Phys 256
Problem Assignment 10
Due Wednesday, November 28th, 2012 4pm 64 marks

1) Cemented doublet: Two thin lenses are in contact such that the combination has a focal length of 50 cm. The focal length of the second lens is -50 cm. If the V number of the negative lens is 30, what does the V number of the positive lens need to be to make this an achromatic doublet? See pg 272-273 for a sample calculation.

See pg 272-273 for a sample calculation. 4 marks

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{50} = \frac{1}{f_1} - \frac{1}{50} \quad ; f_1 = 25\,\text{cm}$$

$V_2 = 30 \qquad V_1 = ?$

$$f_1 V_1 + f_2 V_2 = 0$$

$$V_1 = \frac{-f_2 V_2}{f_1}$$

$$= \frac{-(-50)30}{25}$$

$$V_1 = 60$$

2) Hecht 10.25. **4 marks**

Circular aperture diffraction Hecht 10.25:

$2a = 2\,mm$

$\lambda_0 = 632.84\,nm$

Angular size to first zero $= \dfrac{1.22\lambda}{2a}$

Angular spot diameter $= \dfrac{2.44\lambda}{2a}$

Size on the moon at $376 \times 10^3\,km = \dfrac{2.44\lambda R}{2a}$

$$= \frac{2.44(6.3284 \times 10^{-7})(3.76 \times 10^{8})}{2 \times 10^{-3}} \qquad \cancel{= 2.44(6.32}$$

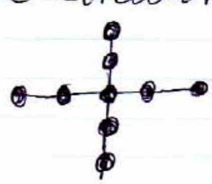
$= 2.9 \times 10^5\,m. \qquad = 290\,km.$

3) Monochromatic aberrations: Hecht 6.27. Explain the answer in less than 2 sentences. **3 marks**

Hecht 6.27

All spots should be equally blurred since SA does not depend on object position

(4) All spots should have circular blur



4) Monochromatic aberrations: Hecht 6.28 **6 marks**

a) Airy disc is altered- aberration. Circularly symmetric blur-must be spherical aberration (could also be defocus, not covered).
b) Asymmetric, comet like shape-coma.
c) Asymmetric with mirror symmetry in two axes-looks like Fig. 6.27-astigmatism.

5) Monochromatic aberrations: Hecht 6.29 **4 marks**

a) Reflection (mirror) symmetry in two perpendicular axes-astigmatism.

b) Comet like shape, similar to Fig. 6.23- coma.

6) I create a combination of two thin lenses of opposite powers which satisfies the Petzval condition and has a focal length of 20 cm.

a) How are the refractive indices of the two lenses related? **3 marks**

$1/n_1f_1+1/n_2f_2=0$

$f_1=-f_2$

$\rightarrow n_1=n_2$

b) If the distance between the two lenses is 5 cm, what are the focal lengths of the two lenses? **2 marks**

$1/f= 1/f_1+1/f_2-d/ f_1f_2$

$1/20=-d/f_1(-f_2)$

$1/20=5/f_1^2$

$f_1=10cm$, $f_2=-10cm$

7) a) Hecht 7.6 Optical path difference: (**Review Problem set2 #8a (3.44)**). In 3.44 you calculated the #waves which spanned a material. Looking at the solution, note that this is proportional to the optical path length and the phase difference between waves is proportional to the OPD. Solve 7.6 using OPD. **4 marks**

Hecht: 7.6

$OPL_B = nx = (1.00)\times(100cm.) = 1.0m.$

$OPL_A = \sum_i n_i x_i = (1.00)\times(89cm) + 2(1.52)(0.5cm.) + (1.33)\times(10cm.) = 103.82cm. = 1.0382\ m.$

$\Lambda = OPL_A - OPL_B = OPD = 1.0382-1.0 = 0.0382\ m.$

$\delta = k_0\Lambda = \left(\frac{2\pi}{\lambda_0}\right)\Lambda = \frac{2\pi \times 3.82\times10^{-2} m.}{500\times10^{-9} m.} \simeq 4.80\times10^5$ radians $= 2\pi\,(7.64\times10^4)$

The waves are in phase as the phase difference is an integer multiple of 2π

b) If two coherent lasers of the same frequency and irradiance $200W/m^2$ are in phase (mode locked), and spatially coincidence, what is the measured irradiance from the combination? REMEMBER: Problem set 2#9. **2 marks**

In phase $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$ where $\delta = 0$ ✓

$I= I_1 + I_2 +2(I_1 I_2)^{1/2}$ where $I_1=I_2=200W/m^2$
$I= 200+200+2\ (200*200)^{1/2}= 800\ W/m^2$

This looks like lack of conservation of energy but the two sources are never exactly coincident in space- so here where they are measured in phase, power is higher. At a nearby position they would be out of phase and power would be zero.

What is the irradiance from 3 in phase, coherent sources? **2 marks**

$I=N^2I_0 =3^2(200) = 1800\ W/m^2$

c) If two incoherent sources have the $200W/m^2$ irradiance, what is the measured irradiance of the combination? HINT When the two waves are spatially incoherent, that is (ε_1-ε_2) randomly varies with time, all values of cos δ are possible, what is the time average? **3 marks**
For coherent sources

In phase $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

In general $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle cos\delta\rangle_t$

For an incoherent source, δ varies randomly from 0 to 2π, cosδ varies from -1 to1 with a time average of zero so $I=I_1 + I_2 =400W/m^2$

8) Young's double slit: Hecht 9.10. **3 marks**

Hecht: 9.10

$\theta_m = m\lambda/a$

3 We want $\theta_{1red} = \theta_{2violet}$

$\lambda_{red}/a = 2\lambda_{violet}/a$

$2\lambda_{violet} = 780$

$\boxed{\lambda_{violet} = 390\,nm}$

9) a) Describe the ideal antireflection coating (thickness and index) for a Fabulite (SrTiO3) plate (n=2.409) in air at λ=589 nm. Assume normal incidence. **2 marks**

a) $n_f=(n_s)^{1/2}=(2.409)^{1/2}=1.55$

$$t = \frac{m\lambda_0}{4n_f} = \frac{1(589)}{4(1.55)} = 95nm \text{ for the thinnest coating}$$

b) If the material I have available has a refractive index of 1.5, show that a film of the thickness calculated above does not fulfill the amplitude condition. **4 marks**

$$E_{01} = \left[\frac{n_f - 1}{n_f + 1}\right] E_0 = \frac{1.5 - 1}{1.5 + 1} E_0 = 0.20 E_0$$

$$E_{02} = \left[\frac{n_s - n_f}{n_s + n_f}\right] E_0 = \frac{2.409 - 1.5}{2.409 + 1.5} = 0.23 E_0$$

So the amplitude of the reflection from the 2nd surface is larger than that from the 1st surface by 0.03 times the incident amplitude.

Notice that the amplitude condition is not fulfilled from another point of view, E_{01} and E_{02} are not π radians out of phase, that is the –ve of each other. The OPD is $2n_f t = 2(1.5)(95) = 285$ nm which gives a phase difference of

$$\delta = 285 \frac{2\pi}{\lambda_0}$$

$$\delta = 285 \frac{2\pi}{589} = 0.97\pi$$

c) What % of incident amplitude and % of incident irradiance would be reflected from the uncoated surface? **4 marks**

$$amplitude = r = \frac{2.409 - 1}{2.409 + 1} = 0.413; \%amplitude = 41.3\%$$

c) Proportion of intensity reflected=$r^2 = \frac{(2.409 - 1)^2}{(2.409 + 1)^2} = 0.17\ I_0$

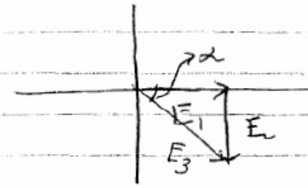
17% of the incident irradiance is reflected from the uncoated surface.

Also review addition of phasors in 7.10.

4) Hecht 7.10

(b) $E_1 = 3\cos\omega t$ $\quad 3\angle 0$

$E_2 = 4\sin\omega t$



$\sin\theta = \cos(\theta - \pi/2)$

So $E_2 = 4\cos(\omega t - \pi/2)$ ✓ $\quad 4\angle -\pi/2$

$E_{30}^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$ Cosine law from phasors

$= (3)^2 + (4)^2 + 2(3)(4)\cos(-\pi/2)$ ✓

$E_{30} = 5$ ✓

OR $E_{30}^2 = E_{01}^2 + E_{02}^2$ from phasor diagram

From phasor diagram

$\tan \cancel{\phi}\ \alpha = -4/3$ ✓

$\cancel{\phi}\ \alpha = -53°$ ✓

E_1 peaks first, then E_1 leads E_3 ✓ (Fig 7.8)