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1.3.2 Experimental verification: Davisson-Germer Experiment

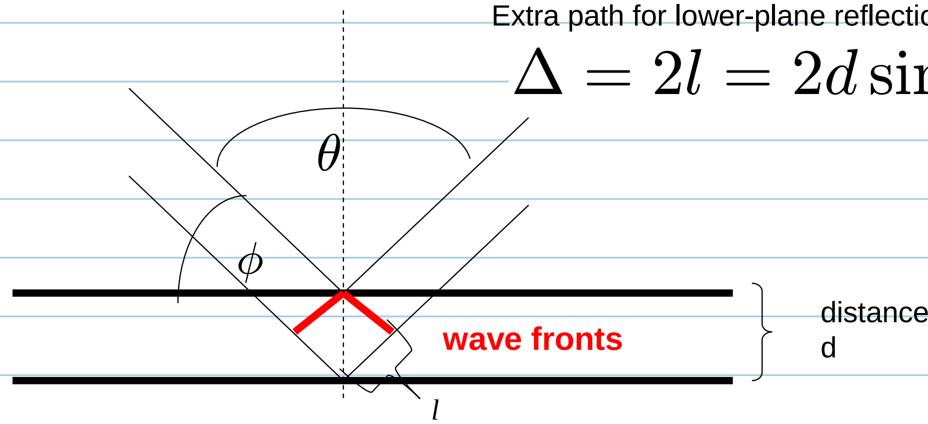
1.3.2.1 Background: Light interference at Bragg plates

Preparation: Let's talk about light waves first

==> Bragg Plates: Think about a stack of partially reflecting surfaces

Extra path for lower-plane reflection:

$$\Delta = 2l = 2d \sin \phi$$



constructive interference: $\Delta = n\,\lambda$

Bragg condition for scattering maxima: $\mathbf{n\,\lambda = 2\,d\,\sin\phi}$

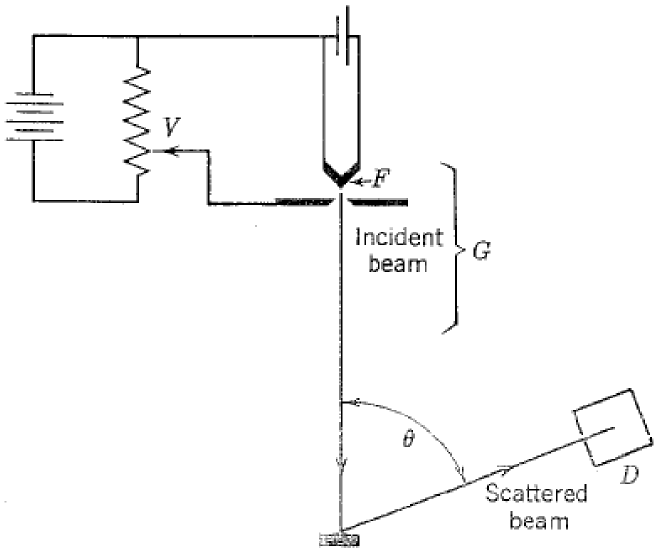
Note:
how many maxima can we see? Well, the extra path can take on any value between 0 (grazing incidence angle) and 2d (light coming in perpendicular to surface)
So the number of maxima is the number of natural numbers in the interval $\left(0, \frac{2d}{\lambda}\right)$

Note:
in Bragg reflection, there are more than two layers! The number of layers does not influence the position and number of maxima, but it influences how much of the incoming wave is reflected in total!
==> actually, there is a growing number of maxima at much lower amplitude, and the amplitude of these secondary maxima gets smaller as the number of layers increases. If we had infinitely narrow spectrum of the source, we would get very sharp main maxima against a mostly dark background.
==> if we use a source with a broader spectrum, the maxima will wash out.

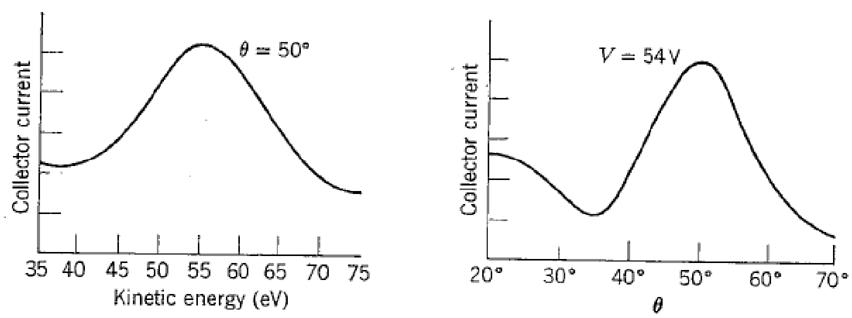
1.3.2.2 Realisation with electrons

The layers of atoms in the crystal lattice serves as partially reflecting surfaces: each layer act like a grating with in itself, which shows the property to be partially reflecting.

Davisson - Germer Experiment

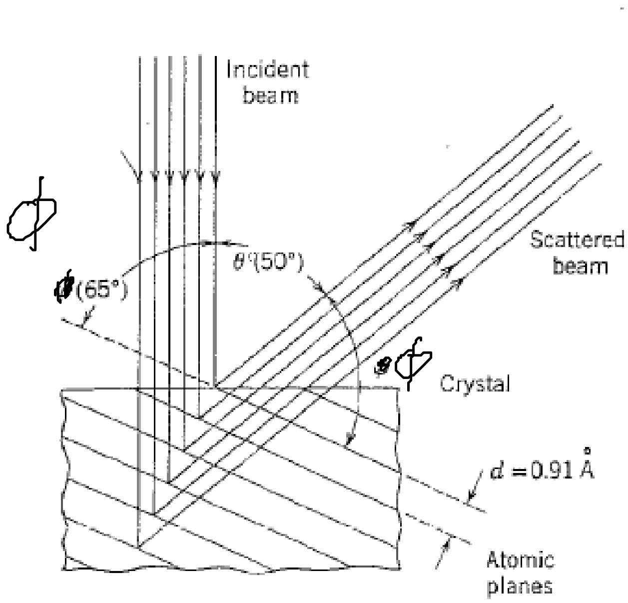


Davisson - Germer Results



Why are the interference patterns not nice sharp lines as predicted by Bragg analysis?

- the analysis neglected that the amplitude of the interfering components differ. Deeper layer reflections will have lower amplitude
- dirt effects in the crystal, e.g. electromagnetic interaction with atoms in the lattice
- the source does not emit monochromatic electrons: the electrons originate from a thermal distribution. After acceleration, it is still a broad distribution. Each energy has the maximum at a different angle. The detectors do not distinguish electrons of different energy, they see only a mixture of events!



Experimental values:

kinetic energy of electrons: $E_{kin} = 54\, eV$

$$= 54 \times 1.6 \times 10^{-19}\, J$$

$$\lambda = 1.65\, \text{Å}$$

$d = 0.91\, \text{Å}$ (nickel)

$$n\lambda = 2d \sin\phi$$

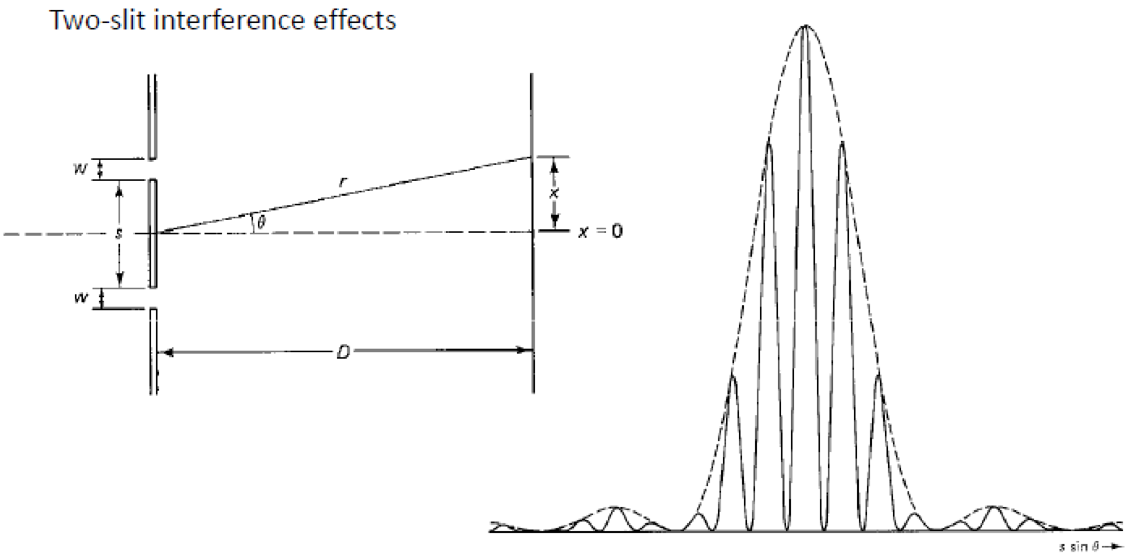
first maxima (n=1):

$$\sin\phi = \frac{\lambda}{2d}$$

$$\Rightarrow \phi = 65^\circ \Rightarrow \theta = 50^\circ$$

Note that it is important not to mix up the two definitions of the scattering angle!

1.4 Young Double-slit Experiment
1.4.1: Double Slit Experiment in Optics
1.4.1.1 Observations

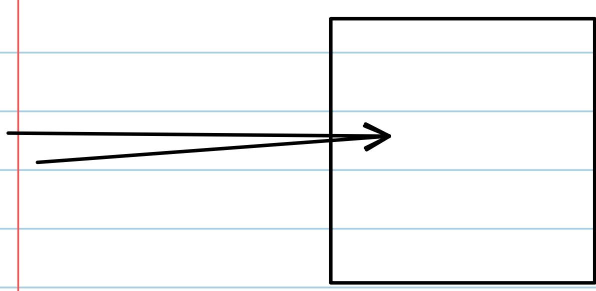


$$I_{\text{double}}(\theta) = 4I_0 \left[\frac{\sin(\pi \frac{\omega \sin\theta}{\lambda})}{(\pi \frac{\omega \sin\theta}{\lambda})} \right]^2 \cos^2\left(\pi \frac{s \sin\theta}{\lambda}\right)$$

Envelope: interference pattern from single slit
internal modulation: interference from two slits

1.4.1.2 Theory: Interference in Optics:

two light beams hitting the same spot on a screen:



$$E_1(x,t) = E_1^0 \cos\left(\frac{2\pi}{\lambda}x - 2\pi\nu t\right)$$

$$E_2(x,t) = E_2^0 \cos\left(\frac{2\pi}{\lambda}(x-\Delta) - 2\pi\nu t\right)$$

Electric field of two light beams at the spot on the screen: $\hat{E}_1 = E_1(x_0, t_0)$
$\hat{E}_2 = E_2(x_0, t_0)$

total electric field at the spot: $\hat{E}_{tot} = \hat{E}_1 + \hat{E}_2$

the light intensity I at the spot is proportional to the square of the electric field:

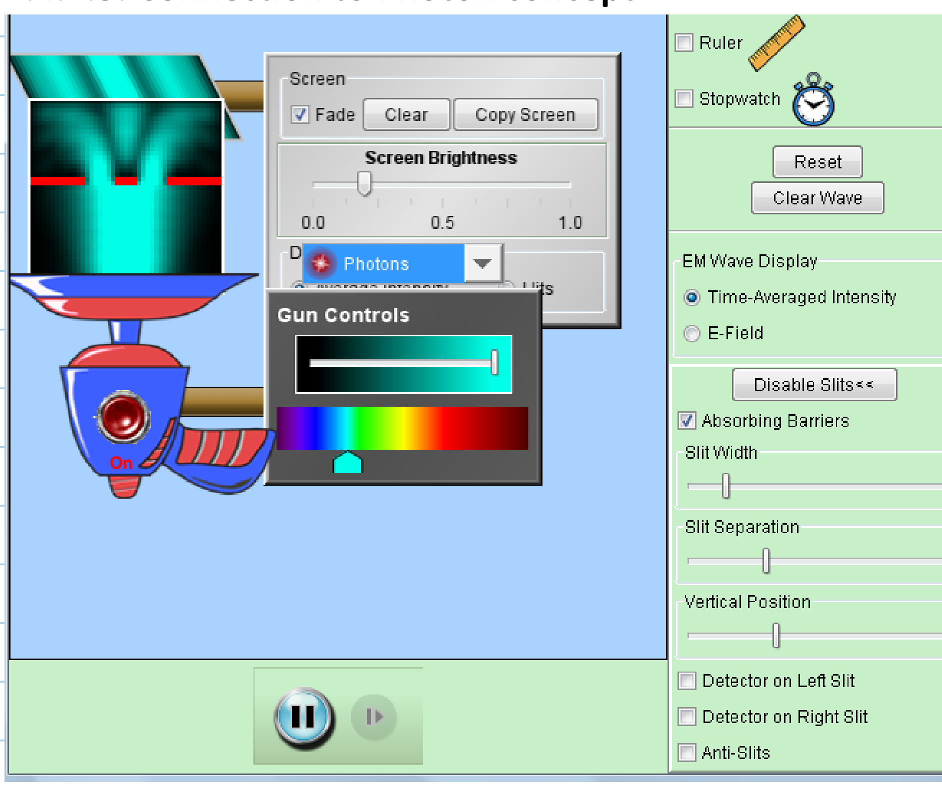
$$I \sim |\hat{E}_{tot}|^2 = |\hat{E}_1 + \hat{E}_2|^2 = |\hat{E}_1|^2 + |\hat{E}_2|^2 + 2\hat{E}_1\hat{E}_2$$

intensity if only light beam 1 would be present: $|\hat{E}_1|^2$
intensity if only light beam 2 would be present: $|\hat{E}_2|^2$
interference correction for presence of both beams: $2\hat{E}_1\hat{E}_2$

Interference term can be negative or positive
(destructive or constructive interference)

Slit, double slit, grating experiments in optics are explained by the interference of many light beams, often as continuous set of light beams corresponding to elementary waves of a wave front.

1.4.1.3: Connection to Photon concept



The light beam is composed of photons (photo-detection works similar to the photo-electric effect!) and the clicks follow the intensity distribution of the classical interference pattern

It even holds once we have only a single photon in the apparatus at any given time!

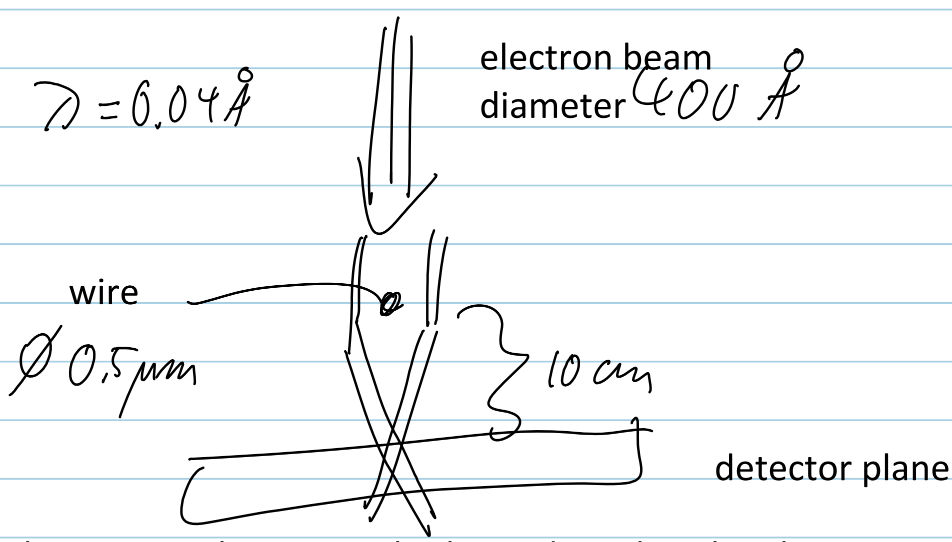
So a photon can interfere with itself! The wave nature of the fields gives rise to a probability distribution where to find the single photon. This probability distribution contains the interference effects.

1.4.2 Double slit experiments with electrons

Electrons can show the same interference pattern!

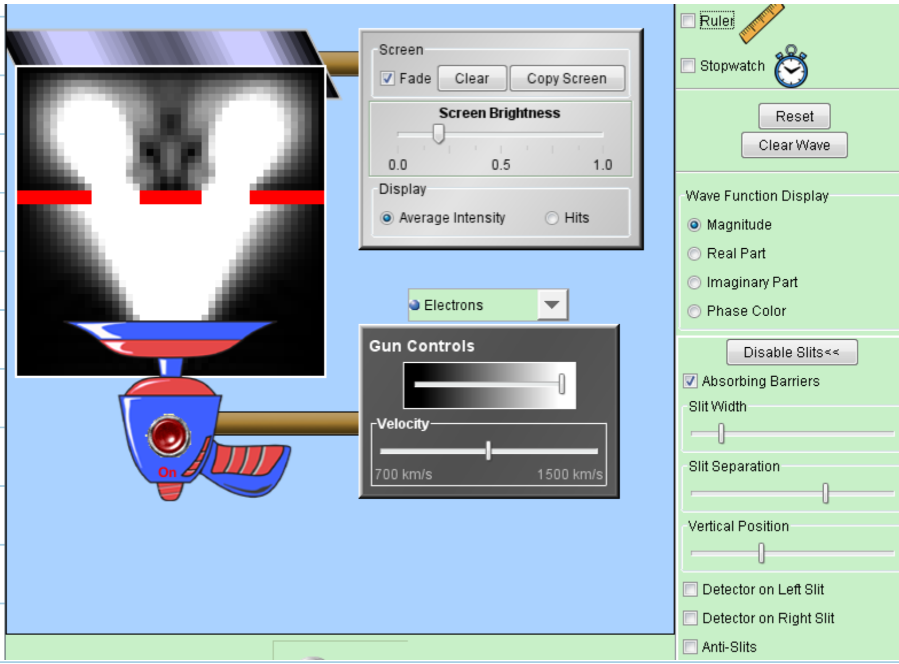
1.4.2.1 Implementation issues

How is it done? Two slits ==> impossible

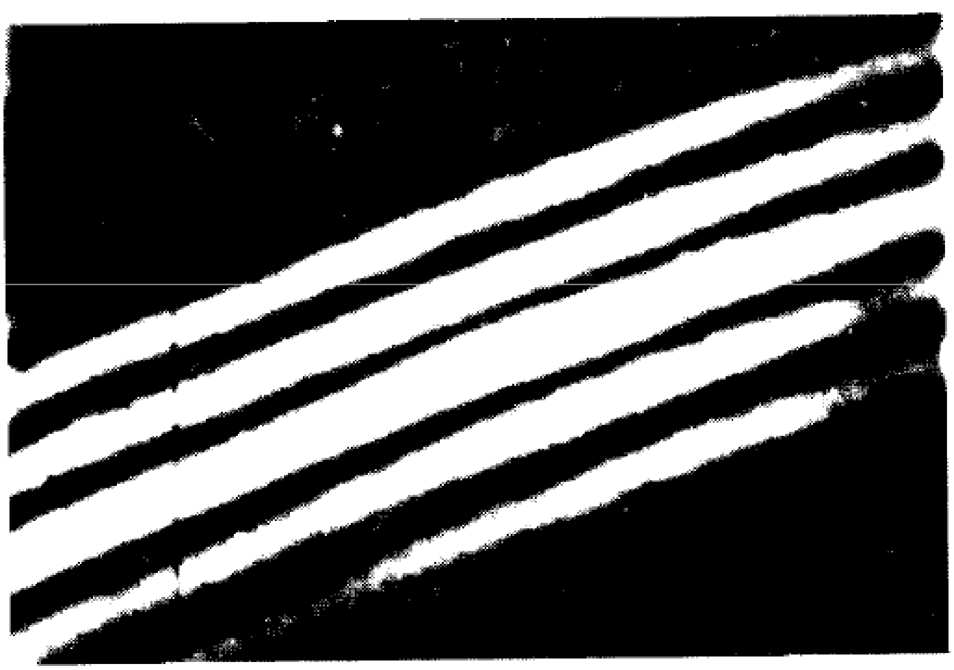


the wire can be positively charged, so that the electron waves bend slightly (depending on the amount of charging). This creates an interference pattern between the right and left beam (controlled by the charge), which is essentially that of a double slit experiment.

1.4.2.2. Observations

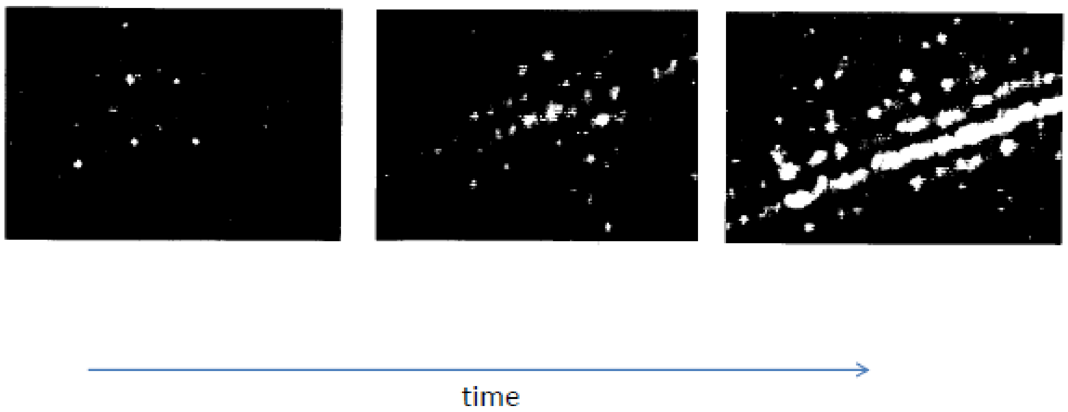


Double-slit electron interference pattern



(actual experiment)

Double-slit electron interference pattern
- one electron at a time



Electron can interfere with itself, even though the slits is separated by larger distances typically attributed to electron sizes. (atom sizes are in the Angstrom regime).

clicker question:

True or False: In the absence of external forces, electrons move along sinusoidal paths.

A) True

B) False

no, electrons do not move along wiggly paths. The wave character tells us where we can detect the photon (like a probability distribution), but it does not say what path the electron takes to the detector. We will actually see in section 1.4.4 that in quantum mechanics, it is very dangerous to talk about paths of objects!

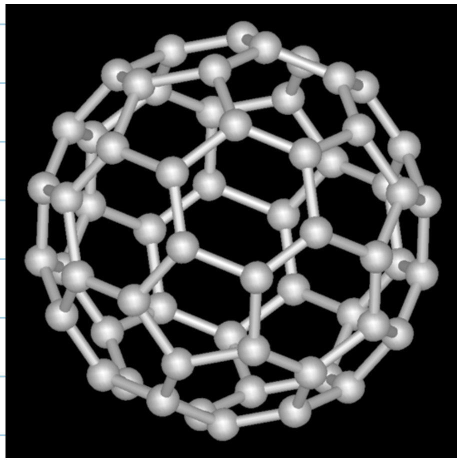
1.4.3: Double Slit experiments with other objects

Experiments done with neutrons

... but also with C-60 molecules!

Current frontier: try to do double-slit experiments with single bacteria

(See article by Nairz/Arnd/Zeilinger DOI Link)



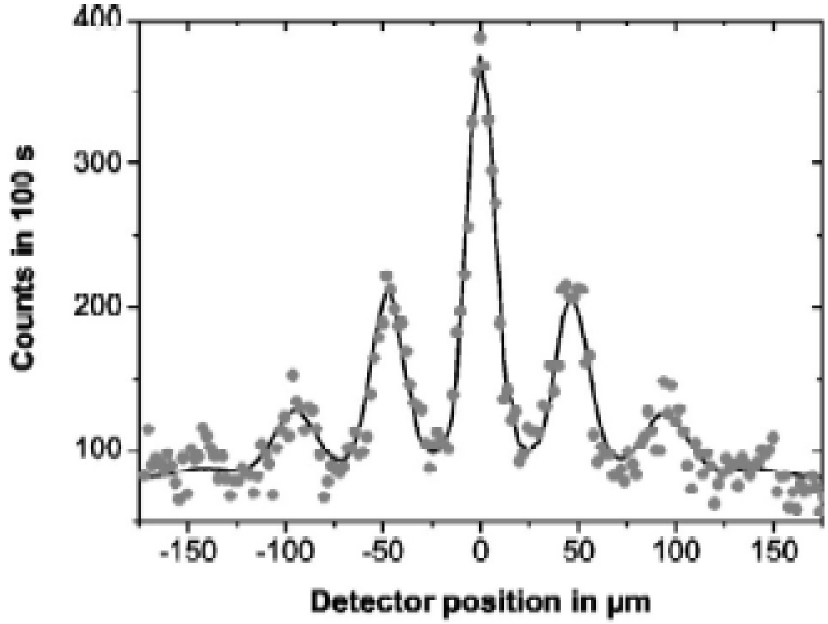


Fig. 7. Far-field diffraction of C_{60} using the slotted disk velocity selector. The mean velocity was $\bar{v} = 117$ m/s, and the width was $\Delta v/v \sim 17\%$. Full circles represent the experimental data. The full line is a numerical model based on Kirchhoff–Fresnel diffraction theory. The van der Waals interaction between the molecule and the grating wall is taken into account in form of a reduced slit width. Grating defects (holes) additionally contribute to the zeroth order.

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In principle, there is limit on how big objects can be to show interference behaviour. It might be however very challenging to observe these! (See discussion in section 1.3.1.2)