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Analysis and parameter estimation of time-varying signals: theory and methods

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Abstract. The need to capture signals, perform classification and make decisions are components of an Industry 4.0 manufacturing system. Since signals in practice are corrupted by noise, it is essential to optimize the signal processing and classification to ensure correct decision making. The time-varying nature of signals requires the use of time-frequency analysis over conventional methods such as spectrum estimation to ensure accurate estimation of signal parameters. The methods presented cover the main classes of time-frequency distributions – linear and quadratic- and compare their strengths and weaknesses for the appropriate choice of application.

1. Background

Industry 4.0 is enabling the current industry revolution that occurred in manufacturing through computers and automation by enhancing it with intelligent and autonomous systems powered by machine learning [1]. Among the developed countries, the German government introduced it as a strategic initiative to transform industrial manufacturing through digitalization and the exploitation of new technologies [2]. Components of Industry 4.0 manufacturing system among others are [3]: Internet of Things (IoT) platforms, system integration, simulation, augmented reality, information security, additive manufacturing, autonomous systems and big data analytics.

IoT offers a platform for sensors and devices to communicate seamlessly within a smart environment and enables multi-platform sharing of information. Various adoptions of IoT includes as smart office, smart retail, smart agriculture, smart water, smart transportation, smart healthcare, and smart energy. Similar to the OSI layers for information exchange, the reference layers for IoT starts from the highest layers [4] that is client/external communications, event processing and analytics (including data storage), aggregation/bus layer, transports, and devices. The cross cutting layer ensures smooth transfer of data between the various layers.

Depending on the application, the signal that is picked by the devices has to be represented in an efficient form to facilitate parameter estimation suitable for transfer (aggregation/bus layer), analysis, storage and decision making at the higher layer. Event processing and analytics often referred as big data analytics [5] forms the main functions of analysis, storage and decision making before further action can be taken at the client/external communication layer. Efficient representation of signal captured by the devices with the



appropriate parameter estimation ensures that the true signal characteristics are correctly represented by increasing the probability of detection or classification, and reduce retrieval time and storage space.

This paper first discusses the different types of signals and their representation in time, frequency and joint time-frequency representation. The spectrum estimation methods are presented by highlighting their strengths and weaknesses. To improve representation of time-varying signals, time-frequency distributions are introduced by presenting their different class and the evolution of the various methods. Finally, the robustness of the methods for instantaneous frequency (IF) estimation in noise are presented.

2. Signal Model

Time-varying continuous-time representation of an arbitrary signal that consists of multiple sinusoids labelled by the k -th subscript can be defined as follows

$$x(t) = \sum_{k=0}^{\infty} a_k(t - T_{d,k}) \cos(2\pi \int_{-\infty}^t f_k(\lambda - T_{d,k}) + \phi_{k,1}(t - T_{d,k})) + v(t) \quad 0 \leq t \leq T$$

$$= 0 \text{ elsewhere} \quad (1)$$

where $a_k(t)$ is the amplitude, $f_k(t)$ is the frequency, $\phi_k(t)$ is the phase, $T_{d,k}$ is the time delay, T is the duration of the signal and $v(t)$ is the background noise in the observed signal typically modelled as zero mean Gaussian random process with standard deviation σ_v . The signal parameters – amplitude, frequency and phase – as functions of time are shown in Equation (1) to represent a time-varying signal. If the signal parameters are constant, the signal observed is considered time-invariant. Noise in the signal makes the signal random instead of deterministic. To process the signal using digital technology and obtain its discrete-time representation, it is appropriate to sample the signal according to the Nyquist theorem.

To demonstrate the strengths and weaknesses of the various signal analysis methods, four different types of signals will be presented similar to those found in applications such as medical, biology, communications and radar: modulated pulse, linear frequency modulation (LFM), 2 component LFM (2LFM), and 4 level frequency-shift keying (4FSK). The signals expressed in continuous-time are

$$x(t) = a \cos(2\pi f_1 t) \prod_T(t) + v(t) \quad (2a)$$

$$x(t) = a \cos(2\pi f_1 t + \alpha t^2) \prod_T(t) + v(t) \quad (2b)$$

$$x(t) = \sum_{k=0}^1 a_k \cos(2\pi f_k t + \alpha_k t^2) \prod_T(t) + v(t) \quad (2c)$$

$$x(t) = \sum_{k=0}^3 a \cos(2\pi f_k(t - kT_b)) \prod_{T_b}(t - kT_b) + v(t) \quad (2d)$$

where the parameters are defined as follows

- Frequency f_1 is fixed for the modulated pulse signal.
- Frequency law α which describes the rate of change in frequency over time is the same for LFM and 2LFM signals. However, the start frequency f_k is different for the 2LFM signal.
- Frequency f_k for the 4FSK signal that varies according to the information bearing data.
- Symbol duration T_b for 4FSK which is much lower compared to the signal duration T .

v. $\prod_T(t)$ is the box function of length T .

Equation (1) - (2) is the representation of signal in time which is often referred as time series representation. Alternately, signals can be represented in frequency and jointly in time and frequency.

3. Frequency Representation of Signals

Besides time representation, signals can be represented in frequency which is often referred as spectrum. From the energy or power spectrum, the signal energy or power could be estimated. The periodogram is a basic method for estimating the power spectrum belonging to the class of non-parametric spectrum estimation. It is based on the Fourier transform and can be calculated as follows

$$S_x(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \left| \int_0^T x(t) e^{-j2\pi f t} dt \right|^2 \quad (3)$$

For discrete-time signals, the periodogram is calculated based on the discrete Fourier transform (DFT). Alternatively, the periodogram can be realized indirectly from the autocorrelation using the Wiener-Khinchine theorem. In noise, the periodogram is not a consistent estimator since the variance in the power spectrum estimate does not reduce signal length. Thus, this is the motivation for developing other methods such as the Bartlett, Welch and Blackman-Tukey methods reduce the variance [6]. Parametric spectrum estimation estimates the power spectrum based on the model of the underlying process that generates the signal [7]. This results in variance reduction with shorter signal duration provided correct assumption on the underlying process. Examples of parametric estimation techniques are covariance, Burg, minimum variance, and the class of sub-space methods.

From the periodogram in Equation (3), the power spectrum estimates for the signals defined in Equation (2) are

$$S_x(f) = \frac{a^2}{4} \left[\text{sinc}^2(f_1 - fT) \right] + \sigma_v^2 \quad 0 \leq f < \infty \quad (4a)$$

$$S_x(f) = \frac{a^2}{4} \left[\text{sinc}^2(f_1 + 0.5T - fT) \right] + \sigma_v^2 \quad 0 \leq f < \infty \quad (4b)$$

$$S_x(f) = \sum_{k=0}^1 \frac{a^2}{4} \left[\text{sinc}^2(f_k + 0.5T - fT) \right] + \sigma_v^2 \quad 0 \leq f < \infty \quad (4c)$$

$$S_x(f) = \sum_{k=0}^3 \frac{a^2}{4} \left[\text{sinc}^2(f_k - fT) \right] + \sigma_v^2 \quad 0 \leq f < \infty \quad (4d)$$

$$S_x(f) = S_x^*(-f) + \sigma_v^2 \quad f < 0 \quad (4e)$$

It can be observed that power spectrum only represents the frequency and not the temporal information as observed for the FM, 2LFM, and 4FSK signals. Therefore, the method would not be suitable to estimate the timing parameters of signals if they are needed for used in a classifier. Alternative methods such as time-frequency analysis has to be used to estimate the parameters of time-varying signals similar to those described in Equation (2).

4. Analysis of Time-Varying Signals

There are two possible approaches for analysing time-varying signals: time-frequency analysis [8] and signal segmentation [9]. Between the two approaches, time-frequency analysis provides a more detail representation of the signal with a joint representation of signal power with respect to time and frequency. From the representation, it is possible to localize each signal component in the time-frequency plane as well as its time duration and bandwidth. For application such as speech recognition and coding, signal segmentation is used based on the assumption of local stationary condition even though there are significant number of works that utilizes time-frequency analysis for speech [10]. The main reason is to facilitate real-time applications such as speech coding and speech recognition where computational complexity is the main focus. From there, signal modelling technique could be adopted which is then used to represent the signal. Linear predictive coding is an example of signal modelling technique [9]. In general, it is crucial to find the suitable mathematical model for the application to allow accurate signal representation with minimum loss of information. From this point onwards, the focus of this paper will be on the use of time-frequency analysis with no further discussion on the signal segmentation method.

There are two general classes of time-frequency analysis methods [8]: linear time-frequency distribution (TFD) and quadratic TFD. Between the two methods, the linear TFD provides an amplitude representation in frequency and time while the quadratic TFD provides a representation in power and energy. Two main classes of linear TFD are the short-time Fourier transform (STFT), Gabor transform and wavelet transform. Examples of quadratic TFD are the spectrogram, Wigner-Ville distribution (WVD), Choi-Williams distribution, and extended modified B distribution (EMBD). Extension to these are the higher order TFDs [11] based on the bispectrum and trispectrum.

5. Linear TFD

The linear TFDs that will be described in this section are the STFT and the wavelet transform, and issues that relates to the resulting time-frequency representation (TFR).

5.1 STFT

Instead of analysing the whole duration of the signal, the STFT performs the spectrum estimate within a window incrementally over time. Thus, the resulting TFR will be able to capture the temporal characteristic of the signal. The STFT is based on the Fourier transform and can be expressed as for continuous-time signal as [12]

$$X_x(t, f) = \int_{-\infty}^{\infty} w(\tau - t) x(\tau) e^{j2\pi f\tau} d\tau \quad (5)$$

where $w(t)$ is the window function. Extending the STFT by its square results in the spectrogram which belongs to the class of quadratic TFD. The resulting TFD is defined as

$$\rho_x(t, f) = |X_x(t, f)|^2 \quad (6)$$

The Gabor transform which also an early form of linear TFD is related to the STFT based on the assumption of a Gaussian window function [8].

The main issue that relates to the TFR obtained from the STFT or spectrogram is the compromise between time and frequency resolution due to the uncertainty principle. It is not possible to get accurate TFR simultaneously in both time and frequency. However, this is crucial if precision is required to estimate the time and frequency parameters of a signal. This is best illustrated by a modulated Gaussian pulse [8]. It is shown that this is only signal where the product $t_{eff} f_{eff}$ (effective time and effective frequency product) is constant at $1/4\pi$ and other signals should conform to this equality $t_{eff} f_{eff} > 1/4\pi$. The result of this inequality is also known as the uncertainty principle similar to the Heisenberg uncertainty principle in quantum

physics. Since the product is a constant, a short duration pulse will have a broad bandwidth and vice versa. Thus, it is not possible to have high resolutions in both time and frequency representation if the STFT or spectrogram is used to obtain the TFR.

5.2 Wavelet Transform

The wavelet transform represents another class of linear TFD that can be used to represent time-varying signals in both time and frequency domain. Unlike the STFT, wavelet transform utilizes basis functions that are characterized by lag and dilation factor. The various wavelet transforms depend on the choice of basis function such as Haar, Daubechies, Morlet and Symlet. The basic formula for the continuous wavelet transform can be expressed as [13]

$$X(t, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(\tau) h\left(\frac{\tau - t}{a}\right) d\tau \quad (7)$$

where $h(t)$ is the basis function, t is the time of interest and a is the scaling factor. The choice of basis function depends on the application. For example, the Haar wavelet has time representation that is square in shape. Therefore, it is more appropriate to select a basis function that is similar in form with the signals related to the application to ensure no loss of representation.

Multiresolution analysis [14] is a computationally efficient structure for implementing the discrete wavelet transform. The process involved decomposing the signal into its high pass and low pass component, where the latter is then decimated by 2 times. For the low pass component, this process is repeated according to the desired decomposition levels resulting in the frequency resolution that is smaller for the lower frequency compared to the higher frequency for a given signal bandwidth. This is unlike the STFT that provides equal frequency resolution for all frequency range. Based on the signals defined in Equation (2), it can be seen that the resulting TFR using the wavelet transform will depend on the frequency content. Signals that have lower frequency contents will be resolved better compared to that with higher frequency contents.

6. Quadratic TFD

The spectrogram derived in the previous section from the STFT belongs to the class of quadratic TFD. Examples of other TFDs includes the Wigner-Ville distribution (WVD), reduced interference distribution (RID), cone kernel distribution (CKD) and the more recent extended modified B distribution (EMBD) [8]. This section will first present the WVD and its characteristics, followed by the quadratic TFD formulation, adaptive TFD and the use of quadratic TFD in parameter estimation for time-varying signals. The quadratic TFD has the advantage of providing high resolution in both time and frequency unlike the STFT but with precaution due to artifacts caused by cross terms.

6.1 WVD

The Wigner function was formulated by E.P. Wigner in 1932 in the context for quantum physics. Extension to wave propagation and signal processing was extended by J. Ville in 1948 [15]. In recognition to the contribution of both researchers, that is the reason why the Wigner function is often referred as the Wigner-Ville distribution (WVD). The WVD can be expressed as [16]

$$W_x(t, f) = \int_{-\infty}^{\infty} K_x(t, \tau) e^{-j2\pi f\tau} d\tau \quad (8)$$

where $K_x(t, \tau)$ is the bilinear product of the analytical signal $x(t)$ which is a complex representation of a signal that can be obtained by applying the Hilbert transform. The bilinear product is produced as a result of the transformation signal in the time representation into the time-lag representation that is defined as

$$K_x(t, \tau) = x(t + \tau/2)x^*(t - \tau/2) \quad (9)$$

Several analysis examples will be used for the WVD expressed in Equation (8) for the signal examples in Equation (2). For the LFM signal in Equation (2b), assuming that it is converted to analytical form at high signal-to-noise ratio (SNR), the resulting time-frequency representation is

$$W_x(t, f) = a^2 T \sin \pi (f_0 + \alpha t - f) T \quad 0 \leq t \leq T \quad (10)$$

From the representation, it is observed that the peaks of the signal appear along the frequency law of α with a mainlobe width in frequency of $1/T$. However, a different picture appears for the 2LFM signal which can be described as the following TFR

$$W_x(t, f) = [a^2 T \text{sinc}(\pi(f_0 + \alpha t - f)T) + a^2 T \text{sinc}(\pi(f_1 + \alpha t - f)T) + 2a^2 \cos(2\pi(f_0 - f_1)t) T \text{sinc}(2\pi(0.5(f_0 + f_1) + \alpha t - f)T)] \quad (11)$$

The three components instead of two observed is due to the nonlinear nature of the bilinear product in Equation (9). The first two terms are the auto terms which represents the true signal characteristics and the third component referred as the cross terms is due to the interaction between the two signal components. The cross terms have a high amplitude time oscillatory term at frequency of $f_0 - f_1$ and peaks at frequency law of α at starting frequency of $(f_0 + f_1)/2$. Without its removal, the true characteristics of this signal could be misinterpreted. Similar results will also be observed if the WVD is applied to the 4FSK signal. Thus, there is a need to reduce the cross terms to ensure accurate TFR of signal.

6.2 Quadratic TFD Formulation

Improving the TFR using kernel function leads to in the formulation for quadratic TFD through earlier works described in [17][18]. The formulation also linked all the previous quadratic TFDs such as the CW distribution, RID distribution, spectrogram, WVD and many more into one single relationship linked by a kernel function. When represented with respect to the Wigner-Ville distribution, the quadratic time-frequency distribution can be defined as [8]

$$\rho_x(t, f) = G(t, f) *_{(t, f)} W_x(t, f), \quad (12)$$

where $G(t, f)$ is the time-frequency kernel function.

To allow flexibility in terms of accuracy and computational complexity, the QTFD can be realized based on the time-lag, Doppler-lag and Doppler-frequency representations as [8]

$$\rho_x(t, f) = FT_{\tau \rightarrow f} \left[G(t, \tau) *_{(t)} K_x(t, \tau) \right] \quad (13a)$$

$$\rho_x(t, f) = FT_{\tau \rightarrow f} \left[g(t, \tau) A_x(v(\tau)) \right] \quad (13b)$$

$$\rho_x(t, f) = IFT_{v \rightarrow t} \left[g(v, f) *_{(f)} r_x(v, f) \right] \quad (13c)$$

where $FT[\]$ represents a Fourier transform operation, $IFT[\]$ represents an inverse Fourier transform operation, and $G(t, \tau)$, $g(v, \tau)$ and $g(v, f)$ are the kernel functions in their respective domains that is designed to minimize cross terms and improve the time-frequency representation. The bilinear product $K_x(t, \tau)$ in Equation (9) when defined in the Doppler-lag and Doppler-frequency domains are

$$A_x(v, \tau) = FT_{t \rightarrow v} [K_x(t, \tau)] \quad (14a)$$

$$r_x(v, f) = FT_{\substack{t \rightarrow v \\ \tau \rightarrow f}} [K_x(t, \tau)] \quad (14b)$$

The Doppler-lag representation of the bilinear product is often referred as the ambiguity function. The effectiveness of kernel functions can be accessed and new kernel functions can be designed based on the analysis of signal in this domain [8]. It is also used in the analysis and design of radar waveforms to assess the resolution in Doppler and time-lag [19]. An area of application for the Doppler-frequency representation is in cyclostationary signal processing [20] where the fundamental assumption on the signal is cyclic stationarity.

In Doppler-lag representation, the auto terms which are located within the origin while the cross terms are located further away [8]. The kernel function can be viewed as a 2-dimensional low-pass filter where the cut-off frequencies are set to attenuate the cross terms and preserve the auto terms. The conversion between the various representation in relation to the TFR as described by Equation (13) to (14).

A separable kernel offers independent control smoothing functions which can be defined in time-lag representation as [21]

$$G(t, \tau) = G_1(t)g_2(\tau) \quad (15)$$

where $G_1(t)$ is the smoothing function in time and $g_2(\tau)$ is the smoothing function in lag. By using a separable kernel, the quadratic TFD in Equation (12) described the time-lag, Doppler-lag, Doppler-frequency and time-frequency domains are

$$\rho_x(t, f) = FT_{\tau \rightarrow f} \left[G_1(t) *_{(t)} K_x(t, \tau) g_2(\tau) \right] \quad (16a)$$

$$\rho_x(t, f) = FT_{\substack{\tau \rightarrow f \\ t \rightarrow v}} [g_1(v) A_x(v, \tau) g_2(\tau)] \quad (16b)$$

$$\rho_x(t, f) = IFT_{v \rightarrow t} \left[G_2(f) *_{(f)} r_x(v, f) g_1(v) \right] \quad (16c)$$

$$\rho_x(t, f) = G_1(t) *_{(t)} G_2(f) *_{(f)} W_x(t, f) \quad (16d)$$

Based on the separable kernel quadratic TFD, the necessary conditions for suppressing cross terms will be described for the 2LFM signals and 4FSK signals. Equation (16a) that is based on the time-lag representation when applied to the signal defined in Equation (2c) results in

$$\begin{aligned} \rho_x(t, f) = FT_{\tau \rightarrow f} \left[G_1(t) *_{(t)} K_x(t, \tau) g_2(\tau) \right] &= FT_{\tau \rightarrow f} \left[G_1(t) *_{(t)} (a^2 e^{j2\pi(f_0 + \alpha t)\tau} + a^2 e^{j2\pi(f_1 + \alpha t)\tau} \right. \\ &\quad \left. + 2a^2 \cos(2\pi(f_0 - f_1)t) e^{j2\pi((f_0 + f_1) + \alpha t)\tau}) K_{\Pi}(t, \tau) g_2(\tau) \right] \end{aligned} \quad (17)$$

where $K_{\Pi}(t, \tau)$ is the bilinear product of the box function. Since the bilinear product auto terms lie within lag $\tau=0$, a window width of T for $g_2(\tau)$ would be sufficient to preserve all terms. The cross terms can be suppressed by selecting $G_1(t)$ such that it is a low pass filter with a cut-off frequency of less than (f_0-f_1) [22]. The resulting TFR is

$$\rho_x(t, f) = a^2 T \text{sinc}(\pi(f_0 + \alpha t - f)T) + a^2 T \text{sinc}(\pi(f_1 + \alpha t - f)T) \quad (18)$$

Similar approach when applied to the 4FSK signal defined in Equation (2d) results in a TFR that is described as

$$\rho_x(t, f) = \sum_{k=0}^3 a^2 \text{sinc}(2\pi(f_k - f)T_b) \prod_{T_b} (t - kT_b) \quad (19)$$

For both TFRs obtained, the auto terms are preserved and the cross terms are attenuated. Thus, accurate TFRs are obtained and signal parameters can be estimated.

6.3 Cross TFD

The introduction to the cross TFD begins with the cross WVD. It differs from the WVD defined in Equation (8) by performing the computation based on two signals $x(t)$ and $y(t)$ resulting in a TFD that is defined as follows [23]

$$W_{xy}(t, f) = \int_{-\infty}^{\infty} K_{xy}(t, \tau) e^{-j2\pi f \tau} d\tau \quad (20)$$

where $K_{xy}(t, \tau)$ is the cross-bilinear product of the analytical signal $x(t)$ and $y(t)$. The cross-bilinear product is produced as a result of the transformation signal from time to the time-lag representation defined as

$$K_{xy}(t, \tau) = x(t + \tau/2) y^*(t - \tau/2) \quad (21)$$

Similar to the quadratic TFD, the cross TFD formulation based on cross-bilinear product can be expressed as [24]

$$\rho_{xy}(t, f) = FT_{\tau \rightarrow f} \left[G(t, \tau) *_{(t)} K_{xy}(t, \tau) \right] \quad (22)$$

where $G(t, \tau)$ is the time-lag kernel that can be designed to remove cross terms. For separable kernel similar to Equation (15), the cross TFD formulation can be expressed as

$$\rho_{xy}(t, f) = FT_{\tau \rightarrow f} \left[G_1(t) *_{(t)} K_{xy}(t, \tau) g_2(\tau) \right] \quad (23)$$

Unlike the quadratic TFD, phase information is represented on the TFR. Furthermore, the estimation of signal parameters such as instantaneous frequency (IF) can be performed at lower SNR compared to the quadratic TFD [23]. Despite its advantages, the most critical issue in the cross WVD is to find the suitable reference signal $y(t)$ if the signal of interest is $x(t)$.

6.4 Adaptive Quadratic TFD

It is shown in Section 6.2 that the appropriate selection of the kernel parameters for a quadratic TFD is necessary to obtain accurate TFR for a given signal. The selection of kernel parameters would be simpler if priori analysis is performed in the Doppler-lag representation or the characteristics of the signal is known.

If this is not available, an adaptive procedure is required to estimate the kernel procedure and obtained the TFR according to Equation (13). Thus, the main objective of the adaptive quadratic TFD is to find the location of the auto terms and cross terms, and automatically set-up the kernel parameters to preserve the auto terms and attenuate the cross terms.

An early work presented in [25] described an adaptive quadratic TFD for multi-component signals similar to 2LFM. The distribution is signal-dependent based on radially Gaussian kernel that adapts over time. This is an iterative process where the immediate step is the short-time Doppler-lag representation in the kernel optimization process. A more recent work based on the adaptive directional time-frequency distribution (ATFD) was developed by the local adaptation of the kernel function at each of the TFR [26]. The objective of the algorithm developed is to adapt in the direction of the energy distribution in the TFR. From the TFR, the instantaneous frequency (IF) is estimated for multi-component signals similar to 2LFM and provides improvement over existing quadratic TFD at SNR of 12 dB.

Other adaptive quadratic TFD was developed for signals that are used in radar and digital communications such as 4FSK, and a sequence of modulated pulses and LFM signals. Global adaptation is used to estimate the kernel parameters applied for digital communication signals similar to 4FSK and a sequence of modulated pulses [27]. A separable kernel similar to Equation (15) is used where the parameter of $G_1(t)$ is obtained from the power spectrum of the signal while the parameter $g_2(t)$ is derived from the smoothed autocorrelation estimate of the signal. Further work based on the cross WVD utilized local adaptation to estimate the kernel parameter $g_2(t)$ at every time instants [24]. For this particular work, local adaptation is required instead of global adaptation due to the characteristics of the signal in the time-lag representation. Between the work reported in [27] and [24], the later gave better in the estimation of instantaneous frequency. Most recent work [22] provides improvement by having an adaptive quadratic TFD that covers a broader class of signals covering commonly used signals in radar and just digital communication signal. The adaptive quadratic TFD operates in the Doppler-lag which not only provides accurate TFR and also performs IF estimation with minimum mean square error meeting the Cramer-Rao lower bound at low SNR of 2 dB.

6.5 Higher Order TFD

Most of the concepts used in power spectrum analysis and quadratic TFD are based on second order statistics. Since both are related to the autocorrelation function, they are considered as functions in the one dimension. Higher order statistics involved generalization of these concepts, then higher order moments in time and frequency will involve higher order functions in time and frequency. Among the reason for exploring higher order functions are: preservation of phase information, detection of phase coupling, and insensitivity to Gaussian noise.

A general formulation for a M order moment function of a random process $X(t)$ can be expressed as [28]

$$m_X(\tau_1, \tau_2, \dots, \tau_{M-1}) = E[X(t)X(t+\tau_1)X(t+\tau_2)\dots X(t+\tau_{M-1})] \quad (24)$$

where τ_k represents the k -th lag. The mean, autocorrelation, bicornelation and tricornelation derived from this formulation are m_X , $m_X(\tau_1)$, $m_X(\tau_1, \tau_2)$ and $m_X(\tau_1, \tau_2, \tau_3)$ respectively. As the order increases, the comparison of the random process is made with reference to more time function. The choice of lag also determines the application if the interest is for example to reduce Gaussian noise or detect phase coupling or signal estimation in multiplicative noise. The result is better estimation in Gaussian noise for the bicornelation and multiplicative noise for the tricornelation.

Taking the FT in multi-dimension results in the higher order spectrum that can be calculated as

$$S_X(f_1, f_2, \dots, f_{M-1}) = \underset{\substack{\tau_1 \rightarrow f_1 \\ \tau_2 \rightarrow f_2 \\ \vdots \\ \tau_{M-1} \rightarrow f_{M-1}}}{FT} [m_X(\tau_1, \tau_2, \dots, \tau_{M-1})] \quad (25)$$

where the power spectrum, bispectrum and trispectrum are $S_X(f_1)$, $S_X(f_1, f_2)$ and $S_X(f_1, f_2, f_3)$ respectively.

Extending to time-varying signals, the generalized higher order time-frequency distribution is defined in the time-lag representation as [29]

$$\rho_x^M(t, f) = \underset{\tau \rightarrow f}{FT} \left[G(t, \tau) *_{(t)} K_x^M(t, \tau_1, \tau_2, \dots, \tau_{M-1}) \right] \quad (26)$$

where the M order time-varying moment is

$$K_x^M(t, \tau_0, \tau_1, \dots, \tau_M) = x(t)x(t+\tau_0)x(t+\tau_1)\dots x(t+\tau_M) \quad (27)$$

It can be observed that for $M=2$, the formulation presented in Equation (26) reduces to become the quadratic TFD in Equation (13a). For $M=3$ and $M=4$, the formation defined the bispectrum TFD [30] and trispectrum TFD [31] respectively. The problems addressed are similar to the higher order spectrum described in Equation (28) except that the signals are time-varying.

7. Parameter Estimation

The IF could be the first parameter that could be estimated from the TFR prior to further estimation. Example of application could be found in [32] for detection and classification of radar signals. The notion of instantaneous frequency was described in [33] and the various estimation method were presented in [34] with more recent work found in [22][24][35].

Using the peak of the TFR to estimate the IF is found to be efficient meeting the theoretical limit defined by the Cramer-Rao lower bound (CRLB) [33]. The IF estimated every time instant as given by

$$f_i(t) = \arg \left\{ \max_f (\rho_x(t, f)) \right\} \quad 0 \leq t \leq T \quad (28)$$

where $f_i(t)$ is the estimated IF from the peaks of the TFR that corresponds to the frequency of the signal. Based on Monte-Carlo simulation based on 100 realizations for SNR range from -5 to 16 dB, the variance in the IF estimate meets the CRLB at the cut-off SNR of 0 dB for pulse signals, 4FSK and LFM signals [22]. This is similar to earlier work for LFM signals [34]. Signals which are more complex such as continuous-wave LFM has a higher cut-off SNR of 5 dB.

8. Conclusion

Time-frequency analysis can be used to analyse time-varying signals and estimate its parameter for use as inputs to a classifier. Quadratic TFD in general provides accurate TFR compare to linear TFD provided there is effective removal of cross terms and preservation of auto terms. Estimation of signal parameters based on the IF shows that the variance in the estimate meets the CRLB at the SNR cut-off range of between 0 to 5 dB.

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