Transient Signal Detection Using Wigner-Ville Distribution and Wavelet Denoising

Nawal Kishor Mishra and G.V.Anand
Department of Electrical Communication Engineering
Indian Institute of Science, Bangalore-560012, India
anandgv@ece.iisc.ernet.in

Abstract

The problem of detecting an unknown transient signal in noise is considered. The SNR of the observed data is first enhanced using wavelet domain filter. The output of the wavelet domain filter is then transformed using a Wigner-Ville transform,which separates the spectrum of the observed signal into narrow frequency bands. Each subband signal at the output of the Wigner-ville block is subjected to wavelet based level dependent denoising (WBLDD)to supress colored noise .A weighted sum of the absolute value of outputs of WBLDD is passed through an energy detector,whose output is used as test statistic to take the final decision.By assigning weights proportional to the energy of the corresponding subband signals,the proposed detector approximates a frequency domain matched filter. Simulation results are presented to show that the performance of the proposed detector is better than that of the wavelet packet transform based detector.

1. Introduction

The problem addressed in this paper is the detection of a transient signal of unknown waveform and arrival time, in noise. Detection of transients plays an important role in many applications such as, in the detection of transient acoustic signals in the ocean by sonar, in the detection and localization of transients in ECG and EEG and in automatic machinery monitoring[1].Detection involves the following hypothesis test

$$H_0 : x(n) = w(n), \qquad n = 0, 1, 2, ..., N-1$$
$$H_1 : x(n) = s(n) + w(n), n = 0, 1, 2, ..., N-1.$$

The hypothesis H_1 implies the presence of signal plus noise while hypothesis H_0 implies noise alone.The problem is to decide which hypothesis is true,based on the observation samples $\{x(n)\}_{n=0}^{N-1}$.Most of the energy of a transient signal is confined to a small number of samples.Normally,the signal shape and arrival time are unknown,and the signal energy is small compared to the noise energy in an observation frame of N samples.Hence,classical detection techniques[2] such as matched filtering,generalized likelihood ratio test(GLRT) or energy detector are not suitable for transient signal detection.Most of the transient detection techniques are based on the use of the wavelet transform or wavelet packet transform which can achieve compaction of the signal into a small number of large coefficients[3, 4].Recently,Sattar and Salomonsson[5] have proposed a detector based on the time-frequency representation of the signal,using filter bank and higher order statistics.In this paper a new method of transient detection based on the use of wavelet domain filtering,Wigner-Ville distribution,and wavelet denoising is proposed.Reciever operating characteristics(ROC) are presented to illustrate the improved performance of the new detector compared to other transient detectors.

2. Proposed Detector

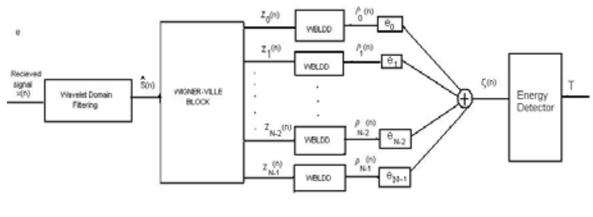

Figure 1. Block diagram of proposed detector

A schematic diagram of the proposed detector is shown in Fig1.The data *x(n)* is passed through an adaptive wavelet domain filter[6] to obtain an estimate $\hat{s}(n)$ of the signal

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s(n).This is a nonlinear filter that also provides an effective SNR gain.A Wigner-Ville transformation of $\hat{s}(n)$ is carried out to decompose $\hat{s}(n)$ into N subband signals $z_k(n), k = 0, 1, ..., N-1$.The signals $z_k(n)$ are subjected to wavelet based level dependent denoising(WBLDD) to obtain the denoised subband signals $\rho_k(n)$.The weighted sum $\zeta(n) = \sum_{k=0}^{k=N-1} e_k|\rho_k(n)|$,where the weight e_k is equal to the energy of the subband signal $\rho_k(n)$,approximates the output of a frequency domain matched filter.The energy of $\zeta(n)$ is used as the test statistic for arriving at the final decision.

2.1. Wavelet Domain Filter

Consider a data vector $\boldsymbol{x} = [x[1] \;\; x[2] \; ... \; x[N]]^T$, where $N = 2^J$ and J is a positive integer. $\boldsymbol{x} = \boldsymbol{s} + \boldsymbol{w}$ if the signal is present and $\boldsymbol{x} = \boldsymbol{w}$ otherwise, where $\boldsymbol{s}$ is the transient signal and $\boldsymbol{w}$ is i.i.d noise with mean 0 and variance σ^2. Consider wavelet decomposition of $\boldsymbol{x}$ upto a certain level $J_0 (0 < J_0 < J)$ to get noisy scaling and wavelet coefficients.The scaling and wavelet coefficients are collectively denoted by the vectors $\mathbf{c}$ and $\mathbf{d}$ respectively.The goal of wavelet domain filtering is to estimate the signal wavelet coefficients by filtering $\mathbf{d}$.For this purpose,each detail coefficient d_I is multiplied by a weighting factor α_I where $0 \leq \alpha_I \leq 1$, to get

$$\hat{d}_I = \alpha_I d_I \tag{1}$$

as the estimate of the corresponding coefficient of the noise-free signal. Taking the inverse DWT of the filtered coefficient vector $\hat{\mathbf{d}}$ and the scaling vector $\mathbf{c}$, an estimate $\hat{\mathbf{s}}$,of the noise free signal is obtained.The estimate $\hat{\mathbf{s}}$ is called a *wavelet-domain filtered* version of $\mathbf{x}$.The collection of weights $\boldsymbol{\alpha}$=$\{\alpha_I\}$ is the *wavelet domain filter*.

The crucial issue in wavelet-domain filtering is the design of filter $\boldsymbol{\alpha}$.It is assumed that each noisy wavelet coefficient is an unbiased estimator of the signal's wavelet coefficient.That is, $\delta_I = E[d_I]$ is the wavelet coefficient of the noise free signal.Then the filter weight that minimizes the mean square error $E[(\delta_I - \alpha_I d_I)^2]$ of the I^{th} coefficient is

$$\alpha_I^{MSE} = \frac{\delta_I^2}{\delta_I^2 + \sigma^2} \;\; , \tag{2}$$

where σ^2 is the variance of the noisy coefficient d_I. The filtering operation may be interpreted as a wavelet-domain analog of the classical Wiener filter.Unfortunately,this optimal filter weight requires δ_I and σ^2.It is not unreasonable to assume that σ^2 is known;excellent estimates are usually easily obtained, e.g., from the MAD estimate[7].The difficulty lies with the unknown value of δ_I.

The problem can be solved by using an estimate of δ_I^2 in place of its true value.Suppose that $d_I \sim \mathcal{N}(0,\gamma^2)$,i.e., zero-mean Gaussian distributed with variance γ^2, with $\gamma^2 = \delta_I^2 + \sigma^2$.Then,given n iid $\mathcal{N}(0,\gamma^2)$ observations $x_1, x_2, ..., x_n$ the minimum mean square error(MSE) estimator of γ^2 of the form $\hat{\gamma}^2 = k(x_1^2 + x_2^2 + ... + x_n^2)$ is achieved for k=1/(n+2).In our case n=1,hence k=1/3,$\hat{\gamma}^2 = d_I^2/3$, and

$$\hat{\delta}_I^2 = (\frac{1}{3}d_I^2 - \sigma^2)_+ \;\; , \tag{3}$$

where $(.)_+$ stands for "the positive part of", i.e., $(x)_+ = x$, if $x \geq 0$, and $(x)_+ = 0$, if $x < 0$.Replacing δ_I^2 by $\hat{\delta}_I^2$ in Eq(2) produces

$$\alpha_I = (\frac{d_I^2 - 3\sigma^2}{d_I^2})_+. \tag{4}$$

This operation can be viewed as a data-adaptive,wavelet domain,Wiener filter.The action of Eq(4) is to set small wavelet coefficients,with squared magnitude less than three times the estimated variance, to zero and to leave the larger coefficients approximately unaltered.

Wavelet domain filtering yields significant SNR gain as shown in Fig2.The signal is a damped sinusoid of the form

$$s[n] = A \exp(-\alpha(n - N_0)) \cos(2\pi f(n - N_0) + \phi), \quad n = N_0, N_0 + 1, ...N \tag{5}$$

with time of arrival $N_0 = 80$ and frame size $N = 128$.The other signal parameters are amplitude $A = 1.5$,frequency $f = 0.1$ cycles/sample,phase $\phi = 0$ radians,decay parameter $\alpha = 0.1$/sample.The noise *w(n)* has independent and identically distributed(iid) samples with mean 0,variance σ^2,and Generalized Gaussian(GG) pdf defined as

$$f(w) = a \exp(-b|w|^p) \text{ for } p > 0, \tag{6}$$

where

$$a = \frac{p\Gamma^{\frac{1}{2}}(3/p)}{2\sigma\Gamma^{\frac{3}{2}}(1/p)} \, , b = \sigma^{-p} \left\{ \frac{\Gamma(3/p)}{\Gamma(1/p)} \right\}^{p/2} . \tag{7}$$

These pdfs are chosen since they provide good models for a wide variety of noise environments in the ocean[8].The noise variance is $\sigma^2 = 1$ in Fig2.Daubechies-8(db 8) wavelets are used in the construction of the wavelet filter,and 3 levels of decomposition are carried out.It is seen that the SNR gain is higher for weaker signals and for noise pdf with lighter tails(larger values of p).

2.2. Wigner-Ville Distribution

The Wigner-Ville block in Fig1 decomposes $\hat{s}(n)$ into N subband signals

$$z_k(n) = P_V \hat{s}(n,k) = \sum_{p=-N}^{p=N-1} \hat{s}(n + p/2)\hat{s}^*(n - p/2) \; exp(-\frac{i2\pi kp}{N}), 0 \leq n, k \leq N-1. \tag{8}$$

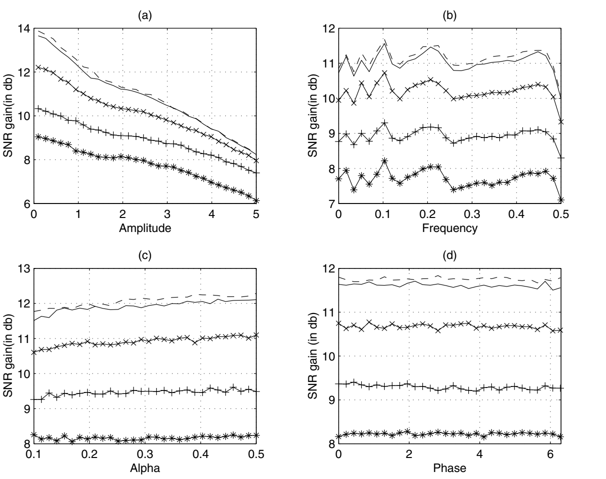

Figure 2. Variation of SNR gain with each of the signal parameters for GG pdfs with p= 0.5(star), 1(plus), 2(cross), 8(solid),30(dashed)

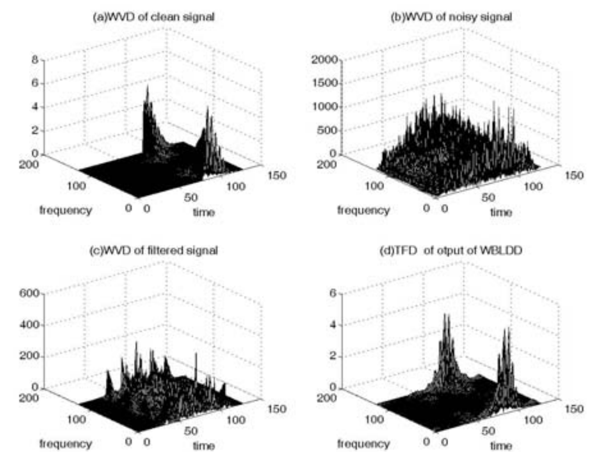

Figure 3. Absolute value of (a) WVD of clean signal *s(n)* **(b)WVD of noisy signal** *x(n)* **for GG noise with p=1(c)WVD of filtered signal$\hat{s}(n)$(d)TFD of output of WBLDD**

The expression on the right hand side of Eq(8) represents the discrete Wigner-Ville distribution (WVD) of $\hat{s}(n)$.Figures3(a)-(c)show the absolute value of WVD of the clean signal *s(n)*, the noisy signal *x(n)*, and the filtered signal $\hat{s}(n)$ respectively.Although the noise *w(n)* is uncorrelated,the residual noise in the filtered signal $\hat{s}(n)$ is correlated.Comparison of Fig3(a) and 3(c) reveals that noise has been partially removed.Further suppression of the correlated noise in each filtered subband signal $z_k(n)$ is achieved by the WBLDD blocks described in the next section.

2.3. Wavelet Based Level Dependent Denoising

The simplest wavelet thresholding techniques use a common threshold $\eta = \sigma\sqrt{2ln(N)}$to denoise all the coefficients in the wavelet transform of the noisy signal.These methods work well only if the noise is white.A different approach,based on level dependent thresholds,is required to suppress coloured noise.The WBLDD described here is motivated by the following observations:

(1)Since the filters used to construct the discrete wavelet transform are time invariant, for signal corrupted with stationary but correlated noise,the variance of the wavelet coefficients will depend on the level in the wavelet decomposition,but will be constant within each level.

(2) Even though the original data may be highly correlated,the correlation within each level,for many models likely to be of relevance in practice, dies away rapidly.Qualitatively,this is a consequence of the fact that wavelets are 'almost eigenfunctions' of many operators.

(3)Since the mapping from the original time domain samples to wavelet coefficients at different levels is obtained by the action of bandpass filters,there is little or no correlation between the wavelet coefficients at different levels.

Observations(1) and (2) imply that we can treat noise at each level as almost white and hence can apply a fixed threshold within each level.Observation(3) implies that different levels can be treated independently.

Thus WBLDD involves taking the wavelet transform of a subband signal,estimating the standard deviation σ_j and applying soft thresholding [9] with threshold $\eta_j = \sigma_j\sqrt{2ln(2^j)}$ to the transformed data at level j, and finally taking the inverse wavelet transform.

Let $\rho_k(n)$ denote the output of the WBLDD block used to denoise the k^{th} subband signal $z_k(n)$.The collection $\{\rho_k(n)\}_{k=0}^{N-1}$ constitutes the time-frequency distribution(TFD) of the denoised version of $\hat{s}(n)$.This TFD is shown in Fig3(d) .A comparison of Fig3(a),3(c) and 3(d)indicates that a significant amount of correlated residual noise present in the filtered signal $\hat{s}(n)$ is removed by the combination of WVD and WBLDD.

3. Test Statistic

The test statistic T used for detection is the energy of the weighted absolute sum of the denoised subband signals $\rho_k(n)$,i.e.,

$$T = \Sigma_n \zeta^2(n), \tag{9}$$

$$\zeta(n) = \sum_{k=0}^{k=N-1} e_k|\rho_k(n)|, \quad 0 \le n \le N-1 \qquad (10)$$

and the weight e_k is equal to the energy of the k^{th}sub-band signal $\rho_k(n)$,i.e

$$e_k = \sum_{n=0}^{n=N-1} \rho_k^2(n) \qquad (11)$$

This choice of weights emphasizes subbands with higher energy content.Hence the weighted sum in Eq(10) mimics a frequency domain matched filter. The decision rule is:decide H_1 if $T > \gamma$ and decide H_0 if $T \le \gamma$, where γ is the detection threshold.

4. Simulation Results

Simulations were carried out using the signal and noise models described in Section 2.1.The number of observed samples was N=128 and 2000 different realizations each of noise and signal-plus-noise were used.For each realization of the transient, the parameters were chosen as follows: $A/\sigma \sim \mathcal{U}[2, 2.6]$, $\phi \sim \mathcal{U}[0, 2\pi]$, $\alpha \sim \mathcal{U}[0.05, 0.1]$, $f \sim \mathcal{U}[0.05, 0.1]$ (where $x \sim \mathcal{U}[a, b]$ means that x is uniformly distributed between a and b).For WBLDD 3-level decomposition was carried out using the db8 wavelet.ROCs of the proposed detector,a similar detector without the wavelet domain filter,and the wavelet packet detector[4] are presented in Fig4.It is seen that even without wavelet domain filter,the proposed detector performs better than the wavelet packet detector.Addition of the wavelet domain filter leads to a further improvement in performance.

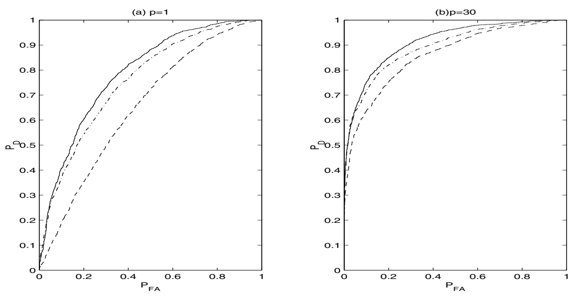

Figure 4. ROCs for GG noise with (a)p=1, (b)p=30, solid line:Proposed detector;Dashed dot:Proposed detector without wavelet domain filter,Dashed:WP detector

5. Conclusion

A novel time-frequency domain transient detector comprising an adaptive wavelet domain filter,a bank of subband WBLDD processors,and a frequency domain matched filter is proposed.The wavelet domain filter and the WBLDD processor bank complement each other,and each of them constributes to noise removal.The proposed detector does not require any knowledge of the signal shape,signal arrival time or noise pdf.The proposed detector gives a significantly better performance compared to the wavelet packet detector over a range of noise pdfs belonging to the generalized Gaussian family.

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