

**Physics 256 Assignment 1 Fall 2012**  
**Wednesday September 19th, 2012**  
**75 marks**

Hecht question 3.14.

irradiance

$$I = \frac{\text{Power}}{A} = \frac{20 \text{ W}}{4\pi(1)^2 \text{ m}^2} = \frac{5 \text{ W}}{\pi \text{ m}^2} \approx 1.59 \text{ W/m}^2$$

Note:  $A=4\pi r^2$

5marks

2.2  $\lambda = c/\nu$

$$\lambda_{\text{light}} = \frac{3.0 \times 10^8 \text{ m/s}}{5 \times 10^{14} \text{ s}^{-1}} = 6 \times 10^{-7} \text{ m}$$

$$\lambda_{\text{EH}} = \frac{3.0 \times 10^8 \text{ m/s}}{60 \text{ s}^{-1}} = 5 \times 10^6 \text{ m}.$$

3 marks

2.4 The time between crests = the period,  $\therefore T = 1/2 \text{ s}$  2

$$\nu = \frac{1}{T} = \frac{1}{0.5} = 2 \text{ s}^{-1} = 2 \text{ Hz} \quad (\text{frequency}) \quad 1$$

$$v = \frac{\Delta d}{\Delta t} = \frac{4.5 \text{ m}}{1.5 \text{ s}} = 3.0 \text{ m/s} \quad (\text{speed})$$

$$\lambda = \frac{v}{\nu} = \frac{3.0 \text{ m/s}}{2.0 \text{ s}^{-1}} = 1.5 \text{ m} \quad \text{wavelength} \quad 3$$

TOTAL 6 marks-must have SI units lose 1 mark if unit missing/ incorrect

2.12 Comparing  $y$  with Eq. (2.13) tells us that  $A = 0.02 \text{ m}$ . Moreover,  $2\pi/\lambda = 157 \text{ m}^{-1}$  and so  $\lambda = 2\pi/(157 \text{ m}^{-1}) = 0.0400 \text{ m}$ . The relationship between frequency and wavelength is  $v = \nu\lambda$ , and so  $\nu = v/\lambda = 1.2 \text{ m/s}/0.0400 \text{ m} = 30 \text{ Hz}$ . The period is the inverse of the frequency, and therefore  $\tau = 1/\nu = 0.033 \text{ s}$ .

TO

TAL: 6 marks

Then if this is the form of the wave at  $t=0$ , and the wave is travelling leftward, write out the wave equation.

Form of the wave in  $x$  was given. Given the speed of  $1.2 \text{ m/s}$ , travelling leftward, the wave equation is:

$$Y(x,t) = (0.02 \text{ m}) \sin(157 \text{ m}^{-1})(x + (1.2 \text{ m/s})t).$$

5 marks

$$2.26 \quad k = \pi 3 \times 10^6 \text{ m}^{-1}, \omega = \pi 9 \times 10^{14} \text{ Hz}, v = \omega/k = 3 \times 10^8 \text{ m/s}.$$

4

marks

Direction of motion is leftwards 1 mark

Redo using equation 2.34:

$$v = - \frac{\left. \frac{\partial \psi}{\partial t} \right|_y}{\left. \frac{\partial \psi}{\partial y} \right|_t} = \frac{(-) - A\pi \sin(z)(9 \times 10^{14})}{-A\pi \sin(z)(3 \times 10^6)} = -3 \times 10^8 \text{ m/s} \text{ indicates leftwards}$$

5 marks

2.32a)

Can do as functional form or ratio of partial derivatives of phi or of psi

$$\psi(y, t) = \exp\left[-a\left(y - \frac{b}{a}t\right)^2\right] = g\left[f\left(y - \frac{b}{a}t\right)\right] \text{ where } g(z) = \exp(-az^2),$$

therefore Conclusion  $v=b/a$  travelling in the +y direction.

MARKS: 6 marks

b)  $\psi(z, t) = A \sin(az^2 - bt^2) = a \sin[(\sqrt{a}z - \sqrt{b}t)(\sqrt{a}z + \sqrt{b}t)]$  is not an  $f(x \pm vt)$ , not a travelling wave 3 marks

$$c) \psi(x, t) = A \sin 2\pi\left(\frac{x}{a} + \frac{t}{b}\right)^2 = G\left[f\left(\frac{1}{a}h\left\{x + \frac{at}{b}\right\}\right)\right] \text{ where } f(z)=z^2,$$

$G(y) = A \sin 2\pi y$  gives conclusion  $v=a/b$  in the  $-x$  direction 6 marks

d)  $\psi(x, t) = A \cos^2 2\pi(t - x) = G[f(x - t)]$  where  $G(z)=A \cos^2(-2\pi z)$   
gives the conclusion  $v=1 \text{ m/s}$  in  $+x$  direction 6 marks

Phasor Question

In each part, draw a phasor diagram to represent the sum of the two waves below. Using the diagram, and the resulting trig identities, calculate the equation of the resulting wave.

$$a) \begin{aligned} \psi_1 &= 2 \sin(kx + \omega t) \\ \psi_2 &= 7 \sin(kx + \omega t - \pi/4) \end{aligned}$$

5 marks diagram, 6 marks calculation

$$\psi_1 = 2 \sin(kx + \omega t)$$

$$\psi_2 = 7 \sin(kx + \omega t - \pi/4)$$

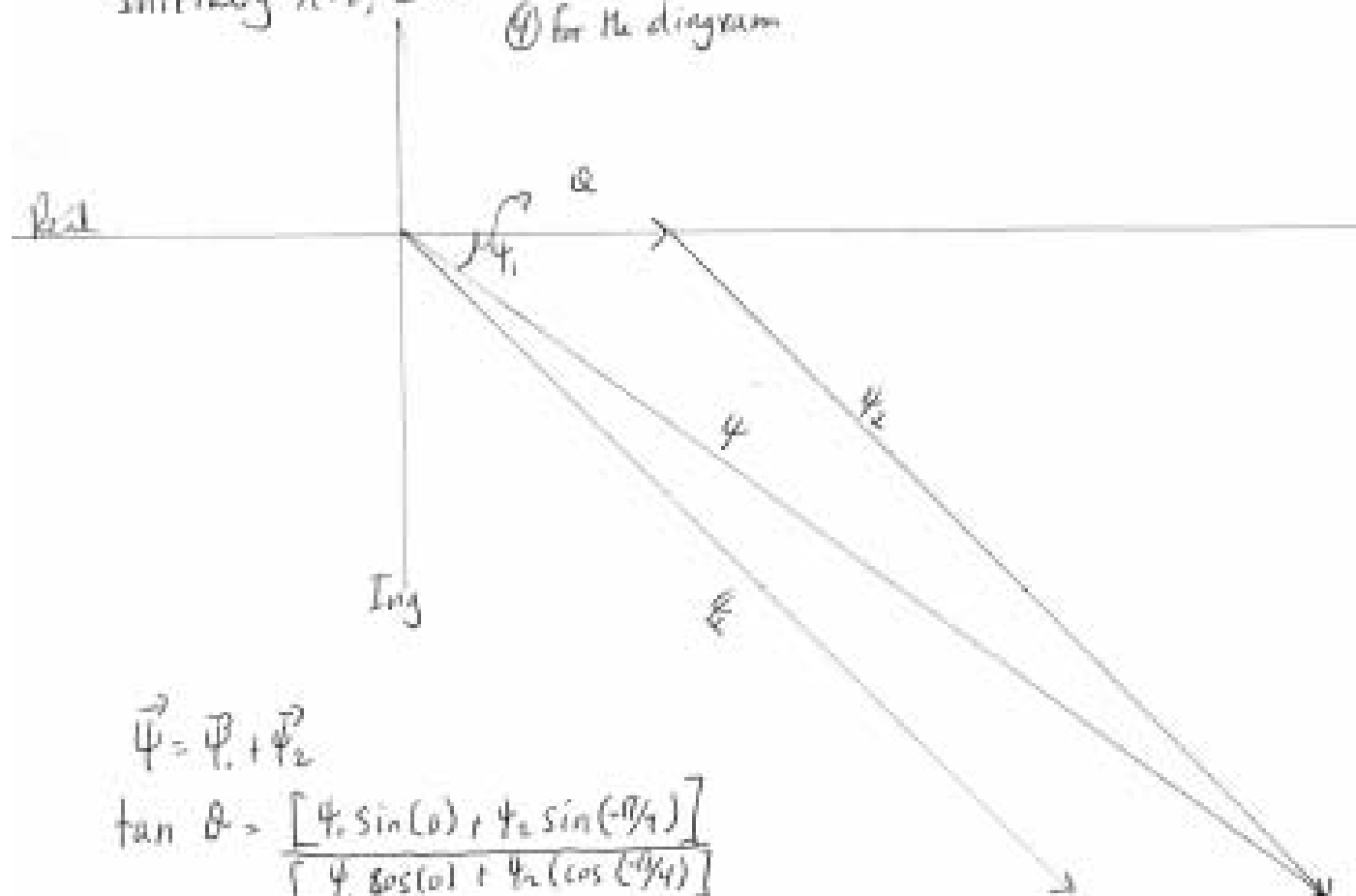
Initially  $x=0, t=0$

$$\psi = A \sin(kx + \omega t + \theta)$$

Phasors:  $2 \angle 0; 7 \angle -\pi/4 (45^\circ)$

$(A_1 \angle \theta_1; A_2 \angle -\pi/4)$

④ for the diagram



$$\vec{\psi} = \vec{\psi}_1 + \vec{\psi}_2$$

$$\tan \theta = \frac{[\psi_1 \sin(0) + \psi_2 \sin(-\pi/4)]}{[\psi_1 \cos(0) + \psi_2 \cos(-\pi/4)]}$$

$$= \frac{-7\sqrt{2}/2}{2 + 7\sqrt{2}/2}$$

$$\theta = \arctan \left[ \frac{-2\sqrt{2}/2}{2 + 7\sqrt{2}/2} \right] = -0.618 \text{ radians OR } \pi - 0.618 \text{ rad}$$

By inspection, angle is in 4th quadrant =  $-0.618$  radians OR  $2\pi - 0.618 \text{ rad} = 5.66 \text{ rad}$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(-\pi/4 - 0)$$

$$= 2^2 + 7^2 + 2(2)(7)(\sqrt{2}/2)$$

$$A = 8.532$$

$$\psi = 8.53 \sin(kx + \omega t - 0.618) \text{ OR } \psi = 8.53 \sin(kx + \omega t + 5.66)$$

$$\psi_1 = 3 \sin(\omega t)$$

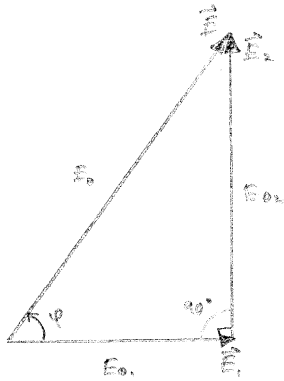
$$\text{b) } \psi_2 = 4 \cos(\omega t) = 4 \sin(\omega t + \frac{\pi}{2}) \text{ OR } \psi_1 = 3 \sin(\omega t) = 3 \cos(\omega t - \frac{\pi}{2})$$

$$\psi_2 = 4 \cos(\omega t)$$

$$\psi_0 = \sqrt{3^2 + 4^2} = 5 \text{ IN BOTH CASES}$$

$$\tan \phi = \frac{4}{3}; \phi = 0.93 \text{ rad} = 53.1 \text{ deg OR } \tan \phi = \frac{-3}{4}; \phi = 0.64 \text{ rad} = -36.9 \text{ deg}$$

$$\psi = 5 \sin(\omega t + 0.93) \text{ OR } \psi = 5 \cos(\omega t - 0.64)$$



4 MARKS CALCULATION, 4 MARKS

DIAGRAM