# SOME USEFUL FORMULAS

$$\Phi_X(u)=\int_{-\infty}^{\infty} f_X(x)\, e^{iux}dx \qquad f_X(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_X(iu)\, e^{-iux}du$$

$$S_X(\omega)=\int_{-\infty}^{\infty} R_X(\tau)e^{-i\omega\tau}d\tau \qquad R_X(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)e^{i\omega\tau}d\omega$$

$$\sum_{n=0}^{\infty} r^n=\frac{1}{(1-r)},\ |r|<1 \qquad \sum_{n=1}^{m} r^n = r\frac{1-r^m}{1-r} \qquad \sum_{n=1}^{\infty} n r^n=\frac{r}{(1-r)^2},\ |r|<1$$

$$\sum_{n=m}^{\infty} r^n=\frac{r^m}{1-r},\ |r|<1 \qquad (A+B)^n=\sum_{k=0}^{n}\frac{n!}{k!(n-k)!}A^{n-k}B^k$$

$$\sum_{k=0}^{n} k\frac{n!}{k!(n-k)!}p^k q^{n-k}=np,\ 0<p<1;\ p+q=1$$

$$2\cos A\cos B=\cos(A-B)+\cos(A+B)$$

$$2\sin A\sin B=\cos(A-B)-\cos(A+B)$$

$$2\sin A\cos B=\sin(A-B)+\sin(A+B)$$

I $$\int_a^b e^{iux}\left\{\frac{1}{b-a}\right\}dx=e^{iu(b+a)/2}\,\frac{\sin u(b-a)/2}{u(b-a)/2}$$

II $$\int_0^{\infty} e^{iux}\left\{\frac{\lambda(\lambda x)^n}{n!}e^{-\lambda x}\right\}dx=\left(\frac{\lambda}{\lambda-iu}\right)^{n+1} \qquad \int_0^{\infty}\alpha^n e^{-\alpha}\,d\alpha=n!$$

III $$\int_{-\infty}^{\infty} e^{iux}\left\{\frac{1}{\sqrt{2\pi}\ \sigma}e^{-(x-m)^2/2\sigma^2}\right\}dx=e^{ium-u^2\sigma^2/2}$$

**Transform Tables** **Probability Equivalents**

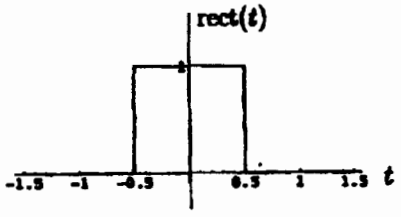
$$\int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt = F(\omega) \qquad t \to x \;\; \omega \to -u \qquad \int_{-\infty}^{+\infty} f(x)e^{iux}dx = \Phi_X(u)$$

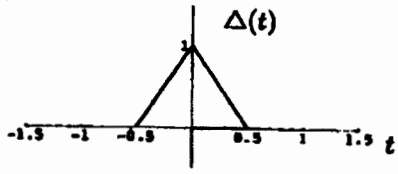
$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{i\omega t}d\omega = f(t) \qquad \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Phi_X(u)e^{-iux}du = f(x)$$

# FOURIER TRANSFORM PAIRS

| | $f(t) \leftrightarrow F(\omega)$ | | |
|---|---|---|---|
| 1 | $e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega}, a>0$ | 14 | $\delta(t) \leftrightarrow 1$ |
| 2 | $e^{at}u(-t) \leftrightarrow \frac{1}{a-j\omega}, a>0$ | 15 | $1 \leftrightarrow 2\pi\delta(\omega)$ |
| 3 | $e^{-a\lvert t\rvert} \leftrightarrow \frac{2a}{a^2+\omega^2}, a>0$ | 16 | $\delta(t-t_o) \leftrightarrow e^{-j\omega t_o}$ |
| 4 | $\frac{a^2}{a^2+t^2} \leftrightarrow \pi a e^{-a\lvert\omega\rvert}, a>0$ | 17 | $e^{j\omega_o t} \leftrightarrow 2\pi\delta(\omega-\omega_o)$ |
| 5 | $te^{-at}u(t) \leftrightarrow \frac{1}{(a+j\omega)^2}, a>0$ | 18 | $\cos(\omega_o t) \leftrightarrow \pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$ |
| 6 | $t^n e^{-at}u(t) \leftrightarrow \frac{n!}{(a+j\omega)^{n+1}}, a>0$ | 19 | $\sin(\omega_o t) \leftrightarrow j\pi[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]$ |
| 7 | $\text{rect}(\frac{t}{\tau}) \leftrightarrow \tau\text{sinc}(\frac{\omega\tau}{2})$ | 20 | $\cos(\omega_o t)u(t) \leftrightarrow \frac{\pi}{2}[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)] + \frac{j\omega}{\omega_o^2-\omega^2}$ |
| 8 | $\text{sinc}(Wt) \leftrightarrow \frac{\pi}{W}\text{rect}(\frac{\omega}{2W})$ | 21 | $\sin(\omega_o t)u(t) \leftrightarrow j\frac{\pi}{2}[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)] + \frac{\omega_o}{\omega_o^2-\omega^2}$ |
| 9 | $\Delta(\frac{t}{\tau}) \leftrightarrow \frac{\tau}{2}\text{sinc}^2(\frac{\omega\tau}{4})$ | 22 | $\text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$ |
| 10 | $\text{sinc}^2(\frac{Wt}{2}) \leftrightarrow \frac{2\pi}{W}\Delta(\frac{\omega}{2W})$ | 23 | $u(t) \leftrightarrow \pi\delta(\omega)+\frac{1}{j\omega}$ |
| 11 | $e^{-at}\sin(\omega_o t)u(t) \leftrightarrow \frac{\omega_o}{(a+j\omega)^2+\omega_o^2}, a>0$ | 24 | $\sum_{n=-\infty}^{\infty}\delta(t-nT) \leftrightarrow \frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-n\frac{2\pi}{T})$ |
| 12 | $e^{-at}\cos(\omega_o t)u(t) \leftrightarrow \frac{a+j\omega}{(a+j\omega)^2+\omega_o^2}, a>0$ | 25 | $\sum_{n=-\infty}^{\infty}f(t)\delta(t-nT) \leftrightarrow \sum_{n=-\infty}^{\infty}\frac{1}{T}F(\omega-n\frac{2\pi}{T})$ |
| 13 | $e^{-\frac{t^2}{2\sigma^2}} \leftrightarrow \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$ | | |

$$\text{sinc}(t) \equiv \frac{\sin(t)}{t}$$



# FOURIER TRANSFORM RELATIONSHIPS

| | Name: | Condition: | Property: |
|---|---|---|---|
| 1 | Amplitude scaling | $f(t) \leftrightarrow F(\omega)$, constant $K$ | $Kf(t) \leftrightarrow KF(\omega)$ |
| 2 | Addition | $f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$, $\cdots$ | $f(t) + g(t) + \cdots \leftrightarrow F(\omega) + G(\omega) + \cdots$ |
| 3 | Hermitian | Real $f(t) \leftrightarrow F(\omega)$ | $F(-\omega) = F^*(\omega)$ |
| 4 | Even | Real and even $f(t)$ | Real and even $F(\omega)$ |
| 5 | Odd | Real and odd $f(t)$ | Imaginary and odd $F(\omega)$ |
| 6 | Symmetry | $f(t) \leftrightarrow F(\omega)$ | $F(t) \leftrightarrow 2\pi f(-\omega)$ |
| 7 | Time scaling | $f(t) \leftrightarrow F(\omega)$ , real $s$ | $f(st) \leftrightarrow \frac{1}{\lvert s \rvert} F(\frac{\omega}{s})$ |
| 8 | Time shift | $f(t) \leftrightarrow F(\omega)$ | $f(t - t_o) \leftrightarrow F(\omega)e^{-j\omega t_o}$ |
| 9 | Frequency shift | $f(t) \leftrightarrow F(\omega)$ | $f(t)e^{j\omega_o t} \leftrightarrow F(\omega - \omega_o)$ |
| 10 | Modulation | $f(t) \leftrightarrow F(\omega)$ | $f(t)\cos(\omega_o t) \leftrightarrow \frac{1}{2}F(\omega - \omega_o) + \frac{1}{2}F(\omega + \omega_o)$ |
| 11 | Time derivative | Differentiable $f(t) \leftrightarrow F(\omega)$ | $\frac{df}{dt} \leftrightarrow j\omega F(\omega)$ |
| 12 | Freq derivative | $f(t) \leftrightarrow F(\omega)$ | $-jtf(t) \leftrightarrow \frac{d}{d\omega}F(\omega)$ |
| 13 | Time convolution | $f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$ | $f(t) * g(t) \leftrightarrow F(\omega)G(\omega)$ |
| 14 | Freq convolution | $f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$ | $f(t)g(t) \leftrightarrow \frac{1}{2\pi}F(\omega) * G(\omega)$ |
| 15 | Compact form | Real $f(t)$ | $f(t) = \frac{1}{2\pi}\int_0^\infty 2\lvert F(\omega)\rvert \cos(\omega t + \angle F(\omega))d\omega$ |
| 16 | Parseval, Energy W | $f(t) \leftrightarrow F(\omega)$ | $W \equiv \int_{-\infty}^{\infty} \lvert f(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert F(\omega)\rvert^2 d\omega$ |