Given a function from $\{0,1\}^n$ to $\{0,1\}$.
We can obtain a boolean expression in DNF.

For each n-tuple with function value 1,
we have a minterm $\tilde{x}_1 \wedge \tilde{x}_2 \wedge \cdots \wedge \tilde{x}_n$ where

$$\tilde{x}_i = \begin{cases} x_i, & \text{if the } i^{th} \text{ component is } 1 \\ \overline{x_i}, & \text{if the } i^{th} \text{ component is } 0 \end{cases}$$

x_1	x_2	x_3	f
0	1	0	1

Similarly, given a function $\{0,1\}^n \to \{0,1\}$.
We can obtain a boolean expression in CNF

For each n-tuple with function value 0, we
have a maxterm $\tilde{x}_1 \vee \tilde{x}_2 \vee \cdots \vee \tilde{x}_n$ where

$$\tilde{x}_i = \begin{cases} x_i ; & \text{if } i^{th} \text{ component is } 0 \\ \overline{x_i} ; & \text{" \quad " \quad " \quad } 1 \end{cases}$$

Q1. Write both CNF and DNF of the boolean expression corresponds to the Boolean function as shown below.

$f: (0\,1)^3 \to (0\,1)$

x_1	x_2	x_3	f
0	0	0	1 ✓
0	0	1	0 ✓
0	1	0	1 ✓
0	1	1	0 ✓
1	0	0	0 ✓
1	0	1	0 ✓
1	1	0	0 ✓
1	1	1	1 ✓

Soln :

CNF : $(x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee x_2 \vee x_3)$
$\wedge (\bar{x}_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3)$

DNF : $(\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3) \vee (\bar{x}_1 \wedge x_2 \wedge \bar{x}_3) \vee (x_1 \wedge x_2 \wedge x_3)$

Q2. Let $E(x_1\ x_2\ x_3) = (x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (\overline{x_2} \wedge x_3)$ be a boolean expression in a 2 valued boolean algebra. Write boolean expression in CNF and DNF.

Soln:

x_1	x_2	x_3	$x_1 \wedge x_2$	$x_1 \cap x_3$	$\overline{x_2} \wedge x_3$	f
0	0	0	0	0	0	0
0	0	1	0	0	1	1
0	1	0	0	0	0	0
1	0	0	0	0	0	0
0	1	1	0	0	0	0
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	1	0	1

DNF: $(\overline{x_1} \wedge \overline{x_2} \wedge x_3) \vee (x_1 \wedge \overline{x_2} \wedge x_3) \vee (x_1 \wedge x_2 \wedge \overline{x_3})$
$\vee (x_1 \wedge x_2 \wedge x_3)$

CNF: $(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3})$
$\wedge (\overline{x_1} \vee x_2 \vee x_3)$

Alternate Method

Consider $(x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (\bar{x}_2 \wedge x_3)$

$$= (x_1 \wedge x_2 \wedge 1) \vee (x_1 \wedge x_3 \wedge 1) \vee (\bar{x}_2 \wedge x_3 \wedge 1)$$

$$= (x_1 \wedge x_2 \wedge (x_3 \vee \bar{x}_3)) \vee (x_1 \wedge x_3 \wedge (x_2 \vee \bar{x}_2))$$

$$\vee (\bar{x}_2 \wedge x_3 \wedge (x_1 \vee \bar{x}_1))$$

$$= (x_1 \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2 \wedge \bar{x}_3) \vee (\cancel{x_1 \wedge x_3 \wedge x_2}) \vee$$

$$(x_1 \wedge x_3 \wedge \bar{x}_2) \vee (\cancel{\bar{x}_2 \wedge x_3 \wedge x_1}) \vee (\bar{x}_2 \wedge x_3 \wedge \bar{x}_1)$$

$$= (x_1 \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2 \wedge \bar{x}_3) \vee (x_1 \wedge x_3 \wedge \bar{x}_2)$$

$$\vee (\bar{x}_2 \wedge x_3 \wedge \bar{x}_1)$$ is in DNF.

To get CNF: write the remaining terms. 8

$$(\bar{x}_1 \wedge x_2 \wedge x_3) \vee (\bar{x}_1 \wedge \bar{x}_3 \wedge x_2) \vee (x_1 \wedge \bar{x}_2 \wedge \bar{x}_3)$$

$$\vee (\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3)$$

Now interchange $\vee$ with $\wedge$ and x_i with $\bar{x}_i$ we get

$$(x_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (x_1 \vee x_3 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee x_2 \vee x_3)$$

$$\wedge (x_1 \vee x_2 \vee x_3)$$ is in CNF.

Q3. Write CNF and DNF.

$$E(x_1\, x_2\, x_3) = \overline{\left(\overline{x_1 \vee x_2}\right) \vee \left(\bar{x}_1 \wedge x_3\right)}$$

Soln :

$$f = \overline{\left(\overline{x_1 \vee x_2}\right) \vee \left(\bar{x}_1 \wedge x_3\right)}$$

x_1	x_2	x_3
0	0	0
0	0	1
0	1	0
1	0	0
0	1	1
1	0	1
1	1	0
1	1	1

DNF :

CNF :

$\overline{a \vee b} = \bar{a} \wedge \bar{b}$

Alternate method:

$$\overline{(\overline{x_1 \vee x_2}) \vee (\overline{x_1} \wedge x_3)} = \overline{\overline{(x_1 \vee x_2)}} \wedge (\overline{\overline{x_1} \wedge x_3}) \quad \text{D'morgan's law.}$$

$$= (x_1 \vee x_2) \wedge (x_1 \vee \bar{x}_3)$$

$$= (x_1 \vee x_2 \vee 0) \wedge (x_1 \vee \bar{x}_3 \vee 0)$$

$$= (x_1 \vee x_2 \vee (x_3 \wedge \bar{x}_3)) \wedge (x_1 \vee \bar{x}_3 \vee (x_2 \wedge \bar{x}_2))$$

$$= (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_3 \vee x_2) \wedge (x_1 \vee \bar{x}_3 \vee \bar{x}_2)$$

$$= (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_3 \vee \bar{x}_2)$$

$\longrightarrow$ CNF.

To obtain DNF

Remaining terms:

$$(\bar{x}_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)$$

DNF:

$$(x_1 \wedge \bar{x}_2 \wedge \bar{x}_3) \vee (\bar{x}_1 \wedge x_2 \wedge \bar{x}_3) \vee (x_1 \wedge x_2 \wedge \bar{x}_3) \vee (x_1 \wedge \bar{x}_2 \wedge x_3) \vee (x_1 \wedge x_2 \wedge x_3)$$

Q4: Write in CNF and DNF

$$E(x_1\ x_2\ x_3\ x_4) = \left[(x_1 \wedge x_2 \wedge \overline{x_3}) \vee (x_1 \wedge \overline{x_2} \wedge x_4)\right]$$
$$\wedge (x_2 \wedge \overline{x_3} \wedge \overline{x_4})$$

DL: $(a \vee b) \wedge c$
$(c \wedge a) \vee (c \wedge b)$

Soln: $\left[(x_1 \wedge x_2 \wedge \overline{x_3}) \wedge (x_2 \wedge \overline{x_3} \wedge \overline{x_4})\right] \vee$

$a \wedge a = a$

$$\left[(x_1 \wedge \overline{x_2} \wedge x_4) \wedge (x_2 \wedge \overline{x_3} \wedge \overline{x_4})\right]$$

$$= (x_1 \wedge x_2 \wedge \overline{x_3} \wedge \overline{x_4}) \vee (x_1 \wedge \underbrace{\overline{x_2} \wedge x_2}_{} \wedge \underbrace{x_4 \wedge \overline{x_4}}_{} \wedge \overline{x_3})$$

$$= (x_1 \wedge x_2 \wedge \overline{x_3} \wedge \overline{x_4}) \vee (x_1 \wedge 0 \wedge 0 \wedge \overline{x_3})$$

$0 \wedge a = 0$

$$= (x_1 \wedge x_2 \wedge \overline{x_3} \wedge x_4) \vee 0$$

$b \vee 0 = b$

$$= (x_1 \wedge x_2 \wedge \overline{x_3} \wedge x_4)$$

is in DNF

Remaining 15 terms:

$(x_1 \wedge x_2 \wedge x_3 \wedge x_4) \vee (\overline{x_1} \wedge x_2 \wedge x_3 \wedge x_4) \vee (x_1 \wedge \overline{x_2} \wedge x_3 \wedge x_4)$
$\vee (x_1 \wedge x_2 \wedge x_3 \wedge \overline{x_4}) \vee (\overline{x_1} \wedge \overline{x_2} \wedge x_3 \wedge x_4) \vee (\overline{x_1} \wedge x_2 \wedge \overline{x_3} \wedge x_4)$
$\vee (\overline{x_1} \wedge x_2 \wedge x_3 \wedge \overline{x_4}) \vee (x_1 \wedge \overline{x_2} \wedge \overline{x_3} \wedge x_4) \vee (x_1 \wedge \overline{x_2} \wedge x_3 \wedge \overline{x_4})$
$\vee (x_1 \wedge x_2 \wedge \overline{x_3} \wedge \overline{x_4}) \vee (\overline{x_1} \wedge \overline{x_2} \wedge \overline{x_3} \wedge x_4) \vee (\overline{x_1} \wedge \overline{x_2} \wedge x_3 \wedge \overline{x_4}) \vee$
$(\overline{x_1} \wedge x_2 \wedge \overline{x_3} \wedge \overline{x_4}) \vee (x_1 \wedge \overline{x_2} \wedge \overline{x_3} \wedge \overline{x_4}) \vee (\overline{x_1} \wedge \overline{x_2} \wedge \overline{x_3} \wedge \overline{x_4})$

In CNF:

$$(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_3 \vee \bar{x}_4) \wedge (\bar{x}_1 \vee x_2 \vee \bar{x}_3 \vee \bar{x}_4)$$
$$\wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3 \vee x_4) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3 \vee x_4) \wedge$$
$$(x_1 \vee x_2 \vee \bar{x}_3 \vee \bar{x}_4) \wedge (x_1 \vee \bar{x}_2 \vee x_3 \vee \bar{x}_4) \wedge$$
$$(x_1 \vee \bar{x}_2 \vee \bar{x}_3 \vee x_4) \wedge (\bar{x}_1 \vee x_2 \vee x_3 \vee \bar{x}_4) \wedge$$
$$(\bar{x}_1 \vee x_2 \vee \bar{x}_3 \vee x_4) \wedge (x_1 \vee x_2 \vee x_3 \vee \bar{x}_4) \wedge$$
$$(x_1 \vee x_2 \vee \bar{x}_3 \vee x_4) \wedge (x_1 \vee \bar{x}_2 \vee x_3 \vee x_4) \wedge$$
$$(\bar{x}_1 \vee x_2 \vee x_3 \vee x_4) \wedge (x_1 \vee x_2 \vee x_3 \vee x_4)$$

In Sem Portion:

Chapter 1 and

Chapter 2.