Lecture 2

1.4 BIODYNAMICS

Biodynamics studies locomotion of living systems

- **Newton's Laws:**
- **Newton's laws of motion** are three physical laws which provide relationships between the forces acting on a body and the motion of the body.
- Newton, in his work *Philosophiae Naturalis Principia Mathematica*, 1687.

- **Newton's First Law:**
- if there is no external force the body stays at rest or moves with a constant speed along the straight line. The net force =0

First Law (Complete Formulation):

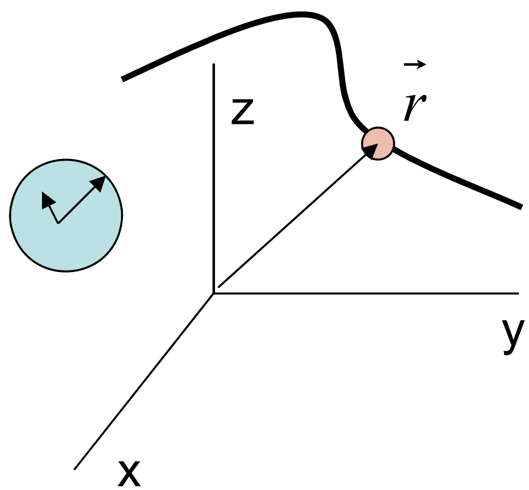
A body in uniform motion remains in uniform motion,
and a body at rest remains at rest, unless acted on a nonzero net force

Valid for Inertial Reference Frames

Reference Frame

we need a coordinate system to describe motion

For example - Cartesian System (x,y,z): Object in it



At any point of time the position vector r of object can be represented by x,y,z coordinates

In Unit vector notation $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$

If object is moving: $\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$

$$\vec{v}(t) = \frac{d\,\vec{r}(t)}{dt} \qquad \vec{a} = \frac{d\,\vec{v}}{dt}$$

So, to describe the motion of an object we need some measure of time

Reference Frame (RF)= some measure of space [coordinate system, x, y, z]
\+ some measure of time [clock, t]

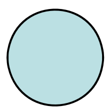
Motion $\{\ \vec{r}(t),\ \vec{v}(t),\ \vec{a}(t)\ \}$ is always described relative to a given RF

Reference Frame can be Inertial (IRF) and Noninertial (NIRF)

Two ways to introduce:

A. “Theoretically”

isolated and nonrotating object



“isolated” - not interacting with other objects (empty space)

RF attached to such object is Inertial Reference Frame

Any RF that moves with constant velocity relative to IRF is also IRF

More physical sense in the following introduction:

B. <u>If First law is valid in some RF then this RF is Inertial RF</u>

Example: Earth (we know that theoretically, in general, it is not IRF), but for a variety of physical phenomena, it can be treated as IRF (Newton’s Laws are valid with a very high accuracy)

- **Newton's Second Law:**
- The force causing motion is equal the rate of momentum change:

$$F_{ext} = \frac{\Delta p}{\Delta t} \quad \text{or} \quad F_{ext} = m\frac{\Delta v}{\Delta t} = ma$$

This law is often stated as
"the net force on an object is equal to the mass of the object multiplied by its acceleration."

Second Law (Complete Formulation):

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

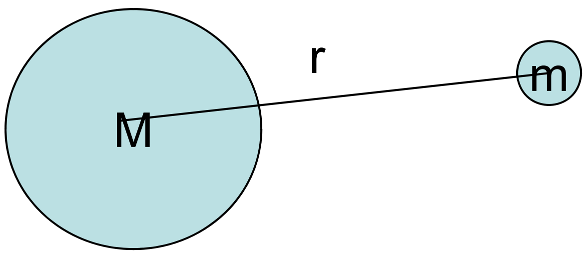
The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object.

Mass and Weight

In the physical sciences, *mass* and *weight* are different properties

mass is an inertial property (<u>scalar</u>)

Weight is the force created when a mass is acted upon by a gravitational field (vector)



$F = G[M\, m/r^2]$ G- gravitational constant

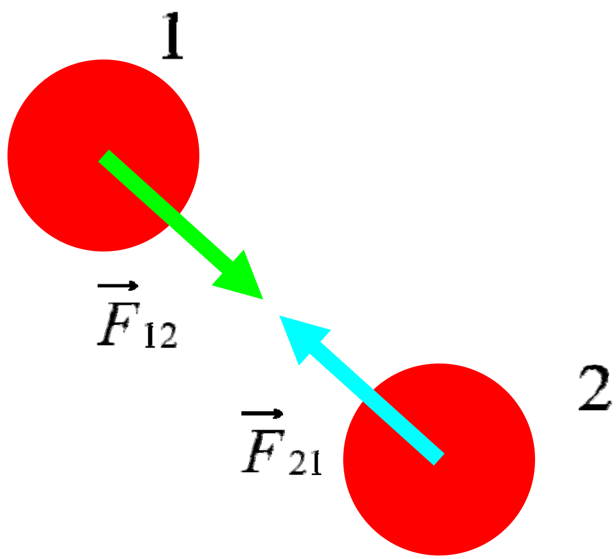
G= 6.67×10^{-11} m^{3} kg^{-1} s^{-2}

on Earth $F = G[M_E\, m/R_E^2] = mg$ $G[M_E\, /R_E^2] = g$

- When the force acts, it transfers momentum to the body = stress or push
- Impulse is the momentum transfer at small Δt

$$impulse = m\Delta v = F\Delta t$$

- Physiological effect of impulse is shock, impact or punch lasting for short duration. This may result in an injury - fracture of bone, rupture of tissues or even death



Newton’s Third Law:

$$\vec{F}_{12} = -\vec{F}_{21}$$

- For every action is an equal and opposite reaction

- Complete formulation

If two objects interact, the force exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on object 1

- This force is important in locomotion

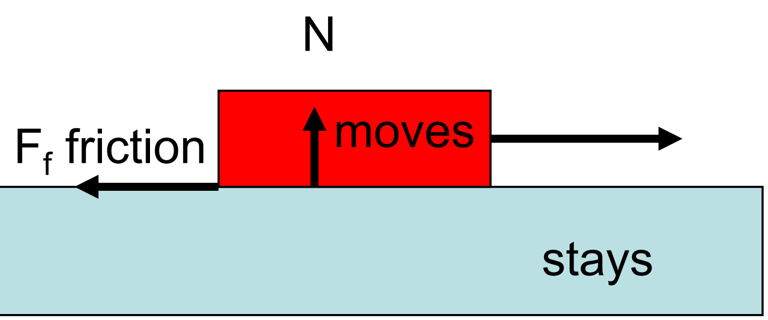
Forces that determine biodynamics

- Gravitational forces
- Elastic forces in muscles and bones
- Frictional forces:
- static friction – locomotion on ground
- viscous forces – motion in fluid

Frictional Forces.

- whenever there is a relative motion between two bodies in contact frictional force appears

$$F_f = \mu N$$

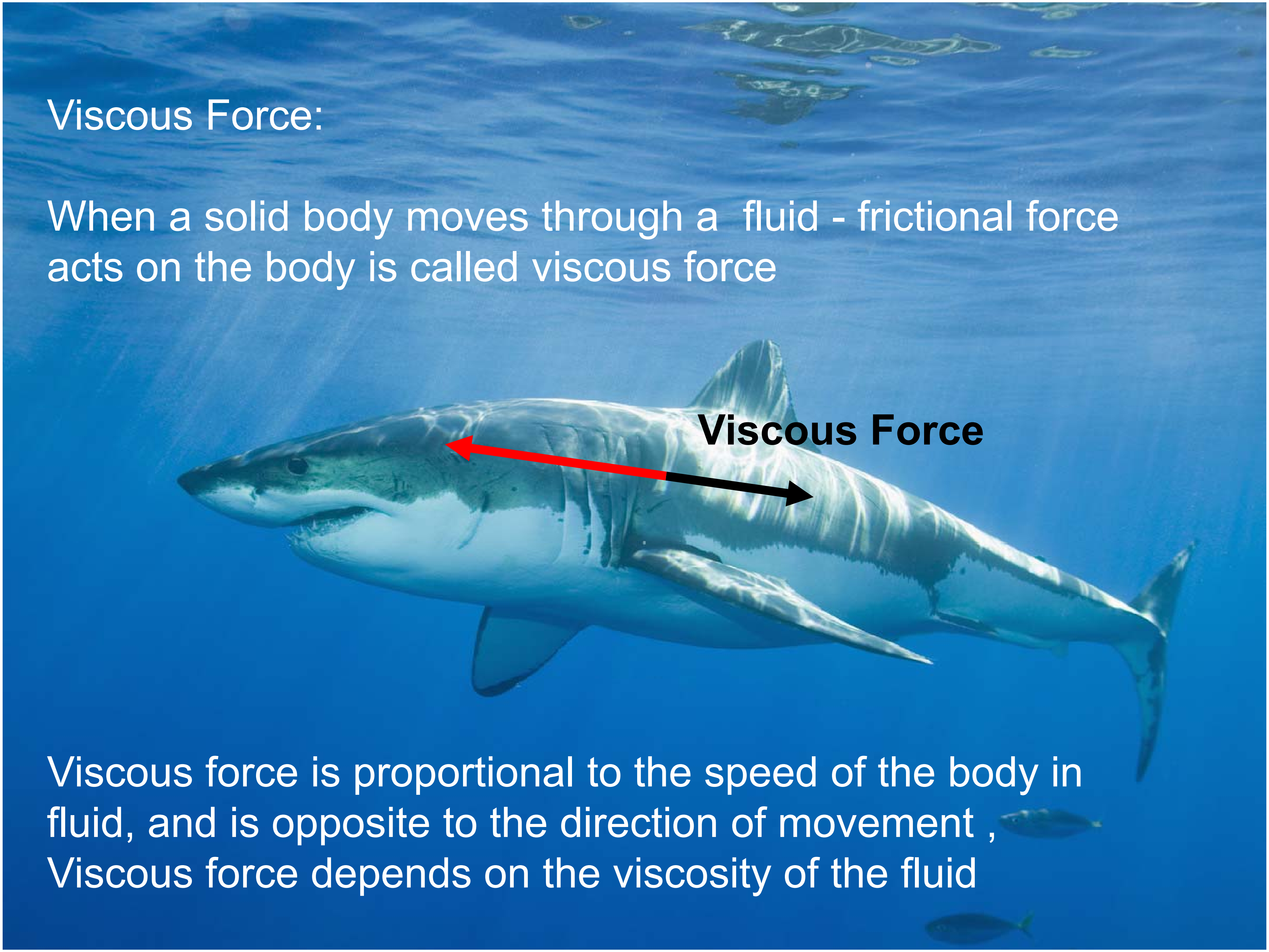


- μ - coefficient of friction

- Friction force acts in opposite to the motion direction
- Friction is independent of the contact area, arises due to intermolecular cohesive forces between two bodies,
- it depends on the roughness of the surface, T, humidity
- N is normal force

Viscous Force:

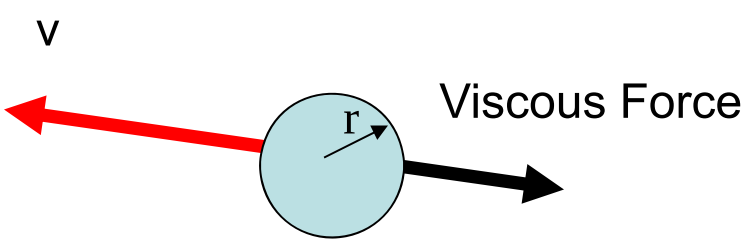
When a solid body moves through a fluid - frictional force acts on the body is called viscous force



Viscous force is proportional to the speed of the body in fluid, and is opposite to the direction of movement , Viscous force depends on the viscosity of the fluid

Stokes Law:

For a sphere moving in fluid:



$$F_v = 6\pi\eta r v$$

η is coefficient of viscosity of fluid

Locomotion on Land

- Nature of forces and energy balance in the whole body

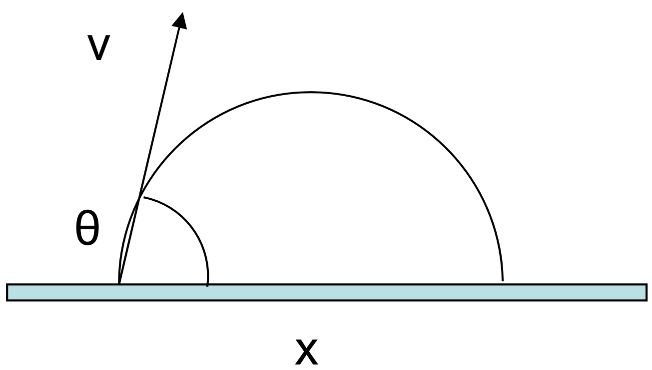
- Newton’s mechanics of inanimate objects:
- Motion is caused by external force

- In living system -locomotion is a result of internal forces applied by various muscles.
- Kinetic energy of locomotion as well as energy lost by friction are supplied by chemical energy stored in the body
- Locomotion on land: walking, jumping, propelling

- Walking – forces are friction and muscle contraction, the feet are always at contact with the ground, muscles apply forces to the ground, and the ground to the body.

Jumping, projectile motion

Projectile motion - Body springs at an angle θ, with initial velocity v. The center of mass moves in a parabolic trajectory and covers a certain distance x , (called range)



$$x = \frac{v^2 \sin 2\theta}{g}$$

$$v^2 = 2as$$

F=Contraction of muscles to acquire the initial velocity v – speed, provides acceleration (a) of the center of mass (CM) - as a result CM rises by a small distance (s), M – mass of the body

a-average acceleration of CM

$$a = F / M$$

Problem. Example 1.3

- Man (M=55kg) jumps from sitting position (CM rises at height 40 cm above the ground) up into the air, when his feet leave the ground height of CM=90cm, and finally rises to 120 cm at the top of jump

- What is the average upward force, exerted by the ground on him and what is the max speed attained?

- F action – push the ground, causes F reaction of the ground
- F reaction is larger than weight

Solution Example 1.3

- F action – push the ground, causes F reaction of the ground
- F reaction is larger than weight
- 1st phase total force rises the CM 90-40=50 cm
- Work is done

$$W = \underbrace{(F - Mg)}_{\text{upward force}} \cdot (h_2 - h_1)$$

This work is amount of internal body energy converted into kinetic energy, which provides an initial upward velocity v and counteracts weight

$$1/2Mv^2 = (F - Mg) \cdot (h_2 - h_1)$$

•With this initial v, the body leaps up to 120 cm, when its all kinetic energy is balanced by gravitational potential energy

$$1/2Mv^2 = Mg(h_3 - h_2)$$

upward force

$$F = \frac{Mg(h_3 - h_1)}{(h_2 - h_1)}$$

initial speed - is also the max speed

$$v = \sqrt{2g(h_3 - h_2)}$$

upward force of 880 N and initial speed of 2.4 m/s are required for man of 55 kg for such a jump

Problem. Example 1.4

- Grasshopper M=4 g, jumps, range 80 cm, starts the leap at 60° to horizontal. Initial translation before taking the leap is 5cm.

- What is the initial speed needed to achieve the given range?
- What is the forward thrust?

Solution. Example 1.4

- Considering projectile motion:

$$x = \frac{v^2 \sin 2\theta}{g}$$

$$v = \sqrt{\frac{xg}{\sin 2\theta}} = \sqrt{\frac{0.8 \cdot 9.8}{0.866}} = 3m/s$$

Starting from rest it attains initial speed after 5 cm, acceleration? time?

Average acceleration $\rightarrow$

$$a = \frac{v^2}{2x} = \frac{9}{2 \cdot 0.05} = 90m/s^2$$

$$v = v_0 + at \qquad t = v/a$$

To attain this acceleration it needs the force

$$F = Ma = 4 \times 10^{-3} \times 90 = 0.36N$$

4 legs therefore 1 leg produces ¼F which is 0.09 N , this is idealized picture, in reality the force is larger around 5 N

- This force produces a different amount of stress on the muscles depending upon their area of cross-section
- D of the muscle fiber is 100μm (S=0.01 mm^2). The stress on this fiber is:

$$stress = \frac{force}{area} = \frac{5}{0.01} = 500 N / m^2$$

- The tensile strength of a tissue is about 600 N/mm^2
- The safety factor in jumping is low:

$$S.F. = \frac{tensile\_strength}{Stress} = \frac{600}{500} = 1.2$$

- Mass specific muscle power of grasshopper
- (leg mass is 5% of the body):

$$P_m = \frac{E_m}{t}$$

$$m = 0.05 \times M$$

$$E = mE_m = kinetic\_energy = 1/2MV^2$$

$$E_m = \frac{1}{2} \cdot \frac{M}{m} V^2 = 1/2 \times 20 \times 9 = 90J/kg$$

$$P_m = \frac{E_m}{t} = \frac{E_m v}{a} = \frac{90 \cdot 3}{90} = 3Watt/kg$$

1Watt= 1kg·m^2/s^3

1 Watt=J/s

Propelling

- Propelling is forward motion of animals like snakes or worms
- Entire body is organ of locomotion

- Lateral Undulation - S-shaped movement – snake bends the front and produces a transverse wave of muscular contractions through the length
- Rectilinear motion - snake pushes front to the ground and pulls the tail close to the head, then pushes the back side to the ground and pushes the front forward.

- Forces – friction and lateral forces of reaction by the ground objects, like pebbles and sand.

Locomotion in air

- More than half of the animal species can fly
- A bird is a living aeroplane, it employs all principles of aerodynamics

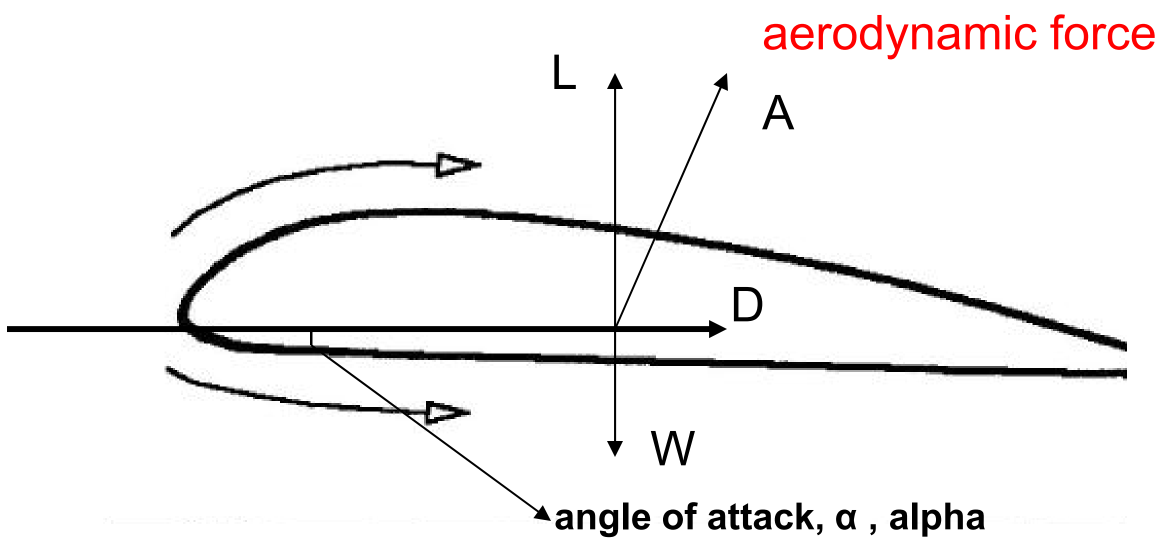
General Aerodynamics: Flying is caused by two groups of forces:

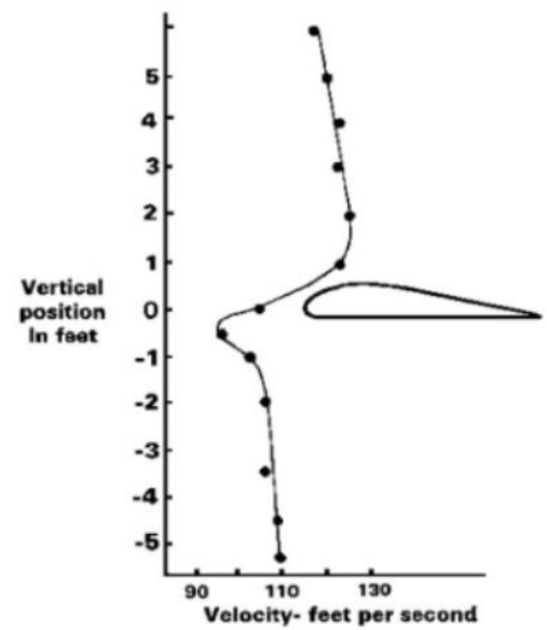
1. A forward thrust, T (generated by propeller) - combats the drag force D (frictional force of the wind)
2. And an upward lift force L which combats the gravity (weight) W

Interplay between these 4 forces and torques determines the dynamics of flying object

Wind force

- The wing is at an angle to the wind, wind produces both drag D and lift L





- The cambered profile of the wing causes the upper stream accelerate and low stream to retard, because the wind blows faster above the wing than below, the air pressure above is less than below, this results in an upward lift force L . The resultant of L and D is called aerodynamic force A

The drug force increases with the angle α until certain value α_m , and then decreases, the maximum effectiveness at angle α_m at which L/D ratio is maximum

the wind force depends on the surface area A of the wing, density of the air ρ and square of wind velocity , v^2

for steady flight:

$$L = C_L \rho A v^2$$

$$D = C_D \rho v^2$$

C_D C_L are drag and lift coefficients,
depend on the angle of attack,
geometry of the wing,
nature of air flow

When the wind is not horizontal
L and D are not vertical and horizontal,
but they have vertical and horizontal
components L_V and D_H
At equilibrium weight W and thrust T
(propeller) balance the lift and drug forces

$$\boxed{\begin{gathered} L_V = W \\ T = D_H \\ L_V / D_H \\ \text{Aerodynamic efficiency} \end{gathered}}$$

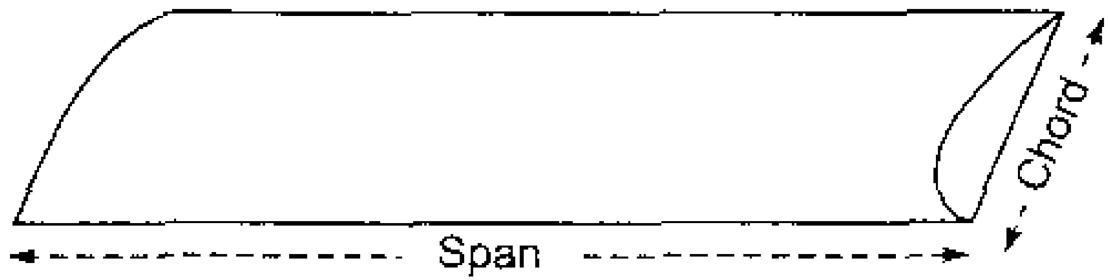


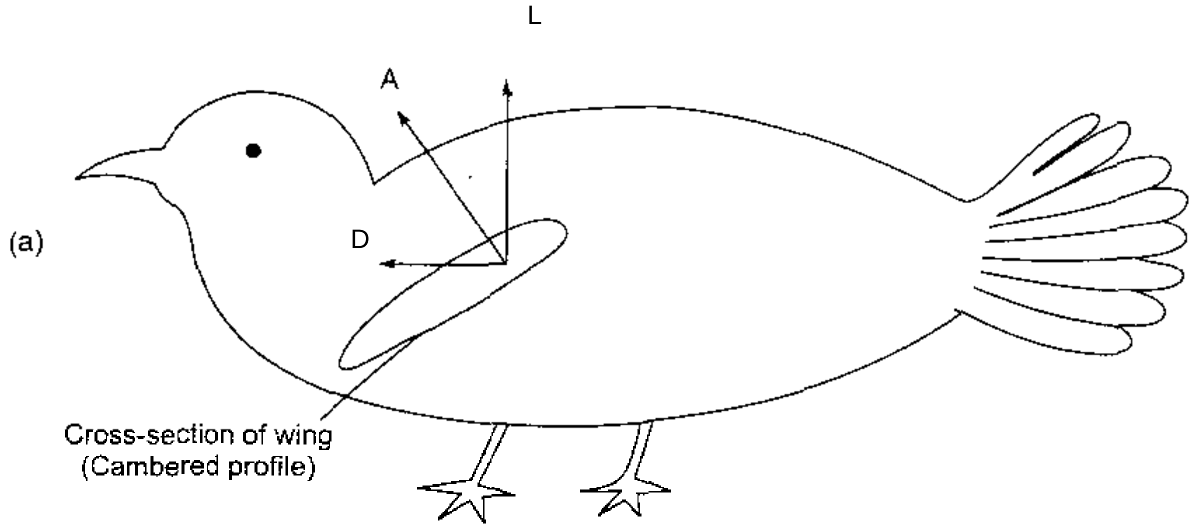
Fig. 1.9 *Aspect ratio is the ratio of span and chord.*

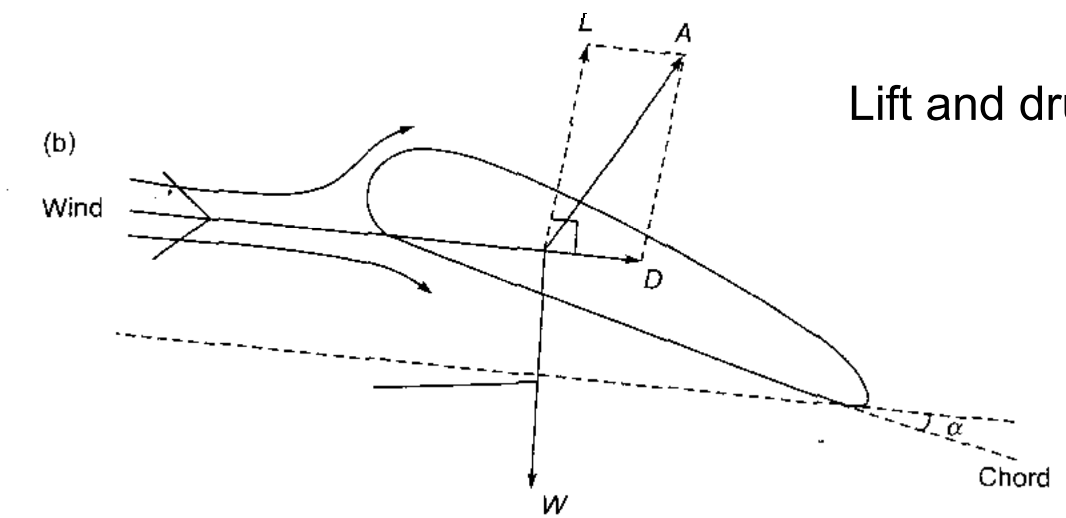
Aspect ratio characterizes geometry of the wing

$$aspect\_ratio = \frac{span}{chord}$$

Aerodynamic efficiency L/D increases with Aspect Ratio (AR)

Bird's aerodynamics





Lift and drug forces during climbing

- Active flight – flapping wings – to get force for taking off or landing
- Passive flight - gliding along the air stream

- The geometry, mass, and the frequency vary between different species , for example frequency varies 10^3 Hz - 1 Hz

- The inner (arm) and the outer part (wrist) move together to produce lift like the wing and propeller in the airplane.

Turbulence

- Energy effective flight require streamline air flow
- The flow can be streamline or turbulent,

- streamline flow - flow in which the velocity at any point is relatively steady
- turbulent flow - flow in which the velocity and a pathway of the flow at any point varies erratically
-
- this is empirically determined
- by the value of Reynold’s number:

$$R_e = \frac{vl}{(\eta / \rho)}$$

- Turbulent flow - Reynold’s number ≥1000

- v –velocity of the air flow, l – length (span) of the wing, η - coefficient of viscosity, ρ – density of the air

- Small insects R_e is low, they do not need streamline flow

- Butterflies, R_e is low than 1000, streamline flow is achieved with flat wings

- Dove, l (span) 10cm, speed 10m/s, η=0.018·10^3 Pa·s, (dynamic viscosity), density ρ=1.29kg/m^3 for the air.

$$R_e = \frac{10 \times 0.1}{0.14 \times 10^{-4}} = 7 \times 10^4$$

- Re is larger than 1000, the flow would be turbulent if wings were uniform
- The wings are arched and are of cambered profile to ensure the streamline flow

Power of flight

- Power of Flight = Work against drug / time

$$P = \frac{\overset{\text{work}}{\Delta W}}{\Delta t} = D\frac{\Delta x}{\Delta t} = Dv$$

- Drug: $D = C_D \rho v^2$

$$P = C_D \rho v^3$$

- For level flight, max distance (range) per unit time = Smax

$$S_{\max} = v_{\max} = \frac{P_{\max}}{D}$$ *(per unit time)* ;

$$S_{\max} = \frac{P_0}{W} \cdot \frac{L_V}{D} = \frac{P_0}{W}\eta$$

When gliding (level flight) - the weight is balanced by upward lift, $W=L_{V,}$

where η –aerodynamic efficiency
P0 – is a constant – for given specie

Locomotion in Water

- Rectilinear movement in viscous fluid
- Is similar to movement in air
- Is similar to the movement of snake on the ground (interaction of body with surrounding)
-
- The weight is balanced by the buoyancy of water for small species, and requires a lift force to balance the weight for large species:

$$lift = V(\rho_b - \rho_w)g = W\left(1 - \frac{\rho_w}{\rho_b}\right)$$

- V –volume of the animal, W – weight, ρ_b - density of the body, ρ_w - water density

- For small animals ($\rho_b = \rho_w$) it is mostly rectilinear motion and only forward thrust is required to balance the drag

- Drag:
- 1. Frictional (surface) Drag - D_f – fluid viscosity against the body
- (depends on the surface area)

- 2. Pressure Drag - D_p – longitudinal stress on the fish by the flowing stream
- (depends on the shape or transverse cross-section)

- Both D_f and D_p depend on size and velocity of the body.

- The forward thrust – flips the tail left and right
- The fastest – dolphins 35 km/h the rest is slower
- Initial acceleration is high even for small fish, $40 m/s^2$

Role of Gravity

- Gravity is everywhere
- It is not uniform (9.77 – 9.83 m/s^2), average 9.8 m/s^2
- Animals adjust to gravity by varying skeletal and cardiac properties
- Plants are affected - directed growth

- Inside the body – particles move down due to gravity according to Stoke's law:

$$\frac{4}{3}\pi r^3(\rho_p - \rho_m)g = 6\pi\eta r v$$

- η - viscosity of fluid
- ρ_p – density of particle
- ρ_m - density of medium

- For small particles gravitation can be balanced by the thermal motion

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$