Project One Template

MAT350: Applied Linear Algebra

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Problem 1

Develop a system of linear equations for the network by writing an equation for each router (A, B, C, D, and E). Make sure to write your final answer as A**x**=**b** where A is the 5x5 coefficient matrix, **x** is the 5x1 vector of unknowns, and **b** is a 5x1 vector of constants.

Solution: For this equation for the network we know that the incoming traffic = outgoing traffic

Node A: $2x_1 + x_2 = 100$

Node B: $x_1 + x_2 - x_3 - x_5 = 0$

Node C: $x_1 - x_3 - x_5 = -50$

Node D: $-x_2 + x_4 + x_5 = 120$

Node E: $x_2 + x_3 - x_4 + x_5 = 0$

Linear Equation in Ax=b form:

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 & -1 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ -50 \\ 120 \\ 0 \end{bmatrix}$$

Problem 2

Use MATLAB to construct the augmented matrix [A **b**] and then perform row reduction using the rref() function. Write out your **reduced matrix and identify the free and basic variables of the system**.

Solution:

```
%code
A = [2, 1, 0, 0, 0;
1, 1, -1, 0, -1;
1, 0, -1, 0, -1;
0, -1, 0, 1, 1;
0, 1, 1, -1, 1;]
```

```
A = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
% Colum Vector of Constant Values
b = [100; 0; -50; -120; 0]
```

```
b = 5×1
   100
     0
   -50
  -120
     0
```

```
% Find x
x = A\b
```

```
x = 5×1
    25
    50
   270
   125
  -195
```

```
% Creating the Augmented Matrix
aug = [A b]
```

```
aug = 5×6
     2     1     0     0     0   100
     1     1    -1     0    -1     0
     1     0    -1     0    -1   -50
     0    -1     0     1     1  -120
     0     1     1    -1     1     0
```

```
%Perform RREF
rref(aug)
```

```
ans = 5×6
     1     0     0     0     0    25
     0     1     0     0     0    50
     0     0     1     0     0   270
     0     0     0     1     0   125
     0     0     0     0     1  -195
```

Problem 3

Use MATLAB to **compute the LU decomposition of A**, i.e., find A = LU. For this decomposition, find the transformed set of equations L**y** = **b**, where **y** = U**x**. Solve the system of equations L**y** = **b** for the unknown vector **y**.

Solution:

```
%code
[L U] = lu(A)
```

```
L = 5×5
    1.0000         0         0         0         0
    0.5000   -0.5000    1.0000    1.0000         0
    0.5000    0.5000    1.0000         0         0
         0    1.0000         0         0         0
         0   -1.0000   -1.0000   -0.5000    1.0000
U = 5×5
    2.0000    1.0000         0         0         0
         0   -1.0000         0    1.0000    1.0000
         0         0   -1.0000   -0.5000   -1.5000
         0         0         0    1.0000    1.0000
         0         0         0         0    1.0000
```

```
%Solve the linear equations Ly = b
y = linsolve(L,b)
```

```
y = 5×1
   100
  -120
   -40
   -70
  -195
```

```
%Find variable x
x = U^-1 * y
```

```
x = 5×1
    25
    50
   270
   125
  -195
```

Problem 4

Use MATLAB to **compute the inverse** of U using the inv() function.

Solution:

```
%code
A = [2, 1, 0, 0, 0;
1, 1, -1, 0, -1;
1, 0, -1, 0, -1;
0, -1, 0, 1, 1;
0, 1, 1, -1, 1;]
```

```
A = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
[L,U] = lu(A);

inv(U)
```

```
ans = 5×5
    0.5000    0.5000         0   -0.5000         0
         0   -1.0000         0    1.0000         0
         0         0   -1.0000   -0.5000   -1.0000
         0         0         0    1.0000   -1.0000
         0         0         0         0    1.0000
```

Problem 5

Compute the solution to the original system of equations by transforming **y** into **x**, i.e., compute **x** = inv(U)**y**.

Solution:

```
%code Computing Solution to original system of equations
A = [2, 1, 0, 0, 0;
1, 1, -1, 0, -1;
1, 0, -1, 0, -1;
0, -1, 0, 1, 1;
0, 1, 1, -1, 1;]
```

```
A = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
b = [100 0 -50 120 0]'
```

```
b = 5×1
   100
     0
   -50
   120
     0
```

```
[L,U] = lu(A);
y = inv(L)*b;
x = inv(U)*y
```

```
x = 5×1
    25
    50
    30
   125
    45
```

```

```

Problem 6

Check your answer for x_1 using Cramer's Rule. Use MATLAB to compute the required determinants using the det() function.

Solution:

```
%code Initializing the matrices for x1, x2, x3, x4,c5
A1 = A
```

```
A1 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A2 = A
```

```
A2 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A3 = A
```

```
A3 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A4 = A
```

```
A4 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A5 = A
```

```
A5 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```

%Replacing columns in A1-A5 with column vectors from b
A1(:,1) = b
```

```
A1 = 5×5
   100     1     0     0     0
     0     1    -1     0    -1
   -50     0    -1     0    -1
   120    -1     0     1     1
     0     1     1    -1     1
```

```
A2(:,2) = b
```

```
A2 = 5×5
     2   100     0     0     0
     1     0    -1     0    -1
     1   -50    -1     0    -1
     0   120     0     1     1
     0     0     1    -1     1
```

```
A3(:,3) = b
```

```
A3 = 5×5
     2     1   100     0     0
     1     1     0     0    -1
     1     0   -50     0    -1
     0    -1   120     1     1
     0     1     0    -1     1
```

```
A4(:,4) = b
```

```
A4 = 5×5
     2     1     0   100     0
     1     1    -1     0    -1
     1     0    -1   -50    -1
     0    -1     0   120     1
     0     1     1     0     1
```

```
A5(:,5) = b
```

```
A5 = 5×5
     2     1     0     0   100
     1     1    -1     0     0
     1     0    -1     0   -50
     0    -1     0     1   120
     0     1     1    -1     0
```

```
%Solution for x1, x2, x3, x4
x1 = det(A1)/det(A)
```

```
x1 = 25.0000
```

```
x2 = det(A2)/det(A)
```

```
x2 = 50
```

```
x3 = det(A3)/det(A)
```

```
x3 = 30.0000
```

```
x4 = det(A4)/det(A)
```

```
x4 = 125.0000
```

```
x5 = det(A5)/det(A)
```

```
x5 = 45
```

Problem 7

The Project One Table Template, provided in the Project One Supporting Materials section in Brightspace, shows the recommended throughput capacity of each link in the network. Put your solution for the system of equations in the third column so it can be easily compared to the maximum capacity in the second column. In the fourth column of the table, provide recommendations for how the network should be modified based on your network throughput analysis findings. The modification options can be No Change, Remove Link, or Upgrade Link. In the final column, explain how you arrived at your recommendation.

Solution:

Fill out the table in the original project document and export your table as an image. Then, use the ***Insert*** *tab in the MATLAB editor to insert your table as an image.*

MAT 350 Project One Tab

Network Link	Recommended Capacity (Mbps)	Solution	Recommendatio
x_1	60	25	No change require
x_2	50	50	No change require
x_3	100	30	No change require
x_4	100	125	Needs Upgrade
x_5	50	45	No change require