University of Tennessee

Electrical Engineering & Computer Science Department

ECE 416 Digital Control

Q-Ball Lab

Due May 12, 2023

Q-Ball System Model:

The Q-Ball is controlled by four motors fitted with counter-rotating propellers. Thus there are 6 degrees of freedom in which the Q-Ball can move.

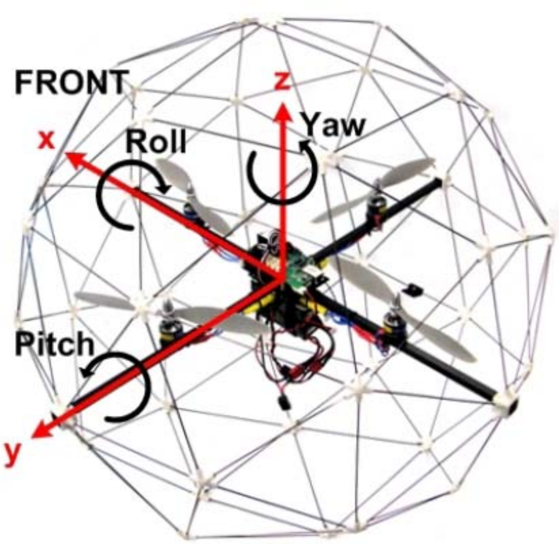

Figure 1: The Q-Ball with translational and rotational axes indicated.

Part I: Use equations below to derive a linearized state space model of the Q-Ball about *pitch=roll=0.0025*.

The nonlinear dynamic equations for the Q-Ball are:

Pitch and Roll:	$J\ddot{\theta} = FL$
Z-Axis:	$M\ddot{Z} = 4F\cos(r)\cos(p) - Mg$
X-Axis:	$M\ddot{X} = 4F\sin(p)$
Y-Axis:	$M\ddot{Y} = -4F\sin(r)$
Yaw:	$J_y\ddot{\theta}_y = K_y\Delta\tau$

Where r = roll angle and p = pitch angle.

The thrust generated by each propeller is modeled using the following first-order system

$$F = K\frac{\omega}{s+\omega}u \tag{1}$$

Where s is the Laplace variable (not to be confused with s below), u is the PWM input to the actuator, ω is the actuator bandwidth and K is a positive gain. These parameters were calculated and verified through experimental studies and are listed in the parameter table below. A state variable, v, will be used to represent the actuator dynamics, which is defined as follows

$$v = \frac{\omega}{s+\omega}u \text{ , where } s \text{ is the Laplace variable here} \tag{2}$$

$$J = J_{roll} = J_{pitch} \tag{3}$$

$$F = K\frac{\omega}{s+\omega}u \text{, where } s \text{ is the Laplace variable here} \tag{4}$$

$$J_y\ddot{\theta} = K_y\Delta\tau \tag{5}$$

$$\dot{v} = \omega\,(u - v) \tag{6}$$

For each axis add a state variable s where,

$\dot{s} = \theta$ for the Pitch Roll axes

$\dot{s} = Z$ for the Z axis

$\dot{s} = X$ for the X axis

$\dot{s} = Y$ for the Y axis

Derive the linearized state space systems for each of the axes to fill in the matrices below.

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{v} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ v \\ s \end{bmatrix} + \begin{bmatrix} \quad \\ \\ \\ \\ \end{bmatrix} u$$

Pitch and Roll

$$\begin{bmatrix} \dot{Z} \\ \ddot{Z} \\ \dot{v} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} Z \\ \dot{Z} \\ v \\ s \end{bmatrix} + \begin{bmatrix} \quad \\ \\ \\ \\ \end{bmatrix} u + \begin{bmatrix} \quad \\ \\ \\ \\ \end{bmatrix}$$

Z-axis

$$\begin{bmatrix} \dot{X} \\ \ddot{X} \\ \dot{v} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \\ v \\ s \end{bmatrix} + \begin{bmatrix} \quad \\ \\ \\ \\ \end{bmatrix} u$$

X-axis

$$\begin{bmatrix} \dot{Y} \\ \ddot{Y} \\ \dot{v} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} Y \\ \dot{Y} \\ v \\ s \end{bmatrix} + \begin{bmatrix} \quad \\ \\ \\ \\ \end{bmatrix} u$$

Y-axis

$$\begin{bmatrix} \dot{\theta}_y \\ \ddot{\theta}_y \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ & \end{bmatrix} \begin{bmatrix} \theta_y \\ \dot{\theta}_y \end{bmatrix} + \begin{bmatrix} \quad \\ \\ \end{bmatrix} u$$

Yaw-Axis

Note that the state 'v' represents the actuator dynamics, and 's' is augmented with the systems to incorporate the use of an integral type control when the control design problem is formulated.

System Parameters:
K = 120 N
ω = 15 rad/sec
J_{roll} = 0.03 kg* m^2
J_{pitch} = 0.03 kg* m^2
M = 1.4 kg
K_y = 4 N*m
J_{yaw} = 0.04 kg*m^2
L = 0.2 m

Part II: Is the linearized system controllable? Observable? (Read that part from the textbook. You can use the Matlab commands ctrl and obsv). Here take the output matrix to be the identity matrix.

Part III: Compute a linear quadratic controller (LQR) that achieve stability and reject disturbances using Matlab.

In this part, we want to design a state feedback controller, $u = -Kx + ref$

where K is gain of the feedback loop and u is the new command input. A new closed-loop state space equation is obtained by substituting u into the original state space equation, which is:

$$\dot{x} = (A - BK)x + B\,ref$$

In the project, LQR is employed to implement the control. The design procedure for the LQR feedback controller K is

1) Select design weight matrices Q and R.

2) Solve the algebraic Riccati equation for P.

3) Find the control gain using: $K = R^{-1}B^T P$.

This is all done in Matlab using the command $lqr(A, B, Q, R)$ to compute the gain K of the LQR controller by choosing the simplest weighs (the positive definite matrices) Q and R first.

Part IV: Simulate the state response of the system using Matlab and check if the system is stable.