INDIAN INSTITUTE OF TECHNOLOGY, PATNA

End Semester Examination 2022

Time: 3 hours | Simulation Lab(MC 503) | Full Marks : 50

Instructions

1. All questions are compulsory.
2. Here you are not supposed to use any R packages.

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1. Generate 100 samples for the unit modified weibull distribution when $(\alpha, \beta, \gamma) = (0.5, 1.2, 3)$ and CDF is give by

$$\text{CDF: } F_Y(y) = e^{\alpha \log y - \beta(-\log y)^\gamma};\ 0 < y < 1; \alpha, \beta, \gamma > 0.$$

Also, draw graph of survival function of given distribution for two sets of parameter $(\alpha, \beta, \gamma) = (0.2, 0.5, 5)$ and $(0.5, 1.2, 3)$.

[4+4]

2. Let X follows the two-parameter Weibull distribution with PDF is given by

$$f(x \mid \alpha, \beta) = \alpha\beta x^{\beta-1} e^{-\alpha x^\beta},\ x > 0,\ \alpha > 0,\ \beta > 0.$$

Find MLE, Bias and MSE of the parameter α and β when $\alpha = 2, \beta = 1.5$.

[4+2+2]

3. Generate progressive censored sample of Weibull distribution using the following algorithm

   (i) Consider value of n, m and $R = (R_1, R_2, \ldots, R_m)$ where $\sum_{i=1}^{m} R_i = n - m$.

   (ii) Generate m independent Uniform$(0, 1)$ observations $W_1, W_2, \ldots, W_m$.

   (iii) Generate $V_i = W_i^{1/(i+R_m+R_{m-1}+\cdots+R_{m-i+1})}$ for $i = 1, 2, \ldots, m$.

   (iv) We set $U_i = 1 - V_m V_{m-1} \ldots V_{m-i+1}$ for $i = 1, 2, \ldots, m$. Then $U_1, U_2, \ldots, U_m$, is the required progressive Type-II censored sample from the Uniform$(0, 1)$ distribution

   (v) Finally, we set $X_i = F^{-1}(U_i)$ for $i = 1, 2, \ldots, m$, where $F^{-1}(\cdot)$ is the inverse CDF of the distribution. Then $X_1, X_2, \ldots, X_m$, is the required progressive Type-II censored sample from the distribution $F(\cdot)$.

Consider $n = 50, m = 40, R = (10, 0^{*39})$ and the CDF of Weibull distribution is $F(x) = 1 - e^{\alpha x^\beta}$ when $\alpha = 2, \beta = 1.5$.

[10]

4. Consider a data represent the strength measured in GPA for single carbon fibers of $10mm$ in gauge lengths with sample size 63 and they are as follows:

1.901,	2.132,	2.203,	2.228,	2.257,	2.350,	2.361,	2.396,	2.397,
2.445,	2.454,	2.474,	2.518,	2.522,	2.525,	2.532,	2.575,	2.614,
2.616,	2.618,	2.624,	2.659,	2.675,	2.738,	2.740,	2.856,	2.917,
2.928,	2.937,	2.937,	2.977,	2.996,	3.030,	3.125,	3.139,	3.145,
3.220,	3.223,	3.235,	3.243,	3.264,	3.272,	3.294,	3.332,	3.346,
3.377,	3.408,	3.435,	3.493,	3.501,	3.537,	3.554,	3.562,	3.628,
3.852,	3.871,	3.886,	3.971,	4.024,	4.027,	4.225,	4.395,	5.020.

Find maximum likelihood estimates (MLEs) of unknown parameters α and λ based on given real data. Also, check the goodness-fit of given data set using generalized logistic distribution by applying the Chi-square test and K-S test.

$$\text{CDF: } F(x) = \left(1 + e^{-\lambda x}\right)^{-\alpha},\ -\infty < x < \infty,\ \alpha > 0, \lambda > 0.$$
$$\text{PDF: } f(x) = \alpha\lambda \left(1 + e^{-\lambda x}\right)^{-\alpha-1} e^{-\lambda x}.$$

[4+4+4]

5. Import data **imdb.csv** which is data related to movies. Find the solution of following questions.

[2+2+2+2+4]

(i). Find the correlation between the IMDB rating and Meta score.

(ii). Find the mean and variance of the IMDB rating of movies released in 2010.

(iii). Find the movies directed by Christopher_Nolan.

(iv). Find the top-10 voted movies.

(v). Draw a bar plot between the years 2001-2010 and the corresponding number of movies released in the years.

——— All the Best ———