

ECE 250 –Random Processes

Homework 1

1. The random variable X has the density

$$f_X(x) = \begin{cases} \alpha e^{-\alpha x}, & x \geq 0 \\ 0, & x < 0. \end{cases}$$

A new random variable is given by $Y = e^{-X}$. Obtain an expression for the n-th moment of Y.

2. Can the following function be the characteristic function of a probability density?

$$G(u) = \frac{1 + iu}{1 + 4u^2}$$

3. The random variable X has the probabilities

$$P(X = n) = (1/2)^n, \quad n = 1, 2, \dots$$

Evaluate the mean and variance of X.

4. Determine the probability density of a random variable X whose moments are given by

$$E[X^n] = \mu^n, \quad n = 0, 1, 2, \dots \quad \mu > 0.$$

5. The independent random variables X and Y have $E[X] = E[Y] = 0$ and $\text{Var}[X] = \text{Var}[Y] = 1$.

Prove that $P(|X-Y| \geq 10) \leq 1/50$.

6. The Poisson variable X has the probabilities

$$P(X = n) = \frac{\lambda^n}{n!} e^{-\lambda}, \quad n = 0, 1, \dots$$

A new random variable Y is defined as follows

$$Y = \begin{cases} 1, & X = \text{even} \\ -1, & X = \text{odd}. \end{cases}$$

Evaluate the probabilities $P(Y = 1)$ and $P(Y = -1)$.