CS 583: Assignment 1

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* **Q1:** $x = [5, -3, -1, 2]^T$

(1) the squared L_2-norm of x,

$$||x||_2^2 = \sum_i x_i^2$$

$$= (5)^2 + (-3)^2 + (-1)^2 + (2)^2$$

$$= 25 + 9 + 1 + 4$$

$$||x||_2^2 = 39$$

(2) the L_1-norm of x,

$$||x||_1 = \sum_i |x_i|$$

$$= |5| + |-3| + |-1| + |2|$$

$$||x||_1 = 11$$

(3) the inner product of x and a, where $a = [4, -2, 6, -1]^T$,

$$a^T x = [4, -2, 6, -1] \begin{bmatrix} 5 \\ -3 \\ -1 \\ 2 \end{bmatrix}$$

$$= (5)(4) + (-2)(-3) + (6)(-1) + (-1)(2)$$

$$= 20 + 6 - 6 - 2$$

$$a^T x = 18$$

* **Q2:** $A = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix}$

(1) the matrix-vector product:

$$Ab = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} (6)(-4) + (5)(1) + (-2)(2) \\ (-5)(-4) + (7)(5) + (9)(2) \end{bmatrix}$$

$$= \begin{bmatrix} -24 + 5 - 4 \\ 20 + 35 + 18 \end{bmatrix}$$

$$\therefore Ab = \begin{bmatrix} -23 \\ 73 \end{bmatrix}$$

(2) the matrix-matrix product:

$$AA^T = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \begin{bmatrix} 6 & -5 \\ 1 & 7 \\ -2 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 36 + 1 + 4 & -30 + 7 - 18 \\ -30 + 7 - 18 & 25 + 49 + 81 \end{bmatrix}$$

$$= \begin{bmatrix} 41 & -41 \\ -41 & 155 \end{bmatrix}$$

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* **Q3:** $x = [x_1, x_2, x_3]$ and $y = \frac{x_1^2}{2} + \log_e x_2 - \frac{x_1}{x_3}$

⇒ dy/dx at $x = [9, 1, 1/2]$.

$\therefore \frac{dy}{dx} = \left[\frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2}, \frac{\partial y}{\partial x_3}\right]$

$= \left[x_1 - \frac{1}{x_3}, \frac{1}{x_2}, \frac{x_1}{x_3^2}\right]$

→ At $x = [9, 1, 1/2]$

$\therefore \frac{\partial y}{\partial x} = \left[9 - \frac{1}{(1/2)}, \frac{1}{1}, \frac{9}{(1/2)^2}\right]$

$\therefore \frac{\partial y}{\partial x} = [7, 1, 36]$

* **Q4:** $F(w) = \|Xw - y\|_2^2 + \lambda \|w\|_2^2$:- $X: n \times d$, $y: n \times 1$, $w: d \times 1$

$\therefore \frac{\partial F(w)}{\partial w} = \frac{\partial}{\partial w}\left[\|Xw - y\|_2^2 + \lambda \|w\|_2^2\right]$

→ Now, squared L_2-norm of x, $\|x\|_2^2 = \Sigma_i x_i^2$

$\therefore \frac{\partial(\|x\|_2^2)}{\partial x_i} = \frac{\partial}{\partial x} \Sigma_i x_i^2$

$= \Sigma_i \frac{\partial}{\partial x_i} x_i^2 = 2x_i$

$$\therefore \frac{\partial (\|x\|_2^2)}{\partial x} = 2x$$

$$\Rightarrow \therefore \frac{\partial (\|w\|_2^2)}{\partial w} = 2w = \begin{bmatrix} 2w_1 \\ 2w_2 \\ \vdots \\ 2w_d \end{bmatrix}_{d \times 1} \quad \ldots (i)$$

$$\Rightarrow \text{Now, } \frac{\partial}{\partial w_i}\left(\|xw - y\|_2^2\right) = \frac{\partial}{\partial w_i}\left[\Sigma_j (x_j w_i - y_j)^2\right]$$

$$= \frac{\partial}{\partial w_i} \Sigma_j (x_j w_i - y_i)^2$$

$$= \Sigma_j \left[\frac{\partial}{\partial w_i} (x_j w_i - y_i)^2\right]$$

$$= \Sigma_j \, 2(x_j w_i - y_i) \cdot x_j$$

$$\therefore \frac{\partial}{\partial w_i}\left(\|xw - y\|_2^2\right) = \Sigma_j \left(2x_j (x_j w_i - y_i)\right)$$

$$\therefore \frac{\partial}{\partial w}\left(\|Xw - y\|_2^2\right) = \begin{bmatrix} \Sigma_j (2x_j (x_j w_1 - y_j)) \\ \Sigma_j (2x_j (x_j w_2 - y_j)) \\ \vdots \\ \Sigma_j (2x_j (x_j w_n - y_j)) \end{bmatrix}_{n \times 1}$$

$$\therefore \frac{\partial f(w)}{\partial w} = \begin{bmatrix} \Sigma_j \left(2x_j \left(x_j w_1 - y_j\right)\right) \\ \Sigma_j \left(2x_j \left(x_j w_2 - y_j\right)\right) \\ \vdots \\ \Sigma_j \left(2x_j \left(x_j w_d - y_i\right)\right) \end{bmatrix} + \begin{bmatrix} 2w_1 \\ 2w_2 \\ \vdots \\ 2w_d \end{bmatrix}$$

$\rightarrow$ Both the first and the second term of $f(w)$ will $d \times 1$ vectors.