Blind Hopping Phase Estimator in Frequency-Hopped FM and BFSK Systems

Myungsup Kim, Jinsuk Seong, and Seong-Ro Lee

A blind hopping phase estimator is proposed for the demodulation of received signals in frequency-hopping spread spectrum systems. The received signals are assumed to be bandwidth limited with a shaping filter, modulated as frequency modulation (FM) or binary frequency shift keying (BFSK), and hopped by predetermined random frequency sequences. In the demodulation procedure in this paper, the hopping frequency tracking is accomplished by choosing a frequency component with maximum amplitude after taking a discrete Fourier transform, and the hopping phase estimator performs the conjugated product of two consecutive signals and moving-average filtering. The probability density function and Cramer-Rao low bound (CRLB) of the proposed estimator are evaluated. The proposed scheme not only is very simple to implement but also performs close to the CRLB in demodulating hopped FM/BFSK signals.

Keywords: Frequency, hopping, spread, spectrum, phase, estimator, blind, demodulator.

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I. Introduction

Transmission technologies to transmit a signal safely, and against jamming signals, by choosing a robust modulation scheme over a channel have been considered in many studies. The frequency modulation (FM) and binary frequency shift keying (BFSK) are known to be very robust to noise with high magnitude since they produce constant enveloped signals. On the other hand, they have the drawback that they require more bandwidth compared with amplitude modulation and phase modulation.

In the meantime, since frequency hopping (FH) technology has high security and strong anti-jamming ability, it has been adopted for commercial and military means for radio communications [1]. Demodulation of frequency-hopping spread spectrum (FHSS) signals is accomplished through signal detection and separation [2]–[3]; parameter estimation — such as hop timing and hopping frequency [4]–[6]; and de-hopping and baseband demodulation. Since these schemes are general approaches toward demodulating FHSS signals, they have high complexity in their implementation; thus, their performances are not usually close to being optimal.

In this paper, we obtain a simple, yet near-optimally performing, demodulation scheme for frequency modulation/frequency hopping (FM FH)/BFSK signals. Specifically, we propose a hopping phase estimator for FM FH/BFSK systems that does not require any information on the hopping frequency when the symbol duration and shaping pulse are known; hence, it is a *blind* hopping phase estimator. We also suggest a tone filter to improve the signal-to-noise ratio (SNR) of the received signal. We transform the hopped signal into instant hopping phases, establish a hopping phase estimator, evaluate the Cramer-Rao low bound (CRLB) performance of the estimator, and then compare it with the result obtained by simulation.

The rest of this paper is organized as follows. In Section II, we introduce a transmitter model and its functions. In Section III, we derive a receiver model and demodulation scheme. In Section IV, the performance of the proposed scheme is analyzed and shown to be efficient.

II. Transmitter

The transmit signal is depicted in Fig. 1. A transmit burst consists of a dwell time in which the information-bearing signal is transmitted and a blanking interval — a pause time. The FM FH signal may be represented as

$$q(t) = A_c \cos\left(2\pi(f_c + f_i)t + 2\pi f_\Delta \int_0^t x_m(\tau)\,d\tau\right) \quad i = 0, 1, 2, \ldots, N-1, \tag{1}$$

where N is the number of hopping frequencies, A_c is the amplitude, f_c is the carrier frequency, f_i is the hopping frequency, f_Δ is the frequency deviation for FM, and

$$x_m(t) = \sum_{k=0}^{\infty} d_k g(t - kT) \tag{2}$$

is the information-bearing signal with $d_k \in \{-1, 1\}$, and

$$g(t) = \begin{cases} 1 & -\frac{T}{2} \le t \le \frac{T}{2}, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

a shaping pulse. Defining

$$x(t) = 2\pi f_\Delta x_m(t), \tag{4}$$

$\omega_c = 2\pi f_c$, and $\omega_i = 2\pi f_i$, the signal model (1) can be represented compactly as

$$q(t) = A_c \cos\left((\omega_c + \omega_i)t + \int_0^t x(\tau)\,d\tau\right) \quad i = 0, 1, 2, \ldots, N-1. \tag{5}$$

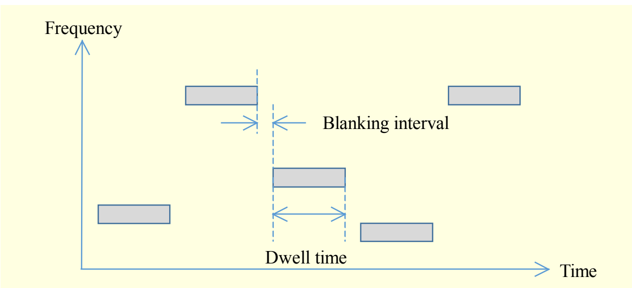

Fig. 1. FHSS signal.

III. Demodulation

The hopping FM signals have very low SNR since they have wide frequency bandwidth compared with their own bandwidth in the baseband, as shown Fig. 2. To demodulate the spread spectrum (SS) signal, the signal contaminated by noise should be converted to a high-SNR baseband signal. The FM-hopping SS signal, which is generated using thousands of hopping carriers, can be regarded as a tone during this short time interval, since the information-bearing signal bandwidth is very narrow compared with that of the hopping bandwidth.

To mitigate the noise included in the wideband signal, we can use a DFT with the samples obtained from the short time interval. After sampling the wideband signal at a sufficiently high rate compared with the baseband symbol transmission speed, we use the DFT of the obtained signal and take a sample of signals having the largest amplitudes among the DFT samples. We then reduce the sampling rate by taking a sample for every DFT size. Thus, we obtain a down-sampled signal; for example, 1,024. These procedures are depicted in Fig. 3.

Dropping the ω_i term in (5) and Hilbert transforming it, we have

$$w(t) = A_c \exp\left(j\omega_c t + j\int_0^t x(\tau)\,d\tau\right). \tag{6}$$

This signal can be separated as a product of two terms as

$$w(t) = c(t)s(t), \tag{7}$$

where

$$c(t) = \exp(j\omega_c t) \tag{8a}$$

is the complex carrier and

$$s(t) = A_c \exp\left(j\int_0^t x(\tau)\,d\tau\right) \tag{8b}$$

is the information-bearing signal. When we sample the signal $w(t)$ at the sampling frequency $f_s = 1/T_s$, we obtain

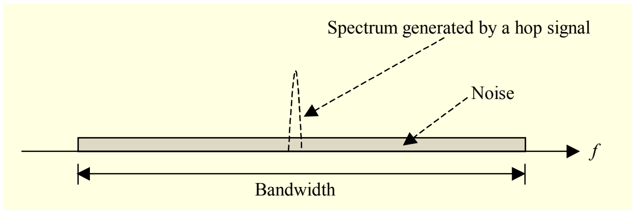

Fig. 2. Hopping signal and noise.

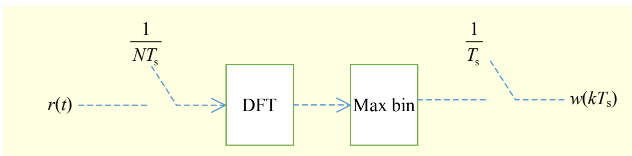

Fig. 3. Noise reduction and conversion to baseband signal.

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$$w(kT_s) = c(kT_s)s(kT_s). \quad (9)$$

The spectrum of $w(kT_s)$, $W(e^{j\omega})$, can be represented as the convolution of two frequency-domain signals $C(e^{j\omega})$ and $S(e^{j\omega})$ as

$$W(e^{j\omega}) = C(e^{j\omega}) \otimes S(e^{j\omega}). \quad (10)$$

We know that the sampled FM baseband signal is modulated by a sampled carrier frequency. On the other hand, since the frequency domain representation of a complex carrier is

$$C(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_c - 2\pi k), \quad (11)$$

the sampled FM signal becomes

$$\begin{aligned} W(e^{j\omega}) &= 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_c - 2\pi k) \otimes S(e^{j\omega}) \\ &= 2\pi \sum_{k=-\infty}^{\infty} S\left(e^{j(\omega-\omega_c-2\pi k)}\right). \end{aligned} \quad (12)$$

Thus, we know that the sampled FM signal is shifted by ω_c and is modulated by a sampled carrier. In general, a complex carrier can be represented as

$$\rho(t) = \exp\left(j\left(2\pi f_c t + \theta_0\right)\right), \quad (13)$$

where θ_0 is the initial phase in the interval $[0, 2\pi)$.

As shown in Fig. 4, from the product of two samples taken at times k–1 and k, we have a phase

$$\begin{aligned} \theta &= \arg\left(\rho(t_k)\rho^*(t_{k-1})\right) = \arg\left\{\exp\left[j2\pi f_c\left(t_k - t_{k-1}\right)\right]\right\} \\ &= \arg\left[\exp\left(j2\pi \frac{f_c}{f_s}\right)\right] = \left(2\pi \frac{f_c}{f_s}\right) \bmod 2\pi, \\ &\qquad -\pi \le \theta \le \pi, \end{aligned} \quad (14)$$

where $t_k - t_{k-1} = T_s = 1/f_s$.

From this phase, we have a frequency as

$$h(f) = \frac{\theta}{2\pi} f_s = \frac{f_s}{2}\left(\left(2\frac{f}{f_s}\right) \bmod 2\right), \quad (15)$$

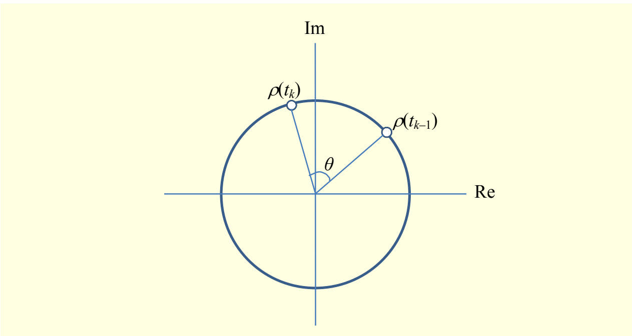

Fig. 4. Sampling of a complex carrier.

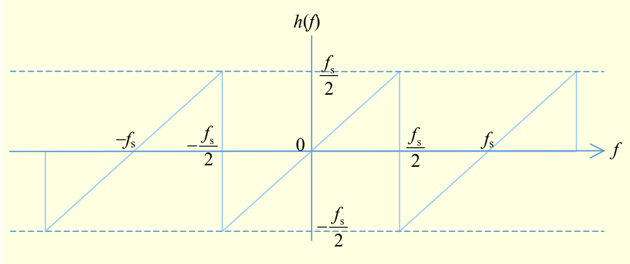

Fig. 5. Frequency characteristics of a sampled complex carrier.

which is depicted in Fig. 5. From Fig. 5, since the sample frequency range is $-f_s/2 \le h(f) \le f_s/2$, when the bandwidth of the message signal is within f_s, the spectrum range of the sampled SS is confined as

$$-\frac{f_s}{2} \le \mathrm{BW}_W \le \frac{f_s}{2}, \quad (16)$$

where BW_W denotes the bandwidth of $W(f)$. Therefore, we know that as long as we sample the SS signal at more than twice the baseband signal bandwidth, there is no information loss and that it will appear as a modulated signal to a sample carrier.

Without loss of generality and for mathematical simplicity, we let $\omega_c = 0$ and $A_c = 1$ in (5); thus, we have

$$w(t) = \exp\left(j\omega_i t + j\int_0^t x(\tau)\,\mathrm{d}\tau\right). \quad (17)$$

We define a product of two functions with a positive value Δt as

$$b(t, \Delta t) \equiv w(t)w^*(t - \Delta t) = c_i(\Delta t)s(t, \Delta t), \quad (18)$$

where

$$c_i(\Delta t) = \exp(j\omega_i \Delta t), \quad (19a)$$

$$s(t, \Delta t) = \exp\left[j\left(\int_{t-\Delta t}^{t} x(\tau)\,\mathrm{d}\tau\right)\right]. \quad (19b)$$

In the function $b(t, \Delta t)$, $c_i(\Delta t)$ is a phasor that has a phase consisting of a hopping frequency ω_i and a time offset Δt, and it appears as a point in the complex plane. The phase in the phasor increases as the hopping frequency increases, and vice versa. The term $s(t, \Delta t)$ in (19b) includes information in the form of a complex exponential function, and the integrand for small Δt can be represented as

$$\int_{t-\Delta t}^{t} x(\tau)\,\mathrm{d}\tau \cong x(t)\Delta t. \quad (20)$$

Substituting (20) with (19) yields

$$b(t, \Delta t) = c_i(\Delta t)\exp\left(jx(t)\Delta t\right). \quad (21)$$

We define a hopping phase estimator to estimate the phase caused by a hopping frequency as

$$\hat{\Phi}_i(\Delta t) \equiv c_i(\Delta t)\frac{1}{N}\sum_{k=0}^{N-1}\exp\left(jx\left(t+kT\right)\Delta t\right). \qquad (22)$$

The term $x(t + kT)$ in the summation term has $2\pi f_\Delta$ when $d_k = 1$, but it has $-2\pi f_\Delta$ when $d_k = -1$. As shown in Fig. 1, if the dwell time $L(=NT)$ is sufficiently long, since the probability that $d_k = 1$ and the probability that $d_k = -1$ are the same, then the summation term in (22) can be written as

$$\begin{aligned}\gamma &\equiv \frac{1}{N}\sum_{k=0}^{N-1}\exp\left(jx\left(t+kT\right)\Delta t\right)\\ &\cong \Pr\{d_k=1\}\cdot\exp\left(j2\pi f_\Delta\Delta t\right)+\Pr\{d_k=-1\}\cdot\exp\left(-j2\pi f_\Delta\Delta t\right)\\ &\cong \frac{1}{2}\cdot\exp\left(j2\pi f_\Delta\Delta t\right)+\frac{1}{2}\cdot\exp\left(-j2\pi f_\Delta\Delta t\right)\\ &=\cos\left(2\pi f_\Delta\Delta t\right). \end{aligned} \qquad (23)$$

Substituting the average term in (22) with (23) yields

$$\hat{\Phi}_i(\Delta t) = \gamma\, c_i(\Delta t). \qquad (24)$$

This hopping phase estimator does not have any hopping phase term. With this estimator and $b(t, \Delta t)$, we define a complex conjugate product as

$$\begin{aligned}\hat{\Phi}_i^*(\Delta t)b(t,\Delta t) &\equiv \gamma\, c_i^*(\Delta t)c_i(\Delta t)\exp\left(jx(t)\Delta t\right)\\ &= \gamma\exp\left(jx(t)\Delta t\right).\end{aligned} \qquad (25)$$

The right term is irrelevant to index i in $\hat{\Phi}_i^*(\Delta t)$; thus, we can drop it and obtain the information

$$x(t) = \frac{1}{\Delta t}\arg\left(b(t,\Delta t)\hat{\Phi}^*(\Delta t)\right). \qquad (26)$$

From this equation, we know that the frequency-hopping SS signal can be demodulated using the two functions $b(t, \Delta t)$ and $\hat{\Phi}(\Delta t)$ without any hopping information.

On the other hand, in the case of including a carrier frequency, (18) can be written as

$$w(t) = \exp\left(j\left(\omega_c+\omega_i\right)t + j\int_0^t x(\tau)\,\mathrm{d}\tau\right). \qquad (27)$$

Through the same procedure as above, we can obtain the total hopping phase estimator

$$\hat{\hat{\Phi}}(\Delta t) = \exp\left(j\omega_c\Delta t\right)\hat{\Phi}(\Delta t). \qquad (28)$$

Since the total hopping phase estimator includes the term $\exp(j\omega_c\Delta t)$, which is generated by a carrier frequency, and $\hat{\Phi}(\Delta t)$, which is a hopping phase estimator, it can estimate the total phase generated by a carrier and a hopping frequency. For BFSK systems, developing the above procedure, we have a hopping phase estimator from (see Appendix, A-9) as

$$\hat{\Phi}_i(\Delta t) \cong A_c^2\exp\left(j\left(\omega_c+\omega_i\right)\Delta t\right)\cos\left(2\pi\Delta f\Delta t\right). \qquad (29)$$

Since A_c^2 and $\cos(2\pi\Delta f\Delta t)$ are real and the phase of $\hat{\Phi}_i(\Delta t)$ is generated by a carrier and a hopping frequency, the phase estimator represented by (29) shows that it can estimate the phase generated by any carrier and any hopping frequency. Therefore, we can demodulate a BFSK-type SS signal in the same way that we can an FM-type signal, by applying the same method.

IV. Performance Analysis

1. Performance of Tone Filter

Let an input signal be $r(k)$, and let us take the DFT over the signal as

$$R(k) = \frac{1}{M}\sum_{l=0}^{M-1} r(l)\exp\left(-j\frac{2\pi kl}{M}\right), \quad k = 0, 1, 2, \ldots, M-1, \qquad (30)$$

where M is the size of the DFT. Then assume that the input signal has only a single frequency component

$$r(k) = \exp\left(j\frac{2\pi mk}{M}\right) + \nu(k), \qquad (31)$$

where $\nu(k)$ is a noise with zero mean and variance σ_ν^2. Inserting (31) into (30), we have

$$R(k) = \delta(k-m) + V(k), \qquad (32)$$

where

$$\delta(k-m) = \begin{cases}1 & k = m,\\ 0 & k \neq m,\end{cases} \qquad (33)$$

and $V(k)$ is the kth frequency component among the DFT outputs of $\nu(k)$. This means that they are all noise terms except when $k = m$ for $V(k)$. We need only the term containing the frequency component, so we take a term with maximum amplitude as

$$w = \max_k R(k). \qquad (34)$$

This proposed scheme is very simple to mitigate the noise effect on the signal for a wideband signal, such as in the case of frequency-hopping SS signals whose baseband signals have constant envelope. The DFT of an FM or FSK signal in a short time interval has a high amplitude component and many small amplitude components contaminated by noise.

Taking a sample with maximum amplitude from the DFT results is equivalent to taking a sample with maximum amplitude from M narrow bandpass filters. Therefore, taking the DFT is regarded as separating the input signal into signal and noise. The relation between input and output with respect to SNR can be written as

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$$\mathrm{SNR}_{\mathrm{out}} = M \cdot \mathrm{SNR}_{\mathrm{in}}. \tag{35}$$

This shows that M, of DFT size, should be set up to be sufficiently big to obtain high gain. Therefore, for FM- and FSK-type SS signals, we can obtain high gain by taking the DFT of the incoming signal contaminated by noise and then taking a frequency component comprising high-amplitude signals.

2. Performance of Hopping Frequency Estimator

In the analysis, since the carrier and the hopping frequency do not affect the performance of a hopping phase estimator, we drop their terms so that $\omega_c = 0$ and $\omega_i = 0$, and add a noise term as

$$q(t) = \exp\left(j\int_0^t x(\tau)\,\mathrm{d}\tau\right) + n(t), \tag{36}$$

where $n(t)$ is the additive white Gaussian noise term with mean 0 and variance σ_n^2. We define a random variable $\Gamma(k)$ as

$$\begin{aligned}\Gamma(k) &\equiv q(t+kT)\,q^*(t+kT-\Delta t)\\ &= \sum_{i=1}^{4} S_i,\end{aligned} \tag{37}$$

where

$$S_1(k) = \exp\left(j\int_{t+kT-\Delta t}^{t+kT} x(\tau)\,\mathrm{d}\tau\right), \tag{38a}$$

$$S_2(k) = \exp\left(j\int_0^{t+kT} x(\tau)\,\mathrm{d}\tau\right) n^*(t+kT-\Delta t), \tag{38b}$$

$$S_3(k) = n(t+kT)\exp\left(-j\int_0^{t+kT-\Delta t} x(\tau)\,\mathrm{d}\tau\right), \tag{38c}$$

and

$$S_4(k) = n(t+kT)\,n^*(t+kT-\Delta t). \tag{38d}$$

From (B-7) in the Appendix, the mean of $\Gamma(k)$ is

$$m_\Gamma \cong \gamma, \tag{39}$$

and it is independent of k. We define a hopping phase estimator as

$$\hat{\Phi}(k) \equiv \frac{1}{N}\sum_{k=0}^{N-1}\Gamma(k). \tag{40}$$

Investigating (37) and (38), we can see that $\Gamma(k)$ and $\Gamma(l)$ for $k \neq l$ are independent, because they are taken at different times from each other. By the central limit theorem, we can regard $\hat{\Phi}(k)$ as having a Gaussian distribution, so the mean of $\hat{\Phi}(k)$ is

$$m_\Phi = \mathrm{E}\left[\hat{\Phi}(k)\right] \equiv \frac{1}{N}\sum_{k=0}^{N-1}\mathrm{E}\left[\Gamma(k)\right] \cong \gamma, \tag{41}$$

and the second moment is

$$\begin{aligned}m_{2,\Phi} &= \mathrm{E}\left[\hat{\Phi}(k)\hat{\Phi}^*(k)\right]\\ &= \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1}\mathrm{E}\left[\Gamma(k)\Gamma^*(l)\right].\end{aligned} \tag{42}$$

Substituting the expectation term in (42) with (B-17) of the Appendix yields

$$m_{2,\Phi} = \frac{1+2\sigma_n^2+\sigma_n^4}{N} + \gamma^2\left(1-\frac{1}{N}\right). \tag{43}$$

With (41) and (43), the standard deviation becomes

$$\sigma_\Phi = \sqrt{\frac{1}{N}\left[1-\left(\gamma^2+\frac{2}{\mathrm{SNR}}+\frac{1}{\mathrm{SNR}^2}\right)\right]}, \tag{44}$$

where $\mathrm{SNR} = 1/\sigma_n^2$. It is clear that a random variable $\hat{\Phi}(k)$ has a real part that has mean γ and standard deviation $\sigma = \sigma_\Phi/\sqrt{2}$ and an imaginary part that has mean zero and standard deviation $\sigma = \sigma_\Phi/\sqrt{2}$.

We define a random variable for the phase of a hopping phase estimator, to analyze the performance, as

$$\hat{\Theta}(k) \equiv \arg\left(\hat{\Phi}(k)\right) = \arg\left(\frac{1}{N}\sum_{k=0}^{N-1}\Gamma(k)\right). \tag{45}$$

We can obtain the PDF of $\hat{\Theta}(k)$ through converting from Cartesain coordinates to polar coordinates as

$$\begin{aligned}f_{\hat{\Theta}}(\varphi) &= \left(\frac{1}{\pi}+\frac{1}{\sqrt{\pi}}\frac{\gamma\cos\varphi}{\sigma_\Phi}\,\mathrm{erf}\left(\frac{\gamma\cos\varphi}{\sigma_\Phi}\right)\exp\left(\frac{\gamma^2\cos^2\varphi}{\sigma_\Phi^2}\right)\right)\\ &\quad\times\exp\left(-\frac{\gamma^2}{\sigma_\Phi^2}\right) \qquad -\pi/2 \le \varphi \le \pi/2,\end{aligned} \tag{46}$$

where $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,\mathrm{d}t.$

The function $f_{\hat{\Theta}}(\phi)$ consists of two terms. The first term is independent of ϕ, but the second term is dependent on ϕ. The first term is a function of γ and σ_Φ, and diminishes as SNR increases and disappears as SNR tends toward infinity. The second term is dependent on ϕ, and as γ increases, the width of $f_{\hat{\Theta}}(\phi)$ is narrower. As the standard deviation σ_Φ increases, the width of $f_{\hat{\Theta}}(\phi)$ becomes wider. When σ_Φ approaches to infinity, $f_{\hat{\Theta}}(\phi)$ approaches to $1/\pi$. This is a special case of uniform density functions. Since both end points of the second term are zero, it is obvious that $f_{\hat{\Theta}}(\pm\pi/2) = (1/\pi)\cdot\exp\left(-\gamma^2/\sigma_\Phi^2\right)$. The figures of $f_{\hat{\Theta}}(\phi)$ with various parameters are depicted in Fig. 6. In these figures, we can see that the PDFs can be well approximated as Gaussian functions for several parameters.

Though (46) is of a closed form, it is still very complicated to integrate. This is unfortunate, because it is essential to do so to evaluate the first and second moments to obtain the phase variance performance. To mitigate this problem, we can write (46) in the form of a power series, to be tractable, as

$$f_{\hat{\Theta}}(\phi)=\frac{1}{\pi}\left(1+2\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^k}{(2k+1)k!l!}\left(\frac{\gamma^2}{\sigma_\Phi^2}\right)^{k+l+1}\cos^{2(k+l+1)}\phi\right) \times\exp\left(-\frac{\gamma^2}{\sigma_\Phi^2}\right) \qquad -\pi/2\le\phi\le\pi/2. \tag{47}$$

The proposed estimator can be evaluated by CRLB, which is a lower bound on the variance of any unbiased estimator. Since the PDF of the proposed estimator can be approximated well as a Gaussian function, as shown in Fig. 6, the CRLB of the proposed estimator becomes the variance of that, approximately. Since $f_{\hat{\Theta}}(\phi)$ is an even function and $\phi f_{\hat{\Theta}}(\phi)$ is an odd function, the mean of $\hat{\Theta}(k)$ is zero, and from (C-9a) the variance is given by

$$\sigma_{\hat{\Theta}}^2=\left(\frac{\pi^2}{12}+\frac{2}{\pi}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{2(k+l+1)}\left(\frac{\gamma^2}{\sigma_\Phi^2}\right)^{k+l+1}\frac{(-1)^k}{(2k+1)k!l!2^{2(k+l+1)}}\binom{2(k+l+1)}{m} \times Q(m,k,l)\right)\exp\left(-\frac{\gamma^2}{\sigma_\Phi^2}\right), \tag{48}$$

where

$$Q(m,k,l)=\begin{cases}\dfrac{\pi^3}{12} & m=k+l+1,\\[2ex] \dfrac{\pi(-1)^{m-k-l-1}}{2(m-k-l-1)^2} & \text{otherwise}.\end{cases}$$

The variance performances of $\hat{\Theta}(k)$ are depicted in Fig. 7, and from them, it is obvious that the proposed estimator is very efficient, since the CRLB and variance obtained by simulation

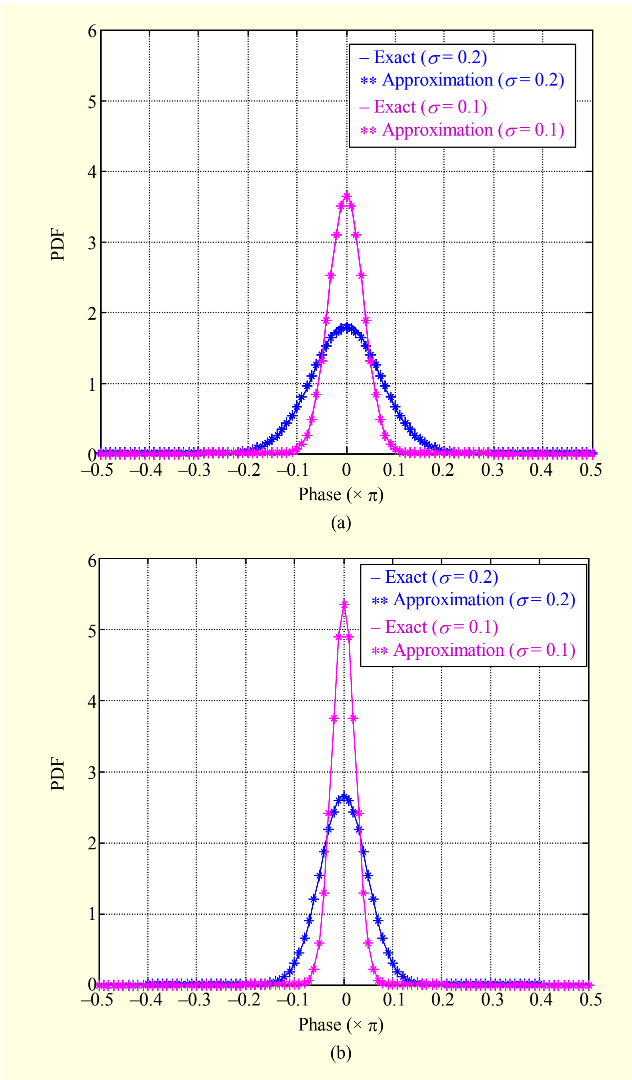

Fig. 6. PDF and Gaussian approximations: (a) $\gamma = 0.65$ and (b) $\gamma = 0.95$.

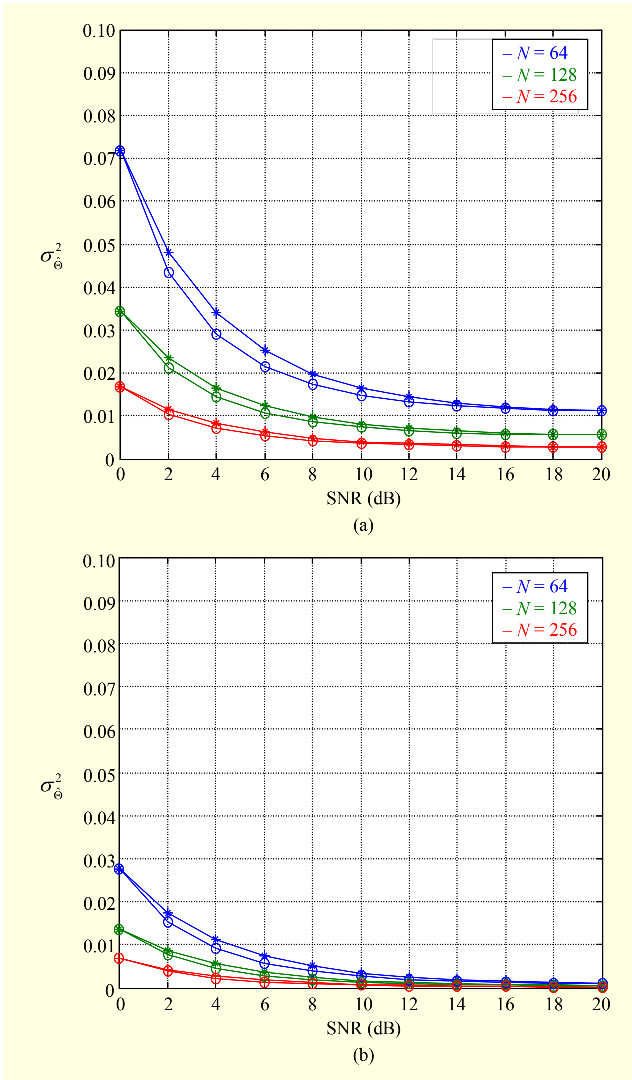

Fig. 7. Phase variance performance (o: CRLB, ∗: simulation): (a) $\gamma = 0.65$ and (b) $\gamma = 0.95$.

show perfect efficiency at low and high SNRs, and they show only a small difference between them for all other SNR values. There is an unusual characteristic in that although SNR increases toward infinity, the variance does not tend toward zero, as shown in Fig. 7. This is because when SNR is infinity, the standard deviation is not zero but $\sqrt{(1-\gamma^2)/N}$. Despite the noise being absent, the standard deviation does not tend toward zero because of self-noise in the proposed estimator. The reduction methods of self-noise are increasing the value of γ (so that it nearly reaches the value of 1) or significantly increasing N. However, these high values increase the implementation complexity, because the sampling rate of the input signal and the taps of an average filter should be increased. Therefore, there is a tradeoff in the proposed scheme between system performance and implementation complexity.

V. Conclusion

We proposed a scheme to improve SNR using the DFT of the incoming signal, which was contaminated by noise over the channel, and investigated its performance. We also proposed and analyzed a hopping phase estimator required to demodulate a hopped frequency modulation/binary frequency shift keying (FM/BFSK) signal. Since the estimator consisted of two consecutive samples and a moving-average filter, its structure was easy to implement and performance easy to analyze.

We obtained a probability density function for the proposed estimator to evaluate the performance and found that it could be approximated by a Gaussian function. To evaluate the performance of the proposed scheme, we obtained the Cramer-Rao low bound (CRLB) of the proposed phase estimator with the Gaussian function and compared it with the result obtained by simulation. It was verified that the proposed estimator was very efficient, since the CRLB and variance obtained by simulation showed perfect efficiency at low and high SNRs and only a small difference between them for all other SNR values.

Since the proposed scheme is very simple to implement and has near-perfect efficiency, it will be useful in demodulating hopped FM/BFSK signals without any prior information for carrier synchronization.

Appendix A: Hopping Phase Estimator for Hopped BFSK Signal

In the case where the information data is modulated by BFSK, the FHSS signal can be represented as

$$q(t) = A_c \cos\left(\omega_c t + \omega_i t + y(t)\right) \quad i = 0, 1, 2, \ldots, M-1, \tag{A-1}$$

where

$$\begin{aligned} y(t) &= \left(\frac{1}{2}(d_k+1)\cdot 2\pi f_H t\right) - \left(\frac{1}{2}(d_k-1)\cdot 2\pi f_L t\right) \\ &= \left[(d_k+1) f_H - (d_k-1) f_L\right]\pi t, \end{aligned} \tag{A-2}$$

where f_H is a selected frequency when $d_k = 1$ and f_L is a selected frequency when $d_k = -1$. Hilbert transforming (A-1), we have

$$w(t) = A_c \exp\left[j\left(\omega_c t + \omega_i t + y(t)\right)\right] \quad i = 0, 1, 2, \ldots, M-1. \tag{A-3}$$

We define a function, $b(t, \Delta t)$, as

$$b(t, \Delta t) \equiv w(t) w^*(t - \Delta t). \tag{A-4}$$

Through the same procedure as a hopped FM system, we can obtain

$$\Lambda(k) \equiv A_c^2 \exp\left(j(\omega_c + \omega_i)\Delta t\right) \exp\left(j dy(t + kT)\right), \tag{A-5}$$

where

$$dy(t) \equiv y(t) - y(t - \Delta t). \tag{A-6}$$

We define a hopping phase estimator as

$$\begin{aligned} \hat{\Phi}_i(\Delta t) &\equiv \frac{1}{N}\sum_{k=0}^{N-1} \Lambda(k) \\ &= A_c^2 \exp\left(j(\omega_c + \omega_i)\Delta t\right) \frac{1}{N}\sum_{k=0}^{N-1} \exp\left(j dy(t + kT)\right). \end{aligned} \tag{A-7}$$

When N is sufficiently large, the probability that f_H is selected when $d_k = 1$ and f_L is selected when $d_k = -1$ is 1/2. Therefore, we have

$$\begin{aligned} &\frac{1}{N}\sum_{k=0}^{N} \exp\left(j dy(t + kT)\right) \\ &\cong \frac{1}{2}\exp\left(j 2\pi f_H \Delta t\right) + \frac{1}{2}\exp\left(j 2\pi f_L \Delta t\right). \end{aligned} \tag{A-8}$$

If we design symmetrically so that $f_L = -f_H = -\Delta f$, then (A-7) becomes

$$\hat{\Phi}_i(\Delta t) \cong A_c^2 \exp\left(j(\omega_c + \omega_i)\Delta t\right) \cos\left(2\pi \Delta f \Delta t\right). \tag{A-9}$$

Appendix B: Expectation of $\Gamma(k)$ and $\Gamma(k)\,\Gamma^*(l)$

We separate $S_i(k)$ into two terms as follows:

$$S_i(k) = G_i(k) Z_i(k), \tag{B-1}$$

where $G_i(k)$ and $Z_i(k)$, except $Z_1(k)$, are the information-bearing signal and noise, respectively. From (42), we have

$$G_1(k)=\exp\left(j\int_{t+kT-\Delta t}^{t+kT}x(\tau)\,\mathrm{d}\tau\right), \tag{B-2a}$$

$$G_2(k)=\exp\left(j\int_{0}^{t+kT}x(\tau)\,\mathrm{d}\tau\right), \tag{B-2b}$$

$$G_3(k)=\exp\left(-j\int_{0}^{t+kT-\Delta t}x(\tau)\,\mathrm{d}\tau\right), \tag{B-2c}$$

$$G_4(k)=1, \tag{B-2d}$$

and

$$Z_1(k)=1, \tag{B-3a}$$

$$Z_2(k)=n^*(t+kT-\Delta t), \tag{B-3b}$$

$$Z_3(k)=n(t+kT), \tag{B-3c}$$

$$Z_4(k)=n(t+kT)n^*(t+kT-\Delta t). \tag{B-3d}$$

We obtain the mean of $\Gamma(k)$ as

$$m_\Gamma=\mathrm{E}\left[\Gamma(k)\right]\equiv\sum_{i=1}^{4}\mathrm{E}\left[S_i(k)\right]. \tag{B-4}$$

Since information-bearing terms are independent of noise terms, it is obvious that

$$\mathrm{E}\left[S_i(k)\right]=\mathrm{E}\left[G_i(k)\right]\mathrm{E}\left[Z_i(k)\right]. \tag{B-5}$$

From (B-3), we know that

$$\mathrm{E}\left[Z_i(k)\right]=0 \quad i=2,\ 3,\ 4. \tag{B-6}$$

Therefore, (B-4) becomes

$$\begin{aligned}m_\Gamma&=\mathrm{E}\left[G_1(k)\right]\\&=\mathrm{E}\left[\exp\left(j\int_{t+kT-\Delta t}^{t+kT}x(\tau)\,\mathrm{d}\tau\right)\right]\cong\mathrm{E}\left[\exp\left(jx(t)\Delta t\right)\right]\\&=\gamma.\end{aligned} \tag{B-7}$$

We obtain the expectation of $\Gamma(k)\Gamma^*(l)$ as follows:

$$\begin{aligned}&\mathrm{E}\left[\Gamma(k)\Gamma^*(l)\right]\\&=\sum_{p=1}^{4}\sum_{q=1}^{4}\mathrm{E}\left[S_p(k)S_q^*(l)\right]\\&=\sum_{p=1}^{4}\sum_{q=1}^{4}\mathrm{E}\left[G_p(k)G_q^*(l)\right]\mathrm{E}\left[Z_p(k)Z_q^*(l)\right].\end{aligned} \tag{B-8}$$

Inspecting noise terms from (B-3a) to (B-3d), since they are independent for $Z_p(k)$ and $Z_q(k)$, we have

$$\mathrm{E}\left[Z_p(k)Z_q^*(l)\right]=\begin{cases}\mathrm{E}\left[Z_p(k)Z_p^*(l)\right] & p=q,\\ 0 & \text{otherwise.}\end{cases} \tag{B-9}$$

On the other hand, for the different times k and l, (B-9) may be written as follows:

$$\mathrm{E}\left[Z_p(k)Z_p^*(l)\right]=\begin{cases}1 & p=1,\\ \sigma_n^2\delta(k-l) & p\neq 1.\end{cases} \tag{B-10}$$

Substituting $\mathrm{E}\left[Z_p(k)Z_q^*(l)\right]$ in (B-1) by (B-10), we have

$$\begin{aligned}&\mathrm{E}\left[\Gamma(k)\Gamma^*(l)\right]\\&=\mathrm{E}\left[G_1(k)G_1^*(l)\right]+\sum_{p=2}^{4}\mathrm{E}\left[\left|G_p(k)\right|^2\right]\mathrm{E}\left[\left|Z_p(k)\right|^2\right].\end{aligned} \tag{B-11}$$

In (B-11), the first term becomes

$$\begin{aligned}&\mathrm{E}\left[G_1(k)G_1^*(l)\right]\\&=\mathrm{E}\left[\exp\left(j\int_{t+kT-\Delta t}^{t+kT}x(\tau)\,\mathrm{d}\tau\right)\exp\left(-j\int_{t+lT-\Delta t}^{t+lT}x(\tau)\,\mathrm{d}\tau\right)\right]\\&=\delta(k-l)+\gamma^2\left(1-\delta(k-l)\right).\end{aligned} \tag{B-12}$$

From (B-2), we know that

$$\begin{aligned}\mathrm{E}\left[\left|G_2(k)\right|^2\right]&=\mathrm{E}\left[\left|G_3(k)\right|^2\right]=\mathrm{E}\left[\left|G_4(k)\right|^2\right]\\&=1.\end{aligned} \tag{B-13}$$

From (B-3), we know that

$$\mathrm{E}\left[\left|Z_1(k)\right|^2\right]=1, \tag{B-14}$$

$$\mathrm{E}\left[\left|Z_2(k)\right|^2\right]=\mathrm{E}\left[\left|Z_3(k)\right|^2\right]=\sigma_n^2, \tag{B-15}$$

$$\mathrm{E}\left[\left|Z_4(k)\right|^2\right]=\sigma_n^4. \tag{B-16}$$

Finally, inserting results from (B-12) to (B-16) into (B-8) yields

$$\begin{aligned}&\mathrm{E}\left[\Gamma(k)\Gamma^*(l)\right]\\&=\left(1+2\sigma_n^2+\sigma_n^4\right)\delta(k-l)+\gamma^2\left(1-\delta(k-l)\right).\end{aligned} \tag{B-17}$$

Appendix C: Derivation of (50)

We define two PDFs as follows:

$$f_{\mathrm{a}}(\varphi)=\begin{cases}\dfrac{1}{\pi} & -\pi/2\leq\varphi\leq\pi/2,\\ 0 & \text{otherwise,}\end{cases} \tag{C-1}$$

$$f_{\mathrm{b}}(\varphi)=\begin{cases}\dfrac{\cos\varphi}{2} & -\pi/2\leq\varphi\leq\pi/2,\\ 0 & \text{otherwise.}\end{cases} \tag{C-2}$$

Then we can write (52) as

$$\begin{aligned}f_{\hat{\Theta}}(\varphi)=&\left(f_{\mathrm{a}}(\varphi)+\frac{4}{\pi}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^k}{(2k+1)k!l!}\left(\frac{\gamma^2}{\sigma_\Phi^2}\right)^{k+l+1}f_{\mathrm{b}}^{2(k+l+1)}(\varphi)\right)\\&\times\exp\left(-\frac{\gamma^2}{\sigma_\Phi^2}\right).\end{aligned} \tag{C-3}$$

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The moment generating function is

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{\infty} f_{\hat{\Theta}}(\varphi) e^{j\omega\varphi} \mathrm{d}\varphi \\ &= \left(F_{\mathrm{a}}(\omega) + \frac{4}{\pi} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{(-1)^k}{(2k+1)k!l!} \left(\frac{\gamma^2}{\sigma_\Phi^2} \right)^{k+l+1} \right. \\ &\quad \left. \times\ F_{\mathrm{b}}\left(\omega, 2(k+l+1)\right) \right) \exp\left(-\frac{\gamma^2}{\sigma_\Phi^2} \right), \end{aligned} \tag{C-4}$$

where

$$F_{\mathrm{a}}(\omega) = \frac{\sin\left(\frac{\pi}{2}\omega\right)}{\pi\omega}, \tag{C-5}$$

$$F_{\mathrm{b}}(\omega, s) = \frac{1}{2^n} \sum_{r=0}^{n} \binom{s}{r} \frac{\sin\left(\frac{\pi}{2}\left[(\omega+s)-2r\right]\right)}{(\omega+s)-2r}. \tag{C-6}$$

The first and second moments for the two functions are

$$m_{f_{\mathrm{a}},1} = -j\frac{\partial}{\partial\omega} F_{\mathrm{a}}(0) = 0, \tag{C-7a}$$

$$m_{f_{\mathrm{a}},2} = -\frac{\partial^2}{\partial\omega^2} F_{\mathrm{a}}(0) = \frac{\pi^2}{12}, \tag{C-7b}$$

$$m_{f_{\mathrm{b}},1} = -j\frac{\partial}{\partial\omega} F_{\mathrm{b}}(0, 2n) = 0, \tag{C-8a}$$

$$\begin{aligned} m_{f_{\mathrm{b}},2} &= -\frac{\partial^2}{\partial\omega^2} F_{\mathrm{b}}(0, 2n), \\ &= \frac{1}{2^n} \sum_{r=0}^{n} \binom{2n}{r} \cdot \begin{cases} \dfrac{\pi^3}{12} & r = n, \\ \dfrac{(-1)^{n-r}}{4(n-r)^2} & \text{otherwise.} \end{cases} \end{aligned} \tag{C-8b}$$

Therefore, the variance for the PDF given by (C-3) becomes

$$\begin{aligned} \sigma_{\hat{\Theta}}^2 &= \left(\frac{\pi^2}{12} + \frac{2}{\pi} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{2(k+l+1)} \left(\frac{\gamma^2}{\sigma_\Phi^2} \right)^{k+l+1} \frac{(-1)^k}{(2k+1)k!l!2^{2(k+l+1)}} \binom{2(k+l+1)}{m} \right. \\ &\quad \left. \times\ Q(m, k, l) \right) \exp\left(-\frac{\gamma^2}{\sigma_\Phi^2} \right), \end{aligned} \tag{C-9a}$$

where

$$Q(m, k, l) = \begin{cases} \dfrac{\pi^3}{12} & m = k+l+1, \\ \dfrac{\pi(-1)^{m-k-l-1}}{2(m-k-l-1)^2} & \text{otherwise.} \end{cases} \tag{C-9b}$$

References

[1] M.K. Simon et al., "*Spread Spectrum Communicaion Handbook*," New York: McGraw Hill, 2002.

[2] C.-Q. Liu, X.-L. Hu, and Y.-Y. Zhang, "Blind Multiuser Detection Algorithm for Asynchronous MFSK/FFH-MA System," *Int. Conf. Wireless Commun., Netw. Mobile Comput.*, Wuhan, China, Sept. 22–24, 2006, pp. 1–4.

[3] H. Fan, Y. Guo, and Y. Xu, "A Novel Algorithm of Blind Detection of Frequency Hopping Signal Based on Second-Order Cyclostationarity," *Congress Image Signal Process.*, vol. 5, Sanya, China, May 27–30, 2008, pp. 399–402.

[4] X.-Q. Liu, J.-L. Li, and X.-L. Ma, "An EM Algorithm for Blind Hop Timing Estimation of Multiple FH Signals Using an Array System with Bandwidth Mismatch," *IEEE Trans. Veh. Technol.*, vol. 56, no. 5, Sept. 2007, pp. 2545–2554.

[5] Y. Lei, Z. Zhong, and Y. Wu, "A New Hop Duration Blind Estimation Algorithm for Frequency-Hopping Signals," *IEEE Pacific-Asia Workshop Comput. Intell. Ind. Appl.*, Wuhan, China, Dec. 19–20, 2008, pp. 695–699.

[6] Z.-C. Sha et al., "Online Hop Timing Detection and Frequency Estimation of Multipath FH Signals," *ETRI J.*, vol. 35, no. 5, Oct. 2013, pp. 748–756.

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