

Composing Meaning via Dependent Type Semantics

Day 1: Overview

Daisuke Bekki

Ochanomizu University
Faculty of Core Research
<https://daisukebekki.github.io/>

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Dependent Type Semantics (DTS) (Bekki 2014; Bekki and Mineshima 2017; Bekki 2021)

- ▶ A framework of natural language semantics
- ▶ Unified approach to (general) inferences and anaphora/presupposition resolution in terms of *type checking* and *proof search*

Main features:

1. **Proof-theoretic semantics:**
From model theory (denotations and models) to proof theory (proofs and contexts)
2. **Anaphora/Presuppositions:** A proof-theoretic alternative to Dynamic Semantics (DRT, DPL, etc.)
3. **Compositionality:** Syntax-semantics interface via categorial grammars (e.g. CCG, TLG, ACG, etc)
4. **Implementation:** Applications to Natural Language Processing.

<https://github.com/DaisukeBekki/lightblue/>

Dependent Types

Per Martin-Löf

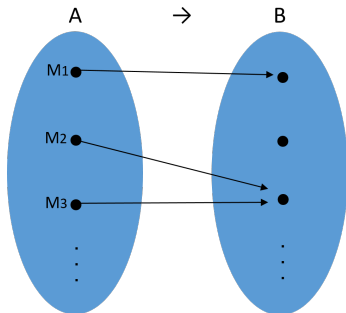


Martin-Löf (1984) “Intuitionistic type theory”

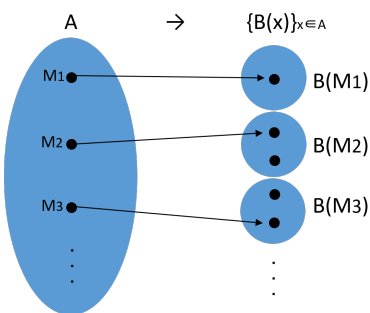
What are Π -types

Π -type is a type of *fibred* functions.

Simple function space



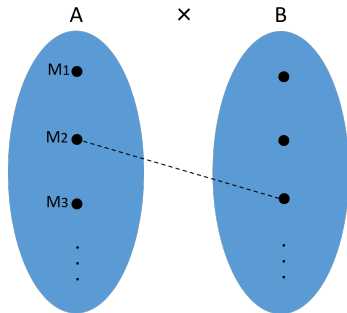
Fibred function space



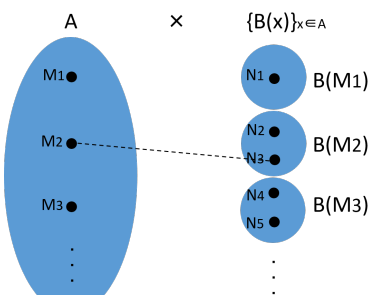
What are Σ -types

Σ -type is a type of *fibred* products.

Simple product space



Fibred product space



Notations

| DTS notation | Standard notation | $x \notin fv(B)$ | $x \in fv(B)$ |
|---|-------------------|-------------------|--------------------|
| $(x : A) \rightarrow B$ | $(\Pi x : A)B$ | $A \rightarrow B$ | $(\forall x : A)B$ |
| $(x : A) \times B$ <i>or</i> $\left[\begin{array}{l} x : A \\ B \end{array} \right]$ | $(\Sigma x : A)B$ | $A \wedge B$ | $(\exists x : A)B$ |

Scope of the variable in Π -types: $(x : A) \rightarrow B$

Scope of the variable in Σ -types: $\left[\begin{array}{l} x : A \\ B \end{array} \right]$

Π -type F/I/E rules

$$\frac{\frac{A : s_1 \quad B : s_2}{(x : A) \rightarrow B : s_2} \quad \frac{\overline{x : A^i} \quad \vdots}{(PI),i}}{(\Pi F),i} \quad \text{where } (s_1, s_2) \in \left\{ \begin{array}{l} (\text{type}, \text{type}), \\ (\text{type}, \text{kind}) \end{array} \right\}.$$

$$\frac{\frac{A : \text{type} \quad M : B}{\lambda x. M : (x : A) \rightarrow B} \quad \frac{\overline{x : A^i} \quad \vdots}{(PI),i}}{(\Pi I),i} \quad \frac{M : (x : A) \rightarrow B \quad N : A}{MN : B[N/x]} (\Pi E)$$

Σ -type F/I/E rules

$$\frac{\begin{array}{c} \overline{}^i \\ x : A \\ \vdots \\ A : \text{type} \quad B : \text{type} \end{array}}{(x : A) \times B : \text{type}} (\Sigma F), i$$

$$\frac{M : A \quad N : B[M/x]}{(M, N) : (x : A) \times B} (\Sigma I)$$

$$\frac{M : (x : A) \times B}{\pi_1(M) : A} (\Sigma E)$$

$$\frac{M : (x : A) \times B}{\pi_2(M) : B[\pi_1(M)/x]} (\Sigma E)$$

Rules of DTS

Rules from Martin-Löf Type Theory

- ▶ Axioms and Structural rules
- ▶ Π -type (Dependent function type) $[F/I/E]$
- ▶ Σ -type (Dependent product type) $[F/I/E]$
- ▶ Intensional equality type $[F/I/E]$
- ▶ Disjoint union type $[F/I/E]$
- ▶ Enumeration type $[F/I/E]$
- ▶ Natural number type $[F/I/E]$

New rule in DTS

- ▶ @ (the 'asperand' operator)
 - ▶ Anaphora and presupposition triggers (linguistically speaking)
 - ▶ Open proofs (logically speaking)

Conjunction, Implication, and Negation

Definition

$$\left[\begin{array}{c} A \\ B \end{array} \right] \stackrel{def}{\equiv} (x : A) \times B \quad \text{where } x \notin fv(B).$$

$$A \rightarrow B \stackrel{def}{\equiv} (x : A) \rightarrow B \quad \text{where } x \notin fv(B).$$

$$\neg A \stackrel{def}{\equiv} (x : A) \rightarrow \perp$$

Anaphora in Natural Language

A theory of anaphora

- ▶ Anaphora representable by a constant symbol:

- ▶ Deictic use:

(1) (*Pointing at John*)

He was born in Detroit.

bornIn(*j*, *d*)

- ▶ Coreference:

(2) John loves a girl who hates him.

$\exists x(\text{girl}(x) \wedge \text{love}(j, x) \wedge \text{hate}(x, j))$

- ▶ Anaphora representable by a variable

- ▶ Bound variable anaphora:

(3) Every boy loves his father.

$\forall x(\text{boy}(x) \rightarrow \text{love}(x, \text{fatherOf}(x)))$

A theory of anaphora

- ▶ Anaphora not representable by FoL:

- ▶ E-type anaphora:

(4) A man entered into the park. He whistled.

- ▶ Donkey anaphora:

(5) Every farmer who owns a donkey beats it.

(6) If a farmer owns a donkey, he beats it.

- ▶ Anaphora not representable by FoL nor dynamic semantics:

- ▶ Syllogistic anaphora:

(7) Every girl received a present. Some girl opened it.

- ▶ Disjunctive antecedent:

(8) If Mary sees a horse or a pony, she waves to it.

Donkey anaphora: Geach (1962)

For the donkey sentences (9), a first-order formula (10), whose truth condition is the same as those of (9), is a candidate of its semantic representation (SR). (We only discuss its *strong reading* here. See Tanaka (2021)

for its *weak reading*.)

(9) a. Every farmer who owns [a donkey]¹ beats it₁.

b. If [a farmer]¹ owns [a donkey]², he₁ beats it₂.

(10) $\forall x(\text{farmer}(x) \rightarrow \forall y(\text{donkey}(y) \wedge \text{own}(x, y) \rightarrow \text{beat}(x, y)))$

But the translation from the sentence (9) to (10) is not straightforward since i) the indefinite noun phrase *a donkey* is translated into a universal quantifier in (10) instead of an existential quantifier, and ii) the syntactic structure of (10) does not corresponds to that of (9).

Donkey anaphora: Geach (1962)

- (9) a. Every farmer who owns [a donkey]¹ beats **it₁** .
b. If [a farmer]¹ owns [a donkey]², he₁ beats **it₂** .

The syntactic parallel of (9) is, rather, the SR (11), in which the indefinite noun phrase is translated into an existential quantification.

$$(11) \quad \forall x(\text{farmer}(x) \wedge \exists y(\text{donkey}(y) \wedge \text{own}(x, y)) \rightarrow \text{beat}(x, y))$$

However, (11) does not represent the truth condition of (9) correctly since the variable y in **beat**(x, y) fails to be bound by \exists . Therefore, neither (10) nor (11) qualifies as the SR of (9).

Various approaches in discourse semantics

Dynamic Semantics

- ▶ Discourse Representation Theory (DRT): Kamp (1981), Kamp and Reyle (1993)
- ▶ Dynamic Predicate Logic (DPL): Groenendijk and Stokhof (1991)
- ▶ Dynamic Plural Predicate Logic (DPPL): van den Berg (1996), Sudo (2012)

Type-theoretical Semantics

- ▶ Analysis of donkey anaphora: Sundholm (1986))
- ▶ Type Theoretical Grammar (TTG): Ranta (1994)
- ▶ Type Theory with Record (TTR): Cooper (2005)
- ▶ MTT-semantics: Luo (1997, 1999, 2010, 2012), Asher and Luo (2012), Chatzikyriakidis (2014)
- ▶ Dependent Type Semantics (DTS): Bekki (2014), Bekki and Mineshima (2017)

Donkey anaphora: Sundholm (1986)

(9a) Every farmer who owns a donkey beats it .

$$\left(u : \left[\begin{array}{l} x : \text{entity} \\ \text{farmer}(x) \\ \left[\begin{array}{l} v : \left[\begin{array}{l} y : \text{entity} \\ \text{donkey}(y) \end{array} \end{array} \right] \\ \text{own}(x, \pi_1 v) \end{array} \right] \right] \right) \rightarrow \text{beat}(\pi_1 u, \pi_1 \pi_1 \pi_2 \pi_2 u)$$

Note: $(x : A) \rightarrow B$ is a type for functions from A to $B[x]$.

From TTG to DTS: Compositionality

- Q: How could one get to these (dependently-typed) representations from arbitrary sentences?
- A: By lexicalization.
- Q: But, how could we lexicalize context-dependent words like pronouns?

$$\left(u : \left[\begin{array}{l} x : \text{entity} \\ \text{farmer}(x) \\ \left[v : \left[\begin{array}{l} y : \text{entity} \\ \text{donkey}(y) \end{array} \right] \\ \text{own}(x, \pi_1 v) \end{array} \right] \right] \right) \rightarrow$$
$$\text{beat}(\pi_1 u, \pi_1 \pi_1 \pi_2 \pi_2 u)$$

From TTG to DTS: Compositionality

Q: How could one get to these (dependently-typed) representations from arbitrary sentences?

A: By lexicalization.

Q: But, how could we lexicalize context-dependent words like pronouns?

A: By using **underspecified types**.

Q: How could we retrieve a context for an underspecified type?

A: By **type checking**.

Summary and History

A Unified, Compositional Theory of *Projective* Meaning

- ▶ DTS provides a unified analysis for (general) inferences and anaphora resolution mechanisms.
- ▶ The background theory for DTS is an extension of DTT with underspecified types and the @-rule .
 - ▶ Lexical items of anaphoric expressions and presupposition triggers are represented by using underspecified types.
 - ▶ Context retrieval in DTS reduces to type checking .
 - ▶ Anaphora resolution and presupposition binding in DTS reduces to proof search .
 - ▶ Type checker translates a proof diagram of DTS into a proof diagram of DTT, by which an SR in DTT is obtained with all anaphora resolved.

Natural language semantics via dependent types:

The first generation

- ▶ Donkey anaphora: Sundholm (1986)
- ▶ Translation from DRS to dependent type representations: Ahn and Kolb (1990)
- ▶ Summation: Fox (1994a,b)
- ▶ Ranta's TTG (Relative and Implicational Donkey Sentences, Branching Quantifiers, Intensionality, Tense): Ranta (1994)
- ▶ Translation from Montague Grammar to dependent type representations: Dávila-Pérez (1995)
- ▶ Presupposition Binding and Accommodation, Bridging: Krahmer and Piwek (1999), Piwek and Krahmer (2000)

Natural language semantics via dependent types: The second generation

- ▶ Type Theory with Record (TTR): Cooper (2005)
- ▶ Modern Type Theory: Luo (1997, 1999, 2010, 2012), Asher and Luo (2012), Chatzikyriakidis (2014)
- ▶ Semantics with Dependent Types: Grudzinska and Zawadowski (2014; 2017)
- ▶ Dynamic Categorical Grammar: Martin and Pollard (2014)
- ▶ **Dependent Type Semantics (DTS): Bekki (2014), Bekki and Mineshima (2017)**

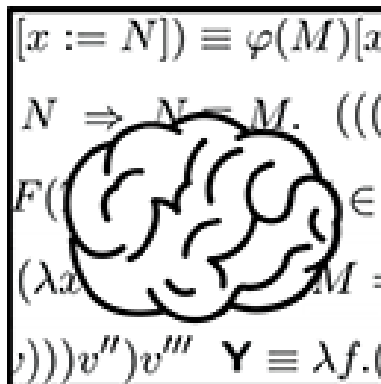
Semantic Analyses by DTS: The third generation

- ▶ Generalized Quantifiers: Tanaka (2014)
- ▶ Honorification: Watanabe et al. (2014)
- ▶ Conventional Implicature: Bekki and McCready (2015), Matsuoka et al. (2023)
- ▶ Factive Presuppositions: Tanaka et al. (2015)
- ▶ Dependent Plural Anaphora: Tanaka et al. (2017)
- ▶ Paycheck sentences: Tanaka et al. (2018)
- ▶ Coercion and Metaphor: Kinoshita et al. (2017, 2018)
- ▶ Questions: Watanabe et al. (2019), Funakura (2022)
- ▶ Comparison with DRT: Yana et al. (2019)
- ▶ The proviso problem: Yana et al. (2021)
- ▶ Weak Crossover: Bekki (2023)

Computational Aspects of DTS

- ▶ Type Checker for (the fragment of) DTS: Bekki and Sato (2015)
- ▶ Development of an automated theorem prover (for the fragment of) DTS: Daido and Bekki (2020)
- ▶ Integrating Deep Neural Network with DTS: Bekki et al. (2023, 2022)

Thank you!



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