With these methods, more points means higher order polynomial …

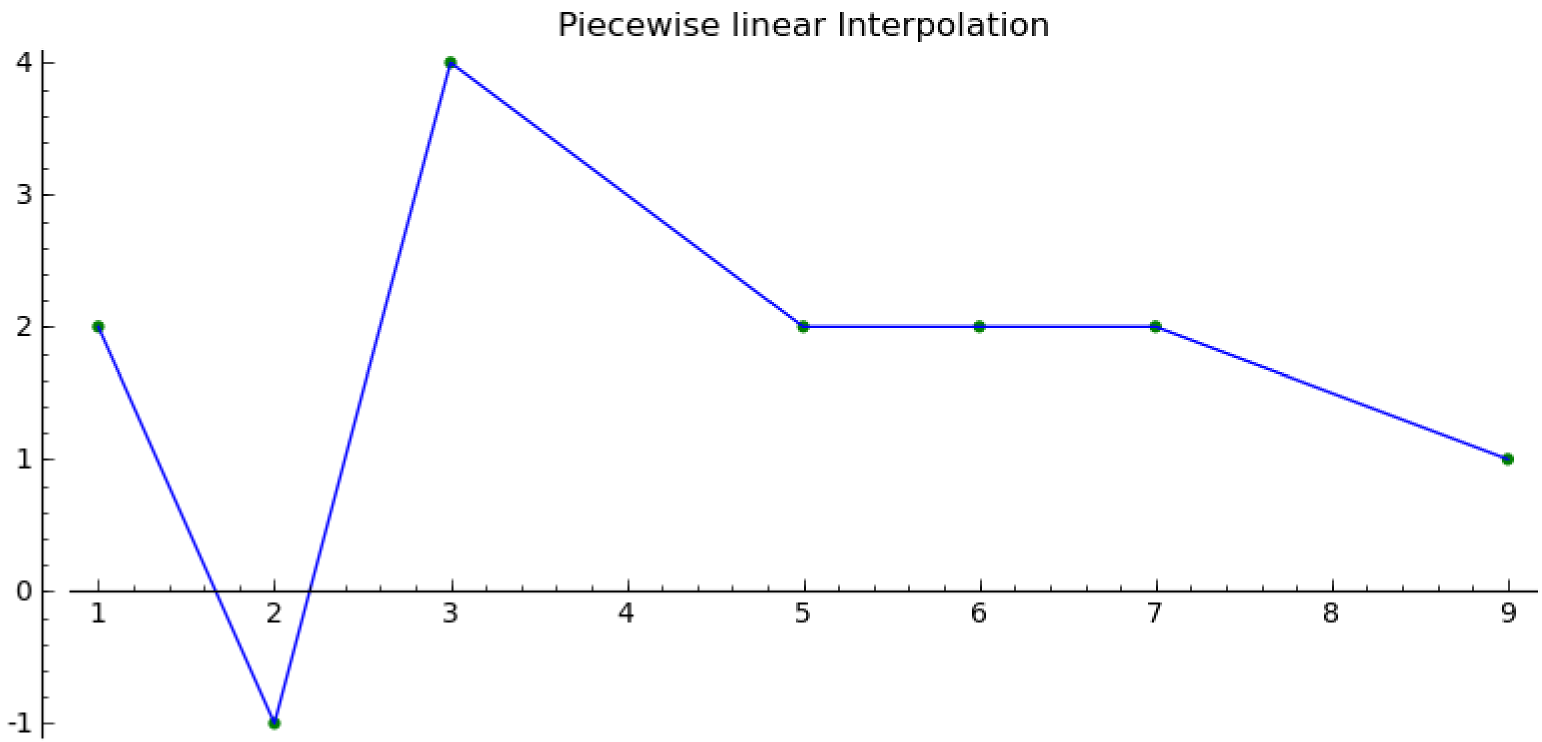
- Is this always better? higher degree

  → more fluctuations in p(x)

  → Less "cost effective" evaluations

- Alternatives?
  - "closest" fit with lower order polynomial over full interval
  - Divide full interval into smaller intervals and fit a low order poly to each interval

Piecewise Linear Interpolation (assume $x_0 < x_1 < \ldots < x_n$)



For each interval, $[x_k, x_{k+1}]$: $p_k(x) = y_k + \frac{y_{k+1} - y_k}{x_{k+1} - x_k}(x - x_k)$

Then, $p(x) = p_k(x)$ where $x_k \leq x \leq x_{k+1}$

Accuracy of Linear Piecewise Interpolation of *f(x)* over $[x_0,x_n]$

- Assume $|f''(x)| \le M$ over $[x_0,x_n]$,
- Let $h_k = x_{k+1} - x_k$ and $h_{max} = max\ h_k$,

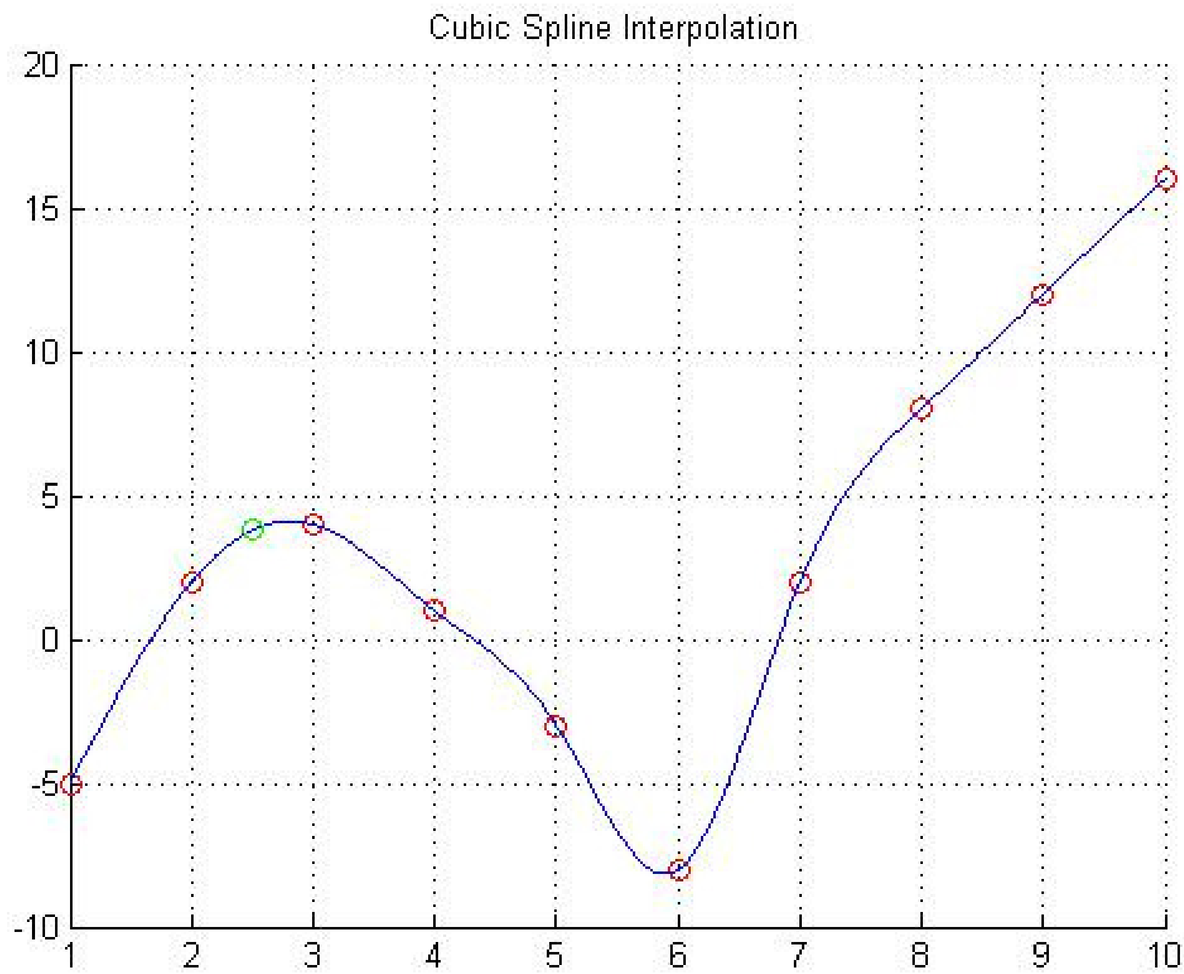
- for all $x \in [x_k,x_{k+1}]$, $\exists \vartheta \in [x_k,x_{k+1}]$, such that

$$|f(x) - p(x)| \le \frac{1}{8} M {h^2}_{max}$$

Comments on Linear Piecewise Interpolation

- Straight line segments may not be a good fit in all intervals
- First derivatives may not be smooth at x_k
- Piecewise quadratic function can be chosen to smooth connections
- In practice, piecewise cubic polynomials work very well

Cubic Spline Interpolation



Set the scene … spline *S(x)*

$S(x) = S_j(x)$ for $x \in [x_j, x_{j+1}]$, where

$$S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3$$

Since j=0:n-1, we have 4n unknowns.

What "knowns" do we have?

After many simplifications ….

We can define the cubic splines by the defining the system: A x = b, where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ h_0 & 2(h_0+h_1) & h_1 & 0 & \cdots & 0 \\ 0 & h_1 & 2(h_1+h_2) & h_2 & \vdots & 0 \\ \vdots & 0 & & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & & 2(h_{n-2}+h_{n-1}) & h_{n-1} \\ 0 & 0 & & & 0 & 1 \end{bmatrix}$$

(Note: A is diagonally dominant)

$$x = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ \vdots \\ c_n \end{bmatrix} \text{and } b = \begin{bmatrix} 0 \\ z_1 \\ \vdots \\ z_{n-1} \\ 0 \end{bmatrix}, \text{ for } z_j = \frac{3}{h_j}\left(a_{j+1} - a_j\right) - \frac{3}{h_{j-1}}\left(a_j - a_{j-1}\right)$$