Causal Discovery from Interventions

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causalcourse.com

Structural Interventions

Single-Node Interventions

Multi-Node Interventions

Parametric Interventions

Interventional Markov Equivalence

Miscellaneous Other Settings

Structural Interventions

Single-Node Interventions

Multi-Node Interventions

Parametric Interventions

Interventional Markov Equivalence

Miscellaneous Other Settings

Intervention in Two-Variable Setting

$A \rightarrow B$ or $A \leftarrow B$

Intervention in Two-Variable Setting

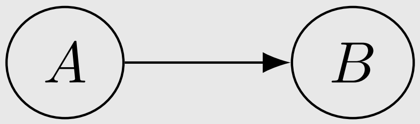
$A \rightarrow B$ or $A \leftarrow B$

Same Markov equivalence class / essential graph

$A - B$

Intervention in Two-Variable Setting

$$A := N_A$$
$$B := f(A, N_B)$$



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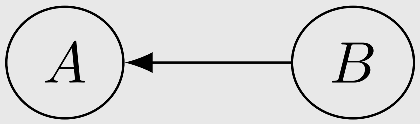
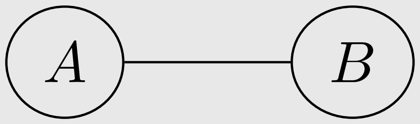


Same Markov equivalence class / essential graph



Intervention in Two-Variable Setting

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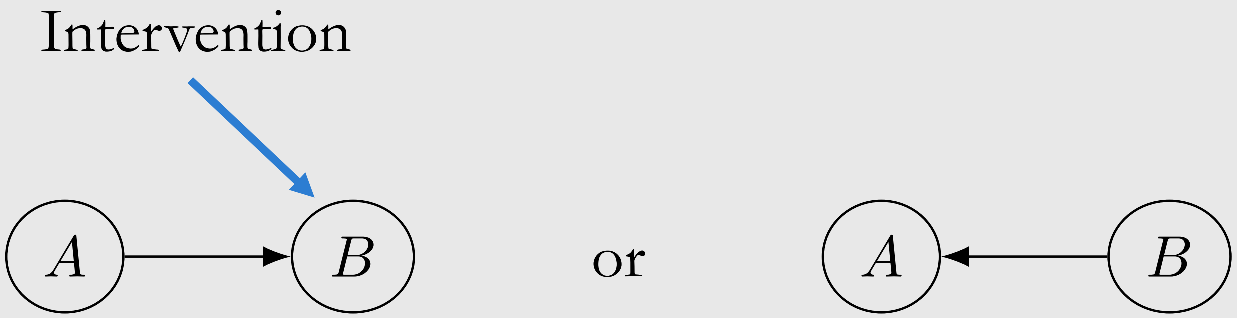

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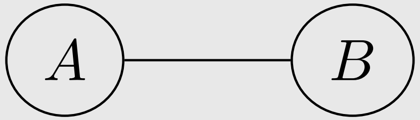

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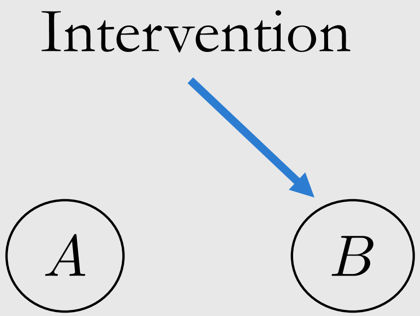

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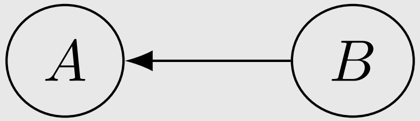
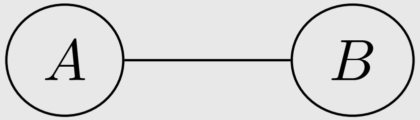


Same Markov equivalence class / essential graph



Intervention in Two-Variable Setting

$$A := N_A$$
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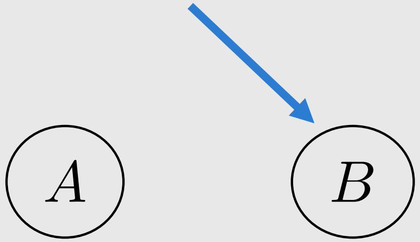
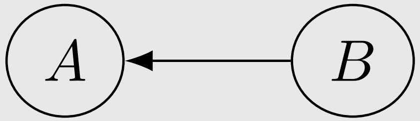
Intervention



Manipulated graph

or



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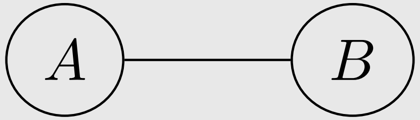
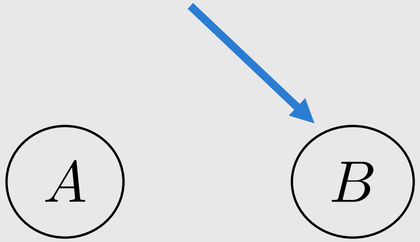


Intervention in Two-Variable Setting

$$A := N_A$$
$$\cancel{B := f(A, N_B)}$$
$$B := N'_B$$

Intervention



Manipulated graph

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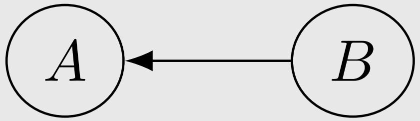

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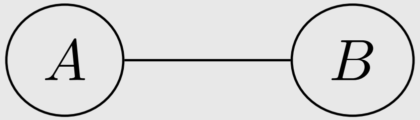

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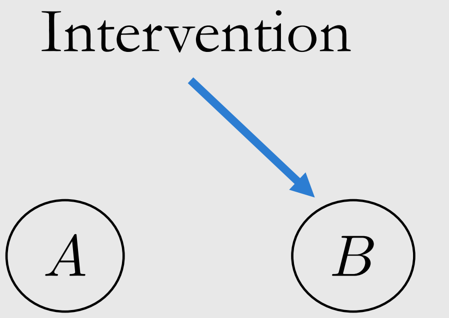
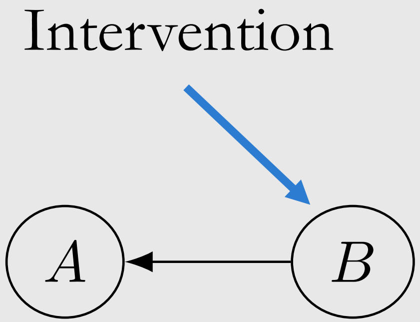


Manipulated graph

or



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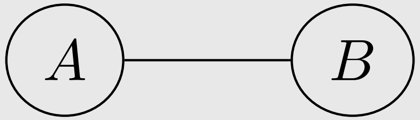

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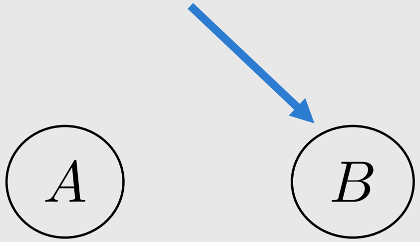

Manipulated graph

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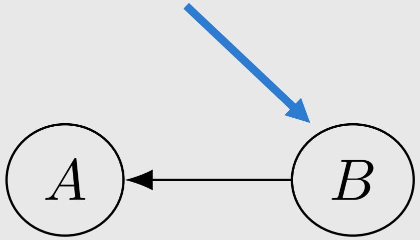
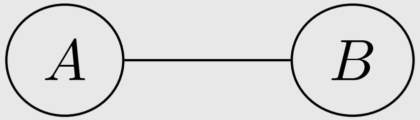


Manipulated graph

Same Markov equivalence class / essential graph



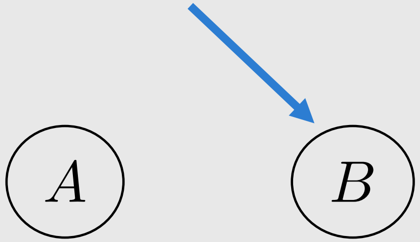
Intervention in Two-Variable Setting

$A := N_A$

$\cancel{B := f(A, N_B)}$

$B := N'_B$

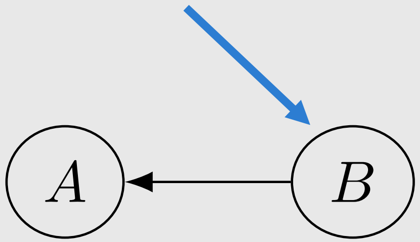
Intervention



Manipulated graph

or

Intervention

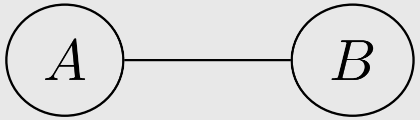


Manipulated graph

$B := N_B$

$A := f(B, N_A)$

Same Markov equivalence class / essential graph



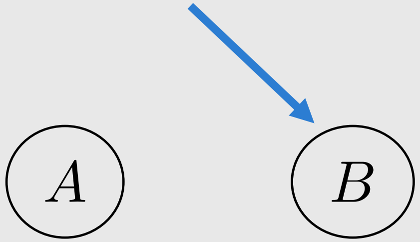
Intervention in Two-Variable Setting

$A := N_A$

$\cancel{B := f(A, N_B)}$

$B := N'_B$

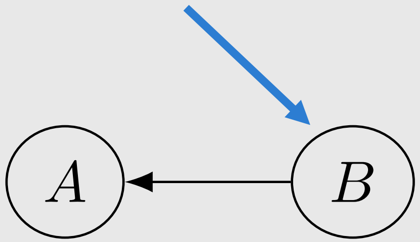
Intervention



Manipulated graph

or

Intervention



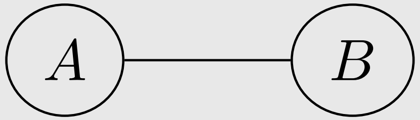
Manipulated graph

$B := N'_B$

$\cancel{B := N_B}$

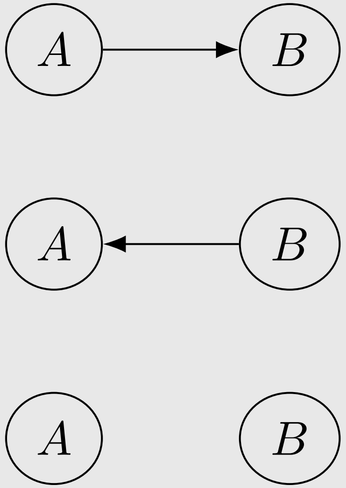
$A := f(B, N_A)$

Same Markov equivalence class / essential graph



Two Variables: Interventional Essential Graphs

True graph

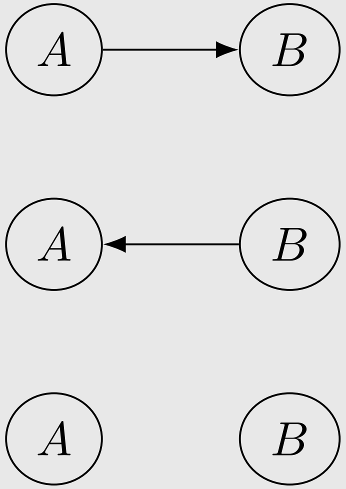


Two Variables: Interventional Essential Graphs

Interventional essential graphs

True graph

$I = \{A\}$ $\quad$ $I = \{B\}$ $\quad$ $I = \{\}$



Two Variables: Interventional Essential Graphs

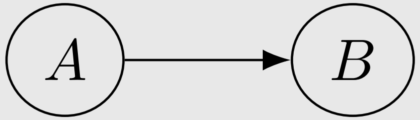
$$A \rightarrow B$$

Interventional essential graphs

True graph	I = {A}	I = {B}	I = {}
$A \rightarrow B$			
$A \leftarrow B$			
$A \qquad B$			

Two Variables: Interventional Essential Graphs

True graph under intervention I = {A}



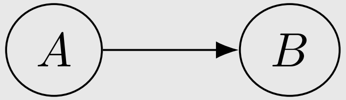
Interventional essential graphs

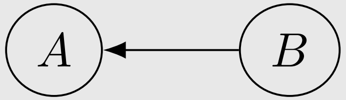
True graph

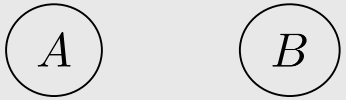
I = {A}

I = {B}

I = {}







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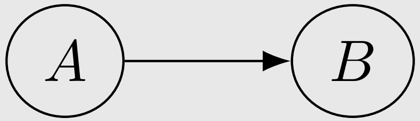
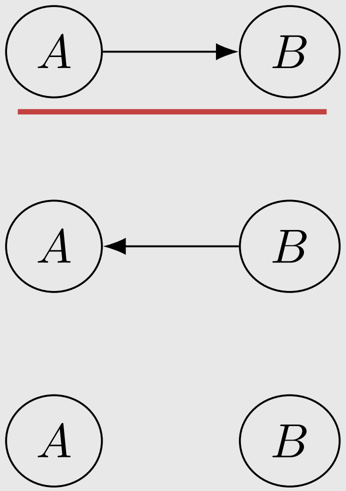


Interventional essential graphs

True graph

I = {A}   I = {B}   I = {}



Two Variables: Interventional Essential Graphs

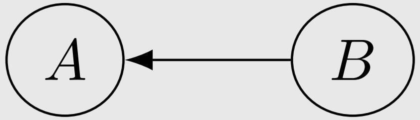
True graph under intervention $\mathrm{I} = \{\mathrm{A}\}$ $A \rightarrow B$

Interventional essential graphs

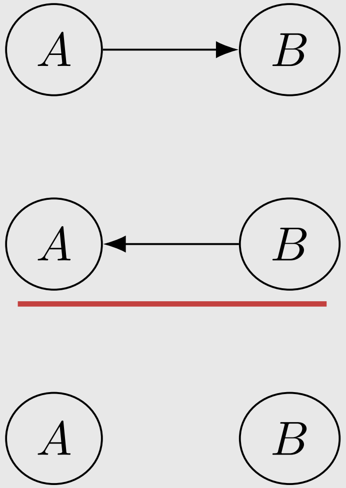
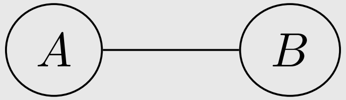
True graph	$\mathrm{I} = \{\mathrm{A}\}$	$\mathrm{I} = \{\mathrm{B}\}$	$\mathrm{I} = \{\}$
$A \rightarrow B$	$A - B$		
$A \leftarrow B$			
$A \quad B$			

Two Variables: Interventional Essential Graphs

True graph under intervention I = {A}

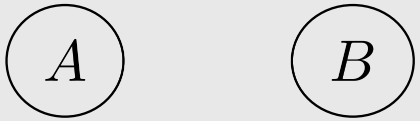


Interventional essential graphs

True graph	I = {A}	I = {B}	I = {}
			

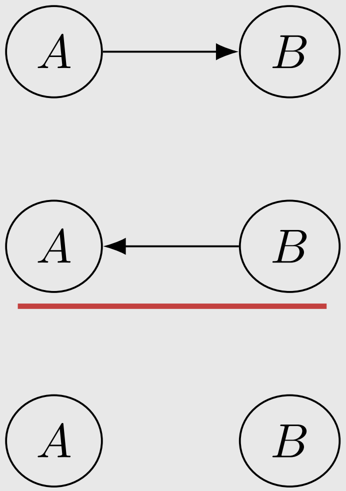
Two Variables: Interventional Essential Graphs

True graph under intervention $\mathrm{I} = \{\mathrm{A}\}$



Interventional essential graphs

True graph



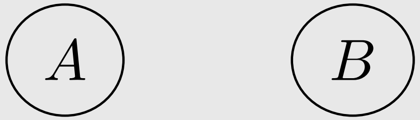
$\mathrm{I} = \{\mathrm{A}\}$

$\mathrm{I} = \{\mathrm{B}\}$

$\mathrm{I} = \{\}$

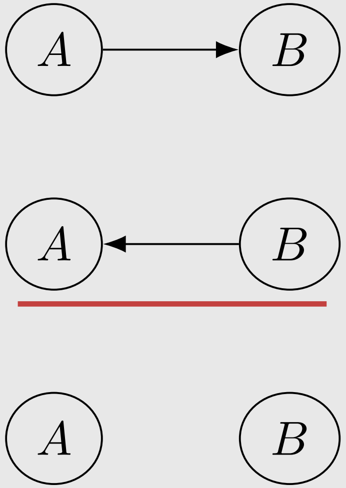
Two Variables: Interventional Essential Graphs

True graph under intervention $\mathrm{I} = \{\mathrm{A}\}$

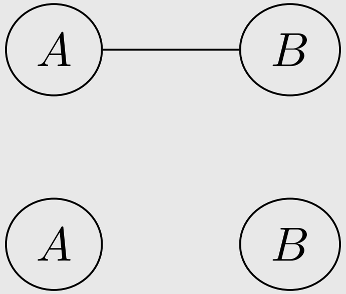


Interventional essential graphs

True graph



$\mathrm{I} = \{\mathrm{A}\}$

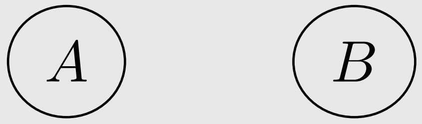


$\mathrm{I} = \{\mathrm{B}\}$

$\mathrm{I} = \{\}$

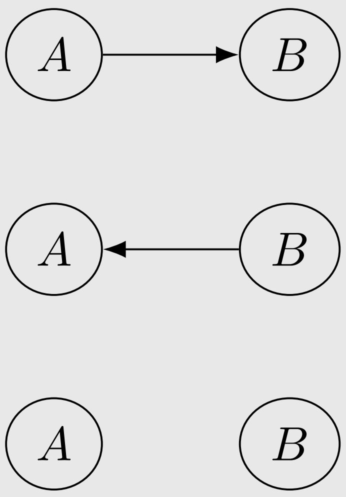
Two Variables: Interventional Essential Graphs

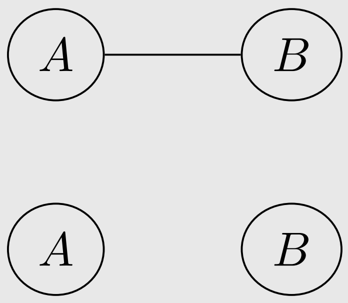
True graph under intervention I = {A}



Interventional essential graphs

True graph	I = {A}	I = {B}	I = {}





Two Variables: Interventional Essential Graphs

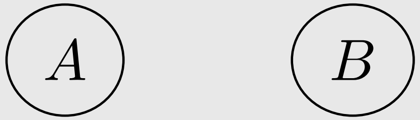
True graph under intervention $\mathrm{I} = \{\mathrm{A}\}$ $\quad A \qquad B$

Interventional essential graphs

True graph	$\mathrm{I} = \{\mathrm{A}\}$	$\mathrm{I} = \{\mathrm{B}\}$	$\mathrm{I} = \{\}$
$A \rightarrow B$	$A - B$		
$A \leftarrow B$	$A \quad B$		
$A \quad B$			

Two Variables: Interventional Essential Graphs

True graph under intervention $\mathrm{I} = \{\mathrm{A}\}$



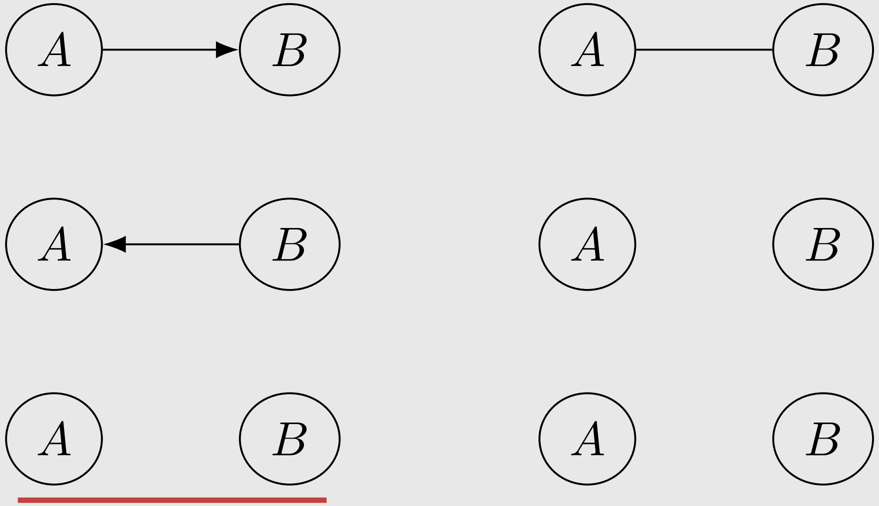
Interventional essential graphs

True graph

$\mathrm{I} = \{\mathrm{A}\}$

$\mathrm{I} = \{\mathrm{B}\}$

$\mathrm{I} = \{\}$



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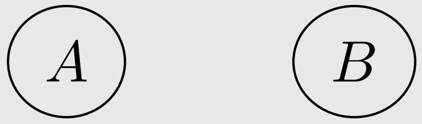
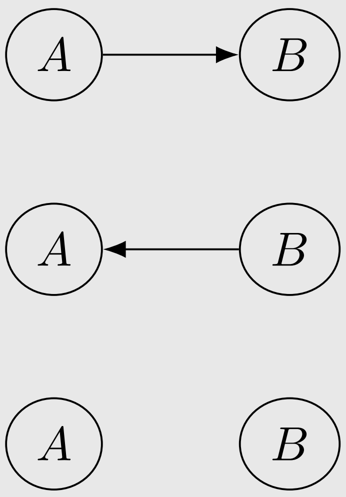
True graph under intervention $\mathrm{I} = \{\mathrm{A}\}$ 

Interventional essential graphs

True graph



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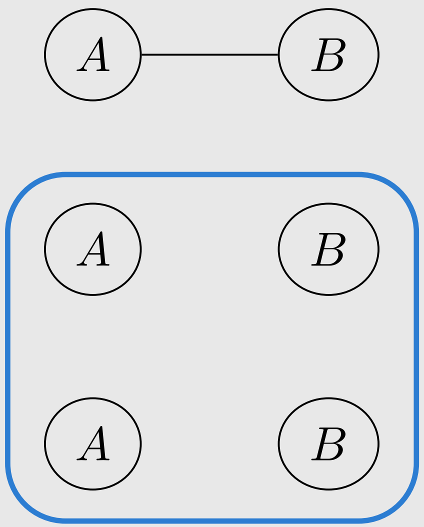
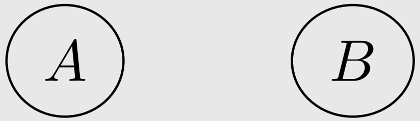


$\mathrm{I} = \{\mathrm{B}\}$

$\mathrm{I} = \{\}$

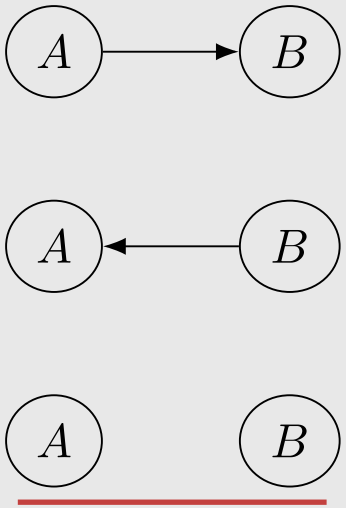
Two Variables: Interventional Essential Graphs

True graph under intervention I = {B}

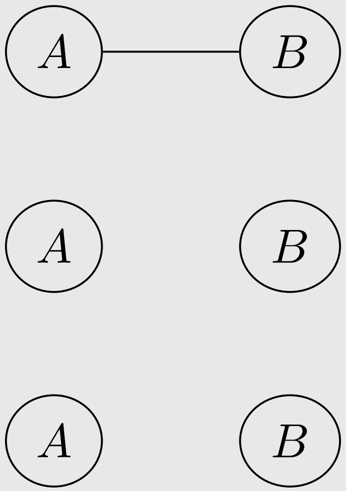


Interventional essential graphs

True graph



I = {A}

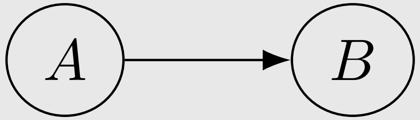


I = {B}

I = {}

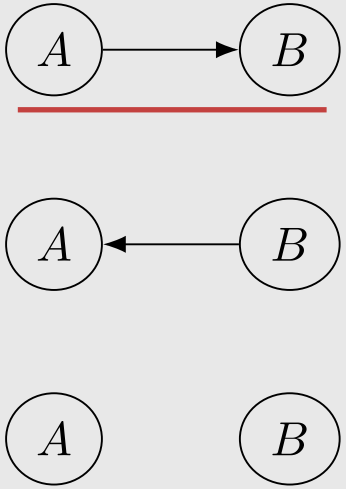
Two Variables: Interventional Essential Graphs

True graph under intervention I = {B}

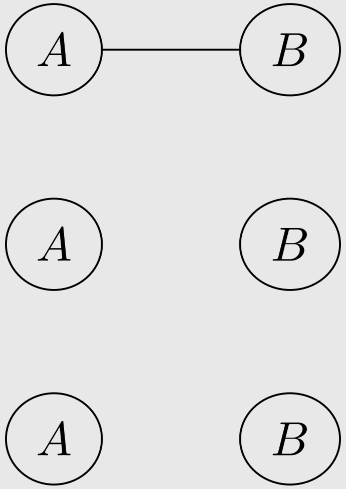


Interventional essential graphs

True graph



I = {A}

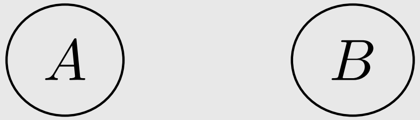


I = {B}

I = {}

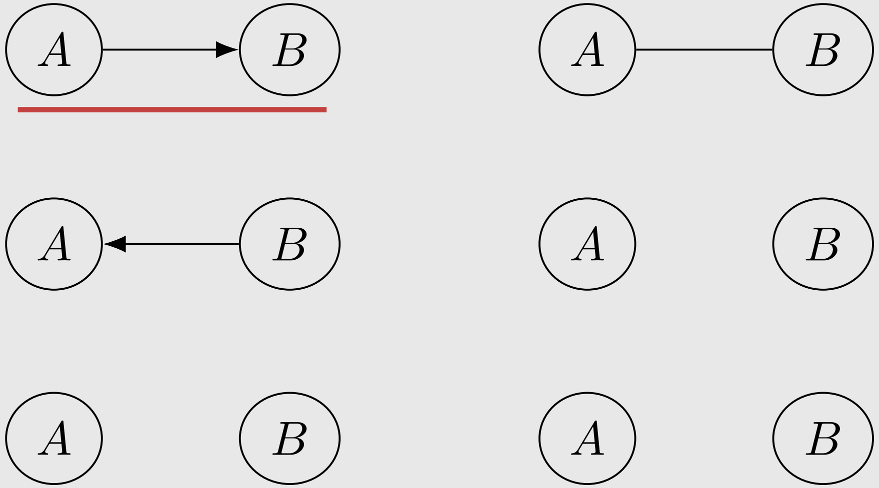
Two Variables: Interventional Essential Graphs

True graph under intervention $\mathrm{I} = \{\mathrm{B}\}$

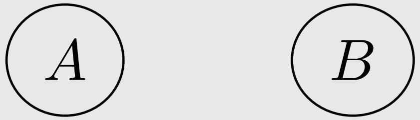


Interventional essential graphs

True graph	$\mathrm{I} = \{\mathrm{A}\}$	$\mathrm{I} = \{\mathrm{B}\}$	$\mathrm{I} = \{\}$

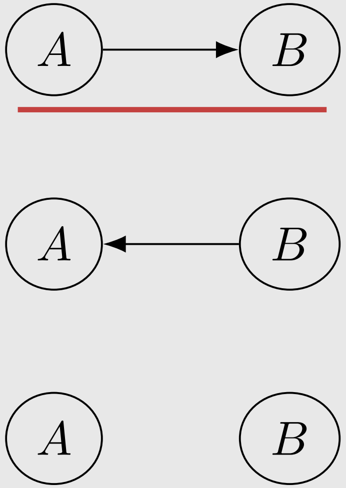


Two Variables: Interventional Essential Graphs

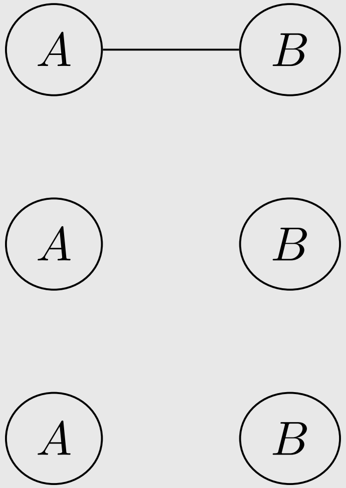
True graph under intervention I = {B} 

Interventional essential graphs

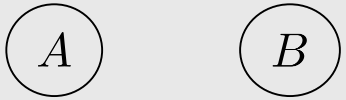
True graph



I = {A}



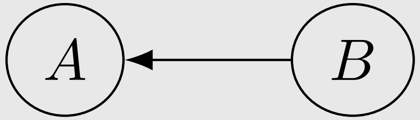
I = {B}



I = {}

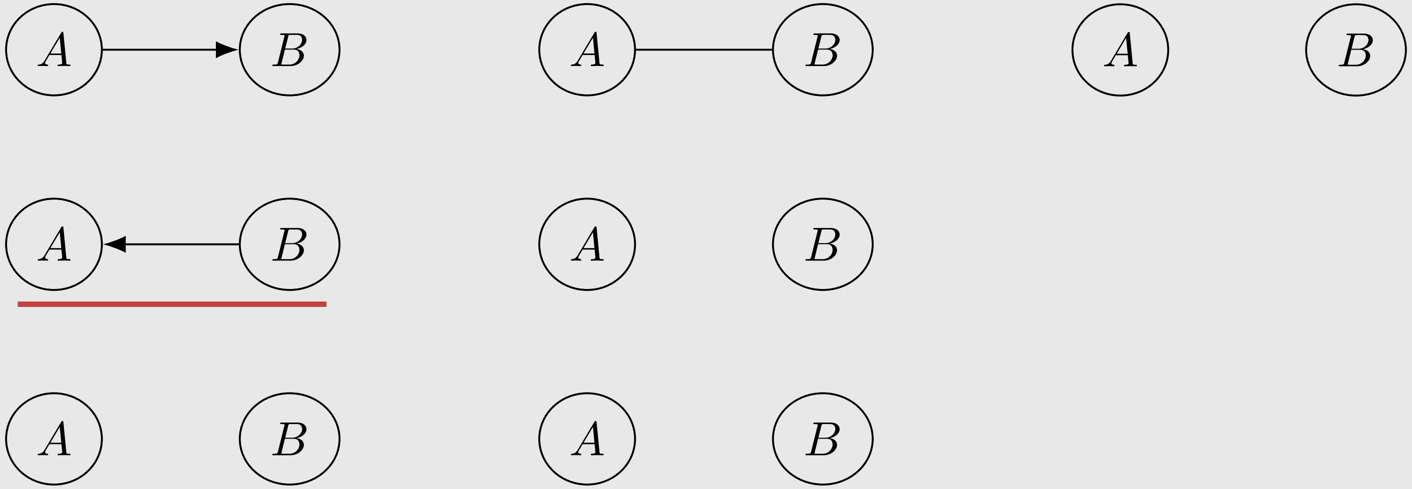
Two Variables: Interventional Essential Graphs

True graph under intervention $\mathrm{I} = \{\mathrm{B}\}$



Interventional essential graphs

True graph | $\mathrm{I} = \{\mathrm{A}\}$ | $\mathrm{I} = \{\mathrm{B}\}$ | $\mathrm{I} = \{\}$



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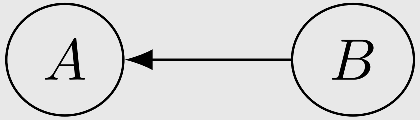

Interventional essential graphs

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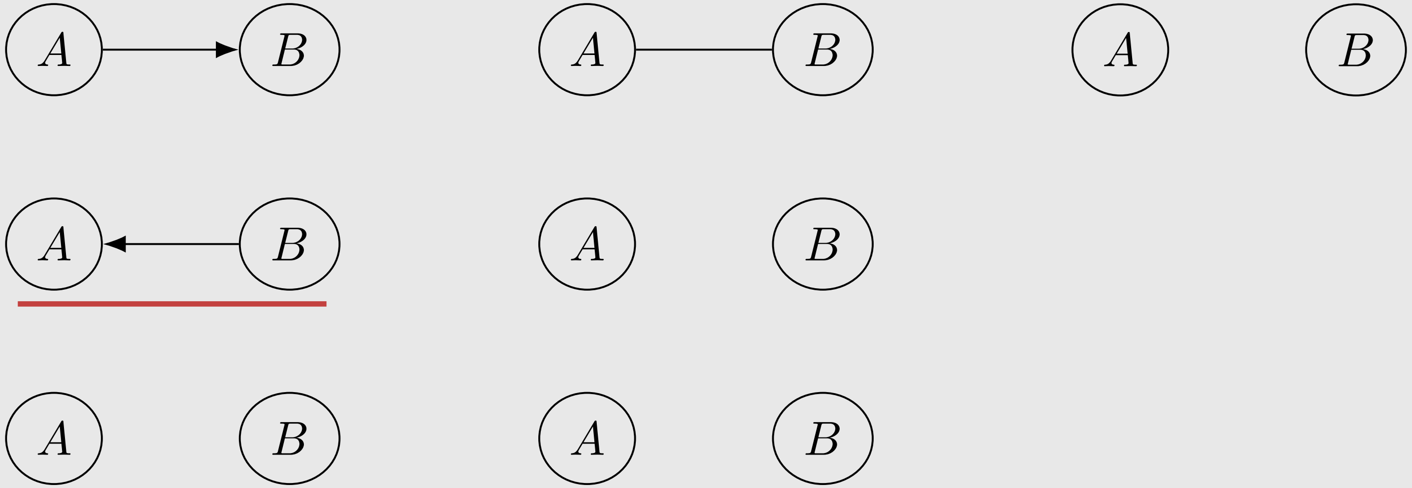

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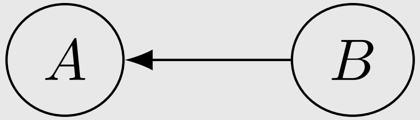

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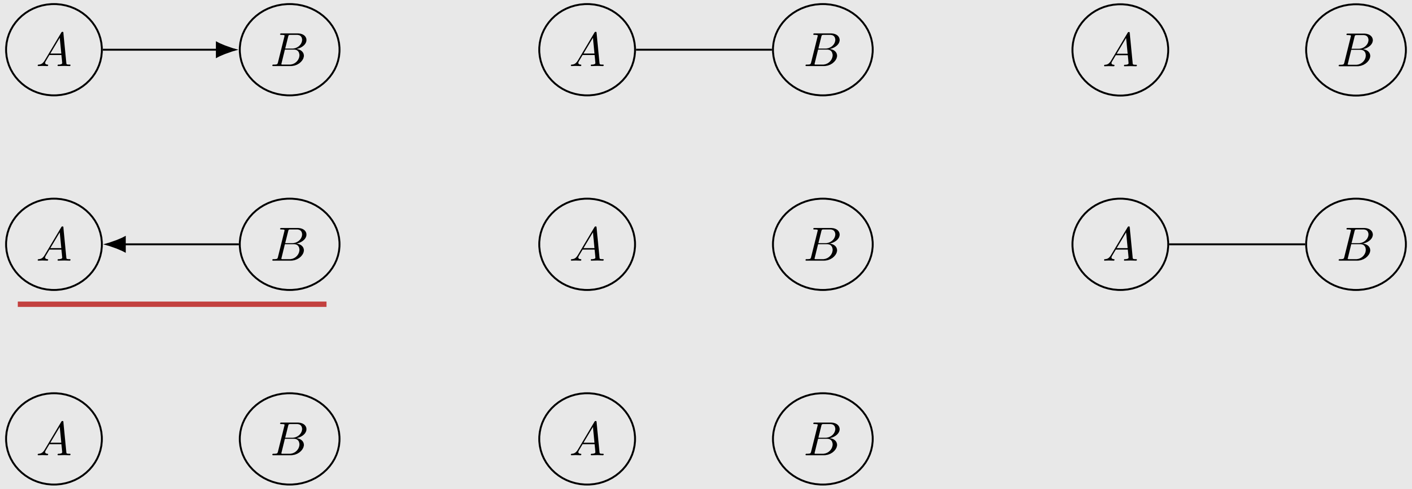

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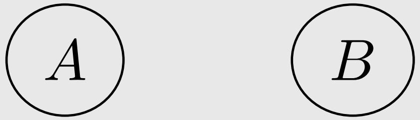

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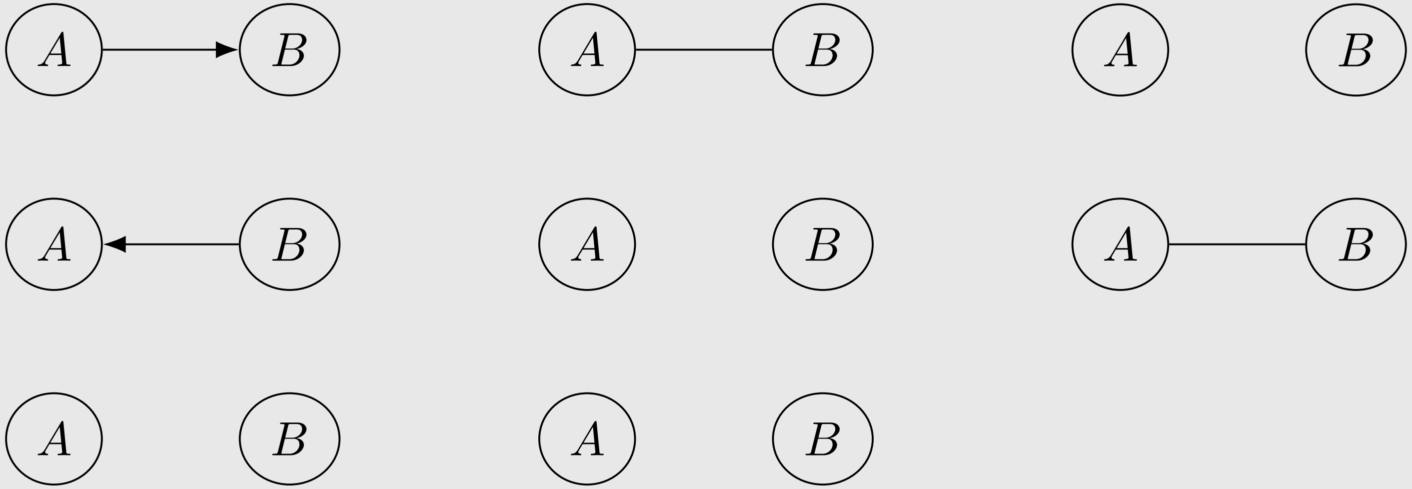

Two Variables: Interventional Essential Graphs

True graph under intervention I = {B} $\quad A \qquad B$

Interventional essential graphs

True graph	I = {A}	I = {B}	I = {}
$A \rightarrow B$	$A - B$	$A \quad B$	
$A \leftarrow B$	$A \quad B$	$A - B$	
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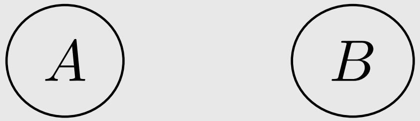

Interventional essential graphs

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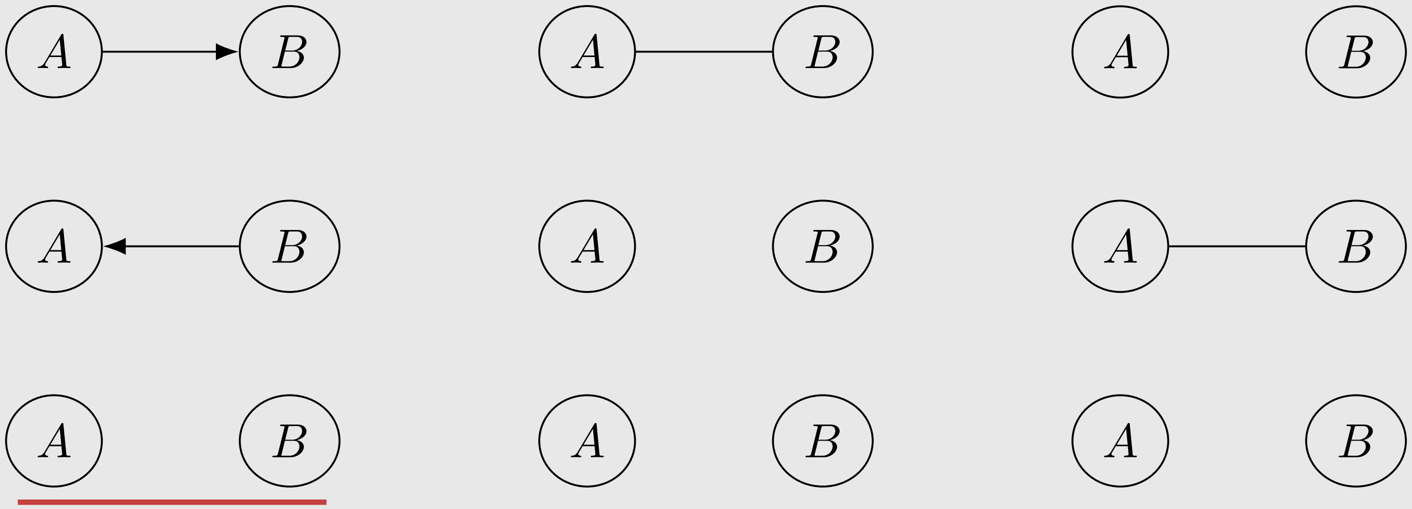

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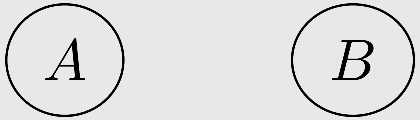

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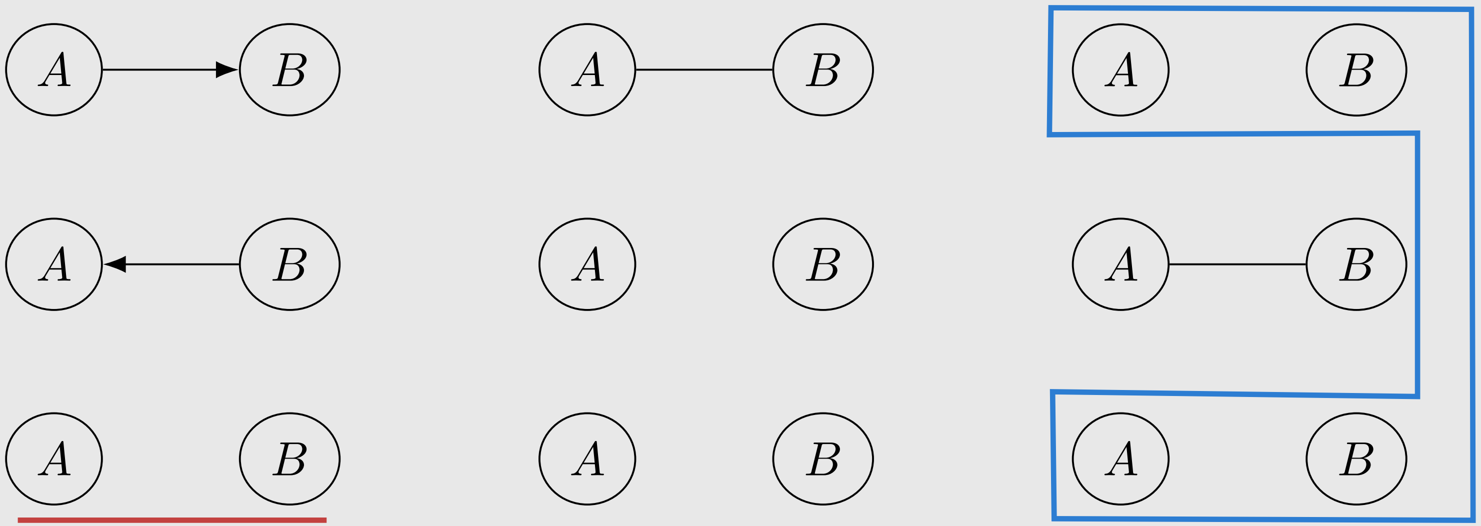

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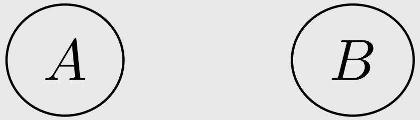

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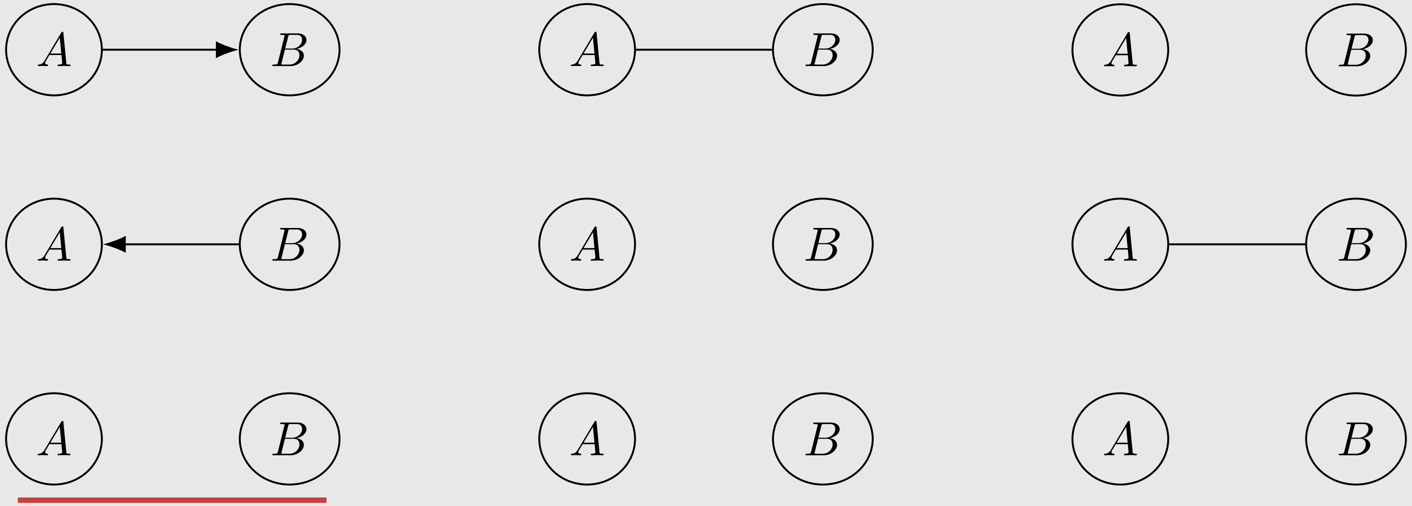

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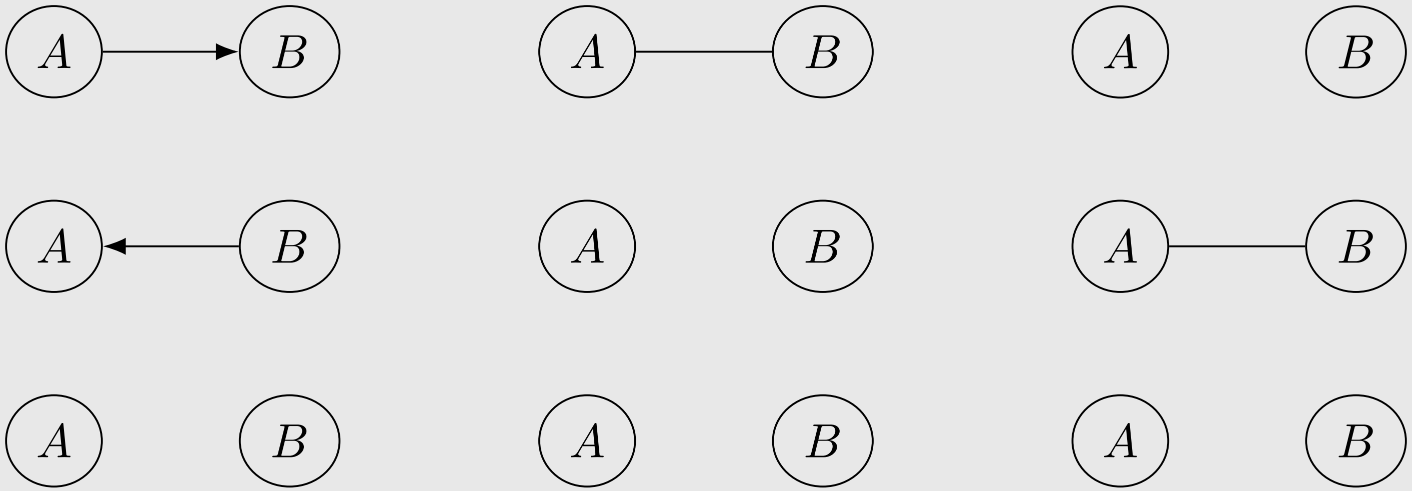

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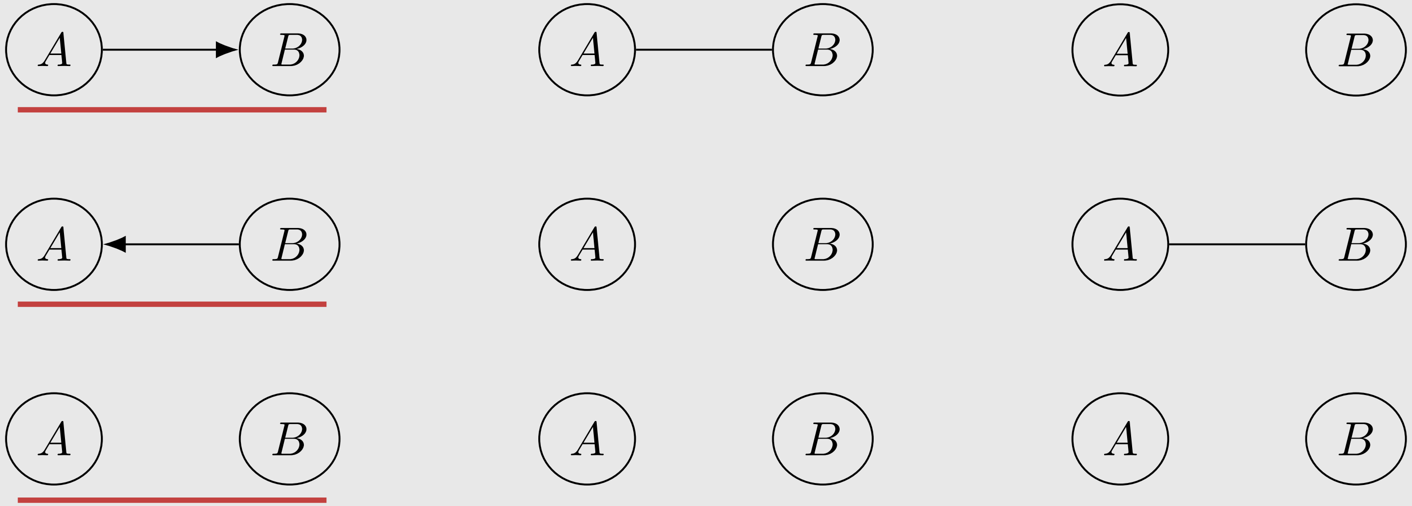

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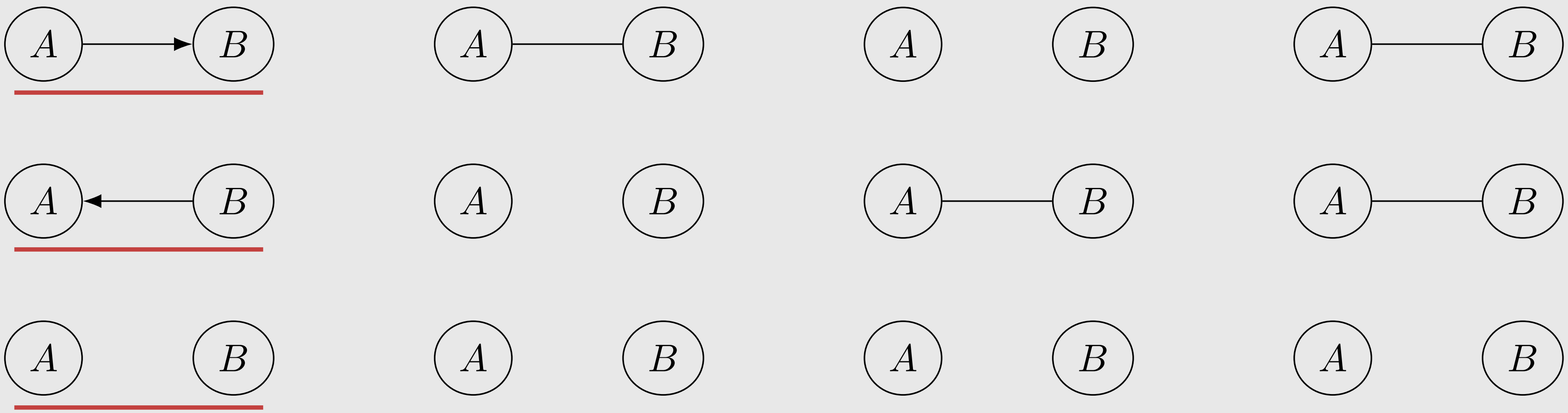

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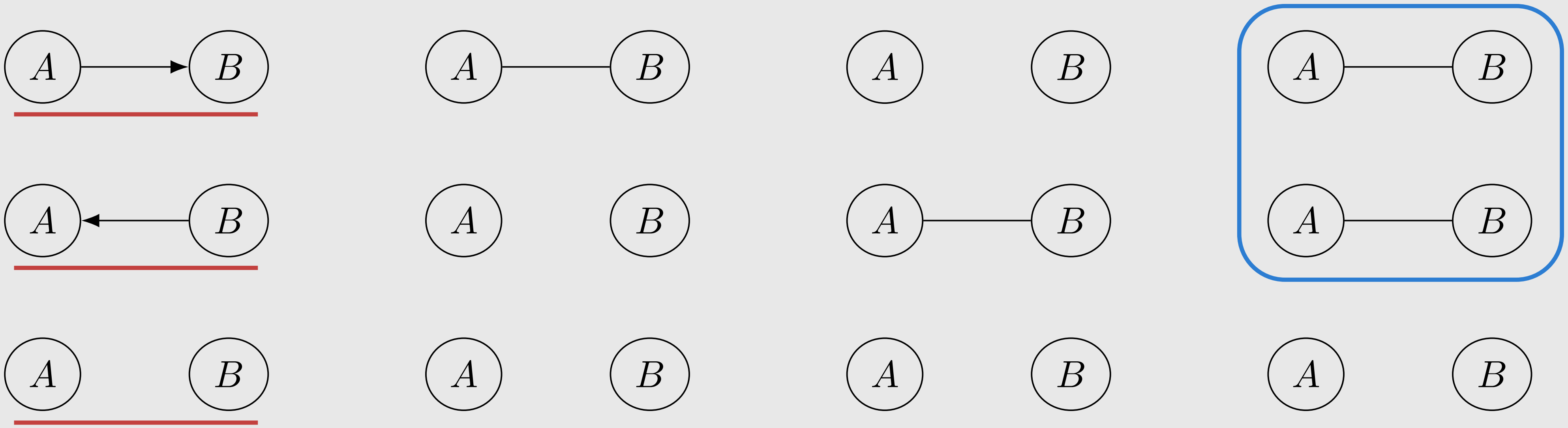

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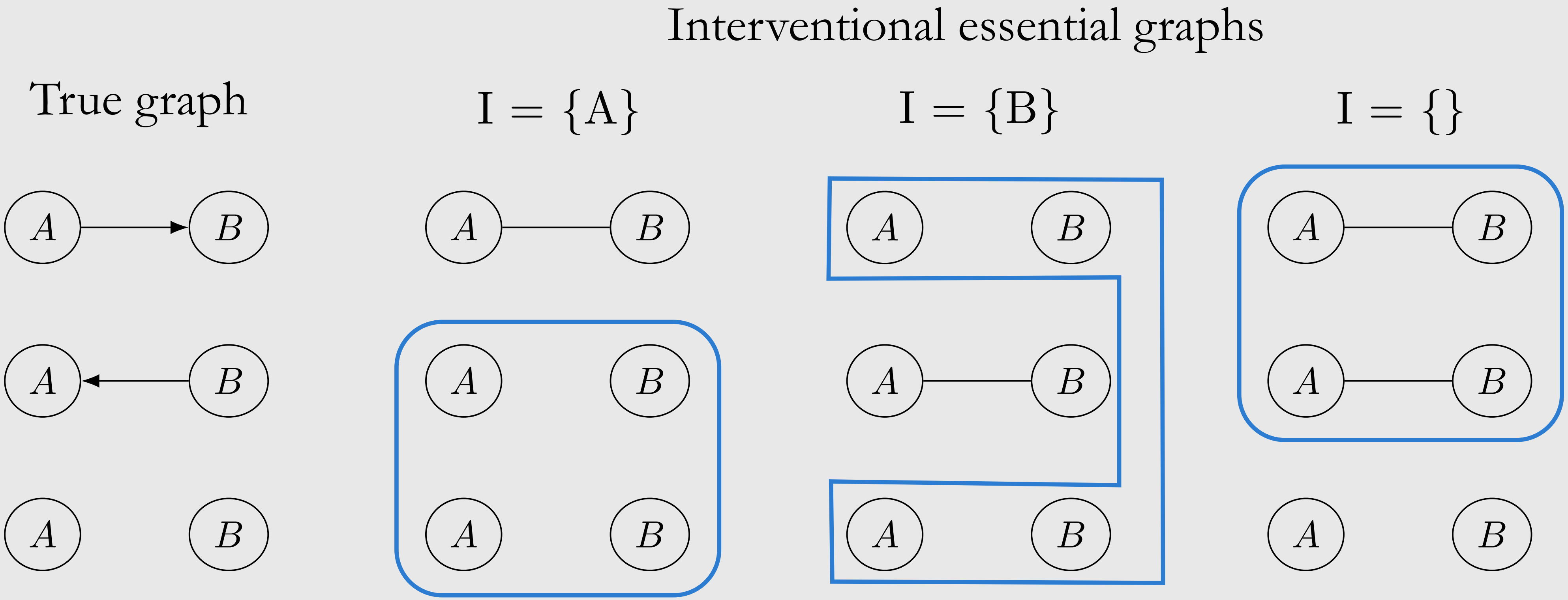
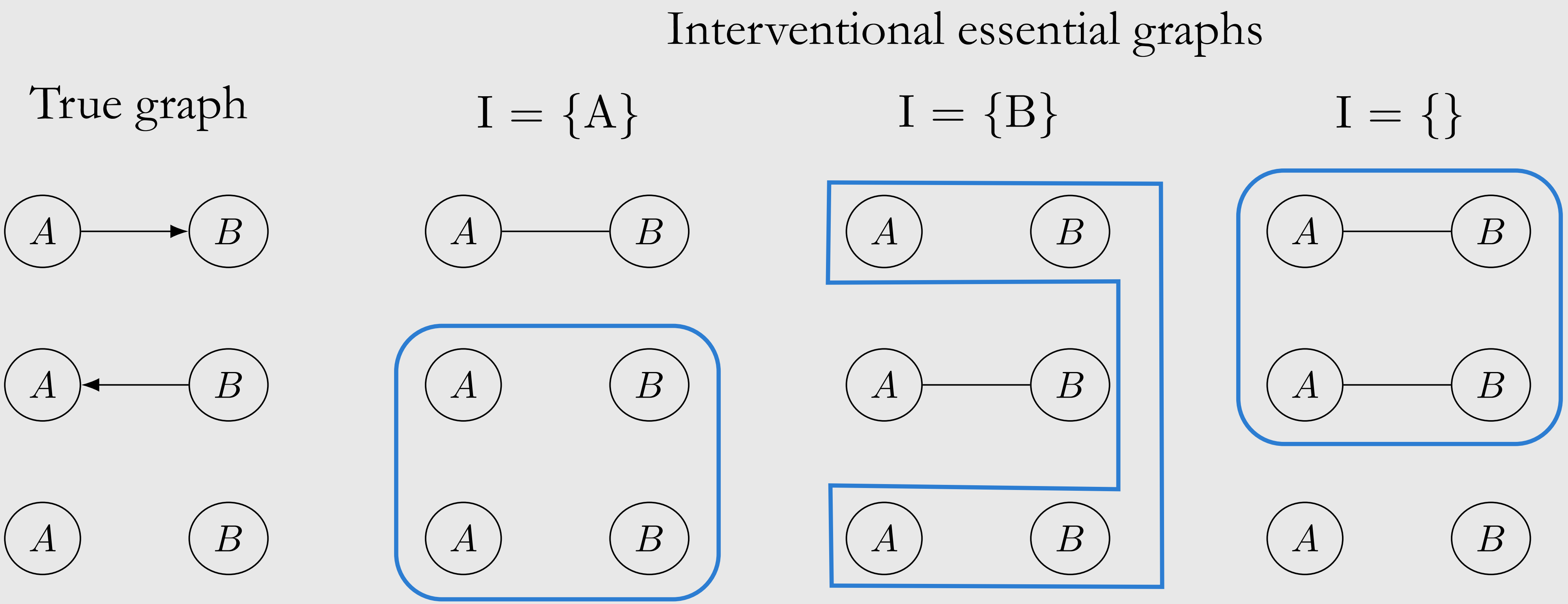


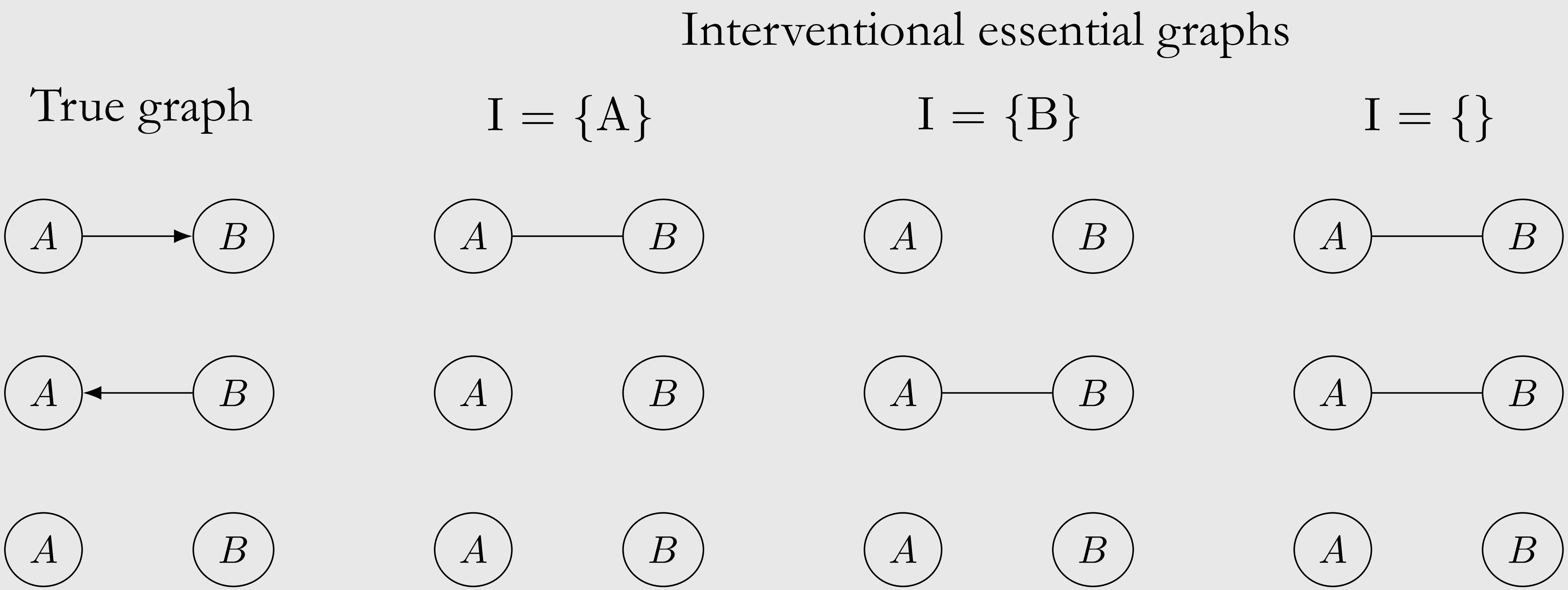
Two Variables: Interventional Essential Graphs

Need more than one intervention to identify the graph



Two Variables: Two Interventions Identify the Graph

Two interventions are sufficient and necessary to identify the graph.



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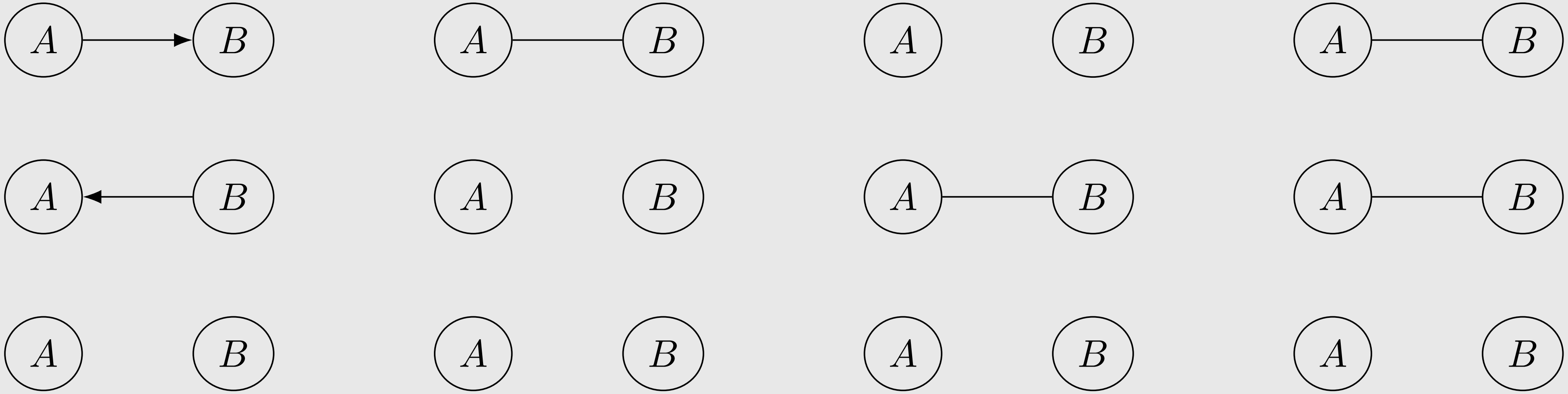

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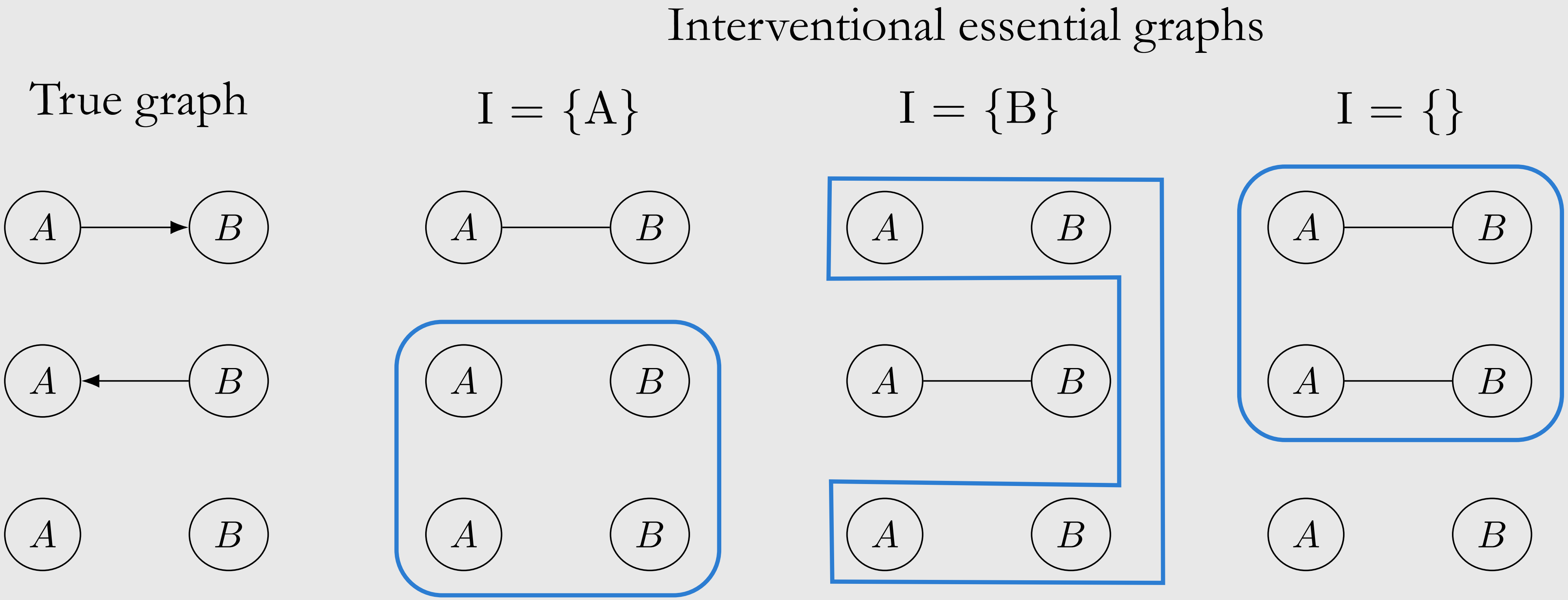

Complete Graphs Are the Worst Case

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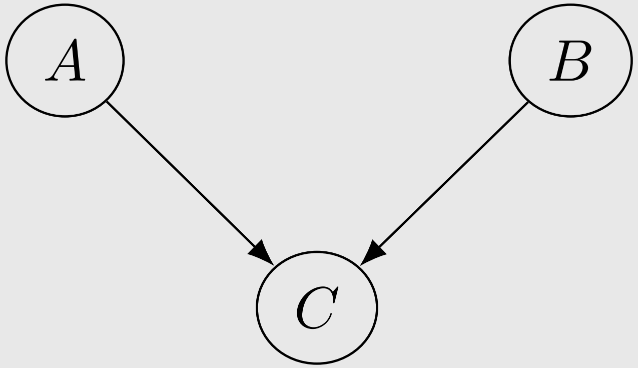
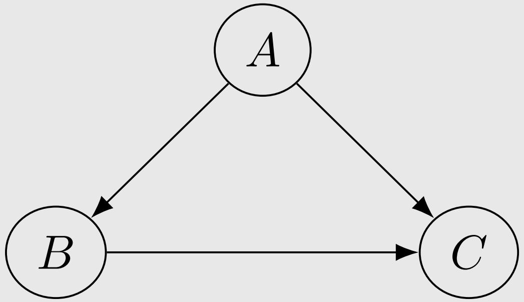
Immorality



Three Variables



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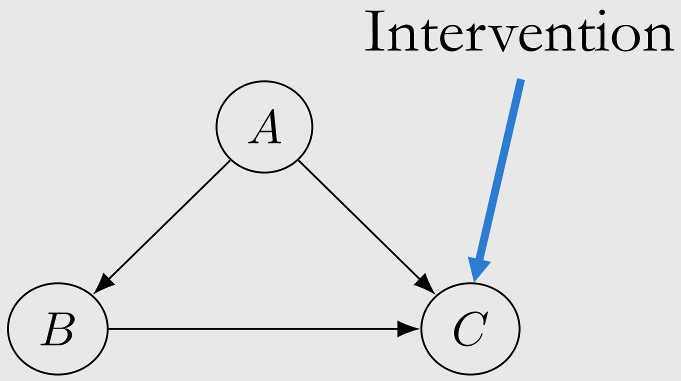

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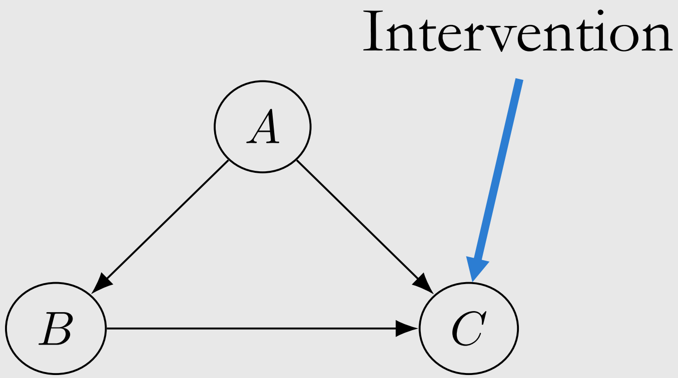
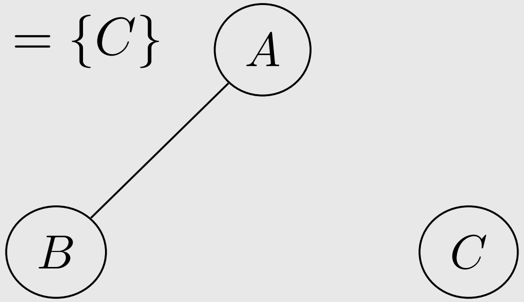


Interventional essential graphs

$I = \{C\}$



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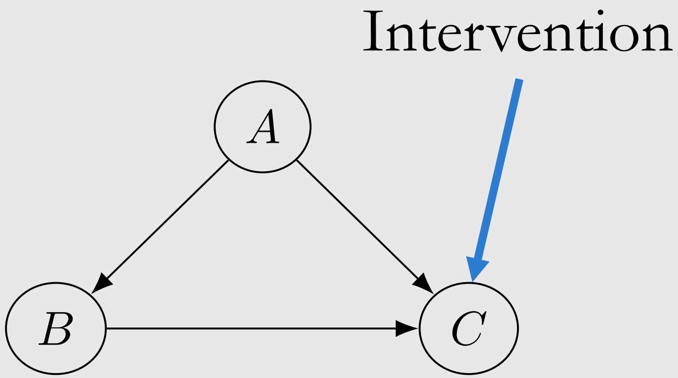

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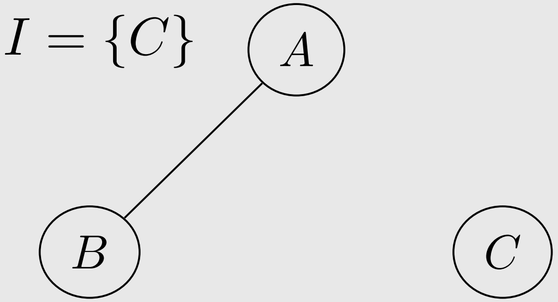
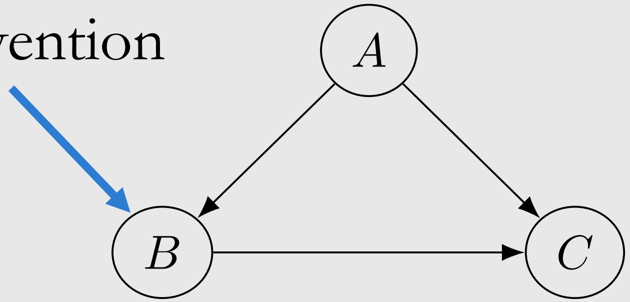


What we've learned:

- No C → A edge
- No C → B edge
- A and B connected

Three Variables

Intervention



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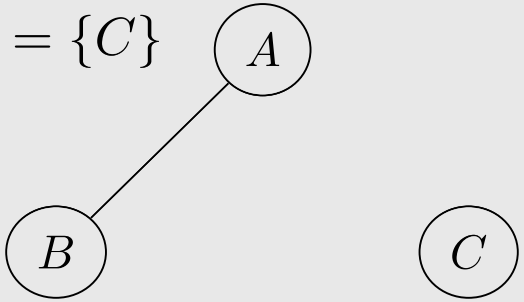

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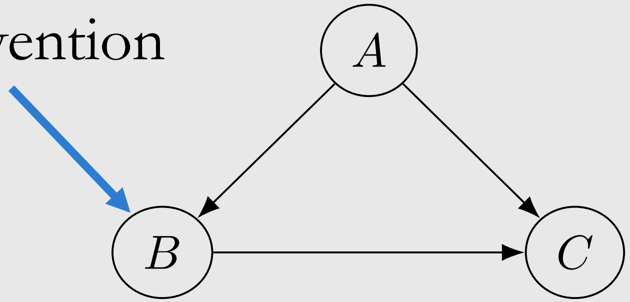

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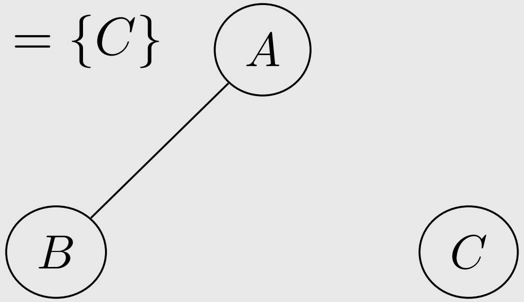

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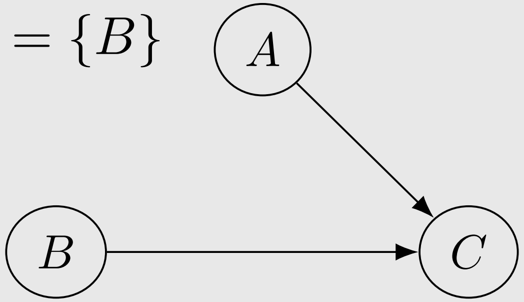

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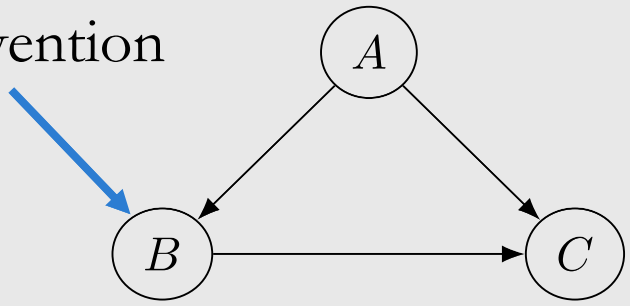

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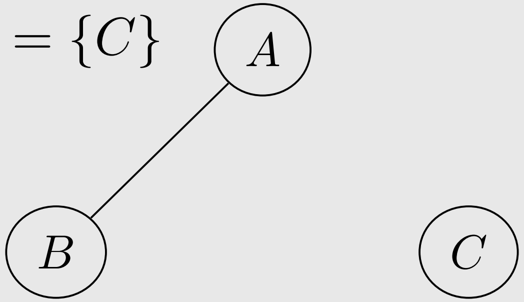

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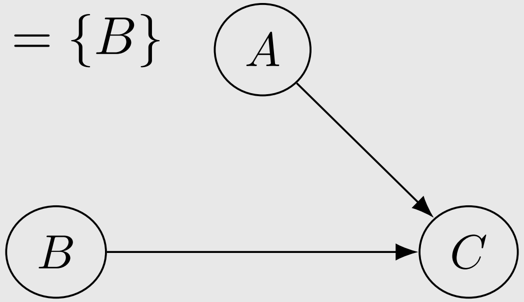

What we've learned:

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- A and B connected
- No B → A edge
- Yes A → C edge
- Yes B → C edge

Single Variable Interventions: n – 1 Are Sufficient for n > 2 (Eberhardt et al., 2006)

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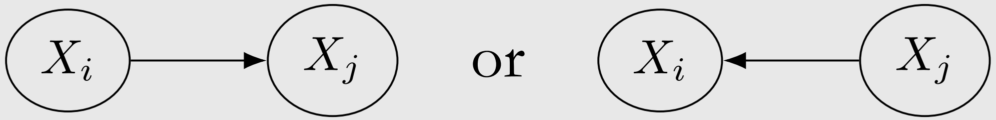

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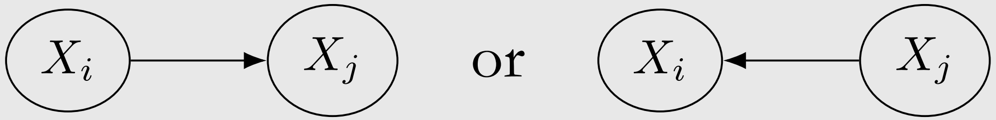

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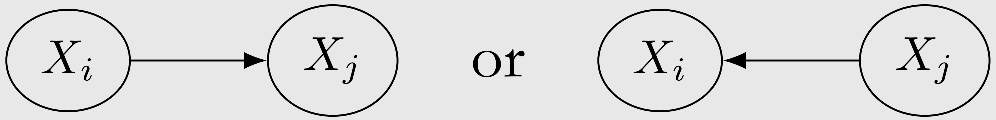

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Note: if you include the empty set (observational), then n are necessary in the worst case (Eberhardt et al., 2006).

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Adaptivity doesn't help in the worst case (Eberhardt et al., 2006).

Questions (where the goal is to identify the graph):

1. Show that n interventions are necessary in the worse case in the three-variable setting when you use the observational data (null intervention) as one of the interventions.
2. What is the minimum number of interventions necessary in the worst case when $n = 3$?
3. What is the minimum number of interventions necessary in the worst case when $n = 2$?

Structural Interventions

Single-Node Interventions

Multi-Node Interventions

Parametric Interventions

Interventional Markov Equivalence

Miscellaneous Other Settings

Multiple-Node Interventions

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Number of Interventions in the Worse Case

Eberhardt et al. (2005) found that $\lfloor \log_2(n) \rfloor + 1$ multi-node interventions are necessary for worst case graphs (complete graph), but what about for non-complete graphs?

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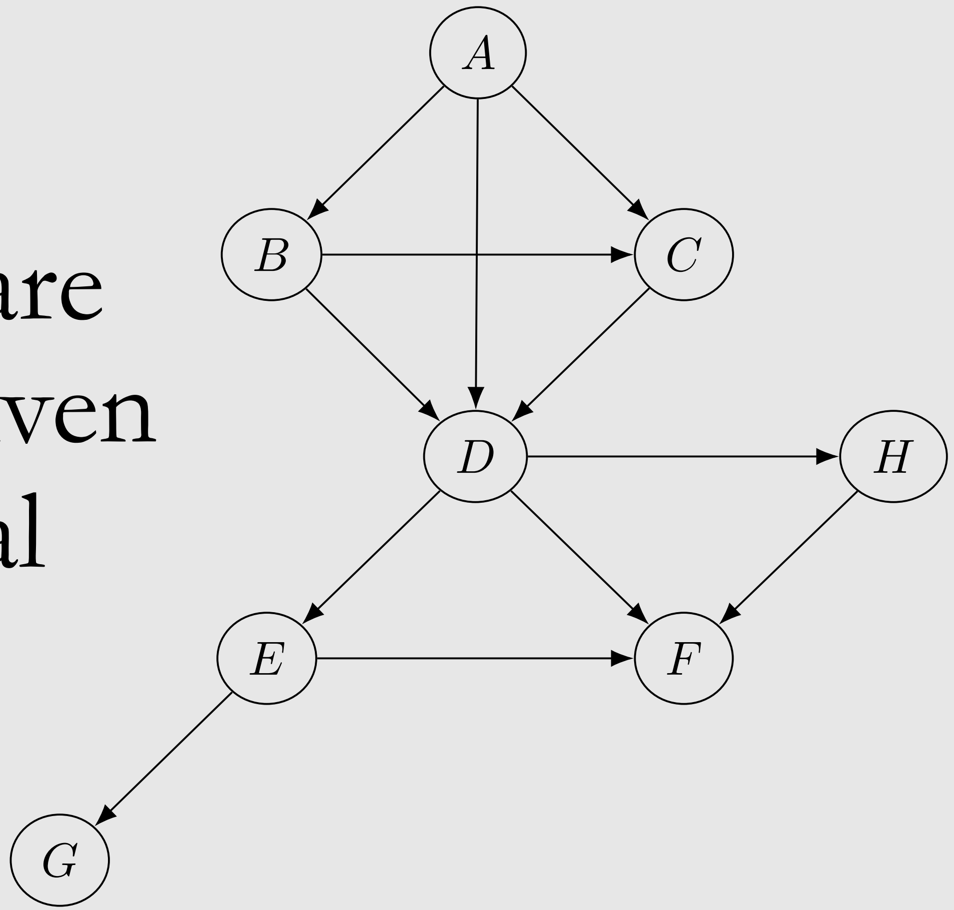
Theorem: $\lceil \log_2(c) \rceil$ multi-node interventions are sufficient and necessary in the worst case, where c is the size of the largest clique (conjectured by Eberhardt (2008)) and proven by Hauser & Bühlmann (2014)).

Question:

In this graph, how many multi-node interventions are sufficient and necessary given that you know the essential graph?



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Structural vs. Parametric Interventions

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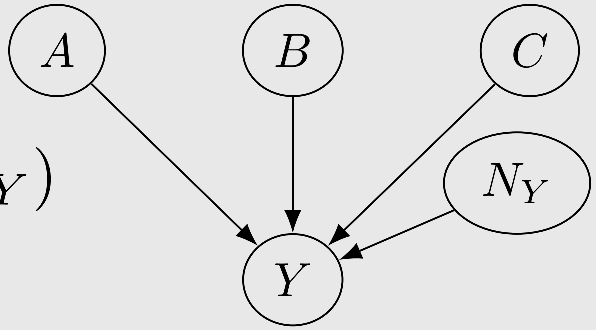
Structural:

$$Y := f_\theta(A, B, C, N_Y)$$



Structural vs. Parametric Interventions

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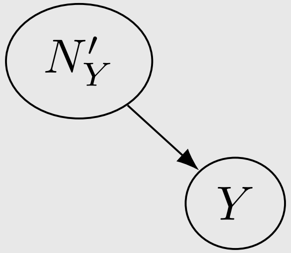

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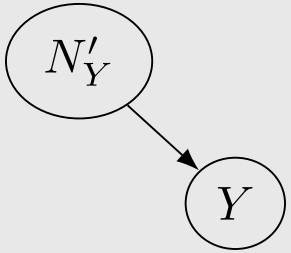

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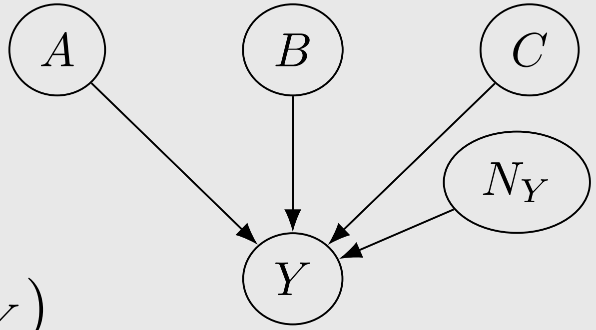

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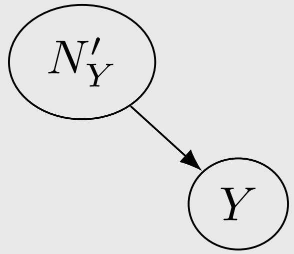

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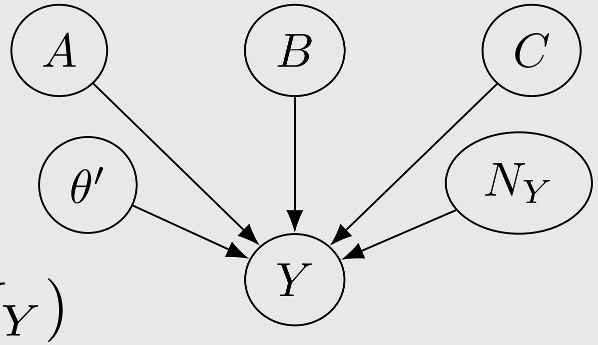

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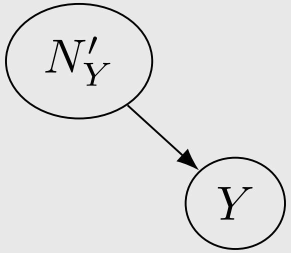

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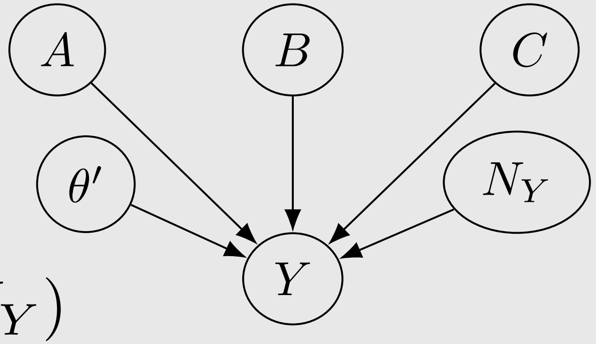

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Other names: hard vs. soft, perfect vs. imperfect, etc.

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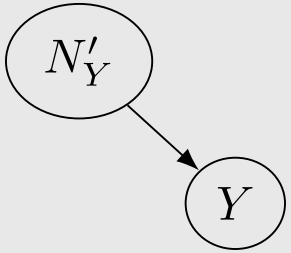

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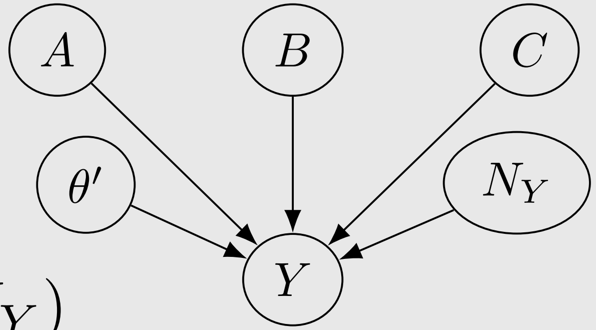

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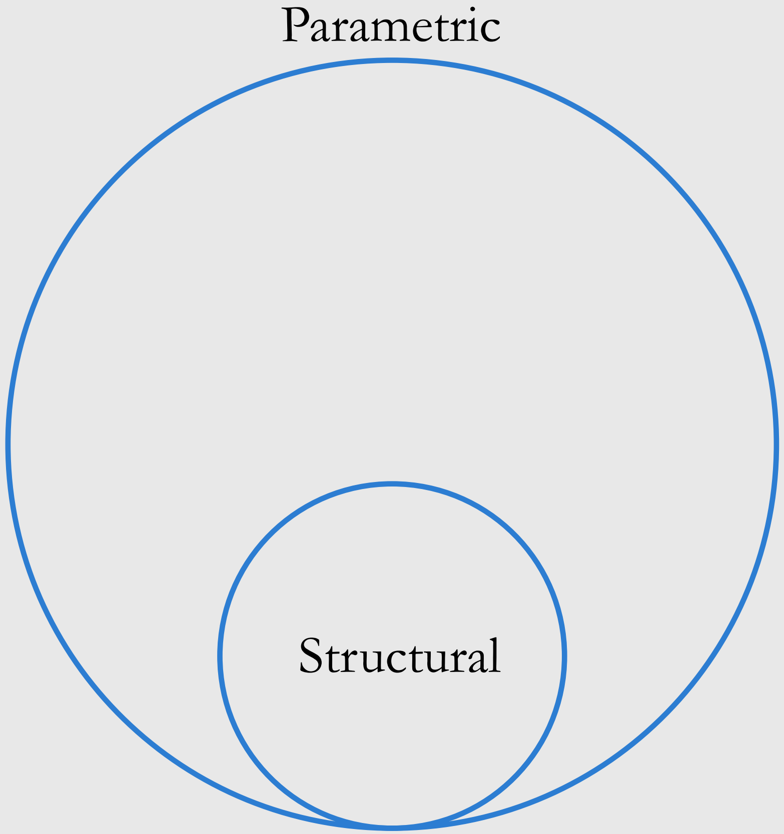

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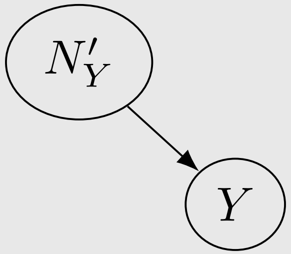

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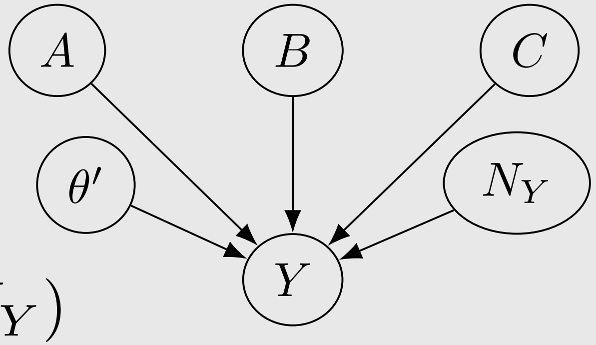

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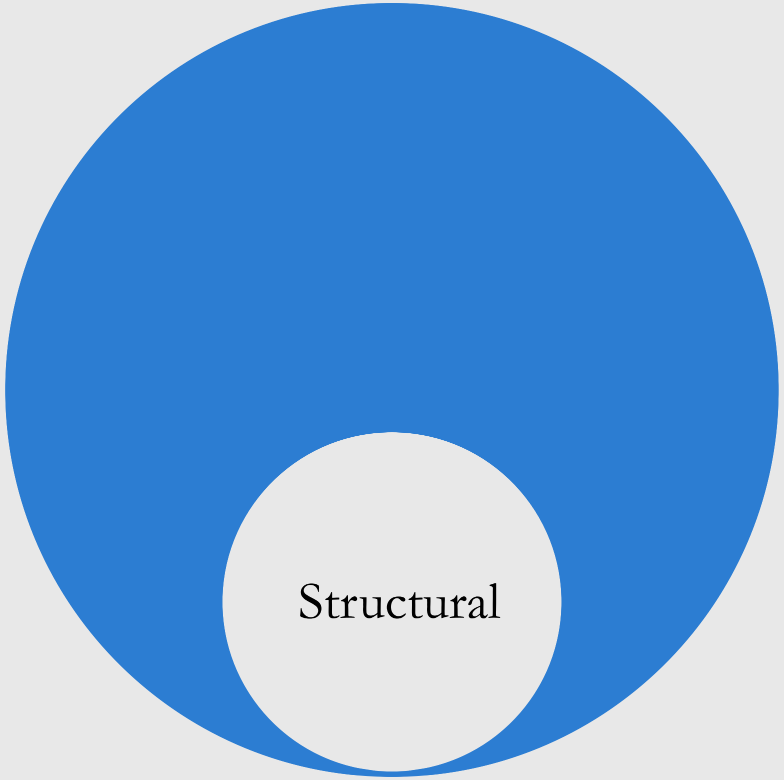

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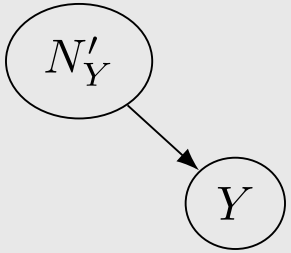

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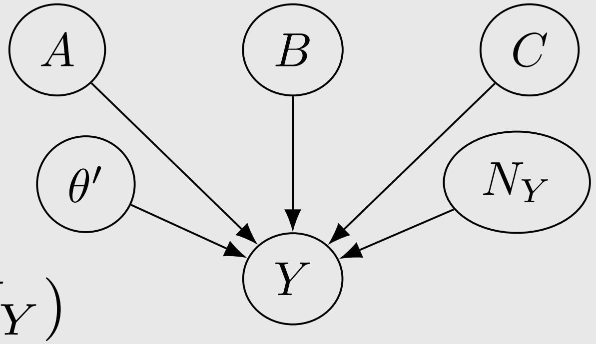

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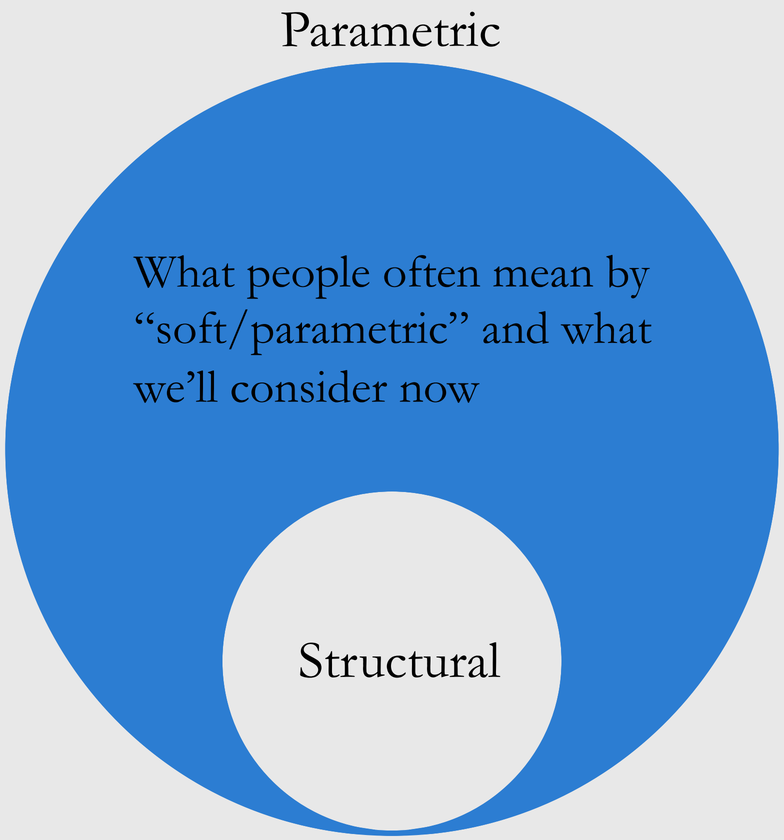

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Number of Parametric Single-Node Interventions

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How about the in-between? How much of the graph can we identify with fewer interventions?

How much of the graph can we identify with a given set of interventions?

Structural Interventions

Single-Node Interventions

Multi-Node Interventions

Parametric Interventions

Interventional Markov Equivalence

Miscellaneous Other Settings

Interventions Introduce Immoralities: Single-Node

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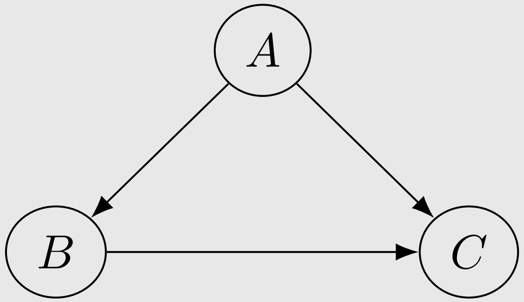
True Graph



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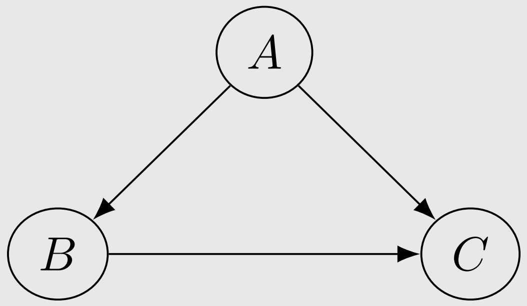
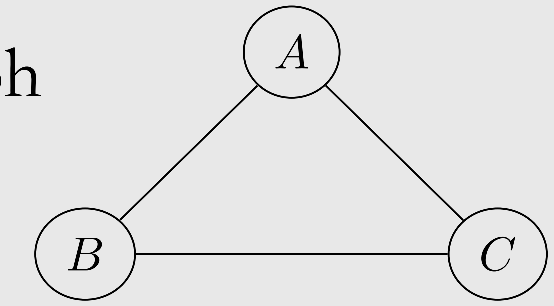


Essential Graph



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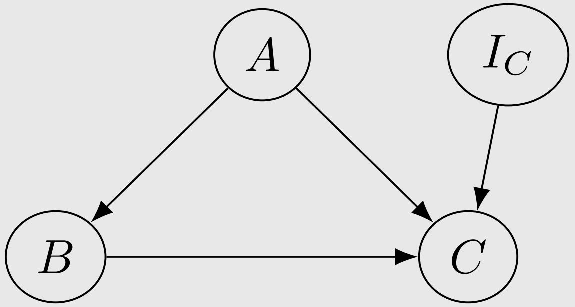
Interventional Graph

True Graph



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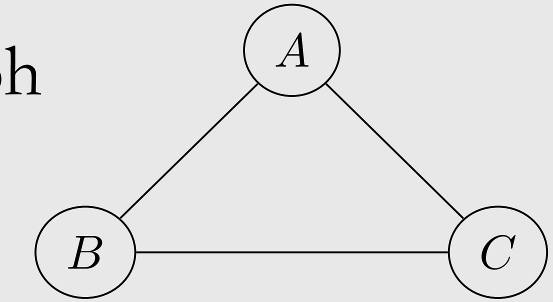

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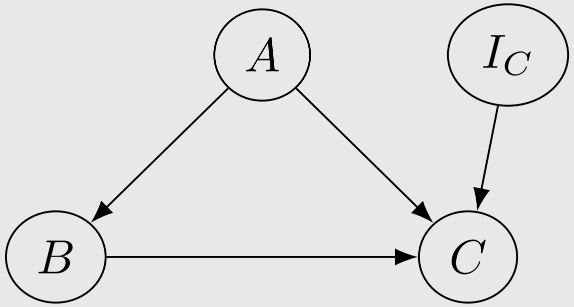

Preserves structure, unlike structural interventions

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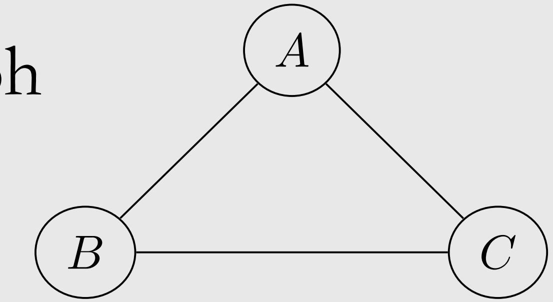

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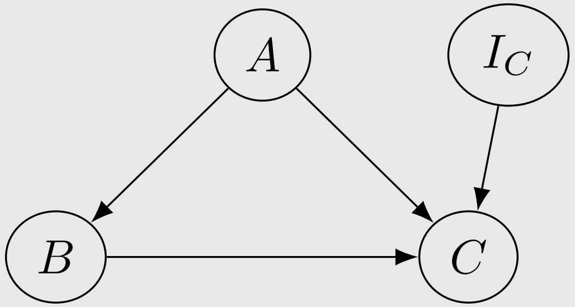

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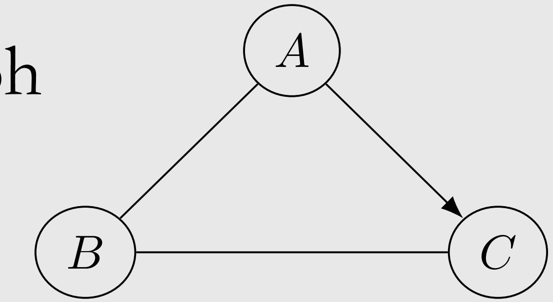

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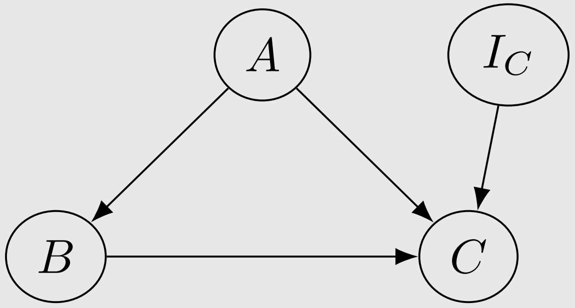

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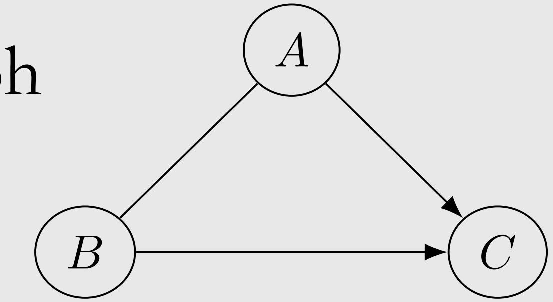

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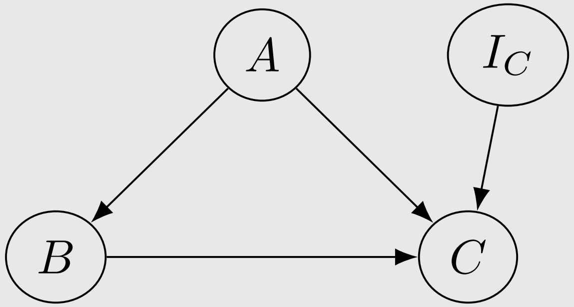

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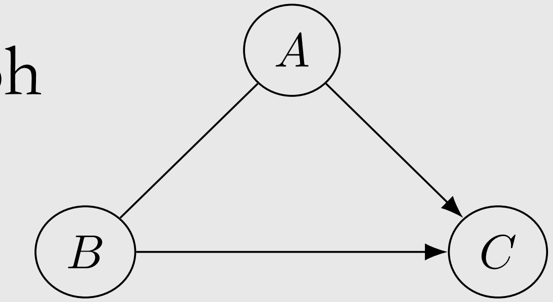

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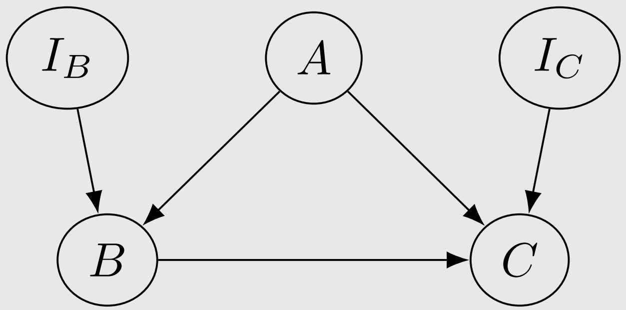

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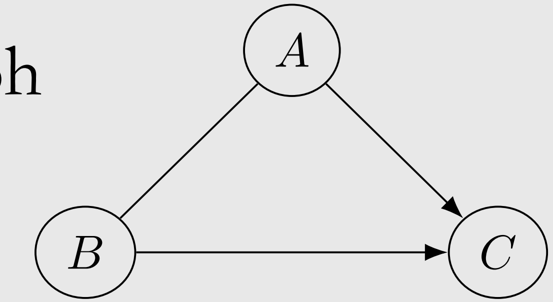

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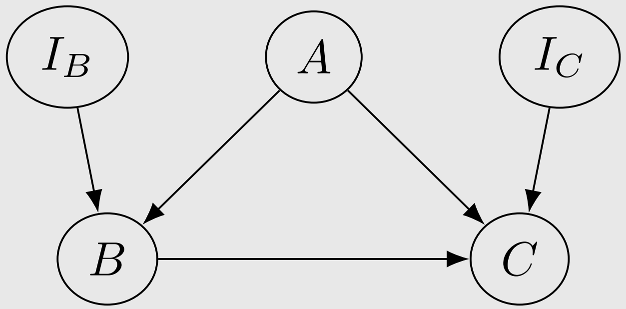

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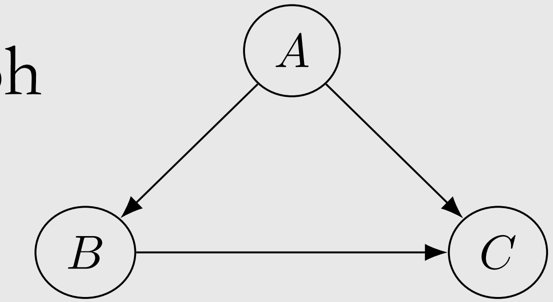

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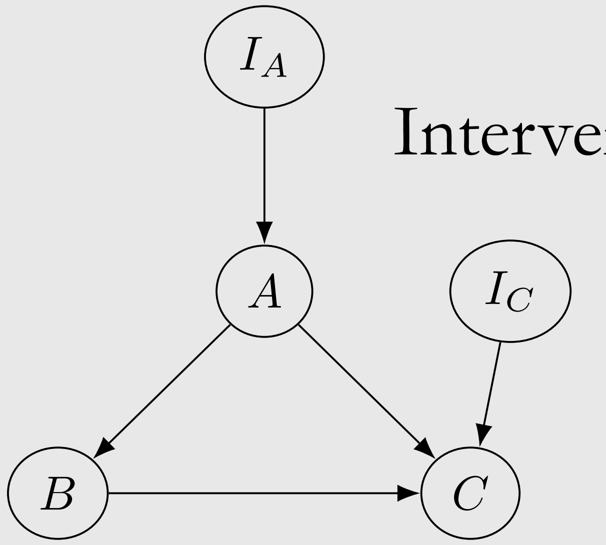

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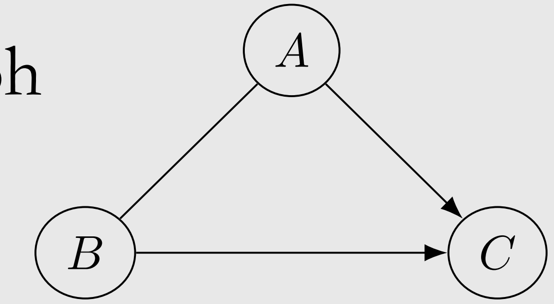

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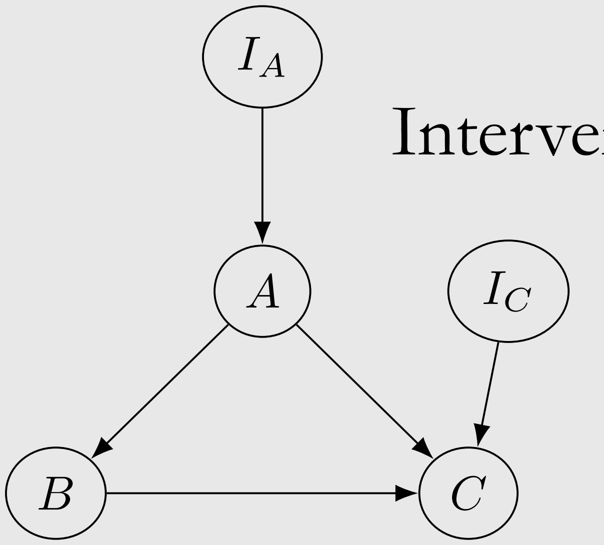

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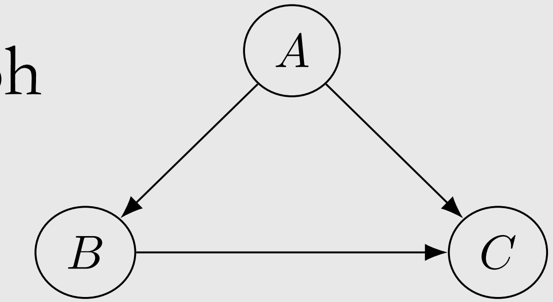

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Interventional Graph: Multi-Node Interventions

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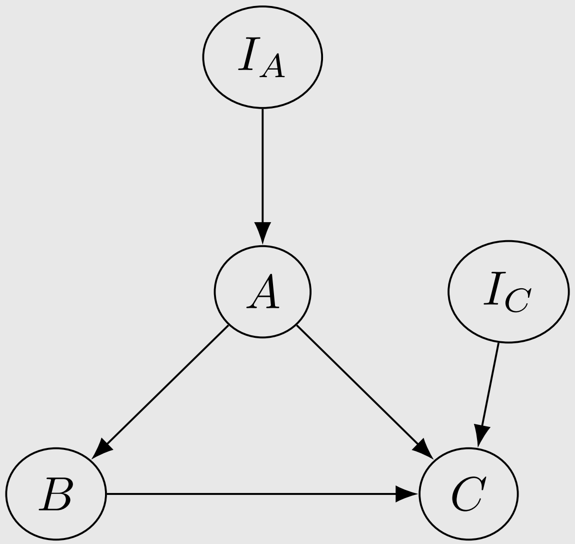

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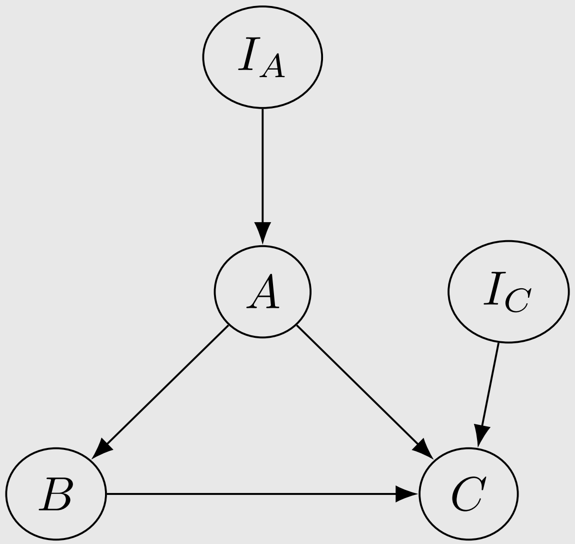

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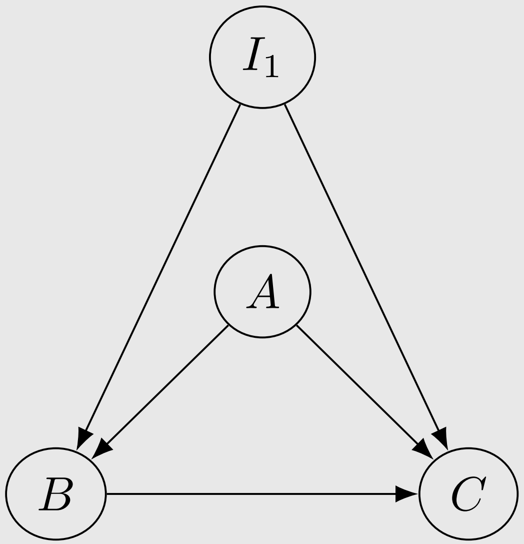

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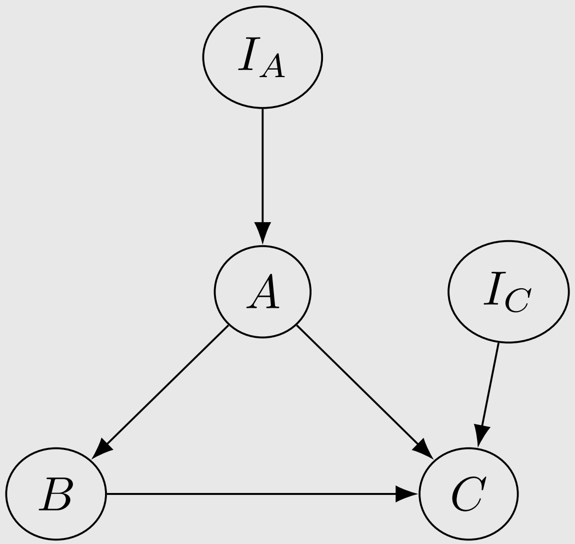

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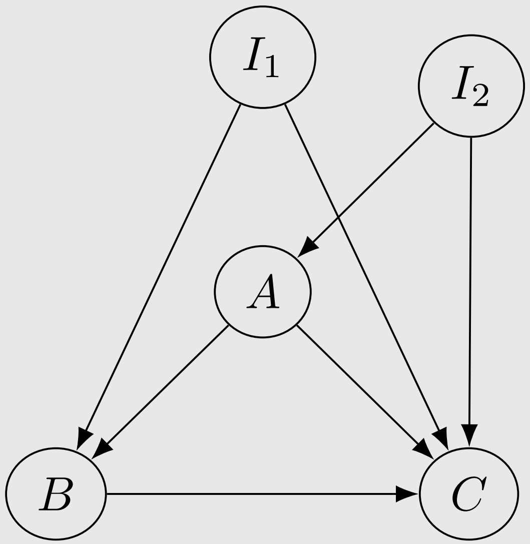

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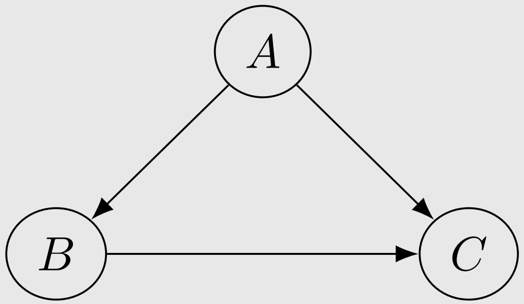

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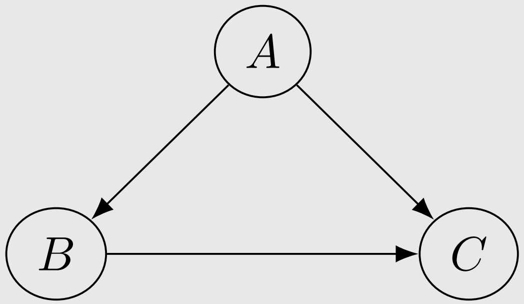

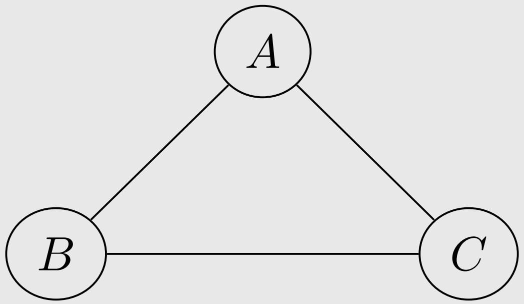

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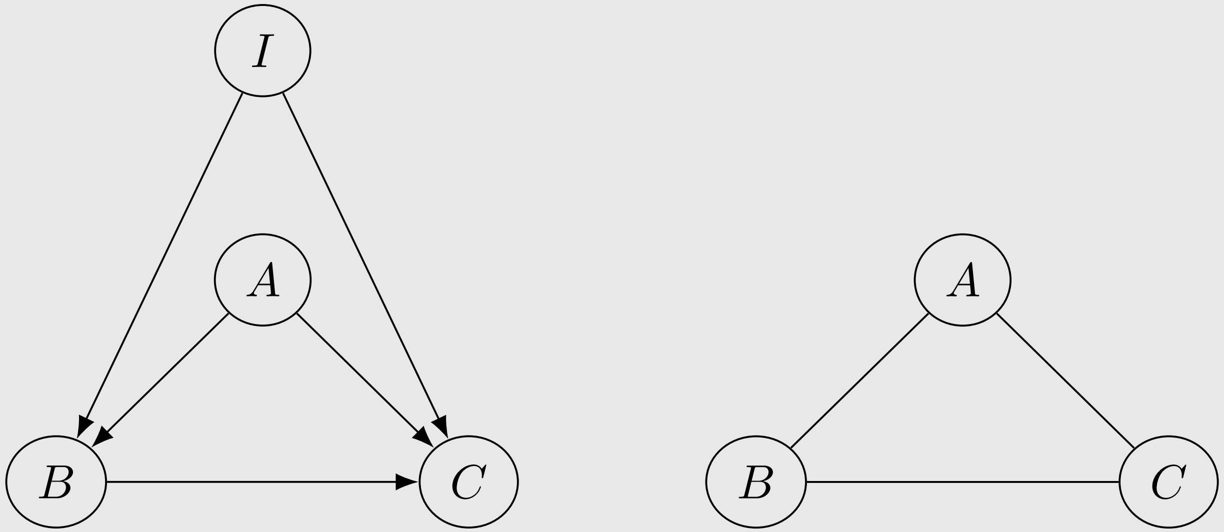

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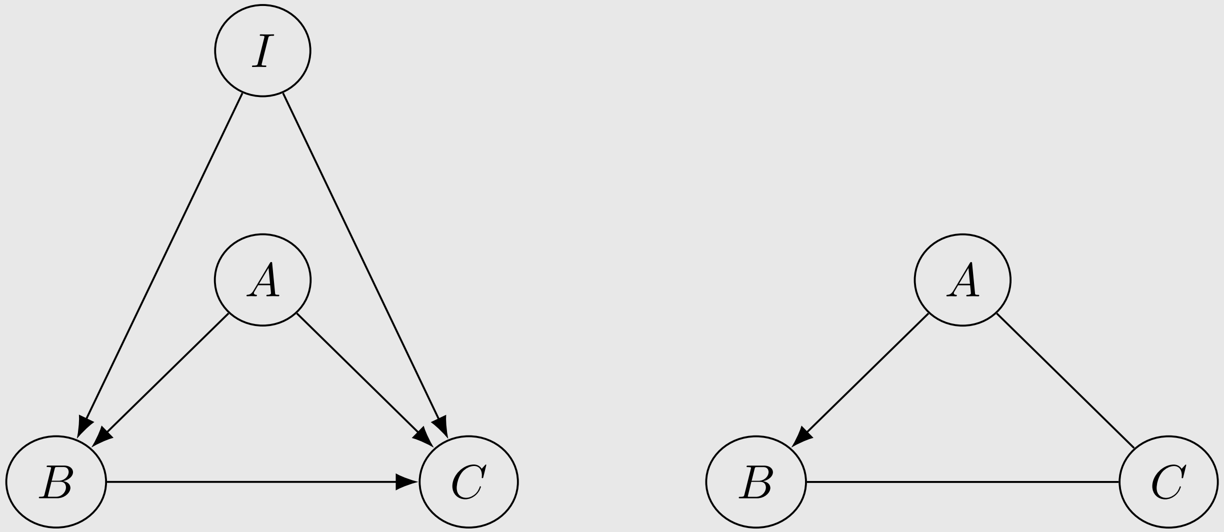

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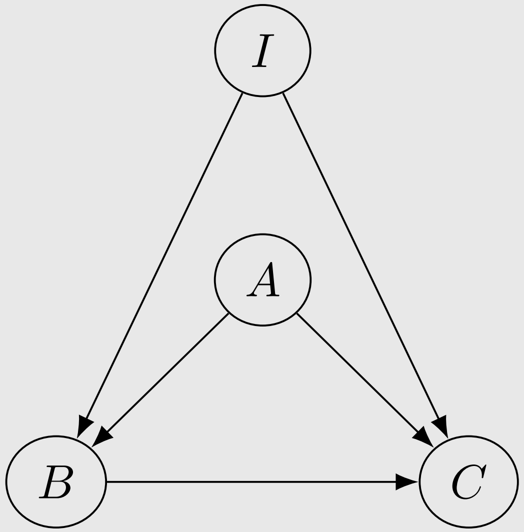

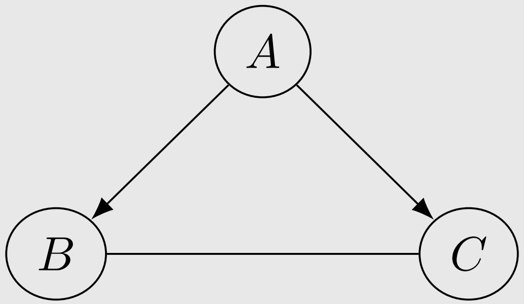

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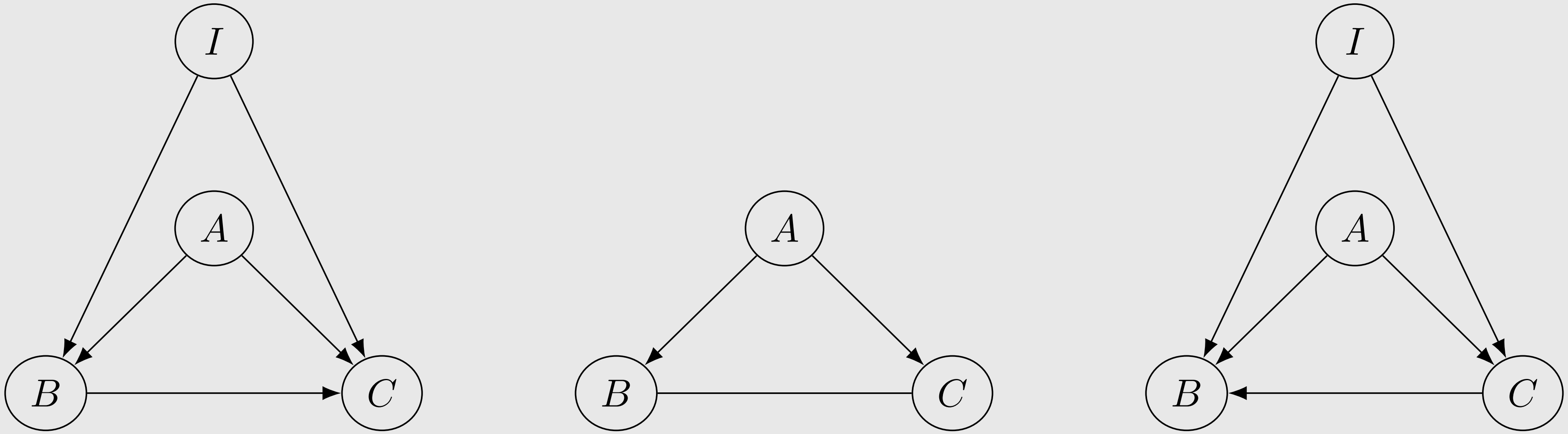

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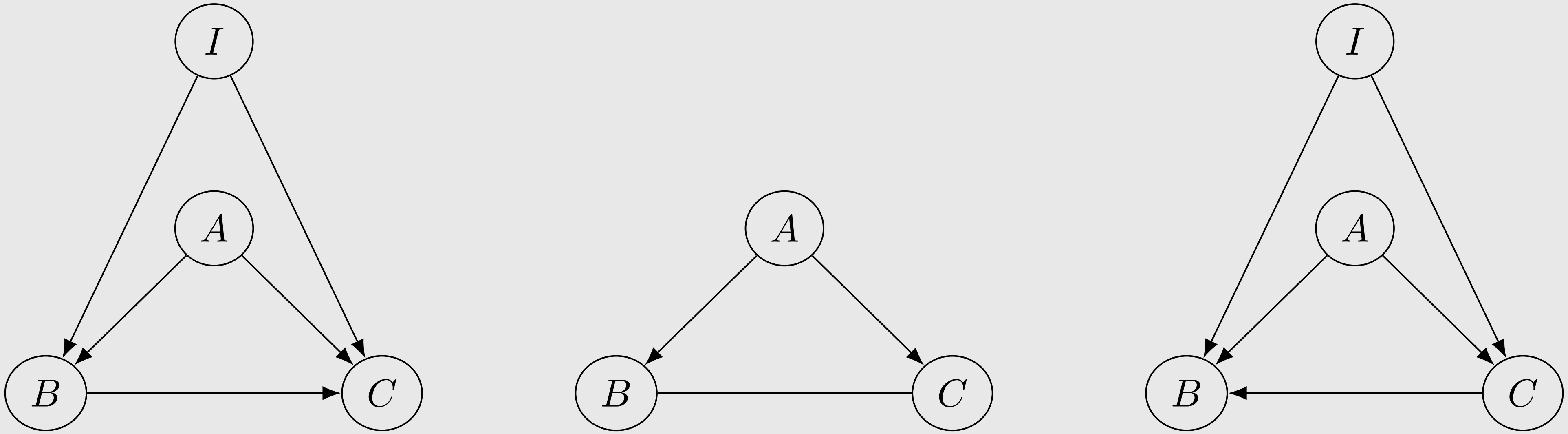
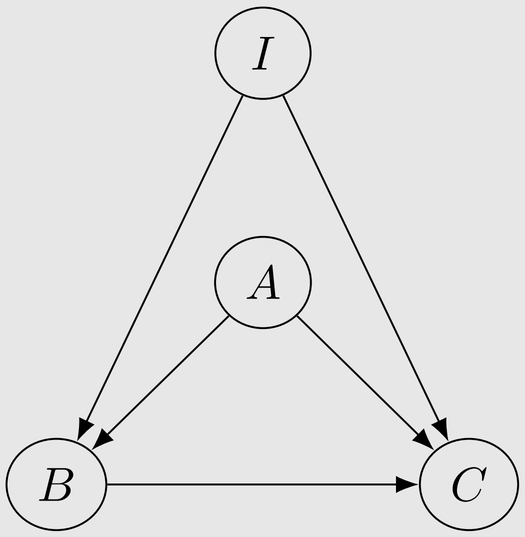
Structural/perfect analog from Hauser & Bühlmann (2012)



Questions

1. How many parametric single-node interventions are necessary and sufficient for identifying a directed acyclic graph?
2. What is the essential graph of the graph on the right (ignoring the intervention node)?
3. What is the interventional essential graph?
4. What is a graph that this graph is interventionally Markov equivalent to?



Structural Interventions

Single-Node Interventions

Multi-Node Interventions

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Interventional Markov Equivalence

Miscellaneous Other Settings

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- Randomized algorithms
  - Only need O(log log n) interventions with high probability (Hu et al., 2014)
  - Better than O(log c) from Eberhardt (2008) and Hauser & Bühlmann (2014)
- Intervene on at most k variables per intervention: Shanmugam et al. (2015)
  - Worst case: $\sim \frac{n}{2k}$
  - Randomized: $O\left(\frac{n}{k} \log \log k\right)$
- Only k interventions: Ghassami et al. (2018)
- Unobserved confounding: Kocaoglu et al. (2017)