L21 Summary

Planck constant
$h = 6.6\ 10^{-34}$ Js
$\hbar = h/(2\ \pi)$

Particle-wave duality

	waves:	**particles:**
Classical physics:	λ wavelength	E energy p momentum
New effects:	$E = h\nu\ = \hbar\omega$ (Photo-Effect) $p = \dfrac{h}{\lambda}$ (Compton Effect)	$\lambda = \dfrac{h}{p}$ (De Broglie hypothesis/ Davisson Germer)

Photo-Effect

$E_\nu = W + E_{kin}$

photon energy

Work function (escape energy)

kinetic energy of released electron

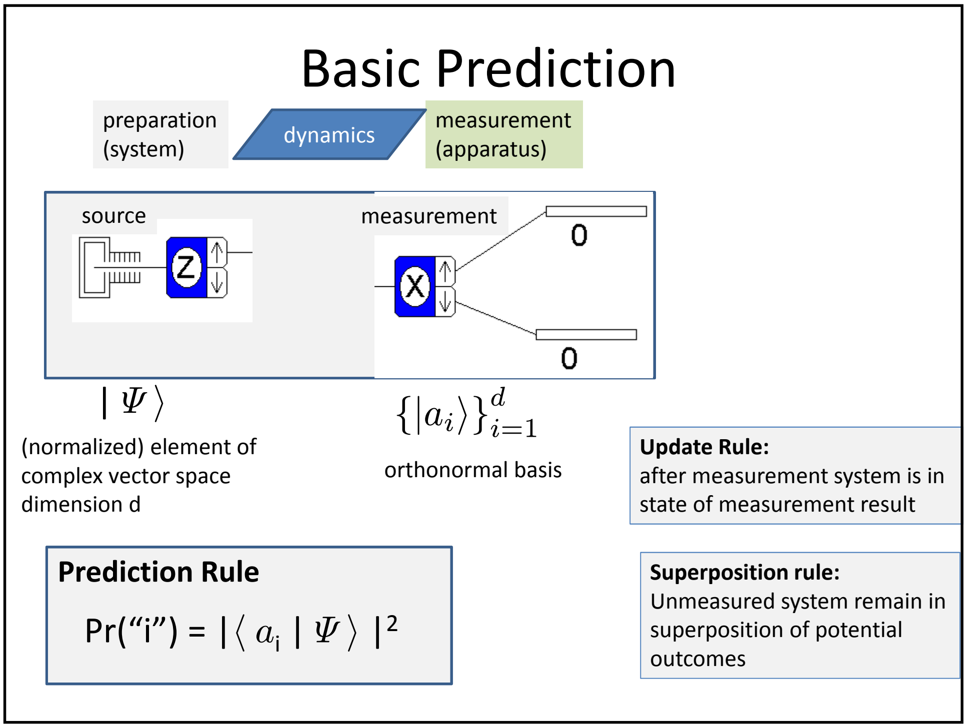
$E_\nu = W + e\ V_0$
stopping voltage

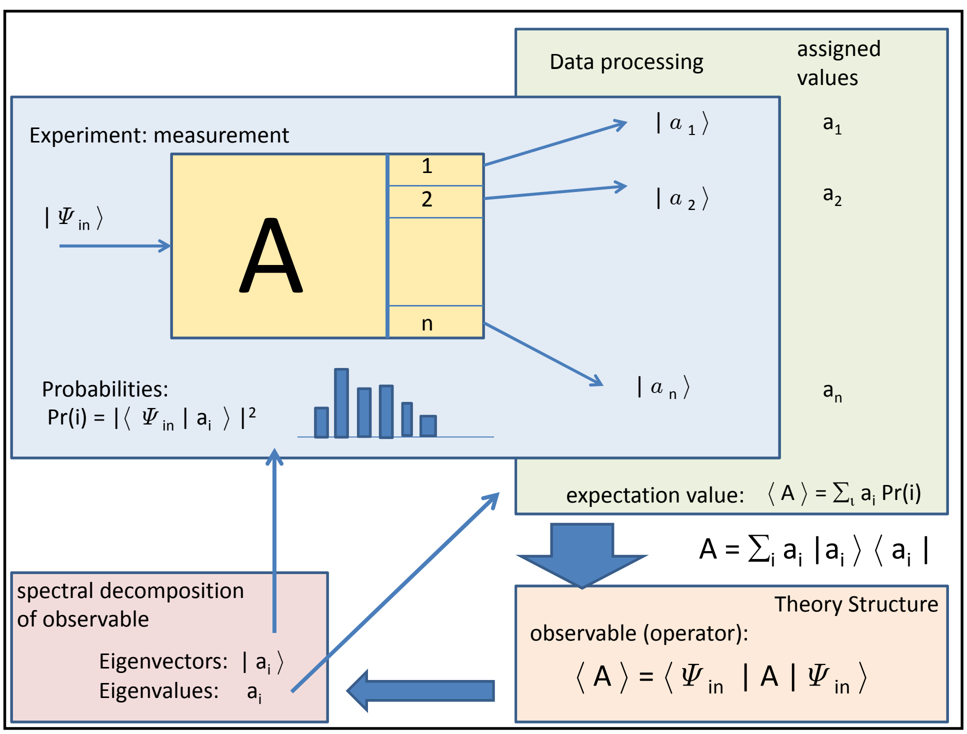
Compton Effect

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

λ_C

Bragg condition for scattering maxima:
n λ = 2 d sin ϕ


Basic Prediction
preparation (system)
dynamics
measurement (apparatus)
source
measurement
Z
X
0
0
$| \Psi \rangle$
(normalized) element of complex vector space dimension d
$\{|a_i\rangle\}_{i=1}^{d}$
orthonormal basis
Update Rule: after measurement system is in state of measurement result
Prediction Rule
$\Pr(\text{“i”}) = |\langle a_i | \Psi \rangle|^2$
Superposition rule: Unmeasured system remain in superposition of potential outcomes


Data processing
assigned values
Experiment: measurement
$| a_1 \rangle$
a_1
$| a_2 \rangle$
a_2
$| \Psi_{in} \rangle$
A
1
2
n
$| a_n \rangle$
a_n
Probabilities:
$\Pr(i) = |\langle \Psi_{in} | a_i \rangle|^2$
expectation value: $\langle A \rangle = \sum_i a_i \Pr(i)$
$A = \sum_i a_i |a_i\rangle\langle a_i|$
spectral decomposition of observable
Eigenvectors: $| a_i \rangle$
Eigenvalues: a_i
Theory Structure
observable (operator):
$\langle A \rangle = \langle \Psi_{in} | A | \Psi_{in} \rangle$

Properties of Density Matrix

density matrix
$$\rho := \sum_s Pr(s)|\Psi_s\rangle\langle\Psi_s|$$

$$\{\mathrm{Pr(signal\ s)}\ \ , |\Psi_s\rangle\}_{s=1}^n$$

eigenvalues λ_i
eigenvectors $|\lambda_i\rangle$

$$\rho := \sum_s Pr(s)|\Psi_s\rangle\langle\Psi_s|$$

ρ

Use:

$$\boxed{\langle A\rangle = Tr\left[\rho A\right]}$$

Mixed State Source

1) Hermitian $\rho = \rho^\dagger$

2) **positive semi definite** $\rho \geq 0$

$$\langle\Psi|\rho|\Psi\rangle \geq 0 \quad \forall|\Psi\rangle$$

3) **Unit Trace**

$$Tr\left[\rho\right] = 1$$

A) The density matrix completely describes the source!

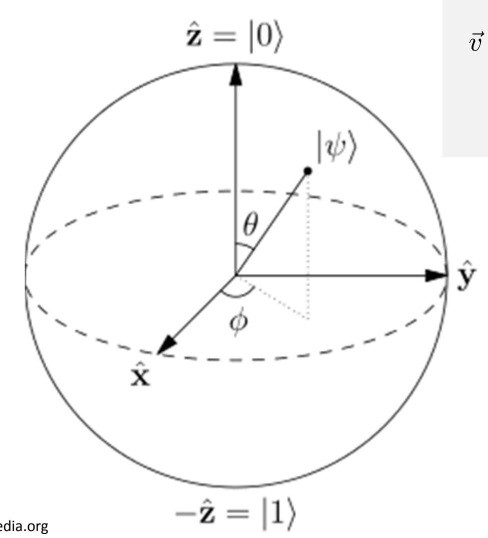
B) several source preparations methods can yield same density matrix:
➔ the sources are then completely equivalent

C) For given ρ, one can always find *one particular* source realization via the spectral decomposition!

Bloch Vector

$$\vec{v} = \begin{pmatrix} Tr[\rho\sigma_x] \\ Tr[\rho\sigma_y] \\ Tr[\rho\sigma_y] \end{pmatrix}$$

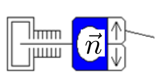
$$\rho = \frac{1}{2}\left(\mathbb{1} + \vec{v}.\vec{\sigma}\right)$$



Summary of Quantum Prediction Rules

	prediction of outcome probabilities measurement $\{ \vert \Phi_i \rangle \}_i$	special case: $A \rightarrow \vert\phi_i\rangle\langle\phi_i\vert$	prediction of expectation value operator A
pure state $\vert\Psi\rangle$	$Pr(i) = \vert\langle\phi_i\vert\Psi\rangle\vert^2$		$\langle A\rangle_{\vert\Psi\rangle} = \langle\Psi\vert A\vert\Psi\rangle$
special case: $\rho \rightarrow \vert\Psi\rangle\langle\Psi\vert$			
mixed state ρ	$Pr(i) = \langle\phi_i\vert\rho\vert\phi_i\rangle$		$\langle A\rangle_\rho = Tr[A\rho]$

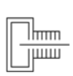
Sources



conditional state (pre-selected source)

$|+\rangle_n$ state preparation by selective measurement

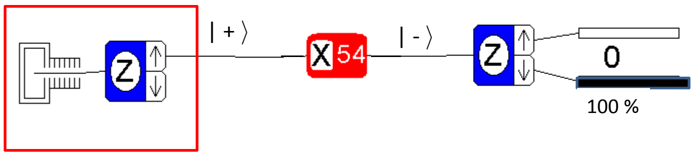
thermal source



$$\rho = \frac{1}{2}|+\rangle\langle+| + \frac{1}{2}|-\rangle\langle-|$$

Measurements:

- X, Y, Z measurements with SG
- SG in general direction



input

$|\Psi\rangle$

output

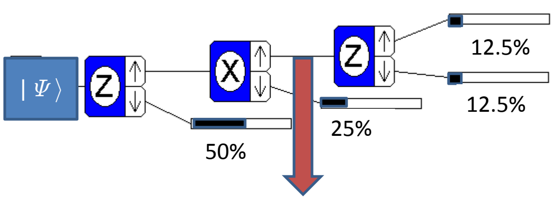
$|\Psi'\rangle = U\,|\Psi\rangle$

Unitary operators:

$$U^\dagger U = UU^\dagger = \mathbb{1}$$

conserve norm of vectors

Projectors and Selective measurements



unnormalized state $P_{|+\rangle_x} P_{|+\rangle_z} |\Psi\rangle = |+\rangle_{xx}\langle +|+\rangle\langle +|\Psi\rangle$

normalized state: $|+\rangle_x$

probability amplitude ${}_x\langle +|+\rangle\langle +|\Psi\rangle$

for prediction of last z-measurement:
(renormalize state)

$$|\Psi\rangle \to |\Psi'\rangle = \frac{P_{|+\rangle_x} P_{|+\rangle}|\Psi\rangle}{\sqrt{\langle\Psi|P_{|+\rangle}P_{|+\rangle_x}P_{|+\rangle}|\Psi\rangle}} = |+\rangle_x$$

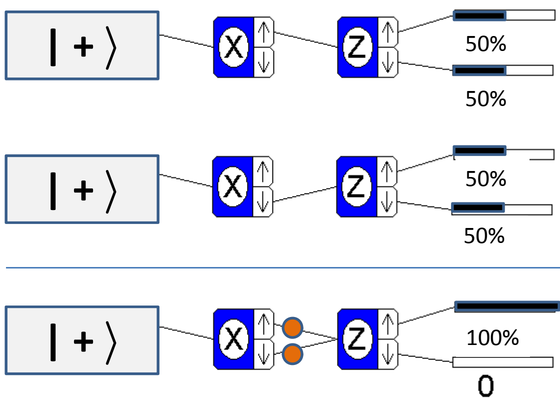
Conditional probabilities:

use $|\Psi'\rangle$: Prob("+") = $|\langle + | \Psi'\rangle|^2 = 1/2$

Prob("-") = $|\langle - | \Psi'\rangle|^2 = 1/2$

Experiment 4

Upper detector:



$P_{|+\rangle} P_{|+\rangle_x} |+\rangle$

$P_{|+\rangle} P_{|-\rangle_x} |+\rangle$

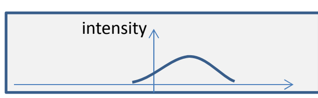
$\left(P_{|+\rangle} P_{|+\rangle_x} + P_{|+\rangle} P_{|-\rangle_x}\right) |+\rangle$

If I know which path has been taken
➔ no interference
Without that information
➔ interference

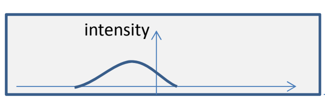
If measurement takes place after SG in X:
mixture of
$$\left.\begin{array}{l} P_{|+\rangle} P_{|+\rangle_x} |+\rangle \\ P_{|+\rangle} P_{|-\rangle_x} |+\rangle \end{array}\right\} \rho$$

Which-path information …

right slit only:

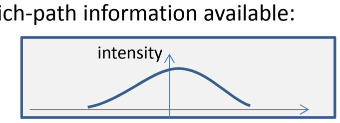


left slit only:



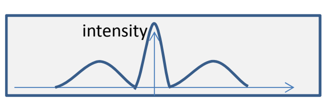
\+ =

which-path information available:

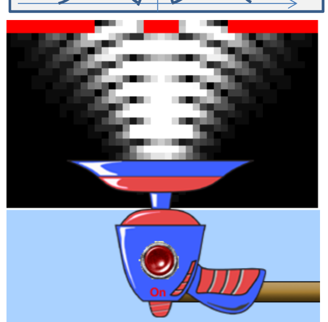


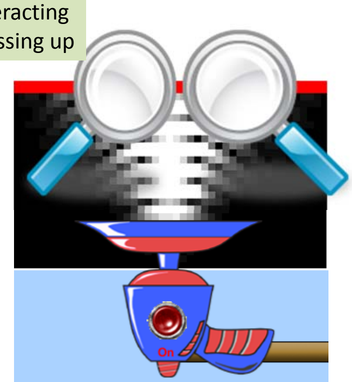
➔ interference effects vanishes

both slits:



Observing:
➔ interacting
➔ messing up





wide slits:
single-slit envelope
very broad …

Time Evolution: Coordinate representation in Energy Eigenbasis (time independent Hamiltonian)

Schrödinger Equation

$$i\,\hbar\,\frac{d}{dt}\,|\Psi(t)\rangle = H\,|\Psi(t)\rangle$$

expansion in eigenstates of H:

$$|\Psi(t)\rangle = \sum_n c_n(t)\,|E_n\rangle$$

dynamical equation: $i\hbar\frac{d}{dt}c_n(t) = E_n c_n(t)$

initial values: $c_n(0) = c_n$

➔ $c_n(t) = c_n\,e^{-i\frac{E_n t}{\hbar}}$

Solution of Schrödinger's equation via Energy Eigenstates:

Step 1: find eigenvectors $|E_n\rangle$ and eigenvalue E_n of H

Step 2: Expand initial state in eigenbasis

$$|\Psi(0)\rangle = \sum_n c_n\,|E_n\rangle$$

Step 3: Write down solution

$$|\Psi(t)\rangle = \sum_n c_n\,e^{-i\frac{E_n t}{\hbar}}\,|E_n\rangle$$