Composing Meaning via Dependent Type Semantics
Day 1: Overview

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Dependent Type Semantics (DTS) (Bekki 2014; Bekki and Mineshima 2017; Bekki 2021)

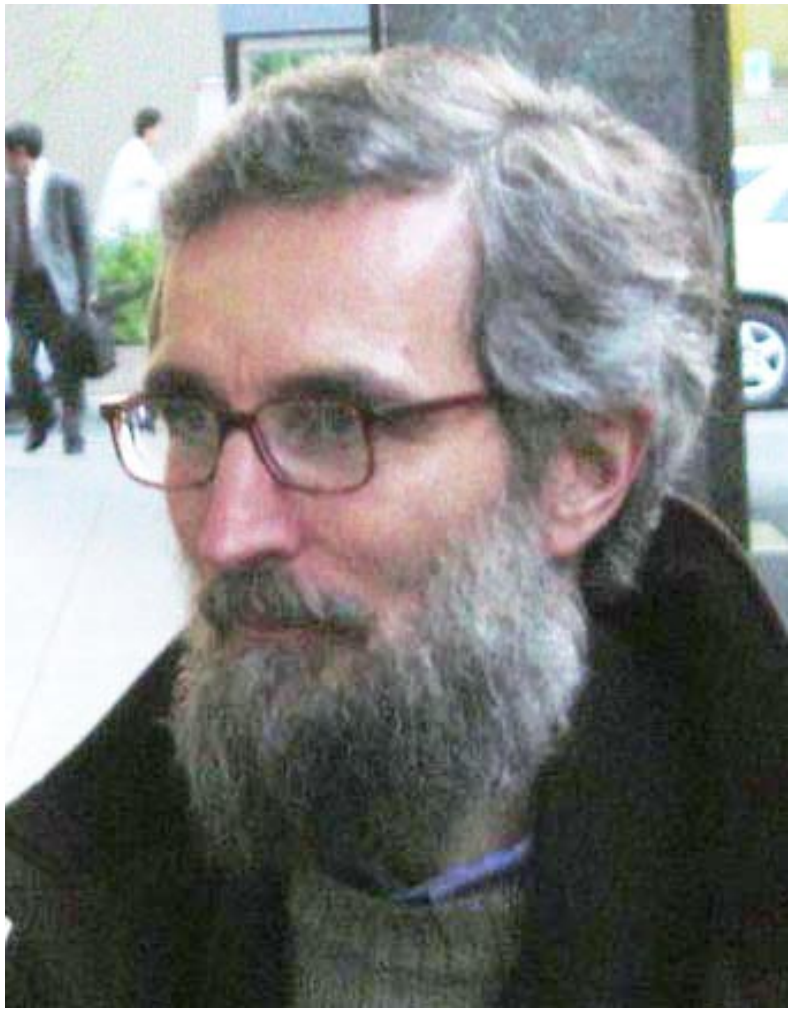
- A framework of natural language semantics
- Unified approach to (general) inferences and anaphora/presupposition resolution in terms of *type checking* and *proof search*

Main features:

1. **Proof-theoretic semantics**:
   From model theory (denotations and models) to proof theory (proofs and contexts)
2. **Anaphora/Presuppositions**: A proof-theoretic alternative to Dynamic Semantics (DRT, DPL, etc.)
3. **Compositionality**: Syntax-semantics interface via categorial grammars (e.g. CCG, TLG, ACG, etc)
4. **Implementation**: Applications to Natural Language Processing.
   `https://github.com/DaisukeBekki/lightblue/`

Dependent Types

Per Martin-Löf

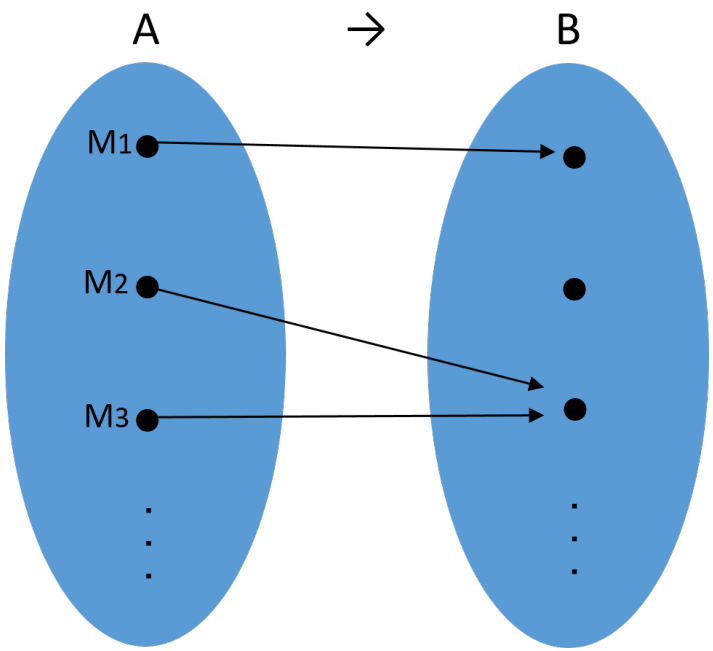


Martin-Löf (1984) “Intuitionistic type theory”

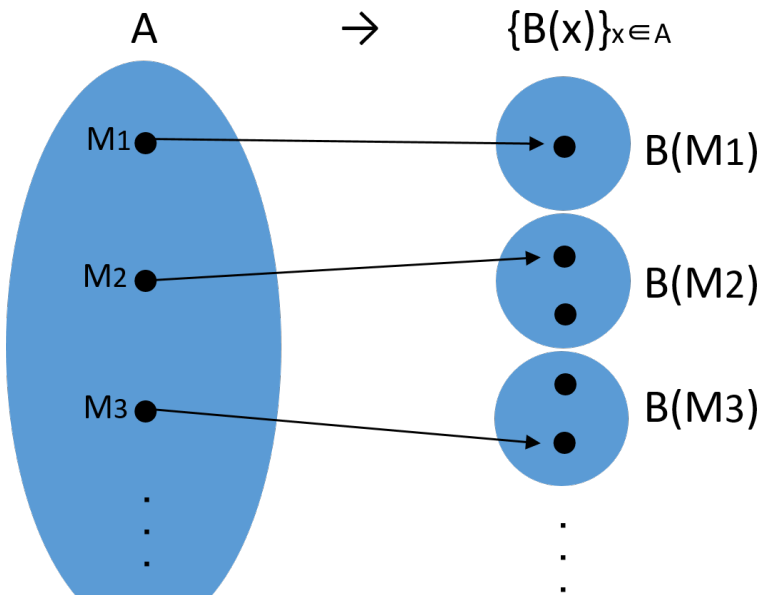
What are Π-types

Π-type is a type of *fibred* functions.

Simple function space



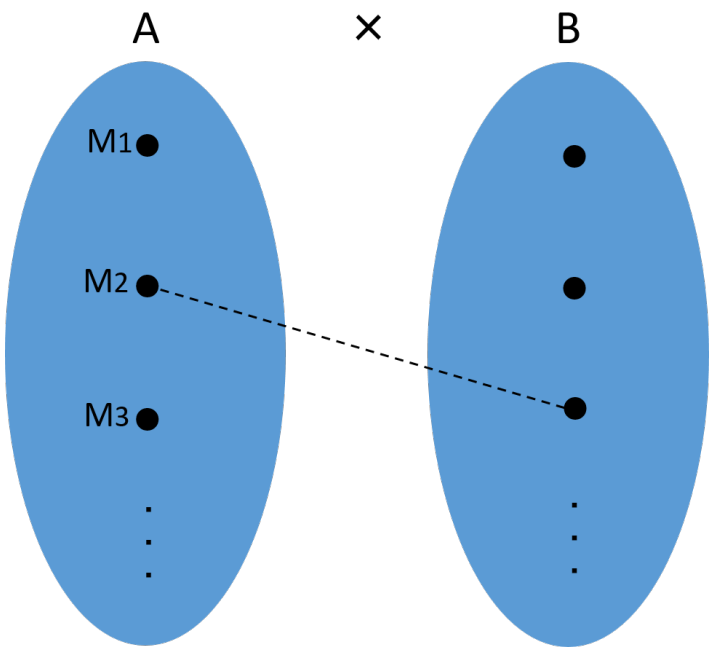
Fibred function space



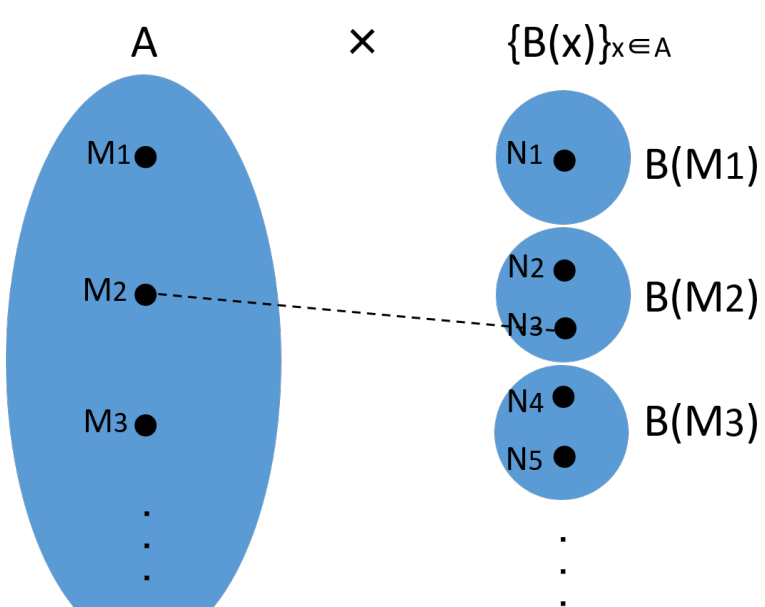
What are Σ-types

Σ-type is a type of *fibred* products.

Simple product space



Fibred product space



Notations

DTS notation	Standard notation	$x \notin fv(B)$	$x \in fv(B)$
$(x : A) \to B$	$(\Pi x : A)B$	$A \to B$	$(\forall x : A)B$
$(x : A) \times B$ *or* $\left[\begin{array}{l} x : A \\ B \end{array}\right]$	$(\Sigma x : A)B$	$A \wedge B$	$(\exists x : A)B$

Scope of the variable in Π-types: $(x : A) \to B$

Scope of the variable in Σ-types: $\left[\begin{array}{l} x : A \\ B \end{array}\right]$

Π-type F/I/E rules

$$\frac{\begin{array}{c}\overline{x : A}^{\,i} \\ \vdots \\ A : \mathsf{s_1} \quad B : \mathsf{s_2}\end{array}}{(x : A) \to B : \mathsf{s_2}}\,(\Pi F),i \qquad \textit{where } (\mathsf{s_1}, \mathsf{s_2}) \in \left\{ \begin{array}{l} (\mathsf{type}, \mathsf{type}), \\ (\mathsf{type}, \mathsf{kind}) \end{array} \right\}.$$

$$\frac{\begin{array}{c}\overline{x : A}^{\,i} \\ \vdots \\ A : \mathsf{type} \quad M : B\end{array}}{\lambda x.M : (x : A) \to B}\,(\Pi I),i \qquad \frac{M : (x : A) \to B \quad N : A}{MN : B[N/x]}\,(\Pi E)$$

Σ-type F/I/E rules

$$\frac{\begin{array}{c}\overline{x : A}^{\,i} \\ \vdots \\ A : \text{type} \quad B : \text{type}\end{array}}{(x : A) \times B : \text{type}}\,(\Sigma F),i$$

$$\frac{M : A \quad N : B[M/x]}{(M, N) : (x : A) \times B}\,(\Sigma I)$$

$$\frac{M : (x : A) \times B}{\pi_1(M) : A}\,(\Sigma E) \qquad \frac{M : (x : A) \times B}{\pi_2(M) : B[\pi_1(M)/x]}\,(\Sigma E)$$

Rules of DTS

Rules from Martin-Löf Type Theory

- Axioms and Structural rules
- Π-type (Dependent function type) [F/I/E]
- Σ-type (Dependent product type) [F/I/E]
- Intensional equality type [F/I/E]
- Disjoint union type [F/I/E]
- Enumeration type [F/I/E]
- Natural number type [F/I/E]

New rule in DTS

- @ (the 'asperand' operator)
  - Anaphora and presupposition triggers (linguistically speaking)
  - Open proofs (logically speaking)

Conjunction, Implication, and Negation

Definition

$$\begin{bmatrix} A \\ B \end{bmatrix} \overset{def}{\equiv} (x : A) \times B \qquad where\ x \notin fv(B).$$

$$A \to B \overset{def}{\equiv} (x : A) \to B \qquad where\ x \notin fv(B).$$

$$\neg A \overset{def}{\equiv} (x : A) \to \bot$$

Anaphora in Natural Language

A theory of anaphora

- Anaphora representable by a constant symbol:
  - Deictic use:

    (1) *(Pointing at John)*

    He was born in Detroit.

    $\mathbf{bornIn}(j, d)$
  - Coreference:

    (2) John loves a girl who hates him.

    $\exists x(\mathbf{girl}(x) \wedge \mathbf{love}(j, x) \wedge \mathbf{hate}(x, j))$
- Anaphora representable by a variable
  - Bound variable anaphora:

    (3) Every boy loves his father.

    $\forall x\,(\mathbf{boy}(x) \rightarrow \mathbf{love}(x, \mathbf{fatherOf}(x)))$

A theory of anaphora

- Anaphora not representable by FoL:
  - E-type anaphora:

    (4) A man entered into the park. He whistled.

  - Donkey anaphora:

    (5) Every farmer who owns a donkey beats it.

    (6) If a farmer owns a donkey, he beats it.

- Anaphora not representable by FoL nor dynamic semantics:
  - Syllogistic anaphora:

    (7) Every girl received a present. Some girl opened it.

  - Disjunctive antecedent:

    (8) If Mary sees a horse or a pony, she waves to it.

Donkey anaphora: Geach (1962)

For the donkey sentences (9), a first-order formula (10), whose truth condition is the same as those of (9), is a candidate of its semantic representation (SR). (We only discuss its *strong reading* here. See Tanaka (2021) for its *weak reading*.)

(9) a. Every farmer who owns [a donkey]^{1} beats it$_{1}$.

b. If [a farmer]^{1} owns [a donkey]^{2}, he$_{1}$ beats it$_{2}$.

(10) $\forall x(\textbf{farmer}(x) \rightarrow \forall y\,(\textbf{donkey}(y) \wedge \textbf{own}(x,y) \rightarrow \textbf{beat}(x,y)))$

But the translation from the sentence (9) to (10) is not straightforward since i) the indefinite noun phrase *a donkey* is translated into a universal quantifier in (10) instead of an existential quantifier, and ii) the syntactic structure of (10) does not corresponds to that of (9).

Donkey anaphora: Geach (1962)

(9) a. Every farmer who owns [a donkey]$^{\mathbf{1}}$ beats $\mathrm{it}_{\mathbf{1}}$.

b. If [a farmer]$^{\mathbf{1}}$ owns [a donkey]$^{\mathbf{2}}$, $\mathrm{he}_{\mathbf{1}}$ beats $\mathrm{it}_{\mathbf{2}}$.

The syntactic parallel of (9) is, rather, the SR (11), in which the indefinite noun phrase is translated into an existential quantification.

(11) $\forall x(\mathbf{farmer}(x) \wedge \exists y(\mathbf{donkey}(y) \wedge \mathbf{own}(x, y)) \rightarrow \mathbf{beat}(x, y))$

However, (11) does not represent the truth condition of (9) correctly since the variable y in $\mathbf{beat}(x, y)$ fails to be bound by $\exists$. Therefore, neither (10) nor (11) qualifies as the SR of (9).

Various approaches in discourse semantics

Dynamic Semantics

- Discourse Representation Theory (DRT): Kamp (1981), Kamp and Reyle (1993)
- Dynamic Predicate Logic (DPL): Groenendijk and Stokhof (1991)
- Dynamic Plural Predicate Logic (DPPL): van den Berg (1996), Sudo (2012)

Type-theoretical Semantics

- Analysis of donkey anaphora: Sundholm (1986))
- Type Theoretical Grammar (TTG): Ranta (1994)
- Type Theory with Record (TTR): Cooper (2005)
- MTT-semantics: Luo (1997, 1999, 2010, 2012), Asher and Luo (2012), Chatzikyriakidis (2014)
- Dependent Type Semantics (DTS): Bekki (2014), Bekki and Mineshima (2017)

Donkey anaphora: Sundholm (1986)

(9a) Every farmer who owns a donkey beats it .

$$\left(u : \left[\begin{array}{l} x : \text{entity} \\ \left[\begin{array}{l} \textbf{farmer}(x) \\ \left[\begin{array}{l} v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array} \right] \\ \textbf{own}(x, \pi_1 v) \end{array} \right] \end{array} \right] \end{array} \right] \right) \rightarrow \textbf{beat}(\pi_1 u, \pi_1 \pi_1 \pi_2 \pi_2 u)$$

Note: $(x : A) \rightarrow B$ is a type for functions from A to $B[x]$.

From TTG to DTS: Compositionality

Q: How could one get to these (dependently-typed) representations from arbitrary sentences?

A: By lexicalization.

Q: But, how could we lexicalize context-dependent words like pronouns?

$$\left(u : \left[\begin{array}{l} x : \text{entity} \\ \left[\begin{array}{l} \textbf{farmer}(x) \\ \left[\begin{array}{l} v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array} \right] \\ \textbf{own}(x, \pi_1 v) \end{array} \right] \end{array} \right] \end{array} \right] \right) \rightarrow$$

$$\textbf{beat}(\pi_1 u,\ \pi_1 \pi_1 \pi_2 \pi_2 u\ )$$

From TTG to DTS: Compositionality

Q: How could one get to these (dependently-typed) representations from arbitrary sentences?

A: By lexicalization.

Q: But, how could we lexicalize context-dependent words like pronouns?

A: By using **underspecified types**.

Q: How could we retrieve a context for an underspecified type?

A: By **type checking**.

Dependent Type Semantics (DTS)

Underspecified types

DTS $=$ DTT $+$ underspecified types

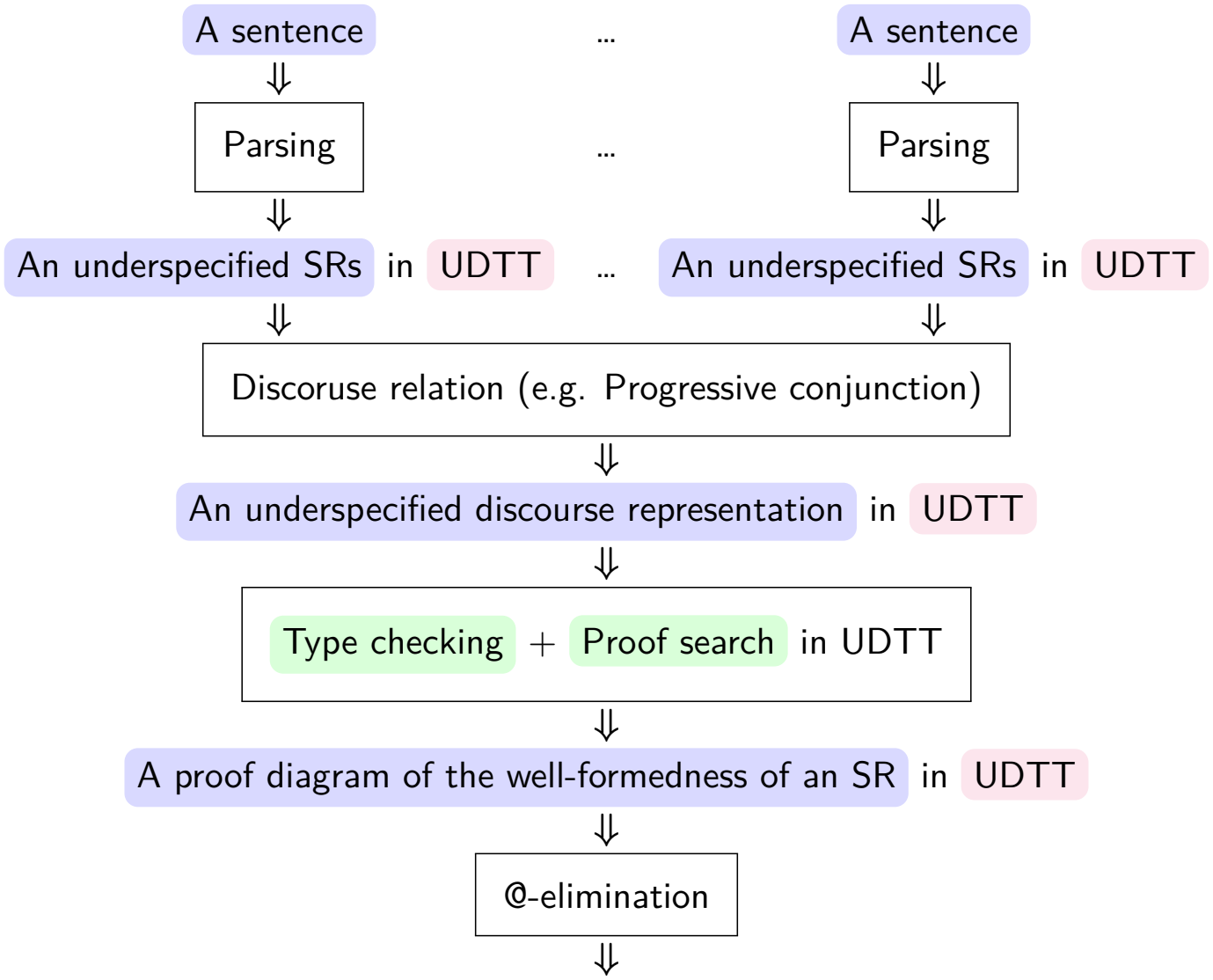
Definition (@-rule)

$$\dfrac{\begin{array}{c}\overline{x : A}^{\,i} \\ \vdots \\ A : \text{type} \quad B : \text{type} \quad A \ \mathit{true}\end{array}}{: \text{type}}\ (@F)$$

- @-rule states that the well-formedness of $(x@A) \times B$ requires:
  - A is a well-formed type
  - the inhabitance of a proof (let it be M) of A, checking of which launches a proof search
  - $B[M/x]$ is a well-formed type

This means that the truth of A is *presupposed*.

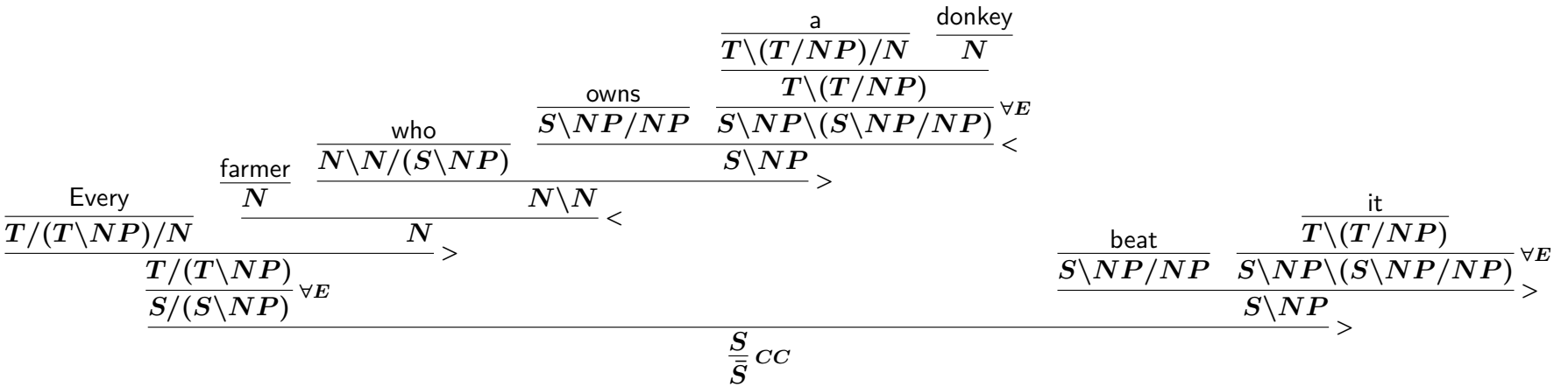
A model of natural language understanding via CCG and DTS



Lexical items in CCG style

Surface form	Syntactic category	Semantic representation
every	$\begin{cases} T/(T\backslash NP)/N \\ T\backslash(T/NP)/N \end{cases}$	$\lambda n.\lambda p.\lambda \vec{x}.\left(u : \begin{bmatrix} x : \text{entity} \\ n(x) \end{bmatrix}\right) \to p(\pi_1 u)\vec{x}$
a, some	$\begin{cases} T/(T\backslash NP)/N \\ T\backslash(T/NP)/N \end{cases}$	$\lambda n.\lambda p.\lambda \vec{x}.\begin{bmatrix} u : \begin{bmatrix} x : \text{entity} \\ n(x) \end{bmatrix} \\ p(\pi_1 u)\vec{x} \end{bmatrix}$
if	$\begin{cases} S/S/S \\ S\backslash S/S \end{cases}$	$\lambda p.\lambda q.\,(u : p) \to q$
who/whom	$\begin{cases} N\backslash N/(S\backslash NP) \\ N\backslash N/(S/NP) \end{cases}$	$\lambda p.\lambda n.\lambda x.\begin{bmatrix} nx \\ px \end{bmatrix}$
farmer	N	**farmer**
donkey	N	**donkey**
owns	$S\backslash NP/NP$	**own**
beats	$S\backslash NP/NP$	**beat**
he/him	$\begin{cases} T/(T\backslash NP) & nom \\ T\backslash(T/NP) & acc \end{cases}$	$\lambda p.\lambda \vec{x}.\begin{bmatrix} u@\begin{bmatrix} x : \text{entity} \\ \textbf{male}(x) \end{bmatrix} \\ p(\pi_1 u)\vec{x} \end{bmatrix}$
it	$\begin{cases} T/(T\backslash NP) & nom \\ T\backslash(T/NP) & acc \end{cases}$	$\lambda p.\lambda \vec{x}.\begin{bmatrix} u@\begin{bmatrix} x : \text{entity} \\ \neg\textbf{human}(x) \end{bmatrix} \\ p(\pi_1 u)\vec{x} \end{bmatrix}$
the	$\begin{cases} T/(T\backslash NP)/N & nom \\ T/(T\backslash NP)/N & acc \end{cases}$	$\lambda n.\lambda p.\lambda \vec{x}.\begin{bmatrix} u@\begin{bmatrix} x : \text{entity} \\ n(x) \end{bmatrix} \\ p(\pi_1 u)\vec{x} \end{bmatrix}$

Donkey anaphora: Syntactic Structure



Donkey anaphora: Semantic Composition (1/2)

$$\dfrac{\dfrac{\text{farmer}}{\mathbf{farmer}} \qquad \dfrac{\dfrac{\dfrac{\text{who}}{\lambda p.\lambda n.\lambda x.\begin{bmatrix} n(x) \\ p(x) \end{bmatrix}} \qquad \dfrac{\dfrac{\text{owns}}{\lambda y.\lambda x.\ \mathbf{own}(x,y)} \qquad \dfrac{\dfrac{\text{a}}{\lambda n.\lambda p.\begin{bmatrix} v : \begin{bmatrix} y : \text{entity} \\ n(y) \end{bmatrix} \\ p(\pi_1 v) \end{bmatrix}} \qquad \dfrac{\text{donkey}}{\mathbf{donkey}}}{\lambda p.\begin{bmatrix} v : \begin{bmatrix} y : \text{entity} \\ \mathbf{donkey}(y) \end{bmatrix} \\ p(\pi_1 v) \end{bmatrix}}>}{\lambda x.\begin{bmatrix} v : \begin{bmatrix} y : \text{entity} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}(x,\pi_1 v) \end{bmatrix}}<}{\lambda n.\lambda x.\begin{bmatrix} n(x) \\ v : \begin{bmatrix} y : \text{entity} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}(x,\pi_1 v) \end{bmatrix}}>}{\lambda x.\begin{bmatrix} \mathbf{farmer}(x) \\ v : \begin{bmatrix} y : \text{entity} \\ \mathbf{donkey}(y) \end{bmatrix} \\ \mathbf{own}(x,\pi_1 v) \end{bmatrix}}<$$

Donkey anaphora: Semantic Composition (2/2)

$$\dfrac{\dfrac{\text{Every}}{\lambda n.\lambda p.\left(u : \left[\begin{array}{l} x : \text{entity} \\ n(x) \end{array}\right]\right) \to p(\pi_1 u)} \quad \dfrac{\text{farmer who owns a donkey}}{\lambda x.\left[\begin{array}{l} \textbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(x, \pi_1 v) \end{array}\right]}}{\lambda p.\left(u : \left[\begin{array}{l} x : \text{entity} \\ \textbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(x, \pi_1 v) \end{array}\right]\right) \to p(\pi_1 u)} > \quad \dfrac{\dfrac{\text{beat}}{\lambda y.\lambda x.\ \textbf{beat}(x, y)} \quad \dfrac{\text{it}}{\lambda p.\lambda x.\left[\begin{array}{l} w@\left[\begin{array}{l} z : \text{entity} \\ \neg\textbf{human}(z) \end{array}\right] \\ p(\pi_1 w) x \end{array}\right]}}{\lambda x.\left[\begin{array}{l} w@\left[\begin{array}{l} z : \text{entity} \\ \neg\textbf{human}(z) \end{array}\right] \\ \textbf{beat}(x, \pi_1 w) \end{array}\right]} >}{\left(u : \left[\begin{array}{l} x : \text{entity} \\ \textbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(x, \pi_1 v) \end{array}\right]\right) \to \left[\begin{array}{l} w@\left[\begin{array}{l} z : \text{entity} \\ \neg\textbf{human}(z) \end{array}\right] \\ \textbf{beat}(\pi_1 u, \pi_1 w) \end{array}\right]} >$$

Donkey anaphora: Semantic Felicity Condition (=Type Checking)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\left[\begin{array}{l} x : \text{entity} \\ \textbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(x, \pi_1 v) \end{array}\right] : \text{type}}}{\left(u : \left[\begin{array}{l} x : \text{entity} \\ \textbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(x, \pi_1 v) \end{array}\right]\right) \to \left[\begin{array}{l} w@\left[\begin{array}{l} z : \text{entity} \\ \neg\textbf{human}(z) \end{array}\right] \\ \textbf{beat}(\pi_1 u, \pi_1 w) \end{array}\right] : \text{type}}\,(\Pi I),1}}{\left[\begin{array}{l} w@\left[\begin{array}{l} z : \text{entity} \\ \neg\textbf{human}(z) \end{array}\right] \\ \textbf{beat}(\pi_1 u, \pi_1 w) \end{array}\right] : \text{type}}}{(\textbf{beat}(\pi_1 u, \pi_1 w))[?/w] : \text{type}}}{? : \left[\begin{array}{l} z : \text{entity} \\ \neg\textbf{human}(z) \end{array}\right]}\,1}{\left[\begin{array}{l} z : \text{entity} \\ \neg\textbf{human}(z) \end{array}\right] : \text{type}}$$

Donkey anaphora: Anaphora Resolusion (=Proof Search)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{u : \left[\begin{array}{l} x : \text{entity} \\ \textbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(x, \pi_1 v) \end{array}\right]}\,1}{\pi_2 u : \left[\begin{array}{l} \textbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(\pi_1 u, \pi_1 v) \end{array}\right]}\,(\Sigma E)}{\pi_2\pi_2 u : \left[\begin{array}{l} v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(\pi_1 u, \pi_1 v) \end{array}\right]}\,(\Sigma E)}{\pi_1\pi_2\pi_2 u : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right]}\,(\Sigma E)}{\pi_1\pi_1\pi_2\pi_2 u : \text{entity}}\,(\Sigma E)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{u : \left[\begin{array}{l} x : \text{entity} \\ \textbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(x, \pi_1 v) \end{array}\right]}\,1}{\pi_2 u : \left[\begin{array}{l} \textbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(\pi_1 u, \pi_1 v) \end{array}\right]}\,(\Sigma E)}{\pi_2\pi_2 u : \left[\begin{array}{l} v : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right] \\ \textbf{own}(\pi_1 u, \pi_1 v) \end{array}\right]}\,(\Sigma E)}{\pi_1\pi_2\pi_2 u : \left[\begin{array}{l} y : \text{entity} \\ \textbf{donkey}(y) \end{array}\right]}\,(\Sigma E)$$

$$\dfrac{\dfrac{}{d : \left(u : \left[\begin{array}{l} x : \text{entity} \\ \textbf{donkey}(x) \end{array}\right]\right) \to \neg\textbf{human}(\pi_1 u)}\,(CON)}{d(\pi_1\pi_2\pi_2 u) : \neg\textbf{human}(\pi_1\pi_1\pi_2\pi_2 u)}\,(\Pi E)$$

$$\dfrac{}{(\pi_1\pi_1\pi_2\pi_2 u, d(\pi_1\pi_2\pi_2 u)) : \left[\begin{array}{l} z : \text{entity} \\ \neg\textbf{human}(z) \end{array}\right]}\,(\Sigma I)$$

Donkey anaphora: Semantic Felicity Condition (=Type Checking) continued

$$\dfrac{\begin{array}{c}\vdots\\ \left[\begin{array}{l} x : \text{entity} \\ \mathbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \mathbf{donkey}(y) \end{array}\right] \\ \mathbf{own}(x, \pi_1 v) \end{array}\right] : \text{type}\end{array} \qquad \begin{array}{c}\overline{\left[\begin{array}{l} x : \text{entity} \\ \mathbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \mathbf{donkey}(y) \end{array}\right] \\ \mathbf{own}(x, \pi_1 v) \end{array}\right]}^{1} \\ \vdots \\ \mathbf{beat}(\pi_1 u, \pi_1\pi_1\pi_2\pi_2 u) : \text{type}\end{array}}{\left(u : \left[\begin{array}{l} x : \text{entity} \\ \mathbf{farmer}(x) \\ v : \left[\begin{array}{l} y : \text{entity} \\ \mathbf{donkey}(y) \end{array}\right] \\ \mathbf{own}(x, \pi_1 v) \end{array}\right]\right) \to \mathbf{beat}(\pi_1 u, \pi_1\pi_1\pi_2\pi_2 u) : \text{type}}\,(\Pi F),1$$

$$\begin{array}{ll} & (\mathbf{beat}(\pi_1 u, \pi_1 w))[\,(\pi_1\pi_1\pi_2\pi_2 u, d(\pi_1\pi_2\pi_2 u))\,/w] : \text{type} \\ \twoheadrightarrow_\beta & \mathbf{beat}(\pi_1 u, \pi_1\pi_1\pi_2\pi_2 u) \end{array}$$

A takeaway message from DTS on Donkey Anaphora

The SR of a pronoun can refer to a donkey in question even from outside of its existential scope, because the donkey can be *constructed from* the variable in the context for the proof search.

In other words, anaphora accessibility is *provability*.

Notes on donkey anaphora

- The same analysis applies to the *implicational* donkey sentence:

  (12) If [a farmer]$^{\mathbf{1}}$ owns [a donkey]$^{\mathbf{2}}$, he$_{\mathbf{1}}$ beats it$_{\mathbf{2}}$.

- This analysis only predicts the *strong reading* for donkey sentences. For deriving both the strong readings and the *weak readings*, we need to refine the semantic representations for quantificational expressions: See Tanaka (2021).
- The refined analysis also explains why the anaphoric link to the *parametrized sum individual* (Krifka, 1996) is allowed (Tanaka, 2021).

  (13) [Every farmer]$^{\mathbf{1}}$ who owns [a donkey]$^{\mathbf{2}}$ loves its$_{\mathbf{2}}$ tail. But they$_{\mathbf{1}}$ beat it$_{\mathbf{2}}$.

Presupposition

Joint work with Koji Mineshima

What is Presupposition? — Background content

(14) It is John who broke my iPhone.

Presupposition: *Someone broke my iPhone*.

- the background content
- its truth is usually taken for granted

Assertion: *John was the one who did it*.

- the foreground content
- the main point of an utterance

Two puzzules of Presupposition – (i) Projection

(14) It was John who broke my iPhone.

(15) Someone broke my iPhone. ((14) **presupposes** (15))

(15) projects out of all the embedded contexts in (16a–e).

(16) a. It wasn't John who broke my iPhone. **negation**

b. Maybe it was John who broke my iPhone. **modal**

c. If it was John who broke my iPhone, then he has to fix it. **the antecedent of a conditional**

d. Was it John who broke my iPhone? **question**

e. Suppose that it was John who broke my iPhone. **hypothetical assumption**

The Case of Entailment

(17) John is an American pianist.

(18) John is American. ((17) **entails** (18))

(18) does not survive in the contexts (19a–e).

(19) a. John is not an American pianist. **negation**

b. Maybe John is an American pianist. **modal**

c. If John is an American pianist, he is skillful.
the antecedent of a conditional

d. Is John an American pianist? **question**

e. Suppose that John is an American pianist.
hypothetical assumption

The Case of Entailment

(17) John is an American pianist.
american(john) ∧ pianist(john)

(19) a. John is not an American pianist.
¬(american(j) ∧ pianist(j))

b. Maybe John is an American pianist.
◇(american(j) ∧ pianist(j))

c. If John is an American pianist, he is skillful.
american(j) ∧ pianist(j) → skillful(j)

Standard semantics correctly predicts these patterns:

- (17) ⊢ **american(john)**
- (19a) ⊬ **american(john)**
- (19b) ⊬ **american(john)**
- (19c) ⊬ **american(john)**

The Case of Presupposition

(14) It was John who broke my iPhone. $\boldsymbol{SR_1}$

(16) a. It wasn't John who broke my iPhone. $\neg \boldsymbol{SR_1}$

b. Maybe it was John who broke my iPhone. $\Diamond \boldsymbol{SR_1}$

c. If it was John who broke my iPhone, he has to fix it. $\boldsymbol{SR_1} \rightarrow \cdots$

What SR accounts for the following inference patterns?

- $\boldsymbol{SR_1} \Vdash \exists x(\textbf{broke}(x, \textbf{my\_iphone}))$
- $\neg \boldsymbol{SR_1} \Vdash \exists x(\textbf{broke}(x, \textbf{my\_iphone}))$
- $\Diamond \boldsymbol{SR_1} \Vdash \exists x(\textbf{broke}(x, \textbf{my\_iphone}))$
- $\boldsymbol{SR_1} \rightarrow \boldsymbol{A} \Vdash \exists x(\textbf{broke}(x, \textbf{my\_iphone}))$

Q: Can "$\Vdash$" be defined as a standard consequence relation "$\vdash$"?
A: No. If that were the case, then $\exists x(\textbf{broke}(x, \textbf{my\_iphone}))$ was a tautology (under the classical setting).

Two puzzules of Presupposition – (ii) Filteration

(20) presupposes that someone broke the window, but the conditional in (21) does not inherit this presupposition.

(20) It was John who broke the window.
$\Rightarrow$ Someone broke the window

(21) If the window was broken, it was John who broke it.
$\not\Rightarrow$ Someone broke the window

Similarly for (22) and (23).

(22) The king of France is wise.
$\Rightarrow$ France has a king.

(23) If France has a king, the king of France is wise.
$\not\Rightarrow$ France has a king.

A presupposition is **filtered** when it occurs in certain contexts.

Presupposition triggers

(24) a. **The elevator** in this building is clean. **Description**

b. There is an elevator in this building.

(25) a. **John's sister** is happy. **Possessive**

b. John has a sister.

(26) a. Bill **regrets** that he lied to Mary. **Factive**

b. Bill lied to Mary.

(27) a. John has **stopped** beating his wife. **Aspectual**

b. John has beaten his wife.

(28) a. Harry **managed** to find the book. **Implicative**

b. Finding the book required some effort.

Presupposition triggers

(29) a. Sam broke the window **again** today. **Iterative**

b. Sam broke the window before.

(30) a. **It was** Sam **who** broke the window. **Cleft**

b. Someone broke the window.

(31) a. **What** John broke **was** his typewriter. **Pseudo-cleft**

b. John broke something.

(32) a. $[\text{Pat}]_F$ is leaving, **too**. (Focus on *Pat*) **Additive**

b. Someone other than Pat is leaving.

For classical examples of presupposition triggers, see Levinson (1983), Soames (1989), Geurts (1999), and Beaver (2001), among others.

“Presupposition Is Anaphora” hypothesis

"Presupposition Is Anaphora" hypothesis

There are striking parallels between anaphoric expressions and presupposition triggers. (van der Sandt, 1992; Geurts, 1999)

Presupposition filtering:

(33) a. John has children and **John's children** are wise.
b. If John has children, **John's children** are wise.

(34) a. The window was broken and **it was** John **who** broke it.
b. If the window was broken, **it was** John **who** broke it.

Compare (33) and (34) with the paradigm examples of anaphora resolution.

Anaphora resolution:

(35) a. John owns a donkey and he beats **it**.
b. If John owns a donkey, he beats **it**.

DTS on Filtering

- The present account can explain the filtering of presupposition without further stipulation.

  (36) If France has a king, **the king of France** is wise.

$$\textbf{SR} \quad \left(u : \left[\begin{array}{l} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{array}\right]\right) \rightarrow \left[\begin{array}{l} v@\left[\begin{array}{l} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{array}\right] \\ \textbf{wise}(\pi_1 v) \end{array}\right]$$

Filtering: Type checking

$$\dfrac{\dfrac{\vdots}{\left[\begin{array}{l} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{array}\right] : \text{type}} \qquad \dfrac{\dfrac{\vdots}{\left[\begin{array}{l} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{array}\right] : \text{type}} \quad \dfrac{}{u : \left[\begin{array}{l} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{array}\right]}\,1 \quad \dfrac{\vdots}{? : \left[\begin{array}{l} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{array}\right]} \quad \dfrac{\vdots}{\textbf{wise}(\pi_1 v)[\,?\,/v]}}{\left[\begin{array}{l} v@\left[\begin{array}{l} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{array}\right] \\ \textbf{wise}(\pi_1 v) \end{array}\right]}\,(@)}{\left(u : \left[\begin{array}{l} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{array}\right]\right) \to \left[\begin{array}{l} v@\left[\begin{array}{l} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{array}\right] \\ \textbf{wise}(\pi_1 v) \end{array}\right] : \text{type}}\,(\Pi F),1$$

DTS on Filtering

$$\dfrac{\dfrac{\begin{bmatrix} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{bmatrix} : \text{type} \quad \dfrac{\textbf{wise} : \text{entity} \to \text{type}}{\textbf{wise}(\pi_1 u) : \text{type}}\,(CON) \quad \dfrac{u : \begin{bmatrix} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{bmatrix}^1}{\pi_1 u : \text{entity}}\,(\Sigma E)}{\left(u : \begin{bmatrix} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{bmatrix}\right) \to \textbf{wise}(\pi_1 u) : \text{type}}\,(\Pi E)}{}\,(\Pi F),_1$$

- Type checking algorithm returns a fully-specified semantic representation.
- Presupposition filtering is performed via exactly the same process as anaphora resolution.

A takeaway message from DTS on Presupposition Filtering

A presupposition is filtered in the same way as an anaphoric expression is resolved.

DTS on Projection

- The projection of presupposition is naturally accounted for using DTS.
- Recall that negation is defined to be an implication of the form $\neg A \equiv A \to \bot$.
- According to the formation rule $(\mathbf{\Pi F})$, the proposition A and its negation $\neg A$ have the same presupposition.

Example:

It is not the case that the king of France is bald.

$$\textbf{SR} \qquad \neg \left[\begin{array}{l} u@\left[\begin{array}{l} x : \text{entity} \\ \textbf{king}(x, fr) \end{array} \right] \\ \textbf{bold}(\pi_1 u) \end{array} \right]$$

Projection: Syntax

Lexical entries:

- It is not the case that: S/S
- the: $S/(S\backslash NP)/N$
- king: N/PP_{of}
- of: PP_{of}/NP
- France: NP
- is: $S\backslash NP/(S\backslash NP)$
- bald: $S\backslash NP$

Derivation:

$$\dfrac{PP_{of}/NP \quad NP}{PP_{of}}\,>$$

$$\dfrac{N/PP_{of} \quad PP_{of}}{N}\,<$$

$$\dfrac{S/(S\backslash NP)/N \quad N}{S/(S\backslash NP)}\,>$$

$$\dfrac{S\backslash NP/(S\backslash NP) \quad S\backslash NP}{S\backslash NP}\,>$$

$$\dfrac{S/(S\backslash NP) \quad S\backslash NP}{S}\,>$$

$$\dfrac{S/S \quad S}{S}\,>$$

Projection: Semantic Composition

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{the}}{\lambda n.\lambda p.\left[\begin{array}{l} u@\left[\begin{array}{l} x : \text{entity} \\ n(x) \end{array}\right] \\ p(\pi_1 u) \end{array}\right]} \quad \dfrac{\dfrac{\text{king}}{\lambda z.\lambda x.\mathbf{kingOf}(x,z)} \quad \dfrac{\dfrac{\text{of}}{id} \quad \dfrac{\text{France}}{fr}}{fr}>}{\lambda x.\mathbf{kingOf}(x,fr)}<}{\lambda p.\left[\begin{array}{l} u@\left[\begin{array}{l} x : \text{entity} \\ \mathbf{kingOf}(x,fr) \end{array}\right] \\ p(\pi_1 u) \end{array}\right]}> \quad \dfrac{\dfrac{\text{is}}{id} \quad \dfrac{\text{bald}}{\mathbf{bald}}}{\mathbf{bald}}>}{\left[\begin{array}{l} u@\left[\begin{array}{l} x : \text{entity} \\ \mathbf{kingOf}(x,fr) \end{array}\right] \\ \mathbf{bald}(\pi_1 u) \end{array}\right]}> \quad \dfrac{\text{It is not the case that}}{\lambda p.\neg p}}{\neg\left[\begin{array}{l} u@\left[\begin{array}{l} x : \text{entity} \\ \mathbf{kingOf}(x,fr) \end{array}\right] \\ \mathbf{bald}(\pi_1 u) \end{array}\right]}>$$

Projection: Type checking

$$\dfrac{\dfrac{\vdots}{\begin{bmatrix} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{bmatrix}} \quad ? : \begin{bmatrix} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{bmatrix} \quad \neg\textbf{bald}(\pi_1 u)[\,?\,/u] : \text{type}}{\begin{bmatrix} u@\begin{bmatrix} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{bmatrix} \\ \neg\textbf{bald}(\pi_1 u) \end{bmatrix} : \text{type}}\,(@) \qquad \dfrac{}{\bot : \text{type}}\,(\{\}F)}{\neg\begin{bmatrix} u@\begin{bmatrix} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{bmatrix} \\ \neg\textbf{bald}(\pi_1 u) \end{bmatrix} : \text{type}}\,(\Pi F)$$

- In order for the sentence "The king of France is bald" to be well-formed, the context Γ must be such that the following type inhabits a proof (namely, there exists a king of France).

$$\Gamma \vdash ? : \begin{bmatrix} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{bmatrix}$$

DTS on Projection

- The same inference is triggered for the antecedent of a conditional sentence like (37):

  (37) If the king of France is wise, people will be happy.

$$\frac{\begin{bmatrix} u@\begin{bmatrix} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{bmatrix} \\ \textbf{wise}(\pi_1 u) \end{bmatrix} : \text{type} \quad \textbf{happy}(people) : \text{type}}{\begin{bmatrix} u@\begin{bmatrix} x : \text{entity} \\ \textbf{kingOf}(x, fr) \end{bmatrix} \\ \textbf{wise}(\pi_1 u) \end{bmatrix} \rightarrow \textbf{happy}(people) : \text{type}} (\Pi F)$$

A takeaway message from DTS on Presupposition Projection

A presupposition projects because it's truth is a requirement for a sentence containing it to be *semantically well-formed*, not to be *true*.

Corollary: Existence of an antecedent is a requirement for a sentence containing anaphora to be semantically well-formed, not to be true.

Summary and History

A Unified, Compositional Theory of *Projective* Meaning

- DTS provides a unified analysis for (general) inferences and anaphora resolusion mechanisms.
- The background theory for DTS is an extention of DTT with underspecified types and the @-rule.
  - Lexical items of anaphoric expressions and presupposition triggers are represented by using underspecified types.
  - Context retrieval in DTS reduces to type checking.
  - Anaphora resolution and presupposition binding in DTS reduces to proof search.
  - Type checker translates a proof diagram of DTS into a proof diagram of DTT, by which an SR in DTT is obtained with all anaphora resolved.

Natural language semantics via dependent types: The first generation

- Donkey anaphora: Sundholm (1986)
- Translation from DRS to dependent type representations: Ahn and Kolb (1990)
- Summation: Fox (1994a,b)
- Ranta's TTG (Relative and Implicational Donkey Sentences, Branching Quantifiers, Intensionality, Tense): Ranta (1994)
- Translation from Montague Grammar to dependent type representations: Dávila-Pérez (1995)
- Presupposition Binding and Accommodation, Bridging: Krahmer and Piwek (1999), Piwek and Krahmer (2000)

Natural language semantics via dependent types: The second generation

- Type Theory with Record (TTR): Cooper (2005)
- Modern Type Theory: Luo (1997, 1999, 2010, 2012), Asher and Luo (2012), Chatzikyriakidis (2014)
- Semantics with Dependent Types: Grudzinska and Zawadowski (2014; 2017)
- Dynamic Categorial Grammar: Martin and Pollard (2014)
- **Dependent Type Semantics (DTS): Bekki (2014), Bekki and Mineshima (2017)**

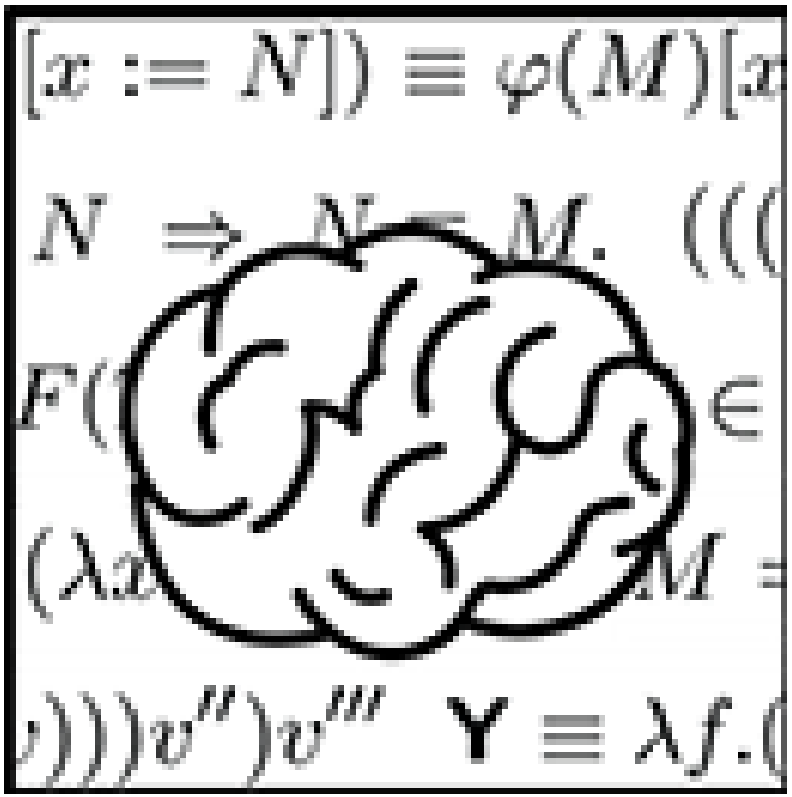
Semantic Analyses by DTS: The third generation

- Generalized Quantifiers: Tanaka (2014)
- Honorification: Watanabe et al. (2014)
- Conventional Implicature: Bekki and McCready (2015), Matsuoka et al. (2023)
- Factive Presuppositions: Tanaka et al. (2015)
- Dependent Plural Anaphora: Tanaka et al. (2017)
- Paycheck sentences: Tanaka et al. (2018)
- Coercion and Metaphor: Kinoshita et al. (2017, 2018)
- Questions: Watanabe et al. (2019), Funakura (2022)
- Comparision with DRT: Yana et al. (2019)
- The proviso problem: Yana et al. (2021)
- Weak Crossover: Bekki (2023)

Computational Aspects of DTS

- Type Checker for (the fragment of) DTS: Bekki and Sato (2015)
- Development of an automated theorem prover (for the fragment of) DTS: Daido and Bekki (2020)
- Integrating Deep Neural Network with DTS: Bekki et al. (2023, 2022)

Thank you!


$[x := N]) \equiv \varphi(M)[x$
$N \Rightarrow N = M.$ (((
$F($ $\in$
$(\lambda x$ $M =$
$v'))))v'')v'''$ $\mathbf{Y} \equiv \lambda f.($

Appendix I: Type System in DTS

Axioms and Structural Rules

$$\frac{A : \text{type}}{x : A}\,(VAR) \qquad \frac{}{c : A}\,(CON) \quad where\ (c : A) \in \sigma.$$

$$\frac{}{\text{type} : \text{kind}}\,(\text{type}F)$$

$$\frac{M : A \quad N : B}{M : A}\,(WK) \qquad \frac{M : A}{M : B}\,(CONV) \quad where\ A =_\beta B.$$

Π-type F/I/E rules

$$\frac{\begin{array}{c}\overline{x : A}^{\,i} \\ \vdots \\ A : \mathsf{s}_1 \quad B : \mathsf{s}_2\end{array}}{(x : A) \to B : \mathsf{s}_2}\,(\Pi F),i \qquad \textit{where } (\mathsf{s}_1, \mathsf{s}_2) \in \left\{ \begin{array}{l} (\mathsf{type}, \mathsf{type}), \\ (\mathsf{type}, \mathsf{kind}) \end{array} \right\}.$$

$$\frac{\begin{array}{c}\overline{x : A}^{\,i} \\ \vdots \\ A : \mathsf{type} \quad M : B\end{array}}{\lambda x.M : (x : A) \to B}\,(\Pi I),i \qquad \frac{M : (x : A) \to B \quad N : A}{MN : B[N/x]}\,(\Pi E)$$

Σ-type F/I/E rules

$$\dfrac{\begin{array}{c}\overline{x : A}^{\,i} \\ \vdots \\ A : \mathsf{type} \quad B : \mathsf{type}\end{array}}{(x : A) \times B : \mathsf{type}}\ (\Sigma F),i$$

$$\dfrac{M : A \quad N : B[M/x]}{(M, N) : (x : A) \times B}\ (\Sigma I)$$

$$\dfrac{M : (x : A) \times B}{\pi_1(M) : A}\ (\Sigma E) \qquad \dfrac{M : (x : A) \times B}{\pi_2(M) : B[\pi_1(M)/x]}\ (\Sigma E)$$

Disjoint Union Type F/I/E rules

$$\frac{A : \mathsf{type} \quad B : \mathsf{type}}{A + B : \mathsf{type}}\,(+F)$$

$$\frac{M : A}{\iota_1(M) : A + B}\,(+I) \qquad \frac{N : B}{\iota_2(N) : A + B}\,(+I)$$

$$\frac{M : A + B \quad P : (A + B) \to \mathsf{type} \quad N_1 : (x : A) \to P(\iota_1(x)) \quad N_2 : (x : B) \to P(\iota_2(x))}{\mathsf{unpack}^P_M\,(N_1, N_2) : P(M)}\,(+E),i$$

Natural Number Type F/I/E rules

$$\frac{}{\mathbb{N} : \mathsf{type}}\ (\mathbb{N}\boldsymbol{F})$$

$$\frac{}{0 : \mathbb{N}}\ (\mathbb{N}\boldsymbol{I}) \qquad \frac{n : \mathbb{N}}{\mathsf{s}(n) : \mathbb{N}}\ (\mathbb{N}\boldsymbol{I})$$

$$\frac{n : \mathbb{N} \quad P : \mathbb{N} \to \mathsf{type} \quad e : P(0) \quad f : (k : \mathbb{N}) \to P(k) \to P(\mathsf{s}(k))}{\mathsf{natrec}_n^P(e, f) : P(n)}\ (\mathbb{N}\boldsymbol{E})$$

Enumeration Type F/I/E rules

$$\frac{}{\{a_1, \ldots, a_n\} : \text{type}} \; (\{\}F)$$

$$\frac{}{a_i : \{a_1, \ldots, a_n\}} \; (\{\}I)$$

$$\frac{M : \{a_1, \ldots, a_n\} \quad P : \{a_1, \ldots, a_n\} \to \text{type} \quad N_1 : P(a_1) \quad \ldots \quad N_n : P(a_n)}{\text{case}^P_M\,(N_1, \ldots, N_n) : P(M)} \; (\{\}E)$$

Intensional Equality Type F/I/E rules

$$\frac{A : \mathsf{type} \quad M : A \quad N : A}{M =_A N : \mathsf{type}}\ (=F)$$

$$\frac{M : A}{\mathsf{refl}_A(M) : M =_A M}\ (=I)$$

$$\frac{E : M =_A N \quad P : (x : A) \to (y : A) \to (x =_A y) \to \mathsf{type} \quad R : (x : A) \to Pxx(\mathsf{refl}_A(x))}{\mathsf{idpeel}^P_E(R) : PMNE}\ (=E)$$

@-rule

$$\dfrac{\begin{array}{c}\overline{x : A}^{\,i} \\ \vdots \\ A : \mathsf{type} \quad B : \mathsf{type} \quad A\ \mathit{true}\end{array}}{: \mathsf{type}}\ (@F)$$

Appendix II: Type Check/Inference Algorithm

Inferable terms (1/2)

The collection of *inferable terms* (notation $M_{\uparrow}$) and the collection of *checkable terms* (notation $M_{\downarrow}$) are simultaneously defined by the following BNF notations, where v, v' are values.

$M_{\uparrow}$	$::=$	x	variable
	$\mid$	c	constant symbol
	$\mid$	type	the type of types
	$\mid$	$(x : M_{\downarrow}) \to M_{\downarrow}$	dependent functional type
	$\mid$	$M_{\uparrow}\ M_{\downarrow}$	functional application
	$\mid$	$(x : M_{\downarrow}) \times M_{\downarrow}$	dependent sum type
	$\mid$	$\pi_i(M_{\uparrow}\ )$	projections
	$\mid$	$M_{\downarrow} + M_{\downarrow}$	disjoint union types
	$\mid$	$\mathrm{unpack}^{M_{\downarrow}}_{M_{\uparrow}}\ (M_{\downarrow}, M_{\downarrow})$	unpack

Inferable terms (2/2)

$\bot$	bottom type
$\top$	top type
$()$	unit
$M_\downarrow =_{M_\downarrow} M_\downarrow$	Intensional Equality types
$\mathsf{refl}_{M_\downarrow}(M_\downarrow)$	reflexive
$\mathsf{idpeel}^{M_\downarrow}_{M_\uparrow}\ (M_\downarrow)$	idpeel
$\mathbb{N}$	natural number types
0	zero
$\mathsf{s}(M_\downarrow)$	successor
$\mathsf{natrec}^{M_\downarrow}_{M_\uparrow}\ (M_\downarrow, M_\downarrow)$	mathematical induction
$M_\downarrow : M_\downarrow$	annotated term
$\left[\begin{array}{l} x@M_\downarrow \\ M_\downarrow \end{array}\right]$	underspecified type

Checkable terms

$M_\downarrow$	$::=$	$M_\uparrow$	inferable terms
	$\mid$	$\lambda x.M_\downarrow$	lambda abstraction
	$\mid$	$(M_\downarrow, M_\downarrow)$	pair
	$\mid$	$\iota_i(M_\downarrow)$	injections

Type Checking/Inference Algorithm

Type inference of a UDTT is a transformation of a UDTT judgment to a set of DTT proof diagrams, recursively defined by the following set of rules.

Definition (Inferable Terms: Structural Rules)

$$[\![\Gamma, x : A, \Delta \vdash x : ?]\!] = \left\{ x : A' \;\middle|\; \begin{array}{c}\vdots \\ A' : \mathsf{type}\end{array} \in [\![\Gamma \vdash A : \mathsf{type}]\!] \right\}$$

$$[\![\Gamma \vdash c : ?]\!] = \left\{ \frac{}{c : A}\,(CON) \;\middle|\; \sigma \vdash c : A \right\}$$

$$[\![\Gamma \vdash \mathsf{type} : ?]\!] = \left\{ \frac{}{\Gamma \vdash \mathsf{type} : \mathsf{kind}}\,(\mathsf{type}F) \;\middle|\; \right\}$$

Type Checking/Inference Algorithm

Definition (Π-types)

$$[\![\Gamma \vdash (x : A) \to B : ?]\!] = \left\{ \frac{\begin{array}{cc}\Gamma & \Gamma, x : A' \\ \mathcal{D}_A & \mathcal{D}_B \\ A' : \text{type} & B' : \text{type}\end{array}}{(x : A') \to B' : \text{type}}\,(\Pi F) \;\middle|\; \begin{array}{l}\mathcal{D}_A \in [\![\Gamma \vdash A : \text{type}]\!] \\ \mathcal{D}_B \in [\![\Gamma, x : A' \vdash B : \text{type}]\!]\end{array} \right\}$$

$$[\![\Gamma \vdash \lambda x.M : (x : A) \to B]\!] = \left\{ \frac{\begin{array}{cc}\Gamma & \Gamma, x : A' \\ \mathcal{D}_A & \mathcal{D}_M \\ A' : \text{type} & M' : B'\end{array}}{\lambda x.M' : (x : A') \to B'}\,(\Pi I) \;\middle|\; \begin{array}{l}\mathcal{D}_A \in [\![\Gamma \vdash A : \text{type}]\!] \\ \mathcal{D}_M \in [\![\Gamma, x : A' \vdash M : B]\!]\end{array} \right\}$$

$$[\![\Gamma \vdash MN : ?]\!] = \left\{ \frac{\dfrac{\begin{array}{cc}\Gamma & \Gamma \\ \mathcal{D}_M & \mathcal{D}_N \\ M' : (x : A) \to B & N' : A\end{array}}{M'N' : B[N'/x]}\,(\Pi E)}{M'N' : B'}\,(CONV) \;\middle|\; \begin{array}{l}\mathcal{D}_M \in [\![\Gamma \vdash M : ?]\!] \\ \mathcal{D}_N \in [\![\Gamma \vdash N : A]\!] \\ B[N'/x] \twoheadrightarrow_\beta B'\end{array} \right\}$$

Type Checking/Inference Algorithm

Definition (Σ-types)

$$\left[\!\left[\Gamma \vdash \begin{bmatrix} x : A \\ B \end{bmatrix} : ?\right]\!\right] = \left\{ \left. \frac{\begin{matrix} \Gamma & \Gamma, x : A' \\ \mathcal{D}_A & \mathcal{D}_B \\ A' : \text{type} & B' : \text{type} \end{matrix}}{\begin{bmatrix} x : A' \\ B' \end{bmatrix} : \text{type}} (\Sigma F) \;\right|\; \begin{matrix} \mathcal{D}_A \in [\![\Gamma \vdash A : \text{type}]\!] \\ \mathcal{D}_B \in [\![\Gamma, x : A' \vdash B : \text{type}]\!] \end{matrix} \right\}$$

$$\left[\!\left[\Gamma \vdash (M, N) : \begin{bmatrix} x : A \\ B \end{bmatrix}\right]\!\right] = \left\{ \left. \frac{\begin{matrix} \Gamma & \Gamma \\ \mathcal{D}_M & \mathcal{D}_N \\ M' : A' & N' : B' \end{matrix}}{(M', N') : \begin{bmatrix} x : A' \\ B' \end{bmatrix}} (\Sigma I) \;\right|\; \begin{matrix} \mathcal{D}_M \in [\![\Gamma \vdash M : A]\!] \\ B[M'/x] \twoheadrightarrow_\beta B' \\ \mathcal{D}_N \in [\![\Gamma \vdash N : B']\!] \end{matrix} \right\}$$

$$[\![\Gamma \vdash \pi_1(M) : ?]\!] = \left\{ \left. \frac{\begin{matrix} \Gamma \\ \mathcal{D}_M \\ M' : \begin{bmatrix} x : A \\ B \end{bmatrix} \end{matrix}}{\pi_1(M') : A} (\Sigma E) \;\right|\; \mathcal{D}_M \in [\![\Gamma \vdash M : ?]\!] \right\}$$

$$[\![\Gamma \vdash \pi_2(M) : ?]\!] = \left\{ \left. \frac{\dfrac{\begin{matrix} \Gamma \\ \mathcal{D}_M \\ M' : \begin{bmatrix} x : A \\ B \end{bmatrix} \end{matrix}}{\pi_2(M') : B[\pi_1 M'/x]} (\Sigma E)}{\pi_2(M') : B'} (CONV) \;\right|\; \begin{matrix} \mathcal{D}_M \in [\![\Gamma \vdash M : ?]\!] \\ B[\pi_1 M'/x] \twoheadrightarrow_\beta B' \end{matrix} \right\}$$

Type Checking/Inference Algorithm

Definition (Disjoint Union types)

$$[\![\Gamma \vdash A + B : ?]\!] = \left\{ \left. \frac{\begin{array}{cc}\Gamma & \Gamma \\ \mathcal{D}_A & \mathcal{D}_B \\ A' : \mathsf{type} & B' : \mathsf{type}\end{array}}{A' + B' : \mathsf{type}}\,(+F) \;\right|\; \begin{array}{l}\mathcal{D}_A \in [\![\Gamma \vdash A : \mathsf{type}]\!] \\ \mathcal{D}_B \in [\![\Gamma \vdash B : \mathsf{type}]\!]\end{array} \right\}$$

$$[\![\Gamma \vdash \iota_1(M) : A + B]\!] = \left\{ \left. \frac{\begin{array}{c}\Gamma \\ \mathcal{D}_M \\ M' : A'\end{array}}{\iota_1(M') : A' + B}\,(+I) \;\right|\; \mathcal{D}_M \in [\![\Gamma \vdash M : A]\!] \right\}$$

$$[\![\Gamma \vdash \iota_2(N) : A + B]\!] = \left\{ \left. \frac{\begin{array}{c}\Gamma \\ \mathcal{D}_N \\ N' : B'\end{array}}{\iota_2(N') : A + B'}\,(+I) \;\right|\; \mathcal{D}_N \in [\![\Gamma \vdash N : B]\!] \right\}$$

$$[\![\Gamma \vdash \mathsf{unpack}^P_L\,(M, N) : ?]\!]$$

$$= \left\{ \left. \frac{\begin{array}{cccc}\Gamma & \Gamma & \Gamma & \Gamma \\ \mathcal{D}_L & \mathcal{D}_P & \mathcal{D}_M & \mathcal{D}_N \\ L' : A + B & P' : (A + B) \to \mathsf{type} & M' : (x : A) \to P'(\iota_1(x)) & N' : (x : B) \to P'(\iota_2(x))\end{array}}{\dfrac{\mathsf{unpack}^P_{L'}\,(M', N') : P'(L')}{\mathsf{unpack}^{PB}_{L'}\,(M', N') : P_L}\,(CONV)}\,(+E),i \;\right|\; \begin{array}{l}\mathcal{D}_L \in [\![\Gamma \vdash L : ?]\!] \\ \mathcal{D}_P \in [\![\Gamma \vdash P : (A + B) \to \mathsf{type}]\!] \\ P'(L') \twoheadrightarrow_\beta P_L \\ P'(\iota_1(x)) \twoheadrightarrow_\beta P_M \\ P'(\iota_2(x)) \twoheadrightarrow_\beta P_N \\ \mathcal{D}_M \in [\![\Gamma \vdash M : (x : A) \to P_M]\!] \\ \mathcal{D}_N \in [\![\Gamma \vdash N : (x : B) \to P_N]\!]\end{array} \right\}$$

Type Checking/Inference Algorithm

Definition (Enumeration types)

$$[\![\Gamma \vdash \{a_1, \ldots, a_n\} : ?]\!] = \left\{ \frac{}{\{a_1, \ldots, a_n\} : \mathsf{type}} \, (\{\}F) \right\}$$

$$[\![\Gamma \vdash a_i : ?]\!] = \left\{ \frac{}{a_i : \{a_1, \ldots, a_n\}} \, (\{\}I) \right\}$$

$$[\![\Gamma \vdash \mathsf{case}^P_M\,(N_1, \ldots, N_n) : ?]\!]$$

$$= \left\{ \frac{\overset{\Gamma}{\mathcal{D}_M} \quad M : \{a_1, \ldots, a_n\} \quad P : \{a_1, \ldots, a_n\} \to \mathsf{type} \quad \frac{\overset{\Gamma}{\mathcal{D}_{P_1}} \quad N_1 : P_1}{N_1 : P(a_1)}\,(CONV) \quad \ldots \quad \frac{\overset{\Gamma}{\mathcal{D}_{P_n}} \quad N_n : P_n}{N_n : P(a_n)}\,(CONV)}{\dfrac{\mathsf{case}^P_M\,(N_1, \ldots, N_n) : P(M)}{\mathsf{case}^P_M\,(N_1, \ldots, N_n) : P_M}\,(CONV)}\,(\{\}E) \right.$$

$$\left| \begin{array}{l} \mathcal{D}_M \in [\![\Gamma \vdash M : ?]\!] \\ \mathcal{D}_P \in [\![\Gamma \vdash P : \{a_1, \ldots, a_n\} \to \mathsf{type}]\!] \\ P(a_1) \twoheadrightarrow_\beta P_1 \\ \mathcal{D}_{P_1} \in [\![\Gamma \vdash N_1 : P_1]\!] \\ \ldots \\ P(a_n) \twoheadrightarrow_\beta P_n \\ \mathcal{D}_{P_n} \in [\![\Gamma \vdash N_n : P_n]\!] \\ P(M) \twoheadrightarrow_\beta P_M \end{array} \right\}$$

Type Checking/Inference Algorithm

Definition (Intensional Equality types)

$$[\![\Gamma \vdash M =_A N : ?]\!] = \left\{ \frac{\begin{array}{ccc}\overset{\Gamma}{\mathcal{D}_A} & \overset{\Gamma}{\mathcal{D}_M} & \overset{\Gamma}{\mathcal{D}_N} \\ A' : \mathsf{type} & M' : A' & N' : A'\end{array}}{M' =_{A'} N' : \mathsf{type}}\,(=F) \;\middle|\; \begin{array}{l}\mathcal{D}_A \in [\![\Gamma \vdash A : \mathsf{type}]\!] \\ \mathcal{D}_M \in [\![\Gamma \vdash M : A']\!] \\ \mathcal{D}_N \in [\![\Gamma \vdash N : A']\!]\end{array} \right\}$$

$$[\![\Gamma \vdash \mathsf{refl}_A(M) : ?]\!] = \left\{ \frac{\begin{array}{cc}\overset{\Gamma}{\mathcal{D}_A} & \overset{\Gamma}{\mathcal{D}_M} \\ A' : \mathsf{type} & M' : A'\end{array}}{\mathsf{refl}_{A'}(M') : M' =_{A'} M'}\,(=I) \;\middle|\; \begin{array}{l}\mathcal{D}_A \in [\![\Gamma \vdash A : \mathsf{type}]\!] \\ \mathcal{D}_M \in [\![\Gamma \vdash M : A']\!]\end{array} \right\}$$

$$[\![\Gamma \vdash \mathsf{idpeel}_E^P(R) : ?]\!]$$

$$= \left\{ \frac{\frac{\begin{array}{ccc}\overset{\Gamma}{\mathcal{D}_E} & \overset{\Gamma}{\mathcal{D}_P} & \overset{\Gamma}{\mathcal{D}_R} \\ E' : M =_A N & P' : (x:A) \to (y:A) \to (x =_A y) \to \mathsf{type} & R' : (x:A) \to P'xx(\mathsf{refl}_A(x))\end{array}}{\mathsf{idpeel}_{E'}^{P'}(R') : P'MNE'}\,(=E)}{\mathsf{idpeel}_{E'}^{P'}(R') : P''}\,(CONV) \;\middle|\; \begin{array}{l}\mathcal{D}_E \in [\![\Gamma \vdash E : ?]\!] \\ \mathcal{D}_P \in [\![\Gamma \vdash P : (x:A) \to (y:A) \to (x =_A y) \to \mathsf{type}]\!] \\ \mathcal{D}_R \in [\![\Gamma \vdash R : (x:A) \to P'xx(\mathsf{refl}_A(x))]\!] \\ PMNE \twoheadrightarrow_\beta P''\end{array} \right\}$$

Type Checking/Inference Algorithm

Definition (Natural Number types)

$$[\![\Gamma \vdash \mathbb{N} : ?]\!] = \left\{ \frac{}{\mathbb{N} : \mathsf{type}}\ (\mathbb{N}F) \right\}$$

$$[\![\Gamma \vdash 0 : ?]\!] = \left\{ \frac{}{0 : \mathbb{N}}\ (\mathbb{N}I) \right\}$$

$$[\![\Gamma \vdash \mathsf{s}(n) : ?]\!] = \left\{ \left. \frac{\begin{array}{c}\Gamma \\ \mathcal{D}_n \\ n : \mathbb{N}\end{array}}{\mathsf{s}(n) : \mathbb{N}}\ (\mathbb{N}I) \;\right|\; \mathcal{D}_n \in [\![\Gamma \vdash n : \mathbb{N}]\!] \right\}$$

$$[\![\Gamma \vdash \mathsf{natrec}_n^P(e,f) : ?]\!]$$
$$= \left\{ \left. \frac{\begin{array}{c}\Gamma \\ \mathcal{D}_n \\ n' : \mathbb{N} \quad P : \mathbb{N} \to \mathsf{type} \quad e : P(0) \quad f : (k : \mathbb{N}) \to P(k) \to P(\mathsf{s}(k))\end{array}}{\dfrac{\mathsf{natrec}_n^P(e,f) : P(n)}{\mathsf{natrec}_n^P(e,f) : P_n}\ (CONV)}\ (\mathbb{N}E) \;\right|\; \begin{array}{l}\mathcal{D}_n \in [\![\Gamma \vdash n : \mathbb{N}]\!] \\ P(n) \twoheadrightarrow_\beta P_n\end{array} \right\}$$

Type Checking/Inference Algorithm

Definition (Underspecified types)

$$\left[\!\left[\Gamma \vdash \left[\begin{array}{c} x@A \\ B \end{array}\right] : ?\right]\!\right] = \left\{ \begin{array}{c} \Gamma \\ \mathcal{D}_B \\ B'' : \text{type} \end{array} \;\middle|\; \begin{array}{l} \vdots \\ A' : \text{type} \end{array} \in [\![\Gamma \vdash A : \text{type}]\!] \right.$$
$$\left. \begin{array}{l} \vdots \\ M : A' \end{array} \in [\![\Gamma \vdash ? : A']\!] \right.$$
$$\left. \begin{array}{l} B[M/x] \twoheadrightarrow_\beta B' \\ \mathcal{D}_B \in [\![\Gamma \vdash B' : \text{type}]\!] \end{array} \right\}$$

Type Checking/Inference Algorithm

Definition (Checkable Terms)

$$[\![\Gamma \vdash M : A]\!] = \left\{ \begin{array}{c} \Gamma \\ \mathcal{D}_M \\ M : A' \end{array} \;\middle|\; \begin{array}{l} \mathcal{D}_M \in [\![\Gamma \vdash M : ?]\!] \\ A =_\beta A' \end{array} \right\} \quad \textit{where } M \textit{ inferable.}$$

β-reduction Rules

$$\frac{M \twoheadrightarrow_\beta \lambda x.v \quad v[N/x] \twoheadrightarrow_\beta v'}{MN \twoheadrightarrow_\beta v'}\ (\beta) \qquad \frac{M \twoheadrightarrow_\beta (v_1, v_2)}{\pi_i M \twoheadrightarrow_\beta v_i}\ (\beta)$$

$$\frac{}{x \twoheadrightarrow_\beta x}\ (VAR=) \qquad \frac{}{c \twoheadrightarrow_\beta c}\ (CON=)$$

$$\frac{}{\mathsf{type} \twoheadrightarrow_\beta \mathsf{type}}\ (\mathsf{type}=) \qquad \frac{}{\mathsf{kind} \twoheadrightarrow_\beta \mathsf{kind}}\ (\mathsf{kind}=)$$

β-reduction Rules

$$\frac{M \twoheadrightarrow_\beta v \quad N \twoheadrightarrow_\beta v'}{(x : M) \to N \twoheadrightarrow_\beta (x : v) \to v'}\ (\Pi=) \qquad \frac{M \twoheadrightarrow_\beta v}{\lambda x.M \twoheadrightarrow_\beta \lambda x.v}\ (\lambda=)$$

$$\frac{M \twoheadrightarrow_\beta v \quad N \twoheadrightarrow_\beta v'}{\begin{bmatrix} x : M \\ N \end{bmatrix} \twoheadrightarrow_\beta \begin{bmatrix} x : v \\ v' \end{bmatrix}}\ (\Sigma=) \qquad \frac{M \twoheadrightarrow_\beta v \quad N \twoheadrightarrow_\beta v'}{(M, N) \twoheadrightarrow_\beta (v, v')}\ (PAIR=)$$

$$\frac{}{() \twoheadrightarrow_\beta ()}\ (()=) \qquad \frac{}{\top \twoheadrightarrow_\beta \top}\ (\top=) \qquad \frac{}{\bot \twoheadrightarrow_\beta \bot}\ (\bot=)$$

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