Principle of inclusion & exclusion:-

$N(a_1'a_2'\dots a_n')$ ← No of cases where none of the prop a_i are satisfied

$$N(a_1'a_2'\dots a_n') = T - \sum N(a_i) + \sum N(a_i a_j) - \sum N(a_i a_j a_k) + \dots + (-1)^n N(a_1 a_2 \dots a_n)$$

* No of derrangements of n digits 1, 2, ... n

$$\boxed{n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}}$$, But its $\frac{n!}{e}$ when n is large

* No of arrangements of the digits 1, 2, 3, 4, 5 s.t none of the dig is in its proper positⁿ

soln

ex:- 21435 ✗ 21534 ✓

$$n! \sum_{k=0}^{n} \frac{(-1)^k}{k!} \Rightarrow 5! \sum_{k=0}^{5} \frac{(-1)^k}{k!} = 120\left[\frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \dots + \frac{(-1)^5}{5!}\right]$$

$$= 120\left[1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right] = 120 \times \frac{11}{30}$$

$$= 44$$

① How many +ve integers ≤ 70 are relatively prime to 70?

solⁿ

The integers (a,b) are relatively prime, $\gcd(a,b)=1$

They don't have any common divisor other than 1.

ex:- $(3,70)=1$

prime factorization of $70 \Rightarrow 2,5,7$

The nos which are not divisible by 2,5,7 are all relatively prime to 70.

Thus, I've to calculate the no of elts which are not divisible 2,5,7

$a_1 \rightarrow$ the prop that the no is $\div$ble by 2

$a_2 \rightarrow$ $\div$ble by 5

$a_3 \rightarrow$ $\div$ble by 7

$$N(a_1'a_2'a_3') = ?$$

$N(a_1)$ = No of elts $\div$ble by 2

$$= \left\lfloor \frac{70}{2} \right\rfloor = 35$$

$N(a_2)$ = No of elts $\div$ble by 5 $= \left\lfloor \frac{70}{5} \right\rfloor = 14$

$$N(a_3) = \left\lfloor \frac{70}{7} \right\rfloor = 10$$

$N(a_1 a_2)$ = Nos which are $\div$ble by 2 & 5

$$= \left\lfloor \frac{70}{2\cdot 5} \right\rfloor = 7$$

$N(a_2 a_3)$ = Nos which are $\div$ble by 5 & 7

$$= \left\lfloor \frac{70}{5\cdot 7} \right\rfloor = 2$$

$$N(a_1 a_2 a_3) = \left\lfloor \frac{70}{2\cdot 5\cdot 7} \right\rfloor = 1$$

$$N(a_3 a_1) = \left\lfloor \frac{70}{7\cdot 2} \right\rfloor = 5$$

$$N(a_1' a_2' a_3') = T - \{N(a_1) + N(a_2) + N(a_3)\}$$
$$+ \{N(a_1 a_2) + N(a_2 a_3) + N(a_1 a_3)\}$$
$$- N(a_1 a_2 a_3)$$

$$= 70 - \{10 + 14 + 35\} + \{7 + 2 + 5\}$$
$$- \{1\}$$

$$= \underline{\underline{24}}$$

(ans)

* S.T the propotion of the permutations of $1, 2, \ldots n$ which contains no consecutive pair $(i, i+1)$ for any i, is approximately equal to $\frac{n+1}{e}$

Soln

Permutatns in which $(i, i+1)$ are not adjacent for every (i)

The elt $(i+1)$ is never next to the elt i

$\underline{\underline{1, 2345}}$ $\begin{cases} 13245 \times \\ 14325 \checkmark \end{cases}$

$N(a_1' a_2' \ldots a_{n-1}')$

$a_i \leftarrow$ the prop that $(i, i+1)$ occur consecutively

$N(a_i) \leftarrow$ No of arrangements of $1, 2, \ldots n$ where $(i, i+1)$ are together and adjacent

i	$i+1$			$\ldots$

$N(a_i) = (n-1)!$ {considering $(i, i+1)$ as one block and then arrange the rem $(n-1)$ elts}

$N(a_i a_j) = (n-2)!$

$N(a_i a_j a_k) = (n-3)!$

$\vdots$

$N(a_1 a_2 \cdots a_n) = 1$

$\therefore$

$$N(a_1' a_2' \cdots a_n') = T - \sum N(a_i) + \sum N(a_i a_j) \cdots + (-1)^n N(a_1 \cdots a_n)$$

$(1,2)\ (3,4)$
$(2,3 \cdot (n-1, n)$

$$= n! - {}^{n-1}C_1 (n-1)! + {}^{n-1}C_2 (n-2)! - {}^{n-1}C_3 (n-3)! \cdots + (-1)^n 1$$

$$= n! - (n-1)(n-1)! + \frac{(n-1)(n-2)}{2!}(n-2)! - \frac{(n-1)(n-2)(n-3)}{3!}(n-3)! \cdots + (-1)^n$$

$$= (n-1)! \left[n - (n-1) + \frac{(n-2)}{2!} - \frac{(n-3)}{3!} \cdots + \frac{(-1)^n}{(n-1)!} \right]$$

$$= (n-1)!\left[\cancel{n} - (\cancel{n}-1) + \frac{(n-2)}{2!} - \frac{(n-3)}{3!} + \cdots + \frac{(-1)^n}{n!}\right]$$

$$= (n-1)!\left[1 + \left(\frac{n}{2!} - \frac{2}{2!}\right) - \left(\frac{n}{3!} - \frac{3}{3!}\right) + \cdots + (-1)^n\left(\frac{n}{(n-1)!} - \frac{(n-1)}{(n-1)!}\right)\right]$$

As $n \to \infty$

$$= (n-1)!\left[\left(1 - \frac{2}{2!} + \frac{3}{3!} \cdots\right) + n\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \cdots\right)\right]$$

$$= (n-1)!\left[\left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} \cdots\right) + n\left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} \cdots\right)\right]$$

$$= (n-1)!\left\{e^{-1} + n e^{-1}\right\}$$

$$= (n-1)!\left(\frac{1}{e} + \frac{n}{e}\right)$$

$$= \frac{(n+1)}{e}(n-1)!$$

$= RHS$

PARTITIONS AND COMPOSITIONS

6 → 6

1+5 , 5+1 , 3+3 4+2 , 2+4

1+2+3

Types :- ① Partition → unordered division

② Composition → ordered division

Partitions of 5 :

5, 1+4, 2+3, 1+1+3, $\underbrace{2+2+1}_{221}$, 1+1+1+2

1+1+1+1+1

Here 2+3 and (3+2) are same, as the order is not considered

Notation

The partion 2+3+1 the integer 6 can be written as 6 = 321

i) omit the + sign

ii) largest part is written first.

6 = 21111

partitions of 4 ⟹ one part → 4.

two part → 22, 31.

threeparts → 211

four parts → 111.1

∴ There are 5 partitions of the integer '4' compositnˢ of 4 ⇒ into 1 part : 4

2 parts : 14, 41, 2, 2

3 parts : 112, 211, 121

4 parts : 1111

* For this kind of representatn

→ Do we need order

→ How many parts do you nee.

Partition of the integer 7 into 3 parts

421, 511, 322,

Compositn of the integer 7 into 3 parts :

422, 242, 224, 511, 151, 115, 322, 223, 232

Count the no of partitions of 'n'

" " compositn of n

} logical

gf

Counting the no of compositions of the integer n

Composit^n of 5 :-
Thought as writing 1 (5 times)

|{1 1|{1 {1 ⇒ 1 2 1 1

1 1{1{1{1 ⇒ 2 1 1 1

{place the marker
&
dont place

∴ The total no of compositions of the integer 'n' is equal to :- 2^{n-1}

The no of composit^ns of 5 ⇒ 2^4 ⇒ 16

Counting the no of compositions of the integer 'n' into m parts

I have to place (m-1) markers, which will give rise to m parts
∴ I've to select (m-1) posit^ns to place the (m-1) markes out of total of (n-1) places

$${}^{n-1}C_{m-1}$$

Composit^n of 10 into 4 parts :-

ex :- 2251

|1|2|3{|4|5|6{|7|8{|9|

select 3 places to place the markes

Compositions

① No of compositions of the integer 'n'

$$\Rightarrow 2^{n-1}$$

② No of compositions of the integer 'n' into 'm' parts

$$\Rightarrow {}^{n-1}C_{m-1}$$