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Problem Set 8

Problem 1

a)

Country 1:

$$\begin{aligned}
Q_D &= 20 - P \\
\epsilon_D &= \frac{dQ}{dP} \cdot \frac{P}{Q} \\
&= -1(\frac{P}{20-P}) \\
10 &= P^*(1 - \frac{20-P^*}{P^*}) \\
&= P^* - 20 + P^* \\
&= 2P^* - 20 \\
P^* &= 15
\end{aligned}$$

Country 2:

$$\begin{aligned}
Q_D &= (\frac{A}{P})^2 \\
\epsilon_D &= (-\frac{2A^2}{P^3})(\frac{P^3}{A^2}) \\
&= -2 \\
10 &= P^*(1 - \frac{1}{2}) \\
P^* &= 20
\end{aligned}$$

Country 3:

$$
\begin{aligned}
Q_D &= (\frac{A}{P})^3 \\
\epsilon_D &= (-\frac{3A^3}{P^4})(\frac{P^4}{A^3}) \\
&= -3 \\
10 &= P^*(1 - \frac{1}{3}) \\
P^* &= 15
\end{aligned}
$$

We can compute the new price that the firm has by taking $MR = 11$.

b)

Country 1:

$$
\begin{aligned}
Q_D &= 20 - P \\
\epsilon_D &= \frac{dQ}{dP} \cdot \frac{P}{Q} \\
&= -1(\frac{P}{20 - P}) \\
11 &= P^*(1 - \frac{20 - P^*}{P^*}) \\
&= P^* - 20 + P^* \\
&= 2P^* - 20 \\
P^* &= 15.5
\end{aligned}
$$

The new price is increased by $\frac{1}{2}$ of the tax, so the incidence is $50 - 50$ on the consumers and the firm.

Country 2:

$$
\begin{aligned}
Q_D &= (\frac{A}{P})^2 \\
\epsilon_D &= (-\frac{2A^2}{P^3})(\frac{P^3}{A^2}) \\
&= -2 \\
11 &= P^*(1 - \frac{1}{2}) \\
P^* &= 22
\end{aligned}
$$

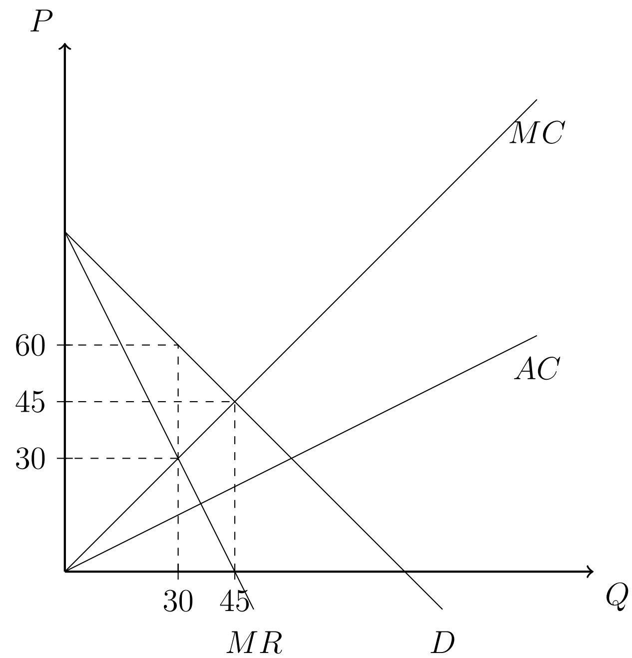
The new price is increased by twice the tax, so the incidence is 200% on consumers.
Country 3:

$$
\begin{aligned}
Q_D &= (\frac{A}{P})^3 \\
\epsilon_D &= (-\frac{3A^3}{P^4})(\frac{P^4}{A^3}) \\
&= -3 \\
11 &= P^*(1 - \frac{1}{3}) \\
P^* &= 16.5
\end{aligned}
$$

The new price is increased by 1.5 times the tax, so the incidence is 150% on consumers.

Problem 2

a), b)



The socially optimal price is 45, the socially optimal quantity is 45. The profit maximizing price is 60, the profit maximizing quantity is 30.

c)

The imposition of a tax on profits does not actually change the equilibrium (the profit maximizing price and quantity) as the firm considers it much like a lump sum tax as it does not change MC or MR. Then, it also does not reduce deadweight loss.

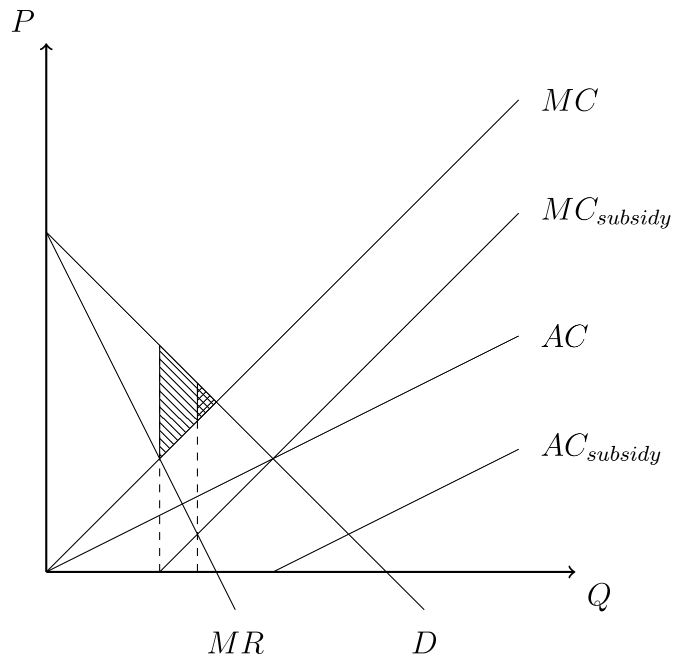
d)

Perfect price discrimination would allow the firm to capture all of consumer surplus, meaning that they would produce at the socially optimal level in order to maximize total surplus.

Thus, the price charged would be 45, selling also 45, for a profit of $\frac{1}{2}(90)(45) = 2025$ (1012.5 if taxed at 50%). The deadweight loss would be 0.

e)

To minimize deadweight loss, the government ought to use a subsidy; specifically, a per-unit subsidy should increase production and lower prices, bringing equilibrium closer to socially optimal levels.



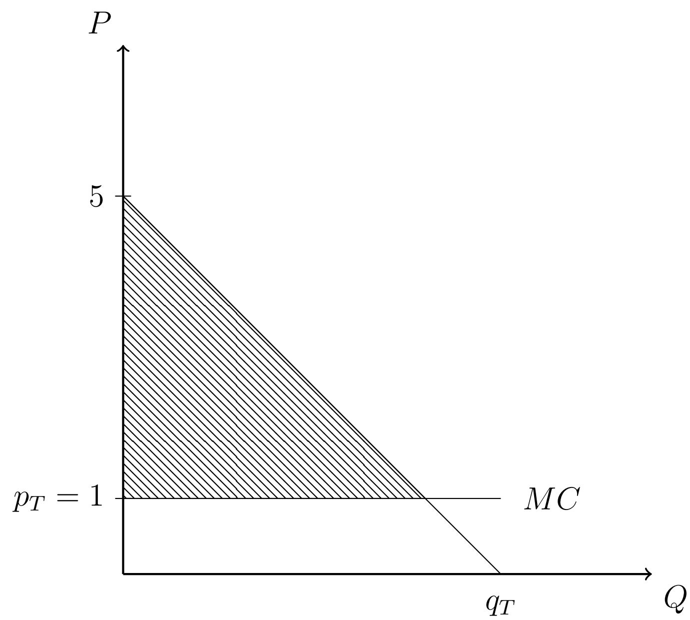
The area in northwest lines is old deadweight loss; the area in northeast lines new deadweight loss after a subsidy of 30.

Problem 3

a)

The profit maximizing price is equal to MC, so $p = 1$. The maximizing fee is equal to consumer surplus, so $F = \frac{1}{2}(4)(4) = 8$. They ought to charge F slightly less than this, so about 7.99.

b)



The fee F is equal to the shaded area.

c)

Similar to above, profit maximizing $p_S = 1$, $F_S = \frac{1}{2}(3)(3) = 4.5$, or slightly less at 4.49.

d)

The total profit is $2\frac{1}{2}(4-p)^2 + (p-1)((4-p)+(5-p)) = (4-p^2) + (p-1)(9-2p)$. The FOC simplifies to $2p = 3 \implies p = 1.5$, with a profit maximizing fee of $\frac{1}{2}(4-1.5)^2 = 3.125$, with a total of $9 - 2(1.5) = 6$ rides.

The senior's consumer surplus is 0. The teen's consumer surplus is $\frac{1}{2}(5-1.5)^2 - 3.125 = 3$.

The deadweight loss is increased as the socially optimal price and quantity is 1 and 7; thus, we have that DWL $= \frac{1}{2}(1)(0.5) = 0.25$.

e)

Problem 4

a)

a.1

We want that $MR(q_L) = MR(q_O + q_L), MR(q_O) = MC(q_O + q_L)$. We have that $P_L = 30 - q_L$, $MR_L = 30 - 2q_L = 10 = MC$, so $q_L = 10$. Similarly, $P_O = 60 - 5q_O, MR_O = 60 - 10q_O = 10 = MC$, so $q_O = 5$.

a.2

$P_L = 30 - 10 = 20, P_O = 60 - 5(5) = 35$.

b)

b.1

Let Q be total demand, Q^O total occasional demand, and Q_L the total lovers demand. Then, $Q^O = 100q_O = 1200 - 20P_O, Q^L = 100q_L = 3000 - 100P_L$.

$$Q = \begin{cases} Q^O & 30 < P \leq 60 \\ Q^O + Q^L & 0 \leq P \leq 30 \end{cases} = \begin{cases} 1200 - 20P & 30 < P \leq 60 \\ 4200 - 120P & 0 \leq P \leq 30 \end{cases}$$

b.2,b.3

We need that $MR = MC$. Since

$$P = \begin{cases} 60 - \frac{Q}{20} & 0 \leq Q \leq 600 \\ 35 - \frac{Q}{120} & Q \geq 600 \end{cases}$$

,
we have that

$$MR = \begin{cases} 60 - \frac{Q}{10} & 0 \leq Q \leq 600 \\ 35 - \frac{Q}{60} & Q \geq 600 \end{cases}$$

We have two possibilities; if they only sell to occasional customers, then $60 - \frac{1}{10}Q = 10 \implies Q = 500, P = 60 - \frac{500}{20} = 35$. If the sell to both, then $35 - \frac{1}{60}Q = 10 \implies Q = 1500, P = 35 - \frac{1500}{120} = 22.5$.

Since $\pi_O = 35(500) - 500(10) - FC = 12500 - FC, \pi_B = 22.5(1500) - 1500(10) - FC = 18750 - FC$. Thus, they sell to both and $P^* = 22.50, Q^* = 1500$.

b.4

In tne price monopoly, $P = 22.5, Q = 1500, \pi = 18750 - FC$. In market segmentation, $P_O = 35, Q_O = 500, P_L = 20, Q_L = 1000, \pi_{seg} = (20)1000 + 35(500) - 10(1500) - FC = 22500 - FC$.

We can see that segmenting the market has the occasional customers facing a higher price, so they lose, but the opera lovers gain because they face a lower price. The monopolist also gains because they increase total profit.

c)

c.1

The profit maximizing behavior of the firm is that they will actively target the high paying audience with tickets and the more frequent, poorer customers the subscription.

The subscription will be a two-part tariff with constant marginal cost; as before there will be 20 such members in the market, so $n = 20$. The amount charged, then will be capturing the totality of surplus, which is computed to be $20(10) + \frac{1}{2}20(20) = 400$. Since they want have opera lovers be better off than just buying nothing, they charge $S = 399$.

Then, as before, we have that the profit maximizing price in the infrequent customer market is $P = 35$; further, this price means that opera lovers are priced out, and therefore is the actual ticket price in the presence of the subscription.

c.2

We see that since occasional customers only gain surplus by buying tickets and opera lovers only gain surplus by buying the subscription, the occasional customers buy tickets and the lovers buy subscriptions.

Problem 5

a)

$\pi(Q_{US}, Q_A) = p_{US}Q_{US} + p_AQ_A - 10(Q_{US} + Q_A) = p_{US}(60 - p_{US}) + p_A(60 - 2p_A) - 10(120 - p_{US} - 2p_A)$

In terms of quantity,

$$\pi(Q_{US}, Q_A) = p_{US}Q_{US}+p_AQ_A-10(Q_{US}+Q_A) = Q_{US}(60-Q_{US})+Q_A(30-\frac{1}{2}Q_A)-10(Q_{US}+Q_A)$$

b), c), d)

Are b) and c) not the same problem?

$$\frac{\partial \pi}{\partial p_{US}} = 60 - 2p_{US} + 10 = 0 \implies p^*_{US} = 35, Q^*_{US} = 25$$

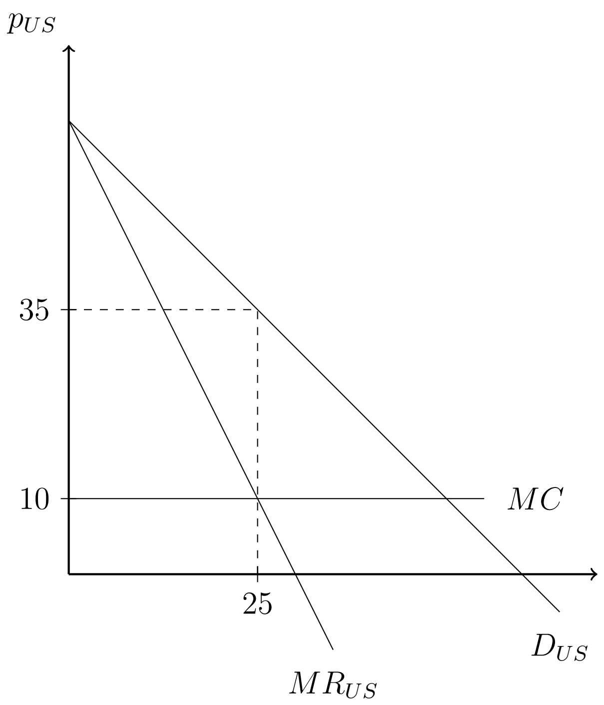
$$\frac{\partial \pi}{\partial p_A} = 60 - 4p_A + 20 = 0 \implies p^*_A = 20, Q^*_A = 20$$

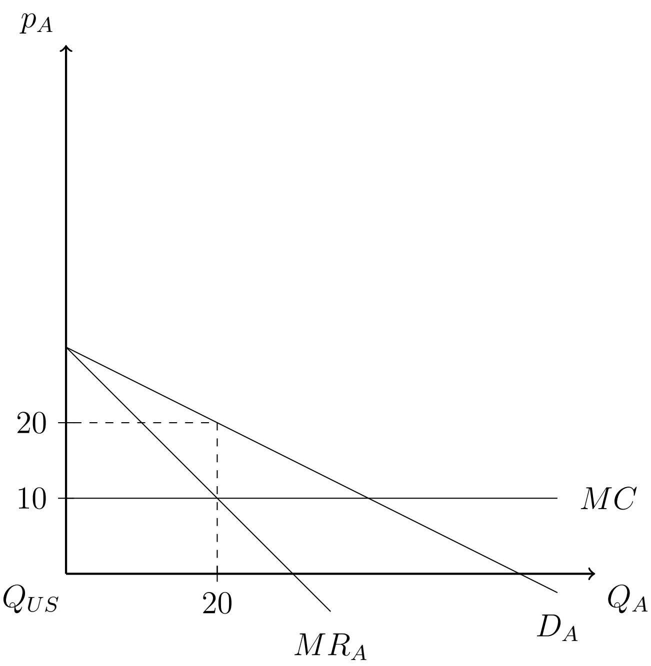
In terms of quantity,

$$\frac{\partial \pi}{\partial p_{US}} = 60 - 2Q_{US} - 10 = 0 \implies Q^*_{US} = 25, p^*_{US} = 35$$

$$\frac{\partial \pi}{\partial p_A} = 30 - Q_A - 10 = 0 \implies Q^*_A = 20, p^*_A = 20$$

e)





The profit maximizing quantities and prices are marked with dashed lines.

f)

If the WTO bans region codes, then the US will benefit and Aus will suffer, as the company is incentivized to hold prices somewhere inbetween p_{US} and p_A.

More specifically, we have that once solved, $Q_T = 120 - 3P \implies Q^* = 45, P^* = 30$ if they sell to both, making $\pi = 900$ compared to $\pi = 625$ in solely the American market. This then hurts the Australians and helps Americans.