7.3 а) $Y = \sqrt{X}$

$X \sim Exp(\lambda)$

$y \sim Exp(\lambda)$

xy - хүрэлцэх магадлал (доголдох)

$X \sim Exp(\lambda) \Rightarrow E(x) = \frac{1}{\lambda} = 7 \quad \lambda = \frac{1}{7}$

$f_x(x) = f_y(x) = \lambda e^{-\lambda x} = \frac{1}{7} \cdot e^{-x/7} \quad x > 0$

$F_x(x) = F_y(x) = 1 - e^{-\lambda x} = 1 - e^{-x/7}$

б) $Z = \max(X, Y)$ → Серверийн магадлах хуваарь

$X \sim Exp(\lambda)$

$y \sim Exp(\lambda)$ $\Rightarrow F_Z(z) = (1 - e^{-z/7})^2$

7.2 а) $Y = \sqrt{x}$

$g(x) = \sqrt{x}$

$F_y(x) = P(y < x) = P(\sqrt{x} < x) = P(x < x^2) = F_x(x^2)$

$= 1 - e^{-\lambda x^2}$

$f_y(y) = (F_y(x))' = (1 - e^{-\lambda x^2})' = 2\lambda x e^{-\lambda x^2}, \quad x > 0$

б) $g(x) = \frac{1}{\lambda} \ln x = g^{-1}(y) = e^{\lambda y} \quad (e^{\lambda y})' = \lambda e^{\lambda y}$

$f_y(y) = \lambda e^{-\lambda e^{\lambda y}} \lambda e^{\lambda y} = \lambda^2 e^{\lambda(y - e^{\lambda y})} \quad y \in \mathbb{R}$

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7.1.a $Y = 2X - 1$

x	-2	-1	0	1	2
$f_x(x)$	0.1	0.2	0.3	0.3	0.1

$g(x) = 2x - 1$	-5	-3	-1	1	3
$f_x(x)$	0.1	0.2	0.3	0.3	0.1

$P(Y=-5) = P(2x-1=-5) = P(X=-2) = 0.1$

$P(Y=-3) = P(X=-1) = 0.2$

$P(Y=-1) = P(X=0) = 0.3$

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$P(Y=1) = P(X=1) = 0.3$

$P(Y=3) = P(X=2) = 0.1$

b) $Y = x^2 + 1$

x	-2	-1	0	1	2
$g(x)$	5	2	1	2	5
$f_x(x)$	0.1	0.2	0.3	0.3	0.1

$P(Y=5) = (P X=\pm 2) = 0.1+0.1 = 0.2$

$P(Y=2) = P(X=\pm 1) = 0.2+0.3 = 0.5$

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$P(Y=1) = P(X=0) = 0.3$

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7.4 $f_x(x) = \frac{1}{\pi(1+x^2)}$

а) $y = 1/x$

$$F_x(x) = \int_{-\infty}^{x} f_x(x)\,dx = \int_{-\infty}^{x} \frac{1}{\pi(1+x^2)}\,dx$$

$$= \int_{-\infty}^{x} \frac{1}{\pi(1+x^2)}\,dx = \frac{1}{\pi}\int_{-\infty}^{x} d(\operatorname{arctg} x) = \frac{1}{\pi}\left(\operatorname{arctg} x + \frac{\pi}{2}\right)$$

$$F_y(y) = P(y < x) = P(1/x < x) = 1 - F_x\left(\frac{1}{x}\right)$$

$$F_y(y) = \frac{1}{1} - \frac{1}{\pi}\left(\operatorname{arctg} 1/x + \pi/2\right) = \frac{1}{2} - \frac{1}{\pi}\operatorname{arctg} 1/x,$$

б) $y = \operatorname{arctg} x$

$$F_y(y) = \frac{1}{\pi(1+\operatorname{tg}^2 y)} \cdot \frac{1}{\cos^2 y}$$