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Blind Parameter Estimation of Frequency-Hopping Signals Based on Atomic Decomposition

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***Abstract*—The parameter estimation of frequency hopping signals under non-cooperative conditions is a key problem to which more attentions are being paid in the field of signal processing. According to the structure of frequency hopping signals, three-parameter time-frequency atoms were obtained from Gabor functions by successive applications of scaling, time-shift and frequency-shift operators, and a new atomic decomposition based parameter estimation method of frequency-hopping signals was developed. Using this method, the parameters of frequency hopping signals, such as hop duration, hopping time and hopping frequencies, can be estimated without knowing any priori knowledge. Simulation results show that the method is effective.**

Keywords-frequency hopping; parameter estimation; atomic decomposition

I. INTRODUCTION

Frequency hopping (FH) signal is a typical kind of non-stationary signal, its parameters usually can be obtained by time-frequency analysis methods. An algorithm to estimate hop duration, hopping time and hopping frequencies by using WVD is proposed in [1]. PWVD and SPWVD are proposed respectively in [2-4] in place of WVD to effectively reduce influence of cross-term. These algorithms have better estimation performance, however, they all need multiple complete hop durations as analysis basis. With the request of real-time detection, it means that these algorithms can only be applied to the blind parameter estimation for frequency hopping signals with high hop rate, as for frequency hopping signals with low hop rate, because the number of complete hop duration is very limited in the sample, these algorithms will lead to worse performance.

Atomic decomposition (AD), which can also be called as matching pursuit, is an adaptive approximation technology, it can obtain the sparse representation of signals by decomposing the signals into a cluster of time-frequency atoms. AD has been widely used in the time-frequency analysis and parameter estimation of multi-component signals. Each hop of FH signal can be treated as a signal component, and FH signal can be regarded as a linear combination of multiple signal components. Considering the characteristics of the FH signals, an algorithm of parameter estimation of FH signal based on AD is proposed. The advantages of the algorithm proposed can be listed as follow: ① no cross-term interference; ② strong anti-noise ability; ③ less complete hop duration is needed.

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II. AD OF FH SIGNAL

The basic idea of atomic decomposition is to iterative approximate signal $f(t)$ by orthogonal projection of $f(t)$ on time-frequency atoms in redundant function dictionary D. Suppose that the function dictionary is $D=\{g_{\gamma_0}, g_{\gamma_1}, \ldots, g_{\gamma_n}\}$, where $\|g_{\gamma_i}\|=1$ (γ_i is parameter vector, $i\in[0,n]$), and the qth order AD is

$$f(t)=\sum_{i=0}^{q}\left\langle R^i f(t), g_{\gamma_i}\right\rangle g_{\gamma_i}+R^{q+1}f(t) \tag{1}$$

$\langle\bullet\rangle$ denotes inner product. $R^i f(t)$ is residual of the ith decomposition, and $R^0 f(t)=f(t)$.

In the first decomposition, pick out the best match atom g_{γ_0} in accordance with the principle of the largest inner product, and $f(t)$ can be composed as

$$f(t)=R^0 f(t)=\left\langle f(t), g_{\gamma_0}\right\rangle g_{\gamma_0}+R^1 f(t) \tag{2}$$

Where, $R^1 f(t)$ denote residual of approximation in the direction of g_{γ_0}.

Suppose that g_{γ_n} is the best match atom of $R^n f(t)$ in the nth decomposition, then $R^n f(t)$ can be decomposed as

$$\begin{cases} R^n f(t)=\left\langle R^n f(t), g_{\gamma_n}\right\rangle g_{\gamma_n}+R^{n+1}f(t) \\ g_{\gamma_n}=\arg\max\limits_{g_{\gamma_i}\in D}\left|\left\langle R^n f(t), g_{\gamma_n}\right\rangle\right| \end{cases} \tag{3}$$

Where, $|\bullet|$ is modular arithmetic. Due to $R^{n+1}f(t)$ is orthogonal to g_{γ_n}, energy conservation equation of $f(t)$ can be deduced as

$$\begin{aligned} \|R^n f(t)\|^2 &= \left|\left\langle R^n f(t), g_{\gamma_n}\right\rangle\right|^2+\|R^{n+1}f(t)\|^2 \\ \|f(t)\|^2 &= \sum_{i=0}^{M}\left|\left\langle R^i f(t), g_{\gamma_i}\right\rangle\right|^2+\|R^{M+1}f(t)\|^2 \end{aligned} \tag{4}$$

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Where, $\|\bullet\|$ denote norm.

If the dictionary D is complete, the iteration result will converge to $f(t)$ as

$$f(t)=\sum_{i=0}^{\infty}\left\langle R^i f(t),g_{\gamma_i}\right\rangle g_{\gamma_i} \quad (5)$$

Set hop duration of FH signal as T_H, assume that there are K complete hops in the signal sample and the center frequency of the kth complete hop is f_k, the foremost hop is incomplete, whose duration is Δt_0 which equals to the first hopping time t_1, and its center frequency is f_0, the mth hopping time is $t_1+(m-1)T_H$, and the aftermost hop is also incomplete whose duration is Δt_E and its center frequency is f_E. Thus, the FH sample can be expressed as:

$$f(t)=s(t)\begin{pmatrix}\exp\left[j\left(2\pi f_0 t+\phi_0\right)\right]rect\left(\dfrac{t}{\Delta t_0}\right)\\ +\sum_{k=1}^{K}\exp\left[j\left(2\pi f_k t+\phi_k\right)\right]rect\left(\dfrac{t-(k-1)T_H-\Delta t_0}{T_H}\right)\\ +\exp\left[j\left(2\pi f_E t+\phi_E\right)\right]rect\left(\dfrac{t-KT_H-\Delta t_0}{\Delta t_E}\right)\end{pmatrix} \quad (6)$$

$$0<t\le L$$

Where, $s(t)$ is baseband complex envelope, ϕ_0 and ϕ_E is the initial phase of the foremost hop and the aftermost hop respectively, and ϕ_k is the initial phase of the kth complete hop, and the rectangular window $rect(t)$ can be expressed as

$$rect(t)=\begin{cases}1 & , \quad t\in(0,1]\\ 0 & , \quad \text{else}\end{cases}$$

As shown in (6), actually, all of the hops of FH sample are limited sinusoidal signals without overlapping each other. Therefore, Gabor function dictionary is selected to decompose FH signal sample in this paper. The Gabor function can be expressed as

$$g_\gamma(t)=\left(\frac{\alpha}{\pi}\right)^{1/4}e^{-\frac{\alpha}{2}(t-T)^2}\cdot e^{j[2\pi f(t-T)]} \quad (7)$$

The parameter vector γ is

$$\gamma=[\alpha,T,f]^T \quad (8)$$

Where, α is a parameter which has relation to the duration of atom, d, as

$$d=\sqrt{2/\alpha} \quad (9)$$

T and f is the mid-point of time and center frequency respectively.

III. STOPPING CRITERION

For FH signals without any priori knowledge, the number of hops in samples cannot be predicted. If the iterations of AD are less than the number of hops, some hops will be missed. On the other hand, if the iterations are more than the number of hops, fake hops will be decomposed which may influence the precision of parameter estimation. Thereby, it is a very important factor to find an appropriate stopping criterion. In this paper, a stopping criterion is presented according to the variation law of residual energy.

Assume that there are M hops in the signal sample $\{f(t)\}_{t\in[0,L]}$. According to energy conservation equation (4), after n iteration, energy ratio between $\sum_{i=0}^{n-1}\left\langle R^i f(t),g_{\gamma_i}\right\rangle g_{\gamma_i}$ and residual $R^n f(t)$ can be expressed as

$$\begin{aligned}RE_n &\triangleq \frac{\sum_{i=0}^{n-1}\left|\left\langle R^i f(t),g_{\gamma_i}\right\rangle g_{\gamma_i}\right|^2}{\left\|R^n f(t)\right\|^2}=\frac{\|f(t)\|^2-\left\|R^n f(t)\right\|^2}{\left\|R^n f(t)\right\|^2}\\ &=\frac{\|f(t)\|^2}{\left\|R^n f(t)\right\|^2}-1\end{aligned} \quad (10)$$

Due to $\quad \left|R^{n-1}f(t)\right|^2>\left|R^n f(t)\right|^2>\left|R^{n+1}f(t)\right|^2$

thus $\quad RE_{n-1}<RE_n<RE_{n+1}$

When $n=M$,

$$RE_{M-1}<RE_M<RE_{M+1} \quad (11)$$

Suppose that the result of the first iteration of AD, $\left\langle f(t),g_{\gamma_0}\right\rangle g_{\gamma_0}$, is approximation of the kth ($k\in[1,M]$) hop, energy estimation of the kth hop can be expressed as

$$\hat{P}_{hop_k}=\left|\left\langle f(t),g_{\gamma_0}\right\rangle g_{\gamma_0}\right|^2 \quad (12)$$

Set the part of $R^1 f(t)$ within the duration g_{γ_0}, $t\in[(T_0-d_0/2),(T_0+d_0/2)]$, as $R^1 f_{\gamma_0}(t)$, its energy is

$$P_{R^1 f\gamma_0}=\left|R^1 f_{\gamma_0}(t)\right|^2 \quad (13)$$

Energy ratio between $\left\langle f(t),g_{\gamma_0}\right\rangle g_{\gamma_0}$ and $R^1 f_{\gamma_0}(t)$ is

$$r_1=\frac{\hat{P}_{chip_k}}{P_{R^1 f\gamma_0}}=\frac{\left|\left\langle f(t),g_{\gamma_0}\right\rangle g_{\gamma_0}\right|^2}{\left|R^1 f_{\gamma_0}(t)\right|^2} \quad (14)$$

After the ith iteration, it can be obtained that

$$r_i=\frac{\left|\left\langle f(t),g_{\gamma_{i-1}}\right\rangle g_{\gamma_{i-1}}\right|^2}{\left|R^i f_{\gamma_{i-1}}(t)\right|^2} \quad (15)$$

So the stopping criterion can be expressed as follow: after the *(n+1)*th iteration, if r_i satisfy

$$RE_n\le r_i<RE_{n+1}\ \forall i\in[1,n] \quad (16)$$

Then iteration is stopped, and the *(n+1)*th iteration is deemed to be invalid, and the effective iterations is n.

In practice, in order to reduce the effect of error, stopping criterion ought to be modified as follow: after the *(n+1)*th iteration, if r_i satisfy

$$RE_n\le\frac{1}{n}\sum_{i=1}^{n}r_i<RE_{n+1} \quad (17)$$

Then iteration is stopped, and the *(n+1)*th iteration is deemed to be invalid, and the effective iterations is n.

IV. PARAMETER ESTIMATION

Assume that after the *(n+1)*th iteration, AD is stopped and the effective iterations is *n*. The parameters of FH signal can be estimated as follow:

Step1. Estimate hop duration T_H and the first hopping time t_1

Sort the atoms $\{g_{\gamma_0}, g_{\gamma_1}, \ldots, g_{\gamma_{n-1}}\}$ in the numerical order of $\{T_0, T_1, \ldots, T_{n-1}\}$. Calculate the arithmetic average of durations of these atoms in the middle, which can be defined as $\bar{d}$, and $\bar{d}$ is the estimation of hop duration T_H, it can be expressed as $\widehat{T_H} = \bar{d}$. Supposed the foremost atom is g_{γ_p} (where $p \in [0, n-1]$), its duration d_p is the estimation of the first hopping time t_1, which can be expressed as $\hat{t}_1 = d_p$.

Step2. Estimate hopping frequencies

Assume that the *m*th atom in the numerical order of $\{T_0, T_1, \ldots, T_{n-1}\}$ is g_{γ_q}, and its carrier frequency f_q is the *m*th hopping frequency f_m, it can be expressed as $\hat{f}_m = f_q$.

Thus, hop duration, hopping time and hopping frequencies have been estimated.

V. PERFORMANCE ANALYSIS

A FH signal sample, mixed with white gauss noise, is used in simulations. Hopping frequencies are 13Hz, 20Hz, 50Hz, 37Hz, 26Hz, 43Hz, 10Hz, 22Hz, 33Hz, 45Hz, 8Hz and 15Hz respectively. Sampling frequency is 200Hz. Discrete hop duration is 90 points. The first discrete hopping time is 80 points. Genetic algorithm is applied as global search algorithm in AD. The population is 200, the maximal generation number is 20 and crossover rate is 0.7.

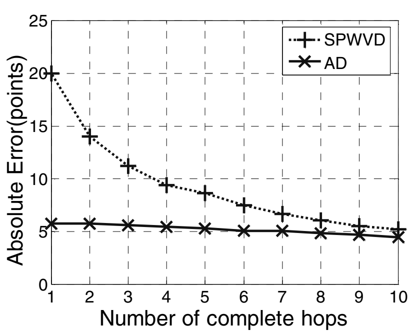

Figure 1. The influence to hop duration estimation from the number of complete hops

The relation between the number of complete hops and estimation error of discrete hop duration under SNR=3dB is shown in fig.1. "SPWVD" denotes the algorithm in [2], "AD" denotes the algorithm proposed in this paper. From fig. 1, it can be seen that, with the reduction of the number of complete hops, estimation errors of the algorithm in [2] increase rapidly, but estimation errors of the proposed algorithm change very little. This shows that the proposed algorithm is more suitable for the estimation of hop duration of FH sample with very few complete hops than the algorithm in [2].

Performance curves of discrete hop duration estimation of FH samples with only one complete hop under different SNR is shown in Fig.2, the algorithm proposed is much better than the algorithm in [2]. When SNR reaches 5dB, performance of the algorithm in [2] closes the performance of the algorithm proposed. When SNR is less than 4dB, performance of the algorithm in [2] deteriorates rapidly.

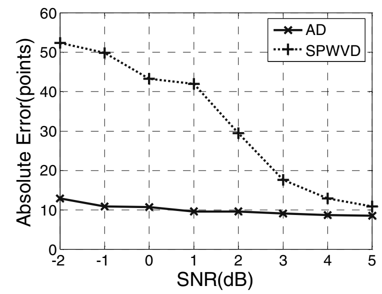

Figure 2. The curve of estimation errors of discrete hop duration

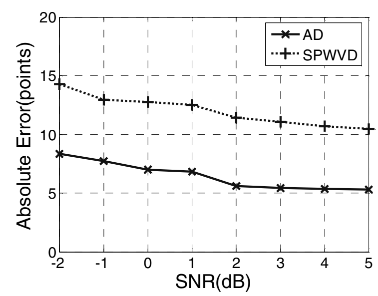

Figure 3. the curve of estimation errors of the first discrete hopping time

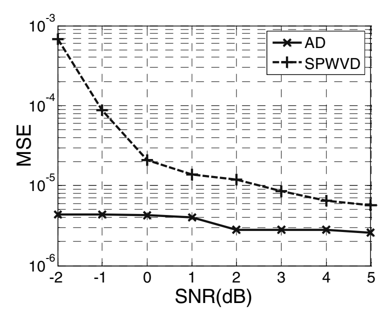

Figure 4. MSE curves of estimation of normalized hopping frequencies

The curves of estimation errors of the first discrete hopping time using the two algorithms are given respectively in Fig.3. Because the estimation performance of hopping time using the algorithm in [2] is influenced by the estimation of hop duration, in order to analyze it alone, the estimation of discrete hop duration is equaled to the actual value. MSE curve of normalized hopping frequencies is shown in Fig. 4. It can be seen that, in the estimation of discrete hopping time and hopping frequencies, the algorithm proposed is better than the algorithm in [2].

VI. CONCLUSIONS

In this paper, atomic decomposition is introduced to time-frequency decomposition of FH signal, and a new parameter estimation algorithm is presented to estimate hop duration, hopping time and hopping frequencies. The algorithm proposed does not need any prior knowledge of signals. Comparing with algorithms[1-4] based on quadratic time-frequency distribution, the advantage of the proposed algorithm lies on that it need less complete hops, therefore it more satisfies real-time application.

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