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The Research on Frequency-Hopping Signals Analysis methods Based on Adaptive Optimal Kernel Time-Frequency Representation

Li Nan
School of Information Engineering,
Northeast Dianli University JiLin
Jilin, China
jllinan@163.com

Dong Shuang
Power Transmission and
Transformation Technology college
Jilin China
Dongshuang_1015@126.com

Yang Dongmei，Hao Zhonghua
Gongzhuling City Rural Power
Limited Company, Gongzhuling,
Gongzhuling China

Abstract— **It is inevitable to be interfered by cross-term interference when frequency-hopping signal parameters are estimated using time-frequency distribution, how to curb cross-term signal interference has become a main problem of time-frequency analysis. Based on studying of adaptive optimal kernel time-frequency representation（AOK TFR）, a new time-frequency method is put forward. This kind of AOK TFR can tune its kernel ambiguity function shape by two-dimensional optimization procedure according to the characteristics of the analyzed signal. A frequency-hopping signal is analyzed with four time-frequency analysis methods, and simulation results used AOK TFR shows that better time-frequency and adaptability are obtained, and anti-noise and cross-term interference are represented.**

Keywords* — *time-frequency distribution; cross-term; adaptive optimal kernel function; frequency-hopping signals; parameter estimation

I. INTRODUCTION

Frequency-hopping communication has shown great advantages in the field of the current modern electronic war for it has good capabilities of anti-interference, anti-multipath fading, the low rate of intercepted communications, and multi-site communications. It is an important premise for implementing frequency-tracking interference to detect the frequency-hopping law of the received frequency-hopping signals, so how to accurately predict the parameters of frequency-hopping signal has become one of focus studies in modern electronic warfare. Frequency-hopping signal is a typical time-varying multi-component non-stationary signal due to its constantly changing over time, so the time-frequency analysis theory is required to be adopted.

At present, the common time-frequency analysis methods of frequency-hopping signal include: short-time Fourier transform (STFT)[1], Wigner-Ville distribution (WVD), pseudo-Wigner-Ville distribution (PWVD), and smooth pseudo-Wigner-Ville distribution (SPWVD) [2-4]. However, these methods have significant drawbacks. STFT can not obtain a good resolution ratio in time-domain and frequency-domain at the same time because of the restraint of the uncertainty principle. It is inevitable that the result was interfered by the cross-term interference (CTI) when the multi-component analysis of frequency hopping signals is analyzed using WVD, PWVD and SPWVD. Although the impact of crossing-term interference is reduced by the designing of kernel function and adopting the smooth technology according to distribution characteristics of the signal and cross-term in the ambiguity-domain, the received signal is always random frequency-hopping signals, and different signals require different kernel function, so the fixed kernel function can not change adaptively with signal to realize the greatest degree inhibition for cross-term.

In this paper, a time-frequency analysis method based on adaptive optimal kernel (AOK FTR) time-frequency representation is put forward. The interference of cross-term can be suppressed maximum degree by adopting signal-based nuclear function, which changes adaptively with signals. Four time-frequency analysis methods are used to analyze simulation and performance comparison of the time-frequency signals. For multi-component random frequency-hopping signals, adaptive kernel time-frequency analysis method has not only good inhibition for cross-terms, but also good aggregation for self-component of signal.

II. ADAPTIVE KERNEL TIME-FREQUENCY DISTRIBUTION

Time-frequency Distribution

Time-frequency analysis describes the signal frequency changes over time, which maps one-dimensional time-domain signal to the two-dimensional time-frequency plane, and it is the main tool of non-stationary signals analyzing.

Among the time-frequency analysis methods, Wigner-Ville distribution is the most basic method, for it has many attractive qualities such as high time-frequency resolution and time-bandwidth product reaching the lower bound given by Hisen-berg's uncertainty principle[5]. However, Wigner-Ville distribution is bilinear transformation, so the scope of its application is limited because of the more serious cross-term interference existence for the multi-component signal.

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At present, general form of Cohen's bilinear class time-frequency distribution is given by

$$P(t,w)=\frac{1}{2\pi}\iint A(\theta,\tau)\phi(\theta,\tau)e^{-j\theta t-j\tau\omega}d\theta d\tau \qquad (1)$$

$A(\theta,\tau)$ is the ambiguity function (AF) of signal $s(t)$ and is defined as:

$$A(\theta,\tau)=\int s(t+\frac{\tau}{2})s^{*}(t-\frac{\tau}{2})e^{j\theta t}dt \qquad (2)$$

Weighting function $\phi(\theta,\tau)$ is defined as kernel function. Different choices for the Kernel function yield widely different time-frequency distribution, which determines the nature of time-frequency distribution.

Optimal Kernel Time-frequency Distribution

Radial Gaussian kernel time-frequency distribution is a kind of ideal time-frequency distribution with a lot of good characteristics, such as good cross-term suppression, higher aggregation of time-frequency distribution, the optimized algorithm achieved through simple iteration, and the same amount of calculation between the entire time-frequency distribution and fixed kernel time-frequency distribution. The signal-dependent TFR proposed is based on kernels with Gaussian radial cross sections

$$\phi(\theta,\tau)=\exp(-\frac{\theta^{2}+\tau^{2}}{2\sigma^{2}(\psi)}) \qquad (3)$$

The function $\sigma(\psi)$ controls the spread of the Gaussian at radial angle ψ; we will call $\sigma(\psi)$ the spread function. Clearly, if σ is smooth, then ϕ is also smooth. The angle $\psi=\arctan(\tau/\theta)$ is measured between the radial line through the point (θ,τ) and the θ axis. It is natural to express radial Gaussian Kernels in polar coordinates, using $r=\sqrt{\theta^{2}+\tau^{2}}$ as the radius variable

$$\phi(r,\psi)=\exp(-\frac{r^{2}}{2\sigma^{2}(\psi)}) \qquad (4)$$

A high-quality time-frequency representation results when the kernel is well matched to the components of a given signal. The radial Gaussian kernel is adapted to a signal by solving the following optimization problem:

$$\max_{\phi}\int_{0}^{2\pi}\int_{0}^{\infty}|A(r,\psi)\phi(r,\psi)|^{2}rdrd\psi \qquad (5)$$

Subject to

$$\phi(r,\psi)=\exp(-\frac{r^{2}}{2\sigma^{2}(\psi)}) \qquad (6)$$

$$\frac{1}{2\pi}\int_{0}^{2\pi}\int_{0}^{\infty}|A(r,\psi)\phi(r,\psi)|^{2}rdrd\psi \qquad (7)$$

$$=\frac{1}{2\pi}\int_{0}^{2\pi}\sigma^{2}(\psi)d\psi\le\alpha,\alpha\ge 0$$

Here, $A(r,\psi)$ represents the AF of the signal in polar coordinates. The first constrain (6) limits the scope of the optimization to the class of radial Gaussian kernels. Thus, the kernel is constrained to be a low-pass filter. Since the AF auto-components are centered at the origin, this encourages the kernel to preferentially pass auto-components. The second constrain (7) limits the volume of the kernel to α so that cross-components are suppressed. An advantage of this formulation is that the constraints are insensitive to both the time scale and orientation of the signal in time-frequency.

The performance measure (5) determines the shape of the pass-band of the optimal radial Gaussian kernel; it expressed a desire to minimize auto-component distortion by passing as much auto-component energy as possible into the TFR for a kernel of fixed volume. Clearly, in order to maximize the performance measure, $\phi(r,\psi)$ should be large where $A(r,\psi)$ is large, regardless that whether the peaks correspond to auto or cross-components. However, the radial Gaussian constraint favors kernels that pass components concentrated at the origin, which are the auto-components, because a large spread in the direction of cross-components requires excessive kernel volume.

Adaptive Optimal Kernel Time-frequency Distribution

The signal-dependent TFR with radial Gaussian kernel has many attractive qualities, but it is based on a block algorithm that designs only one Kernel for the entire signal. For signals with characteristics that change over time, for real-time, on-line operation or for very long signal, an adaptive signal-dependent TFR is more desirable. An adaptive algorithm alters the kernel at each time to achieve optimal local performance, thus better tracking changes in a signal. R.G.Baraniuk and D.L.Jones designed the adaptive kernel with two-dimensional radial Gaussian using constrained optimization methods [6、7].

We define the *symmetrical* STAF $A(t;\theta,\tau)$ as the AF of a windowed signal, with the window centered at time *t*. the STAF is given by

$$A(t;\theta,\tau)=\int_{-\infty}^{\infty}s^{*}(u-\frac{\tau}{2})\omega^{*}(u-t-\frac{\tau}{2}) s(u+\frac{\tau}{2})\omega(u-t+\frac{\tau}{2})e^{j\theta u du} \qquad (8)$$

where $\omega(u)$ is a symmetrical window function equal to zero for $|\mu|>T$. The variables τ and θ are the usual ambiguity plane parameters; the variable *t* indicates the center position of the signal window. Only the portion of the signal in the interval [$\tilde{t}T$, tT] is incorporated into $A(t;\theta,\tau)$ and the range of τ for which the windowed instantaneous correlation function is nonzero is $|\tau|<2T$.

Given a time-localized STAF, the adaptive signal-dependent TFR with radially Gaussian kernel is straightforward to define. Conceptually, the algorithm proceeds as follows. At each time *t,* we compute the STAF centered at time *t* in both rectangular and polar coordinates and solve the optimization problem (5)-(7) to obtain the optimal kernel $\phi_{opt}(t;\theta,\tau)$. Since the STAF varies with time, so does the optimal signal-dependent kernel. Once the optimal kernel has been determined, a single, current-time slice of the adaptive optimal-kernel (AOK) TFR is computed as one slice (at time *t* only) of the 2-D Fourier transform of the STAF-kernel product: The AOK procedure for discrete-time data is defined similarly.

$$P_{AOK}(t,\omega) = \frac{1}{2\pi}\iint A(t;\theta,\tau)\phi_{opt}(t;\theta,\tau)e^{-j\theta t - j\tau\omega}\,d\theta\,d\tau \qquad (9)$$

III. Simulation and Performance Analysis

Numerical Simulation

Define frequency-hopping signal model as:

$$s(t) = A\sum_k rect_{T_H}(t - kT_H)e^{j2\pi f_k(t-kT_H)+j\theta} + n(t) \qquad (10)$$

$$0 < t \le T$$

Here A is signal amplitude, T_H is dwell time for each jump, $rect_{TH}$ is the width of the rectangular window, f_k is frequency hopping, θ is random phase, $n(t)$ is additive noise. In order to simplify the simulation, we let $\theta = 0$, and assume that the observation time is 8 hopping-cycles, the sampling frequency fs = 100Hz, hopping-cycle TH is 0.64s and its frequencies-hopping followed by f_k (5, 45, 20, 10, 35, 25, 40, 5). 512 samples have been obtained by sampling.

When signal to noise ratio SNR is 8dB, analysis methods with WVD, PWVD, SPWVD and AOK TFR are utilized to analyze frequency-hopping signals. Fig.1 shows the diagram of time-domain signal wave, Fig.2 illustrates time-frequency analysis diagram of the WVD, PWVD, SPWVD and AOK TFR.

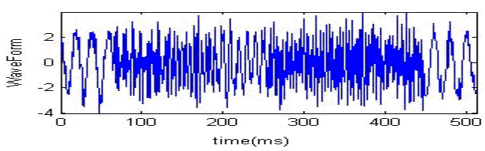

Figure 1. Wave figure of frequency-hopping signals

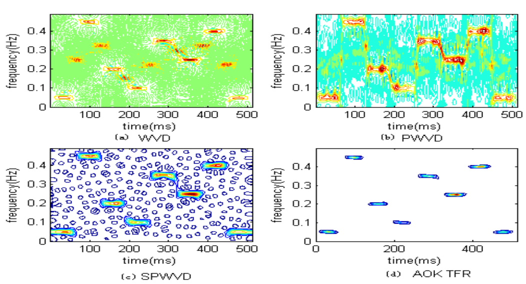

Figure 2. Time-frequency distribution of frequency-hopping

As we can see from the Fig.2(a), outline of the frequency-hopping pattern shown blurry using WVD, which is due to cross-interference existence. Fig.2 (b) is a once smooth PWVD map in the frequency domain. Although oscillation interference along with the frequency axis has been eliminated, it is not very reliable to estimate frequency signal parameters using PWVD because of the interference oscillations along the timeline. Fig.2(c) is time-frequency distribution graph which is smooth in the time domain and frequency domain. The cross-term interference and a clear outline of the frequency-hopping patterns are reduced shown in the map, but the estimated accuracy of signal parameters will be reduced to a certain extent because of the kernel function smoothing effect which makes auto-component expend and aggregation of time-frequency loss. Fig.2(d) is the time-frequency distribution map using adaptive kernel for smoothing, which automatically selects the appropriate kernel for smoothing filter according to the analysis signals, so the CAI has been largely suppressed. We can accurately recognize frequency-hopping pattern. Its time-frequency performance can be comparable with WVD and has high-resolution time-frequency.

Though observing the above-mentioned time-frequency distribution, there is a peak altitude in center frequency hopping time. To take maximum frequency along the frequency axis, we would obtain a significant periodic oscillating map. Shown in Fig.3, the oscillation frequency is the rate of hopping, and its countdown is hopping cycle. Oscillation frequency can be easily obtained using the FFT transform. Shown in Fig.4, the peak position of spectrum is the value of frequency-hopping rate.

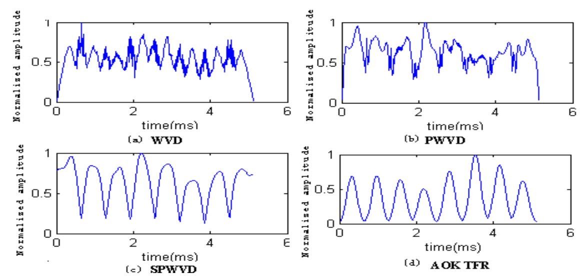

Figure.3 Max value of time-frequency cure

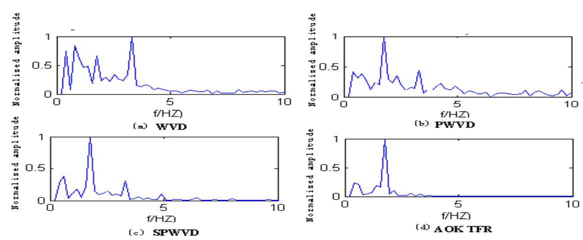

Figure.4 Spectrum of time-frequency series in time-frequency distribution

Fig.3(a) shows the curve is similar to the noise because of the cross-term interference, so it is difficult to visually determine its periodicity. Fig.3 (b), (c), (d) illustrate that the cross-term interference has been eliminated, and the curve has obvious periodicity because of utilizing smoothing and filtering techniques in the time-frequency as well as time-frequency domain. The cyclical nature in Fig.3(d) is stronger than Fig (b) and Fig(c), and it is closer to sine wave with frequency-hopping period. It can be seen from that the spectrum in Fig.4(d) is cleaner than it in Fig.4(a), (b), (c), so the higher accuracy of parameters estimates can be obtained.

Performance Analysis

Based on the above-mentioned frequency signal model, Tab.1 gives the estimated values considering different signal-to-noise ratios and different hopping periods on condition that repeatedly tried 400 times and utilized four kinds of analysis methods.

TABLE I. ESTIMATION OF FREQUENCY-HOPPING DURATION AT DIFFERENT SIGNAL-NOISE RATE

SNR（dB）			-4	-2	0	2	4	6	8
Frequency-hopping Period T_h(MS)	0.32	WVD	1.954	1.723	1.551	1.196	1.602	1.212	0.998
		PWVD	1.438	1.348	1.257	0.776	0.787	0.320	0.320
		SPWVD	1.435	1.342	1.252	0.771	0.780	0.320	0.320
		AOK TFR	1.291	1.046	0.994	0.320	0.320	0.320	0.320
	0.24	WVD	1.141	1.056	1.018	0.753	0.224	0.963	1.320
		PWVD	1.073	0.957	0.964	0.923	0.471	0.240	0.240
		SPWVD	1.071	0.955	0.962	0.922	0.470	0.240	0.240
		AOK TFR	0.913	0.755	0.714	0.722	0.240	0.240	0.240
	0.64	WVD	3.394	3.817	2.715	3.563	3.624	2.622	0. 321
		PWVD	1.894	1.618	1.217	0.712	0.640	06.40	0. 640
		SPWVD	1.821	1.613	1.212	0.640	0.640	0.640	0. 640
		AOK TFR	1.673	1.278	0.640	0.640	0.640	0.640	0. 640

As shown in Tab.1, the frequency-hopping period can not accurately estimate because of the serious cross-term using WVD, even if the signal-to-noise ratio is 8dB. Accurate estimates of the frequency-hopping period can be obtained using PWV, SPWV analysis methods when signal-to-noise ratio is 2dB and 4dB. In this paper, a more accurate valuation of the frequency-hopping period can be obtained when signal-to-noise ratio is 0dB using AOK TFR method. In addition, the simulation results show that the calculation precision is associated with the number of observation signals, observation time, and the number of samples involved in computing. The higher accuracy is estimated, but calculation time is longer when number of signal observation and samples involved in computing are more and observation time is longer.

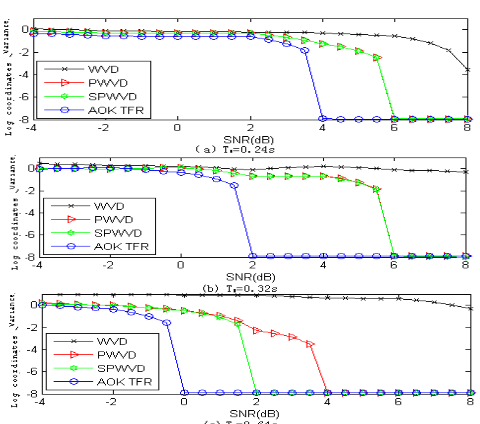

Figure.5 Estimation variance curve of hop duration in different signal-noise rate

Fig.5 gives the average variance curve of hopping period trying 200 times under different signal-to-noise ratio. Compared with the estimated performance curve, we can see that different frequency-hopping period has the different anti-noise performance: the longer frequency-hopping period has the stronger capacity of anti-noise. When frequency-hopping period is fixed, frequency variance will decrease with the increasing of signal-to-noise ratio and a value of the jump takes place at some place of value. The reason is that power of signal increases when the signal-to-noise ratio is low, then frequency components of harmonic signal and noise after FFT in the same part of the stack lead to beyond signal peaks, and become the largest peak, so relative valuation errors show very large distinction between hopping-frequency period and the actual value, and the accuracy degree of estimates cuts down quickly.

IV. CONCLUSION

Based on studying of adaptive optimal kernel time-frequency representation(AOK TFR）, this paper puts forward a new time-frequency method. Adaptation of the kernel over time is beneficial because it permits the kernel to match the local signal characteristics**.** The AF is utilized in the optimization formulation because of its important separation property. The auto and cross-components separate somewhat in the AF plane, with the auto-components lying centered at the origin and the cross-components lying away from the origin. The simulation shows that accuracy of extraction frequency-hopping period is better than WVD, PWVD, SPWVD, and other analytical methods. It is not sensitive to noise, and it still maintains a higher time-frequency resolution when signal-to-noise ratio is very low.

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