Optimization Models in Engineering—Homework 1

1. The Fruit Computer company produces two types of computers: Pear computers and Apricot computers. The following table shows the number of hours and the number of chips needed to make a computer as well as the equipment cost and selling price:

Computer	Labor	Chips	Equipment cost ($)	Selling price ($)
Pear	1 hour	2	50	400
Apricot	2 hours	5	100	900

   A total of 3000 chips and 1200 hours of labor are available. The company needs to decide how many computers from each type should be made in order to maximize the profit. Formulate this problem as an optimization problem.

2. The Paradise City Police Department employs 28 police officers. Each officer works 5 days per week. The crime rate fluctuates with the day of the week, so the minimum number of police officers required each day depends on which day of the week it is: Saturday, 24; Sunday, 13; Monday, 15; Tuesday, 25; Wednesday, 26; Thursday, 18; Friday, 22. The police department wants to schedule police officers to minimize the number of officers whose days off are not consecutive. Formulate this problem as an optimization problem (hint: define the variable x_{ij}, where $i, j \in \{1, 2, ..., 7\}$ and $i \neq j$, to be the number of officers who are scheduled to not work on days i and j).

3. A taxi company has n taxies available and n customers to be picked up as soon as possible. For every $i, j \in \{1, ..., n\}$, if taxi i decides to pick up customer j, the amount of time (delay) to pick up the customer is d_{ij}. Each taxi is allowed to pick up only one customer. The goal is to assign each customer to a taxi so that the total delay (i.e., sum of the delays for all customers) is minimized. Formulate this assignment problem as an optimization problem.

4. Consider the set $\mathcal{S}$ defined as

   $$\mathcal{S} = \{x \in \mathbb{R}^3 | x_1 + 2x_2 + 3x_3 = 0, \;\; 3x_1 + 2x_2 + x_3 = 0\} \tag{1}$$

   Show that $\mathcal{S}$ is a subspace. Determine its dimension and find a basis for it.

5. Consider the set $\mathcal{P}$ defined as

   $$\mathcal{P} = \{x \in \mathbb{R}^3 | x_1 + 2x_2 + 3x_3 = 1\} \tag{2}$$

   Show that $\mathcal{P}$ is an affine set of dimension 2. To this end, express it as $x^{(0)} + \text{span}(x^{(1)}, x^{(2)})$, where $x^{(0)} \in \mathcal{P}$ and $x^{(1)}, x^{(2)}$ are linearly independent vectors.