CS 105: Department Introductory Course on Discrete Structures

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Lecture 02 – Propositions, Predicates, Boolean Algebra

Recap: Logistics

Course hours: Slot 10;
Tue 14:00-15:25, Thu 14:00-15:25
Office hours: To be announced.
Problem Solving/Help Session (Optional): One hour per week, run by teaching assistants. (Time and Venue to be decided)

Course material, references will be posted at

- `http://www.cse.iitb.ac.in/~akshayss/teaching.html`
- Announcements, Problem sheets and Online Discussion: Piazza
  - `https://piazza.com/iit_bombay/fall2025/cs105`
- One problem sheet will be posted (almost) every week!

Attendance
As per Institute rules: SAFE

Recap: More Logistics

Evaluation

- Quizzes: 30%
- Midsem: 25%
- Endsem: 40%
- Other {participation, pop quizzes, assignments}: 5%

Minimum requirements (Tentative)

10/40 in endsem + 15/60 in remaining.

How to reach me after class?

- Send a message on piazza
- Drop by my office...
  - CS 507 (5th floor of New CSE/CC building)
  - Temporarily CC 313 (3rd floor!)

Course Outline

What we will cover in this course

1. Mathematical reasoning: proofs and structures
2. Counting and combinatorics
3. Elements of graph theory

Textbooks

- Discrete Mathematics and its Applications with Combinatorics and Graph Theory, by Kenneth H Rosen.
- Introduction to Graph theory by Douglas B West.
- Discrete Mathematics by Norman Biggs.
- More will be listed on webpage as we go along.

Chapter 1: Proofs and Logical reasoning

Outline of next few classes

- Propositions, statements
- What/why of proofs and some generic proof strategies
- Mathematical induction

Propositional calculus and Boolean algebra

A proposition is a statement that is either true or false (but not both).

- It is raining

Combining propositions

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \rightarrow q$: If it is raining then it will be wet.
- $p \leftrightarrow q$: p if and only if q (also written p iff q)
- p, q are called atomic propositions (basic propositions that can't be further divided) and $\vee, \neg, \wedge, \rightarrow, \leftrightarrow$ are Boolean connectives.
- Ex: You can go for excursion only if you are in CSE and if you aren't a freshie. $e \rightarrow (c \wedge \neg f)$.

Truth Tables

p	$\neg p$
T	F
F	T

p	q	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$p \leftrightarrow q$
F	F	F	F	T	T
F	T	T	F	T	F
T	F	T	F	F	F
T	T	T	T	T	T

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \rightarrow q$: If I am elected then I will lower taxes.

Logical Equivalence: Truth tables are identical!

- $p \rightarrow q$ is "same as" or logically equivalent to $\neg p \vee q$
- $p \leftrightarrow q$ is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$
- You will pass this course iff you score $\geq 10/40$ in endsem and $\geq 15/60$ in rest.

Some more exercises

Precedence of Operators

$\neg$ then $\wedge$ then $\vee$ then $\rightarrow$ then $\leftrightarrow$

Exercise: Construct the truth table of the compound proposition

$(p \vee \neg q) \rightarrow (p \wedge q)$

Exercise: Show that the following are equivalent

1. De-Morgan's Laws
   - 1.1 $\neg(p \wedge q) \equiv \neg p \vee \neg q$
   - 1.2 $\neg(p \vee q) \equiv \neg p \wedge \neg q$
2. $\neg\neg p \equiv p$ (Double negation)
3. $\neg(p \rightarrow q) \equiv p \wedge \neg q$

So idea: just use previous proved equivalences! When in doubt, fall back to truth tables.

Recap: Negation, Converse and Contrapositive

Consider the proposition: If it rains today, the match is cancelled. $(p \to q)$

1. What is its converse? $(q \to p)$
2. What is its contra-positive? $(\neg q \to \neg p)$
3. What is its negation? $\neg(p \to q)$

Which of these are equivalent to the original proposition? Why?

Exercise

Consider the proposition: If I will eat samosa or bhel puri, then I will not eat rice.

1. Write it as a Boolean combination of atomic propositions.
2. Write its converse, negation and contra-positive, both in plain english and formally.

Predicates and quantifiers

Can all mathematical statements be written as above?

Consider again...

- $\forall n \in \mathbb{N}\ (n+1)(n-1) = (n^2-1)$
- $\forall x, \exists y, x, y \in \mathbb{Z}\ x = y + 8$

- $\forall n$ stands for all values of n in a given domain
- $\exists n$ stands for exists n
- $\in$ is the element of symbol
- $\mathbb{N}$ stands for all natural numbers
- $\mathbb{Z}$ stands for all integers
- $\mathbb{R}$, $\mathbb{Q}$, ...

Some propositions are not so easy to "determine"...
– e.g., $2^{67} - 1$ is not a prime.

A bit more about quantifiers

Negating Quantifiers

- What is the negation of "All students have MA105"?

  There exists a student who does not have MA105.
- $\neg\forall x P(x) \equiv \exists x \neg P(x)$ [1]
  - $\neg\forall x P(x)$ is true iff $\forall x P(x)$ is false
  - $\forall x P(x)$ is false iff there is some x s.t. $\neg P(x)$ is true.
  - i.e., $\exists x \neg P(x)$.
- $\neg\exists x P(x) \equiv \forall x \neg P(x)$

Exercises

1. What is the negation of $\forall x(x^2 > x)$? $\exists x(x^2 \leq x)$
2. Show that $\neg\forall x(P(x) \to Q(x)) \equiv \exists x(P(x) \wedge \neg Q(x))$.
3. Use predicates and quantifiers to say "Every email larger than 10MB will be compressed." $\forall M(S(M, 10) \to C(M))$

[1] $\equiv$ stands for "is logically equivalent to".