MA-105 Tutorial-2

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1. Let
$$x_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$$
(x_n) is monotonically increasing but not bounded above and hence cannot converge.

   Show that $x_4 > 2$, $x_8 > \frac{5}{2}$ and $x_{16} > 3$

2. Let $a > 0$ and
$$x_n = 1 + a + \frac{a^2}{2} + \cdots + \frac{a^n}{n!}$$
Show that x_n is monotonically increasing and bounded above

3. Is the function $f(x) = \frac{\log(x+1)}{\sin(x)}$ defined on $\left(-\frac{1}{2}, \frac{1}{2}\right)$ differentiable at the origin, given $f(0) = 1$? Calculate $f'(0)$.

4. Compute $D^n f(1)$ when $f(x) = \frac{(x^2-1)^n}{2^n n!}$.

5. Prove that if f, g are two n-times differentiable functions at p then
$$(D^n fg)(p) = \sum_{k=0}^{n} \binom{n}{k} D^k f(p) D^{n-k} g(p)$$

6. Prove that $D^n \sin x = \sin\left(x + \frac{n\pi}{2}\right)$

7. Find the n^{th} derivative of $\sin^{-1} x$ at $x = 0$.

   To begin, set $y = \sin^{-1} x$, then $y''(1 - x^2) - xy' = 0$.

8. Suppose given that $y(x)$ is infinitely differentiable and satisfies

   $$(1 - x^2)y'' - 2xy' + p(p+1)y = 0$$

   Prove that if $y(1) = 0$, then $D^n y(1) = 0 \ \forall n \in \mathbb{N}$.