INTERCEPTION OF FREQUENCY HOPPED SPREAD SPECTRUM SIGNALS

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ABSTRACT **— A frequency-hopped, spread spectrum signal is modelled as a sinusoid that has one of N random, discrete frequencies. Based on this model, Neyman-Pearson detection theory is used to derive optimum coherent and noncoherent receiver structures. It is shown that the probability of detection, P_D, approaches, the probability of false alarm, P_F, as N gets large. For large N simple formula that relate P_D to P_F at a given SNR are given. For a given N, we determine the SNR below which interception is impractical as $P_D \simeq P_F$. Both single hop and multiple hop per detection interval situations are considered.[1]**

I. INTRODUCTION

During the last fifteen years, spread spectrum communications has been receiving increased interest in the open literature. In addition to military applications, numerous civilian uses are being developed. Examples can be found in [1–5]. Tutorials on spread spectrum communications can be found in [1] and [2].

In this paper, we model a frequency hopped signal as a sinusoidal signal with a random frequency that has a discrete probability distribution. The number of possible frequencies in this distribution is denoted by N. Neyman-Pearson detection theory is used to establish the constant false alarm rate (CFAR) interception receiver. For a given false alarm probability, the detection probability of this receiver represents a fundamental detection performance limit for any receiver. The frequency hopping rate is first assumed to be slow so that the modulated signal is not allowed to change in frequency over the detection interval. The analysis is then generalized to fast hopping where the signal hops M times in one detection interval.

The receiver structure derived for slow hopping consists of a summation of the outputs from a bank of correlators, one correlator for each discrete frequency in the random frequency distribution. Such a receiver can be realized using surface acoustic wave filters [6]. The resulting receiver is called the average likelihood (AL) receiver. The output from each correlator in the bank could also be tested. This receiver, the maximum likelihood (ML) receiver [7], is easily analyzed and has very similar performance to the AL receiver. This is fortunate, as the AL receiver is not as easily implemented or analyzed. The near equivalence of the two receivers in a radar detection problem with unknown Doppler frequency was established by Brennan et al [8]. A fast hopping signal receiver is also derived and has the following conceptual structure. There is a bank of M AL receivers where M is the number of hops per detection interval. Each AL receiver is an optimum receiver for detecting the presence or absence of the signal on one of the interval chips corresponding to one hop. The logarithms of the M chip receiver outputs are summed and thresholded. This may be implemented using one AL receiver and M consecutive outputs. That is, the AL receiver is matched to the hop interval and the output is used M times. This receiver, the multihop average likelihood (MAL) receiver is approximated by a multihop maximum likelihood (MML) receiver. Coherent and noncoherent MML receivers are analyzed. Analogous results to those obtained for the slow hop ML receiver are obtained. The main references on the problem we consider can be found in [7–13].

II. SINGLE HOP CASE

The signal is assumed to change frequency once per detection interval. The detection problem is, $H_1 : r(t) = s(t) + n(t)$, $0 \le t \le T$, $H_0 : r(t) = n(t)$, where

$$s(t) = A \cos(\omega t + \theta) \tag{1}$$

with $n(t)$, white Gaussian noise with spectral height, $N_0/2$ W/Hz. The signal, when present, has amplitude, angular frequency, and phase constant A, ω and θ, respectively. The signal duration is denoted as T. In the coherent case, $\theta = 0$. In the noncoherent case, the phase is uniform in $[0, 2\pi]$. The angular frequency ω is assumed to have the discrete distribution

$$P_\omega(\omega) = \frac{1}{N} \sum_{i=1}^{N} \delta(\omega - \omega_i) \tag{2}$$

where $\delta(x)$ is the Dirac-delta function. The frequencies are spaced multiples of $\frac{1}{T}$ or $\frac{1}{2T}$ apart for noncoherent and coherent systems, respectively, making the signals orthogonal.

The likelihood ratio for this problem is given by [14]

$$L[r(t)] = E_{\omega,\theta} \left\{ \exp\left\{ \frac{2}{N_0} \int_0^T r(t)\, s(t)\, dt - \frac{E}{N_0} \right\} \right\} \tag{3}$$

where E is the signal energy and $s(t)$ is given in (1). Here $E_{\omega,\theta}$ represents expectation with respect to frequency and phase. The signal energy is assumed to be independent of these signal parameters.

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For the coherent case, the expectation in θ in (3) does not occur as it is assumed to be perfectly known. The unknown frequency has the pdf in (2) and for $s(t)$ in (1) the AL ratio test is

$$L = \sum_{i=1}^{N} \ell_i \underset{H_0}{\overset{H_1}{\gtrless}} \eta N e^{d^2/2} = \eta' \tag{4}$$

where $\ell_i = e^{\alpha_i}$, the signal-to-noise ratio (SNR) is $d^2 = 2E/N_0$, and

$$\alpha_i = \frac{2A}{N_0} \int_0^T r(t) \cos\omega_i t \, dt \,. \tag{5}$$

The constant η sets the false alarm rate and η' is then the detection threshold. One notes that α_i is Gaussian and that the sufficient statistic is a sum of, independent, lognormal random variables, e^{α_i}. The independence follows from the frequency spacing assumed following (2) as a consequence of the orthogonality of the signals [15, p.151]. The receiver structure is shown in Figure 1.

For the noncoherent case, the phase is uniform in $[0, 2\pi]$. Performing the expectations in (3) the AL ratio test is

$$L = \sum_{i=1}^{N} I_0(q_i) \underset{H_0}{\overset{H_1}{\gtrless}} \eta N e^{d^2/2} = \eta' \tag{6}$$

where $I_0(x)$ is the modified Bessel function of zero order and

$$q_i^2 = L_{c_i}^2 + L_{s_i}^2 \,, \tag{7}$$

$$L_{c_i} = \frac{2A}{N_0} \int_0^T r(t) \cos \omega_i t \, dt \,, \tag{8}$$

and

$$L_{s_i} = \frac{2A}{N_0} \int_0^T r(t) \sin \omega_i t \, dt \,. \tag{9}$$

The independence of the summands in (6) again follows from the assumed frequency spacing [15, p.210]. The receiver structure is shown in Figure 2.

The coherent ML receiver is a suboptimum, hardlimited version of the AL receiver shown in Figure 1. In words, the test is as follows. An optimal test to determine the presence or absence of signal at each of the N frequencies is performed ignoring the other frequencies. The overall decision as to whether signal is present at any of the N frequencies is made by declaring signal present if signal has been declared present for one or more of the N frequencies. The, optimum, AL receiver may then be viewed as a "soft" combining of the signal tests for the individual frequencies where the combining is done with exponential weighting. The optimal detection of the presence or absence of signal for the i-th frequency is obtained by setting $N = 1$ in (4),

$$e^{\alpha_i} \underset{H_0}{\overset{H_1}{\gtrless}} \eta e^{d^2/2},$$

or

$$A \int_0^T r(t) \cos \omega_i t \, dt \underset{H_0}{\overset{H_1}{\gtrless}} \frac{N_0}{2} \ell n\, \eta + \frac{E}{2} = \gamma \,.$$

The ML receiver statistic may be written in the form of (4) by letting ℓ_i be a random variable (R.V.) that assumes the values 0 and 1 as signal is declared absent or present at the i-th frequency. Then,

$$L = \sum_{i=1}^{N} \ell_i \underset{H_0}{\overset{H_1}{\gtrless}} 1/2 \tag{10}$$

with ℓ_i determined by

$$A \int_0^T r(t) \cos \omega_i t \, dt \underset{\ell_i=0}{\overset{\ell_i=1}{\gtrless}} \frac{N_0}{2} \ell n\, \eta + \frac{E}{2} = \gamma, \tag{11}$$

and where the threshold for L is chosen arbitrarily to be 1/2. Any value between 0 and 1 may be used. The ML receiver structure is shown in Figure 3. The probability of detection of signal at the i-th frequency can be shown to be

$$Q_D = Q\left(\frac{\ell n\, \eta}{d} - \frac{d}{2}\right) \tag{12}$$

and is independent of i. The probability of false alarm is

$$Q_F = Q\left(\frac{\ell n\, \eta}{d} + \frac{d}{2}\right). \tag{13}$$

Here $Q(x)$ is the area under the unit variance, zero mean, Gaussian probability density function from x to ∞. The ML receiver detection and false alarm probabilities are, respectively,

$$P_D = 1 - (1 - Q_D)(1 - Q_F)^{N-1} \tag{14}$$

and

$$P_F = 1 - (1 - Q_F)^N \,. \tag{15}$$

The noncoherent, ML receiver is the analogous suboptimum version of the noncoherent AL receiver of Figure 2. That is, ℓ_i denotes a R.V. whose outcome depends upon an optimum test for signal at the i-th frequency according to

$$q_i \underset{\ell_i=0}{\overset{\ell_i=1}{\gtrless}} I_0^{-1}\left(\eta\, e^{d^2/2}\right) = \gamma \tag{16}$$

which follows from (6) by setting $N = 1$. The overall test is then

$$L = \sum_{i=1}^{N} \ell_i > 1/2$$

in analogy to the coherent ML test. The receiver structure is shown in Figure 4. It follows from [14] that

$$Q_F = \exp\left[-\gamma^2/2d^2\right] \tag{17}$$

and

$$Q_D = Q_M(d, \gamma/d) \tag{18}$$

where $Q_M(\alpha, \beta)$ is Marcum's Q-function,

$$Q_M(\alpha,\beta) = \int_{\beta}^{\infty} x \exp\left\{-\frac{x^2+\alpha^2}{2}\right\} I_0(\alpha x)\, dx. \quad (19)$$

The overall, noncoherent ML detector performance is given by (14) and (15) with Q_F and Q_D given in (17) - (19).

III. MULTIPLE HOP CASE

In this section the signal is assumed to change frequency a number of times, M, per detection interval. This may be the case where the detection interval is the same as the symbol interval and the signal hops M times per symbol. It may also be the case where the signal hops once per symbol interval and the receiver integrates over M symbol intervals before making a decision. In this way, an interceptor concerned only with the presence or absence of the signal may gain an advantage over a friendly receiver. In general, let M_s denote the number of symbol periods and M_H the number of hops per symbol interval. Then in one detection interval the signal is permitted to hop $M = M_s \cdot M_H$ times. That is, the MML receiver may integrate over M_s symbols a signal that hops M_H times per symbol. For the multiple hop case $M = M_s \cdot M_H$.

3.1 Interception Receiver Structures

We again consider the binary hypothesis problem in the previous section, but now

$$s(t) = \sum_{j=1}^{M} s^j(t) \quad (20)$$

where

$$s^j(t) = \begin{cases} A\cos(\omega t + \theta), & (j-1)T/M \le t \le jT/M \\ 0, & \text{elsewhere.} \end{cases} \quad (21)$$

The angular frequency ω can assume one of N values $\omega_i, i = 1,\ldots,N$, in each hop interval of width T/M. That is, there are N hopping frequencies for each chip. The discrete distribution of ω is again given by (2). Now however, the frequencies are spaced multiples of $1/MT$ or $2/MT$ apart for noncoherent and coherent systems, respectively. Let $\bar{\omega} = (\omega^1,\ldots,\omega^M)$ be the random vector whose component $\omega^j, j = 1,\ldots,M$ is a random variable that represents the angular frequency during the j-th hop. Then $\bar{\omega}$ can assume one of N^M values under hypothesis H_1. Each outcome of $\bar{\omega}$ is assumed to be equally likely. Similarly, let $\bar{\theta} = (\theta^1,\ldots,\theta^M)$ be a random vector whose component θ^j is a random variable that represents the phase constant for the j-th hop. In the coherent case θ^j is known, whereas in the noncoherent case, it is uniformly distributed on $[0,2\pi]$. The components of $\bar{\theta}$ and $\bar{\omega}$ are assumed mutually independent and are also independent of the noise $n(t)$.

From (20), (21), and our analysis in the previous section, the likelihood ratio is

$$L[r(t)] = E_{\bar{\omega},\bar{\theta}}\left\{\exp\left(\frac{2}{N_0}\int_0^T r(t)\, s(t)\, dt - \frac{E}{N_0}\right)\right\}$$

$$= E_{\bar{\omega},\bar{\theta}}\left\{\prod_{j=1}^{M}\exp\left(\frac{2}{N_0}\int_{(j-1)\frac{T}{M}}^{j\frac{T}{M}} r(t)\, s^j(t)\, dt\right)\exp\left(\frac{-E}{N_0}\right)\right\}$$

$$= \exp\left(-\frac{E}{N_0}\right)\prod_{j=1}^{M} E_{\omega^j,\theta^j}\left\{\exp\left(\frac{2}{N_0}\int_{(j-1)\frac{T}{M}}^{j\frac{T}{M}} r(t)\, s^j(t)\, dt\right)\right\}$$

$$= \exp\left(-\frac{E}{N_0}\right)\prod_{j=1}^{M} E_{\theta^j}\left\{\frac{1}{N}\sum_{i=1}^{N}\exp\left(\frac{2A}{N_0}\int_{(j-1)\frac{T}{M}}^{\frac{T}{M}} r(t)\cos(\omega_i t + \theta^j)\, dt\right)\right\}. \quad (22)$$

The logarithm of the likelihood ratio is thus

$$\ell n\{L[r(t)]\} = \sum_{j=1}^{M}\ell n\left\{E_{\theta^j}\left\{\frac{1}{N}\sum_{i=1}^{N} \bullet \exp\left(\frac{2A}{N_0}\int_{(j-1)\frac{T}{M}}^{\frac{T}{M}} r(t)\cos(\omega_i t + \theta^j)\, dt\right)\right\}\right\} - \frac{E}{N_0}. \quad (23)$$

For the coherent case, the log AL ratio test is

$$\ell n\{L\} = \sum_{j=1}^{M}\ell n\left\{\sum_{i=1}^{N}\exp\left(\frac{2A}{N_0}\int_{(j-1)\frac{T}{M}}^{j\frac{T}{M}} r(t)\cos(\omega_i t + \theta^j)\, dt\right)\right\} \underset{H_0}{\overset{H_1}{\gtrless}} \ell n\,\eta + \frac{E}{N_0} + M\,\ell n\, N = \eta'$$

which may be written as

$$\ell n\{L\} = \sum_{i=1}^{N}\ell n\left\{\sum_{i=1}^{N}\ell_i^j\right\} \underset{H_0}{\overset{H_1}{\gtrless}} \eta' \quad (24)$$

where

$$\ell_i^j = e^{\alpha_i^j} \quad (25)$$

and

$$\alpha_i^j = \frac{2A}{N_0}\int_{(j-1)\frac{T}{M}}^{j\frac{T}{M}} r(t)\cos(\omega_i t + \theta^j)\, dt\,. \quad (26)$$

The receiver statistic is a logarithmic sum of sums of lognormal random variables, $e^{\alpha_i^j}$. Note the relation of the optimum coherent multiple hop average likelihood (CMAL) receiver to the optimum coherent single hop average likelihood (CAL) receiver. Conceptually, the optimum multi-hop receiver may be implemented by adding the logarithms of the outputs of M receivers. Each of these is the optimum AL receiver for detecting the presence of a single signal chip. This may be implemented using one AL receiver that is used M times, each consecutive time corresponding to one hop. The noncoherent case is derived in [17].

We have previously noted the similar performances of the single hop AL and ML receivers. Motivated by this similarity, and by the form of (24), we propose the following suboptimum statistic for detecting multi-hop FH signals;

$$L = \sum_{j=1}^{N}\ell^j \underset{H_0}{\overset{H_1}{\gtrless}} T_0\,. \quad (27)$$

In (27), $\ell^j = 0, 1$ according to whether signal is declared absent or present during the j-th hop by a ML receiver for the j-th hop. That is, $\ell^j = 0,1$ according to whether a ML receiver for the $j-$th hop declares H_0 or H_1, respectively. Observe that the multi-hop statistic (27) has an extra parameter, T_0.

In the coherent case, each chip receiver is a CML receiver. Let

$$\beta_i^j = A \int_{(j-1)\frac{T}{M}}^{j\frac{T}{M}} r(t) \cos\left(\omega_i t + \theta^j\right) dt, \tag{28}$$

then the random variable ℓ_i^j assumes the values 0,1 according to

$$\beta_i^j = \underset{\ell_i^j=0}{\overset{\ell_i^j=1}{\gtrless}} \frac{N_0}{2} \ell n\, \eta + \frac{E}{2M} = \gamma_M$$

and

$$\ell^j = \max_{i=1,\ldots,N} \left\{\ell_i^j\right\}. \tag{29}$$

The probability of detection for each hop detector is

$$Q_D = Q\left(\frac{\ell n\, \eta}{d_M} - \frac{d_M}{2}\right) \tag{30}$$

where $d_M^2 = d^2/M = 2E/MN_0$. The probability of hop false alarm is

$$Q_F = Q\left(\frac{\ell n\, \eta}{d_M} + \frac{d_M}{2}\right). \tag{31}$$

In the noncoherent case, each hop receiver is a NCML receiver. One has

$$q_i^j \underset{\ell_i^j=0}{\overset{\ell_i^j=1}{\gtrless}} I_0^{-1}\left(\eta\, e^{d_M^2/2}\right) = \gamma_M \tag{32}$$

and, again

$$l^j = \max_{i=1,\ldots,N} \left\{\ell_i^j\right\}. \tag{33}$$

The noncoherent hop probabilities of detection and false alarm are

$$Q_D = Q_M\left(d_M, \gamma_M/d_M\right) \tag{34}$$

and

$$Q_F = \exp\left[-\gamma_M^2/2d_M^2\right], \tag{35}$$

respectively.

The MML receiver detection and false alarm probabilities are, respectively

$$P_D = 1 - \sum_{j=0}^{T_0-1} \binom{M}{j} \left\{1 - (1-Q_D)(1-Q_F)^{N-1}\right\}^j \bullet \left\{(1-Q_D)(1-Q_F)^{N-1}\right\}^{M-j}, \tag{36}$$

$$P_F = 1 - \sum_{j=0}^{T_0-1} \binom{M}{j} \left\{1 - (1-Q_F)^N\right\}^j \left\{(1-Q_F)^N\right\}^{M-j}. \tag{37}$$

IV. PERFORMANCE

The detection performance of ML and MML receivers is straightforward. For the single-hop case this performance is given by equations (12) through (15). For the multiple-hop case we have equations (34) through (37). On the other hand, the analysis of the AL receiver is very complicated. We have only analyzed the AL receivers for the single-hop case. The small$N$ case was given in [16]. For moderate and large N, as well as a fuller treatment of all the results of this paper please see [17]. In our analysis of the Al receiver we have always found that it performs similarly to the ML receiver.

4.1 Analytical Results

We begin our examination of the CMML receiver performance by showing that $P_D \to P_F$ as $N \to \infty$ regardless of the value of T_0. The single hop case then follows as a special case. In order to show this, we first show that for a fixed P_F, $Q_D \to 0$ as $N \to \infty$. Equations (30) and (31) give

$$Q_D = Q\left(Q^{-1}(Q_F) - d_M\right) \tag{38}$$

so that $Q_D \to 0$ if $Q_F \to 0$ as $N \to \infty$. The latter is proved by contradiction. Consider $0 < P_F < 1$ and assume that $\lim_{N\to\infty} Q_F \neq 0$. Then (37) leads to $\lim_{N\to\infty} P_F = P_F = 1$ which contradicts the original assumption that $P_F < 1$. Hence, $\lim_{N\to\infty} Q_F = 0$ and $Q_D \to 0$ as $N \to \infty$. Interpret now equations (36) and (37) in the following manner,

$$P_D = 1 - \sum_{j=0}^{T_0-1} \binom{M}{j} \left\{1 - \frac{1-Q_D}{1-Q_F}(1-Q_F)^N\right\}^j \bullet \left\{\frac{1-Q_D}{1-Q_F}(1-Q_F)^N\right\}^{M-j},$$

$$P_F = 1 - \sum_{j=0}^{T_0-1} \binom{M}{j} \left\{1 - (1-Q_F)^N\right\}^j \left\{(1-Q_F)^N\right\}^{M-j}.$$

Since $Q_D \to 0$ and $Q_F \to 0$ as $N \to \infty, P_D \to P_F$ as $N \to \infty$ for any choice of T_0.

It can also be shown that $P_D \to P_F$ as $N \to \infty$ for the NCMML receiver. The proof is similar to that for the CMML receiver except that Q_D and Q_F are given by (34) and (35) rather than (30) and (31). Again, this is true for any value of T_0.

To provide some motivation for our next analytical result we need a computational result. More computational results are given in the next section. Figure 5 presents an evaluation of (12) through (15) for the coherent ML receiver. We plot P_D as a function of SNR, $2E/N_0$, for practical P_F with $\gamma = \log_{10} N$ as a parameter. Note that there is an SNR below which interception is impractical as $P_D \simeq P_F$ and the intercepter has a problem distinguishing detections from false alarms. We now derive a simple formula for this SNR.

From equations (14) and (15),

$$P_D = \frac{1-Q_D}{1-Q_F} P_F + \frac{Q_D - Q_F}{1-Q_F}. \tag{39}$$

It has been established that $Q_D \to 0$ and $Q_F \to 0$ as $N \to \infty$

(Q_D and Q_F depend indirectly on N through η) for a constant false alarm detection probability P_F. Furthermore, using (12), (13) and an asymptotic expansion for the $Q(\cdot)$ function, it can be shown that $Q_D/Q_F \to \infty$ as $N \to \infty$. Therefore, for large N

$$P_D \approx P_F + Q_D \tag{40}$$

or using (12) $\to$ (15),

$$P_D \approx P_F + Q\left\{Q^{-1}\left[1-(1-P_F)^{\frac{1}{N}}\right] - d\right\} . \tag{41}$$

Now we wish to find the approximate SNR where P_D becomes close to P_F. Setting $P_D = 1.2P_F$ in (41) and using

$$(1-P_F)^{\frac{1}{N}} = e^{\frac{1}{N}\ell n(1-P_F)} \approx 1 + \frac{\ell n\,(1-P_F)}{N} \tag{42}$$

yields for large N,

$$d = \sqrt{2E/N_0} \approx Q^{-1}\left(\frac{-\ell n\,(1-P_F)}{N}\right) - Q^{-1}\left(P_F/5\right) . \tag{43}$$

To obtain a similar result for the noncoherent ML case, note that (39) is still valid for the present consideration. We show that $Q_D \to 0$ and $Q_F \to 0$ as $N \to \infty$ for fixed P_F in the noncoherent case. Using an asymptotic expansion [18:F.17] for the Marcum $Q-$function with (17) and (18) it can be shown that $Q_D/Q_F \to \infty$ as $N \to \infty$ and hence, for large N

$$P_D \approx P_F + Q_D \tag{44}$$

or

$$P_D \approx P_F + Q_M\left\{d, \left\{-2\ell n\left[1-(1-P_F)^{\frac{1}{N}}\right]\right\}^{1/2}\right\} . \tag{45}$$

Equation (45) may be used to find the approximate SNR where P_D becomes close to P_F for the noncoherent case. In the noncoherent case, this SNR is about 1 dB greater than that determined using (43) for the coherent case.

In analogy to equation (43) for the single-hop ML receiver, one may also derive an expression for the SNR at which P_D becomes approximately equal to P_F for the M hop/symbol case. Setting $P_D = 1.2P_F$ and $T_0 = 1$ in (37), and using (30) and (42) gives for large N,

$$d = \sqrt{2E/N_0} \approx \sqrt{M}\left[Q^{-1}\left(-\frac{\ell n\,(1-P_F)}{N\,M}\right) - Q^{-1}\left(P_F/5M\right)\right] .$$

4.2 Computational Results

We now consider the AL receiver. Results for a small number of frequencies can be found in [16]. Herein, and in [17], we treat the case of large N via the central limit theorem. Consider the coherent case where the likelihood ratio test is given by (4) where $\ell_i = e^{\alpha_i}$ and as such is a log normal random variable. The performance of this test estimated by assuming that $L = \sum_{i=1}^{N} \ell_i$ is normal is called the CLT approximation. When the signal-present hypothesis is enabled, there is only one term in this sum with a non-zero mean. This is a situation that is not good for the CLT approximation. As such we can invoke the CLT for $N-1$ terms and then average over the single non-zero mean term. We call this the $CLT+1$ approximation. This final averaging requires a numerical integration [17]. Finally the Wilkinson approximation [16] assumes that ℓnL is normal and is treated in [16]. Table 1 presents data on P_D, P_F and N for these approximations. The $CLT+1$ is the best. Note that the AL and ML receivers have similar performance. We have found similar [17] behaviours for the noncoherent case.

4.2.1 Detection Threshold

The intuitive trade-off involved in the choice of T_0 is the following. The greater the value of T_0, the smaller the likelihood of a false alarm, since more hop receivers must erroneously declare signal present. At the same time, the smaller the probability of detection, since T_0 or more hop receivers must declare signal present. That is, a greater T_0 leads to a smaller P_F but at the same time leads to smaller P_D. One wants to select T_0 to achieve the greatest P_D for a given P_F.

Figure 6 for the CMML receiver shows the ration P_D/P_F plotted as a function of P_F for $M = 3, T_0 = 1-3$ and $N = 10000$. Analogous results for the NCMML receiver for $M = 3$ and SNR $= 13$ dB were obtained [17]. Interestingly, the best performance is achieved for $T_0 = 1$. We have examined a number of cases for $M \leq 10$ and $T_0 = 1$ was best choice every time [17].

4.2.2. Multiple Hops per Symbol

The spread spectrum user may elect to hop several times per symbol to make the jamming of each symbol more difficult. That is, in some cases the symbol may be recovered because the jamming has only wiped out a part of the symbol. Hopping several times during the duration of a symbol is also desirable from the point of view of defeating an unauthorized intercepter. This is shown in Figure 7 where the ratio of the detection probability using $(N, M) = (10^4, M)$ to the detection probability using $(N, M) = (M \cdot 10^4, 1)$ is plotted as a function of P_F at SNR $= 13$ dB. The coherent case is shown and the noncoherent case is similar. Observe, for example, that hopping 10 times per bit results in a P_D that is 4.4×10^{-6} times the P_D that results from hopping once per bit over 10 times as many frequencies. This is for $P_F = 10^{-9}$, SNR $= 13$ dB, and $N = 10000$ in the coherent case. One also notes from observation of these Figures, that P_D approaches P_F very rapidly as M is increased. Corresponding to (40) and (44), one can show that for large N and M hops/symbol

$$P_D = (M \text{ hops/symbol}) \approx P_F + M\,Q_D \tag{46}$$

for both coherent and noncoherent cases. Comparison of (46) with (30) and (40) shows why hopping M times per symbol is so much more effective in reducing P_D to P_F than hopping M times as many frequencies once per symbol.

4.2.3 Integration Over Several Symbols

The interceptor may gain an advantage by integrating over

M_s symbols before deciding whether signal is present. In this section, the case where the transmitter hops M_H times per symbol and the MML receiver integrates over M_s symbol periods is examined. In this case, the energy E per detection interval and symbol energy E_s are related by $E = M_s \cdot E_s$. The MML receiver performance is given by (36) and (37) with Q_D and Q_F given by (30), (31) and (34), (35) for the coherent and noncoherent cases, respectively. However, now

$$d_M = \frac{2E}{M\, N_0} = \frac{2\, M_s E_s}{M_H M_s N_0} = \frac{2E_s}{M_H N_0}\,. \qquad (47)$$

Note that the energy per hop d_M in (47) does not depend on M_s. As a consequence, the probability of detection increases slowly as M_s increases. This is seen in Figure 8 where P_D is plotted versus P_F for SNR = 5 dB and $M_H = 1$. That is, the signal hops once per symbol interval and the receiver integrates over M_s symbols to improve the detection probability. Curves for $M_s = 1, 10, 100$ and 1000 are presented and P_F ranges from 10^{-3} to 10^{-9}. Observe that P_D is increased by less than a factor of 4.4 by integrating over 1000 symbols. Contrast these results shown in Figure 8 with those shown in Figure 9 concerning the influence of M_H. Note that d_M in (47) is inversely proportional to M_H. One sees from Figure 9 that for a receiver that integrates over 1000 symbols, the probability of detection is decreased by a factor of 4.4 if the transmitter hops three times per symbol rather than once per symbol. This is at a false alarm probability, $P_F = 10^{-9}$.

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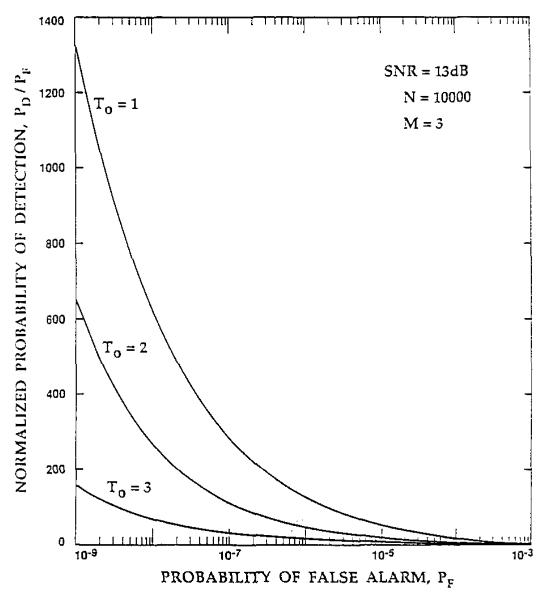

Figure 6: The normalized detection probability P_D/P_F as a function of P_F for the coherent MML receiver with M = 3.

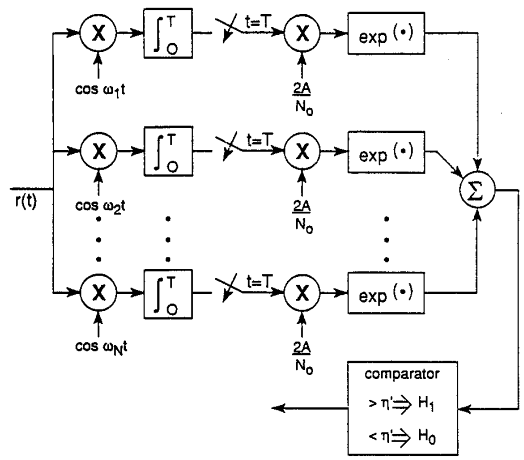

Figure 1: Optimum coherent receiver.

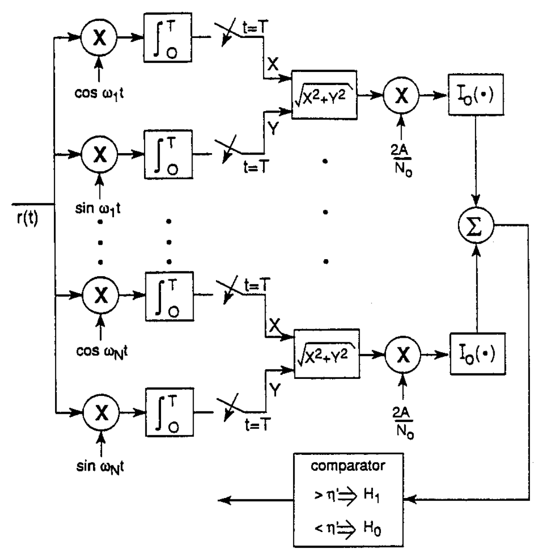

Figure 2: Optimum noncoherent receiver.

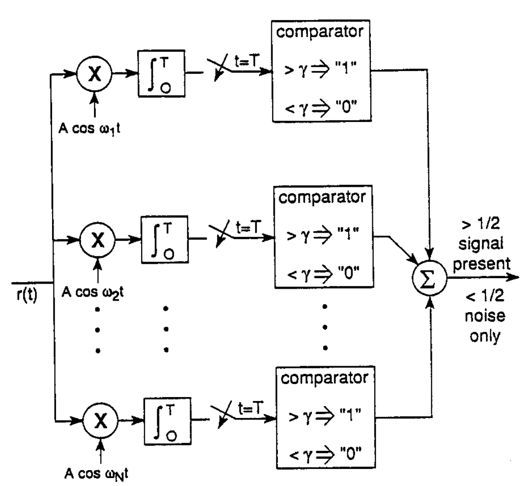

Figure 3: Coherent maximum likelihood receiver.

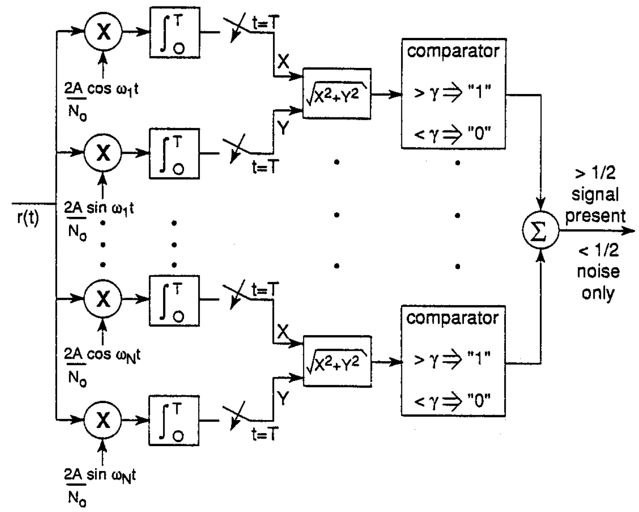

Figure 4: Noncoherent maximum likelihood receiver.

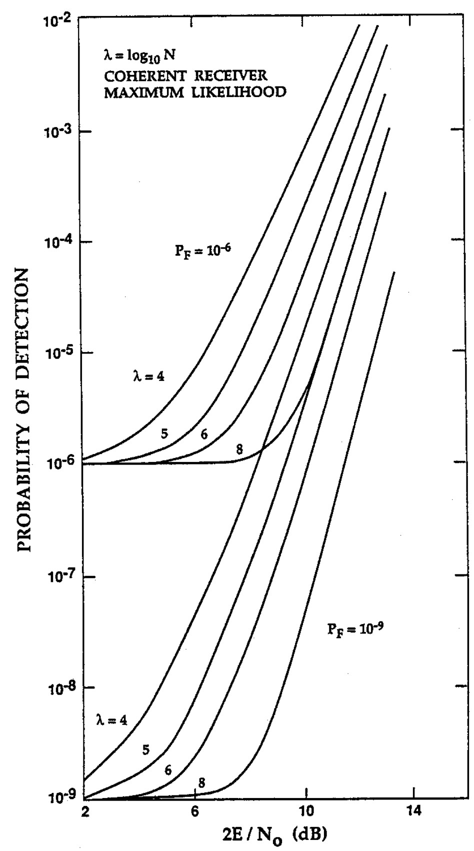

Figure 5: Probability of detection as a function of $2E/N_0$ for $P_F = 10^{-6}, 10^{-9}$ for the coherent ML receiver.

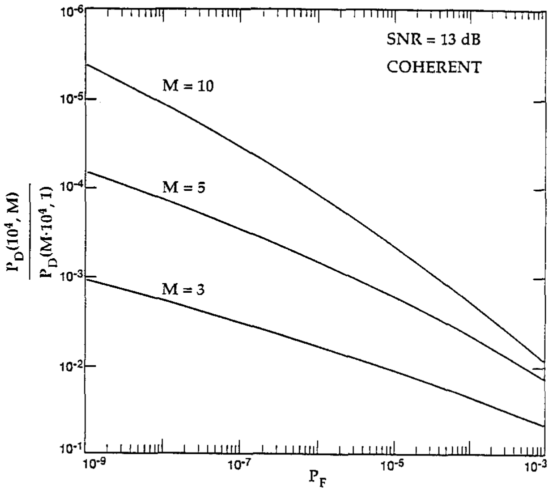

Figure 7: The ratio of P_D for a coherent MML receiver with $(N,M) = (10^4,M)$ to P_D for a coherent ML receiver with $(N,M) = (M \bullet 10^4, 1)$.

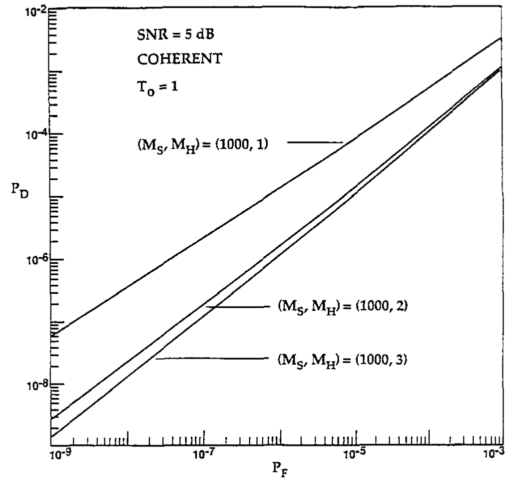

Figure 9: The detection probability, P_D, as a function of the false alarm probability, P_F, for a coherent MML receiver that integrates over 1000 symbols. The signal hops M_H times per symbol.

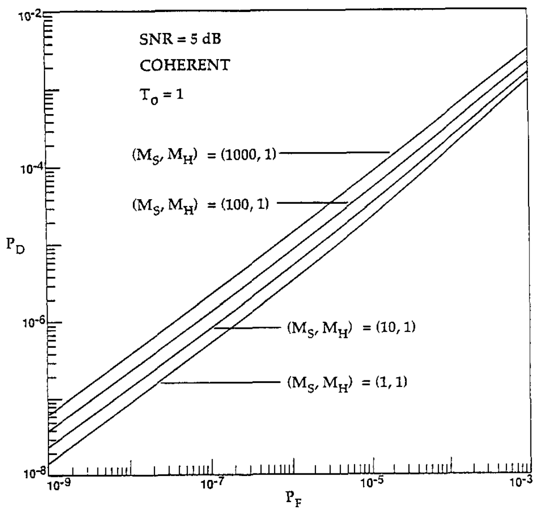

Figure 8: The detection probability, P_D, as a function of the false alarm probability, P_F. The signal hops once per symbol interval and the coherent MML receiver integrates over M_s symbols before making a detection decision.

	P_F					
$N = 10^4$	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
WLKN	1×10^{-0}	9×10^{-1}	8×10^{-1}	7×10^{-1}	5×10^{-1}	3×10^{-1}
CLT	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}
ML	4×10^{-1}	2×10^{-1}	1×10^{-1}	6×10^{-2}	3×10^{-2}	1×10^{-2}
CLT + 1	1×10^{-1}	1×10^{-1}	9×10^{-2}	9×10^{-2}	8×10^{-2}	8×10^{-2}
$N = 10^5$						
WLKN	9×10^{-1}	8×10^{-1}	7×10^{-1}	5×10^{-1}	3×10^{-1}	2×10^{-1}
CLT	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}
ML	2×10^{-1}	1×10^{-1}	6×10^{-2}	3×10^{-2}	1×10^{-2}	5×10^{-3}
CLT + 1	8×10^{-2}	7×10^{-2}	6×10^{-2}	5×10^{-2}	5×10^{-2}	5×10^{-2}
$N = 10^6$						
WLKN	9×10^{-1}	7×10^{-1}	5×10^{-1}	3×10^{-1}	2×10^{-1}	1×10^{-1}
CLT	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}
ML	1×10^{-1}	6×10^{-2}	3×10^{-2}	1×10^{-2}	5×10^{-3}	2×10^{-3}
CLT + 1	5×10^{-2}	4×10^{-2}	3×10^{-2}	3×10^{-2}	3×10^{-2}	3×10^{-2}
$N = 10^7$						
WLKN	8×10^{-1}	5×10^{-1}	4×10^{-1}	2×10^{-1}	1×10^{-1}	6×10^{-2}
CLT	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}
ML	7×10^{-2}	3×10^{-2}	1×10^{-2}	5×10^{-3}	2×10^{-3}	7×10^{-4}
CLT + 1	4×10^{-2}	2×10^{-2}	2×10^{-2}	2×10^{-2}	2×10^{-2}	2×10^{-2}
$N = 10^8$						
WLKN	6×10^{-1}	4×10^{-1}	2×10^{-1}	1×10^{-1}	6×10^{-2}	3×10^{-2}
CLT	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}	5×10^{-1}
ML	4×10^{-2}	1×10^{-2}	5×10^{-3}	2×10^{-3}	7×10^{-4}	3×10^{-4}
CLT + 1	2×10^{-2}	1×10^{-2}	1×10^{-2}	9×10^{-3}	8×10^{-3}	8×10^{-3}

TABLE 1

WLKN, CLT and CLT + 1 Approximations to P_D for Coherent Optimum AL Receiver and P_D for ML Receiver, SNR = 13 dB