Hop Period Estimation for Frequency Hopping Signals Based on Hilbert-Huang Transform

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Abstract

A method for estimating the hop period of frequency hopping (FH) signals based on Hilbert-Huang transform (HHT) is proposed. This method iteratively decomposes the FH signal into intrinsic mode functions (IMFs) and applies Hilbert transform to each IMF to yield Hilbert spectrum, whose maxima for each time instant are extracted to form a sequence. A simple Fourier transform based analysis of the obtained sequence provides the estimate of hop period for a FH signal. The proposed method is fully cross-term free, and it does not make any assumption about the parameter information. The performance of the method is evaluated in terms of estimation variances in the presence of Gaussian white noise.

1. Introduction

Frequency hopping signals are widely used in communications due to many good properities such as low probability of interception, code division multiple access, immunity against interferences. The hop period accurate estimation is one of the key problems in the interception, acquisition or synchronization of FH signals. Therefore, substantial amount of research work has been conducted on this problem [1][2]. Most of these methods focus on using Wigner-Ville distribution (WVD) to localize the FH signals in time frequency plane [3][4][5]. However, common WVD based methods suffer from a trade-off between time frequency resolution and cross-term suppression [6]. The recently presented HHT, which consists of empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA), is potentially viable for nonlinear and nonstationary signals analysis, especially for time-frequency-energy representations. The HHT can give results much sharper than those from any of the traditional analysis method in time frequency domain [7]. In this paper, we propose a new hop period estimation method for the FH signal using the adaptive signal analysis technique, HHT. By decomposing a FH signal into its IMFs, the decomposition will allow the construction of a cross-term free time frequency distribution by combing the Hilbert spectrum of the individual intrinsic function generated by the decomposition. The proposed method can achieve high time frequency resolution since the HHT is based on an adaptive basis, and the frequency is defined through the Hilbert transform, and consequently there is no uncertainty principle limitation on time or frequency resolution.

2. Hilbert-Huang transform

By definition an IMF satisfies two conditions: 1) The number of extrema and zero-crossings of the function must be equal or differ by no more than one . 2)The mean at any point of the envelope defined by the local maxima and local minima is zero. The procedures to decompose a given signal $s(t)$ into IMFs can be summarized as: identify all the local extrema from the given signal, then connect all the local maxima by a cubic spline as the upper envelope. Repeat the previous step for the local minima to produce the lower envelope. The upper and lower envelope should cover the signal between them. Their mean is designated as $m_1(t)$, and the difference between the signal and $m_1(t)$ is the first component $h_1(t)$, i.e.,

$$h_1(t) = s(t) - m_1(t) . \tag{1}$$

To eliminate riding waves and make the wave profiles more symmetric, the sifting process of Eq.(1)

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should be repeated as many times as required to reduce the extracted signal to an IMF. In the next step $h_1(t)$ is treated as the signal , and

$$h_1(t)-m_{11}(t)=h_{11}(t). \tag{2}$$

After repeated siftings in this manner up to k times , $h_{1k}(t)$ becomes an IMF; that is,

$$h_{1(k-1)}(t)-m_{1(k-1)}(t)=h_{1k}(t). \tag{3}$$

then it is designated as

$$c_1(t)=h_{1k}(t), \tag{4}$$

the first IMF component from the signal. [7] suggested a Cauchy type of convergence test for the stoppage criterion of the sifting process, which is calculated from two consecutive sifting results as

$$SD_k=\frac{\sum_{t=0}^{T}\left|h_{1k-1}(t)-h_{1k}(t)\right|^2}{\sum_{t=0}^{T}h_{1k-1}^2(t)}, \tag{5}$$

where T is the time span. If the evaluated SD_k is smaller than a predetermined limitation , the sifting process will be stopped.

Generally speaking, $c_1(t)$ contains the finest scale or the shortest period component of the signal. Separating $c_1(t)$ from the rest of the signal by

$$r_1(t)=s(t)-c_1(t), \tag{6}$$

then the residue of the signal $r_1(t)$, which contains longer period variations in the signal is obtained. Treating $r_1(t)$ as a new signal and subjecting it to the same sifting process as described above, the second IMF $c_2(t)$ is obtained. This procedure can be repeated with all the subsequent $r_n(t)$'s, and the result is

$$r_2(t)=r_1(t)-c_2(t), \tag{7}$$

$$\dots$$

$$r_N(t)=r_{N-1}(t)-c_N(t). \tag{8}$$

By summing up equations (6), (7), and (8), the given signal $s(t)$ is reconstructed by

$$s(t)=\sum_{n=1}^{N}c_n(t)+r_N(t). \tag{9}$$

When leaving out the residue and $r_N(t)$ and performing the Hilbert transform on each IMF component, the original signal can be expressed as the real part $\Re$ in the following form:

$$s(t)=\Re\left[\sum_{n=1}^{N}a_n(t)\exp\left(i2\pi\int f_n(t)dt\right)\right] \tag{10}$$

where $a_n(t)=\left[c_n^2(t)+\mathcal{H}^2(c_n(t))\right]^{1/2}$,

$\varphi_n(t)=\arctan\left(\mathcal{H}(c_n(t))/c_n(t)\right)$, $f_n(t)=\frac{1}{2\pi}\frac{d\varphi_n(t)}{dt}$,

and $\mathcal{H}[c_n(t)]$ is the Hilbert Transform of $c_n(t)$. Eq.(10) gives both the amplitude and frequency of each component as functions of time. This time-frequency distribution of the amplitude is designated as the Hilbert spectrum $H(t,f)$ as follows

$$H(t,f)=\sum_{n=1}^{N}w_{n,f}(t)a_n(t), \tag{11}$$

where the weight factor $w_{n,f}(t)$ takes 1 if $f_n(t)$ in Eq.(10) equals to f , otherwise is 0.

3. Estimation algorithm

Step 1: Model the observed signal $x(t)$ as a sum of the IMF components using EMD mentioned in section 2:

$$x(t)=\sum_{n=1}^{N}c_n(t), \tag{12}$$

then apply Hilbert transform to each component in (12) to obtain the Hilbert spectrum $H_x(t,f)$ of $x(t)$ in the form of (11).

Step 2: Compute the maxima of $H_x(t,f)$ for each time instant to obtain a sequence $y(t)$. The maxima of $H_x(t,f)$ reveals a periodicity of the observed signal.

Step 3: Apply Fourier transform to $y(t)$, thus yielding its Fourier spectrum $Y(f)$.

Step 4: Determine the hop rate $\hat{f}_H$ corresponding to the maximum of $|Y(f)|$. Let

$$\hat{f}_H=\arg\max_f|Y(f)|, \tag{13}$$

the hop period is then estimated as

$$\hat{T}_H=\frac{1}{\hat{f}_H}. \tag{14}$$

4. Simulation experiments

The simulation experiments presented in this section are based on the following FH signal model:

$$x(t)=A\sum_k rect_{T_H}(t-kT_H-\theta)e^{j2\pi f_k(t-kT_H-\theta)}+w(t) \tag{15}$$

where $rect_{T_H}(t)$ is equal to one for $t\in(-T_H/2,T_H/2]$ and zero elsewhere, f_k is the generic hop frequency, belonging to a given finite alphabet, T_H is the hop period to be estimated, and $w(t)$ is additive Gaussian white noise. For simplicity sake, the time offset θ is assumed to be equal to zero.

In the simulation, the observed signal is composed of 1024 samples. There are 8 hops within the observation interval, which means there are 128 samples for each sinusoid. The frequency alphabet is given by $\{0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45\}$. Figure 1 shows the Hilbert-Huang transform of a FH signal embedded in additive Gaussian white noise.

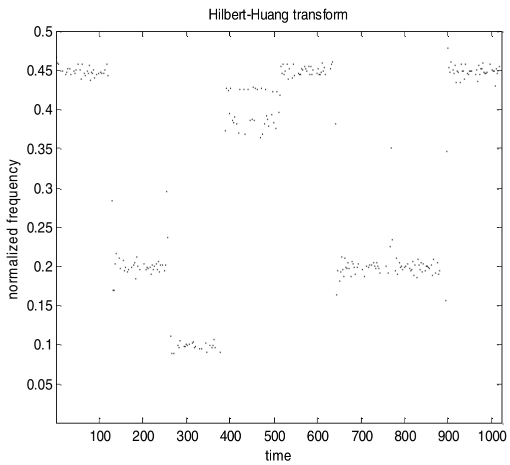

Figure 1. Hilbert-Huang transform of observed signal

The corresponding function $y(t)$, computed at step 3 of estimation algorithm, is shown in Figure 2.

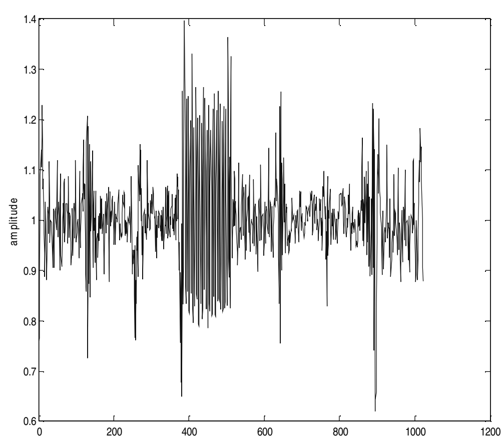

Figure 2. Maxima of the Hilbert-Huang spectrum versus time

The Fourier tranform amplitude spectrum of $y(t)$ depicted in Figure 2 is reported in Figure 3.

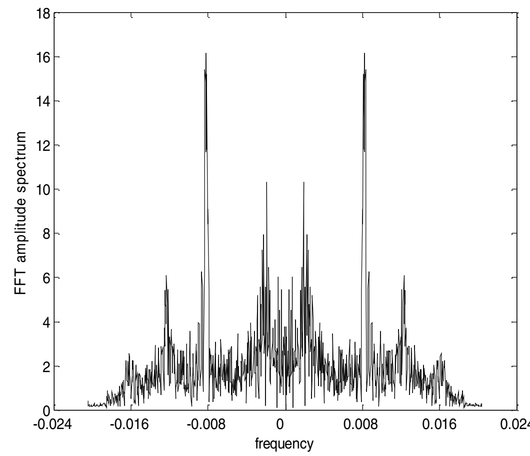

Figure 3. Fourier transform amplitude spectrum

To measure the performance of the method developed in this paper, we assume the number of independent trials is 500, and evaluate the variance of $\hat{T}_H$ at different SNR values in the range of -10 dB to 30dB . Figure 4 shows the estimation variance of $\hat{T}_H$ vs. SNR.

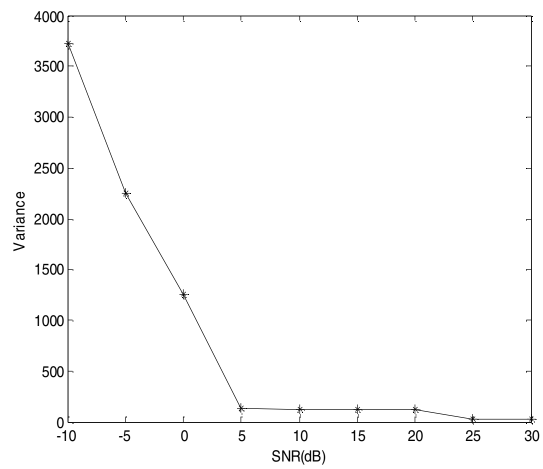

Figure 4. Hop period estimation variance vs. SNR

As seen from Figure 4, the estimation performance has deteriorated with SNR below about 5dB. Above the threshold, the variance is approximately inversely proportional to the SNR, and keeps a very low level, which implies the estimate of hop period is reliable.

5. Conclusions

In this paper, we aim at the hop period accurate estimation for the frequency hopping signals without knowing any a priori knowledge. The proposed method provides advantages with respect to other time-frequency distribution based techniques for the estimate of hop period, concerning the following aspects: 1) The HHT is cross-term free, and it doesn't suffer from the trade-off between time frequency resolution and cross-term suppression. 2) The HHT can give results much sharper than those from any of the traditional analysis method in time-frequency domain. Simulation results show that the proposed method is effective and efficient.

6. References

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