L06 Stern Gerlach Experimental Evidence and Explanations

September-21-11
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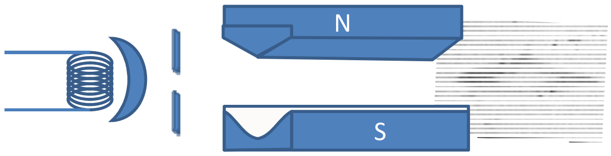
Stern Gerlach: Basic Observations



Abstraction:
atoms go *either* up *or* down
(but not anything inbetween)
ignore remaining spatial distribution

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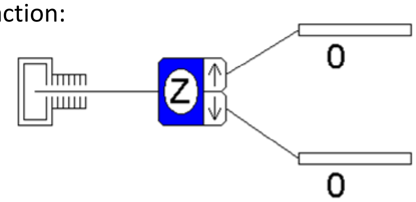


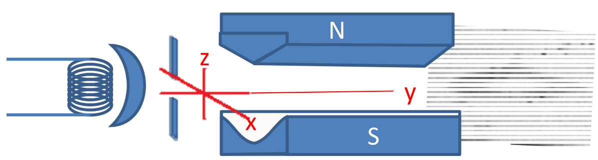
Observation 1: (thermal source)
Atoms are either pulled up or pulled down (two discrete values) with equal probability

This observation could be explained if the source emits atoms that have with 50/50 chance their dipolemoment pointed straight up or straight down (with respect to the z-direction of the magnetic field)
BUT: why would there be a preferred direction for the source? It seems no dipole moment is oriented perpendicular to the magnetic field... Strange!

2.2.5 Rotated Stern-Gerlach

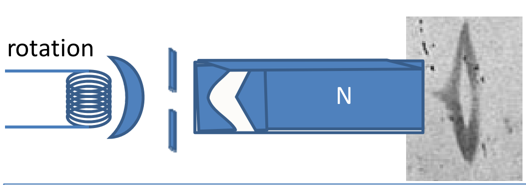
Stern Gerlach: Basic Observations

a)



Abstraction:
atoms go *either* up *or* down
(but not anything inbetween)
ignore remaining spatial distribution

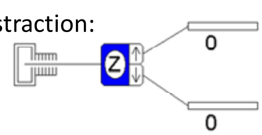
b) rotation



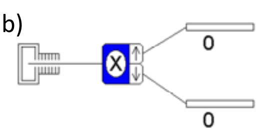
Abstraction:
atoms go *either* right *or* left
(but not anything inbetween)
ignore remaining spatial distribution

Observation 2 for thermal source:
no matter how you orient the (inhomogeneous) magnetic field:
the atoms go *either* towards the stronger field *or* towards the weaker field!
Quantization of dipole moment with respect to applied magnetic field!

a) Abstraction:



b)



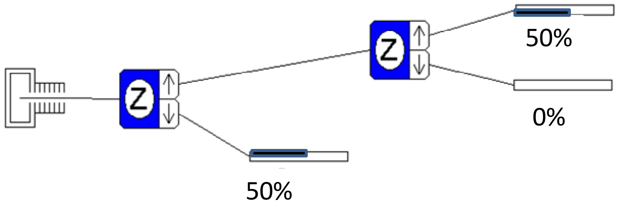
all outcomes are "binary" for all possible orientations of the device!
(at least for silver atoms)

if the source emits up- or down-ward oriented dipoles, as observation a) would force us to believe, then we run into trouble with set-up b, because then those up and down orientations (in z-direction) of the source dipoles would give no force on the atoms in the SG device, and the atoms should come out in the middle, without any deflection. But that does not happen.
In b) we would need to think about the dipole to point up or down with respect to the X-direction!!!
Classically, it can be only one or the other, but not both ...

2.3 Explorative Experiments with Stern Gerlach Set-up

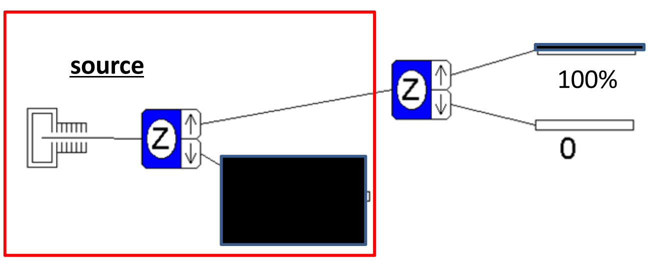
2.3.1 Experiment 1 and Effective Sources

Experiment 1



Clicker Question:
What do you expect to happen in the second SG?
the incoming atoms
A) all go *up*
B) 50% go *up*, 50% go *down*
C) all go *down*

Observation 3: Sequential Stern-Gerlach devices I
two identically oriented Stern-Gerlach devices in sequence always give the same result:

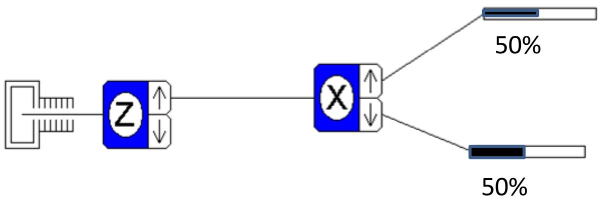


small print:
we talk now about probabilities ***conditioned*** on the fact that an atom leaves the source!

Effective Source: The oven (thermal source) together with the first Stern-Gerlach device form an effective source. We take only atoms emerging from one arm of the SG-Set-up for further experiments, the other arm is blocked off.

2.3.2 Experiment 2 with crossed SG devices

Experiment 2



<u>Clicker Question:</u>
What do you expect to happen in the second SG?
the incoming atoms
A) all go *up*
B) 50% go *up*, 50% go *down*
C) all go *down*
(*with respect to the X-direction)*

Observations 4: Sequential Stern-Gerlach devices II
- still only two possible outcomes in second SG
- although the source gives deterministc outcome for Z-measurements, in X-direction we get only probabilistic outcomes (here 50/50 chance)
- It turns out that the Z-orientation of the SG is the ONLY orientation (measurement direction) which gives a deterministic outcome!

Quantitative observation:
whenever the two SG involved have orthogonally orientation to each other
➔ 50/50 outcome

This runs strongly against the intuition of the classical picture: after the first measurement 'revealed' an up-direction of the magnetic dipole moment, then this dipole moment should not be deflected in a second, perpendicular oriented Stern-Gerlach Device!

2.4 Structure of Theory and naming quantum states

Structure of Theory

preparation (system) → dynamics → measurement (apparatus)

<u>Example:</u>
billard ball on table

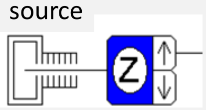
system state:
(phase space)
position x
momentum p
...

measurement description
e.g position measurement:
positions x_1, x_2, x_3 ...

all you need to know about system to predict measurement result!

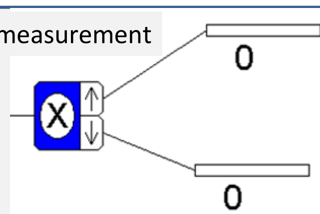
Stern-Gerlach Experiment

source



state description?
ket $|\Psi\rangle$

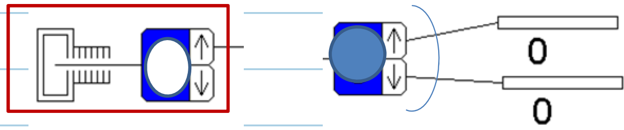
measurement



pre- Postulate 1:
The state of a quantum mechanical system is represented mathematically by a symbol ket $|\Psi\rangle$. This symbol
- summarizes everything you can know about the system
- and everything you need to know to predict measurement results

Naming the States ...

so what characterizes a source for the SG experiment?



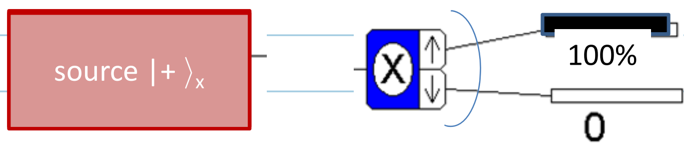
A source of this type is fully characterized by stating for which measurement orientation it will create

rotate until 100% in one outcome!

We introduce labels for the state of atoms leaving a Stern-Gerlach device.
The label corresponds to a promise by physics that such an atom will always give a deterministic result when queried in a Stern-Gerlach setting with the same direction that is indicated by the label!

We will later see that this label corresponds to some complex valued vector space! But for now, it is just a name ...

so what characterizes a source for the SG experiment?



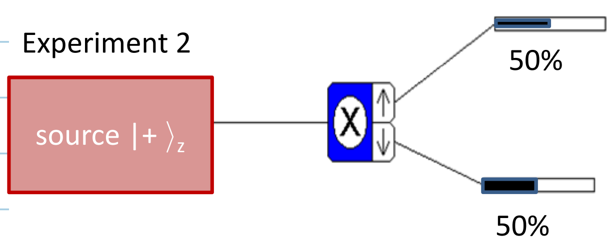
A source of this type is fully characterized by stating for which measurement orientation it will create

give source label of deterministic outcome: $|+\rangle_x$

rotate until 100% in one outcome!

The only thing that we can say about this type of source is that for a particular SG measurement it will give a deterministic outcome!

Experiment 2



But wait:
what tells us how the system behaves in a different measurement?
There must be more information?
Sorry, no, that's it ...
Our theory will only predict probabilities!

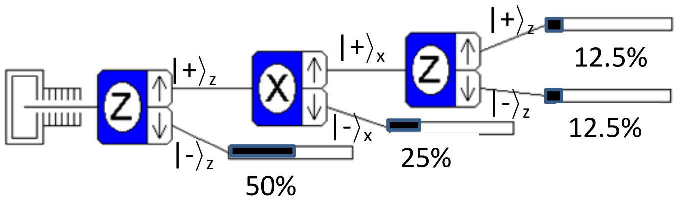
2.5 Explorative Experiments (Part 2)

2.5.1 Experiment 3 with Three SG devices

After Experiment 2: maybe the X and Z measurements reveal just independent properties of the atoms?

... No, as we see now:

Experiment 3



Clicker Question:
What do you expect to happen in the last SG?
the incoming atoms
A) all go *up*
B) 50% go *up*, 50% go *down*
C) all go *down*
(*with respect to the z-direction)*

Observation 5:
The measurement outcome depends only on the preceding measurement.

This justifies again the naming idea of our states and the Postulate 1!

The measurement in X made the atoms 'forget' what z-direction they had!
==> this is a justification for our label idea:
the label gives a promise for subsequent measurements, the history of the system is irrelevant!

Apparently, the measurements of X and Z are incompatible! They do not correspond to independent properties, but access the same system, but in an incompatible way. Measuring one property, say X, messes up the property with respect to the Z measurement!

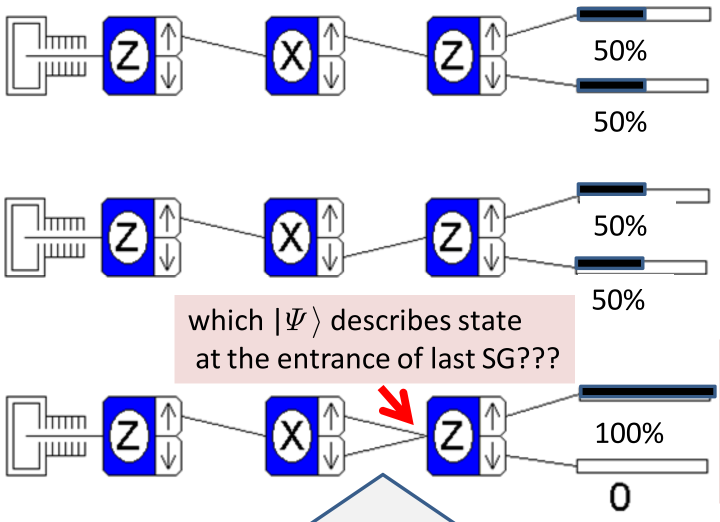
It is apparently wrong to think about the atoms to 'have' a specific dipole moment with a specific orientation, such that the Stern-Gerlach device simply has to reveal this strength and orientation...

But how should we think about it?

2.5.2 Experiment 4 with Interference

Experiment 4

Clicker Question:
What do you expect to happen in the last SG?
the incoming atoms
A) all go *up*
B) 50% go *up*, 50% go *down*
C) all go *down*



which $|\Psi\rangle$ describes state at the entrance of last SG???

If I know which path has been taken
➔ no interference
Without that information
➔ interference

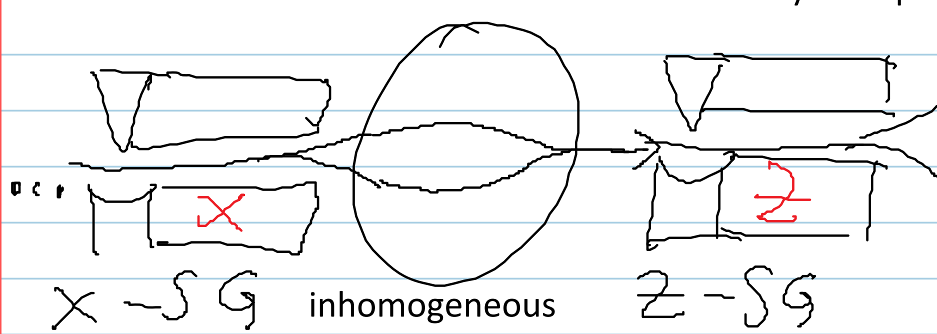
recombination with additional inhomogeneous fields
➔ atoms from lower and upper arm of middle SG must end up in the same input beam for last SG

Apparently, allowing more pathes (path a) and b) together can lead to effects looking like interference!
It looks as if the X device would not be there!

Note: in this set-up, there is no knowledge which path the atoms took in the X-device, so in the middle there is no measurement!

In quantum mechanics, measurement is an interaction that enables someone to determine which path a system took! Same as in optics (double slit).

Combiner:
The combiner in front of the last SG device is tricky to implement



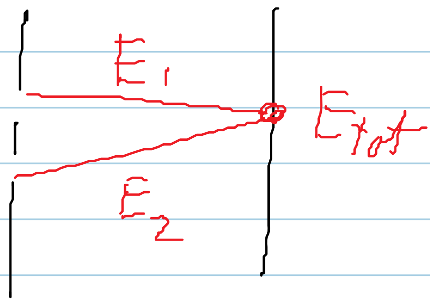
inhomogeneous
magnetic field

one would need to engineer the right inhomogeneous magnetic field, such that the atoms in the upper arm come in the same position and the same angle in the last SG-device as the atoms that take the lower path. We omit all details here.
This is tricky to realize, so treat it a bit more like a thought-experiment!

2.6 Starting point of Interpretation via Amplitudes

Experiment 4 suggest that there are interference effects at work here!

two-slit experiment



two-slit screen

for each location on the screen there is a total amplitude from which the probability/intensity is calculated to find light there

$$I \sim |E_{tot}|^2$$

the total amplitude has contributions from each path the light could have taken

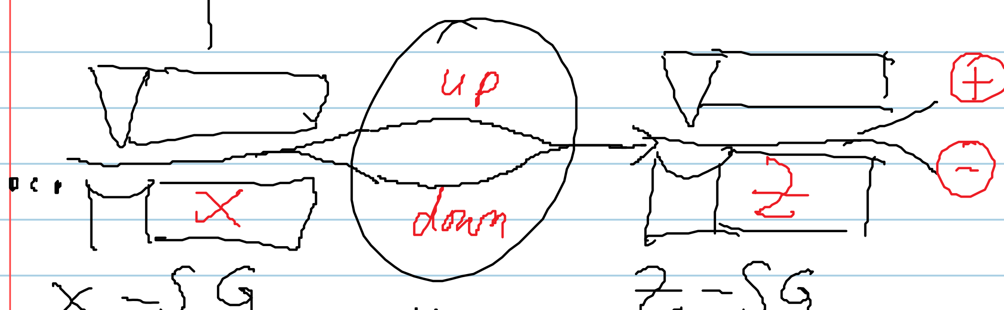
$$E_{tot} = E_1 + E_2$$

actual sign/phase depends on the geometry, that is on the location on the screen

A Stern Gerlach device then seems to act similarly:

The probability to detect an atom in one of the outcomes is associated with a total amplitude to appear there. Different paths contribute to this amplitude.

outcome in Z-ZG	amplitudes Path up X	down X	total
+	A_+^{up}	A_+^{down}	$A_+^{up} + A_+^{down}$
-	A_-^{up}	A_-^{down}	$A_-^{up} + A_-^{down}$



combiner

By combining the two beams (without retaining which-path information) we need to add amplitudes to total amplitudes.

We haven't said much about how to connect amplitudes to count-probabilities, but for now it sufficies to say that amplitude zero means no counts ....

Then it is clear that each path (up or down) can separately give 50/50 results, while if properly combined can lead to interference effects so that only one outcome result will be possible (constructive interference), while the other result will no longer occur (destructive interference)

The result smells of vector spaces, if we consider all possible outcomes:

$$\begin{pmatrix} A_+^{up} \\ A_-^{up} \end{pmatrix} + \begin{pmatrix} A_+^{down} \\ A_-^{down} \end{pmatrix} = \begin{pmatrix} A_+^{up} + A_+^{down} \\ A_-^{up} + A_-^{down} \end{pmatrix}$$

let's study vector spaces!

M2: Vector Spaces

M2.1 Abstract properties

Vector Spaces

Basic Elements:

set of vectors $\{|\alpha\rangle, |\beta\rangle, |\gamma\rangle \ldots\}$

set of scalars (field) {a,b,c, ...} [relevant case: complex numbers]

Vector Addition:

closure:

$|\alpha\rangle + |\beta\rangle = |\gamma\rangle$

commutative:

$|\alpha\rangle + |\beta\rangle = |\beta\rangle + |\alpha\rangle$

associative:

$|\alpha\rangle + (|\beta\rangle + |\gamma\rangle) = (|\alpha\rangle + |\beta\rangle) + |\gamma\rangle$

unique null vector $|0\rangle$: $\forall\, |\alpha\rangle$

$|\alpha\rangle + |0\rangle = |\alpha\rangle$

inverse vector: $\forall\, |\alpha\rangle$ exists a unique $|-\alpha\rangle$

$|\alpha\rangle + |-\alpha\rangle = |0\rangle$

Scalar Multiplication:

closure:

$a\,|\alpha\rangle = |\gamma\rangle$

distributive 1:

$a\,(|\alpha\rangle + |\beta\rangle) = a\,|\alpha\rangle + a\,|\beta\rangle$

distributive 2:

$(a + b)\,|\alpha\rangle = a\,|\alpha\rangle + b\,|\alpha\rangle$

associative:

$a\,(b\,|\alpha\rangle) = (a\,b)\,|\alpha\rangle$

connection to special scalars:

null element of multiplication:

$0\,|\alpha\rangle = |0\rangle$

neutral element of multiplication:

$1\,|\alpha\rangle = |\alpha\rangle$

inverse element of addition:

$(-1)\,|\alpha\rangle = |-\alpha\rangle$

Example: vectors $\begin{pmatrix} a \\ b \end{pmatrix}$

addition: $\binom{a}{b} + \binom{c}{d} = \binom{a+c}{b+d}$

scalar multiplication: $c\binom{a}{b} = \binom{ca}{cb}$

M2.2 Bases and Representation

dimension d: Choose the number of mutually exclusive outcomes in a measurement and check that for each such outcome, there exists a source which always triggers that outcome.

In our cases: we see that we should choose

d = 2

NOTE: in this section, we will work with a 2-dim vector space, but of course in general we will encouter any dimension. The extension to higher dimensions is straightforward ..

Basis states:

any set of mutually exclusive labels can be associated with an orthonormal basis set

examples

$$\{|+\rangle_z, |-\rangle_z\}$$

or

$$\{|+\rangle_x, |-\rangle_x\}$$

Notation: if no direction is given, we mean the z-direction

$$|+\rangle_z \equiv |+\rangle$$

$$|-\rangle_z \equiv |-\rangle$$

Completeness:

Any state vector can be expanded in a set of basis vectors

$$|\psi\rangle = a|+\rangle + b|-\rangle$$

Coordinate representation of vectors (kets)

The vectors can be represented by the coefficients of the basis states.

To do this, a basis must be chosen, and that choice must be kept in mind!

We represent abstract vectors (kets) as column vectors in the coordinate representation.

$$|\psi\rangle = a|+\rangle + b|-\rangle$$

$$|\psi\rangle \doteq \begin{pmatrix} a \\ b \end{pmatrix}$$

The symbol $\doteq$ indicates that the right hand side is a representation of the abstract vector. The representation depends on the basis: in another choice of basis the numbers in the column will be different! Keep that in mind: whenever you use the coordinate representation, you have to make sure you know to which basis it refers, otherwise it will create havoc with your results.

M2.3 Dual vector space:

With the vector space of 'kets' $|\psi\rangle$

we also associate a dual vector space with elements denoted as 'bra' $\langle\psi|$

Each element of the original vector space is one-to-one assigned with an element of the dual vector space. This relationship can be defined using any basis (but turns out to be basis independent)

As a first step, we assign the basis states of the vector space to the basis states of the dual vector space:

$$|+\rangle \Longleftrightarrow \langle +|$$

$$|-\rangle \Longleftrightarrow \langle -|$$

The other assignments are done using the linear relation:

vector:

$$|\psi\rangle = a\,|+\rangle + b\,|-\rangle$$

dual vector

$$\langle\psi| = a^*\langle +| + b^*\langle -|$$

Coordinate representation of dual vectors (bras):

We can represent a dual vector (bra vector) again by the coefficients of the basis vector. By convention, we write these coefficients as row vector.

$$\langle\psi| = \alpha\langle +| + \beta\langle -|$$

$$\langle\psi| \doteq (\alpha, \beta)$$

Connection between coordinate representation of vectors and their dual correspondence

$$|\psi\rangle = \alpha|+\rangle + \beta|-\rangle \doteq \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

$$\updownarrow \qquad\qquad\qquad \updownarrow$$

$$\langle\psi| = \alpha^*\langle +| + \beta^*\langle -| \doteq (\alpha^*, \beta^*)$$