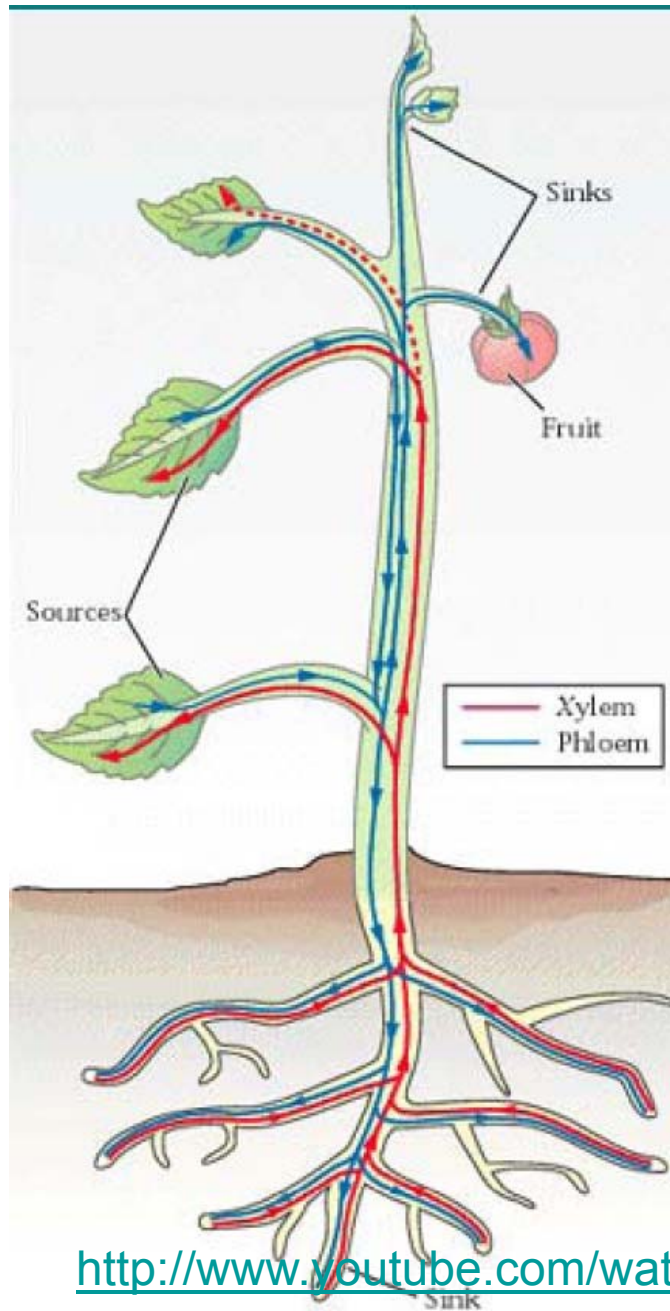


Lecture 4

Biophysics of fluid flow: Fluid in Plants

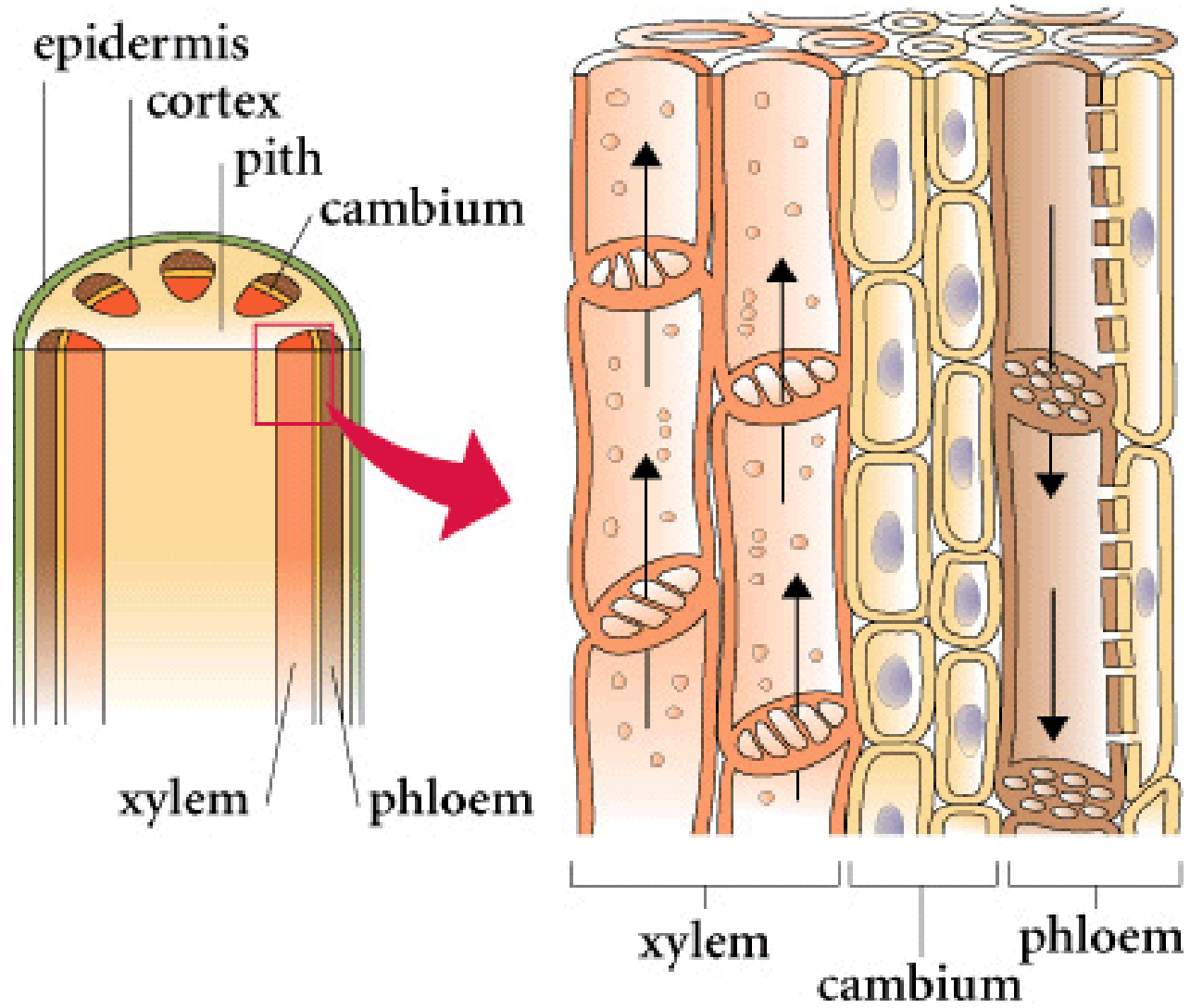
Biophysics and Gas Transport:
The Ideal Gas

Fluid in Plants



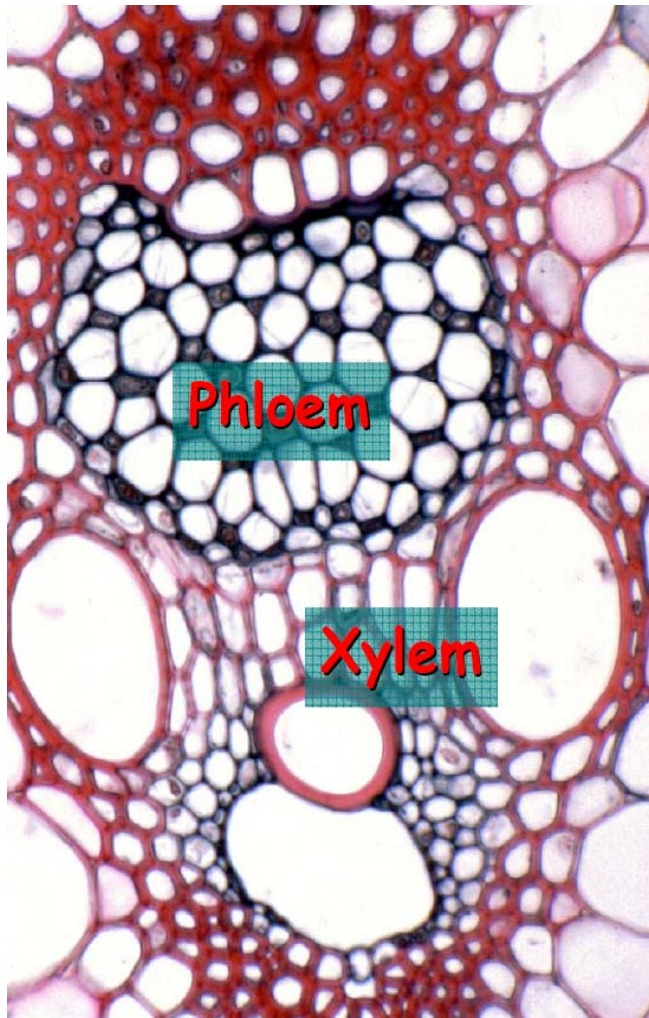
- An example of fluid flow in living systems – **xylem** and **phloem** in plants
- **Xylem** - water conducting channels in plants, that transport water upwards, as high as 200 feet above the ground
- **Phloem** - channels that transport products of photosynthesis from sites of production (leaves) to sites of utilization (roots and expanding or reproductive sites). Moves fluids down

<http://www.youtube.com/watch?v=w6f2BiFiXiM>

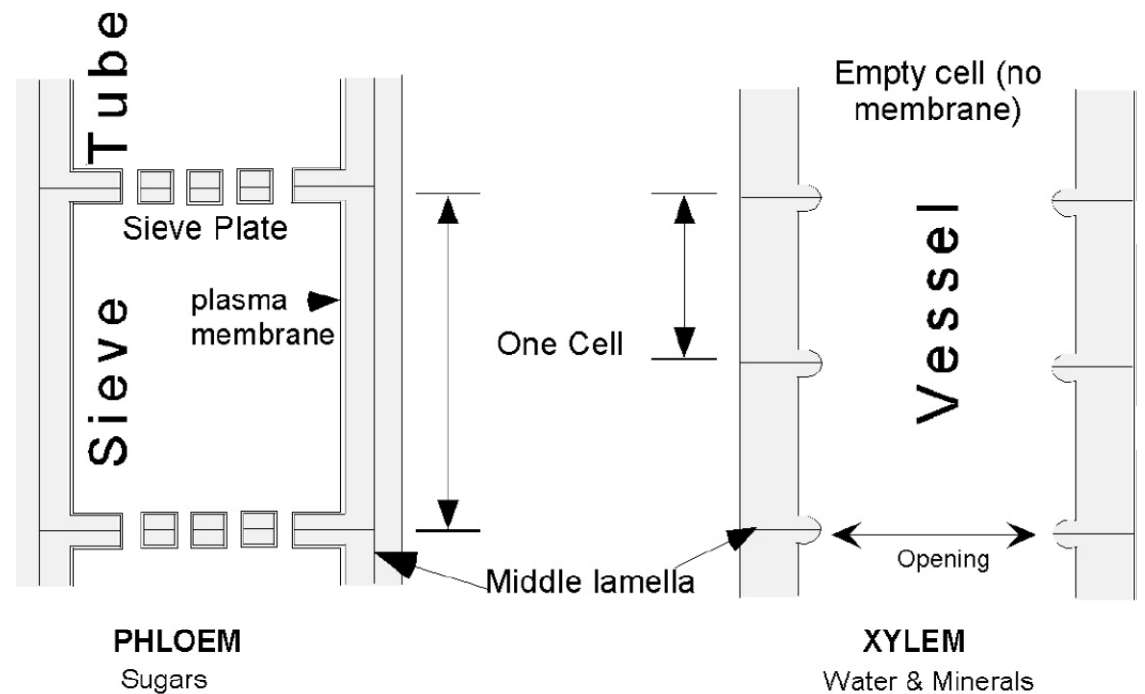


Elizabeth Morales

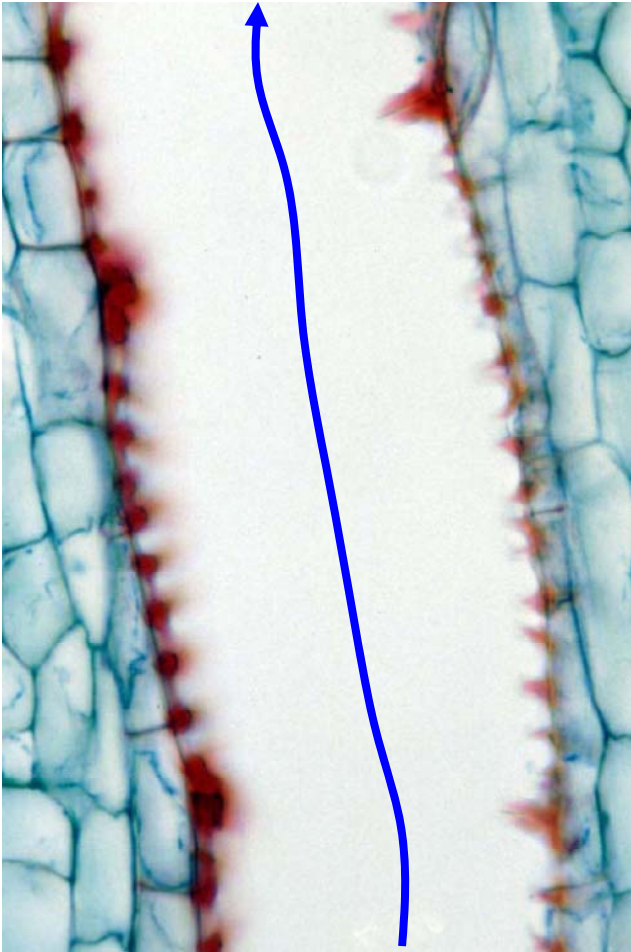
xylem and phloem channels



Xylem – dead cells joined together to form rigid water conducting channels



Xylem transport



- Xylem –rigid water conducting channels
- Mechanics of fluid flow through xylem:
 - Transport water upwards:
 - (hydrodynamics what forces move water up)
 - No upward flow during the night
 - (hydrostatics – what forces hold the water)

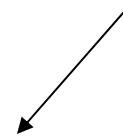
Xylem transport

- Hydrodynamics:
- Xylem transport theoretically can be considered as a laminar flow of a viscous fluid in a pipe
- Poiseuille's relation:

$$\Delta V = \frac{\pi r^4}{8\eta} \cdot \frac{\Delta P}{l}$$

- ΔV - volume per unit time
- r - radius
- $\Delta P/l$ - pressure gradient
- η - coefficient of viscosity

$$v_{\max} = \frac{r^2}{4\eta} \cdot \frac{\Delta P}{l}$$



Maximum velocity of the flow in the middle of the tube

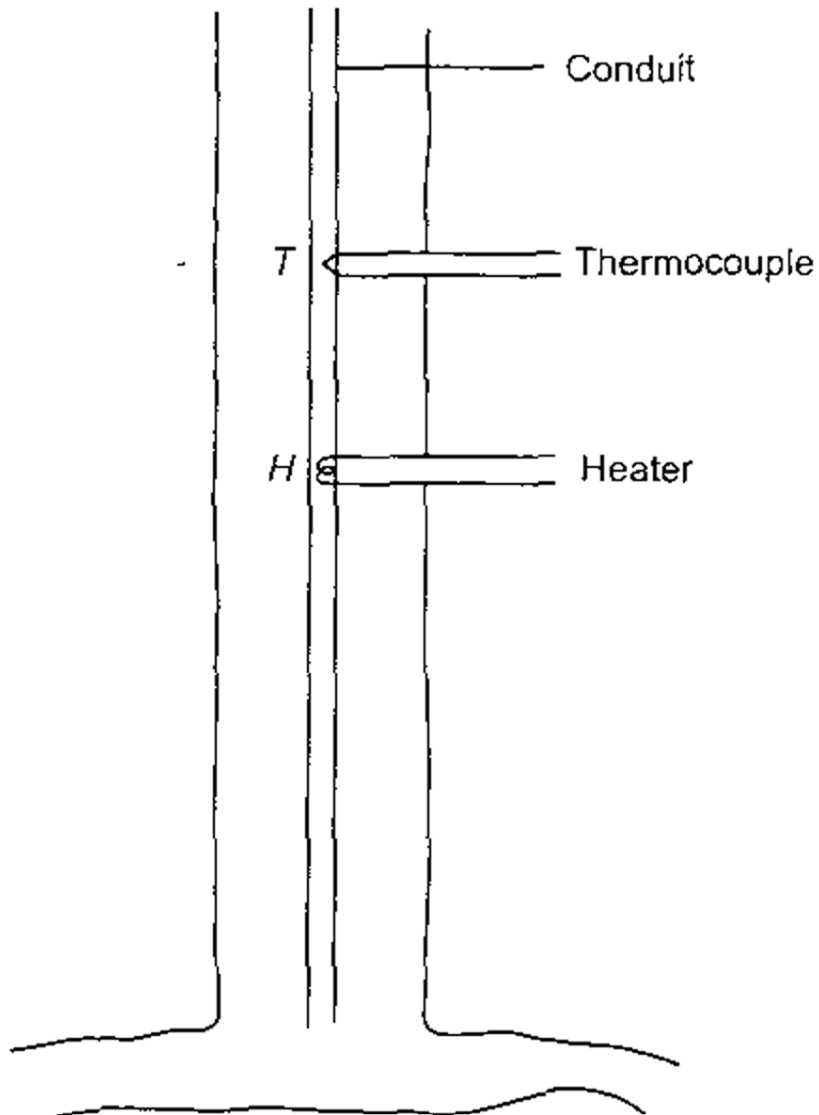


Fig. 2.5 Measurement of fluid velocity

velocity measurements using thermocouple:

the current is passed in the element for brief time 1-3 sec, which heats the moving fluid at point H. the hot fluid moves upward to the point T, chosen at known distance from H. When heated fluid reaches the point T, thermocouple registers the higher T of the heated fluid. From the time during which the fluid moves from point H to point T, the speed is determined

Speed varies from 1-2 m/hr (conifers), to 150 m/hr (vines).

Resistivity

- Resistivity = resistance per unit length, can be calculated from r of channel assuming it is ideal capillary
- For a channel of diameter 100 μm , taking $\eta(\text{water})=10^{-3} \text{ Pa}\cdot\text{s}$ (or 10^{-2} Poise) we get:

$$R = \frac{8\eta l}{\pi r^4} = \frac{8 \times 10^{-3} \times 1}{3.14 \times (50 \times 10^{-6})^4} \approx 10^5 \text{ Pa} \cdot \text{s} / \text{m}^3$$

Resistivity calculated in vines agrees well with theory, it is not the case for other plants

Forces –pressure gradient

- Pressure gradient = pressure difference per unit length of xylem channels depends on resistivity , and is different in different plants
- For vines:

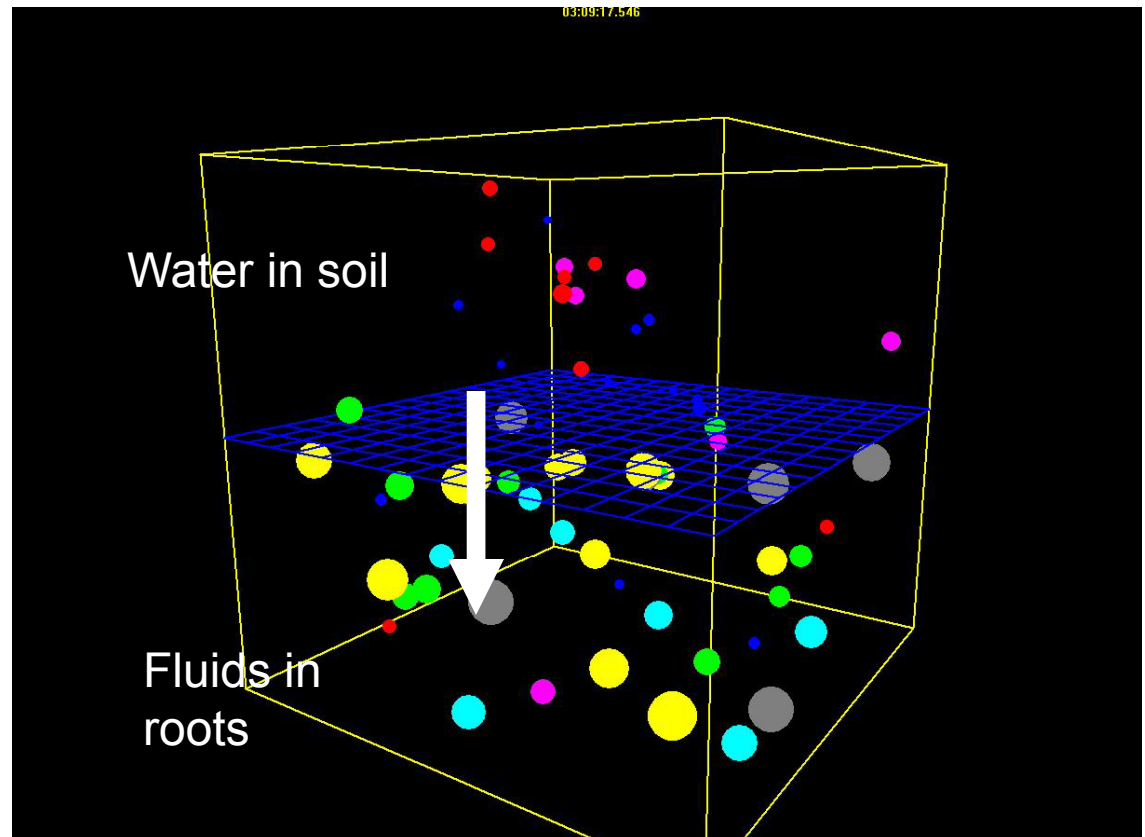
$$\frac{\Delta P}{l} = \frac{4\eta v_{\max}}{r^2} \approx \frac{4 \times 10^{-3} \times 4 \times 10^{-2}}{(10^{-4})^2} \approx 160 Pa / cm$$

- Hydrodynamic pressure gradient of 150-200 Pa/cm is necessary to raise water in plants
- Pressure gradient originates from two sources:
- **Positive pressure** from the roots –smaller contribution (balances gravity)- hydrostatics
- **Negative pressure** from the leaves transpiration – larger contribution (moves water upwards)-hydrodynamics

- The transport in xylem is not powered by energy spent by the channel elements themselves, which are dead at maturity and no longer have living contents. The transport is due to positive and negative pressure:
- **Positive pressure – root pressure:** If the water potential of the root cells is more negative than the soil, usually due to high concentrations of solute, water can move by **osmosis** into the root. This may cause a positive pressure that will force sap up the xylem towards the leaves.

- **Osmosis**

- is the spontaneous net movement of water through a semi-permeable membrane (permeable to the solvent, but not the solute) from a region of low solute concentration to a solution with a high solute concentration.
- It is a physical process in which a solvent moves without input of energy.



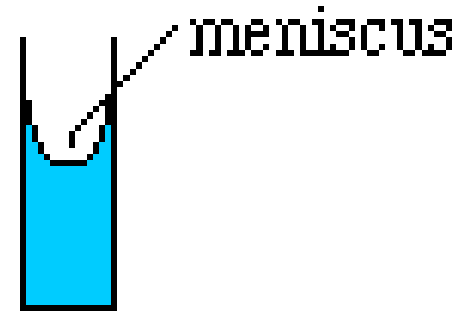
<http://www.youtube.com/watch?v=sdiJtDRJQEc&feature=related>

Forces – negative pressure

- **Negative pressure –transpiration pull:**
- the most important cause of xylem sap flow, is caused by the evaporation of water from the surface mesophyll cells to the atmosphere.
- This creates a tension in the surface cells that draw water from xylem of the nearest vein
- And in turn, establishes tension on the entire volume of water in the xylem, so the column is drawn up from the roots. This is equivalent to a suction pump at the top

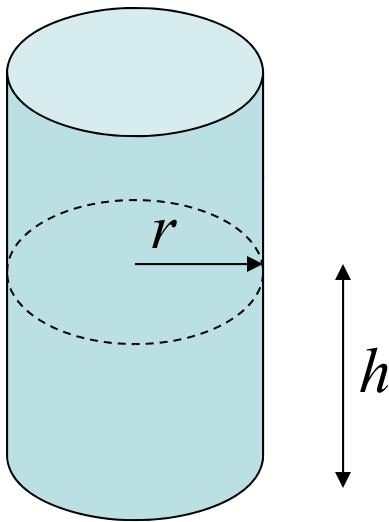
Small contribution from capillary forces

- surface tension allow water to climb the walls of small diameter tubes like xylem.
- This is called capillary action. The U shaped surface formed by water as it climbs the walls of the tube is called a meniscus.



Example. fluid flow due to the surface tension

- Assuming xylem tube $R=100\text{ }\mu\text{m}$, how high water will rise due to surface tension alone?



$$F_{gravity} = mg = Sh\rho g = \pi r^2 hg$$

$$F_{tension} = \gamma \cdot 2\pi r$$

$$\gamma \cdot 2\pi r = \pi r^2 hg\rho$$

we need to calculate h at which
F gravity = F tension

$$\text{density} = \rho = 1000\text{kg} / \text{m}^3$$

$$\text{surface _ tension} = \gamma = 72 \times 10^{-3} \text{ N} / \text{m}$$

$$h = \frac{2\gamma}{\rho g r} \approx 0.14\text{m}$$

- How narrow the tubes should be to transport water to the top of 100m tree due to the surface tension alone?

$$h = \frac{2\gamma}{\rho g r}$$

$$r = \frac{2 \times (72 \times 10^{-3} \text{ N / m})}{(10^3 \text{ kg / m}^3)(10 \text{ m / s}^2)(100 \text{ m})} = 0.14 \times 10^{-6} \text{ m} = 140 \text{ nm}$$

- Tubes 140nm are needed to transport water to the 100 m tree by surface tension alone, real xylem channels are 100 to 1000 times larger

Phloem transport

Sugars, amino acids, minerals and other solutes are transported in phloem from sources to sinks

Source – organ producing more sugars, than it requires -leaves

Sink – organ consuming sugars, root, flower or developing fruit

food in phloem sap can be transported in any direction needed so long as there is a source of sugar and a sink able to use, store or remove the sugar.

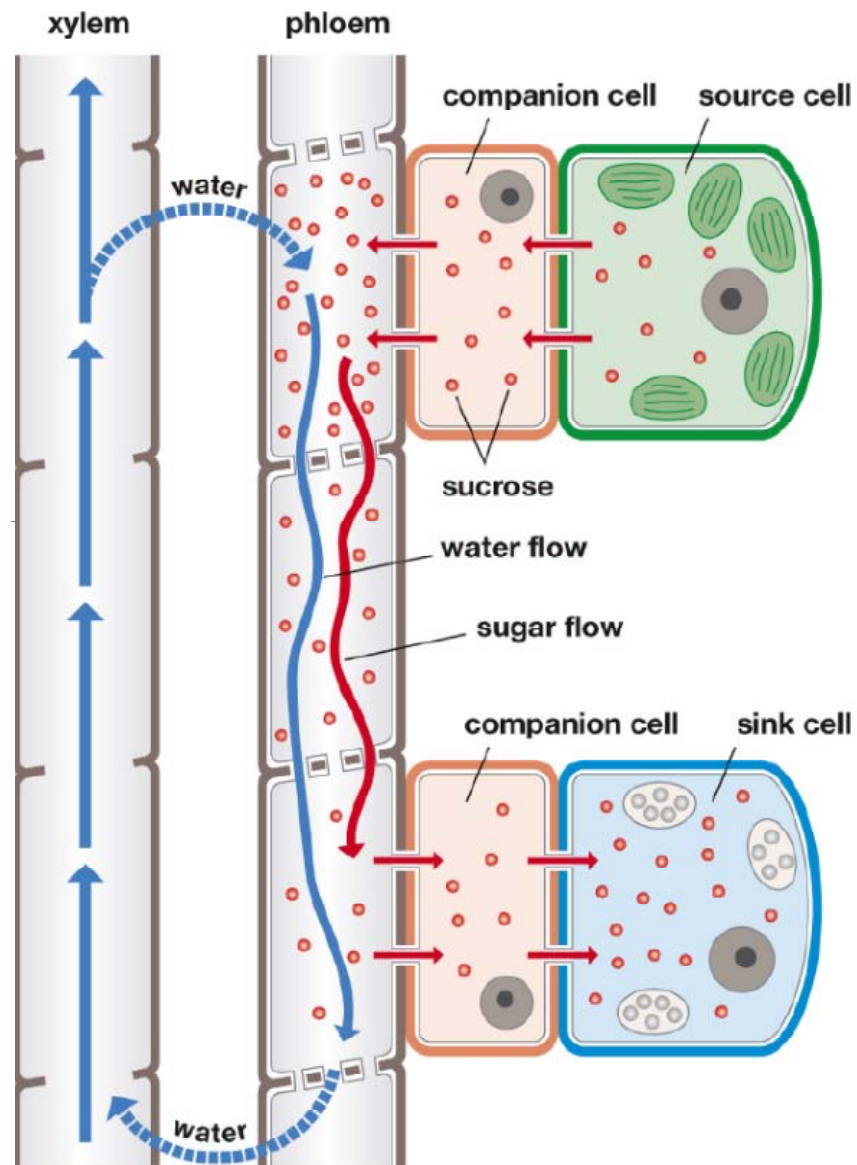


Figure 9.14 Plant Biology, 2/e

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Phloem transport- pressure flow model

- The phloem transport flows at constant speed, $\sim 1\text{m/hr}$, no acceleration due to gravity
- The phloem transport is thought to be due to osmotic pressure gradient:
- Source cells (leaves) have a greater sucrose concentration than surrounding cells
 - water enters source cells by osmosis,
 - causes greater pressure at the source, so that the sap moves by bulk flow towards the sink
- Lower osmotic pressure is created in the sink (root) because of withdrawal of the sucrose
- The difference of pressure between root and leaves moves the fluids with constant speed

- The amount of substance transported per unit time is proportional to the tube area and concentration gradient along the tube –
- use Fick's law:

$$\frac{\Delta M}{\Delta t} = DA \frac{\Delta C}{\Delta l}$$

- D –diffusion coefficient, ΔM – amount of substance moved by diffusion in time Δt
- Specific Mass Transfer (SMT) – amount of material in g moving per unit time hr per unit area 1cm^2 is a stable value for many types of trees
- Deciduous trees SMT=10-25 g/cm²hr,
- Conifers SMT=2.2-2.5 g/cm²hr

- An estimate of the average speed of mass flow can be made if concentration of phloem solution is known:

$$SMT = C \times v$$

- $v = 1m / hr$ the flow across unit area 1 cm^2

- C is concentration in g/cm^3
- v is speed of the solution in cm/hr
- for deciduous trees: $SMT=15\text{g/cm}^2\text{hr}$ and $C=0.15\text{g/cm}^3$
- Considering steady flow in a pipe – average speed = $1/2 v_{max}$
- $v_{max} = 2\text{m/hr}$ - this agrees with experimental data for these trees.

Biophysics and Gas Transport

Diffusion and gas transport in biosystems

- Breathing or external respiration in animals transports oxygen from and carbon dioxide into the atmosphere through respiratory canals or airways
- Oxygen and CO₂ diffuse across membranes of lungs or other external respiratory systems.
- Later blood transports these gases between lungs and other cells
- Exchange of oxygen and carbon dioxide between blood and body occurs by diffusion.
- In living organisms the gas transport occurs through:
 - Convective transport and
 - Diffusion of gases

The Ideal Gas

- physical characteristic parameters of gas: pressure, volume, temperature
- Ideal Gas – gas of low density, no interactions between molecules
- **Equation of state or Ideal Gas Law:**

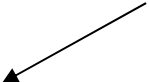
valid also for
gases of low
densities, where
intermolecular
interactions are
not significant

$$PV = nRT$$

R=8.31 J/mol·K – universal Gas constant
n-number of moles, V- volume
P – pressure, T - temperature

- Amount of gas in a volume is expressed in number of moles n of gas:
- 1 mole of any substance is amount of substance that contains N_A atoms or molecules
- N_A is Avogadro's number , $N_A = 6.022 \times 10^{23}$

Number of moles n :

$$n = \frac{m}{M} \quad \text{or} \quad n = \frac{N_{total}}{N_A}$$


Molar mass = atomic mass from periodic table expressed in g/mole

Ideal gas law

can be expressed in terms of the total number of molecules N:

$$PV = nRT = \frac{N}{N_A} RT$$

$$PV = Nk_B T$$

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J / K}$$

**Boltzmann's constant**

For ideal gas

- **Boyle's Law** – volume varies inversely with pressure at constant temperature

At const T

$$PV = \text{const}$$
$$\text{const} = nRT$$

- **Charles' law** – at constant pressure, volume of a given quantity of an ideal gas varies directly with temperature

At const P

$$V = \text{const} \cdot T$$
$$\text{const} = \frac{nR}{P}$$

For ideal gas

- **Gay-Lussac' law** –
when volume is kept
constant the pressure of
an ideal gas is directly
proportional to the
temperature

At const V

$$P = \textit{const} \times T$$

$$\textit{const} = \frac{nR}{V}$$

Example 1

- Scuba diver fills his lung to full capacity (5.5 L) at 12 m below the surface of water in the pool.
- To what volume the lung will expand if he quickly rises to the surface ?

- Assume $T = \text{const}$
- Apply Boyle's Law:

$$P_0 V_0 = \text{const} = P_1 V_1$$

Solution Example 1

- Assume $T = \text{const}$
- Apply Boyle's Law:

$$P_0 V_0 = \text{const} = P_1 V_1$$

- The pressure inside the pool at 12 m = atmospheric pressure + water pressure of the 12 m column

$$P = \frac{F}{S} = \frac{mg}{S} = \frac{V\rho g}{S} = \rho gh$$

The pressure
of the water
column

At 12 m below the surface the pressure is:

$$\begin{aligned} P_{12m} &= P_{atm} + \rho gh = 10^5 \text{ N / m}^2 + (1000 \text{ kg / m}^3 \times 9.8 \text{ m / s}^2 \times 12 \text{ m}) = \\ &= 2.17 \times 10^5 \frac{\text{kg}}{\text{ms}^2} = 2.17 \times 10^5 \frac{\text{N}}{\text{m}^2} \end{aligned}$$

Assuming that body temperature is constant, we use Boyle's Law

$$\begin{aligned} P_0 V_0 &= P_{12m} V_{12m} \\ V_0 &= \frac{P_{12m} V_{12m}}{P_0} = \frac{2.17 \times 10^5 \text{ N / m}^2 \times 5.5 \text{ m}^3}{10^5 \text{ N / m}^2} \approx 12 \text{ m}^3 \end{aligned}$$

The lung will rupture on such large and sudden expansion!

Dalton's Law of partial pressure

- Gas is a mixture of ideal gases
- Total number of moles $n = n_1 + n_2 + n_3 + \dots$

$$PV = nRT$$

P_i - partial pressure

$$PV = n_1RT + n_2RT + \dots$$

$$P_i = \left\{ \frac{n_i}{n_1 + n_2 + \dots} \right\} P$$

$$PV = P_1V + P_2V + \dots$$

$$P = P_1 + P_2 + \dots$$

The pressure of mixture of gases = sum of partial pressures of individual gases

- For example dry atmospheric air contains:
- Air = 78% N₂ + 21% O₂ + 0.04% CO₂
- With partial pressures:
- P_{air} = 1 atm, P_{O₂} = 0.21 atm, P_{CO₂} = 4 × 10⁻⁴ atm
- **Example 2: What is the % of CO₂ in the lungs :**
- P_{CO₂} = 35 Torr in the lungs (much higher than in atmosphere)
- What is the % of CO₂ in the lungs?
- Assuming P_{air} inside = P_{air} outside the lungs = 760 mmHg
- 760 mmHg = 760 torr
- % of P_{CO₂} in the lungs = 35/760 × 100 = 4.6% of P_{air}
- concentration of CO₂ in the lungs = 4.6 % of the air inside the lungs

Pressure units

- **1 atmosphere = 760 mmHg = 29.92 inHg = 14.7 lb/in² = 101.3 KPa or KN/m²**
- The fundamental SI unit of pressure is the Pascal (Pa) $\text{Pa} = \text{N/m}^2$,
- Since the static fluid pressure is dependent only upon density and depth, choosing a liquid of standard density like mercury or water allows you to express the pressure in units of height or depth, e.g., mmHg or inches of water. The mercury barometer is the standard instrument for atmospheric pressure measurement in weather reporting. The decrease in atmospheric pressure with height can be predicted from the barometric formula.
- The unit mmHg is often called torr, particularly in vacuum applications: $760 \text{ mmHg} = 760 \text{ torr}$
- For weather applications, the standard atmospheric pressure is often called 1 bar or 1000 millibars.

Vapor Pressure

- Phases of matter: **Liquid** \xrightleftharpoons{T} **Gas**
- Evaporation occurs at defined T (T of evaporation) at a given P
- Even at lower T there is an exchange between gas and liquid, evaporation and condensation – balanced at equilibrium.
- **Saturated vapor** - vapor in equilibrium with its liquid,
- Characterized by saturated vapor pressure (SVP)
- Saturated vapor pressure rises with T
- Saturated vapor pressure does not depend on the volume of the container
- Water SVP=17.5 mmHg at 20°C, and 47.1 mmHg at 37°C

- Air normally contains water vapor from evaporation
- In respiratory system in the body all gases are in contact with water and are saturated with water vapor at body T
- The presence of water vapor changes the partial pressures of all other component gases in the system
- At body T:
- Partial P of water vapor – 47, O₂ – 100, CO₂ – 40 mmHg
- In Biosystems gases are considered under two conditions:
 1. at **B**ody **T**emperature, T=37°C, atm **P**ressure (760 mmHg) and **S**aturated with water vapor - **BTPS**
 2. at **S**tandard **T** (0°C) and **P**ressure (760 mmHg), and 0 water vapor pressure or **D**ry - **STPD**

Example 3

- **Air expired from lungs is saturated with water vapor.**
- **The volume expired at room T (20°C) is $V=500\text{mL}$.**
- **What is the volume of the same expired air within the lungs?**

Solution Example 3

- **Air expired from lungs is saturated with water vapor. The volume expired at room T (20°C) is $V=500\text{mL}$. What is the volume of the same expired air within the lungs?**
- Expired air is saturated with water vapor
- Total Pressure P inside the lung = atmospheric pressure = 760mmHg
- **Partial Pressure of air inside:**
- $P_i = \text{Total } P - P_w(37^\circ\text{C}) = 760 - 47.1 = 712.9 \text{ mmHg}$
- Saturated water vapor pressure P_w at $37^\circ\text{C} = 47.1 \text{ mmHg}$
- **Partial pressure of air outside:**
- $P_o = \text{Total } P - P_w(20^\circ\text{C}) = 760 - 17.5 = 742.5 \text{ mmHg}$
- P_w -saturated water vapor pressure P_w at $20^\circ\text{C} = 17.5 \text{ mmHg}$

- The same amount of air (n-number of moles) inside and outside has different V, T, P

$$PV = nRT \qquad \frac{P_i V_i}{T_i} = \frac{P_o V_o}{T_o}$$

- Volume of expired air (V inside):

$$V_i = \frac{P_o V_o}{T_o} \times \frac{T_i}{P_i} = \frac{(760 - 17.5)}{(760 - 47.1)} \times \frac{(273 + 37)}{(273 + 20)} \times 500 = 551 \text{ ml}$$

Inside V 551 ml reduces to 500ml outside – due to condensation of saturated water vapor

Solutions and Henry's Law

- **Solutions** – homogeneous mixtures of two or more substances
- **Solvent** – liquid substance present in very large amount compared to others
- Other substances – **solutes**
- **Solubility** – ability of a solute to dissolve in a solvent – is limited and is a defined amount for a specific solute and solvent and T.
- **Saturated solution** – solute present at the maximum possible concentration
- **Diluted or weak solution** - solute is in a small amount

- Gases dissolve in liquids – form gas-liquid solutions
- Solubility depends on gas, solvent, P and T of the system
- When there is a chemical similarity –solubility is high
- N_2 , H_2 , O_2 – small solubility in water, CO_2 , NH_3 – high solubility in water

Henry's Law

- Often dissolution of gas in liquid leads to evolution of heat
- If the heat is evolved – the reaction rate decreases with T rise, hence solubility of gases decreases with T increase
- At const T the solubility depends upon the P of gas above the liquid
- Henry's Law: the solubility of a gas is linearly proportional to its partial pressure above the liquid (at low pressures)

$$C = \alpha \cdot P$$

C-molar concentration = number of moles
per unit volume
 α -solubility coefficient,
P-pressure above the liquid
at constant T

- At equilibrium: evaporation = condensation per unit time
- Evaporation \sim concentration (or solubility of gas) in liquid
- Condensation \sim density (pressure) above the liquid

- Solubility coefficient α depends on T
- α decreases when T increases
- At body T (37C) and atmospheric pressure solubility of respiratory gases:
 - CO₂ 0.57
 - O₂ 0.024
 - CO 0.018
 - N₂ 0.012
 - He 0.008
- In the mixture of several gases the total P is the sum of partial P_i of each gas, the solubility of each gas is proportional to its partial P_i, each gas solubility is independent of other gases present.
- This is not the case in the body – increase in Partial P of CO₂ requires higher partial P of O₂ to saturate blood with oxygen – **Bohr's effect**

Pressure Units

$$1 \text{ pascal (Pa)} \equiv 1 \text{ N}\cdot\text{m}^{-2} \equiv 1 \text{ J}\cdot\text{m}^{-3} \equiv 1 \text{ kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}$$

	pascal (Pa)	bar (bar)	technical atmosphere (at)	atmosphere (atm)	torr (Torr)	pound-force per square inch (psi)
1 Pa	$\equiv 1 \text{ N/m}^2$	10^{-5}	1.0197×10^{-5}	9.8692×10^{-6}	7.5006×10^{-3}	145.04×10^{-6}
1 bar	100,000	$\equiv 10^6 \text{ dyn/cm}^2$	1.0197	0.98692	750.06	14.504
1 at	98,066.5	0.980665	$\equiv 1 \text{ kgf/cm}^2$	0.96784	735.56	14.223
1 atm	101,325	1.01325	1.0332	$\equiv 1 \text{ atm}$	760	14.696
1 torr	133.322	1.3332×10^{-3}	1.3595×10^{-3}	1.3158×10^{-3}	$\equiv 1 \text{ Torr}; = 1 \text{ mmHg}$	19.337×10^{-3}
1 psi	6,894.76	68.948×10^{-3}	70.307×10^{-3}	68.046×10^{-3}	51.715	$\equiv 1 \text{ lbf/in}^2$

Example reading: $1 \text{ Pa} = 1 \text{ N/m}^2 = 10^{-5} \text{ bar} = 10.197 \times 10^{-6} \text{ at} = 9.8692 \times 10^{-6} \text{ atm}$

Note: mmHg is an abbreviation for millimeters of mercury.