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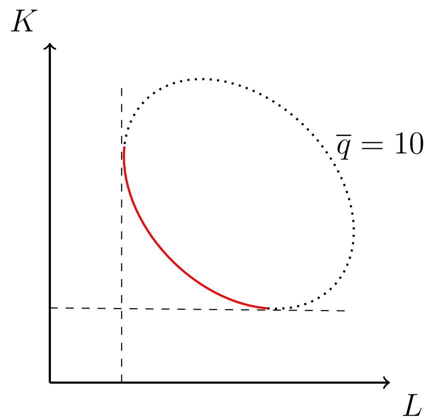
Problem Set 5

ECON 3211
October 17, 2019

Problem 1

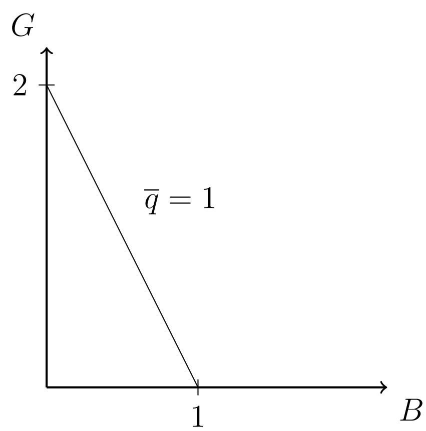
a)



The isoquant here only considers techniques that are efficient, such that any techniques with strictly more labor or equipment consumption are not highlighted. Thus, only the downward sloping piece of the oval that is closest to the axes is solid.

b)

b.1



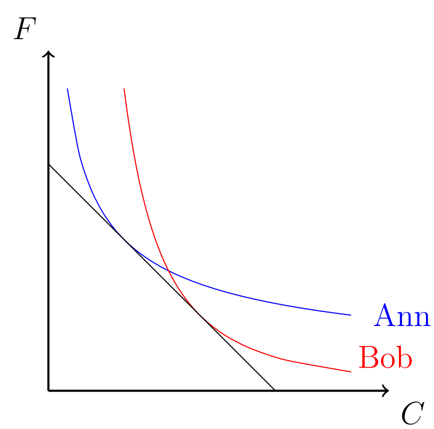
b.2

$$MP_B = \frac{\delta q}{\delta B} = 1$$
$$MP_G = \frac{\delta q}{\delta G} = 0.5$$
$$MRTS_{G,B} = \frac{MP_B}{MP_G} = \frac{1}{0.5} = 2$$

The marginal rate of technical substitution is 2 units of G for a single unit of B.

c)

c.1



c.2

For Ann, we want that since she is better at the theory of the firm, that if $C = F$, $MP_F > MP_C$. Then, for a general Cobb-Douglas $q(C, F) = C^\alpha F^\beta$, we have that $MP_F = \beta C^\alpha F^{\beta-1}, MP_C = \alpha C^{\alpha-1} F^\beta$. When $C = F$,

$$MP_F > MP_C \implies \beta C^\alpha F^{\beta-1} > \alpha C^{\alpha-1} F^\beta \implies \beta > \alpha$$

For Bob, we want that since he is better at the theory of consumer choice, that if $C = F$, $MP_F < MP_C$.

$$MP_F > MP_C \implies \beta C^\alpha F^{\beta-1} < \alpha C^{\alpha-1} F^\beta \implies \beta < \alpha$$

In order to reflect that they are equally good students, we want that their total production should be equal if they have the same amount of time to spend. Since Cobb-Douglass production functions yield the optimal technique as spending $\frac{\alpha}{\alpha+\beta}$ of the total time spent on C and $\frac{\beta}{\alpha+\beta}$ of the total time spent on F, we want that for both Ann and Bob that $\alpha_{Ann} = \beta_{Bob}, \beta_{Ann} = \alpha_{Bob}$.

Thus, we can take $q_{Ann}(C, F) = CF^2, q_{Bob}(C, F) = C^2F$.

This is also probably not a good way to actually study, in the sense that this calculation of production is not reflective of how learning is quantified on exams.

Problem 2

a)

$$\begin{aligned} q(\lambda L, \lambda K, \lambda CR) &= (\lambda L)^{0.55}(\lambda K)^{0.04}(\lambda CR)^{0.41} \\ &= \lambda^{0.55+0.04+0.41} L^{0.55} K^{0.04} CR^{0.41} \\ &= \lambda L^{0.55} K^{0.04} CR^{0.41} \\ &= \lambda q(L, K, CR) \end{aligned}$$

We then see constant returns to scale.

However, if you only consider returns to scale relative to L, K, then we see that

$$q(\lambda L, \lambda K) = \lambda^{0.59} q(L, K) < \lambda q(L, K)$$

and we get decreasing returns to scale.

b)

$$\begin{aligned} MP_{CR} &= \frac{dq}{dCR} \\ &= 0.41 L^{0.55} K^{0.04} CR^{0.41-1} \\ &= 0.41 L^{0.55} K^{0.04} CR^{-0.59} \end{aligned}$$

c)

$$\begin{aligned} MRTS_{K,L} &= \frac{MP_L}{MP_K} \\ &= \frac{0.55L^{0.55-1}K^{0.04}CR^{0.41}}{0.04L^{0.55}K^{0.04-1}CR^{0.41}} \\ &= \frac{0.55K}{0.04L} \end{aligned}$$

This does not change with the level of cash reserves.

Problem 3

a)

$$\begin{aligned} q(\lambda L, \lambda K) &= (\lambda L)^{0.5}\lambda K \\ &= \lambda^{1.5}L^{0.5}K \\ &> \lambda L^{0.5}K \\ &= \lambda q(L, K) \end{aligned}$$

We see increasing returns to scale, as $q(\lambda L, \lambda K) > \lambda q(L, K)$.

b)

$$\begin{aligned} MP_L &= \frac{dq}{dL} = 0.5L^{-0.5}K \\ MP_K &= \frac{dq}{dK} = L^{0.5} \end{aligned}$$

b.1

$$\min_{[L,K]} 4000L + 8000K \text{ s.t. } q = L^{0.5}K = 1000$$

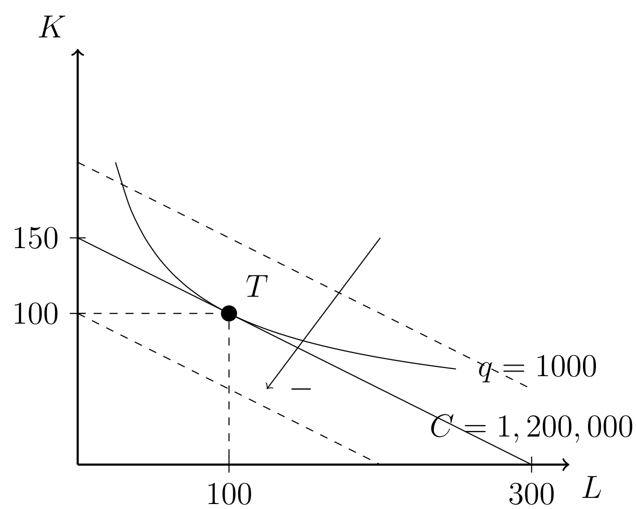
b.2,b.3,b.4

$$
\begin{aligned}
\mathcal{L}(L, K, \lambda) &= 4000L + 8000K - \lambda[L^{0.5}K - 1000] \\
\frac{\delta \mathcal{L}}{\delta L} &= 4000 - 0.5\lambda L^{-0.5}K = 0 \\
\frac{\delta \mathcal{L}}{\delta K} &= 8000 - \lambda L^{0.5} = 0 \\
\frac{\delta \mathcal{L}}{\delta \lambda} &= L^{0.5}K - 1000 = 0 \\
\implies \qquad L^{-1}K &= 1 \\
\implies \qquad L &= K \\
\implies \qquad K &= 1000^{0.67} = 100 \\
\implies \qquad L &= 1000^{0.67} = 100
\end{aligned}
$$

b.5

To see 1000 patients, we have that the minimum cost is $4000(100) + 8000(100) = 1,200,000$.

c)



d)

The share on labor is equal to

$$\frac{\text{total spent on labor}}{\text{total costs}} = \frac{wL}{TC} = \frac{4000(100)}{1200000} = \frac{1}{3}$$

Similarly, the share on capital is equal to

$$\frac{\text{total spent on capital}}{\text{total costs}} = \frac{wL}{TC} = \frac{8000(100)}{1200000} = \frac{2}{3}$$

We notice that $q(L, K) = L^{0.5}K$, and that the share spent on labor is $\frac{0.5}{0.5+1} = \frac{1}{3}$, and that the share spent on capital is $\frac{1}{0.5+1} = \frac{2}{3}$.

In general, for $q(L, K) = L^{\alpha}K^{\beta}$, we have that the share spent on labor is $\frac{\alpha}{\alpha+\beta}$ and the share spent on capital is $\frac{\beta}{\alpha+\beta}$, matching what we know about Cobb-Douglas functions from consumer choice.

e),f),g)

These do not exist, for some reason.

h)

We solve for cost minimizing L, K as functions of w, r, q (these are conditional demands for L, K) and substitute these into the cost function.

i)

$$\begin{aligned}
\frac{MP_L}{MP_K} &= \frac{w}{r} \\
\frac{0.5L^{-0.5}K}{L^{0.5}} &= \frac{w}{r} \\
\implies \frac{K}{2L} &= \frac{w}{r} \\
\implies K &= \frac{2wL}{r} \\
\implies q &= L^{0.5}\frac{2wL}{r} \\
&= L^{1.5}\frac{2wL}{r} \\
\implies L &= (\frac{rq}{2w})^{\frac{2}{3}} \\
\implies q &= (\frac{rq}{2w})^{\frac{1}{3}}K \\
\implies K &= q(\frac{2w}{rq})^{\frac{1}{3}} \\
&= (\frac{2wq^2}{r})^{\frac{1}{3}}
\end{aligned}$$

j)

The clinic's total cost function is then $C(w,r,q) = w(\frac{rq}{2w})^{\frac{2}{3}} + r(\frac{2wq^2}{r})^{\frac{1}{3}} = (\frac{1}{2}rwq^{\frac{1}{2}})^{\frac{2}{3}} + (2wq^2r^2)^{\frac{1}{3}}$.

k)

We have that

$$\begin{aligned}
AC &= \frac{C(w,r,q)}{q} \\
&= \frac{1}{q}[(\frac{1}{2}rwq^{\frac{1}{2}})^{\frac{2}{3}} + (2wq^2r^2)^{\frac{1}{3}}] \\
&= q^{-\frac{1}{3}}(\frac{1}{2}rw^{\frac{1}{2}})^{\frac{2}{3}} + q^{-\frac{1}{3}}(2wr^2)^{\frac{1}{3}} \\
&= q^{-\frac{1}{3}}((\frac{1}{2}rw^{\frac{1}{2}})^{\frac{2}{3}} + (2wr^2)^{\frac{1}{3}}) \\
\implies \frac{dAC}{dq} &= -\frac{1}{3}q^{-\frac{4}{3}}((\frac{1}{2}rw^{\frac{1}{2}})^{\frac{2}{3}} + (2wr^2)^{\frac{1}{3}}) < 0
\end{aligned}$$

Thus, AC is decreasing with increased output, agreeing with our initial finding of economies of scale.

Problem 4

This does not exist.

Problem 5

a)

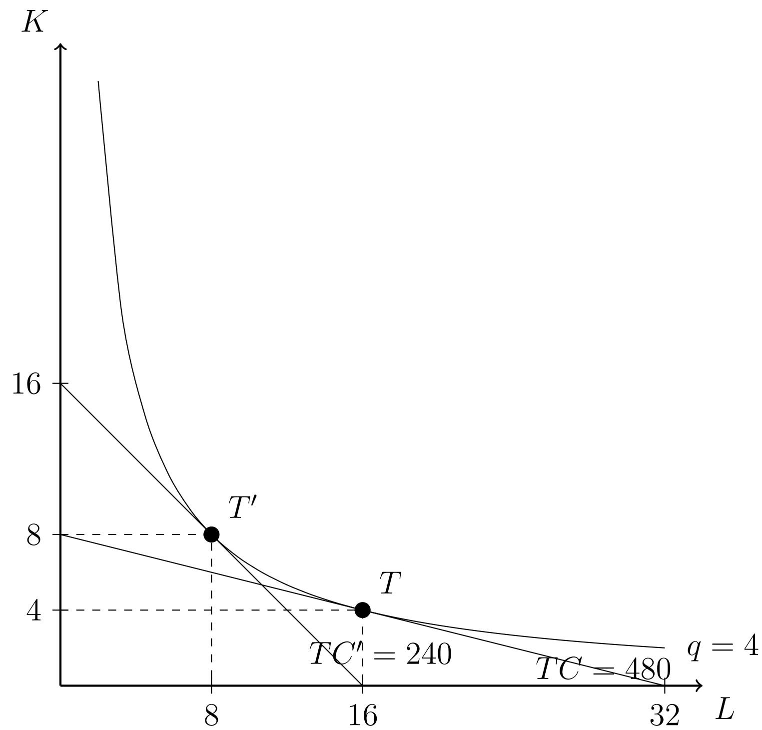
The problem here is to minimize $15L + 60K$ such that $q(L,K) = L^{\frac{1}{3}}K^{\frac{1}{3}} = 4$.

$$\begin{aligned}
\mathcal{L}(L,K,\lambda) &= 15L + 60K - \lambda(L^{\frac{1}{3}}K^{\frac{1}{3}} - 4) \\
\frac{\delta\mathcal{L}}{\delta L} &= 15 - \frac{1}{3}\lambda L^{\frac{-2}{3}}K^{\frac{1}{3}} = 0 \\
\frac{\delta\mathcal{L}}{\delta L} &= 60 - \frac{1}{3}\lambda L^{\frac{1}{3}}K^{\frac{-2}{3}} = 0 \\
\frac{\delta\mathcal{L}}{\delta \lambda} &= L^{\frac{1}{3}}K^{\frac{1}{3}} - 4 = 0 \\
\implies \quad \frac{60}{15} &= \frac{L}{K} \\
\implies \quad 4K &= L \\
\implies \quad 4^{\frac{1}{3}}K^{\frac{2}{3}} &= 4 \\
\implies \quad K &= 4, L = 16
\end{aligned}$$

b)

The firm's total cost is then $15(16) + 60(4) = 240 + 240 = \$480$.

c),d)



Note that after the subsidy, we have that $\frac{MU_K}{MU_L} = \frac{15}{15} \implies \frac{L}{K} = 1 \implies L = K = 8$.

e)

The total cost is diminished to $15L + 15K = 15(8) + 15(8) = \$240$.

f)

This situation is different from consumer choice in the sense that the firm's output does not actually change with the subsidy, meaning that the government is just paying for no extra production. The welfare benefit to the producer then is simply the reduction in costs of \$240.

This subsidy's cost is given by the fact that the firm utilizes 16 units of K, each subsidized at \$45 each, for a total cost of $16(45) = 720$. The final excess burden then is $720 - 240 = 480$.

Morally, this is also the same as excess burden in consumer choice as that considered the situation where a lump sum incurring the same welfare loss would raise more tax revenue; in this case, a flat subsidy of \$240 would have sufficed, but the per unit subsidy on K incurs an extra \$480 of government spending.

g)

If we have a long run evolution of factor productivity like this, then we would see that the subsidy increases total factor productivity to a higher degree than without the subsidy. For example, after the first year ($q_0 = L^{\frac{1}{3}}K^{\frac{1}{3}}$), we would see that $q_1 = 8L^{\frac{1}{3}}k^{\frac{1}{3}}$ in the presence of the subsidy and $q_1 = 4L^{\frac{1}{3}}K^{\frac{1}{3}}$ without.

The case with the subsidy, the firm would spend for $L = K = \frac{1}{2\sqrt{2}}$ units for a total cost of $\frac{1}{\sqrt{2}} \cdot 15 = \frac{15}{\sqrt{2}}$, whereas without the subsidy the firm would spend for $L = K = 1$ for a total cost of $15 \cdot 2 = 30$.

Then, the subsidy has now totaled spending of $720 + 45(\frac{1}{2\sqrt{2}}) = 736$, a small increase after the first year. Comparatively, the firm has only spent $240 + \frac{15}{\sqrt{2}} = 251$, as compared to $480 + 30 = 510$ without the subsidy. The excess burden is then $736 - (510 - 251) = 477$, which has diminished from before, indicating that the subsidy is actually positive for the second year, drastically increasing the efficacy of the policy.

After the first few years, q_t is harder to predict as it is not $q_t = \frac{K_{t-1}}{2}q_{t-1}$, which would make it easier to solve, but still generally have the subsidy as more effective than in the first year (i.e. incurring less excess burden).