Chapter 22

Coreference and Bound Variable Anaphora

In this chapter, I will give an overview of the two different use of pronouns, deictic and anaphoric, in preparation for the discussion that will follow in the next few chapters.

22.1 Deictic and Anaphoric Use of Pronouns

Evans (1980) classified the use of pronouns into deictic use and anaphoric use. The difference between the two lies in whether the referent of the pronoun has been linguistically introduced in the preceding context. If no, it is called deictic use, and if yes, it is called anaphoric use (this difference, however, is later shown not to be so clear cut).

22.1.1 Deictic Use

The deictic use of pronouns is known to be discussed by Kaplan (1989) and is also called *indexical* or *demonstrative* use, but the typical usage is with ostension, as in (501).

(501) *(Pointing at John)*

He is born in Detroit.

In other words, in a situation where John is in front of the speaker, by pointing to John while speaking, the speaker is specifying that the *He* refers to John. In this case, in terms of truth conditions, uttering (501) is almost the same as uttering (502).

(502) John is born in Detroit.

The difference between (501) and (502) is the pathway by which the hearer "reaches" John. In (501), the path taken is that of selecting the person being pointed to from the situation at hand, and then reaching the entity representing that person, *john*. In contrast, in (502), the hearer selects a proper name with the surface form *John* from the lexicon and arrives at its semantic representation, *john*.

There can be intermediate cases between them, such as when John is not in front of the speaker, but the speaker utters (501) while pointing to a photograph of John, which is also classified as a deictic use. In this case, the image being pointed to is selected from the picture in the situation in front of us, and from the person from whom the image originated, we arrive at the entity representing that person, *john*.

A further intermediate case is when several people, including John, are having coffee and John happens to leave the table, and the topic is "who is from which city," and the speaker points to John's coffee cup and utters "He is born in Detroit." In this case, the cup being pointed to is selected from the table in the situation at hand, and from the person who was using the cup, we arrive at the entity representing that person, *john*.

One feature of deictic use that is common in all the above cases is that it constructs the referent of the pronoun from what is observed in the situation. In this respect, it differs from anaphoric use, which constructs the referent from the linguistic expression in the preceding context. However, as already implied in the discussion above, it is important to note that the referent entity itself need not be present in the situation. Also, to "construct a referent" means to construct a chain of relations that reaches to the referent, as the case of the photograph suggests, which will be discussed later.

22.1.2 Anaphoric Use

In contrast to deictic use, anaphoric use is the use in which the pronoun's referent is constructed from the preceding context or by a linguistic expression that provides a context for the pronoun. Let us take the example of the pronoun *her* in (503).

(503) Every girl enjoys a hobby that excites her.

There are at least two different readings for *her* in (503). The first is a reading in which *her* refers to someone in particular. For example, in a context where all the girls want to please Mary because all of them love her, (503) can be uttered with the same truth conditions as (504).

(504) Every girl enjoys a hobby that excites Mary.

Such a reading of *her* in (503) is called *coreference reading*. It has the name *coreference* since the proper name *Mary* and the *her* in (503) refer to the same entity.

Note that the truth condition of (503) is the same as the FoL formula (505), meaning that the semantic representation of pronouns when having coreference readings are *names* in FoL.

(505) $\forall x(\mathbf{girl}(x) \rightarrow \exists y(\mathbf{hobby}(y) \wedge \mathbf{enjoy}(x, y) \wedge \mathbf{excite}(y, mary)))$

The second reading of *her* in (503) is a reading in which, for each girl, *her* is that girl. Such a reading of *her* in (503) is called *bound variable anaphora reading, or BVA reading*. The name BVA comes from the fact that, under a BVA reading, the truth condition for (503) is the same as the following FoL formula, where *her* corresponds to a bound variable x in terms of FoL.

(506) $\forall x(\mathbf{girl}(x) \rightarrow \exists y(\mathbf{hobby}(y) \wedge \mathbf{enjoy}(x, y) \wedge \mathbf{excite}(y, x)))$

Remark 442. How can coreference and BVA be distinguished? One answer is to distinguish by inference. Consider the following inference (507).

(507) Every girl enjoys a hobby that excites her. Sue is a girl.

$\Longrightarrow$ Sue enjoys a hobby that excites Sue.

When *her* in the first sentence has a coreference reading and refers to *Mary*, the inference is not valid. However, when it has a BVA reading, the inference holds. Analysis by FoL (although our final analysis is not FoL) correctly predicts this fact. Taking the case where *her* in the first sentence has a coreference reading and refers to *Mary*, (507) would be as follows in FoL, which is not valid.

(508) $\forall x(\mathbf{girl}(x) \rightarrow \exists y(\mathbf{hobby}(y) \wedge \mathbf{enjoy}(x,y) \wedge \mathbf{excite}(y, mary))),\ \mathbf{girl}(sue)$
$\vdash \exists y(\mathbf{hobby}(y) \wedge \mathbf{enjoy}(sue, y) \wedge \mathbf{excite}(y, sue)))$

On the other hand, if *her* the first sentence has a BVA reading, then this is valid, although the following is true in FoL: (507).

(509) $\forall x(\mathbf{girl}(x) \rightarrow \exists y(\mathbf{hobby}(y) \wedge \mathbf{enjoy}(x,y) \wedge \mathbf{excite}(y, x))),\ \mathbf{girl}(sue)$
$\vdash \exists y(\mathbf{hobby}(y) \wedge \mathbf{enjoy}(sue, y) \wedge \mathbf{excite}(y, sue)))$

Exercise 443. Prove (508) and (509).

Conversely, the following inference is valid when *her* in the first sentence has a coreference reading and refers to *Mary*. However, it does not hold when it has a BVA reading.

(510) Every girl enjoys a hobby that excites her. Sue is a girl.
$\Longrightarrow$ Sue enjoys a hobby that excites Mary.

Therefore, it can be seen that the coreference and BVA in the interpretation of the *her* of (503) are independent readings of each other.

Remark 444. BVA reading is also possible for quantificational expressions other than *Every*. Although (511) is the case of *No*, there is also coreference reading and BVA reading for *her* in this sentence, and in BVA reading, for each girl, it means that no girl enjoys a hobby that bores the girl herself. In BVA reading, it means that for each girl, there is no girl who enjoys a hobby that bores her.

(511) No girl enjoyes a hobby that makes her bored.

Exercise 445. For (511), consider an inference that distinguishes between coreference and BVA. Also show that the inference can be predicted by the semantic representation by FoL.

Remark 446. This way of stating that "a reading that produces such an inference is a BVA reading" by listing a number of inferences to a single sentence does not define what a BVA reading is. Instead, it might be tempting to "define" BVA reading as, for example, "a reading in which the indicative object of the pronoun changes in conjunction with the indicative object of the antecedent." However, this definition contains imprecise notions. For example, what is the "denotation" of the antecedent *every girl*? As we have seen in the Chapter 16, quantificational expressions are functions with higher-order types, so there is nothing that "changes in conjunction" with them. More to the point, there is no such denotation here for *no girl*, so this argument does not apply to examples such as (511).

What this circumstance tells us is that, in fact, the notion of BVA cannot be defined empirically. Therefore, the only possible position for the concept of BVA is to characterise it with a pattern of inference that the sentence follows, such as (507) or (510).

Remark 447. The pronoun used in anaphoric use is called anaphora (*her* in (503)(511)) and the linguistic expression introducing the object to which anaphora refers is called antecedent (*every girl* in (503) and *no girl* in (511)). The relationship established between anaphora and its antecedent is called anaphoric link (but this relationship itself is sometimes called anaphra).

BVA is one of the most common examples of anaphoric use of pronouns, along with E-type anaphora, which will be discussed in the next chapter.

Remark 448. The BVA antecedent must be in the same sentence. In the example below, the BVA reading is not present in the second sentence textither.*1

(512) Every girl enjoys a hobby. It excites her.

Even in the same sentence, BVA reading is said not to occur if the antecedent does not C-synthesise the pronoun. There is a BVA reading in the *his* of (513a), but there is no BVA reading in the *his* of (513b). This is an effect called the weak crossover effect (WCO), but the WCO is discussed again in Chapter **??**.*2

(513) a. Every boy praised his father.
b. * His father praised every boy.

Remark 449. The meaning of sentences containing pronouns of Deictic use is identical to that of sentences in which pronouns are replaced by proper nouns, as shown by (502). However, it is important to note that the meaning of anaphora is not identical to that of the antecedent. In other words, anaphora is not a copy of the antecedent. For example, the truth condition of (514a) differs from that of (503) and the truth condition of (514b) differs from that of (511).

(514) a. Every girl enjoyes a hobby that excites every girl.
b. No girl enjoyes a hobby that makes no girl bored.

In contrast, the meaning of sentences containing coreference is identical to that of sentences in which pronouns are replaced by proper nouns. In this sense, the question arises as to whether coreference should be categorised as an independent use of a pronoun as a type of anaphora, or whether it is more akin to deictic use. This issue is discussed further in the next section.

22.1.3 More on Coreference

Typical examples of Coreference are as follows, where a noun phrase that appears in the preceding context or in the same sentence (hereafter *John*) and a pronoun (hereafter *he*) refer to the same object. (515) is an example where *John* appears in

*1 The following exception to this is known to exist, which is called *telescoping*. Telescoping is discussed again in Subsection 24.7.1.

(636) Each candidate for the space misson meets all our requirements. He has a Ph.D. in Astrophysics and extensive prior flight experience.

*2 For more information on the WCO acceptability swing and control methods in experiments, see Hoji (2015), Plesniak (2022), Fukushima et al. (2023).

the same sentence as *he*, and (516) is an example where *John* appears in the preceding sentence.

(515) John claimed that he was sick.

(516) John is a student. He is a smart guy.

In both (515) and (516), the proper name *John* and the pronoun *he* refer to the same individual (i.e., the actual John). However, there has been considerable debate regarding whether the pronoun (in this instance, *he*) refers to the linguistic expression *John* (found in the preceding context or within the same sentence) or to John himself, akin to a deictic use. The former perspective posits that the pronoun refers to the preceding noun phrase. The latter, in contrast, suggests that the pronoun and the preceding noun phrase happen to refer to the same extralinguistic entity. *3

In the former view, coreference is the linguistic relationship between a pronoun and its antecedent, traditionally discussed as the operation of replacing one with the other when the same noun phrase appears in two places in a sentence (**?**). In this case, the antecedent must be present in the sentence. In particular, some literature distinguishes between cases where the antecedent literally precedes the pronoun by the antecedent and cases where the antecedent appears after the pronoun by the postcedent.

In the latter view, the denotation of the pronoun is in the knowledge or immediate situation, and the coreference is the denoting relation between the pronoun and the denotation. In this view, the antecedent (and its indicating object) is incidental and it does not matter whether the antecedent appears before or after the pronoun. The second view also means that there is no theoretical distinction between deictic use and coreference of pronouns.

In fact, there are some usages that can clearly be said to be deictic use of *her* in (503). For example, it is possible to make the following utterance while pointing to Mary nearby.

(517) *(Pointing at Mary)*

Every girl enjoyes a hobby that excites her!

When taken together with the fact that in deictic use, as discussed in the previous section, the object of instruction does not necessarily have to be present in front of us, it is questionable whether this usage can be distinguished from the coreference reading of (**??**).

Also, if we argue that the *He* in *he was sick* and *He is a smart guy* refer to the preceding *He* (not John himself) respectively, when we consider the following example If this is the case, how does the preceding *He* itself reach John?

*3 In the first place, a proper noun refers to an object in verificationist semantics because 1) the proper noun contributes to the verification condition of the sentence containing it by being associated in the lexical entry of the proper noun in the form that it is mapped to a semantic representation containing the constructor of a particular Entity, and 2) the constructor contributes to the verification condition of the sentence containing it by being associated with the constructor of the Entity. This means that the constructor contributes to the validation conditions of the sentence containing it. Here, the proper noun *John* is mapped to the semantic representation $\lambda p.p(john)$, so the semantic representation of the sentence containing it contains *john*, but in that semantic representation *john* is one of the e-type It means that *john* behaves as one of the *constructors* of type e.

(518) a. He claimed that he was sick.
b. He is a student. He is a smart guy.

If we put these things together, there is merit in the analysis that the use of the pronoun *coreference* should be categorised in the same bracket as deictic use, and that there is or is not ostension depending on whether there happens to be an indicating object in front of us or not. The analysis would have advantages.

Put another way, the linguistic relation assumed by the first view is surplus. If there is deictic use in pronoun and a relation between pronoun and denotation established there, then pronoun should be able to establish that relation with denotation in the syntax of coreference, therefore, because any theory of deictic use would also explain the case of coreference. For these reasons, the DTS takes the position that coreference is not to be distinguished from deictic use.

22.1.4 Toward a Unified Theory of Pronouns

The pronouns provide many challenges to the theory of meaning. This is because the usage of pronouns is ambiguous in most cases and thereby the way the denotations/antecedents are looked for and the way they are represented within the theory.

For each sentence containing an anaphoric expression, A theory of anaphora has to provide the following:

1. A proper semantic representation that captures its meaning
2. An algorithm for determining its antecedent with:
   - An algorithm to enumerate its accessible antecedents
   - An algorithm to disambiguate between accessible antecedents

It has not been clear for many years what theory could represent this in an integrated way. The theory that resolves this is UDTT, introduced in the previous section. Up to this point, the preterm of DTT has been used for semantic representation, but from the next section onwards this will be changed to the preterm of UDTT. The semantic felicity condition is then required as a judgment in UDTT.

With this change, semantic representations containing underspecified types (i.e. semantic representations of sentences containing pronouns) will invoke @-rule in the middle of a proof diagram to show that the semantic felicity condition is satisfied. This requires that the proof (in that context) of the proposition required by the prounoun is valid. This is linguistically synonymous with the existence of an (accessible) antecedent in prounoun, so in DTS, the existence of an antecedent in prounoun is part of the SFC.

On the other hand, semantic representations that do not contain underspecified types are not affected by this change. This is because the SFC for semantic representations that do not contain underspecified types by refthUDTT3 is still a proof diagram for DTT.

From the next section onwards, the semantic representations of pronouns are given as UDTT preterms, while looking at the process by which the meanings of the pronouns discussed so far are interpreted through synthesis from syntactic structures to semantic representations.

22.2 Lexicalizing Pronouns

With underspecified types, the lexical items for pronouns in $\star$-CCG are specified as follows.[*4]

(519) a. $$\llbracket \text{he} \vdash NP^\star \rrbracket \stackrel{def}{\equiv} \left[\begin{array}{l} u@\left[\begin{array}{l} x:\mathrm{e} \\ \mathbf{male}(x)\end{array}\right] \\ \boxed{\pi_1 u}\end{array}\right]$$

b. $$\llbracket \text{she} \vdash NP^\star \rrbracket \stackrel{def}{\equiv} \left[\begin{array}{l} u@\left[\begin{array}{l} x:\mathrm{e} \\ \mathbf{female}(x)\end{array}\right] \\ \boxed{\pi_1 u}\end{array}\right]$$

c. $$\llbracket \text{it} \vdash NP^\star \rrbracket \stackrel{def}{\equiv} \left[\begin{array}{l} u@\left[\begin{array}{l} x:\mathrm{e} \\ \neg\mathbf{human}(x)\end{array}\right] \\ \boxed{\pi_1 u}\end{array}\right]$$

The semantic representation in (519a) is an example of the use of an underspecified type, assuming that it has a proof, that is, a pair of some entity and a proof of its being a male, under the given context, which the presupposition triggered by the use of the pronoun he.

The key insight in the analysis presented in (519) is that the pronouns are inherently *scope takers*. This property is captured within the framework of $\star$-CCG in which their semantic representations take the form of a 2-level box, exhibiting underspecification at the second level. This proves crucial for an analysis of the Weak Crossover (WCO) phenomenon, to which we shall later turn our attention. Pronouns beyond the nominative case can be similarly defined, adhering to this schema.

(520) a. $$\llbracket \text{him} \vdash NP^\star \rrbracket \stackrel{def}{\equiv} \left[\begin{array}{l} v@\left[\begin{array}{l} y:\mathrm{e} \\ \mathbf{male}(y)\end{array}\right] \\ \boxed{\pi_1 v}\end{array}\right]$$

b. $$\llbracket \text{her} \vdash NP^\star \rrbracket \stackrel{def}{\equiv} \left[\begin{array}{l} v@\left[\begin{array}{l} y:\mathrm{e} \\ \mathbf{female}(y)\end{array}\right] \\ \boxed{\pi_1 v}\end{array}\right]$$

22.3 Anaphora Resolution as Proof Construction

(ToDo: Add historical remarks on Krause (1995), Krahmer and Piwek (1999) and Piwek and Krahmer (2000))

[*4] The square brackets for underspecified types are syntactically distinguished from those of the Σ-type notation, but when these square brackets nest, they may be omitted except for the outermost one in the same way as brackets in Σ-type notation, as follows.

$$\left[\begin{array}{l} x:A \\ y@B \\ C(x,y)\end{array}\right] \equiv \left[\begin{array}{l} x:A \\ \left[\begin{array}{l} y@B \\ C(x,y)\end{array}\right]\end{array}\right]$$

22.3.1 Case of Deictic Use

We begin by examining the mechanism through which DTS resolves reference of deictic pronouns.

(501) *(Pointing at John)*

He is born in Detroit.

The syntactic structure and semantic composition within $\star$-CCG is presented as follows. For simplicity, *born in Detroit* is here treated as a single predicate, a simplification that does not impact the analysis.

(521) Syntactic structure of (501)

$$\cfrac{\cfrac{\cfrac{\text{He}}{NP^\star} \qquad \cfrac{\cfrac{\text{is}}{(S\backslash NP/(VP\backslash NP))^\star} \quad \cfrac{\text{born in Detroit}}{(VP\backslash NP)^\star}}{(S\backslash NP)^\star}>^\star}{S^\star}<^\star}{S}\checkmark$$

(522) Semantic composition of (501)

$$\cfrac{\cfrac{\cfrac{\text{He}}{\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \boxed{\pi_1 u} \end{bmatrix}} \qquad \cfrac{\cfrac{\text{is}}{\boxed{\lambda p.\lambda x.\begin{bmatrix} s:\mathsf{e} \\ pxs \end{bmatrix}}} \quad \cfrac{\text{born in Detroit}}{\boxed{\lambda x.\lambda s.\mathbf{bornInDetroit}(s,x)}}}{\begin{array}{l}\boxed{\lambda x.\begin{bmatrix} s:\mathsf{e} \\ \mathbf{bornInDetroit}(s,x) \end{bmatrix}} \\ \equiv \boxed{\lambda x.\mathbf{bornInDetroit}^*_s(x)}\end{array}}>^\star}{\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \boxed{\mathbf{bornInDetroit}^*_s(\pi_1 u)} \end{bmatrix}}<^\star}{\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{bornInDetroit}^*_s(\pi_1 u) \end{bmatrix}}\checkmark$$

Recall the definition of the $*$-operator from Section 19.3.5. This operator will be employed as illustrated above for reasons of conciseness. The semantic felicity condition for (501) within a given context Γ is thus:

(523) $$\Gamma \vdash \begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{bornInDetroit}^*_e(\pi_1 u) \end{bmatrix} : \mathsf{type}$$

Note that (523) is a UDTT judgment, not one of DTT, as elaborated in Subsection 22.1.4. Consequently, throughout this and subsequent chapters, SFC will be stated as a UDTT judgment. Recall that UDTT's signatures and contexts consist of DTT preterms.

Consider, then, an enumeration type $\mathsf{e} \stackrel{def}{\equiv} \{john, bill, \ldots\}$. Let us assume the provision of the following signature: **male** is a one-place predicate, and m_j and m_b serve as proofs attesting to John and Bill, respectively, as male persons.

(524) $\sigma \overset{def}{\equiv} \mathbf{male} : \mathsf{e} \to \mathsf{type}, m_j : \mathbf{male}(john),\ m_b : \mathbf{male}(bill)$

This minimal configuration immediately highlights an ambiguity in the deictic use of *he* between *john* and *bill*.

Exercise 450. Check the c.o.v. of the signature σ above.

When (523) is presented as a type check query, it yields the following proof diagram. The lowest line of this diagram corresponds to the (@)-rule, with $\mathcal{D}_1$, $\mathcal{D}_2$, and $\mathcal{D}_3$ representing UDTT proof diagrams.

(525) Type check diagram of (501)

$$\frac{\overset{\mathcal{D}_1}{\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} : \mathsf{type}} \quad \overset{\mathcal{D}_2}{? : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix}} \quad \overset{\mathcal{D}_3}{\mathbf{bornInDetroit}^*_s(\pi_1 u)[?/u] : \mathsf{type}}}{\begin{bmatrix} u@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{bornInDetroit}^*_s(\pi_1 u) \end{bmatrix} : \mathsf{type}}\ (@)$$

The configuration of $\mathcal{D}_1$ is straightforward, as shown below.

(526) Proof diagram $\mathcal{D}_1 \equiv$

$$\frac{\dfrac{}{\mathsf{e} : \mathsf{type}}\,(\{\}F) \quad \dfrac{\dfrac{}{\mathbf{male} : \mathsf{e} \to \mathsf{type}}\,(CON) \quad \dfrac{}{x : \mathsf{e}}\,{}^{1}}{\mathbf{male}(x) : \mathsf{type}}\,(\Pi E)}{\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} : \mathsf{type}}\ (\Sigma F)$$

The immutability of the proof diagram under @-elimination (cf. Theorem 440) entails that the preterms on the respective right and left-hand sides of the bottom lines of $\mathcal{D}_2$ and $\mathcal{D}_1$ are equivalent. Consequently, for (501) to satisfy SFC, a proof $\mathcal{D}_2$ of this type must be constructed under the specified context.[*5]

The linguistic derivation of $\mathcal{D}_2$ is directly correlated with the search for a referent of *He* in (501). In deictic use, the referent is constructed from the signature σ and the context Γ. Given the current signature, which contains two male entities, namely John and Bill, the aforementioned proof search yields the following two distinct proof diagrams.

(527) Proof diagram $\mathcal{D}_j \equiv$

$$\frac{\dfrac{}{john : \mathsf{e}}\,(\{\}I) \quad \dfrac{}{m_j : \mathbf{male}(john)}\,(CON)}{(john, m_j) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix}}\ (\Sigma I)$$

[*5] The very definition of the @-rule implies that type inference and checking within UDTT necessitate proof construction, thereby leading to undecidability. While this presents a potential engineering challenge for DTS, it aligns felicitously with a linguistic perspective, given the inherent undecidability of human natural language understanding. Indeed, the construction of such a proof precisely mirrors the cognitive computation a hearer undertakes to resolve pronominal antecedents.

(528) Proof diagram $\mathcal{D}_b \equiv$

$$\dfrac{\dfrac{}{bill : \mathsf{e}}\,(\{\}I) \qquad \dfrac{}{m_b : \mathbf{male}(bill)}\,(CON)}{(bill, m_b) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix}}\,(\Sigma I)$$

The two proof diagrams possess distinct proof terms. Since $\mathcal{D}_3$ requires the substitution of u in $\mathbf{bornInDetroit}^*_s(\pi_1 u)$, the conclusion of $\mathcal{D}_3$ is contingent upon whether $\mathcal{D}_2$ equals to $\mathcal{D}_j$ or $\mathcal{D}_b$. Through β-reduction, the subsequent calculation illustrates this for each aforementioned instance.

(529) $$\begin{aligned} \mathbf{bornInDetroit}^*_s(\pi_1 u)[(john, m_j)/u] &\equiv \mathbf{bornInDetroit}^*_s(\pi_1(john, m_j)) \\ &\to_\beta \mathbf{bornInDetroit}^*_s(john) \end{aligned}$$

(530) $$\begin{aligned} \mathbf{bornInDetroit}^*_s(\pi_1 u)[(bill, m_b)/u] &\equiv \mathbf{bornInDetroit}^*_s(\pi_1(bill, m_b)) \\ &\to_\beta \mathbf{bornInDetroit}^*_s(bill) \end{aligned}$$

In each case, the overall proof diagram is as follows.

(531) Type check diagram of (501) for (529)

$$\dfrac{\begin{matrix} \mathcal{D}_1 \\ \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} : \mathsf{type} \end{matrix} \quad \begin{matrix} \mathcal{D}_j \\ (j, m_j) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \end{matrix} \quad \dfrac{\dfrac{\dfrac{}{\begin{matrix}\mathbf{bornInDetroit}^*_s \\ : \mathsf{e} \to \mathsf{type}\end{matrix}}\,(CON) \quad \dfrac{}{john : \mathsf{e}}\,(\{\}F)}{\mathbf{bornInDetroit}^*_s(john) : \mathsf{type}}\,(\Pi E)}{\mathbf{bornInDetroit}^*_s(\pi_1 u)[(j, m_j)/u] : \mathsf{type}}\,(CONV)}{\begin{bmatrix} u@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{bornInDetroit}^*_s(\pi_1 u) \end{bmatrix} : \mathsf{type}}\,(@)$$

(532) Type check diagram of (501) for (530)

$$\dfrac{\begin{matrix} \mathcal{D}_1 \\ \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} : \mathsf{type} \end{matrix} \quad \begin{matrix} \mathcal{D}_b \\ (b, m_b) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \end{matrix} \quad \dfrac{\dfrac{\dfrac{}{\begin{matrix}\mathbf{bornInDetroit}^*_s \\ : \mathsf{e} \to \mathsf{type}\end{matrix}}\,(CON) \quad \dfrac{}{bill : \mathsf{e}}\,(\{\}F)}{\mathbf{bornInDetroit}^*_s(bill) : \mathsf{type}}\,(\Pi E)}{\mathbf{bornInDetroit}^*_s(\pi_1 u)[(b, m_b)/u] : \mathsf{type}}\,(CONV)}{\begin{bmatrix} u@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{male}(x) \end{bmatrix} \\ \mathbf{bornInDetroit}^*_s(\pi_1 u) \end{bmatrix} : \mathsf{type}}\,(@)$$

(531) and (532) guarantee that SFC is met in every instance. For any UDTT proof diagram $\Gamma \vdash M : \mathsf{type}$, applying the (@)-elimination yields a corresponding DTT proof diagram of the form $\Gamma \vdash M' : \mathsf{type}$.

(533) @-elimination of (501) for (529)

$$\dfrac{\dfrac{}{\begin{matrix}\mathbf{bornInDetroit}^*_s \\ : \mathsf{e} \to \mathsf{type}\end{matrix}}\,(CON) \quad \dfrac{}{john : \mathsf{e}}\,(\{\}F)}{\mathbf{bornInDetroit}^*_s(john) : \mathsf{type}}\,(\Pi E)$$

(534) @-elimination of (501) for (530)

$$\dfrac{\dfrac{}{\mathbf{bornInDetroit}^*_s : \mathsf{e} \to \mathsf{type}}\,(CON) \qquad \dfrac{}{\mathit{bill} : \mathsf{e}}\,(\{\}F)}{\mathbf{bornInDetroit}^*_s(\mathit{bill}) : \mathsf{type}}\,(\Pi E)$$

Consequently, the semantic representation of (501) is either $\mathbf{bornInDetroit}^*_s(john)$ or $\mathbf{bornInDetroit}^*_s(bill)$, each of which corresponds to a distinct referent for the pronoun *He*. Thus, (501) possesses two separate deictic readings, each being represented by a unique semantic representation in DTT. The first reading is equivalent to *John is born in Detroit*, and the second to *Bill is born in Detroit*.

Remark 451. In DTS, the well-formedness of a semantic representation, obtained through semantic composition, is contingent upon it being a preterm of type. Consequently, if no proof diagram satisfies the second premise of the (@)-rule – that is, if the pronoun lacks a satisfying referent – the sentence or discourse is not merely truth-valueless but is instead judged to be semantically ill-formed.

Remark 452. As demonstrated by (501), multiple proof diagrams can satisfy the conditions imposed by the (@)-rule. This corresponds to the existence of several potential referents for a pronoun, grounded in the assumption that a theory of meaning must enumerate all possible readings.

In actual linguistic use, however, the selection of the appropriate referent from a set of candidates is determined by contextual and personal preferences. This process is analogous to selecting the most suitable proof diagram from multiple possibilities derived from a proof search and ranking them accordingly. However, this task falls within the domain of pragmatics and thus lies outside the scope of DTS as a semantic theory. Such computations have been investigated in natural language processing under the name *anaphora resolution*. It is crucial to recognize that DTS provides the set of possible referents (as semantic representations that satisfy SFC) from which an anaphora resolution process must select a solution.

Remark 453. In the context of the utterance in (501), the speaker's act of pointing at John disambiguates the intended referent. To formally integrate this aspect into our theoretical framework, we can explicitly incorporate information about ostension into the signature and context.

(535) $\sigma,\ \mathbf{point} : \mathsf{e} \to \mathsf{e} \to \mathsf{type}$

(536) $\Gamma,\ ost : \mathbf{point}(speaker, john)$

Let us define $\mathbf{point}(x, y)$ as a proposition that entity x points to entity y. The proof of ostension, *ost*, would then be that the speaker is pointing to John. Consequently, the lexical entry for pronouns with ostension should be modified as shown in (537).

(537) $$[\![\text{he} \vdash NP^\star]\!] \stackrel{def}{\equiv} \left[\begin{array}{l} u@\left[\begin{array}{l} x : \mathsf{e} \\ \mathbf{male}(x) \end{array}\right] \\ v@\mathbf{point}(speaker, \pi_1 u) \\ \boxed{\pi_1 u} \end{array}\right]$$

Exercise 454. An alternative approach is to treat ostension as a unary rule that applies to constituents of the syntactic type $NP^\star$. While this rule could, without further constraints, apply to quantifiers such as *every boy*, the empirical consequences of such an application must be carefully considered.

22.3.2 Case of Coreference/BVA

A core objective in syntactic theory is the analysis of coreference and BVA readings of pronouns. For this, we take example (503) as our point of departure.

(503) Every girl enjoys a hobby that excites her.

We assume that **entity** $\stackrel{def}{\equiv} \{mary, susan, \ldots\}$, along with the following signature:

(538)
$$\sigma \stackrel{def}{\equiv} \begin{array}{rcl} \textbf{female} & : & \mathrm{e} \to \mathbf{type}, \\ \textbf{hobby} & : & \mathrm{e} \to \mathbf{type}, \\ \textbf{enjoy} & : & \mathrm{e} \times \mathrm{e} \times \mathrm{e} \to \mathbf{type}, \\ \textbf{excite} & : & \mathrm{e} \times \mathrm{e} \times \mathrm{e} \to \mathbf{type}, \\ f_m & : & \textbf{female}(mary), \\ f_s & : & \textbf{female}(susan), \\ gf & : & (u : (x : \mathrm{e}) \times \textbf{girl}) \to \textbf{female}(\pi_1 u) \end{array}$$

This signature includes the function gf, which represents the world knowledge that all girls are female.

(539) Syntactic structure of (503)

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{Every}}{NP^\star/N}\quad\dfrac{\text{girl}}{N}}{NP^\star}> \quad \dfrac{\dfrac{\text{enjoys}}{(S\backslash NP/NP)^\star}\quad \dfrac{\dfrac{\text{a}}{NP^\star/N}\quad \dfrac{\dfrac{\text{hobby}}{N}\quad \dfrac{\dfrac{\text{that}}{N\backslash N/(S\backslash NP)^\star}\quad \dfrac{\dfrac{\text{excites}}{(S\backslash NP/NP)^\star}\quad\dfrac{\text{her}}{NP^\star}}{(S\backslash NP)^\star}>^\star}{N\backslash N}>}{N}<}{NP^\star}>}{(S\backslash NP)^\star}>^\star}{S^\star}<^\star}{S}\checkmark$$

(540) Semantic composition of (503)

$$\dfrac{\dfrac{\text{excites}}{\boxed{\begin{array}{l}\lambda y.\lambda x.\\ \mathbf{excite}^*_{e_2}(x,y)\end{array}}} \quad \dfrac{\text{her}}{\left[\begin{array}{l} w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \boxed{\pi_1 w}\end{array}\right]}}{\left[\begin{array}{l} w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \boxed{\lambda x.\mathbf{excite}^*_{e_2}(x,\pi_1 w)}\end{array}\right]}>^{\star}$$

$$\dfrac{\dfrac{\text{that}}{\lambda q.\lambda n.\lambda x.\left[\begin{array}{l}nx\\ q(\lambda p.px)\end{array}\right]} \quad \left[\begin{array}{l} w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \boxed{\lambda x.\mathbf{excite}^*_{e_2}(x,\pi_1 w)}\end{array}\right]}{\lambda n.\lambda x.\left[\begin{array}{l}nx\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(x,\pi_1 w)\end{array}\right]}>$$

$$\dfrac{\dfrac{\text{hobby}}{\mathbf{hobby}} \quad \lambda n.\lambda x.\left[\begin{array}{l}nx\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(x,\pi_1 w)\end{array}\right]}{\lambda x.\left[\begin{array}{l}\mathbf{hobby}(x)\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(x,\pi_1 w)\end{array}\right]}<$$

$$\dfrac{\dfrac{\text{a}}{\lambda n.\left[\begin{array}{l}v:\left[\begin{array}{l}y:\mathrm{e}\\ n(y)\end{array}\right]\\ \boxed{\pi_1 v}\end{array}\right]} \quad \lambda x.\left[\begin{array}{l}\mathbf{hobby}(x)\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(x,\pi_1 w)\end{array}\right]}{\left[\begin{array}{l}v:\left[\begin{array}{l}y:\mathrm{e}\\ \mathbf{hobby}(y)\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{array}\right]\\ \boxed{\pi_1 v}\end{array}\right]}>$$

(541) Semantic composition of (503)

$$\dfrac{\dfrac{\text{enjoys}}{\boxed{\begin{array}{l}\lambda y.\lambda x.\\ \mathbf{enjoy}^*_{e_1}(x,y)\end{array}}} \quad \dfrac{\text{a hobby that excites her}}{\left[\begin{array}{l}v:\left[\begin{array}{l}y:\mathrm{e}\\ \mathbf{hobby}(y)\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{array}\right]\\ \boxed{\pi_1 v}\end{array}\right]}}{\left[\begin{array}{l}v:\left[\begin{array}{l}y:\mathrm{e}\\ \mathbf{hobby}(y)\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{array}\right]\\ \boxed{\lambda x.\mathbf{enjoy}^*_{e_1}(x,\pi_1 v)}\end{array}\right]}>^{\star}$$

$$\dfrac{\dfrac{\text{Every}}{\lambda n.\left(u:\left[\begin{array}{l}x:\mathrm{e}\\ n(x)\end{array}\right]\right)\to\boxed{\pi_1 u}} \quad \dfrac{\text{girl}}{\mathbf{girl}}}{\left(u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{girl}(x)\end{array}\right]\right)\to\boxed{\pi_1 u}}>$$

$$\dfrac{\left(u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{girl}(x)\end{array}\right]\right)\to\boxed{\pi_1 u} \quad \left[\begin{array}{l}v:\left[\begin{array}{l}y:\mathrm{e}\\ \mathbf{hobby}(y)\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{array}\right]\\ \boxed{\lambda x.\mathbf{enjoy}^*_{e_1}(x,\pi_1 v)}\end{array}\right]}{\left(u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{girl}(x)\end{array}\right]\right)\to\left[\begin{array}{l}v:\left[\begin{array}{l}y:\mathrm{e}\\ \mathbf{hobby}(y)\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{array}\right]\\ \boxed{\mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)}\end{array}\right]}<^{\star}$$

$$\dfrac{\left(u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{girl}(x)\end{array}\right]\right)\to\left[\begin{array}{l}v:\left[\begin{array}{l}y:\mathrm{e}\\ \mathbf{hobby}(y)\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{array}\right]\\ \boxed{\mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)}\end{array}\right]}{\left(u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{girl}(x)\end{array}\right]\right)\to\left[\begin{array}{l}v:\left[\begin{array}{l}y:\mathrm{e}\\ \mathbf{hobby}(y)\\ w@\left[\begin{array}{l}z:\mathrm{e}\\ \mathbf{female}(z)\end{array}\right]\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{array}\right]\\ \mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)\end{array}\right]}\checkmark$$

The semantic felicity condition under a given context requires the following:

(542) Type check diagram of (503)

$$
\dfrac{
\dfrac{\overline{\mathsf{e}:\mathsf{type}}\,(\{\}F)
\qquad
\dfrac{
\begin{array}{c}\overline{y:\mathsf{e}}^{\,3}\\ \vdots \\ \mathbf{hobby}(y):\mathsf{type}\end{array}
\qquad
\dfrac{
\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}:\mathsf{type}
\quad
\begin{array}{c}\overline{u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}}^{\,1}\quad \overline{y:\mathsf{e}}^{\,3}\\ \mathcal{D}_1\\ ?:\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\end{array}
\quad
\begin{array}{c}\mathcal{D}_2\\ \mathbf{excite}^*_e(y,\pi_1 w)[?/w]:\mathsf{type}\end{array}
}{
\begin{bmatrix} w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_e(y,\pi_1 w)\end{bmatrix}:\mathsf{type}
}(@)
}{
\begin{bmatrix} \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_e(y,\pi_1 w)\end{bmatrix}:\mathsf{type}
}(\Sigma F)
}{
\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}:\mathsf{type}
}(\Sigma F),3
\qquad
\begin{array}{c}
\overline{u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}}^{\,1}
\quad
\overline{v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}}^{\,2}\\
\vdots\\
\mathbf{enjoy}^*_e(\pi_1 u,\pi_1 v):\mathsf{type}
\end{array}
}{
\begin{array}{c}\vdots\\ \begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}:\mathsf{type}\end{array}
\quad
\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}\\ \mathbf{enjoy}^*_e(\pi_1 u,\pi_1 v)\end{bmatrix}:\mathsf{type}
}(\Sigma F),2
$$

$$
\dfrac{
\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}:\mathsf{type}
\quad
\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}\\ \mathbf{enjoy}^*_e(\pi_1 u,\pi_1 v)\end{bmatrix}:\mathsf{type}
}{
\left(u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}(x)\end{bmatrix}\right)\to\begin{bmatrix} v:\begin{bmatrix} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ w@\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}\\ \mathbf{excite}^*_{e_2}(y,\pi_1 w)\end{bmatrix}\\ \mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)\end{bmatrix}:\mathsf{type}
}(\Pi F),1
$$

$\mathcal{D}_1$ corresponds to the search for the antecedent of her in (503). There are at least two valid proof terms: $(mary, f_m)$ and $(\pi_1 u, gf(u))$, which correspond to the proof diagrams $\mathcal{D}_m$ and $\mathcal{D}_u$, respectively.

(543) Proof diagram $\mathcal{D}_m \equiv$

$$
\dfrac{\overline{mary:\mathsf{e}}\,(\{\}I)\qquad \overline{f_m:\mathbf{female}(mary)}\,(CON)}{(mary,f_m):\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}}(\Sigma I)
$$

(544) Proof diagram $\mathcal{D}_u \equiv$

$$
\dfrac{
\dfrac{\overline{u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}\end{bmatrix}}^{\,1}}{\pi_1 u:\mathsf{e}}(\Sigma E)
\qquad
\dfrac{\overline{gf:\left(u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}\end{bmatrix}\right)\to\mathbf{female}(\pi_1 u)}\,(CON)\qquad u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{girl}\end{bmatrix}}{gf(u):\mathbf{female}(\pi_1 u)}(\Pi E)
}{
(\pi_1 u, gf(u)):\begin{bmatrix} z:\mathsf{e}\\ \mathbf{female}(z)\end{bmatrix}
}(\Sigma I)
$$

The proof in (543) represents the coreference reading where *her* is resolved to Mary, while (544) represents the BVA reading where *her* is bound by *every girl*. In the former proof, the entity originates from a constructor of an enumeration type; in the latter, it comes from a variable in the context. The corresponding $\mathcal{D}_2$ for each case is as follows.

(545) Type check diagram of (503): $\mathcal{D}_2$ for Coreference reading via (543)

$$\dfrac{\dfrac{\dfrac{}{\mathbf{excite}^*_{e_2} : \mathsf{e} \to \mathsf{e} \to \mathsf{type}}{\scriptstyle (CON)} \qquad \dfrac{\begin{array}{c}\mathcal{D}_m \\ (mary, f_m) : \begin{bmatrix} z : \mathsf{e} \\ \mathbf{female}(z) \end{bmatrix}\end{array}}{\pi_1(mary, f_m) : \mathsf{e}}{\scriptstyle (\Sigma E)}}{\mathbf{excite}^*_{e_2}(\pi_1(mary, f_m)) : \mathsf{e} \to \mathsf{type}}{\scriptstyle (\Pi E)} \qquad \overline{y : \mathsf{e}}^{3}}{\begin{array}{c}\mathbf{excite}^*_{e_2}(y, \pi_1 w)[(mary, f_m)/w] \\ \equiv \mathbf{excite}^*_{e_2}(y, \pi_1(mary, f_m)) : \mathsf{type}\end{array}}{\scriptstyle (\Pi E)}$$

(546) Type check diagram of (503): $\mathcal{D}_2$ for BVA reading via (544)

$$\dfrac{\dfrac{\dfrac{}{\mathbf{excite}^*_{e_2} : \mathsf{e} \to \mathsf{e} \to \mathsf{type}}{\scriptstyle (CON)} \qquad \dfrac{\begin{array}{c} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{girl}(x) \end{bmatrix} \\ \mathcal{D}_u \\ (\pi_1 u, gf(u)) : \begin{bmatrix} z : \mathsf{e} \\ \mathbf{female}(z) \end{bmatrix}\end{array}}{\pi_1(\pi_1 u, gf(u)) : \mathsf{e}}{\scriptstyle (\Sigma E)}}{\mathbf{excite}^*_{e_2}(\pi_1(\pi_1 u, gf(u))) : \mathsf{e} \to \mathsf{type}}{\scriptstyle (\Pi E)} \qquad y : \mathsf{e}}{\begin{array}{c}\mathbf{excite}^*_{e_2}(y, \pi_1 w)[(\pi_1 u, gf(u))/w] \\ \equiv \mathbf{excite}^*_{e_2}(y, \pi_1(\pi_1 u, gf(u))) : \mathsf{type}\end{array}}{\scriptstyle (\Pi E)}$$

Applying @-elimination to these diagrams yields two DTT proof diagrams.

(547) @-elimination of (503) for Coreference reading via (543)

$$\dfrac{\begin{array}{c}\vdots \\ \begin{bmatrix} x : \mathsf{e} \\ \mathbf{girl}(x) \end{bmatrix} : \mathsf{type}\end{array} \qquad \dfrac{\dfrac{\dfrac{}{\mathsf{e} : \mathsf{type}}{\scriptstyle (\{\}F)} \qquad \dfrac{\begin{array}{c}\overline{y : \mathsf{e}}^{3} \\ \vdots \\ \mathbf{hobby}(y) : \mathsf{type}\end{array} \quad \begin{array}{c}\overline{y : \mathsf{e}}^{3} \\ \mathcal{D}_2 \\ \mathbf{excite}^*_{e_2}(y, mary)\end{array}}{\begin{bmatrix} \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, mary) \end{bmatrix} : \mathsf{type}}{\scriptstyle (\Sigma F)}}{\begin{bmatrix} y : \mathsf{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, mary) \end{bmatrix} : \mathsf{type}}{\scriptstyle (\Sigma F),3} \qquad \begin{array}{c}\overline{u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{girl}(x) \end{bmatrix}}^{1} \quad \overline{v : \begin{bmatrix} y : \mathsf{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, mary) \end{bmatrix}}^{2} \\ \vdots \\ \mathbf{enjoy}^*_{e}(\pi_1 u, \pi_1 v) : \mathsf{type}\end{array}}{\begin{bmatrix} v : \begin{bmatrix} y : \mathsf{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, mary) \end{bmatrix} \\ \mathbf{enjoy}^*_{e}(\pi_1 u, \pi_1 v) \end{bmatrix} : \mathsf{type}}{\scriptstyle (\Sigma F),2}}{\left(u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{girl}(x) \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \mathsf{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, mary) \end{bmatrix} \\ \mathbf{enjoy}^*_{e_1}(\pi_1 u, \pi_1 v) \end{bmatrix} : \mathsf{type}}{\scriptstyle (\Pi F),1}$$

(548) @-elimination of (503) for BVA reading via (544)

$$
\dfrac{
\begin{array}{c}\vdots\\ \left[\begin{array}{l} x:\mathsf{e}\\ \mathbf{girl}(x)\end{array}\right]:\mathsf{type}\end{array}
\qquad
\dfrac{
\dfrac{
\dfrac{}{\mathsf{e}:\mathsf{type}}(\{\}F)
\quad
\dfrac{
\begin{array}{c}\overline{y:\mathsf{e}}^{3}\\ \vdots\\ \mathbf{hobby}(y):\mathsf{type}\end{array}
\quad
\begin{array}{c}\overline{y:\mathsf{e}}^{3}\ \ \overline{u:\left[\begin{array}{l} x:\mathsf{e}\\ \mathbf{girl}(x)\end{array}\right]}^{1}\\ \mathcal{D}_2\\ \mathbf{excite}^*_{e_2}(y,\pi_1 u)\end{array}
}{
\left[\begin{array}{l}\mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,\pi_1(\pi_1 u, gf(u)))\end{array}\right]:\mathsf{type}
}(\Sigma F)
}{
\left[\begin{array}{l} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,\pi_1(\pi_1 u, gf(u)))\end{array}\right]:\mathsf{type}
}(\Sigma F),3
\qquad
\begin{array}{c}
\overline{u:\left[\begin{array}{l} x:\mathsf{e}\\ \mathbf{girl}(x)\end{array}\right]}^{1}
\quad
\overline{v:\left[\begin{array}{l} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,\pi_1 u)\end{array}\right]}^{2}\\
\vdots\\
\mathbf{enjoy}^*_{e}(\pi_1 u,\pi_1 v):\mathsf{type}
\end{array}
}{
\left[\begin{array}{l} v:\left[\begin{array}{l} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,\pi_1 u)\end{array}\right]\\ \mathbf{enjoy}^*_{e}(\pi_1 u,\pi_1 v)\end{array}\right]:\mathsf{type}
}(\Sigma F),2
}{
\left(u:\left[\begin{array}{l} x:\mathsf{e}\\ \mathbf{girl}(x)\end{array}\right]\right)\to\left[\begin{array}{l} v:\left[\begin{array}{l} y:\mathsf{e}\\ \mathbf{hobby}(y)\\ \mathbf{excite}^*_{e_2}(y,\pi_1 u)\end{array}\right]\\ \mathbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)\end{array}\right]:\mathsf{type}
}(\Pi F),1
$$

The semantic representations for the sentence (503) are located at the bottom-left of the proof diagrams in (547) and (548). The first representation corresponds to a coreference reading where the referent is *mary*, and the second to a BVA reading where the antecedent is *every girl*. These readings differ solely in the proof construction of a female entity, as mandated by the (@)-rule. This divergence arises from the use of constant symbols in one proof versus a variable introduced by the c-commanding quantifier *every girl* in the other. Otherwise, the derivation process for the semantic representation remains identical. This unified analysis of coreference and BVA readings is made possible by representing pronominal lexical items with underspecified types.

22.4 Discussion

22.4.1 Anaphoric expression as Variable?

As established in this chapter, DTS uses underspecified types to represent anaphoric expressions. Anaphora resolution is then achieved through a proof search triggered by type checking of the semantic representations that include these underspecified types.

This approach prompts a fundamental question: is it not sufficient to simply represent anaphoric expressions as variables? The alternative would find an antecedent in the context and assign it to the variable, using a cut rule. The core question is whether the extended mechanism of underspecified types is truly necessary. We will argue why representing anaphoric expressions as variable, in its various forms, is inadequate as a theory of anaphora.

Let us revisit an example, (503), to illustrate the problem.

(503) Every girl enjoys a hobby that excites her.

While there are several ways to formalize the analysis of anaphora as variable, we will first consider a policy where the pronoun *him* has the syntactic type NP and is represented semantically by a free variable, z. The term corresponding to the antecedent is assigned to z at a later stage.

(549) Syntactic structure of (503)

$$
\dfrac{\dfrac{S^\star}{S}\,\checkmark}{}
\quad
\dfrac{
\dfrac{\dfrac{\text{Every}}{NP^\star/N} \quad \dfrac{\text{girl}}{N}}{NP^\star}\,{>}
\qquad
\dfrac{\dfrac{\text{enjoys}}{(S\backslash NP/NP)^\star} \quad
\dfrac{\dfrac{\text{a}}{NP^\star/N} \quad
\dfrac{\dfrac{\text{hobby}}{N} \quad
\dfrac{\dfrac{\text{that}}{N\backslash N/(S\backslash NP)^\star} \quad
\dfrac{\dfrac{\text{excites}}{(S\backslash NP/NP)^\star} \quad \dfrac{\text{her}}{NP^\star}}{(S\backslash NP)^\star}\,{>^\star}}
{N\backslash N}\,{>}}
{N}\,{<}}
{NP^\star}\,{>}}
{(S\backslash NP)^\star}\,{>^\star}}
{\dfrac{S^\star}{S}\,\checkmark}\,{<^\star}
$$

(550) Semantic composition of (503)

$$
\dfrac{
\dfrac{\text{a}}{\lambda n.\left[\begin{array}{l} v : \begin{bmatrix} y : \mathrm{e} \\ n(y) \end{bmatrix} \\ \boxed{\pi_1 v} \end{array}\right]}
\quad
\dfrac{
\dfrac{\text{hobby}}{\textbf{hobby}}
\quad
\dfrac{
\dfrac{\text{that}}{\lambda q.\lambda n.\lambda x.\begin{bmatrix} nx \\ q(\lambda p.px) \end{bmatrix}}
\quad
\dfrac{\dfrac{\text{excites}}{\boxed{\begin{array}{l}\lambda y.\lambda x. \\ \textbf{excite}^*_{e_2}(x,y)\end{array}}} \quad \dfrac{\text{her}}{\boxed{z}}}{\boxed{\lambda x.\textbf{excite}^*_{e_2}(x,z)}}\,{>}
}{\lambda n.\lambda x.\begin{bmatrix} nx \\ \textbf{excite}^*_{e_2}(x,z) \end{bmatrix}}\,{>}
}{\lambda x.\begin{bmatrix} \textbf{hobby}(x) \\ \textbf{excite}^*_{e_2}(x,z) \end{bmatrix}}\,{<}
}{\left[\begin{array}{l} v : \begin{bmatrix} y : \mathrm{e} \\ \textbf{hobby}(y) \\ \textbf{excite}^*_{e_2}(y,z) \end{bmatrix} \\ \boxed{\pi_1 v} \end{array}\right]}\,{>}
$$

(551) Semantic composition of (503)

$$
\dfrac{
\dfrac{
\dfrac{\text{Every}}{\lambda n.\left(u : \begin{bmatrix} x : \mathrm{e} \\ n(x) \end{bmatrix}\right) \to \boxed{\pi_1 u}}
\quad
\dfrac{\text{girl}}{\textbf{girl}}
}{\left(u : \begin{bmatrix} x : \mathrm{e} \\ \textbf{girl}(x) \end{bmatrix}\right) \to \boxed{\pi_1 u}}\,{>}
\quad
\dfrac{
\dfrac{\text{enjoys}}{\boxed{\begin{array}{l}\lambda y.\lambda x. \\ \textbf{enjoy}^*_{e_1}(x,y)\end{array}}}
\quad
\dfrac{\text{a hobby that excites her}}{\left[\begin{array}{l} v : \begin{bmatrix} y : \mathrm{e} \\ \textbf{hobby}(y) \\ \textbf{excite}^*_{e_2}(y,z) \end{bmatrix} \\ \boxed{\pi_1 v} \end{array}\right]}
}{\left[\begin{array}{l} v : \begin{bmatrix} y : \mathrm{e} \\ \textbf{hobby}(y) \\ \textbf{excite}^*_{e_2}(y,z) \end{bmatrix} \\ \boxed{\lambda x.\textbf{enjoy}^*_{e_1}(x,\pi_1 v)} \end{array}\right]}\,{>^\star}
}{
\dfrac{\left(u : \begin{bmatrix} x : \mathrm{e} \\ \textbf{girl}(x) \end{bmatrix}\right) \to \left[\begin{array}{l} v : \begin{bmatrix} y : \mathrm{e} \\ \textbf{hobby}(y) \\ \textbf{excite}^*_{e_2}(y,z) \end{bmatrix} \\ \boxed{\textbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v)} \end{array}\right]}
{\left(u : \begin{bmatrix} x : \mathrm{e} \\ \textbf{girl}(x) \end{bmatrix}\right) \to \left[\begin{array}{l} v : \begin{bmatrix} y : \mathrm{e} \\ \textbf{hobby}(y) \\ \textbf{excite}^*_{e_2}(y,z) \end{bmatrix} \\ \textbf{enjoy}^*_{e_1}(\pi_1 u,\pi_1 v) \end{array}\right]}\,\checkmark
}\,{>}
$$

The resulting semantic representation includes z as a free term. Substituting $\pi_1 u$, the term for the antecedent, for z yields the following result (assuming w is an appropriately chosen variable not already present).

$$
\left(\left(u : \begin{bmatrix} x : \mathrm{e} \\ \mathbf{girl} \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_e(y, z) \end{bmatrix} \\ \mathbf{carry}^*_e(\pi_1 u, \pi_1 v) \end{bmatrix}\right)[\pi_1 u / z]
$$

$$
\equiv \left(w : \begin{bmatrix} x : \mathrm{e} \\ \mathbf{girl} \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_e(y, \pi_1 u) \end{bmatrix} \\ \mathbf{carry}^*_e(\pi_1 w, \pi_1 v) \end{bmatrix}
$$

The reason why we have to change the variable u of type Π to w here is that in original substitution, $u \notin fv(\Pi_1 u)$ is in violation of the condition set by the substitution rule defined in Definition 132. Therefore, this approach is not a semantic representation for BVA reading.

An alternative approach is to represent the pronoun with its antecedent's term from the outset. This raises the question of how lexical entries are initially defined. For the purpose of this discussion, we will assume a specific set of semantic representations, but the underlying issue remains.

(552) Semantic composition of (503)

$$
\dfrac{
\dfrac{\text{a}}{\lambda n.\begin{bmatrix} v : \begin{bmatrix} y : \mathrm{e} \\ n(y) \end{bmatrix} \\ \boxed{\pi_1 v} \end{bmatrix}}
\quad
\dfrac{\dfrac{\text{hobby}}{\mathbf{hobby}} \quad \dfrac{\dfrac{\text{that}}{\lambda q.\lambda n.\lambda x.\begin{bmatrix} nx \\ q(\lambda p.px) \end{bmatrix}} \quad \dfrac{\dfrac{\text{excites}}{\boxed{\lambda y.\lambda x.\,\mathbf{excite}^*_{e_2}(x, y)}} \quad \dfrac{\text{her}}{\boxed{\pi_1 u}}}{\boxed{\lambda x.\mathbf{excite}^*_{e_2}(x, \pi_1 u)}}\,{>}}{\lambda n.\lambda x.\begin{bmatrix} nx \\ \mathbf{excite}^*_{e_2}(x, \pi_1 u) \end{bmatrix}}\,{>}}{\lambda x.\begin{bmatrix} \mathbf{hobby}(x) \\ \mathbf{excite}^*_{e_2}(x, \pi_1 u) \end{bmatrix}}\,{<}
}{
\begin{bmatrix} v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, \pi_1 u) \end{bmatrix} \\ \boxed{\pi_1 v} \end{bmatrix}
}\,{>}
$$

(553) Semantic composition of (503)

$$\dfrac{\dfrac{\dfrac{\text{Every}}{\lambda n.\left(u : \begin{bmatrix} x : \mathrm{e} \\ n(x) \end{bmatrix}\right) \to \boxed{\pi_1 u}} \quad \dfrac{\text{girl}}{\mathbf{girl}}}{\left(u : \begin{bmatrix} x : \mathrm{e} \\ \mathbf{girl}(x) \end{bmatrix}\right) \to \boxed{\pi_1 u}}> \qquad \dfrac{\dfrac{\text{enjoys}}{\boxed{\begin{array}{l} \lambda y.\lambda x. \\ \mathbf{enjoy}^*_{e_1}(x, y) \end{array}}} \quad \dfrac{\text{a hobby that excites her}}{\begin{bmatrix} v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, \pi_1 u) \end{bmatrix} \\ \boxed{\pi_1 v} \end{bmatrix}}}{\begin{bmatrix} v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, \pi_1 u) \end{bmatrix} \\ \boxed{\lambda x.\mathbf{enjoy}^*_{e_1}(x, \pi_1 v)} \end{bmatrix}}>^\star}{\left(w : \begin{bmatrix} x : \mathrm{e} \\ \mathbf{girl}(x) \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \mathrm{e} \\ \mathbf{hobby}(y) \\ \mathbf{excite}^*_{e_2}(y, \pi_1 u) \end{bmatrix} \\ \boxed{\mathbf{enjoy}^*_{e_1}(\pi_1 w, \pi_1 v)} \end{bmatrix}}>$$

This second approach also encounters a significant problem. It necessitates assigning the verb phrase's semantic representation, with u as a free variant, into the scope of the variable u introduced by the Π-type that results from universal quantification. Consequently, the variable introduced by the Π-type must be changed from u to something else, say w. This change means that $\pi_1 u$ is no longer the term representing the antecedent. This, too, prevents the system from yielding a BVA reading.

We have considered two versions of the "anaphora as a variable" approach and found that in both cases, the bound variable of the quantifier must be externally inserted into the quantifier's scope, an operation that is not permissible. This failure clearly demonstrates the central challenge that any theory of anaphora must address in its semantic composition. The DTS framework, with its underspecified types, offers a minimal but sufficient mechanism for solving this very problem.

22.4.2 Cataphora

(or backward pronominalization)

(554)

M.A.K. Halliday and Ruqaiya Hasan (1976) "Cohesion in English"

History and Further Readings

For more information on the research history of Coreference and BVA, examine the following references

Postal (1971), Lasnik (1976), Reinhart (1976), Hankamer and Sag (1976), Williams (1977), Partee (1976), Evans (1980), Lasnik (1981), Reinhart (1981), Reinhart (1983a), Reinhart (1983b), Higginbotham (1983b), Reinhart (1986), Lasnik (1989a), Lasnik (1989b), Lebeaux (1990), Hoji (1991), Grodzinsky and Reinhart (1993), Speas (1993), Hornstein (1994), Fox (1995), Hoji (1995), Safir (1997), Ueyama (1998)

The version of DTS adopted in **?**, Kubota et al. (2019), and Tanaka (2021), which is slightly different from the one adopted in Bekki and Mineshima (2017) that uses the *context-passing* mechanism.