Lecture 3

Biophysics and Fluid Flow

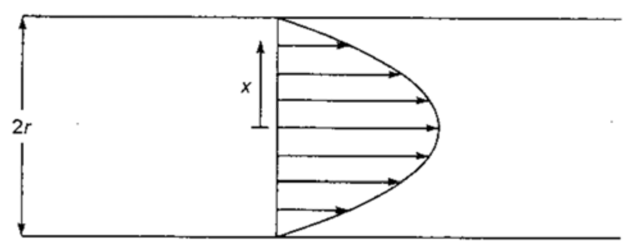
- BIOL 280 presentation topic **Electrophoresis**.

- Sonica Sharma, biology looking for a partner
- s34sharm@gmail.com

Viscosity

- Fluid moves if force is applied
- One part of the fluid gains velocity another opposes (this is not a an equilibrium flow)
- Internal friction between two parts transfers momentum from faster to slower part of the flow in order to establish equilibrium- viscosity

- Fluid – solid contact force of adhesion stronger than in fluid
- The speed is larger in the middle of the pipe



- Examples:
- blood flow,
- water in plants, air flow in respiration

Dynamics of steady laminar flow

- Steady laminar flow - the velocity at any point is constant (or relatively steady)
- Incompressible flow – density remains constant (in reality there are always variations)
- if variations are small the fluid can be considered as steady laminar flow
- Steady flow in pipe – the pressure difference which pumps the fluid is constant
- Steady laminar flow can be visualized as layers of fluid (planes or lamina) molecules moving parallel to each other
- Each particle follows a smooth path
- Neighboring layers path each other smoothly

Coefficient of viscosity

Steady laminar flow on a stationary solid surface

Force required to move a layer at a distance y from the surface with velocity v

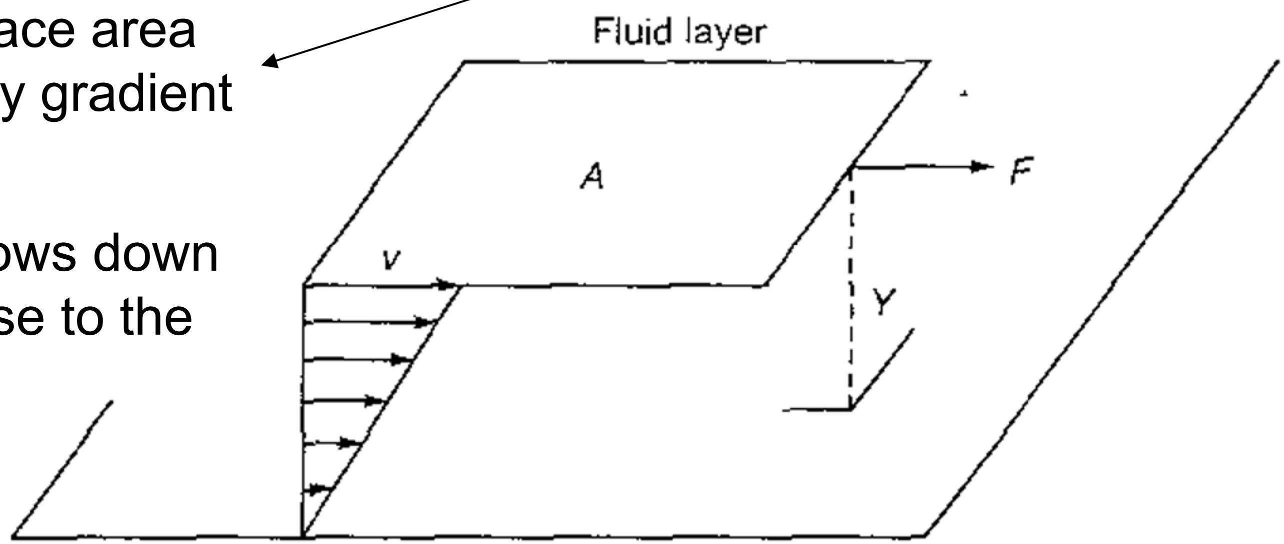
$$F = \eta A \frac{\Delta v}{\Delta y}$$

η-coefficient of viscosity

A-surface area

velocity gradient

Viscosity slows down the fluid close to the surface



Coefficient of viscosity

η-coefficient of viscosity (dynamic viscosity) is a quantitative measure of viscosity of the fluid, the greater the viscosity of the fluid - the greater the force required to move it

Units:

$$P(poise) = \frac{dyne \cdot \sec}{cm^2}$$

in CGS

$$Pa \cdot s = N \cdot s / m^2 = 10P$$

in MKS

Dynamic and kinematic viscosity

η - dynamic viscosity

$$P(poise) = \frac{dyne \cdot \sec}{cm^2}$$

$$\nu = \frac{\eta}{\rho}$$

- kinematic viscosity

$$[\nu] = \frac{cm^2}{\sec}$$

Physically - kinematic viscosity is a rate at which velocity equalizes during laminar fluid flow

Table. Dynamic and kinematic viscosities of some liquids

at 37° C

	η (Pa.s)	ν (cm^2/sec)
Hydrogen	0.91×10^{-5}	
Air	1.90×10^{-5}	16.92×10^{2}
Water	0.69×10^{-3}	0.69×10^{2}
Glycerine	0.35	
Whole Blood	$3\text{–}4\times10^{-3}$	
Blood Plasma	$\sim1.4\times0^{-3}$	

Dynamic viscosity for water is higher than air,
Kinematic viscosity for water is lower than air

Temperature dependence - liquids

The viscosity of liquids decreases when the T rises – movement of molecules becomes easier.

For organic liquids with high viscosity – decreases exponentially:

$$\eta(T) \sim e^{-E/RT}$$

E – activation energy per mole

$$\eta(T) = A + \frac{B}{T}$$

A and B constants, determined empirically

Temperature dependence - gases

- Viscosity of gas increases when T increases (opposite to liquids)
- At high T collisions are more frequent and internal friction is larger
- According to kinetic theory:

$$\eta = \frac{1}{3} v l \rho$$

v-speed, l –mean free path of molecule, ρ – density of gas

$$\eta \propto v = \sqrt{\frac{3RT}{M}}$$

- for ideal gas

For real gases viscosity decreases faster than $\sqrt{T}$ with T decrease

- In ideal case – viscosity is a constant at a given T
- in reality viscosity depends upon velocity of the flow

- Newtonian Fluid – viscosity can be taken as a constant at a given T during laminar flow

- Non-Newtonian Fluid – viscosity is a function of velocity
- Example - blood

Poiseuille's Formula

Steady laminar flow in a rigid cylindrical tube of uniform cross-section

$$\Delta V = \frac{\pi r^4}{8\eta l}\Delta P$$

$$\Delta V = \frac{\Delta P}{R}$$

Poiseuille's Formula

Viscosity leads to friction between the fluid and walls –pressure is required to maintain the flow

Pressure difference between two ends of a tube

Volume discharge (current) - amount of fluid moving per unit time

$$R = \frac{8\eta l}{\pi r^4}$$

Resistance of the pipe to the flow

l –length, r– radius of a tube

η – fluid viscosity

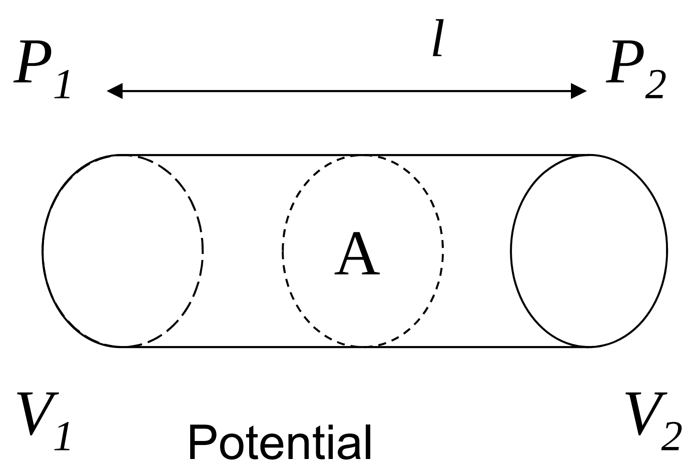
$$\Delta V = \frac{\Delta P}{R}$$

$$\Delta P = R \Delta V$$

$$R = \frac{8\eta l}{\pi r^4}$$

Resistance of the pipe to the flow

Poiseuille’s Formula is analogue to Ohm’s law of electrical circuits



$$\Delta V = IR$$

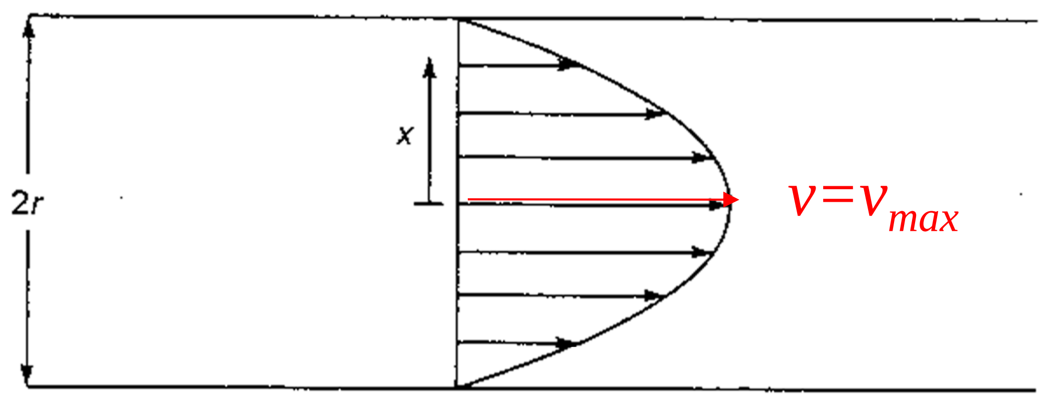
$$\Delta V = \left(\frac{l}{\sigma A}\right) \cdot I = RI$$

conductivity

resistance

Velocity Profile

$$v = \frac{\Delta P}{4\eta l}\left(r^2 - x^2\right) \qquad v_{\max} = \frac{r^2}{4\eta l}\Delta P$$



Velocity profile along a pipe for laminar flow of a viscous fluid is a parabola

Velocity is max at the middle of the pipe (x=0),

v=0 at the wall, where x=r

Continuity Equation

- Continuous flow – no leak in the pipe,
- Example blood flow in vessels
- The amount of mass entering = amount of mass leaving the pipe
- The velocity depends on cross-section of the pipe or vessel - continuity equation:

for incompressible fluids

$$\boxed{A_1 v_1 = A_2 v_2}$$

A_1 and A_2 – cross-sections of the pipe at point 1 and 2

v_1 and v_2 – velocities at the point 1 and 2

Av the volume rate of flow (volume of fluid flowing per unit time) across area A of the pipe

Flow Network and Equivalent Resistance

Poiseuille's Formula and Ohm's law of electrical circuits both describe flow of substance (fluid or charge) under potential gradients through rigid pipe or wire – similar resistance

In series

$$\boxed{R_S = R_1 + R_2 + \ldots.}$$

$$R_i = \frac{8\eta l_i}{\pi r_i^{\,4}}$$

$$\Delta Q = \Delta P / R_S$$

Fluid rate in series

In parallel:

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots..}$$

Energetics of fluid flow

- Moving fluids contains both potential and kinetic energy
-
  - Both Potential and Kinetic energy – change because of the net work done by external pressure gradient and gravity

  - Viscous forces work in opposite direction to the movement – dissipation of energy

  - If we disregard viscosity - Energy conservation law:

    Potential + Kinetic energy = constant

Bernoulli's equation

Potential + Kinetic energy = constant

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g y = const}$$

Describes:
Steady
Streamline
Incompressible flow
With no viscosity

$\frac{1}{2}\rho v^2 =$ Kinetic energy of a fluid element of unit volume moving with velocity v

$\rho g y =$ Gravitational potential energy of a fluid element at the height y

$P =$ Pressure of the fluid element which dimensionally, gives a measure of work done by external pressure

Turbulence in fluids

- Laminar flow (steady) – ordered, smooth, and silent, the velocity at any point is relatively steady
- velocity ~pressure

$$v = \frac{\Delta P}{4\eta l}\left(r^2 - x^2\right)$$

- Turbulent flow – disordered, irregular paths, the velocity at any point varies erratically. At high velocities a flow becomes turbulent
- In a pipe an average velocity of a turbulent flow is constant for most of the cross-section except for the pipe surface.

average velocity ~ $\sqrt{pressure}$

$$v^2 = C\frac{r}{\rho l}\Delta P$$

Empirical constant

Reynolds Number, fluids

- What factors define when the flow becomes turbulent?
- 1. velocity
- 2. density
- 3. viscosity
- 4. radius of the pipe

$$\boxed{R_e = \frac{2r\rho v}{\eta}}$$

- Reynolds Number is dimensionless

$$\Delta V = \pi r^2 v = \frac{\pi r R_e \eta}{2\rho}$$

Flow rate = volume flowing per unit time

$$R_e = \frac{2\rho}{\pi r \eta}\Delta V$$

$v = \eta/\rho$ - Kinematic viscosity

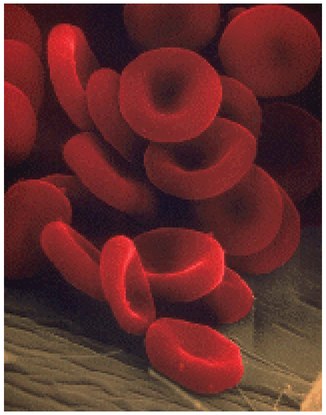
Hemodynamics

- Hemodynamics
- is a study of blood flow in closed arterial to venous circulation
- Blood is considered as viscous flow in pipelines

- Blood is composed of cells and plasma,
- Viscosity of plasma =1.8 viscosity of water,

- Viscosity of blood is 3-4 times more viscous than water

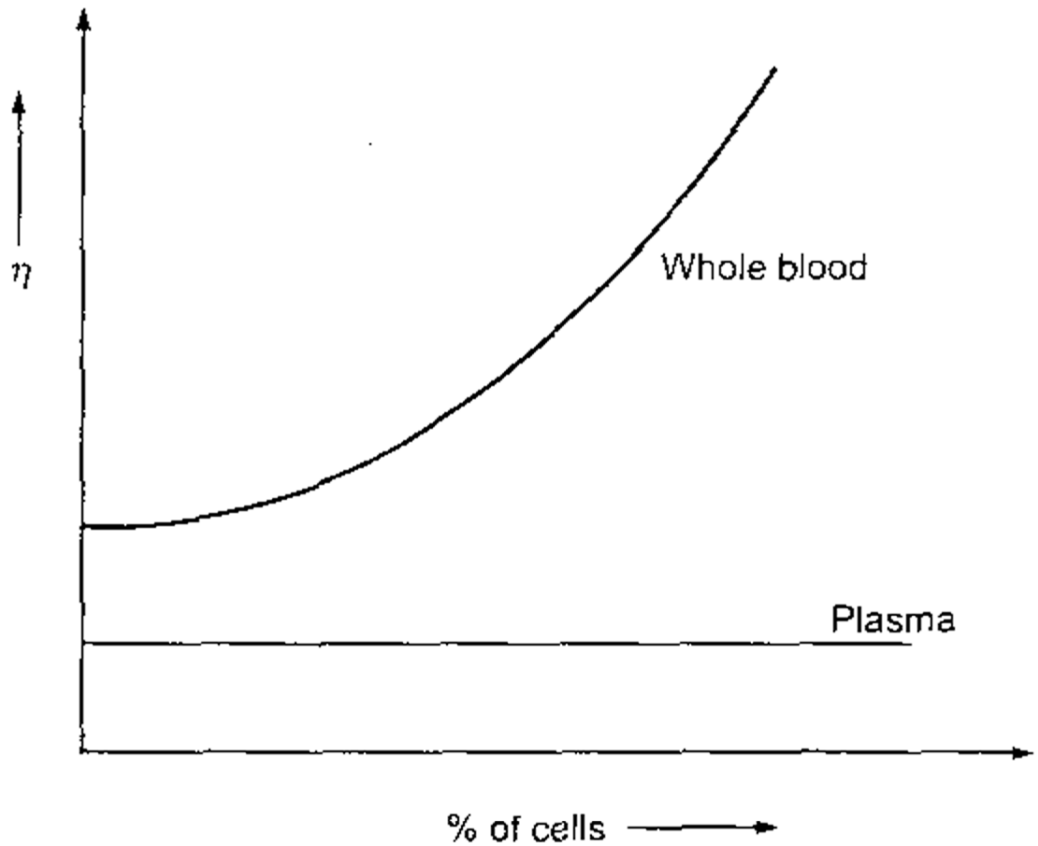
– larger forces required to move blood than water



- Blood viscosity is due to the cells (red blood cells)

– increase internal friction

% of the blood cells = hematocrit
Normal – 42% cells, 58 % plasma
Viscosity increases with the increase in cell %



Blood circulation

- The blood circulates throughout the body. It carries nutrients (food) and oxygen to all the cells of the body. And carries away waste products so that they can be removed from the body. Without access to the blood, cells and body tissues die.

  The blood moves around the body inside the circulatory system. This is made up of blood vessels (tubes) called arteries, veins and capillaries. The blood moves through these vessels because it is being pumped by the heart.

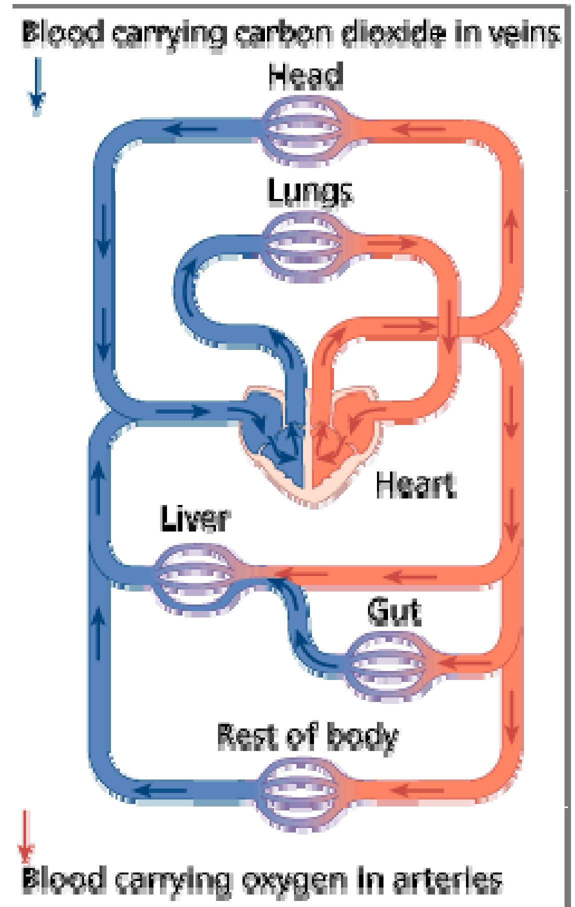
  Arteries carry blood that is full of oxygen from the heart to all parts of the body. As the arteries get further and further away from the heart, they get smaller and smaller.

  Eventually they turn into capillaries. These are the smallest blood vessels. They go right into the tissues. Here the blood in the capillaries gives oxygen to the cells and picks up the waste gas, carbon dioxide, from the cells.

  The capillaries are connected to the smallest veins in the body. The veins get bigger and bigger as they carry the blood back towards the heart.

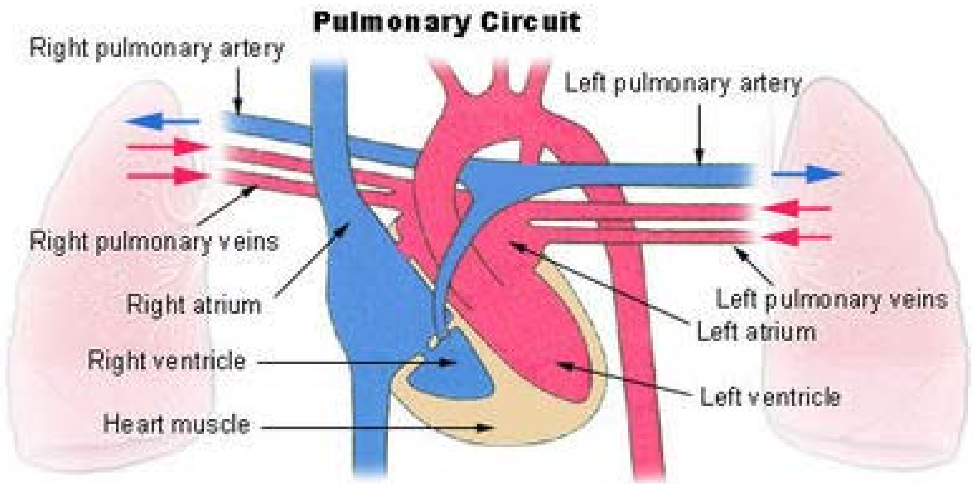
  The blood passes through the right side of the heart and goes to the lungs where it gets rid of carbon dioxide and picks up more oxygen.

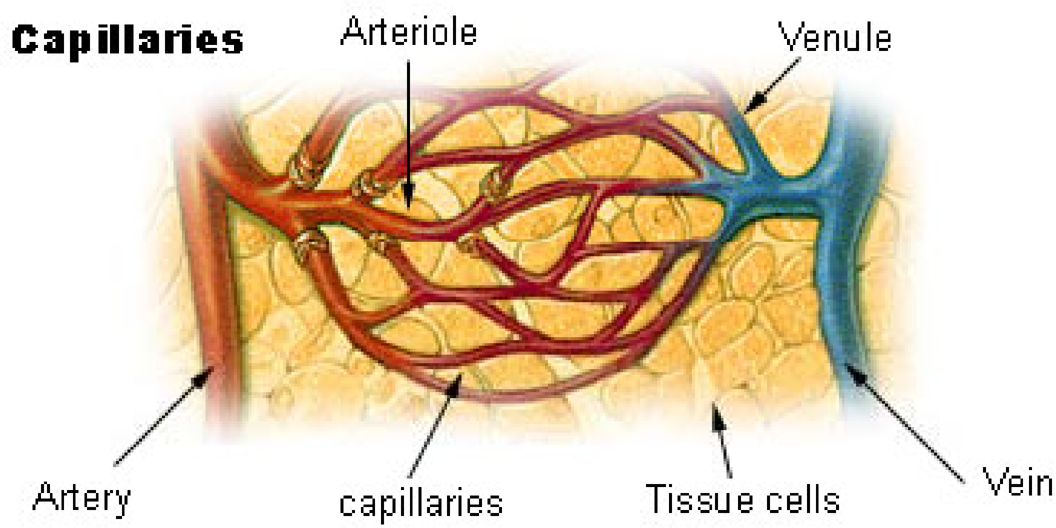
  It then passes through the left side of the heart and is pumped back around the body.

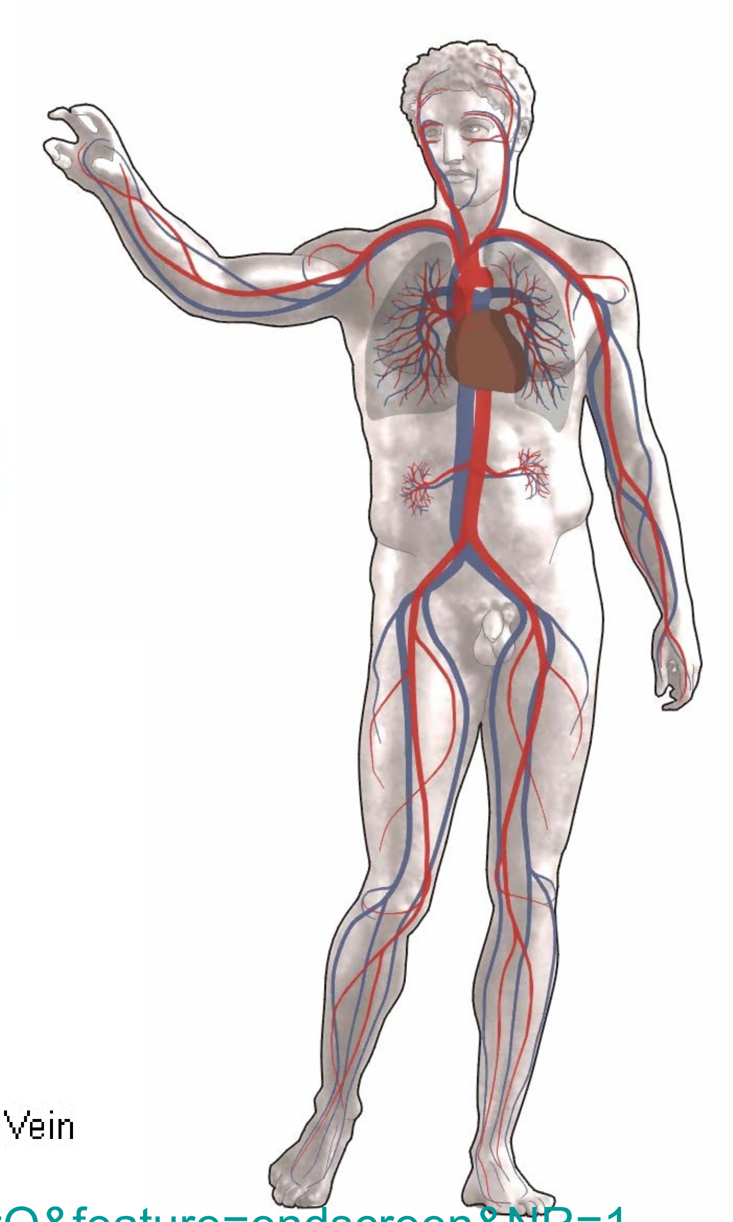


•The blood always circulates through the body in the same direction. As well as oxygen and carbon dioxide, many other substances are carried in the blood. The blood circulating through the digestive system picks up digested food products and carries them to the liver to be used or stored.

http://www.youtube.com/watch?v=oE8tGkP5_tc





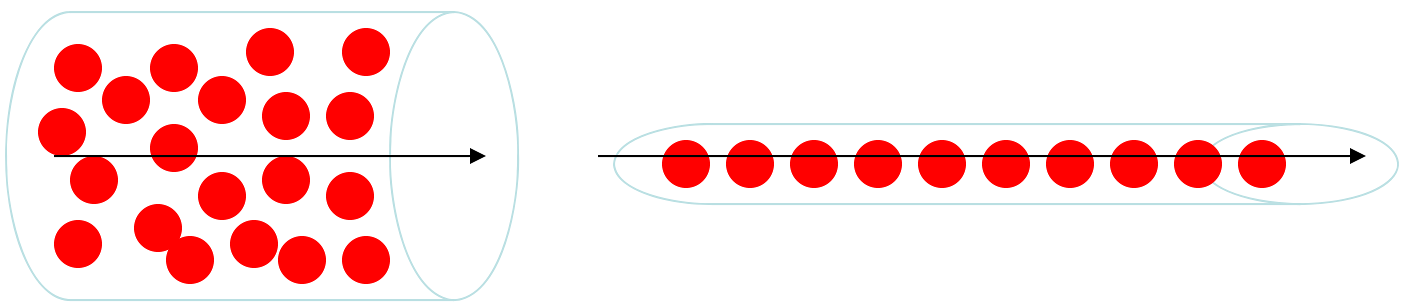


http://www.youtube.com/watch?v=whtNDBIhczQ&feature=endscreen&NR=1

Fahraeus-Lindqvist effect

- Blood flowing in narrow vessels shows less relative viscosity than blood flowing in larger vessels.

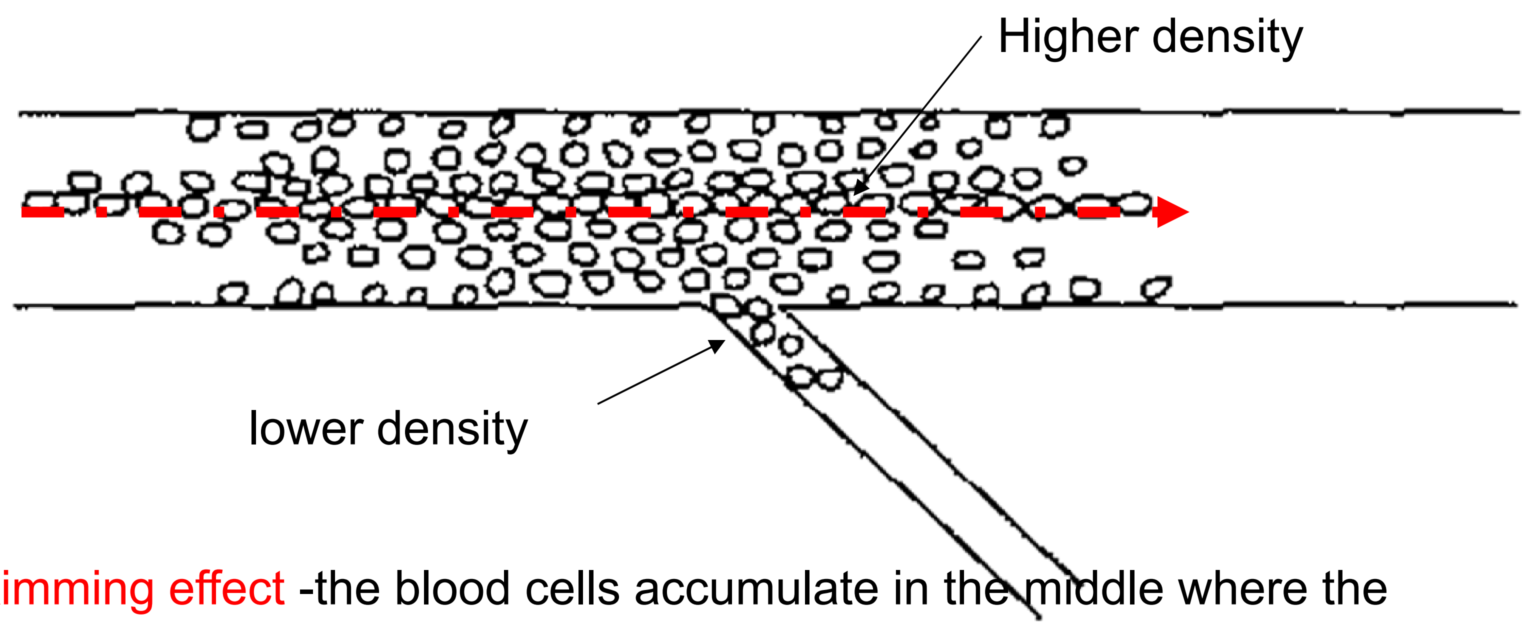
- Relative viscosity of blood decreases with the diameter, for capillaries less than 1mm, it is close to the viscosity of plasma alone.
- This effect is caused by alignment of red blood cells in narrow vessel



- The effect appears both in living systems (blood flow in the capillaries) and in mechanical, non-biological suspensions like glass beads in salt solution

Plasma skimming

- Blood flow is laminar – flow velocity is max in the middle of the pipe and slows down along the walls



Skimming effect -the blood cells accumulate in the middle where the velocity is max, close to the walls there are less blood cells – viscosity is lower – close to plasma

Velocity profile is more flat, velocity at the cross-section is the same

Turbulence in hemodynamics

- Blood flow is mostly laminar
- Turbulence occurs if there is an obstruction in a vessel or sharp turn
- At the branches of arterial tree, in aorta and pulmonary artery the blood flow is turbulent
- Turbulence – sounds, laminar flow is silent. Sound indicates the places of turbulent flow

Pressure-flow relation in blood

- **Poiseulle's formula**
- for laminar blood flow
- R=vascular resistance

$$\Delta V = \frac{\Delta P}{R}$$

- Resistance of blood in peripheral circulation is measured in peripheral resistance units (PRU)

- peripheral resistance unit (PRU) = resistance of a flow of ΔV=1ml/s, when pressure difference is ΔP=1 mm Hg

$$1PRU = \frac{1mmHg}{1ml/s} = 1300 dyne \cdot \sec/cm^5$$

- Poiseulle’s equation is applicable to steady laminar flow in a rigid pipe. Cardio-vascular blood flow is different:

- 1. not steady, Contraction of the heart – pressure and flow oscillations. Oscillations are larger at the ventricle and damp out as wave travels away from the heart, almost steady at the periphery

- 2. blood vessels are not rigid but distensible. Blood vessels are elastic and can stretch when the blood pressure increases.

$$compliance = \frac{\Delta V}{\Delta P}$$

- Compliance depends on size and elasticity
- Venous system is more compliant than arterial due to the larger volume

- Arterial system is more elastic – helps to damp oscillations

Example 1

- The nature of the flow is determined by Reynolds number
- In human system :
- Radius of aorta 1cm,
- Average speed of blood flow 30 cm/s
- Density ρ=1.05 gm/cm^2
- Viscosity η=4X10^{-2} Poise

Calculate Re:

$$R_e = \frac{2rv\rho}{\eta} = \frac{2\times1\times30\times1.05}{4\times10^{-2}} = 1575$$

It is close to 1700 – the flow is close to turbulent, but can be considered as laminar

Problem Example 2

- The blood from aorta flows into capillaries
- In human system there are 4 billion capillaries, radius ~$4X10^{-4}$ cm
- What is speed of blood in capillaries?

Solution Example 2

- The blood from aorta flows into capillaries
- In human system there are 4 billion capillaries, radius ~$4X10^{-4}$ cm
- What is speed of blood in capillaries?
- Assume that the blood is incompressible fluid

- Continuity equation: $$v_a A_a = N v_c A_c$$
- $V_a = v_a A_a$ is the volume of blood flowing in aorta per sec
- (v_a=30 cm/s and $A_a = \pi r_a^2 = \pi (1cm)^2$
- Total blood flowing into N capillaries is $V_c = N v_c A_c$, $N = 4\times 10^9$

$$A_c = \pi r_c^2 = \pi (4\times 10^{-4} cm)^2$$

Blood flow in
capillaries is slow
~ 1mm/sec - laminar

$$v_c = \frac{30}{4\times 10^9 \times (4\times 10^{-4})^2} \approx 0.5 mm/s$$

Problem Example 3

Blood is given to a patient through a needle inserted in vein, needle l = 3cm long, r = 0.25 mm.

How high should be the bottle placed to achieve the steady flow of 0.1 cm^3/sec?

Blood pressure of patient is 20 mmHg, viscosity of blood is $4x10^{-3}$ Pa.s

Solution Example 3

Blood is given to a patient through a needle inserted in vein, needle l = 3cm long, r = 0.25 mm. How high should be the bottle placed to achieve the steady flow of 0.1 cm^3/sec. Blood pressure of patient is 20 mmHg, viscosity of blood is 4x10^{-3} Pa.s

use Poisseulle's eq:

The flow rate: $$\Delta V = \frac{\Delta P}{R} \longrightarrow \Delta P = R\Delta V = \frac{8\eta l}{\pi r^4}\Delta V$$

$$R = \frac{8\eta l}{\pi r^4}$$

$$\Delta P = \frac{8\times 4\times 10^{-3}\times 3\times 10^{-2}}{3.14\times\left(0.25\times 10^{-3}\right)^4}\times\left(0.1\times 10^{-6}\right) = 7826.75 N/m^2$$

Pressure difference in the needle

$$\Delta P = \left(P_{atm} + h\rho g\right) - \left(P_{atm} + 20mmHg\right) = \left(h\rho g - 20\times 133\right) N/m^2$$

blood pressure in beginning of needle

blood pressure at the end of needle = in vein

$$h = \frac{7826.75 + 20\times 133}{1\times 10^3\times 10} = 1.0m$$

1mmHg=133N/m^2,

density=1x10^{3}kg/m^3

g=10m/s