Reg.No										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER B.E DEGREE END SEMESTER EXAMINATION

SUB: ENGG. MATHEMATICS I (MAT – 101)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All questions carry equal marks

1A. Find the n^{th} derivative of

i) $\dfrac{x^2}{(x-1)^2(x+2)}$ (ii) $\cos^2 x.\ \sin^3 x$

1B. Find the evolute of the curve $y^2 = 4ax$

1C. Find the distance of the point (1, –2, 3) from the plane measured parallel to the line $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{(-6)}$.

(4 + 3 + 3)

2A. If $\cos^{-1}\left(\dfrac{y}{b}\right) = \log_e\left(\dfrac{x}{n}\right)^n$, then prove $x^2 y_{n+2} + (2n+1)\, xy_{n+1} + 2n^2 y_n = 0$

2B. Find the radius of curvature at any point on the curve $x = a\ (\theta + \sin\theta),\quad y = a\ (1 + \cos\theta)$.

2C. Find the entire length of the cardioid $r = a(1 + \cos\theta)$

(4 + 3 + 3)

3A. Show that the tangents to the cardioide $r = a\ (1+\cos\theta)$ at the points $\theta = \left(\dfrac{\pi}{3}\right)$ and $\theta = \left(\dfrac{2\pi}{3}\right)$ are parallel and perpendicular to the initial line.

3B. Find the nature of the series $\sum \dfrac{(\lfloor n)^2}{\lfloor 2n} x^{2n}$.

3C. The period of simple pendulum is given by $T=2\pi\sqrt{l/g}$. If T is computed using $l = 80$cm and $g = 981\text{cm}/\sec^2$, find the approximate error in T, if the true values are 80.2 cm and $g = 981.4/\sec^2$. Also find percentage error.

(4 + 3+ 3)

P.T.O

4A. Sketch and find the area enclosed by the loop of the curve $3ay^2 = x(x-a)^2$.

4B. Find the radius of curvature at the point (-2a, 2a) on the curve $x^2y = a(x^2 + y^2)$

4C. (i) Define absolute and conditional convergence of an infinite series.

(ii) Prove that the series $\frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3}$..... converges absolutely.

(4 + 3+ 3)

5A. (i) Evaluate $\int_3^5 \sqrt{(x-3)^9(5-x)^8}\,dx$

(ii) Obtain the reduction formula for $\int(\cos^n x)dx$ & hence evaluate $\int_0^{\pi/2} \sin^n x\, dx$.

5B. (i) Prove that if $a_0, a_1, a_2, \ldots\ldots, a_n$ are real numbers such that $\frac{a_0}{n+1} + \frac{a_1}{n} + \ldots\ldots\ldots + \frac{a_{n-1}}{2} + a_n = 0$, then there exists at least one real number x between 0 and 1 such that $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n = 0$.

(ii) Verify Cauchy's mean value theorem for the functions $f(x) = \log_e x$, $F(x) = \frac{1}{x}$ in the interval [1, e].

5C. Trace with explanation : $a^2(y^2 - x^2) + x^4 = 0$.

(4 + 3+ 3)

6A. Find the equation of the right circular cone generated when the straight line $2y + 3z = 6$ and $x = 0$ revolves about z – axis.

6B. Obtain first three non-zero terms in the Maclaurin's series expansion of $f(x) = \tan x$.

6C. Find the value of

(i) $\lim_{x\to 1} \frac{x^x - x}{x - 1 - \log x}$

(ii) $\lim_{x\to 0} \frac{\log x}{\cot x}$ (4 + 3+ 3)

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1A. Find the n^{th} derivative of the following

(i) $\dfrac{x}{x^2+3x+2}$ (ii) $\sin x \cos 2x \sin 3x$.

1B. Show that the radius of curvature at any point on the catenary is proportional to the square of the ordinate at that point.

1C. A line makes an angle α, β, γ and δ with the diagonals of a cube.
Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$.

(4 + 3+ 3)

2A. If $v = r^m$ where $r^2 = x^2 + y^2 + z^2$ show that
$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=m(m+1)r^{m-2}.$$

2B. If ρ_1 and ρ_2 be the radii of curvature at the extremities of any chord through the pole of the cardioide $r = a(1 + \cos\theta)$, then prove that $\rho_1^2+\rho_2^2=\dfrac{16a^2}{9}$.

2C. Find the entire length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

(3 + 4+ 3)

3A. Prove that the evolute of the rectangular hyperbola $2xy = a^2$ is $(x+y)^{2/3}-(x-y)^{2/3}=2a^{2/3}$.

P.T.O

3B. A variable plane which lies at a constant distance p from the origin meets the axes at A, B, C. Through A, B, C planes are drawn parallel to coordinate planes. Show that the locus of their point of intersection is given by $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

3C. Find the nature of the series (i) $\sum_{1}^{\infty} \frac{2^n}{n!}$ (ii) $\sum_{2}^{\infty} \left(\frac{1}{n \log n}\right)$

(3 + 4 + 3)

4A. If u is a homogenous function of order n then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u$$

4B. Find the area enclosed by the curve $r = a(1 + \cos\theta)$.

4C. If $y = a \cos(\log x) + b \sin(\log x)$ then show that

$x^2(y_{n+2}) + (2n+1)x\, y_{n+1} + (n^2+1)y_n = 0$

(4 + 3 + 3)

5A. Obtain the formula for $\int_0^{\pi/2} \sin^m x \, dx, \quad m > 1$.

5B. Find the angle of intersection of the curves
$r^m = a^m \cos m\theta$ and $r^m = a^m \sin m\theta$

5C. Sketch and find the volume formed by revolution of the loop of the curve $y^2(a+x) = x^2(a-x)$ about the x – axis.

(4 + 3 + 3)

6A. Find the points on the lines $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ which are nearest to each other. Hence find the shortest distance between the lines and its equation.

6B. Find the equation of the right circular cone with vertex at the origin and axis along the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$, which has semi vertical angle of 30°.

6C. State and prove Cauchy's mean value theorem.

(4 + 3 + 3)

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