Phys 256
Problem Assignment 11
Due Monday, Decmber 3rd, 2012 4pm
Question 5 is a bonus, 6 and 7 are extra relevant to the labs
Total marks: =34+8 bonus marks

1) A soap film is formed in a wire frame held vertically. It is illuminated normally with a HeNe beam of 633 nm. There are 15 black fringes per cm.

a) 3 marks If the soap film is as thin as possible at the first black fringe, what is its thickness?

6) $\lambda_0 = 633\text{nm}$ $n = 1.33$ 15 fringes/cm vertically

π relative phase change on reflection in the soap film

Destructive interference
OPD+Δr=(m+1/2)λ
OPD +λ/2=(m+1/2)λ
OPD=mλ

$2n_f t \cos\theta_t = m\lambda$ destructive interference

thinnest film is $m=1$

$2n_f t \cos\theta_t = \lambda$ $\theta_t = 0$

$2n_f t = \lambda_0$

$t = \frac{633}{(1.33)(2)}$

$= 234\ nm.$ $\boxed{\text{OR 237nm}}$

b) 2 marks At the next black fringe downwards, what is the thickness?

Assume the angle of ~~the~~ viewing of the next fringe is still $\approx 0°$

At the 2nd fringe $m=2$

$2n_f t = 2(\lambda_0)$

$t = \frac{633(2)}{(1.33)(2)}$

$= 468\ nm.$ $\boxed{\text{OR 475nm}}$

c) 3 marks What angle do the two edges of the wedge form?

15 fringes/cm. allow us to calculate the angle of the wedge formed:

Thickness change in 1 fringe = 237 nm . over $\frac{1}{15}$ cm. =

$$\tan\theta = \frac{237\ nm}{0.667\ mm} = 3.5\times10^{-4}\ \text{radians} = 0.02\ \text{degrees}$$

2) 4 marks Michelson interferometer: Hecht 9.37

Interferometer: Hecht 9.37

chamber = 10.0 cm. $\lambda = 600\ nm$

Change in OPD with air removal $n_{air} = 1.00029$

$$= (1.00029 - 1)10\ cm. = (2.9\times10^{-4})(10\times10^{-2})$$
$$= 2.9 \times 10^{-5}\ m.$$

The total path length change = 2OPD

$2OPD = N\lambda_0$ OR $OPD = N\lambda_0/2$

Dark fringe at [2OPD (chamber)+2xother OPD in interferometer]=$m\lambda_0$

$\Delta m = 2OPD/\lambda_0$

$$\#\ \text{fringes} = \Delta m = \frac{2OPD}{\lambda_0}$$

$$= \frac{2(2.9\times10^{-5}\ m.)}{600\times10^{-9}\ m.}$$

$$= 96.7\ \text{fringes} \simeq 97\ \text{fringes}$$

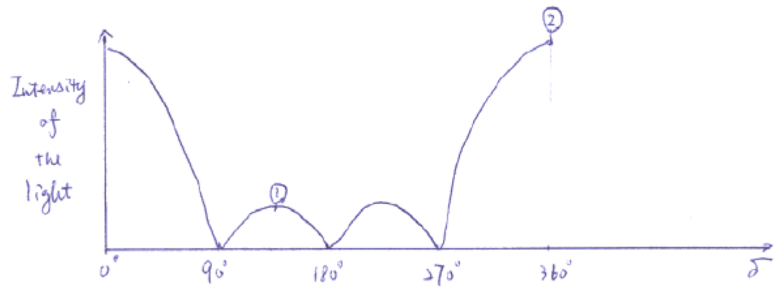
Accept 96 or 97. Or 96.5 fringes

(Subtract 2 for 48 or 49)

3) Phasor diagrams for thin slits- 4 slits: a) 6 marks Sketch phasor diagrams for 4 equally spaced thin slits at the first maximum (the central peak), the first secondary maximum and the first minimum. Sketch the pattern on the screen 3 marks.

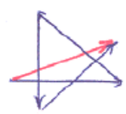
a) see http://mail.rdcrd.ab.ca/~smolesky/FOV1-00028FA7/LTU/physlets/main/phasorslits.shtml

7 a)



phasor diagram:

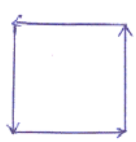
maximum at 1 : $\delta = 135°$ or maximum at 3 : $\delta = 360°$



(1) Secondary maximum

(2) Central or other maximum

minimum at $\delta = 90°$



First minimum

(3)

b) 5 marks At what phase difference do the central maximum, secondary maximum and minimum occur?

b) Central max at 0 radians (**1 mark)** (and agaian a primary maximum at 2π radians (360 deg) marked (2)- phasor diagrams identical). Because there are 3 minima between the two primary maxima, the first minimum is at phase=$2\pi/4$ or $\pi/2$ radians (90 deg)- marked as (3) (**2 marks).** Secondary maximum is ½ way between the minima at $\pi/2$ and π radians at $3\pi/4$ radians= 135 degrees (1). Note the Java applet gives 133 deg (**2 marks)**.

c) 8 marks If the screen is 1 m away, and the slit to slit distance is 0.05 mm, what angle to the centre of the slits are these minimum and maximum for a 500 nm wavelength?

c) **8 marks** $\delta = \frac{d \sin\theta}{\lambda} 2\pi$

$$first\ minimum;\ \sin\theta = \frac{\delta\lambda}{2\pi d} = \frac{(\pi/2)5X10^{-7}}{2\pi(5x10^{-5}} = 2.5\frac{1x10^{-3}}{};\theta = \quad 0.0025\ rad$$

$$distance = \theta L = \quad 0.0025m$$

$$for\ maximum;\ \sin\theta = \frac{\delta\lambda}{2\pi d} = \frac{(3\pi/4)5X10^{-7}}{2\pi(5x10^{-5}} = \frac{3.75x}{10^{-3}};\theta = 3.75x10^{-3}$$

$$distance = \theta L = 3.75x10^{-3}m$$

The central maximum is at the centre.

4) 5 marks Single slit diffraction Hecht 10.8 **These are bonus marks.**

7) Single slit diffraction Hecht 10.8
Assume Fraunhofer diffraction.

In air:

$$b \sin\theta_m = m\lambda \checkmark$$

$$b = \frac{m\lambda}{\sin\theta_m} = \frac{10(1.1522 \times 10^{-6} m)}{\sin(6.2°)} = 1.07 \times 10^{-4} m. \checkmark$$

In water:

$$b \sin\theta = \frac{m\lambda_0}{n_{water}} = m\lambda \checkmark$$

$$\sin\theta_m = \frac{m\lambda_0}{n_{water} b} \checkmark$$

$$\theta_m = \sin^{-1}\left[\frac{10(1)(1.1522 \times 10^{-6})}{1.33(1.07 \times 10^{-4})}\right] = 4.7° \checkmark$$

OR $\sin\theta_w = \sin\theta_m / n_w = 6.2/1.33 = 4.7°$

5) **This is a bonus:** 3 marks Superposition of many sources Hecht 10.2. Remember the chord length of a circle, c=2rsin(θ/2) where r is the radius, θ is the angle subtended. Apply the chord length to get an expression for $\vec{E}$ then apply the chord length equation to get an expression for $\vec{E_0}$ and combine these expressions. **See set of slides on multiple slits.**

c=2rsin(θ/2)

The irradiance for N coherent source (or N thin slits)
(Hecht 10.2 – see diagram)

$$\vec{E}_0/2 = R\sin\delta/2, \quad \vec{E} = 2R\sin(N\delta/2)$$

$$\frac{\vec{E}}{\vec{E}_0} = \frac{2R\sin(N\delta/2)}{2R\sin\delta/2}$$

$$\vec{E} = \frac{\vec{E}_0\sin(N\delta/2)}{\sin\delta/2}$$

$$I = E^2 = \vec{E}\cdot\vec{E} = \frac{E_0^2\sin^2(N\delta/2)}{\sin^2(\delta/2)}$$

6) **Extra good practice for the lab:** Two slit diffraction: a) Hecht 10.11.

3) Hecht 10.11

$R = 2.5m.$

Is it Fraunhofer - Is $R > b^2/\lambda$ $b = 0.10$ mm $\lambda = 500$ nm

$b^2/\lambda = \frac{(1\times10^{-1})^2}{5\times10^{-4}} = 20$ mm $= 0.02m.$

1) 5 $0.02 << 2.5m.$ Yes it is Fraunhofer

Distance from the peak to the first zero $\beta = \pi = \frac{kb}{2}\sin\theta_1$

$\sin\theta_1 = 2\pi/kb = \lambda/b$

OR $b\sin\theta = m\lambda$ $m=1$

$\sin\theta_1 = \lambda/b = \frac{5\times10^{-7}}{1\times10^{-4}}$

$\theta_1 = 0.29°$

In the double slit pattern, what order does this correspond to?

For maxima:

4 $\alpha = m'\pi = \frac{ka}{2}\sin\theta_1$ $m' = \frac{ka}{2\pi}\sin\theta_1$

OR $a\sin\theta_1 = m'\lambda$ $m' = \frac{a\sin\theta_1}{\lambda} = \frac{5\times10^{-7}}{1\times10^{-4}}\frac{(2\times10^{-4})}{5\times10^{-7}}$

$= 2$

7) **Extra good practice for the lab:** A double slit diffraction pattern is formed with light of 546.1 nm slit widths of 0.100 mm. The 4th order interference maxima are missing from the pattern.

a) What is the slit separation?

b) What is the irradiance of the 1st order maximum relative to the central peak?

Pedrotti and Pedrotti: Question

$\lambda = 546.1$ nm., double slit diffraction pattern with slit width of 0.1 mm.
The 4th order is missing in the pattern, what is the slit separation?
What is the irradiance relative to the centre of the first 3 orders?

$$I = \cancel{I(0)}\, I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2 \quad \text{where } N = 2$$

$$\text{OR} \quad I(\theta) = 4I_0\left(\frac{\sin^2\beta}{\beta^2}\right)\cos^2\alpha$$

L'Hopital's Rule $I(0) = 4I_0$

I_0 is the intensity from 1 slit

The term $\left(\frac{\sin\beta}{\beta}\right)^2$ depends on the slit width with zero at $\beta = \pm\pi, \pm 2\pi, \pm 3\pi$ $\quad \beta = \frac{kb}{2}\sin\theta$

at $\beta = +\pi$, $\frac{kb}{2}\sin\theta = \pi$ $\quad \sin\theta = \frac{2\pi}{2\pi}\frac{\lambda}{b} = \lambda/b$

The fourth peak occurs at this zero so that the 4th peak is mostly missing Slides 40-42,47, a/b=4

$\therefore$ There are $m = 0, \pm1, \pm2, \pm3 + 2\times\frac{1}{2}$ peaks $= 8$ peaks within the central diffraction peak
$2m = 8 >$ # of peaks, $m = 4$
where $a = mb$ $\quad a = 4b = 4 \times 0.1 = 0.4$ mm is the slit separation.

Another approach

$$\alpha = \frac{\pi a \sin\theta}{\lambda} = \frac{ka\sin\theta}{2} = 4\pi \text{ } \textit{the 4th interference peak}$$

$$\sin\theta = \frac{4\lambda}{a} \textit{ at 4th peak} = \frac{\lambda}{b} \textit{ at diffraction } 0$$

$$\textit{Therefore } a = 4b = 0.4mm$$

* Maxima of the interference pattern ocurr at:

$$\frac{\sin 2\alpha}{\sin \alpha} = 2 \quad \text{(Show by L'Hopital's Rule or } \sin 2\alpha \text{ expansion)}$$

* $\alpha = 0, \pm\pi, \pm 2\pi, \ldots$ OR $\cos^2\alpha = \pm 1$

* Primary Minimum ocurr at $\alpha = \pm \pi/2, \pm 3\pi/2, \pm 5\pi/2 \ldots$

So the primary maxima under the diffraction pattern ocurr at

$$\alpha = \frac{ka}{2}\sin\theta = 0, \pm\pi, \pm 2\pi, \ldots$$

$$\cancel{\text{or } \sin\theta = \ldots = \pm\pi/2, \pm\pi, \pm\frac{3\pi}{2}}$$

$$\sin\theta = \cancel{X} = 0, \pm\frac{2\pi}{ka}, \pm\frac{4\pi}{ka}, \pm\frac{6\pi}{ka}$$

$$= 0, \pm\lambda/a, \pm 2\lambda/a, \pm 3\lambda/a$$

$$\theta = \pm \cancel{0.078^\circ}\ 0.078^\circ, \pm 0.16^\circ, \pm 0.234^\circ$$

Now $I(\theta) = \cancel{I(0)}\ I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$

For interference maxima without diffraction $\cancel{I(0)} \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2 = 4$

so with diffraction, the maxima are

$$I(\theta) = 4 I_0 \left(\frac{\sin\beta}{\beta}\right)^2$$

$$I(\theta) = I(0) \left(\frac{\sin\beta}{\beta}\right)^2$$

$$\beta = \frac{kb}{2}\sin\theta$$

For $\theta = 0.078^\circ, \pm 0.16^\circ, \pm 0.234^\circ$

$$\beta = \frac{2\pi(0.1mm)}{2(546.1nm)}\sin\theta = 5.753\times10^{-4}\sin\theta$$

$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2$$

$$= I(0)\left[\frac{\sin(1/4\,\pi)}{1/4\,\pi}\right]^2 \text{ for first order peak}$$

$$\beta = \frac{kb}{2}\sin\theta = \pm\frac{kb}{2}\left(\frac{2\pi}{ka}\right), \pm\frac{kb}{2}\left(\frac{4\pi}{ka}\right)$$

$$\pm\left(\frac{c\pi}{ka}\right)\left(\frac{kb}{2}\right)$$

$$= \pm b/a\,\pi, \pm b/a\,2\pi), (\pm b/a\,3$$

$$= I(0)(0.8105)$$

In the same way $I(\theta) = I(0)(0.4053)$ for $\beta = \pm b/a\, 2\pi$

and $I(\theta) = I(0)(0.09)$ for $\beta = \pm b/a\, 3\pi$