dc3451 STAT 4001
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Assignment 1

1.4.1

Consider $x \in B^C$. Assume that $x \notin A^C$. Then, we have that $x \notin A^C \implies x \in A \implies x \in B$. $\Rightarrow\Leftarrow$, so $x \in A^C$.

1.4.8

Blood type	Set representation
A	$A \cap B^C$
B	$B \cap A^C$
AB	$A \cap B$
O	$A^C \cap B^C$

1.5.12

Note that the B_i are all disjoint; any B_i contains elements in A_i that are not seen before as elements of $A_1, A_2, ..., A_{i-1}$.

More formally, suppose that for $i \neq j, x \in B_i, B_j$. Without loss of generality, take $j > i$. Since $x \in B_j, x \in A_i^C$, and since $x \in B_i, x \in A_i$. $\Rightarrow\Leftarrow$, so B_i, B_j are disjoint.

Further, we have that for any n, $\bigcup_{i=1}^n A_i = \bigcup_{i=1}^n B_i$. For any $x \in \bigcup_{i=1}^n A_i$, let k be the least index such that $x \in A_k$. Then, $x \in B_k$ as well; similarly, since $B_k \subset A_k$, we have that $\bigcup_{i=1}^n A_i = \bigcup_{i=1}^n B_i$.

We have now that

$$\sum_{i=1}^n \Pr(B_i) = \Pr\left(\bigcup_{i=1}^n B_i\right) = \Pr\left(\bigcup_{i=1}^n A_i\right)$$

1.5.14.a

Since there are only four possibilities, we have that $\Pr(\text{AB}) = 1 - (0.5 + 0.12 + 0.34) = 0.04$.

The events below are disjoint, as one can only have a single blood type.

Antigen	Probability
anti-A	$\Pr(\text{A or AB}) = 0.34 + 0.04 = 0.38$
anti-B	$\Pr(\text{B or AB}) = 0.12 + 0.04$

1.5.14.b

This probability is exactly $\Pr(AB) = 0.04$.

1.6.6

Each outcome of faces is equally likely by assumption, where there are 2^3 total possibilities. There are only two outcomes where all are the same (all heads, all tails), so the probability is $\frac{1}{4}$.

1.7.3

This is simply just $5! = 120$ total ways.

1.7.6

There are in total $6!$ ways to order $1, 2, 3, 4, 5, 6$; there are 6^6 total ways to get a sequence of rolls. The odds are $\frac{6!}{6^6} = \frac{5}{324} \approx 0.0154$.

1.7.10.abc

In any given ordering of the balls, the odds that any given position is occupied by a red ball is $\frac{r}{100}$. Thus, the odds for all three parts are all $\frac{r}{100}$.

1.7.11

$$P_{n,k} = \frac{(2\pi)^{\frac{1}{2}} n^{n+\frac{1}{2}} e^{-n}}{(2\pi)^{\frac{1}{2}} (n-k)^{n-k+\frac{1}{2}} e^{k-n}} = \frac{n^{n+\frac{1}{2}}}{e^k (n-k)^{n-k+\frac{1}{2}}}$$

1.8.15.a

$$\sum_{i=0}^{n} \binom{n}{i}$$

is simply the total amount of subsets of any size of a size of set n. This can be counted by considering that either an element is in a subset or not, leading to a total of 2^n.

Alternatively, consider that for $f(x, y) = (x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$ has that $f(1, 1) = 2^n = \sum_{i=0}^{n} \binom{n}{i} 1^i 1^{n-i}$.

1.8.15.b

Take the same function $f(x, y) = (x + y)^n$ again; we see that $f(-1, 1) = 0^n = 0 = \sum_{i=0}^{n} \binom{n}{i} (-1)^i = 0$.

1.8.18

There are $\binom{20}{2}$ ways to choose the two recipients for each class; there are $\binom{100}{10}$ ways to choose the ten recipients.

The probability is then

$$\frac{\binom{20}{2}^5}{\binom{100}{10}} \approx 0.014$$

1.9.4

There is only one correct spelling of statistics; the total amount of ways to do such an ordering is

$$\binom{10}{3,3,2,1,1} = \frac{10!}{3!3!2!1!1!} = 50400$$

The probability of an ordering spelling statistics is then

$$\frac{1}{\binom{10}{3,3,2,1,1}} = \frac{1}{50400} \approx 0.0000198$$

1.10.6

Put R, W, B as the odds of picking no red, white, and blue balls, respectively. We then need to compute $\Pr(R \cup W \cup B) = \Pr(R) + \Pr(W) + \Pr(B) - \Pr(R, W) - \Pr(R, B) - \Pr(W, B)$. We have that $\Pr(R) = \Pr(W) = \Pr(B) = \frac{\binom{60}{10}}{\binom{90}{10}}$, and that $\Pr(R, W) = \Pr(R, B) = \Pr(W, B) = \frac{\binom{30}{10}}{\binom{90}{10}}$, so that

$$\Pr(R \cup W \cup B) = 3\frac{\binom{60}{10} - \binom{30}{10}}{\binom{90}{10}}$$

1.10.10

The probability of exactly the first envelope being correct is $\frac{1}{6!}$, as there is only one such ordering that places only the first envelope correctly out of six total orderings; this is disjoint with only the second and third envelopes each being correct, so the total probability is $3\frac{1}{6} = \frac{1}{2}$.