Problem 2

a)

Increasing RTS (consider that $\gamma q(L,K) > q(\gamma L, \gamma K)$)

b)

$$\frac{dq}{dL} = MP_L = 0.5L^{-0.5}K, \frac{dq}{dK} = L^{0.5}$$

b.1

$$\min_{[L,K]} 4000L + 8000K \text{ s.t. } q = L^{0.5}K = 1000$$

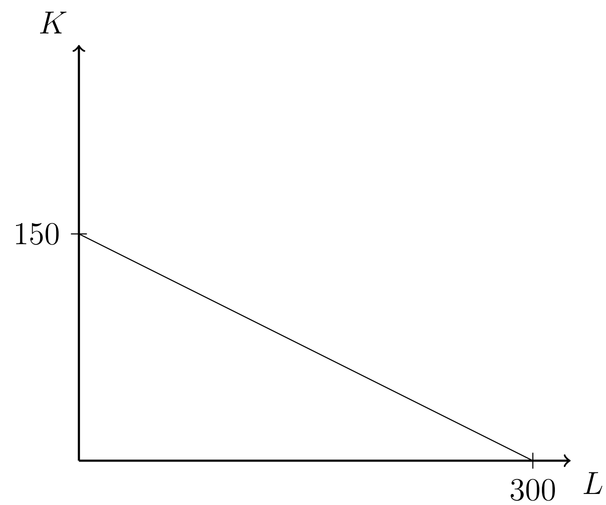
b.2,b.3,b.4

$$\begin{aligned}
\mathcal{L} &= 4000L + 8000K - \lambda[L^{0.5}K - 1000] \\
\frac{\delta\mathcal{L}}{\delta L} &= 4000 - 0.5\lambda L^{-0.5}K = 0 \\
\frac{\delta\mathcal{L}}{\delta K} &= 8000 - \lambda L^{0.5} = 0 \\
\frac{\delta\mathcal{L}}{\delta \lambda} &= L^{0.5}K - 1000 = 0 \\
\implies L^{-1}K &= 1 \\
\implies L &= K \\
\implies K &= 1000^{0.67} = 100 \\
\implies L &= 1000^{0.67} = 100
\end{aligned}$$

b.5

To see 1000 patients, we have that the minimum cost is $4000(100) + 8000(100) = 1,200,000$.

c)



d)

The share on labor is equal to

$$\frac{\text{total spent on labor}}{\text{total costs}} = \frac{wL}{TC} = \frac{4000(100)}{1200000} = \frac{1}{3}$$

Similarly, the share on capital is equal to

$$\frac{\text{total spent on capital}}{\text{total costs}} = \frac{wL}{TC} = \frac{8000(100)}{1200000} = \frac{2}{3}$$

We notice that $q(L,K) = L^{0.5}K$, and that the share spent on labor is $\frac{0.5}{0.5+1} = \frac{1}{3}$, and that the share spent on capital is $\frac{1}{0.5+1} = \frac{2}{3}$.
In general, for $q(L,K) = L^{\alpha}K^{\beta}$, we have that the share spent on labor is $\frac{\alpha}{\alpha+\beta}$ and the share spent on capital is $\frac{\beta}{\alpha+\beta}$.

e),f),g)

These do not exist, for some reason.

h)

We solve for cost minimizing L, K and plug in the cost function.

i)

$$
\begin{aligned}
\frac{MP_L}{MP_K} &= \frac{w}{r} \\
\frac{0.5L^{-0.5}K}{L^{0.5}} &= \frac{w}{r} \\
\implies \frac{K}{2L} &= \frac{w}{r} \\
\implies K &= \frac{2wL}{r} \\
\implies q &= L^{0.5}\frac{2wL}{r} \\
&= L^{1.5}\frac{2wL}{r} \\
\implies L &= (\frac{rq}{2w})^{\frac{2}{3}} \\
\implies q &= (\frac{rq}{2w})^{\frac{1}{3}}K \\
\implies K &= q(\frac{2w}{rq})^{\frac{1}{3}} \\
&= (\frac{2wq^2}{r})^{\frac{1}{3}}
\end{aligned}
$$

j)

The clinic's total cost function is then $C(w,r,q) = w(\frac{rq}{2w})^{\frac{2}{3}} + r(\frac{2wq^2}{r})^{\frac{1}{3}} = (\frac{1}{2}rwq^{\frac{1}{2}})^{\frac{2}{3}} + (2wq^2r^2)^{\frac{1}{3}}$.

k)

We have that

$$
\begin{aligned}
AC &= \frac{C(w,r,q)}{q} \\
&= \frac{1}{q}[(\frac{1}{2}rwq^{\frac{1}{2}})^{\frac{2}{3}} + (2wq^2r^2)^{\frac{1}{3}}] \\
&= q^{-\frac{1}{3}}(\frac{1}{2}rw^{\frac{1}{2}})^{\frac{2}{3}} + q^{-\frac{1}{3}}(2wr^2)^{\frac{1}{3}} \\
&= q^{-\frac{1}{3}}((\frac{1}{2}rw^{\frac{1}{2}})^{\frac{2}{3}} + (2wr^2)^{\frac{1}{3}}) \\
\implies \frac{dAC}{dq} &= -\frac{1}{3}q^{-\frac{4}{3}}((\frac{1}{2}rw^{\frac{1}{2}})^{\frac{2}{3}} + (2wr^2)^{\frac{1}{3}}) < 0
\end{aligned}
$$

Thus, AC is decreasing with increased output, agreeing with our initial finding of economies of scale.