dc3451 | ECON 3213
David Chen | February 11, 2020

Problem Set 3

Problem 1

Part 1

$$
\begin{aligned}
Y^{us}_{2014} &= A(K^{us}_{2014})^{0.3}(L^{us}_{2014})^{0.7} \\
\implies \quad Y^{us}_{2014} &= A(2.5Y^{us}_{2014})^{0.3}(L^{us}_{2014})^{0.7} \\
\implies \quad (Y^{us}_{2014})^{0.7} &= A2.5^{0.3}(L^{us}_{2014})^{0.7} \\
\implies (Y^{us}_{2014}/L^{us}_{2014})^{0.7} &= A2.5^{0.3} \\
\implies \quad A &= \frac{46405.25^{0.7}}{2.5^{0.3}} = 1403.5
\end{aligned}
$$

Part 2

For both the US and China,

$$
\begin{aligned}
Y_t &= AK^{\alpha}Y^{1-\alpha} \\
\implies \frac{Y_t}{AL_t} &= (\frac{K_t}{L_t})^{\alpha} \\
\implies \frac{K_t}{L_t} &= (\frac{Y_t}{AL_t})^{\frac{1}{\alpha}}
\end{aligned}
$$

In China, we have that

$$\frac{K^{ch}_{2014}}{L^{ch}_{2014}} = 29.22$$

and in the US

$$\frac{K^{us}_{2014}}{L^{us}_{2014}} = 116015.40$$

Part 3, 4

We have the following:

$$
\begin{aligned}
Y_t &= AK^{\alpha}L^{1-\alpha} \\
\implies \quad y_t &= Ak_t^{\alpha}
\end{aligned}
$$

$$\implies k_{t+1} = (1-\delta)k_t + \sigma A k_t^\alpha$$

The following `R` code was used to compute and export the table.

```
library(xtable)

A_computed <- 1403.5
us_k <- 116015.40
ch_k <- 29.22

part_three_data <- function(k, alpha, delta, sigma, A, years) {
  ## Preallocate size because R copies the entire vector...
  k_t <- double(years)
  y_t <- double(years)
  i_t <- double(years)

  k_t[1] <- k
  y_t[1] <- A * (k ^ alpha)
  i_t[1] <- sigma * y_t[1]

  for (year in 2:years) {
    k_t[year] <- k_t[year - 1] * (1 - delta) + i_t[year - 1]
    y_t[year] <- A * (k_t[year] ^ alpha)
    i_t[year] <- sigma * y_t[year]
  }

  return (list(capital=k_t, invest_per_capital=(i_t / k_t), output=y_t))
}

us_data <- part_three_data(us_k, 0.3, 0.1, 0.25, A_computed, 40)
ch_data <- part_three_data(ch_k, 0.3, 0.1, 0.25, A_computed, 40)

part_three <- data.frame(us_data$capital, ch_data$capital,
                         us_data$invest_per_capital, ch_data$invest_per_capital,
                         us_data$output, ch_data$output, us_data$output / ch_data$output

colnames(part_three) <- c("US k_i", "CH k_t",
                          "US i_t/k_t", "CH i_t/k_t",
                          "US y_t", "CH y_t", "y^{us}_t / y^{ch}_t")

part_four <- data.frame(us_data$output / ch_data$output)
```

```
colnames(part_four) <- c("y^{us}_t / y^{ch}_t")

xtable(part_three, type="latex")
```

Chinese output seems to converge after about 25 years, or about 2038.

Year	US k_i	CH k_t	US i_t/k_t	CH i_t/k_t	US y_t	CH y_t	y_t^{us}/y_t^{ch}
1	116015.40	29.22	0.10	33.05	46405.25	3862.93	12.01
2	116015.17	992.03	0.10	2.80	46405.22	11121.67	4.17
3	116014.96	3673.24	0.10	1.12	46405.20	16471.28	2.82
4	116014.76	7423.74	0.10	0.69	46405.17	20342.26	2.28
5	116014.58	11766.93	0.10	0.50	46405.15	23356.69	1.99
6	116014.41	16429.41	0.10	0.39	46405.13	25816.60	1.80
7	116014.25	21240.62	0.10	0.33	46405.11	27884.48	1.66
8	116014.10	26087.68	0.10	0.28	46405.09	29658.08	1.56
9	116013.97	30893.43	0.10	0.25	46405.08	31201.27	1.49
10	116013.84	35604.40	0.10	0.23	46405.06	32558.44	1.43
11	116013.72	40183.57	0.10	0.21	46405.05	33761.91	1.37
12	116013.61	44605.69	0.10	0.20	46405.03	34836.10	1.33
13	116013.51	48854.15	0.10	0.18	46405.02	35799.99	1.30
14	116013.41	52918.73	0.10	0.17	46405.01	36668.68	1.27
15	116013.32	56794.03	0.10	0.16	46405.00	37454.43	1.24
16	116013.24	60478.23	0.10	0.16	46404.99	38167.36	1.22
17	116013.17	63972.25	0.10	0.15	46404.98	38815.92	1.20
18	116013.09	67279.00	0.10	0.15	46404.97	39407.26	1.18
19	116013.03	70402.92	0.10	0.14	46404.96	39947.50	1.16
20	116012.97	73349.50	0.10	0.14	46404.96	40441.90	1.15
21	116012.91	76125.03	0.10	0.13	46404.95	40895.04	1.13
22	116012.86	78736.29	0.10	0.13	46404.94	41310.92	1.12
23	116012.81	81190.39	0.10	0.13	46404.94	41693.06	1.11
24	116012.76	83494.61	0.10	0.13	46404.93	42044.57	1.10
25	116012.72	85656.30	0.10	0.12	46404.93	42368.22	1.10
26	116012.68	87682.72	0.10	0.12	46404.92	42666.46	1.09
27	116012.64	89581.06	0.10	0.12	46404.92	42941.51	1.08
28	116012.61	91358.33	0.10	0.12	46404.91	43195.34	1.07
29	116012.57	93021.34	0.10	0.12	46404.91	43429.74	1.07
30	116012.54	94576.64	0.10	0.12	46404.91	43646.31	1.06
31	116012.52	96030.55	0.10	0.11	46404.90	43846.53	1.06
32	116012.49	97389.13	0.10	0.11	46404.90	44031.71	1.05
33	116012.47	98658.14	0.10	0.11	46404.90	44203.06	1.05
34	116012.44	99843.09	0.10	0.11	46404.89	44361.66	1.05
35	116012.42	100949.20	0.10	0.11	46404.89	44508.53	1.04
36	116012.40	101981.41	0.10	0.11	46404.89	44644.58	1.04
37	116012.39	102944.42	0.10	0.11	46404.89	44770.64	1.04
38	116012.37	103842.63	0.10	0.11	46404.89	44887.47	1.03
39	116012.35	104680.24	0.10	0.11	46404.88	44995.78	1.03
40	116012.34	105461.16	0.10	0.11	46404.88	45096.22	1.03

Problem 2

We have the following:

$$\begin{aligned} Y_t &= AK_t^\alpha L_t^{1-\alpha} \\ \implies \quad y_t &= Ak_t^\alpha \end{aligned}$$

Since we have at equilibrium that $i_t = \delta k^*$,

$$\delta k^* = \sigma A(k^*)^\alpha$$

Further, since we have that $c^* = y^* - i^*$,

$$c^* = A(k^*)^\alpha - \delta k^*$$

Taking the first order condition,

$$\begin{aligned} \frac{dc^*}{dk^*} &= \alpha A(k^*)^{\alpha-1} - \delta = 0 \\ \implies \quad \delta &= A(k^*)^{1-\alpha} \\ \implies \sigma_{GR} &= \frac{\alpha A(k^*)^{\alpha-1}k^*}{A(k^*)^\alpha} \\ &= \alpha \end{aligned}$$

Countries that are saving less than $\alpha = 0.26$ are undersaving (GBR, USA, PHL, HKG) and the rest are oversaving.

Problem 3

The above relation still holds; the optimal growth rate per this version of the Solow model would be that $\sigma = \alpha = 0.26$; however, China continually saves a lot, over 30% every year and over 40% every year beyond 2004.

Problem 4

This is equivalent to stating that $\frac{dc^*}{d\sigma} < 0$. Taking just boundary conditions we already see that this is untrue, as we have non-zero consumption at $\sigma \in (0, 1)$ but zero consumption at $\sigma = 0, 1$.

Further, in general if we parameterize k^* as $k^*(\sigma)$, we have:

$$c^* = Af(k^*(\sigma)) - \delta k^*(\sigma)$$

Taking the first order condition,

$$\frac{dc^*}{d\sigma} = Af'(k^*(\sigma))(k^*)'(\sigma) - \delta(k^*)'(\sigma) = 0$$

Thus we have that when

$$\delta = Af'(k^*(\sigma))$$

consumption is maximized.

Note that the above is equivalent to σ_{GR}, but also shows that for $\sigma < \sigma_{GR}$, $\frac{dc^*}{d\sigma} > 0$, and for $\sigma > \sigma_{GR}$, $\frac{dc^*}{d\sigma} < 0$, and thus the statement is true in the case that $\sigma > \sigma_{GR}$, and false $\sigma < \sigma_{GR}$