

Exercise 5.2

$$\begin{array}{c|c|c} A_{11} & d_{m1} & \emptyset \\ \hline d_{m1}^T & d_{mm} & d_{m1}^T \\ \hline \emptyset & \emptyset & A_{ff} \end{array} \rightarrow \begin{array}{c|c|c|c} A_{00} & d_{01} & \emptyset & \emptyset \\ \hline d_{01}^T & d_{11} & d_{21} & \emptyset \\ \hline \emptyset & d_{11} & d_{22} & d_{32}^T \\ \hline \emptyset & \emptyset & d_{22} & A_{33} \end{array}$$

$$\begin{array}{c|c} A_{00} & d_{01} \\ \hline d_{01}^T & d_{11} \end{array} = \begin{array}{c|c|c|c} X & X & & \\ \hline X & X & d_{10} & \\ \hline d_{10} & d_{11} & d_{21} & \\ \hline & d_{21} & d_{22} & d_{32} \\ \hline & & d_{32} & X & + \\ & & & X & X & X \\ & & & & X & X \end{array} =$$

$$\begin{array}{c|c} u_{11} d_{11} u_{11}^T & u_{12} d_{22} \\ \hline u_{12} d_{21} & d_{22} \\ \hline X & d_{22} \end{array} \quad \boxed{
 \begin{aligned} d_{21} &= d_{21} / d_{22} \\ d_{11} &= d_{11} - d_{22} d_{21}^2
 \end{aligned}
 }$$

$$d_{22} + d_{22} = d_{22} \sim \text{no change}$$

$$d_{01} = u_{12} d_{22}$$

$$u_{12} = d_{01} / d_{22} = \left(\frac{0}{d_{11}} \right) / d_{22} =$$

$$d_{21} = d_{21} / d_{22}$$

$$A_{002} = \begin{pmatrix} A_{00} & d_{10} \\ X & d_{11} \end{pmatrix}$$

$$A_{22} = A_{22} - d_{22} d_{21} d_{21}^T$$

$$\begin{pmatrix} A_{00} & d_{10} \\ X & d_{11} \end{pmatrix} - d_{22} \begin{pmatrix} \emptyset \\ d_{21} \end{pmatrix} \begin{pmatrix} \emptyset \\ d_{21} \end{pmatrix}^T =$$

$$\begin{pmatrix} A_{00} & d_{10} \\ X & d_{11} \end{pmatrix} - d_{22} \begin{pmatrix} 0 & 0 \\ 0 & d_{21}^2 \end{pmatrix} = \begin{pmatrix} A_{00} & d_{10} \\ X & d_{11} - d_{22} d_{21}^2 \end{pmatrix}$$