Performance Analysis of a Maximum-Likelihood FFH/MFSK Receiver with Partial-Band-Noise Jamming over Frequency-Selective Fading Channels

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***Abstract*— A maximum-likelihood (ML) receiver is proposed for a fast frequency-hopping M-ary frequency-shift-keying system. The effects of frequency-selective Rayleigh-fading channels, partial-band-noise jamming, and additive white Gaussian noise are considered. It is shown that side information on the noise variance, the jamming variance, as well as the power levels of the correlated paths, is required to implement the ML receiver. Analytical bit-error rate expressions are derived for the ML receiver. Our analysis also shows that there exists an optimum diversity order under certain channel conditions.**

***Index Terms*— Frequency-selective fading, maximum-likelihood, partial-band-noise jamming.**

I. Introduction

In a fast frequency-hopping (FFH) M-ary frequency-shift-keying (MFSK) system, the carrier frequency is pseudo-randomly hopped over the total spread-spectrum (SS) bandwidth under the control of a pseudo-noise code. The FFH/MFSK transmitter is assumed to perform L hops per data symbol with $L \geq 1$ being the diversity order.

The performance of a square-law nonlinear combining soft-decision receiver for FH binary FSK (BFSK) systems under PBNJ was studied in [1]. In [2], the maximum-likelihood (ML) receiver was proposed for an FFH/MFSK system under PBNJ and additive white Gaussian noise (AWGN). However, the channel fading effect was not considered. Wu and Hung [3] proposed two sub-optimal ML receivers for FFH/BFSK systems over a frequency-selective Rayleigh-fading channel with multitone jamming. In this letter, we will study the performance analysis of the ML receiver for an FFH/MFSK system over frequency-selective Rayleigh-fading channels with PBNJ and AWGN.

II. System Model

A typical block diagram of the FFH/MFSK receiver structure can be found in [4]. Following [3], the multipath frequency-selective Rayleigh-fading channel could be modeled as a correlated two-path channel model within one hop duration. In other words, we can sum up all the possible multipaths and realize them into two main clumps to form these two paths. Hence, the de-hopped signal $r(t)$ can be expressed as

$$\begin{aligned} r(t) \;=\; & \sqrt{2}|a_{s_1}|\cos(2\pi f_m t + \phi_{s_1}) + \\ & \sqrt{2}|a_{s_2}|\cos(2\pi f_m (t-\tau) + \phi_{s_2}) + w(t) + n_J(t) \end{aligned} \tag{1}$$

where $\sqrt{2}|a_{s_1}|$ and $\sqrt{2}|a_{s_2}|$ denote the two main correlated Rayleigh amplitudes, f_m is the baseband frequency of the desired signal, ϕ_{s_1} and ϕ_{s_2} are the random signal phases uniformly distributed over $[-\pi, \pi]$, and τ is the time delay between the two correlated paths and uniformly distributed over $[0, T_h]$, in which $T_h = T_s/L$ is the hopping duration and T_s is the symbol duration. Note that $f_m = m/T_h$ for $m = 1, 2, \ldots, M$. The term $w(t)$ is the Gaussian noise with zero mean and variance of $\sigma_w^2 = N_0 B$, where N_0 is the one-sided power spectral density (PSD) of the noise and $B = 1/T_h$. In addition, $n_J(t)$ represents the PBNJ with equivalent one-sided PSD of N_J and variance of $\sigma_J^2 = BN_J/\gamma$. The parameter γ $(0 < \gamma \leq 1)$ is the fraction of the total SS bandwidth W_{ss} being jammed by PBNJ. The total variance of the AWGN and PBNJ can be expressed as

$$\sigma_{q_l}^2 \;=\; \sigma_w^2 + q_l \sigma_J^2 \tag{2}$$

where $q_l = 1$ or 0 is an indicator function denoting whether the lth hop is jammed or not. It is assumed that the PBNJ is present simultaneously at all the M branches for the entire hop duration or not at all [1]. The two correlated Rayleigh amplitudes can be modeled as [3]

$$|a_{s_1}| \;=\; \sqrt{2}\sigma_{s_1}|c_1 + id_1| \tag{3}$$

and

$$\begin{aligned} |a_{s_2}| \;=\; & \sqrt{2}\sigma_{s_1}[\epsilon\lambda^2|c_1 + id_1|^2 + \epsilon(1-\lambda^2)|c_2 + id_2|^2 \\ & +2\epsilon\lambda\sqrt{1-\lambda^2}(c_1c_2 + d_1d_2)]^{1/2} \end{aligned} \tag{4}$$

where $i = \sqrt{-1}$ is the complex operator, c_1, c_2, d_1, and d_2 are independent Gaussian random variables with zero mean and variance of $1/2$. The power ratio is defined as $\epsilon = \sigma_{s_2}^2/\sigma_{s_1}^2$ $(0 < \epsilon \leq 1)$, where $2\sigma_{s_1}^2$ and $2\sigma_{s_2}^2$ are the diffused power levels of the two correlated paths, respectively. The parameter λ is a function of the cross-correlation coefficient ρ of the two main correlated paths, and it can be expressed as [3]

$$\lambda^2 \cong 8[\sqrt{(4/\pi - 1)\rho + 1} - 1]. \tag{5}$$

The de-hopped signal $r(t)$ is detected by the square-law detector and the output is denoted as r_{ml,q_l} $(m = 1, 2, \ldots, M; l =$

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$1,2,\ldots,L$). According to [3] and after some modifications, the output r_{ml,q_l} can be expressed as

$$\begin{aligned} r_{ml,q_l} &= [|a_{s_1}|\cos(\phi_{s_1}) + |a_{s_2}|\cos(\phi_{s_2} - 2\pi f_m\tau) + n_I]^2 \\ &+ [|a_{s_1}|\sin(\phi_{s_1}) + |a_{s_2}|\sin(\phi_{s_2} - 2\pi f_m\tau) + n_Q]^2 \end{aligned} \quad (6)$$

where n_I and n_Q are zero mean Gaussian distributions with variance of $\sigma_{q_l}^2$. After that, the ML diversity-combining technique is used to combine the L hop receptions before making a final decision.

III. Probability of Bit Error Analysis

The ML decision statistic in the lth hop is [4]

$$z_{ml,q_l} = \ln\left(\frac{p_{r_{ml,q_l}}(r_{ml,q_l}|\phi_{ss_{ml}},\mathrm{S})}{p_{r_{ml,q_l}}(r_{ml,q_l}|\mathrm{NS})}\right) \quad (7)$$

for $m = 1,2,\ldots,M, l = 1,2,\ldots,\ L$, $\phi_{ss_{ml}} = \phi_{s_1} - \phi_{s_2} + 2\pi f_m\tau$ is the phase difference between the two correlated paths. The notations S and NS denote the existence and non-existence of the useful signal in the square-law detector output, respectively. From [3] and after some modifications, we can obtain the conditional probability density function (pdf) of r_{ml,q_l} as

$$\begin{aligned} & p_{r_{ml,q_l}}(r_{ml,q_l}|\phi_{ss_{ml}},\mathrm{S}) \\ &= \frac{1}{2(\sigma_p^2+\sigma_{q_l}^2)}\exp\left(-\frac{r_{ml,q_l}}{2(\sigma_p^2+\sigma_{q_l}^2)}\right)U(r_{ml,q_l}) \end{aligned} \quad (8)$$

where $U(\cdot)$ is the unit step function, and $\sigma_p^2 = [1 + 2\lambda\sqrt{\epsilon}\cos(\phi_{ss_{ml}})]\sigma_{s_1}^2 + \sigma_{s_2}^2$. By setting $\sigma_{s_1}^2 = \sigma_{s_2}^2 = 0$ in (8), we have

$$p_{r_{ml,q_l}}(r_{ml,q_l}|\mathrm{NS}) = \frac{1}{2\sigma_{q_l}^2}\exp\left(-\frac{r_{ml,q_l}}{2\sigma_{q_l}^2}\right)U(r_{ml,q_l}). \quad (9)$$

Substituting (8) and (9) into (7), we obtain

$$z_{ml,q_l} = A_{ml,q_l}r_{ml,q_l} + B_{ml,q_l} \quad (10)$$

where $A_{ml,q_l} = \frac{\sigma_p^2}{2\sigma_{q_l}^2(\sigma_p^2+\sigma_{q_l}^2)}$ and $B_{ml,q_l} = \ln\left(\frac{\sigma_{q_l}^2}{\sigma_p^2+\sigma_{q_l}^2}\right)$. The final decision statistic is $y_{m,j} = \sum_{l=1}^{L} z_{ml,q_l}$, where $j = \sum_{l=1}^{L} q_l$ denotes the number of hops being jammed out of the L hops. To implement the ML receiver, side information on the noise variance, the jamming variance, as well as the power levels of the two main correlated paths, is required. In practice, the Gaussian noise variance can be estimated from a "noise-only" channel for noise power measure [1], and the jamming variance can be extracted from an energy detector [5]. Without loss of generality, we assume that the desired signal with frequency f_1 was transmitted. The conditional pdf of z_{ml,q_l} is

$$\begin{aligned} & p_{z_{ml,q_l}}(z_{ml,q_l}|\cos(\phi_{ss_{ml}})) \\ &= \frac{1}{2A_{ml,q_l}\sigma_{ml,q_l}^2}\exp\left(-\frac{z_{ml,q_l}-B_{ml,q_l}}{2A_{ml,q_l}\sigma_{ml,q_l}^2}\right) \\ &\times U\left(\frac{z_{ml,q_l}-B_{ml,q_l}}{A_{ml,q_l}}\right) \end{aligned} \quad (11)$$

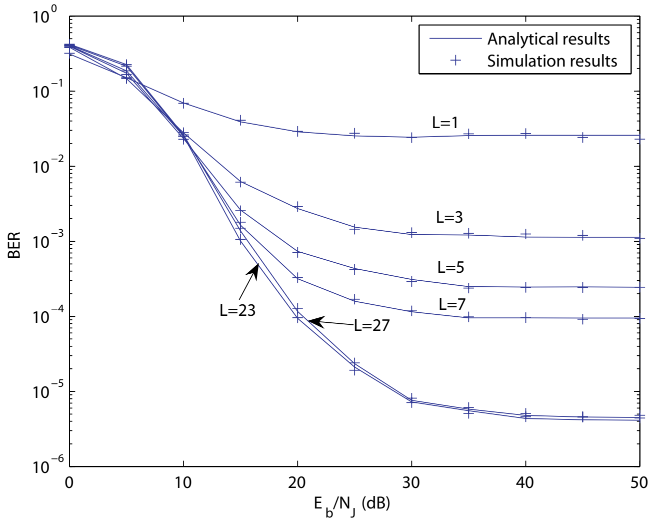

Fig. 1. Worst-case BER results for the ML receiver under different diversity orders with $E_b/N_0 = 13.35$ dB, $M = 32$, $\epsilon = 1$ and $\rho = 0.5$.

where

$$\sigma_{ml,q_l}^2 = \begin{cases} \sigma_p^2 + \sigma_{q_l}^2, & \text{for } m = 1 \\ \sigma_{q_l}^2, & \text{for } m = 2,3,\ldots,M. \end{cases} \quad (12)$$

Applying the approximation based on the Taylor series expansion in central differences [6], we obtain

$$\begin{aligned} & p_{z_{ml,q_l}}(z_{ml,q_l}) \\ &\approx \frac{1}{3}\sum_{b=-1}^{1} p_{z_{ml,q_l}}\left(z_{ml,q_l}|\cos(\phi_{ss_{ml}}) = \frac{\sqrt{3}b}{2}\right). \end{aligned} \quad (13)$$

Following that, the characteristic function of z_{ml,q_l} can be obtained by taking the Fourier transform of its pdf as

$$\Phi_{z_{ml,q_l}}(iv) \approx \frac{1}{3}\sum_{b=-1}^{b=1}\frac{\exp(ivB_{ml,q_l})}{1 - i2vA_{ml,q_l}\sigma_{ml,q_l}^2}\Bigg|_{\cos(\phi_{ss_{ml}})=\frac{\sqrt{3}b}{2}}. \quad (14)$$

Since all the hops are independent of each other, the characteristic function of $y_{m,j}$ is

$$\Phi_{y_{m,j}}(iv) = [\Phi_{z_{ml,1}}(iv)]^j \times [\Phi_{z_{ml,0}}(iv)]^{L-j}. \quad (15)$$

The corresponding pdf of $y_{m,j}$ can be obtained by performing the inverse Fourier transform of $\Phi_{y_{m,j}}(iv)$. An error occurs if $y_{1,j} < y_{m,j}$, for any $m = 2,3,\ldots,M$. Hence, the conditional probability of symbol error given j out of L hops are jammed is

$$P_{s,j} = 1 - \int_{-\infty}^{\infty} p_{y_{1,j}}(y_1)\left[\int_{-\infty}^{y_1} p_{y_{m,j}}(y_m)dy_m\right]^{M-1} dy_1. \quad (16)$$

The average symbol error probability can be obtained as

$$P_s = \sum_{j=0}^{L}\binom{L}{j}\gamma^j(1-\gamma)^{L-j}P_{s,j}. \quad (17)$$

Following that, the average BER is [4]

$$P_b = \frac{M}{2(M-1)}P_s. \quad (18)$$

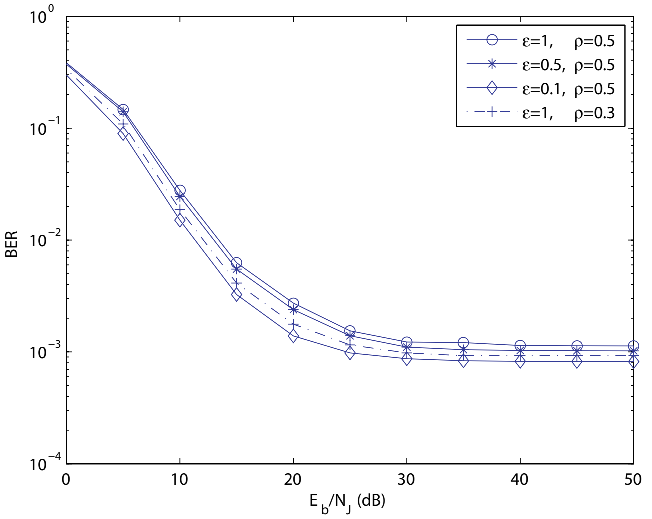

Fig. 2. Performance comparison of the worst-case BER results for the ML receiver under different power ratios and cross-correlation coefficients with $E_b/N_0 = 13.35$ dB, $L = 3$ and $M = 32$.

IV. Numerical Results and Discussions

In this letter, the worst-case BER is considered, while the parameter γ is numerically searched to maximize the BER for each E_b/N_J level. In Fig. 1, the worst-case analytical and simulated BER results of the proposed ML receiver are presented under different diversity orders with $E_b/N_0 = 13.35$ dB, $M = 32$, $\epsilon = 1$ and $\rho = 0.5$. The close match between analytical and simulation results validates the BER expressions derived in Section III. It is also observed that for $E_b/N_J >$ 10 dB, the system performance gradually improves as the diversity order increases initially. This is because a system with a higher diversity order can combat jamming and channel fading better. However, a higher diversity order also leads to a higher non-coherent diversity-combining loss. Hence, by numerically searching for every value of L, we have found an optimum diversity order, namely $L = 23$. The optimum diversity order depends on the modulation level M. We can numerically search the optimum diversity orders for other M values, though only results for $M = 32$ are provided. Fig. 2 illustrates the worst-case BER results under different power ratios ϵ and cross-correlation coefficients ρ. We can observe from Fig. 2 that a lower ϵ leads to better BER performance. This is because in the multipath frequency-selective Rayleigh-fading channel, inter-symbol interference (ISI) can degrade the system performance. Since ISI comes from the second path, as ϵ decreases, the power level of the second path decreases, which results in less severe ISI. Moreover, it is shown that better BER performance can be obtained when ρ becomes lower as these two paths become less correlated.

V. Conclusion

In this letter, the maximum-likelihood receiver has been proposed and studied for an FFH/MFSK system under frequency-selective Rayleigh-fading channels with PBNJ and AWGN. Analytical BER expressions were derived and verified via simulation. Due to the non-coherent combining loss phenomenon, it was observed that an optimum diversity order exists for the proposed ML receiver. Finally, assuming a two-path frequency selective fading channel, the effect of the power ratio and the cross-correlation coefficient on the performance was studied.

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