PROBLEMS ON PROBABILITY

1. Suppose A and B are events such that $P(A) = x, P(B) = y, P(A \cap B) = z$. Express each of the following in terms of x, y, z
   (i) $P(A \cup B)$
   (ii) $P(\bar{A} \cap \bar{B})$
   (iii) $P(\bar{A} \cap B)$
   (iv) $P(\bar{A} \cup \bar{B})$

2. Five letters are selected at random from the 26 letters of English alphabet (a) with replacement (b) without replacement. For each of the cases (a) and (b), find the probability that the word from these letters would contain
   (i) a **Ans** : (a) 0.1644 (b) 0.1923
   (ii) Only vowels **Ans** : (a) 2.6302×10^{-4} (b) 1.5202×10^{-5}
   (iii) The word WOMAN **Ans** : (a) $\frac{1}{26*25*24*23*22}$ (b) $\frac{1}{26^5}$

3. One number is chosen at random from 1,2, 3,…, 50. What is the probability that this number is divisible by 6 or by 8? **Ans** : $\frac{6}{25}$

4. A committee has 9 students: 2 of first year, 3 of second year and 4 of third year. Three students are chosen at random. Find the probability that
   (i) 3 students belong to different classes **Ans** : $\frac{2}{7}$
   (ii) 2 students belong to same class **Ans** : $\frac{55}{84}$
   (iii) 3 students belong to the same class **Ans** : $\frac{5}{84}$

5. 6 girls are to enter a dance with 10 boys to form a ring so that every girl is between 2 boys. What is the probability that
   (i) Some specified boy remains between 2 boys?
   (ii) A certain girl dances with a certain boy?

6. Three winning tickets are to be selected from a box with 100 tickets. What is the probability of a person winning if he buys
   (i) 4 tickets? **Ans** : 0.1164
   (ii) Only one ticket? **Ans** : 0.03

7. n persons will throw their n hats to the center of the hall and picks it later.
   (i) Show that the probability that no one gets their own hat is $\sum_{k=2}^{n} \frac{(-1)^k}{k!}$
   (ii) Find the probability that exactly k persons out of n will get their own hat.

8. Suppose four digits 1,2,3,4 are written in random order. What is the probability that at least one digit will occupy its proper position? **Ans** : $\frac{5}{8}$

9. An inefficient secretary has n letters in n addressed envelope. If she puts letters in random envelopes, what is the probability that atleast one will be placed in proper envelope?

10. A class consists of 10 boys and 20 girls. Half of the boys and half of the girls have brown eyes. Find the probability that a person selected at random is a boy or has brown eyes. **Ans** : $\frac{2}{3}$

11. Two marbles are drawn successively from a box with 10 red, 30 white, 20 black and 15 orange marbles (a) with replacement (b) without replacement. In both the cases (a) and (b), find the probability that
    (i) Both are white. **Ans** : (a) 0.16 (b) 0.1568
    (ii) First ball is black and second is white. **Ans** : (a)0.1067 (b) 0.1082
    (iii) Neither is orange. **Ans** : (a)0.64 (b) 0.6378

12. Two planes bomb a target in succession. The probability of each correctly attaining a hit is 0.3 and 0.2 respectively. The second plane will bomb only if the first one will miss the target. Find the probability that
    (i) The target is hit. **Ans** : 0.44
    (ii) The target is hit by the second plane. **Ans** : 0.14
    (iii) Both fail. **Ans** : 0.56

13. Three newspapers A,B and C are published in a city. Recent survey indicates the following: 20% read A, 16% read B, 14% read C, 8% read A and B, 5% read A and C, 4% read B and C, 2% read A,B and C. One person is chosen at random. Find the probability that he reads
    (i) None of the papers. **Ans** : 0.65
    (ii) Exactly one paper. **Ans** : 0.22
    (iii) Reads A and B if it is known that he reads at least one. **Ans** : 0.2286

14. Two absent minded roommates forget their umbrella in some way or the other. 'A' always takes his umbrella when he goes out. 'B' forgets to take his umbrella with a probability $\frac{1}{2}$. Each of them forget their umbrella at a shop with probability $\frac{1}{4}$. After visiting three shops, they return home. Find the probability that
    (i) Both have umbrella. **Ans** : $\frac{2457}{8192}$
    (ii) Only one has umbrella. **Ans** : $\frac{4366}{8192}$
    (iii) 'B' lost his umbrella given that there is only one umbrella after their return. **Ans** : $\frac{999}{4366}$

15. Coefficients A,B,C of the quadratic equation $Ax^2 + Bx + C = 0$ are obtained by rolling a dice thrice. What is the probability that the roots of this equation are real?

Ans : 0.1991

16. A problem in statistics is given to three students A,B,C whose chances of solving are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?

Ans : $\frac{29}{32}$

17. Each of two persons toss three coins. What is the probability that they get the same number of heads? **Ans** : $\frac{5}{16}$

18. A basket contains 40 mangoes, out of which 15% are overripe. Two mangoes are selected at random. If both are good, then the basket is accepted. If one is good and the other is overripe, then another is picked from the remaining and if this is good, then the basket is accepted. Find the probability of accepting the basket.

Ans: 0.9464

19. There are two groups of subjects, one of which consists of 5 science and 3 engineering subjects, and the other consists of 3 science and 5 engineering subjects. An unbiased d is rolled. If 3 or 5 comes up, a subject is selected at random from the first group, otherwise from the second. Find the probability that an engineering subject is selected.

20. 4 boys and 3 girls stand in a queue for a family picture. Find the probability that they stand in alternate position?

Ans:0.02857

21. Ajay was born between Oct 6th and Oct 10th(excluding 6th and 10th). What is the probability that he was born on Saturday? **Ans:** 0.4285

22. Mother, father and son line up at random for a family picture, find P(E|F), where
E:Son at one end F:father in the middle **Ans:**1

23. Box A has 8W+7B balls and Box B has 9B+7W balls. A ball is randomly drawn from Box A and placed in box B. then a ball is transferred from B to A. Finally one ball is selected from box A. What is the probability that it is white? **Ans:**0.5273

24. A die is rolled 3 times, Find the probability that 6 appears at least once? **Ans:** $\mathbf{1 - \frac{5^3}{6^3}}$

25. Assuming men and women exist equal in number, and assuming 5% of men are color blind and 25% of women are colorblind, evaluate the probability that a person selected at random is colorblind. Also evaluate the probability that, having drawn a color-blind person, this is a male?

26. Two marbles are drawn successively from a box with 10 red, 30 white, 20 black and 15 orange balls with replacement. Find the probability that
    i. Both are white
    ii. First one black and second white
    iii. Neither is orange. Solve the same problem without replacement.

Ans: With rep: i. $\frac{4}{25}$

ii. $\frac{8}{75}$

iii. $\frac{16}{25}$

Without rep: i.$\frac{29}{185}$

ii. $\frac{4}{37}$

iii. $\frac{118}{185}$

27. If A, B and C are 3 independent events, then prove that $A \cup B$ and C are independent.

28. If A and B are two independent events of S such that $P(\overline{A} \cap B) = \frac{2}{15}$, $P(\overline{B} \cap A) = \frac{1}{16}$. Then find $P(B)$? **Ans:**$\frac{1}{6}$ or $\frac{4}{5}$

29. If A and B are two events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{1}{2}$. Then find
    i. $P(A|B)$
    ii. $P(B|A)$
    iii. $P(\overline{B} \cap A)$
    iv. $P(A|\overline{B})$

Ans: i, iv. $\frac{1}{3}$

ii, iii. $\frac{1}{4}$

30. In a certain town, 40% have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. A person is selected at random.
    i. If he has brown hair, what is the probability that he has brown eyes also
    ii. If he has brown eyes, what is the probability that he doesn't have brown hair?
    iii. Determine the probability that he has neither brown hair nor brown eyes?

Ans: i. $\frac{3}{8}$

ii. $\frac{2}{5}$

iii. 0.5

34.Two squares are selected at random from a 8*8 chess board, what is the probability that they have a side in common? Assume that only those squares with the same dimension can share a side in common. **Ans**: $\frac{222}{204_{C_2}}$

35. At an art exhibition, there are 12 paintings of which 10 are original. A visitor selects a painting at random and before he decides to buy, he asks the opinion of an expert about the authenticity of the painting. The expert is right in 9 out of 10 cases on average.

i. Given that the expert decides that the painting is authentic, what is the probability that this is really the case?

ii. If the expert decides that the painting is a copy, then the visitor return it and choses another one, what is the probability that his second choice is original?

Ans: i. $\frac{90}{92}$

ii. $\frac{99}{110}$

36. For a certain binary communication channel, the probability that a transmitted zero is received as zero is 0.95 and the probability that a transmitted one is received as one is 0.90. If the probability that a zero is transmitted is 0.4, find the probability that

i. a one is received

ii. a one was transmitted given that a one was received.

Ans: i)0.56

ii)0.9642

37. Two people A and B toss an unbiased coin alternatively on the understanding that the first one to get head wins the game. If A starts the game, find the probability that B wins?

Ans: 0.66666

38..In a certain town, 40% have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. A person is selected at random

i. If he has brown hair, then what is the probability that he has brown eyes also?

ii. If he has brown eyes, then what is the probability that he doesn't have brown hair?

iii. What is the probability that he has neither brown hair nor brown eyes?

Ans: i)0.375

ii)0.4

iii)0.5

39. Person A tosses a coin and then person B rolls a die. This is repeated independently until a head or one of the numbers 1, 2,3,4 appears, at which time the game is stopped. Person A wins with head and person B wins with one of the numbers 1, 2,3,4. Computes the probability that A wins the game. Also find the probability that A wins if B starts the game?

Ans: i)0.6

ii)0.2

40. Each bag in a large box contains 25 tulip bulbs. It is known that 60% of the bags contain bulbs for 5 red and 20 yellow tulips while the remaining 40% of the bags contain bulbs for 15 red and 10 yellow tulips. A bag is selected at random and a bulb is taken from this bag is planted.

i. What is the probability that it is a yellow tulip?

ii. Given that it is yellow, what is the conditional probability that that it comes from a bag that contained 5 red and 20 yellow bulbs?

Ans: i)0.64
ii)0.75

41. A restaurant serves 2 special dishes A and B to its customers consisting of 60% of men and 40% of women. 80% of men order dish A and rest B. 70% of women order dish B and rest A. In what proportion A:B of dishes A and B should be prepared?

Ans: 6:4