Histogram Processing

- The histogram of a digital image with gray levels in the range [0, L-1] is a discrete function $h(r_k) = n_k$, where r_k is the k-th gray level and n_k is the number of pixels in the image having gray level r_k.
- It is common practice to normalize a histogram by dividing each of its values by the total number of pixels in the image, denoted by n. Thus, a normalized histogram is given by $\boldsymbol{p(r_k) = n_k / n}$ for $\boldsymbol{k} = 0, 1, \ldots, \text{L-1}$.
- Loosely speaking, $\boldsymbol{p(r_k)}$ given an estimate of the probability of occurrence of gray level $\boldsymbol{r_k}$.

Histogram

0	2	1	3	4
1	3	4	3	3
0	1	3	1	4
3	1	4	2	0
0	4	2	4	4

Image

Intensity	Number of Pixels
0	4
1	5
2	3
3	6
4	7

Histogram Table

$h(r_k) = n_k$

Image Histogram

Histogram Processing

- Histograms are the basis for numerous spatial domain processing techniques. Histogram manipulation can be used effectively for image enhancement.
- The information inherent in histograms also is quite useful in other processing applications, such as image compression and image segmentation.
- Because they are simple to calculate in software and also lend themselves to economic hardware implementations. Thus, those advantages making them a popular tool for real time image processing.

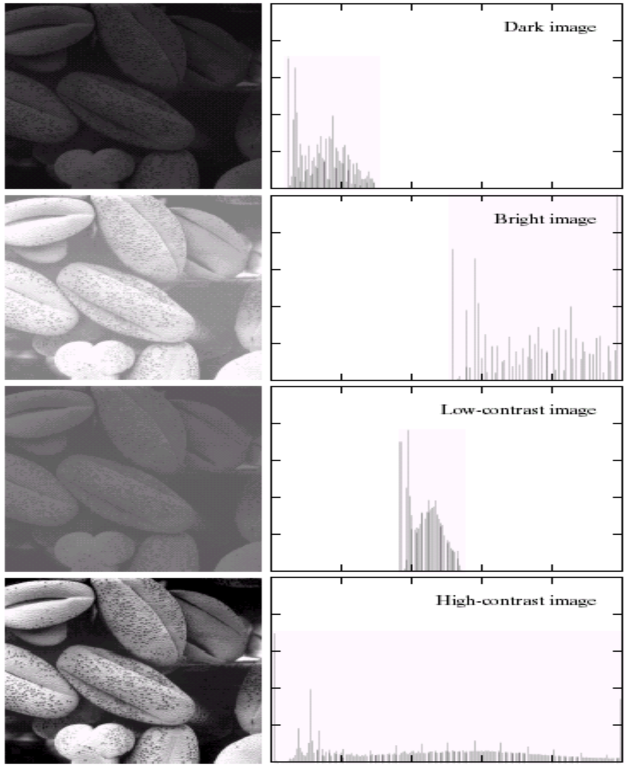
Histogram Processing

- Consider Fig 3.15, which is the pollen image of Fig 3.10 shown in four basis gray-level characteristic : *dark*, *light*, *low contrast* and *high contrast*.
- We note in the dark image that the components of the histogram are concentrated on the low (dark) side of the gray scale. Similarly, the components of the histogram of the bright image are biased toward the high side of the gray scale.
- An image with low contrast has a histogram that will be narrow and will be centered toward the middle of the gray scale. For a monochrome image this implies a dull, wash-out gray look.
- The components of the histogram in the high contrast image cover a broad range of the gray scale. The distribution of pixels is not too far from uniform, with very few vertical lines being much higher than the others.

Histogram Processing

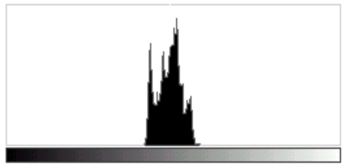
$$T(x) = \frac{x - min}{\max - min} \times 255$$

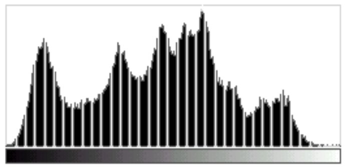
max: biggest value in the image
min: smallest value in the image



a b

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)





Histogram Equalization

- Histogram equalization:
  - To improve the contrast of an image
  - To transform an image in such a way that the transformed image has a nearly uniform distribution of pixel values
- Transformation:
  - Assume r has been normalized to the interval [0,1], with $r = 0$ representing black and $r = 1$ representing white.

$$s = T(r) \qquad 0 \le r \le 1$$

  - The transformation function satisfies the following conditions:
    - $T(r)$ is single-valued and monotonically increasing in the interval $0 \le r \le 1$
    - $0 \le T(r) \le 1$ for $0 \le r \le 1$

Histogram Equalization

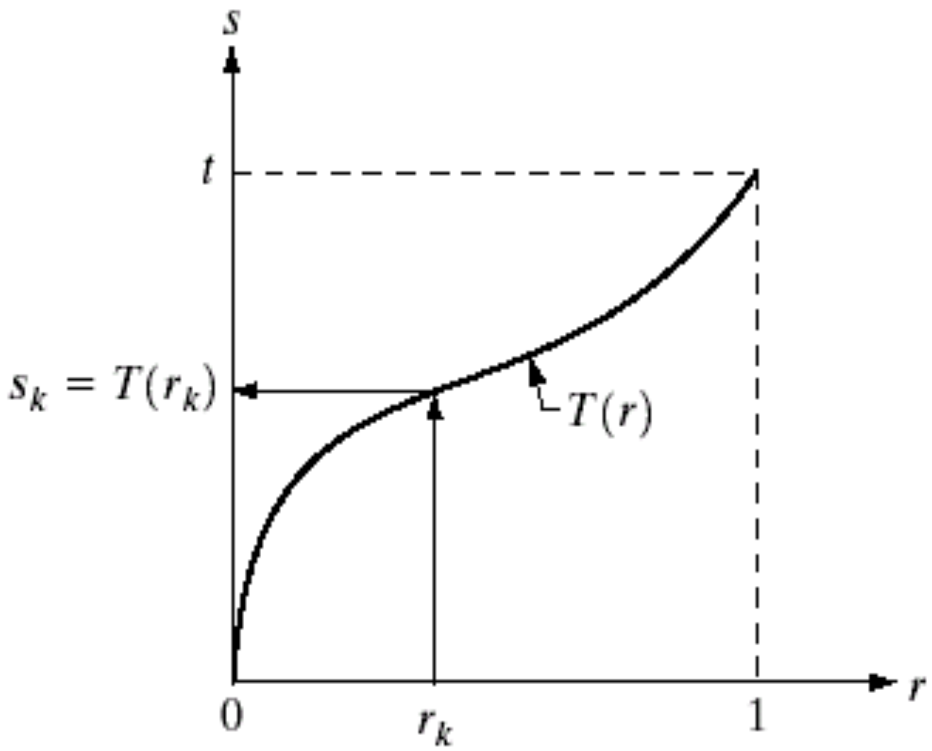


FIGURE 3.16 A gray-level transformation function that is both single valued and monotonically increasing.

Histogram Equalization

- Histogram equalization is based on a transformation of the probability density function of a random variable.
- Let $p_r(r)$ and $p_s(s)$ denote the probability density function of random variable r and s, respectively.
- If $p_r(r)$ and $T(r)$ are known, then the probability density function $p_s(s)$ of the transformed variable s can be obtained

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

- Define a transformation function $s = T(r) = \int_0^r p_r(w)dw$

where w is a dummy variable of integration and the right hand side of this equation is the cumulative distribution function of random variable r.

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Histogram Equalization

- Given transformation function $T(r)$, $s = T(r) = \int_0^r p_r(w)dw$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = \frac{d}{dr}\left[\int_0^r p_r(w)dw\right] = p_r(r)$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = p_r(r)\left|\frac{1}{p_r(r)}\right| = 1 \quad 0 \le s \le 1$$

➡ $p_s(s)$ now is a uniform probability density function.

- $T(r)$ depends on $p_r(r)$, but the resulting $p_s(s)$ always is uniform.

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Histogram Equalization

- For discrete values, we deal with probabilities and summations instead of probability density functions and integrals.
- The probability of occurrence of gray level r_k in an image is approximated by

$$p_r(r_k) = \frac{n_k}{n} \qquad k = 0,1,2,\ldots,L-1$$

where n is the total number of pixels in the image;

n_k is the number of pixels that have gray level r_k;

L is the total number of possible gray levels in the image.

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Histogram Equalization

- The discrete version of the transformation function is

$$\boxed{s_k = T(r_k) = (L-1) \times \sum_{j=0}^{k} p_r(r_j) = (L-1) \times \sum_{j=0}^{k} \frac{n_j}{n}} \qquad k = 0,1,2,\ldots,L-1$$

- Thus, a processed (**output**) image is obtained by mapping each pixel with level r_k in the **input** image into a corresponding pixel with level s_k in the output image via the aforementioned equation.
- As indicated earlier, a plot of $p_r(r_k)$ versus r_k is called a histogram, the transformation (mapping) given in the aforementioned equation is called histogram equalization or histogram linearization.
- It cannot be proved in general that this **discrete** transformation will produce the discrete equivalent of a uniform pdf.

Histogram Equalization

- Example：

0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
1	1	1	2	2	2	3	4
1	1	1	2	2	2	3	4
1	1	1	2	2	3	3	5
1	1	1	2	2	3	3	5
1	1	2	2	2	3	4	6
1	1	2	2	2	3	4	7

Original

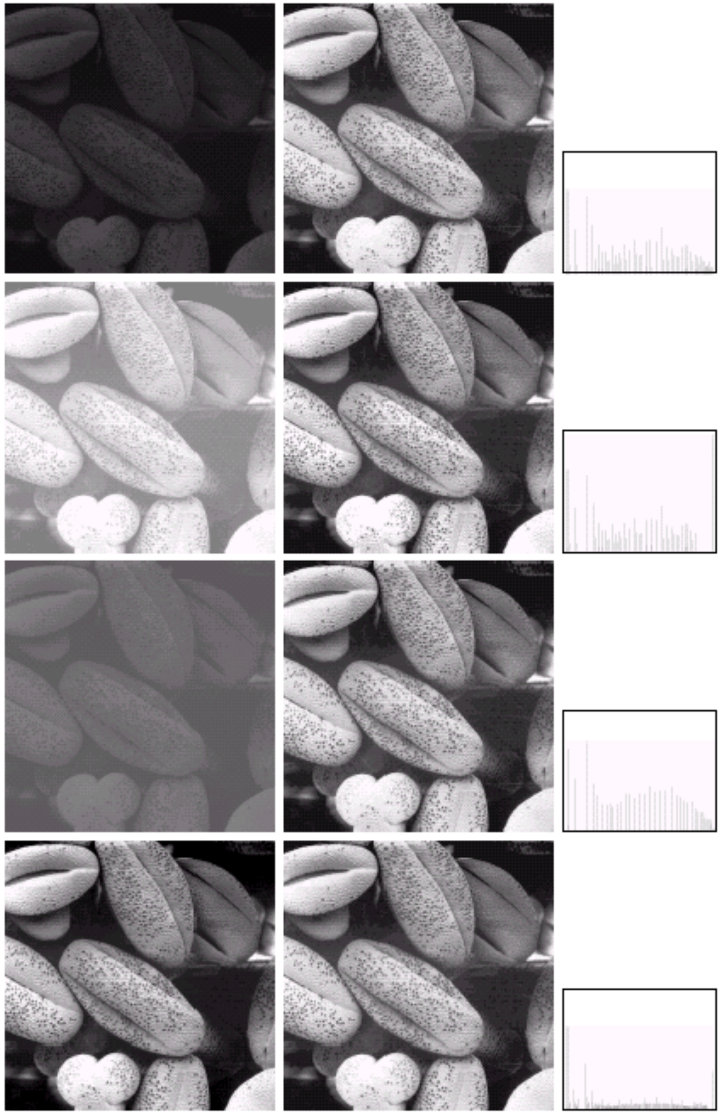
0	0	0	0	0	0	0	0
3	3	3	3	3	3	3	3
3	3	3	5	5	5	6	6
3	3	3	5	5	6	6	6
3	3	3	5	5	6	6	6
3	3	3	5	5	6	6	6
3	3	5	5	5	6	6	6
3	3	5	5	5	6	6	7

Result of histogram equalization

r_i	0	1	2	3	4	5	6	7	
Number	8	24	16	8	4	2	1	1	
Sum	8	32	48	56	60	62	63	64	$\times\frac{7}{64}$
s_i	0	3	5	6	6	6	6	7	

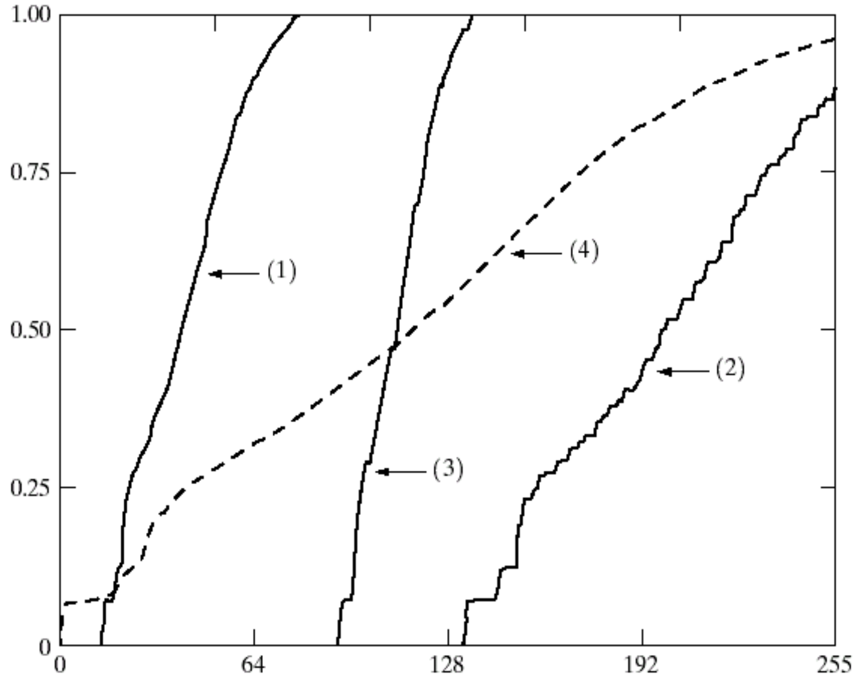
$$s_k = T(r_k) = (L-1)\times\sum_{j=0}^{k} p_r(r_j) = (L-1)\times\sum_{j=0}^{k}\frac{n_j}{n}$$

Histogram Equalization



a b c

FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.



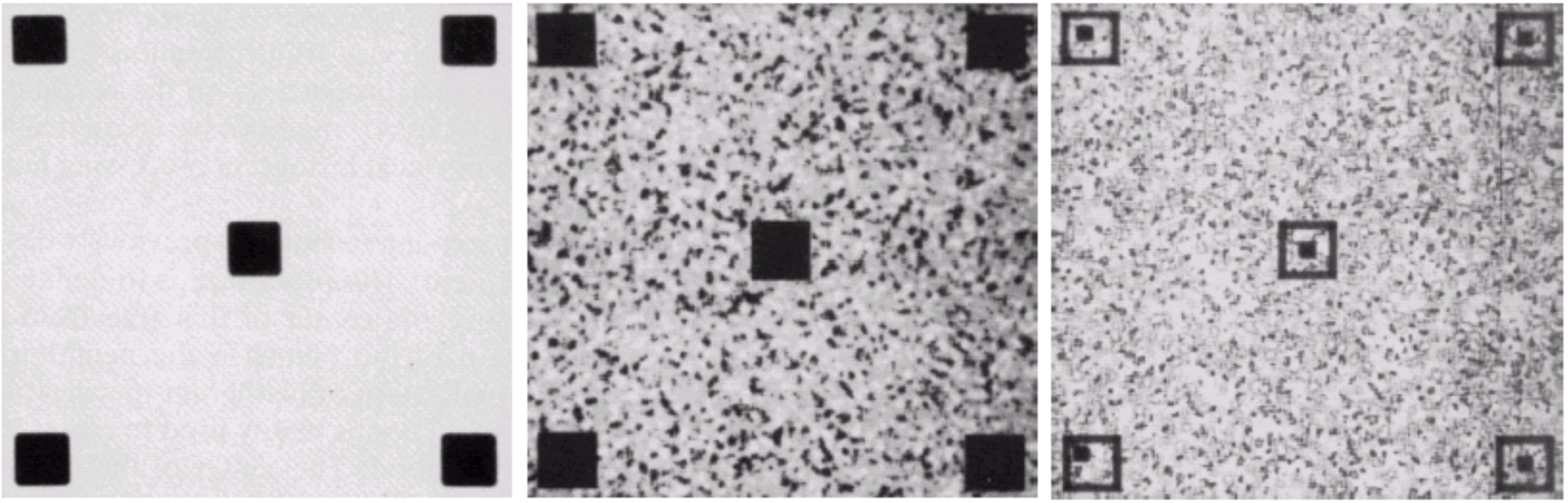
Local Enhancement

- The histogram processing methods discussed above are global, in the sense that pixels are modified by a transformation function based on the gray-level content of an entire image.
- However, there are cases in which it is necessary to enhance details over small areas in an image.
- The procedure is to define a square or rectangular neighborhood and move the center of this area from pixel to pixel.

Local Enhancement

- At each location, the histogram of the points in the neighborhood is computed and either a histogram equalization or histogram specification transformation function is obtained.
- This function is finally used to map the gray level of the pixel centered in the neighborhood, the center of the neighborhood region is then moved to an adjacent pixel location and the procedure is repeated.
- Another approach used some methods to reduce computation is to utilize nonoverlapping regions, but this method usually produces an undesirable checkerboard effect.

Local Enhancement



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7 × 7 neighborhood about each pixel.

Use of Histogram Statistics for Image Enhancement

- Moments (動差) can be determined directly from a histogram much faster than they can from the pixels directly.
- Let $\boldsymbol{r}$ denote a discrete random variable representing discrete gray-levels in the range [0, L-1], and let $p(r_i)$ as an estimate of the probability of occurrence of gray level r_i. The nth moment of $\boldsymbol{r}$ about its mean is defined as

$$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

where m is the mean value of $\boldsymbol{r}$ (its average gray level)

$$m = \sum_{i=0}^{L-1} r_i p(r_i)$$

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Use of Histogram Statistics for Image hancement

- It follows from $\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$ and $m = \sum_{i=0}^{L-1} r_i p(r_i)$ that $\mu_0 = 1$ and $\mu_1 = 0$.
- The second moment is given by

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$$

- We recognize this expression as the variance of ***r***, which is denoted conventionally by $\sigma^2(r)$. The standard deviation is defined simply as the square root of the variance.
- In terms of enhancement, we are interested primarily in the mean, which is a measure of average gray level in an image, and the variance, which is a measure of average contrast.

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Use of Histogram Statistics for Image Enhancement

- We consider the use of the mean and variance (standard deviation) for enhancement purposes.
- The global mean and variance are measured over an entire image and are useful primarily for gross adjustments of overall intensity and contrast.
- The mean and standard deviation for a local region are useful for correcting for large-scale changes in intensity and contrast.

Use of Histogram Statistics for Image hancement

- Let (x,y) be the coordinates of a pixel in an image, and let S_{xy} denote a neighborhood (subimage) of specified size, centered at (x,y). The mean value of the pixel in S_{xy} can be computed by

$$m_{S_{xy}} = \sum_{(s,t)\in S_{xy}} r_{s,t}\, p(r_{s,t}) \qquad (3.3\text{-}21)$$

  where $r_{s,t}$ is the gray level at coordinates (s,t) in the neighborhood, $p(r_{s,t})$ is the neighborhood normalized histogram component corresponding to that value of gray level.

- The gray level variance of the pixels in region S_{xy} is given by

$$\sigma^2_{S_{xy}} = \sum_{(s,t)\in S_{xy}} [r_{s,t} - m_{S_{xy}}]^2 p(r_{s,t}) \qquad (3.3\text{-}22)$$

Use of Histogram Statistics for Image hancement

- The local mean is a measure of average gray level in neighborhood S_{xy}, and the variance is a measure of contrast in that neighborhood.
- An important aspect of image processing using the local mean and variance is the flexibility they afford in developing simple, yet powerful enhancement techniques based on statistical measures that have a close, predictable correspondence with image appearance.

Use of Histogram Statistics for Image hancement

FIGURE 3.24 SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

Use of Histogram Statistics for Image hancement

- A summary of the enhancement method is as follows :

Let $f(x,y)$ represent the value of an image pixel at any image coordinates (x,y), and let $g(x,y)$ represent the corresponding enhanced pixel at those coordinates.

Then,

$$g(x,y) = \begin{cases} E \cdot f(x,y) & if \;\; m_{S_{xy}} \leq k_0 M_G \; AND \; k_1 D_G \leq \sigma_{S_{xy}} \leq k_2 D_G \\ f(x,y) & otherwise \end{cases}$$

where E, k_0, k_1, and k_2 are specified parameters;

M_G is the global mean of the input image;

D_G is its global standard deviation.

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Use of Histogram Statistics for Image hancement

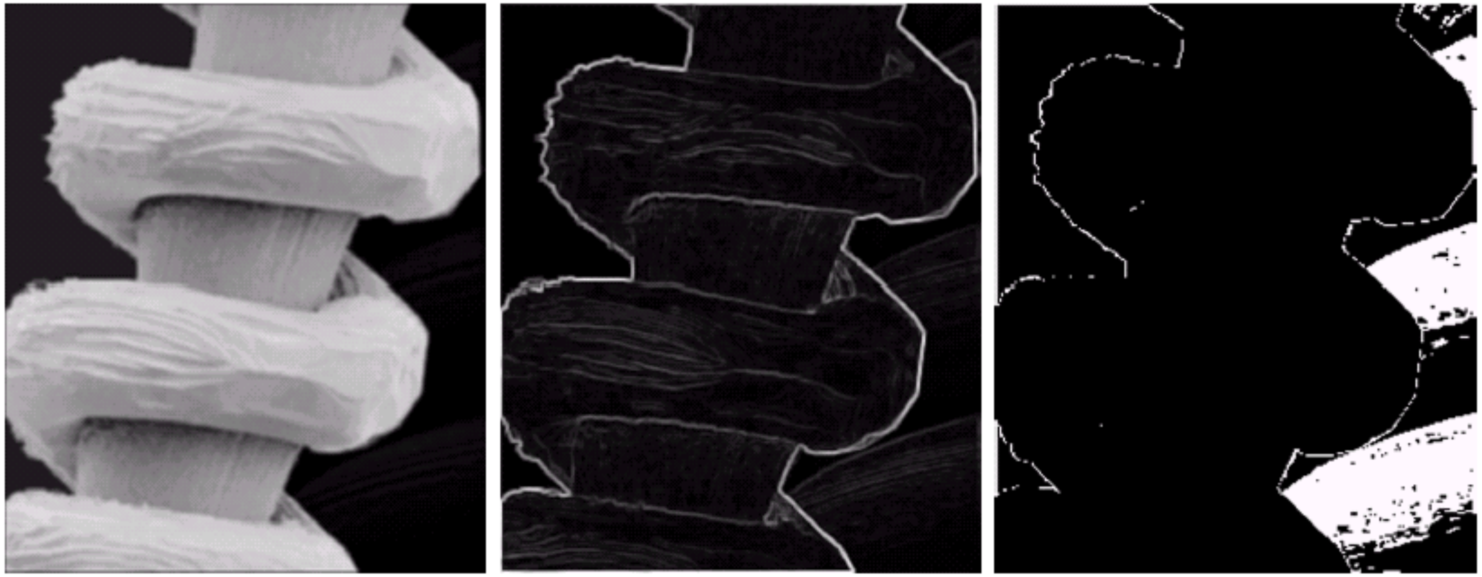

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

$$g(x,y) = \begin{cases} E \cdot f(x,y) & if \;\; m_{S_{xy}} \le k_0 M_G \;\; AND \;\; k_1 D_G \le \sigma_{S_{xy}} \le k_2 D_G \\ f(x,y) & otherwise \end{cases}$$

Use of Histogram Statistics for Image hancement

FIGURE 3.26 Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

$$g(x,y) = \begin{cases} E \cdot f(x,y) & if \;\; m_{S_{xy}} \le k_0 M_G \;\; AND \;\; k_1 D_G \le \sigma_{S_{xy}} \le k_2 D_G \\ f(x,y) & otherwise \end{cases}$$

$E=4.0$, $k_0=0.4$, $k_1=0.02$, and $k_2=0.4$