Computational Efficiency of Modified DFT Polyphase Filter Banks

N. J. Fliege

Hamburg University of Technology
Eissendorfer Strasse 40, D-21071 Hamburg, Germany

Abstract

In this paper, the so-called modified DFT polyphase filter bank is introduced. Main purpose of the paper is a comparison between the computational efficiency of some versions of the modified DFT polyphase filter bank and that of the well-known cosine-modulated filter bank. Additionally, some remarkable differences between both filter bank types are discussed.

1 Introduction: Modified DFT filter banks

In what follows, we will consider subband coding (SBC) filter banks which are built as a cascade of an analysis and a synthesis filter bank. Among the variety of known SBC filter bank structures we will select those with M-channel parallel structure and maximum decimation. Again, we have the choice between several classes of filter banks which differ by the design method. Among these, modulated filter banks have become very popular because of their facile design and the possibility to implement them by means of highly efficient DFT polyphase filter bank structures.

When we talk about a modulated filter bank we have a cosine-modulated filter bank in mind. Historically, the first filter banks of this kind were pseudo-QMF filter banks providing almost perfect reconstruction (Nussbaumer 1981, Rothweiler 1983, Chu 1985, Masson and Picel 1985, Cox 1986). Later, cosine-modulated filter banks with perfect reconstruction were proposed (Malvar 1990, Ramstad 1991, Koilpillai and Vaidyanathan 1991).

There is another class of modulated filter banks providing highest computational efficiency: the so-called DFT polyphase filter banks proposed by Bellanger et al. in 1974. These filter banks use complex modulation instead of cosine-modulation. Unfortunately, the original DFT polyphase filter bank provides no mechanism to cancel adjacent-spectrum aliasing. That is why this kind of filter bank is considered as nonsuitable for subband coding in literature, e.g. [Vai 90, Mal 92].

By means of a slight modification, it is possible to establish the almost perfect reconstruction property in DFT filter banks. Fig. 1 shows the modified DFT filter bank (MDFT filter bank).

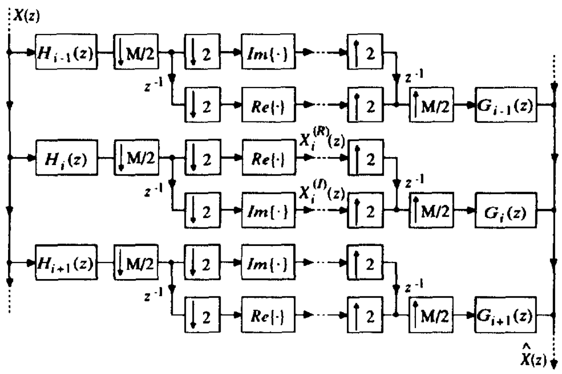

Figure 1: Modified DFT filter bank for SBC

The following two modifications are made to the MDFT filter bank:

- Subsampling, with and without a phase offset. By a two-stage subsampling the subband signals are firstly subsampled taking samples without shifting the phase, i.e. at times $n = m \cdot M$, and secondly subsampled taking samples with a phase offset of $M/2$, i.e. at times $n = n \cdot M + M/2$.
- Formation of the real and imaginary parts of the

1058-6393/93 $03.00 © 1993 IEEE

subband signals in the time-domain. The corresponding z-transforms are denoted by $X_i^{(R)}(z)$ and $X_i^{(I)}(z)$. The mapping of real and imaginary parts onto subsampling with and without a phase offset varies from channel to channel, see Fig. 1.

This structure is able to cancel exactly the adjacent-spectrum aliasing. The nonadjacent-spectrum aliasing is suppressed by an appropriately low stop band gain of the analysis and synthesis filters. The proof to this will be published elsewhere.

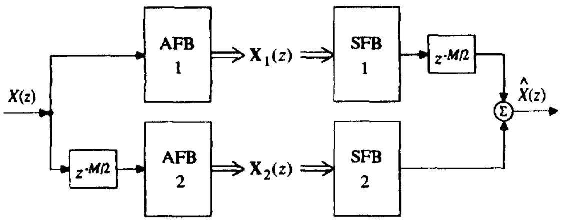

Figure 2: Equivalent structure of the MDFT filter bank, AFB = analysis filter bank, SFB = synthesis filter bank

Instead of subsampling the signals in each channel with and without a phase offset as shown in Fig. 1, we can use two separate filter banks. These both perform subsampling with no phase offset, but are driven by signals with the corresponding phase offset, see Fig. 2. The alternating formation of real and imaginary parts of the subband signal is not changed. For reasons of computational efficiency, the analysis and synthesis filter banks are realized as DFT polyphase filter banks. Additionally, M is chosen to be a power of 2 resulting in the efficient FFT algorithm for the filter banks.

2 Filter bank design

The transfer functions $H_i(z)$ of the analysis filter bank and $G_i(z)$ of the synthesis filter bank, $i = 0, 1, 2, \dots M-1$, where M is the number of channels, is derived from a suitable prototype baseband filter $H(z)$ by frequency shifting, i.e. $H_i(z) = H(zW_M^i)$ and $G_i(z) = M \cdot H(zW_M^i)$ with $W_M = \exp(-j2\pi/M)$. Then the transfer function of the filter bank (distortion transfer function) is given by

$$F(z) = \sum_{k=0}^{M-1} H^2(zW_M^k). \qquad (1)$$

Fig. 3 shows a scheme of the frequency characteristic of the prototype filter H and of the filters H_i derived by frequency shifts of multiples of $2\pi/M$. The prototype filter is assumed to be effectively bandlimited to $2\pi/M$. If the transfer function $H(z)$ is power-complementary with respect to a reduced sampling rate $2\pi/M$, then the overall transfer function $F(z)$ is a pure delay.

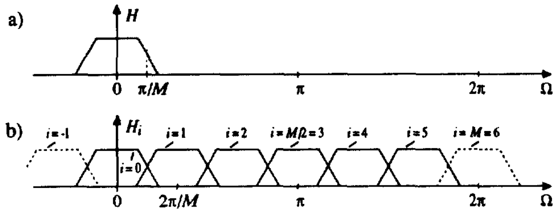

Figure 3: Course frequency characteristic of the prototype filter (a) and the uniform filter bank (b)

In [Fli 93] a closed form design of the prototype transfer function is proposed. It is based on linear phase FIR square root raised-cosine functions and provides an approximate power-complementary prototype filter. This kind of prototype can be used for complex modulated as well as for cosine-modulated subband coding as well as transmultiplexer filter banks. In each case we obtain a filter bank with almost perfect reconstruction, i.e. linear distortion (or intersymbol interference) and aliasing (or crosstalk) can be kept arbitrarily small by means of an appropriate length of the prototype impulse response.

If we like to talk about the computational efficiency of modulated filter banks we must be able to estimate the length of the prototype filter. Although not equally approximated in the pass and in the stop band it has been proved that the length of the square root raised-cosine filters can be estimated by a similar expression as that proposed by Bellanger [Bel 84]: If we accept a maximum ripple of 0.2 dB in the pass band and a maximum gain of -80 dB in the stop band we can use the following estimate for the number of coefficients:

$$N \approx 3 \cdot \frac{1}{b} \qquad (2)$$

with b being the relative transition bandwidth. In case of a square root raised-cosine characteristic and a frequency scheme as shown in Fig. 3a this parameter is given by $b = r/M$ with r being the rolloff factor of the filter. Substituting this expression for b in (2) yields

$$N \approx 3 \cdot \frac{M}{r}. \qquad (3)$$

Subsequently, for comparison of the computational efficiency of different filter banks we will use a prototype filter with rolloff factor $r = 0.5$. In case of a cutoff frequency of π/M, see Fig. 3a, the prototype design will result in an impulse response with $N = 6 \cdot M$ coefficients. If we double the bandwidth of the prototype, the number N is halved. If we halve the bandwidth, N is doubled.

3 Efficiency of MDFT filter banks (real signals)

In the following, the sampling rate of the input and output signals is referred to as system sampling rate f_{ss}. The sampling rate of the subband signals may be called subband sampling rate f_{sb}, where $f_{ss} = M \cdot f_{sb}$, M being the number of subbands.

Furtheron, let us define a signal processing operation (SPO) as a multiplication together with an accumulation (addition) and some memory access operations. Modern signal processing architectures can carry out these operation in parallel within one instruction cycle. For simplicity, we will regard the multiplications in the filters and fast transform algorithms as SPO. (Additions could be counted as extra SPO. However, this leads to similar results).

In case of the considered filter banks we will distinguish between the SPO for filtering and SPO for modulation. Dealing with a DFT polyphase analysis filter bank and with real input signals we have $N \cdot f_{sb}$ SPO per second performed by the polyphase filters. The MDFT filter bank consists of two analysis filter banks and needs therefore $N \cdot 2f_{sb}$ SPO per second, see Fig. 2. With $N = 6M$, $f_{ss} = M \cdot f_{sb}$, and $f_{ss} = 1$ we obtain the efficiency of the polyphase filters as

$$\mathcal{N}_{filter} = 6 \cdot 2 = 12 \qquad (4)$$

SPO per input sample.

The modulation is done by means of the FFT algorithm. If we use real input signals we need $(M/2 \cdot \log_2 M - 3M/2 + 2) \cdot 2f_{sb}$ SPO per second [Mal 92] for modulation. (The factor 2 in $2f_{sb}$ indicates the number of analysis filter banks). Again, with $f_{ss} = M \cdot f_{sb}$ and $f_{ss} = 1$ we obtain the efficiency of the modulation as

$$\mathcal{N}_{RMDFT} = \log_2 M - 3 + \frac{4}{M} \qquad (5)$$

SPO per input sample.

It may be noted that the analysis filter bank provides $2M$ subband signals (M complex signals). However, due to the real input signal half of them are conjugates of the other half, so that we have M different real subband signals.

4 Efficiency of MDFT filter banks (complex signals)

A MDFT polyphase filter bank with M channels driven by a complex-valued input signal leads to M complex or $2M$ real subband signals. Due to the complex samples and the real coefficients the polyphase filters need twice the number of SPO, namely $2N \cdot 2f_{sb}$ SPO per second.

The FFT algorithm applied to complex input values needs more SPO than in case of real values, namely $M \cdot \log_2 M - 3M + 4$ [Mal 92]. Therefore, instead of (5) we need $2 \cdot \log_2 M - 6 + \frac{8}{M}$ SPO per input sample if we use complex input signals.

As it will be shown later, we can achieve the highest efficiency by converting a real input signal into a complex one with half sampling rate and by applying a MDFT filter bank with a decimation of $M/2$. This will result into M real subband signals each decimated by M. Fig. 4 shows a block diagram of the analysis filter bank.

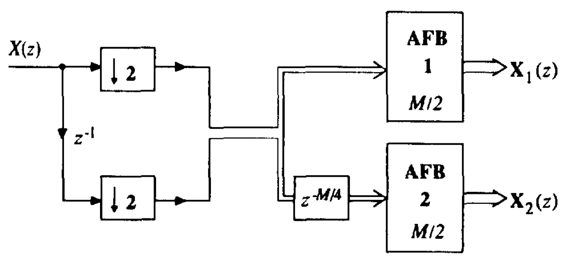

Figure 4: MDFT analysis filter bank with real input signals and complex signal processing

By changing the number of filter bank channels from M to $M/2$ we double the bandwidth of the filters. We can derive the new filter with unchanged characteristics (pass band ripple, stop band gain) from the old filter by taking every second coefficient of the impulse response. Thus, the filter length is halved, i.e. N=3M. Therefore, with $f_{ss} = M \cdot f_{sb}$, and $f_{ss} = 1$ the efficiency of the polyphase filters is given by

$$\mathcal{N}_{filter} = 2 \cdot N \cdot 2\frac{f_{ss}}{M} = 12 \qquad (6)$$

SPO per input sample. This is the same efficiency as in case of real signals, see (4).

A certain efficiency improvement is obtained from the modulation overhead. One FFT of length $M/2$ with complex inputs needs $[(M/2) \cdot \log_2(M/2) - (3M/2) + 4]$ SPO. This leads to

$$\mathcal{N}_{CMDFT} = \log_2 \frac{M}{2} - 3 + \frac{8}{M} \qquad (7)$$

SPO per input sample.

5 Efficiency of cosine-modulated filter banks

The cosine-modulated filter bank is composed of $2M$ filters, each of them with half the bandwidth of the complex modulated filter bank. The prototype filter can be derived from the prototype of the DFT filter bank without changing the filter characteristics by interpolation with a factor of 2. Therefore we obtain a comparable filter of length $N = 12M$. The polyphase filter bank is operated only once in one subband sampling space. All multiplications are real. Thus, the efficiency of the filter part of the cosine-modulated filter bank is given by

$$\mathcal{N}_{filter} = 1 \cdot 12 \cdot 1 = 12 \qquad (8)$$

SPO per input sample, which is the same result as in case of the two other filter banks considered above.

The modulation can be done by means of some simple diagonal matrices and a DCT of type 4 [Vai 93]. The DCT type 4 algorithm needs $(M/2) \cdot \log_2 M + M$ SPO [Mal 92] and has to be performed $f_{sb} = M/f_{ss}$ times a second. Without taking the diagonal matrices into account we obtain a modulation overhead efficiency of

$$\mathcal{N}_{COS} = \frac{1}{2} \log_2 M + 1 \qquad (9)$$

SPO per input sample.

6 Comparison

In the following, we first will evaluate the efficiency expressions derived above. Fig. 5 shows in a diagram the signal processing operations per input sample as a function of the decimation factor M. The different efficiency expressions have been evaluated for $M = 16, 32, 64, 128, 256$, and 512.

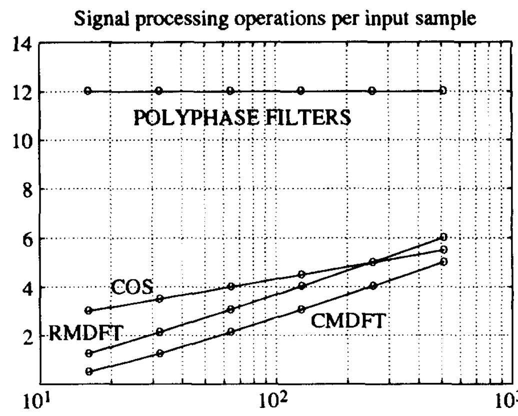

Figure 5: Computational efficiency in form of signal processing operation per input sample of the polyphse filter part and the modulation overhead of some filter banks: cosine modulated filter bank (COS), real-valued MDFT filter bank (RMDFT), and complex-valued MDFT filter bank (CMDFT)

One fundamental result is that according to (4), (6), and (8) the filter parts of the three filter banks possess the same constant efficiency independent of the decimation factor or number of subband signals, respectively. Assuming the above mentioned filter characteristics we always need 12 signal processing operation (multiplications plus accumulations plus some memory access operations) per input sample, see Fig. 5.

There are slight differences in the efficiency of the modulation overhead. The overhead ranges from less than 10% until 50% of the filter operations. The differences between the considered filter bank types are not substantial. The cosine-modulated filter bank needs the highest number of operations. (The additional overhead in form of diagonal matrices is not taken into account in Fig. 5). The MDFT filter bank with complex signal processing needs always somewhat less operations.

Besides the computational efficiency, in the following some other properties may be noticed:

- Complexed-valued input signals can also be mapped into complex subband signals using two cosine-modulated filter banks. There is, however, a difference in the kind of mapping: the cosine-modulated filter bank maps the real part of the input signal into the real parts of the subband signals. The corresponding holds for the imaginary parts. The MDFT filter bank maps the real part of the input signal into the real as well as into the imaginary parts of the subband signals. The same holds for the imaginary part. This fact might be exploited to eliminate correlated signal components in coding applications.
- The MDFT filter bank structure is more straight forward, it consists of multiplexers, demuliplexers, FIR filters, and FFT or IFFT blocks. All these parts are very regular and hence suitable for designing an efficient integrated circuit architecture. In case of a chip design it is not necessary to double the analysis and synthesis filter banks as shown in Fig. 2. Only the state variable memories must be doubled.
- Designing the prototype filter of the MDFT filter bank we have already the final characteristic of the analysis and synthesis filters. This is different from the cosine-modulated filter bank where the analysis filters are derived from a sum of scaled complex conjugate prototypes.
- The considered three filter banks have different filter bandwidths. The bandwidth may have different (positive or negative) significance in different applications. For example, if we use the transposed structure of the SBC filter bank we obtain a transmultiplexer filter bank which can be used for multicarrier data transmission [Fli 90, Fli 92]. The MDFT filter bank with complex signals provides the highest bandwidth und thus the lowest transmission delay, which is a significant property in data transmission systems.

7 Conclusions

The performance characteristics of the new MDFT filter bank have been presented. The filter bank provides highest computational efficiency which is at least not lower than that of the well-known cosine-modulated filter banks. Due to some specific properties the MDFT polyphase filter bank may open new ways in certain applications.

References

[Bel 74] M.G. Bellanger, J.L. Daguet: *TDM-FDM Transmultiplexer: Digital Polyphase and FFT.* IEEE Trans. Commun., vol. COM-22, pp. 1199-1204, Sept. 1974.

[Bel 84] M. Bellanger: *Digital Processing of Signals.* New York: John Wiley & Sons, 1984.

[Cox 86] R.V. Cox: *The Design of Uniformly and Nonuniformly Spaced Pseudoquadrature Mirror Filters.* IEEE Trans. ASSP, vol. 34, pp. 1090-1096, October 1986.

[Fli 90] N.J. Fliege: *Polyphase FFT Filter Bank for QAM Data Transmission.* Proc. IEEE ISCAS'90, pp. 654-657, 1990.

[Fli 92] N.J. Fliege: *Orthogonal multiple carrier data transmission.* European Trans. on Telecomm. ETT, vol. 3, pp. 255-265, May 1992.

[Fli 93] N.J. Fliege: *Closed Form Design of Prototype Filters for Linear Phase DFT Polyphase Filter Banks.* Proc. IEEE ISCAS'93, pp. 651-654, 1993.

[Koi 92] R.D. Koilpillai, P.P. Vaidyanathan: *Cosine-Modulated FIR Filter Banks Satisfying Perfect Reconstruction.* IEEE Trans. SP, vol. 40, pp. 770-783, April 1992.

[Mal 90] H.S. Malvar: *Modulated QMF Filter Banks with Perfect Reconstruction.* Electr. Letters, vol. 26, pp. 906-907, June 1990.

[Mal 92] H.S. Malvar: *Signal Processing with Lapped Transforms.* Norwood: Artech House, 1992.

[Mas 85] J. Masson, Z. Picel: *Flexible design of computationally efficient nearly perfect QMF filter banks.* Proc. IEEE ICASSP'85, pp. 14.7.1-14.7.4, March 1985.

[Ngu 93]
T.Q. Nguyen: *Near-Perfect-Reconstruction Pseudo-QMF Banks.* Will be published in IEEE Trans. on SP (and in Proc. Asilomar Conf. 1993).

[Ram 91] T.A. Ramstad, J.P. Tanem: *Cosine-Modulated Analysis-Synthesis Filter Bank with Critical Sampling and Perfect Reconstruction.* Proc. IEEE ICASSP'91, pp. 1789-1792, May 1991.

[Rot 83] J.H. Rothweiler: *Polyphase Quadrature Filters - A New Subband Coding Technique.* IEEE ICASSP'83, pp. 1280-1283, 1983.

[Vai 90] P.P. Vaidyanathan: *Multirate Digital Filters, Filter Banks, Polyphase Networks and Applications: A Tutorial.* Proc. IEEE, vol. 78, pp. 56-93, January 1990.

[Vai 93] P.P. Vaidyanathan: *Multirate Systems and Filter Banks.* Englewood Cliffs: Prentice-Hall, 1993.