Signal Processing

Action & Cognition: Computational Cognition
Online Course 2022

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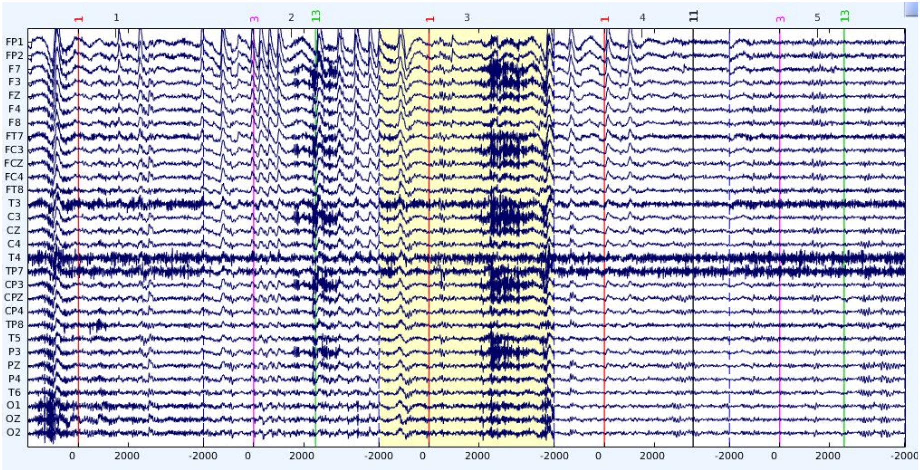
It’s All About The Sensors

Pupil Player: 2018_06_18 - pi_data
CPU
58 FPS
id0 conf: 1.00
id1 conf: 1.00
Pupil Diameter [px]
left
right
Pupil Confidence
left
right
Recorded FPS
world FPS
01:56.951 - 6355
0.25x00:00.000
23:24

It’s All About The Sensors

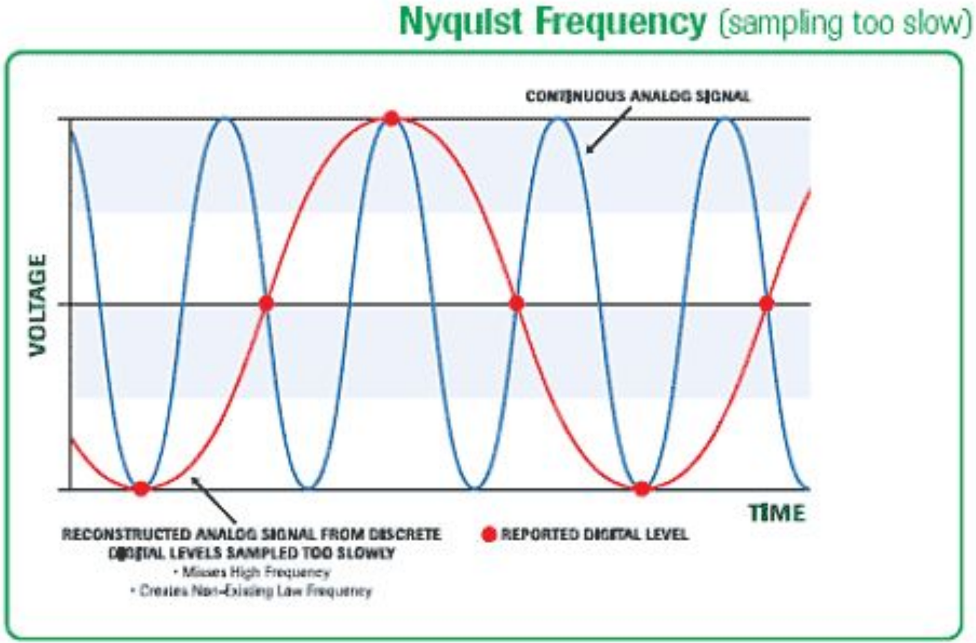


Sensors measure physical, continuous quantities.

Sampling & Nyquist Frequency

Sampling frequency Fs: Number of times a sample is taken from a signal. Measured in Hz.

Nyquist Freq is twice the highest frequency contained in your signal. If the sampling frequency is lower than nyquist freq then we lose information about the original signal. This is called *aliasing*.



Types of Signals

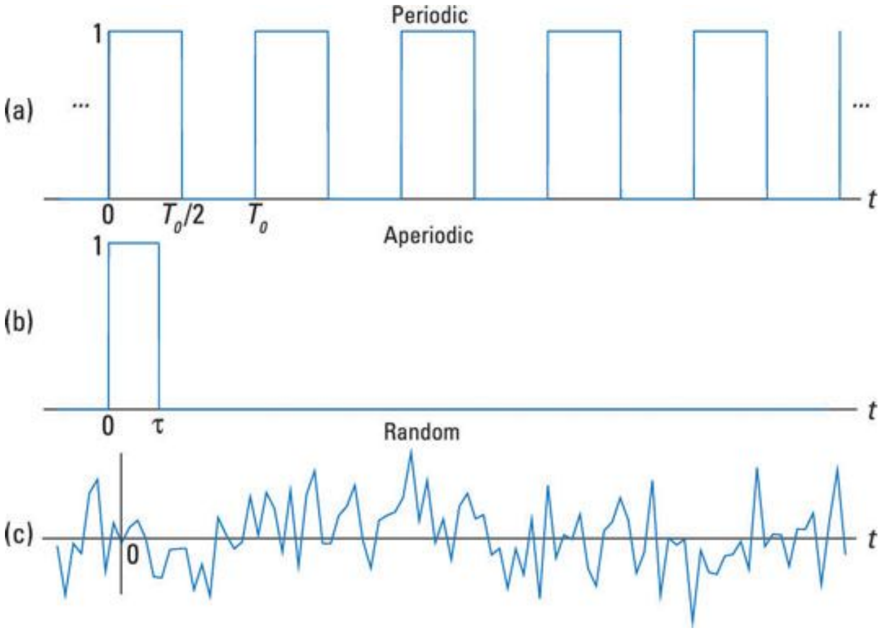
Static:

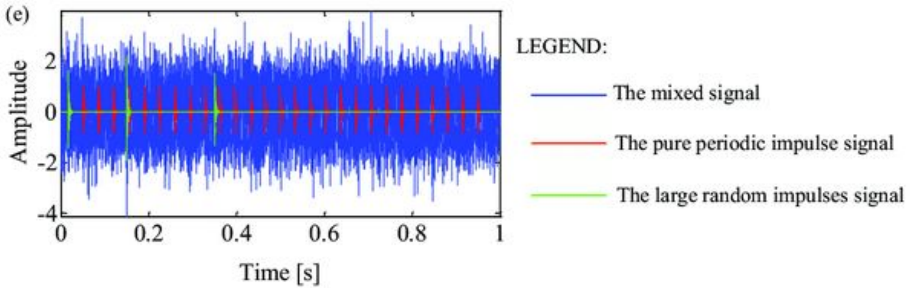
- Where temporal information is not important
- The output at time **t** is only dependent on the input at time **t**
- Measuring body weight or temperature

Dynamic:

- The output at time **t** depends on the input at time **t** as well as the input at other times (i.e. in the past or future)
- A temporal structure is always maintained

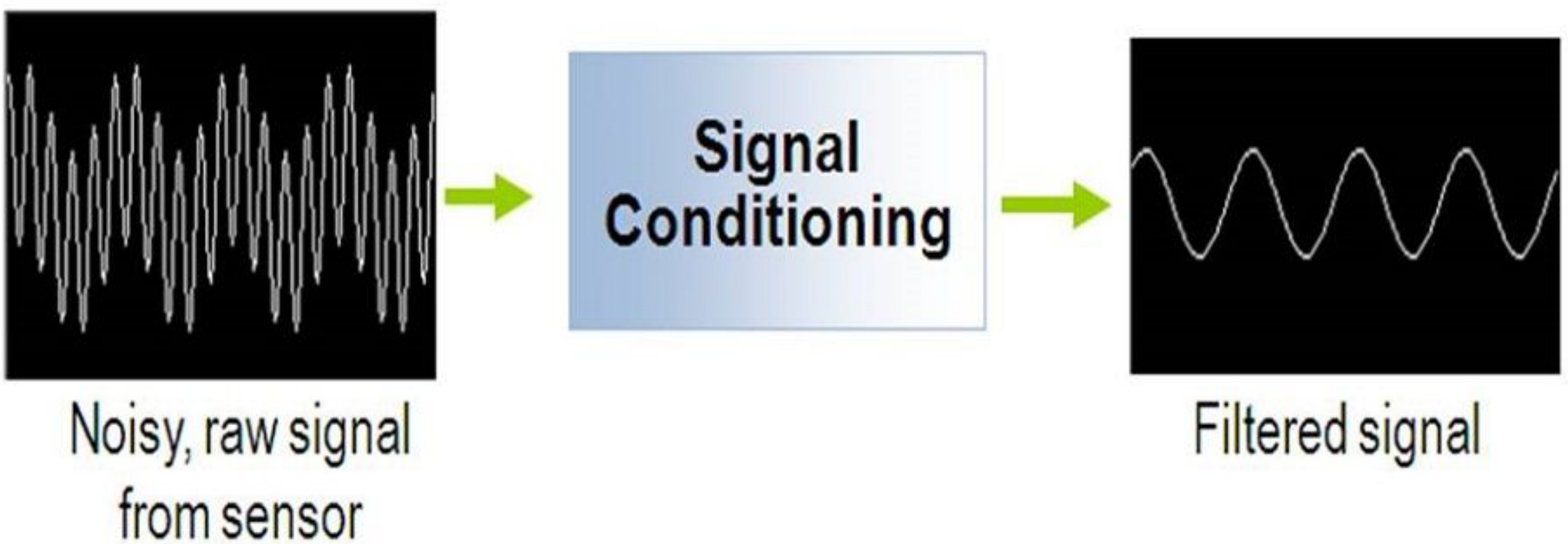
Types of Signals





For our purpose, we assume a given signal is composed of finite number of periodic signals with some random noise that we cannot account for. Most of signal processing involves doing *some detective work* on the signal regularities.

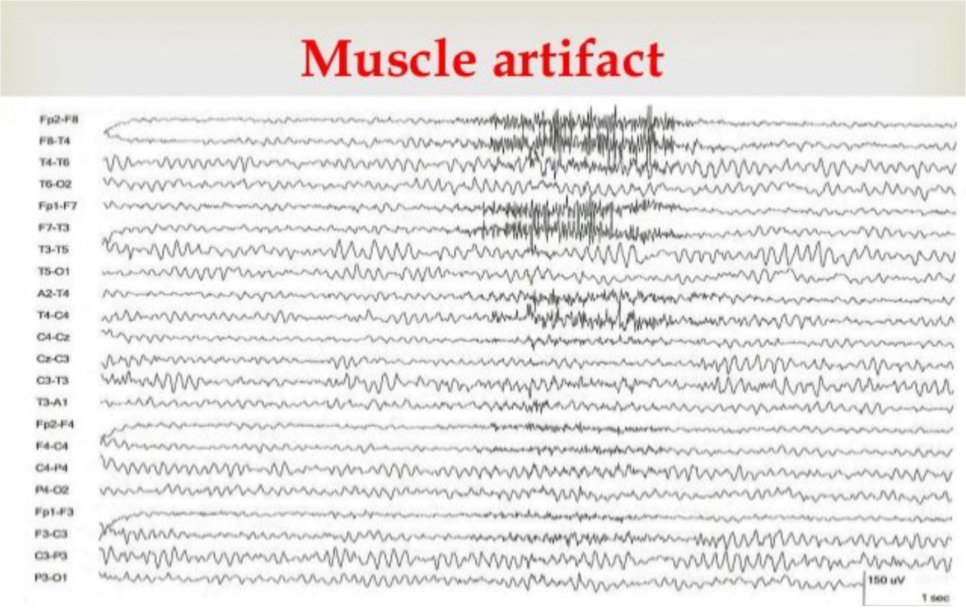
Signal Conditioning



Signal out of the sensor is usually not readily usable and we need to either filter it in post-processing or calibrate our sensors before measurements. We try to do both.

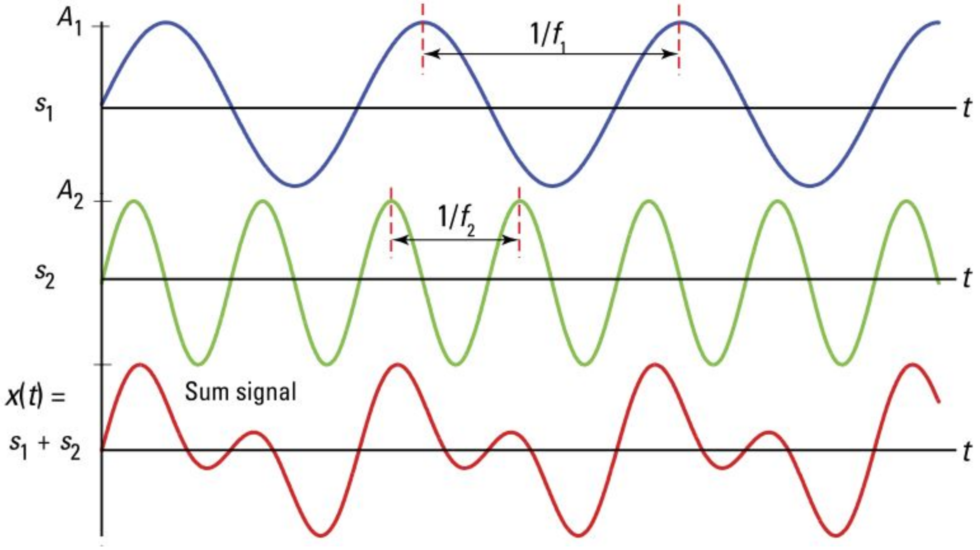
Dealing with Noise

- Filtering the data
  - Remove components by eyeballing
  - Remove outliers by calculating summary statistics on the data
  - Transform time domain data to frequency domain and check for “irregularities” there.



Describing a Signal

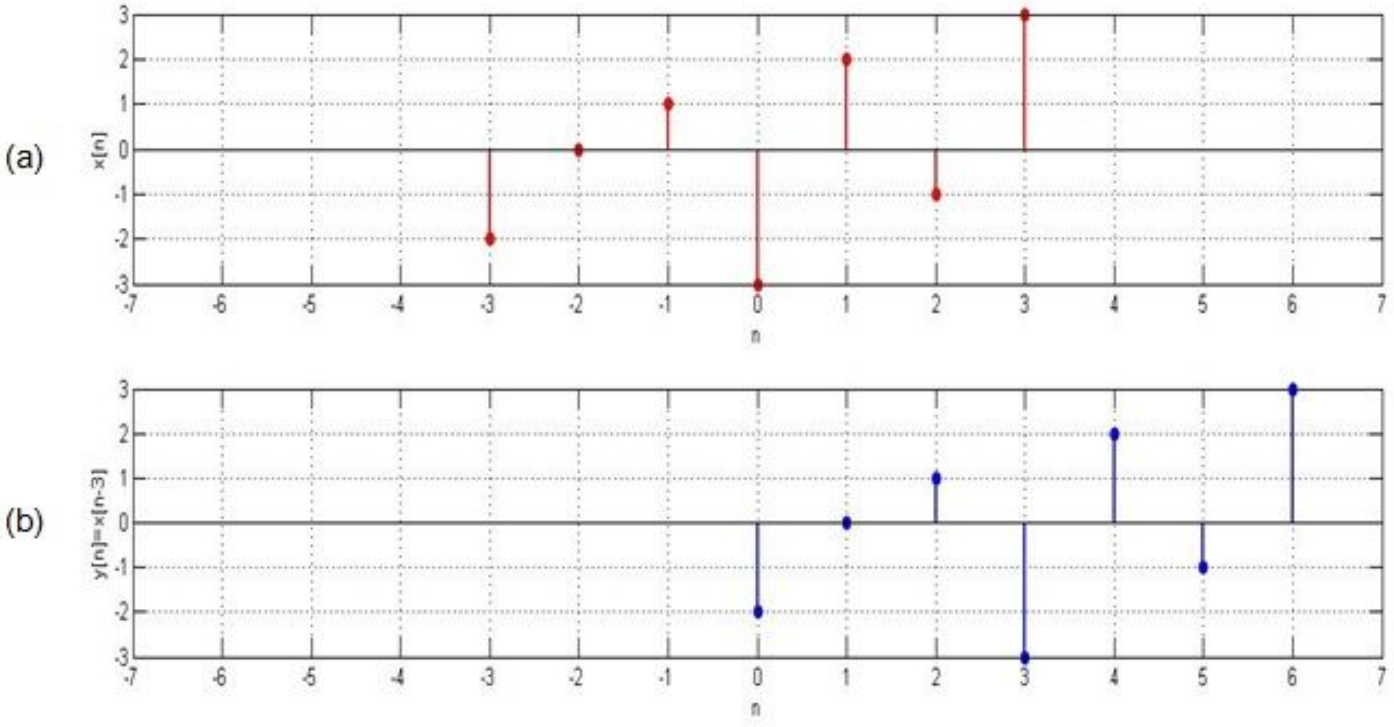
Mathematically, a signal can be described as a sum of sinusoids of different frequencies.



$$y(t) = A\sin(2\pi ft + \varphi) = A\sin(\omega t + \varphi)$$

Basic Operations On A Signal

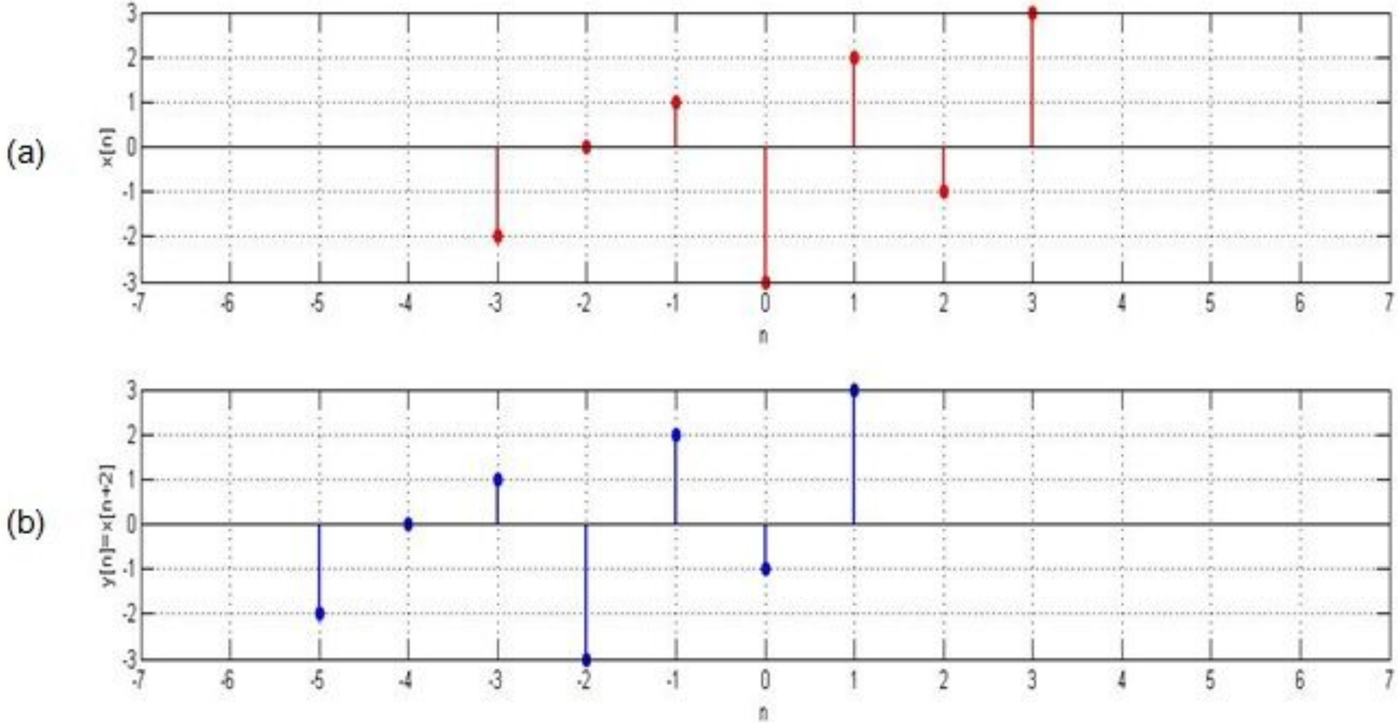
Time
Delayed
Signal



We’re only shifting the signal here. There’s no change in the amplitude or span of the signal

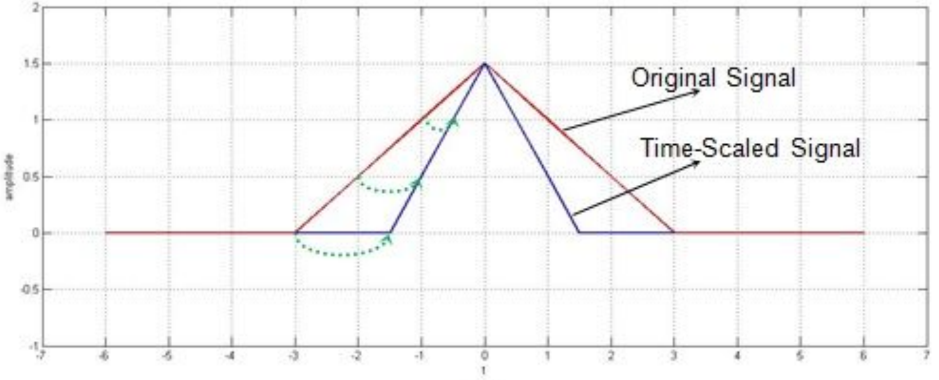
Basic Operations On A Signal

Time Advanced Signal

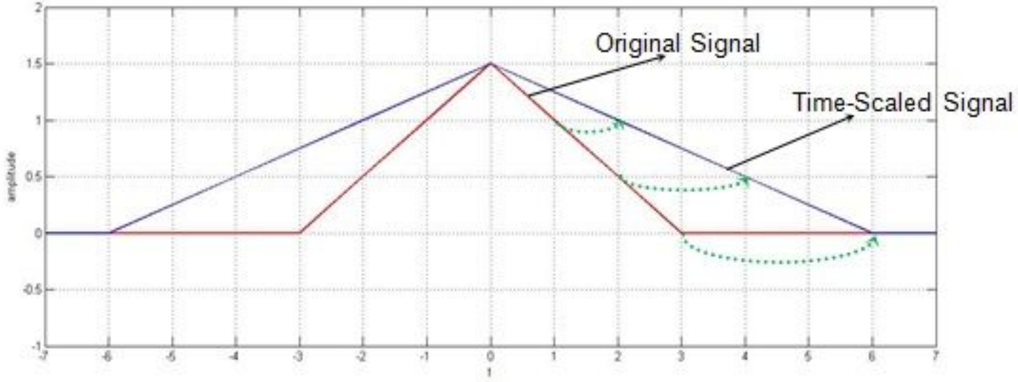


Basic Operations On A Signal

Time Scaling



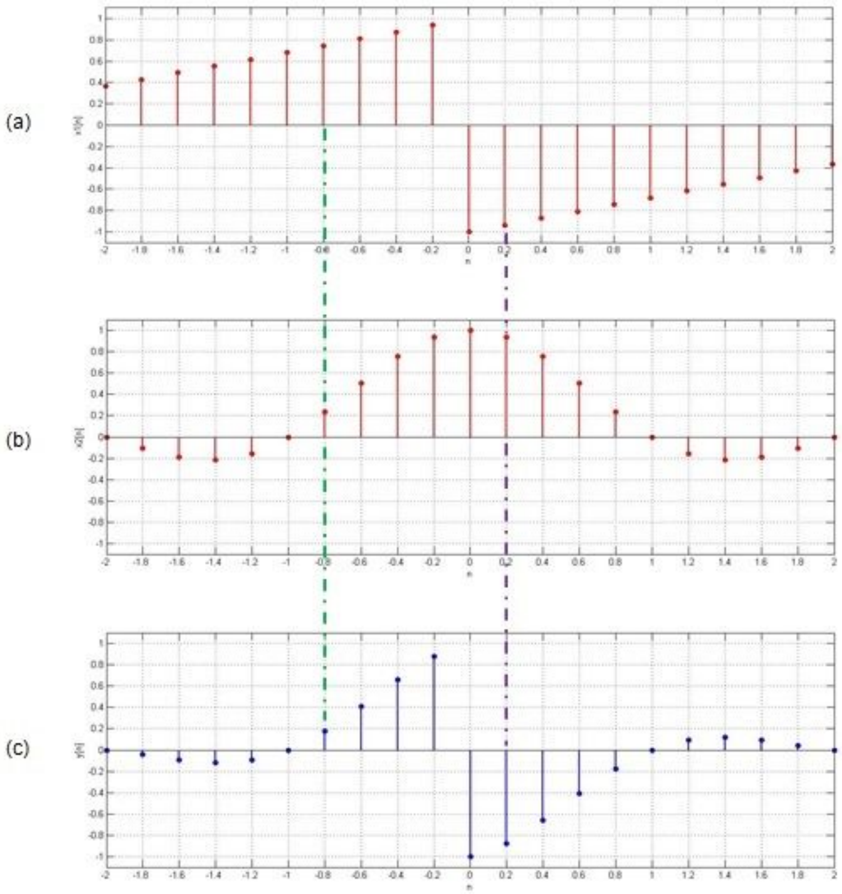
(a)



(b)

Basic Operations On A Signal

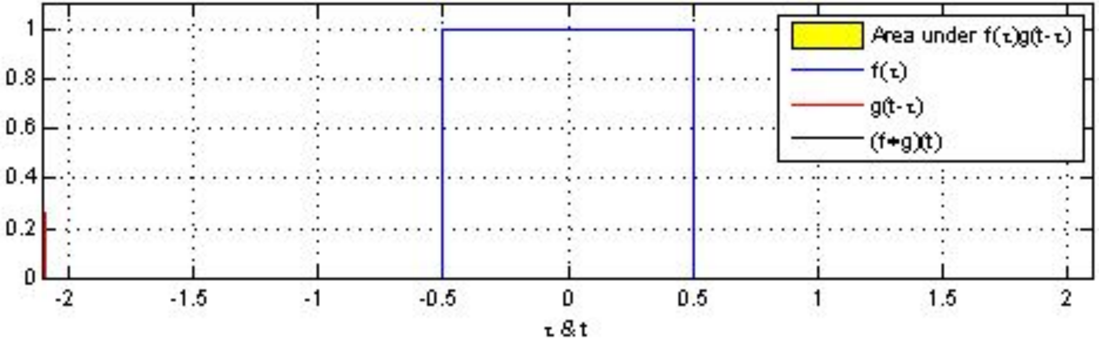
Multiplication

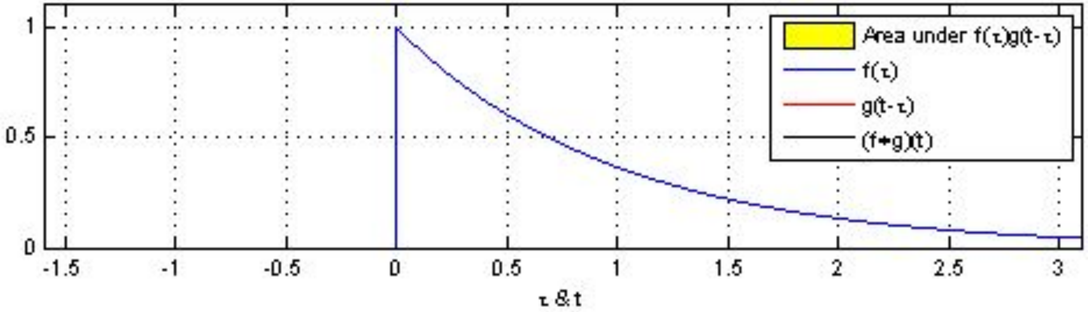


Basic Operations On A Signal

Convolution

$$f(t) * g(t) \triangleq \underbrace{\int_{-\infty}^{\infty} f(\tau) g(t-\tau)\, d\tau}_{(f*g)(t)},$$

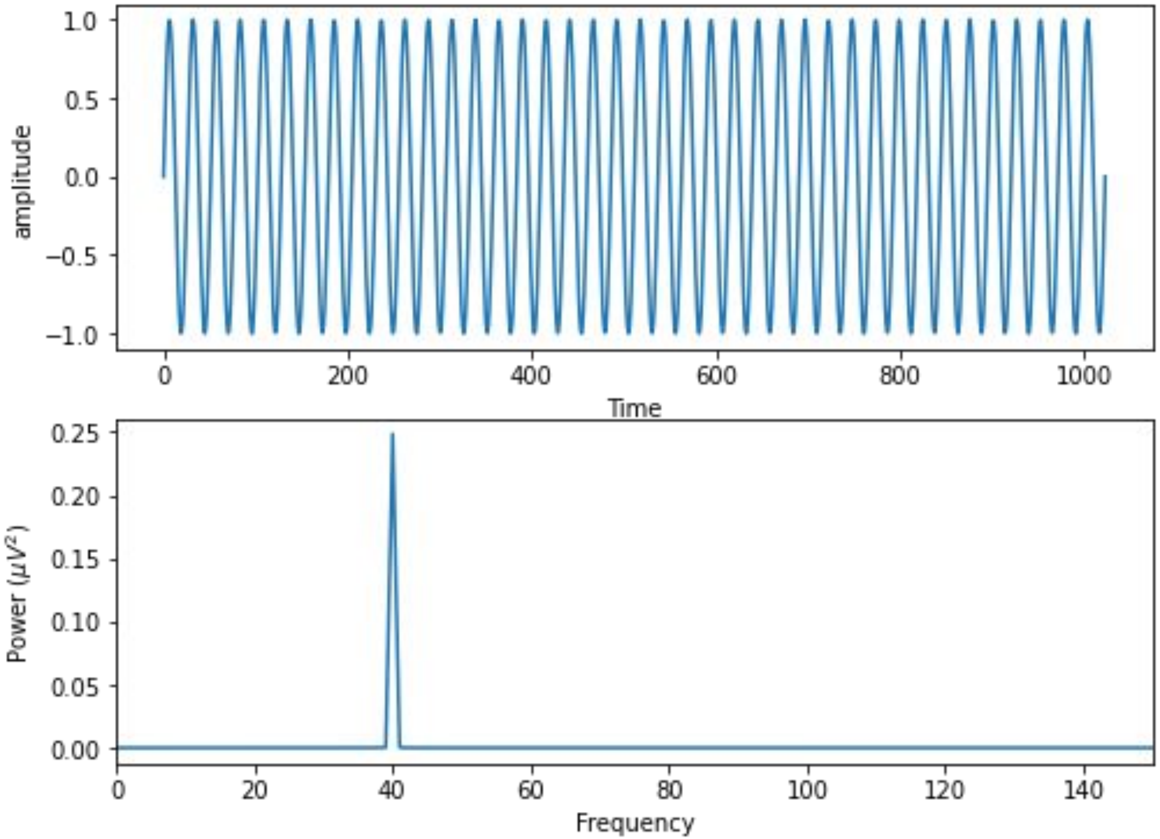




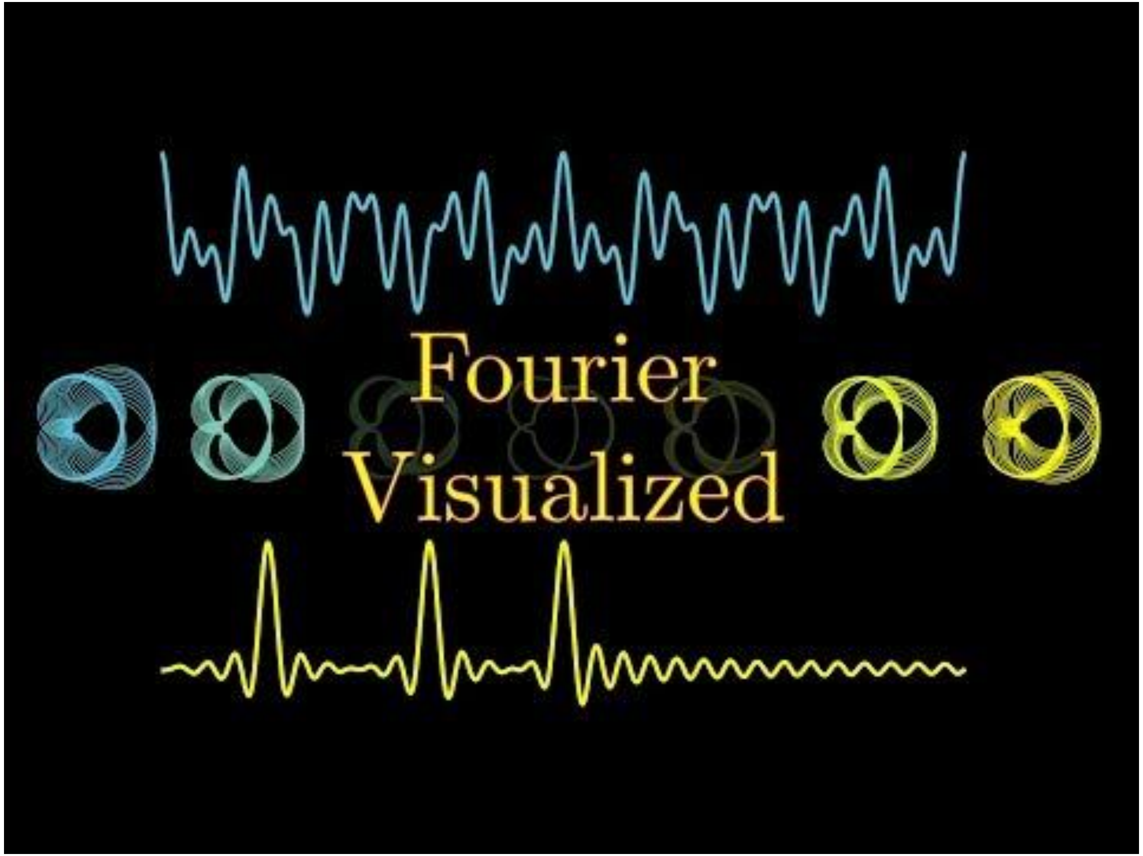
Fourier Transform

Instead of looking at a signal from the time domain, we can learn more about it (often) by looking at its decomposition in frequency domain

Signal in time and frequency domain



Fourier Transform

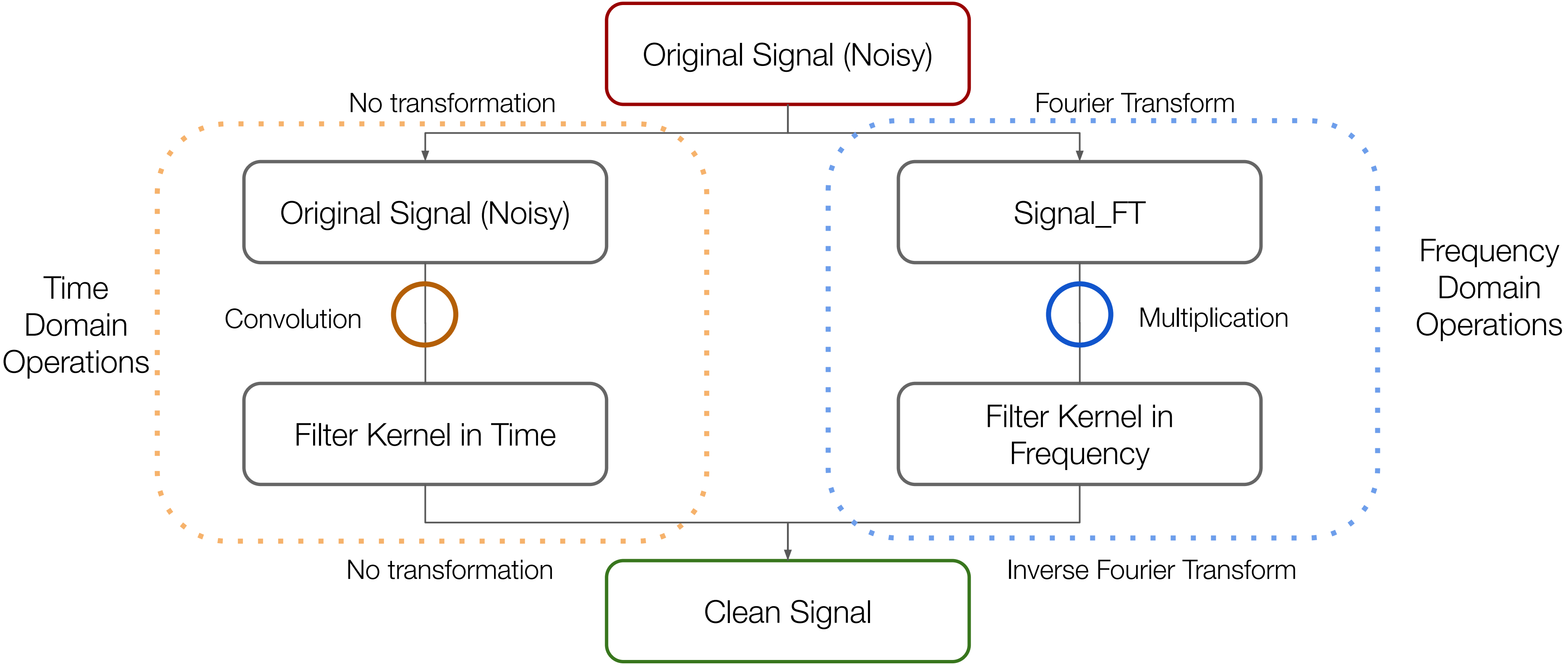


Relationship between Time and Freq. Domain

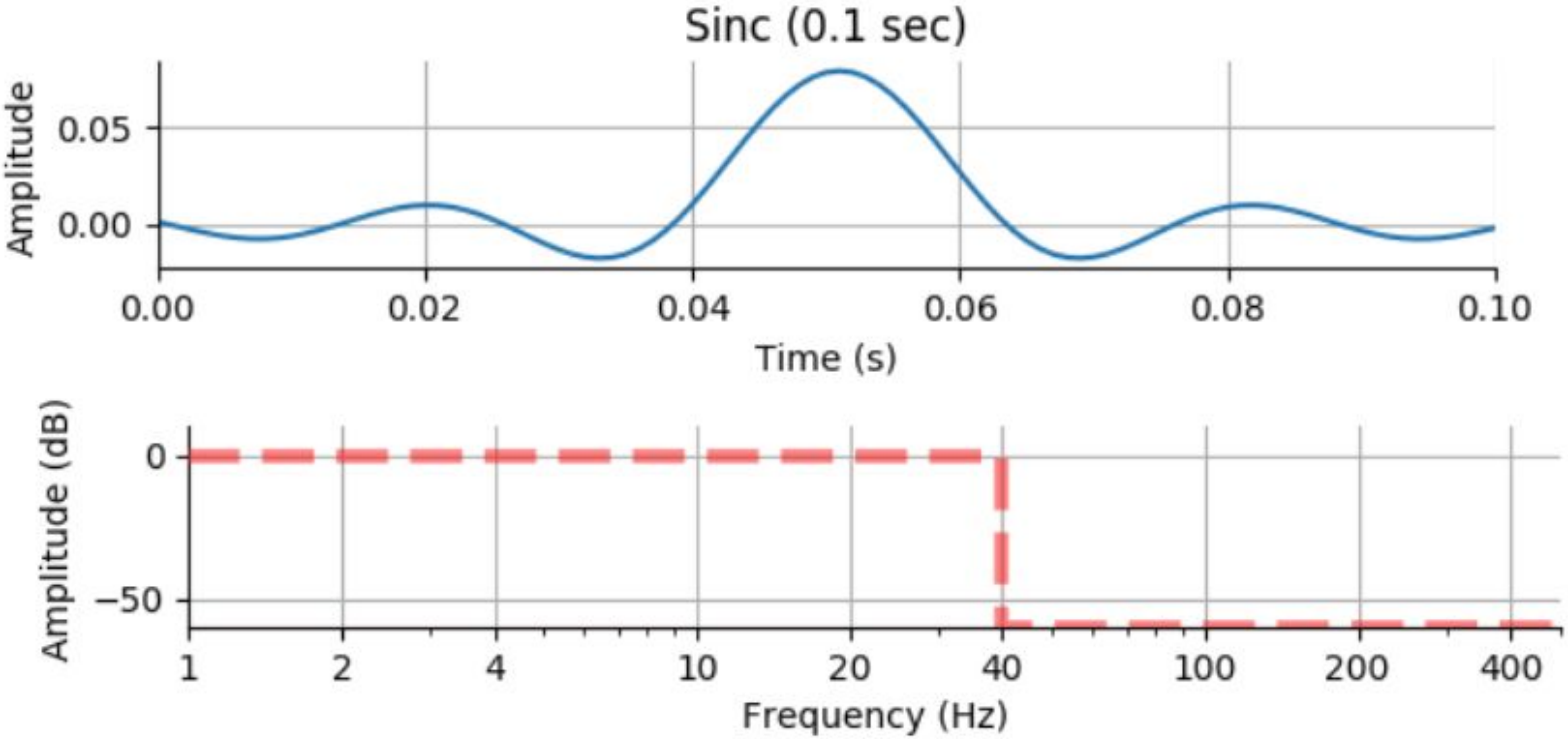
TABLE 7.2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\, du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

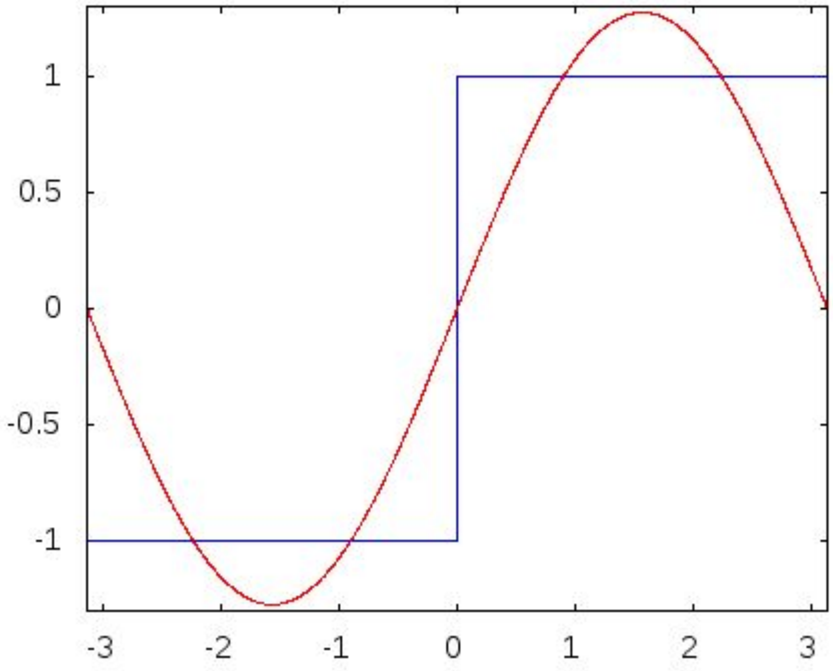
Filtering The Data Using What We Know Now



Filtering The Data Using What We Know Now



A Filter <--> Step Function



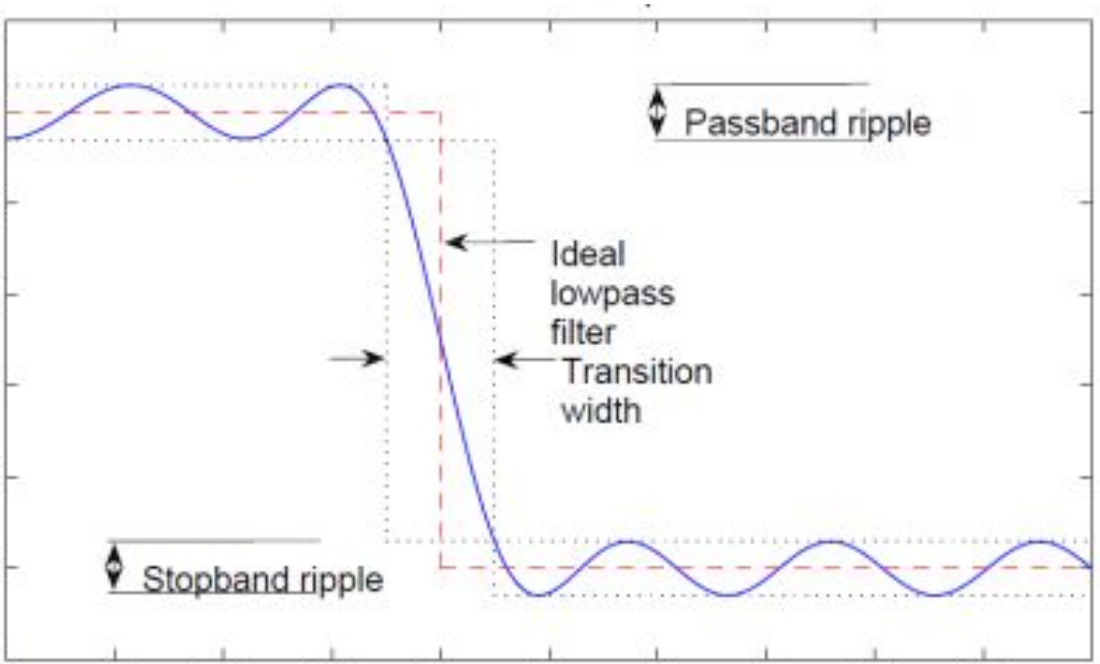
A step function

Fourier approximation of a step function with increasing harmonics

An ideal square wave has instantaneous transitions between the high and low levels. In practice, this is never achieved because of physical limitations of the system that generates the waveform

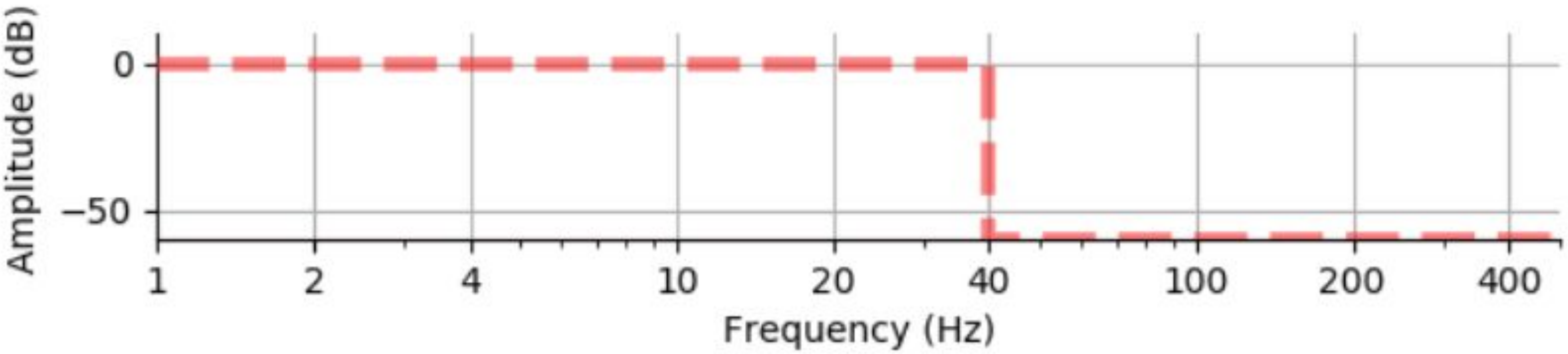
Filtering Keywords

- Cut off frequency
- Pass band gain
- Stop band gain
- Transition width
- Half amplitude cutoff frequency

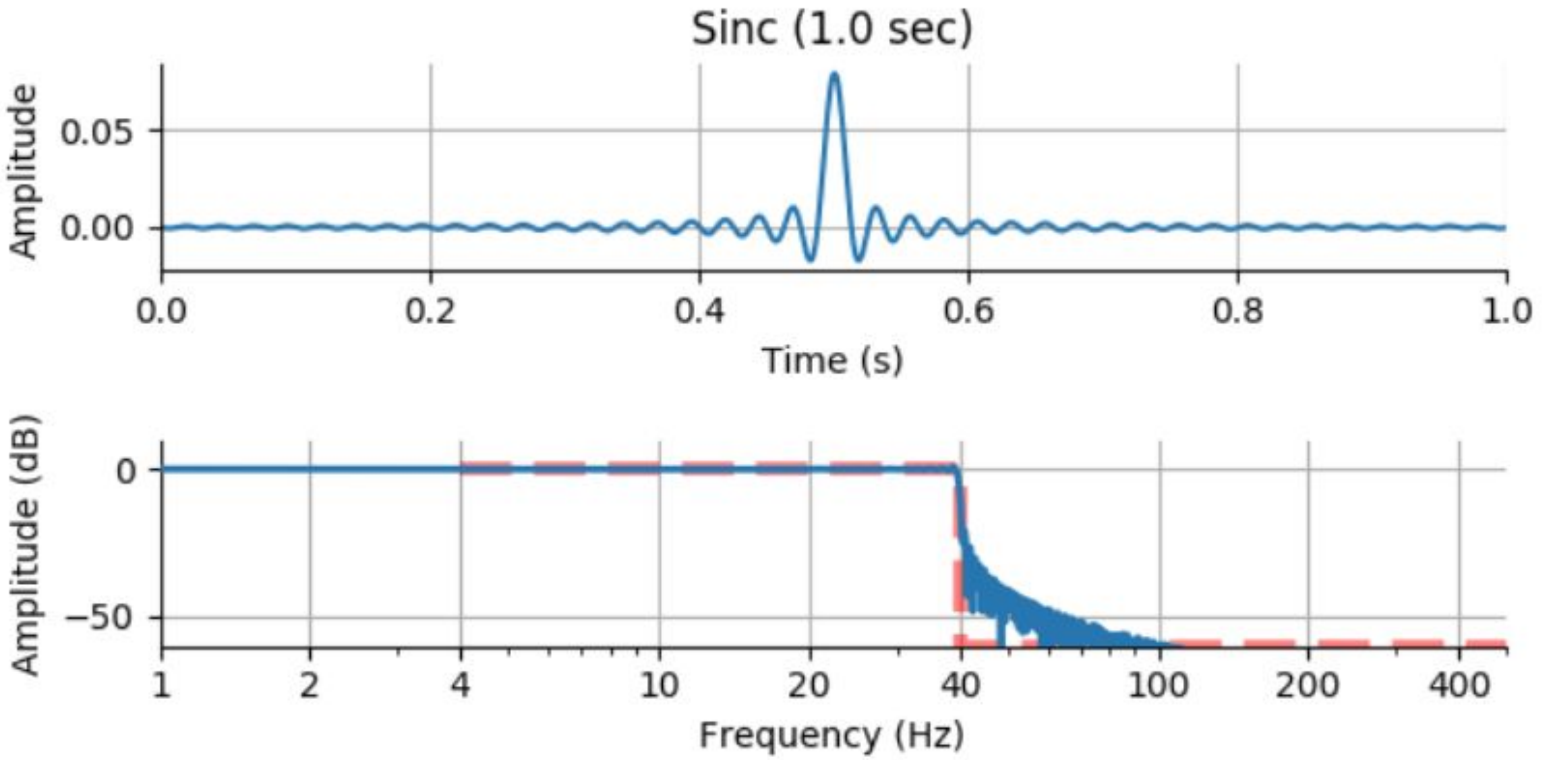


Examples

Ideal Low Pass filter:

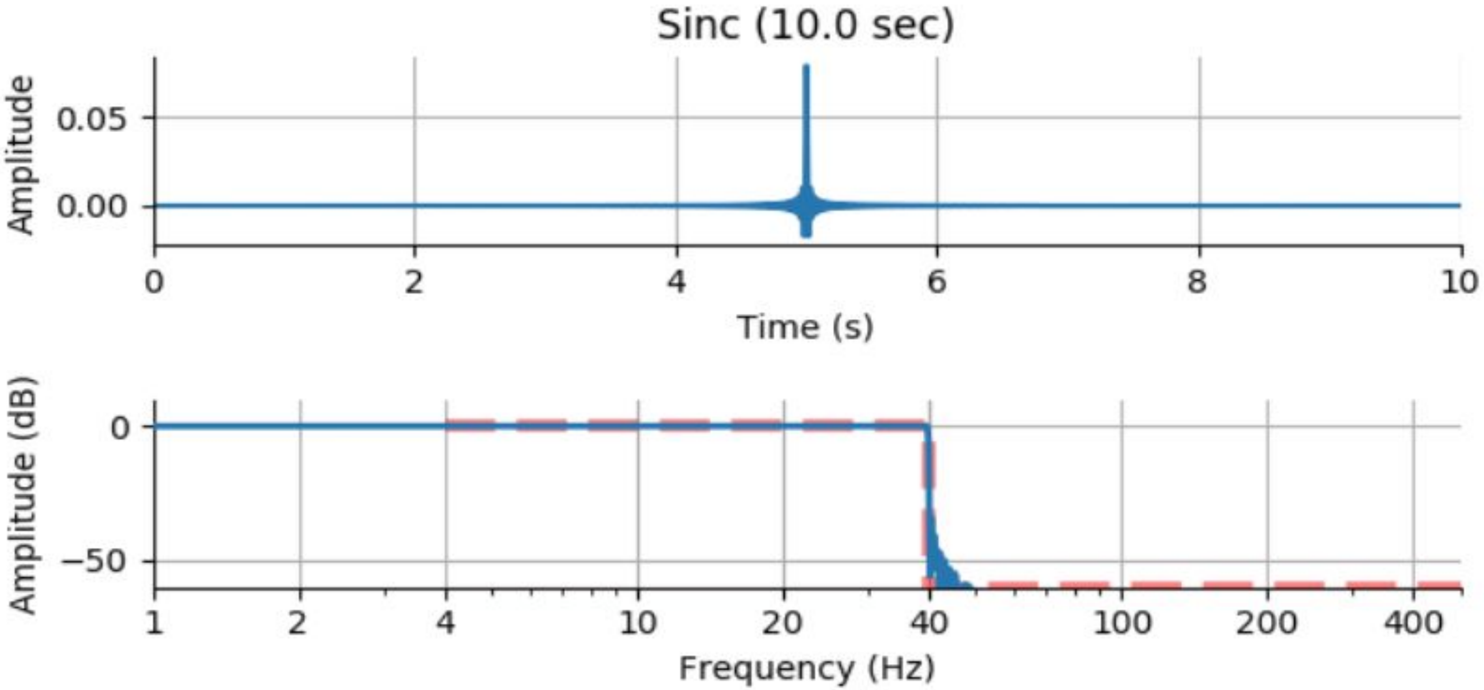


A longer Filter (1 sec)

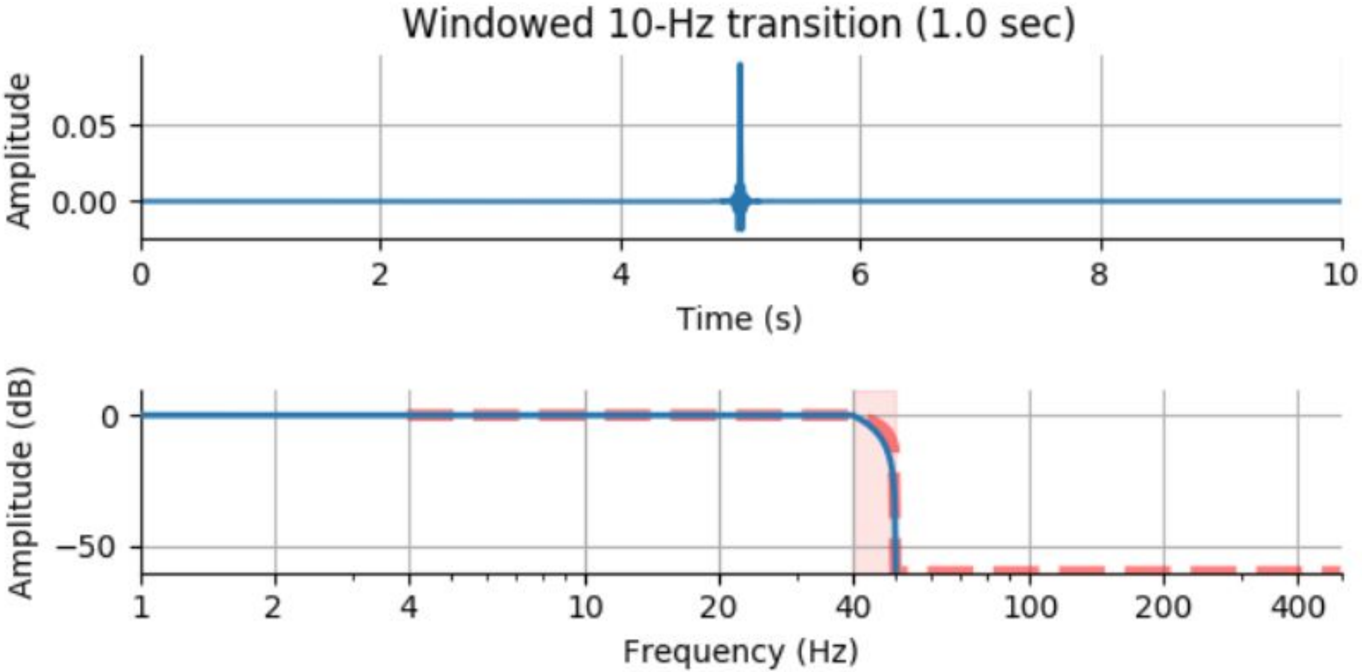


Even Longer Filter

(good attenuation in stop band, but 1 sec ringing in time domain)



Filter with transition band (it's a bit better)



Which do you prefer? Distortions in time domain or distortions in frequency domain. Pick your poison!

Filter Tradeoffs

- Ripple in the pass-band
- Attenuation of the stop-band
- Steepness of roll-off
- Filter order (i.e., length for FIR filters)
- Time-domain ringing

Questions?