MA-105 Tutorial-4

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1. **Find the volume of the solid obtained by revolving** $g(x) = \frac{\tan^2 x}{x}$**,** $g(0) = 0$**, for** $0 \leq x \leq \frac{\pi}{4}$**.**

2. **Prove that given** $m, n \in \mathbb{N}$**,** $m \neq n$**:**

$$\int_{-1}^{1} D^m(x^2-1)^m D^n(x^2-1)^n dx = 0$$

3. **Let** $f : \mathbb{I} \to \mathbb{R}$ **be a convex function, where** $\mathbb{I}$ **is an open interval. Can** f **have a local maximum? Can it have a local minimum? Can it have two distinct local minima?**

4. **Let** $f : [0, \infty) \to \mathbb{R}$**. We say** $\lim_{x\to\infty} f(x) = l$ **if given** $E > 0$**,** $\exists x_0 \in \mathbb{R}$ **such that** $f(x) \in (l - E, l + E)\ \forall\ x > x_0$**.**

   Show that $\lim_{x\to\infty} e^{-x^2} = 0$**,** $\lim_{x\to\infty} \frac{1}{1+x^2} = 0$

   and $\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}$**. Also calculate the value of** $\int_0^\infty \frac{dx}{1+x^2}$