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Exercise sheet 4
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Neurodynamics 2023

HOMEWORK: STABILITY IN 2-D DYNAMICAL SYSTEMS.

To explore the mathematical tools of stability analysis via linearization, we will be looking at two systems outside neuroscience in this homework sheet: The Lotka-Volterra model of competition and the Lotka-Volterra predator-prey model. **Hand-in for this sheet is end of day on June, 16th**

Question 1. First, take a look at the competitive Lotka-Volterra model.

$$\dot{x} = r_x x(1 - \frac{x + a_{xy}y}{K_x})$$
$$\dot{y} = r_y y(1 - \frac{y + a_{yx}x}{K_y})$$

Think of x as a population of rabbits, and y as a population of sheep – $x, y \leq 0$ –, with respective growth rates r_x and r_y. Individually, each population can grow logistically to a capacity K_x or K_y, but they compete for the same food resources, grass, modelled the terms $a_{xy}y$ and $a_{yx}x$.
Parameters are: $r_x = 3$, $r_y = 2$, $K_x = 3$, $K_y = 2$, $a_{xy} = 2$, $a_{yx} = 1$.
Give a full phase portrait analysis of the system including:

- The nullclines of the system.
- Fixed Points of the system.
- A stability analysis of the fixed points of the system via linearization.

For each, show how you arrived at each answer analytically in the notebook file by using a markdown cell and latex mathematics syntax. Plot the nullclines, indicate fixed points in the plot, and indicate the flow of the vector field. Give an interpretation of the fixed points of the system.
Pick 2 exemplary starting values with different dynamic behavior and simulate the two trajectories. Plot them both in phase space and the solutions for x and y independently as functions of time.

Question 2. Next, investigate the Lotka-Volterra predator-prey model.

$$\dot{x} = \alpha x - \beta xy$$
$$\dot{y} = \delta xy - \gamma y$$

Here, x are rabbits, but y are foxes, eating rabbits. Rabbits naturally reproduce at a rate α, but get eaten according at a rate depending on the number of foxes and β. The reproduction rate of the fox population depends on the number of available rabbits and δ, and on the rate of death in the fox population γ.
Please note, that this model makes a number of strong, simplifying assumptions and isn't applicable to real predator-prey systems. Nevertheless, the system is a great toy model.
$x, y \leq 0$ and parameters are: $\alpha = 0.1, \beta = 0.02, \gamma = 0.4, \delta = 0.02$.
Again, give a full phase portrait analysis of the system including:

- The nullclines of the system.
- Fixed Points of the system.
- A stability analysis of the fixed points of the system via linearization.

For each, show how you arrived at each answer analytically in the notebook file by using a markdown cell and latex mathematics syntax. Plot the nullclines, indicate fixed points in the plot, and indicate the flow of the vector field. Give an interpretation of the fixed points of the system.
Pick 1 exemplary starting value (for example $(x = 10, y = 10)$ and simulate the trajectory. Plot it in phase space and also the solutions for x and y as functions of time.