6.1 a

$$\begin{pmatrix} A_{00} & a_{10}I & 0 \\ a_{10}I^T & a_{11} & a_{21}e_F^T \\ \emptyset & a_{21}e_F & A_{22} \end{pmatrix} = \begin{pmatrix} L_{00} & \emptyset & \emptyset \\ \lambda_{10}I^T & 1 & \emptyset \\ \emptyset & \lambda_{21}e_F & L_{22} \end{pmatrix} \begin{pmatrix} D_{00} & \emptyset & \emptyset \\ \emptyset & d_1 & \emptyset \\ \emptyset & \emptyset & D_{22} \end{pmatrix} \begin{pmatrix} L_{00}^T & a_{10}I & \emptyset \\ \emptyset & 1 & \lambda_{21}e_F^T \\ \emptyset & \emptyset & L_{22}^T \end{pmatrix}$$

$$A_{00} = L_{00} D_{00} L_{00}^T$$

$$\begin{pmatrix} D_{00}L_{00}^T & D_{00}\lambda_{10}I & \emptyset \\ \emptyset & d_1 & d_1\lambda_{21}e_F^T \\ \emptyset & \emptyset & D_{22}L_{22}^T \end{pmatrix}$$

$$\begin{pmatrix} A_{00} & a_{10}I & \emptyset \\ a_{10}I^T & a_{11} & a_{21}e_F^T \\ \emptyset & a_{21}e_F & A_{22} \end{pmatrix} - \begin{pmatrix} L_{00}D_{00}L_{00}^T & L_{00}D_{00}\lambda_{10}I & \emptyset \\ \lambda_{10}I^T D_{00} L_{00}^T & \lambda_{10}I^T D_{00}\lambda_{10}I + d_1 & d_1\lambda_{21}e_F^T \\ \emptyset & \lambda_{21}e_F d_1 & L_{22}D_{22}L_{22}^T \end{pmatrix}$$

$$\begin{pmatrix} A_{00} & a_{10}I & \emptyset \\ a_{10}I^T & a_{11} & a_{21}e_F^T \\ \emptyset & a_{21}e_F & A_{22} \end{pmatrix} = \begin{pmatrix} V_{00} & v_{01}e_I & \emptyset \\ \emptyset & 1 & v_{21}e_F^T \\ \emptyset & \emptyset & V_{22} \end{pmatrix} \begin{pmatrix} E_{00} & \emptyset & \emptyset \\ \emptyset & \varepsilon_1 & \emptyset \\ \emptyset & \emptyset & E_{22} \end{pmatrix} \begin{pmatrix} V_{00}^T & \emptyset & \emptyset \\ v_{01}e_I^T & 1 & \emptyset \\ \emptyset & v_{21}e_F^T & V_{22}^T \end{pmatrix}$$

$$\begin{pmatrix} V_{00}E_{00}V_{00}^T & \varepsilon_1 V_{01}e_I & \emptyset \\ \varepsilon_1 v_{01}e_I^T & \varepsilon_1 + v_{21}e_F^T E_{22} v_{21}e_F & v_{21}e_F^T E_{22} V_{22}^T \\ \emptyset & V_{22}E_{22}v_{21}e_F & V_{22}E_{22}V_{22}^T \end{pmatrix} \begin{pmatrix} E_{00}V_{00}^T & \emptyset & \emptyset \\ \varepsilon_1 V_{01}e_I^T & \varepsilon_1 & \emptyset \\ \emptyset & E_{22}v_{21}e_F^T & E_{22}V_{22}^T \end{pmatrix}$$

$$\begin{pmatrix} L_{00} & \emptyset & \emptyset \\ \lambda_{10e_L}^T & 1 & v_{12e_F}^T \\ \emptyset & \emptyset & U_{22} \end{pmatrix} \begin{pmatrix} D_{00} & 0 & 0 \\ \emptyset & \phi_1 & \emptyset \\ \emptyset & \emptyset & E_{22} \end{pmatrix} \begin{pmatrix} L_{00}^T & \lambda_{10e_L} & \emptyset \\ \emptyset & 1 & \emptyset \\ \emptyset & v_{12e_F} & U_{22}^T \end{pmatrix}$$

$$\star \begin{pmatrix} D_{00} L_{00}^T & D_{00}\lambda_{10e_L} & \emptyset \\ \emptyset & \phi_1 & \emptyset \\ \emptyset & E_{22} v_{12e_F} & E_{22} U_{22}^T \end{pmatrix}$$

$$\begin{pmatrix} L_{00} D_{00} L_{00}^T & L_{00} D_{00} \lambda_{10e_L} & \emptyset \\ \lambda_{10e_L}^T D_{00} L_{00}^T & \lambda_{10e_L}^T D_{00} \lambda_{10e_L} + \phi + v_{12e_F}^T E_{22} v_{12e_F} & v_{12e_F}^T E_{22} U_{22}^T \\ \emptyset & U_{22} E_{22} v_{12e_F} & U_{22} E_{22} U_{22}^T \end{pmatrix}$$

$$(0\,0\,0 \ldots \lambda_{10}) \begin{pmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \lambda_{10} \end{pmatrix}$$

$$(0\,0\,0 \ldots \lambda_{10}) \begin{pmatrix} 0 \\ 0 \\ 0 \\ d_n \lambda_{10} \end{pmatrix}$$

α

$$\alpha_{11} = \lambda_{10e_L}^T D_{00} \lambda_{10e_L} + d_1$$

$$\alpha_{11} = \varepsilon_1 + v_{12e_F}^T E_{22} v_{12e_F}$$

$$\alpha_{11} = \lambda_{10e_L}^T D_{00} \lambda_{10e_L} + \phi + v_{12e_F}^T E_{22} v_{12e_F}$$

$$\alpha_{11} = \alpha_{11} - d_1 + \phi + \alpha_{11} - \varepsilon_1$$

$$\alpha_{11} - \alpha_{11} - \alpha_{11} = -d_1 + \phi - \varepsilon_1$$

$$-\alpha_{11} = -d_1 + \phi - \varepsilon_1$$

$$\boxed{\phi = d_1 + \varepsilon_1 - \alpha_{11}}$$

justifying ✓

6.1b

$$\phi = d_1 + \varepsilon_1 - \alpha_{11}$$

1 flop (for $\varepsilon_1 - \alpha_{11}$), 1 flop.

$$\alpha_{11} = \lambda_{10} d_{10} \lambda_{10} + d_1$$

$$\alpha_{11} = \varepsilon_1 + v_{12}^T E_2 v_{12}$$

3 flops $= d_1 = \alpha_{11} - (\lambda_{10} d_{10} \lambda_{10})$

3 flops $= \varepsilon_1 = \alpha_{11} - v_{12} E_2 v_{12}$

Cost of computing one twisted factorization: $O(1)$

6.1 c

$$\begin{pmatrix} D_{00} & \phi & \phi \\ \phi & \phi & \phi \\ \phi & \phi & F_{22} \end{pmatrix}$$

n potential ϕ

cost of computing them all

$O(n)$