Reg.No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER B.Tech DEGREE END SEMESTER EXAMINATION – DECEMBER, 2012

Sub: MAT 101 - ENGG. MATHEMATICS I

(REVISED CREDIT SYSTEM - 2011)

Time: 3 Hrs. **Max. Marks: 50**

Note: a). Answer any FIVE full questions b). All questions carry equal marks

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1A. Find the n^{th} derivatives of the following

(i) $\dfrac{x^2}{2x^2+7x+6}$ (ii) $\cosh x.\cos 3x$

1B. If $y=\sin\left(m\sin^{-1}x\right)$, show that $(1-x^2)y_{n+2} = (2n+1)\,x\,y_{n+1} + (n^2-m^2)\,y_n$

1C. Obtain the reduction formula for $\int \sin^n x\,dx$ and hence evaluate $\int_0^{\pi/2} \cos^n x dx$.

(3+3+4)

2A. Evaluate:

(i) $\int_0^1 x^2 \sin^{-1} x dx$. (ii) $\int_0^\infty \dfrac{x^4}{\left(1+x^2\right)^{7/2}} dx$

2B. Show that plane $x + 2y - 2z - 8 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x4y - 6z + 5 = 0$. Find the point of contact.

2C. Trace the curve $y^2(a^2+x^2) = x^2(a^2-x^2)$, $a>0$

(3+3+4)

3A. Test the Nature of the following series

(i) $\dfrac{3}{4}+\dfrac{3.6}{4.7}+\dfrac{3.6.9}{4.7.10}+\ldots$

(ii) $\left(\dfrac{2^2}{1^2}-\dfrac{2}{1}\right)^{-1}+\left(\dfrac{3^3}{2^3}-\dfrac{3}{2}\right)^{-2}+\left(\dfrac{4^4}{3^4}-\dfrac{4}{3}\right)^{-3}+\ldots\ldots$

3B. Find the reflection of the point $(2,-1,3)$ in the plane $3x-2y-z-9=0$.

3C. Using Maclaurin's series, expand the function $\log(1+\sin x)$ upto first three non - zero terms. **(3+3+4)**

4A. Obtain the interval of convergence for $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n} x^{n}$

4B. If u = f(r), where $r^2 = x^2 + y^2 + z^2$, prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f''(r)+\frac{2}{r}f'(r)$

4C. Find the evolute of the curve $y^2 = 4ax$. **(3+3+4)**

5A. Find the area common to the cardioids $r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$.

5B. Find the angle of intersection between the curves $r^2 \sin 2\theta = 4$ and $r^2 = 16 \sin 2\theta$.

5C. Sketch and find the perimeter of the curve $r = a(1+\cos\theta),\ a > 0$.

(3+3+4)

6A. Find the points on the lines $\frac{x-6}{3}=\frac{y-7}{-1}=\frac{z-4}{1}$ and $\frac{x}{-3}=\frac{y+9}{2}=\frac{z-2}{4}$ which are nearest to each other. Hence find the shortest distance between the lines.

6B. Find the volume of the solid obtained by revolving the curve $y^2(2a - x) = x^3$ about its asymptote.

6C. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ show that

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(1-4\sin^2 u)\sin 2u$

(3+3+4)

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