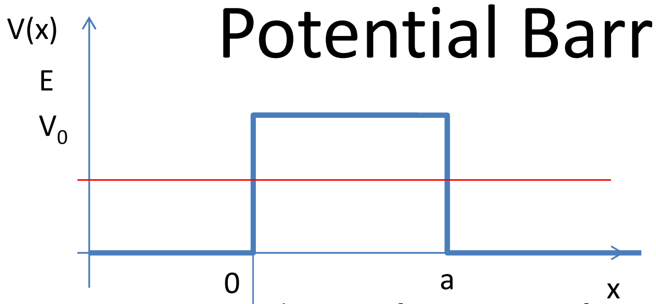
L31 Tunneling

7.6.4 Solutions for E<V

Potential Barrier ($E< V_0$)



$$k = \frac{1}{\hbar}\sqrt{2mE}$$

$$\kappa = \frac{1}{\hbar}\sqrt{2m(V_0 - E)}$$

1)Ansatz:
$$\Psi_E(x) = \begin{cases} A_+\, e^{ikx} +\, A_- e^{-ikx} & x \leq 0 \\ B_+\, e^{\kappa x} +\, B_- e^{-\kappa x} & 0 \leq x \leq a \\ C_+\, e^{ikx} +\, C_- e^{-ikx} & a < x \end{cases}$$

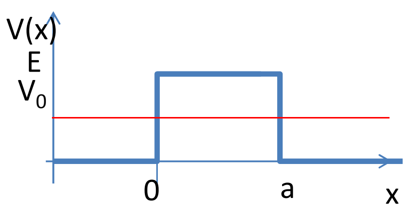
2) Choice: only incoming waves from -∞ ➔ $C_- = 0$

<u>3)Continuity of Ψ(x) and d/dx Ψ(x) at x=0 and x=a:</u>

$$\begin{array}{rrcl} \Psi:(x=0) & A_+ + A_- & = & B_+ + B_- \\ \frac{d}{dx}\Psi:(x=0) & ikA_+ - ikA_- & = & \kappa B_+ - \kappa B_- \\ \Psi:(x=a) & B_+\, e^{\kappa a} + B_-\, e^{-\kappa a} & = & C_+ e^{ika} \\ \frac{d}{dx}\Psi:(x=b) & \kappa\, B_+ e^{\kappa a} - \kappa\, B_- e^{-\kappa a} & = & ikC_+ e^{ika} \end{array}$$

4) Solve for C_+ as function of incoming amplitude A_+

Comparison



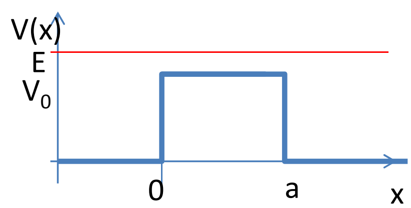
Transmission:

$$T = \frac{1}{1 + \frac{1}{4\epsilon(1-\epsilon)}\sinh^2(K_0 a\sqrt{1-\epsilon})}$$

$$\epsilon = \frac{E}{V_0}$$

$$K_0 = \frac{1}{\hbar}\sqrt{2mV_0}$$

Reflection:
R = 1-T



Transmission:

$$T = \frac{1}{1 + \frac{1}{4\epsilon(\epsilon-1)}\sin^2(K_0 a\sqrt{\epsilon-1})}$$

$$\epsilon = \frac{E}{V_0}$$

$$K_0 = \frac{1}{\hbar}\sqrt{2mV_0}$$

Reflection:
R = 1-T

Approximation:

$$T = \frac{1}{1 + \frac{1}{4\varepsilon(1-\varepsilon)}\underbrace{\sinh^2(\kappa a)}_{\approx \left[\frac{1}{2}e^{\kappa a}\right]^2}}$$

neglect

for $\kappa a \gg 1$ (wide barrier)

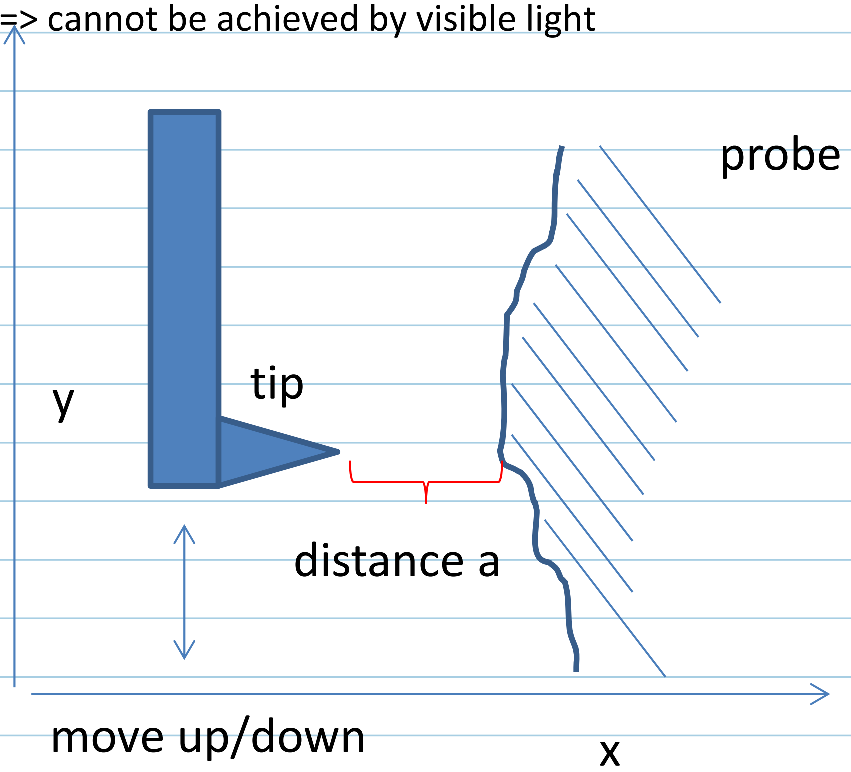
$$T \approx \frac{4\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)}{\left[\frac{1}{2}e^{\kappa a}\right]^2} = 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2\kappa a}$$

$$\kappa = \frac{1}{\hbar}\sqrt{2m(V_0-E)}$$

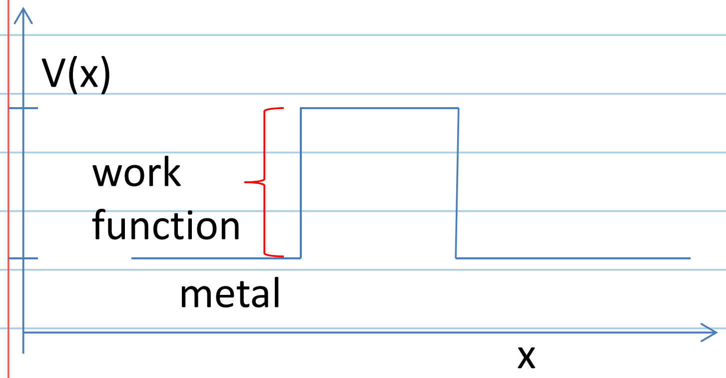
7.5.3 Application of Tunneling Effect: Scanning Tunneling Microscopy

Goal: resolve surface structures of conducting material of order of magnitude of 1 nm (Atomic resolution)

=> cannot be achieved by visible light



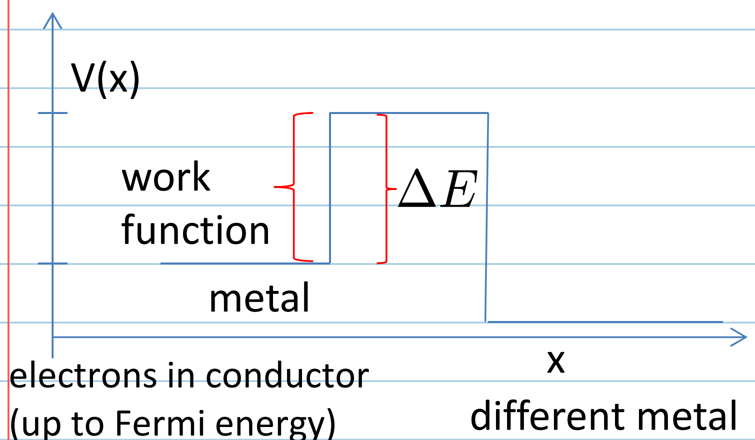
Potential landscape:



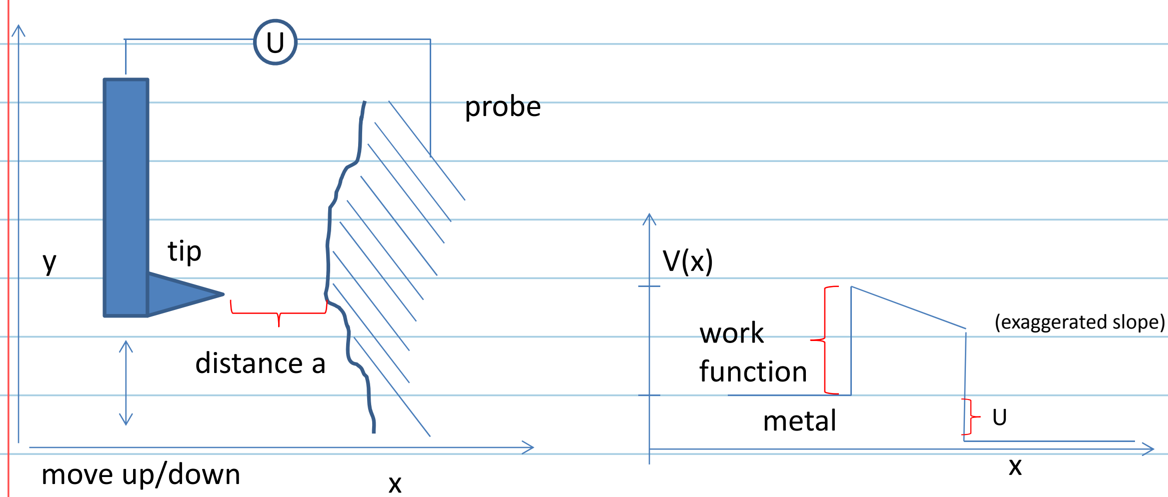
Work function as seen in photo effect!

We can now measure the transmission of the air gap between tip and probe by measuring the current passing through this barrier.
For this to work, we need to modify the potential landscape to create a current:

Method 1: use different materials for probe and tip, with different work functions

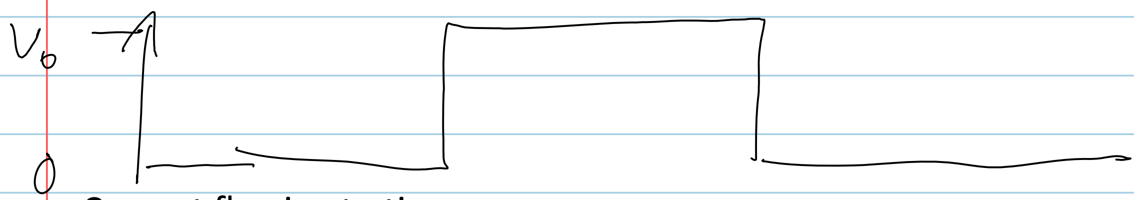


Method 2: Apply external voltage between probe and tip:



In order to calculate the transmission probability for the electrons, we can approximate those potentials roughly again by square potentials.

Abstraction: Barrier



Current flowing to tip:

$$I \sim T$$

work function (Photo-Effect):

$$V_0 - E \sim 4\,eV$$

$$\kappa = \frac{1}{\hbar}\sqrt{2m_e(V_0 - E)} \approx 10^{10}\,m^{-1}$$

==> we are in the right approximation regime

$$\kappa a \approx 10 \gg 1$$

For the two positions:

$$T(a_1) \sim \exp[-2\kappa a_1] = \exp[-20]$$

$$T(a_2) \sim \exp[-2\kappa a_2] = \exp[-26]$$

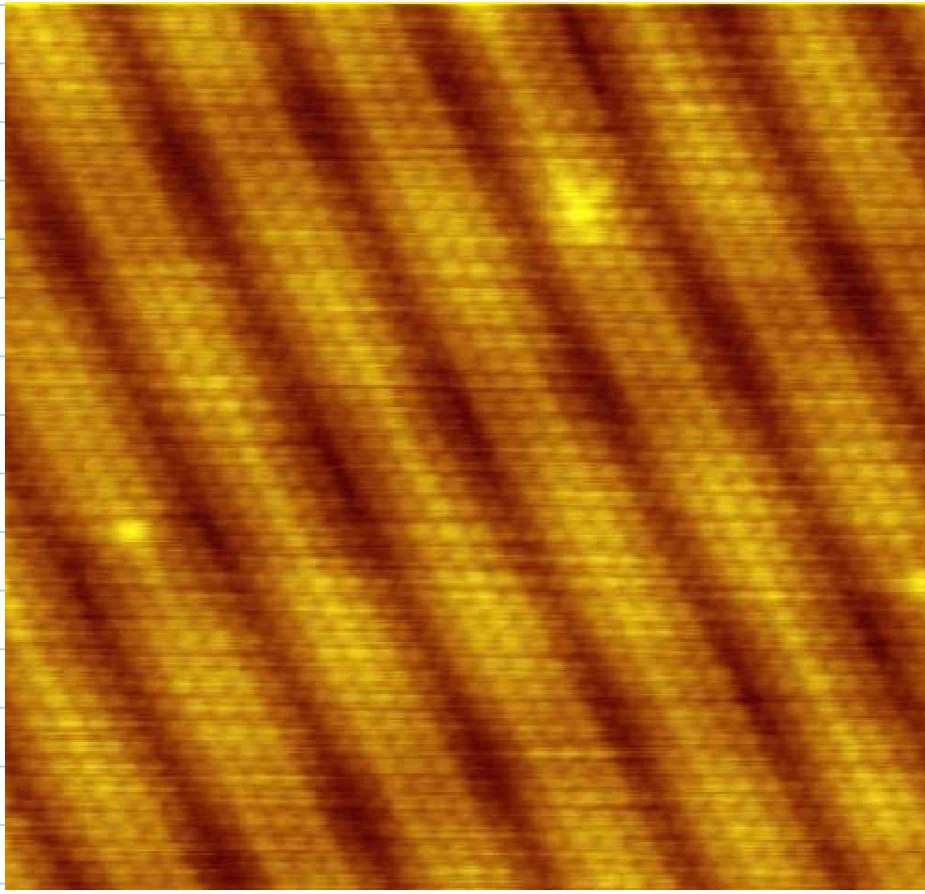
$$\Rightarrow \frac{T(a_1)}{T(a_2)} = e^6 \approx 400$$

The tunneling of electrons corresponds to a current flowing from the metal surface to the probing tip. The current is proportional to the transmittance coefficient!

The current is very sensitive to the distance a between tip and surface!

$\Delta a = 0.3\,nm \qquad \frac{a_2}{a_1} = 1.3 \quad \Rightarrow \quad \frac{T_2}{T_1} = \frac{1}{400}$

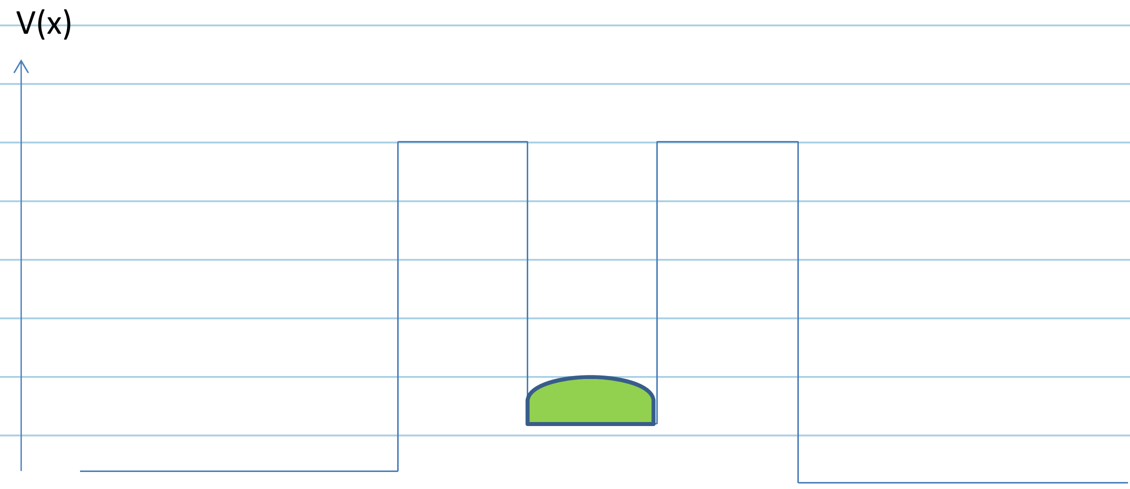
We see that a small change in distance is amplified by the exponential behaviour of the transmission probability to a huge difference in the observed current!



Example: surface scan of a gold surface
(see wikipedia, Scanning Tunneling Microscope)

7.6.6 Application: Radioactive decay

Tunnel can also explain the behaviour of radioactive decay. We use the following step-potential as a toy-model of what is going on in a nucleus



Initially, the nucleus particles are trapped between two potential barriers. The system is set-up so that it would be energetically beneficial for the particle to be outside the barrier. The energy eigenstates corresponding to the initial state have components that have waves going to plus or minus infinity, so at any time there is a probability for the particle to leave the central well and move outside. Over time, the presence of the particle in the potential well decays exponentially, as expected from our observation of radioactive decay. Also, directly from the start there is a probability for the decay to happen. The time scale of the exponential decay depends on the barrier height and width, and also on the difference between the potential in and outside the potential well.