Composing Meaning via Dependent Types
Day 1: Overview

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Dependent Type Semantics (DTS) (Bekki 2014; Bekki and Mineshima 2017; Bekki 2021)

- A framework of natural language semantics
- Unified approach to (general) inferences and anaphora/presupposition resolution in terms of *type checking* and *proof search*

Main features:

1. **Proof-theoretic semantics**:
   From model theory (denotations and models) to proof theory (proofs and contexts)
2. **Anaphora/Presuppositions**: A proof-theoretic alternative to Dynamic Semantics (DRT, DPL, etc.)
3. **Compositionality**: Syntax-semantics interface via categorial grammars (e.g. CCG, TLG, ACG, etc)
4. **Implementation**: Applications to Natural Language Processing.
   `https://github.com/DaisukeBekki/lightblue/`

Dependent Types

Per Martin-Löf

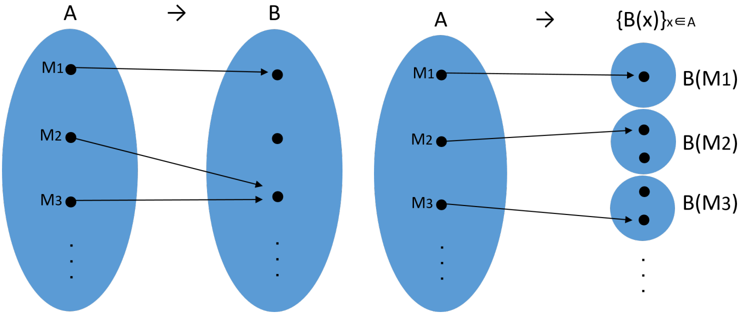
Martin-Löf (1984) “Intuitionistic type theory”

What are Π-types

Π-type is a type of *fibred* functions.

Simple function space

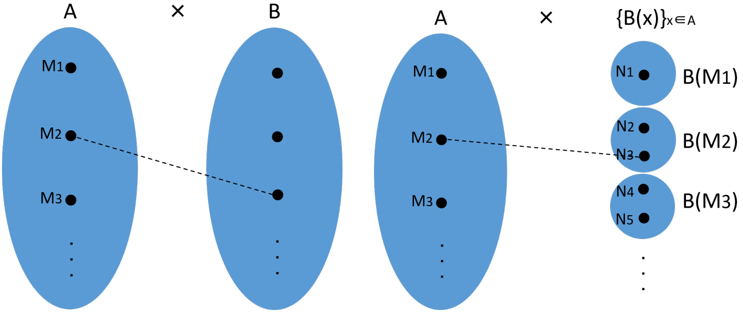
Fibred function space



What are Σ-types

Σ-type is a type of *fibred* products.

Simple product space *Fibred* product space



Notations

DTS notation	Standard notation	$x \notin fv(B)$	$x \in fv(B)$
$(x : A) \to B$	$(\Pi x : A)B$	$A \to B$	$(\forall x : A)B$
$(x : A) \times B$ *or* $\begin{bmatrix} x : A \\ B \end{bmatrix}$	$(\Sigma x : A)B$	$A \wedge B$	$(\exists x : A)B$

Scope of the variable in Π-types: $(x : A) \to \boxed{B}$

Scope of the variable in Σ-types: $\begin{bmatrix} x : A \\ \boxed{B} \end{bmatrix}$

Π-type F/I/E rules

(o)

discharge

$$\dfrac{A : \mathsf{s}_1 \quad \dfrac{\overline{x : A}^{\,i}}{\vdots}\; B : \mathsf{s}_2}{(x : A) \to B : \mathsf{s}_2}\,(\Pi F),i$$

type

kind

M : A type assignment

$$\textit{where } (\mathsf{s}_1, \mathsf{s}_2) \in \left\{ \begin{array}{l} (\text{type}, \text{type}), \\ (\text{type}, \text{kind}) \end{array} \right\}.$$

B[x:=N]

Use condition

$$\dfrac{A : \text{type} \quad \dfrac{\overline{x : A}^{\,i}}{\vdots}\; M : B}{\lambda x.M : (x : A) \to B}\,(\Pi I),i \qquad \dfrac{M : (x : A) \to B \quad N : A}{MN : B[N/x]}\,(\Pi E)$$

verification condition

substitution

Σ-type F/I/E rules

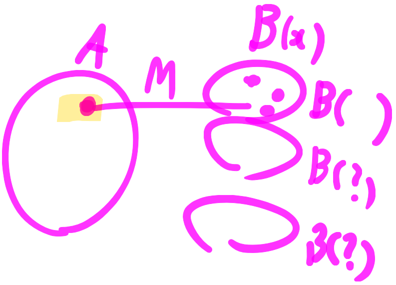
$$\frac{A : \text{type} \qquad \begin{array}{c}\overline{x : A}^{\,i} \\ \vdots \\ B : \text{type}\end{array}}{(x : A) \times B : \text{type}}\ (\Sigma F), i$$

$$\frac{M : A \qquad N : B[M/x]}{(M, N) : (x : A) \times B}\ (\Sigma I)$$

$$\frac{M : (x : A) \times B}{\pi_1(M) : A}\ (\Sigma E) \qquad \frac{M : (x : A) \times B}{\pi_2(M) : B[\pi_1(M)/x]}\ (\Sigma E)$$

first projection

Second projection



Rules of DTS

Rules from Martin-Löf Type Theory

- Axioms and Structural rules
- $\mathbf{\Pi}$-type (Dependent function type) [F/I/E]
- $\mathbf{\Sigma}$-type (Dependent product type) [F/I/E]
- Intensional equality type [F/I/E]
- Disjoint union type [F/I/E]
- Enumeration type [F/I/E]
- Natural number type [F/I/E]

New rule in DTS

- @ (the ‘asperand’ operator)
  - Anaphora and presupposition triggers (linguistically speaking)
  - Open proofs (logically speaking)

Conjunction, Implication, and Negation

Definition

$$\begin{bmatrix} A \\ B \end{bmatrix} \overset{def}{\equiv} (x : A) \times B \qquad where\ x \notin fv(B).$$

$$A \rightarrow B \overset{def}{\equiv} (x : A) \rightarrow B \qquad where\ x \notin fv(B).$$

$$\neg A \overset{def}{\equiv} (x : A) \rightarrow \bot$$

absurdity

bottom { Enumeration

$$\frac{x : A \quad y : \neg A \equiv (x : A) \rightarrow \bot}{y(x) : \bot} (\Pi E)$$

Anaphora in Natural Language

A theory of anaphora

- Anaphora representable by a constant symbol:
  - Deictic use:

    (1) *(Pointing at John)*

    He was born in Detroit.

    $\textbf{bornIn}(j, d)$
  - Coreference:

    (2) John loves a girl who hates him.

    $\exists x(\textbf{girl}(x) \wedge \textbf{love}(j, x) \wedge \textbf{hate}(x, j))$
- Anaphora representable by a variable
  - Bound variable anaphora:

    (3) Every boy loves his father.

    $\forall x\,(\textbf{boy}(x) \rightarrow \textbf{love}(x, \textbf{fatherOf}(x)))$

A theory of anaphora

- Anaphora not representable by FoL:
  - E-type anaphora:

    (4) A man entered into the park. He whistled.
  - Donkey anaphora:

    (5) Every farmer who owns a donkey beats it.

    (6) If a farmer owns a donkey, he beats it.
- Anaphora not representable by FoL nor dynamic semantics:
  - Syllogistic anaphora:

    (7) Every girl received a present. Some girl opened it.
  - Disjunctive antecedent:

    (8) If Mary sees a horse or a pony, she waves to it.

Donkey anaphora: Geach (1962)

For the donkey sentences (9), a first-order formula (10), whose truth condition is the same as those of (9), is a candidate of its semantic representation (SR). (We only discuss its *strong reading* here. See Tanaka (2021) for its *weak reading*.)

(9) a. Every farmer who owns [a donkey]$^{\mathbf{1}}$ beats it$_{\mathbf{1}}$.

b. If [a farmer]$^{\mathbf{1}}$ owns [a donkey]$^{\mathbf{2}}$, he$_{\mathbf{1}}$ beats it$_{\mathbf{2}}$.

(10) $\forall x(\mathbf{farmer}(x) \rightarrow \forall y\,(\mathbf{donkey}(y) \wedge \mathbf{own}(x,y) \rightarrow \mathbf{beat}(x,y)))$

But the translation from the sentence (9) to (10) is not straightforward since i) the indefinite noun phrase *a donkey* is translated into a universal quantifier in (10) instead of an existential quantifier, and ii) the syntactic structure of (10) does not corresponds to that of (9).

Donkey anaphora: Geach (1962)

(9) a. Every farmer who owns [a donkey]^{1} beats $\boxed{\text{it}_{\mathbf{1}}}$.

b. If [a farmer]^{1} owns [a donkey]^{2}, he$_{\mathbf{1}}$ beats $\boxed{\text{it}_{\mathbf{2}}}$.

The syntactic parallel of (9) is, rather, the SR (11), in which the indefinite noun phrase is translated into an existential quantification.

(11) $\forall x(\mathbf{farmer}(x) \wedge \exists y(\mathbf{donkey}(y) \wedge \mathbf{own}(x, y)\cancel{)} \to \mathbf{beat}(x, \boxed{y})))$

$\to$ ex.

However, (11) does not represent the truth condition of (9) correctly since the variable y in $\mathbf{beat}(x, y)$ fails to be bound by $\exists$. Therefore, neither (10) nor (11) qualifies as the SR of (9).

Various approaches in discourse semantics

Dynamic Semantics

- Discourse Representation Theory (DRT): Kamp (1981), Kamp and Reyle (1993)
- Dynamic Predicate Logic (DPL): Groenendijk and Stokhof (1991)
- Dynamic Plural Predicate Logic (DPPL): van den Berg (1996), Sudo (2012)

Type-theoretical Semantics

- Analysis of donkey anaphora: Sundholm (1986))
- Type Theoretical Grammar (TTG): Ranta (1994)
- Type Theory with Record (TTR): Cooper (2005)
- MTT-semantics: Luo (1997, 1999, 2010, 2012), Asher and Luo (2012), Chatzikyriakidis (2014)
- Dependent Type Semantics (DTS): Bekki (2014), Bekki and Mineshima (2017)

Donkey anaphora: Sundholm (1986)

A Type for all entities. {Enumeration type...

(9a) Every farmer who owns a donkey beats it .

$$\left(u : \begin{bmatrix} x : \text{entity} \\ \begin{bmatrix} \mathbf{farmer}(x) \\ \begin{bmatrix} v : \begin{bmatrix} y : \text{entity} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}(x, \pi_1 v) \end{bmatrix} \end{bmatrix} \end{bmatrix}\right) \to \mathbf{beat}(\pi_1 u, \pi_1\pi_1\pi_2\pi_2 u)$$

Note: $(x : A) \to B$ is a type for functions from A to $B[x]$.

Constant symbol of type entity → type

From TTG to DTS: Compositionality

Q: How could one get to these (dependently-typed) representations from arbitrary sentences?

A: By lexicalization.

Q: But, how could we lexicalize context-dependent words like pronouns?

?

$$\left(u : \begin{bmatrix} x : \text{entity} \\ \begin{bmatrix} \textbf{farmer}(x) \\ \begin{bmatrix} v : \begin{bmatrix} y : \text{entity} \\ \textbf{donkey}(y) \end{bmatrix} \\ \textbf{own}(x, \pi_1 v) \end{bmatrix} \end{bmatrix} \end{bmatrix} \right) \to \textbf{beat}(\pi_1 u, \pi_1\pi_1\pi_2\pi_2 u)$$

From TTG to DTS: Compositionality

Q: How could one get to these (dependently-typed) representations from arbitrary sentences?

A: By lexicalization.

Q: But, how could we lexicalize context-dependent words like pronouns?

A: By using **underspecified types**.

Q: How could we retrieve a context for an underspecified type?

A: By **type checking**.

Summary and History

A Unified, Compositional Theory of *Projective* Meaning

- DTS provides a unified analysis for (general) inferences and anaphora resolution mechanisms.
- The background theory for DTS is an extention of DTT with underspecified types and the @-rule.
  - Lexical items of anaphoric expressions and presupposition triggers are represented by using underspecified types.
  - Context retrieval in DTS reduces to type checking.
  - Anaphora resolution and presupposition binding in DTS reduces to proof search.
  - Type checker translates a proof diagram of DTS into a proof diagram of DTT, by which an SR in DTT is obtained with all anaphora resolved.

Natural language semantics via dependent types: The first generation

- Donkey anaphora: Sundholm (1986)
- Translation from DRS to dependent type representations: Ahn and Kolb (1990)
- Summation: Fox (1994a,b)
- Ranta's TTG (Relative and Implicational Donkey Sentences, Branching Quantifiers, Intensionality, Tense): Ranta (1994)
- Translation from Montague Grammar to dependent type representations: Dávila-Pérez (1995)
- Presupposition Binding and Accommodation, Bridging: Krahmer and Piwek (1999), Piwek and Krahmer (2000)

Natural language semantics via dependent types: The second generation

- Type Theory with Record (TTR): Cooper (2005)
- Modern Type Theory: Luo (1997, 1999, 2010, 2012), Asher and Luo (2012), Chatzikyriakidis (2014)
- Semantics with Dependent Types: Grudzinska and Zawadowski (2014; 2017)
- Dynamic Categorial Grammar: Martin and Pollard (2014)
- **Dependent Type Semantics (DTS): Bekki (2014), Bekki and Mineshima (2017)**

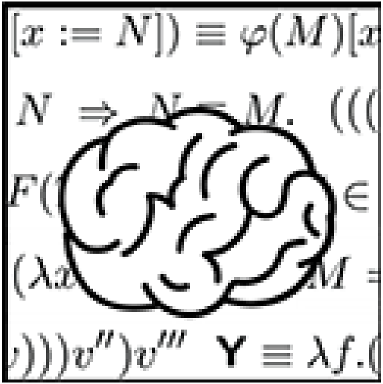
Semantic Analyses by DTS: The third generation

- Generalized Quantifiers: Tanaka (2014)
- Honorification: Watanabe et al. (2014)
- Conventional Implicature: Bekki and McCready (2015), Matsuoka et al. (2023)
- Factive Presuppositions: Tanaka et al. (2015)
- Dependent Plural Anaphora: Tanaka et al. (2017)
- Paycheck sentences: Tanaka et al. (2018)
- Coercion and Metaphor: Kinoshita et al. (2017, 2018)
- Questions: Watanabe et al. (2019), Funakura (2022)
- Comparision with DRT: Yana et al. (2019)
- The proviso problem: Yana et al. (2021)
- Weak Crossover: Bekki (2023)

Computational Aspects of DTS

- Type Checker for (the fragment of) DTS: Bekki and Sato (2015)
- Development of an automated theorem prover (for the fragment of) DTS: Daido and Bekki (2020)
- Integrating Deep Neural Network with DTS: Bekki et al. (2023, 2022)

Thank you!



Reference I

Ahn, R. and H.-P. Kolb. (1990) "Discourse Representation meets Constructive Mathematics", In: L. Kalman and L. Polos (eds.): *Papers from the Second Symposium on Logic and Language*. Akademiai Kiado.

Asher, N. and Z. Luo. (2012) "Formalisation of coercions in lexical semantics", In the Proceedings of *Sinn und Bedeutung 17*. pp.63–80.

Beaver, D. I. (2001) *Presupposition and Assertion in Dynamic Semantics*, Studies in Logic, Language and Information. CSLI Publications & FoLLI.

Reference II

Bekki, D. (2014) “Representing Anaphora with Dependent Types”, In the Proceedings of N. Asher and S. V. Soloviev (eds.): *Logical Aspects of Computational Linguistics (8th international conference, LACL2014, Toulouse, France, June 2014 Proceedings), LNCS 8535*. pp.14–29, Springer, Heiderburg.

Bekki, D. (2023) “A Proof-theoretic Analysis of Weak Crossover”, In: *New Frontiers in Artificial Intelligence (JSAI-isAI 2021 Workshops, JURISIN, LENLS18, SCIDOCA, Kansei-AI, AI-BIZ, Yokohama, Japan, November 13-15, 2021, Revised Selected Papers), LNAI 13856*. Springer, pp.228–241.

Reference III

Bekki, D. and E. McCready. (2015) “CI via DTS”, In: *New Frontiers in Artificial Intelligence (JSAI-isAI 2014 Workshops, LENLS, JURISIN, and GABA, Yokohama, Japan, November 23-24, 2014, Revised Selected Papers)*, Vol. LNAI 9067. Springer.

Bekki, D. and K. Mineshima. (2017) “Context-Passing and Underspecification in Dependent Type Semantics”, In: S. Chatzikyriakidis and Z. Luo (eds.): *Modern Perspectives in Type-Theoretical Semantics*, Studies of Linguistics and Philosophy. Springer, pp.11–41.

Bekki, D. and M. Sato. (2015) “Calculating Projections via Type Checking”, In the Proceedings of *TYpe Theory and LExical Semantics (TYTLES), ESSLLI2015 workshop*.

Reference IV

Bekki, D., R. Tanaka, and Y. Takahashi. (2022) “Learning Knowledge with Neural DTS”, In the Proceedings of *the 3rd Natural Logic Meets Machine Learning (NALOMA III)*. pp.17–25, Association of Computational Linguistics.

Bekki, D., R. Tanaka, and Y. Takahashi. (2023) “Integrating Deep Neural Network with Dependent Type Semantics”, In: R. Loukanova, P. L. Lumsdaine, and R. Muskens (eds.): *Logic and Algorithms in Computational Linguistics 2021 (LACompLing2021)*, Studies in Computational Intelligence 1081. Springer.

Reference V

Chatzikyriakidis, S. (2014) “Adverbs in a Modern Type Theory”, In: N. Asher and S. V. Soloviev (eds.): *Logical Aspect of Computational Linguistics, 8th International Conference, LACL2014, Toulouse, France, June 18-20, 2014 Proceedings*. Springer.

Cooper, R. (2005) “Records and Record Types in Semantic Theory”, *Journal of Logic and Computation* **15**(2), pp.99–112.

Daido, H. and D. Bekki. (2020) “Development of an automated theorem prover for the fragment of DTS”, In the Proceedings of *the 17th International Workshop on Logic and Engineering of Natural Language Semantics (LENLS17)*.

Dávila-Pérez, R. (1995) “Semantics and Parsing in Intuitionistic Categorial Grammar”, Thesis, University of Essex. Ph.D. thesis.

Reference VI

Fox, C. (1994a) "Discourse Representation, Type Theory and Property Theory", In the Proceedings of H. Bunt, R. Muskens, and G. Rentier (eds.): *the International Workshop on Computational Semantics*. pp.71–80.

Fox, C. (1994b) "Existence Presuppositions and Category Mistakes", *Acta Linguistica Hungarica* **42**(3/4), pp.325–339.

Funakura, H. (2022) "Answers, Exhaustivity, and Presupposition of wh-questions in Dependent Type Semantics", In the Proceedings of *Logic and Engineering of Natural Language Semantics 20 (LENLS20)*. pp.72–76.

Geach, P. (1962) *Reference and Generality: An Examination of Some Medieval and Modern Theories*. Ithaca, New York, Cornell University Press.

Reference VII

Geurts, B. (1999) *Presuppositions and pronouns*. Elsevier, Oxford.

Groenendijk, J. and M. Stokhof. (1991) “Dynamic Predicate Logic”, *Linguistics and Philosophy* **14**, pp.39–100.

Kamp, H. (1981) “A Theory of Truth and Semantic Representation”, In: J. Groenendijk, T. M. Janssen, and M. Stokhof (eds.): *Formal Methods in the Study of Language*. Amsterdam, Mathematical Centre Tract 135.

Kamp, H. and U. Reyle. (1993) *From Discourse to Logic*. Kluwer Academic Publishers.

Kinoshita, E., K. Mineshima, and D. Bekki. (2017) “An Analysis of Selectional Restrictions with Dependent Type Semantics”, In: S. Kurahashi, Y. Ohta, S. Arai, K. Satoh, and D. Bekki (eds.): *New Frontiers in Artificial Intelligence. JSAI-isAI 2016, Lecture Notes in Computer Science, vol 10247*. Springer, pp.19–32.

Reference VIII

Kinoshita, E., K. Mineshima, and D. Bekki. (2018) "Coercion as Proof Search in Dependent Type Semantics", In: C. Fabricius-Hansen, B. Behrens, A. Pitz, and H. Petter Helland (eds.): *Possessives in L2 and translation: basic principles and empirical findings, Oslo Studies in Language 10, No 2*. pp.1–20.

Krahmer, E. and P. Piwek. (1999) "Presupposition Projection as Proof Construction", In: H. Bunt and R. Muskens (eds.): *Computing Meanings: Current Issues in Computational Semantics*, Studies in Linguistics Philosophy Series. Dordrecht, Kluwer Academic Publishers.

Krifka, M. (1996) "Parametrized Sum Individuals for Plural Anaphora", *Linguistics and Philosophy* **19**, pp.555–598.

Reference IX

Levinson, S. (1983) *Pragmatics*. Cambridge, Cambridge University Press.

Luo, Z. (1997) "Coercive subtyping in type theory", In: D. van Dalen and M. Bezem (eds.): *CSL 1996. LNCS, vol. 1258*. Heidelberg, Springer.

Luo, Z. (1999) "Coercive subtyping", *Journal of Logic and Computation* **9**(1), pp.105–130.

Luo, Z. (2010) "Type-theoretical semantics with coercive subtyping", In the Proceedings of *Semantics and Linguistic Theory 20 (SALT 20)*.

Luo, Z. (2012) "Formal Semantics in Modern Type Theories with Coercive Subtyping", *Linguistics and Philosophy* **35**(6).

Reference X

Martin, S. and C. J. Pollard. (2014) "A dynamic categorial grammar", In the Proceedings of *Formal Grammar 19, LNCS 8612*.

Matsuoka, D., D. Bekki, and H. Yanaka. (2023) "Appositive Projection as Implicit Context Extension in Dependent Type Semantics", In the Proceedings of *the 20th International Workshop on Logic and Engineering of Natural Language Semantics (LENLS20)*. pp.82–87.

Piwek, P. and E. Krahmer. (2000) "Presuppositions in Context: Constructing Bridges", In: P. Bonzon, M. Cavalcanti, and R. Nossum (eds.): *Formal Aspects of Context*, Applied Logic Series. Dordrecht, Kluwer Academic Publishers.

Reference XI

Ranta, A. (1994) *Type-Theoretical Grammar*. Oxford University Press.

Soames, S. (1989) “Presupposition”, In: D. Gabbay and F. Guenthner (eds.): *Handbook of Philosophical Logic*, Vol. 4. Dordrecht, Reidel, pp.553–616.

Sudo, Y. (2012) “On the Semantics of Phi Features on Pronouns”, Thesis, MIT. Doctoral dissertation.

Sundholm, G. (1986) “Proof theory and meaning”, In: D. Gabbay and F. Guenthner (eds.): *Handbook of Philosophical Logic*, Vol. III. Reidel, Kluwer, pp.471–506.

Reference XII

Tanaka, R. (2014) “A Proof-Theoretic Approach to Generalized Quantifiers in Dependent Type Semantics”, In the Proceedings of R. de Haan (ed.): *the ESSLLI 2014 Student Session, 26th European Summer School in Logic, Language and Information*. pp.140–151.

Tanaka, R. (2021) “Natural Language Quantification and Dependent Types”, Thesis, Ochanomizu University. Doctoral Dissertation.

Tanaka, R., K. Mineshima, and D. Bekki. (2015) “Factivity and Presupposition in Dependent Type Semantics”, In the Proceedings of *TYpe Theory and LExical Semantics (TYTLES), ESSLLI2015 workshop*.

Reference XIII

Tanaka, R., K. Mineshima, and D. Bekki. (2017) "On the Interpretation of Dependent Plural Anaphora in a Dependently-Typed Setting", In: S. Kurahashi, Y. Ohta, S. Arai, K. Satoh, and D. Bekki (eds.): *New Frontiers in Artificial Intelligence. JSAI-isAI 2016, Lecture Notes in Computer Science, vol 10247*. Springer, pp.123–137.

Tanaka, R., K. Mineshima, and D. Bekki. (2018) "Paychecks, presupposition, and dependent types", In the Proceedings of *the Fifth Workshop on Natural Language and Computer Science (NLCS2018), Preprint no.215*. Oxford University.

van den Berg, M. (1996) "Some aspects of the internal structure of discourse – the dynamics of nominal anaphora –", Thesis, University of Amsterdam.

Reference XIV

van der Sandt, R. (1992) "Presupposition projection as anaphora resolution", *Journal of Semantics* **9**, pp.333–377.

Watanabe, K., K. Mineshima, and D. Bekki. (2019) "Questions in Dependent Type Semantics", In the Proceedings of *Proceedings of the Sixth Workshop on Natural Language and Computer Science (NLCS'19)*. pp.23–33.

Watanabe, N., E. McCready, and D. Bekki. (2014) "Japanese Honorification: Compositionality and Expressivity", In the Proceedings of S. Kawahara and M. Igarashi (eds.): *FAJL 7: Formal Approaches to Japanese Linguistics, the MIT Working Papers in Linguistics 73*. pp.265–276.

Reference XV

Yana, Y., D. Bekki, and K. Mineshima. (2019) “Variable Handling and Compositionality: Comparing DRT and DTS”, *Journal of Logic, Language and Information* **28**(2), pp.261–285.

Yana, Y., K. Mineshima, and D. Bekki. (2021) “The proviso problem from a proof-theoretic perspective”, In the Proceedings of *Logical Aspects of Computational Linguistics (LACL) 2021*. pp.159–176.