ECE 250 –Random Processes

Homework 3

1. The process Z(t) is given by $Z(t) = X\cos\omega t + Y\sin\omega t.$

   Here X and Y are independent random variables with identical densities

   $$f_X(x) = \begin{cases} 1-|x|, & |x| \le 1 \\ 0, & otherwise \end{cases} \quad ; \quad f_Y(y) = \begin{cases} 1-|y|, & |y| \le 1 \\ 0, & otherwise. \end{cases}$$

   Is Z(t) wide sense stationary? Is it strict sense stationary?

2. Let X_k, k = 1, 2, ... be independent, identically-distributed random variables with

   $E[X_k] = 0$ and $E[X_k^2] = 1$.

   A discrete-time process is defined by

   $$Y_n = \sum_{k=1}^{n} X_k$$

   Is Y_n wide sense stationary? Is it strict sense stationary?

3. A zero-mean random process X(t) has correlation function

   $R_X(t, s) = \min[s, t]$.

   Evaluate the correlation function of $Y(t) = e^t X(e^{-2t})$.

4. The random variable T has the distribution $F_T(\tau)$. A random process is defined by

   $$X(t) = u(t-T) \quad \text{where} \quad u(t) = \begin{cases} 1, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

   Obtain expressions for the mean and correlation function of X(t) in terms of the distribution of T.

5. N(t) is a classical Poisson process (N (0) = 0, independent increments, constant rate λ). Evaluate

   Cov [N(t) N(s)].

6. Define

   $$M_T = \frac{2}{T^2}\int_0^T N(t)dt$$

   where N(t) is a classical Poisson process (N (0) = 0, independent increments, constant rate λ). Show that

   $\lim_{T\to\infty} P(|M_T - \lambda| \ge \varepsilon) = 0, \quad \varepsilon > 0.$

   [HINT: You may find the result of problem 5 helpful.]