MODULE SIX PROBLEM SET

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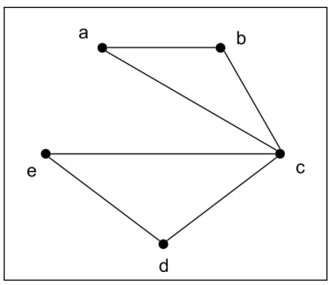
Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

For parts (a) and (b), indicate if each of the two graphs are equal. Justify your answer.

(a)



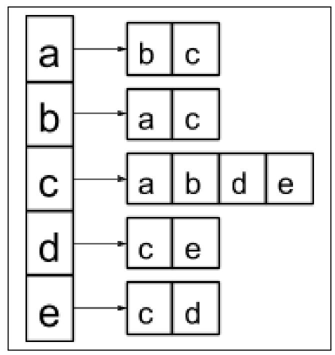
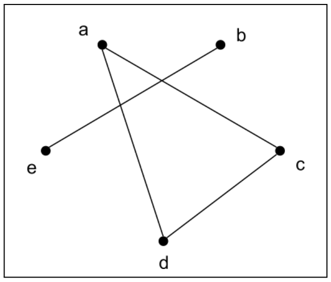


Figure 1: *Left: An undirected graph has 5 vertices. The vertices are arranged in the form of an inverted pentagon. From the top left vertex, moving clockwise, the vertices are labeled: a, b, c, d, and e. Undirected edges, line segments, are between the following vertices: a and b; a and c; b and c; c and d; e and d; and e and c.*

Figure 2: *Right: The adjacency list representation of a graph. The list shows all the vertices, a through e, in a column from top to bottom. The adjacent vertices for each vertex in the column are placed in a row to the right of the corresponding vertexs cell in the column. An arrow points from each cell in the column to its corresponding row on the right. Data from the list, as follows: Vertex a is adjacent to vertices b and c. Vertex b is adjacent to vertices a and c. Vertex c is adjacent to vertices a, b, d, and e. Vertex d is adjacent to vertices c and e. Vertex e is adjacent to vertices c and d.*

The adjacency list shows us which verteces are connected together. The graph that corresponds to the adjacency list looks how it should so the the two are in fact equal.

(b)



$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Figure 3: *An undirected graph has 5 vertices. The vertices are arranged in the form of an inverted pentagon. Moving clockwise from the top left vertex a, the other vertices are, b, c, d, and e. Undirected edges, line segments, are between the following vertices: a and c; a and d; d and c; and e and b.*

The adjacency matrix shows us if there is an edge present between 2 vertexes. Denoted with a 1 or 0 in position (v_i, v_j) according to whether v_i and v_j are adjacent or not. Therefore, the adjacency matrix is not equal to the graph in this problem.

(c) Prove that the two graphs below are isomorphic.

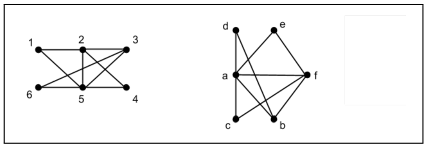


Figure 4: *Two undirected graphs. Each graph has 6 vertices. The vertices in the first graph are arranged in two rows and 3 columns. From left to right, the vertices in the top row are 1, 2, and 3. From left to right, the vertices in the bottom row are 6, 5, and 4. Undirected edges, line segments, are between the following vertices: 1 and 2; 2 and 3; 1 and 5; 2 and 5; 5 and 3; 2 and 4; 3 and 6; 6 and 5; and 5 and 4. The vertices in the second graph are a through f. Vertices d, a, and c, are vertically inline. Vertices e, f, and b, are horizontally to the right of vertices d, a, and c, respectively. Undirected edges, line segments, are between the following vertices: a and d; a and c; a and e; a and b; d and b; a and f; e and f; c and f; and b and f.*

A graph $G_1 = (V_1, E_1)$ is said to be isomorphic to a graph $G_2 = (V_2, E_2)$ if there is a one-to-one correspondence between the vertices $V_1 and V_2$ and a one-to-one correspondence between the edges $E_1 and E_2$ Given two graphs $G_1 = (V_1, E_1) and G_2 = (V_2, E_2, where$
$V_1 = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{(1, 2), (1, 5), (2, 3), (2, 4), (2, 5), (3, 5), (3, 6), (4, 5), (5, 6)\}$

$V_2 = \{a, b, c, d, e, f\}$

$E_2 = \{(a, b), (a, c), (a, d), (a, e), (a, f), (b, d), (b, f), (c, f), (e, f)\}$ Therefore, $G_1 and G_2$ are isomorphic.

(d) Show that the pair of graphs are not isomorphic by showing that there is a property that is preserved under isomorphism which one graph has and the other does not.

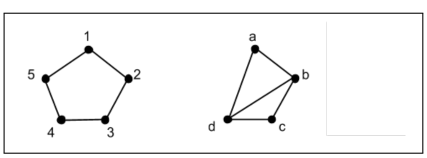


Figure 5: *Two undirected graphs. The first graph has 5 vertices, in the form of a regular pentagon. From the top vertex, moving clockwise, the vertices are labeled: 1, 2, 3, 4, and 5. Undirected edges, line segments, are between the following vertices: 1 and 2; 2 and 3; 3 and 4; 4 and 5; and 5 and 1. The second graph has 4 vertices, a through d. Vertices d and c*

are horizontally inline, where vertex d is to the left of vertex c. Vertex a is above and between vertices d and c. vertex b is to the right and below vertex a, but above the other two vertices. Undirected edges, line segments, are between the following vertices: a and b; b and c; a and d; d and c; d and b.

Here degree of each vertex in graph $H_1 is 2$. But H_2 has two vertices of degree 3 (b, d). So, we cannot get a one-to-one correspondence between the vertices $H_1 and H_2$ with a preserved degree of vertices. Since isomorphism preserves the degree of vertices, $H_1 and H_2$ are not isomorphic.

Problem 2

Refer to the undirected graph provided below:

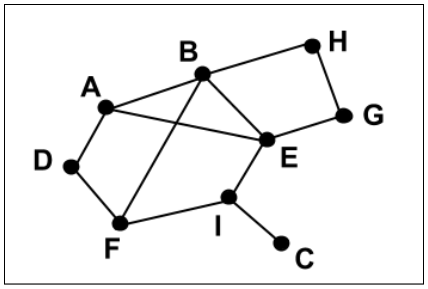


Figure 6: *An undirected graph has 9 vertices. 6 vertices form a hexagon, which is tilted upward to the right. Starting from the leftmost vertex, moving clockwise, the vertices forming the hexagon shape are: D, A, B, E, I, and F. Vertex H is above and to the right of vertex B. Vertex G is the rightmost vertex, below vertex H and above vertex E. Vertex C is the bottommost vertex, a little to the right of vertex E. Undirected edges, line segments, are between the following vertices: A and D; A and B; B and F; B and H; H and G; G and E; B and E; A and E; E and I; I and C; I and F; and F and D.*

(i) What is the maximum length of a path in the graph? Give an example of a path of that length.

A path is a open walk in which no edge or vertex is repeated. A walk is called open-walk if source vertex and destination vertex are not the same. The maximum length of the path in this graph is 8. $C- > I- > E- > G- > H- > B- > A- > D- > F$

(ii) What is the maximum length of a cycle in the graph? Give an example of a cycle of that length.

A cycle in a graph is the set of edges and vertices that make the source vertex and destination vertex the same. The maximum length of the cycle in the graph is also 8. $I- > E- > G- > H- > B- > A- > D- > F- > I$

(iii) Give an example of an open walk of length five in the graph that is a trail but not a path.

A trail is defined as an open walk where no edges are repeated but there can be vertex repetition. $E- > B- > A- > E- > I- > F$

(iv) Give an example of a closed walk of length four in the graph that is not a circuit.

A closed walk is a walk with the source and destination vertex are the same. $B- > A- > D- > A- > B$ A circuit must be trail and can have repitiion of vertex but not edge.

(v) Give an example of a circuit of length zero in the graph.

C is an isolated vertex and has a length of 0.

Problem 3

(a) Find the connected components of each graph.

(i) $G = (V, E)$. $V = \{a, b, c, d, e\}$. $E = \emptyset$

By the definition each vertex is a component, because there are no edges in the graph. So there are five components in this graph (a, b, c, d, e)

(ii) $G = (V, E)$. $V = \{a, b, c, d, e, f\}$. $E = \{\{c, f\}, \{a, b\}, \{d, a\}, \{e, c\}, \{b, f\}\}$

By the given definition, since the graph itself is connected there is only one component. All the verteces are connected by a path to any other verteces.

(b) Determine the edge connectivity and the vertex connectivity of each graph.

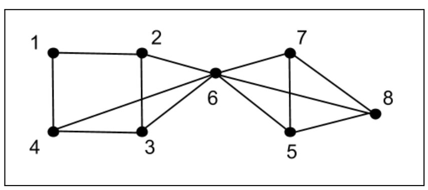
(i) 

Figure 7: *An undirected graph has 8 vertices, 1 through 8. 4 vertices form a rectangular-shape on the left. Starting from the top left vertex and moving clockwise, the vertices of the rectangular shape are, 1, 2, 3, and 4. 3 vertices form a triangle on the right, with a vertical side on the left and the other vertex on the extreme right. Starting from the top vertex and moving clockwise, the vertices of the triangular shape are, 7, 8, and 5. Vertex 6 is between the rectangular shape and the triangular shape. Undirected edges, line segments, are between the following vertices: 1 and 2; 2 and 3; 3 and 4; 4 and 1; 2 and 6; 4 and 6; 3 and 6; 6 and 7; 6 and 8; 6 and 5; 7 and 5; 7 and 8; and 5 and 8.*

Clearly, this graph is connected so $(K(G) > 0)$ Let us find the vertex connectivity $(K(G))$ If you number all the verteces from $1-8$ you can see if 6 was removed the graph would become disconncected. Next we find the edge connectivity $A((G))$ if we remove any edge the graph will not be disconnected, so edge connectivity $(A(G)) = 2$ and vertex connectivity $(K(G) = 1)$

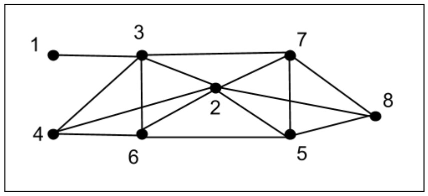
(ii) 

Figure 8: *An undirected graph has 8 vertices, 1 through 8. 4 vertices form a rectangular shape in the center. Starting from the top left vertex and moving clockwise, the vertices of the rectangular shape are, 3, 7, 5, and 6. Vertex 2 is at about the center of the rectangular shape. Vertex 8 is to the right of the rectangular shape. Vertex 1 and 4 are to the left of the rectangular shape, horizontally in-line with vertices 3 and 6, respectively. Undirected edges, line segments, are between the following vertices: 1 and 3; 3 and 7; 3 and 4; 3 and 6; 3 and 2; 4 and 2; 4 and 6; 6 and 2; 6 and 5; 2 and 5; 2 and 7; 2 and 8; 7 and 5; 7 and 8; and 5 and 8.*

Clearly this graph is connected. If we were to remove vertex three the graph would then become disconnected, because there would no longer be a path to from vertex one to other vertices. Vertex connectivity ($K(G) = 1$) Similarily, if we remove edge 13 the graph will no longer be connected from vertex one to other vertices. So the edge connectivity ($A(G) = 1$)

Problem 4

For parts (a) and (b) below, find an Euler circuit in the graph or explain why the graph does not have an Euler circuit.

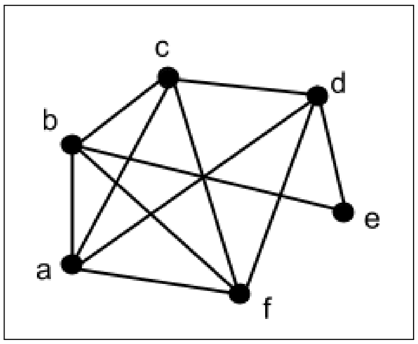
(a) 

Figure 9: *An undirected graph has 6 vertices, a through f. 5 vertices are in the form of a regular pentagon, rotated 90 degrees clockwise. Hence, the top vertex becomes the rightmost vertex. From the bottom left vertex, moving clockwise, the vertices in the pentagon shape are labeled: a, b, c, e, and f. Vertex d is above vertex e, below and to the right of vertex c. Undirected edges, line segments, are between the following vertices: a and b; a and c; a and d; a and f; b and f; b and c; b and e; c and d; d and e; and d and f. Edges c f, a d, and b e intersect at the same point.*

AN Euluer circuit is a circuit that uses every edge of the graph only once. In this given graph we find a Euler circuit starting and ending at vertex e. Euler circuit is $(e, d, f, a, b, c, d, a, c, f, b, e)$

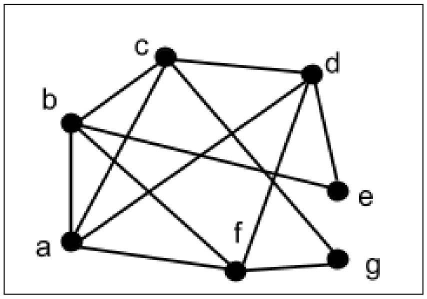
(b) 

Figure 10: *An undirected graph has 7 vertices, a through g. 5 vertices are in the form of a regular pentagon, rotated 90 degrees clockwise. Hence, the top vertex becomes the rightmost vertex. From the bottom left vertex, moving clockwise, the vertices in the pentagon shape are labeled: a, b, c, e, and f. Vertex d is above vertex e, below and to the right of vertex c. Vertex g is below vertex e, above and to the right of vertex f. Undirected edges, line segments, are between the following vertices: a and b; a and c; a and d; a and f; b and f; b and c; b and e; c and d; c and g; d and e; d and f; and f and g.*

For this graph we find the Euler circuit by starting and ending at vertex e. The Euler circuit is $(e, d, c, b, a, f, g, c, a, d, f, b, e)$

(c) For each graph below, find an Euler trail in the graph or explain why the graph does not have an Euler trail.

(Hint: One way to find an Euler trail is to add an edge between two vertices with odd degree, find an Euler circuit in the resulting graph, and then delete the added edge from the circuit.)

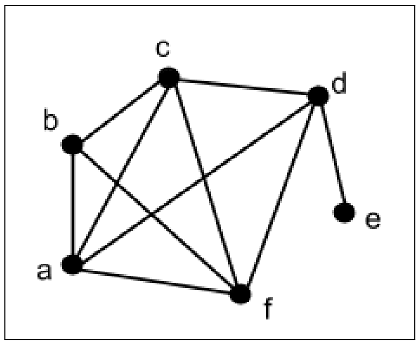
(i) 

Figure 11: *An undirected graph has 6 vertices, a through f. 5 vertices are in the form of a regular pentagon, rotated 90 degrees clockwise. Hence, the top vertex becomes the rightmost vertex. From the bottom left vertex, moving clockwise, the vertices in the pentagon shape are labeled: a, b, c, e, and f. Vertex d is above vertex e, below and to the right of vertex c. Undirected edges, line segments, are between the following vertices: a and b; a and c; a and d; a and f; b and f; b and c; c and d; c and f; d and e; and d and f.*

This graph has 6 vertices with two vertices b and e with an odd degree. A Euluer graph can have at most two odd degree so this is in fact an Euler graph. $(e, d, c, b, a, f, d, a, c, f, b)$

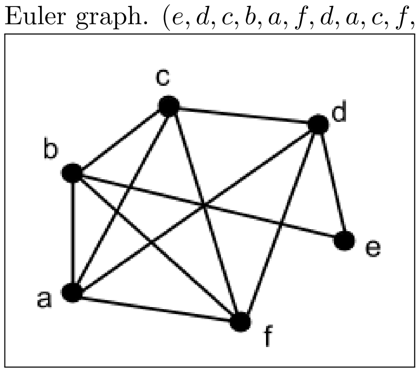
(ii) 

Figure 12: *An undirected graph has 6 vertices, a through f. 5 vertices are in the form of a regular pentagon, rotated 90 degrees clockwise. Hence, the top vertex becomes the rightmost vertex. From the bottom left vertex, moving clockwise, the vertices in the pentagon shape are labeled: a, b, c, e, and f. Vertex d is above vertex e, below and to*

the right of vertex c. Undirected edges, line segments, are between the following vertices: a and b; a and c; a and d; a and f; b and f; b and c; b and e; c and d; d and e; and d and f. Edges c f, a d, and b e intersect at the same point.

The graph has 6 vertices and zero odd vertices. A connected graph has a Euler trail if has at most, two odd degrees, so this does have a trail. $(e, d, c, b, a, f, d, a, c, f, e)$ This is a closed Euler trail so it would also be a Euler circuit.

PROBLEM 5

Consider the following tree for a prefix code:

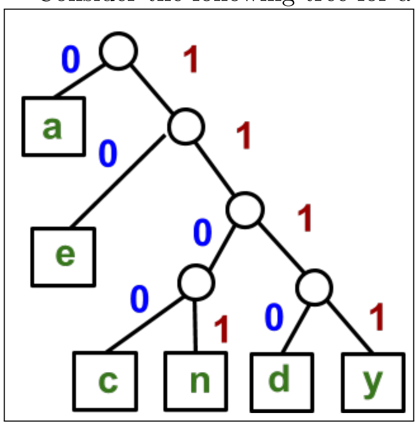


Figure 13: *A tree with 5 vertices. The top vertex branches into character, a, on the left, and a vertex on the right. The vertex in the second level branches into character, e, on the left, and a vertex on the right. The vertex in the third level branches into two vertices. The left vertex in the fourth level branches into character, c, on the left, and character, n, on the right. The right vertex in the fourth level branches into character, d, on the left, and character, y, on the right. The weight of each edge branching left from a vertex is 0. The weight of each edge branching right from a vertex is 1.*

(a) Use the tree to encode "day".

111001111

(b) Use the tree to encode "candy".

11000110111101111

(c) Use the tree to decode "1110101101″.

The word is den.

(d) Use the tree to decode "111001101110010″.

The word is dance.

PROBLEM 6

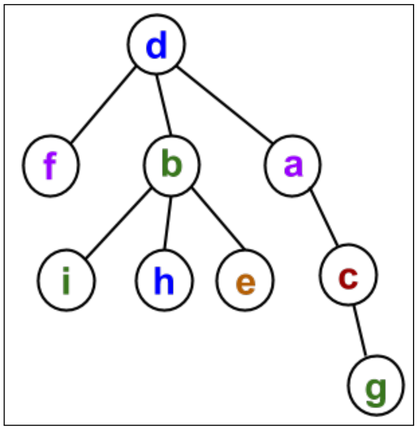


Figure 14: *A tree diagram has 9 vertices. The top vertex is d. Vertex d has three branches to vertices, f, b, and a. Vertex b branches to three vertices, i, h, and e. Vertex a branches to vertex c. Vertex c branches to vertex g.*

(a) Give the order in which the vertices of the tree are visited in a post-order traversal.

Post order traversal visits child first from left to right and then the node. The post-order traversal would be ($fihebgcad$)

(b) Give the order in which the vertices of the tree are visited in a pre-order traversal.

Pre-order traversal visits the node first, followed by the child. So the pre-order would be ($dfbiheacg$)

Problem 7

Consider the following tree. Assume that the neighbors of a vertex are considered in alphabetical order.

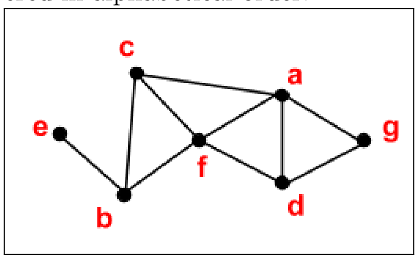


Figure 15: *A graph has 7 vertices, a through g, and 10 edges. Vertex e on the left end is horizontally inline with vertex g on the right end. Vertex b is below and to the right of vertex e. Vertex c is above vertex e and to the right of vertex b. Vertex f is between and to the right of vertices c and b. Vertex f is horizontally inline with vertices e and g. Vertex a is above and to the right of vertex f. Vertex d is below and to the right of vertex f. Vertex a is vertically inline with vertex d. Vertex g is between and to the right of vertices a and d. The edges between the vertices are as follows: e and b; b and c; c and f; c and a; a and d; b and f; f and a; f and d; a and g; and d and g.*

(a) Give the tree resulting from a traversal of the graph below starting at vertex a using BFS.

BFS's concept is to visit all the neighboring vertices before visiting other neighbor vertices of neighbor vertices. First in BFS all the neighbors will need to be traversed. $(a->, c->, d->, f->, g)$ The traverse the neighbors of the neighbors $(b-> e)$ So the final BFS traversal will be $(a->, c->, d->, f->, g->, b->, e)$

(b) Give the tree resulting from a traversal of the graph below starting at vertex a using DFS.

The concept of DFS is to visit all the neighbor vertices of a neighbor vertex before visiting the other neighbor vertices. So first we traverse the bneighbors of neighbors. $(a->, c->, b->, e)$ Next we traverse the neighbors of the main vertex $(d->, f-> g)$ So the full traversal of DFS will be $(a->, c->, b->, e->, d->, f->, g)$

Problem 8

An undirected weighted graph G is given below:

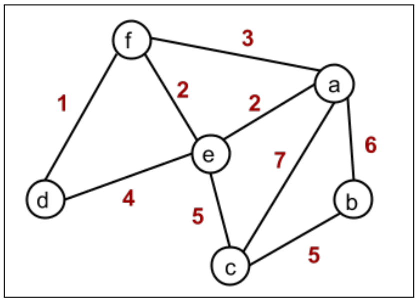


Figure 16: *An undirected weighted graph has 6 vertices, a through f, and 9 edges. Vertex d is on the left. Vertex f is above and to the right of vertex d. Vertex e is below and to the right of vertex f, but above vertex d. Vertex c is below and to the right of vertex e. Vertex a is above vertex e and to the right of vertex c. Vertex b is below and to the right of vertex a, but above vertex c. The edges between the vertices and their weight are as follows: d and f, 1; d and e, 4; f and e, 2; e and a, 2; f and a, 3; e and c, 5; c and a, 7; c and b, 5; and a and b, 6.*

(a) Use Prim's algorithm to compute the minimum spanning tree for the weighted graph. Start the algorithm at vertex a. Show the order in which the edges are added to the tree.

$(A- > E), (A- > E- > F), (A- > E- > F- > D), (A- > E- > F- > D- > C), (A- > E- > F- > D- > C- > B)$. So if we add the values $1 + 2 + 2 + 5 + 5 = 15$ 15 is the minimum spanning tree

(b) What is the minimum weight spanning tree for the weighted graph in the previous question subject to the condition that edge $\{d, e\}$ is in the spanning tree?

If we need to find the minimum spanning tree that includes edge (d, e) we will chose (d, e) instead of backtracking from (d, f) in the previous answer. If calculate the values $4 + 2 + 2 + 5 + 5 = 18$. 18 will be the minimum spanning tree that include edgtes (d, e)

(c) How would you generalize this idea? Suppose you are given a graph G and a particular edge $\{u, v\}$ in the graph. How would you alter Prim's algorithm to find the minimum spanning tree subject to the condition that $\{u, v\}$ is in the tree?

Generalization that edge (U, V) should be present in the graph can be modified using Prim's algorithm. You start with the given vertices and go until you do not get any of the vertices from (U, V) When you finally get (U, V) you can draw a line from edges that connect $(U, V) or (V, U)$