L03 - 16.09

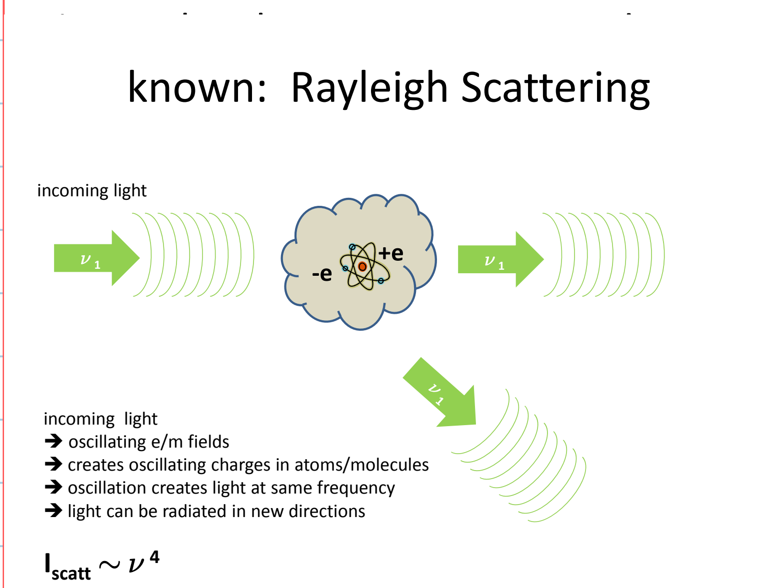
September-21-09

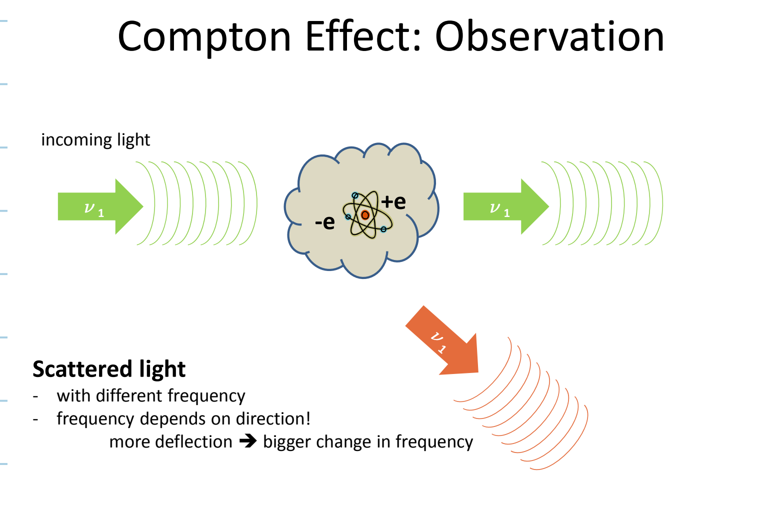
8:00 AM

1.2.2 Compton Effect (see Zettili, or other text books)

In the photoelectric effect, we treated light as being composed of individual light particles (photons) that carry some energy. We take this concept now more seriously and will see that it makes also sense to

1.2.2.1 Basic Observation

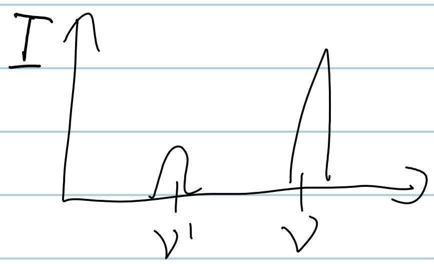




Electromagnetic radiation is scattered by a target object. In classical theory, the charges in the target object will respond to the incoming wave and start to oscillate. All oscillating charges emit radiation at the frequency of oscillation, and this newly generated generation can also be detected at some angle θ with respect to the incoming wave.

This classical model explains via Rayleigh scattering why the sky is not black etc. But an essential feature is that the scattering process itself does not change the frequency of in- and outgoing radiation.

In experiments with x-ray radiation on a graphene target, one observes, however, a second frequency at any given angle θ

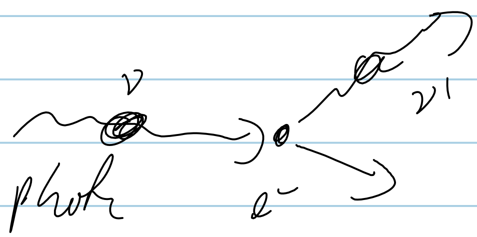


The basic effect is independent of the material, though the intensities might vary.

1.2.2.2 Explanation via Quantum Theory:

Some photons of the incoming light collide with electrons in the material as particles! For this, we need electrons that are loosely bound to the nuclei and can be pushed around a bit ... For this, graphene is a good object.

The whole process is similar to a game of billard, where one moving ball (photon) hits a resting ball (electron). After the collision, the electron moves, and therefore carries some energy. This energy is missing from the photon. Following Einstein's postulate, that means that the frequency of the outgoing photon is lower than that of the incoming photon.



1.2.2.3 Quantitative Analysis:

The exact amount of energy difference depends on the angle under which the new photon is observed, and it can be calculated by momentum and energy conservation alone.

The calculation itself is not too tricky, but must make use of relativistic formulation of the energy of the electron, so I omit it here. (you can find more e.g in Zettili)

For the photon, we have to assign the momentum as

$$p_\gamma = \frac{h\nu}{c}$$ c: vacuum speed of light

Momentum of a photon

classical electromagnetic theory:

fields can carry energy and momentum

classical relationship for energy and momentum density:

$$\mathcal{P} = \frac{\mathcal{E}}{c}$$

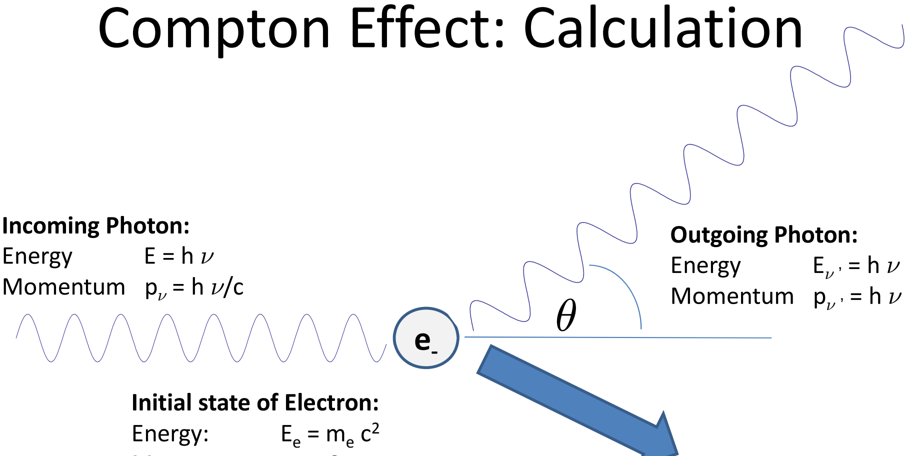
if energy comes in lumps, then it makes sense that momentum comes in lumps:

for a single photon momentum:

$$\boxed{p_\nu = \frac{E_\nu}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}}$$

We have the following initial and final states of electron and photon:

Compton Effect: Calculation



Incoming Photon:
Energy $E = h\nu$
Momentum $p_\nu = h\nu/c$

Outgoing Photon:
Energy $E_{\nu'} = h\nu'$
Momentum $p_{\nu'} = h\nu'/c$

Initial state of Electron:
Energy: $E_e = m_e c^2$
Momentum: $p_e = 0$

Final state of Electron:
Energy: $E_e = \sqrt{p_e^2 c^2 + m_e^2 c^4}$ (relativistic)
Momentum: p_e

Apply:
Energy conservation
Moment conservation (vector)

$\lambda = \frac{c}{\nu}$

$$\boxed{\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)}$$

The resulting energy loss is determined by the change in frequency

$$\Delta\nu = \nu - \nu'$$

however, the result is better expressed in terms of the corresponding change in wavelength, using $\nu\lambda = c$

We obtain for the Compton shift

$$\Delta\lambda = \lambda_c (1 - \cos\theta)$$

with $\lambda_c = \frac{h}{m_e c} = 2{,}43 \times 10^{-12}\, m$

where m_e : rest mass of the electron

Effect negligible in the optical domain (300-700 nm $\approx 10^{-7} m$)

==> shift relevant for X-ray domain!

(higher energy than visible light ==> smaller wavelength)

1.3 Particles as waves:

1.3.1 De Broglie Wave Hypothesis

1.3.1.1 Basic Hypothesis

so far: wave phenomena (light) can exhibit particle features

energy: $E = h\nu$

momentum: $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

In the formulation $p = \frac{h}{\lambda}$ there is no reference anymore to the speed of light, or actually anything that has to do with light!

De Broglie (1924) postulate:

particles can exhibit wave-like properties

assigned wave-length: $\lambda = \frac{h}{p}$

In order to see wave phenomena, two things have to be of the same order of magnitude

- **wavelength**
- **spacial structure (grating, hole, whatever ...)**

!

1.3.1.2 Examples of wavelength

Expected wave length:

a) baseball (fast-ball pitch)

mass: 0.1456 kg

velocity: 90 mph = 40.2 m/s

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{0.1456 \times 40.2} \approx 10^{-34}\ m$$

Problem: wave like property exhibited by interaction with structure on the length-scale of the wavelength ... subatomic!!!

b) slow electrons (kinetic energy in 100 eV)

$$E_{kin} = \frac{1}{2} m v^2$$

$$2 m E_{kin} = p^2$$

$$p = \sqrt{2 m E_{kin}}$$

$$E_{kin} = 100\,eV = 100 \times 1.6 \times 10^{-19}\ J$$

$$= 1.6\ 10^{-17}\ J$$

$$m = 9 \cdot 10^{-31}$$

$$\Rightarrow p = 5.4 \cdot 10^{-24}\ kg\ \frac{m}{s}$$

$$\left(v = 6.6 \cdot 10^{6}\ \frac{m}{s}\right) \approx 0.02 \cdot c$$

$$\Rightarrow \lambda = \frac{h}{p} = 1.2 \; 10^{-10} \, m$$
$$\approx 1 \, \text{Å}$$

This is a good wavelength! we can use atomic lattices as grating!

1.3.2 Experimental verification: Davisson-Germer Experiment

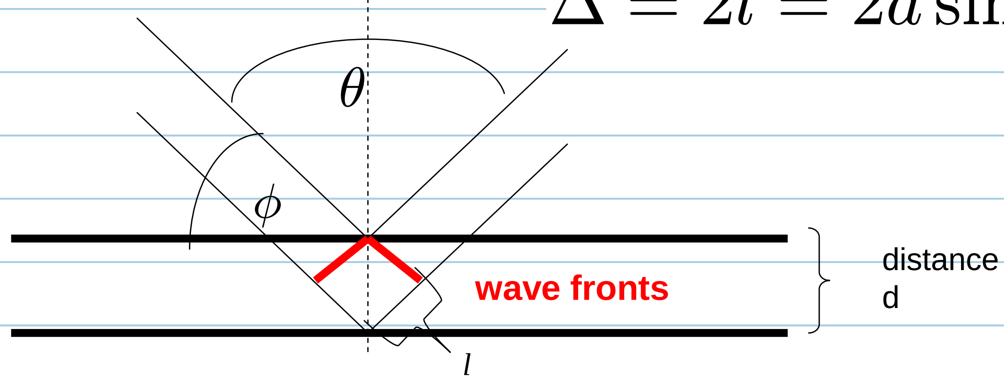
1.3.2.1 Background: Light interference at Bragg plates

Preparation: Let's talk about light waves first

==> Bragg Plates: Think about a stack of partially reflecting surfaces

Extra path for lower-plane reflection:

$$\Delta = 2l = 2d \sin \phi$$



constructive interference: $\Delta = n \lambda$

Bragg condition for scattering maxima: $\mathbf{n \lambda = 2 d \sin \phi}$

Note:

how many maxima can we see? Well, the extra path can take on any value between 0 (grazing incidence angle) and 2d (light coming in perpendicular to surface)

So the number of maxima is the number of natural numbers in the interval $\left(0, \frac{2d}{\lambda}\right)$

Note:

in Bragg reflection, there are more than two layers! The number of layers does not influence the position and number of maxima, but it influences how much of the incoming wave is reflected in total!