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Exercise sheet 5
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Neurodynamics 2023

Homework: The Izhikevich model of spiking neurons.

In this last exercise of the course, we will explore the connection of different spiking behaviors to different phase plan geometries. In particular, we will be looking at the simple 2-D spiking model Izhikevich proposed in 2003 [1]. In only two dynamical equations and 4 parameters, the model manages to reproduce a wide variety of different spiking behavior. **Hand-in for this sheet is end of day on June, 23rd**

Question 1. We are looking at the Izhikevich model as presented in the original publication:

$$\begin{aligned}\dot{V} &= 0.04V^2 + 5V + 140 - u + I \\ \dot{u} &= a(bV - u) \\ &\text{if } V \geq 30, \text{ then } V \leftarrow c \text{ and } u \leftarrow u + d\end{aligned}$$

with four parameters a, b, c and d. Here, we are treating the entire equation as dimensionless, even though the specific coefficient form is chosen so that V is in a typical range for real membrane potentials.
Make sure that you can solve the system in `brian2` given a set of parameters, an initial condition, and a stepcurrent I.
Next, also make sure that you can plot the model's nullclines in a phase portrait.

Question 2. For each of the following parameterisations: plot the nullclines in the phase plane, the

- Simulate for 300ms, switching on the step current to the specified value at $t = 10$ms for the rest of the experiment. The starting value for V will be given, the starting value for u is the asymptotic $u_o = b * V$.
- Plot the trace of V over time.
- For up to a maximum of three spikes, plot the trajectories in phase space.

Here are the four parameter sets to look at:

1) $a = 0.02$, $b = 0.2$, $c = -65$, $d = 6$, with $V_0 = -70$ and the applied current $I = 14$
2) $a = 0.02$, $b = 0.25$, $c = -65$, $d = 6$, with $V_0 = -64$ and the applied current $I = 0.5$
3) $a = 0.02$, $b = 0.2$, $c = -50$, $d = 2$, with $V_0 = -70$ and the applied current $I = 15$
4) $a = 0.02$, $b = 0.25$, $c = -55$, $d = 0.05$, with $V_0 = -64$ and the applied current $I = 0.6$

Describe the qualitative differences in spiking behavior for each case, and how they are reflected in phase space.

References

[1] Eugene M Izhikevich. Simple model of spiking neurons. *IEEE Transactions on neural networks*, 14(6):1569–1572, 2003.