Kolmogorov Forward Equation and its Application In Proving Dupire's Formula

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TODO

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1. Introduction

2. Kolmogorov Forward Equation

2.1. Background

2.2. Kolmogorov Forward Equation (KFE)

THEOREM 1 **(Kolmogorov Forward Equation)**. *The transition probability density $p(t,y|s,x)$ defined for $s<t$ as $P_{x,s}(X_t \in dy) = p(t,y|s,x)dy$ satisfies the partial differential equation, for every fixed $x \in \mathbb{R}^n$:*

$$\frac{\partial p(t,y|s,x)}{\partial t} = \mathcal{A}_t^* p(t,y|s,x), \ (t,y) \in (s,\infty) \times \mathbb{R}^n$$
$$\lim_{t \to s} p(t,y|s,x)dy = \delta(y-x)$$

where $\delta_x(\cdot)$ is the Dirac delta function.

Proof. TODO

2.3. Notes on KFE

3. Dupire's Formula: An Application of KFE

3.1. Introduction to Local Volatility

3.2. Dupire's Formula