# Discrete Cosine Transform

# Transform Coding

- Spatial image data (image or motion-compensated residual image) are transformed into a different representation, transform domain.
  - Make the image data easy to be compressed
- Techniques:
  - Discrete Cosine Transform (DCT)
    - Usually applied to small regular blocks of image, ex. 8 x 8 squares
    - JPEG, H.26x, MPEG-x
  - Discrete Wavelet Transform (DWT)
    - Usually applied to larger image section, ex. Tiles, or to complete image
    - JPEG 2000, MPEG-4 still texture

# Basic Transformation Forms

- 2-D forward transforms

$$T(u,v) = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} \underbrace{g(x,y)}_{\text{Original signal}} f(x,y,u,v),$$

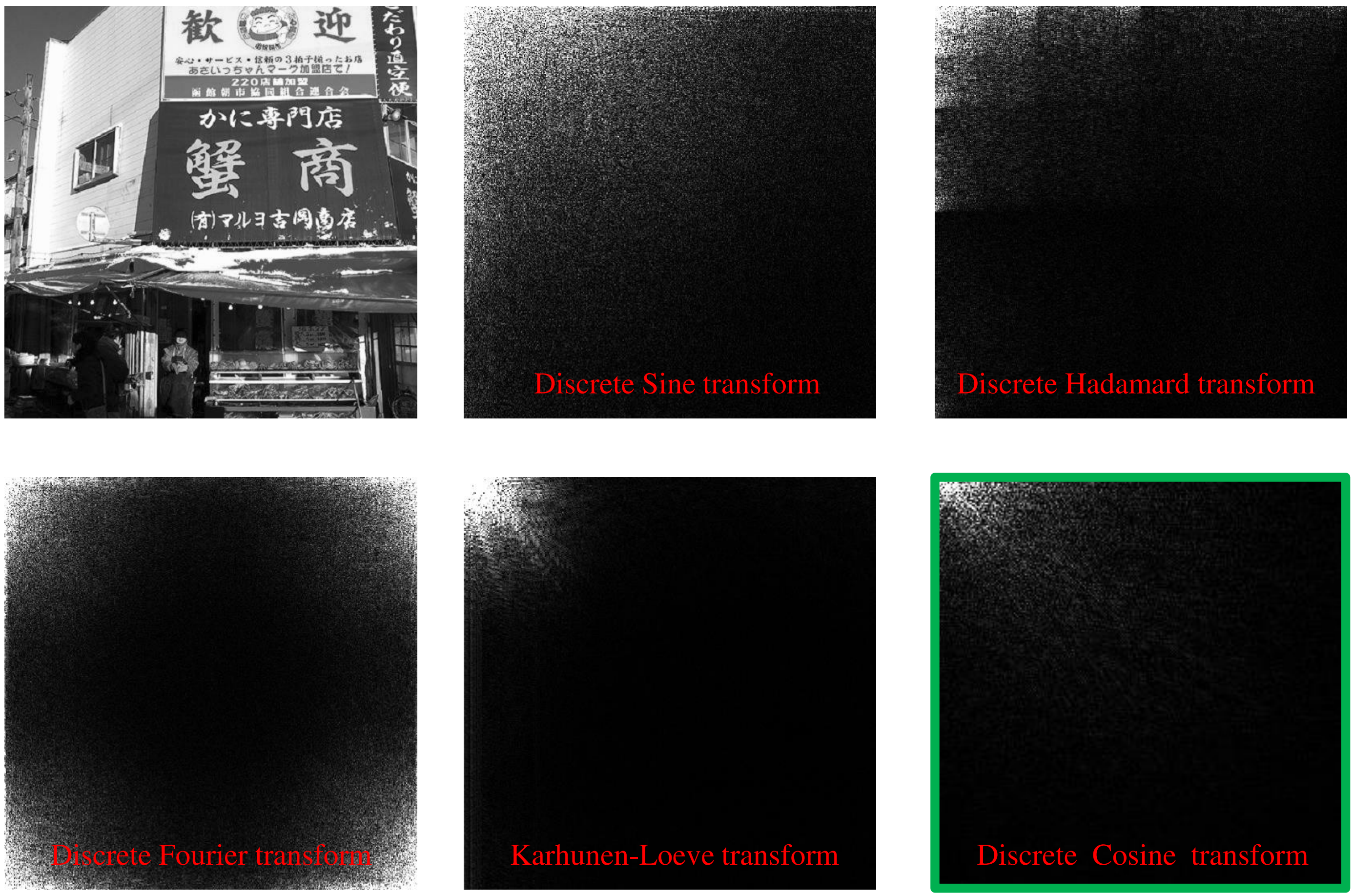
2-D inverse transforms

$$g(x,y) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} \underbrace{T(u,v)}_{\text{Transformed coefficients}} i(x,y,u,v),$$

where f (x, y, u, v) and i(x, y, u, v) are referred to as the forward and inverse transformation kernels, respectively.

- DFT, DHT, DCT, DST, KLT, DWT, …..

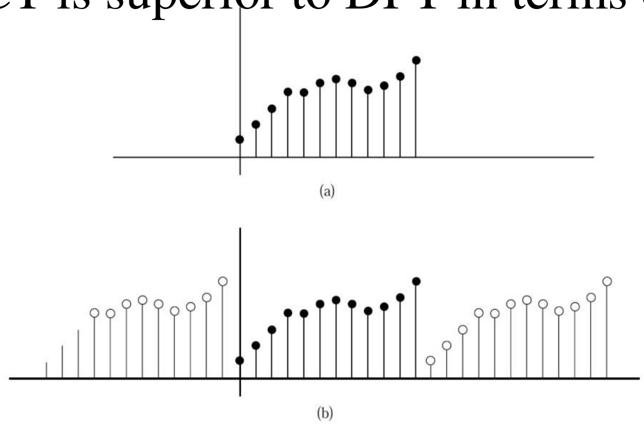
# Compaction Efficiency For Various Image Transforms



# DCT-Based Coding

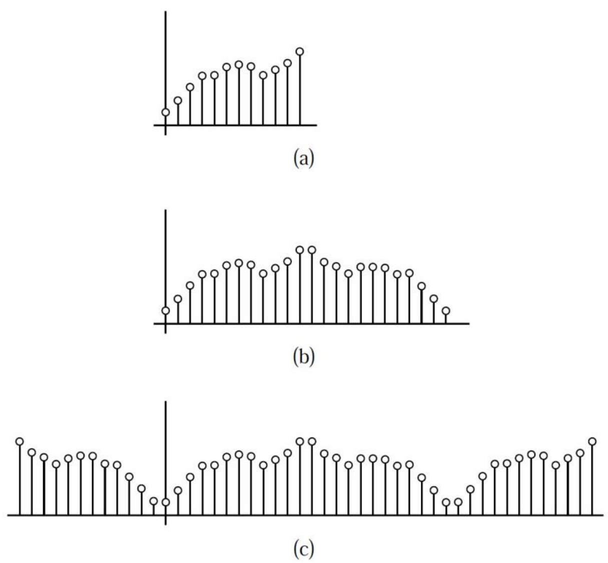
- Optimal transform is KLT, but
  - KLT is image dependent
  - complex computing complexity
- DCT-based coding,
  - is image independent, unlike KLT for highly correlated image data,
  - DCT compaction efficiency is close to KLT.
  - Computations of DCT can be performance with fast algorithms which can be easily implemented on parallel architectures.

# Reason why DCT is superior to DFT in terms of compression




← Original 1-D input sequence

← Considered 1-D input sequence for DFT




← Original 1-D input sequence

← Considered 1-D input sequence for DCT



Jing-Ming Guo (jmguo@seed.net.tw), Dept. EE, NTUST, Multimedia Signal Processing Lab.

# Implementation of the DCT

- DCT-based codecs use a two-dimensional version of the transform.
- The 2-D DCT and its inverse (IDCT) of an N x N block are shown below:

  - 2-D DCT: $F(u,v) = \frac{2}{N} C(u)C(v) \sum_{y=0}^{N-1} \sum_{x=0}^{N-1} f(x,y) \cos[\frac{(2x+1)u\pi}{2N}] \cos[\frac{(2y+1)v\pi}{2N}]$
  - 2-D IDCT: $f(x,y) = \frac{2}{N} \sum_{v=0}^{N-1} \sum_{u=0}^{N-1} C(u)C(v)F(u,v) \cos[\frac{(2x+1)u\pi}{2N}] \cos[\frac{(2y+1)v\pi}{2N}]$

$$C_u, C_v = \frac{1}{\sqrt{2}} for u, v = 0$$
$$C_u, C_v = 1 otherwise$$

$$F(i,j)=\frac{2}{N}C(i)C(j)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}f(x,y)\cos\left[\frac{(2x+1)i\pi}{2N}\right]\cos\left[\frac{(2y+1)j\pi}{2N}\right]$$

f(x,y)

| | 0 | 1 |
|---|---|---|
| 0 | 8 | 5 |
| 1 | 3 | 4 |

F(i,j)

| | 0 | 1 |
|---|---|---|
| 0 | 10 | |
| 1 | | |

$$F(0,0)=\frac{2}{2}\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\times \begin{matrix} 8\times\cos\left(\frac{(2\times 0+1)\times 0\times\pi}{2\times 2}\right)\times\cos\left(\frac{(2\times 0+1)\times 0\times\pi}{2\times 2}\right)+ \\ 5\times\cos\left(\frac{(2\times 0+1)\times 0\times\pi}{2\times 2}\right)\times\cos\left(\frac{(2\times 1+1)\times 0\times\pi}{2\times 2}\right)+ \\ 3\times\cos\left(\frac{(2\times 1+1)\times 0\times\pi}{2\times 2}\right)\times\cos\left(\frac{(2\times 0+1)\times 0\times\pi}{2\times 2}\right)+ \\ 4\times\cos\left(\frac{(2\times 1+1)\times 0\times\pi}{2\times 2}\right)\times\cos\left(\frac{(2\times 1+1)\times 0\times\pi}{2\times 2}\right) \end{matrix}$$

**= 10**

# IDCT – 2D

$$f(x,y)=\frac{2}{N}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}C(i)C(j)F(i,j)\cos\left[\frac{(2x+1)i\pi}{2N}\right]\cos\left[\frac{(2y+1)j\pi}{2N}\right]$$

f(x,y)

| | 0 | 1 |
|---|---|---|
| 0 | 8 | |
| 1 | | |

F(i,j)

| | 0 | 1 |
|---|---|---|
| 0 | 10 | 1 |
| 1 | 3 | 2 |

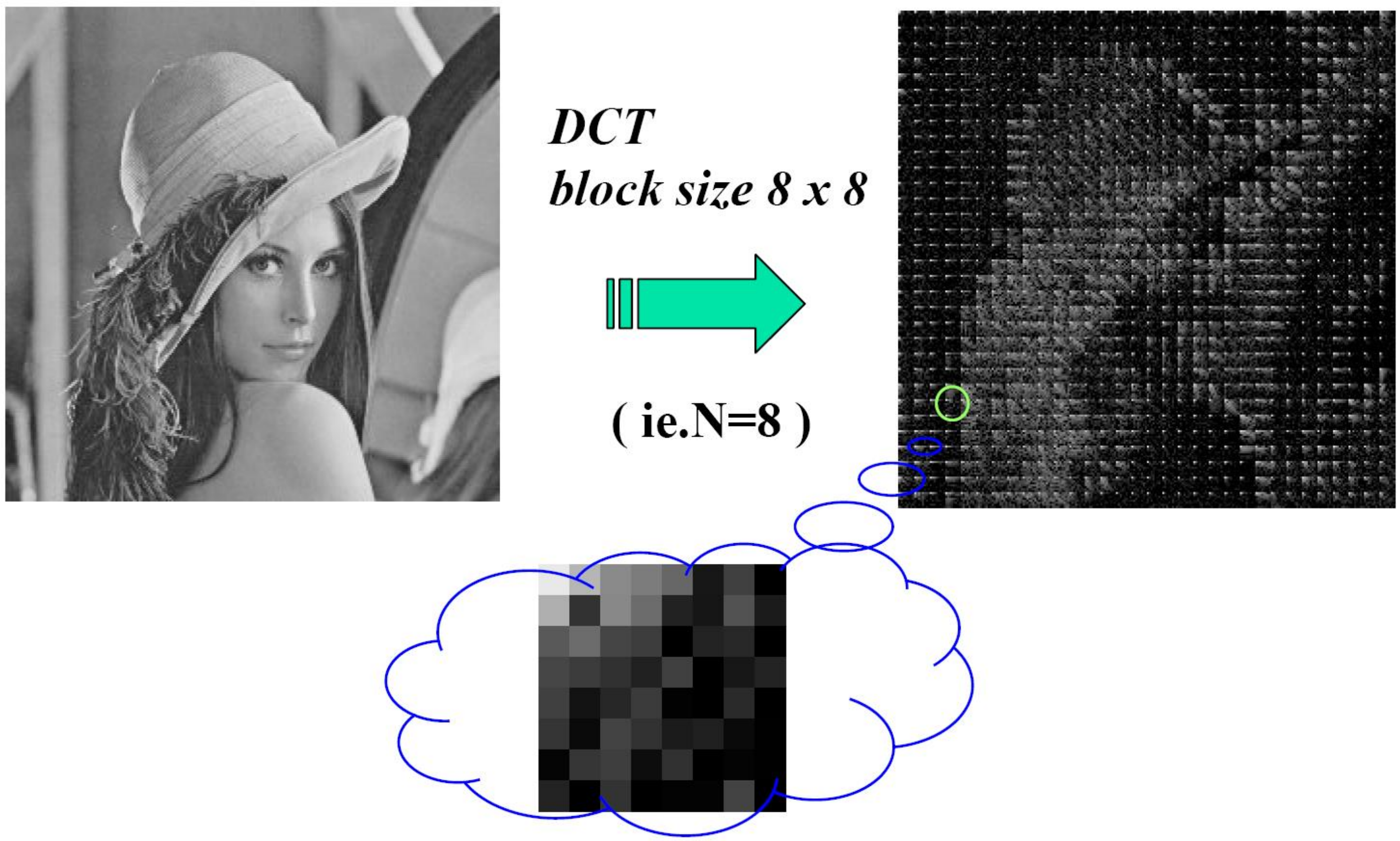
$$\begin{aligned} f(0,0)=\frac{2}{2}\times\; & \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\times 10\times\cos\left(\frac{(2\times 0+1)\times 0\times\pi}{2\times 2}\right)\times\cos\left(\frac{(2\times 0+1)\times 0\times\pi}{2\times 2}\right)+\\ & \frac{1}{\sqrt{2}}\times 1\times 1\times\cos\left(\frac{(2\times 0+1)\times 0\times\pi}{2\times 2}\right)\times\cos\left(\frac{(2\times 0+1)\times 1\times\pi}{2\times 2}\right)+\\ & 1\times\frac{1}{\sqrt{2}}\times 3\times\cos\left(\frac{(2\times 0+1)\times 1\times\pi}{2\times 2}\right)\times\cos\left(\frac{(2\times 0+1)\times 0\times\pi}{2\times 2}\right)+\\ & 1\times 1\times 2\times\cos\left(\frac{(2\times 0+1)\times 1\times\pi}{2\times 2}\right)\times\cos\left(\frac{(2\times 0+1)\times 1\times\pi}{2\times 2}\right)\end{aligned}$$

**= 8**

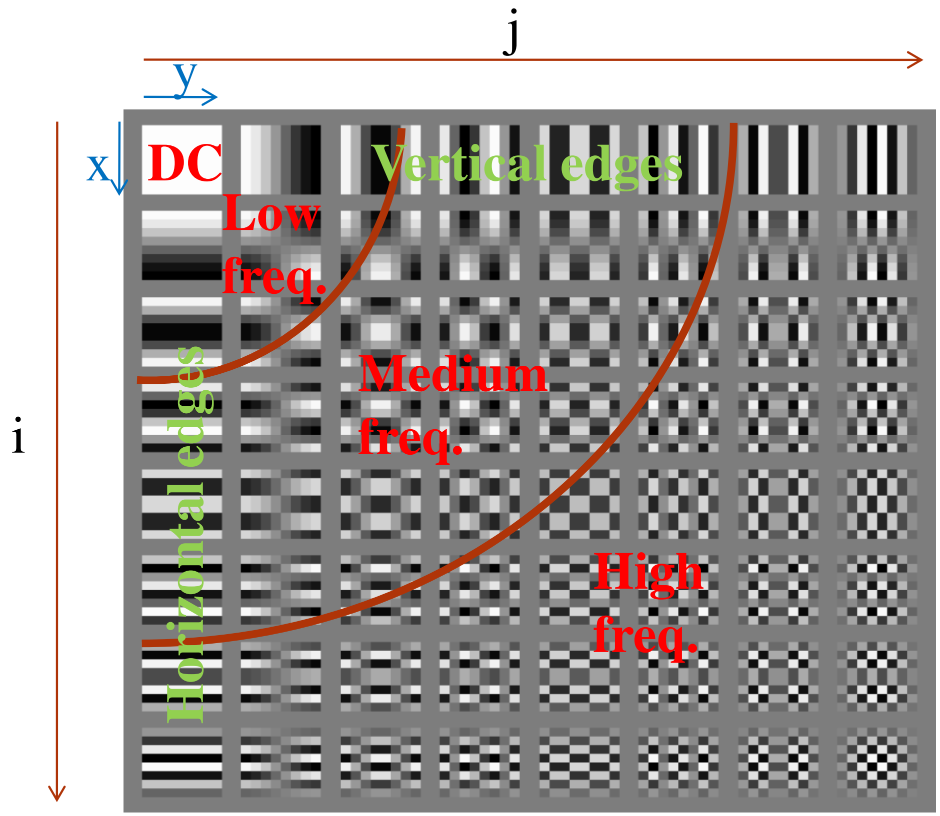


Jing-Ming Guo (jmguo@seed.net.tw), Dept. EE, NTUST, Multimedia Signal Processing Lab.

# Discrete Cosine Transform



# DCT Basis




$$F(i,j) = \frac{2}{N} C(i)C(j) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{(2x+1)i\pi}{2N}\right] \cos\left[\frac{(2y+1)j\pi}{2N}\right]$$

# 1D-DCT

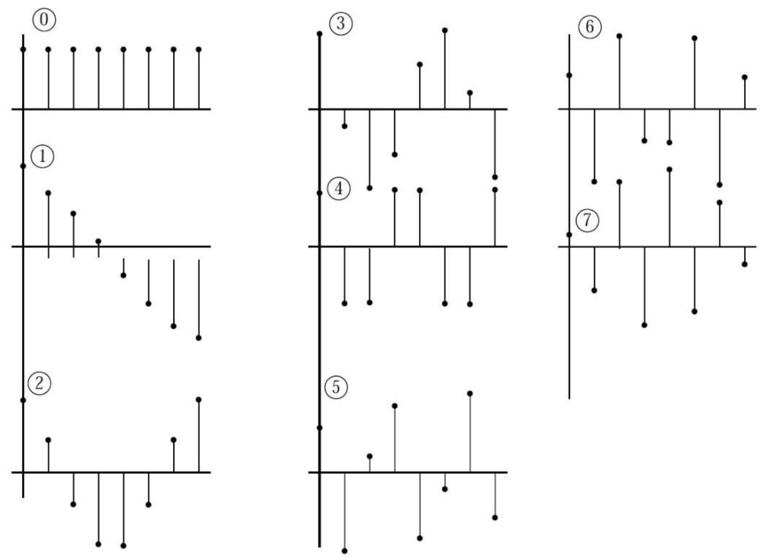
- 1-D
- 8-point DCT

$$F(u) = C(u) \sum_{x=0}^{N-1} f(m) \cos\left[\frac{(2m+1)u\pi}{16}\right]$$
$u = 0,1,\ldots,7.$

$$f(m) = \sum_{x=0}^{N-1} C(u)F(u) \cos\left[\frac{(2m+1)u\pi}{16}\right]$$
$m = 0,1,\ldots,7.$

*where*

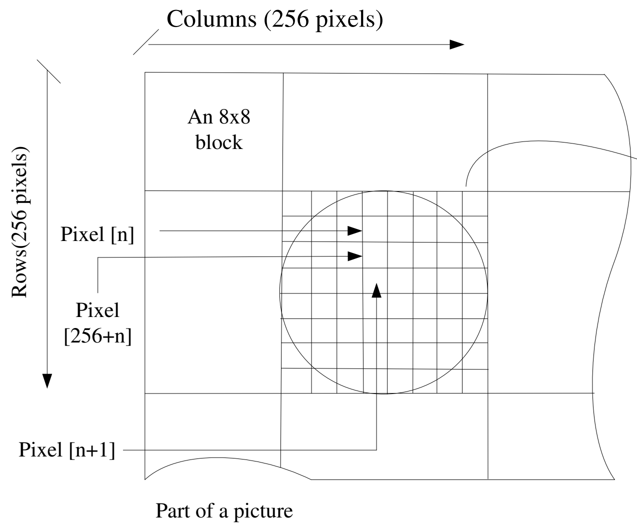
$$C(0) = \sqrt{\frac{1}{8}}, \quad C(u) = \sqrt{\frac{2}{8}}, \quad u = 1,2,\ldots,M-1$$



# 2-D DCT using a 1-D DCT Pair

- One of the properties of the 2-D DCT is that it is separable, meaning that it can be separated into a pair of 1-D DCTs.
- To obtain the 2-D DCT of a block, a 1-D DCT is first performed on the rows of the block then a 1-D DCT is performed on the columns of the resulting block.
- The same applies to the IDCT.
- This process is illustrated on the following slide.

# 2-D DCT using a 1-D DCT Pair




**Raw Block of pixels**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 128 | 127 | 130 | 128 | 134 | 130 | 128 | 128 |
| 128 | 127 | 130 | 128 | 134 | 130 | 128 | 128 |
| 125 | 126 | 130 | 127 | 130 | 131 | 128 | 127 |
| 124 | 128 | 127 | 126 | 129 | 130 | 127 | 127 |
| 131 | 132 | 130 | 129 | 132 | 128 | 129 | 131 |
| 129 | 131 | 134 | 131 | 133 | 129 | 131 | 131 |
| 129 | 134 | 134 | 136 | 136 | 137 | 134 | 132 |
| 133 | 135 | 136 | 138 | 137 | 140 | 136 | 132 |

**Result after DCT by rows**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 356 | -1 | -4 | 1 | 1 | -3 | 1 | 3 |
| 356 | -1 | -4 | 1 | 1 | -3 | 1 | 3 |
| 352 | -3 | -4 | 0 | -3 | -1 | 2 | 1 |
| 399 | -3 | -3 | 0 | -3 | -3 | 0 | -1 |
| 358 | 1 | 1 | -1 | 1 | -3 | -2 | 1 |
| 370 | 0 | -3 | -3 | -1 | -1 | -1 | 2 |
| 378 | -3 | -6 | 0 | -3 | -2 | -1 | -2 |
| 383 | -1 | -6 | 2 | -3 | 0 | 0 | -3 |

**Final result after DCT by columns**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 102 | -4 | -11 | 0 | -4 | -6 | 0 | 1 |
| -12 | -1 | 1 | 0 | 2 | -2 | 2 | 4 |
| 11 | -1 | -5 | 2 | 3 | 0 | 1 | -1 |
| 0 | 2 | 2 | -3 | 3 | -2 | -2 | 3 |
| 0 | 1 | 1 | 1 | 0 | -1 | -1 | -2 |
| -4 | -3 | -3 | -1 | -3 | -1 | 0 | -2 |
| -2 | 0 | -1 | -2 | -1 | 1 | 1 | 0 |
| 2 | 0 | 1 | 0 | 0 | -1 | -1 | -1 |

# Fast Algorithms For The DCT

64x64

- An F(i,j) requires 64 multiplications and 64 additions.
- 4096 multiply accumulate operations are needed for each 8x8 block.
- Use the row-column decomposition, only 16 1-D DCTs (8 for row and 8 for column) is needed (total 1024 multiply-accumulate operations)

8x8x8x2

$$F(i,j) = \frac{2}{N} C(i)C(j) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{(2x+1)i\pi}{2N}\right] \cos\left[\frac{(2y+1)j\pi}{2N}\right]$$

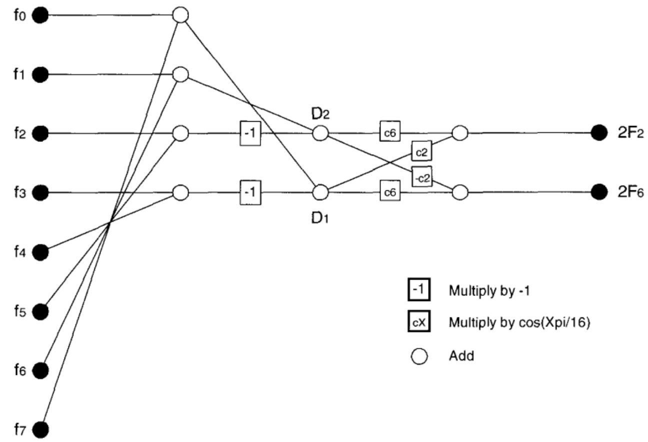


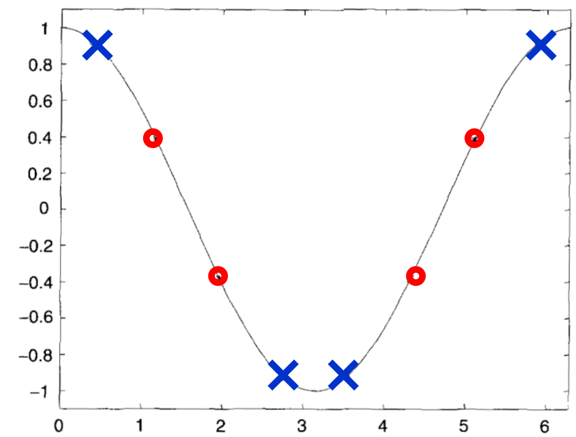
Jing-Ming Guo (jmguo@seed.net.tw), Dept. EE, NTUST, Multimedia Signal Processing Lab.

# Fast DCT algorithms

$$F(k) = \frac{c(k)}{2}\sum_{n=0}^{7} f(n)\cos\left(\frac{2n+1}{16}\cdot k\pi\right), c(0) = \frac{1}{\sqrt{2}}, c(x) = 1$$

- Flowgraph algorithm




$$F_2 = \frac{1}{2}\sum_{i=0}^{7} f_i \cos\left(\frac{(2i+1).2\pi}{16}\right)$$

$$= \frac{1}{2}\left[f_0\cos\left(\frac{\pi}{8}\right) + f_1\cos\left(\frac{3\pi}{8}\right) + f_2\cos\left(\frac{5\pi}{8}\right) + f_3\cos\left(\frac{7\pi}{8}\right) + f_4\cos\left(\frac{9\pi}{8}\right) + f_5\cos\left(\frac{11\pi}{8}\right) + f_6\cos\left(\frac{13\pi}{8}\right) + f_7\cos\left(\frac{15\pi}{8}\right)\right]$$

$$2F_2 = \left[(f_0 - f_4 + f_7 - f_3)\cdot\cos\left(\frac{\pi}{8}\right) + (f_1 - f_2 - f_5 + f_6)\cdot\cos\left(\frac{3\pi}{8}\right)\right]$$

$$D_1 = (f_0 - f_4 + f_7 - f_3) \quad and \quad D_2 = (f_1 - f_2 + f_5 + f_6)$$

$$2F_2 = \left[D_1\cos\left(\frac{\pi}{8}\right) + D_2\cos\left(\frac{3\pi}{8}\right)\right] \quad and \quad 2F_6 = \left[D_1\cos\left(\frac{3\pi}{8}\right) + D_2\cos\left(\frac{\pi}{8}\right)\right]$$