

$$9.5 \quad E(Y|X) = 7 - \frac{X}{4} \quad \delta_a \quad E(X|Y) = 10 - Y \quad \delta_{01} \quad p(X, Y) \quad 0.1$$

$$E(X) = E(10) - E(Y)$$

$$E(Y) = E(7) - E\left(\frac{X}{4}\right)$$

$$\left\{ \begin{array}{l} E(X) + E(Y) = 10 \\ E(X) = 4 \end{array} \right.$$

$$\left\{ \begin{array}{l} E(Y) + E\left(\frac{X}{4}\right) = 7 \\ E(Y) = 6 \end{array} \right.$$

Homework

$$9.1 \quad a) \operatorname{cov}(X, X) = D(X)$$

$$\operatorname{cov}(X, X) = E(X^2) - E(X)E(X) = E(X^2) - (E(X))^2$$

$$\Rightarrow \operatorname{cov}(X, X) = D(X)$$

$$b) \operatorname{cov}(X, Y) = 0$$

$$\operatorname{cov}(X, Y) = E(XY) - E(X)E(Y) = E(X)E(Y) - E(X)E(Y) = 0$$

$$9.3 \quad X_2 = E(X_2|X_1) + U \quad \delta_{01} \quad E(U) = 0$$

$$E(E(X_2|X_1)) = \int E(X_2|X_1) f_{X_1}(x_1) dx_1 =$$

$$\int x_2 \left(\int f_{X_2|X_1}(x_2, x_1) dx_1 \right) dx_2 = \int x_2 f_{X_2}(x_2) dx_2 = E(X_2)$$

$$X_2 = E(X_2|X_1) + U \Rightarrow E(X_2) = E(E(X_2|X_1) + U)$$

$$\Rightarrow E(X_2) = E(E(X_2|X_1)) + E(U) \quad \Rightarrow E(U) = 0$$

9.2 $x = (x_1, x_2, x_3)$

$\mu = (1, 2, -1)$

$\Sigma = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{pmatrix}$ $f_{x_3|x_1, x_2}(x_3|x_1, x_2) = ?$

$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{pmatrix}$

$f_{x_3|x_1, x_2}(x_3|x_1, x_2) = \frac{1}{\sqrt{6\sqrt{2\pi}}} \cdot e^{-\frac{(x_3 - \mu_3)^2}{6}}$

4.4.2020 a-2 enbel

$\omega = \mu_2 + \Sigma_{21} \Sigma_{11}^{-1} ((x_1, x_2) - (1, 2)) = -1 + \frac{1}{6} \cdot 0 \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} = -1$

$\theta = 1$

$f_{x_3|x_1, x_2}(x_3|x_1, x_2) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(x_3 - \mu_3)^2}{2}}$

9.4 $\Sigma = \begin{pmatrix} D(x_1) & \text{cov}(x_1, x_2) \\ \text{cov}(x_2, x_1) & D(x_2) \end{pmatrix}$

$\Sigma_{21} = \text{cov}(x_2, x_1)$
 $\Sigma_{11} = D(x_1)$

$f(x_1, x_2) = \frac{\text{cov}(x_1, x_2)}{D(x_1) D(x_2)}$

$\Sigma_{21} \Sigma_{11}^{-1} = \text{cov}(x_2, x_1) \cdot (D(x_1))^{-1} = \frac{\text{cov}(x_2, x_1)}{D(x_1)} \Rightarrow$

$= \frac{\text{cov}(x_1, x_2)}{D(x_1)} \cdot \frac{\sqrt{D(x_2)}}{\sqrt{D(x_1)}} = \frac{\text{cov}(x_1, x_2)}{\sqrt{D(x_1) D(x_2)}} \cdot \frac{\sqrt{D(x_2)}}{\sqrt{D(x_1)}}$

$E(x_2|x_1) = x_1 = \mu_2 + \frac{\text{cov}(x_1, x_2)}{\sqrt{D(x_1)}} \cdot \frac{\sqrt{D(x_2)}}{\sqrt{D(x_1)}} \cdot (x_1 - \mu_1)$