Angular Diameter: Worksheet

Getting started:

Go to: https://ccnmtl.github.io/astro-simulations/small-angle-demo/

Warming up:

Before beginning the questions below, take a minute to familiarize yourself with the simulator by experimenting with the different controls and options. You can drag the sliders or make smaller changes by clicking on the bars. You can also input a custom position or diameter directly into the relevant field, or simply drag the ball to any desired position.

Note that the demo calculates the angular diameter (labeled with the Greek letter alpha) in arcseconds, which are equivalent to 1/3600 degrees.

Once you have spent some time experimenting with the simulation, click the ‘reset’ button at the top right of the screen and proceed to the questions below.

Questions:

1. For a beach ball of diameter 1.0 unit, how far away must the ball be to subtend an angle of 1°? We call this angle the *angular diameter* of the ball.

   See below for all work. 57.3 units

2. What happens to the angular diameter if you double the actual diameter of the beach ball?

   Angular diameter doubles.

3. Set the distance to 60. If you halve the distance, what happens to the angular diameter?

   Angular diameter doubles.

4. If the linear diameter and the distance were equal, what would be the angular diameter ***in degrees***? Does this number look familiar?

   57.3 degress.

5. Multiply this number by 2π. Does this number look familiar?

   360 degrees.

6. The angular diameter of the Moon is 0.518 degrees (1865 arcseconds). Using the demo, what can you say, qualitatively, about the distance to the Moon as measured in Moon diameters?

   110 moon diameters.

7. The Moon's radius is 1737.5 km. What is its distance?

   382250 km

8. During a solar eclipse, it is evident that the Moon is just about the right size in the sky to entirely block out the Sun. What can you infer from this about their angular diameters?"

   See below

9. If the Sun's diameter is 1.39 million km, how far away is it?

152 million km

ASTR 1764 Angular Diameter WS

David Chen, Lara Kemper, Scott Rosati, William Meyer

January 20, 2021

Question 1

Abbreviate 206265 to C for brevity. We get

$$\alpha = C \cdot \frac{\text{linear diameter}}{\text{distance}} \implies \text{distance} = \frac{C \cdot \text{linear diameter}}{\alpha}$$

Taking the linear diameter to be 1 unit and $\alpha = 1^\circ = 3600$ arcseconds, we get

$$\text{distance} = C \cdot 1/3600 = 57.3 \text{ units}$$

Question 2

Since we have

$$\alpha = C \cdot \frac{\text{linear diameter}}{\text{distance}}$$

it is clear that $\alpha \propto$ linear diameter, so we get that the angular diameter doubles when the linear diameter doubles.

Question 3

Since we have

$$\alpha = C \cdot \frac{\text{linear diameter}}{\text{distance}}$$

it is clear that $\alpha \propto \frac{1}{\text{distance}}$, so we get that the angular diameter doubles when the distance halves.

Question 4

Since we have

$$\alpha = C \cdot \frac{\text{linear diameter}}{\text{distance}}$$

taking the proportion to be 1, we get $\alpha = C$, and converting to degrees we get $C/3600 = 57.3°$, the same (numerical) result as in Question 1.

Question 5

$$57.3 \cdot 2\pi = 360°$$

the amount of degrees in a full circle.

Question 6

Since we have

$$\alpha = C \cdot \frac{\text{linear diameter}}{\text{distance}} \implies \text{distance} = \frac{C \cdot \text{linear diameter}}{\alpha}$$

taking $\alpha = 1865$, we get that

$$\text{distance} = \frac{C}{1865} \text{ moon diameters} = 110 \text{ moon diameters}$$

Question 7

$$110 \text{ moon diameters } = 110 \text{moon diameters} \cdot \left(2 \cdot 1737.5 \text{ km } \cdot \text{ moon diameter }^{-1}\right) = 382250 \text{ km}$$

Question 8

Playing with the model, we can see that they have identical angular diameters (or very close) since they occupy the same size in the sky (so the angles they subtend to an earthly observer are nearly the same).

Question 9

Since we have from Question 8 that the sun and moon have the same angular diameter,

$$\text{distance}_{\text{sun}} = 110 \cdot (1.39\text{million km}) = 152.9\text{million km}$$

or a distance of $1.53 \cdot 10^8$ km.