LECTURE 21

Stresses due to fluid pressure in thin cylinders

- **Introduction**
- **Circumferential Stress or Hoop Stress.**
- **Longitudinal Stress**
- **Maximum Shear Stress**
- **Evaluation of Strain**

INTRODUCTION:

In many engineering applications, cylinders are frequently used for transporting or storing of liquids, gases or fluids.

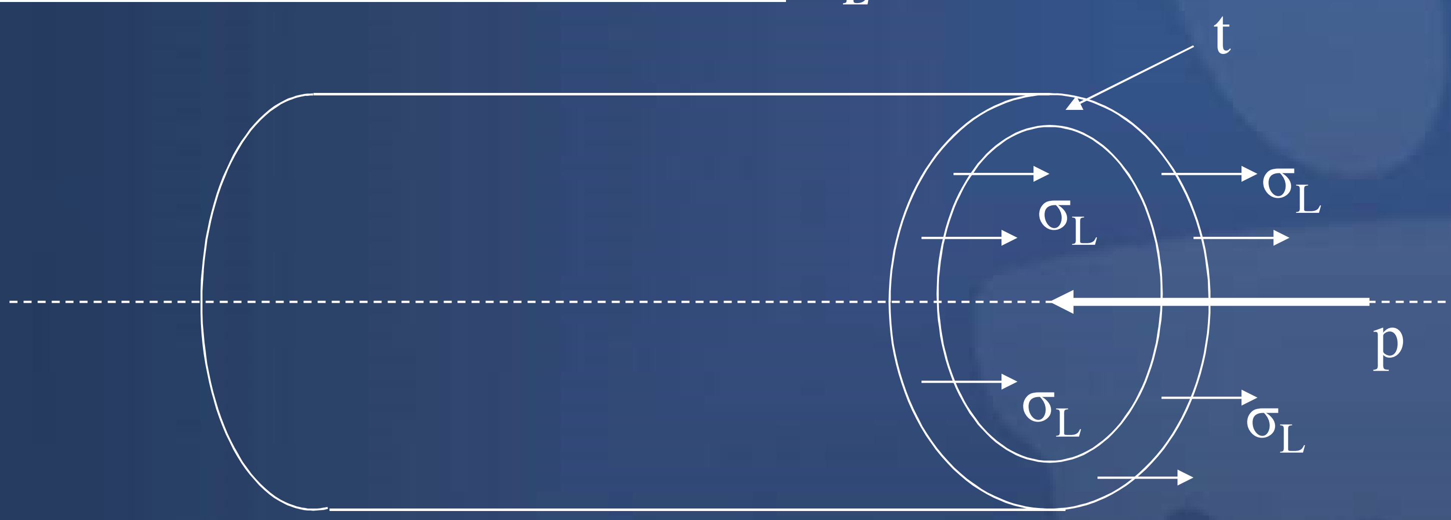
Eg: Pipes, Boilers, storage tanks etc.

These cylinders are subjected to fluid pressures. When a cylinder is subjected to internal pressure, at any point on the cylinder wall, A cylinder or spherical shell is considered to be thin when the metal thickness is small compared to internal diameter. i. e., when the wall thickness, ‘t’ is equal to or less than ‘d/20’, where ‘d’ is the internal diameter of the cylinder or shell, we consider the cylinder or shell to be thin, otherwise thick.

1. CIRCUMFERENTIAL or HOOP STRESS (σ_C):

$$\sigma_c = \frac{pd}{2t}$$

2. LONGITUDINAL STRESS (σ_L):

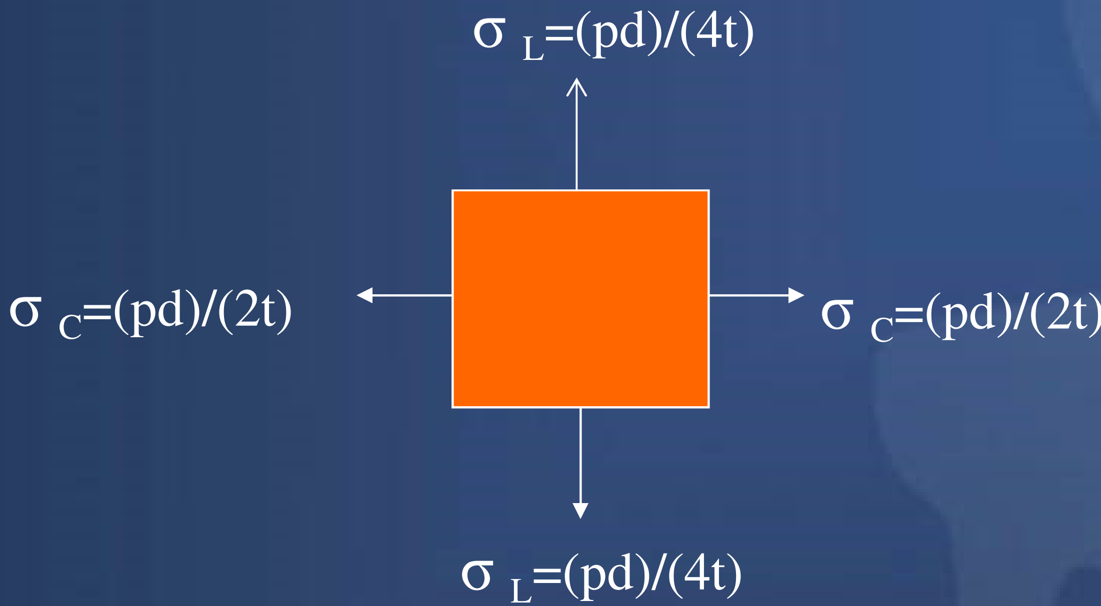


$$\sigma_L = \frac{pd}{4t}$$

3.MAXIMUM SHEAR STRESS:

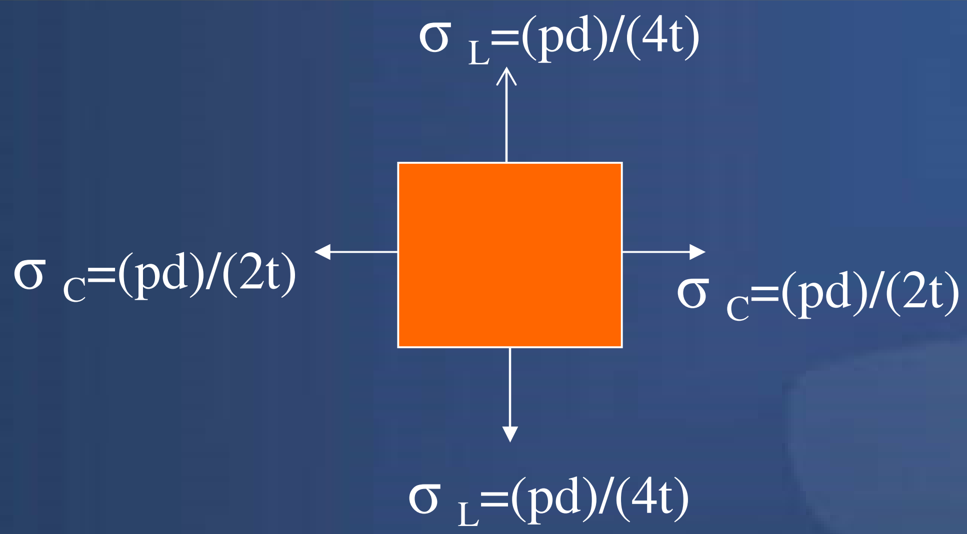
$$\tau_{\max} = \frac{pd}{8t}$$

EVALUATION OF STRAINS



A point on the surface of thin cylinder is subjected to biaxial stress system, (Hoop stress and Longitudinal stress) mutually perpendicular to each other, as shown in the figure. The strains due to these stresses i.e., circumferential and longitudinal are obtained by applying Hooke's law and Poisson's theory for elastic materials.

EVALUATION OF STRAINS



Circumferential strain= $$\varepsilon_c = \frac{\delta_d}{d} = \frac{pd}{4tE}(2-\mu)$$

Longitudinal strain = $$\varepsilon_L = \frac{\delta_L}{L} = \frac{pd}{4tE}(1-2\mu)$$

Volumetric strain = $$\varepsilon_V = \frac{\delta_V}{V} = \frac{pd}{4tE}(5-4\mu)$$

JOINT EFFICIENCY

The cylindrical shells like boilers are having two types of joints namely Longitudinal and Circumferential joints. Due to the holes for rivets, the net area of cross section decreases and hence the stresses increase. If the efficiencies of these joints are known, the stresses can be calculated as follows.

Let η_L=Efficiency of Longitudinal joint

and η_C=Efficiency of Circumferential joint.

Circumferential stress is given by,

$$\sigma_c = \frac{p \times d}{2 \times t \times \eta_L}$$

Longitudinal stress is given by,

$$\sigma_L = \frac{p \times d}{4 \times t \times \eta_C}$$

Note: In longitudinal joint, the circumferential stress is developed and in circumferential joint, longitudinal stress is developed.

Illustrative Problems

Q.9.1

A thin cylindrical shell is 3m long and 1m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12mm, find the circumferential stress, longitudinal stress, changes in diameter, length and volume . Take E=200 GPa and μ= 0.3.

SOLUTION:

1. Circumferential stress, σ_C:

$$\sigma_C = (p \times d) / (2 \times t) = (1.2 \times 1000) / (2 \times 12)$$
$$= \underline{50 \text{ MPa}} \text{ (Tensile).}$$

2. Longitudinal stress, σ_L:

$$\sigma_L = (p \times d) / (4 \times t) = \sigma_C/2 = 50/2$$
$$= \underline{25 \text{ MPa}} \text{ (Tensile).}$$

3. Circumferential strain, ε_c:

$$\varepsilon_c = \frac{(p \times d)}{(4 \times t)} \times \frac{(2-\mu)}{E} = \frac{(1.2 \times 1000)}{(4 \times 12)} \times \frac{(2-0.3)}{200 \times 10^3}$$

$$= \frac{25 \times (2-0.3)}{(200 \times 10^3)} = \underline{2.125 \times 10^{-04}} \text{ (Increase)}$$

Change in diameter, $\delta d = \varepsilon_c \times d = 2.125 \times 10^{-04} \times 1000 = \underline{0.2125}$ mm (Increase).

4. Longitudinal strain, ε_L:

$$\varepsilon_L = \frac{(p \times d)}{(2 \times t)} \times \frac{(1-2 \times \mu)}{E} = \frac{(1.2 \times 1000)}{(2 \times 12)} \times \frac{(1-2 \times 0.3)}{200 \times 10^3}$$

$$= \frac{50 \times (1-2 \times 0.3)}{(200 \times 10^3)} = \underline{5 \times 10^{-05}} \text{ (Increase)}$$

Change in length $= \varepsilon_L \times L = 5 \times 10^{-05} \times 3000 = \underline{0.15}$ mm (Increase).

Volumetric strain, $\frac{dv}{V}$:

$$\frac{dv}{V} = \frac{(p \times d)}{(4 \times t) \times E} \times (5 - 4 \times \mu) = \frac{(1.2 \times 1000)}{(4 \times 12) \times 200 \times 10^{3}} \times (5 - 4 \times 0.3)$$

$$= 4.75 \times 10^{-4} \text{ (Increase)}$$

$$\therefore \text{Change in volume, } dv = 4.75 \times 10^{-4} \times V = 4.75 \times 10^{-4} \times \frac{\pi}{4} \times 1000^{2} \times 3000$$

$$= 1.11919 \times 10^{6} \text{ mm}^{3} = 1.11919 \times 10^{-3} \text{ m}^{3}$$

$$= \underline{1.11919 \text{ Litres}}.$$

Q.9.2

A cylindrical boiler is 800mm in diameter and 1m length. It is required to withstand a pressure of 100m of water. If the permissible tensile stress is 20N/mm^2, permissible shear stress is 8N/mm^2 and permissible change in diameter is 0.2mm, find the minimum thickness of the metal required. Take E = 89.5GPa, and $\mu = 0.3$.

SOLUTION:

$$\text{Fluid pressure, } p = 100\text{m of water} = 100\times9.81\times10^3 \text{ N/m}^2$$
$$= 0.981\text{N/mm}^2 .$$

1. Thickness from Hoop Stress consideration: (Hoop stress is critical than long. Stress)

$$\sigma_C = (p\times d)/(2\times t) \text{ i. e., } 20 = (0.981\times800)/(2\times t)$$
$$\text{Therefore, } t = \underline{19.62} \text{ mm}$$

2. Thickness from Shear Stress consideration:

$$\tau_{max} = \frac{(p \times d)}{(8 \times t)} \quad \text{i.e.,} \quad 8 = \frac{(0.981 \times 800)}{(8 \times t)}$$

$$\therefore t = \underline{12.26mm.}$$

3. Thickness from permissible change in diameter consideration

($\delta d = 0.2mm$):

$$\frac{\delta d}{d} = \frac{(p \times d)}{(2 \times t)} \times \frac{(2 - \mu)}{E}$$

$$\frac{0.2}{800} = \frac{(0.981 \times 800)}{(2 \times t)} \times \frac{(2 - 0.3)}{200 \times 10^3}$$

$$t = \underline{6.67mm}$$

Therefore, required thickness, t = 19.62 mm.

Additional Tutorial Problems

AT1. A cylindrical boiler is 800 mm in diameter and 1 m length. If the permissible tensile stress is 15 N/mm^2, permissible shear stress is 10 N/mm^2 and permissible change in diameter is 0.25 mm, find the pressure to be borne by the cylinder if the thickness of the metal is 10mm. Take E = 90 GPa, and $\mu = 0.28$. Compute the change in length for the pressure determined. (Ans: 0.0367 mm)

AT2. At a point in a thin cylinder subjected to internal fluid pressure, the value of hoop strain is 600×10^{-4} (tensile). Compute the hoop and longitudinal stresses. How much is the percentage change in the volume of the cylinder? Take E=200 GPa and $\mu = 0.28$.

Summary

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