

# ISAR SHIP IMAGING BASED ON REASSIGNED SMOOTHED PSEUDO WIGNER-VILLE DISTRIBUTION

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**Abstract**—According to the difference of motion characteristics between ship targets and the targets that move stationary, the application of a time-frequency distribution—Reassigned Smoothed Pseudo Wigner-Ville Distribution (RSPWVD) in ISAR imaging spectrum at ship target is studied. The high resolution instantaneous Doppler frequency spectra at a certain time instant is obtained by processing the radar echo data with RSPWVD, then the instantaneous ISAR image from that time instant is obtained. Results of real data show the validity of the method.

**Keywords**- Inverse Synthetic Aperture Radar; ship imaging; time-frequency distribution; reassignment method

## I. INTRODUCTION

Inverse Synthetic Aperture Radar (ISAR) is a well-known technology to obtain radar images of long-distance non-cooperative moving targets. The basic algorithm of ISAR imaging is Range-Doppler (RD) algorithm. RD algorithm use discrete Fourier transform (DFT) to analyze Doppler spectrum of radar signals. Doppler frequency is a constant in observation period is assumed. This assumption is satisfied when the moving of the target is stationary. However, for ship target, the sway caused by sea wave is the main source for imaging [2]. Because of sea condition and motion state of ship is unknown in practical imaging environment and their variation is difficult to be predicted, the Doppler frequency of radar echoes are time-varying. In this condition, time-frequency analysis towards radar echoes is needed to obtain Doppler frequency at different time instant.

According to ISAR instantaneous imaging, many imaging algorithms have been put forward at present. Such as joint time-frequency algorithm [3], adaptive chirplet decomposition algorithm [4][5] and imaging algorithm that based on parameter estimation of linear frequency modulated (LFM) signal[6][7]. But these algorithms need great amount of calculation and high calculation precision, especially for the last two algorithms.

In this paper a time-frequency distribution: Reassigned Smoothed Pseudo Wigner-Ville Distribution (RSPWVD) [8] has been applied to ISAR imaging of ship target. To a great extent, this distribution depresses the cross terms that caused by multi-component of signal and it has very high time-frequency concentration at the same time.

## II. ISAR IMAGING THEORY BASED ON TIME-FREQUENCY DISTRIBUTION

When the target moves stationary, the Doppler frequency of radar echoes that caused by the target rotating is a constant. Doppler spectrum can be obtained from the data sequence by DFT. But for ship targets, their Doppler frequency is time-varying. So the Doppler spectrum of whole course can not represent the distribution of scattering points in cross direction. Time-frequency analysis is needed to obtain high resolution instantaneous Doppler spectrum of each time instant. By Range-Instant-Doppler (RID) method we can get images of target in different time instant and view the target's attitude variation in the whole process.

## III. REASSIGNED SMOOTHED PSEUDO WIGNER-VILLE DISTRIBUTION

Reassigned Smoothed Pseudo Wigner-Ville Distribution is the reassigned form of quadric time-frequency distribution: Smoothed Pseudo Wigner-Ville Distribution (SPWVD). SPWVD is the Wigner-Ville Distribution which added a smooth window to depress cross-term. The definition formulae of these distributions and the reassignment method will be given as follows.

### A. Wigner-Ville Distribution

Signal  $x(t)$ 's Wigner-Ville Distribution (WVD) is defined as

$$W(t, f) = \int_{-\infty}^{\infty} x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau \quad (1)$$

For multi-component signal

$$x(t) = \sum_{i=1}^p x_i(t) \quad (2)$$

$p$  is the number of signal components in (2). Its WVD is

$$W(t, f) = \sum_{i=1}^p W_{x_i}(t, f) + \sum_{i,j=1, i \neq j}^p W_{x_i, x_j}(t, f) \quad (3)$$

The first term of (3) is the signal themselves WVD which is needed. The second term is cross-terms, which will interfere the signal term greatly. A feasible method to depress the influence of the cross-terms is to add windows in WVD, educe the concept of SPWVD.

#### B. Smoothed Pseudo Wigner-Ville Distribution

Signal  $x(t)$ 's Smoothed Pseudo Wigner-Ville Distribution (SPWVD) is defined as

$$SPWVD(t, f) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(u)h(\tau)x\left(t-u+\frac{\tau}{2}\right) \times x^*\left(t-u-\frac{\tau}{2}\right)e^{-j2\pi f\tau}dud\tau \quad (4)$$

$g(u)$  and  $h(\tau)$  are two real even window functions, and  $g(0)=h(0)=1$ . After adding windows, SPWVD greatly depressed the influence of the cross-terms of multi-component signal.

#### C. Reassignment method and Reassigned Smoothed Pseudo Wigner-Ville Distribution

Compared with WVD, SPWVD greatly depresses the influence of the cross-terms of multi-component signal. But its time-frequency concentration decreased somewhat. For depresses the influence of the cross-terms and keep high frequency concentration at the same time, people have made some effort in this aspect, obtained a general method called Reassignment method.

Write the general expression of Cohen type time-frequency distribution of signal  $x(t)$  as two dimensional convolution form of  $x(t)$ 's WVD as

$$C_x(t, f; \Pi) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Pi(t-s, f-\xi)W_x(s, \xi)dsd\xi \quad (5)$$

In which  $\Pi(s, \xi)$  is the kernel of Cohen type time-frequency distribution,  $W_x(s, \xi)$  is signal  $x(t)$ 's WVD. From (5) we can see  $\Pi(t-s, f-\xi)$  determined a neighborhood nearby point  $(t, f)$  to assign the weighted mean of the signal's WVD. However, there isn't any reason indicating that those mean values symmetrically distribute around the geometric center of the region  $(t, f)$ . Hence we assume that the mean values of the signal's WVD don't distribute around  $(t, f)$ , but around the gravity center of the region. It can represent the distribution of local energy better.

Reassignment method is a method which puts values of time-frequency distribution at any point  $(t, f)$  to another point  $(\hat{t}, \hat{f})$  which is the signal's energy gravity center around point  $(t, f)$ , as

$$\begin{aligned} \hat{t}(x; t, f) &= \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} s\Pi(t-s, f-\xi)W_x(s, \xi)dsd\xi}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Pi(t-s, f-\xi)W_x(s, \xi)dsd\xi} \\ \hat{f}(x; t, f) &= \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \xi\Pi(t-s, f-\xi)W_x(s, \xi)dsd\xi}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Pi(t-s, f-\xi)W_x(s, \xi)dsd\xi} \end{aligned} \quad (6)$$

$$C_x^{(r)}(t', f'; \Pi) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} C_x(t, f; \Pi)\delta(t'-\hat{t}(x; t, f))\delta(f'-\hat{f}(x; t, f))dtdf \quad (7)$$

$C_x^{(r)}(t', f'; \Pi)$  is the Cohen type time-frequency distribution after reassignment in (7). If we choose a suitable smoothing kernel, the distributions after reassignment will depress cross-terms and keep very high time-frequency concentration at the same time. Specially, for smoothed Pseudo Wigner-Ville distribution (SPWVD), Reassigned Smoothed Pseudo Wigner-Ville distribution (RSPWVD) is defined as

$$SPWV_x^{(r)}(t', f'; g, h) = \iint_{-\infty}^{+\infty} SPWV(t, f)\delta(t'-\hat{t}(x; t, f))\delta(f'-\hat{f}(x; t, f))dtdf \quad (8)$$

$$\begin{aligned} \hat{t}(x; t, f) &= t - \frac{SPWV_x(t, f; \tau_g, h)}{2\pi SPWV_x(t, f; g, h)} \\ \hat{f}(x; t, f) &= f + j \frac{SPWV_x(t, f; g, \frac{dh}{dt}(t))}{2\pi SPWV_x(t, f; g, h)} \end{aligned} \quad (9)$$

$SPWV_x^{(r)}(t', f'; g, h)$  is RSPWVD of signal  $x(t)$  in (8).

RSPWVD defined by (8) and (9) will be used to form ISAR ship images in this paper.

## IV. NUMERICAL EXAMPLES

#### A. Example1: Consider three linear frequency modulation signal

$$\begin{aligned} x_1(t) &= \exp[j(-0.5 \times 0.3589t^2 + 0.5\pi)] \\ x_2(t) &= \exp[j(0.5 \times 0.4589t^2 + 3\pi)] \\ x_3(t) &= 0.5 \exp(-j2\pi) \\ x(t) &= x_1(t) + x_2(t) + x_3(t) \end{aligned}$$

The SPWVD and RSPWVD of LFM signal  $x(t)$  are shown in figure 1

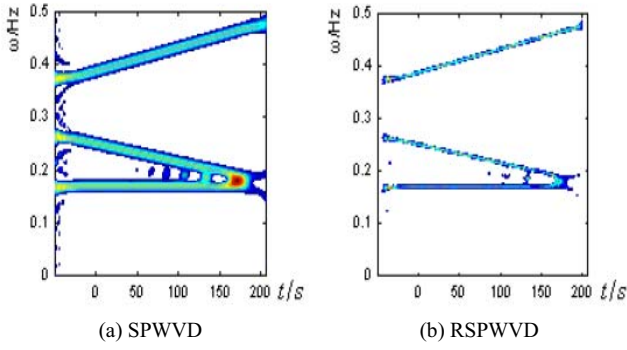


Figure 1. Time-frequency distributions of simulation signals

Fig.1(a) is SPWVD of  $x(t)$ , Fig.1(b) is RSPWVD of  $x(t)$ . We can see in Fig.1 that the time-frequency concentration of SPWVD after reassigned has greatly improved and the cross-terms are well depressed.

#### B. Example2:Real data of ISAR ship target

Using a piece of ISAR real data of ship target to verify RSPWVD method in ISAR ship imaging.

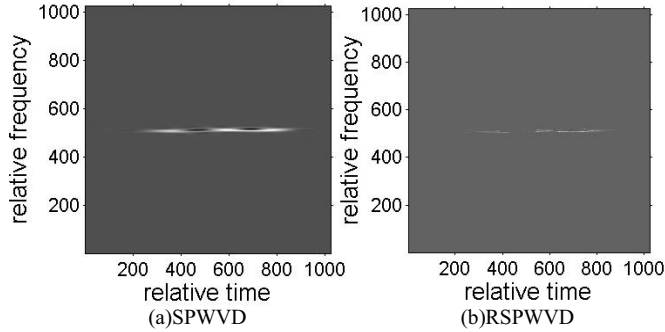


Figure 2. Time-frequency distributions of 400th range cell of ISAR ship data

Fig.2 is time-frequency distribution of 400th range cell of ISAR ship data after motion compensation. Fig.2(a) is the result of SPWVD, Fig.2(b) is the result of RSPWVD. From these two figures we can see the time-frequency concentration of RSPWVD is higher.

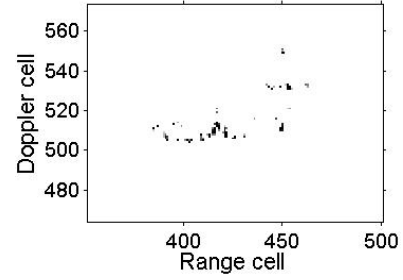


Figure 3. Result of ISAR ship target imaging

Fig.3 is the result of ISAR ship target imaging using RSPWVD. From Fig.3 we can see because of high time-frequency concentration of RSPWVD, a distinct image is given.

#### V. CONCLUSION

The application of Reassigned Smoothed Pseudo Wigner-Ville Distribution (RSPWVD) in ship target ISAR imaging is studied. The results of real data processing shows that by using RSPWVD, we can obtain better instantaneous images of ship target than by DFT and traditional Cohen type time-frequency distributions.

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