# Chapter 25

# Disjunction and Anaphora

## 25.1 Dynamics of Disjunction

Disjunction has a semantic representation of the form $A + B$ using the disjoint union type. The behavior for anaphora is as seen below: first, pronouns in $B$ cannot access antecedents in $A$.

(648) Either [a woman]$^1$ walked in or #she$_1$ sat down.

Second, the pronoun in the following sentence of $A+B$ cannot access the antecedent in $A$ nor the antecedent in $B$.

(649) a. Either [a woman]$^1$ walked in or John sat down. #She$_1$ smiled.

b. Either John sat down or [a woman]$^1$ walked in. #She$_1$ smiled.

Therefore, the disjunction is internally and externally static. Below we will see how the accessibility of this disjunction is explained in DTS. The meaning of the disjunctive sentence is expressed by the disjoint union type in DTS, as described in Section 20.1. First of all, the syntactic structure and semantic composition of the (**??**) are as follows.

(650) Syntactic structure of (648)

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{Either}}{S/S/CONJ_S/S} \quad \dfrac{\text{a woman walked in}}{S}}{S\backslash S/CONJ_S}{>} \quad \dfrac{\text{or}}{CONJ_S}}{S/S}{>} \quad \dfrac{\text{she sat down}}{S}}{S}{>}$$

(651) Semantic composition of (648)

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{Either}}{\lambda p.\lambda c.\lambda q.c(p)(q)} \quad \dfrac{\text{a woman walked in}}{\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e_1}(\pi_1 u) \end{bmatrix}}}{\lambda c.\lambda q.c\left(\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e}(\pi_1 u) \end{bmatrix}\right)(q)}> \quad \dfrac{\text{or}}{\mathbf{or}_S}}{\lambda q.\mathbf{or}_S\left(\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e}(\pi_1 u) \end{bmatrix}\right)(q)}> \quad \dfrac{\text{she sat down}}{\begin{bmatrix} v@ \begin{bmatrix} y : \mathsf{e} \\ \mathbf{female}(y) \end{bmatrix} \\ \mathbf{satDown}^*_{e_2}(\pi_1 v) \end{bmatrix}}}{\begin{array}{c} \mathbf{or}_S\left(\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e}(\pi_1 u) \end{bmatrix}\right)\left(\begin{bmatrix} v@ \begin{bmatrix} y : \mathsf{e} \\ \mathbf{female}(y) \end{bmatrix} \\ \mathbf{satDown}^*_{e}(\pi_1 v) \end{bmatrix}\right) \\ \equiv \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e}(\pi_1 u) \end{bmatrix} + \begin{bmatrix} v@ \begin{bmatrix} y : \mathsf{e} \\ \mathbf{female}(y) \end{bmatrix} \\ \mathbf{satDown}^*_{e}(\pi_1 v) \end{bmatrix} \end{array}}>$$

Since this semantic representation is a disjoint union type of the form $A + B$, the semantic representation $A$ cannot serve as an assumption to prove the well-formedness of $B$. This is the explanation for the fact that the disjunction is internally static. The semantic composition process of the first sentence of (649a) is as follows. The syntactic structure is almost identical to that of (648), so it is omitted here.

(652) Semantic composition of (649a)

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{Either}}{\lambda p.\lambda c.\lambda q.c(p)(q)} \quad \dfrac{\text{a woman walked in}}{\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e}(\pi_1 u) \end{bmatrix}}}{\lambda c.\lambda q.c\left(\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e}(\pi_1 u) \end{bmatrix}\right)(q)}> \quad \dfrac{\dfrac{\text{or}}{\mathbf{or}_{\boldsymbol{T}}}}{\mathbf{or}_S}{\scriptstyle \forall E}}{\lambda q.\mathbf{or}_S\left(\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e}(\pi_1 u) \end{bmatrix}\right)(q)}> \quad \dfrac{\text{John sat down}}{\mathbf{satDown}^*_e(john)}}{\begin{array}{c} \mathbf{or}_S\left(\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e}(\pi_1 u) \end{bmatrix}\right)(\mathbf{satDown}^*_e(john)) \\ \equiv \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e}(\pi_1 u) \end{bmatrix} + \mathbf{satDown}^*_e(john) \end{array}}>$$

Therefore, the semantic representation of the discourse (649a) is as follows.

(653) $$\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_{e}(\pi_1 u) \end{bmatrix} + \mathbf{satDown}^*_e(john); \begin{bmatrix} v@ \begin{bmatrix} y : \mathsf{e} \\ \mathbf{female}(y) \end{bmatrix} \\ \mathbf{smile}^*_{e}(\pi_1 v) \end{bmatrix}$$

$$\twoheadrightarrow_\beta \begin{bmatrix} w : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_e(\pi_1 u) \end{bmatrix} + \mathbf{satDown}^*_e(john) \\ v@\begin{bmatrix} y : \mathsf{e} \\ \mathbf{female}(y) \end{bmatrix} \\ \mathbf{smile}^*_e(\pi_1 v) \end{bmatrix}$$

(654) Type check diagram of (649a)

$$\dfrac{\overset{\vdots}{\begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_e(\pi_1 u) \end{bmatrix} + \mathbf{satDown}^*_e(john) : \mathsf{type}} \qquad \dfrac{\overset{\mathcal{D}_1}{\begin{bmatrix} y : \mathsf{e} \\ \mathbf{female}(y) \end{bmatrix} : \mathsf{type}} \qquad \overset{\substack{w : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_e(\pi_1 u) \end{bmatrix} + \mathbf{satDown}^*_e(john) \\ \mathcal{D}_2}}{? : \begin{bmatrix} y : \mathsf{e} \\ \mathbf{female}(y) \end{bmatrix}} \qquad \mathcal{D}_3}{\begin{bmatrix} v@\begin{bmatrix} y : \mathsf{e} \\ \mathbf{female}(y) \end{bmatrix} \\ \mathbf{smile}^*_e(\pi_1 v) \end{bmatrix} : \mathsf{type}}{\scriptstyle (@)}}{\begin{bmatrix} w : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{woman}(x) \end{bmatrix} \\ \mathbf{walkIn}^*_e(\pi_1 u) \end{bmatrix} + \mathbf{satDown}^*_e(john) \\ v@\begin{bmatrix} y : \mathsf{e} \\ \mathbf{female}(y) \end{bmatrix} \\ \mathbf{smile}^*_e(\pi_1 v) \end{bmatrix} : \mathsf{type}}{\scriptstyle (\Pi F),1}$$

Here, the above proof diagram $\mathcal{D}_2$ is searched, which corresponds to looking for the antecedent of the second sentence of (649a) in *she*. The proof term $w$ has disjoinnt union type, and the existence of a female entity cannot be attributed from $V$ (because even if $A \to C$ can be shown, $A + B \to C$ cannot be shown).[*1] Therefore, we can see that a woman cannot be the antecedent of she. Almost the same argument (even if $B \to C$ can be shown, $A + B \to C$ cannot be shown) holds for (649b). This explains why the disjunction is externally static.

## 25.2 Disjunctive Antecedents

As far as the examples (649a)(649b) are concerned, disjunctions are externally static. However, it is known that under certain conditions, a pronoun in a subsequent sentence can reach an antecedent in disjunction. Let's look at the discourse (655). [*2]

(655) a. [[A professor] or [an assistant professor]]$^i$ will attend the meeting of the university board.

b. She$_i$ will report to the faculy.

We can read *she* in the second sentence of (655) to refer to the one, whether it is a professor or an assistant professor, who makes *will attend the meeting of the university*

[*1] Although it is difficult to show that $A + B \to C$ cannot be shown, I would like to present the following argument. Suppose $A \to C \vdash A + B \to C$ holds for any type $A, B, C$, then $\bot \to \bot \vdash \bot + \top \to \bot$ holds. Together with $\vdash \bot \to \bot$, $\vdash \top$, and $\top \vdash \bot + \top$, we have $\vdash \bot$. Therefore, we can say that if DTT is consistent, it is inconsistent with this result.

[*2] Example sentences are from the Groenendijk and Stokhof (1991)

*board* true. [*3]

More generally speaking, when the meaning of the first sentence is a disjunction of the form $A$ or $B$, it is known that if both $A$ and $B$ provide antecedents (in this case *a professor* and *an assistant professor*) and it is known that anaphoric link can be established from outside the disjunction if the pronoun (here *she*) can refer to both of them. We also give examples of anaphora within the same sentence, where a similar generalization applies. [*4].

(656) If Mary sees [[a horse] or [a pony]]$^i$, she waves to it$_i$.

**Remark 462.** The fact that she is NOT reachable to a woman in (649a), while she is reachable to a professor or an assistant professor in (655), means that the disjunction is neither externally static nor externally dynamic (or both), while the fact that she IS reachable to a professor or an assistant professor in (655) implies that the disjunction is neither externally static nor externally dynamic (or both). This shows that the static/dynamic dichotomy is not necessarily an appropriate concept for describing and classifying the properties of E-type anaphora.

Moreover, the notion of a pronoun antecedent, a pronoun antecedent, is no longer self-evident. In (655), is the antecedent of she a professor, an assistant professor, both, or either, but it is unclear which, or is it a professor or an assistant profeossor noun phrase?

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[*3] There seem to be two different situations that would make this reading true. First, a situation in which we know that a particular faculty member will be attending the meeting, but we do not know whether that faculty member's position is professor or assistant professor. Second, there is a situation where there is a professor and an assistant professor, and we know that one of them will attend the meeting, but we do not know which one will attend. The question of whether these two correspond to different "readings" is also the question of whether discourse (655) has two different semantic indications corresponding to these two. The semantic indications corresponding to the first "reading" and the second "reading" would be written as follows.

(1) a. $\begin{bmatrix} x : \mathrm{e} \\ \mathbf{prof}(x) + \mathbf{assitantProf}(x) \\ \mathbf{wAtMotUB}(x) \end{bmatrix}$

b. $\begin{bmatrix} x : \mathrm{e} \\ \mathbf{prof}(x) \\ \mathbf{wAtMotUB}(x) \end{bmatrix} + \begin{bmatrix} x : \mathrm{e} \\ \mathbf{assisttantProf}(x) \\ \mathbf{wAtMotUB}(x) \end{bmatrix}$

However, these two semantic indications are equivalent on DTT (an example of a distributive law that holds between $\Sigma$ type and disjoint union type, but the proof is an exercise), and thus there is no situation that makes one of these two readings true and the other false. This is not a situation unique to DTT, but is also true in FoL and dynamic semantics.

These two semantic indications have different anaphoric potential in many dynamic semantics other than DTS, such as DRT (Kamp and Reyle, 1993) and DPL (Groenendijk and Stokhof, 1991). That is, the former provides antecedents to the subsequent context, while the latter does not. Empirically, however, it would have to be explained that in both "readings" the second sentence of (655), *she*, is capable of participating in the anaphoric link as discussed above. It is also not obvious how the first semantic marking can be synthesized from the syntactic structure of (655).

Therefore, the position that there is only one semantic indication of (655) (and not two different readings) is favorable. The two situations described above do not imply the existence of two "readings," but rather two different pieces of evidence that make the semantic indication true.

[*4] Example sentence is by (Elbourne, 2011)

However, DTS can give the correct analysis of (655) without assuming anything other than the previous mechanism. The mechanism is anaphpra resolution using the disjunction elimination rule ($+E$), which states that if $A \to C$ and $B \to C$, then $A + B \to C$.

Let's look at the actual analysis. First, the syntactic structure and semantic composition of the two sentences of (655) are as follows. The internal structure of "will attend the meeting of the university board" is omitted.

(657) Syntactic structure of (655a)

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{A}}{NP^\star/N}\quad\dfrac{\text{professor}}{N}}{NP^\star}{>}\quad\dfrac{\text{or}}{CONJ_{NP^\star}}\quad\dfrac{\dfrac{\text{an}}{NP^\star/N}\quad\dfrac{\text{assistant professor}}{N}}{NP^\star}{>}}{NP^\star}{(\Phi)}\quad\dfrac{\text{will attend the meeting of the university board}}{(S\backslash NP)^\star}}{\dfrac{S^\star}{S}\checkmark}{>}$$

(658) Semantic composition of (655a)

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{A}}{\lambda n.\left[\begin{array}{l}u:\left[\begin{array}{l}x:\mathrm{e}\\ n(x)\end{array}\right]\\ \boxed{\pi_1 u}\end{array}\right]}\quad\dfrac{\text{professor}}{\mathbf{prof}}}{\left[\begin{array}{l}u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{prof}(x)\end{array}\right]\\ \boxed{\pi_1 u}\end{array}\right]}{>}\quad\dfrac{\text{or}}{\mathbf{or}_{NP^\star}}\quad\dfrac{\dfrac{\text{an}}{\lambda n.\left[\begin{array}{l}u:\left[\begin{array}{l}x:\mathrm{e}\\ n(x)\end{array}\right]\\ \boxed{\pi_1 u}\end{array}\right]}\quad\dfrac{\text{assistant professor}}{\mathbf{assistantProf}}}{\left[\begin{array}{l}u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{assistantProf}(x)\end{array}\right]\\ \boxed{\pi_1 u}\end{array}\right]}{>}}{\left[\begin{array}{l}u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{prof}(x)\end{array}\right]\\ \boxed{\pi_1 u}\end{array}\right]+\left[\begin{array}{l}u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{assistantProf}(x)\end{array}\right]\\ \boxed{\pi_1 u}\end{array}\right]}{(\Phi)}\quad\dfrac{\text{will attend the meeting of the university board}}{\boxed{\lambda x.\mathbf{wAtMotUB}^*_{e_1}(x)}}}{\dfrac{\left[\begin{array}{l}u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{prof}(x)\end{array}\right]\\ \boxed{\mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)}\end{array}\right]+\left[\begin{array}{l}u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{assistantProf}(x)\end{array}\right]\\ \boxed{\mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)}\end{array}\right]}{\left[\begin{array}{l}u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{prof}(x)\end{array}\right]\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{array}\right]+\left[\begin{array}{l}u:\left[\begin{array}{l}x:\mathrm{e}\\ \mathbf{assistantProf}(x)\end{array}\right]\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{array}\right]}\checkmark}{>}$$

(659) Syntactic structure of (655b)

$$\dfrac{\dfrac{\text{she}}{NP^\star}\quad\dfrac{\text{will report to the faculty}}{(S\backslash NP)^\star}}{\dfrac{S^\star}{S}\checkmark}{>}$$

(660) Semantic composition of (655b)

$$
\cfrac{\cfrac{\dfrac{\text{she}}{\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e}\\ \mathbf{female}(x)\end{bmatrix}\\ \boxed{\pi_1 u}\end{bmatrix}}\qquad\dfrac{\text{will report to the faculty}}{\boxed{\lambda x.\mathbf{wRttF}^*_{e_2}(x)}}}{\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e}\\ \mathbf{female}(x)\end{bmatrix}\\ \boxed{\mathbf{wRttF}^*_{e_2}(\pi_1 u)}\end{bmatrix}}\,>}{\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e}\\ \mathbf{female}(x)\end{bmatrix}\\ \mathbf{wRttF}^*_{e_2}(\pi_1 u)\end{bmatrix}}\,\checkmark
$$

(661) Progressive conjunction for (655b)

$$
\begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{prof}(x)\end{bmatrix}\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{bmatrix}+\begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{assistantProf}(x)\end{bmatrix}\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{bmatrix}\ ;\ \begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e}\\ \mathbf{female}(x)\end{bmatrix}\\ \mathbf{wRttF}^*_{e_1}(\pi_1 u)\end{bmatrix}
$$

$$
\equiv\begin{bmatrix} v:\begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{prof}(x)\end{bmatrix}\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{bmatrix}+\begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{assistantProf}(x)\end{bmatrix}\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{bmatrix}\\ u@\begin{bmatrix} x:\mathsf{e}\\ \mathbf{female}(x)\end{bmatrix}\\ \mathbf{wRttF}^*_{e}(\pi_1 u)\end{bmatrix}
$$

The semantic felicity condition is checked as follows.

(662) Type check diagram of (655)

$$
\cfrac{\begin{matrix}\vdots\\ \begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{prof}(x)\end{bmatrix}\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{bmatrix}+\\ \begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{assistantProf}(x)\end{bmatrix}\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{bmatrix}:\mathsf{type}\end{matrix}\qquad \cfrac{\begin{matrix}\vdots\\ \begin{bmatrix} x:\mathsf{e}\\ \mathbf{female}(x)\end{bmatrix}:\mathsf{type}\end{matrix}\quad \begin{matrix} v:\begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{prof}(x)\end{bmatrix}\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{bmatrix}+\begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{assistantProf}(x)\end{bmatrix}\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{bmatrix}\\ \mathcal{D}_1\\ ?:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{female}(x)\end{bmatrix}\end{matrix}\quad \mathcal{D}_2}{\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e}\\ \mathbf{female}(x)\end{bmatrix}\\ \mathbf{wRttF}^*_{e}(\pi_1 u)\end{bmatrix}:\mathsf{type}}\,(@)}{\begin{bmatrix} v:\begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{prof}(x)\end{bmatrix}\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{bmatrix}+\begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \mathbf{assistantProf}(x)\end{bmatrix}\\ \mathbf{wAtMotUB}^*_{e_1}(\pi_1 u)\end{bmatrix}\\ u@\begin{bmatrix} x:\mathsf{e}\\ \mathbf{female}(x)\end{bmatrix}\\ \mathbf{wRttF}^*_{e_2}(\pi_1 u)\end{bmatrix}:\mathsf{type}}\,(\Sigma F)
$$

The proof search for pronoun *she* is invoked in the proof diagram $\mathcal{D}_1$ construction. The key point is that the disjunction appears in the premise of the inference, from which the $(+E)$ rule can be deduced as follows.

(663) Proof diagram of (655a)

$$\dfrac{v : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{prof}(x) \end{bmatrix} \\ \mathbf{wAtM}^*_e(\pi_1 u) \end{bmatrix} + \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{aProf}(x) \end{bmatrix} \\ \mathbf{wAtM}^*_e(\pi_1 u) \end{bmatrix} \quad \mathcal{D}_P \quad \mathcal{D}_{N_1} \quad \mathcal{D}_{N_2}}{\mathbf{unpack}_v^{\lambda z.\begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix}}(\lambda w.(\pi_1\pi_1 w, \mathrm{pf}(\pi_1 w)), \lambda w.(\pi_1\pi_1 w, \mathrm{af}(\pi_1 w))) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix}}\ {\scriptstyle (+E)}$$

The following two functions are used for inference from $\pi_1 v$. They are the knowledge corresponding to *Every professor is female* and *Every assistant professor is female*, respectively.

(664) a. $\mathrm{pf} : \left(u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{prof}(x) \end{bmatrix}\right) \to \mathbf{female}(\pi_1 u)$

b. $\mathrm{af} : \left(u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{aProf}(x) \end{bmatrix}\right) \to \mathbf{female}(\pi_1 u)$

Also, the proof diagram of $\mathcal{D}_P$ is as follows.

(665) Proof diagram $\mathcal{D}_P \equiv$

$$\dfrac{\dfrac{\dfrac{}{\mathsf{e} : \mathsf{type}}\ {\scriptstyle (\{\}I)} \qquad \dfrac{\dfrac{}{\mathbf{female} : \mathsf{e} \to \mathsf{type}}\ {\scriptstyle (CON)} \qquad \dfrac{}{x : \mathsf{e}}^{1}}{\mathbf{female}(x) : \mathsf{type}}\ {\scriptstyle (\Pi E)}}{\begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix} : \mathsf{type}}\ {\scriptstyle (\Sigma F),1}}{\lambda x.\begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix} : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{prof}(x) \end{bmatrix} \\ \mathbf{wAtM}^*_e(\pi_1 u) \end{bmatrix} + \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{aProf}(x) \end{bmatrix} \\ \mathbf{wAtM}^*_e(\pi_1 u) \end{bmatrix} \to \mathsf{type}}\ {\scriptstyle (\Pi I)}$$

(666) Proof diagram $\mathcal{D}_{N_1} \equiv$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{w : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{prof}(x) \end{bmatrix} \\ \mathbf{wAtM}^*_e(\pi_1 u) \end{bmatrix}}^{1}}{\pi_1 w : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{prof}(x) \end{bmatrix}}\ {\scriptstyle (\Sigma E)}}{\pi_1\pi_1 w : \mathsf{e}}\ {\scriptstyle (\Sigma E)} \qquad \dfrac{\dfrac{\dfrac{}{w : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{prof}(x) \end{bmatrix} \\ \mathbf{wAtM}^*_e(\pi_1 u) \end{bmatrix}}^{1}}{\pi_1 w : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{prof}(x) \end{bmatrix}}\ {\scriptstyle (\Sigma E)} \qquad \mathrm{pf} : \begin{matrix} \left(u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{prof}(x) \end{bmatrix}\right) \\ \to \mathbf{female}(\pi_1 u) \end{matrix}}{\mathrm{pf}(\pi_1 w) : \mathbf{female}(\pi_1\pi_1 w)}\ {\scriptstyle (\Pi E)}}{(\pi_1\pi_1 w, \mathrm{pf}(\pi_1 w)) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix}}\ {\scriptstyle (\Sigma I),1}}{(\pi_1\pi_1 w, \mathrm{pf}(\pi_1 w)) : \left(\lambda z.\begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix}\right)(\iota_1(w))}\ {\scriptstyle (CONV)}}{\lambda w.(\pi_1\pi_1 w, \mathrm{pf}(\pi_1 w)) : \left(x : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{prof}(x) \end{bmatrix} \\ \mathbf{wAtM}^*_e(\pi_1 u) \end{bmatrix}\right) \to \left(\lambda z.\begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix}\right)(\iota_1(w))}\ {\scriptstyle (\Pi I),1}$$

(667) Proof diagram $\mathcal{D}_{N_2} \equiv$

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{}{w : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{aProf}(x) \end{bmatrix} \\ \mathbf{wAtM}^*_e(\pi_1 u) \end{bmatrix}}{}^{1}
}{\pi_1 w : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{aProf}(x) \end{bmatrix}}(\Sigma E)
}{\pi_1\pi_1 w : \mathsf{e}}(\Sigma E)
\quad
\dfrac{
\dfrac{
\dfrac{}{w : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{aProf}(x) \end{bmatrix} \\ \mathbf{wAtM}^*_e(\pi_1 u) \end{bmatrix}}{}^{1}
}{\pi_1 w : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{aProf}(x) \end{bmatrix}}(\Sigma E)
\quad
\mathsf{af} : \begin{array}{c} \left(u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{aProf}(x) \end{bmatrix}\right) \\ \to \mathbf{female}(\pi_1 u) \end{array}
}{\mathsf{af}(\pi_1 w) : \mathbf{female}(\pi_1\pi_1 w)}(\Pi E)
}{
\dfrac{(\pi_1\pi_1 w, \mathsf{af}(\pi_1 w)) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix}}{(\pi_1\pi_1 w, \mathsf{af}(\pi_1 w)) : \left(\lambda z. \begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix}\right)(\iota_2(w))}(CONV)
}(\Sigma I),1
$$

$$
\dfrac{(\pi_1\pi_1 w, \mathsf{af}(\pi_1 w)) : \left(\lambda z. \begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix}\right)(\iota_2(w))}{\lambda w.(\pi_1\pi_1 w, \mathsf{af}(\pi_1 w)) : \left(x : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{aProf}(x) \end{bmatrix} \\ \mathbf{wAtM}^*_e(\pi_1 u) \end{bmatrix}\right) \to \left(\lambda z. \begin{bmatrix} x : \mathsf{e} \\ \mathbf{female}(x) \end{bmatrix}\right)(\iota_2(w))}(\Pi I),1
$$

A crucial data exist to support this analysis: When $A \to C$ but not $B \to C$, $A + B$ does not provide a antecedent to $C$, as shown in (668).

(668) [A professor][1] attended the meeting or the meeting was cancelled.

# She$_1$ will report to the faculty.

Although *A professor* can be denoted by *she* in (668), an anaphoric link cannot be established because there is no entity that can be denoted by *she* in the disjunction's antecedent. It should be emphasized that, in (668), there is no reading in which *she* links only to *A professor*.

To begin with, static/dynamic is defined as the property of reachability of the antecedent and pronoun. In this case, can we really assume that the antecedent is the disjunctive NP *a professor or an assistant professor*?

This is made more evident by the following example. There is a fact that the antecedent in $A$ can be referred to by assuming $\neg B$ in the second sentence, even in the first sentence with a similar disjunctive structure $A + B$, as shown in (669).

(669) (Rothschild, 2017)

a. Either it's a holiday or [a customer][1] will come in.

b. And if it's not a holiday, they$_1$ will want to be served.

In this case, can we say that the antecedent is *a customer*? (669) is highly problematic for dynamic semantics, but DTS allows for a straightforward analysis.

(670) Syntactic structure of (669a)

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{\dfrac{\text{Either}}{\boldsymbol{T}/\boldsymbol{T}/CONJ_{\boldsymbol{T}}/\boldsymbol{T}}}{S/S/CONJ_S/S}\forall E
\quad
\dfrac{\text{it's a holiday}}{S}
}{S/S/CONJ_S}>
\quad
\dfrac{\dfrac{\text{or}}{CONJ_{\boldsymbol{T}}}}{CONJ_S}\forall E
}{S/S}>
\quad
\dfrac{\text{a customer will come in}}{S}
}{S}>
$$

(671) Semantic composition of (669a)

$$
\dfrac{
\dfrac{
\dfrac{\dfrac{\text{Either}}{\lambda p.\lambda c.\lambda q.c(p)(q)}}{\lambda p.\lambda c.\lambda q.c(p)(q)}\,\forall E
\qquad
\dfrac{\text{it's a holiday}}{\lambda k.\begin{bmatrix} s:\mathsf{e}\\ \textbf{holiday}(s,today)\\ k(s)\end{bmatrix}}
}{\lambda c.\lambda q.c\left(\lambda k.\begin{bmatrix} s:\mathsf{e}\\ \textbf{holiday}(s,today)\\ k(s)\end{bmatrix}\right)(q)}>
\qquad
\dfrac{\dfrac{\text{or}}{\mathbf{or}_{\boldsymbol{T}}}}{\mathbf{or}_S}\,\forall E
}{\lambda q.(\mathbf{or}_S)\left(\lambda k.\begin{bmatrix} s:\mathsf{e}\\ \textbf{holiday}(s,today)\\ k(s)\end{bmatrix}\right)(q)}>
$$

$$
\dfrac{
\lambda q.(\mathbf{or}_S)\left(\lambda k.\begin{bmatrix} s:\mathsf{e}\\ \textbf{holiday}(s,today)\\ k(s)\end{bmatrix}\right)(q)
\qquad
\dfrac{\text{a customer will come in}}{\lambda k.\begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \textbf{customer}(x)\end{bmatrix}\\ e:\mathsf{e}\\ \textbf{WCI}(e,\pi_1 u)\\ k(e)\end{bmatrix}}
}{(\mathbf{or}_S)\left(\lambda k.\begin{bmatrix} s:\mathsf{e}\\ \textbf{holiday}(s,today)\\ k(s)\end{bmatrix}\right)\left(\lambda k.\begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \textbf{customer}(x)\end{bmatrix}\\ e:\mathsf{e}\\ \textbf{WCI}(e,\pi_1 u)\\ k(e)\end{bmatrix}\right)}>
$$

$$
\equiv \lambda k.\left(\begin{bmatrix} s:\mathsf{e}\\ \textbf{holiday}(s,today)\\ k(s)\end{bmatrix} + \begin{bmatrix} u:\begin{bmatrix} x:\mathsf{e}\\ \textbf{customer}(x)\end{bmatrix}\\ e:\mathsf{e}\\ \textbf{WCI}(e,\pi_1 u)\\ k(e)\end{bmatrix}\right)
$$

(672) Syntactic structure of (669b)

$$
\dfrac{
\dfrac{\dfrac{\text{if}}{S/S/S}\qquad\dfrac{\text{it's not a holiday}}{S}}{S/S}>
\qquad
\dfrac{\dfrac{\dfrac{\text{they}}{\boldsymbol{T}/(\boldsymbol{T}\backslash NP)}}{S/(S\backslash NP)}\,\forall E\qquad\dfrac{\text{will want to be served}}{S\backslash NP}}{S}>
}{S}>
$$

(673) Semantic composition of (669b)

$$
\dfrac{
\dfrac{\dfrac{\text{if}}{\lambda q.\lambda p.\lambda k.q(\lambda e.\top)\to p(k)}\qquad\dfrac{\text{it's not a holiday}}{\lambda k.\neg\begin{bmatrix} s:\mathsf{e}\\ \textbf{holiday}(s,today)\\ k(s)\end{bmatrix}}}{\lambda p.\lambda k.\neg\textbf{holiday}^*_e(today)\to p(k)}>
\qquad
\dfrac{
\dfrac{\dfrac{\text{they}}{\left|\lambda p.\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e}\\ \textbf{human}(x)\end{bmatrix}\\ p(\pi_1 u)\end{bmatrix}\right|^q_{\boldsymbol{T}}}}{\lambda p.\lambda k.\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e}\\ \textbf{human}(x)\end{bmatrix}\\ p(\pi_1 u)k\end{bmatrix}}\,\forall E
\qquad
\dfrac{\text{will want to be served}}{\lambda x.\lambda k.\begin{bmatrix} e:\mathsf{e}\\ \textbf{WWtbS}(e,x)\\ k(e)\end{bmatrix}}
}{\lambda k.\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e}\\ \textbf{human}(x)\end{bmatrix}\\ e:\mathsf{e}\\ \textbf{WWtbS}(e,\pi_1 u)\\ k(e)\end{bmatrix}}>
}{\lambda k.\neg\textbf{holiday}^*_e(today)\to\begin{bmatrix} u@\begin{bmatrix} x:\mathsf{e}\\ \textbf{human}(x)\end{bmatrix}\\ e:\mathsf{e}\\ \textbf{WWtbS}(e,\pi_1 u)\\ k(e)\end{bmatrix}}>
$$

(674)
$$\mathbf{holiday}^*_e(\mathit{today}) + \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{customer}(x) \end{bmatrix} \\ \mathbf{WCI}^*_e(\pi_1 u) \end{bmatrix} ; \neg\mathbf{holiday}^*_e(\mathit{today}) \to \begin{bmatrix} u@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{human}(x) \end{bmatrix} \\ \mathbf{WWtbS}^*_e(\pi_1 u) \end{bmatrix}$$
$$\equiv \begin{bmatrix} v : \left( \mathbf{holiday}^*_e(\mathit{today}) + \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{customer}(x) \end{bmatrix} \\ \mathbf{WCI}^*_e(\pi_1 u) \end{bmatrix} \right) \\ \neg\mathbf{holiday}^*_e(\mathit{today}) \to \begin{bmatrix} u@\begin{bmatrix} x : \mathsf{e} \\ \mathbf{human}(x) \end{bmatrix} \\ \mathbf{WWtbS}^*_e(\pi_1 u) \end{bmatrix} \end{bmatrix}$$

(675) Proof diagram $\mathcal{D}_P \equiv$

$$\dfrac{\dfrac{\dfrac{}{\mathsf{e} : \mathsf{type}}\,(\{\}I) \qquad \dfrac{\dfrac{}{\mathbf{human} : \mathsf{e} \to \mathsf{type}}\,(CON) \qquad \dfrac{}{x : \mathsf{e}}{}^{1}}{\mathbf{human}(x) : \mathsf{type}}\,(\Pi E)}{\begin{bmatrix} x : \mathsf{e} \\ \mathbf{human}(x) \end{bmatrix} : \mathsf{type}}\,(\Sigma F),1}{\lambda z.\begin{bmatrix} x : \mathsf{e} \\ \mathbf{human}(x) \end{bmatrix} : \mathbf{holiday}^*_e(\mathit{today}) + \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{customer}(x) \end{bmatrix} \\ \mathbf{WCI}^*_e(\pi_1 u) \end{bmatrix} \to \mathsf{type}}\,(\Pi I)$$

(676) Proof diagram $\mathcal{D}_{N_1} \equiv$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{h : \mathbf{holiday}^*_e(\mathit{today})}{}^{1} \qquad w : \neg\mathbf{holiday}^*_e(\mathit{today})}{w(h) : \bot}\,(\Pi E)}{\mathsf{case}_M(w(h)) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{human}(x) \end{bmatrix}}\,(EFQ)}{\mathsf{case}_M(w(h)) : \left(\lambda z.\begin{bmatrix} x : \mathsf{e} \\ \mathbf{human}(x) \end{bmatrix}\right)(\iota_1(h))}\,(CONV)}{\lambda h.\mathsf{case}_M(w(h)) : (h : \mathbf{holiday}^*_e(\mathit{today})) \to \left(\lambda z.\begin{bmatrix} x : \mathsf{e} \\ \mathbf{human}(x) \end{bmatrix}\right)(\iota_1(h))}\,(\Pi I),1$$

(677) Proof diagram $\mathcal{D}_{N_2} \equiv$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{h : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{customer}(x) \end{bmatrix} \\ \mathbf{WCI}^*_e(\pi_1 u) \end{bmatrix}}{}^{1}}{\pi_1 h : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{customer}(x) \end{bmatrix}}\,(\Sigma E)}{\pi_1\pi_1 h : \mathsf{e}}\,(\Sigma E) \qquad \dfrac{\dfrac{\dfrac{}{h : \begin{bmatrix} u : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{customer}(x) \end{bmatrix} \\ \mathbf{WCI}^*_e(\pi_1 u) \end{bmatrix}}{}^{1}}{\pi_1 h : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{customer}(x) \end{bmatrix}}\,(\Sigma E)}{\mathsf{ch}(\pi_1 h) : \mathbf{human}(\pi_1\pi_1 h)}\,(\Pi E)}{(\pi_1\pi_1 h, \mathsf{ch}(\pi_1 h)) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{human}(x) \end{bmatrix}}\,(\Sigma I)}{(\pi_1\pi_1 h, \mathsf{ch}(\pi_1 h)) : \left(\lambda z.\begin{bmatrix} x : \mathsf{e} \\ \mathbf{human}(x) \end{bmatrix}\right)(\iota_1(h))}\,(CONV)}{\lambda h.(\pi_1\pi_1 h, \mathsf{ch}(\pi_1 h)) : \left(h : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{customer}(x) \\ \mathbf{WCI}^*_e(x) \end{bmatrix}\right) \to \left(\lambda z.\begin{bmatrix} x : \mathsf{e} \\ \mathbf{human}(x) \end{bmatrix}\right)(\iota_1(h))}\,(\Pi I),1$$