Physics 256 Assignment 5 Fall 2012 62 marks

1) a) 4.40, **8 marks**

4.40 From Snell's law $\theta_t = 12.748°$; from Eq. (4.43),

$$r_{\|} = \tan 7.252° / \tan 32.748° = 0.1978;$$

using Eq. (4.42),

$$r_{\perp} = -\sin 7.252° / \sin 32.748° = -0.2352;$$

$$[E_{or}]_{\|} = r_{\|}[E_{oi}]_{\|} = 1.98\text{V/m};$$

$$[E_{or}]_{\perp} = r_{\perp}[E_{oi}]_{\perp} = -4.70 \text{ V/m}.$$

b) 4.45: Field amplitudes and irradiances. **8 marks Final field amplitudes are given by the product of the r's and initial amplitudes.**

4.45 $r \simeq n_t - n_i/n_t + n_i$. Air-water: $r = \frac{4/3-1}{4/3+1} = 1/7 = 0.14$. Air-crown glass: $r = \frac{3/2-1}{3/2+1} = 1/5 = 0.20$. More reflectance for glass. From (4.54) and (4.56 $I_r/I_i = R = r^2$. Air-water: $R = (1/7)^2 = 0.02$. Air-crown glass: $R = (1/5)^2 = 0.04$.

c) If a beam of quasimonochromatic light with an irradiance of 400W/m^2 is incident in each case, what is the transmitted irradiance in each case? **8 marks**

The reflected irradiance is RI_i= 0.02*400= 8W/m^2 and 0.04*400=16W/m^2. Since 1=R+T, Since the incidence is near normal, the transmitted irradiance is T*I_i. By conservation of power, T*I_i= (1-R)*I_i=0.98*400 W/m^2 and 0.96*400 W/m^2 or 392 W/m^2 and 384 W/m^2.

Alternately

$$\frac{I_t}{I_i} = \frac{n_t}{n_i} t^2 = \frac{4/3}{1}\left(\frac{2n_i}{n_i + n_t}\right)^2 = \frac{4/3}{1}\left(\frac{2}{1+4/3}\right)^2 = 0.98 \; OR$$

$$\frac{I_t}{I_i} = \frac{n_t}{n_i}\left(\frac{2n_i}{n_i + n_t}\right)^2 = \frac{3/2}{1}\left(\frac{2}{1+3/2}\right)^2 = 0.96$$

And I_t=0.98*400=392 W/m^2 or I_t=0.96*400=384 W/m^2

d) If the beam diameter is 1 cm, what is the transmitted power? **5 marks**

Beam diameter= 1 cm; area= $\pi(0.5^2) = 0.785cm^2$

Transmitted power= transmitted irradiance x area =

$$392\boldsymbol{W}/\boldsymbol{m}^2 \ \boldsymbol{and} \ 384\boldsymbol{W}/\boldsymbol{m}^2\boldsymbol{x}7.85\boldsymbol{X}10^{-5} = 3.08\boldsymbol{X}10^{-2}\boldsymbol{Wand}3.01\boldsymbol{x}10^{-2}\boldsymbol{W}$$

OR P_t=T*P_i=0.98*400*7.85X10^{-5} and 0.96*400*7.85X10^{-5}=3.08X10^{-2} and 3.01X10^{-2}

2) Waveguide Question Slide 15: Reflection notes: Cone inner segments at the rear of the eye are waveguides and their internal refractive index is 1.353. They are surrounded by a refractive index of 1.339 and the refractive index in front of them is also 1.339. From geometrical theory, what is the critical angle leaving the fibre? Calculate the numerical aperture of the fibre and the fibre acceptance angle, θ_{max}. How large a cone of light is accepted? 10 marks

Calculate the critical angle, the numerical aperture and the fibre acceptance angle for cone inner segment with the given parameters.

Parameter	Cone inner segment
n_f	1.353
n_c	1.339
n_0	$=n_c$=1.339

Answers:

$\varphi_c=\sin^{-1}(n_c/n_f)= \sin^{-1}(1.339/1.353)$=81.75 deg The critical angle is 81.75 deg.

N.A.= $n_0 \sin \theta_m=(n_f^2-n_c^2)^{1/2}=(1.353^2-1.339^2)^{1/2}$= 0.194 The numerical aperture is 0.194.

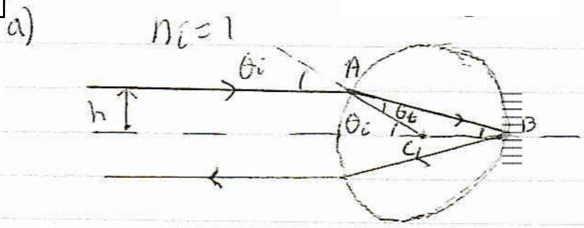
$1.339\sin\theta_m=0.194$; θ_m=8.34 deg The aceptance angle is 8.34 deg so the cone of acceptance is 16.68 deg. (approx gives 8.3 °)

The light is accepted in a cone of 2X8.34° =16.68° across.

3) Retroreflector Problem: A transparent sphere can act as a retroreflector. It has an index of refraction of 1.8 and a radius of 1.0 cm. At what distance from the centre line must a ray be sent in parallel to the line to exit the sphere parallel to the incoming beam? Use Snell's Law. **8 marks**

8

a)



$n_i = 1$ $n_t = 1.8$ $R = 1.0$ cm

Consider 

The requirement is that B is on a diameter so $CB = r$

A C also $= R$

This is an isosceles triangle so $AC = AB$ and $\angle CBA = \theta_t$

$$\theta_i = \theta_t + \theta_t$$

$$\theta_t = \theta_i / 2$$

$$n_i \sin\theta_i = n_t \sin\theta_t \quad \text{(Snell's Law)}$$

$$n_i \sin 2\theta_t = n_t \sin\theta_t$$

$$\sin 2\theta_t = 2\sin\theta_t \cos\theta_t$$

$$2\cancel{\sin\theta_t}\cos\theta_t = 1.8\cancel{\sin\theta_t}$$

$$\cos\theta_t = 0.9$$

$$\theta_t = 25.84^\circ$$

$$\theta_i = 2\theta_t = 51.68^\circ$$

$$\boxed{h = R\cancel{\tan\theta_t}\sin\theta_i = \cancel{1.265}\ 0.784\ \text{cm}}$$

4) Brewster's angle: 8.28 a) At what angle will the reflection of the sky coming off the surface of the pond, completely vanish when seen through a Polaroid filter? These sunglasses preferentially absorb one polarization of light in order to reduce the effects of glare from reflecting surfaces like water.

b) Should the glasses absorb vertical or horizontally polarized light?

a) At Brewster's angle there is only one polarization reflected so the Polaroid axis can be set to absorb this. This is at Brewster's angle

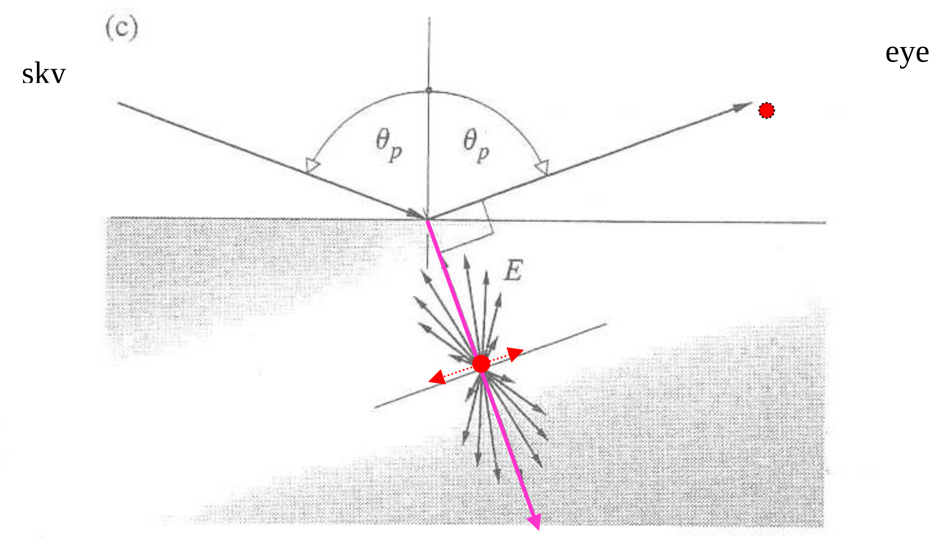
$$\tan\theta_p = \frac{n_t}{n_i} = \frac{1.333}{1} = 1.333$$

$$\theta_p = 53.123^\circ$$

the incident angle of the light from the sky to the normal. The angle of the light from the sky to the water would be

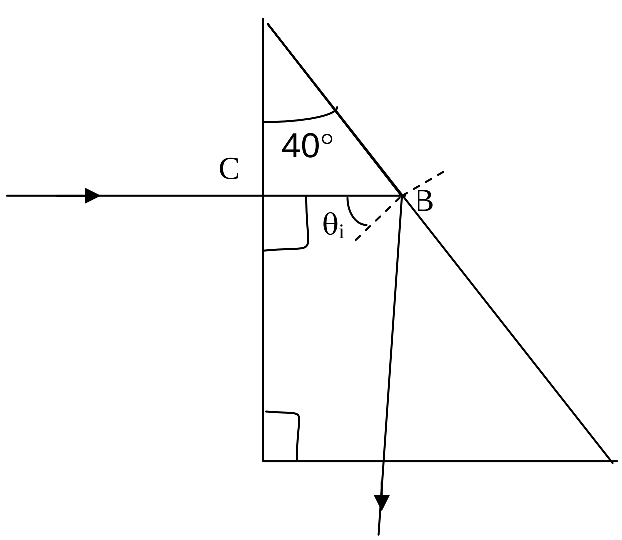
90-53.12=46.88 deg. **3 marks**

b) Light along the perpendicular polarization is always reflected from the water (vibration out of the plane below). At Brewster's angle, this is the only light reflected. If you want to absorb this reflected light, the Polaroid glasses should absorb this polarization which is horizontal (parallel to the water) **3 marks**



5) Reflecting prism: Light traverses the following prism.
a) What is the minimum refractive index of the prism in order for the light to reflect?
b) What is the disadvantage of this ray path for a reflecting prism?

a) The third angle of the triangle angle ABC is 180-(90+40)=50 deg. This means that the angle of the ray to the normal to the second surface θ_i is 90-50=40 deg. This must be larger than the critical angle or alternately the critical angle must be smaller than 40deg for TIR. A



$$\theta_c = \sin^{-1}\frac{n_t}{n_i} < 40 \text{ deg}$$

$$\sin^{-1}\frac{1}{n_i} < 40\,\text{deg}$$

$$\frac{1}{n_i} < \sin(40)$$

$$n_i > \frac{1}{\sin(40)}$$

$$n_i > 1.56$$

The minimum prism refractive index is 1.56. **6 marks**

b) In a reflecting prism, we usually want a range of wavelengths to traverse the same path. Although the reflection will be wavelength independent, the refraction at the second surface for the non normal incidence will depend on wavelength.
By equal angles of reflection and the normal at 90 deg, angle FBD= 50 deg.
Angle EDA= angle CBA= 50 deg.
Therefore angle EFB =80 deg not 90 deg as needed. **3 marks**