

- 1.
1.  $p \rightarrow q$  | given
  2.  $p \rightarrow r$  | given
- } Prove:  $p \rightarrow (q \text{ AND } r)$
3.  $p \rightarrow (q \text{ AND } r)$  | conjunction

Prove  $p \rightarrow q$

- 2.
1.  $p \rightarrow (q \vee r)$  | given
  2.  $p \rightarrow (q \vee \sim r)$  | given
  3.  $p \rightarrow (q \vee r) \wedge (q \vee \sim r)$  | conjunction
  4.  $p \rightarrow q \vee (r \wedge \sim r)$  | distributive
  5.  $p \rightarrow q \vee F$  | negation
  6.  $p \rightarrow q$  | elimination

II.

p	q	r	$p \rightarrow q$	$p \rightarrow r$	$p \rightarrow (q \wedge r)$
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	0	1	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	0
1	1	0	1	0	0
1	1	1	1	1	1

III.


1. Satisfiable
2. Satisfiable
3. Tautology
4. Contradiction

IV.  $p \vee (q \wedge \sim(r \wedge (s \rightarrow t)))$

$(p \vee q) \wedge (p \vee \sim(r \wedge (s \rightarrow t)))$  | distributive

$(p \vee q) \wedge (p \vee \sim r)$  | specialization

V.



$p$	$q$	$r$	
1	0	0	1

} This short certificate proves the formula from IV.