18/10/2021

ORDINARY DIFFERENTIAL EQUATION

Differential eqⁿ:- A d.e is an eqⁿ that relates one ore more functions and their derivaties.

(i) $\frac{dy}{dx} = f(x)$ (ii) $\frac{dy}{dx} = g(x,y)$

(ii) $(1-x)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 5y = \cos x$

(iii) $\frac{d^2y}{dx^2} = \sqrt{1 - \left(\frac{dy}{dx}\right)^2}$

ODE:- is a d.e. containing one or more functions of one independent variable and the derivaties of those functions.

Eg:- $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0$

Types of d.e's,

(i) ODE

(ii) PDE (partial diff. eqⁿ)

Recall the following definitions

* ORDER of a d.e.
* DEGREE of a d.e.
* SOLUTION of a d.e.

PARTICULAR SOLUTION

GENERAL SOLUTION

General form of an ODE

The nth order ODE is given as,

$$F(x, y, y', y'', \dots y^n) = 0$$

LINEAR ODE :- An n^{th} order ODE is said to be linear if it can be written as

$$a_0(x) y^n + a_1(x) y^{n-1} + a_2(x) y^{n-2} + \dots + a_n(x) y = R(x) \quad (*)$$

where $a_j(x)$ for $0 \leq j \leq n$ are called the coefficients of the eq^n.

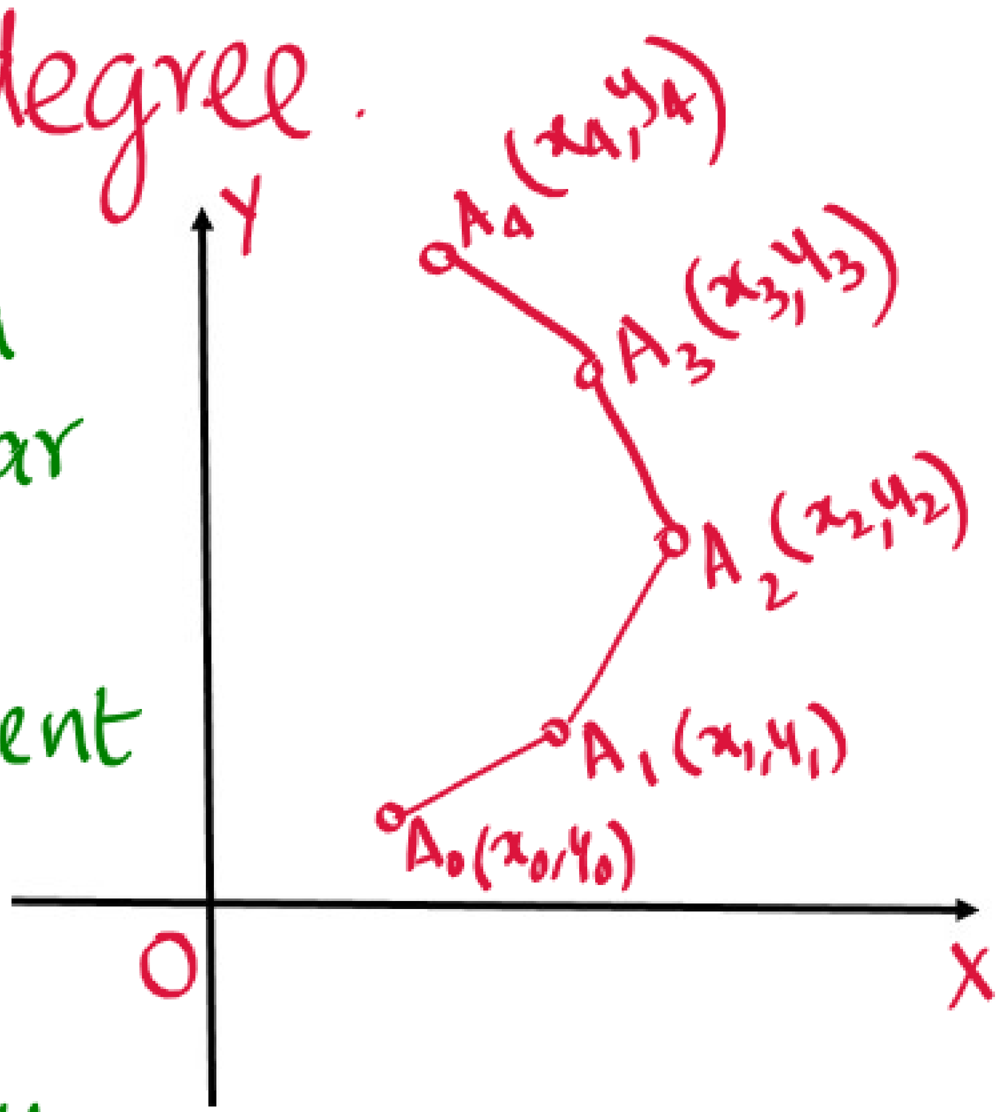
In $(*)$, if $R(x) = 0$ then $(*)$ is called a homogenous l.d.e.

In $(*)$ if $R(x) \neq 0$ then $(*)$ is called non-homogenous d.e.

Geometrical meaning of first order first degree differential eqⁿ.

Let $f\left(x, y, \frac{dy}{dx}\right) = 0$ be the d.e ——— (*)

of Iˢᵗ order Iˢᵗ degree.



Note:- The direction of a curve at a particular point is determined by drawing a tangent line at that point.

ie; its slope is given by $\frac{dy}{dx}$ at that particular point.

Let $A_0(x_0, y_0)$ be any point in the plane.

Let $m_0 = \left(\frac{dy}{dx}\right)_{(x_0,y_0)} = \frac{dy_0}{dx_0}$ be the slope of the curve at A_0 derived from (*).

Let $A_1(x_1, y_1)$ be a neighbouring point to A_0 such that the slope $(A_0A_1) = m_0$.

Let $m_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{dy_1}{dx_1}$ be the slope of the curve at A_1 derived from $\circledast$.

Let $A_2(x_2,y_2)$ be a neighbouring point of A_1 such that slope $(A_1A_2) = m_1$

Continuing like this, we get a succession of points say, $A_0, A_1, A_2, A_3, A_4, \ldots$

If the points are chosen sufficiently close to each other, they approximate a smooth curve $C_1 : y = \phi(x)$, which is a solution of $\circledast$, corresponds to the starting point $A_0(x_0,y_0)$.

Any point on C_1 and the slope at that point satisfies $\circledast$.

If we start from a point (not on C_1) and moves as before, it will describe a new curve C_2.

The eq^n of each such curve is called a <u>PARTICULAR SOLUTION</u> of $\circledast$.

Formation of a d.e.

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→ by eliminating arbitrary constants.

Eg:- Eliminate the arbitrary constants and form the d.e. from the eqⁿ

$$y = e^{x}(A\cos x + B\sin x).$$

Ans:-

$$\frac{dy}{dx} = e^{x}(A\cos x + B\sin x) + e^{x}(-A\sin x + B\cos x)$$

$$\Rightarrow \frac{dy}{dx} = y + e^{x}(-A\sin x + B\cos x)$$

$$\Rightarrow e^{x}(-A\sin x + B\cos x) = \frac{dy}{dx} - y \qquad (1)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^{x}(-A\sin x + B\cos x) + e^{x}(-A\cos x - B\sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^{x}(-A\sin x + B\cos x) - y$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{dy}{dx} - y - y$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 2y \Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$