L08 Remarks on

The story so far:

Postulate 1:

The state of a quantum mechanical system is represented mathematically by a *normalized* vector, a symbol ket $|\Psi\rangle$. This symbol

- summarizes everything you can know about the system
- and everything you need to know to predict measurement results
- corresponds to an element of a complex vector space of suitable dimension.

Postulate 3:

A measurement with mutually exclusive outcomes can be described by a set of orthonormal basis vectors $\{ |\phi_i\rangle \}$, i = 1, … , d

Postulate 4

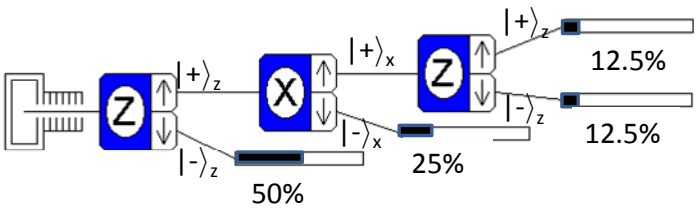
For an input state described by (normalized) ket $|\Psi\rangle$
and a measurement with mutually exclusive events "i" described by elements of an orthonormal basis $\{ |\phi_i\rangle \}$, i = 1, … , d
the probability Pr("i") to observe outcome "i" is given by

$$\Pr(\text{"i"}) = |\langle \phi_i | \Psi \rangle|^2$$

We already saw that these postulates allow us to assign kets (complex vectors) to sources and measurements, and to understand the relative structure of these kets, e.g. how $|+\rangle_x$ is expressed in the z-basis ...

3.4 Experiment 3 and the Update-Postulate

Experiment 3



Pre-Postulate 5 (update rule)

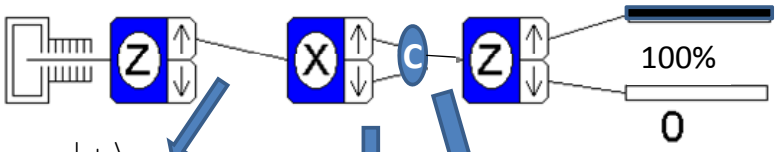
After a measurement on an input state described by state vector $|\Psi\rangle$
with outcome "i" associated with ket $|\phi_i\rangle$
the outgoing state is described by state vector ket $|\phi_i\rangle$

the measurement gives new knowledge
➔ update our summary of our knowledge given by the state vector!

The update rule expresses the fact that for our simple measurements only the measurement result of the preceding step characterizes all future predictions, not the steps before that measurement.
(In statistics this type of process is referred to as Markov process)

3.5 Experiment 4 and Superposition States

Experiment 4



$$|+\rangle_z = \frac{1}{\sqrt{2}}(|+\rangle_x + |-\rangle_x)$$

after combiner:

$$\frac{1}{\sqrt{2}}(|+\rangle_x + |-\rangle_x) = |+\rangle_z$$

before combiner:
total state
(no measurement)

$$\frac{1}{\sqrt{2}}(|+\text{, upper path}\rangle_x + |-\text{, lower path}\rangle_x)$$

Rules to Remember:
amplitudes remain amplitudes until a measurement takes place! (superposition of possibilities)

One set of amplitudes for one set of events can be converted into
amplitudes for another set of events! (Example: Z-basis to X-Basis)
➔ it is only a description level (see input)

measurement turn amplitudes into probabilities: (turn possibilities into realities)
only one of the possibilities is realized!

For the atoms emitted by the source (after Z-measurement) only the internal state is important, with amplitudes for different possible events.
The X-Stern-Gerlach device introduces now a second degree of freedom: the atoms can be located in the upper path, or the lower path. Without measurement (That is: without involving detectors) the inner degree of freedom (amplitude for x-direction SG measurement) and outer degree of freedom (which path) appear now in only two combinations. They are still only possibilities …
After the combiner, the path degree of freedom is removed quantum mechanically leaving still the amplitudes as possibilities, not realized events! All atoms are again in one location, but still have their internal degree of freedom.

without measurement of location after X-measurement:
➔ the two independent paths from source to detector interfere
(amplitude contributions from each path add up)
➔ the particle is in *superposition* of being on one or the other path

3.6 Mixtures versus Superpositions

3.6.1 Overview

Mixture versus superposition

A source emits the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_x + |-\rangle_x\right) = |+\rangle_z$$

Superposition

isn't that the same as if the source emits with
- probability ½ the state $|+\rangle_x$
- probability ½ the state $|-\rangle_x$???

Mixed State Source

$\{\Pr(\text{signal s}), |\Psi_s\rangle\}_{s=1}^{n}$

Prediction rules for superposition states:

Pr("i") = $|\langle \phi_i | \Psi \rangle|^2$

$$\Pr(+ \text{ in z}) = |_z\langle +|\Psi\rangle|^2 \quad =1$$

$$\Pr(\text{- in z}) = |_z\langle -|\Psi\rangle|^2 \quad =0$$

Prediction rules for mixed states sources:

$$\Pr(\text{"i"}) = \sum_{s=1}^{n} \Pr(\text{signal s}) \; |\langle \phi_i|\Psi_s\rangle|^2$$

$$\Pr(+ \text{ in z}) = \frac{1}{2}|_z\langle +|+\rangle_x|^2 + \frac{1}{2}|_z\langle +|-\rangle_x|^2 = \frac{1}{2}$$

$$\Pr(\text{- in z}) = \frac{1}{2}|_z\langle -|+\rangle_x|^2 + \frac{1}{2}|_z\langle -|-\rangle_x|^2 = \frac{1}{2}$$

different paths in experiments (Exp. 4 or Double slit):

which-path information
- unavailable ➔ superposition of paths
- available ➔ mixture

The rule for mixture can be easily understood once we realize the that there are two random variables involved:
- index s to describe which state was prepares
- index i to describe the measurement outcome

The source description gives the probability over the signal space

$$\Pr(signal = s)$$

Quantum Mechanics gives the conditional probabilities that outcome i is triggered given that state $|\psi_s\rangle$ has been prepared:

$$\Pr(\text{outcome "i"} \mid signal = s) = |\langle \phi_i | \psi_s \rangle|^2$$

Then we have the joint probability of having prepared signal and obtained outcome "i" as

$$\Pr(\text{outcome "i"}, signal = s) = \Pr(signal = s)\, |\langle \phi_i | \psi_s \rangle|^2$$

In our experiment, we do not know which signal is being prepared in each run of the experiment, but we can observe the overall probability that outcome "i" is begin triggered, this is given by the marginal distribution

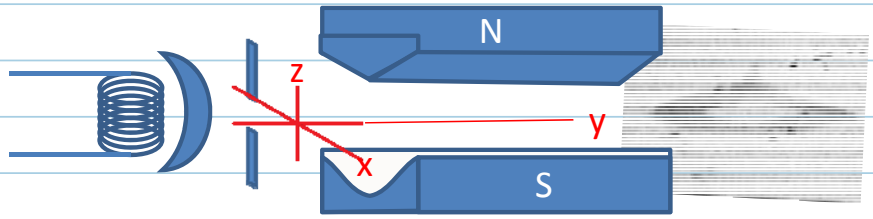
$$\Pr(\text{"i"}) = \sum_{s=1}^{n} \Pr(\text{signal s}) \; |\langle \phi_i | \Psi_s \rangle|^2$$

$\Pr(\text{outcome "i"})$

3.6.2 Example of Thermal Source

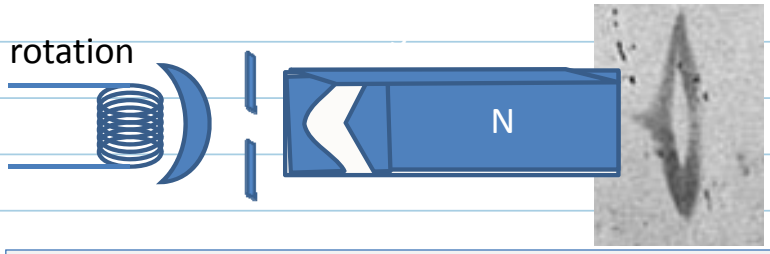
Stern Gerlach: Basic Observations

a)



Abstraction:
atoms go *either* up *or* down
(but not anything inbetween)
ignore remaining spatial distribution

b) rotation



Abstraction:
atoms go *either* right *or* left
(but not anything inbetween)
ignore remaining spatial distribution

Observation 2 for thermal source:
no matter how you orient the (inhomogeneous) magnetic field:
the atoms go *either* towards the stronger field *or* towards the weaker field!
Quantization of dipole moment with respect to applied magnetic field!

Experiment a) could be explained by a pure state description of the source, for example as ket $|+\rangle_x$, but this would not sit well with observation in experiment b)

Other obvious explanation attempts would be a mixed source

- probability ½ the state $|+\rangle_z$
- probability ½ the state $|-\rangle_z$

Similarly, one could try to explain the source as

- probability ½ the state $|+\rangle_x$
- probability ½ the state $|-\rangle_x$

Other possibilities would involve a random mixture of all possible states ...

What is correct?

Well, correct means: it gives the right predictions for all possible experiments. We just go here for the X and Z measurement, but take my word for it that it works for all measurements:

Example of mixed states source: thermal source

Thermal Source

$$\{\Pr(\text{signal s}) \ , |\Psi_s\rangle\}_{s=1}^{n}$$

Prediction rules for mixed states sources:

$$\Pr(\text{"i"}) = \sum_{s=1}^{n} \Pr(\text{signal s}) \ |\langle\phi_i|\Psi_s\rangle|^2$$

source description	measurement prediction Z	X
Experiment a) ➔ - prob. ½ $\|+\rangle_z$ - prob. ½ $\|-\rangle_z$	$\Pr(+ \text{ in z}) = \frac{1}{2}$ $\Pr(- \text{ in z}) = \frac{1}{2}$	$\Pr(+ \text{ in x}) = \frac{1}{2}\|_x\langle+\|+\rangle_z\|^2 + \frac{1}{2}\|_x\langle+\|-\rangle_z\|^2 = \frac{1}{2}$ $\Pr(- \text{ in x}) = \frac{1}{2}\|_x\langle-\|+\rangle_z\|^2 + \frac{1}{2}\|_x\langle-\|-\rangle_z\|^2 = \frac{1}{2}$
Experiment b) ➔ - prob. ½ $\|+\rangle_x$ - prob. ½ $\|-\rangle_x$	$\Pr(+ \text{ in z}) = \frac{1}{2}\|_z\langle+\|+\rangle_x\|^2 + \frac{1}{2}\|_z\langle+\|-\rangle_x\|^2 = \frac{1}{2}$ $\Pr(- \text{ in z}) = \frac{1}{2}\|_z\langle-\|+\rangle_x\|^2 + \frac{1}{2}\|_z\langle-\|-\rangle_x\|^2 = \frac{1}{2}$	$\Pr(+ \text{ in x}) = \frac{1}{2}$ $\Pr(- \text{ in x}) = \frac{1}{2}$

Thermal source can be written as 50/50 mixture of ANY orthonormal basis states!
➔ all descriptions are quantum mechanically equivalent! (more later …)

So **mixed sources** allow many different descriptions which are all equivalent, that is, they give the same prediction of measurement outcome probabilities.

If the source is described by only one ket $|\psi\rangle$, characterized by a measurement where it will give always one specific outcome, then this is the only way to characterize the source.

Such sources are generated for example by a thermal source followed by a SG device where one output arm is blocked.

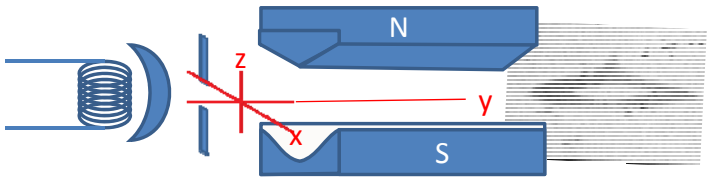
We refer so these sources as **Pure State Sources.**

3.7 Spins and Quantization of Dipole Moments

3.7.1 Observations for Silver atoms

Spins and Dipole Moments

a)



How big is the separation on screen? … depends which force acted on the atom

➔ how big is the force on the atom

➔ how big is the dipole moment of the atom

3.7.2 Classical Part

Let's return first to **classical physics**

Energy:

$$E_{mag} = -\vec{\mu} \cdot \vec{B}$$

here specialization: magnetic field in z-direction

$$\vec{B} = B_z \vec{e}_z$$

$$\vec{\mu} = \mu_x \vec{e}_x + \mu_y \vec{e}_y + \mu_z \vec{e}_z$$

$$\Rightarrow E_{mag}(z) = -\mu_z B_z(z)$$

Here $\vec{e}_x, \vec{e}_y, \vec{e}_z$ are unit vectors for the three spatial direction. (For spatial direction I use the usual vector notation with an arrow on top, to separate it from the abstract vectors of type $|\rangle$ that characterize quantum states!)

Force:

A force follows from a gradient of the energy.

In general we have:

$$\vec{F} = \left(\frac{\partial}{\partial x} E_{mag}\right)\vec{e}_x + \left(\frac{\partial}{\partial y} E_{mag}\right)\vec{e}_y + \left(\frac{\partial}{\partial z} E_{mag}\right)\vec{e}_z$$

"partial derivatives" with respect to x, y, z

In our case the energy depends only on the z-coordinate, so everything simplifies to

$$\vec{F} = \left(\frac{d}{dz} E_{mag}(z)\right)\vec{e}_z$$

$$= -\mu_z \left(\frac{d}{dz} B_z(z)\right)\vec{e}_z$$

usual derivate
(only one variable left in the game: z)

From classical considerations, one can split up the dipole moment of an electron in the following way (see McIntyre!)

$$\vec{\mu} = -g\frac{e}{m_e}\vec{S}$$

with

g as some geometric factor
e as the elementary charge of an electron
m as the mass of the electron
$\vec{S}$ as a new variable callen SPIN

(Note: we have not really done anything: we just switched from one quantity, the dipole moment $\vec{\mu}$ to another quantity, the spin $\vec{S}$)

3.7.3 Quantum Mechanical Part:

in quantum mechanics we learned that we cannot talk about classical dipole moments represented by three-dimensional vectors, and for the same reason we also cannot talk about some 3-dim vectors called spin.

What makes sense is to talk about effective dipole moments (or spins) for given directions:

$$\mu_z = -g \frac{e}{m_e} S_z$$

Note: only the outer-lying electron of the silver atom plays a role. Its dipole moment pulls the whole atom with it.

We can do this for all directions, for example for x, y ,z, resulting in

$$\mu_x, \mu_y, \mu_z$$

but our systems does not have pre-described values for all three directions, as would be the case if they were components of a 3-dim vector.
Instead, the system has amplitudes to manifest particular values of the dipole moments in each separate experiment with probabilities related to the probability amplitudes of the corresponding ket-vector description of the systems.

So we can ask about the possible values, say of S_z in a given measurement:

Experiment:

Using the thoughts above, one can use the experimental data to assign values for the dipole moment, and thus for the spin variable.

Results:

We already know that only two different values will be found in each experiment, which is known as quantization of the spin.
The value one finds is

$$S_z = \pm \frac{\hbar}{2}$$

where

$$\hbar = \frac{h}{2\pi}$$

contains the Planck Constant h

Similarly, we find for the other measurement directions for example

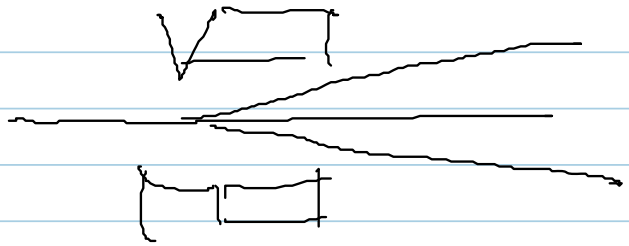
$$S_x = \pm \frac{\hbar}{2}$$

The behaviour of showing two splittings with these values leads to the name

Spin 1/2 particles

3.7.4 Other Spin particles: more splittings and higher dimensional spaces

If we use different type of atoms, these atoms can have different values for the spin, but the spin is always quantized!
For example, one could observe a splitting like this



where one will find in corresponding calculations
the values

$$S_z \in \{-\hbar, 0, +\hbar\}$$

These types of systems are called Spin-1 particles, and their internal state needs to be described by three-dimensional complex vector spaces

Systems with more splittings exists, requiring higher-dimensional complex vector spaces for their quantum mechanical description.