopoly unless absolutely convinced for external reasons that they will stay in business while taking a loss economically (or they don't care that no output will be produced, such as if this market is untenable socially in the monopoly case).