MAT 230 EXAM TWO

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Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

PROBLEM 1

This question has 2 parts.

Part 1: Suppose that F and X are events from a common sample space with $P(F) \neq 0$ and $P(X) \neq 0$.

(a) Prove that $P(X) = P(X|F)P(F) + P(X|\bar{F})P(\bar{F})$. Hint: Explain why $P(X|F)P(F) = P(X \cap F)$ is another way of writing the definition of conditional probability, and then use that with the logic from the proof of Theorem 4.1.1.

$P(A|B) = P(A \cap B)/P(B).Therefore, P(A \cap B) = P(A|B).P(B)$ Hence , $P(X \cap F) = P(X|F).P(F)$

(b) Explain why $P(F|X) = P(X|F)P(F)/P(X)$ is another way of stating Theorem 4.2.1 Bayes Theorem.

$P(A|B) = P(A \cap B)/P(B)$ So, $Therefore,$P(A $\cap B) = P(A|B)P(B).LHS = P(X|F)P(F) + P(X|F)P(F).RHS = P(X \cap F) + P(X \cap F) = P(X)$ $P(F|X) = P(F \cap X)/P(x), when P(x) \neq 0$

Part 2: A website reports that 70% of its users are from outside a certain country. Out of their users from outside the country, 60% of them log on every day. Out of their users from inside the country, 80% of them log on every day.

(a) What percent of all users log on every day? Hint: Use the equation from Part 1 (a).

Total login users is $24 + 42 = 66$ Total users is 100. So $66/100 = 66$

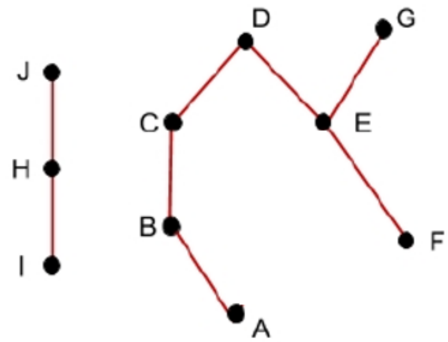
$Using Bayes Theorem, out of users who log on every day, what is the probability that they are from inside the country?$

$P(Login/own country)$ =24/30 $= 4/5 P(Own country)$ $=1/2 P(login/other country)$ $=3/5 So$P(login / own country) = 4/7

Problem 2

This question has 2 parts.

Part 1: The drawing below shows a Hasse diagram for a partial order on the set:
$\{A, B, C, D, E, F, G, H, I, J\}$



(b)

Figure 1: *A Hasse diagram shows 10 vertices and 8 edges. The vertices, represented by dots, are as follows: vertex J is upward of vertex H; vertex H is upward of vertex I; vertex B is inclined upward to the left of vertex A; vertex C is upward of vertex B; vertex D is inclined upward to the right of vertex C; vertex E is inclined upward to the left of vertex F; vertex G is inclined upward to the right of vertex E. The edges, represented by line segments between the vertices are as follows: 3 vertical edges connect the following vertices: B and C, H and I, and H and J; 5 inclined edges connect the following vertices: A and B, C and D, D and E, E and F, and E and G.*

Determine the properties of the Hasse diagram based on the following questions:

(a) What are the minimal elements of the partial order?

I, A, F

(b) What are the maximal elements of the partial order?

J, D, G

(c) Which of the following pairs are comparable?

$$(A, D),\ (J, F),\ (B, E),\ (G, F),\ (D, B),\ (C, F),\ (H, I), (C, E)$$

(A,D), (G, F), (D,B), (H,I)

Part 2: Consider the partial order with domain {3, 5, 6, 7, 10, 14, 20, 30, 60, 70} and with $x \leq y$ if x evenly divides y. Select the correct Hasse diagram for the partial order.

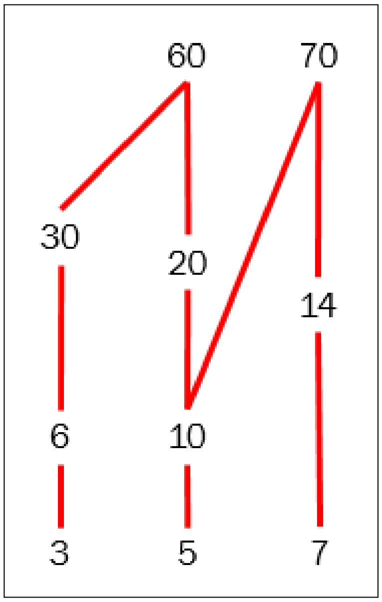
(a) 

Figure 2: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 20, 20 and 60, 10 and 70, 7 and 14, 14 and 70.*

10 equally divides 30 but the relationship is missing link.

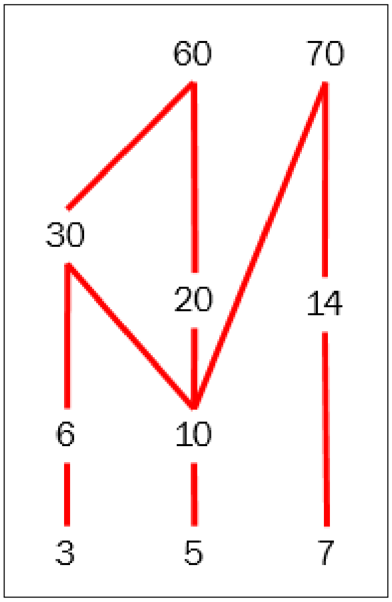
(b) 

Figure 3: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 30, 10 and 20, 20 and 60, 10 and 70, 7 and 14, 14 and 70.*

All of the relations are present in this diagram. So this is the correct Hasse diagram.

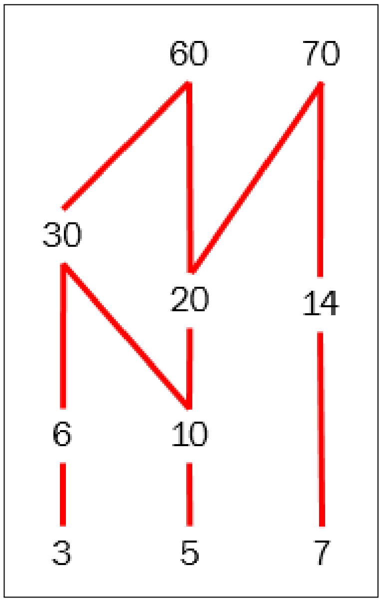
(c) 

Figure 4: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 30, 10 and 20, 20 and 60, 20 and 70, 7 and 14, 14 and 70.*

10 does equally divide 70, but there is missing a link so no relation is present.

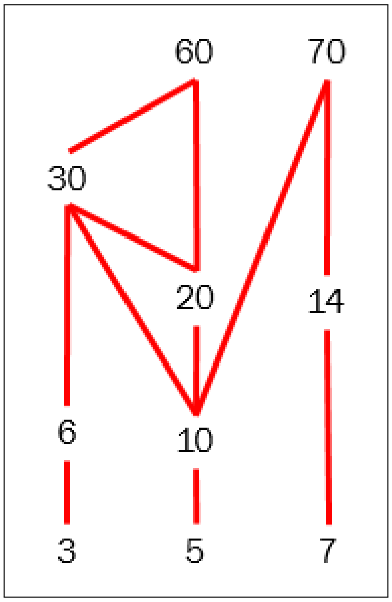
(d) 

Figure 5: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 30, 10 and 20, 20 and 30, 20 and 60, 10 and 70, 7 and 14, 14 and 70.*

20 does not equally divide 30 but the relation is still present. So this is the wrong link

Problem 3

A car dealership sells cars that were made in 2015 through 2020. Let the cars for sale be the domain of a relation R where two cars are related if they were made in the same year.

(a) Prove that this relation is an equivalence relation.

Reflexive x is related to x. $So, (car(x), car(x)) isinR$ Symmetric relation:. x is related to y implies y is related to x. Here (car(x) , car(y)) is in R , Therefore car(x) and car(y) are made the same year. Transitive, x is related y and y is related to z then x is related to z. Car(x), car(y) and car(z) are made in same year, so car(x) is in car(z). This relation obeys symettric, transitive, and reflexive properties.

(b) Describe the partition defined by the equivalence classes.

In the question about all the cars made in the same year are in one equivalence class. All the cars are made throughout 6 years (2015-2020), so there are 6 classes. The car can only be in one partition from those 6 years.

Problem 4

Analyze each graph below to determine whether it has an Euler circuit and/or an Euler trail.

- If it has an Euler circuit, specify the nodes for one.
- If it does not have an Euler circuit, justify why it does not.
- If it has an Euler trail, specify the nodes for one.
- If it does not have an Euler trail, justify why it does not.

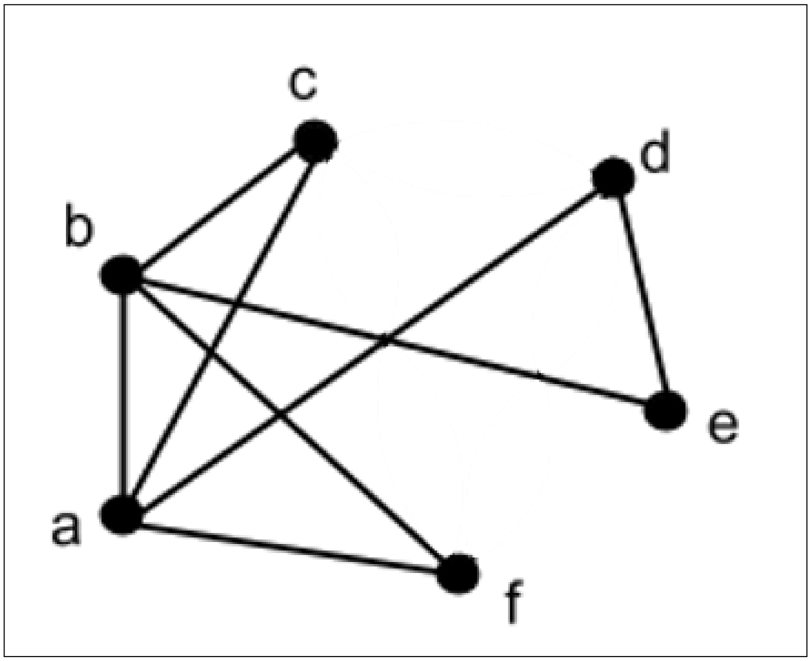
(a) 

Figure 6: *An undirected graph has 6 vertices, a through f. There are 8-line segments that are between the following vertices: a and b, a and c, a and d, a and f, b and c, b and e, b and f, d and e.*

Euler trail is not possible. This is a Euler circuit (a, b, c, a, f, b, e, d, a). Not a trail because the degree of the verticies are all even.

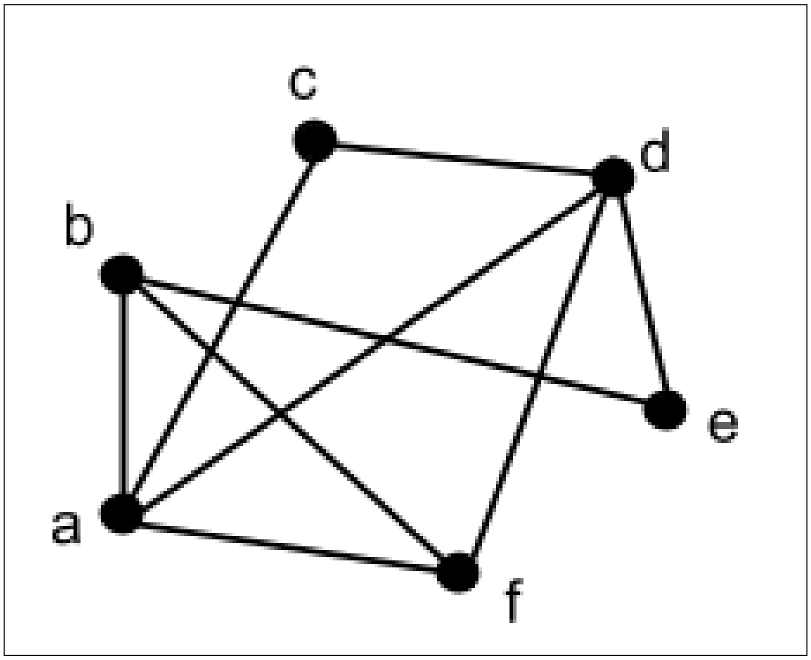
(b) 

Figure 7: *An undirected graph has 6 vertices, a through f. There are 9-line segments that are between the following vertices: a and b, a and c, a and d, a and f, b and e, b and f, c and d, d and e, d and f.*

In this example, it contains verticies with an odd degree so a Euler circuit is not possible. The Euler trail is (a, b, e, d, c, a, d, f, b)

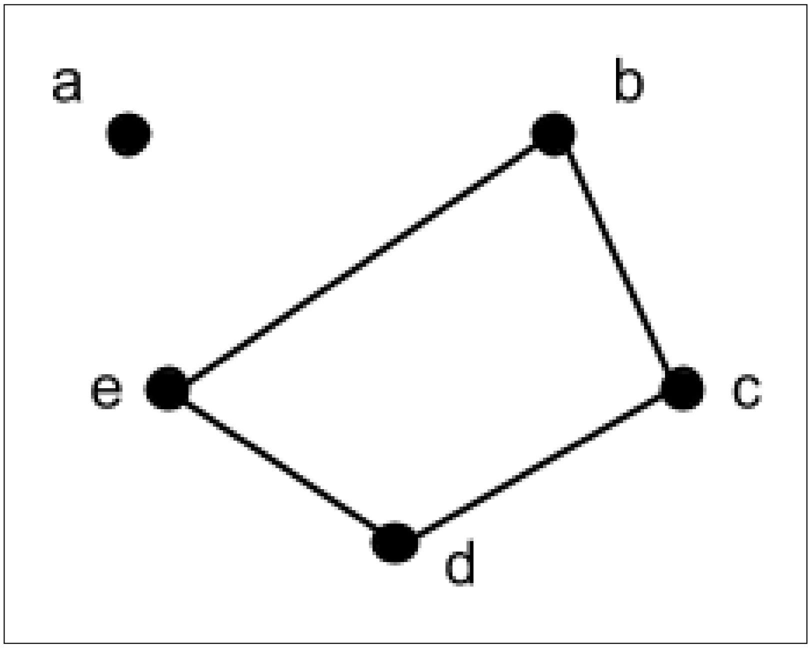
(c) 

Figure 8: *An undirected graph has 5 vertices, a through e. There are 4-line segments that are between the following vertices: b and c, b and e, c and d, d and e.*

This example all the vertices have an even degree so a Euler trail is not possible. The Euler circuit is (d, e, b, c, d)

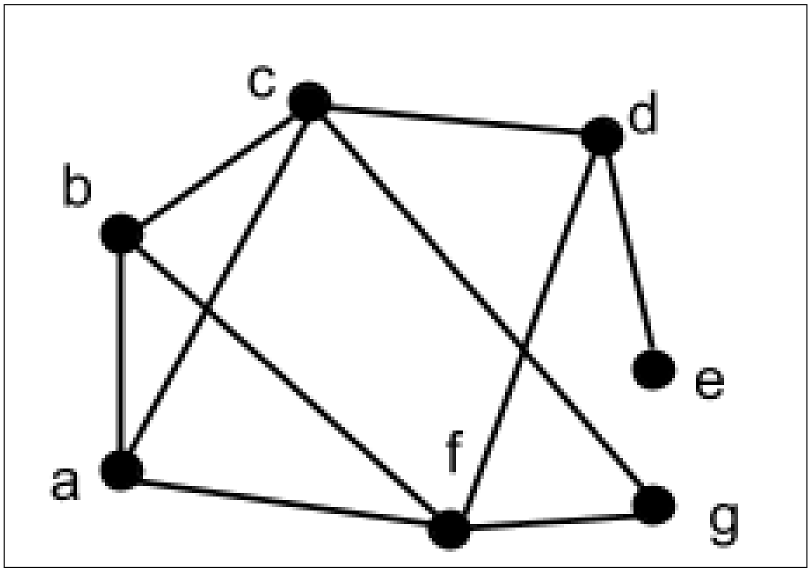
(d) 

Figure 9: *An undirected graph has 7 vertices, a through g. There are 10-line segments that are between the following vertices: a and b, a and c, a and f, b and c, b and f, c and d, c and g, d and e, d and f, f and g.*

This final example, has odd degree of verticies so a Euler circuit is not possible. The example has more than 2 odd verticies so this is not a Euler trail either.

Problem 5

Use Prim's algorithm to compute the minimum spanning tree for the weighted graph. Start the algorithm at vertex A. Explain and justify each step as you add an edge to the tree.

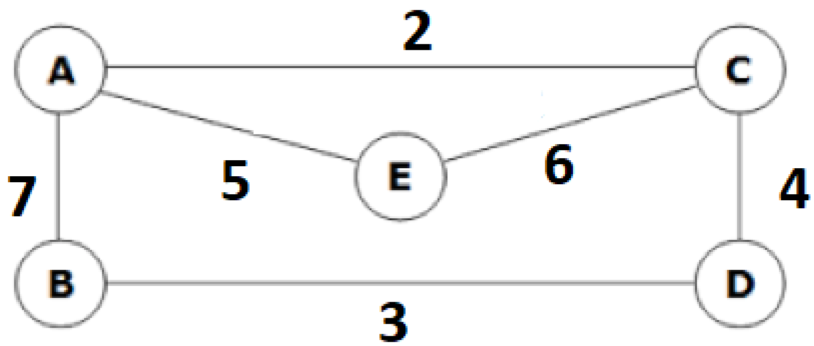


Figure 10: *A weighted graph shows 5 vertices, represented by circles, and 6 edges, represented by line segments. Vertices A, B, C, and D are placed at the corners of a rectangle, whereas vertex E is at the center of the rectangle. The edges, A B, B D, A C, C D, A E, and E C, have the weights, 7, 3, 2, 4, 5, and 6, respectively.*

We start from vertex A and find the smallest Vertex connected to A which is C. (A, C). The next lowest value connected to C would be D. (A, C, D). The next lowest value would be B (A,C,D,B). The final value would be E. So by Primm's algorithms the minimum spanning tree will be (A, C, D, B, E) with values (2, 4, 3, 5)

Problem 6

A lake initially contains 1000 fish. Suppose that in the absence of predators or other causes of removal, the fish population increases by 10% each month. However, factoring in all causes, 80 fish are lost each month.

Give a recurrence relation for the population of fish after n months. How many fish are there after 5 months? If your fish model predicts a non-integer number of fish, round down to the next lower integer.

If the initial population is 1000 a(0) and it increases by 10 percent each month. $a(n) = a(n-1) + 0.1a(n-1).A(n) = 1 * 1a(n-1) - 80$ for the fish lost each month. So $a(5) = 1 * 1a(4-80)$. So we get $a(5) = 1 * 1(1092 - 82) - 80 = 1122$ is the population of fish after 5 months.