Problem Set 2: Phys 256 Fall 2012
Total marks:91

1) Do 3.7a), b) and c) The wave has a wavelength of 550nm. In part d), write $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$**.** Assume that the fields are zero at y=0 and t=0.

a) c = $\nu\lambda$, so ν = c/λ = (3*10^8 m/s) / (550*10^-9 m) = 5.45 *10^14Hz 2 MARKS
b) ω= 2$\pi\nu$ = 2π(5.45*10^14Hz)=3.43*10^15rad/s 2 MARKS
k=2π/λ=2π/(550*10^-9 m)=1.14*10^7m^-1 2 MARKS
c) E_0=cB_0, B_0=E_0/c=(600V/m) /(3*10^8 m/s)=2.00*10^-6 T 2 MARKS NOTE 1T =1Vs/m^2
d) $\vec{E}(\vec{r},t)$=E(y,t) = $\vec{E}_0$sin(ky- ωt + ε); E(0,0) = 0 = E_0sin(ε); ε=0;
E(y,t) = (600V/m) $\hat{k}$ sin((1.14*10^7m^-1)y- (3.43*10^15rad/s)t 2 MARKS
$\vec{B}(\vec{r},t)$=B(y,t) = $\vec{B}_0$ sin(ky- ωt + ε); B(0,0) = 0 = B_0sin(ε); ε=0;
B(y,t) = (2*10^-6 T) $\hat{i}$ sin((1.14*10^7m^-1)y- (3.43*10^15rad/s)t); 2 MARKS
e) Flux density I=$\frac{1}{2}$ c$\varepsilon_0 E_0^2$ = $\frac{1}{2}$*3*10^8 m/s*8.85*10^{-12} C^2/Nm^2*$(600V/m)^2$=4.78*10^2 W/m^2 2 MARKS

Energy density=$\frac{I}{c}$=$\frac{4.78\times10^2\ W/m^2}{3\times10^8 m/s}$=1.59*$10^{-6}$ J/m^3 2 MARKS

2) Write an expression in Cartesian coordinates for a harmonic plane **E** wave with E_x=0, with a frequency ω=3 X$0^{15}s^{-1}$, k=1X10^7m^{-1} for which $\vec{k}$ is along a line from the origin through the point (1,-1,2). 5 MARKS

$$k = \left(k_x{}^2 + k_y{}^2 + k_z{}^2\right)^{1/2} = a\left((1)^2 + (-1)^2 + (2)^2\right)^{1/2} = \sqrt{6}a = 1X10^7\,m^{-1}$$

$$\vec{k} = k\left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right) = 1X10^7\left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right) where \left(\left(\frac{1}{\sqrt{6}}\right)^2, \left(\frac{-1}{\sqrt{6}}\right)^2, \left(\frac{2}{\sqrt{6}}\right)^2\right)^{1/2} = 1$$

$$\vec{E} \perp \vec{k};\ \vec{E} \bullet \vec{k} = 0;\ let\ E_z = 0;\ then\ by\ inspection\ E_x = E_y = \frac{E_0}{\sqrt{2}}$$

$$OR\ k\frac{E_x}{\sqrt{6}} + k\frac{E_y}{\sqrt{6}} = 0; E_0 = \left(E_x + E_y\right)^{1/2} = \left(2E_x\right)^{1/2}; E_x = E_y = \frac{E_0}{\sqrt{2}}$$

$$\vec{E}(x,y,z,t) = \vec{E}_0 cos\left(k\left(\frac{k_x}{k}x + \frac{k_y}{k}y + \frac{k_z}{k}z\right) - 3\ X10^{15}t + \varepsilon\right) The\ minus\ sign\ in\ front\ of\ t\ is\ because$$

the wave is travelling with the vector defined

$$\vec{E}(x,y,z,t) = \frac{E_0}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)cos\left(k\left(\frac{1}{\sqrt{6}}x + \frac{-1}{\sqrt{11}}y + \frac{3}{\sqrt{11}}z\right) - 3\ X015s^{\cdot}\right)$$ ε is not necessary

OR **E**(***r*,**t) = $E_0(\hat{i} + \hat{j})$ sink/$\sqrt{6}$/(x-y+2z-3X10^{15}t+ε) Note that it is a plane wave so **r** is a vector. The minus sign in front of ωt is because the wave is travelling with the vector defined (not in the opposite direction).

3) For $\vec{E} = \left(0.866\hat{i} + 0.5\hat{j}\right)\left(3X10^3 V/m\right)\exp\left[i\left[1.114X10^7\right]\left(z - 2X10^8 t\right)\right]$. Assume SI units.
a) Find the direction along which the electric field oscillates;

The electric field oscillates along the direction $0.866\hat{i} + 0.5\hat{j}$, that is along a vector (0.866 ,0.5,0). If θ is the angle to the x axis, tan (θ)=0.5/0.866. θ = 30 deg to the x axis
2MARKS
b) the scalar value of the amplitude of the electric field;

$\vec{E}_0 = E_0$ (0.866 ,0.5,0) $=\sqrt{0.866^2 + 0.5^2}$ *3*10^3 OR Double check that (0.866 ,0.5,0) is a unit vector: $0.866^2+0.5^2=1$. Then $E_0=3X10^3$ V/m. 1MARK

c) the direction of propagation of the wave;

The wave propagates along the +ve z direction. 1 MARK

d) the propagation constant and wavelength;

k=$1.114X10^7$ m^{-1} is the propagation constant. $\lambda = \frac{2\pi}{k} = \frac{2(3.141)}{1.114X10^7} = 5.63X10^{-7}\,m = 563nm$
1 MARK EACH

e) the speed and refractive index of the medium;
v=$2X10^8$ m/s 1MARK
n=$3X10^8/2X10^8$=1.5 1 MARK

f) the frequency and angular frequency;
$\nu = \frac{c}{\lambda} = \frac{2X10^8}{5.63X10^{-7}} = 3.55X10^{14}\,Hz;\ \omega = 2\pi\nu = 2.23X10^{15}\,rad/s$ 1 MARK EACH
g) the direction of oscillation of the magnetic field and its direction of propagation.

The direction of propagation of the B field is the same as the E field. The wave propagates along the +ve z direction. 1MARK
The direction of oscillation is perpendicular to the propagation (the z axis). It is therefore in the x-y plane perpendicular to the E field oscillation. It must be 90 deg to the E field at 120 deg to the x axis, in a direction (-0.5, 0.866 ,0) OR $0.5\hat{x} + 0.866\hat{y}$. 1MARK

4) a)3.19

I=$c\varepsilon_0 E_0^2/2$; $E_0=(2I/c\varepsilon_0)$^(1/2)= $\sqrt{\frac{2\times10^{20}}{3\times10^{8}\times8.86\times10^{-12}}}$ = (7.535*10^22)^(1/2) = 2.7*10^11V/m
2MARKS

b) What is the energy density and pressure of the pulse?

energy density=pressure=I/c = (10^20W/m^2)/(3*10^8m/s) = 3.3*10^11 J/m^3 2 MARKS

c) If the peak wavelength is 800nm **(this is not needed at this point)**, what is the length of the pulse in space?

length of the pulse = ct = (3*10^8m/s)*(10^-12s) = 3*10^-4m 2MARKS

d) What is the total energy of the pulse assuming a single wavelength **(this then uses the pulse length calculated above)** and a diameter of 2μm?

energy = energy density*A*Δd= (3.3*10^11 J/m^3)*(π*(10^-6m)^2)*(3*10^-4m) = 3.14*10^-4 J OR $IAt=10^{20} W/m^2 *\pi*(10^{-6})^2\ m^2*10^{-12}s=3.14*10^{-4}$ J 2 MARKS

e) What is the force applied on a reflecting surface?

reflecting surface

$F = \frac{2Power}{c}$ = 2IA/c = 2*(10^20W/m^2)*(π*(10^-6m)^2)/(3*10^8m/s) = 2.09 N 2 MARKS

f) What momentum is carried per unit volume of the wave?

momentum p_v=I/(c^2) = (1X10^20W/m^2)/(3*10^8m/s)^2 = 1.1*10^3 Ws^2/m^3 2 MARKS

g) What momentum is transferred by the pulse to a reflecting surface?

p=FΔt=(2.09 N)*10^-12 = 2.09 *10^-12 Ns This could also be calculated from p_{v*} A*Δd OR

$p=\frac{2Energy}{c}=\frac{2IAt}{c}=\frac{2*10^{20}\ W/m^2*\pi*(10^{-6})^2\ m^2*10^{-12}s}{3*10^8\ m/s}=2.09*10^{-12}$ Ns 2 MARKS

h) Calculate the power of the pulse, the photon flux and the photon flux density.

h) Power = IA = (10^20W/m^2)*(π*(10^-6m)^2) = 3.14*10^8W 2 MARKS

5) 3.38

a) 2πν= π(10^15) thus ν = 5*10^14Hz 2 MARKS NOTE FORM OF EQUATION BOTTOM PG 16
b) λ = v/ν = 0.65c /ν = 3.9*10^-7 m 2 MARKS
c) n = c/v = c/0.65c = 1.54 2 MARKS

6) 3.39

$v=c/n = \frac{3*10^8\ m/s}{2.42}$ = 1.24*10^8 m/s 2 MARKS

7) 3.40

$\lambda=\lambda_0/n$ =540nm/1.33=406nm 2 MARKS

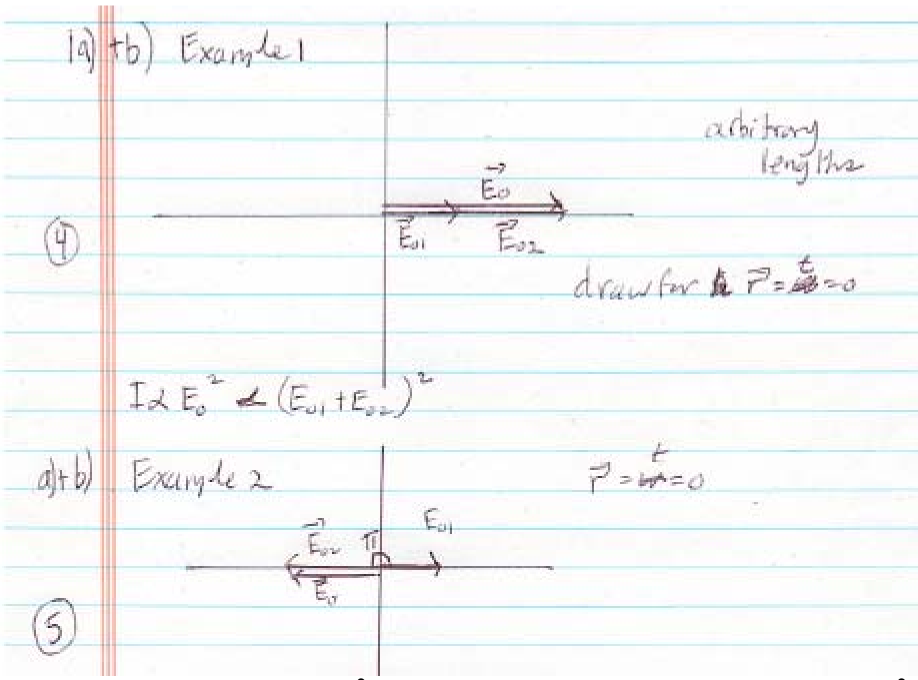
8) a) 3.44 4 MARKS
b) If a parallel, adjacent wave traverses the vacuum, what is the phase difference between them when the first one exits the glass? Will the two waves interfere constructively or destructively? 5 MARKS

a) $\lambda=\lambda_0/n$ =500nm/1.60=312.5nm 2 MARKS
in glass: (1.00*10^-2m)/(312.5nm)=3.2*10^4 waves 2 MARKS

b) in vacuum: (1.00*10^-2m)/(500nm)=2*10^4 waves 2 MARKS
The waves were in phase at the beginning of the plate. At the other end of the glass plate, the two waves are out of phase by (3.2-2)X10^4 waves, a multiple of a wavelength (1 MARK) which means that they appear in phase, and will constructively interfere. 2 MARKS

9) a) Add these two fields together in a phasor diagrams for $\varepsilon=0$ (in phase) and $\varepsilon=\pi$ (out of phase) in order to find $\vec{E}_T(\vec{r},t)=\vec{E}_0 e^{i(\vec{k}\bullet\vec{r}-\omega t+\phi)}$.

$$\vec{E}_1(\vec{r},t)=\vec{E}_{01}e^{i(\vec{k}\bullet\vec{r}-\omega t)}$$
$$\vec{E}_2(\vec{r},t)=\vec{E}_{02}e^{i(\vec{k}\bullet\vec{r}-\omega t+\varepsilon)}$$



b) Given that irradiance, $I\alpha\ E_0^2$, show that, in the first case, $I\alpha(E_{01}+E_{02})^2$ and in the second case, $I\alpha(E_{01}-E_{02})^2$ which does not give $I=I_1+I_2$.

First case: $E_0=E_1+E_2$; $E_0^2=(E_1+E_2)^2$ **2 marks**

Secondcase **2 marks**:

$I \propto (E_0)^2 \propto (\vec{E}_{01}+\vec{E}_{02})\bullet(\vec{E}_{01}+\vec{E}_{02}) \propto E_{01}^2+E_{02}^2-2E_{01}E_{02}$
$\propto (E_{01}-E_{02})^2$

OR $I \propto E_0^2 \propto E_{01}^2+E_{02}^2+2E_{01}E_{02}\cos(\pi) \propto E_{01}^2+E_{02}^2-2E_{01}E_{02}$
$\propto (E_{01}-E_{02})^2$

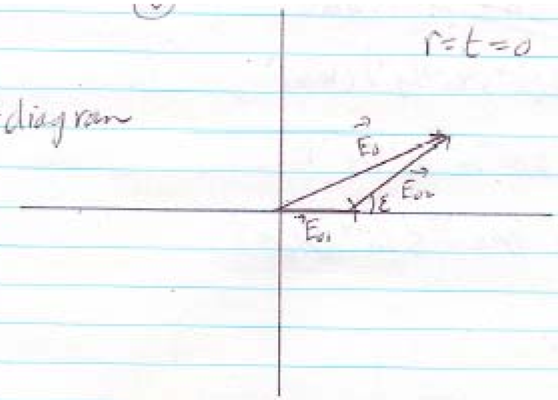
OR $I \propto E_0^2 \propto E_0$ where $|\vec{E}_0|=|\vec{E}_{01}|-|\vec{E}_{02}|$ by inspection

c) Draw the general phasor diagram for a phase difference of ε. In this case, $I=I_1+I_2+I_{12}$.

1c)

6 for diagram

$r = t = 0$



d) From the diagram, give a value of I_{12} in terms of E_{01}, E_{02} and ε, the phase difference.

d) $E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\varepsilon)$ from the cosine law

$I \propto E_0^2 \propto (E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\varepsilon))$

$\cancel{\propto I_1 + I}$

5 $I_1 \propto E_{01}^2$; $I_2 \propto E_{02}^2$

$I = I_1 + I_2 + I_{12}$

$I_{12} \propto 2E_{01}E_{02}\cos(\varepsilon) = 2AE_{01}E_{02}\cos(\varepsilon)$

$I_1 \propto E_{01}^2; I_2 \propto E_{02}^2; I \propto E_0^2$

$I = I_1 + I_2 - 2AE_{01}E_{02} \neq I_1 + I_2 incase2$

In general $I = I_1 + I_2 + 2AE_{01}E_{02}cos(\alpha_2 - \alpha_1) = I_1 + I_2 + 2AE_{01}E_{02}cos(\boldsymbol{\varepsilon})$