Chapter 23

E-type Anaphora

E-type anaphora is a linguistic phenomenon involving anaphora in which the dependent types are particularly valuable. In this chapter, we will discuss what E-type anaphora is, why it was a major problem in formal semantics from the 1980s to the 2000s, and how DTS solves this problem. Two typical examples of E-type anaphora are discourse anaphora and donkey anaphora. I will explain them in the following order.

23.1 Discourse Anaphora

23.1.1 A Puzzle

The principle of compositionality, discussed in Section 14.2, faced a problem when Evans discovered *E-type anaphora*, as a third case of anaphora that is distinguished from coreference and BVA (Evans (1977, 1980)). In the mini-discouse (555), for example, which consists of two sentences (555a) and (555b), contains anaphoric link between the pronoun *He* in (555b) and its antecedent *A man* in (555a).

(555) a. [A man]$_i$ walked into the park.
b. He$_i$ whistled.

The subscript i attached to *A man* and *He* is intended to indicate that we are focusing on a particular reading in which the antecedent of *He* is (a witness of) *A man*, namely, an anaphoric link is established between the pronoun *He* and a quantifier *A man*. This anaphoric link has the following features:

1. The anaphoric link is *inter-clausal*, namely, the pronoun and its antecedent belong to different clauses.
2. The antecedent is a quantificational expression, including indefinite noun phrase (or existential quantifier, semantically).

Anaphora with these features is called *E-type anaphora*, and a pronoun used in E-type anaphora (such as *He* in (555b)) is called *E-type pronoun*. Empirically, the mini-discourse (555) participate in the entailment relations, not exhaustive though, listed in (556).

(556) a. [A man]$_i$ walked into the park. He$_i$ whistled. $\Longrightarrow$ A man walked into the park.
b. [A man]$_i$ walked into the park. He$_i$ whistled. $\Longrightarrow$ A man whistled.

c. $[\text{A man}]_i$ walked into the park. He$_i$ whistled. $\Longrightarrow$ A man walked into the park and whistled.

d. A man walked into the park and whistled. $\Longrightarrow$ $[\text{A man}]_i$ walked into the park. He$_i$ whistled.

(556a) obtains obviously by the conjunction elimination rule. (556b) is not so obvious, but (556c) and (556d) mean that the mini-discourse (555) and the sentence *A man walked into the park and whistled* are mutually deducibile[*1], from which (556a) and (556b) are also deducible. Thus, one reasonable candicate of the semantic representation of the mini-discourse (555) is that of *a man walked into the park and whistled*, which is (557) if described in first-order logic (and most of its extentions).

(557) $\exists x(\mathbf{man}(x) \wedge \mathbf{walk}(x) \wedge \mathbf{whistle}(x))$

However, the following questions immediately arise, for which it does not seem straightforward to give a single solution.

(i) What are the semantic representations of the sentences (555a) and (555b)?
(ii) How the semantic representation (557) of the mini-discourse (555) is compositionaly obtained from those of (555a) and (555b)?
(iii) What is the semantic representation of *He* in (555b)?

The problem (i) and (ii) are entangled with each other. To demonstrate it, let us start from the following three näive assumptions:

Assumption 1: The SR of (555a) is $\exists x(\mathbf{man}(x) \wedge \mathbf{walk}(x))$
Assumption 2: The SR of (555b) is $\mathbf{whistle}(x)$
Assumption 3: The SR of two assertive sentences is obtained by conjoining their SRs by $\wedge$.

Putting these assumptions together, we obtain (558) as the SR of (555).

(558) $\exists x(\mathbf{man}(x) \wedge \mathbf{walk}(x)) \wedge \mathbf{whistle}(x)$

The variable x in $\mathbf{whistle}(x)$ occurs free, thus (558) is true under the interpretation (M,g) when $[\![\exists x(\mathbf{man(x)} \wedge \mathbf{walk}(x))]\!]_{M,g} = 1$ and $g(x) \in [\![\mathbf{whistle}]\!]_{M,g}$. This is not truth-conditionally equivalent to (557), since (558) is true even under the interpretation that some man walked into the park, and no man but some woman whistled.

Therefore, we have to abandon at least one of Assumption 1, 2, 3, and/or other hidden assumptions behind this näive anaysis.[*2]

[*1] This generalization holds only when the antecedent is a singular indefinite noun phrase. The original example given in Evans (1980) includes a discouse such as (1).

(1) Few congressmen admire Kennedy. They are all junior. (Evans (1980, p.339))

In the cases like (**??**) where the antecedent is a numeral expression (see Chapter **??**) or a generalized quantifier (see Chapter **??**) s, it shows slightly different entailsement patterns from the one given in (556). We will discuss these cases later in Chapter **??** and Chapter **??**.

[*2] For example, the classical DRT (Kamp and Reyle, 1993) abandon Assumption 2 and 3, and the direct compositoinality of meaning. DPL (Groenendijk and Stokhof, 1991) abandon Assumption 1, and the standard model-theoretic interpretation of first-order logic, in which (557) and (558) are not equivalent.

Remark 455. The name discourse anaphora literally means anaphora in which the antecedent and prounoun appear in separate sentences, but that description would also cover the case of coreference. Discourse anaphora as a separate category from coreference or BVA, is more essentially a case where the antecedent is a quantifier and the prounoun is outside its scope.

23.1.2 Semantic Representation

In contrast, DTS can give a semantic representation that is compatible with both the dynamics and compositionality of E-type anaphora. The semantic equivalence of (557) in DTT is one of the following.

(559) $$\left[\begin{array}{l} v : \left[\begin{array}{l} u : \left[\begin{array}{l} x : \mathsf{e} \\ \mathbf{man}(x) \end{array}\right] \\ \mathbf{walk}(\pi_1 u) \end{array}\right] \\ \mathbf{whistle}(\pi_1\pi_1 v) \end{array}\right]$$

(560) $$\left[\begin{array}{l} v : \left[\begin{array}{l} x : \mathsf{e} \\ \left[\begin{array}{l} \mathbf{man}(x) \\ \mathbf{walk}(x) \end{array}\right] \end{array}\right] \\ \mathbf{whistle}(\pi_1 v) \end{array}\right]$$

(561) $$\left[\begin{array}{l} x : \mathsf{e} \\ \left[\begin{array}{l} \mathbf{man}(x) \\ \left[\begin{array}{l} \mathbf{walk}(x) \\ \mathbf{whistle}(x) \end{array}\right] \end{array}\right] \end{array}\right]$$

Exercise 456. Prove the equivalence between (559), (560), and (561).

Since they are equivalent on DTT, if one is valid as a semantic representation of (555), then the others are also valid. The structure of (559), however, is parallel to the syntactic structures of (555), therefore we will consider the semantic composition toward (559).

In the semantic representation (559), the position of $\pi_1\pi_1 v$, which is the entity corresponding to *He*, is not in the scope of the variable x brought about by the quantificational expression *a man*, and this is the same situation as the one in which the **whistle**(x) in the FoL formula (557) is not in the scope of x. However, $\pi_1\pi_1 v$ is in the scope of v of the outer Σ-type, and thus it is in the position in which it can use v.

This v corresponds to the proof of the semantic representation of the first sentence. In other words, performing type checking of the semantic representation of the second sentence in the position where v is available means that in the felicity condition of the second sentence, we can assume that the first sentence has a proof. And since the first sentence is an existential quantification, its semantic representation is of type Σ, and its proof is a pair. Then, from the pair, we can extract the elements by applying the (ΣE) rule, i.e., projections.

23.1.3 Semantic Composition

In what followw, it is explained in turn how the semantic representation (559) is obtained from the mini-discourse (555) in DTS. The syntactic structure and semantic composition of each of the (555) is shown in (562) and (563)(564).

(562) Syntactic structure of (555)

$$\dfrac{\dfrac{\dfrac{\text{A}}{NP^\star/N} \quad \dfrac{\text{man}}{N}}{NP^\star}> \quad \dfrac{\dfrac{\text{walked}}{S\backslash NP}}{(S\backslash NP)^\star}\uparrow}{\dfrac{S^\star}{S}\downarrow}<^\star \qquad \dfrac{\dfrac{\text{He}}{NP^\star} \quad \dfrac{\dfrac{\text{whistled}}{S\backslash NP}}{(S\backslash NP)^\star}\uparrow}{\dfrac{S^\star}{S}\downarrow}<^\star$$

(563) Semantic composition of (555a)

$$\dfrac{\dfrac{\dfrac{\text{A}}{\lambda n.\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ n(x) \end{bmatrix} \\ \boxed{\pi_1 u} \end{bmatrix}} \quad \dfrac{\text{man}}{\mathbf{man}}}{\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \boxed{\pi_1 u} \end{bmatrix}}<^\star \quad \dfrac{\text{entered}}{\boxed{\lambda x.\mathbf{walk}^*_{e_1}(x)}}}{\dfrac{\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \boxed{\mathbf{walk}^*_{e_1}(\pi_1 u)} \end{bmatrix}}{\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_{e_1}(\pi_1 u) \end{bmatrix}}\checkmark}>$$

(564) Semantic composition of (555b)

$$\dfrac{\dfrac{\text{He}}{\begin{bmatrix} u@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \boxed{\pi_1 u} \end{bmatrix}} \quad \dfrac{\text{whistled}}{\boxed{\lambda x.\mathbf{whistle}^*_{e_2}(x)}}}{\dfrac{\begin{bmatrix} u@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \boxed{\mathbf{whistle}^*_{e_2}(\pi_1 u)} \end{bmatrix}}{\begin{bmatrix} u@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{whistle}^*_{e_2}(\pi_1 u) \end{bmatrix}}\checkmark}<^\star$$

Then, we assume that the semantic representation of the two sentences in discourse is connected by the following rule. [*3]

[*3] Progressive conjunction is a common assumption in the literature made by **?**. From the perspective of discourse theory, however, this assumption might be too simplistic. Two sentences in a discourse can have various relations, and not all of them are in conjunction. In that sense, the assumption here can be paraphrased as applying when the relation between two sentences

Definition 457 (Progressive conjunction)**.**

$$M;N \stackrel{def}{\equiv} \begin{bmatrix} u : M \\ N \end{bmatrix} \qquad \text{where } u \notin fv(N)$$

This connects the semantic representation of (**??**) and (**??**) as follows.

(565) Semantic composition of (555)

$$\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_{e_1}(\pi_1 u) \end{bmatrix} ; \begin{bmatrix} w@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{whistle}^*_{e_2}(\pi_1 v) \end{bmatrix} = \begin{bmatrix} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_{e_1}(\pi_1 u) \end{bmatrix} \\ w@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{whistle}^*_{e_2}(\pi_1 w) \end{bmatrix}$$

The semantic felicity condition (SFC) for this mini-discourse is that a UDTT proof diagram of the following form exists.

(566) Type check diagram of (555)

$$\dfrac{\begin{array}{c} \mathcal{D}_1 \\ \begin{bmatrix} \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_{e}(\pi_1 u) \end{bmatrix} : \mathsf{type} \end{array} \quad \dfrac{\begin{array}{c} \mathcal{D}_2 \\ \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} : \mathsf{type} \end{array} \quad \begin{array}{c} \dfrac{}{v : \begin{bmatrix} \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_{e}(\pi_1 u) \end{bmatrix}}{\scriptstyle 1} \\ \mathcal{D}_3 \\ ? : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \end{array} \quad \mathcal{D}_4}{\begin{bmatrix} w@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{whistle}^*_{e}(\pi_1 w) \end{bmatrix} : \mathsf{type}}{\scriptstyle (@)}}{\begin{bmatrix} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_{e}(\pi_1 u) \end{bmatrix} \\ w@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{whistle}^*_{e}(\pi_1 w) \end{bmatrix} : \mathsf{type}}{\scriptstyle (\Sigma F),1}$$

The identification of $\mathcal{D}_3$ requires a proof search. This proof is based on the assumption that the first sentence has an evidence for the existence of a male person, the search for which is linguistically equivalent to the operation of determining the antecedent of *He* in the second sentence. Here, we assume that there is prior world knowledge that all men are male persons. Within DTS theory, this is treated as a following type assignnment in the context.

(567) $\mathbf{m} : \left(u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \right) \to \mathbf{male}(\pi_1 u)$

Using this function $\mathbf{m}$, it can be show that $(\pi_1\pi_1 v, \mathbf{m}(\pi_1 v))$ is a proof term that we have serched for as follows.

is *narration* (cf. Asher and Lascarides (2003)). For other discourse relations, rules other than progressive conjunction should apply, but further integration of discourse theory and DTS should be an open issue.

(568) Proof diagram $\mathcal{D}_3 \equiv$

$$\dfrac{\dfrac{\dfrac{v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1 u) \end{bmatrix}}{\pi_1 v : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix}}\,(\Sigma E)}{\pi_1\pi_1 v : \mathsf{e}}\,(\Sigma E) \quad \dfrac{\dfrac{v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1 u) \end{bmatrix}}{\pi_1 v : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix}}\,(\Sigma E) \quad \dfrac{}{\mathbf{m} : \left(u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix}\right) \to \mathbf{male}(\pi_1 u)}\,(CON)}{\mathbf{m}(\pi_1 v) : \mathbf{male}(\pi_1\pi_1 v)}\,(\Pi E)}{(\pi_1\pi_1 v, \mathbf{m}(\pi_1 v)) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix}}\,(\Sigma I)$$

In this $\mathcal{D}_3$ using (566), $\mathcal{D}_4$ is as follows.

(569) Proof diagram $\mathcal{D}_4 \equiv$

$$\dfrac{\dfrac{}{\begin{array}{l}\mathbf{whistle}^*_e \\ : \mathsf{e} \to \mathsf{type}\end{array}}\,(CON) \quad \dfrac{\begin{array}{c} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1 u) \end{bmatrix} \\ \mathcal{D}_3 \\ (\pi_1\pi_1 v, \mathbf{m}(\pi_1 v)) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \end{array}}{\pi_1(\pi_1\pi_1 v, \mathbf{m}(\pi_1 v)) : \mathsf{e}}\,(\Sigma E)}{\begin{array}{l}\mathbf{whistle}^*_e(\pi_1 w)[(\pi_1\pi_1 v, \mathbf{m}(\pi_1 v))/w] \\ \equiv \mathbf{whistle}^*_e(\pi_1(\pi_1\pi_1 v, \mathbf{m}(\pi_1 v))) : \mathsf{type}\end{array}}\,(\Pi E)$$

Applying @-elimination to (566), we obtain the following DTT proof diagram.

(570) @-elimination of (555)

$$\dfrac{\begin{array}{c} \mathcal{D}_1 \\ \begin{bmatrix} \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_{e_1}(\pi_1 u) \end{bmatrix} : \mathsf{type} \end{array} \quad \begin{array}{c} \dfrac{}{v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_{e_1}(\pi_1 u) \end{bmatrix}}\,1 \\ \mathcal{D}_4 \\ \mathbf{whistle}^*_{e_2}(\pi_1\pi_1 v) : \mathsf{type} \end{array}}{\begin{bmatrix} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_{e_1}(\pi_1 u) \end{bmatrix} \\ \mathbf{whistle}^*_{e_2}(\pi_1\pi_1 v) \end{bmatrix} : \mathsf{type}}\,(\Sigma F),1$$

The left-hand side of the bottom node is the semantic representation of the mini-discourse when interpreting *He* in (555b) as an E-type anaphora with *A man* in (555a) as the antecedent. It is noteworthy that the E-type reading was derived by exactly the same process as both coreference and BVA readings, in the previous chapter.

There can be readings other than E-type anaphora for *He* in (555b); it could be a case of deictic use, coreference, or an E-type anaphora with other antecedents (and there is no BVA reading in (555b)). . Such a case (as shown in the previous section) is represented as $\mathcal{D}_3$ being another proof diagram.

Remark 458. In general it is assumed that indefinite noun phrases in English such as *a man* are considered to have no denotation. However, in (555), the pronoun *He*, whose antecedent is *a man*, seems to denote some man, namely, the man who walked into the park. Evans descrived it as “E-type pronoun denotes a set whose member satisfies the antecedent-containing clause” (Evans (1980)). But this set is hardly representable in the first-order logic and many of its extentions. In the above semantic representation, $\pi_1\pi_1 v$ is the *witness* of the first sentence, which is close to Evans' intuition.

23.1.4 Verification Condition

In the same way, the semantic representations of (555a) and the sentence *a man whistled* are (571) and (572), following the argument in Section 16.2.2.

(571)
$$\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1(u)) \end{bmatrix}$$

(572)
$$\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{whistle}^*_e(\pi_1(u)) \end{bmatrix}$$

Thus (556a) and (556b) are reduced to the following DTT inferences, respectively.

(573) a. $$\Gamma, \begin{bmatrix} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1 u) \end{bmatrix} \\ \mathbf{whistle}^*_e(\pi_1\pi_1 v) \end{bmatrix} \vdash \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1(u)) \end{bmatrix}$$

b. $$\Gamma, \begin{bmatrix} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1 u) \end{bmatrix} \\ \mathbf{whistle}^*_e(\pi_1\pi_1 v) \end{bmatrix} \vdash \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{whistle}^*_e(\pi_1(u)) \end{bmatrix}$$

DTS correctly predicts the entailsment patterns in (556a) in a proof-theoretic manner.

(574) Proof diagram of (556a)

$$\dfrac{t : \begin{bmatrix} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1(u)) \end{bmatrix} \\ \mathbf{whistle}^*_e(\pi_1(\pi_1(v))) \end{bmatrix}}{\pi_1(t) : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1(u)) \end{bmatrix}}\,(\Sigma E)$$

The entailsment in (556b) is more complex, but simple enough as in (**??**), where the use of the $(CONV)$ rule (see Subsection **??**) is somewhat abused.

(575) Proof diagram of (556b)

$$
\dfrac{
\dfrac{
\dfrac{
t : \begin{bmatrix} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1(u)) \end{bmatrix} \\ \mathbf{whistle}^*_e(\pi_1\pi_1(v)) \end{bmatrix}
}{
\pi_1(t) : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}^*_e(\pi_1(u)) \end{bmatrix}
}
}{
\pi_1\pi_1(t) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix}
}{\scriptstyle (\Sigma E)}
\qquad
\dfrac{
t : \begin{bmatrix} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{walk}(\pi_1(u)) \end{bmatrix} \\ \mathbf{whistle}^*_e(\pi_1\pi_1(v)) \end{bmatrix}
}{
\pi_2(t) : \begin{aligned} & \mathbf{whistle}^*_e(\pi_1\pi_1(v))[\pi_1(t)/v] \\ \equiv\ & \mathbf{whistle}^*_e(\pi_1\pi_1\pi_1(t)) \\ \equiv\ & \mathbf{whistle}^*_e(\pi_1(u))[\pi_1\pi_1(t)/u] \end{aligned}
}{\scriptstyle (\Sigma E)}
}{
(\pi_1\pi_1(t), \pi_2(t)) : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{man}(x) \end{bmatrix} \\ \mathbf{whistle}^*_e(\pi_1(u)) \end{bmatrix}
}{\scriptstyle (\Sigma I)}
$$

Thus, both inferences in (573) hold, which correctly accounts for the data in (555).

Exercise 459. Confirm that DTS gives a correct predictions for (556c) and (556d).

23.2 Donkey Anaphora

This section will demonstrate a derivation of donkey sentence (Geach (1962)) shown in (578a), as one of the canonical benchmark tests that we expect any new semantic/discourse theory to cover.

23.2.1 A Puzzle

Compositionality in semantic theory is such that it provides a way to calculate the *meaning* of any sentence from the *meanings* of its parts. As has been discussed in many literature in the field of formal semantics, *donkey sentences* in (578a) and (576b) are problematic in terms of compositionality, pointed out originally by Geach (1962).

(576) a. Every farmer who owns [a donkey]$_i$ beats it$_i$. (*relative donkey sentence*)

b. If [a farmer]$_i$ owns [a donkey]$_j$, he$_i$ beats it$_j$. (*conditional donkey sentence*)

The structural analogue of (576a) and (576b) in FoL, which allows us to design a straightforward compositional theory, is (577a) and (577b).

(577) a. $\forall x(\mathbf{farmer}(x) \wedge \exists y(\mathbf{donkey}(y) \wedge \mathbf{own}(x, y)) \rightarrow \mathbf{beat}(x, y))$

b. $\exists x(\mathbf{farmer}(x) \wedge \exists y(\mathbf{donkey}(y) \wedge \mathbf{own}(x, y))) \rightarrow \mathbf{beat}(x, y)$

However, they are not well-formed semantic representations, since the variable y in $\mathbf{beat}(x, y)$ occurs as a free variable in (577a), and both x and y in $\mathbf{beat}(x, y)$ occur as free variables in (577b).

23.2.2 Semantic Representation

For the donkey sentences (578), a first-order formula (579), whose truth condition is the same as those of (578), is a candidate of its SR.

(578) a. Every farmer who owns [a donkey]^1 beats it$_1$.

b. If [a farmer]^1 owns [a donkey]^2, he$_1$ beats it$_2$.

(579) $\forall x(\mathbf{farmer}(x) \to \forall y(\mathbf{donkey}(y) \wedge \mathbf{own}(x,y) \to \mathbf{beat}(x,y)))$

But the translation from the sentence (578) to (579) is not straightforward since i) the indefinite noun phrase *a donkey* is translated into a universal quantifier in (579) instead of an existential quantifier, and ii) the syntactic structure of (579) does not corresponds to that of (578).

(578) a. Every farmer who owns [a donkey]^1 beats it$_1$.

b. If [a farmer]^1 owns [a donkey]^2, they$_1$ beats it$_2$.

The syntactic parallel of (578) is, rather, the SR (580), in which the indefinite noun phrase is translated into an existential quantification.

(580) $\forall x(\mathbf{farmer}(x) \wedge \exists y(\mathbf{donkey}(y) \wedge \mathbf{own}(x,y)) \to \mathbf{beat}(x,y))$

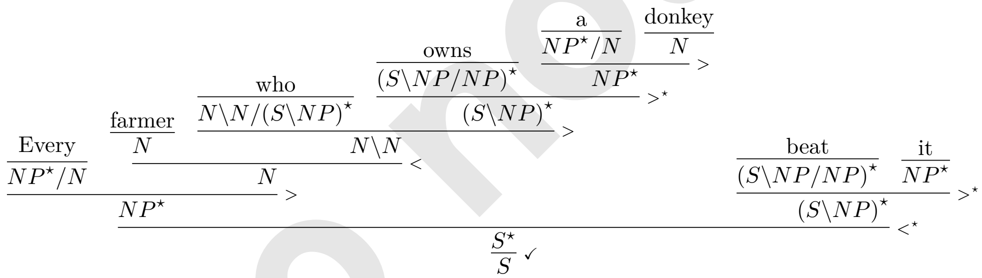
However, (580) does not represent the truth condition of (578) correctly since the variable y in $\mathbf{beat}(x,y)$ fails to be bound by $\exists$.

Therefore, neither (579) nor (580) qualifies as the SR of (578).

23.2.3 Semantic Composition

The derivation of the relative donkey sentence (576a) is as follows.

(581) Syntactic structure of (579a)



(582) Semantic composition of (579a)

$$
\dfrac{\dfrac{\text{farmer}}{\mathbf{farmer}} \qquad
\dfrac{\dfrac{\text{who}}{\lambda q.\lambda n.\lambda x.\begin{bmatrix} n(x) \\ q(\lambda p.px) \end{bmatrix}} \quad
\dfrac{\dfrac{\text{owns}}{\boxed{\begin{array}{l}\lambda y.\lambda x.\\ \mathbf{own}^{*}_{e_1}(x,y)\end{array}}} \quad
\dfrac{\dfrac{\text{a}}{\lambda n.\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ n(y)\end{bmatrix}\\ \boxed{\pi_1 v}\end{bmatrix}} \quad \dfrac{\text{donkey}}{\mathbf{donkey}}}{\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{donkey}(y)\end{bmatrix}\\ \boxed{\pi_1 v}\end{bmatrix}}{>^{\star}}}
{\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{donkey}(y)\end{bmatrix}\\ \boxed{\lambda x.\mathbf{own}^{*}_{e_1}(x,\pi_1 v)}\end{bmatrix}}{>^{\star}}}
{\lambda n.\lambda x.\begin{bmatrix} n(x)\\ v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{donkey}(y)\end{bmatrix}\\ \mathbf{own}^{*}_{e_1}(x,\pi_1 v)\end{bmatrix}}{>}}
{\lambda x.\begin{bmatrix} \mathbf{farmer}(x)\\ v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{donkey}(y)\end{bmatrix}\\ \mathbf{own}^{*}_{e_1}(x,\pi_1 v)\end{bmatrix}}{<}
$$

(583) Semantic composition of (579a)

$$
\dfrac{\dfrac{\dfrac{\dfrac{\text{Every}}{\left(u:\begin{bmatrix} x:\mathsf{e}\\ n(x)\end{bmatrix}\right)\to\boxed{\pi_1 u}} \quad
\dfrac{\text{farmer who owns a donkey}}{\lambda x.\begin{bmatrix} \mathbf{farmer}(x)\\ v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{donkey}(y)\end{bmatrix}\\ \mathbf{own}^{*}_{e_1}(x,\pi_1 v)\end{bmatrix}}}
{\left(u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{farmer}(x)\\ v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{donkey}(y)\end{bmatrix}\\ \mathbf{own}^{*}_{e_1}(x,\pi_1 v)\end{bmatrix}\right)\to\boxed{\pi_1 u}}{<^{\star}}
\quad
\dfrac{\dfrac{\text{beat}}{\boxed{\begin{array}{l}\lambda y.\lambda x.\\ \mathbf{beat}^{*}_{e_2}(x,y)\end{array}}} \quad
\dfrac{\text{it}}{\begin{bmatrix} @\begin{bmatrix} z:\mathsf{e}\\ \neg\mathbf{human}(z)\end{bmatrix}\\ \boxed{\pi_1 w}\end{bmatrix}}}
{\lambda x.\begin{bmatrix} @\begin{bmatrix} z:\mathsf{e}\\ \neg\mathbf{human}(z)\end{bmatrix}\\ \boxed{\lambda x.\mathbf{beat}^{*}_{e_2}(x,\pi_1 w)}\end{bmatrix}}{>^{\star}}}
{\left(u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{farmer}(x)\\ v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{donkey}(y)\end{bmatrix}\\ \mathbf{own}^{*}_{e_1}(x,\pi_1 v)\end{bmatrix}\right)\to\begin{bmatrix} @\begin{bmatrix} z:\mathsf{e}\\ \neg\mathbf{human}(z)\end{bmatrix}\\ \boxed{\mathbf{beat}^{*}_{e_2}(\pi_1 u,\pi_1 w)}\end{bmatrix}}{<^{\star}}}
{\left(u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{farmer}(x)\\ v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{donkey}(y)\end{bmatrix}\\ \mathbf{own}^{*}_{e_1}(x,\pi_1 v)\end{bmatrix}\right)\to\begin{bmatrix} @\begin{bmatrix} z:\mathsf{e}\\ \neg\mathbf{human}(z)\end{bmatrix}\\ \mathbf{beat}^{*}_{e_2}(\pi_1 u,\pi_1 w)\end{bmatrix}}{\checkmark}
$$

Thus we obtain the following semantic representation for the (quantificational) donkey sentence.

(584) Semantic composition of (579a)

$$
\Gamma\vdash\left(u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{farmer}(x)\\ v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{donkey}(y)\end{bmatrix}\\ \mathbf{own}^{*}_{e_1}(x,\pi_1 v)\end{bmatrix}\right)\to\begin{bmatrix} v@\begin{bmatrix} y:\mathsf{e}\\ \neg\mathbf{human}(y)\end{bmatrix}\\ \mathbf{beat}^{*}_{e_2}(\pi_1 u,\pi_1 v)\end{bmatrix} : \mathsf{type}
$$

The felicity condition requires that it has a type type. We assume that the signature includes the following entries:

(585)
$$\begin{array}{rcl}
\mathbf{farmer} & : & \mathrm{e} \to \mathrm{type}, \\
\mathbf{donkey} & : & \mathrm{e} \to \mathrm{type}, \\
\mathbf{own} & : & \mathrm{e} \to \mathrm{e} \to \mathrm{e} \to \mathrm{type}, \\
\mathbf{beat} & : & \mathrm{e} \to \mathrm{e} \to \mathrm{e} \to \mathrm{type} \\
d & : & \left(u : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix} \right) \to \neg\mathbf{human}(\pi_1 u)
\end{array}$$

The function D represents the common sense that DONKEY is not HUMAN.

(586) Type check diagram of (579a)

$$\dfrac{\overset{\mathcal{D}_1}{\begin{bmatrix} x : \mathrm{e} \\ \mathbf{farmer}(x) \\ v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}^*_{e_1}(x, \pi_1 v) \end{bmatrix}} : \mathrm{type} \qquad \dfrac{\overset{\mathcal{D}_2}{\begin{bmatrix} y : \mathrm{e} \\ \neg\mathbf{human}(y) \end{bmatrix}} : \mathrm{type} \qquad \overset{\overline{u : \begin{bmatrix} x : \mathrm{e} \\ \mathbf{farmer}(x) \\ v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}^*_{e_1}(x, \pi_1 v) \end{bmatrix}}^{\,1}}{\overset{\mathcal{D}_3}{? : \begin{bmatrix} y : \mathrm{e} \\ \neg\mathbf{human}(y) \end{bmatrix}}} \qquad \mathcal{D}_4}{\begin{bmatrix} v@\begin{bmatrix} y : \mathrm{e} \\ \neg\mathbf{human}(y) \end{bmatrix} \\ \mathbf{beat}^*_{e_2}(\pi_1 u, \pi_1 v) \end{bmatrix} : \mathrm{type}}\,(@)}{\left(u : \begin{bmatrix} x : \mathrm{e} \\ \mathbf{farmer}(x) \\ v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}^*_{e_1}(x, \pi_1 v) \end{bmatrix} \right) \to \begin{bmatrix} v@\begin{bmatrix} y : \mathrm{e} \\ \neg\mathbf{human}(y) \end{bmatrix} \\ \mathbf{beat}^*_{e_2}(\pi_1 u, \pi_1 v) \end{bmatrix} : \mathrm{type}}\,(\Pi F),1$$

The construction of $\mathcal{D}_3$ requires a PROOF SEARCH. As the following proof diagram shows, $(\pi_1\pi_1\pi_2\pi_2 u, d(\pi_1\pi_2\pi_2 u))$ is a proof term of this type, which corresponds to the reading where t refers to the donkey.

(587) Proof diagram $\mathcal{D}_3 \equiv$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{u : \begin{bmatrix} x : \mathrm{e} \\ \mathbf{farmer}(x) \\ v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}(x, \pi_1 v) \end{bmatrix}}{\pi_2 u : \begin{bmatrix} \mathbf{farmer}(x) \\ v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}(\pi_1 u, \pi_1 v) \end{bmatrix}}\,(\Sigma E)}{\pi_2\pi_2 u : \begin{bmatrix} v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}(\pi_1 u, \pi_1 v) \end{bmatrix}}\,(\Sigma E)}{\pi_1\pi_2\pi_2 u : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix}}\,(\Sigma E)}{\pi_1\pi_1\pi_2\pi_2 u : \mathrm{e}}\,(\Sigma E) \qquad \dfrac{\dfrac{\dfrac{\dfrac{u : \begin{bmatrix} x : \mathrm{e} \\ \mathbf{farmer}(x) \\ v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}(x, \pi_1 v) \end{bmatrix}}{\pi_2 u : \begin{bmatrix} \mathbf{farmer}(x) \\ v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}(\pi_1 u, \pi_1 v) \end{bmatrix}}\,(\Sigma E)}{\pi_2\pi_2 u : \begin{bmatrix} v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}(\pi_1 u, \pi_1 v) \end{bmatrix}}\,(\Sigma E)}{\pi_1\pi_2\pi_2 u : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{donkey}(y) \end{bmatrix}}\,(\Sigma E) \qquad \dfrac{}{d : \begin{array}{l} \left(u : \begin{bmatrix} x : \mathrm{e} \\ \mathbf{donkey}(x) \end{bmatrix} \right) \\ \to \neg\mathbf{human}(\pi_1 u) \end{array}}\,(CON)}{d(\pi_1\pi_2\pi_2 u) : \neg\mathbf{human}(\pi_1\pi_1\pi_2\pi_2 u)}\,(\Pi E)}{(\pi_1\pi_1\pi_2\pi_2 u, d(\pi_1\pi_2\pi_2 u)) : \begin{bmatrix} z : \mathrm{e} \\ \neg\mathbf{human}(z) \end{bmatrix}}\,(\Sigma I)$$

By using this proof search result, the proof search (586) continues as follows.

(588) Proof diagram $\mathcal{D}_4 \equiv$

$$\dfrac{\dfrac{\dfrac{}{\mathbf{beat}^*_{e_2} : \mathsf{e} \to \mathsf{e} \to \mathsf{type}}(CON) \quad \dfrac{\begin{array}{c}\mathcal{D}_3 \\ (\pi_1\pi_1\pi_2\pi_2 u, d(\pi_1\pi_2\pi_2 u)) \\ : \left[\begin{array}{l} y : \mathsf{e} \\ \neg\mathbf{human}(y)\end{array}\right]\end{array}}{\pi_1(\pi_1\pi_1\pi_2\pi_2 u, d(\pi_1\pi_2\pi_2 u)) : \mathsf{e}}(\Sigma E)}{\mathbf{beat}^*_{e_2}(\pi_1((\pi_1\pi_1\pi_2\pi_2 u, d(\pi_1\pi_2\pi_2 u)))) : \mathsf{e} \to \mathsf{type}}(\Pi E) \quad \dfrac{u : \left[\begin{array}{l} x : \mathsf{e} \\ \mathbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \mathsf{e} \\ \mathbf{donkey}(y)\end{array}\right] \\ \mathbf{own}^*_{e_1}(x, \pi_1 v)\end{array}\right]}{\pi_1 u : \mathsf{e}}(\Sigma E)}{\begin{array}{l}\mathbf{beat}^*_{e_2}(\pi_1 u, \pi_1 v)[(\pi_1\pi_1\pi_2\pi_2 u, d(\pi_1\pi_2\pi_2 u))/v] \\ \equiv \mathbf{beat}^*_{e_2}(\pi_1 u, \pi_1((\pi_1\pi_1\pi_2\pi_2 u, d(\pi_1\pi_2\pi_2 u)))) : \mathsf{type}\end{array}}(\Pi E)$$

Applying @-elimination to this UDTT proof diagram, we obtain the following DTT proof diagram (and semantic representation of).

(589) @-elimination of (579a)

$$\dfrac{\begin{array}{c}\mathcal{D}_1 \\ \left[\begin{array}{l} x : \mathsf{e} \\ \mathbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \mathsf{e} \\ \mathbf{donkey}(y)\end{array}\right] \\ \mathbf{own}^*_{e_1}(x, \pi_1 v)\end{array}\right] : \mathsf{type}\end{array} \quad \begin{array}{c}\overline{u : \left[\begin{array}{l} x : \mathsf{e} \\ \mathbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \mathsf{e} \\ \mathbf{donkey}(y)\end{array}\right] \\ \mathbf{own}^*_{e_1}(x, \pi_1 v)\end{array}\right]}^{\,1} \\ \mathcal{D}_4 \\ \begin{array}{l}\equiv \mathbf{beat}^*_{e_2}(\pi_1 u, \pi_1((\pi_1\pi_1\pi_2\pi_2 u, d(\pi_1\pi_2\pi_2 u)))) : \mathsf{type} \\ \to_\beta \mathbf{beat}^*_{e_2}(\pi_1 u, \pi_1\pi_1\pi_2\pi_2 u) : \mathsf{type}\end{array}\end{array}}{\left(u : \left[\begin{array}{l} x : \mathsf{e} \\ \mathbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \mathsf{e} \\ \mathbf{donkey}(y)\end{array}\right] \\ \mathbf{own}^*_{e_1}(x, \pi_1 v)\end{array}\right]\right) \to \mathbf{beat}^*_{e_2}(\pi_1 u, \pi_1\pi_1\pi_2\pi_2 u) : \mathsf{type}}(\Pi F), 1$$

As a result of this process, we obtain the *resolved* semantic representation below.

(590) Semantic composition of (579a)

$$\left(u : \left[\begin{array}{l} x : \mathsf{e} \\ \mathbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \mathsf{e} \\ \mathbf{donkey}(y)\end{array}\right] \\ \mathbf{own}^*_{e_1}(x, \pi_1 v)\end{array}\right]\right) \to \mathbf{beat}^*_{e_2}(\pi_1 u, \pi_1\pi_1\pi_2\pi_2 u)$$

23.2.4 Implicational Donkey Sentence

Recall that the implicational donkey setentence (**??**) has the same meaning as the quantificational one (576a). DTS can correctly predict it (but under the assumption that they in the main clause are anaphora resolved with a farmer as antecedent).

(576b) If [a farmer]$_i$ owns [a donkey]$_j$, they$_i$ beats it$_j$.

Let us follow the process. First, the inside of the conditional clause is a transitive sentence with two existential quantifiers, which we have already dealt with in , but just to be sure, we will start with the derivation.