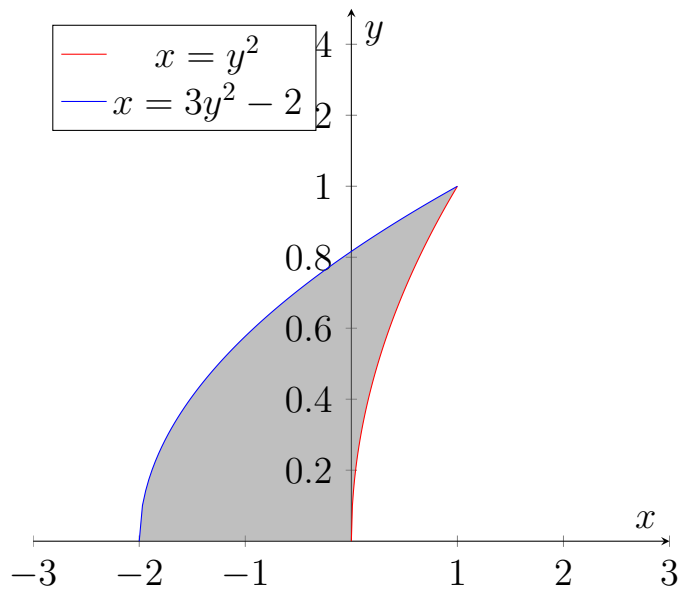


# MA-105 Tutorial-5

Daksh Maahor

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1. Find the volume of the solid obtained by revolving the given shaded region about the x-axis.



2. Prove that if  $f : [a, b] \rightarrow \mathbb{R}$  is continuous  $\exists c \in [a, b]$  such that

$$\int_a^b f(x)dx = (b - a)f(c)$$

Further deduce that  $\frac{d}{dx} \int_a^x f(t)dt = f(x)$

3. Prove that the set  $\left\{ (x, y) \mid \frac{x^2}{44} - \frac{y^2}{37} < 1 \right\}$  is an open set.
4. Is the set  $\left\{ (x, y) \mid \frac{x^2}{4} + \frac{y^2}{9} < 1 \right\}$  convex?
5. Prove that a polynomial in two variables is a continuous function.
6. Prove that if  $f(x, y)$  is a continuous real valued function  $\mathbb{R}^2 \rightarrow \mathbb{R}$  then the level set  $\{(x, y) \in \mathbb{R}^2 \mid f(x, y) = c\}$  is a closed set.
7. Sketch the level sets of  $f(x, y, z) = x^2 + y^2 - z^2$ .