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Problem Set 10

ECON 3211
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Problem 1

a)

The game in normal form is as follows:

Players: Xavier, Yvonne

Strategy Sets: Xavier chooses $p_x \geq 0$, Yvonne chooses $p_y \geq 0$.

Playoffs: Xavier has profit $\pi_x(p_x, p_y) = (44-2p_x+p_y)(p_x-8)$, Yvonne has profit $\pi_y(p_x, p_y) = (44-2p_y+p_x)(p_y-8)$

b)

We have that

$$
\begin{aligned}
\frac{\partial \pi_x}{\partial p_x} &= 44 - 4p_x + p_y + 16 = 0 \\
\implies \quad p_x &= 15 - \frac{p_y}{4} \\
\frac{\partial \pi_y}{\partial p_y} &= 44 - 4p_y + p_x + 16 = 0 \\
\implies \quad p_y &= 15 - \frac{p_y}{4}
\end{aligned}
$$

c)

We have that

$$
\begin{aligned}
p_x &= 15 + \frac{1}{4}(15 + \frac{p_x}{4}) \\
&= \frac{75}{4} + \frac{p_x}{16} \\
&= \frac{75}{4}\frac{16}{15} \\
&= 20 \\
p_y &= 15 + \frac{1}{4}(15 + \frac{p_y}{4}) \\
&= \frac{75}{4} + \frac{p_y}{16}
\end{aligned}
$$

$$= \frac{75}{4}\frac{16}{15}$$
$$= 20$$

d)

Thus, we have that $q_x = 44 - 2(20) + 20 = 24, q_y = 44 - 2(20) + 20 = 24$. Total quantity is $Q^S = 24 + 24 = 48$.

e)

We have that $MC = 8 < 20$, so we have that $MC \neq P$.

f)

The total demand is $Q^D = (44 - 2p_x + p_y) + (44 - 2p_y + p_x) = 88 - p_x - p_y$.
Then, we have that the socially optimal amount is $P = MC \implies Q^* = 88 - 8 - 8 = 72$.

g)

Deadweight loss is $\frac{1}{2}(20 - 8)(72 - 48) = 144$.

Problem 2

a)

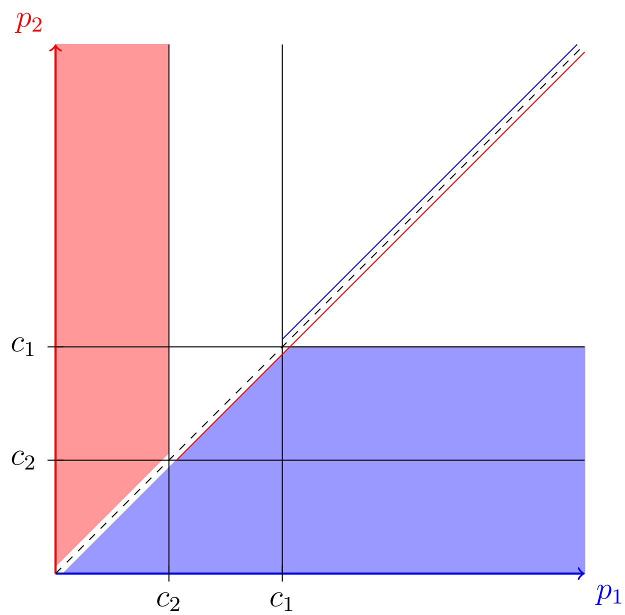
Players: Firm 1, Firm 2
Strategy Sets: Firm 1 chooses $p_1 \geq 0$, Firm 2 chooses $p_2 \geq 0$.
Payoffs:
Firm 1 faces the following, assuming that they face inverse market demand $p = A - bQ$:

$$\pi_1 = \begin{cases} 0 & p_1 > p_2 \\ \frac{1}{2}(p_1 - c)(\frac{A-p_1}{b}) & p_1 = p_2 \\ (p_1 - c)(\frac{A-p_1}{b}) & p_1 < p_2 \end{cases}$$

Firm 2 faces the following:

$$\pi_2 = \begin{cases} 0 & p_2 > p_1 \\ \frac{1}{2}(p_2 - c)(\frac{A-p_2}{b}) & p_2 = p_1 \\ (p_2 - c)(\frac{A-p_2}{b}) & p_2 < p_1 \end{cases}$$

b)



c)

The most likely outcome is that Firm 2 sets price at the profit maximizing price in the range (c_2, c_1). This would then make Firm 1's best response be to simply not produce (i.e. by setting price below p_2), meaning that Firm 2 can make actual profit in this price range, greater than when they split the market if they charge $p_2 \geq c_1$.

Problem 3

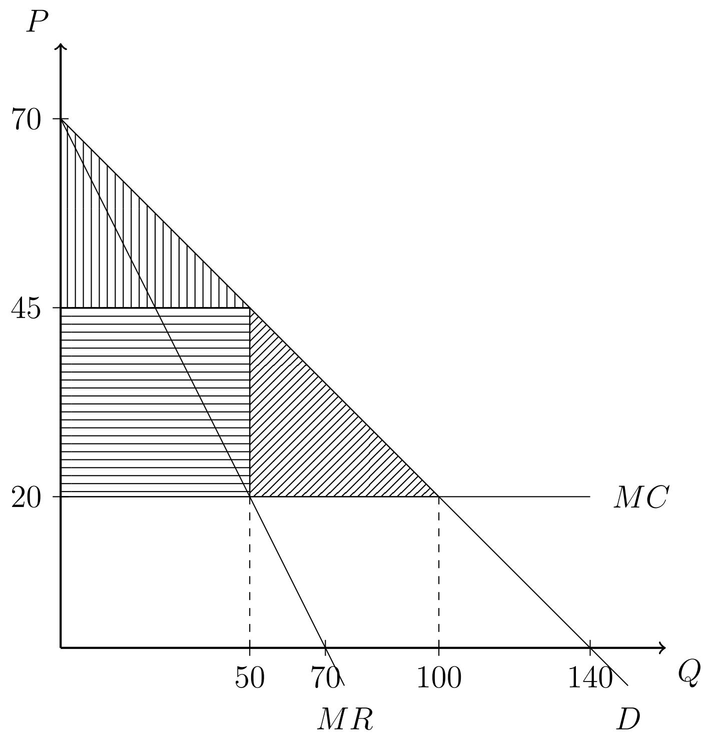
a)

If they cannot price discriminate, then $MR = \frac{d}{dQ}(70 - \frac{1}{2}Q)Q = 70 - Q$.

b)

The profit maximizing output of the one price monopolist is then $70 - Q = 20 \implies Q = 50 \implies 50 = 140 - 2P \implies P = 45$.

c)



Consumer surplus ($= \frac{1}{2}(25)(50) = 625$) is in vertical lines, producer surplus ($= 25(50) = 1250$) in horizontal lines, and deadweight loss ($= \frac{1}{2}(25)(50) = 625$) in northeast lines.

d)

Players: Incumbent, Entrant

Strategy Sets: Incumbent picks $q_I \geq 0$, Entrant picks $q_E \geq 0$.

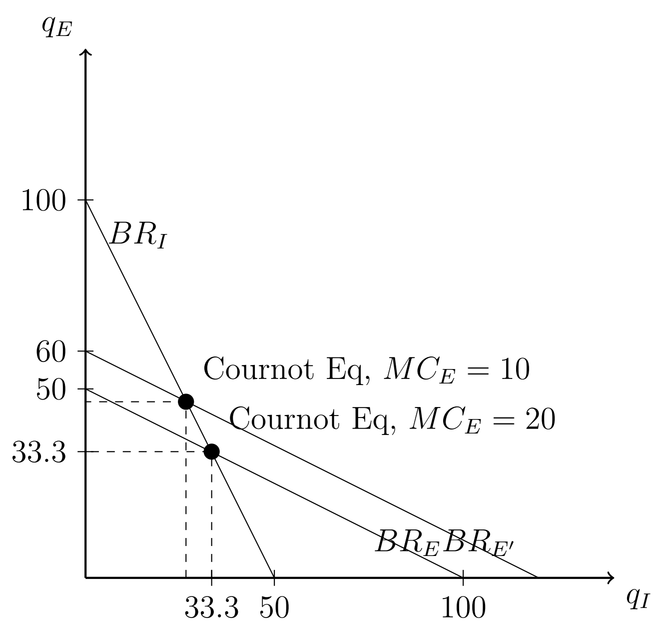
Payoffs: Incumbent has profit $\pi_I(q_I, q_E) = (70 - \frac{1}{2}q_I - \frac{1}{2}q_E)q_I - 200 - 20q_I$, Entrant has profit $\pi_E(q_I, q_E) = (70 - \frac{1}{2}q_I - \frac{1}{2}q_E)q_E - 200 - 20q_E$.

e),f)

The two firms have best response functions given by

$$
\begin{aligned}
\frac{\partial \pi_I}{\partial q_I} &= 70 - q_I - \frac{1}{2}q_E - 20 = 0 \\
\implies \quad q_I &= 50 - \frac{q_E}{2} \\
\frac{\partial \pi_E}{\partial q_E} &= 70 - q_E - \frac{1}{2}q_I - 20 = 0 \\
\implies \quad q_E &= 50 - \frac{q_I}{2} \\
\implies \quad q_I &= 50 - \frac{1}{2}(50 - \frac{1}{2}q_I) \\
&= 25\frac{4}{3} = \frac{100}{3} \\
\implies \quad q_E &= \frac{100}{3}
\end{aligned}
$$

g), i)



h)

The market output is $Q^S = \frac{100}{3} + \frac{100}{3} = \frac{200}{3}$ at a price of $\frac{200}{3} = 140 - 2P \implies P = \frac{110}{3}$.

h) (again)

The Entrant's best response function is now given by

$$\begin{aligned}
\pi_E(q_I, q_E) &= (70 - \frac{1}{2}q_I - \frac{1}{2}q_E)q_E - 200 - 10q_E \\
\frac{\partial \pi_E}{\partial q_E} &= 70 - q_E - \frac{1}{2}q_I - 10 = 0 \\
\implies \qquad q_E &= 60 - \frac{q_I}{2}
\end{aligned}$$

j)

The Cournot equilibrium is now given by

$$\begin{aligned}
\frac{\partial \pi_I}{\partial q_I} &= 70 - q_I - \frac{1}{2}q_E - 20 = 0 \\
\implies \quad q_I &= 50 - \frac{q_E}{2} \\
\frac{\partial \pi_E}{\partial q_E} &= 70 - q_E - \frac{1}{2}q_I - 10 = 0 \\
\implies \quad q_E &= 60 - \frac{q_I}{2} \\
\implies \quad q_I &= 50 - \frac{1}{2}(60 - \frac{1}{2}q_I) \\
&= 20\frac{4}{3} = \frac{80}{3} \\
\implies \quad q_E &= 60 - \frac{1}{2}(50 - \frac{1}{2}q_I) \\
&= 35\frac{4}{3} = \frac{140}{3}
\end{aligned}$$

k)

The Entrant is unable to run the Incumbent out of business, as the Incumbent still makes $\pi_I = (70 - \frac{1}{2}(\frac{220}{3}))\frac{80}{3} - 200 - 20\frac{80}{3} = \frac{1400}{9}$.