ECE 250 –Random Processes

Homework 2

1. The joint probability of the random variables X and Y is given by

$$P(X=n, Y=m) = \frac{\alpha^m \beta^{n-m}}{m!(n-m)!} e^{-(\alpha+\beta)} \qquad \begin{matrix} m = 0, 1, 2\ldots \\ n = m, m+1, \ldots \end{matrix}$$

   Evaluate P (X = n) and P (Y = m). Are X and Y independent?

2. The discrete random variables X and Y have the joint probability mass function

$$P(X=n\ , Y=m) = (1/6)(2n+m) \qquad \text{n = 0, 1 and m = 0, 1.}$$

   Evaluate the correlation and covariance of X and Y.

3. Let Θ be a uniform random variable with density

$$f_\Theta(\vartheta) = \begin{cases} \frac{1}{2\pi}, & 0 \le \vartheta \le 2\pi \\ 0, & otherwise. \end{cases}$$

   Define $X = cos\Theta$ and $Y = sin\Theta$. Show that X and Y are uncorrelated but not independent.

4. The random variables X and Y have the joint density

$$f_{X,Y}(x,y) = \begin{cases} (1/8)(x+y), & 0 \le x \le 2,\ 0 \le y \le 2 \\ 0, & otherwise. \end{cases}$$

   Evaluate the conditional densities

$$f_{X|Y}(x|y) \quad \text{and} \quad f_{Y|X}(y|x).$$

5. The random variables X and Y are independent with probabilities

   P (X = n) = $(1/2)^{n+1}$ and P(Y = m) = $(1/2)^{m+1}$, m, n=0, 1…

   Evaluate the probability P (X = Y).

6. Let X and Y be independent random variables with densities

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \ge 0 \\ 0, & x < 0 \end{cases} \qquad f_Y(y) = \begin{cases} \mu e^{-\mu y}, & y \ge 0 \\ 0, & y < 0 \end{cases}$$

   where λ and μ are positive constants. Evaluate the probability density of Z = X + Y if $\mu \ne \lambda$ and if $\mu = \lambda$.