* Problem 4:-

(I) → Equation of SVM Hyperplane $h(x)$:-

⇒ ~~From~~ From the given table, points having $\alpha_i = 0$ are not support vectors.

→ Only non-zero α_i (Lagrangian multipliers) determine the support vectors. i.e x_1, x_4, x_7, x_9

→ weight vector, $w = \sum_{\alpha_i > 0} \alpha_i y_i x_i$

$$= (0.414)\begin{pmatrix}4\\2.9\end{pmatrix} - 0.018\begin{pmatrix}2.5\\1\end{pmatrix} + 0.018\begin{pmatrix}3.5\\4\end{pmatrix} - 0.414\begin{pmatrix}2\\2.1\end{pmatrix}$$

$$= \begin{pmatrix}0.846\\0.385\end{pmatrix}$$

→ Bias of each support vector (b_i):-

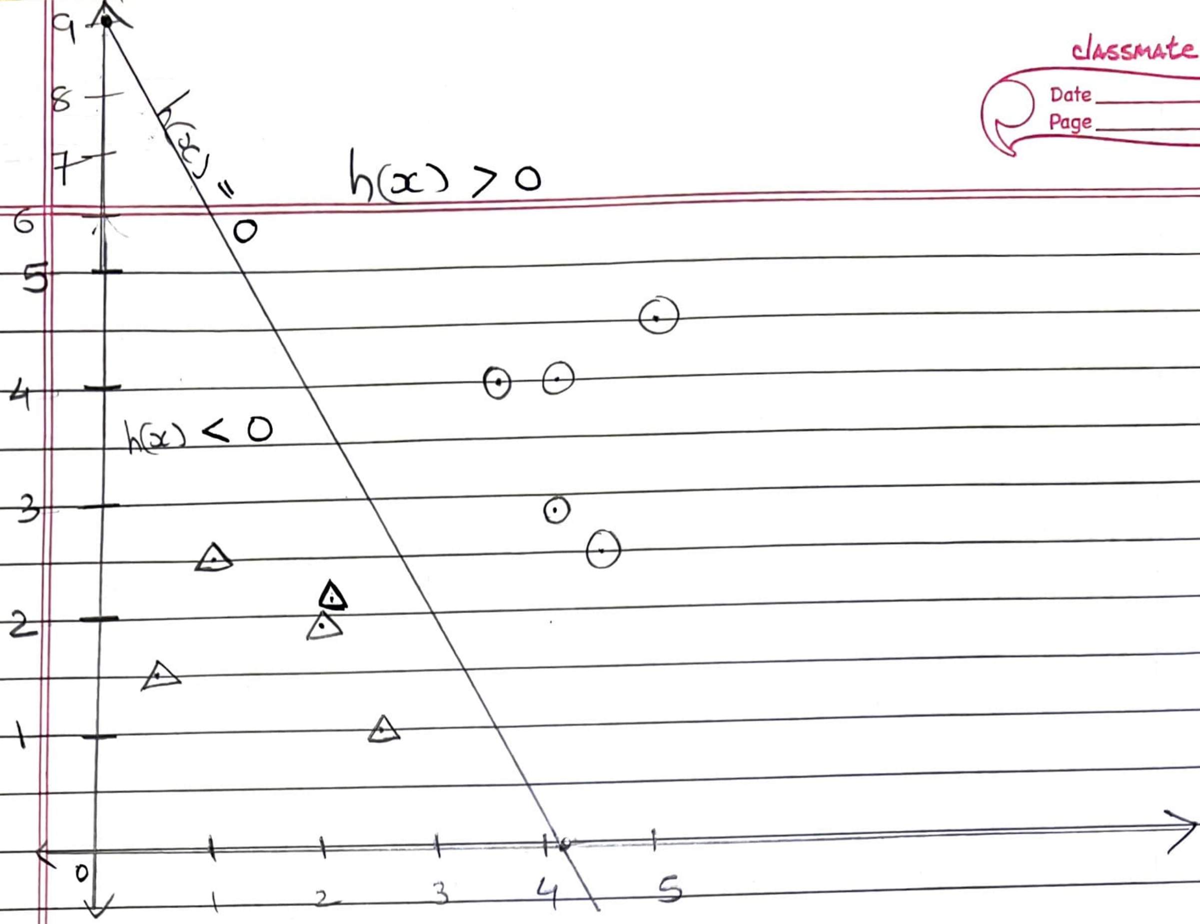
x_i	$w^T x_i$	$b_i = y_i - w^T x_i$
x_1	4.5005	-3.5005
x_4	2.5000	-3.5000
x_7	4.5010	-3.5010
x_9	2.5005	-3.5005

⇒ Final bias is average of above bias

$\therefore b = 3.5005$

⇒ The optimal hyperplane $h(x)$:-

$$h(x) = \begin{pmatrix}0.846\\0.385\end{pmatrix}^T x - 3.5005 = 0$$



→ Given 10 points are plotted. Points labelled +1 are shown as circles and those labelled −1 are shown as triangles.

⇒ we have $h(x) = \begin{pmatrix} 0.846 \\ 0.385 \end{pmatrix}^T x - 3.5005 = 0$

$\therefore \Rightarrow \begin{pmatrix} 0.846 \\ 0.385 \end{pmatrix}^T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - 3.5005 = 0$

$\therefore \Rightarrow 0.846\, x_1 + 0.385\, x_2 = 3.5005$

⇒ Keeping $x_1 = 0$, we get $x_2 = 9.092$ and then $x_2 = 0$, we get $x_1 = 4.1377$. which satisfy the hyperplane $h(x) = 0$.

(2) Distance of x_6 from hyperplane (δ):-

$$\therefore \delta = yr = \frac{y\, h(x)}{\|w\|}$$

$\rightarrow$ Now for $x_6 = \begin{pmatrix} 1.9 \\ 1.9 \end{pmatrix}$

$$\therefore h(x_6) = \begin{pmatrix} 0.846 \\ 0.385 \end{pmatrix}^T \begin{pmatrix} 1.9 \\ 1.9 \end{pmatrix} - 3.5005$$

$$= 2.3389 - 3.5005$$

$$= -1.1616.$$

$\therefore$ for x_6, $y = -1$ and $\|w\| = \sqrt{7.22}$

$$\Rightarrow \delta = yr = \frac{y\, h(x)}{\|w\|}$$

$$= \frac{(-1)(-1.1616)}{\sqrt{7.22}}$$

$$= \frac{1.1616}{2.687} = \boxed{0.432}$$

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(2) Margin of the classifier is minimum distance of a point from the hyperplane $h(x)$:

$$\delta^* = \min_{x_i} \left\{ \frac{y_i (w^T x_i + b)}{\|w\|} \right\}$$

$\therefore \delta_{x_1} = \frac{(1)(4.5005 - 3.5005)}{\sqrt{24.41}}$
$= 0.2024$

$\delta_{x_6} = \frac{(-1)(2.3389 - 3.5005)}{\sqrt{7.22}}$
$= 0.4323$

$\therefore \delta_{x_2} = \frac{(1)(4.924 - 3.5005)}{\sqrt{32}}$
$= 0.2516$

$\delta_{x_7} = \frac{(1)(4.5010 - 3.5005)}{\sqrt{28.25}}$
$= 0.1882$

$\therefore \delta_{x_3} = \frac{(-1)(1.8085 - 3.5005)}{\sqrt{7.25}}$
$= 0.6285$

$\delta_{x_8} = \frac{(-1)(1.0005 - 3.5005)}{\sqrt{2.5}}$
$= 1.581$

$\therefore \delta_{x_4} = \frac{(-1)(2.5 - 3.5005)}{\sqrt{7.25}}$
$= \cancel{0.6285}\ 0.3716$

$\delta_{x_9} = \frac{(-1)(2.5005 - 3.5005)}{\sqrt{8.41}}$
$= 0.3448$

$\therefore \delta_{x_5} = \frac{(1)(5.8779 - 3.5005)}{\sqrt{44.26}}$
$= 0.3573$

$\delta_{x_{10}} = \frac{(1)(4.7695 - 3.5005)}{\sqrt{26.5}}$
$= 0.2465$

$\Rightarrow$ Thus $\min(\delta_{x_i}) = 0.1882$ (of point x_7)

$\therefore \boxed{\delta^* = 0.1882}$

$\Rightarrow$ Now distance of x_6 from $h(x)$ is 0.432, So it is not within margin of classifier

(3) Classify the point $z = (3,3)^T$ using above $h(x)$.

⇒ we classify the point z by,

$$\hat{y} = \text{sign}(h(z)) = \text{sign}(w^T z + b)$$

; where sign(·) functions returns +1 if argument is positive and −1 if argument is negative.

$$\therefore \hat{y} = \text{sign}\left[\begin{pmatrix} 0.846 \\ 0.385 \end{pmatrix}^T \begin{pmatrix} 3 \\ 3 \end{pmatrix} - 3.5005\right]$$

$$= \text{sign}\left[3.693 - 3.5005\right]$$

$$= \text{sign}(0.1925)$$

⇒ As argument in sign(·) function is positive, we classify point z as +1