Reference Book: Discrete Mathematical Structures with Applications to Computer Science - Tremblay and Manohar

Proposition Calculus-Introduction

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Example

It rained yesterday.

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Example

-3 is a natural number.

Definition

Two propositions p and q are said to be **Logically Equivalent** if when p is T, q is also T and when p is F, q is also F and conversely.

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q: He'll be 60 years old in 1994

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p: He was born in 1934
q: He'll be 60 years old in 1994
p and q are equivalent

Example

p: x is a prime number
q: x is not divisible by 2
p and q are not equivalent, as x not divisible by 2 does'nt mean its prime

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Let p be a proposition, we define **Negation** of p denoted by $\neg \mathbf{p}$ or $\sim \mathbf{p}$ to be a proposition which is true when p is false and is false when p is true.

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$\sim$	p	$\sim p$
	T	F
	F	T

Definition

Let p and q be two propositions. The **Disjunction** of two propositions is denoted by $\mathbf{p} \vee \mathbf{q}$(**read as p or q**)

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$\vee$	p	q	$p \vee q$
	T	T	T
	T	F	T
	F	T	T
	F	F	F

Definition

Let p and q be two propositions. The **Conjunction** of two propositions is denoted by $\mathbf{p} \wedge \mathbf{q}$(**read as p and q**)

$\wedge$	p	q	$p \wedge q$
	T	T	T
	T	F	F
	F	T	F
	F	F	F

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$\to$	p	q	$p \to q$
	T	T	T
	T	F	F
	F	T	T
	F	F	T

Definition

The **Conditional** statement is denoted by $\mathbf{p} \rightarrow \mathbf{q}$(**read as if p then q**)

$\rightarrow$	p	q	$p \rightarrow q$
	T	T	T
	T	F	F
	F	T	T
	F	F	T

Check whether $p \to q$ is logically equivalent to $\sim p \vee q$?
Show that $(p \to q) \to (\sim p \vee q)$ is a tautology.

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Show that $(p \to q) \to (\sim p \vee q)$ is a tautology.

Note: For the conditional $p \to q$,

(i) $q \to p$ is called "converse"

p	q	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$	$\neg p \vee q$
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Note: For the conditional $p \to q$,

(i) $q \to p$ is called "converse"

(ii) $\neg p \to \neg q$ is called " inverse"

p	q	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$	$\neg p \vee q$
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Note: For the conditional $p \to q$,

(i) $q \to p$ is called "converse"

(ii) $\neg p \to \neg q$ is called " inverse"

(iii) $\neg q \to \neg p$ is called "contrapositive"

p	q	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$	$\neg p \vee q$
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

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p	q	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$	$\neg p \vee q$
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

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p	q	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$	$\neg p \vee q$
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Note:

(i) $p \to q$ and $\neg q \to \neg p$ are logically equivalent

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p	q	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$	$\neg p \vee q$
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Note:

(i) $p \to q$ and $\neg q \to \neg p$ are logically equivalent

(ii) Inverse and Converse are logically equivalent. i.e., $q \to p$ and $\neg p \to \neg q$ are logically equivalent

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(i) $q \to p$ is called "converse"

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p	q	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$	$\neg p \vee q$
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
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Note:

(i) $p \to q$ and $\neg q \to \neg p$ are logically equivalent

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(iii) $p \to q$ and $\neg p \vee q$ are logically equivalent

Problem 1

Question: There are two restaurants next to each other. One has a sign that says " Good food is not cheap". The other has a sign that says " Cheap food is not good". Are the signs saying the same thing?

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Solution : Let A: Food is good

B: Food is cheap

We have to examine $A \rightarrow \neg B$ and $B \rightarrow \neg A$

A	B	$A \rightarrow \neg B$	$B \rightarrow \neg A$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

Inference: Both are saying the same thing.

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$\leftrightarrow$	p	q	$p \leftrightarrow q$
	T	T	T
	T	F	F
	F	T	F
	F	F	T

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$\leftrightarrow$	p	q	$p \leftrightarrow q$
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A proposition having one or more logical connectivities is called a **Compound Proposition**. Otherwise is called **Simple**

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iii) If A and B are WFF's, $A \vee B$, $A \wedge B$, $A \rightarrow B$, $A \leftrightarrow B$ are also WFFs.

iv) A string of symbols consisting of statement variables, connectivities and parenthesis is a WFF iff it can be obtained by finite many applications of the rules (i), (ii) and (iii)

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i) A statement variable is a WFF
ii) If A is WFF, then $\neg A$ is also a WFF
iii) If A and B are WFF's, $A \vee B$, $A \wedge B$, $A \rightarrow B$, $A \leftrightarrow B$ are also WFFs.
iv) A string of symbols consisting of statement variables, connectivities and parenthesis is a WFF iff it can be obtained by finite many applications of the rules (i), (ii) and (iii)

Example

(1) $p \wedge q$, $\neg(p \wedge q)$, $(\neg(p \rightarrow q)) \vee r$, $((p \rightarrow q) \rightarrow r$ are WFFs.
(2) $p \wedge q \rightarrow r$ is not a WFF as it can be $(p \wedge q) \rightarrow r$ or $p \wedge (q \rightarrow r)$

Equivalence of formulas

Definition

Let A and B be two statement formulas and $P_1, P_2, \cdots P_n$ denote all the variables occurring in A and B. If the truth value of A is same as that of B for each of 2^n possible set of assignments to the variables $P_1, P_2, \cdots P_n$, then A and B are said to be equivalent.

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Two statement formulas A and B are equivalent iff $A \leftrightarrow B$ is a Tautology.

Table of equivalence

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(1) $\neg\neg p \Leftrightarrow p$

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(2) Commutative: (a) $p \vee q \Leftrightarrow q \vee p$
(b) $p \wedge q \Leftrightarrow q \wedge p$

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(2) Commutative: (a) $p \vee q \Leftrightarrow q \vee p$
(b) $p \wedge q \Leftrightarrow q \wedge p$

(3) Associative: (a) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$
(b) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

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(b) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

(4) Distributive: (a) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
(b) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

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(b) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

(5) Absorption: (a) $p \vee (p \wedge q) \Leftrightarrow p$
(b) $p \wedge (p \vee q) \Leftrightarrow p$

Table of equivalence

(1) $\neg\neg p \Leftrightarrow p$

(2) Commutative: (a) $p \vee q \Leftrightarrow q \vee p$
(b) $p \wedge q \Leftrightarrow q \wedge p$

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(b) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

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(b) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

(5) Absorption: (a) $p \vee (p \wedge q) \Leftrightarrow p$
(b) $p \wedge (p \vee q) \Leftrightarrow p$

(6) Idempotent: (a) $(p \wedge p) \Leftrightarrow p$
(b) $(p \vee p) \Leftrightarrow p$

Table of equivalence

(7) (a) $p \wedge (\neg p) \Leftrightarrow F$
(b) $p \vee (\neg p) \Leftrightarrow T$

Table of equivalence

(7) (a) $p \wedge (\neg p) \Leftrightarrow F$
(b) $p \vee (\neg p) \Leftrightarrow T$

(8) (a) $p \vee F \Leftrightarrow p$
(b) $p \wedge F \Leftrightarrow F$

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(8) (a) $p \vee F \Leftrightarrow p$
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(9) (a) $p \vee T \Leftrightarrow T$
(b) $p \wedge T \Leftrightarrow p$

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(8) (a) $p \vee F \Leftrightarrow p$
    (b) $p \wedge F \Leftrightarrow F$

(9) (a) $p \vee T \Leftrightarrow T$
    (b) $p \wedge T \Leftrightarrow p$

(10) (a) $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$
     (b) $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

Table of equivalence

(7) (a) $p \wedge (\neg p) \Leftrightarrow F$
(b) $p \vee (\neg p) \Leftrightarrow T$

(8) (a) $p \vee F \Leftrightarrow p$
(b) $p \wedge F \Leftrightarrow F$

(9) (a) $p \vee T \Leftrightarrow T$
(b) $p \wedge T \Leftrightarrow p$

(10) (a) $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$
(b) $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

(11) (a) $p \rightarrow q \Leftrightarrow \neg p \vee q$
(b) $\neg(p \rightarrow q) \Leftrightarrow p \wedge \neg q$

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(7) (a) $p \wedge (\neg p) \Leftrightarrow F$
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(8) (a) $p \vee F \Leftrightarrow p$
(b) $p \wedge F \Leftrightarrow F$

(9) (a) $p \vee T \Leftrightarrow T$
(b) $p \wedge T \Leftrightarrow p$

(10) (a) $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$
(b) $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

(11) (a) $p \to q \Leftrightarrow \neg p \vee q$
(b) $\neg(p \to q) \Leftrightarrow p \wedge \neg q$

(12) (a) $p \to q \Leftrightarrow (\neg q \to \neg p)$
(b) $q \to p \Leftrightarrow (\neg p \to \neg q)$

Problem 2

Question: Show that $p \to (q \to r) \Leftrightarrow p \to (\neg q \vee r) \Leftrightarrow (p \wedge q) \to r$

Problem 3

Question: Show that $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

Problem 3

Question: Show that $(\neg p \land (\neg q \land r)) \lor (q \land r) \lor (p \land r) \Leftrightarrow r$

$$(\neg p \land (\neg q \land r)) \lor (q \land r) \lor (p \land r) \quad \Leftrightarrow$$

Problem 3

Question: Show that $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

$$(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \quad \Leftrightarrow \quad (\neg p \wedge (\neg q \wedge r)) \vee$$

Problem 3

Question: Show that $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

$$\begin{aligned}(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) &\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee (r \wedge (q \vee p)) \\ &\Leftrightarrow\end{aligned}$$

Question: Show that $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

$$
\begin{aligned}
(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) &\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee (r \wedge (q \vee p)) \\
&\Leftrightarrow ((\neg p \wedge \neg q) \wedge r) \vee (r \wedge (p \vee q)) \\
&\Leftrightarrow
\end{aligned}
$$

Problem 3

Question: Show that $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

$$\begin{aligned}
(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) &\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee (r \wedge (q \vee p)) \\
&\Leftrightarrow ((\neg p \wedge \neg q) \wedge r) \vee (r \wedge (p \vee q)) \\
&\Leftrightarrow (\neg (p \vee q) \wedge r) \vee (r \wedge (p \vee q)) \\
&\Leftrightarrow
\end{aligned}$$

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$$\begin{aligned}
(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) &\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee (r \wedge (q \vee p)) \\
&\Leftrightarrow ((\neg p \wedge \neg q) \wedge r) \vee (r \wedge (p \vee q)) \\
&\Leftrightarrow (\neg(p \vee q) \wedge r) \vee (r \wedge (p \vee q)) \\
&\Leftrightarrow r \wedge [(p \vee q) \vee \neg(p \vee q)] \\
&\Leftrightarrow
\end{aligned}$$

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$$
\begin{aligned}
(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) &\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee (r \wedge (q \vee p)) \\
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&\Leftrightarrow (\neg (p \vee q) \wedge r) \vee (r \wedge (p \vee q)) \\
&\Leftrightarrow r \wedge [(p \vee q) \vee \neg (p \vee q)] \\
&\Leftrightarrow r \wedge T \\
&\Leftrightarrow
\end{aligned}
$$

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(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) &\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee (r \wedge (q \vee p)) \\
&\Leftrightarrow ((\neg p \wedge \neg q) \wedge r) \vee (r \wedge (p \vee q)) \\
&\Leftrightarrow (\neg (p \vee q) \wedge r) \vee (r \wedge (p \vee q)) \\
&\Leftrightarrow r \wedge [(p \vee q) \vee \neg (p \vee q)] \\
&\Leftrightarrow r \wedge T \\
&\Leftrightarrow r
\end{aligned}
$$

Problem 4

Question: Show that
$((p \vee q) \wedge \neg[(\neg p) \wedge (\neg q \vee \neg r)]) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$ **is a tautology**.

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$\Leftrightarrow T$

Tautological Implications

Definition

A is said to tautologically imply to statement B if $A \to B$ is a tautology. In this case, we write $A \Rightarrow B$ (read as A implies B)

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Problem 5

Question: Show that $\neg(p \to q) \implies \neg q$

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p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p \to q) \to \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

Problem 6

Question: Show that $\neg q \wedge (p \to q) \implies \neg p$

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Solution: Suppose $\neg q \wedge (p \to q)$ is true.

Question: Show that $\neg q \land (p \to q) \implies \neg p$

Solution: Suppose $\neg q \land (p \to q)$ is true.
$\neg q$ is true and $p \to q$ is true

Problem 6

Question: Show that $\neg q \wedge (p \rightarrow q) \implies \neg p$

Solution: Suppose $\neg q \wedge (p \rightarrow q)$ is true.
$\neg q$ is true and $p \rightarrow q$ is true
q is false and $p \rightarrow q$ is true

Problem 6

Question: Show that $\neg q \wedge (p \to q) \implies \neg p$

Solution: Suppose $\neg q \wedge (p \to q)$ is true.
$\neg q$ is true and $p \to q$ is true
q is false and $p \to q$ is true
$\implies p$ is false

Problem 6

Question: Show that $\neg q \wedge (p \rightarrow q) \implies \neg p$

Solution: Suppose $\neg q \wedge (p \rightarrow q)$ is true.
$\neg q$ is true and $p \rightarrow q$ is true
q is false and $p \rightarrow q$ is true
$\implies p$ is false
$\implies \neg p$ is true

Problem 6

Question: Show that $\neg q \wedge (p \to q) \implies \neg p$

Solution: Suppose $\neg q \wedge (p \to q)$ is true.
$\neg q$ is true and $p \to q$ is true
q is false and $p \to q$ is true
$\implies$ p is false
$\implies$ $\neg p$ is true
$\therefore \neg q \wedge (p \to q) \implies \neg p$

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Question: Show that $\neg q \wedge (p \to q) \implies \neg p$

Solution: Suppose $\neg q \wedge (p \to q)$ is true.
$\neg q$ is true and $p \to q$ is true
q is false and $p \to q$ is true
$\implies p$ is false
$\implies \neg p$ is true
$\therefore \neg q \wedge (p \to q) \implies \neg p$

Table of Tautological Implications

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(1) $p \wedge q \implies p$

$p \wedge q \implies q$

Table of Tautological Implications

(1) $$\begin{aligned} p \wedge q &\implies p \\ p \wedge q &\implies q \end{aligned}$$

(2) $$\begin{aligned} p &\implies p \vee q \\ q &\implies p \vee q \end{aligned}$$

Table of Tautological Implications

(1) $p \wedge q \implies p$

$p \wedge q \implies q$

(2) $p \implies p \vee q$

$q \implies p \vee q$

(3) $\neg p \implies p \to q$

Table of Tautological Implications

(1) $p \wedge q \implies p$

$p \wedge q \implies q$

(2) $p \implies p \vee q$

$q \implies p \vee q$

(3) $\neg p \implies p \to q$

(4) $q \implies p \to q$

Table of Tautological Implications

(1) $p \land q \implies p$

$p \land q \implies q$

(2) $p \implies p \lor q$

$q \implies p \lor q$

(3) $\neg p \implies p \to q$

(4) $q \implies p \to q$

(5) $\neg(p \to q) \implies p$

Table of Tautological Implications

(1) $p \wedge q \implies p$

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$q \implies p \vee q$

(3) $\neg p \implies p \to q$

(4) $q \implies p \to q$

(5) $\neg(p \to q) \implies p$

(6) $\neg(p \to q) \implies \neg q$

Table of Tautological Implications

(7) $p \wedge (p \to q) \implies q$

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(8) $\neg q \wedge (p \to q) \implies \neg p$

(9) $\neg p \wedge (p \vee q) \implies q$

Table of Tautological Implications

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\begin{aligned}
(7)\quad & p \wedge (p \to q) \implies q \\
(8)\quad & \neg q \wedge (p \to q) \implies \neg p \\
(9)\quad & \neg p \wedge (p \vee q) \implies q \\
(10)\quad & (p \to q) \wedge (q \to r) \implies p \to r
\end{aligned}
$$

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&(7)\quad p \wedge (p \to q) \implies q\\
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Problem 7

Question: Show that $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \implies r$

Rules of Inference

To demonstrate that a particular formula is valid consequence of a given set of premises, we use the follow rules of inference.

Rule P : *A premise may be introduced at any point in the derivation*

Rule T : *A formula S may be introduced in a derivation if S is tautologically implied by any one or more of the preceding formulas in the derivation*

Question : Demonstrate that r is a valid inference from the premises $p \to q,\ \ q \to r\ \ and\ \ p$

Solution :

$$p \to q \qquad (Rule\ P)$$

Problem 8

Question : Demonstrate that r is a valid inference from the premises $p \to q, \;\; q \to r \;\; and \;\; p$
Solution :

$$\begin{aligned} p \to q & \qquad (Rule\ P) \\ p & \qquad (Rule\ P) \end{aligned}$$

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Question : Demonstrate that r is a valid inference from the premises $p \to q, \quad q \to r \quad and \quad p$

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$$\begin{aligned} p \to q & \qquad (Rule\ P) \\ p & \qquad (Rule\ P) \\ q & \qquad (Rule\ T,\ p \wedge (p \to q) \implies q) \end{aligned}$$

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p \to q &\qquad (Rule\ P) \\
p &\qquad (Rule\ P) \\
q &\qquad (Rule\ T,\ p \wedge (p \to q) \implies q) \\
q \to r &\qquad (Rule\ P)
\end{aligned}$$

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p & \qquad (Rule\ P) \\
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q \rightarrow r & \qquad (Rule\ P) \\
r & \qquad (Rule\ T, q \wedge (q \rightarrow r) \implies r)
\end{aligned}$$

Problem 9

Question : RVS follows logically from the premises $C \wedge D$, $C \vee D \to \neg H$, $\neg H \to (A \wedge \neg B)$, $(A \wedge \neg B) \to (R \vee S)$.
Solution :

Problem 9

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Solution :

$$C \vee D \rightarrow \neg H \qquad (Rule\ P)$$

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Solution :

$$
\begin{aligned}
C \vee D \to \neg H & \qquad (Rule\ P) \\
\neg H \to A \wedge \neg B & \qquad (Rule\ P)
\end{aligned}
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C \vee D \to A \wedge \neg B & \qquad (Rule\ T) \\
A \wedge \neg B \to R \vee S & \qquad (Rule\ P) \\
C \vee D \to R \vee S & \qquad (Rule\ T)
\end{aligned}$$

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C \vee D \to \neg H &\qquad (Rule\ P)\\
\neg H \to A \wedge \neg B &\qquad (Rule\ P)\\
C \vee D \to A \wedge \neg B &\qquad (Rule\ T)\\
A \wedge \neg B \to R \vee S &\qquad (Rule\ P)\\
C \vee D \to R \vee S &\qquad (Rule\ T)\\
C \vee D &\qquad (Rule\ P)\\
R \vee S &\qquad (Rule\ T)
\end{aligned}$$

Problem 10

Question : Show that $S \vee r$ is tautologically implied by $(p \vee q) \wedge (p \to r) \wedge (q \to s)$

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Solution :

$$\begin{aligned} p \vee q \qquad & (Rule\ P) \\ \neg p \to q \qquad & (Rule\ T\ i.e.,\ p \to q \Leftrightarrow \neg p \vee q) \end{aligned}$$

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Solution :

$$
\begin{array}{rl}
p \vee q & (Rule\ P) \\
\neg p \to q & (Rule\ T\ i.e.,\ p \to q \Leftrightarrow \neg p \vee q) \\
q \to S & (Rule\ P) \\
\neg p \to S & (Rule\ T) \\
\neg S \to p & (Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p)
\end{array}
$$

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Solution :

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Question : Show that $S \vee r$ is tautologically implied by $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow s)$
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p \vee q & (Rule\ P) \\
\neg p \rightarrow q & (Rule\ T\ i.e.,\ p \rightarrow q \Leftrightarrow \neg p \vee q) \\
q \rightarrow S & (Rule\ P) \\
\neg p \rightarrow S & (Rule\ T) \\
\neg S \rightarrow p & (Rule\ T, p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p) \\
p \rightarrow r & (Rule\ P) \\
\neg S \rightarrow r & (Rule\ T)
\end{array}
$$

Problem 10

Question : Show that $S \vee r$ is tautologically implied by $(p \vee q) \wedge (p \to r) \wedge (q \to s)$
Solution :

$$\begin{array}{rl}
p \vee q & (Rule\ P) \\
\neg p \to q & (Rule\ T\ i.e.,\ p \to q \Leftrightarrow \neg p \vee q) \\
q \to S & (Rule\ P) \\
\neg p \to S & (Rule\ T) \\
\neg S \to p & (Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p) \\
p \to r & (Rule\ P) \\
\neg S \to r & (Rule\ T) \\
S \vee r & (Rule\ T, p \to q \Leftrightarrow \neg(\neg p \vee q))
\end{array}$$

Problem 11

Question : If A works hard, then either B or C will enjoy themselves. If B enjoys himself, then A will not work hard. If D enjoys himself, then C will not, therefore prove that if A works hard, D will not enjoy himself.

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$$\begin{aligned} A &: \quad A \text{ works hard} \\ B &: \quad B \text{ will enjoy himself} \\ C &: \quad C \text{ will enjoy himself} \\ D &: \quad D \text{ will enjoy himself} \end{aligned}$$

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To prove: $A \to \neg D$ follows from $A \to B \vee C$, $B \to \neg A$ and $D \to \neg C$

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A \to B \vee C &\qquad (Rule\ P) \\
\neg(B \vee C) \to \neg A &\qquad (Rule\ T\ i.e.,\ p \to q \Leftrightarrow \neg q \to \neg p) \\
(\neg B \wedge \neg C) \to \neg A &\qquad (Rule\ T)
\end{aligned}
$$

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(\neg B \wedge \neg C) \to \neg A & (Rule\ T) \\
\neg B \to (\neg C \to \neg A) & (Rule\ T, (p \wedge q) \to r \Leftrightarrow (q \to r)
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\end{array}
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\neg B &\to (A \to C) && (Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p)\\
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\neg B \to (A \to C) & (Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p) \\
B \to \neg A & (Rule\ P) \\
A \to \neg B & (Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p) \\
A \to (A \to C) & (Rule\ T, p \to q, q \to r \Leftrightarrow p \to r)
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B \to \neg A & (Rule\ P) \\
A \to \neg B & (Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p) \\
A \to (A \to C) & (Rule\ T, p \to q, q \to r \Leftrightarrow p \to r) \\
(A \wedge A) \to C & (Rule\ T)
\end{array}
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A \to (A \to C) & (Rule\ T, p \to q, q \to r \Leftrightarrow p \to r) \\
(A \wedge A) \to C & (Rule\ T) \\
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A \to (A \to C) & (Rule\ T, p \to q, q \to r \Leftrightarrow p \to r) \\
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B \to \neg A & (Rule\ P) \\
A \to \neg B & (Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p) \\
A \to (A \to C) & (Rule\ T, p \to q, q \to r \Leftrightarrow p \to r) \\
(A \wedge A) \to C & (Rule\ T) \\
A \to C & (Rule\ P) \\
D \to \neg C & (Rule\ P) \\
C \to \neg D & (Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p)
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To prove, $A \to \neg D$ follows from $A \to B \vee C$, $B \to \neg A$ and $D \to \neg C$

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$\neg B \to (\neg C \to \neg A)$	$(Rule\ T, (p \wedge q) \to r \Leftrightarrow (q \to r)$
$\neg B \to (A \to C)$	$(Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p)$
$B \to \neg A$	$(Rule\ P)$
$A \to \neg B$	$(Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p)$
$A \to (A \to C)$	$(Rule\ T, p \to q, q \to r \Leftrightarrow p \to r)$
$(A \wedge A) \to C$	$(Rule\ T)$
$A \to C$	$(Rule\ P)$
$D \to \neg C$	$(Rule\ P)$
$C \to \neg D$	$(Rule\ T, p \to q \Leftrightarrow \neg q \to \neg p)$
$A \to \neg D$	$(Rule\ T, p \to q, q \to r \Leftrightarrow p \to r)$

Exercise : $R \wedge (p \vee q)$ is a valid conclusion from the premises $p \vee q$, $q \to r$, $p \to M$ and $\neg M$.