Physics 256 Assignment 9

Due: Wednesday, November 21st, 2012 4:00 pm in the drop box Physics 2nd floor or electronically 87 marks

1a) If a telescope is astronomical has a tube length of 12.5 cm, and an angular magnification of (-)4X, what are the focal lengths of the objective and eyepiece lengths? **3 marks**

$$\textit{tube length} = d = f_o + f_e = 12.5cm$$
$$M_\alpha = \frac{\alpha_a}{\alpha} = \frac{-f_o}{f_e} = \pm 4X = -4X \textit{ astronomical}$$
$$f_o = 4f_e$$
$$4f_e + f_e = 12.5cm$$
$$f_e = 2.5cm; f_o = 10cm$$

b) If the exit pupil is 1 cm in diameter, what size is the entrance pupil? **2 marks**

$$M_\alpha = \frac{D_o}{D_e} = \frac{-f_o}{f_e} = \frac{-10}{2.5} = -4$$
$$\frac{D_o}{D_e} = -4$$
$$D_o = 4(1cm) = 4cm$$

Negative sign indicates that a ray path crosses from the top to bottom of the pupil

2) Determine the power of the eyepiece and the lens spacing (tube length) to provide a Galilean telescope with a magnification of 4X with Galileo's objective with a focal length of 1.7 m. Notice that in the Galilean telescope one can use lower powered lenses for the same magnification and length as the astronomical or similar powers with a more compact size. **3 marks**

$$M_\alpha = \frac{-f_o}{f_e} = \frac{-1.7}{f_e} = 4$$
$$f_e = \frac{-1.7}{4} = -0.425m = -42.5cm$$
$$D = \frac{1}{f_e} = \frac{1}{-0.425} = -2.35D$$
$$\textit{tube length} = d = f_o + f_e = 1.7 + (-0.425) = 1.275m$$

b) If the objective is 5.6 cm in diameter, what is the minimum eyepiece diameter? **2 marks**

$$M_\alpha = \frac{D_o}{D_e} = 4$$

$$D_e = \frac{5.6}{4} = 1.4cm$$

Since the light exits parallel, 1.4 cm is also the minimum eyepiece size

3) Telescope used as a reverse laser beam expander: Hecht 5.40. **4 marks**

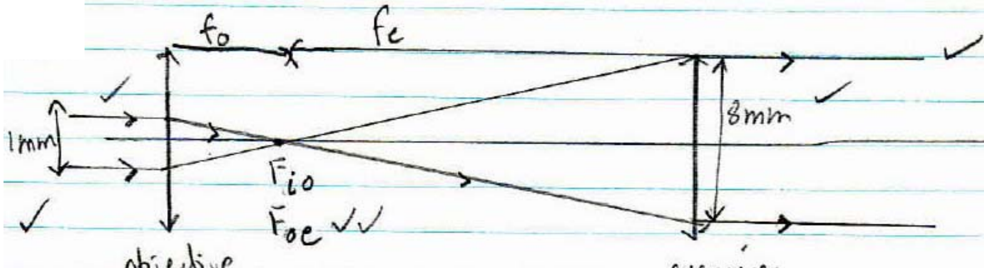
For the orientation of the reversed telescope (diagram not necessary, 2 marks each feyepiece and tubelength:

$$M_{ang} = \frac{D_{en}}{D_{exit}} = \pm\frac{1\,mm}{8\,mm} = -\tfrac{1}{8}\times \quad \text{(Astronomical)}$$

$$M_{ang} = -\frac{f_o}{f_e} = -\frac{5.0\,cm}{f_e} = -\frac{1}{8} \qquad f_e = 40.0\,cm.$$

$$f_e = 40.0\,cm.$$

$$\text{tube length} = f_o + f_e = 5.0\,cm. + 40.00\,cm. = 45\,cm.$$



Note: reversed from usual telescope - shorter focal length is first.

For the classic telescope orientation Give ½ the marks for this solution:

$$M_P = \frac{D_o}{D_e} = \frac{-f_o}{f_e} = \frac{+8}{-1}$$

D_e is inverted relative to D_o

$$\frac{-f_o}{f_e} = \frac{8}{-1} = -8$$

$$-f_o = 50(-8) = -400\text{ mm}$$

3) In the diagram below, which surface is the aperture stop and which is the field stop. Justify your answer. **4 marks** From Smith: Modern Optical Engineering:

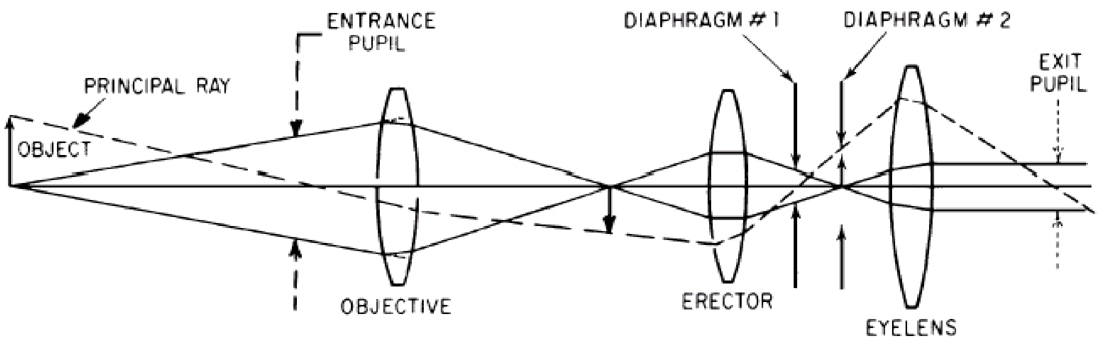
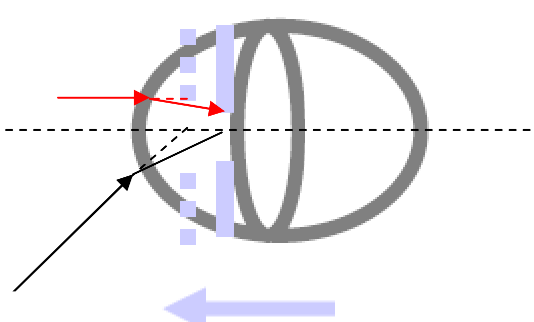


Figure 6.2 Schematic sketch of an optical system to illustrate the relationships between pupils, stops, and fields.

Diaphragm 1 is the aperture stop. Among all surfaces and apertures, it is the one that is limiting the rays from the axial object. Diaphragm 2 is the field stop. The principle ray from the edge of the object (dotted) just grazes the edge of this field stop and clears all other stops and surfaces.

4) The entrance pupil of the eye is the image of the iris seen by an observer looking through the cornea. If the cornea is considered to be a single surface of R=7.8 mm with air in front and water (n=1.333) behind, and the iris is located 3.6 mm behind the cornea, what is the size and magnification of the entrance pupil? Show that the entrance pupil is 3.0mm behind the cornea and is 13% larger than the iris. HINT: Assume that the iris is the object and light comes from the object. (see slide 26). **4 marks**



b) If the iris is 3.5 mm in diameter, what size is the entrance pupil? Is the entrance pupil real or virtual? **2 marks**

c) Trace a marginal and a chief ray through the pupil. **2 marks**

The iris is an object giving off light located +3.6mm in front of the surface. $n_o = 1.333$, $n_i = 1$ $R = -7.8$mm.

(4) $\frac{n_o}{S_o} + \frac{n_i}{S_i} = \frac{(n_i - n_o)}{R}$

$\frac{1.333}{3.6} + \frac{1}{S_i} = \frac{(1-1.333)}{-7.8}$

$\frac{1}{S_i} = \frac{+0.333}{7.8} - \frac{1.333}{3.6}$; $S_i = -3.0$ mm. which indicates it is behind the surface

Magnification of the entrance pupil $= -\frac{n_o S_i}{n_i S_o} = -\frac{1.333(-3.05)}{1\,(3.6)}$

$= \cancel{1.19}$ 1.13

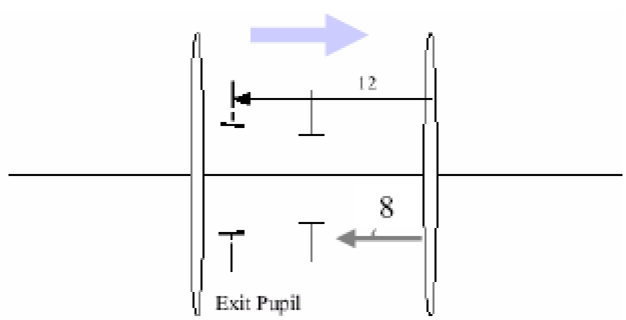
= 13% larger

b) size of pupil=3.5mm*1.13=3.96mm

The entrance pupil is virtual

c) see diagram-the red ray is a marginal ray (from a distance object), the dotted optical axis is a principal (chief ray) from an axial object OR the solid black ray and dotted direction is an off axis principal ray.

5) Calculate the exit pupil position and size: In the picture opposite, f_i of lens 2 is 24 cm. and the aperture stop is 8 cm from the 2nd lens. It is also 8 cm from the 1st lens which has a focal length of 4 cm.



a) Show that the exit pupil is 12 cm from the second lens and,

4) Aperture distance = $S_o = 8$ cm.

Exit pupil distance = S_i

$$\frac{1}{8} + \frac{1}{S_i} = \frac{1}{24}$$

$$\frac{1}{S_i} = -\frac{1}{12}$$

$S_i = -12$ cm. - The exit pupil is 12 cm. from the 2nd lens (behind)

$$M_T = -S_i/S_o = \frac{-(-12)}{8} = 3/2$$

b) if the aperture is 2 cm diameter, the exit pupil is 3 cm. HINT: This will only involve the second lens. **6 marks**

4) Aperture distance = $S_o = 8$ cm.

Exit pupil distance = S_i

$$\frac{1}{8} + \frac{1}{S_i} = \frac{1}{24}$$

$$\frac{1}{S_i} = -\frac{1}{12}$$

$S_i = -12$ cm. - The exit pupil is 12 cm. from the 2nd lens (behind)

$$M_T = -S_i/S_o = \frac{-(-12)}{8} = 3/2$$

Size of the exit pupil= 3/2X(2)=3 cm

6) a) Hecht 5.80 (f# of a camera). **4 marks**

4) Hecht 5.80

$$I \propto \frac{1}{f\#^2} \qquad I \propto \text{time}$$

4

For fixed flux density to the film, time $\times \left(\frac{1}{f\#^2}\right)$ must be fixed

Initially $t_1 = 1/30\,s.$, $f\#_1 = 11$

then $t_2 = \frac{1}{120}\,s.$ $f\#_2 = ?$

$$\frac{t_1}{f_1^{\#2}} = \frac{t_2}{f_2^{\#2}}$$

$$f\#_2^2 = \frac{t_2 f_1^{\#2}}{t_1} = \frac{\left(\frac{1}{120}\right)(11)^2}{(1/30)} = \frac{(11)^2}{4}$$

$$f\#_2 = 11/2 = 5.5 \qquad f/5.5$$

If the speed has increased by a factor of 4, the f# decrease by a factor of 2 to f/5.5

b) By how much does the irradiance of an image change if the f# goes from f/8 to f/2? **2 marks**

f# becomes 4 times smaller, and irradiance should become 16 times larger

c) In red light (λ=632.84 nm), what is the diameter of the image of a point source on the CCD plane assuming only diffraction blur (the "diffraction-limited case") if the entrance pupil is 1 cm and the camera lens has a focal length of 150 mm? **2 marks**

$$Blur = 2.44\lambda \mathbf{f\#} = 2.44\lambda \frac{f}{D}$$

$$Blur = 2.44(632.84)\frac{150}{10}$$

$$Blur = 23{,}161.9nm = 23.2\mu m$$

d) What is the angular extent of the diffraction blur at the entrance pupil plane for the point source in c) above? **2 marks**

$$\textbf{Angular blur} = 2.44\frac{\lambda}{D} = \frac{2.44(.63284)}{10X10^{+3}} = 1.54X10^{-4}\,radians$$

7) a) 10.28: telescope resolution compared to the eye's resolution- the objective mirror is also the entrance pupil. This resolution would be reduced by wavefront aberrations caused by the atmosphere unless adaptive optics are used. **11 marks**

Hecht 10.28 $\quad D_{mirror} = 2a = 5.08$ cm. $\quad \lambda = 550$ nm.

$\Delta\theta_{min} = \frac{1.22\lambda}{D}$

Angular limit to resolution $= \frac{1.22(5.50 \times 10^{-7})}{5.08}$ radian

$=$

[illegible]

$\Delta\theta_{min} = 1.32 \times 10^{-7}(360)/2\pi$

$= 7.57 \times 10^{-6}$ degrees $\qquad 1° = 3600$ sec.

$\Delta\theta_{min} = \frac{7.57 \times 10^{-6}}{\cancel{3600}} \times 3600$ sec/degree

11

$= \cancel{2.10 \times} \; 2.72 \times 10^{-2}$ arc sec

Two objects ~~7.57 x 10^-6~~ 1.32×10^{-7} radian apart

are $s = (1.32 \times 10^{-7})(3.844 \times 10^{8})$ m.

apart at ~~a poi~~ on the moon

$= 5.07 \times 10$ m.

$= 50.7$ m.

For the eye with 4 mm pupil, the distance apart on the moon for just resolvable objects is

$$\frac{1.22(5.50 \times 10^{-7})(3.844 \times 10^{8})}{4 \times 10^{-3}}$$

$= 6.45 \times 10^{4}$ m. $= 64.5$ km.

b) What is the radius of the central portion of the Airy disk formed by a small star at the focal plane of the objective mirror if its f# is 3.3? **3 marks** **Note**: In the case of an in-focus image for a distant source (in air), the image will be located at the focal point of the optical system. In most systems it can be approximated that the entrance pupil of the system is close to the principal plane.

$$radius = 1.22\lambda f\# = 1.22(5.5X10^{-7})3.3 = 2.21x10^{-6}\,m = 2.21 microns$$

8) Answer the questions on slide 74 of the Depth of Focus slides: How does the change in f# from f/22 to f/2 change each of the following? Quantify your answer where possible. **9 marks**

- Flux density? $\boxed{\frac{1}{f/\#^2}; \textit{increased by } 11^2 \textit{ or } 121x}$
-
- Diffraction? $\boxed{\propto f\#; \textit{decreased by } 11x}$
-
- Aberrations?- increase with decreasing f#, increasing pupil

- Depth of field? $\boxed{\propto f\#; \textit{decreased by} \sim 11x}$

9) Depth of Focus and Depth of Field: a) If the acceptable blur diameter in the film plane of a 50 mm focal length camera is 0.01 mm, and the f/# is set at f/16, what is the depth of focus in the camera? Assume that the camera is focused at distance. **2 marks**

6) Depth of Focus problem

a) $f = 50mm$, $B = \pm 0.01mm$ $f_{\#} = 16$

(2) $\delta' = \pm B f_{\#}$

$= (0.01)(16)$

$= 0.16mm$

b) If the camera is sharply focused an object at 1 m from the lens, how close to the camera can the object be moved before it appears blurred? The distance that the object is moved is δ, the depth of field. First the camera lens is moved so that the image is in sharp focus for an object at 1 m. Find the lens to image plane distance in this case by assuming that the focal length does not change. Then find the depth of field in object space approximately by calculating the transverse magnification M_T. δ will be approximately δ'/M_T^2 where δ' is the depth of focus calculated previously. Note that this calculation gives symmetric δ and δ' towards and away from the best focus. **6 marks**

b) $S_o = 1m.$ $f_i = 50mm.$ $S_i = ?$

$$\frac{1}{S_i} + \frac{1}{S_o} = \frac{1}{f_i}$$

$$\frac{1}{S_i} = \frac{1}{50\times10^{-3}} - \frac{1}{1}$$

$S_i = 52.63\,mm$ which is the lens to image plane distance

EITHER

$$M_T = -S_i/S_o = \frac{-52.63\times10^{-3}}{1} = -0.05263$$

(4) $$\delta = \delta'/M_T^2 = \frac{0.16}{(-0.05263)^2} = 57.76\text{ mm} = 5.78\text{ cm}$$

OR Exact calculation (not responsible)

OR S_{i1} = 52.63+0.16=52.79

S_{i2} = 52.63-0.16=52.47

Using $\frac{1}{S_i} + \frac{1}{S_o} = \frac{1}{F_i}$ calculate

$$\frac{1}{s_{o1}} = \frac{1}{50} - \frac{1}{52.79}; \ s_{o1} = 946mm$$

$$\delta_{near} = 1000 - 946 = 54mm$$

(4).

$$\frac{1}{s_{o2}} = \frac{1}{50} - \frac{1}{52.47}; \ s_{o2} = 1062mm$$

$$\delta_{far} = 1062 - 1000 = 62mm$$

c) How does your answer compare to the calculation on the web page that is linked to slide 67? What is the δ away from the object at 1 m? Inwards from the object? **2 marks**

(4) c) The answer on the site is a near focus at 0.94 m, δnear=6 cm depth, and a far focus at 1.06 m, δfar=6 cm depth ✓

Our first answer is not different from these and is symmetrical. The online answer indicates a small asymmetry, similar to that in the second more exact calculation.

d) If the f# becomes f/8, what happens to the depth of focus? Depth of field? **4 marks**

d) If the f# → f/8

$\delta' \propto f\#$ ✓

(3)

δ' will be ½ of the original δ'

depth of focus will half. ✓

depth of field will also approx. half ✓