Basic Electrical Technology

[ELE 1051]

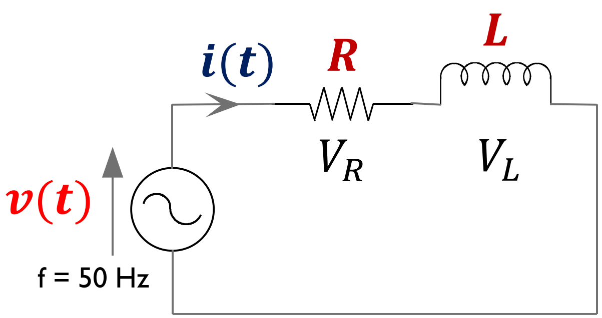
SINGLE PHASE AC CIRCUITS

L16,L17, RL,RC,RLC series circuit

Topics covered…

- AC response of
  - Series RL
  - Series RC
  - Series RLC

RL circuit analysis



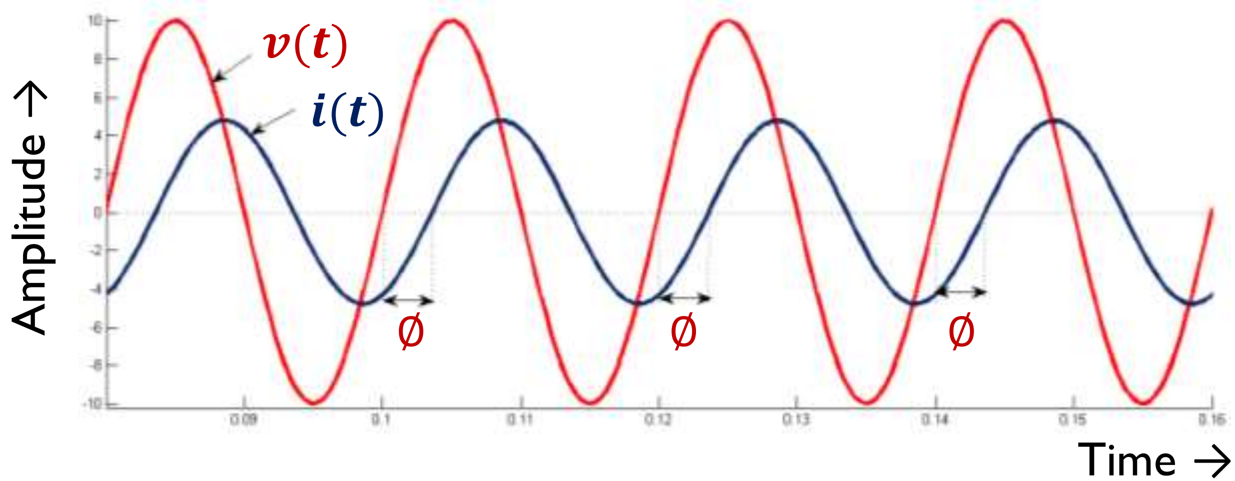
Let $\bar{I}$ *be along the reference*

$$\overline{V_R} = \bar{I}R$$

$$\overline{V_L} = j\bar{I}X_L$$

$$\bar{V} = \overline{V_R} + \overline{V_L} = |V|\angle\emptyset$$

Graphical Representation



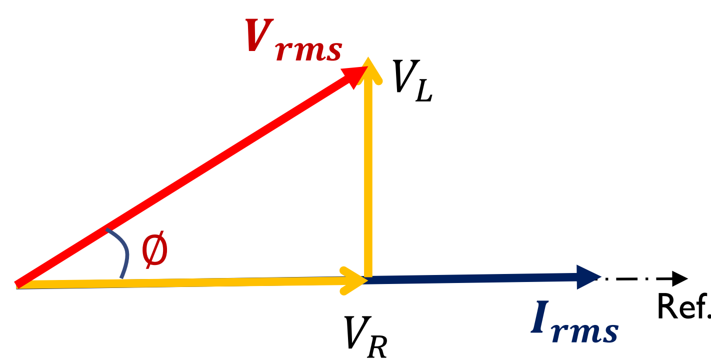
Mathematical Representation

$$i(t) = I_m \sin(\omega t)$$

$$v(t) = V_m \sin(\omega t + \emptyset)$$

$\emptyset$ – *Phase Angle*

Phasor Representation



Impedance

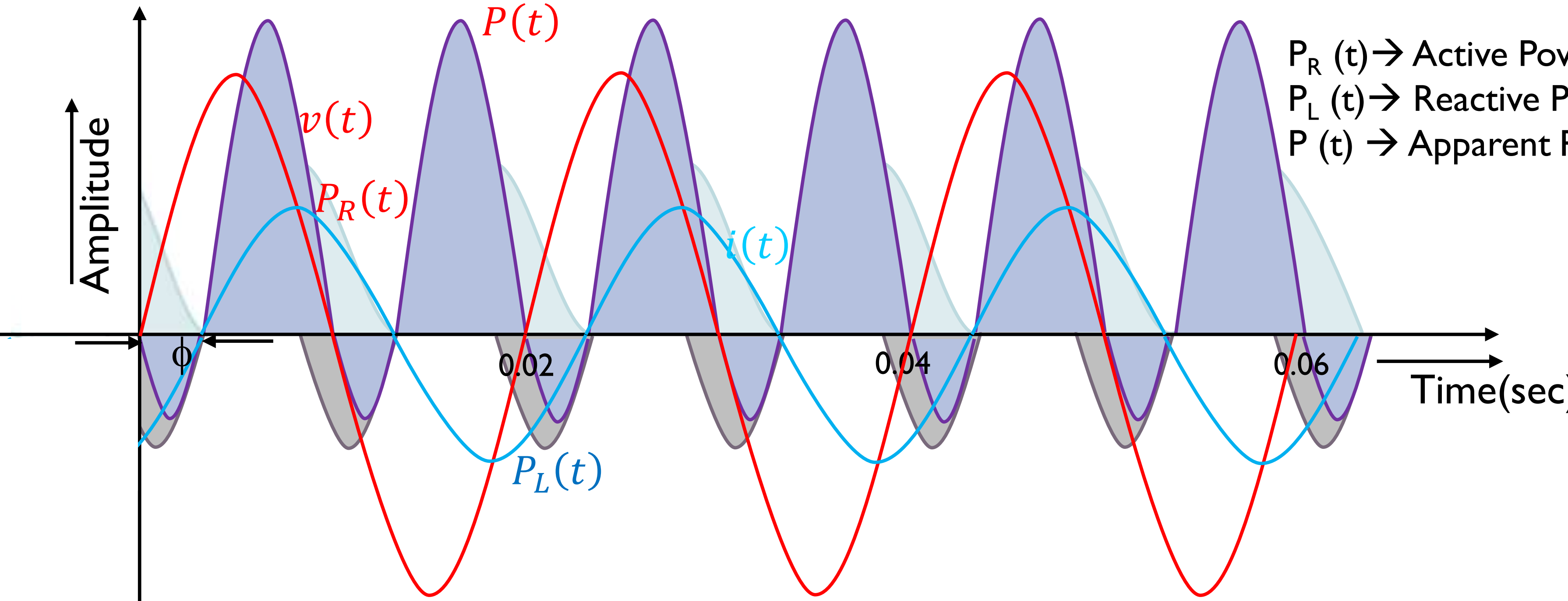
$$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R + jX_L)}{\bar{I}} = R + jX_L = |Z|\angle\emptyset$$

Z – *Impedance of the circuit*

$$\therefore R = |Z|\cos\emptyset \qquad X_L = |Z|\sin\emptyset$$

$$|Z| = \sqrt{R^2 + X_L^2} \qquad \emptyset = \tan^{-1}\frac{X_L}{R}$$

Power associated - RL circuit



P_R (t)→ Active Power
P_L (t)→ Reactive Power
P (t) → Apparent Power

Instantaneous power,

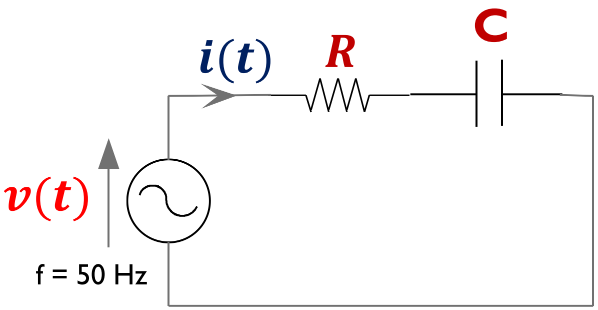
$$p(t) = v(t).\,i(t)$$
$$= V_m I_m \sin\omega t\,.\sin(\omega t + \emptyset)$$
$$= V_{rms} I_{rms}[\cos\emptyset - \cos(2\omega t + \emptyset)]$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2}\cos\emptyset}$$

$$\boxed{\mathbf{P_{avg} = V_{rms} I_{rms} \cos\emptyset}}$$

cos ∅ is called the Power Factor

RC circuit analysis



Let $\bar{I}$ *be along the reference*

$$\overline{V_R} = \bar{I}R$$

$$\overline{V_C} = -j\bar{I}X_C$$

$$\boldsymbol{\bar{V} = \overline{V_R} + \overline{V_C} = |V|\angle - \emptyset}$$

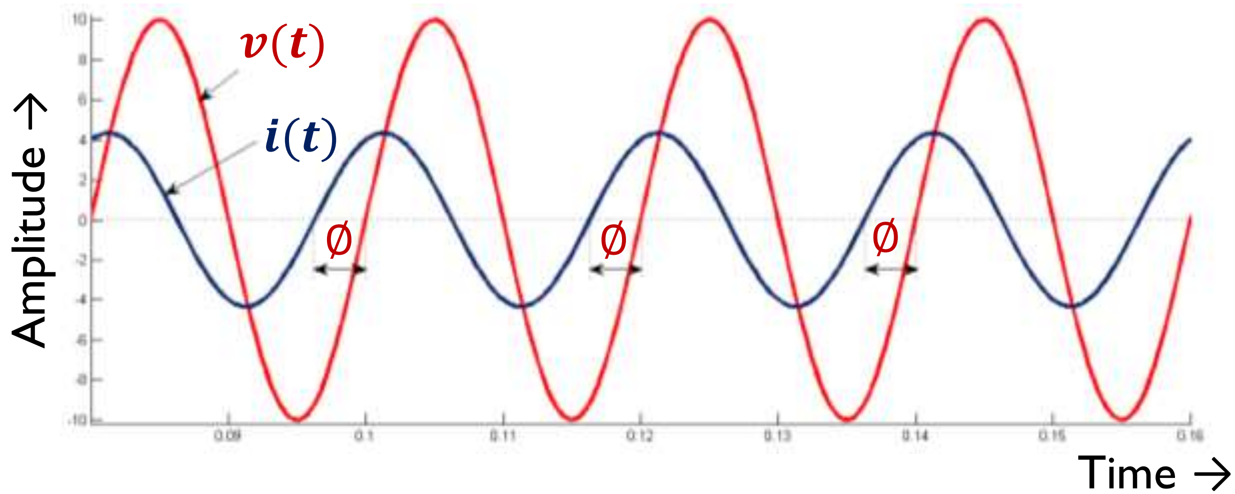
<u>*Mathematical Representation*</u>

$$i(t) = I_m \sin(\omega t)$$

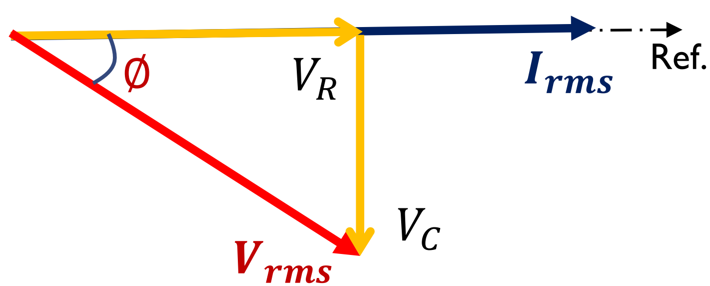
$$v(t) = V_m \sin(\omega t - \emptyset)$$

$\emptyset$ – *Phase Angle*

<u>*Graphical Representation*</u>



<u>*Phasor Representation*</u>



<u>*Impedance*</u>

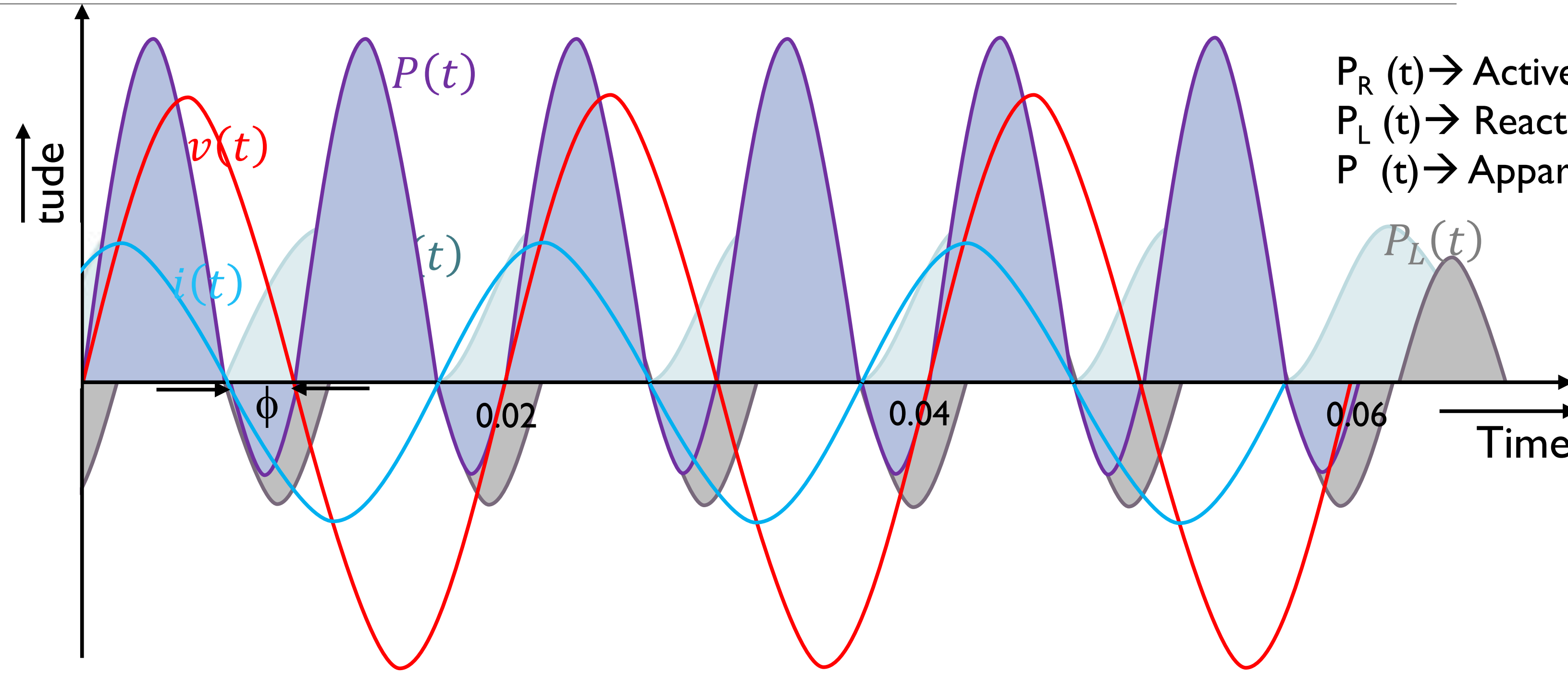
$$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R - jX_L)}{\bar{I}} = R - jX_L = |Z|\angle - \emptyset$$

Z – *Impedance of the circuit*

$$\therefore \boldsymbol{R = |Z|\cos\emptyset} \qquad \boldsymbol{X_C = |Z|\sin\emptyset}$$

$$\boldsymbol{|Z| = \sqrt{R^2 + X_C^2}} \qquad \boldsymbol{\emptyset = \tan^{-1}\frac{X_C}{R}}$$

Power associated - RC circuit



P_R (t)→ Active Power
P_L (t)→ Reactive Power
P (t)→ Apparent Power

Instantaneous power,

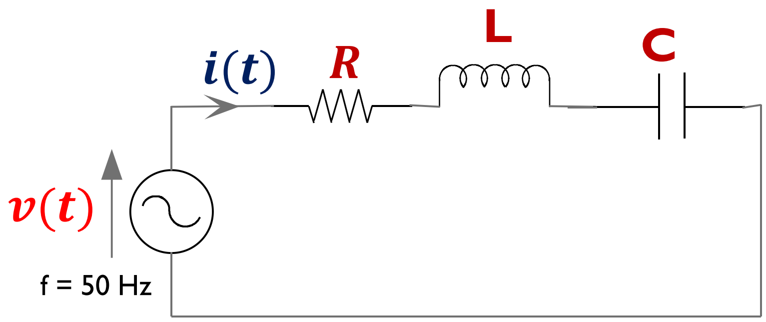
$$p(t) = v(t).\, i(t)$$
$$= V_m I_m \sin \omega t \,.\sin(\omega t - \emptyset)$$
$$= V_{rms} I_{rms}[\cos \emptyset - \cos(2\omega t - \emptyset)]$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2}\cos \emptyset}$$

$$\boxed{\mathbf{P_{avg} = V_{rms} I_{rms} \cos \emptyset}}$$

Ratio of Active power to apparent power is called the ***Power Factor***

RLC circuit



$Let\ i(t)\ be\ the\ reference$

$$\boldsymbol{Impedance, Z = R + j(X_L \sim X_c)}$$

$if\ X_L = X_C \quad \Longrightarrow \quad Resistive\ circuit\ (Resonance\ condition)$

$if\ X_L > X_C \quad \Longrightarrow \quad RL\ series\ circuit$

$if\ X_L < X_C \quad \Longrightarrow \quad RC\ series\ circuit$

Illustration 1

A resistance of 50Ω is connected in series with an inductance of 200mH and capacitance of 101.321μF across a 230V, 50 Hz, single phase AC supply. Obtain,

a) *Impedance of the circuit*
b) *Current drawn*
c) *Power factor*
d) *Power consumed*
e) *Phasor diagram*

Ans:
$59.050\angle 32.14°\ \Omega$
$3.898\angle -32.14°\ A$
$0.846\ lag$
$759.15\ W$

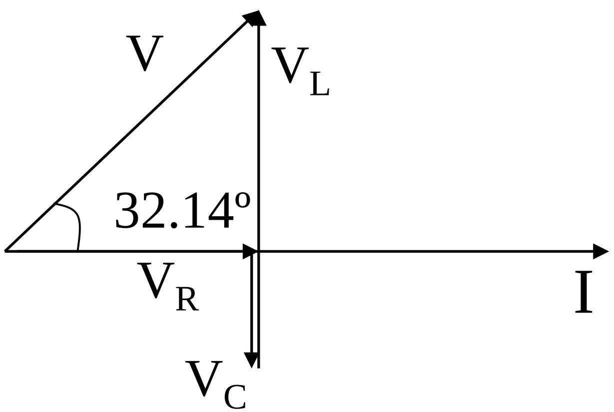
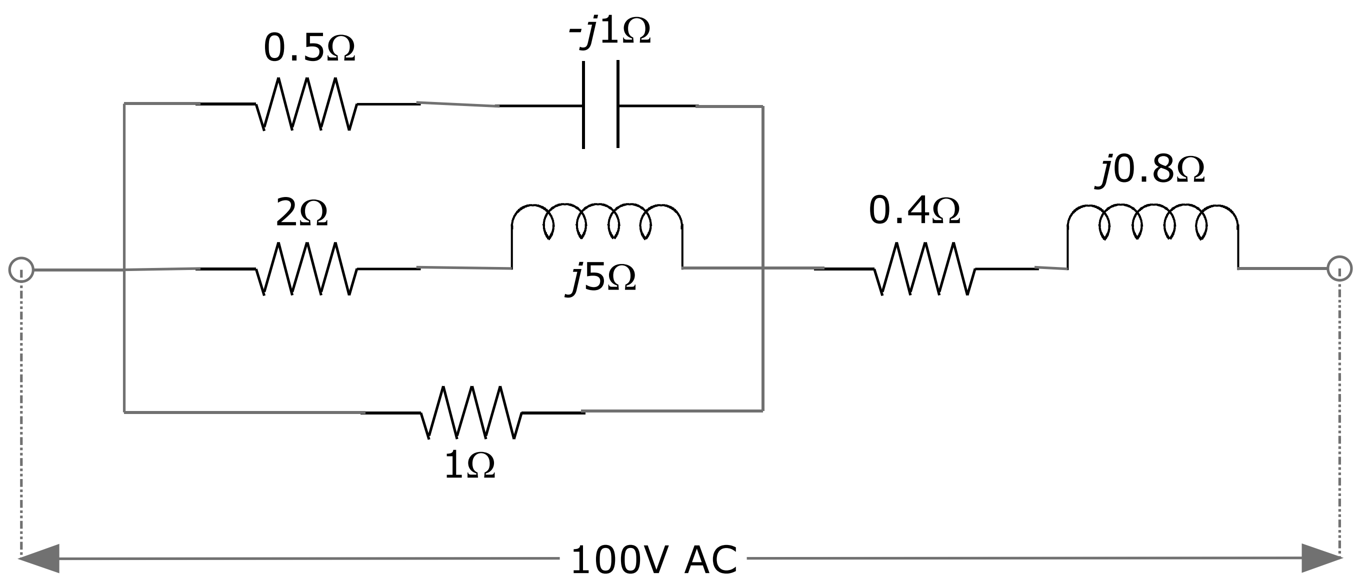


Illustration 2

Determine the impedance of the circuit shown and the power consumed in each branch



$$Ans:$$

$$Z = 1.12\angle 29.5°\Omega$$

$$1.25\ kW;\ \ 0.216\ kW; 3.12\ kW; 3.19\ kW$$

Summary

- **Define:** Impedance
- **Define:** Active Power; Reactive Power; Apparent Power; Power Factor

	RL	RC
Voltage, current relationship	i(t) lags v(t) by angle θ	i(t) leads v(t) by angle θ
Power associated	$S = VI$ $P = VI\cos\theta$ $Q = VI\sin\theta$	$S = VI$ $P = VI\cos\theta$ $Q = -VI\sin\theta$

Basic Electrical Technology

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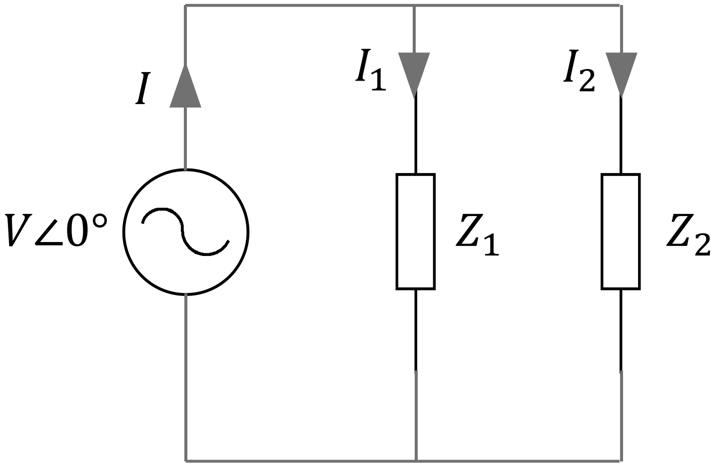
SINGLE PHASE AC CIRCUITS

L18 –Parallel circuits

Topics covered…

- Parallel circuit
  - Admittance
  - Conductance
  - Susceptance

Impedance in parallel



Let $Z_1 = R_1 + jX_1$ $\quad Z_2 = R_2 + jX_2$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad I_1 = I * \frac{Z_2}{Z_1 + Z_2} \qquad I_2 = I * \frac{Z_1}{Z_1 + Z_2}$$

$Y_{eq} = Y_1 + Y_2$ $\quad$ ***Y: Admittance***

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_1} = \frac{1}{(R_1 + jX_1)} * \frac{(R_1 - jX_1)}{(R_1 - jX_1)} = \frac{R_1}{(R_1^2 + X_1^2)} - j\frac{X_1}{(R_1^2 + X_1^2)} = G_1 - jB_1$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 + jX_2} = \frac{1}{(R_2 + jX_2)} * \frac{(R_2 - jX_2)}{(R_2 - jX_2)} = \frac{R_2}{(R_2^2 + X_2^2)} - j\frac{X_2}{(R_2^2 + X_2^2)} = G_2 - jB_2$$

G: Conductance $\qquad$ ***B: Susceptance***

$$B_{eq} = \frac{X_1 + X_2}{(R_1^2 + X_1^2)\,(R_2^2 + X_2^2)}$$

$$\mathbf{Y_{eq} = (G_1 + G_2) - j(B_1 + B_2) = G_{eq} - jB_{eq}}$$

$$G_{eq} = \frac{R_1 + R_2}{(R_1^2 + X_1^2)(R_2^2 + X_2^2)}$$

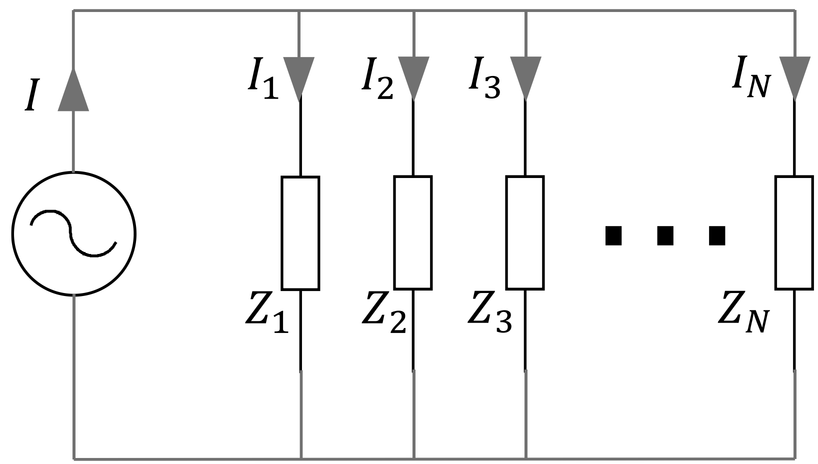
Impedance in parallel

For 'N' impedances connected in parallel,

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}$$

$$Y_{eq} = Y_1 + Y_2 + Y_3 + \cdots + Y_N$$

$$\mathbf{Y_{eq} = G_{eq} \pm \mathit{j}B_{eq}}$$



$$I_1 = VY_1; I_2 = VY_2; I_3 = VY_3; \ldots\ldots I_N = VY_N$$

$$I = I_1 + I_2 + I_3 + \cdots + I_N = VY_{eq}$$

Network equations for AC circuits

KVL Equation

(Matrix form)

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

$$[V] = [Z][I]$$

KCL Equation

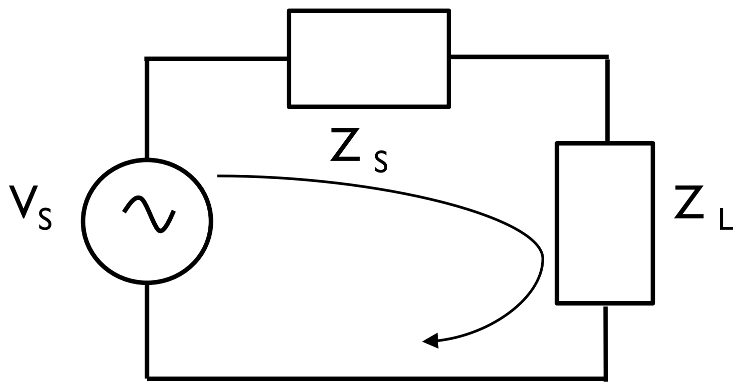
(Matrix form)

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

$$[I] = [Y][V]$$

All the other theorems are applicable to the AC circuits

Maximum power transfer theorem



	Type of load	Condition of maximum power transfer
Case 1	Load is purely resistive	$R_L = \sqrt{R_S^2 + X_S^2}$
Case 2	Both R_L & X_L are variable	$Z_L = Z_{TH}^*$
Case 3	X_L is fixed & R_L is variable	$R_L = \sqrt{R_S^2 + (X_s + X_L)^2}$

Crammers rule

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

Solution for the linear simultaneous equations above is as follows

__Step 1__: finding the determinant

$$\Delta = \begin{vmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{vmatrix}$$

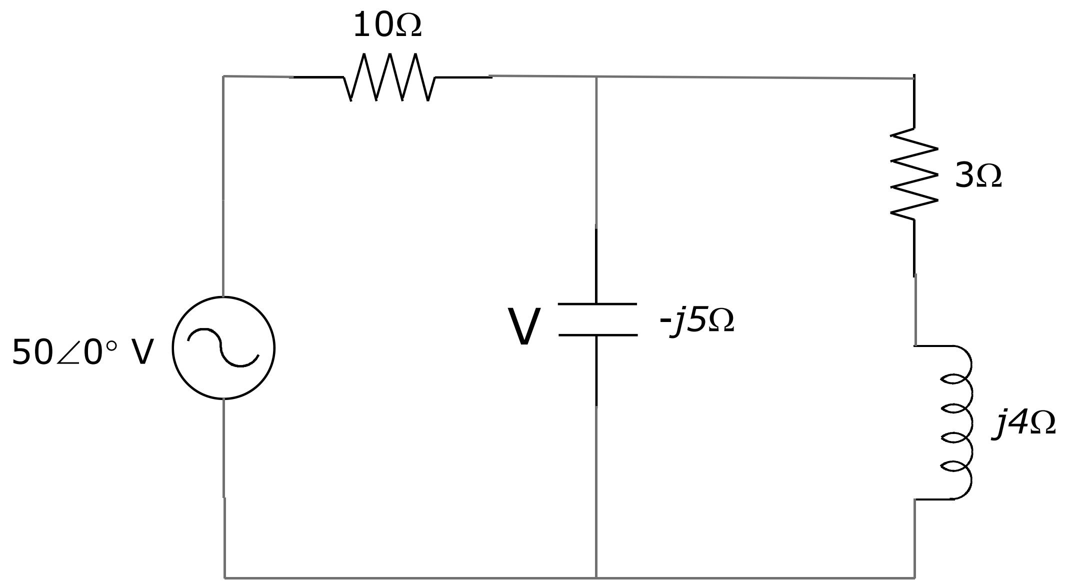
__Step 2__: finding the determinant after substituting first column with RHS column matrix

$$\Delta_1 = \begin{vmatrix} \boldsymbol{V_1} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ \boldsymbol{V_N} & \cdots & Z_{NN} \end{vmatrix}$$

__Step 3__ :Solution for I_1 $\quad I_1 = \frac{\Delta_1}{\Delta}$

Illustration 1

Assigning two mesh currents, find the voltage V across the capacitor in the following circuit

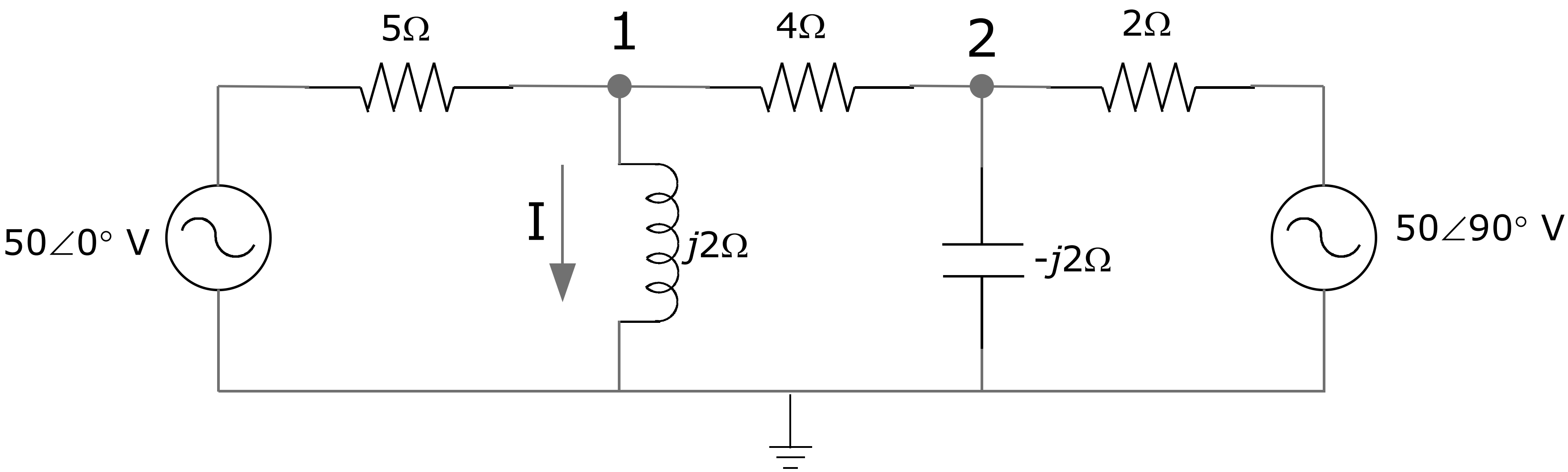


Ans:

$$V = 22.36\angle -10.30°V$$

Illustration 2

Use node voltage method to obtain the current I in the network



Ans:

$$I = 12.38\angle - 17.75^\circ\ A$$

Summary

- **Define:** Conductance; Susceptance; Admittance
- All network equations & theorem are applicable to AC circuits