10.4 $X \sim Exp(\lambda)$ ба $Y \sim U(0,1)$ $Z = X+Y$

$0 \le z < 1$ тохиолдолд

Хуниасын томъёо ашиглавал

$$f_{X+Y}(z) = \int_0^z f_X(z-y) f_Y(y) dy = e^{-\lambda z} \int_0^z e^{\lambda y} d(\lambda y) = 1 - e^{-\lambda z}$$

$z \geq 1$ тохиолдолд

$$f_{X+Y}(z) = e^{-\lambda z} \int_0^1 e^{\lambda y} d(\lambda y) = e^{-\lambda z}(e^{\lambda} - 1)$$

Иймээс

$$f_{X+Y}(z) = \begin{cases} e^{-\lambda z}(e^{\lambda}-1), & 1 \le \\ 1 - e^{-\lambda z}, & 1 \le z < 1 \\ 0, & z < 0 \end{cases}$$

Нэмэлт

10.1 $f_{X+Y}(z=0) = \sum_{k \in \{0\}} f_X(z-k) f_Y(k) = p^0 (1-p)^1 q^0 (1-q) = (1-p)(1-q)$

$$f_{X+Y}(z=1) = p^1(1-p)^0 q^0 (1-q)^1 + p^0 (1-p)^1 q^1 (1-q)^0 = p(1-q) + q(1-p)$$

$$f_{X+Y}(z=2) = p^1(1-p)^0 \cdot q^1 (1-q)^0 = pq$$

Иймээс

$$f_Z(z) = \begin{cases} pq & z=2 \\ p(1-q) + q(1-p) & z=1 \\ (1-p)(1-q) & z=0 \\ 0 & \text{бусад үед} \end{cases}$$

10.6 $\bar{S}_n = \dfrac{X_1 + \dots + X_n}{n}$ иймээс $X \sim N(\mu, \sigma^2)$ $Y \sim N(a + b\mu, b^2\sigma^2)$

$$S_n = X_1 + \dots + X_n = 0 + n\bar{S}_n = a + b\tilde{S}_n \sim N(n\mu, n\sigma^2)$$

10.3 $X \sim U(0,1) \quad Y \sim U(0,2) \quad Z = X + Y$

$$X \sim U(0,1) \quad f_X(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x \notin [0,1] \end{cases}$$

$$Y \sim U(0,2) \quad f_Y(y) \begin{cases} 1/2 & 0 \le y \le 2 \\ 0 & y \notin [0,2] \end{cases}$$

$$f_Z(z) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{z-y} f_X(x)\,dx \right) f_Y(y)\,dy \Rightarrow$$

$$= f_{X+Y}(z) = \int_0^z f_X(z-y)\, f_Y(y)\,dy$$

Үүнээс

$$f_{X+Y}(z) = \begin{cases} \frac{z}{2}, & 0 < z < 1 \\ 1/2, & 1 < z < 2 \\ \frac{3-z}{2}, & 2 < z < 3 \\ 0, & \text{бусад завсарт} \end{cases}$$