Module 07: Finding eigenvalues and eigenvectors

Starting Monday, March 24

Eigenvalues and Eigenvectors: Basic terminology

- A is a nxn matrix
- Suppose A v = λ v, for some nonzero vector v, then
  - λ is called an eigenvalue of A
  - v is called the associated eigenvector.
- Rewriting gives: (A- λI) v = 0
- If $(A- \lambda I)^{-1}$ exists, then x = 0 is the only solution
  - → Find λ such that det(A- λI) = 0
  - → Called "characteristic equation"

Example

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & 1 \end{bmatrix}$$

Some properties of eigenvalues and eigenvectors

- $\det(A-\lambda I) = 0$ is a polynomial of degree n
- If $\lambda_1, \lambda_2, \ldots, \lambda_k$ are distinct eigenvalues of A, then the corresponding eigenvectors $v^{(1)}, v^{(2)}, \ldots, v^{(k)}$ are linearly independent.
- A is positive definite $\leftrightarrow$ all $\lambda_i > 0$.
- A and A^T have the same eigenvalues
- If A^{-1} exists, its eigenvalues are $1/\lambda_i$
- Eigenvalues of A^p are λ_i^p
- If λ is an eigenvalue and v is an associated eigenvalue, then αv is also an eigenvalue, for all $\alpha \neq 0$

Finding eigenvalues and eigenvectors

We will introduce the following techniques:

- Power method
  - Deflation techniques
- Householder transformations
  - QR Algorithm
- Singular Value Decomposition (SVD)

Power Method

- Approximates the eigenvalue with the largest absolute value (*dominant eigenvalue*)
- Also approximates associated eigenvalue
- Assume A satisfies the following:
  - n eigenvalues satisfy
    - $\lambda_1 > \lambda_2 \geq \lambda_2 \geq \ldots \geq \lambda_k$
  - n linearly independent eigenvectors
    - $v^{(1)}, v^{(2)}, \ldots, v^{(n)}$