

# An Efficient Real-Time Implementation of the Wigner-Ville Distribution

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**Abstract**—The Wigner-Ville distribution (WVD) is a valuable tool for time-frequency signal analysis. In order to implement the WVD in real time, an efficient algorithm and architecture have been developed which may be implemented with commercial components. This algorithm successively computes the analytic signal corresponding to the input signal, forms a weighted kernel function, and analyzes the kernel via a discrete Fourier transform (DFT). To evaluate the analytic signal required by the algorithm, it is shown that the time domain definition implemented as a finite impulse response (FIR) filter is practical and more efficient than the frequency domain definition of the analytic signal. The windowed resolution of the WVD in the frequency domain is shown to be similar to the resolution of a windowed Fourier transform.

A real-time signal processor has been designed for evaluation of the WVD analysis system. The system is easily paralleled and can be configured to meet a variety of frequency and time resolutions. The arithmetic unit is based on a pair of high-speed VLSI floating-point multiplier and adder chips.

## I. INTRODUCTION

THE magnitude squared of the Fourier transform (FT) is the classical method used to represent the frequency domain information or spectrum of a stationary signal. For a continuous time signal  $x(t)$ , the FT is defined as

$$X(f) = F(x(t)) = \int_{-\infty}^{\infty} x(t) \exp[-j2\pi ft] dt. \quad (1.1)$$

Clearly, the frequency domain information is defined over an infinite period of time. For nonstationary signals, the time variation of frequency information in  $x(t)$  will be obscured since the spectrum is  $S(f) = |X(f)|^2$ .

An alternative representation is the short time Fourier transform (STFT) which is evaluated by applying a suitable windowing function to the original signal and evaluating the conventional Fourier transform of the resulting finite length sequence. The STFT of the signal  $x(t)$  is given by

$$X_w(t, f) = \int_{t-T/2}^{t+T/2} x(\tau) w(\tau - t) \exp[-j2\pi f\tau] d\tau \quad (1.2)$$

Manuscript received December 23, 1985; revised May 14, 1987. This work was supported in part by the Australian Research Grant Scheme and the Australian Telecommunication and Electronics Research Board.

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IEEE Log Number 8716504

where  $w(\tau)$  is the windowing function, and satisfies  $w(\tau) = 0$  for  $|\tau| > T/2$ .

Using this technique, an approximation to the spectral content at the midpoint of the window interval can be achieved by computing  $S_w(t, f) = |X_w(t, f)|^2$ . It is assumed that the signal is stationary during the short time interval  $T$ . The time-frequency resolution of the STFT technique is inversely related to the window length. Increasing the window length increases the frequency resolution, while at the same time it reduces the frequency tracking capability of the representation.

In recent years, alternative time-frequency representations have been investigated. The conclusion of these investigations is that the Wigner-Ville distribution (WVD) is the most powerful and fundamental time-frequency representation [1]–[5].

The superior properties of the WVD over the STFT technique make it ideal for signal processing in such diverse fields as radar, sonar, speech, seismic, and biomedical analysis [6], [7]. All of these applications usually operate under real-time constraints with varying processing rates and resolutions. For these applications, there is a need for a flexible Wigner-Ville analyzer which can be used with a variety of real-time data rates and resolutions.

## II. THE WINDOWED WIGNER-VILLE DISTRIBUTION

Let  $x(t)$  be restricted to a complex continuous time analytic signal with Fourier transform  $X(f)$ . The WVD distribution is defined in the time domain as

$$W_x(t, f) = \int_{-\infty}^{\infty} x(t + \tau/2) x^*(t - \tau/2) \cdot \exp[-j2\pi f\tau] d\tau \quad (2.1)$$

where  $t$  is the time domain variable and  $f$  is the frequency domain variable.

In the frequency domain, the corresponding definition is

$$W_x(t, f) = \int_{-\infty}^{\infty} X(f + \xi/2) X^*(f - \xi/2) \cdot \exp[j2\pi t\xi] d\xi. \quad (2.2)$$

Both definitions are equivalent, however, the frequency domain definition requires knowledge of the Fourier transform of the signal and hence is computationally more complex, except for band-limited signals [8], [9]. On this

basis, the following discussion will be restricted to the time domain definition. The important properties of the WVD can be found in [3], [6], and [7].

Equation (2.1) implies that evaluation of the WVD is a noncausal operation, that is, the value of the signal for all time must be known before the WVD can be evaluated. Such an expression does not lend itself to real-time evaluation when only a finite delay can be tolerated. This limitation is overcome by applying the Wigner-Ville analysis to a windowed version of the signal. If the WVD is required at time  $t_a$ , the windowed signal is

$$x_w(t, t_a) = x(t) \cdot w(t - t_a) \quad (2.3)$$

where  $w(t)$  is the window function and satisfies  $w(t) = 0$  for  $|t| > Tw/2$ . The WVD of the windowed signal is given by [3]:

$$W_{xw}(t, f) = W_x(t, f)_f^* W_w(t, f). \quad (2.4)$$

The effect of windowing is to smear the WVD representation in the frequency direction only, hence, the frequency resolution can be increased by using a longer window without affecting the time resolution.

By evaluating  $W_x(t, f)$  at time  $t_a$ , a cross section of the WVD at time  $t_a$  can be approximated. With a change of variable from  $t_a$  to  $t$ , the windowed version of the WVD is

$$W_x(t, f) = \int_{-T_w}^{T_w} x(t + \tau/2) x^*(t - \tau/2) \cdot w(\tau/2) w^*(-\tau/2) \exp[-j2\pi f\tau] d\tau. \quad (2.5)$$

Using the shift invariance property, the WVD at any point in time can be evaluated by shifting the signal  $x(t)$  so that time  $t$  is mapped to the time origin. Therefore, the only form of (2.5) which must be evaluated is

$$W_x(0, f) = W_x(f) = \int_{-T_w}^{T_w} x(\tau/2) x^*(-\tau/2) \cdot w(\tau/2) w^*(-\tau/2) \exp[-j2\pi f\tau] d\tau. \quad (2.6)$$

From (2.6) a "frequency spectrum" of the WVD is evaluated about a point in time using

$$W_{xw}(0, f) = \int_{-T_w}^{T_w} \tilde{x}(\tau) \tilde{w}(\tau) \exp[-j2\pi f\tau] d\tau \quad (2.7)$$

where

$$\tilde{x}(\tau) = x(\tau/2) x^*(-\tau/2) \quad (2.8)$$

$$\tilde{w}(\tau) = w(\tau/2) w^*(-\tau/2). \quad (2.9)$$

From (2.7) it can be seen that the WVD is effectively the FT of a signal  $\tilde{x}(\tau)$  which has been windowed by  $\tilde{w}(\tau)$ . Even though the theoretical window function is  $w(\tau)$ , the effective window is  $\tilde{w}(\tau)$  and, hence, the resulting frequency resolution is directly related to the spec-

trum of  $\tilde{w}(\tau)$ . When a window is specified for the WVD it will be implied that this is the function  $\tilde{w}(\tau)$ .

By definition, the real part of  $\tilde{w}(\tau)$  is even and the imaginary part is odd, which is more concisely written as

$$\tilde{w}(\tau) = \tilde{w}^*(-\tau). \quad (2.10)$$

If  $\tilde{W}(f)$  is the FT of  $\tilde{w}(\tau)$ , then using the symmetry properties of the FT,

$$\begin{aligned} \tilde{W}(f) &= \text{FT}(\text{real}(\tilde{w}(\tau))) - \text{FT}(\text{imag}(\tilde{w}(\tau))) \\ &= (\text{even function in } f) - (\text{odd function in } f). \end{aligned} \quad (2.12)$$

Multiplication in the time domain is equivalent to convolution in the frequency domain, hence, symmetrical spectra for signals such as pure sinusoids will only result if  $\tilde{W}(f)$  is even. Therefore, the effective window must be a real function.

As long as  $\tilde{w}(\tau)$  is chosen to be real and symmetric, the theoretical window  $w(\tau)$  will always exist and can be calculated if necessary. Since (2.7) is of the same form as the STFT, the resulting frequency resolution will be proportional to that obtained using identical windows for the calculation of the STFT. Hence, the wealth of knowledge on window functions for the FT [10] may be directly applied to the windowed Wigner-Ville distribution.

### III. AN ALGORITHM FOR THE DISCRETE TIME WVD (DWVD)

The discrete time equivalent of (2.5) is

$$W_x(nT, f) = 2T \sum_{l=-L}^L x(nT + lT) x^*(nT - lT) \cdot w(l) w^*(-l) \exp[-j4\pi flT] \quad (3.1)$$

where  $T$  is the sampling period and  $w(lT) = 0$ , for  $|l| > L$ ;  $L \in \mathbb{Z}^+$ , positive integers.

Adopting the convention that the sampling period is normalized to unity, (3.1) can be written as

$$W_x(n, f) = 2 \sum_{l=-L}^L x(n + l) x^*(n - l) \cdot w(l) w^*(-l) \exp[-j4\pi fl]. \quad (3.2)$$

The properties of the DWVD are similar to the continuous time case except for the periodicity in the frequency variable. A complete summary of the properties can be found in [8], [11], and [12].

As for the continuous time case, it is necessary only to evaluate the DWVD at time zero. Hence, the DWVD becomes

$$W_x(0, f) = 2 \sum_{l=-L}^L K(l) \cdot \exp[-j4\pi fl] \quad (3.3)$$

where  $K(l) = x(l) x^*(-l) w(l) w^*(-l)$  and is called the DWVD kernel sequence. From (3.3) it can be shown that

$$W_x(n, f) = W_x(n, f + \frac{1}{2}). \quad (3.4)$$

Hence, the DWVD will be periodic in the frequency domain with period equal to one-half the sampling frequency.

If  $x(t)$  was a real signal, this would imply that to avoid aliasing, the sampling rate constraint should be [10]

$$f_s \geq 4B \quad (3.5)$$

where  $B$  is the bandwidth of the signal. In this case, we must transform the real signal into an analytic signal as follows [9].

The analytic signal  $z(n)$  corresponding to  $x(n)$  is defined in the time domain as

$$z(n) = x(n) + j \cdot H[x(n)] \quad (3.6)$$

where  $H[x(n)]$  represents the Hilbert transform of  $x(n)$ . Alternatively, the analytic signal can be defined in the frequency domain as

$$Z(f) = \begin{cases} 2X(f), & 0 < f < 1/2 \\ X(f), & f = 0 \\ 0 & -1/2 < f < 0. \end{cases} \quad (3.7)$$

If the real signal  $x(n)$  is replaced by the analytic signal  $z(n)$ , the periodicity is unchanged, however, the absence of a negative frequency spectrum eliminates the problem of aliasing which would otherwise occur if the data are sampled at the Nyquist rate [9].

Hence, if the analytic signal is used, the sampling rate constraint becomes

$$f_s \geq 2B \quad (3.8)$$

which is the well-known Nyquist rate. A detailed discussion can be found in [9].

To apply digital processing techniques to (3.3), the frequency variable must be sampled. The most convenient sampled form is

$$W_x(0, m\Delta f) = 2 \sum_{l=-L}^L K(l) W_4^{ml} \quad (3.9)$$

where  $\Delta f = \frac{1}{N} = \frac{1}{2L+2}$   
and  $W_4 = \exp[-j4\pi/N]$ .

Standard efficient discrete Fourier transform (DFT) algorithms require the time sequence to be indexed from 0 to  $N-1$ . This can be achieved by making a periodic extension of the kernel sequence [8]

The modified kernel sequence is defined as follows:

$$\begin{aligned} K(l), & \quad 0 \leq l \leq N/2 - 1 \\ K_n(l) = K(l - N), & \quad N/2 + 1 \leq l \leq N - 1 \\ 0, & \quad l = N/2 \end{aligned}$$

$$\text{where } N = 2L + 2. \quad (3.10)$$

The DWVD can now be written as

$$W_x(0, n\Delta f) = 2 \sum_{l=0}^{N-1} K(l) W_4^{nl}. \quad (3.11)$$

Equation (3.11) matches the standard form of a DFT except that the so-called twiddle factor is normally defined as  $W_2 = \exp[-j2\pi/N]$ . The additional power of 2 represents a scaling of the frequency axis by a factor of 2 and can be neglected in the calculations. Equation (3.11) can be evaluated efficiently using standard fast Fourier transform (FFT) algorithms.

#### Optimization of the Wigner-Ville Algorithm

The symmetry properties of the Wigner-Ville kernel sequence based on a real signal were used in [4] and [12] to increase the computational efficiency of the algorithm. This technique can also be applied to the complex kernel sequence generated from the analytic signal.

It has been shown that the window product  $w(l) \cdot w^*(-l)$  can be replaced by a single effective sequence  $\tilde{w}(l)$  which is real and symmetric. In the following derivation, the effective window has been neglected as it is a common factor throughout.

The symmetry properties of the kernel (3.10) can be summarized by

$$K(l) = K^*(-l). \quad (3.12)$$

Using the properties of the DFT [7]–[9], the DFT of the kernel will be real. If two successive kernel sequences  $K_1(l)$  and  $K_2(l)$  are combined as follows:

$$K_{\text{comb}}(l) = K_1(l) + jK_2(l), \quad (3.13)$$

then the DFT of the individual sequences can be evaluated using a single DFT as follows:

$$\begin{aligned} W_1(0, m\Delta f) &= \text{DFT}(K_1(l)) \\ &= \text{real}(\text{DFT}(K_{\text{comb}}(l))) \end{aligned} \quad (3.14)$$

$$\begin{aligned} W_2(0, m\Delta f) &= \text{DFT}(K_2(l)) \\ &= \text{imag}(\text{DFT}(K_{\text{comb}}(l))). \end{aligned} \quad (3.15)$$

Hence, by combining successive pairs of kernel sequences, the number of final FFT's will be halved.

The symmetry property of the kernel can also be used to reduce the number of computations required to evaluate  $K_{\text{comb}}(l)$ . Using (3.12), the combined kernel can be written as

$$\begin{aligned} K_{\text{comb}}(l) &= K_1(l) + jK_2(l) \\ K_{\text{comb}}(-l) &= K_1^*(l) + jK_2^*(l) \end{aligned} \quad (3.16)$$

$$\text{for } l = 0, 1, \dots, L. \quad (3.17)$$

Hence, because the symmetry property is preserved, the combined kernel sequence can be evaluated using only the positive time values of the original sequence, thus halving the number of complex multiplications required as indicated at the beginning of this section. The effective win-

dow will be applied to the combined sequence only once, again reducing the number of calculations. Note that no calculations are necessary to form the effective window, as its coefficients are stored.

The logical implementation of this algorithm in a real-time environment is to calculate two Wigner-Ville plots in a repeat loop. The flow diagram for the algorithm is shown in Fig. 1.

#### Evaluation of the Analytic Signal

Evaluation of the WVD using the analytic signal allows sampling at the Nyquist rate and eliminates the concentration of energy around the frequency origin due to cross products between negative and positive frequencies [9].

The easiest and most accurate method for calculating the analytic signal is to use the frequency domain definition [14]. From the definition, the analytic signal  $z(n)$  can be evaluated by calculating the FFT of the signal  $x(n)$ , then setting the negative frequency spectrum to zero and evaluating the inverse FFT. This technique is adequate when the whole signal is known, however, for real-time evaluation only a finite signal delay can be tolerated.

An alternative is to approximate the analytic signal over the region of interest by applying the frequency domain definition to a windowed version of the signal. Because there is no simple method of overlapping the individual segments of the analytic signal, this process must be repeated for each Wigner-Ville plot. The minimum number of samples that can be used is  $N$ , where  $N$  is the number of samples in the kernel sequence. The number of arithmetic operations required to calculate the analytic signal via the FFT algorithm per Wigner-Ville plot is

$$N \log_2 \text{ real multiplications/plot} \quad (3.18)$$

$$4N \log_2 \text{ real additions/plot.} \quad (3.19)$$

The other alternative is to use the time domain definition [9]:

$$z(n) = x(n) + j \cdot x(n) * h(n) \quad (3.20)$$

where  $h(n)$  is the impulse response of an ideal Hilbert transformer which is defined as

$$h(n) = \begin{cases} 2(\sin^2(\pi n/2))/\pi n, & \text{for } n \neq 0 \\ 0, & \text{for } n = 0 \end{cases} \quad (3.21)$$

and is plotted in Fig. 2.

The impulse response of the discrete Hilbert transform is noncausal and infinite in extent, however, by truncating the sequence to  $LF$  samples using a suitable window and introducing a delay of  $(LF - 1)/2$  samples,  $h(n)$  can be implemented using an FIR filter. The number of arithmetic operations per Wigner-Ville plot is related to the number of samples between plots ( $NS$ ) and is given by

$$(LF + 1)NS/2 \quad \text{real multiplications} \quad (3.22)$$

$$(LF - 1)NS/2 \quad \text{real additions.} \quad (3.23)$$

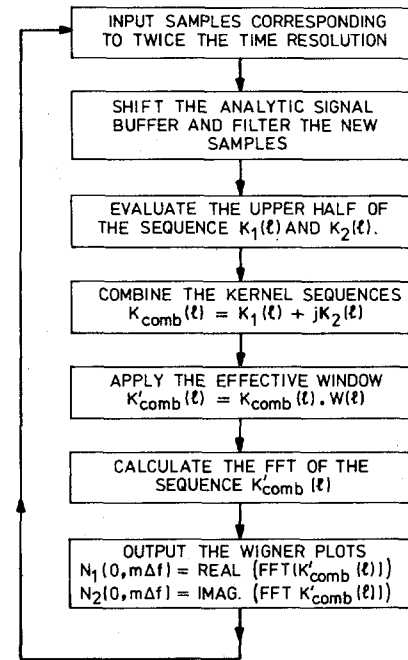


Fig. 1. Flow diagram of program to calculate the DWVD.

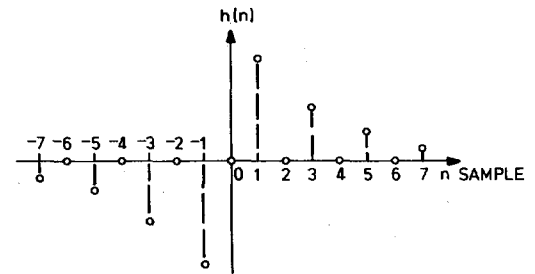


Fig. 2. Impulse response of an ideal Hilbert transformer.

The method chosen will depend on the values of the constants  $N$ ,  $LF$ , and  $NS$ . Using a filter length of 79 and a time resolution of 16 samples, the computational load of the two methods is given in Table I.

Evaluating the analytic signal using an FIR filter is computationally more efficient and technically easier to implement than the FFT-based frequency domain method.

#### IV. EVALUATION OF THE PROPOSED DWVD ALGORITHM

The FIR filter implementation of the Hilbert transform discussed previously involves windowing the ideal impulse response. An alternative design technique is the Remez-exchange algorithm which designs optimal filters that satisfy the so-called minimax error criterion [15].

Optimal FIR Hilbert transforms are characterized by a frequency response:

$$H(f) = \begin{cases} -j & f_L \leq f \leq f_H \\ j & (1 - f_H) \leq f \leq (1 - f_L) \end{cases} \quad (4.1)$$

where  $f_L$  and  $f_H$  are the lower and upper cutoff frequencies.

It can be shown [15] that if  $f_L = 0.5 - f_H$ , the frequency response will be symmetrical and every second filter coef-

TABLE I  
CALCULATIONS REQUIRED TO EVALUATE THE ANALYTIC SIGNAL

Method	Real Operations per Wigner-Ville Plot	
	Multiplications	Additions
FFT, $N = 128$	896	3584
FFT, $N = 256$	2048	8192
FIR filter	640	640

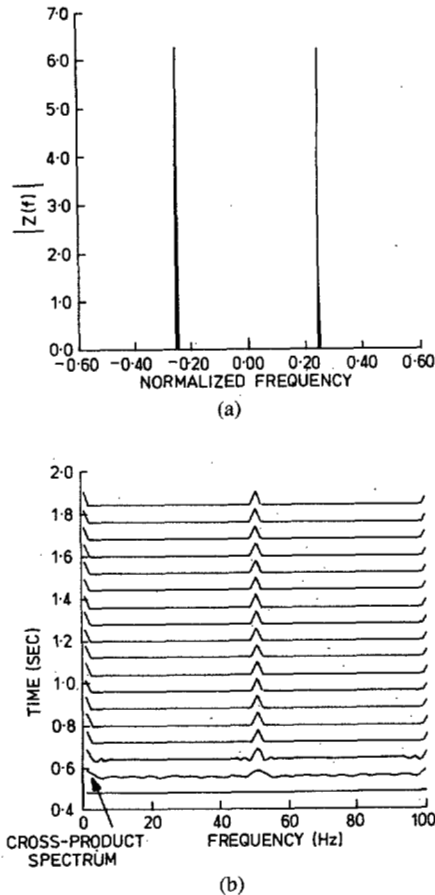


Fig. 3. (a) Spectrum of a real sinusoidal signal at 50 Hz sampled at 200 Hz. (b) DWVD of the real signal as in (a).

ficient will be zero. The filter coefficients were generated using the routines given in [15].

Examples of the DWVD of a pure sinusoid based on filters of various lengths show that as the filter length increases the concentration of unwanted energy about the frequency origin decreases (see Figs. 3–5).

Using the conventional frequency spectrum, the concentration of negative frequency energy for various filter lengths and signals has been determined (see Table II).

The results indicate that the performance of the filter is dependent on the filter length as well as the signal type. A filter of length 79 should be adequate for most applications.

Due to the periodicity of the Wigner-Ville distribution, the  $1/2$  point on the frequency axis also corresponds to the zero frequency of the next periodic extension. The cross-product terms generate both positive and negative

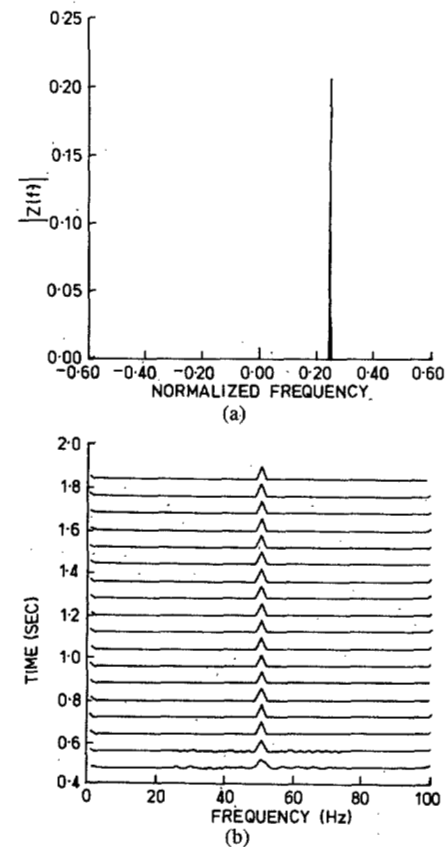


Fig. 4. (a) Spectrum of the analytic signal corresponding to a sinusoid at 50 Hz sampled at 200 Hz, evaluated using an FIR filter length of 39. (b) DWVD of the analytic signal as in (a).

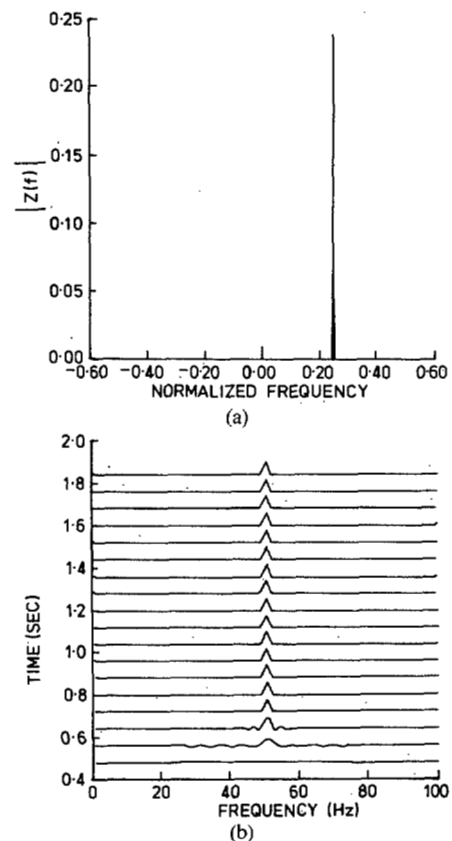


Fig. 5. (a) Spectrum of the analytic signal corresponding to a sinusoid at 50 Hz sampled at 200 Hz, evaluated using an FIR filter length of 79. (b) DWVD of the analytic signal as in (a).

TABLE II  
PERCENTAGE OF NEGATIVE FREQUENCY ENERGY

Signal Type	FIR Filter Length				$\lambda$
	39	47	59	67	
Pure Sinusoid	0.96	0.48	0.16	0.09	0.04
Linear FM	0.40	0.21	0.08	0.05	0.02
Sinusoidal FM	0.39	0.20	0.08	0.04	0.02
Hyperbolic FM	0.40	0.21	0.08	0.06	0.02

frequency components, hence, a poor approximation to the analytic signal will produce energy about the conventional frequency origin and just below the  $1/2$  point. Aliasing occurs about the frequency origin only, hence, a concentration of energy below this point will imply a longer filter should be used.

#### DWVD Algorithm

To test the validity of the proposed algorithm, the resulting Wigner-Ville plots were compared to those generated by a proven program written by Boashash [3]. Fig. 6 shows the comparison for a sinusoidal FM signal.

#### V. SYSTEM PERFORMANCE

The most important factor in designing any real-time signal processor is the computational load. With respect to evaluating the WVD, the following four parameters completely determine the load:

- 1) the number of samples between Wigner-Ville plots which defines the time resolution;
- 2) the length of the final FFT which determines the frequency resolution;
- 3) the length of the FIR filter; and
- 4) the type of arithmetic used; namely, fixed point or floating point.

The upper limit on the time resolution is one sample, however, in typical applications such as speech recognition, a resolution of 16–64 samples would be adequate. To design a system which will meet all possible time resolutions would be inefficient and expensive because, for the majority of applications, the system would be producing redundant information.

A solution to this problem is to design a highly efficient Wigner-Ville analyzer which takes as input the real signal and outputs the Wigner-Ville distribution, with a time resolution of, for example, 16 samples. To increase system throughput, identical systems can be paralleled as shown in Fig. 7. All analyzers run concurrently so that a common control can be used. By adding different delays to the input signals, each analyzer will produce a different set of spectrums at spacing of 16 samples.

The required frequency resolution will be determined by the application [10]. Since the design is for a general purpose Wigner-Ville analyzer, a resolution suitable for most applications must be chosen, with provision for an increase in resolution when necessary. With respect to the STFT, a typical FFT length is 256 points, which implies

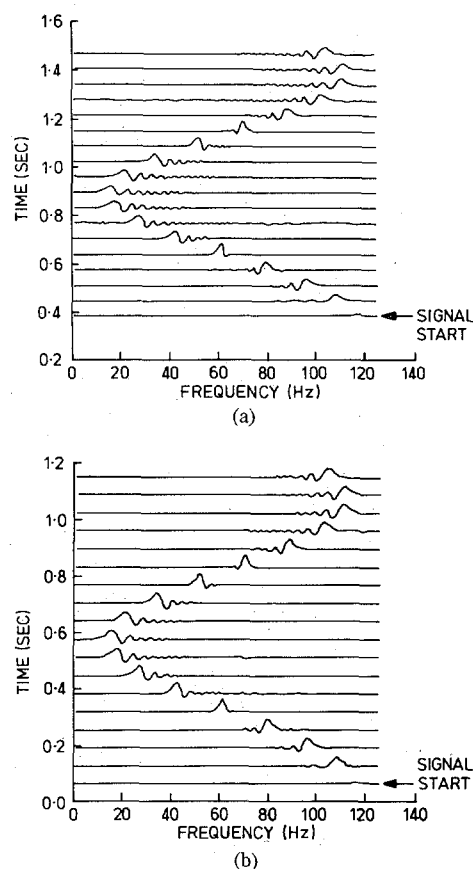


Fig. 6. DWVD of a sinusoidal FM signal computed using the algorithm shown in Fig. 1. (b) DWVD of the same signal as in (a) using a proven Boashash program.

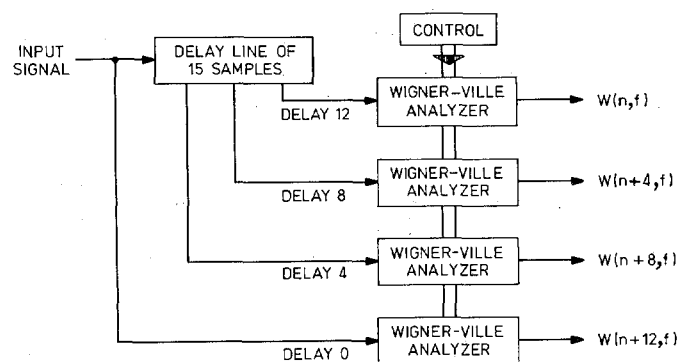


Fig. 7. Example of paralleling a 16 sample time resolution analyzer to yield a time resolution of 4 samples.

a frequency resolution of 128 points. The WVD utilizes all of the spectrum, thus, a comparable Wigner-Ville analyzer requires only a 128 point FFT.

As discussed previously, an FIR filter of length 79 appears adequate, but again provision must be made for an increase in filter length.

As an indication of system performance, the variation in speed as a function of frequency resolution, time resolution, and filter length is given in Fig. 8. The speed is normalized to that of an analyzer which has the following parameters:

- 1) frequency resolution = 128;

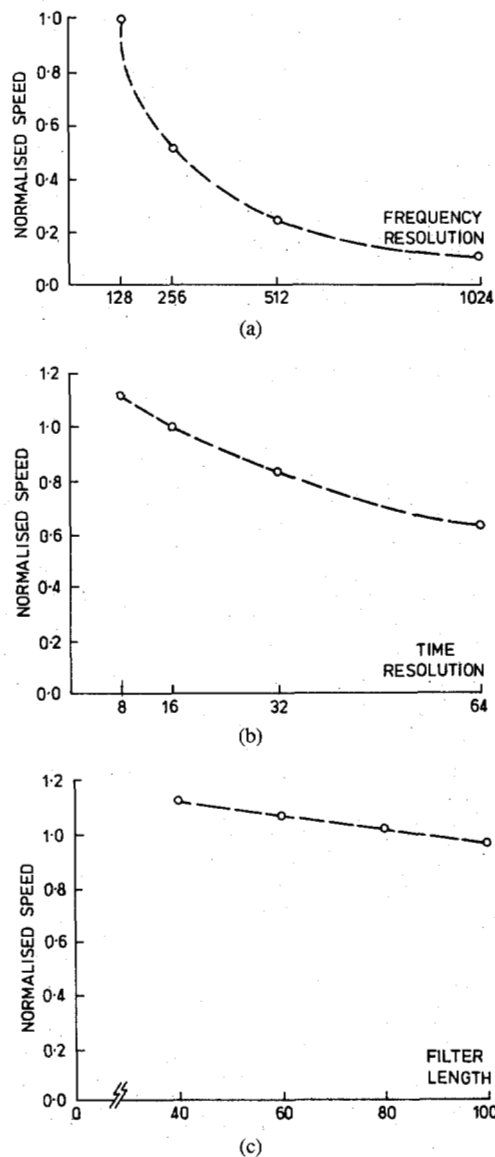


Fig. 8. (a) Performance of the WVD analyzer: speed versus frequency resolution, (b) performance of the WVD analyzer: speed versus time resolution, (c) performance of the WVD analyzer: speed versus filter length.

- 2) time resolution = 16;
- 3) filter length = 79; and
- 4) number of microinstructions = 6530.

Clearly, the dominant factor is the frequency resolution.

The number of Wigner-Ville analyzers which must be paralleled is given by the following:

Number of analyzers is:

$$NA = \text{INT}(NI * T_{\min} * f_s / 2 / TR + 1) \quad (6.1)$$

The time resolution of each analyzer is given by the following:

$$\text{Analyzer time resolution (ATR)} = TR * NA \quad (6.2)$$

where

NI = number of microinstructions  
TR = time resolution required

TABLE III  
MAXIMUM SAMPLING FREQUENCY IN KILOHERTZ WHICH CAN BE HANDLED BY (a) A DWVD SINGLE ANALYZER AND (b) 10 PARALLEL ANALYZERS (FIR FILTER LENGTH: 79)

TR	Frequency Resolution		
	128	256	512
1	1.5	0.7	0.3
16	19.6	9.8	4.9
32	31.3	15.6	7.8

TR	Frequency Resolution		
	128	256	512
1	15.3	7.6	3.8
16	196.0	98.0	49.0
32	313.7	156.8	78.4

$T_{\min}$  = minimum system clock period

$f_s$  = sampling frequency.

The maximum sampling frequency which can be handled in real time by a single analyzer and 10 devices running in parallel is summarized in Table III.

## VI. CONCLUSION

The Wigner-Ville distribution of a real signal contains an unwanted cross-product term about the frequency origin. These terms can be eliminated by replacing the real signal with the analytic equivalent. It was shown that evaluation of the analytic signal using the time domain definition involving the Hilbert transform is the most efficient algorithm for continuously processing the signal.

The Hilbert transform was implemented as an FIR filter and designed using the Remez-exchange algorithm. The results indicated that a filter of length 79 samples would be adequate for most applications.

To implement the Wigner-Ville distribution in real time, an efficient algorithm was developed which exploits the symmetry properties of the Wigner-Ville kernel sequence. It was shown that two complex kernel sequences can be combined so that the real part of the DFT of the combined sequence is the DFT of the first sequence and the imaginary part is the DFT of the second. The combined DWVD kernel sequence can be evaluated using only the positive time values of the original sequences. By applying the effective window function to the combined sequence, the number of window calculations is also halved. Overall, these modifications result in a 50 percent reduction in the number of calculations as compared to direct evaluation of the Wigner-Ville distribution. It was concluded that to produce symmetrical spectrums for signals such as pure sinusoids, the effective window must be a real and even function.

To evaluate the Wigner-Ville distribution in real time for a variety of frequency and time resolutions, it was

concluded that a microprogrammed system which could be easily paralleled would be the best solution.

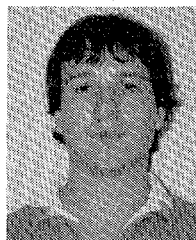
# REFERENCES

- [1] J. Ville, "Theories et application de la notion de signal analytique," *Cables et Transmission*, vol. A(1), pp. 61-74, 1948.
- [2] B. Bouachache and F. Rodriguez, "Recognition of time-varying signals in the time-frequency domain by means of the Wigner distribution," in *IEEE ICASSP 84 Proc.*, San Diego, CA, 1984, pp. 22.5.1-22.5.4.
- [3] B. Bouachache, "Sur la Possibilite d'Utilizer la Representation Conjointe en Temps et Frequence de Ville aux Signaux Vibrosismiques," presented at the Colloque Nat Trait Sign, Grets 121/1, 121/6, Nice, France, 1979.
- [4] T. A. C. M. Claasen and W. F. G. Mecklenbrauker, "The Wigner distribution—A tool for time frequency signal analysis, Part II: Discrete time signals," *Philips J. Res.*, vol. 35, no. 4/5, pp. 276-300, 1980.
- [5] B. Bouachache and P. Flandrin, "Wigner-Ville analysis of time-varying signals," in *Proc. ICASSP 82*, Paris, France, May 1982, pp. 1329-1332.
- [6] B. Bouachache, "Wigner analysis of time-varying signals—An application in seismic prospecting," in *Signal Processing II: Theories and Appl.*, *EUSIPCO 83*, 1983, pp. 703-706. B. Boashash and B. Escudie, "Wigner-Ville analysis of asymptotic signals," *Signal Processing*, vol. 8, no. 3, pp. 315-323, 1985. B. Boashash and H. J. Whitehouse, "Seismic applications of the Wigner-Ville distribution," in *1986 Int. Symp. Circuit and Syst. Proc.*, San Jose, CA, May 1986, pp. 34-37.
- [7] L. Cohen, "A critical review of the fundamental ideas of joint time-frequency distributions," in *1986 Int. Symp. Circuit and Syst. Proc.*, San Jose, CA, May 1986, pp. 28-33.
- [8] D. Chester, "Discrete Wigner implementations," in *1986 Int. Symp. Circuit and Syst. Proc.*, San Jose, CA, May 1986, pp. 38-41.
- [9] B. Boashash, "On the anti-aliasing and computational properties of the Wigner-Ville distribution," in *IASTED Int. Symp. Signal Processing*, Paris, France, M. H. Hamza, Ed. Anaheim, CA: Acta, June 1985, pp. 290-293.
- [10] J. Harris, "On the use of windows for harmonic analysis with the DFT," *Proc. IEEE*, vol. 66, pp. 51-83, 1978.
- [11] T. A. C. M. Claasen and W. F. G. Mecklenbrauker, "The aliasing problem in discrete-time Wigner distributions," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-31, pp. 1067-1072, Oct. 1983.
- [12] W. Martin and P. Flandrin, "Wigner-Ville analysis of nonstationary processes," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-33, pp. 1461-1470, Dec. 1985.
- [13] B. Bouachache, "Représentations temps-frequence," These de Docteur-Ingenieur, Univ. Grenoble, INPG, Grenoble, France, 1982.
- [14] C. D. McGillem and G. R. Cooper, *Continuous and Discrete Signal and System Analysis*. New York: Holt, Rinehart, Winston, 1984.
- [15] L. R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975.



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