250 –Random Processes

Homework 4

1. The input, X(t), and output, Y(t), of a linear system are related via the differential equation

$$\frac{d^2}{dt^2}Y(t)+2\frac{d}{dt}Y(t)+Y(t)=2\frac{d}{dt}X(t)+2X(t).$$

If the input is a zero-mean, wide sense stationary process with correlation

$$R_X(\tau)=(3/4)e^{-2|\tau|},$$

determine the correlation function of Y(t).

2. The input, X(t), and output, Y(t), of a system are related via the equation

$$\mathrm{Y(t) = X(t) +} \int_{-\infty}^{\infty} h(t-\alpha)X(\alpha)\,d\alpha.$$

Here h(t) = $\begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$ and $R_X(\tau)=e^{-2|\tau|}$

Determine the correlation function of Y(t).

3. Let $R(\tau)=\dfrac{\sin \pi\tau}{\pi\tau}$ be the correlation function of a real, wide sense stationary process.

For a fixed $T_o > 0$ consider the function

$$\hat{R}(\tau)=R(\tau-T_o)+R(\tau)+R(\tau+T_o).$$

Can $\hat{R}(\tau)$ be the correlation function of a real wide sense stationary process?

4. The random telegraph signal is defined by

$$X(t)=X(0)(-1)^{N(t)},\quad t\geq 0$$

where N(t) is a classical Poisson process (N (0) = 0, independent increments, constant rate λ).
Evaluate the conditional expectation

$$E[X(t)\,|\,X(s)=1],\quad t\geq s.$$

5. Let X(t) be a random telegraph signal

$$X(t) = X(0)(-1)^{N(t)}$$

N(t) is a classical Poisson process (N (0) = 0, independent increments, constant rate = λ) and P (X (0) = 1) = (1/2). This is used to modulate the phase of a sinusoidal carrier

$$Y(t) = \cos(\omega_o t + \phi - \frac{\pi}{2} X(t)).$$

Here ϕ is a random phase, uniformly distributed in the interval [0, 2π] and independent of X(t).

Show that Y(t) is wide sense stationary and evaluate its power spectral density.

6. A random process is defined by $X(t) = u(Z - t)$ where

$$u(t) = \begin{cases} 0\ , & t < 0 \\ 1\ , & t \geq 0 \end{cases}$$

and Z is a random variable with distribution

$$F_Z(z) = \begin{cases} 1 - e^{-z}, & z \geq 0 \\ 0, & z < 0. \end{cases}$$

It is desired to estimate a future value of X(t) with its value at an earlier time. Determine the value of λ that will minimize the mean square error

$$\mathcal{E} = E[(X(t_o) - \lambda X(t))^2], \quad t_o > t > 0.$$