

Chapter 24

Accessibility

Let us check some of the classical paradigms of accessibility and how dependent type semantics gives them a new life.

24.1 Universal Quantification

The first case is universal quantification in the subject position, which generally blocks an anaphoric link to indefinites within its scope from subsequent sentences, as in (604).^{*1}

- (604) a. Everybody bought [a car]_i.
 b. *It_i is parked outside.

The lexical item for “Everybody” is given by means of the Π -operator as follows.

- (605) Syntactic structure of (604)

$$\begin{array}{c}
 \text{Everybody} \quad \frac{\text{bought} \quad \frac{\frac{a}{NP^*/N} \quad \frac{\text{car}}{N}}{NP^*}}{(S \backslash NP / NP)^*} \quad \frac{}{NP^*} >^* \\
 \hline
 \frac{S^*}{S} \checkmark
 \end{array}
 \quad
 \begin{array}{c}
 \text{It} \quad \frac{\text{is} \quad \text{parked outside}}{\frac{(S \backslash NP / (S \backslash NP))^* \quad (S \backslash NP)^*}{(S \backslash NP)^*}} >^* \\
 \hline
 \frac{S^*}{S} \checkmark
 \end{array}$$

^{*1} The *telescoping*, or *quantificational subordination*, is known to be an exception for this case. See the discussion in Subsection 24.7.1.

(606) Semantic composition of (604a)

$$\begin{array}{c}
\text{Everybody} \\
\hline
\left(u : \left[\begin{array}{c} x : e \\ \mathbf{human}(x) \end{array} \right] \right) \rightarrow \boxed{\pi_1 u} \\
\hline
\left(u : \left[\begin{array}{c} x : e \\ \mathbf{human}(x) \end{array} \right] \right) \rightarrow \left[\begin{array}{c} v : \left[\begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \\ \boxed{\lambda x. \mathbf{buy}_{e_1}^*(x, \pi_1 v)} \end{array} \right] <^* \\
\hline
\left(u : \left[\begin{array}{c} x : e \\ \mathbf{human}(x) \end{array} \right] \right) \rightarrow \left[\begin{array}{c} v : \left[\begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \\ \boxed{\mathbf{buy}_{e_1}^*(\pi_1 u, \pi_1 v)} \end{array} \right] \checkmark \\
\hline
\left(u : \left[\begin{array}{c} x : e \\ \mathbf{human}(x) \end{array} \right] \right) \rightarrow \left[\begin{array}{c} v : \left[\begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \\ \mathbf{buy}_{e_1}^*(\pi_1 u, \pi_1 v) \end{array} \right]
\end{array}$$

$\frac{\text{bought} \quad \frac{\lambda y. \lambda x. \mathbf{buy}_{e_1}^*(x, y)}{\boxed{\lambda y. \lambda x. \mathbf{buy}_{e_1}^*(x, y)}} \quad \frac{\text{a} \quad \frac{\lambda n. \left[v : \left[\begin{array}{c} y : e \\ n(y) \end{array} \right] \right] \quad \frac{\mathbf{car}}{\mathbf{car}}}}{\boxed{\pi_1 v}}}{\left[v : \left[\begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \right] \quad \boxed{\pi_1 v}} >^*$

(607) Semantic composition of (604b)

$$\begin{array}{c}
\text{It} \\
\hline
\left[\begin{array}{c} w @ \left[\begin{array}{c} x : e \\ \neg \mathbf{human}(x) \end{array} \right] \\ \boxed{\pi_1 w} \end{array} \right] \\
\hline
\left[\begin{array}{c} w @ \left[\begin{array}{c} x : e \\ \neg \mathbf{human}(x) \end{array} \right] \\ \boxed{\mathbf{parkedOutside}_{s_1}^*(\pi_1 w)} \end{array} \right] \checkmark \\
\hline
\left[\begin{array}{c} w @ \left[\begin{array}{c} x : e \\ \neg \mathbf{human}(x) \end{array} \right] \\ \mathbf{parkedOutside}_{s_1}^*(\pi_1 w) \end{array} \right]
\end{array}$$

$\frac{\text{is} \quad \frac{\text{parked outside} \quad \frac{\lambda x. \mathbf{parkedOutside}_{s_1}^*(x)}{\boxed{\lambda x. \mathbf{parkedOutside}_{s_1}^*(x)}}}{\boxed{\lambda x. \mathbf{parkedOutside}_e^*(x)}} >^*$

(608) Progressive conjunction between (606) and (607)

$$\begin{array}{c}
\left(u : \left[\begin{array}{c} x : e \\ \mathbf{human}(x) \end{array} \right] \right) \rightarrow \left[\begin{array}{c} v : \left[\begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \\ \mathbf{buy}_{e_1}^*(\pi_1 u, \pi_1 v) \end{array} \right] ; \left[\begin{array}{c} w @ \left[\begin{array}{c} x : e \\ \neg \mathbf{human}(x) \end{array} \right] \\ \mathbf{parkedOutside}_e^*(\pi_1 w) \end{array} \right] \\
\equiv \left[\begin{array}{c} p : \left(u : \left[\begin{array}{c} x : e \\ \mathbf{human}(x) \end{array} \right] \right) \rightarrow \left[\begin{array}{c} v : \left[\begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \\ \mathbf{buy}_{e_1}^*(\pi_1 u, \pi_1 v) \end{array} \right] \\ w @ \left[\begin{array}{c} x : e \\ \neg \mathbf{human}(x) \end{array} \right] \\ \mathbf{parkedOutside}_e^*(\pi_1 w) \end{array} \right]
\end{array}$$

(612) Syntactic structure of (611a)

$$\frac{\frac{\text{If}}{(S/S)^*/S} \quad \frac{\text{John bought a car}}{S} \quad \frac{\text{it is parked outside}}{S^*}}{(S/S)^*} > \frac{S^*}{S} >^* \checkmark$$

(613) Semantic composition of (611a)

$$\frac{\frac{\text{If}}{\lambda p. (u : p) \rightarrow \boxed{id}} \quad \frac{\text{John bought a car}}{\left[v : \begin{array}{l} y : e \\ \mathbf{car}(y) \\ \mathbf{buy}_{e_1}^*(john, \pi_1 v) \end{array} \right]}}{\left(u : \left[v : \begin{array}{l} y : e \\ \mathbf{car}(y) \\ \mathbf{buy}_{e_1}^*(john, \pi_1 v) \end{array} \right] \right) \rightarrow \boxed{id}} > \frac{\text{it is parked outside}}{\left[w @ \begin{array}{l} x : e \\ \neg \mathbf{human}(x) \\ \mathbf{parkedOutside}_{s_1}^*(\pi_1 w) \end{array} \right]}} >^* \frac{\left(u : \left[v : \begin{array}{l} y : e \\ \mathbf{car}(y) \\ \mathbf{buy}_{e_1}^*(john, \pi_1 v) \end{array} \right] \right) \rightarrow \boxed{id}}{\left(u : \left[v : \begin{array}{l} y : e \\ \mathbf{car}(y) \\ \mathbf{buy}_{e_1}^*(john, \pi_1 v) \end{array} \right] \right) \rightarrow \left[w @ \begin{array}{l} x : e \\ \neg \mathbf{human}(x) \\ \mathbf{parkedOutside}_{s_1}^*(\pi_1 w) \end{array} \right]} >^* \checkmark$$

The antecedent *a car* corresponds to a variable *y* that satisfies **car**(*y*), so *it_i* is accessible to the proof by applying (ΣE)-rules to *u*. On the other hand, *It_i* in (611b) is not accessible to the proof *y*.

(614) Progressive Conjunction of (611)

$$\equiv \left[\left(u : \left[v : \begin{array}{l} y : e \\ \mathbf{car}(y) \\ \mathbf{buy}_{e_1}^*(john, \pi_1 v) \end{array} \right] \right) \rightarrow \left[w @ \begin{array}{l} x : e \\ \neg \mathbf{human}(x) \\ \mathbf{parkedOutside}_{s_1}^*(\pi_1 w) \end{array} \right] \right]; \left[t @ \begin{array}{l} x : e \\ \neg \mathbf{human}(x) \\ \mathbf{bePorsche}_{e_1}^*(\pi_1 t) \end{array} \right] \right]$$

As can be seen from the derivation in (614), the underspecified variable *t* has to pick the antecedent from the local context: However, the functional proof of the first sentence encapsulates the participants therein, which includes a variable *y* : *e*. Thus, the accessibility constraint in conditional sentences are predicted via deducibility without any ad-hoc assumptions.

24.3 Negation

The third case is negation. Negations are known to block accessibility, as in (615a).

- (615) a. John didn't buy [a car]¹.
 b. *It₁ is parked outside.

In DTS, this is due to the definition of negation via implication, i.e. $\neg A \stackrel{def}{=} (x : A) \rightarrow \perp$. First, the syntactic structure and the semantic composition of (615a) go as follows.

(616) Syntactic structure of (615)

$$\frac{\frac{\text{John}}{NP^*} \quad \frac{\frac{\text{didn't}}{(S \setminus NP / (S \setminus NP))^*} \quad \frac{\text{buy a car}}{(S \setminus NP)^*}}{S \setminus NP^*} >}{\frac{S^*}{S} \checkmark} < \quad \frac{\frac{\text{It}}{NP^*} \quad \frac{\frac{\text{is}}{(S \setminus NP / (S \setminus NP))^*} \quad \frac{\text{parked outside}}{(S \setminus NP)^*}}{(S \setminus NP)^*} >^*}{\frac{S^*}{S} \checkmark} <^*$$

(617) Semantic composition of (615a)

$$\frac{\frac{\frac{\text{didn't}}{\neg id} \quad \left[v : \begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \quad \left[\lambda x. \mathbf{buy}_{e_1}^*(x, \pi_1(v)) \right]}{>} \quad \frac{\frac{\text{John}}{j} \quad \neg \left[v : \begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \quad \left[\lambda x. \mathbf{buy}_{e_1}^*(x, \pi_1(v)) \right]}{<}}{\neg \left[v : \begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \quad \left[\mathbf{buy}_{e_1}^*(j, \pi_1(v)) \right]} < \quad \frac{\neg \left[v : \begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \quad \left[\mathbf{buy}_{e_1}^*(j, \pi_1(v)) \right]}{\neg \left[v : \begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \quad \left[\mathbf{buy}_e^*(j, \pi_1(v)) \right]} \checkmark$$

According to the definition of negation, the semantic representation in the last line, is a functional type

$$(618) \quad \left[v : \begin{array}{c} y : e \\ \mathbf{car}(y) \end{array} \right] \quad \left[\mathbf{buy}_e^*(j, \pi_1(v)) \right] \rightarrow \perp$$

which is the type of *functions* from the proof of *John bought a car* to \perp . Therefore, it is not possible to pick up *a car* from its proof by means of a composition of projections. In other words, negation blocks accessibility due to its implicational nature.

Notice that the fact that universal quantifications, conditionals and negations block accessibility is not only correctly predicted but also explained in dependent type semantics in a uniform way. In other words, they are all represented by the Π -operator, whose proofs are functions, from which a mere composition of projections cannot pick up the elements involved. This is a deeper explanation on accessibility, which relies only on the structures of proofs between the Σ -operator and the Π -operator, which is widely adopted in type theory, without resource to any ad-hoc assumptions.

24.4 Conjunction

Conjunction is both internally and externally dynamic, as we have already seen in Section 23.1. The first *it* and the second *it* in (619) show that conjunction is internally

dynamic and externally dynamic, respectively.

(619) John bought a car and it is super-expensive. It is parked outside.

This is an inevitable result if we analyze sentence-sentence conjunction by progressive conjunction as in Definition 457. Those anaphora are derived by the conjunction elimination rules.

On the other hand, there are also forms of anaphora in conjunction that we have not dealt with so far. An example is an anaphoric link over a nominal conjunction, as in (620).

(620) [A monk]_i and his_i apprentice walked back to the abbey.

We assume the following lexical item for the possessive pronoun *his*. Recall that *RN* is a syntactic type for relative nouns.

$$(621) \quad \llbracket \text{his} \vdash NP^*/RN \rrbracket \stackrel{def}{=} \lambda r. \left[\begin{array}{c} u @ \left[\begin{array}{c} x : e \\ \mathbf{male}(x) \end{array} \right] \\ v @ \left[\begin{array}{c} y : e \\ r(x)(y) \end{array} \right] \end{array} \right] \boxed{\pi_1 v}$$

The syntactic structure and semantic composition of (620) are as follows. However, the structure inside the *walked back to the abbey* is omitted for the sake of space.

(622) Syntactic structure of (620)

$$\frac{\frac{\frac{A}{NP^*/N} \quad \frac{\text{monk}}{N}}{NP^*} > \quad \frac{\text{and} \quad \frac{\frac{\text{his}}{NP^*/RN} \quad \frac{\text{apprentice}}{RN}}{NP^*} > \quad \frac{\text{walked back to the abbey}}{(S \setminus NP)^*}}{NP^*} \langle \Phi \rangle \quad \frac{S^*}{S} \checkmark <^*$$

(623) Semantic composition of (620)

$$\frac{\frac{\frac{\text{A monk}}{\left[\begin{array}{c} u : \left[\begin{array}{c} x : e \\ \mathbf{monk}(x) \end{array} \right] \end{array} \right] \boxed{\pi_1 u}}}{\text{and}_{NP^*} \quad \frac{\frac{\text{his apprentice}}{\left[\begin{array}{c} v @ \left[\begin{array}{c} z : e \\ \mathbf{male}(z) \end{array} \right] \\ w @ \left[\begin{array}{c} a : e \\ \mathbf{apprenticeOf}(a, \pi_1 v) \end{array} \right] \end{array} \right] \boxed{\pi_1 w}}}{\langle \Phi \rangle}}{\left[\begin{array}{c} u : \left[\begin{array}{c} x : e \\ \mathbf{monk}(x) \end{array} \right] \\ \boxed{\pi_1 u} \\ v @ \left[\begin{array}{c} z : e \\ \mathbf{male}(z) \end{array} \right] \\ w @ \left[\begin{array}{c} a : e \\ \mathbf{apprenticeOf}(a, \pi_1 v) \end{array} \right] \end{array} \right] \boxed{\pi_1 w}}}{\text{walked back to the abbey}} \quad \boxed{\lambda x. \mathbf{WBttA}_e^*(x)} <^*$$

$$\left[\begin{array}{c} u : \left[\begin{array}{c} x : e \\ \mathbf{monk}(x) \end{array} \right] \\ \boxed{\mathbf{WBttA}_{e_1}^*(\pi_1 u)} \\ v @ \left[\begin{array}{c} z : e \\ \mathbf{male}(z) \end{array} \right] \\ a : e \\ w @ \left[\begin{array}{c} \mathbf{apprenticeOf}(a, \pi_1 v) \end{array} \right] \\ \boxed{\mathbf{WBttA}_{e_2}^*(\pi_1 w)} \end{array} \right]$$

As will become clear when type check is performed on this semantic representation, in the proof search for v , u is present in the context and can be used to construct the proof. Thus (if the world knowledge that *every monk is a male* is available) *his* can refer to A monk. This is consistent with an empirical judgment.

Thus, combining the analysis of generalized conjunction in DTS (Chapter ??) and the analysis of E-type anaphora (Chapter 23), predictions for E-type anaphora in nominal conjunctions are bourn out, which are supported by empirical facts.*²

24.5 Anaphoric Potential and Intuitionism

The notion of anaphoric potential raised in dynamic semantics indicates that what we regard as the “meaning of a sentence” includes what cannot be captured by truth conditions (cf. Kamp et al. (2011)). Examples are given below.

- (624) a. Some company is not rich.
b. It is not the case that every company is rich.

The semantic representation in FoL of the sentences (624a) and (624b) are (in logic textbooks) (625a) and (625b) respectively.

- (625) a. $\exists x(\text{company}(x) \wedge \neg \text{rich}(x))$
b. $\neg \forall x(\text{company}(x) \rightarrow \text{rich}(x))$

If we assume classical FoL as a semantic theory, the truth conditions of (624a) and (624b) are equal because the two expressions are semantically and proof-theoretically equivalent. Thus, if the meaning of a sentence is its truth condition, then the meaning of (624a) and (624b) is the same.

However, the two sentences (624a) and (624b) behave differently with respect to the accessibility of the anaphora from subsequent sentences. For example, when a sentence containing a pronoun follows, as in (626) and (627) below, (624a) can provide a antecedent for it, while (624b) cannot.

- (626) a. [Some company]¹ is not rich.
b. It₁ does not have enough money to replace the system.
(627) a. It is not the case that every company is rich.
b. #It does not have enough money to replace the system.

Therefore, the two sentences (624a) and (624b) differ in their ability to provide antecedents to the pronouns in the subsequent sentence. This ability is called *anaphoric potential*.

If we consider anaphoric potential to be a part of semantic information of a sentence, the theory of meaning must be able to distinguish the anaphoric potentials of the two sentences (624a) and (624b). Therefore, there is a limit to the hypothesis that the meaning of a sentence is its truth condition, and one requirement for the theory of meaning is to explain its anaphoric potential. Now, in DTS, the semantic representation of the two sentences (624a) and (624b) is as follows (syntactic structure and semantic composition are left as exercises to the readers).

*² See, however, the open issue discussed in Subsection 24.7.2.

$$(628) \quad \begin{array}{l} \text{a.} \quad \left[\begin{array}{l} f : \left[\begin{array}{l} u : \left[\begin{array}{l} x : e \\ \mathbf{company}(x) \end{array} \right] \\ \neg \mathbf{rich}_e^*(\pi_1 u) \end{array} \right] \\ v @ \left[\begin{array}{l} y : e \\ \neg \mathbf{human}(y) \end{array} \right] \\ \neg \mathbf{haveMoney}_e^*(\pi_1 v) \end{array} \right] \\ \text{b.} \quad \left[\begin{array}{l} f : \neg \left(u : \left[\begin{array}{l} x : e \\ \mathbf{company} \end{array} \right] \right) \rightarrow \mathbf{rich}_e^*(\pi_1 u) \\ v @ \left[\begin{array}{l} y : e \\ \neg \mathbf{human}(y) \end{array} \right] \\ \neg \mathbf{haveMoney}_e^*(\pi_1 v) \end{array} \right] \end{array}$$

In (628a), the antecedent of *it* in the second sentence can be resolved as $\pi_1 v = \pi_1 \pi_1(f)$.^{*3} On the other hand, in (624b), the first sentence does not provide an *it* antecedent for the reasons stated in Section 24.1 and Section 24.3. Thus, in the DTS, the semantic indications (624a) and (624b), (628a) and (628b), even represent a difference in anaphoric potential. This is one of the advantages of DTS over truth-conditional semantics.

However, it is important to note that the DTS is based on an intuitionistic type theory in order for such an explanation of anaphoric potential in the DTS to be valid. As already mentioned, if we add the following double negative elimination (DNE) rule to DTS, DTS becomes a system of classical type theory.

$$\frac{M : \neg \neg A}{\text{dne}(M) : A} \text{ (DNE)}$$

However, if the following inference had a proof term fu , then the anaphoric link in question would be wrongly predicted to be licenced.

$$(629) \quad u : \neg(x : e) \rightarrow \mathbf{L}(x) \rightarrow \neg \mathbf{R}(x) \vdash \left[\begin{array}{l} x : e \\ \left[\begin{array}{l} \mathbf{L}(x) \\ \mathbf{R}(x) \end{array} \right] \end{array} \right] \text{ true}$$

^{*3} More precisely, the knowledge that “company is not human” is resolved as $v = (\pi_1 \pi_1(f), \mathbf{CnH}(\pi_1 f))$, using axioms such as \mathbf{CnH} defined below.

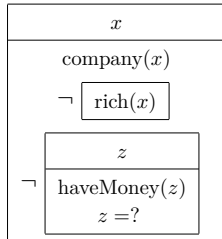
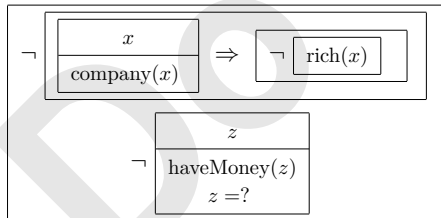
$$\mathbf{CnH} : \left(u : \left[\begin{array}{l} x : e \\ \mathbf{company}(x) \end{array} \right] \right) \rightarrow \neg \mathbf{human}(\pi_1 u)$$

(630) Proof diagram of (627)

$$\begin{array}{c}
\frac{\frac{\frac{}{u : \left[\begin{array}{c} x : e \\ \text{company}(x) \end{array} \right]}^2}{\frac{}{r : \neg \text{rich}_e^*(\pi_1 u)}^1} (\Sigma I)}{\frac{}{(u, r) : \left[\begin{array}{c} u : \left[\begin{array}{c} x : e \\ \text{company}(x) \end{array} \right] \\ \neg \text{rich}_e^*(\pi_1 u) \end{array} \right]} (\rightarrow E)} \quad \frac{}{s : \neg \left[\begin{array}{c} x : e \\ \text{company}(x) \end{array} \right]}^3 \\
\frac{}{s(u, r) : \perp} (\neg I), 1 \\
\frac{}{\lambda r. s(u, r) : \neg \neg \text{rich}_e^*(\pi_1 u)} (\neg I), 1 \\
\frac{}{\text{dne}(\lambda r. s(u, r)) : \text{rich}_e^*(\pi_1 u)} (DNE) \\
\frac{}{f : \neg \left(\left(u : \left[\begin{array}{c} x : e \\ \text{company} \end{array} \right] \right) \rightarrow \text{rich}_e^*(\pi_1 u) \right)} (\rightarrow I), 2 \\
\frac{}{\lambda u. \text{dne}(\lambda r. s(u, r)) : \left(u : \left[\begin{array}{c} x : e \\ \text{company}(x) \end{array} \right] \right) \rightarrow \text{rich}_e^*(\pi_1 u)} (\rightarrow E) \\
\frac{}{f(\lambda u. \text{dne}(\lambda r. s(u, r))) : \perp} (\rightarrow E) \\
\frac{}{\lambda s. f(\lambda u. \text{dne}(\lambda r. s(u, r))) : \neg \left[\begin{array}{c} u : \left[\begin{array}{c} x : e \\ \text{company}(x) \end{array} \right] \\ \neg \text{rich}_e^*(\pi_1 u) \end{array} \right]} (\neg I), 3 \\
\frac{}{\text{dne}(\lambda s. f(\lambda u. \text{dne}(\lambda r. s(u, r)))) : \left[\begin{array}{c} u : \left[\begin{array}{c} x : e \\ \text{company}(x) \end{array} \right] \\ \neg \text{rich}_e^*(\pi_1 u) \end{array} \right]} (DNE)
\end{array}$$

Letting $F(f) \equiv \text{dne}(\lambda s. f(\lambda u. \text{dne}(\lambda r. s(u, r))))$, we see that the variant v in the underspecified type introduced by the second sentence, $V = (\pi_1 \pi_1(F(f)), \mathbf{CnH}(\pi_1 F(f)))$. In other words, the classical-type theory with (DNE) added to DTS loses its ability to explain the difference in anaphoric potential of the two statements (624a) and (624b). Thus, the explanatory capacity of DTS for anaphoric potential depends on the fact that DTS is an intuitionistic system.

What does this imply? In theories such as DRT, the difference in anaphoric potential is the difference in accessibility resulting from the difference in DRS structure. The DRSs of (626)(627) will be (631)(632), respectively, by the accessibility rules set by DRT. In (631), z to x are accessible, but not in (632).

(631) DRS of (624a) where x is accessible from z .(632) DRS of (624b) where x is not accessible from z .

And this is precisely the reason why DRT claims that an intermediate representation such as DRS is necessary.