

Physics 256 Assignment 8

Due: Wednesday, November 14th, 2012 4:00 pm in the drop box in Physics or electronically 89 marks

1) Magnification thin lens in air: 5.26. **Distance between the object and screen image is 60 cm, image real, image can be upright or inverted. 6 marks**

$$|s_i + s_o| = 0.6m$$

$$\left| \frac{s_i}{s_o} \right| = 5; M_T < 0, M_T = \frac{-s_i}{s_o} = -5 \quad \text{This is a real image through a positive lens so the image could}$$

be (and is) inverted and $s_i > 0$

$$s_i + s_o = 0.6$$

$$s_i = 0.6 - s_o$$

$$\frac{-(0.6 - s_o)}{s_o} = -5$$

$$s_o = 0.1m; s_i = 0.5m$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right); \text{equiconvex: } R_2 = -R_1$$

$$\frac{1}{0.1} + \frac{1}{0.5} = 0.5 \frac{2}{R_1}$$

$$R = \frac{1}{12}m = 0.08m = 8cm$$

2) Longitudinal magnification: A camera lens has an $f=50\text{mm}$ focal length lens and is focused 2 m in front of the lens. That is, an object at 2 m is sharply focussed on the film plane.

a) What is the lateral magnification for this lens? **4 marks**

$f_c = 50 \text{ mm}$. If the camera focusses on the object at 2 m in front of the lens, then $s_o = 2 \text{ m}$.

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f_c} \quad \checkmark$$

$$\frac{1}{s_i} = \frac{1}{50 \times 10^{-3}} - \frac{1}{2} \approx 0.0513 \text{ m}^{-1}$$

$$s_i = 51.3 \text{ mm} \quad \checkmark$$

$$M_T = \frac{-s_i}{s_o} = \frac{-51.3 \times 10^{-3}}{2} = -0.026 \quad \checkmark$$

b) If a boy 1.5 m in height stands at 2 m looking towards the camera, how tall is his image? **2 marks**

$$y_i/y_o = M_T \quad \checkmark \quad y_i = y_o M_T = (1.5)(-0.026) = -0.038 \text{ m} \quad \checkmark$$

(inverted)

c) If the boy is holding a 1 m hockey stick out horizontally in front of him, how long will the image be? The hockey stick is straight out in front along the optical axis. What is the longitudinal magnification? **6 marks Assume that you can't use an approximation.**

For each end of the image $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$ For the end at the boy at 2 m, $s_i = 51.28 \text{ mm}$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

For the end 1 m from the lens, $\frac{1}{1} + \frac{1}{s_i} = \frac{1}{5 \times 10^{-3}}$

$$s_i = 52.63 \times 10^{-3} \text{ m}$$

For $\Delta x_i = 2 - 1 = 1 \text{ m}$ directed towards F_o , for the image $\Delta x_i = 51.28 - 52.63 = -1.3 \text{ mm}$ is the length of the image directed away from F_i .

$$\text{EXTRA: } M_L = \frac{\Delta x_i}{\Delta x_o} = \frac{-1.3}{1000} = -1.3 \times 10^{-3} \quad \checkmark$$

EXTRA: The approximation is quite inaccurate because the hockey stick is long, see below approx gives $M_L = -6.76 \times 10^{-4}$

d) If the boy sticks out his finger (2 cm long), use the approximation to calculate the length and orientation of the image. **3 marks**

For a small longitudinal object, $M_L = -M_T^2 = -0.026^2 = -0.000676$

length of the image $= -0.000676 \times 2\text{cm} = -0.001352\text{ cm}$ or -0.01352 mm or -13.5 microns .

Orientation: The negative sign means that if the boy sticks out his finger towards the lens (-ve direction), the image will be in a positive direction which is pointing away from the lens.

3) Lens/ mirror problem: 5.66 **The ray is reversible. The spherical mirror has to create an image at the same position as its object. 6 marks**

For a spherical mirror

$$1/s_o + 1/s_i = -2/r$$

For a concave spherical mirror $r < 0$

$$\text{So } s_o = s_i$$

$$2/s_i = -2/r$$

$$s_i = -r > 0$$

The concave spherical mirror is placed so that its centre of curvature is at the image formed by the lens, which is a real image. The object and image distances from the mirror are then positive.

For a convex spherical mirror $r > 0$

$$\text{So } s_o = s_i$$

$$2/s_i = -2/r$$

$$s_i = -r < 0$$

The convex spherical mirror is placed so that its centre of curvature is at the image formed by the lens. The object and image distances from the mirror are negative and virtual.

4) a) Two thin lenses: 5.38 **Do the calculation through the first lens and then the second lens. Describe position, orientation and real or virtual image. 10 marks including a description of the image.**

For L1:

$$1/s_o + 1/s_i = 1/f_1$$

$$1/25 + 1/s_2 = 1/15$$

$$1/s_i = 2/75$$

$$s_i = 37.5\text{cm}$$

$$M_{T1} = -s_i/s_o = -37.5/25 = -1.5$$

For L2:

$$s_{o2} = d - s_{i1} = 60 - 37.5 = 22.5\text{cm}$$

$$1/s_{o2} + 1/s_{i2} = 1/f_2$$

$$1/22.5 + 1/s_{i2} = -1/15$$

$$1/s_{i2} = -1/9$$

$$s_{i2} = -9\text{cm}$$

$$M_{T2} = -s_{i2}/s_{i1} = -(-9)/22.5 = 0.4$$

$$M_T = M_{T1} * M_{T2} = -1.5 * 0.4 = -0.6$$

The image is an **inverted, minified** (from mag below) image, located between the two lenses.

b) What is the transverse magnification of the print? Use the product of magnifications through the two lenses. **4 marks**

$$M_T = M_{T1} * M_{T2} = -1.5 * 0.4 = -0.6$$

$$M_T = -0.6X$$

c) Repeat 5.38 using a thick lens calculation. Determine the focal length, the back focal length and the front focal length of the combination. Sketch the system. The diagram should show the cardinal points of the system and the focal lengths. **11 marks + diagram 10 marks**

For L1 and L2 combination as a thick lens:

$$1/f = 1/f_1 + 1/f_2 - d/f_1 f_2$$

$$1/f = 1/15 - 1/15 - 60/15(-15)$$

$$\text{Focal length} = f = 3.75 \text{ cm}$$

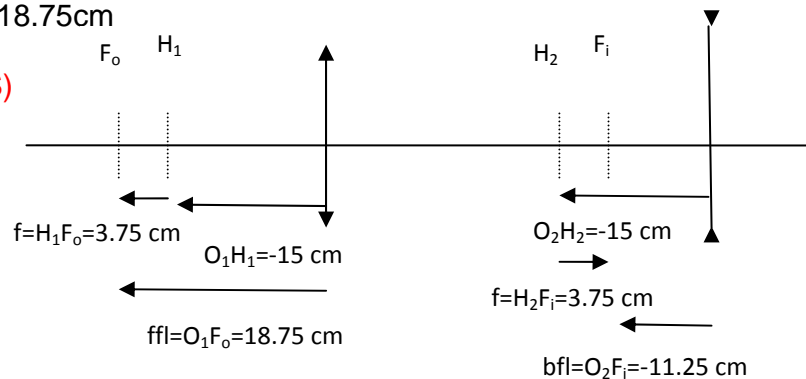
$$O_1 H_1 = f d / f_2 = 3.75 * 60 / -15 = -15 \text{ cm}$$

$$O_2 H_2 = -f d / f_1 = -3.75 * 60 / 15 = -15 \text{ cm}$$

$$\text{bfl} = f + O H_2 = 3.75 + (-15) = -11.25 \text{ cm}$$

$$\text{ffl} = f - O H_1 = 3.75 - (-15) = 18.75 \text{ cm}$$

SKETCH (10 MARKS)



d) Use thick lens optics to calculate the image position. **5 marks**

$$s_o = 25 + O_1 H_1 = 10 \text{ cm}$$

$$1/s_o + 1/s_i = 1/f$$

$$1/10 + 1/s_i = 1/3.75$$

$$1/s_i = 0.1667$$

$$s_i = 6 \text{ cm}$$

The image is located at s_i from the 2nd principal plane = $s_i + O_2 H_2 = 6 - 15 = -9 \text{ cm}$ from the second lens which is 9 cm left of the 2nd lens. This is the same answer as in part a).

5) Hecht 6.8 (spherical thick lens). Find the focal length first. Note the result from question 6.6 that the principal points of a sphere, acting as a thick lens, are both located at the centre of the sphere.

10 marks

$$R_1 = -R_2 = 0.1\text{m}$$

$$\mathcal{D} = \mathcal{D}_1 + \mathcal{D}_2 - \frac{d}{n_L} \mathcal{D}_1 \mathcal{D}_2$$

$$\frac{1}{f} = \frac{n_L - 1}{R_1} + \frac{1 - n_L}{R_2} + \frac{(n_L - 1)^2 d}{n_L R_1 R_2}$$

$$d = 2R = 0.2\text{m}$$

$$\frac{1}{f} = \frac{2(1.4 - 1)}{0.1} - \frac{(0.4)^2 0.2}{1.4(0.1)^2}$$

$$\frac{1}{f} = 8 - 2.286$$

$$f = 0.175\text{m}$$

The principal planes of a spherical lens are located at the centre of the sphere.

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

$$s_o = 4\text{m}$$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o}$$

$$\frac{1}{s_i} = \frac{1}{0.175} - \frac{1}{4}$$

$$s_i = 0.183 = 18.3\text{cm}$$

$$M_T = \frac{-s_i}{s_o} = \frac{-0.183}{4} = -0.046X$$

$$\frac{y_i}{y_o} = -0.046$$

The image is real, inverted and minified.

6) a) Hecht 6.10-locate the image by constructing to scale three rays through the system. Start by marking to scale the cardinal points. **8 marks**

b) Calculate f_i and the distance of the image from the second lens vertex. **4 marks**

5) Hecht 6:16 a) locate image by construction

$s_0 = 50 \text{ cm}$

d can be any distance

$s_i = 75 \text{ cm}$

$H_1 F_1 = f = 30 \text{ cm}$

$H_2 F_2 = f = 30 \text{ cm}$

b) Calculation: $f' = b f_1 l - V_2 H_2 = 29.6 - (-0.4) = 30 \text{ cm}$

$S_0 = 49.8 \text{ cm} + 0.2 \text{ cm} = 50 \text{ cm}$

$\frac{1}{s_0} + \frac{1}{s_i} = \frac{1}{f}$

$\frac{1}{s_i} = \frac{1}{30} - \frac{1}{50}$

$s_i = 75 \text{ cm}$ - from H_2
and 74.6 cm from V_2

If V_1 is marked, it should be 0.2 cm to the left of H_1 (H here). If V_2 is marked, it should be 0.4 cm to the right of H_2 (H' here). P_1 and P_2 are coincident with N_1 and N_2 .