PERFORMANCE ANALYSES OF REED-SOLOMON CODED FFH/BFSK LINEAR-COMBINING RECEIVER OVER MULTITONE JAMMING AND AWGN CHANNELS

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ABSTRACT

Bit-error rate (BER) performance analyses are presented for Reed-Solomon coded fast frequency-hopping binary frequency-shift-keying (FFH/BFSK) spread-spectrum (SS) systems under the conditions of additive white Gaussian noise (AWGN) and the worst-case band multitone jamming (MTJ). The FFH system employs the soft-decision linear-combining receiver to obtain the diversity-combining output. The corresponding hard decision of the diversity-combining output is then used to decode the Reed-Solomon codes. The analytical expressions of the BER performance of the coded FFH systems are derived and shown to be applicable to higher diversity levels without much more computational complexity. Numerical results show that there is an optimum code rate for the Reed-Solomon coded system under the worst-case multitone jamming conditions.

INTRODUCTION

Over the past few decades, fast frequency-hopping (FFH) binary frequency-shift-keying (BFSK) spread-spectrum (SS) communication systems have attracted considerable interests in both commercial and military applications. The use of error-correcting codes provides another effective way of improving the performance of FFH systems. The effects of partial-band noise jamming (PBNJ) on the performance of various coded FFH systems have been investigated in [1]-[5]. In particular, the use of Reed-Solomon codes in an FFH M-ary FSK (MFSK) receiver under the conditions of PBNJ is discussed in [1] with perfect jammer side-information (JSI) and hard-decision error-erasure-correction decoding. In [2], the performance analyses of the coded FFH/MFSK square-law adaptive gain control (AGC) receiver are presented with both PBNJ and additive white Gaussian noise (AWGN). Both block codes and convolutional codes are considered. Similarly, the effects of the PBNJ and AWGN on the coded FFH/BFSK self-normalizing receiver are investigated in [3] under the conditions of Rician-fading channels. In [4], bit-error rate (BER) performance is presented for Reed-Solomon coded FFH/MFSK noise-normalizing receiver with PBNJ and AWGN over Rician-fading channels. In [5], the effects of PBNJ on the FFH/MFSK linear-combining receiver with various error-correcting codes are studied. Both hard-decision decoding without JSI and soft-decision with perfect JSI are investigated.

In contrast to the coded FFH systems with PBNJ, there are very few theoretical analyses available for the performance of coded FFH/FSK systems with multitone jamming (MTJ) [6, 7]. By ignoring the effect of AWGN, the performance analyses of FFH/MFSK systems with MTJ and various error-correcting codes are presented in [6]. It is assumed that the decoder has the perfect JSI and the parallel error-erasure-correction decoding with hard-decision is employed. The combination of optimum diversity and error-correcting codes has been presented and its effectiveness over the uncoded case has also been demonstrated. It has been shown in [6] that the single tone per band MTJ (where there is at most one jamming tone at one of the two frequency slots in a particular frequency-hopped (FH) band) gives the worst-case BER performance among different types of MTJ. Under the same jamming conditions with no AWGN, a simplified BER expression for the Reed-Solomon coded FFH/MFSK linear-combining receiver with single tone per band MTJ has been presented in [7]. The results show that when the redundancy is not large, MTJ is more harmful to the systems than the PBNJ. However, when a large redundancy can be provided by both diversity and Reed-Solomon codes, the worst-case MTJ can be nullified with a properly designed system.

From the above literature survey, it is observed that the BER performance of coded FFH/BFSK systems has not been investigated under the effect of AWGN and MTJ. In this paper, we analyze the BER of the FFH/BFSK system with Reed-Solomon codes under the conditions of MTJ and AWGN. The soft-decision linear-combining receiver for combining the L FFH diversity receptions is studied. We investigate the effect of the single tone per band MTJ since it has been shown in [6, 7] that this type of MTJ gives the worst-case BER performance. In our analyses, we have also taken into account the effect of AWGN since the AWGN will greatly affect the system performance and the optimum diversity gain of the system [2]-[4]. Noncoherent BFSK demodulation is employed here to detect the FFH outputs because it is difficult for the frequency synthesizers to main-

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tain phase coherence between successive hops over a wide SS bandwidth. The commonly used Reed-Solomon codes with hard-decision decoding are considered in this paper due to their good distance properties as well as the existence of efficient hard-decision decoding algorithms, which make it possible to implement relatively long codes in many practical applications where coding is desirable [1, 3, 4, 5, 7]. No JSI is assumed for decoding as JSI is not reliable and difficult to obtain when AWGN is present.

SYSTEM MODELS

The FFH/BFSK transmitter block diagram is shown in Figure 1. The incoming binary data sequences with a bit rate of R_b are mapped to Q-ary symbols with rate $R_q = R_b/q$ where $q = \log_2(Q)$. The resultant symbols are applied to the (n, k) Reed-Solomon encoder. The Reed-Solomon encoder outputs n coded Q-ary symbols for every block of k Q-ary input symbols with a code rate of k/n. The resultant encoder output has a rate of $R_c = nR_q/k$. The coded Q-ary symbols are then mapped to binary words at a rate of $R_d = qR_c = nR_b/k$. In this paper, we consider the case where $Q = n + 1$ [2, 8, 9]. The binary words are then applied to the BFSK modulator which selects one of the two baseband frequencies, f_1 or f_2, according to the incoming binary words. The frequency separation between f_1 and f_2 is $B = 1/T_h$, where $T_h = T_d/L$ is the hopping duration and $T_d = 1/R_d$ is the duration of each binary word and L is the diversity level of the FFH systems. The modulator output is then mixed and aligned with the hopping frequency f_h controlled by a pseudo-noise (PN) code generator. The frequency synthesizer selects f_h from one of the total N_h available frequency-hopped bands pseudo-randomly at a hopping rate of $1/T_h$, which is L times faster than the BFSK modulator. The bandwidth of each frequency-hopped band is $2B$. The output of the frequency hopper is passed through a bandpass filter (BPF) of bandwidth $W_{ss} = 2N_hB$. The hopped signal is then translated by a radio frequency (RF) oscillator before transmission.

The FFH/BFSK receiver block diagram of the linear-combining is shown in Figure 2. At the receiver front end, the received signal consists of the desired signal corrupted by MTJ and AWGN. This received signal is down-converted by an RF oscillator and passed through a BPF with bandwidth equal to W_{SS}. It is then dehopped by a frequency synthesizer which is in turn controlled by a PN code generator. The PN code generator reproduces the exact PN codes used in the transmitter so that the desired signal is dehopped back to the baseband. The resultant signal over one hop duration can be expressed as

$$r(t) = \sqrt{2}a_S \cos(2\pi f_m t + \phi_S) + J(t) + w(t), \quad m = 1 \text{ or } 2, \tag{1}$$

where $\sqrt{2}a_S$ is the amplitude of the desired signal, ϕ_S is the random phase uniformly distributed over $[-\pi, \pi]$, f_m is the baseband frequency depending on the binary code word, $J(t)$ is the multitone jammer, and $w(t)$ is the noise term due to AWGN. Note that $w(t)$ is a Gaussian process with mean zero and variance $\sigma_w^2 = N_0 B$, where N_0 is the one-sided power spectral density of AWGN and B is the bandwidth of the band-pass filters. Each MTJ tone can be expressed as

$$J(t) = \sqrt{2}a_J \cos(2\pi f_J t + \phi_J), \tag{2}$$

where ϕ_J is the random phase of the jamming tone. The jamming frequency f_J is assumed to coincide exactly with one of the hopping frequencies. In this paper, only the case of at most a jamming tone at one of the two possible orthogonal signaling frequencies in a specific FH band is considered, which is called the single tone per band MTJ [6]. We assume that there are N_h non-overlapping FH bands, each with bandwidth $2B$. The MTJ is assumed to have a total power of P_{J_T} and its strategy is to distribute the total jamming power over Q_T equal-power interfering tones so as to maximize the BER performance of the FFH system. The resultant BER performance is called the worst-case BER performance. In this paper, we define an equivalent MTJ power spectral density as $N_J \triangleq P_{J_T}/W_{SS}$ and the resultant signal-to-jamming ratio (SJR) of the system is given by

$$\text{SJR} = E_b/N_J, \tag{3}$$

where E_b is the energy per information bit.

The signal $r(t)$ is passed through two square-law detectors matched to frequencies f_1 and f_2. The two square-law detector outputs of the lth hop are denoted as $r_{m,p_{m,l}}$, where $p_{m,l} = 0$ or 1 is an indicator function representing the jamming state of the lth hop with frequency f_m, $m = 1, 2$. The square-law detector outputs are summed over L diversity receptions to produce

$$r_{j_m} = \sum_{l=1}^{L} r_{m,p_{m,l}}, \quad m = 1, 2 \tag{4}$$

where the subscript $j_m \triangleq \sum_{l=1}^{L} p_{m,l}$ denotes the number of hops jammed out of the total L diversity receptions. The two outputs r_{j_1} and r_{j_2} are then compared to make a binary decision. These binary decisions are then mapped to Q-ary symbols and a hard decision is made for the Q-ary symbols. Each block of n Q-ary symbols are fed into the errors-only hard-decision Reed-Solomon decoder. The Q-ary Reed-Solomon decoder outputs are converted into binary data bit to reconstruct the received estimate of the original binary data sequence. In order to have a fair comparison, the following parameters should be held constant:

- energy per information bit E_b,
- total SS bandwidth W_{SS},
- information bit rate R_b,
- total MTJ power P_{J_T}.

In order to keep the parameters W_{SS} and R_b to be constant simultaneously, the number of FH bands N_h for different diversity level and code rate will be changed accordingly.

In addition, by fixing the energy per information bit at E_b, the corresponding energy per transmitted hop is then given by

$$E_h = \frac{kE_b}{nL}. \quad (5)$$

PROBABILITY OF BIT ERROR

Without loss of generality, we assume that f_1 was transmitted. By using the Taylor-series approximation as proposed in [10], it can be shown that the probability density function (pdf) of $r_{m,p_m,l}$ is given by [10]

$$p_{r_{1,0}}(r_1) = \frac{1}{2\sigma_w^2}\exp\left(-\frac{r_1+2a_S^2}{2\sigma_w^2}\right) I_0\left(\frac{\sqrt{2a_S^2 r_1}}{\sigma_w^2}\right) U(r_1), \quad (6)$$

$$p_{r_{1,1}}(r_1) \approx \sum_{p=-1}^{1} \frac{1}{6\sigma_w^2}\exp\left(-\frac{r_1+2(a_S^2+a_J^2+\sqrt{3}pa_S a_J)}{2\sigma_w^2}\right) \times I_0\left(\frac{\sqrt{2r_1(a_S^2+a_J^2+\sqrt{3}pa_S a_J)}}{\sigma_w^2}\right) U(r_1), \quad (7)$$

$$p_{r_{2,0}}(r_2) = \frac{1}{2\sigma_w^2}\exp\left(-\frac{r_2}{2\sigma_w^2}\right) U(r_2), \quad (8)$$

and

$$p_{r_{2,1}}(r_2) = \frac{1}{2\sigma_w^2}\exp\left(-\frac{r_2+2a_J^2}{2\sigma_w^2}\right) I_0\left(\frac{\sqrt{2a_J^2 r_2}}{\sigma_w^2}\right) U(r_2), \quad (9)$$

where $U(\cdot)$ is the unit step function and $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. It has been shown in [10] that the characteristic function of $r_{m,p_m,l}$ can be expressed as

$$\Phi_{r_{1,0}}(iv) = \frac{1}{1-2iv\sigma_w^2}\exp\left(\frac{2iva_S^2}{1-2iv\sigma_w^2}\right), \quad (10)$$

$$\Phi_{r_{1,1}}(iv) \approx \sum_{p=-1}^{1} \frac{1}{3(1-2iv\sigma_w^2)}\exp\left(\frac{2iv(a_S^2+a_J^2+p\sqrt{3}a_S a_J)}{1-2iv\sigma_w^2}\right), \quad (11)$$

$$\Phi_{r_{2,0}}(iv) = \frac{1}{1-2iv\sigma_w^2}, \quad (12)$$

and

$$\Phi_{r_{2,1}}(iv) = \frac{1}{1-2iv\sigma_w^2}\exp\left(\frac{2iva_J^2}{1-2iv\sigma_w^2}\right), \quad (13)$$

where $i = \sqrt{-1}$ is the complex operator. Since all the L diversity receptions are independent of each other, the characteristic function of the decision statistics r_{j_m} as given in (4) can be expressed as

$$\Phi_{r_{j_m}}(iv) = \left[\Phi_{r_{m,0}}(iv)\right]^{L-j_m}\left[\Phi_{r_{m,1}}(iv)\right]^{j_m}, \quad m=1,2. \quad (14)$$

The corresponding pdf, conditioned on j_m hops being jammed for frequencies $f_m, m=1,2$, is given by

$$p_{r_{j_m}}(r_m|j_m) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ivr_m}\Phi_{r_{j_m}}(iv)dv, \quad m=1,2. \quad (15)$$

Note that the computational complexity of the above expression is independent of the diversity level L. Hence, this expression is applicable to higher diversity levels without much more computational complexity. The average probability of bit error P_b' at the output of the linear diversity combiner is

$$P_b' = \sum_{j=0}^{L}\binom{L}{j}\left(\frac{Q_T}{N_h}\right)^j\left(1-\frac{Q_T}{N_h}\right)^{L-j} \times \left[\sum_{h=0}^{j}\left(\frac{1}{2}\right)^j\binom{j}{h}P_e(h,j-h)\right], \quad (16)$$

where

$$\binom{a}{b} = \begin{cases} \frac{a!}{b!(a-b)!}, & \text{if } a\geq b, a\geq 0, b\geq 0, \\ 0, & \text{otherwise,}\end{cases} \quad (17)$$

and $P_e(\alpha,\beta)$ is the conditional probability of bit error given that α and β hops are being jammed by the single tone per band MTJ for frequencies f_1 and f_2, respectively. Since an error occurs if $r_{j_2} > r_{j_1}$, the expression for $P_e(\alpha,\beta)$ can be shown to be

$$P_e(\alpha,\beta) = \int_0^{\infty}\int_{r_1}^{\infty} p_{r_{j_1}}(r_1|j_1=\alpha)p_{r_{j_2}}(r_2|j_2=\beta)dr_2\,dr_1, \quad (18)$$

where $p_{r_{j_m}}(r_m|j_m), m=1,2$, is given by (15). Since an Q-ary symbol consists of q binary bits, the symbol error probability of the Q-ary symbol is given by

$$P_s = 1-(1-P_b')^q. \quad (19)$$

The symbol error probability at the output of hard-decision Reed-Solomon decoder is given by [2, 11]

$$P_s' \approx \frac{d}{n}\sum_{i=t+1}^{d}\binom{n}{i}P_s^i(1-P_s)^{n-i} + \frac{1}{n}\sum_{i=d+1}^{n} i\binom{n}{i}P_s^i(1-P_s)^{n-i}, \quad (20)$$

where $d = n-k+1$ is the minimum distance of the Reed-Solomon codes and $t = \lfloor (d-1)/2 \rfloor$ is the error-correction capability of the Reed-Solomon codes where $\lfloor x \rfloor$ denotes the integer part of x. Finally, the bit error probability of the resultant binary data is equal to [8]

$$P_b = \frac{Q}{2(Q-1)} P_s'. \quad (21)$$

NUMERICAL RESULTS AND DISCUSSION

In this section, the analytical BER results based on the analytical expressions derived previously are presented. The examples chosen are intended to illustrate general trends instead of being exhaustive. Hence, only BER results of the (n, k) Reed-Solomon codes with $n = 31$ and various values of k are examined. The worst-case BER results are presented which are obtained via a numerical search over the parameter Q_T for each specified value of E_b/N_0 and signal-to-jamming ratio. These worst-case BER results are plotted against the SJR for different diversity levels L. Examples with signal-to-noise ratio of $E_b/N_0 = 13.35$ dB are considered.

The worst-case BER results of the linear-combining receiver with $E_b/N_0 = 13.35$ dB are presented in Figures 3, 4, and 5 for diversity levels $L = 1, 3$, and 5, respectively. We consider the (n, k) Reed-Solomon codes with $n = 31$ and different values of k. We observe from Figures 3-5 that the optimum values of k in order to achieve the minimum BER results are 19, 21, and 23 for diversity levels $L = 1, 3$, and 5, respectively. In general, we conclude that for the (n, k) Reed-Solomon codes with a fixed value of n, the optimum values of k are higher as the diversity level increases. The above conclusion is also applicable to different levels of signal-to-noise ratio although only some typical results with $E_b/N_0 = 13.35$ dB are presented.

Figure 6 shows the worst-case BER results of the linear-combining receiver with $E_b/N_0 = 13.35$ dB for diversity levels of $L = 1, 3$, and 5. The BER results with the optimum code rate are presented. We observe that there is no diversity gain for the linear-combining receiver, i.e., the BER results for the linear-combining receiver are degraded as the diversity level is increased. This is due to the non-coherent combining loss of the system [10].

CONCLUSION

In this paper, we have derived the bit-error rate expressions of the Reed-Solomon coded FFH/BFSK systems using the soft-decision linear-combining receiver. The effects of AWGN and the worst-case single tone per band MTJ are considered. The analytical BER expressions derived in this paper are easy to compute and applicable to higher diversity levels. Numerical results show that the optimum code rate increases as the diversity is increased for both diversity-combining techniques. It has also been shown that there is no diversity gain for the linear-combining receiver.

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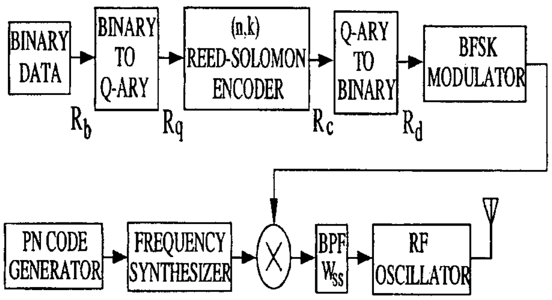

Figure 1: Block diagram of the Reed-Solomon coded FFH/BFSK transmitter.

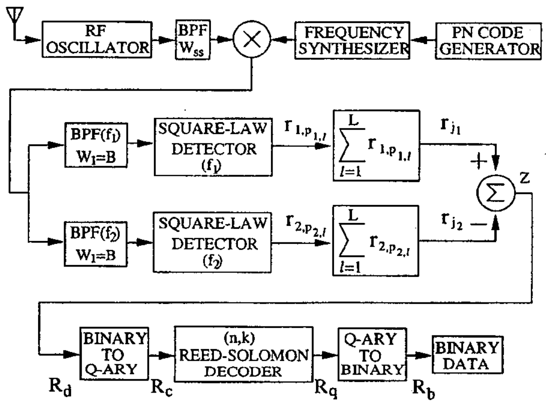

Figure 2: Block diagram of the Reed-Solomon coded FFH/BFSK linear-combining receiver.

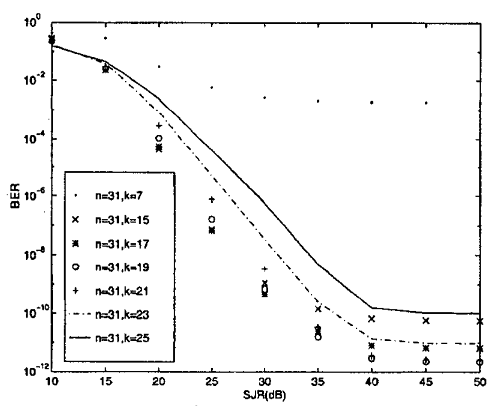

Figure 3: Worst-case BER results of the linear-combining receiver with $L = 1$, $E_b/N_0 = 13.35$ dB, and different code rates.

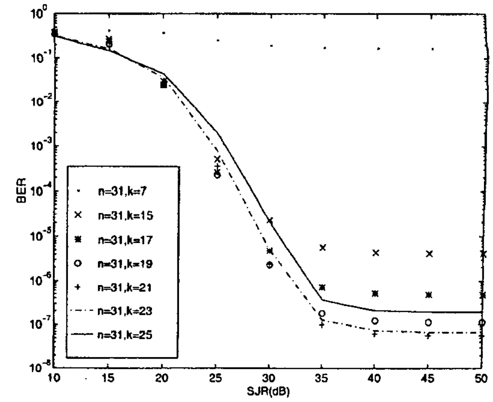

Figure 4: Worst-case BER results of the linear-combining receiver with $L = 3$, $E_b/N_0 = 13.35$ dB, and different code rates.

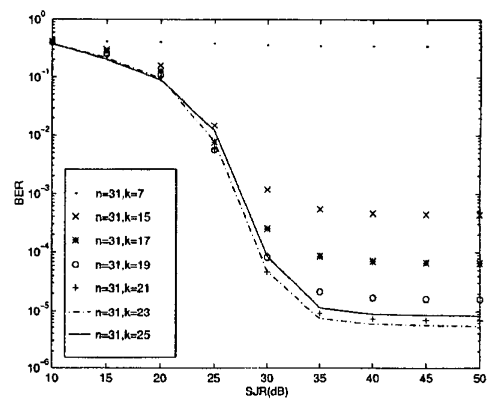

Figure 5: Worst-case BER results of the linear-combining receiver with $L = 5$, $E_b/N_0 = 13.35$ dB, and different code rates.

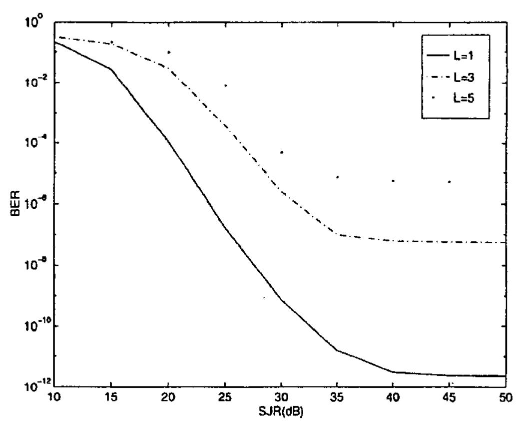

Figure 6: Comparisons of BER results of the linear-combining receiver with the optimum code rate and $E_b/N_0 = 13.35$ dB for different diversity levels.