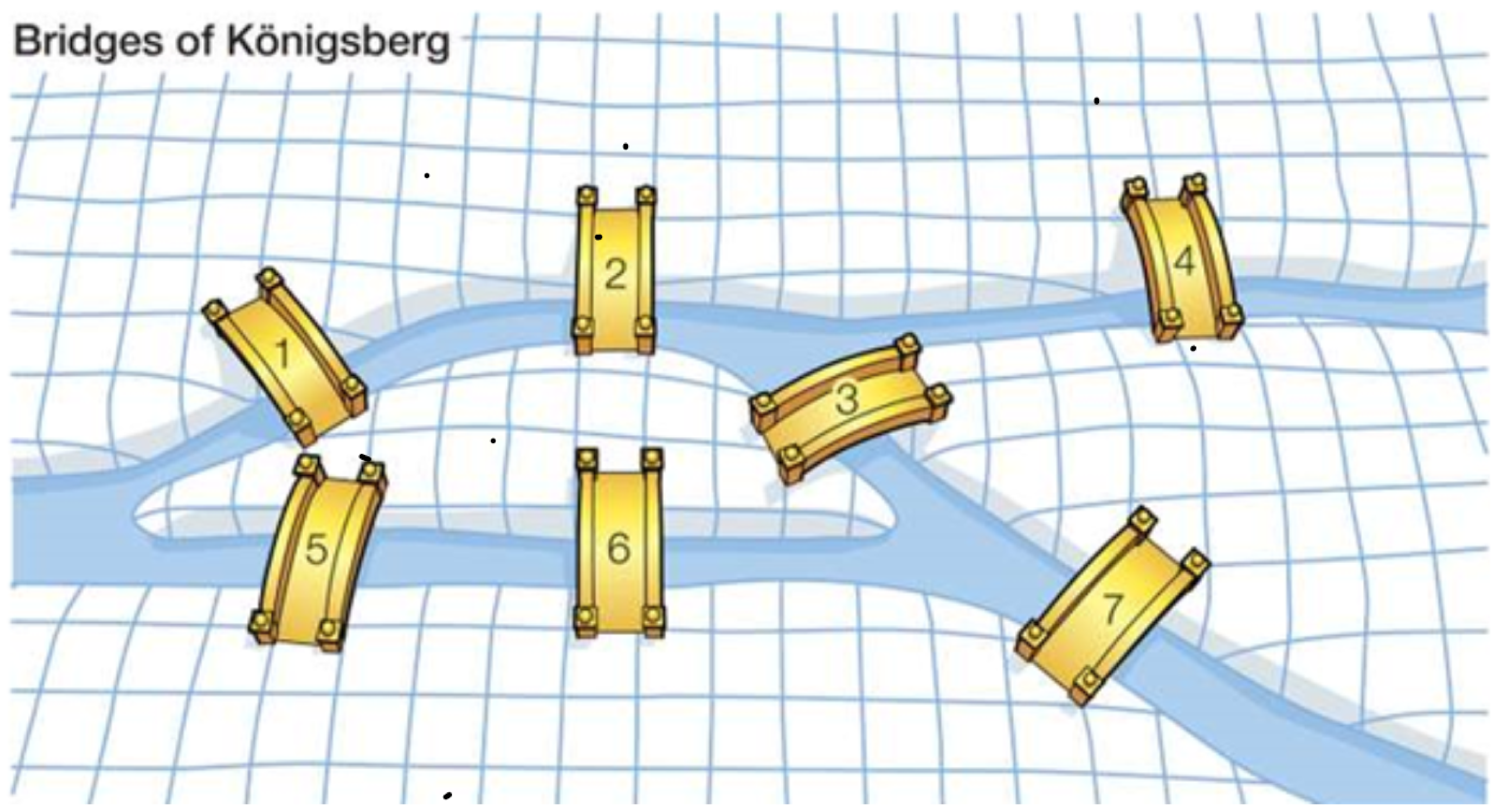
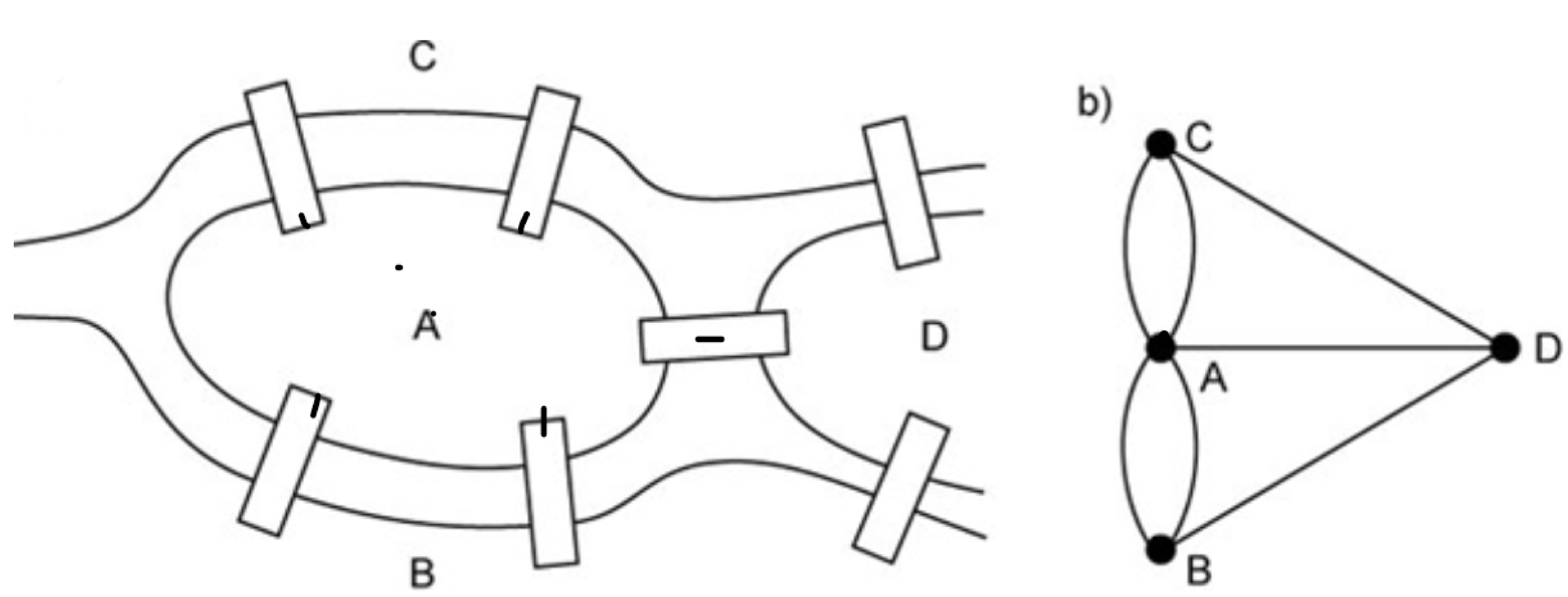
Chapter 3 : Graph Theory

Graph theory was originated from the Konigsberg Bridge Problem, where two islands linked to each other and to the banks of the Pregel River by seven bridges.



The problem was to begin at any of the four land areas, walk across each bridge exactly once and return to the initial vertex.

This problem was solved in 1736 by Euler.



A graph G consists of a finite nonempty set V = V(G) of p points (known as vertices) together with a prescribed set E(G) of q unordered pairs of distinct vertices of V whose elements are called the edges of graph G.

A graph with p vertices and q edges is called a (p, q) graph.

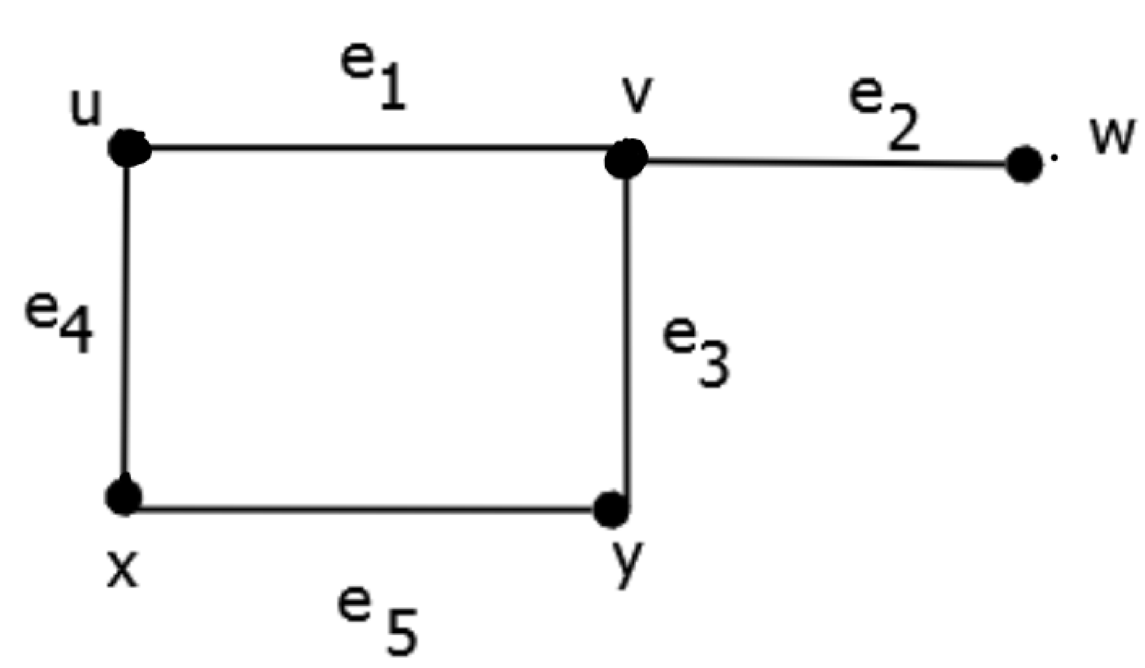
A graph with a single vertex and no edge is called trivial.

A graph is finite if both its vertex set and edge set are finite.

For a graph G, the number of elements in V(G) is called order of the graph G and denoted by |V(G)| and the number of elements in E(G) is called the size of the graph G denoted by |E(G)|.

An edge e is said to join u and v or e is incident with u and v if e = (u, v).
When e = uv = (u, v), we say that u and v are adjacent vertices, denoted by $u \sim v$.
If two distinct edges e and f are incident with a common vertex, then they are adjacent edges.

$e_1 = (u, v)$ $\quad e_5 = (x, y)$



$|V(G)| = 5$. $p = 5$

order = 5

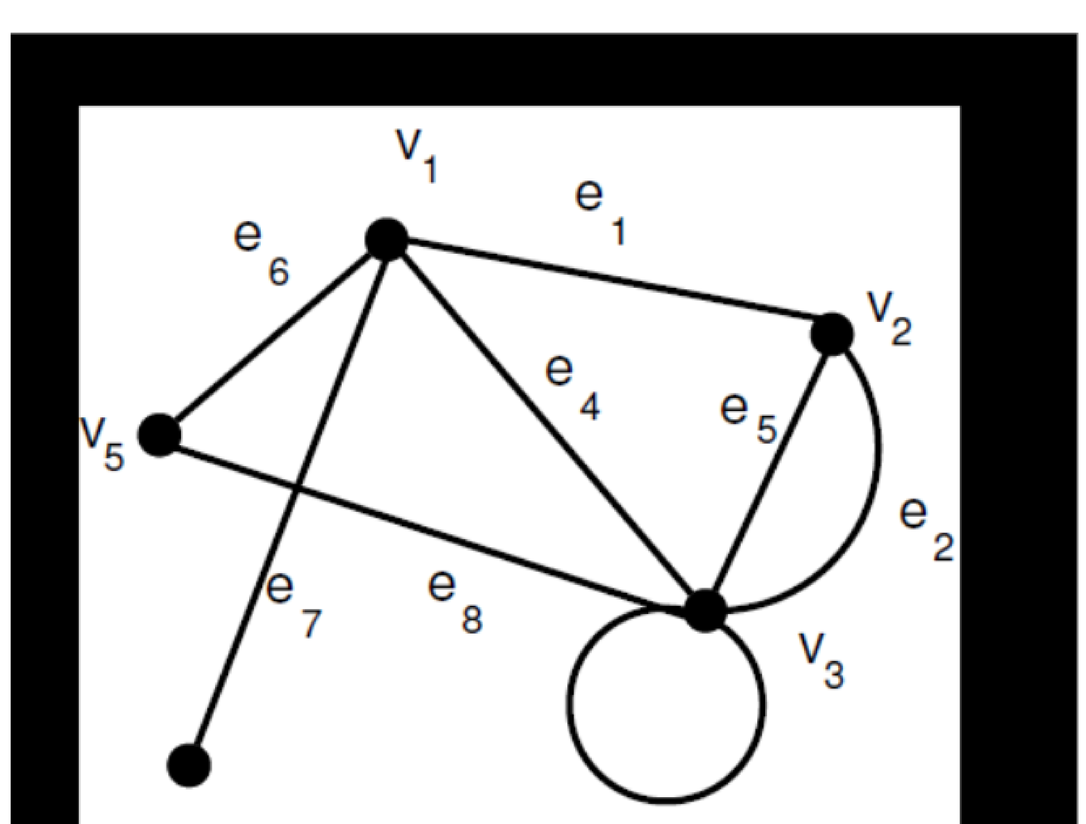
$|E(G)| = 5$, $\quad q = 5$, size = 5

e_5 is incident with x & y.

x and v are not adjacent, i.e., $x \sim v$.

e_5 and e_3 → adj edges

An edge with identical ends is called a loop and two edges with same end vertices are called parallel (multiple) edges.
A graph is simple if it has no loops or parallel edges.



pseudograph.

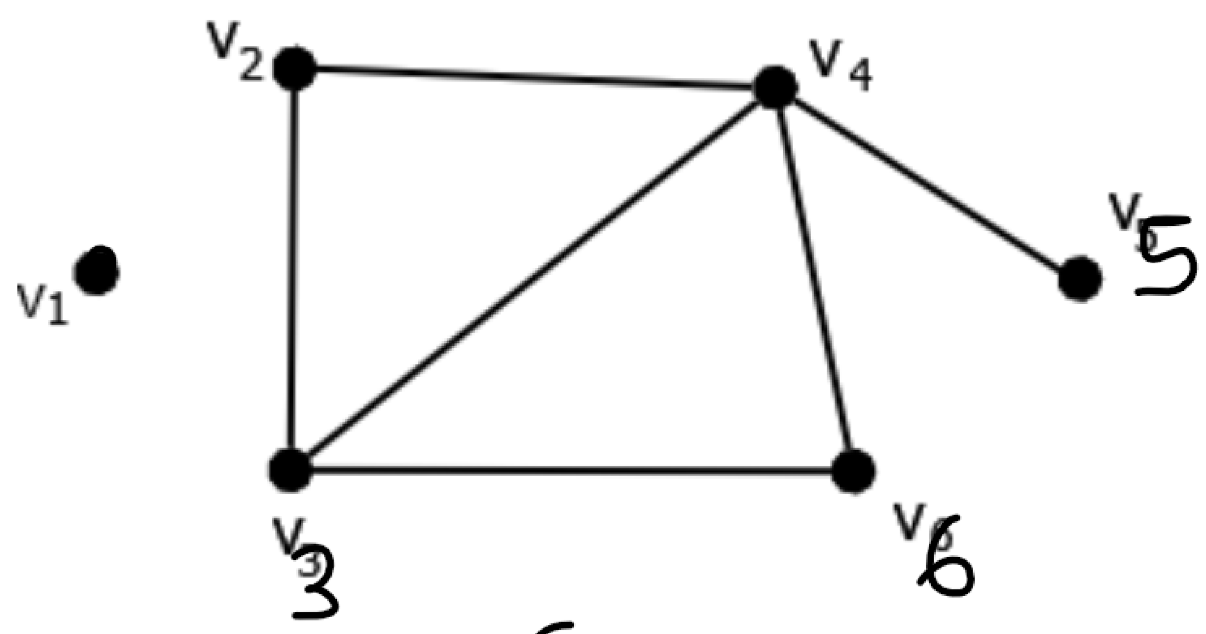
edge e_3 → loop

edge e_5 and edge e_2 → parallel edges

In a multigraph, no loops are allowed but parallel edges are permitted. If both loops and parallel edges are permitted then we have a pseudograph.

A graph is simple if it has no loops or parallel edges.

Degree of a vertex v in G, denoted by deg(v) is number of vertices adjacent to v.



G

$deg(v_1) = 0$ $\quad deg(v_5) = 1$

$deg(v_2) = 2$ $\quad deg(v_4) = 4$

$deg(v_3) = 3$ $\quad deg(v_6) = 2$

$v_1 \to$ isolated, v_5 - pendant

A vertex in a graph G is said to be isolated when its degree is zero.

A vertex is said to be a pendant vertex if its degree is 1.

The minimum degree among the vertices of G is denoted by $\delta(G) = 0$

The maximum degree among the vertices of G is denoted by $\Delta(G) = 4$

In a (p, q) graph, we note that

$0 \leq \deg v \leq p-1$

Degree sequence of the graph G $(0, 1, 2, 2, 3, 4)$