Fourier Series: Complex Form

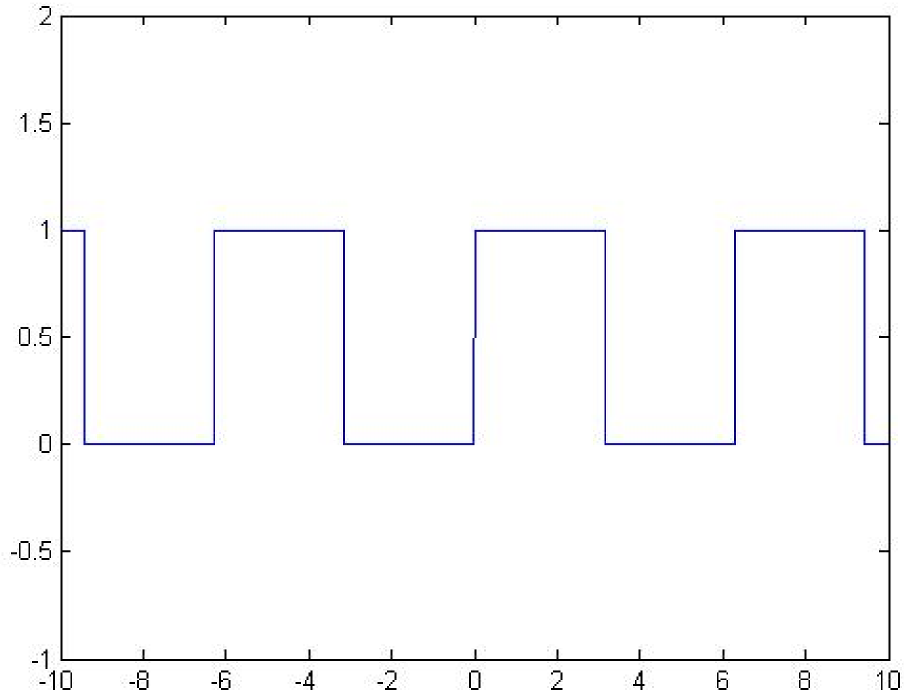
For a function *f(t),* with period T= 2π, there exists values c_k such that

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{ikt}$$

How do we calculate c_k?

Example

- Find the matching Fourier series for the square step function with period T = 2π



Convert to Fourier Series form

- Find a_k, b_k such that

$$f(t) = \ a_0 + \sum_{k=1}^{\infty} a_k \cos\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{\infty} b_k \sin\left(\frac{2\pi kt}{T}\right)$$

- Find c_k such that

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{ikt}$$

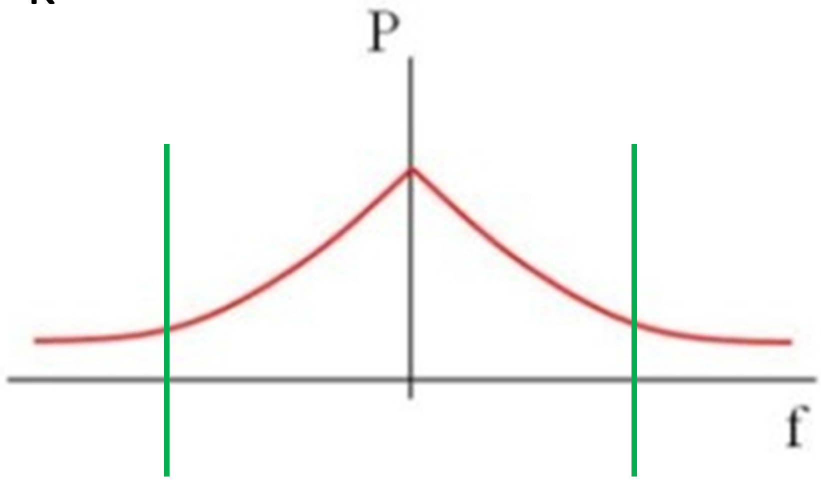
- Exercise (for you): Show that the two representations are equivalent.

Approximating f(t)

- For most f(t), there is little information in high frequency harmonics
  - "noise" in a audio signal
  - Edges in an image
  - "power" in the frequency k/T for electrical signal

Power Spectrum of $f(t) = \sum_{k=-\infty}^{\infty} c_k e^{ikt}$

Plot $|c_k|^2$ = power of k^{th} harmonic



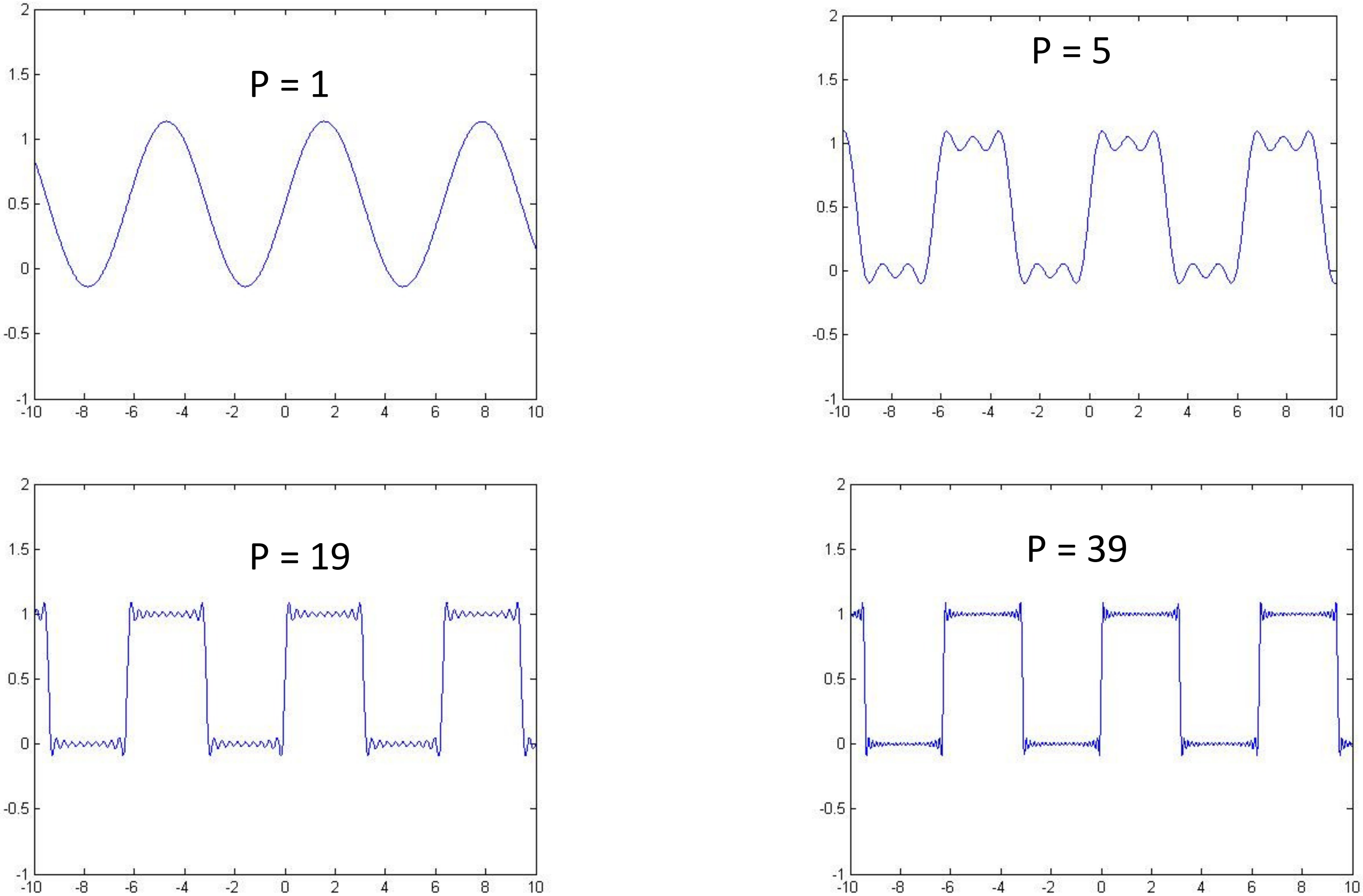
As $|k| \rightarrow \infty$, terms are less significant, and can be dropped.

Approximating f(t)

- Truncate the sum for some M>0:

$$f(t) \cong \sum_{k=-M}^{M} c_k e^{ikt}$$

How many terms in the series do we "need"?
Using a_0 and $b_1,b_3,b_5, \ldots, b_P$ terms



How much is gained using more terms?