SOEN 6481

Repository Details:

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URL: https://github.com/DaitaHemanth/SOEN6481

Liouville's Constant Introduction

Liouville 's constant, sometimes also called Liouville's number, is the real number defined by

$L = \sum_{n=1}^{\infty} 10^{-n!} = 0.110001000000000000000001.......$

Liouville's constant is a decimal fraction with a 1 in each decimal place corresponding to a factorial n! and zeros everywhere else. Liouville constructed an infinite class of transcendental numbers using continued fractions, but the above number was the first decimal constant to be proven transcendental. However, Cantor subsequently proved that "almost all" real numbers are in fact transcendental. Liouville's constant nearly satisfies

$10x^6 - 75x^3 - 190x + 21 = 0,$

which has solution 0.1100009999..., but plugging x= L into this equation gives -0.0000000059 instead of 0.

The existence of Liouville numbers (Liouville's constant):

Characteristics of Liouville's constant:

Here we show that Liouville numbers exist by exhibiting a construction that produces such numbers. For any integer b 2, and any sequence of integers (a1, a2, . . . ,), such that ak 0, 1, 2, . . . , b 1 k 1, 2, 3, . . . and there are infinitely many k with ak 0, define the number

$x = \sum_{k=1}^{\infty} \frac{a_k}{b^{k!}}$

In the special case when b = 10, and ak = 1, k, the resulting number x is called Liouville's constant:

L = 0.1100010000000000000000010000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000001...
It follows from the definition of x that its base-b representation is

$$x = (0.a_1a_2000a_3 00000000000000000 a_4 00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000 a_5 \ldots)$$

where the nth term is separated from the next term by (nn!-1) zeros.

Since this base-b representation is non-repeating it follows that x cannot be rational. Therefore, for any rational number p/q, we have |x - p/q | >0.

Now, for any integer $n \geq 1$, define q_n and p_n as follows:

$$q_n = b^{n!}; \quad p_n = q_n \sum_{k=1}^{n} \frac{a_k}{b^{k!}} = \sum_{k=1}^{n} a_k b^{n!-k!}.$$

Then

$$0 < \left| x - \frac{p_n}{q_n} \right| = \left| x - \frac{q_n \sum_{k=1}^{n} \frac{a_k}{b^{k!}}}{q_n} \right| = \left| x - \sum_{k=1}^{n} \frac{a_k}{b^{k!}} \right| = \left| \sum_{k=1}^{\infty} \frac{a_k}{b^{k!}} - \sum_{k=1}^{n} \frac{a_k}{b^{k!}} \right| = \left| \left(\sum_{k=1}^{n} \frac{a_k}{b^{k!}} + \sum_{k=n+1}^{\infty} \frac{a_k}{b^{k!}} \right) - \sum_{k=1}^{n} \frac{a_k}{b^{k!}} \right| = \sum_{k=n+1}^{\infty} \frac{a_k}{b^{k!}}$$

$$\leq \sum_{k=n+1}^{\infty} \frac{b-1}{b^{k!}} < \sum_{k=(n+1)!}^{\infty} \frac{b-1}{b^{k}} = \frac{b-1}{b^{(n+1)!}} + \frac{b-1}{b^{(n+1)!+1}} + \frac{b-1}{b^{(n+1)!+2}} + \ldots = \frac{b-1}{b^{(n+1)!}b^{0}} + \frac{b-1}{b^{(n+1)!}b^{1}} + \frac{b-1}{b^{(n+1)!}b^{2}} + \ldots = \frac{b-1}{b^{(n+1)!}} \sum_{k=0}^{\infty} \frac{1}{b^{k}}$$

$$= \frac{b-1}{b^{(n+1)!}} \cdot \frac{b}{b-1} = \frac{b}{b^{(n+1)!}} \leq \frac{b^{n!}}{b^{(n+1)!}} = \frac{1}{b^{(n+1)!-n!}} = \frac{1}{b^{(n+1)n!-n!}} = \frac{1}{b^{n(n!)+n!-n!}} = \frac{1}{b^{(n!)n}} = \frac{1}{{q_n}^{n}}$$

Therefore, we conclude that any such x is a Liouville number.

Establishing that a given number is a Liouville number provides a useful tool for proving a given number is transcendental. However, not every transcendental number is a Liouville number. The terms in the continued fraction expansion of every Liouville number are unbounded; using a counting argument, one can then show that there must be uncountably many transcendental numbers which are not Liouville. Using the explicit continued fraction expansion of e, one can show that e is an example of a transcendental number that is not Liouville.

The proof proceeds by first establishing a property of irrational algebraic numbers. This property essentially says that irrational algebraic numbers cannot be well approximated by rational numbers, where the condition for "well approximated" becomes more stringent for larger denominators. A Liouville number is irrational but does not have this property, so it can't be algebraic and must be transcendental.

Interview Details

Interviewee:Vedant Saini

Interviewer:Venkata Sai Parthasarathi Hemanth

- Interviewer: Do you have any idea about Liouville's constant?
  Interviewee: it is a constant represented as $L = \sum_{n=1}^{\infty} 10^{-n!}$.

- Interviewer: Are there any uses of this constant practically?
  Interviewee: I know that by establishing that a given number is a Liouville number, it provides a useful tool for proving a given number is transcendental.

- Interviewer: Is there anything interesting to know about this number?
  Interviewee: Yes. The number was the first ever decimal constant to be proven transcendental.

- Interviewer: How frequently is this constant used in daily life?
  Interviewee: This is not used frequently in everyday scenarios.

- Interviewer: What would you like the calculator size to be?
  Interviewee: I would like if the calculator could fit in my palm so that it is easy to hold.

- Interviewer: Would you prefer having the basic operators at the same place or at random places?
  Interviewee: Since these operators are frequently used in calculations, I would want these operators to be in the same place.

- Interviewer: What additional button/buttons would you prefer in the calculator?
  Interviewee: It would be helpful if there is a history button so that I can track the operations made.

  Interview Analysis: Vedant was clear in what he wanted and the way he wanted to use the calculator. He wanted the calculator to be according to his comforts. He wants to calculate the Liouville's constant using the calculator. Vedant wants all the basic and frequently used operator to be in the same place.

PERSONA DETAILS

Private Information

1) Name: Vedant Saini
2) Occupation: Embedded Software Engineer
3) Qualification: Pursued Masters in Electrical And Computer Engineering
4) University: McGill University, Montreal, Canada
Vedant lives in Toronto with his family.
Vedant is a professional football player and he loves singing.

Use of louville's constant

1) Vedant doesn't use Louville's number in his workplace or daily life.

2) Their significance is relatively esoteric mathematical theory which really has no practical use.

Vedant's work or daily life

1) Vedant completed his Masters in Electrical and Computer Engineering.

2) Vedant does not have a fair knowledge about the Liouville's Constant.

3) If he finds anything related to the Liouville's number in or outside the workplace, he tries to find the usage and applications of the number.

Other uses or relations to the Number

1) According to Vedant, by establishing that a given number is a liouville number, it provides a useful tool for proving a given number is transcendental.

2) There are no practical uses of this Liouville's number.

Influencers that surround the persona and that may influence choices

1) Team mates

2) Friends

3) Managers

Domain Model

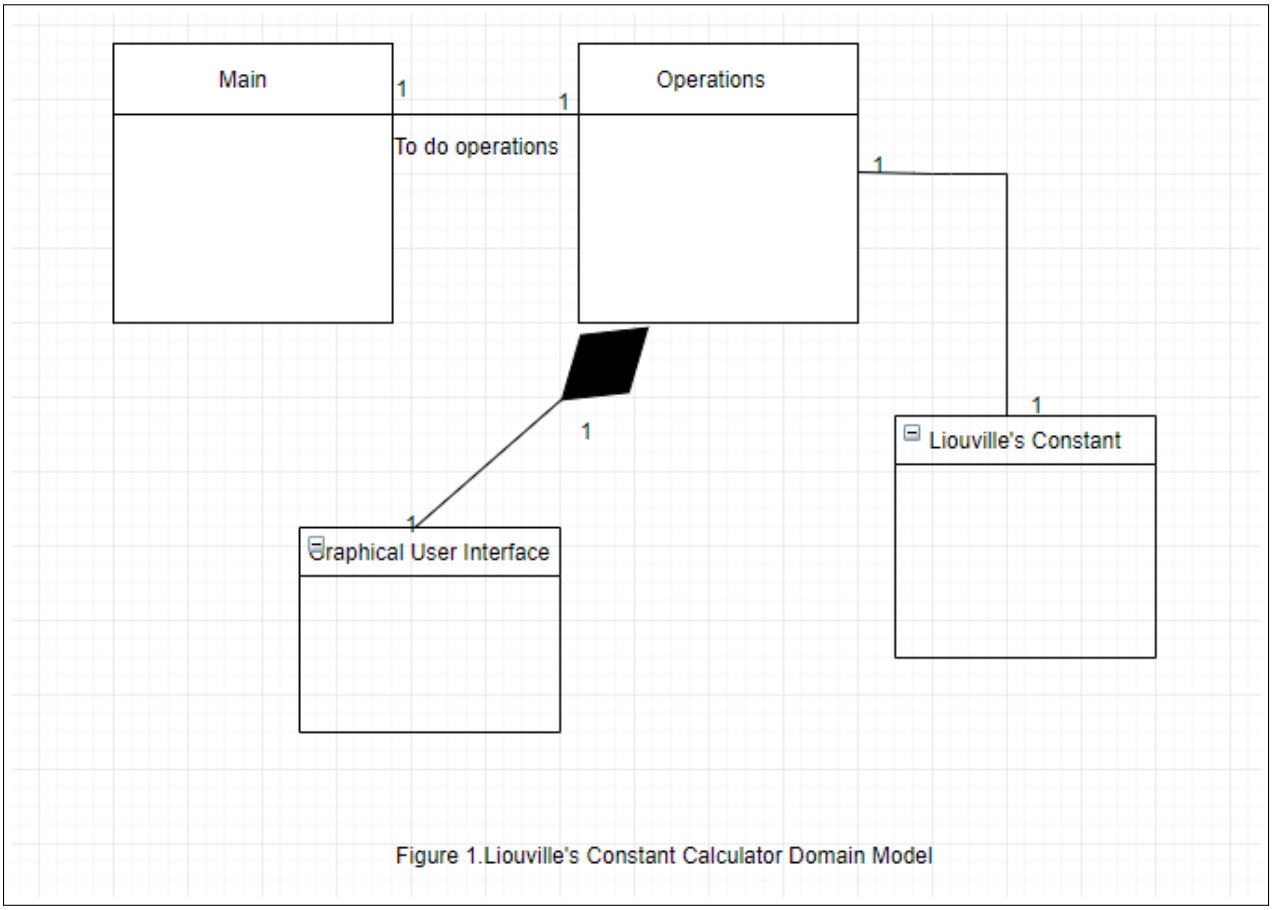
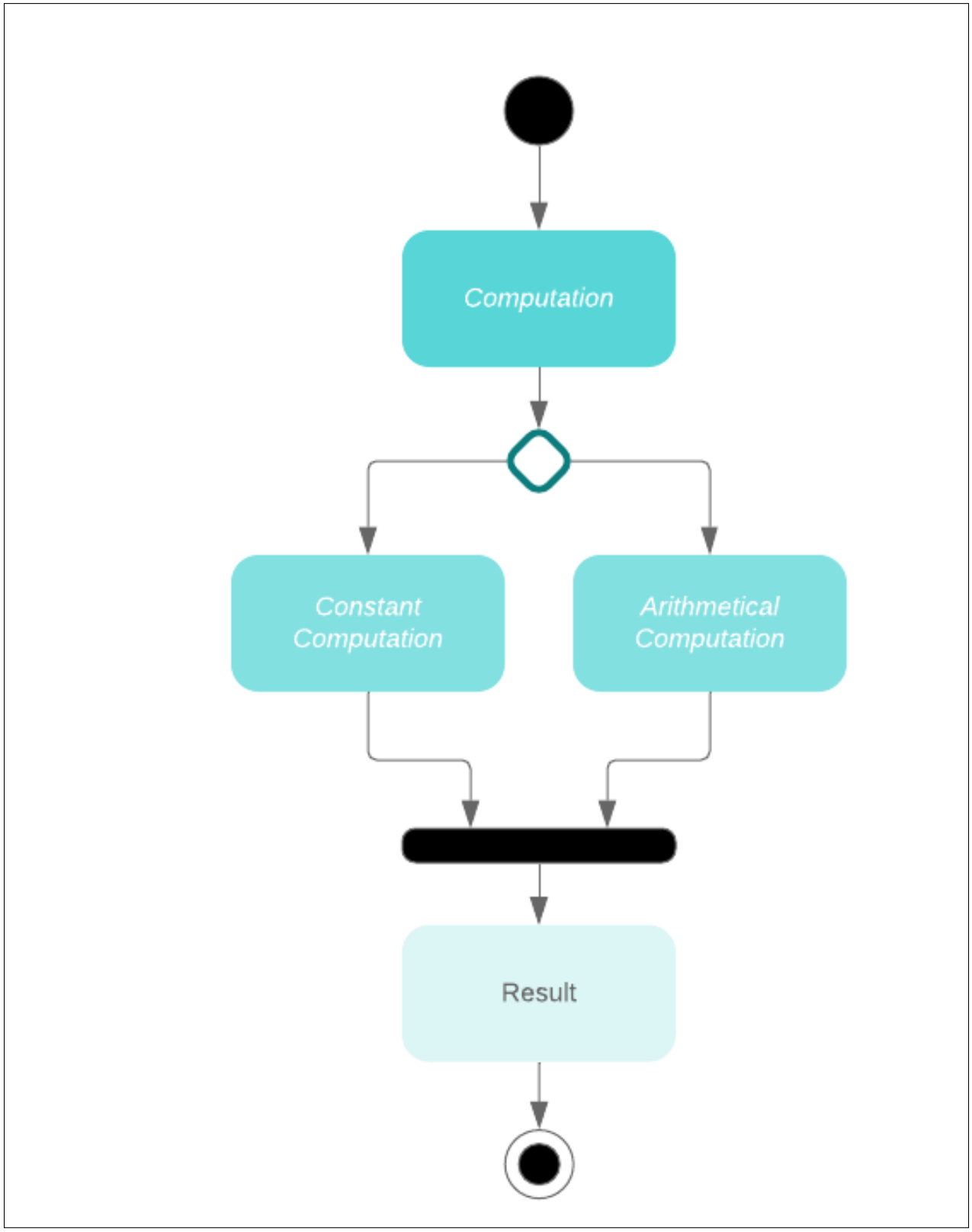


Figure 1.Liouville's Constant Calculator Domain Model

The above domain model shows functioning of the Calculator which does the basic operations addition,subtraction,division,multiplication etc. It also provides the liouville's constant value when the user needs it.

It displays the results using Graphical User Interface.

Activity Diagram



Use Case Diagram

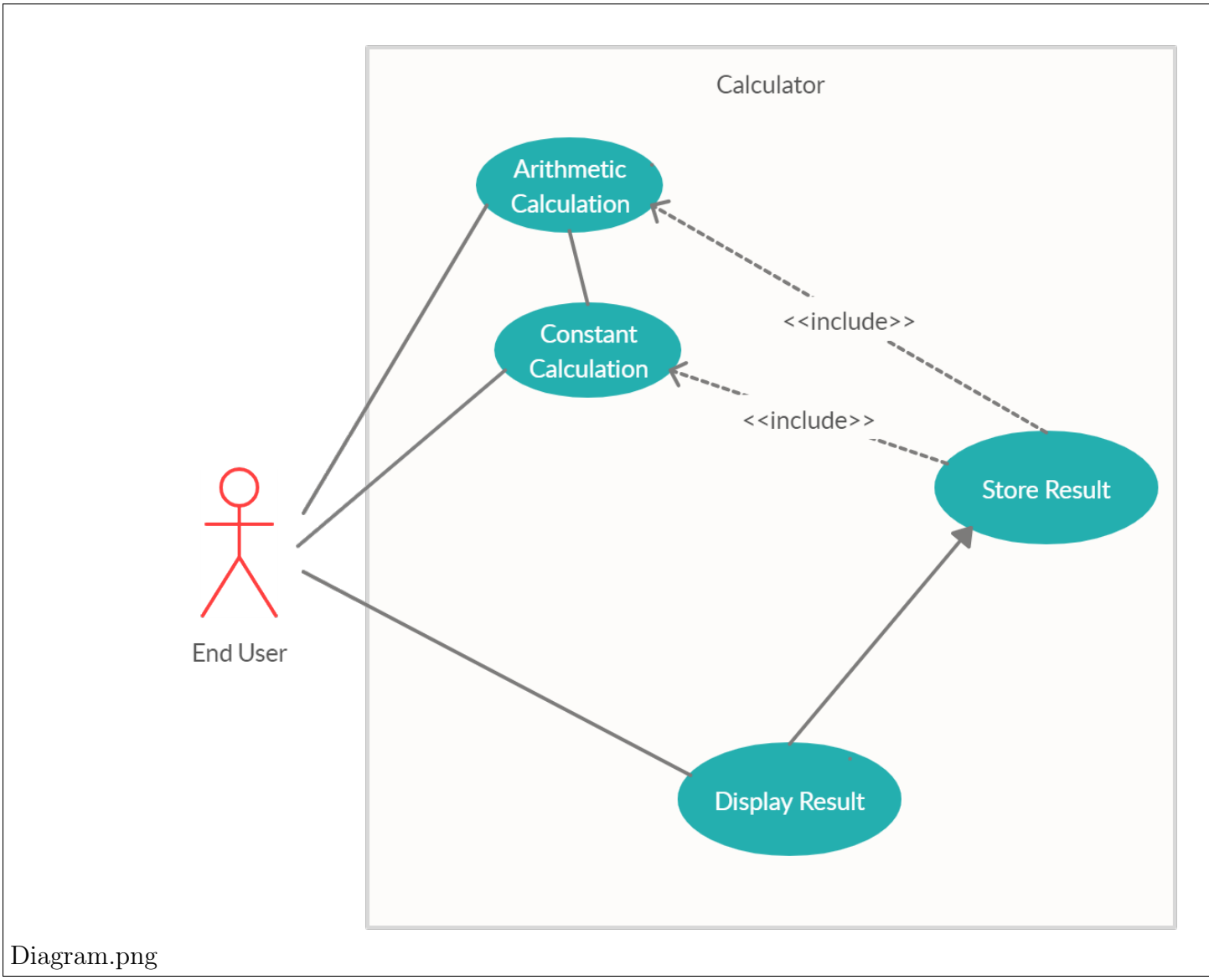


Diagram.png

The End User uses the calculator to do operations. The End user can know the liouville's constant by entering the number. After the calculations are completed the end result is displayed.

User Story: Input of Operator

Story ID	US1
Description	As a user, I would like to perform basic arithmetic operations in the calculator so that I can see the results(Multiplication, addition, division, subtraction).
Priority	High
Story Points	5
Constraints	

Story ID	US2
Description	As a user, I would like to have Liouville's constant button/key(Whenever the constant key is pressed, it asks the user to enter the number for which the user would like to see the liouville's number generated).
Priority	Very High
Story Points	8
Constraints	The calculator displays the Liouville's number upto 5040 decimal digits(i.e., generates for n value between 1 and 7,7!=5040)

Story ID	US3
Description	As a user, I would like to have a natural logarithmic key which calculates and displays the natural log value of input given/number.
Priority	Medium
Story Points	3
Constraints	

Story ID	US4
Description	As a user, I would like to have a memory button so that it keeps a track of the value it displayed previously. given/number.
Priority	Medium
Story Points	3
Constraints	May store 1 value.

Story ID	US5
Description	As a user, I would like to include a clickable element/button with name All Clear so that I can clear the values at any time.
Priority	Medium
Story Points	3
Constraints	

Story ID	US6
Description	As a user, I would like my calculator to have 10 clickable elements/buttons with one number on each from 0-9 which helps me in entering numbers for any operation.
Priority	Medium
Story Points	3
Constraints	

Story ID	US7
Description	As a user, I would like my calculator to provide a button which checks if the entered number is a transcendental number or not. A number is entered for liouville's calulation. The generated sequence is then check if it's matching with liouville number.
Priority	High
Story Points	5
Constraints	Minimum number to be entered is 3 for liouville sequence generation or a decimal sequence(say 0.110001) should be entered and check for transcendental check.

Backward Trace ability Matrix for User Stories

	Use Case	**Interview**	**Surveys**	**Global**	**Persona**
US1-	Arithmetic Calculation	X			
US2- Liouville's Constant	Constant Calculation	X			
US3-	Arithmetic Calculation	X			
US4-	Display Result,Arithmetic Calculation	X			
US5-Clear functionality	Arithmetic Calculation		X	X	
US6-Clickable Elements	Arithmetic Calculation	X			
US7-Application	Arithmetic Calculation	X			X

References :

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https://en.wikipedia.org/wiki/Liouville_number
https://www.hotjar.com/blog/user-personas
https://www.instructables.com/id/How-to-Make-a-Simple-Calculator-in-Java/