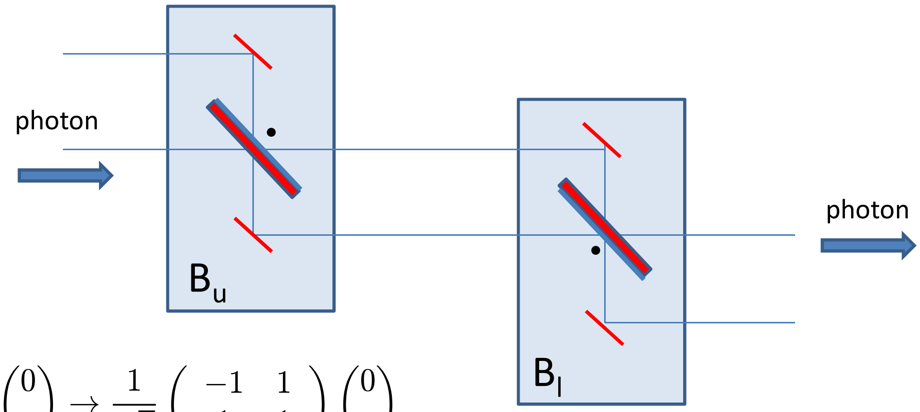
L14 Bomb Detection

Optical Interferometer



$$\begin{pmatrix}0\\1\end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$= \begin{pmatrix}1\\0\end{pmatrix}$$

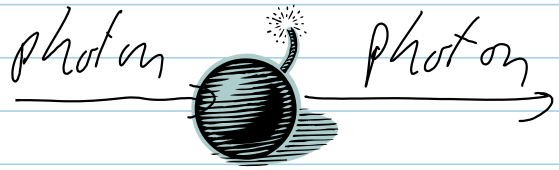
4.6.2.3 Bomb model

working bomb

single photon absorbed
by trigger, bombs get off
==> working bomb
corresponds to a very loud
single photon detector!

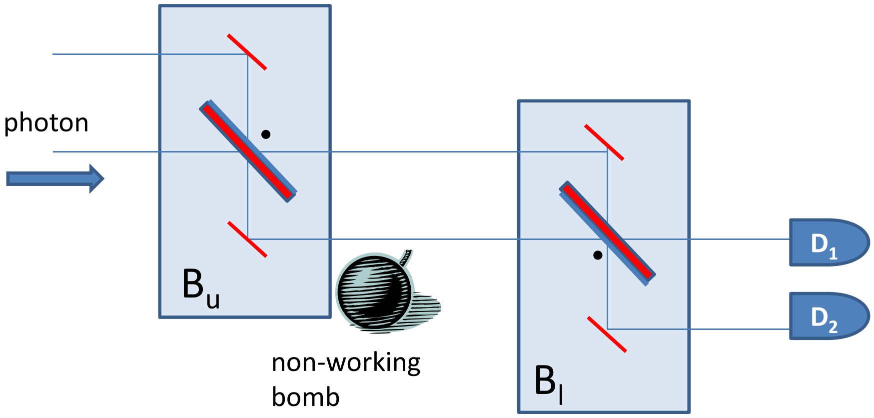
non-working bomb

single photon can pass the fuse the bomb without any effect



4.6.2.4 Interferometer for Bomb Detection

Bomb detection: non-working bomb

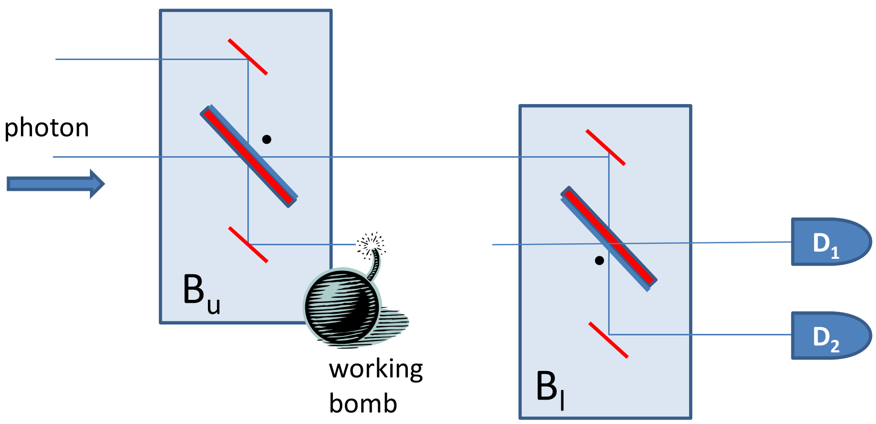


Three possibilities:

1) Test set-up blows up, but no photon in either detector
2) Test set-up does NOT blow up, one photon in detector D_1
3) Test set-up does NOT blow up, one photon in detector D_2

"Boom"	D_1	D_2
-	100 %	-

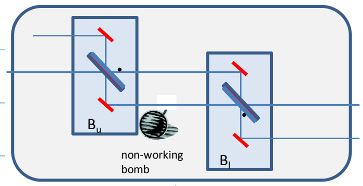
Bomb detection: working bomb

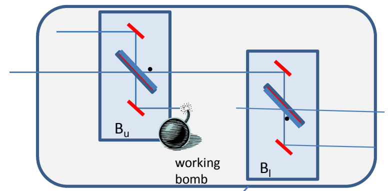


Three possibilities:

1) Test set-up blows up, but no photon in either detector
2) Test set-up does NOT blow up, one photon in detector D_1
3) Test set-up does NOT blow up, one photon in detector D_2

"Boom"	D_1	D_2
50 %	25 %	25 %





Bomb status	"Boom"	D_1	D_2
defective	-	100 %	-
working	50 %	25 %	25 %

If you do not know whether the bomb works or not:

upper detector clicks: still no clue, but can repeat process
lower detector clicks: for sure a working bomb, bomb still intact
no clicks, but loud 'BOOOOM': it was a working bomb ...

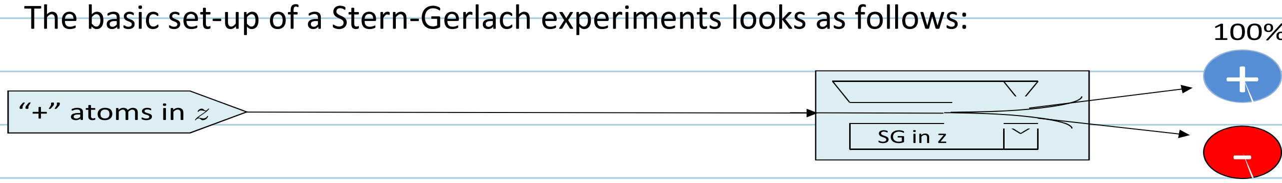
If initially 70% of bombs are working, one iteration gives

(1/4 x 70% =)	17.5 % of guaranteed working bombs
(1 x 30% + 1/4 x 70% =)	47.5 % of still undetermined bombs,
(1/2 x 70% =)	35% of exploded bombs

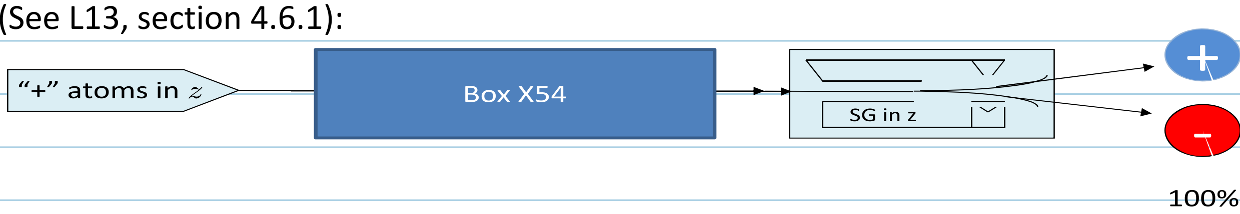
4.7 Quantum Zeno Effect

4.7.1 Preparation and Basic Observation

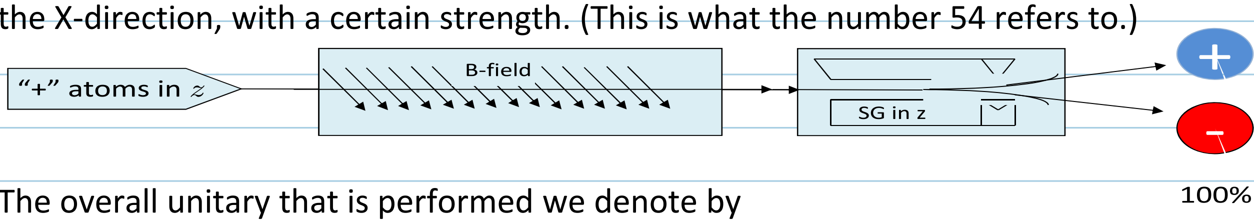
The basic set-up of a Stern-Gerlach experiments looks as follows:



We have previously introduced a box which flips "up" and "down" via a unitary operation (See L13, section 4.6.1):



Now we look inside the box and note that it contains a homogeneous magnetic field directed in the X-direction, with a certain strength. (This is what the number 54 refers to.)



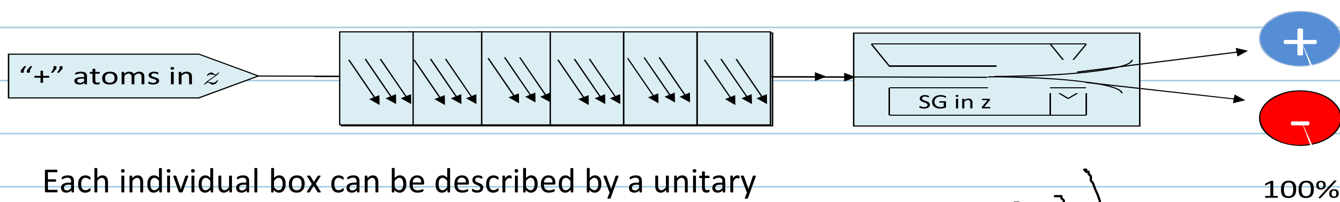
The overall unitary that is performed we denote by

$$U_L = i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Note that I include here a global phase I which is needed for global consistency.

We will learn in the later section about Dynamics in Quantum Mechanics what the physical connection is between the B-Field and how the spin direction of the state is rotated. We suspect for now that this rotation must be a quantum analogue of the rotation of a classical

Next, we can divide the whole length L of our box into smaller boxes of length l. Obviously, this division is more a thought construct that does not influence the result:



Each individual box can be described by a unitary

$$U_l = \begin{pmatrix} \cos\left(\frac{l}{L}\frac{\pi}{2}\right) & i\sin\left(\frac{l}{L}\frac{\pi}{2}\right) \\ i\sin\left(\frac{l}{L}\frac{\pi}{2}\right) & \cos\left(\frac{l}{L}\frac{\pi}{2}\right) \end{pmatrix}$$

(Think about the subscript as a parameter!)

Note the two extreme cases:

$$l \to 0 \qquad U_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

identity operator

$$l = L \qquad U_L = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

as before

We have also the interesting property

$$\mathcal{U}_\ell \, \mathcal{U}_{\ell'} = \mathcal{U}_{\ell+\ell'}$$

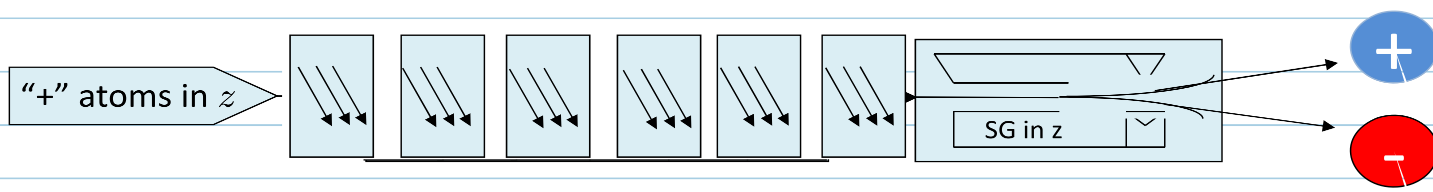
which can be checked using trigonometric identities (or Mathematica/Maple ...)
If we divide the length L into n equal parts, then we have

$$\mathcal{U}_{\frac{L}{n}} = \begin{pmatrix} \cos\frac{\pi}{2n} & i\sin\frac{\pi}{2n} \\ i\sin\frac{\pi}{2n} & \cos\frac{\pi}{2n} \end{pmatrix}$$

and we have

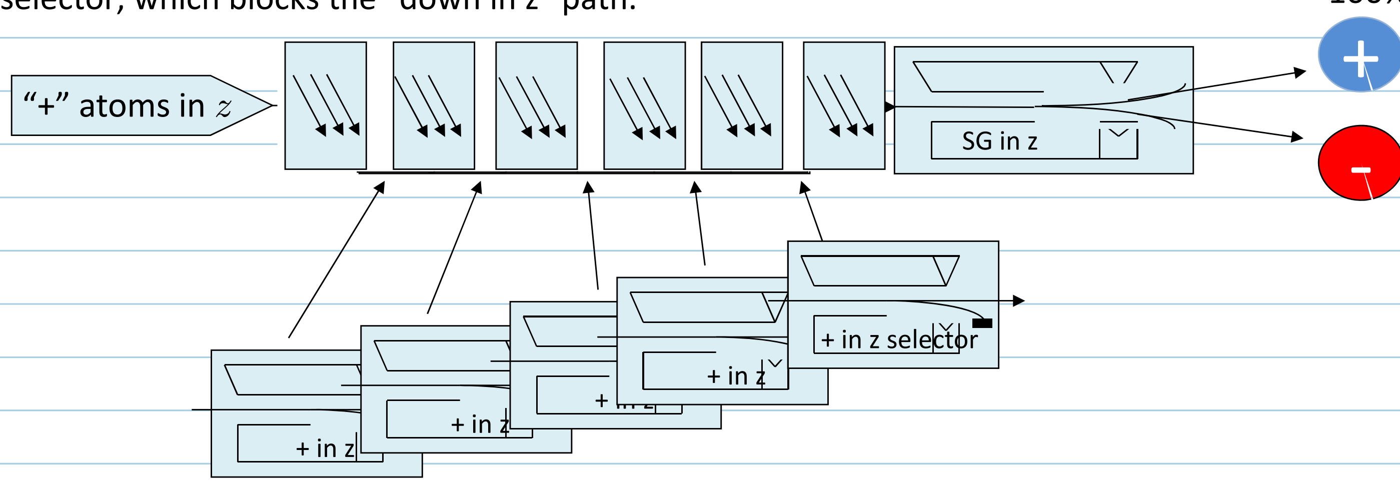
$$\left(\mathcal{U}_{\frac{L}{n}}\right)^n = \mathcal{U}_L$$

Next, we can also separate the individual boxes a bit from each other, and again, this does not influence the outcome of the experiment:



100%
100%

Finally, we insert after each magnetic field stretch a Stern-Gerlach selector, which blocks the "down in z" path.



We find

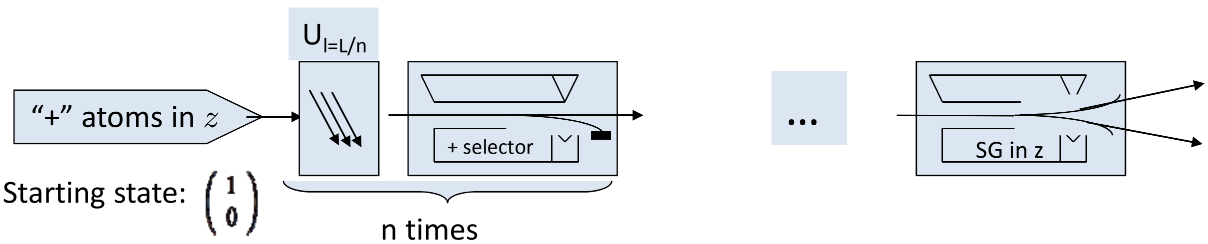
a) that all atoms that emerge on the right hand side are up in z
b) as the number of slices n increases, all atoms make it through the set-up!

Finding a) is easily explained, as just before the final Stern-Gerlach experiment, we made a selector "+ in z" measurement (and the little rotation of the B field section after that will hardly make a difference).

Finding b) is harder to see. Certainly, for a small area of B-field, the atom will rotate only a small amount, and thus basically never gives the spin-down result, passing almost all atoms on. But then we have many such steps. How do they add up?

So let us do the calculations:

4.7.2 Quantitative Analysis



<u>after one iteration:</u>

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\frac{\pi}{2n} & i\sin\frac{\pi}{2n} \\ i\sin\frac{\pi}{2n} & \cos\frac{\pi}{2n} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \cos\frac{\pi}{2n} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$Prob('' + '') = \cos^2\frac{\pi}{2n}$$

<u>after all *n* iterations:</u>

$$\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\frac{\pi}{2n} & i\sin\frac{\pi}{2n} \\ i\sin\frac{\pi}{2n} & \cos\frac{\pi}{2n} \end{pmatrix} \right]^n \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \left[\cos\frac{\pi}{2n}\right]^n \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

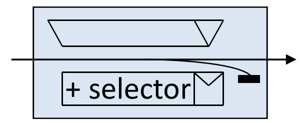
$$Prob('' + '') = \left[\cos^2\frac{\pi}{2n}\right]^n \sim 1 - \frac{\pi^2}{4n}$$

Therefore for n $\rightarrow \infty$: we find (asymptotically) always "+ in z"

"How": the mathematical answer

4.7.3 Physical Interpretation

Measurement is interaction!



Same effect would have worked for the following set-up

here all atoms always pass, but we notice which path they take …

Measurements in quantum mechanics have a
back-reaction
onto the state of the system

Constant observation of a system
➔ constant interaction/disturbance of a system
➔ different behaviour as compared to no observation!

"How": the physical answer

The physical view is the following:
The magnetic field tries to start to turn the spin, but is constantly interrupted by a measurement which turns the probability amplitudes into probabilities. This freezes the evolution of the system in the initial state.

For this note that what happens at the intermediate Stern-Gerlach devices and the direct or indirect observation which path the atoms are taking is an interaction with the atom. So we should not surprised that the outcome of the experiment changes if we introduce this extra interaction.

Note further, that it is not important to learn what the outcome of the Stern-Gerlach measurement is! It is important that the interaction by the observer would in principle allow do distinguish which path has been taken.