

2) Prove that  $\text{rev-append}$  and  $\text{rev-append}'$  are equivalent.

Using induction on  $l_1$

Base Case:  $l_1 = []$

$\text{rev-append } [] \ l_2$   
by  $\text{rev-append}$   
 $\Rightarrow l_2$

$\text{rev-append}' [] \ l_2 = \text{append } (\text{rev } []) \ l_2$   
 $\Rightarrow \text{append } (\text{rev } []) \ l_2$   
by  $\text{rev}$   
 $\Rightarrow \text{append } [] \ l_2$   
by  $\text{append}$   
 $\Rightarrow l_2$

$\therefore$  Base Case proven.

Induction Hypothesis:

$\forall l_2 \quad \text{rev-append} + l_2 = \text{rev-append}' + l_2$



Step Case  $l_1 = h :: t$

Assume The Induction Hypothesis

$\text{rev\_append } h :: t \quad l_2$

$\Rightarrow \text{rev\_append } t \quad (h :: l_2)$  by program

$\Rightarrow \text{rev\_append}' \quad t \quad (h :: l_2)$  by I.H

①  $\Rightarrow (\text{rev } t) \quad @ \quad (h :: l_2)$  by program

$\text{rev\_append}' \quad (h :: t) \quad l_2$

$\rightarrow \text{rev}' \quad (h :: t) \quad @ \quad l_2$  by evaluation  
of  $\text{rev\_append}'$

$\rightarrow (\text{rev } t) @ [h] @ l_2$  by rev

$\rightarrow (\text{rev } t) @ ([h] @ l_2)$  by asso. of @

②  $\Rightarrow (\text{rev } t) @ (h :: l_2)$  by lemma

① = ② QED