Relationship bw Modulus of Elasticity & Rigidity

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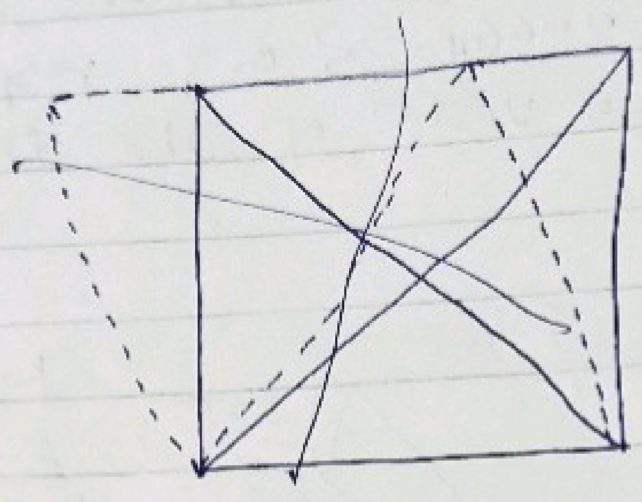
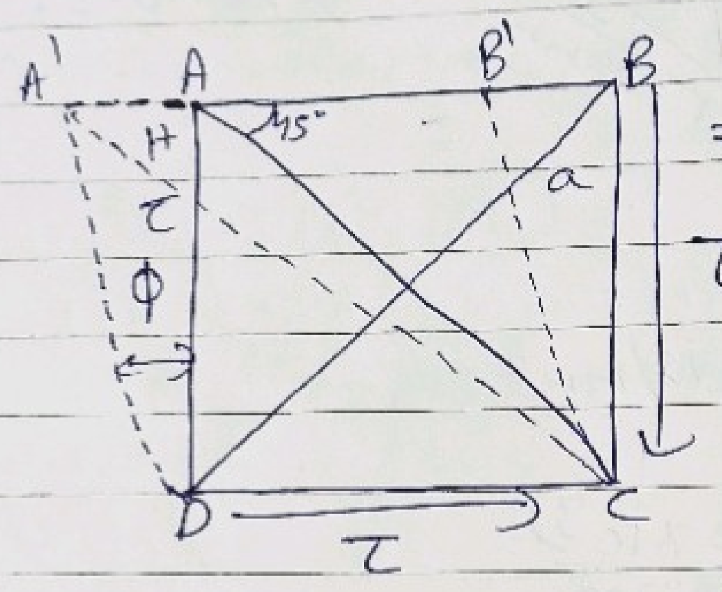
A. Relationship b/w modulus of elasticity (E) & modulus of rigidity (G).





→ Strain along AC

$= \frac{\tau}{E} - \mu\left(-\frac{\tau}{E}\right) = \frac{\tau}{E}(1+\mu)$ — ①

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→ Strain along AC

$= \frac{A'H}{AC} = \frac{A'H}{\sqrt{2}a}$ — ②

from Δ^{le} $AA'H$; $\cos 45° = \frac{A'H}{AA'}$

$\therefore A'H = \frac{AA'}{\sqrt{2}}$

A. ⇒ $\boxed{E = 2G(1+\mu)}$ from derivation

from Δ^{le} $AA'D$; $\tan\phi \simeq \phi = \frac{A'A}{AD} = \frac{AA'}{a}$

$\therefore AA' = a\phi$

eq^n (2) strain along $AC = \frac{AA'}{2a} = \frac{a\phi}{2a} = \frac{\phi}{2}$ — ③

$\rightarrow \frac{\tau}{E}(1+\mu) = \phi/2$

We know that, $G = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi} \Rightarrow \phi = \frac{\tau}{G}$

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$\rightarrow \frac{\tau}{E}(1+\mu) = \frac{\tau}{2G} \Rightarrow E = 2G(1+\mu)$.

⇒ Relationship b/w E, G & K.

$\rightarrow E = 2G(1+\mu)$ .. ①   & ③ ⇒ $E = \frac{9KG}{3K+G}$

$\rightarrow E = 3K(1-2\mu)$ ... ②.

$2G(1+\mu) = 3K(1-2\mu)$

~~2G(2μ~~ $2G + 2\mu G = 3K - 6K\mu$

$2\mu G + 6K\mu = 3K - 2G$

$\mu(2G + 6K) = 3K - 2G$

$\mu = \frac{3K-2G}{2G+6K}$ — ③

substitute ③ in ① $E = 2G\left(1 + \frac{3K-2G}{2G+6K}\right)$

$E = 2G\left(\frac{\cancel{2G} + 6K + 3K - \cancel{2G}}{2G+6K}\right)$

$= \frac{\cancel{2}G(9K)}{\cancel{2}(G+3K)} = \frac{9KG}{3K+G}$

$\Rightarrow E = \frac{9KG}{3K+G}$

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