Physics 256 Assignment 7

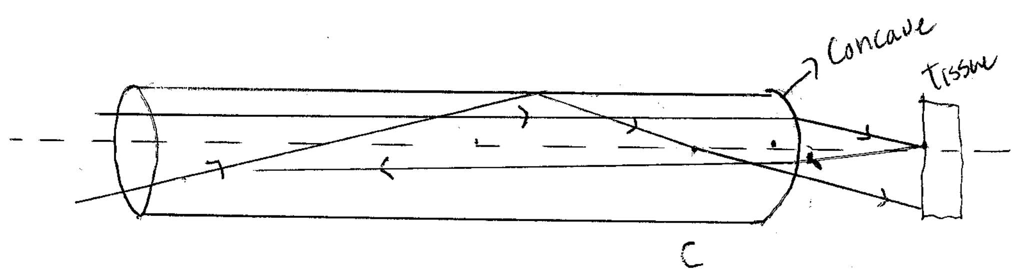
Due: Due: Monday, November 5th, 2012 4:00 pm 106 MARKS

1) a) An endoscope (glass fibre optic, n=1.8) is designed to focus light, from distance, through water onto tissue (n=1.333) 2 cm from its curved tip. What radius of curvature must be placed onto the tip? **Acting as a single surface refractor.** *When this illuminating light is diffusely reflected, ray reversal shows that this illuminated plane is imaged to a collimated beam in the rod which is then captured by a camera at the other end of the rod.* **4 marks**

$$\frac{n_2}{f_i} = \frac{(n_2 - n_1)}{R};$$
$$f_i = \frac{n_2 R}{(n_2 - n_1)} = \frac{1.333(R)}{(1.333 - 1.8)} = 2cm$$
$$R = \frac{(1.333 - 1.8)(2cm)}{1.333} = -0.7cm$$

Note that this is concave (not needed in answer).

b) Draw a sketch of the system and the incident light. **4 marks**



2) Single refracting surface: Hecht 5.7. **The result of question 5.6 is the magnification of a single refracting surface as given in the slides. 7 marks**

a)

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{(n_2 - n_1)}{R_1} = \mathscr{D}_1$$
$$\frac{1}{30} + \frac{1.333}{s_i} = \frac{(1.333 - 1)}{5.0}$$

S_i=40.07 cm so the image is 40.07 cm behind the surface.2 marks

b)

$$M_T = \frac{-n_1 s_i}{n_2 s_0}$$ **single surface**

$$M_T = \frac{-(1)40.07}{1.333*30} = 1.002X$$ **The frog is inverted 1.002X the original height (slightly larger). 3 marks (must include inverted, larger**

$$M_T = \frac{y_i}{y_0}$$

$$y_i = y_o M_T = 3cm(-1.002) = -3.006cm$$

2 marks

3) a) Retroreflector Paraxial Problem: 5.57 A transparent sphere can act as a retroreflector. A retroreflector is a transparent sphere, the back surface of which is silvered to reflect light. As seen in a previous Problem set, parallel incident light is refracted at the front surface, reflected at the rear surface and refracted again at the front surface. This retroreflector has an index of refraction of n and is in air. What refractive index must it have so that the **paraxial approximation** for refraction gives rays that focus at the back of the sphere? That is, so that the focal point to the second surface? **5 marks**

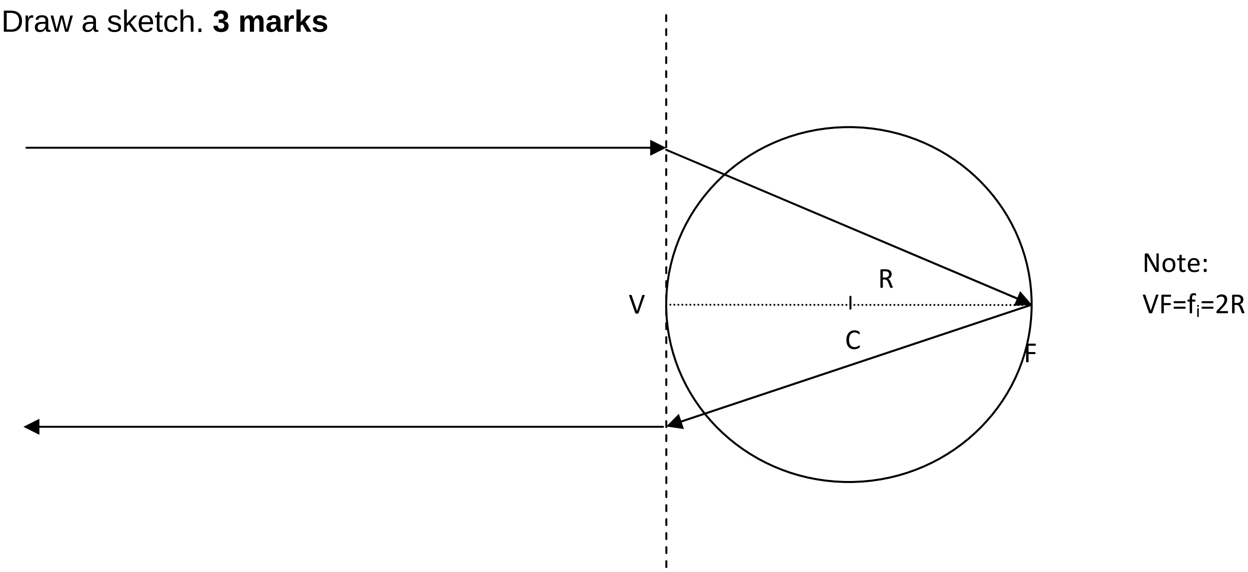
The radius of the sphere is R. The focal length, f_i must be from the vertex to the rear surface=2R (see previous problem set)

$$f_i = \frac{n_2 R}{(n_2 - n_1)} = \frac{n(R)}{(n-1)} = 2R$$

$$n(R) - 2nR = -2R$$

$$n = 2$$

b) Draw a sketch. **3 marks**



Hint: This is the design shown in Assignment 5 but now paraxial. The ray is initially parallel to the optical axis. The refracted ray must be incident on the vertex of the back surface, reflected at an equal angle and refracted out of the sphere parallel to the original ray. Thus the ray must be focused at the rear surface.

4) A simple model of the human eye: A simple model of the human eye consists of a single surface separating air in front from water (n=1.333) behind.

a) The 2nd focal length of the eye must place the focal point on the retina 22.27 mm behind the surface. What is the radius of curvature of the surface? **4 marks**

4

$n_1 = 1$, $n_2 = 1.333$, $f_i = 22.27$ mm.

$$\frac{n_2}{f_i} = \frac{(n_2 - n_1)}{R}$$

$$R = \frac{(n_2 - n_1) f_i}{n_2}$$

$$= \frac{(1.333 - 1)(22.27)}{1.333}$$

$R = 5.56$ mm.

b) Using a ray through the centre of curvature of the model, (called the nodal point), what is the size and orientation of an image of the sun subtending 0.5 degrees at the eye? That is one edge of the sun is on axis, the other is 0.5 deg above the axis. The sun is a distant object so wavefronts will be planar at the eye. Assume that you are still in water at the second focal point. **6 marks You will have two pencils of parallel rays coming from 2 different points.**

b) Surface to centre of curvature $= R = 5.56$ mm. $\theta = 0.5°$

b

$CF_i = f_i - r = 22.27 - 5.56 = 16.71$ mm.

- note that this is also f_o where $\frac{1}{f_o} = \frac{(n_2 - n_1)}{R}$

$- y_i = CF_i \times \tan\theta \simeq CF_i\,\theta$ (when θ is in rad) $\theta(\text{deg}) = \frac{\pi}{180} \times \theta$ radian

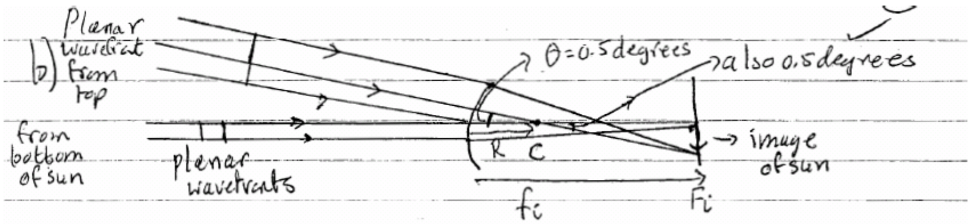
$y_i = -16.71 \times \tan(0.5°)$

$= -0.146$ mm. $= -146\ \mu m.$

The image is 0.146 mm and inverted.

c) Sketch (not to scale) a collimated beam from each edge of the sun and their images at the rear of the eye. **See slides 30, 38-40. 6 marks**

ONLY SKETCH NEEDED



Consider the triangle with C at the apex, the image as one side. The distance from C to the image is $F_i - R = 22.27 - 5.56 = 16.71$ mm.

$$\frac{\text{image size}}{16.71} = \tan(0.5)$$

image size $= 0.146$ mm.

5) a) Single refracting surface construction: a) Construct to scale the paths of three rays showing formation of an image for an object in air 1.5 cm high 10 cm in front of a surface with radius = 4 cm and n =1.5 behind the surface. The surface separates air from glass and refracts the light. You will need to calculate the focal point positions. **Choose different vertical and horizontal scales. If the ray does not appear to hit the refracting surface in a construction, then extend a plane from the surface vertex and construct using this. 9 marks**

1 a)

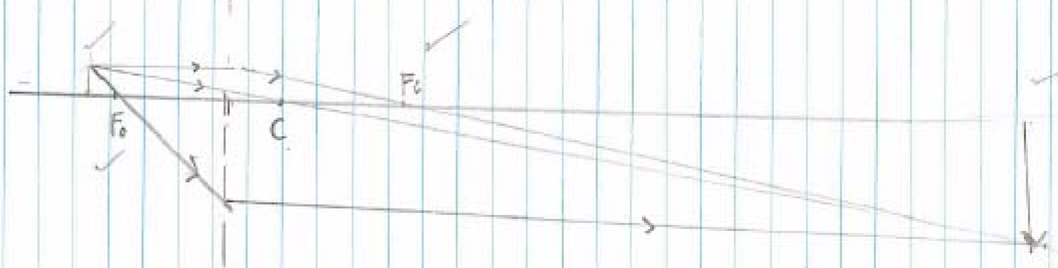
Scale: 1 cm = 3 cm.

$n_1 = 1$ $n_2 = 1.5$

$$f_o = \frac{n_1}{(n_2 - n_1)} R_1 = \frac{1}{(1.5-1)} (4\text{cm}) = 8\text{cm}.$$

$$f_i = \frac{(n_2 - n_1)}{n_2} R_1 = \frac{1.5}{(1.5-1)} (4\text{cm}) = 12\text{cm}.$$

(13)



By construction the image is 57 cm from the surface with a height of 9 cm. It is real and inverted.

2 marks for each of 3 rays

13 marks for part (a)

You may get slightly different results from your construction.

NOTE: Object and image are twice the size as in the assigned problem.

b) What is the height of the image by construction? Is the image real or virtual, upright or inverted? **3 marks**

Should be about 6 cm. It is real and inverted.

c) By calculation, what is the magnification of the system? What is the size of the image? *Note that the thin lens equations do not apply because of the change in refractive index and thus refraction angle***.** **See Slide 36**.**5 marks**

4) b) $M_T = \frac{-n_o s_i}{n_i s_o}$

$\frac{n_i}{s_i} + \frac{n_o}{s_o} = \frac{(n_i - n_o)}{R_1}$ $\quad s_o = 10 cm, \; r = 4 cm, \; n_o = 1, \; n_i = 1.5$

$\frac{1}{s_i} = \frac{1}{1.5}\left[\frac{(1.5-1)}{4} - \frac{1}{10}\right]$

$s_i = \frac{.025}{1.5}$

$= 60 cm$

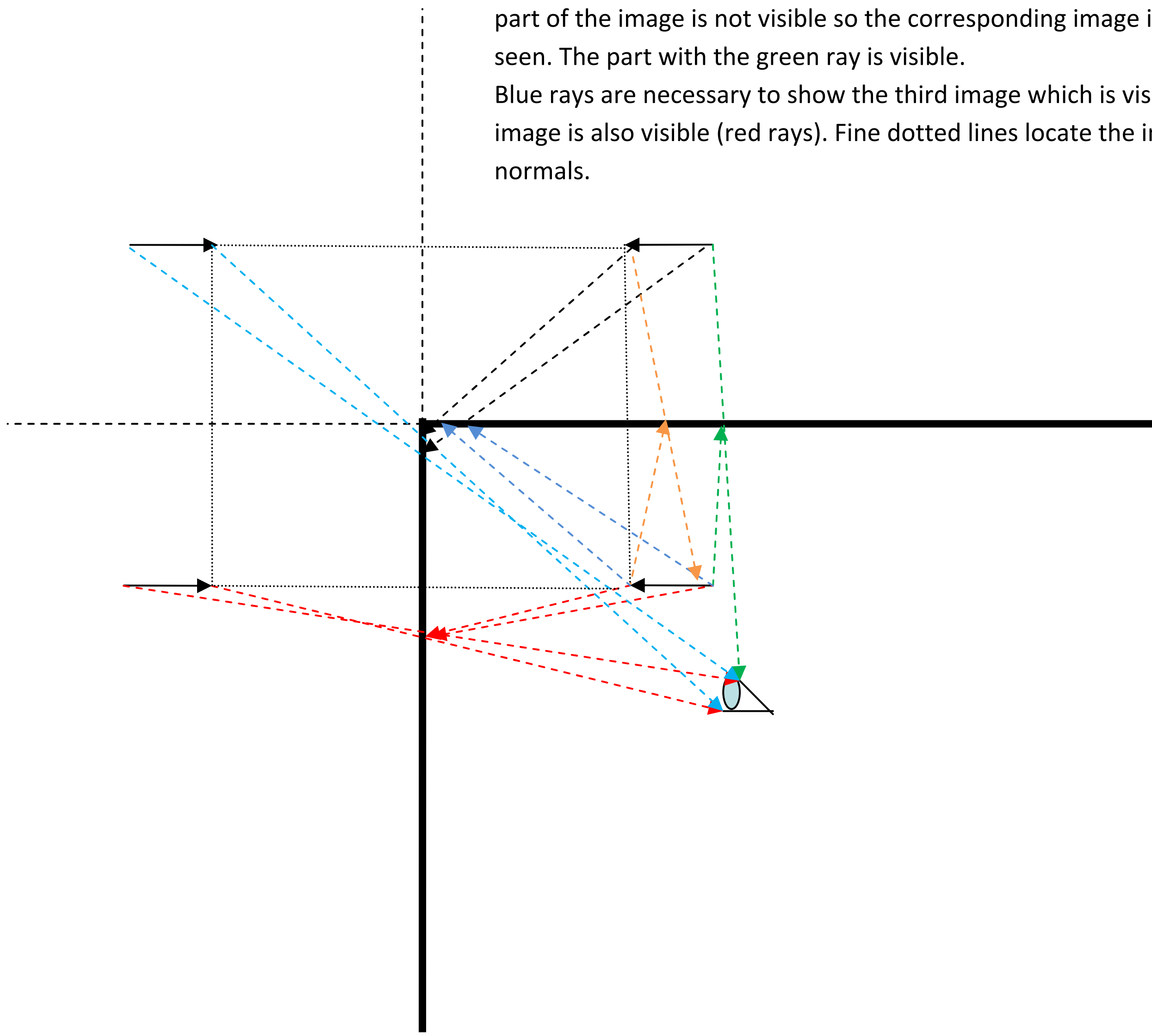
$\frac{y_i}{y_o} = M_T = -\frac{(1)(60)}{1.5(10)} = -4x$

Y_i=(1.5)(-4)=-6cm. The image by calculation is -6 cm.

6) Planar mirror: As per question 7 on assignment 6, construct the three reflections seen in the two mirrors below. Construct rays to show which images are visible to the eye in the position given. **Solve this problem in 2D on the diagram below. 8 marks**

The orange ray is blocked by the arrowhead and doesn't reach the eye. This part of the image is not visible so the corresponding image is not completely seen. The part with the green ray is visible.

Blue rays are necessary to show the third image which is visible. The second image is also visible (red rays). Fine dotted lines locate the images with normals.



7) a) Thin lens 5.10 **TOTAL 16 marks** Calculation **5 marks** Description of image **3 marks** (Assume spider is 0.5 cm high in diagram) Diagram **6 marks**.

8

5.10 (5.16) $1/f = (n-1)(1/R_1 - 1/R_2)$ where $R_2 = -R_1$, so

$$1/f = (n-1)(2/R_1)$$

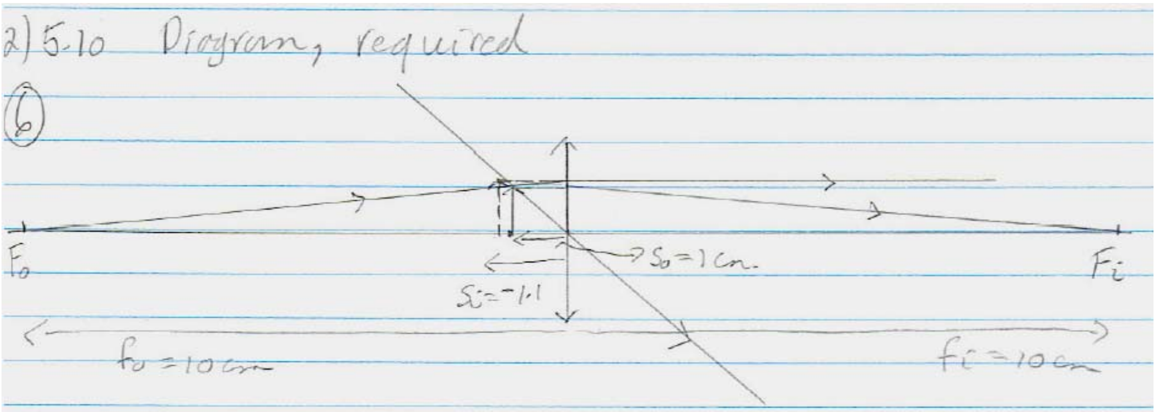
$$R_1 = (n-1)(2)(f) = (1.5-1)(2)(+10.0 \text{ cm})$$

$$= 10.0 \text{ cm}$$

(5.17) $1/s_o + 1/s_i = 1/f$; $s_o = 1.0$ cm;

$1/s_i = 1/f - 1/s_o = 1/10.0 \text{ cm} - 1/1.0 \text{ cm} = -9.0/10.0$; $s_i = -1.1$ cm.

(5.25) $M_T = -s_i/s_o = -(-1.1 \text{ cm})/(1.0 \text{ cm}) = +1.1$. Image is virtual, erect, and larger than the object.



b) What is the power of the lens? **2 marks**
Lens power=n_i/f_i =1/0.1m=10D

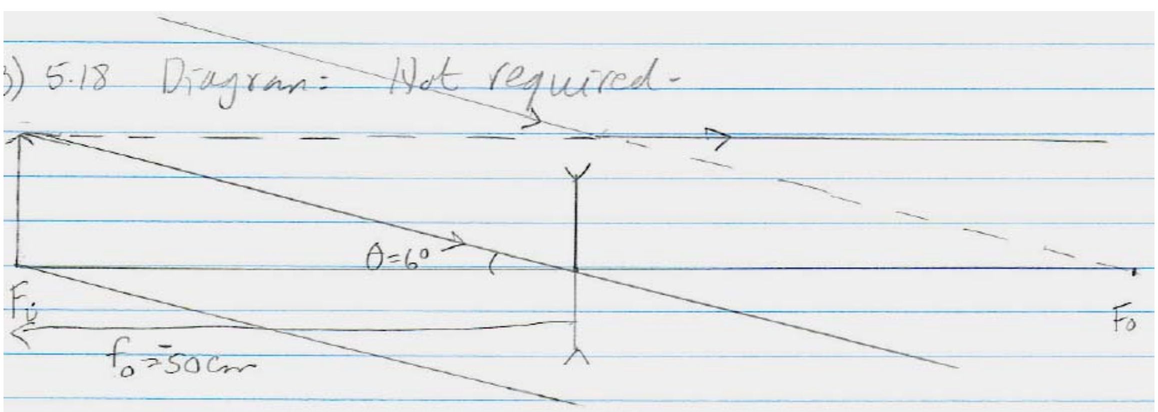
8) a) Negative thin lens: Hecht 5.18, off axis distant object. **5 marks**

Part a) asks for the image of a point source located 6 deg off axis. The image is a single point located in the image focal plane 50 cm in front of the lens. The image will be above the axis if the object is above the optical axis. The distance above the axis is given by y_i=-ftan (6deg)=-(-50)tan(6 deg)=5.3 cm (see diagram below, in part c)).

b) What size is the image of an extended object when the object subtends 6 deg? **2 marks**

The image is 50 cm in front of the lens and extends 5.3 cm from the axis to above the axis.

c) Draw to scale the image of the off axis point using at least 2 rays **4 marks**



9) Choice of positive thin lens, 5.23. **2 marks**

3

5.23 (5.17) $1/f = 1/s_o + 1/s_i = 1/45 + 1/90 = 3/90;\ f = +30$ cm.

10) Negative thin lens plus diagram 5.12. **Total 13 marks: 4 marks + 3 marks for an image description + 6 marks for scale diagram The negative lens in 5.12 is a meniscus form given in slide 14 of the Thin lens notes. In the orientation on the left the back surface has the shorter radius of curvature and both radii are positive (as given in the problem). For the orientation on the right, the first surface has the shorter radius of curvature and both radii are negative as suggested in the hint. Either approach gives the same power and focal length. Use any size object.**

$1/s_o+1/s_{i1}=1/f=(n-1)(1/R_1-1/R_2)=(1.5-1)(1/0.2-1/0.1)=-2.5=1/f$

OR $=(n-1)(1/R_1-1/R_2)=(1.5-1)(\ 1/(-0.1)-1/(-0.2))=-2.5=1/f$

$f= -0.4\ m=-40cm$

$1/0.2+1/\ s_i=-2.5D\ (m^{-1})$

$1/\ s_i=-7.5\ D$

$s_i=-0.133\ m^{-1}=-13.3cm$

$Mt=-\ s_i/\ s_o=-(-13.3)/20=0.665$

The image is virtual, minified, and upright