L05 Stern-Gerlach

September-27-09
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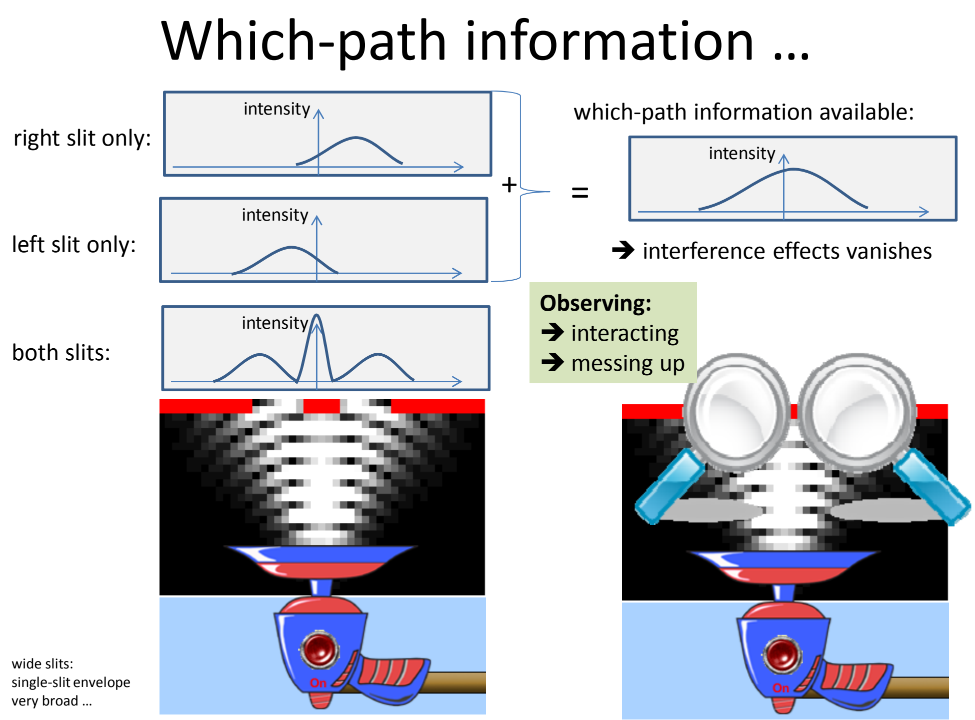
1.4.4 Double Slit Experiments with Which-Path Information

Sneak preview: (we will have to come back to that later to really understand it)
monitoring through which slit the electron goes destroys the interference pattern!



Interference is due to us now knowing which path the particles take.

If we learn the path (detectors at slits) we no longer add amplitude, but just add intensity on the screen for the two possible slits individually!

Why the difference? Well, we will learn and understand later that observing a particle (photon, electron etc) means to interact with it. In a way we 'touch' it, and that changes the behaviour of the system.

PART II: Basic Formalism of Quantum Mechancis

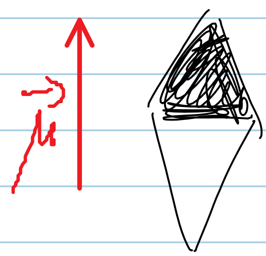
In this section, we study the Stern-Gerlach Experiment to build up the structure of the mathematical description of quantum mechanics. What we want to do is to develop understanding in the sense of rigorous quantitative prediction of effects!

2. Stern Gerlach Experiment

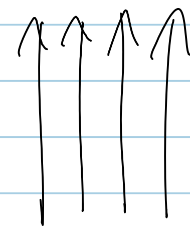
2.1. Magnetic Dipoles in Magnetic Fields

2.1.1. Dipoles and Magnetic fields

$\vec{\mu}$



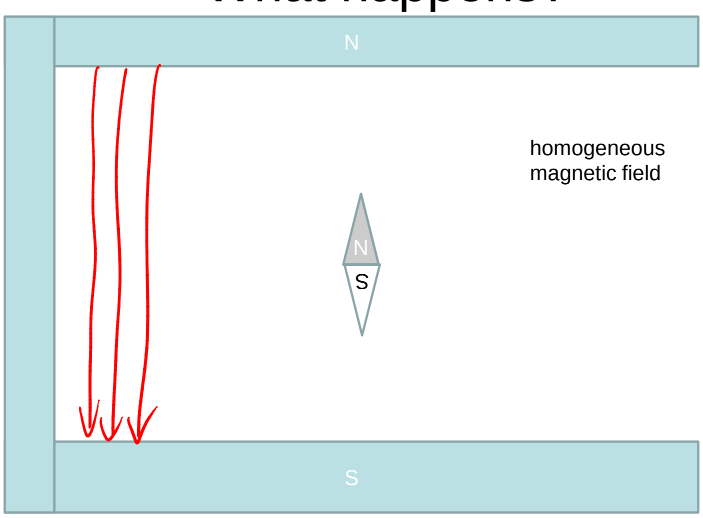
Compass needle
magnetic dipole
moment $\vec{\mu}$



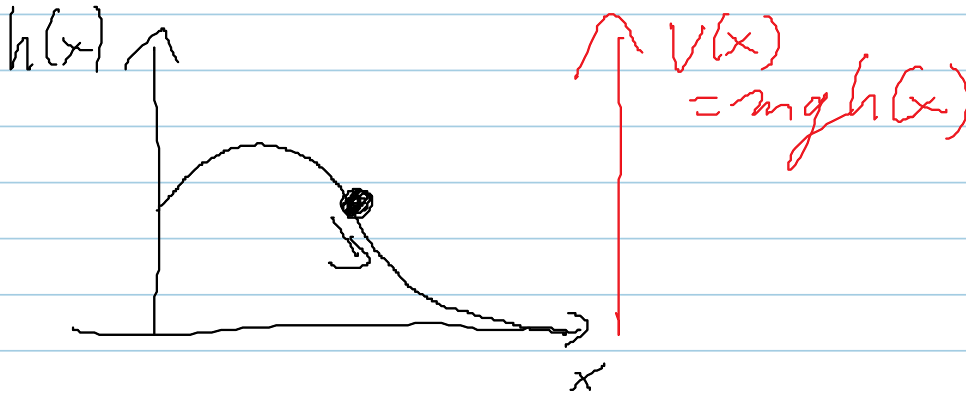
magnetic
field $\vec{B}$

2.1.2. Potential Energy

What happens?



Thinking about how objects react is easily done if we think how the potential energy depends on the configuration of the object (angle, location). This works best if the object is original in rest. For example, in classical mechanics you have no difficulty to see what will happen to the ball in the following landscape:



the ball will tend to the region of lower potential energy!

The potential energy for a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given

$$E_{mag} = -\vec{\mu} \cdot \vec{B}$$

(standard inner product [dot product] for the real-valued 3-dim vectors $\vec{\mu}$ and $\vec{B}$)

2.1.3 Homogeneous field:

set compass needle in homogeneous field. What happens?

- it might rotate (torque)
- there is no net-force pulling the whole needle in either direction

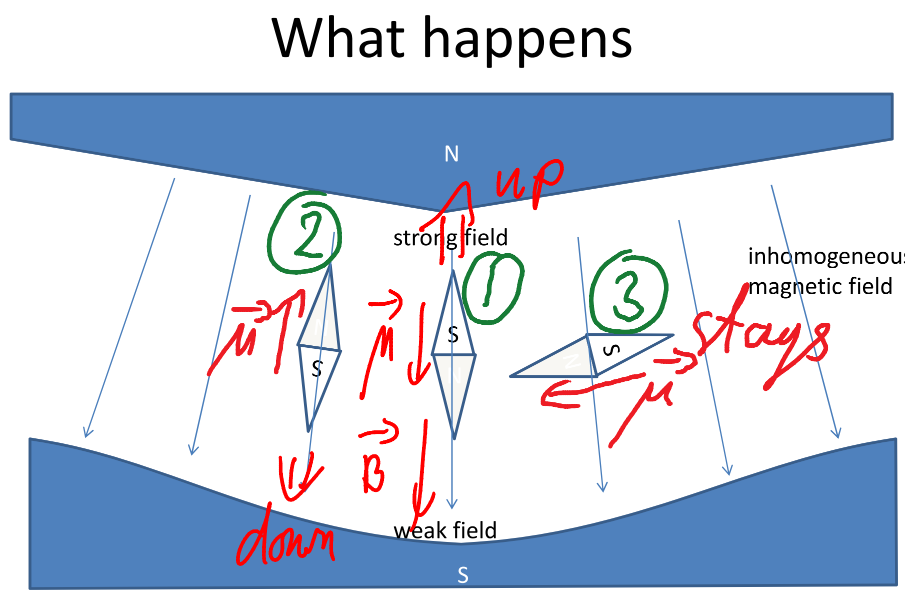
Why?

- At any fixed location, the energy depends on the orientation between the magnetic field and the dipole, so there is a torque to get the dipole moment aligned with the B field to minimize the potential energy
- the magnetic field is constant everywhere in space, so the potential energy does not depend on the location in space (homogeneous field) ==> no reason for the dipole to be pushed around in space!

2.1.4 Inhomogeneous field:

Still same energy expression, but now the magnetic field depends on the position, and thus the potential energy can also depend on the position.

The dipole will tend to move to the direction of lower potential energy.
(Compare this to a particle in a gravitational field and some mountain/valley potential landscape.)



Rules of the game:

- ignore that the magnetic field tries to turn the dipole (just make the overall amplitude of the magnetic field very weak)
- no glue: the dipoles can move
- no gravity: we do this in outer space, so dipole can feely move under the influence of the magnetic field only

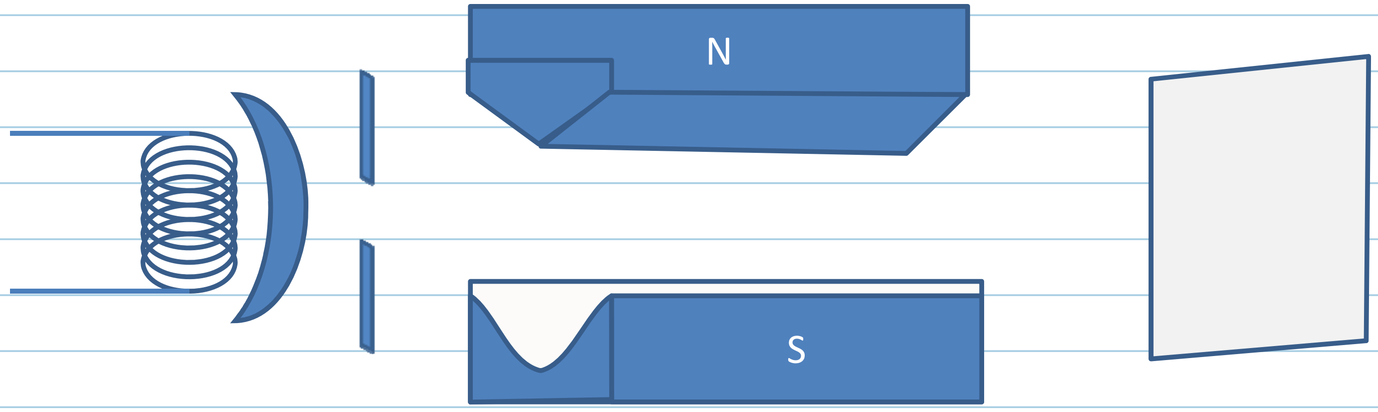
Scenario 1:
magnetic dipole and magnetic field are parallel
==> potential energy is always negative
potential energy gets more negative (decreases) with increasing field
==> magnetic dipole will be drawn towards the area with higher fields (up)

Scenario 2:
magnetic dipole and magnetic field are anti-parallel
==> potential energy is always positive
potential energy gets more positive (increases) with increasing field
==> magnetic dipole will be drawn towards the area with lower fields (down)

Scenario 3:
magnetic dipole and magnetic field are orthogonal
==> potential energy is always zero
potential energy does not depend on the position
==> magnetic dipole will be not be moving anywhere (stays)

2.2 Basic Stern-Gerlach Exerpiment
2.2.1. Set-up

Stern-Gerlach Apparatus: Observation



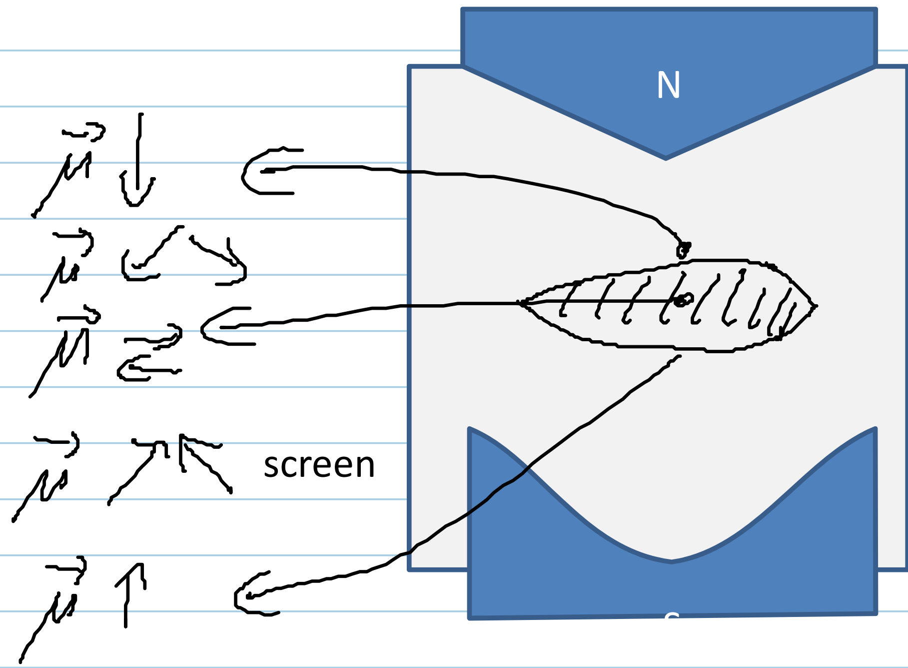
1) user silver atoms:
have significant magnetic moment from outer electron
electrically neutral (no Lorentz force effects)
2) adjust parameters that inhomogeneous effect outweighs homogeneous effect
➔ neglect rotations of dipoles in the magnetic field
➔ only forces pulling in or out of the region of stronger magnetic field

Setup: silver is heated up and evaporates. Through a hole, only a thin beam of electrons excapes and flies in a beam through the magnets. In the end, the silver atoms are deposited on a screen and one can see where the silver builds up.

Where does the magnetic dipole come from?
Actually it comes from the one electron in the outer shell. It is easy to see that atoms can have magnetic momentum, as we like to think about atoms as electrons circling around a positive nucleus, which looks like a current that generates a magnetic field ==> resulting dipole.

2.2.2 Expected Result in Classical Physics

What do you expect?



you would atoms with magnetic dipole moment

- "up" to go to the lower edge
- "down" to go to the upper edge
- all other orientations somewhere between.

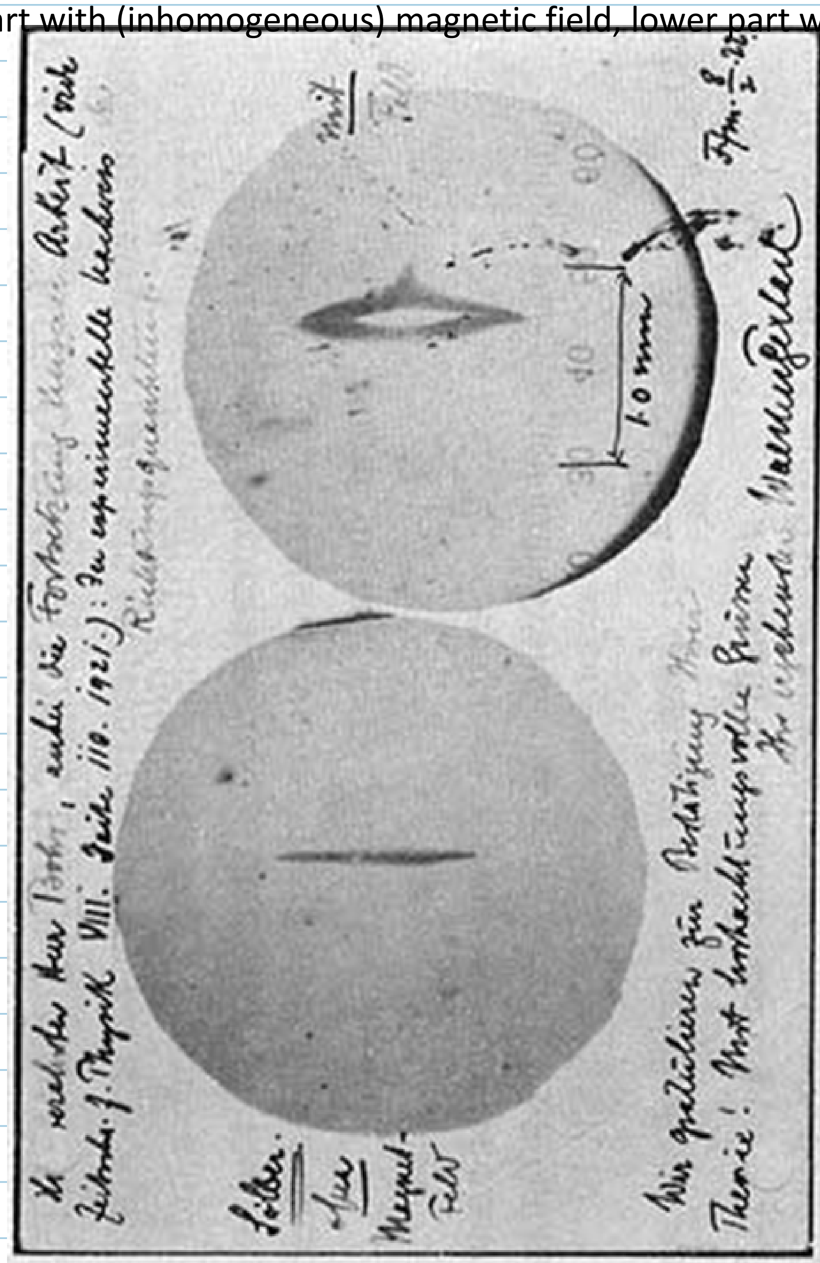
Since the field gradient is the strongest in the middle, there the separation is strongest. To the right and left, the field gradient weakens, and the separation gets smaller.

The atoms are generated by heating (evapourating) silver, so you would expect that the orientations of the dipole are all over the place, so you expect atoms to appear all over the place between the upper and lower boundary.

2.2.3 Actual Experimental Result

What do we see in the actual experiment?

See the postcard that Gerlach wrote to Bohr:
upper part with (inhomogeneous) magnetic field, lower part without.

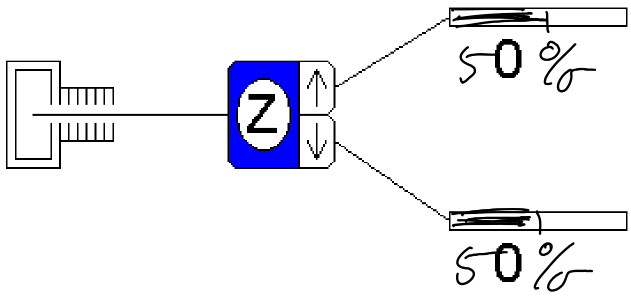


All atoms go EITHER up OR down, but now halfways.

2.2.4 Abstraction of Experiment

We can put a more abstract picture of this experiments, by ignoring the linewidth (spread of spead of atoms from the thermal source) and the right-left deviation of the path through regions with lower field gradient and end up with the following experiments http://phet.colorado.edu/simulations/index.php?cat=Quantum_Phenomena

	counts: run 1	run 2	run 3	
Basic Pre-Experiment A:	508	493	494	50%
	492	517	506	50%



We will from now on only deal with this abstraction. We make from our experiment our first observation

Observation 1: (thermal source)
Atoms are either pulled up or pulled down (two discrete values) with equal probability

Note: each individual run of the experiment (e.g. with 1000 atoms) will show statistical fluctuations. It is the mean that shows the 50/50 splitting in probability!

M1: Before we go on, it is time to do a Math inset:

Basic Probability Theory

fair coin-toss: (colloquial)

fair coin: “head” and “tail” equally likely

mathematically:

assign probabilities

p(“head”) = ½

p(“tail”) = ½

What does it mean?

throw the coin n times

➔

$$\lim_{n\to\infty} \frac{\text{number of "head" outcomes}}{n} = \frac{1}{2}$$

$$\lim_{n\to\infty} \frac{\text{number of "tail" outcomes}}{n} = \frac{1}{2}$$

(assumes independence of events, which is implicit part of the definition of ‘fair coin’)

Definition of probability distribution comprises of

1) Set of events (event space) $X = \{x_1, x_2, \dots, x_m\}$

2) probability of event $x \in X$ to occur: $p(x)$

Properties of probability distributions:

1) $0 \leq p(x) \leq 1$

2) $\sum_{x\in X} p(x) = 1$

Joint & Marginal Probability Distribution

Example:
Drawing objects out of a urn:
- either balls or cubes
- yellow or green

➔ when you draw one object it is

either a ball OR a cube

AND it is

either yellow OR green

OR here as the exclusive version
e.g. cannot be a ball and a cube at the same time

➔ four possibilities
X = {(ball, yellow),
(ball, green)
(cube, yellow)
(cube, green)}

example probabilities
p(ball, yellow)=1/3
p(ball, green) = 1/4
p(cube, yellow)=1/6
p(cube, green)= 1/4

we can also define two sets
Y = {ball, cube}
Z = {yellow, green}

then each event X consists of combination of one event in Y and one in Z:
X = Y × Z
with **joint probability distribution**
p_{YZ}(y,z) for x ∈ X, and y ∈ Y

normalization $$\sum_{y \in Y} \sum_{z \in Z} p_{YZ}(y,z) = 1$$

marginal distribution
$$p_Y(y) = \sum_{z \in Z} p_{YZ}(y,z)$$
example: probability to grab a ball
p_Y(ball) =1/3 + 1/4 = 7/12

normalization $$\sum_{y \in Y} p_Y(y) = 1$$

Conditional Probability Distribution

➔ four possibilities
X = {(ball, yellow),
(ball, green)
(cube, yellow
(cube, green)

example probabilities
p(ball, yellow)=1/3
p(ball, green) = 1/4
p(cube, yellow)=1/6
p(cube, green)= 1/4

then each event X consists of combination of one event in Y and one in Z:
X = Y × Z
with **joint probability distribution**
p_{YZ}(y,z) for x ∈ X, and y ∈ Y

marginal distribution
$$p_Y(y) = \sum_{z \in Z} p_{YZ}(y,z)$$
example: probability to grab a ball
p_Y(ball) =1/3 + 1/4 = 7/12

conditional probability (Bayes theorem)
$$p_{Y|Z}(y|z) := \frac{p_{YZ}(y,z)}{p_Z(z)}$$
probability distribution over Y!!!
z ∈ Z plays here only the role of a parameter

example: probability that you grabbed a ball given that the object is found to be yellow
$$p_{Y|Z}(\text{ball}|\text{yellow}) = \frac{1/3}{1/3+1/6} = \frac{2}{3}$$

normalization $$\sum_{y \in Y} p_{Y|Z}(y|z) = 1$$
for all z ∈ Z!