

3B. Find :

$$(i) \quad L\{\cos t - \cos 2t + \cos 3t\} \quad (ii) \quad L^{-1}\left\{\frac{e^{-4s}}{(s+1)^3}\right\}$$

3C. Find the volume inside the cone $x^2 + y^2 = z^2$ bounded by the sphere $x^2 + y^2 + z^2 = a^2$.

(4 + 3 + 3)

4A. Solve :

$$(i) \quad x \, dx = y (x^2 + y^2 - 1) \, dy$$

$$(ii) \quad y \, e^{x/y} dx = \left(x \, e^{x/y} + y^2 \right) dy$$

4B. Find the extreme values of $x^2 + y^2 + z^2$ subject to the conditions

$$\frac{x^2}{1} + \frac{y^2}{2} + \frac{z^2}{3} = 2 \text{ and } 3x + 2y + z = 0.$$

4C. Evaluate : $\int_a^b (x-a)^p (b-x)^q \, dx$ and hence find $\int_2^4 (x-2)^6 (4-x)^4 \, dx$.

(4 + 3 + 3)

5A. Using Gram - Schmidt orthogonalisation process find an orthonormal basis from $(1, 1, 1), (0, 1, 1), (1, 2, 3)$ in E^3 .

5B. A circuit consists of resistance R , an induction L and a constant e.m.f E switch is closed at $t = 0$ and removed at $t = T$. Find current at any time t .

5C. Test for consistency. If consistent solve by Gauss Elimination

$$4x - 2y + 6z = 8$$

$$x + y + 3z = -1$$

$$15x - 3y + 9z = 21$$

(4 + 3 + 3)

6A. Solve : $\omega''(x) + 2\omega'(x) + \omega(x) = x$ $\omega(0) = -3$ and $\omega(1) = -1$ using Laplace transform method.

6B. Expand $f(x,y) = xy^2 + \cos xy$ about the point $\left(1, \frac{\pi}{2}\right)$ upto second degree terms.

6C. Solve : $2xy'' + 3y' - \frac{y}{x} = 5 - \frac{\sin(\log x)}{x^2}$.

(4 + 3 + 3)

4A. Solve :

- (i) $ydx - x dy + 3x^2y^2e^{x^3} dx = 0$.
(ii) $dy - \sin(x + y) dx = 0$

4B. Expand $f(x, y) = \sin xy$ in powers of $(x - 1)$ and $\left(y - \frac{\pi}{2}\right)$ upto second degree terms.

4C. Prove that : $\gamma(m) \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \gamma(2m)$.

(4 + 3 + 3)

5A. A spring of stiffness $k = 32$ dynes / cm is suspended from a fixed point and carries a mass of 200 gm at the other end. The mass is pulled down from its neutral position to a depth of 10 cm and released from the rest. Find the displacement at any time t .

5B. Find the values of λ and μ for which the system $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ has

- (i) unique solution (ii) no solution (iii) infinite solution

5C. Define : (i) Basis (ii) orthonormal basis , for E^n .

Find the rank of $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ using row transformations.

(3 + 3+ 4)

6A. Solve : $y''(x) + 2y'(x) + y(x) = x$; $y(0) = -3$ and $y(1) = -1$ using Laplace transform method.

6B. Find the double integration the area inside the circle $r = a \sin\theta$ and outside the cardioid $r = a(1 - \cos\theta)$.

6C. Solve :

$$\frac{dx}{dt} + 2x + 3y = 0$$

$$\frac{dy}{dt} + 3x + 2y = 2e^{2t}$$

(3 + 3+ 4)
