

4/15/14

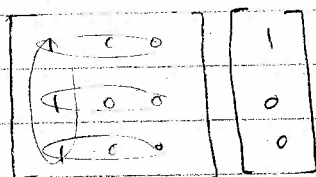
Math explanation

- never a sweater two days in a row

Basic example:

$$L = \begin{matrix} & \begin{matrix} T & S & C \end{matrix} \\ \begin{matrix} T \\ S \\ C \end{matrix} & \begin{bmatrix} 1/5 & 2/3 & 1/4 \\ 1/5 & 0 & 1/4 \\ 3/5 & 1/3 & 1/2 \end{bmatrix} \end{matrix}$$

→ describe what transitions mean (given that he wore — yesterday, this is the chance of him wearing each thing today)



Absorbing example:

$$A = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 1 & 2/3 & 1/4 \\ 0 & 0 & 1/4 \\ 0 & 1/3 & 1/2 \end{bmatrix} \end{matrix}$$

← [regular/irregular; set of states & transition matrix; absorbing (state vs. chain) vs. transi

Canonical form:

$$A = \begin{bmatrix} Q & 0 \\ R & I \end{bmatrix} \rightarrow \begin{matrix} & \begin{matrix} B & C & A \end{matrix} \\ \begin{matrix} B \\ C \\ A \end{matrix} & \begin{bmatrix} 0 & 1/4 & 0 \\ 1/3 & 1/2 & 0 \\ 2/3 & 1/4 & 1 \end{bmatrix} \end{matrix} \rightarrow \begin{matrix} Q = \begin{bmatrix} 0 & 1/4 \\ 1/3 & 1/2 \end{bmatrix} \\ R = \begin{bmatrix} 2/3 & 1/4 \end{bmatrix} \end{matrix}$$

$$A^{k+1} = \begin{bmatrix} Q^k & 0 \\ * & I \end{bmatrix} \leftarrow \text{understand this.}$$

Fundamental matrix:

Define $N = (I - Q)^{-1}$

$$N = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1/4 \\ 1/3 & 1/2 \end{bmatrix} \right)^{-1}$$

Proof:

$$\left\{ \begin{aligned} (I - Q)(I + Q + Q^2 + \dots + Q^n) &= I - Q^{n+1} \\ N[I - Q^{n+1}] &= N(I - Q^{n+1}) \\ I + Q + Q^2 + \dots + Q^n &= N \end{aligned} \right.$$

$$N = \left(\begin{bmatrix} 1 & -1/4 \\ -1/3 & 1/2 \end{bmatrix} \right)^{-1} = \frac{1}{(1)(1/2) - (-1/3)(1/4)} \begin{bmatrix} 1/2 & 1/4 \\ 1/3 & 1 \end{bmatrix} = \frac{12}{5} \begin{bmatrix} 1/2 & 1/4 \\ 1/3 & 1 \end{bmatrix}$$

With N , we can determine how many time steps it takes to get to the absorbing state from a given state

N also gives the expected number of times that the process is in a transient state (s_i) if it started in the transient state s_i

$$N = \begin{pmatrix} S & C \\ \begin{pmatrix} 6/5 & 3/5 \\ 4/5 & 12/5 \end{pmatrix} & \begin{matrix} S \\ C \end{matrix} \end{pmatrix}$$

How to interpret: It takes 3 days for him to wear a turtleneck if he start out wearing a collared shirt.

	S	C	A
S	6/5	3/5	0
C	4/5	12/5	0
A	0	0	1

$$N = (I - Q)^{-1}$$

$$N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 6/5 & 3/5 \\ 4/5 & 12/5 \end{pmatrix} = \begin{pmatrix} -1/5 & -3/5 \\ -4/5 & -7/5 \end{pmatrix}$$

$$N = \begin{pmatrix} 3 & 1 \\ 4 & 3 \end{pmatrix}$$

	S	C	A
S	3	1	0
C	4	3	0
A	0	0	1

$$N = (I - Q)^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 6/5 & 3/5 \\ 4/5 & 12/5 \end{pmatrix} = \begin{pmatrix} -1/5 & -3/5 \\ -4/5 & -7/5 \end{pmatrix}$$

$$N = \begin{pmatrix} 3 & 1 \\ 4 & 3 \end{pmatrix}$$