Chapter 22

Coreference and Bound Variable Anaphora

In this chapter, I will give an overview of the two different use of pronouns, deictic and anaphoric, in preparation for the discussion that will follow in the next few chapters.

22.1 Deictic and Anaphoric Use of Pronouns

Evans (1980) classified the use of pronouns into deictic use and anaphoric use. The difference between the two lies in whether the referent of the pronoun has been linguistically introduced in the preceding context. If no, it is called deictic use, and if yes, it is called anaphoric use (this difference, however, is later shown not to be so clear cut).

22.1.1 Deictic Use

The deictic use of pronouns is known to be discussed by Kaplan (1989) and is also called *indexical* or *demonstrative* use, but the typical usage is with ostension, as in (501).

(501) *(Pointing at John)*
He is born in Detroit.

In other words, in a situation where John is in front of the speaker, by pointing to John while speaking, the speaker is specifying that the *He* refers to John. In this case, in terms of truth conditions, uttering (501) is almost the same as uttering (502).

(502) John is born in Detroit.

The difference between (501) and (502) is the pathway by which the hearer "reaches" John. In (501), the path taken is that of selecting the person being pointed to from the situation at hand, and then reaching the entity representing that person, *john*. In contrast, in (502), the hearer selects a proper name with the surface form *John* from the lexicon and arrives at its semantic representation, *john*.

There can be intermediate cases between them, such as when John is not in front of the speaker, but the speaker utters (501) while pointing to a photograph of John, which is also classified as a deictic use. In this case, the image being pointed to is

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selected from the picture in the situation in front of us, and from the person from whom the image originated, we arrive at the entity representing that person, *john*.

A further intermediate case is when several people, including John, are having coffee and John happens to leave the table, and the topic is "who is from which city," and the speaker points to John's coffee cup and utters "He is born in Detroit." In this case, the cup being pointed to is selected from the table in the situation at hand, and from the person who was using the cup, we arrive at the entity representing that person, *john*.

One feature of deictic use that is common in all the above cases is that it constructs the referent of the pronoun from what is observed in the situation. In this respect, it differs from anaphoric use, which constructs the referent from the linguistic expression in the preceding context. However, as already implied in the discussion above, it is important to note that the referent entity itself need not be present in the situation. Also, to "construct a referent" means to construct a chain of relations that reaches to the referent, as the case of the photograph suggests, which will be discussed later.

22.1.2 Anaphoric Use

In contrast to deictic use, anaphoric use is the use in which the pronoun's referent is constructed from the preceding context or by a linguistic expression that provides a context for the pronoun. Let us take the example of the pronoun *her* in (503).

(503) Every girl enjoys a hobby that excites her.

There are at least two different readings for *her* in (503). The first is a reading in which *her* refers to someone in particular. For example, in a context where all the girls want to please Mary because all of them love her, (503) can be uttered with the same truth conditions as (504).

(504) Every girl enjoys a hobby that excites Mary.

Such a reading of *her* in (503) is called *coreference reading*. It has the name *coreference* since the proper name *Mary* and the *her* in (503) refer to the same entity.

Note that the truth condition of (503) is the same as the FoL formula (505), meaning that the semantic representation of pronouns when having coreference readings are *names* in FoL.

(505) $\forall x(\mathbf{girl}(x) \to \exists y(\mathbf{hobby}(y) \land \mathbf{enjoy}(x,y) \land \mathbf{excite}(y, mary)))$

The second reading of *her* in (503) is a reading in which, for each girl, *her* is that girl. Such a reading of *her* in (503) is called *bound variable anaphora reading, or BVA reading*. The name BVA comes from the fact that, under a BVA reading, the truth condition for (503) is the same as the following FoL formula, where *her* corresponds to a bound variable x in terms of FoL.

(506) $\forall x(\mathbf{girl}(x) \to \exists y(\mathbf{hobby}(y) \land \mathbf{enjoy}(x,y) \land \mathbf{excite}(y, x)))$

Remark 442. How can coreference and BVA be distinguished? One answer is to distinguish by inference. Consider the following inference (507).

(507) Every girl enjoys a hobby that excites her. Sue is a girl.

$\Longrightarrow$ Sue enjoys a hobby that excites Sue.

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When *her* in the first sentence has a coreference reading and refers to *Mary*, the inference is not valid. However, when it has a BVA reading, the inference holds. Analysis by FoL (although our final analysis is not FoL) correctly predicts this fact. Taking the case where *her* in the first sentence has a coreference reading and refers to *Mary*, (507) would be as follows in FoL, which is not valid.

(508) $\forall x(\mathbf{girl}(x) \to \exists y(\mathbf{hobby}(y) \wedge \mathbf{enjoy}(x, y) \wedge \mathbf{excite}(y, mary))),\ \mathbf{girl}(sue)$
$\vdash \exists y(\mathbf{hobby}(y) \wedge \mathbf{enjoy}(sue, y) \wedge \mathbf{excite}(y, sue)))$

On the other hand, if *her* the first sentence has a BVA reading, then this is valid, although the following is true in FoL: (507).

(509) $\forall x(\mathbf{girl}(x) \to \exists y(\mathbf{hobby}(y) \wedge \mathbf{enjoy}(x, y) \wedge \mathbf{excite}(y, x))),\ \mathbf{girl}(sue)$
$\vdash \exists y(\mathbf{hobby}(y) \wedge \mathbf{enjoy}(sue, y) \wedge \mathbf{excite}(y, sue)))$

Exercise 443. Prove (508) and (509).

Conversely, the following inference is valid when *her* in the first sentence has a coreference reading and refers to *Mary*. However, it does not hold when it has a BVA reading.

(510) Every girl enjoys a hobby that excites her. Sue is a girl.
$\Longrightarrow$ Sue enjoys a hobby that excites Mary.

Therefore, it can be seen that the coreference and BVA in the interpretation of the *her* of (503) are independent readings of each other.

Remark 444. BVA reading is also possible for quantificational expressions other than *Every*. Although (511) is the case of *No*, there is also coreference reading and BVA reading for *her* in this sentence, and in BVA reading, for each girl, it means that no girl enjoys a hobby that bores the girl herself. In BVA reading, it means that for each girl, there is no girl who enjoys a hobby that bores her.

(511) No girl enjoyes a hobby that makes her bored.

Exercise 445. For (511), consider an inference that distinguishes between coreference and BVA. Also show that the inference can be predicted by the semantic representation by FoL.

Remark 446. This way of stating that "a reading that produces such an inference is a BVA reading" by listing a number of inferences to a single sentence does not define what a BVA reading is. Instead, it might be tempting to "define" BVA reading as, for example, "a reading in which the indicative object of the pronoun changes in conjunction with the indicative object of the antecedent." However, this definition contains imprecise notions. For example, what is the "denotation" of the antecedent *every girl*? As we have seen in the Chapter 16, quantificational expressions are functions with higher-order types, so there is nothing that "changes in conjunction" with them. More to the point, there is no such denotation here for *no girl*, so this argument does not apply to examples such as (511).

What this circumstance tells us is that, in fact, the notion of BVA cannot be defined empirically. Therefore, the only possible position for the concept of BVA is to characterise it with a pattern of inference that the sentence follows, such as (507) or (510).

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Remark 447. The pronoun used in anaphoric use is called anaphora (*her* in (503)(511)) and the linguistic expression introducing the object to which anaphora refers is called antecedent (*every girl* in (503) and *no girl* in (511)). The relationship established between anaphora and its antecedent is called anaphoric link (but this relationship itself is sometimes called anaphra).

BVA is one of the most common examples of anaphoric use of pronouns, along with E-type anaphora, which will be discussed in the next chapter.

Remark 448. The BVA antecedent must be in the same sentence. In the example below, the BVA reading is not present in the second sentence textither.*[1]

(512) Every girl enjoys a hobby. It excites her.

Even in the same sentence, BVA reading is said not to occur if the antecedent does not C-synthesise the pronoun. There is a BVA reading in the *his* of (513a), but there is no BVA reading in the *his* of (513b). This is an effect called the weak crossover effect (WCO), but the WCO is discussed again in Chapter **??**.*[2]

(513) a. Every boy praised his father.
b. * His father praised every boy.

Remark 449. The meaning of sentences containing pronouns of Deictic use is identical to that of sentences in which pronouns are replaced by proper nouns, as shown by (502). However, it is important to note that the meaning of anaphora is not identical to that of the antecedent. In other words, anaphora is not a copy of the antecedent. For example, the truth condition of (514a) differs from that of (503) and the truth condition of (514b) differs from that of (511).

(514) a. Every girl enjoyes a hobby that excites every girl.
b. No girl enjoyes a hobby that makes no girl bored.

In contrast, the meaning of sentences containing coreference is identical to that of sentences in which pronouns are replaced by proper nouns. In this sense, the question arises as to whether coreference should be categorised as an independent use of a pronoun as a type of anaphora, or whether it is more akin to deictic use. This issue is discussed further in the next section.

22.1.3 More on Coreference

Typical examples of Coreference are as follows, where a noun phrase that appears in the preceding context or in the same sentence (hereafter *John*) and a pronoun (hereafter *he*) refer to the same object. (515) is an example where *John* appears in

*[1] The following exception to this is known to exist, which is called *telescoping*. Telescoping is discussed again in Subsection 24.7.1.

(636) Each candidate for the space misson meets all our requirements. He has a Ph.D. in Astrophysics and extensive prior flight experience.

*[2] For more information on the WCO acceptability swing and control methods in experiments, see Hoji (2015), Plesniak (2022), Fukushima et al. (2023).

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the same sentence as *he*, and (516) is an example where *John* appears in the preceding sentence.

(515) John claimed that he was sick.

(516) John is a student. He is a smart guy.

In both cases, the proper noun (515)(515) and the pronoun (516) refer to the same object (i.e. the actual (515) *he*). However, there has been a debate as to whether the pronoun (here *he*) refers to the linguistic expression John (in the preceding context or in the same sentence) or to John itself, as in deictic use. The former view is that the pronoun refers to the preceding noun phrase, while the latter view is that the pronoun and the preceding noun phrase (by chance) denote the same object.[*3]

In the former view, coreference is the linguistic relationship between a pronoun and its antecedent, traditionally discussed as the operation of replacing one with the other when the same noun phrase appears in two places in a sentence (**?**). In this case, the antecedent must be present in the sentence. In particular, some literature distinguishes between cases where the antecedent literally precedes the pronoun by the antecedent and cases where the antecedent appears after the pronoun by the postcedent.

In the latter view, the denotation of the pronoun is in the knowledge or immediate situation, and the coreference is the denoting relation between the pronoun and the denotation. In this view, the antecedent (and its indicating object) is incidental and it does not matter whether the antecedent appears before or after the pronoun. The second view also means that there is no theoretical distinction between deictic use and coreference of pronouns.

In fact, there are some usages that can clearly be said to be deictic use in refex-EGEaHtEH textither. For example, it is possible to make the following utterance while pointing to Mary nearby.

(517) *(Pointing at Mary)*
Every girl enjoyes a hobby that excites her!

When taken together with the fact that in deictic use, as discussed in the previous section, the object of instruction does not necessarily have to be present in front of us, it is questionable whether this usage can be distinguished from the coreference reading of (**??**).

Also, if we argue that the *He* in *he was sick* and *He is a smart guy* refer to the preceding *He* (not John himself) respectively, when we consider the following example If this is the case, how does the preceding *He* itself reach John?

[*3] In the first place, a proper noun refers to an object in verificationist semantics because 1) the proper noun contributes to the verification condition of the sentence containing it by being associated in the lexical entry of the proper noun in the form that it is mapped to a semantic representation containing the constructor of a particular Entity, and 2) the constructor contributes to the verification condition of the sentence containing it by being associated with the constructor of the Entity. This means that the constructor contributes to the validation conditions of the sentence containing it. Here, the proper noun *John* is mapped to the semantic representation $\lambda p.p(john)$, so the semantic representation of the sentence containing it contains *john*, but in that semantic representation *john* is one of the e-type It means that *john* behaves as one of the *constructors* of type e.

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(518) a. He claimed that he was sick.
b. He is a student. He is a smart guy.

If we put these things together, there is merit in the analysis that the use of the pronoun *coreference* should be categorised in the same bracket as deictic use, and that there is or is not ostension depending on whether there happens to be an indicating object in front of us or not. The analysis would have advantages.

Put another way, the linguistic relation assumed by the first view is surplus. If there is deictic use in pronoun and a relation between pronoun and denotation established there, then pronoun should be able to establish that relation with denotation in the syntax of coreference, therefore, because any theory of deictic use would also explain the case of coreference. For these reasons, the DTS takes the position that coreference is not to be distinguished from deictic use.

22.1.4 Toward a Unified Theory of Pronouns

The existence of pronouns is a challenge to the theory of meaning. This is because the usage of pronouns is ambiguous in most cases and thereby the way the indicative object/ antecedent is looked for and the way it is represented in the theory.

For each sentence containing an anaphoric expression, A theory of anaphora has to provide the following:

1. A proper semantic representation that captures its meaning
2. An algorithm for determining its antecedent with:
   - An algorithm to enumerate its accessible antecedents
   - An algorithm to disambiguate between accessible antecedents

It has not been clear for many years what theory could represent this in an integrated way. The theory that resolves this is UDTT, introduced in the previous section. Up to this point, the preterm of DTT has been used for semantic representation, but from the next section onwards this will be changed to the preterm of UDTT. The semantic felicity condition is then required as a judgment in UDTT.

With this change, semantic representations containing underspecified types (i.e. semantic representations of sentences containing pronouns) will invoke @-rule in the middle of a proof diagram to show that the semantic felicity condition is satisfied. This requires that the proof (in that context) of the proposition required by the prounoun is valid. This is linguistically synonymous with the existence of an (accessible) antecedent in prounoun, so in DTS, the existence of an antecedent in prounoun is part of the SFC.

On the other hand, semantic representations that do not contain underspecified types are not affected by this change. This is because the SFC for semantic representations that do not contain underspecified types by refthUDTT3 is still a proof diagram for DTT.

From the next section onwards, the semantic representations of concrete pronouns are given as UDTT preterms, while looking at the process by which the meanings of the pronouns discussed so far are interpreted through synthesis from syntactic structures to semantic representations.

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22.2 Lexicalizing Pronouns

With underspecified types, the lexical items for pronouns in $\star$-CCG are given as follows.[*4]

(519) a. $[\![\text{he} \vdash NP^{\star}]\!] \stackrel{def}{\equiv} \left[\begin{array}{l} u@\left[\begin{array}{l} x:\mathsf{e} \\ \mathbf{male}(x) \end{array}\right] \\ \boxed{\pi_1 u} \end{array}\right]$

b. $[\![\text{she} \vdash NP^{\star}]\!] \stackrel{def}{\equiv} \left[\begin{array}{l} u@\left[\begin{array}{l} x:\mathsf{e} \\ \mathbf{female}(x) \end{array}\right] \\ \boxed{\pi_1 u} \end{array}\right]$

c. $[\![\text{it} \vdash NP^{\star}]\!] \stackrel{def}{\equiv} \left[\begin{array}{l} u@\left[\begin{array}{l} x:\mathsf{e} \\ \neg\mathbf{human}(x) \end{array}\right] \\ \boxed{\pi_1 u} \end{array}\right]$

The semantic representation in (519a) is an example of the use of an underspecified type, assuming that it has a proof, that is, a pair of some entity and a proof of its being a male, which is a witness of the presupposition triggered by the use of the pronoun he.

The key point in (519) is that the pronouns are scope takers. In other words, their semantic representations are in the form of a 2-level box in $\star$-CCG, with underspecification at the second level. This is important feature when analysing the WCO phenomenon, which will later be discussed. Pronouns other than nominative cases can be defined in the same way as follows.

(520) a. $[\![\text{him} \vdash NP^{\star}]\!] \stackrel{def}{\equiv} \left[\begin{array}{l} v@\left[\begin{array}{l} y:\mathsf{e} \\ \mathbf{male}(y) \end{array}\right] \\ \boxed{\pi_1 v} \end{array}\right]$

b. $[\![\text{her} \vdash NP^{\star}]\!] \stackrel{def}{\equiv} \left[\begin{array}{l} v@\left[\begin{array}{l} y:\mathsf{e} \\ \mathbf{female}(y) \end{array}\right] \\ \boxed{\pi_1 v} \end{array}\right]$

22.3 Anaphora Resolution as Proof Construction

(ToDo: Historical remarks on Krause (1995), Krahmer and Piwek (1999) and Piwek and Krahmer (2000))

[*4] The square brackets for underspecified types are syntactically distinguished from those of the Σ-type notation, but when these square brackets nest, they may be omitted except for the outermost one in the same way as brackets in Σ-type notation, as follows.

$$\left[\begin{array}{l} x:A \\ y@B \\ C(x,y) \end{array}\right] \equiv \left[\begin{array}{l} x:A \\ \left[\begin{array}{l} y@B \\ C(x,y) \end{array}\right] \end{array}\right]$$

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22.3.1 Case of Deictic Use

First, let us look at the process by which DTS searches for the referent of a pronoun in deictic use.

(501) *(Pointing at John)*

He is born in Detroit.

The syntactic structure and semantic composition in $\star$-CCG is as follows. Here, for the sake of simplicity, *born in Detroit* is treated as a one-word predicate, but this reduction does not affect the analysis.

(521) Syntactic structure of (501)

$$\dfrac{\dfrac{\text{He}}{NP^\star} \qquad \dfrac{\dfrac{\text{is}}{(S\backslash NP/(VP\backslash NP))^\star} \quad \dfrac{\text{born in Detroit}}{(VP\backslash NP)^\star}}{(S\backslash NP)^\star}\,{>^\star}}{\dfrac{S^\star}{S}\,\checkmark}\,{<^\star}$$

(522) Semantic composition of (501)

$$\dfrac{\dfrac{\dfrac{\text{He}}{\left[\begin{array}{l} u@\left[\begin{array}{l} x:\mathsf{e}\\ \mathbf{male}(x)\end{array}\right] \\ \boxed{\pi_1 u}\end{array}\right]} \qquad \dfrac{\dfrac{\text{is}}{\boxed{\lambda p.\lambda x.\left[\begin{array}{l} s:\mathsf{e}\\ pxs\end{array}\right]}} \quad \dfrac{\text{born in Detroit}}{\boxed{\lambda x.\lambda s.\mathbf{bornInDetroit}(s,x)}}}{\begin{array}{l}\boxed{\lambda x.\left[\begin{array}{l} s:\mathsf{e}\\ \mathbf{bornInDetroit}(s,x)\end{array}\right]}\\ \equiv \boxed{\lambda x.\mathbf{bornInDetroit}^*_s(x)}\end{array}}\,{>^\star}}{\left[\begin{array}{l} u@\left[\begin{array}{l} x:\mathsf{e}\\ \mathbf{male}(x)\end{array}\right] \\ \boxed{\mathbf{bornInDetroit}^*_s(\pi_1 u)}\end{array}\right]}\,{<^\star}}{\left[\begin{array}{l} u@\left[\begin{array}{l} x:\mathsf{e}\\ \mathbf{male}(x)\end{array}\right] \\ \mathbf{bornInDetroit}^*_s(\pi_1 u)\end{array}\right]}\,\checkmark$$

Recall that $*$-operator defined in Section 19.3.5. We will use the $*$-operator as above for the sake of space. The semantic felicitiy condition for (501) under a given context Γ is the following.

(523) $$\Gamma \vdash \left[\begin{array}{l} u@\left[\begin{array}{l} x:\mathsf{e}\\ \mathbf{male}(x)\end{array}\right] \\ \mathbf{bornInDetroit}^*_e(\pi_1 u)\end{array}\right] : \mathsf{type}$$

Note that (523) is a UDTT judgment, not a DTT judgment, as described in Subsection 22.1.4. In the discussion in this and subsequent chapters, SFC will be stated as a judgment of UDTT. Recall that UDTT's signuatures and contexts consist of DTT preterms.

Now, let us assume that e is an enumeration type defined as $\mathsf{e} \stackrel{def}{\equiv} \{john, bill, \ldots\}$ and that the following signature is given, namely, **male** is a one-place predicate, and m_j and m_b are proofs of John and Bill being male persons, respectively.

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(524) $\sigma \overset{def}{\equiv} \mathbf{male} : \mathsf{e} \to \mathsf{type}, m_j : \mathbf{male}(john),\ m_b : \mathbf{male}(bill)$

This setting provides a minimal situation where the referent of *he* in deictic use is ambiguous between *john* and *bill*.

Exercise 450. Check the c.o.v. of the signature σ above.

If (523) is given as a type check query, it returns a proof diagram as follows, such that the bottom line is the (@)-rule, where $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ are UDTT proof diagrams.

(525) Type check diagram of (501)

$$\dfrac{\overset{\mathcal{D}_1}{\begin{bmatrix} x:\mathsf{e} \\ \mathbf{male}(x)\end{bmatrix} : \mathsf{type}} \quad \overset{\mathcal{D}_2}{?:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{male}(x)\end{bmatrix}} \quad \overset{\mathcal{D}_3}{\mathbf{bornInDetroit}^*_s(\pi_1 u)[?/u] : \mathsf{type}}}{\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e}\\ \mathbf{male}(x)\end{bmatrix} \\ \mathbf{bornInDetroit}^*_s(\pi_1 u)\end{bmatrix} : \mathsf{type}}\,(@)$$

The configuration of $\mathcal{D}_1$ is straightforward, as shown below.

(526) Proof diagram $\mathcal{D}_1 \equiv$

$$\dfrac{\dfrac{}{\mathsf{e}:\mathsf{type}}\,(\{\}F) \qquad \dfrac{\dfrac{}{\mathbf{male}:\mathsf{e}\to\mathsf{type}}\,(CON) \quad \dfrac{}{x:\mathsf{e}}{}^{1}}{\mathbf{male}(x):\mathsf{type}}\,(\Pi E)}{\begin{bmatrix}x:\mathsf{e}\\ \mathbf{male}(x)\end{bmatrix}:\mathsf{type}}\,(\Sigma F)$$

Since this proof diagram is not changed by @-elimination (by Theorem 440), the preterm on the right-hand side of the bottom line of $\mathcal{D}_2$ equals to the preterm (= type) on the left-hand side of the bottom line of $\mathcal{D}_1$. For (501) to satisfy SFC, a proof $\mathcal{D}_2$ of this type must be constructed under the given context.[*5]

Linguistically, the construction of $\mathcal{D}_2$ corresponds to searching for a referent in (501)'s *He*. Deictic use corresponds to constructing the referent from the signature σ: Currently, there are two male entities, *john* and *bill*, in the assumed signature. Therefore, The proof search above returns the following two proof diagrams.

(527) Proof diagram $\mathcal{D}_j \equiv$

$$\dfrac{\dfrac{}{john:\mathsf{e}}\,(\{\}I) \qquad \dfrac{}{m_j:\mathbf{male}(john)}\,(CON)}{(john, m_j):\begin{bmatrix}x:\mathsf{e}\\ \mathbf{male}(x)\end{bmatrix}}\,(\Sigma I)$$

[*5] This definition of (@)-rule implies that the type inference/checking of UDTT involves a construction of a proof, which gives rise to the undecidability of type inference/checking. This may be regarded as a potential problem of DTS from the engeneering perspective. However, from the linguistic perspective, this is on the right track because the human process of natural language understanding is undecidable by itself. The construction of a proof is exactly what a hearer has to calculate to determine the antecedent of pronouns.

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(528) Proof diagram $\mathcal{D}_b \equiv$

$$\dfrac{\dfrac{}{bill : \mathsf{e}}\,(\{\}I) \qquad \dfrac{}{m_b : \mathbf{male}(bill)}\,(CON)}{(bill, m_b) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix}}\,(\Sigma I)$$

The two proof diagrams have different proof terms, but in $\mathcal{D}_3$, the proof is supposed to replace u in $\mathbf{bornInDetroit}^*_s(\pi_1 u)$, so depending on whether $\mathcal{D}_2 = \mathcal{D}_j$ or $\mathcal{D}_2 = \mathcal{D}_b$, the conclusion of $\mathcal{D}_3$ will change. By β-reducing them, the calculation proceeds as follows for each of the above examples.

(529)
$$\begin{aligned} \mathbf{bornInDetroit}^*_s(\pi_1 u)[(john, m_j)/u] &\equiv \mathbf{bornInDetroit}^*_s(\pi_1(john, m_j)) \\ &\to_\beta \mathbf{bornInDetroit}^*_s(john) \end{aligned}$$

(530)
$$\begin{aligned} \mathbf{bornInDetroit}^*_s(\pi_1 u)[(bill, m_b)/u] &\equiv \mathbf{bornInDetroit}^*_s(\pi_1(bill, m_b)) \\ &\to_\beta \mathbf{bornInDetroit}^*_s(bill) \end{aligned}$$

In each case, the overall proof diagram is as follows.

(531) Type check diagram of (501) for (529)

$$\dfrac{\begin{matrix} \mathcal{D}_1 \\ \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} : \mathsf{type} \end{matrix} \quad \begin{matrix} \mathcal{D}_j \\ (j, m_j) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \end{matrix} \quad \dfrac{\dfrac{\dfrac{}{\begin{matrix} \mathbf{bornInDetroit}^*_s \\ : \mathsf{e} \to \mathsf{type} \end{matrix}}\,(CON) \quad \dfrac{}{john : \mathsf{e}}\,(\{\}F)}{\mathbf{bornInDetroit}^*_s(john) : \mathsf{type}}\,(\Pi E)}{\mathbf{bornInDetroit}^*_s(\pi_1 u)[(j, m_j)/u] : \mathsf{type}}\,(CONV)}{\begin{bmatrix} u@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{bornInDetroit}^*_s(\pi_1 u) \end{bmatrix} : \mathsf{type}}\,(@)$$

(532) Type check diagram of (501) for (530)

$$\dfrac{\begin{matrix} \mathcal{D}_1 \\ \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} : \mathsf{type} \end{matrix} \quad \begin{matrix} \mathcal{D}_b \\ (b, m_b) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \end{matrix} \quad \dfrac{\dfrac{\dfrac{}{\begin{matrix} \mathbf{bornInDetroit}^*_s \\ : \mathsf{e} \to \mathsf{type} \end{matrix}}\,(CON) \quad \dfrac{}{bill : \mathsf{e}}\,(\{\}F)}{\mathbf{bornInDetroit}^*_s(bill) : \mathsf{type}}\,(\Pi E)}{\mathbf{bornInDetroit}^*_s(\pi_1 u)[(b, m_b)/u] : \mathsf{type}}\,(CONV)}{\begin{bmatrix} u@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{bornInDetroit}^*_s(\pi_1 u) \end{bmatrix} : \mathsf{type}}\,(@)$$

This ensures that the SFC is satisfied in each case, because for any UDTT proof diagram $\Gamma \vdash M : \mathsf{type}$, we obtain a DTT proof diagram of the form $\Gamma \vdash M' : \mathsf{type}$ by applying (@)-elimination. In this caes, by applying @-elimination to the above UDTT proof diagrams, the following DTT proof diagrams are obtained.

(533) @-elimination of (501) for (529)

$$\dfrac{\dfrac{}{\begin{matrix} \mathbf{bornInDetroit}^*_s \\ : \mathsf{e} \to \mathsf{type} \end{matrix}}\,(CON) \qquad \dfrac{}{john : \mathsf{e}}\,(\{\}F)}{\mathbf{bornInDetroit}^*_s(john) : \mathsf{type}}\,(\Pi E)$$

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(534) @-elimination of (501) for (530)

$$\dfrac{\dfrac{}{\begin{array}{l}\mathbf{bornInDetroit}^*_s\\ : \mathsf{e} \to \mathsf{type}\end{array}}{\scriptstyle (CON)} \qquad \dfrac{}{bill : \mathsf{e}}{\scriptstyle (\{\}F)}}{\mathbf{bornInDetroit}^*_s(bill) : \mathsf{type}}{\scriptstyle (\Pi E)}$$

Therefore, the semantic representation of (501) is $\mathbf{bornInDetroit}^*_s(john)$ or $\mathbf{bornInDetroit}^*_s(bill)$, which correspond to different referents of the pronoun *He*. In this way, (501) has two different deictic readings, each of which is represented as a distinct semantic representation in DTT: the first means the same as *John is born in Detroit*, while the second means the same as *Bill is born in Detroit*.

Remark 451. SFC in DTS requires that a semantic representation obtained as a result of semantic composition is a preterm of type, which is imposed as a condition for the well-formedness of semantics in DTS. Therefore, if there is no proof diagram that satisfies the second premise of the (@)-rule, that is, if the pronoun does not find a reference satisfying the condition, then the sentence/discourse is neither true nor false, but *semantically ill-formed*.

Remark 452. As the example (501) shows, multiple proof diagrams may be found as proof diagrams that (@)-rule requries. This corresponds to the existence of multiple candidates as the referents of a pronoun, which is based on the assumption that the theory of meaning should be able to enumerate all possible readings.

In actual linguistic usage, however, even when a pronoun has more than one referent candidates, the appropriate referent is selected by contextual and personal preferences. This corresponds to the selection of the most appropriate proof diagram under a context from the multiple candidates obtained as a result of proof search, and ranking them in some way or another, but this task rather belongs to pragmatics and is outside the scope of DTS as a semantic theory. Such computation has also been studied in natural language processing as a task called *anaphora resolution*. But it is important to note that DTS provides a set of possible referents (as semantic representations satisfying SFC) that anaphora resolution should find a solution from.

Remark 453. In the situation where (501) is uttered, the speaker's pointing to John dismbiguates the referent. If we integrate this aspects into the theory, the information about ostention could be added explicitly to the signature and the context as follows.

(535) $\sigma,\ \mathbf{point} : \mathsf{e} \to \mathsf{e} \to \mathsf{type}$

(536) $\Gamma,\ ost : \mathbf{point}(speaker, john)$

Let $\mathbf{point}(x, y)$ be a propositon that the entity x points to y, then ost is the proof that the speaker is pointing to John. The lexical entry for pronouns with ostension should then be changed as in (537).

(537) $$[\![\text{he} \vdash NP^\star]\!] \stackrel{def}{\equiv} \left[\begin{array}{l} u@\left[\begin{array}{l} x : \mathsf{e} \\ \mathbf{male}(x) \end{array}\right] \\ v@\mathbf{point}(speaker, \pi_1 u) \\ \boxed{\pi_1 u} \end{array}\right]$$

Exercise 454. As an alternative to the above analysis, define ostention as a unary rule applied to constituents of the syntactic type $NP^\star$. Note that such an unary rule

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can be applied to quantifiers such as *every boy* if no other constraints are imposed, but examine its empirical consequences.

22.3.2 Case of Coreference/BVA

Next, we consider coreference and BVA readings of pronouns. Again, we take (503) as an example.

(503) Every girl enjoys a hobby that excites her.

We assume that **entity** $\stackrel{def}{\equiv} \{mary, susan, \ldots\}$, and the following signature.

(538)
$$\sigma \stackrel{def}{\equiv} \begin{array}{rcl} \textbf{female} & : & \mathrm{e} \to \mathrm{type}, \\ \textbf{hobby} & : & \mathrm{e} \to \mathrm{type}, \\ \textbf{enjoy} & : & \mathrm{e} \times \mathrm{e} \times \mathrm{e} \to \mathrm{type}, \\ \textbf{excite} & : & \mathrm{e} \times \mathrm{e} \times \mathrm{e} \to \mathrm{type}, \\ f_m & : & \textbf{female}(mary), \\ f_s & : & \textbf{female}(susan), \\ gf & : & (u : (x : \mathrm{e}) \times \textbf{girl}) \to \textbf{female}(\pi_1 u) \end{array}$$

gf is a function, representing the world knowledge that every girl is female (by definition of the word). The syntactic structure and semantic composition of (503) is as follows.

(539) Syntactic structure of (503)

$$\begin{prooftree}
\AxiomC{Every}\UnaryInfC{$NP^\star/N$}
\AxiomC{girl}\UnaryInfC{N}
\RightLabel{$>$}\BinaryInfC{$NP^\star$}
\AxiomC{enjoys}\UnaryInfC{$(S\backslash NP/NP)^\star$}
\AxiomC{a}\UnaryInfC{$NP^\star/N$}
\AxiomC{hobby}\UnaryInfC{N}
\AxiomC{that}\UnaryInfC{$N\backslash N/(S\backslash NP)^\star$}
\AxiomC{excites}\UnaryInfC{$(S\backslash NP/NP)^\star$}
\AxiomC{her}\UnaryInfC{$NP^\star$}
\RightLabel{$>^\star$}\BinaryInfC{$(S\backslash NP)^\star$}
\RightLabel{$>$}\BinaryInfC{$N\backslash N$}
\RightLabel{$<$}\BinaryInfC{N}
\RightLabel{$>$}\BinaryInfC{$NP^\star$}
\RightLabel{$>^\star$}\BinaryInfC{$(S\backslash NP)^\star$}
\RightLabel{$<^\star$}\BinaryInfC{$S^\star$}
\RightLabel{$\checkmark$}\UnaryInfC{S}
\end{prooftree}$$

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(540) Semantic composition of (503)

$$\dfrac{\dfrac{\text{excites}}{\boxed{\begin{array}{l}\lambda y.\lambda x.\\ \mathbf{excite}^*_{e_2}(x,y)\end{array}}} \quad \dfrac{\text{her}}{\begin{bmatrix} w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \boxed{\pi_1 w}\end{bmatrix}}}{\begin{bmatrix} w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \boxed{\lambda x.\mathbf{excite}^*_{e_2}(x,\pi_1 w)}\end{bmatrix}}\ {>^\star}$$

$$\dfrac{\dfrac{\text{that}}{\begin{array}{l}\lambda q.\lambda n.\lambda x.\\ \begin{bmatrix} nx\\ q(\lambda p.px)\end{bmatrix}\end{array}} \quad \begin{bmatrix} w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \boxed{\lambda x.\mathbf{excite}^*_{e_2}(x,\pi_1 w)}\end{bmatrix}}{\lambda n.\lambda x.\begin{bmatrix} nx\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(x,\pi_1 w)\end{bmatrix}}\ {>}$$

$$\dfrac{\dfrac{\text{hobby}}{\mathbf{hobby}} \quad \lambda n.\lambda x.\begin{bmatrix} nx\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(x,\pi_1 w)\end{bmatrix}}{\lambda x.\begin{bmatrix} \mathbf{hobby}(x)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(x,\pi_1 w)\end{bmatrix}}\ {<}$$

$$\dfrac{\dfrac{\text{a}}{\lambda n.\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ n(y)\end{bmatrix}\\ \boxed{\pi_1 v}\end{bmatrix}} \quad \lambda x.\begin{bmatrix} \mathbf{hobby}(x)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(x,\pi_1 w)\end{bmatrix}}{\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}\\ \boxed{\pi_1 v}\end{bmatrix}}\ {>}$$

(541) Semantic composition of (503)

$$\dfrac{\dfrac{\text{enjoys}}{\boxed{\begin{array}{l}\lambda y.\lambda x.\\ \mathbf{enjoy}^*_{e_1}(x,y)\end{array}}} \quad \dfrac{\text{a hobby that excites her}}{\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}\\ \boxed{\pi_1 v}\end{bmatrix}}}{\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}\\ \boxed{\lambda x.\mathbf{enjoy}^*_{e_1}(x,\pi_1 v)}\end{bmatrix}}\ {>^\star}$$

$$\dfrac{\dfrac{\text{Every}}{\lambda n.\left(u:\begin{bmatrix} x:\mathsf{e}\\ n(x)\end{bmatrix}\right)\to\boxed{\pi_1 u}} \quad \dfrac{\text{girl}}{\mathbf{girl}}}{\left(u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}\right)\to\boxed{\pi_1 u}}\ {>}$$

$$\dfrac{\dfrac{\left(u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}\right)\to\boxed{\pi_1 u} \quad \begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}\\ \boxed{\lambda x.\mathbf{enjoy}^*_{e_1}(x,\pi_1 v)}\end{bmatrix}}{\left(u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}\right)\to\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}\\ \boxed{\mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)}\end{bmatrix}}\ {<^\star}}{\left(u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}\right)\to\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}\\ \mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)\end{bmatrix}}\ \checkmark$$

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The semantic felicity condition under a given context requires the following:

(542) Type check diagram of (503)

$$
\dfrac{
\begin{array}{c}\vdots \\ \begin{bmatrix} x:\mathrm{e} \\ \mathbf{girl}(x) \end{bmatrix}:\mathrm{type}\end{array}
\quad
\dfrac{
\dfrac{
\dfrac{}{\mathrm{e}:\mathrm{type}}{\scriptstyle(\{\}F)}
\quad
\dfrac{
\begin{array}{c}\dfrac{}{y:\mathrm{e}}{}^{3} \\ \vdots \\ \mathbf{hobby}(y):\mathrm{type}\end{array}
\quad
\dfrac{
\begin{bmatrix} z:\mathrm{e} \\ \mathbf{female}(z) \end{bmatrix}:\mathrm{type}
\quad
\begin{array}{c} \dfrac{}{u:\begin{bmatrix} x:\mathrm{e} \\ \mathbf{girl}(x) \end{bmatrix}}{}^{1} \quad \dfrac{}{y:\mathrm{e}}{}^{3} \\ \mathcal{D}_1 \\ ?:\begin{bmatrix} z:\mathrm{e} \\ \mathbf{female}(z) \end{bmatrix}\end{array}
\quad
\begin{array}{c}\mathcal{D}_2 \\ \mathbf{excite}^*_e(y,\pi_1 w)[?/w]:\mathrm{type}\end{array}
}{
\begin{bmatrix} w@\begin{bmatrix} z:\mathrm{e} \\ \mathbf{female}(z) \end{bmatrix} \\ \mathbf{excite}^*_e(y,\pi_1 w) \end{bmatrix}:\mathrm{type}
}{\scriptstyle(@)}
}{
\begin{bmatrix} \mathbf{hobby}(y) \\ w@\begin{bmatrix} z:\mathrm{e} \\ \mathbf{female}(z) \end{bmatrix} \\ \mathbf{excite}^*_e(y,\pi_1 w) \end{bmatrix}:\mathrm{type}
}{\scriptstyle(\Sigma F)}
}{
\begin{bmatrix} y:\mathrm{e} \\ \mathbf{hobby}(y) \\ w@\begin{bmatrix} z:\mathrm{e} \\ \mathbf{female}(z) \end{bmatrix} \\ \mathbf{excite}^*_{e_2}(y,\pi_1 w) \end{bmatrix}:\mathrm{type}
}{\scriptstyle(\Sigma F),3}
\quad
\begin{array}{c}
\dfrac{}{u:\begin{bmatrix} x:\mathrm{e} \\ \mathbf{girl}(x) \end{bmatrix}}{}^{1}
\quad
\dfrac{}{v:\begin{bmatrix} y:\mathrm{e} \\ \mathbf{hobby}(y) \\ w@\begin{bmatrix} z:\mathrm{e} \\ \mathbf{female}(z) \end{bmatrix} \\ \mathbf{excite}^*_{e_2}(y,\pi_1 w) \end{bmatrix}}{}^{2}
\\ \vdots \\
\mathbf{enjoy}^*_e(\pi_1 u,\pi_1 v):\mathrm{type}
\end{array}
}{
\begin{bmatrix} v:\begin{bmatrix} y:\mathrm{e} \\ \mathbf{hobby}(y) \\ w@\begin{bmatrix} z:\mathrm{e} \\ \mathbf{female}(z) \end{bmatrix} \\ \mathbf{excite}^*_{e_2}(y,\pi_1 w) \end{bmatrix} \\ \mathbf{enjoy}^*_e(\pi_1 u,\pi_1 v) \end{bmatrix}:\mathrm{type}
}{\scriptstyle(\Sigma F),2}
}{
\left(u:\begin{bmatrix} x:\mathrm{e} \\ \mathbf{girl}(x) \end{bmatrix}\right)\to\begin{bmatrix} v:\begin{bmatrix} y:\mathrm{e} \\ \mathbf{hobby}(y) \\ w@\begin{bmatrix} z:\mathrm{e} \\ \mathbf{female}(z) \end{bmatrix} \\ \mathbf{excite}^*_{e_2}(y,\pi_1 w) \end{bmatrix} \\ \mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v) \end{bmatrix}:\mathrm{type}
}{\scriptstyle(\Pi F),1}
$$

The $\mathcal{D}_1$ corresponds to the search for the antecedent of *her* in (503). There are at least two solutions to the proof term that satisfy it, $(mary, f_m)$ and $(\pi_1 u, gf(u))$. The following $\mathcal{D}_m$ and $\mathcal{D}_u$ are the respective proof diagrams.

(543) Proof diagram $\mathcal{D}_m \equiv$

$$
\dfrac{\dfrac{}{mary:\mathrm{e}}{\scriptstyle(\{\}I)} \quad \dfrac{}{f_m:\mathbf{female}(mary)}{\scriptstyle(CON)}}{(mary, f_m):\begin{bmatrix} z:\mathrm{e} \\ \mathbf{female}(z) \end{bmatrix}}{\scriptstyle(\Sigma I)}
$$

(544) Proof diagram $\mathcal{D}_u \equiv$

$$
\dfrac{
\dfrac{\dfrac{}{u:\begin{bmatrix} x:\mathrm{e} \\ \mathbf{girl} \end{bmatrix}}{}^{1}}{\pi_1 u:\mathrm{e}}{\scriptstyle(\Sigma E)}
\quad
\dfrac{\dfrac{}{gf:\left(u:\begin{bmatrix} x:\mathrm{e} \\ \mathbf{girl} \end{bmatrix}\right)\to\mathbf{female}(\pi_1 u)}{\scriptstyle(CON)} \quad u:\begin{bmatrix} x:\mathrm{e} \\ \mathbf{girl} \end{bmatrix}}{gf(u):\mathbf{female}(\pi_1 u)}{\scriptstyle(\Pi E)}
}{
(\pi_1 u, gf(u)):\begin{bmatrix} z:\mathrm{e} \\ \mathbf{female}(z) \end{bmatrix}
}{\scriptstyle(\Sigma I)}
$$

(543) corresponds to one of the cases where the antecedent of the *her* is Mary, i.e. one of the coreference readings, while (544) corresponds to the case where the antecedent of the *her* is *every girl*, i.e. the BVA reading. In the proof (543), the entity in question comes from a constructor of the enumeration type, whereas in (544), it comes from a variable in the context. The corresponding $\mathcal{D}_2$ in each case is as follows.

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(545) Type check diagram of (503): $\mathcal{D}_2$ for Coreference reading via (543)

$$\dfrac{\dfrac{\dfrac{}{\mathbf{excite}^*_{e_2} : \mathsf{e} \to \mathsf{e} \to \mathsf{type}}\,(CON) \qquad \dfrac{\overset{\mathcal{D}_m}{(mary, f_m) : \begin{bmatrix} z : \mathsf{e} \\ \mathbf{female}(z) \end{bmatrix}}}{\pi_1(mary, f_m) : \mathsf{e}}\,(\Sigma E)}{\mathbf{excite}^*_{e_2}(\pi_1(mary, f_m)) : \mathsf{e} \to \mathsf{type}}\,(\Pi E) \qquad \overline{y : \mathsf{e}}^{3}}{\begin{array}{l}\mathbf{excite}^*_{e_2}(y, \pi_1 w)[(mary, f_m)/w] \\ \equiv \mathbf{excite}^*_{e_2}(y, \pi_1(mary, f_m)) : \mathsf{type}\end{array}}\,(\Pi E)$$

(546) Type check diagram of (503): $\mathcal{D}_2$ for BVA reading via (544)

$$\dfrac{\dfrac{\dfrac{}{\mathbf{excite}^*_{e_2} : \mathsf{e} \to \mathsf{e} \to \mathsf{type}}\,(CON) \qquad \dfrac{\begin{array}{c} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{girl}(x) \end{bmatrix} \\ \mathcal{D}_u \\ (\pi_1 u, gf(u)) : \begin{bmatrix} z : \mathsf{e} \\ \mathbf{female}(z) \end{bmatrix}\end{array}}{\pi_1(\pi_1 u, gf(u)) : \mathsf{e}}\,(\Sigma E)}{\mathbf{excite}^*_{e_2}(\pi_1(\pi_1 u, gf(u))) : \mathsf{e} \to \mathsf{type}}\,(\Pi E) \qquad y : \mathsf{e}}{\begin{array}{l}\mathbf{excite}^*_{e_2}(y, \pi_1 w)[(\pi_1 u, gf(u))/w] \\ \equiv \mathbf{excite}^*_{e_2}(y, \pi_1(\pi_1 u, gf(u))) : \mathsf{type}\end{array}}\,(\Pi E)$$

Applying @-elimination to the each proof diagram, the following two DTT proof diagrams are obtained.

(547) @-elimination of (503) for Coreference reading via (543)

$$\dfrac{\begin{array}{c}\vdots \\ \begin{bmatrix} x : \mathsf{e} \\ \mathbf{girl}(x) \end{bmatrix} : \mathsf{type}\end{array} \qquad \dfrac{\dfrac{\dfrac{}{\mathsf{e} : \mathsf{type}}\,(\{\}F) \qquad \dfrac{\begin{array}{c}\overline{y : \mathsf{e}}^{3} \\ \vdots \\ \mathbf{hobby}(y) : \mathsf{type}\end{array} \qquad \begin{array}{c}\overline{y : \mathsf{e}}^{3} \\ \mathcal{D}_2 \\ \mathbf{excite}^*_{e_2}(y, mary)\end{array}}{\begin{bmatrix} \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, mary) \end{bmatrix} : \mathsf{type}}\,(\Sigma F)}{\begin{bmatrix} y : \mathsf{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, mary) \end{bmatrix} : \mathsf{type}}\,(\Sigma F),3 \qquad \begin{array}{c}\overline{u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{girl}(x) \end{bmatrix}}^{1} \quad \overline{v : \begin{bmatrix} y : \mathsf{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, mary) \end{bmatrix}}^{2} \\ \vdots \\ \mathbf{enjoy}^*_{e}(\pi_1 u, \pi_1 v) : \mathsf{type}\end{array}}{\begin{bmatrix} v : \begin{bmatrix} y : \mathsf{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, mary) \end{bmatrix} \\ \mathbf{enjoy}^*_{e}(\pi_1 u, \pi_1 v) \end{bmatrix} : \mathsf{type}}\,(\Sigma F),2}{\left(u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{girl}(x) \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \mathsf{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, mary) \end{bmatrix} \\ \mathbf{enjoy}^*_{e_1}(\pi_1 u, \pi_1 v) \end{bmatrix} : \mathsf{type}}\,(\Pi F),1$$

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(548) @-elimination of (503) for BVA reading via (544)

$$
\dfrac{\begin{array}{c}\vdots\\ \begin{bmatrix} x:\mathrm{e}\\ \mathbf{girl}(x)\end{bmatrix}:\mathrm{type}\end{array}
\quad
\dfrac{\dfrac{\dfrac{}{\mathrm{e}:\mathrm{type}}(\{\}F)
\quad
\dfrac{\begin{array}{c}\overline{y:\mathrm{e}}^{3}\\ \vdots\\ \mathbf{hobby}(y):\mathrm{type}\end{array}
\quad
\begin{array}{c}\overline{y:\mathrm{e}}^{3}\ \overline{u:\begin{bmatrix} x:\mathrm{e}\\ \mathbf{girl}(x)\end{bmatrix}}^{1}\\ \mathcal{D}_2\\ \mathbf{excite}^*_{e_2}(y,\pi_1 u)\end{array}}
{\begin{bmatrix}\mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,\pi_1(\pi_1 u, gf(u)))\end{bmatrix}:\mathrm{type}}(\Sigma F)}
{\begin{bmatrix} y:\mathrm{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,\pi_1(\pi_1 u, gf(u)))\end{bmatrix}:\mathrm{type}}(\Sigma F),3
\quad
\begin{array}{c}\overline{u:\begin{bmatrix} x:\mathrm{e}\\ \mathbf{girl}(x)\end{bmatrix}}^{1}\quad \overline{v:\begin{bmatrix} y:\mathrm{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,\pi_1 u)\end{bmatrix}}^{2}\\ \vdots\\ \mathbf{enjoy}^*_{e}(\pi_1 u,\pi_1 v):\mathrm{type}\end{array}}
{\begin{bmatrix} v:\begin{bmatrix} y:\mathrm{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,\pi_1 u)\end{bmatrix}\\ \mathbf{enjoy}^*_{e}(\pi_1 u,\pi_1 v)\end{bmatrix}:\mathrm{type}}(\Sigma F),2}
{\left(u:\begin{bmatrix} x:\mathrm{e}\\ \mathbf{girl}(x)\end{bmatrix}\right)\to\begin{bmatrix} v:\begin{bmatrix} y:\mathrm{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,\pi_1 u)\end{bmatrix}\\ \mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)\end{bmatrix}:\mathrm{type}}(\Pi F),1
$$

In both cases, the semantic representations of the sentence (503) appear at the bottom, left-hand side of the proof diagram (547) and (548). It can be read off that the first semantic representation corresponds to a coreference reading where the referent is *mary*, and the second to a BVA reading where the antecedent is *every girl*. The difference between these readings lies in the different proofs for the proof construction of a female entity required by the (@)-rule, due to the difference between using only constant symbols in the proof, and using the variable introduced by the quantifier *every girl* c-commanding the pronoun.

Otherwise, the process of deriving the semantic representation is exactly the same in these readings. Thus, by representing the lexical items of pronouns by using underspecified types, a unified analysis is given to the coreference reading and the BVA reading.

22.4 Discussion

22.4.1 Anaphoric expression as Variable?

As described in this chapter, DTS provides underspecified types for semantic representation of anaphoric expressions, and anaphora resolution is performed through type checking of semantic representations including underspecified types. The anaphora resolution is performed by a proof search invoked through type checking of the semantic representation including the underspecified type.

In contrast, one may ask whether the semantic representation of anaphoric expressions should not simply be "variable. The question is whether it is not sufficient to find the antecedent in the context and assign it to variable (by cut rule). In other words, is it really necessary to extend DTT with a mechanism like underspecified type? In the following, we will discuss why simply defining anaphoric expressions as VARIABLE is inadequate as a theory of anaphora.

As an example of BVA, we reiterate (503).

(503) Every girl enjoys a hobby that excites her.

Although there are several possibilities to realize the analysis of anaphora as vari-

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able, we will first consider the following policy: the syntactic type of the pronoun *him* is NP, its semantic representation is the (free) variable z, and the term corresponding to the antecedent is assigned to the free variable z later on. The following is a discussion of the policy of substituting the term corresponding to the antecedent into the free-variant Z afterwards.

(549) Syntactic structure of (503)

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{Every}}{NP^{\star}/N} \quad \dfrac{\text{girl}}{N}}{NP^{\star}}{>} \qquad \dfrac{\dfrac{\text{enjoys}}{(S\backslash NP/NP)^{\star}} \quad \dfrac{\dfrac{\text{a}}{NP^{\star}/N} \quad \dfrac{\dfrac{\text{hobby}}{N} \quad \dfrac{\dfrac{\text{that}}{N\backslash N/(S\backslash NP)^{\star}} \quad \dfrac{\dfrac{\text{excites}}{(S\backslash NP/NP)^{\star}} \quad \dfrac{\text{her}}{NP^{\star}}}{(S\backslash NP)^{\star}}{>^{\star}}}{N\backslash N}{>}}{N}{<}}{NP^{\star}}{>}}{(S\backslash NP)^{\star}}{>^{\star}}}{S^{\star}}{<^{\star}}}{S}\checkmark$$

(550) Semantic composition of (503)

$$\dfrac{\dfrac{\text{a}}{\lambda n.\begin{bmatrix} v:\begin{bmatrix} y:\mathrm{e} \\ n(y)\end{bmatrix} \\ \boxed{\pi_1 v}\end{bmatrix}} \quad \dfrac{\dfrac{\text{hobby}}{\mathbf{hobby}} \quad \dfrac{\dfrac{\text{that}}{\lambda q.\lambda n.\lambda x.\begin{bmatrix} nx \\ q(\lambda p.px)\end{bmatrix}} \quad \dfrac{\dfrac{\text{excites}}{\boxed{\begin{matrix}\lambda y.\lambda x. \\ \mathbf{excite}^{*}_{e_2}(x,y)\end{matrix}}} \quad \dfrac{\text{her}}{\boxed{z}}}{\boxed{\lambda x.\mathbf{excite}^{*}_{e_2}(x,z)}}{>}}{\lambda n.\lambda x.\begin{bmatrix} nx \\ \mathbf{excite}^{*}_{e_2}(x,z)\end{bmatrix}}{>}}{\lambda x.\begin{bmatrix}\mathbf{hobby}(x) \\ \mathbf{excite}^{*}_{e_2}(x,z)\end{bmatrix}}{<}}{\begin{bmatrix} v:\begin{bmatrix} y:\mathrm{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^{*}_{e_2}(y,z)\end{bmatrix} \\ \boxed{\pi_1 v}\end{bmatrix}}{>}$$

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(551) Semantic composition of (503)

$$
\dfrac{
  \dfrac{
    \dfrac{\text{Every}}{\lambda n.\left(u:\begin{bmatrix}x:\mathsf{e}\\ n(x)\end{bmatrix}\right)\to\boxed{\pi_1 u}}
    \quad
    \dfrac{\text{girl}}{\mathbf{girl}}
  }{\left(u:\begin{bmatrix}x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}\right)\to\boxed{\pi_1 u}}\,{>}
  \quad
  \dfrac{
    \dfrac{\text{enjoys}}{\boxed{\begin{array}{l}\lambda y.\lambda x.\\ \mathbf{enjoy}^*_{e_1}(x,y)\end{array}}}
    \quad
    \dfrac{\text{a hobby that excites her}}{\begin{bmatrix} v:\begin{bmatrix}y:\mathsf{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,z)\end{bmatrix}\\ \boxed{\pi_1 v}\end{bmatrix}}
  }{\begin{bmatrix} v:\begin{bmatrix}y:\mathsf{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,z)\end{bmatrix}\\ \boxed{\lambda x.\mathbf{enjoy}^*_{e_1}(x,\pi_1 v)}\end{bmatrix}}\,{>^\star}
}{
  \dfrac{\left(u:\begin{bmatrix}x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}\right)\to\begin{bmatrix} v:\begin{bmatrix}y:\mathsf{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,z)\end{bmatrix}\\ \boxed{\mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)}\end{bmatrix}}{\left(u:\begin{bmatrix}x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}\right)\to\begin{bmatrix} v:\begin{bmatrix}y:\mathsf{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,z)\end{bmatrix}\\ \mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)\end{bmatrix}}\,{\checkmark}
}\,{>}
$$

Now, the semantic representation thus obtained includes Z as a free-variant term, and substituting $\pi_1 u$, the term corresponding to the antecedent, for it yields the following (assuming that w is an appropriately chosen variant not included as a free-variant term in the original expression).

$$
\begin{aligned}
&\left(\left(u:\begin{bmatrix}x:\mathsf{e}\\ \mathbf{girl}\end{bmatrix}\right)\to\begin{bmatrix} v:\begin{bmatrix}y:\mathsf{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e}(y,z)\end{bmatrix}\\ \mathbf{carry}^*_{e}(\pi_1 u,\pi_1 v)\end{bmatrix}\right)[\pi_1 u/z]\\
\equiv&\left(w:\begin{bmatrix}x:\mathsf{e}\\ \mathbf{girl}\end{bmatrix}\right)\to\begin{bmatrix} v:\begin{bmatrix}y:\mathsf{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e}(y,\pi_1 u)\end{bmatrix}\\ \mathbf{carry}^*_{e}(\pi_1 w,\pi_1 v)\end{bmatrix}
\end{aligned}
$$

The reason why we have to change the variable u of type Π to w here is that in original substitution, $u \notin fv(\mathit{\Pi}_1 u)$ is in violation of the condition set by the substitution rule defined in Definition 132. Therefore, this approach is not a semantic representation for BVA reading.

Another approach is to use the semantic representation of a pronoun as its antecedent term from the beginning. This is a problem of how the lexical entry is given in the dictionary in the first place, which must first be concretized for verification, but we will ignore this and assume that the following semantic representations are given.

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(552) Semantic composition of (503)

$$
\dfrac{
\dfrac{\text{a}}{\lambda n.\begin{bmatrix} v : \begin{bmatrix} y : \mathsf{e} \\ n(y) \end{bmatrix} \\ \boxed{\pi_1 v} \end{bmatrix}}
\qquad
\dfrac{
\dfrac{\text{hobby}}{\textbf{hobby}}
\qquad
\dfrac{
\dfrac{\text{that}}{\lambda q.\lambda n.\lambda x.\begin{bmatrix} nx \\ q(\lambda p.px) \end{bmatrix}}
\qquad
\dfrac{
\dfrac{\text{excites}}{\boxed{\begin{array}{l}\lambda y.\lambda x. \\ \textbf{excite}^*_{e_2}(x,y)\end{array}}}
\quad
\dfrac{\text{her}}{\boxed{\pi_1 u}}
}{\boxed{\lambda x.\textbf{excite}^*_{e_2}(x,\pi_1 u)}}>
}{\lambda n.\lambda x.\begin{bmatrix} nx \\ \textbf{excite}^*_{e_2}(x,\pi_1 u) \end{bmatrix}}>
}{\lambda x.\begin{bmatrix} \textbf{hobby}(x) \\ \textbf{excite}^*_{e_2}(x,\pi_1 u) \end{bmatrix}}<
}{
\begin{bmatrix} v : \begin{bmatrix} y : \mathsf{e} \\ \textbf{hobby}(y) \\ \textbf{excite}^*_{e_2}(y,\pi_1 u) \end{bmatrix} \\ \boxed{\pi_1 v} \end{bmatrix}
}>
$$

(553) Semantic composition of (503)

$$
\dfrac{
\dfrac{
\dfrac{\text{Every}}{\lambda n.\left(u : \begin{bmatrix} x : \mathsf{e} \\ n(x) \end{bmatrix}\right) \to \boxed{\pi_1 u}}
\quad
\dfrac{\text{girl}}{\textbf{girl}}
}{\left(u : \begin{bmatrix} x : \mathsf{e} \\ \textbf{girl}(x) \end{bmatrix}\right) \to \boxed{\pi_1 u}}>
\qquad
\dfrac{
\dfrac{\text{enjoys}}{\boxed{\begin{array}{l}\lambda y.\lambda x. \\ \textbf{enjoy}^*_{e_1}(x,y)\end{array}}}
\quad
\dfrac{\text{a hobby that excites her}}{\begin{bmatrix} v : \begin{bmatrix} y : \mathsf{e} \\ \textbf{hobby}(y) \\ \textbf{excite}^*_{e_2}(y,\pi_1 u) \end{bmatrix} \\ \boxed{\pi_1 v} \end{bmatrix}}
}{\begin{bmatrix} v : \begin{bmatrix} y : \mathsf{e} \\ \textbf{hobby}(y) \\ \textbf{excite}^*_{e_2}(y,\pi_1 u) \end{bmatrix} \\ \boxed{\lambda x.\textbf{enjoy}^*_{e_1}(x,\pi_1 v)} \end{bmatrix}}>^{\star}
}{
\left(w : \begin{bmatrix} x : \mathsf{e} \\ \textbf{girl}(x) \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \mathsf{e} \\ \textbf{hobby}(y) \\ \textbf{excite}^*_{e_2}(y,\pi_1 u) \end{bmatrix} \\ \boxed{\textbf{enjoy}^*_{e_1}(\pi_1 w,\pi_1 v)} \end{bmatrix}
}>
$$

This last step is problematic, because it means that we must eventually assign the semantic representation of the verb phrase with u as a free variant to the scope of the variant u introduced by the Π-type resulting from the omnisymmetric quantification. As a result, the variant introduced by the Π type must be changed from u to something else (here w is chosen), and as a result $\Pi_1 u$ is no longer a term representing the antecedent. Therefore, it can be seen that this approach also does not yield a semantic representation for BVA reading.

We have considered two versions of the analysis "taking the semantic representation of a pronoun as a declension," and found that in both cases, the declension that the quantifier binds must be inserted from the outside into the scope of the quantifier, and since this is not allowed, the resulting semantic representation does not have a BVA reading. This clearly illustrates the problem that must be solved in the semantic composition of pronouns.

The DTS underspecified type framework provides the minimum mechanism to solve this problem as well as the minimum mechanism to solve this problem.

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