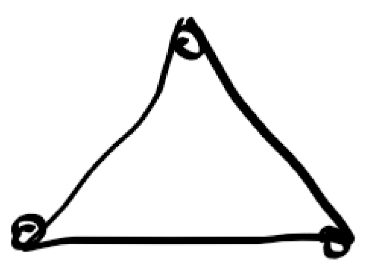
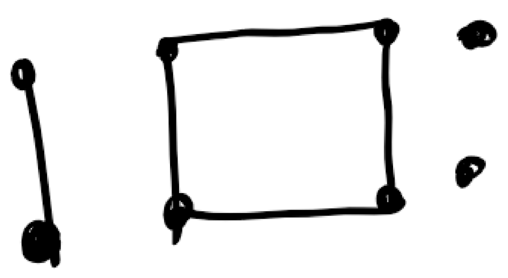
Connected graphs :-

A graph is said to be connected if there is a path b/w every pair of vertices. Else it's called a disconnected graph

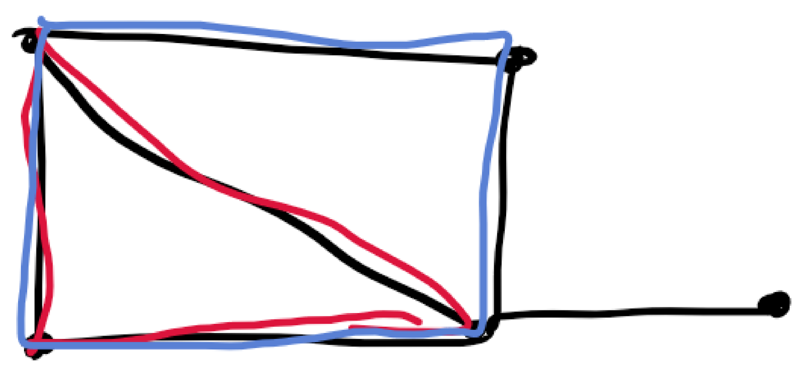
Connected component : of a graph is a maximal connected subgraph

 → connected graph, has only one component

 → Disconnected graph, has 4 components

Girth of a graph : Length of the shortest cycle present in the graph (if any). Denoted by $g(G)$

Circumference : Length of the longest cycle in G. Denoted by $c(G)$



$g(G) = 3$

$c(G) = 4$

(length → no of edges)

Distance $d(u,v)$ b/w the vertices u & v :

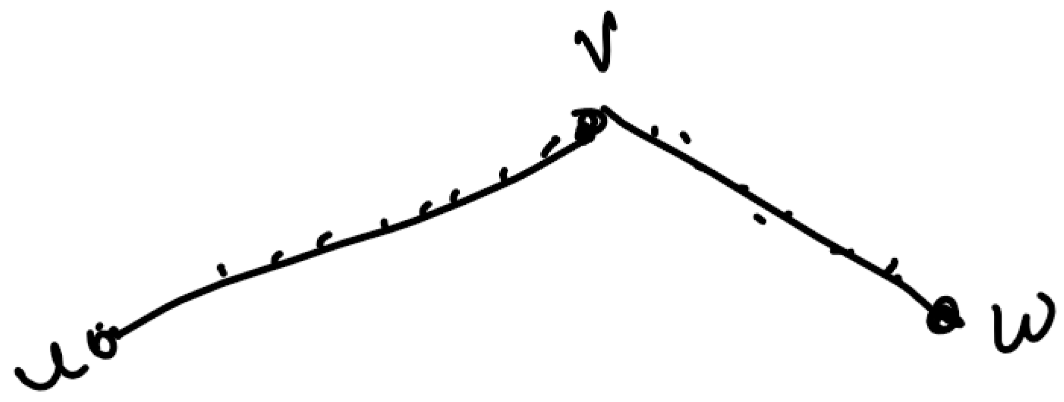
The distance $d(u,v)$ b/w u & v is the length of the shortest path joining them if exists. Otherwise $d(u,v) = \infty$

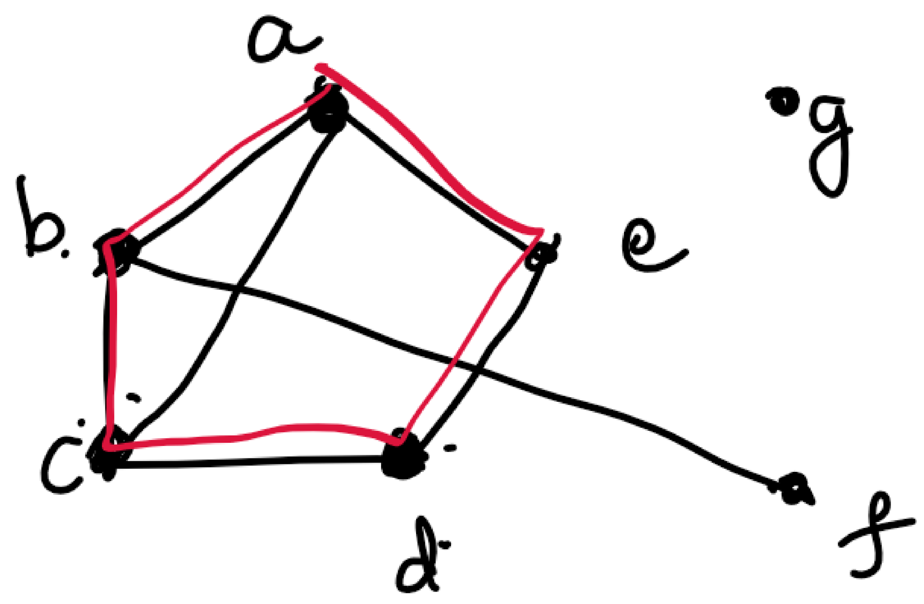
In a connected graph, $d(u,v)$ satisfies

i) $d(u,v) \geq 0$ with $d(u,v) = 0$ iff $u = v$

ii) $d(u,v) = d(v,u)$

iii) $d(u,v) + d(v,w) \geq d(u,w)$





$d(a,b) = 1$ $\quad$ $d(b,f) = 1$

$d(a,d) = 2$ $\quad$ $d(a,g) = \infty$

$\delta(G) = 0$ $\quad$ $\Delta(G) = 3$

girth $g(G) = 3$

circum $c(G) = 5$

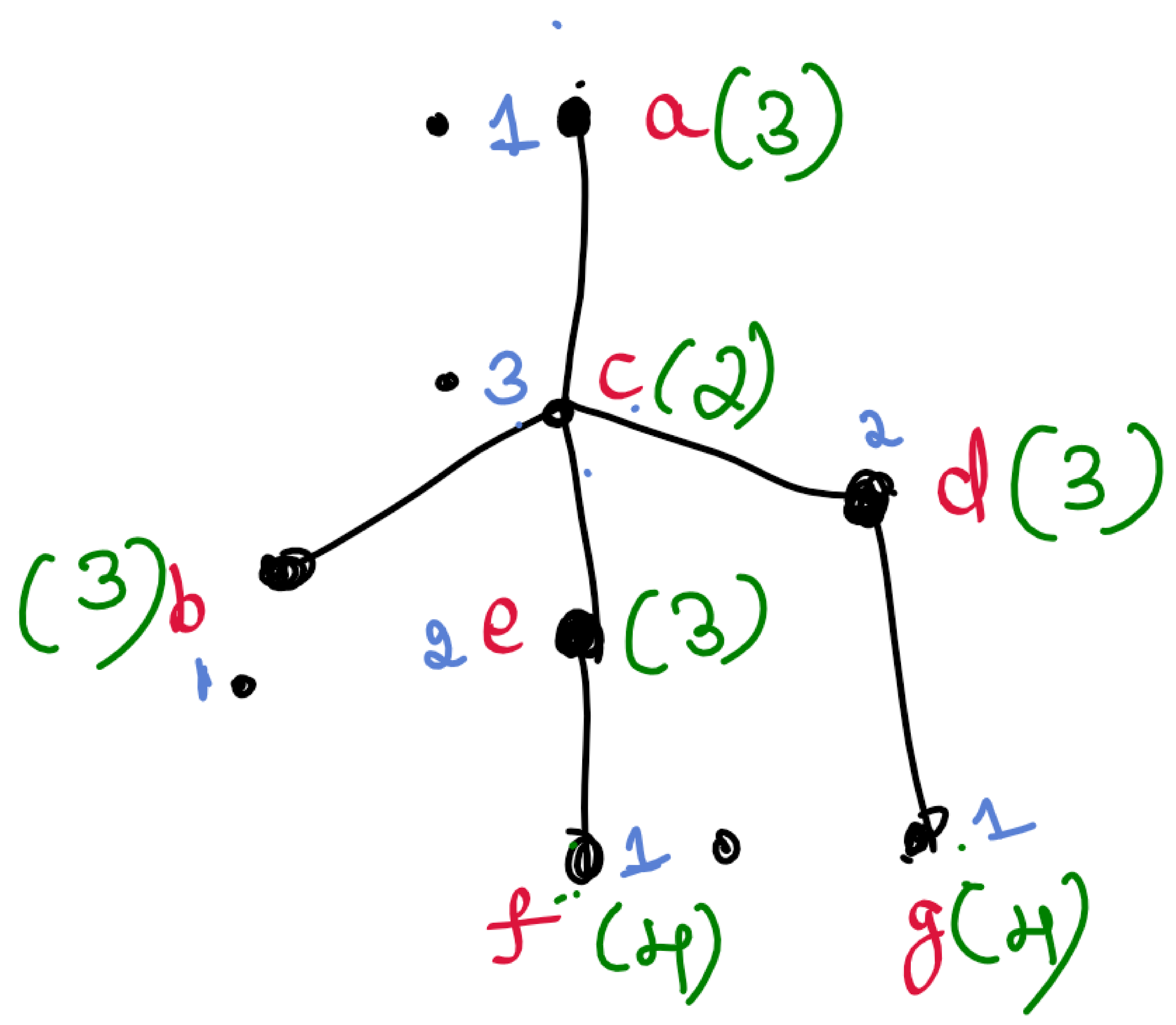
The paths b/wn a & d → abcd
acd
aed

geodesic :- shortest path

Eccentricity of a vertex ($e(v)$) : Distance of v from the farthest vertex

Radius of a graph : minimum eccentricity

Diameter of a graph : maximum eccentricity
or
length of the longest geodesic ie
$\max\{d(u,v)\}$



radius$(G) = 2$

diameter$(G) = 4$

$\delta(G) = 1$

$\Delta(G) = 3$

minimum degree ($\delta(G)$)

maximum degree ($\Delta(G)$).

Theorem

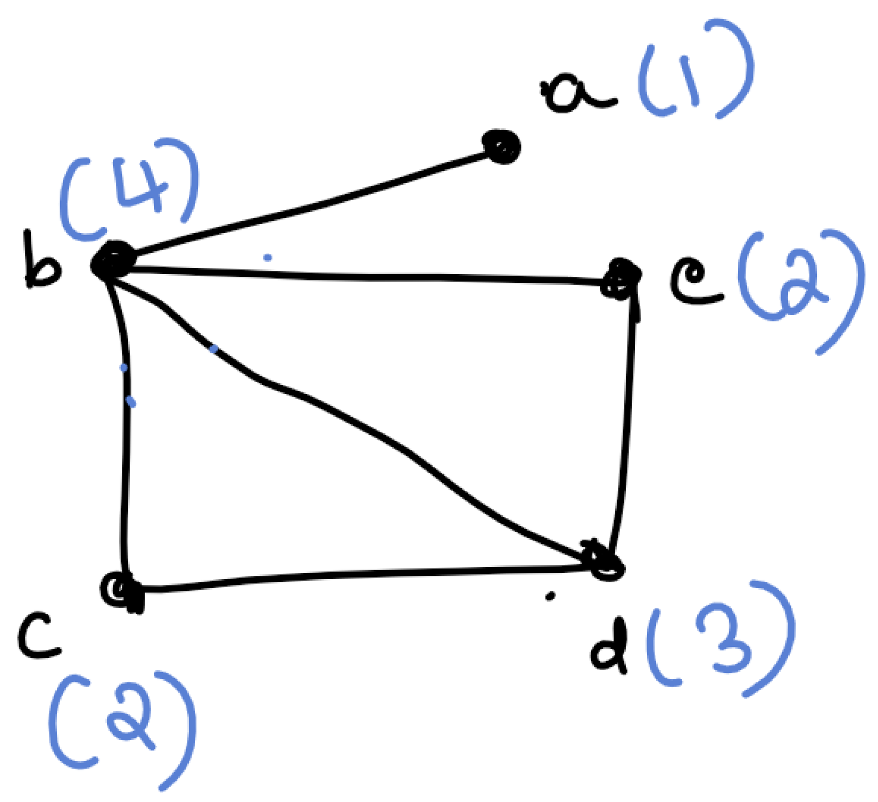
Let G be (p,q) graph, Then $\sum_{v \in V(G)} deg(v) = 2q$

no of vertices → p, no of edges → q

(sum of deg of all vertices = Twice the no of edges)

Proof

Every edge is incident of 2 vertices & every edge contributes degree 2 to the $\sum_{v \in V(G)} deg(v)$



$p = 5$ $\quad q = 6$

$$\sum deg(v) = deg(a) + deg(b) + deg(c) + deg(d) + deg(e)$$
$$= 1 + 4 + 2 + 3 + 2 = 12 = 2(6)$$

Corollary:

In any graph, the no of vertices of degree is even

(No of odd deg vertices in any graph is even)

Proof:-

$S_e \to$ s degrees of all even deg vertices

$S_o \to$ s of degrees of all odd deg vertices

From prev thm, So $\underbrace{S_e}_{\text{even}} = \underbrace{2q}_{\text{even}}$ (∵ prev thm)

$S_o = 2q - S$ $= \text{even} - \text{even}$ $= \text{even}$

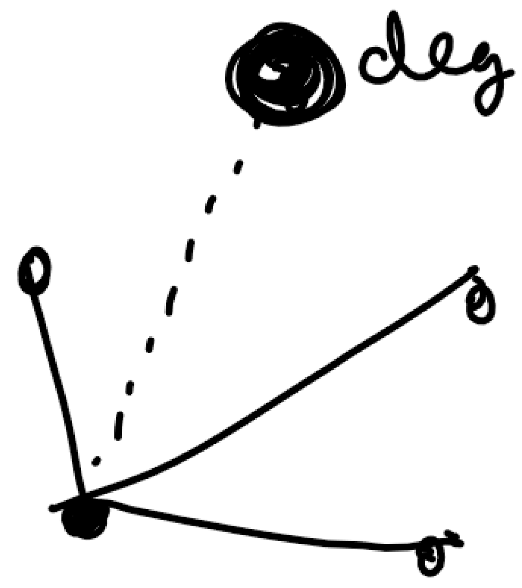
Since S_o is even & each term in S_o is odd. ⇒ there are even of the s

Perfect graph : if no two vertices in the graph are of same degree.

(Draw a graph on 5 vertices s.t each vertex has distinct degree) *.

(Draw a perfect graph on 5 vertices)

deg(?)



$0 \le \deg(v) \le 4$

a b c d

0, 1, 2, 3, 4 (e)

1 2 3 4

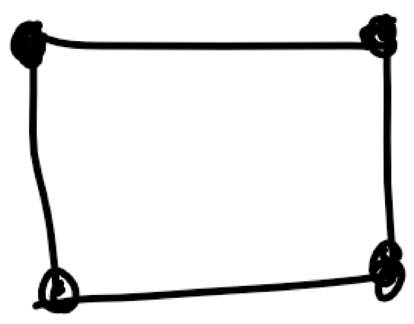
Show that no graph is perfect. Pigeon hole principle

(Try)

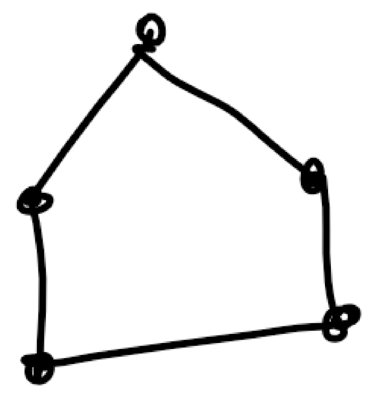
Types of graphs

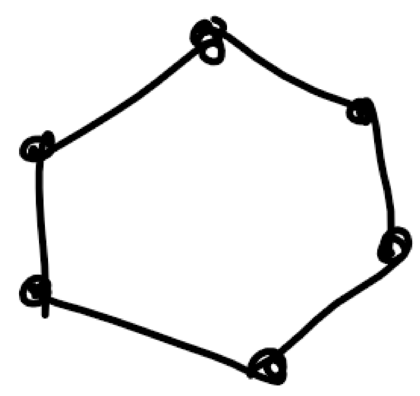
① **Regular graph** :- Every vertex is of same degree.

if every vertex has degree r, then we call it r-regular graph

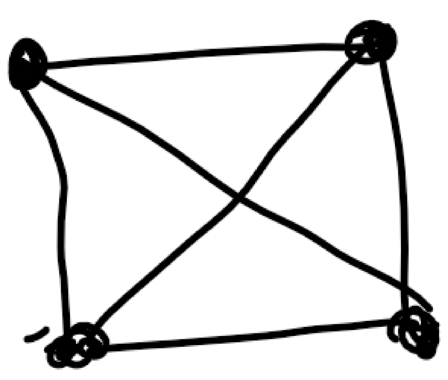


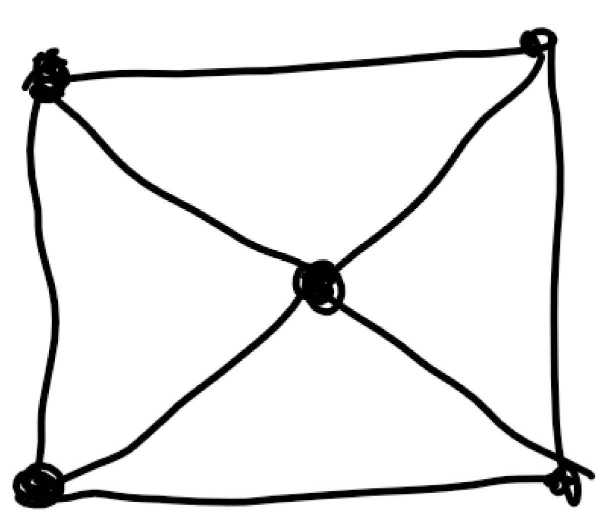
2-regular graph





3-regular graph

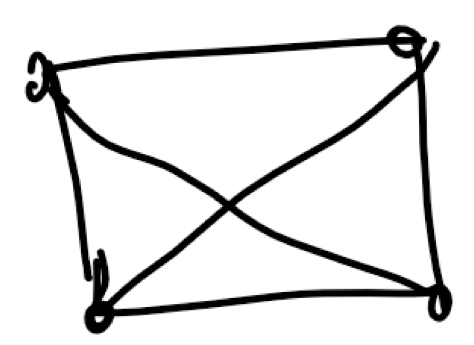




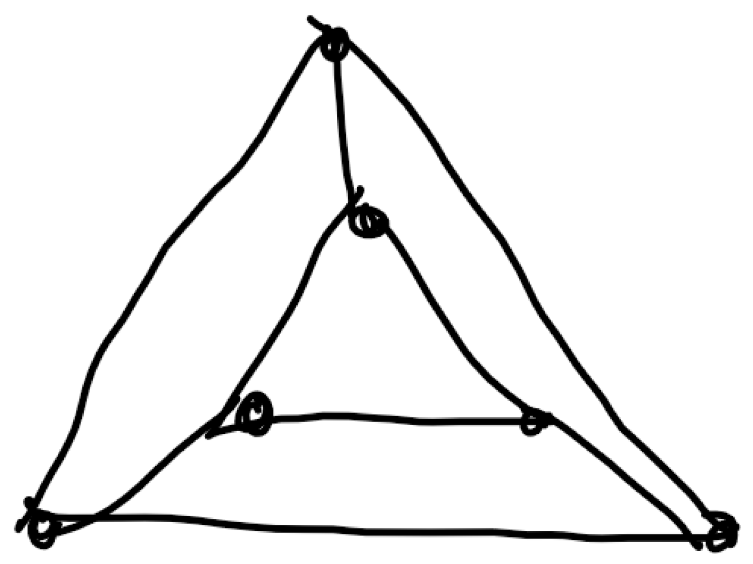
Not a regular graph

All cycles are 2-regular

(2) Cubic graph :- 3-regular graph



cubic graph on 4 vertices



cubic graph on 6 vertices

* (Draw a cubic graph on 5 vertices) $\longrightarrow$ Not possible

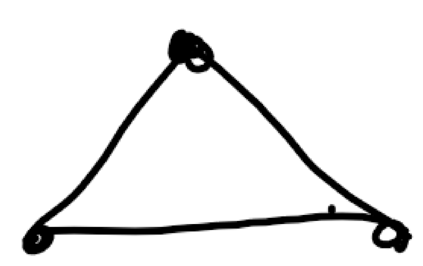
* (No of vertices in any cubic graph must be even

Bcz no of odd deg vertices must be even

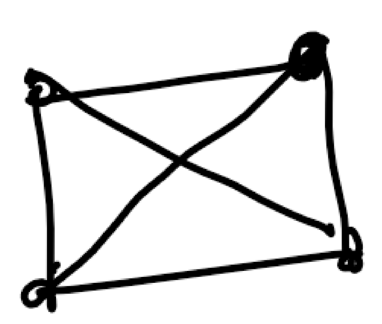
(3) Complete graph; if every pair of vertices are adjacent

A complete graph on p vertices $\longrightarrow$ K_p

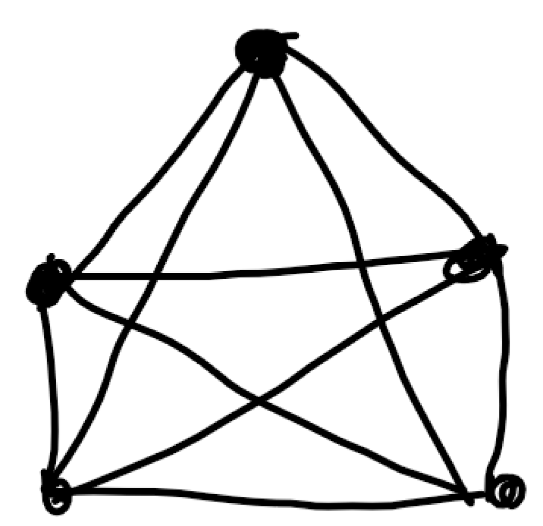
K_2



K_3



K_4



K_5

* K_p is $(p-1)$ - regular graph

* No of edges in K_p = ?

$$2q = \text{Sum of deg}$$

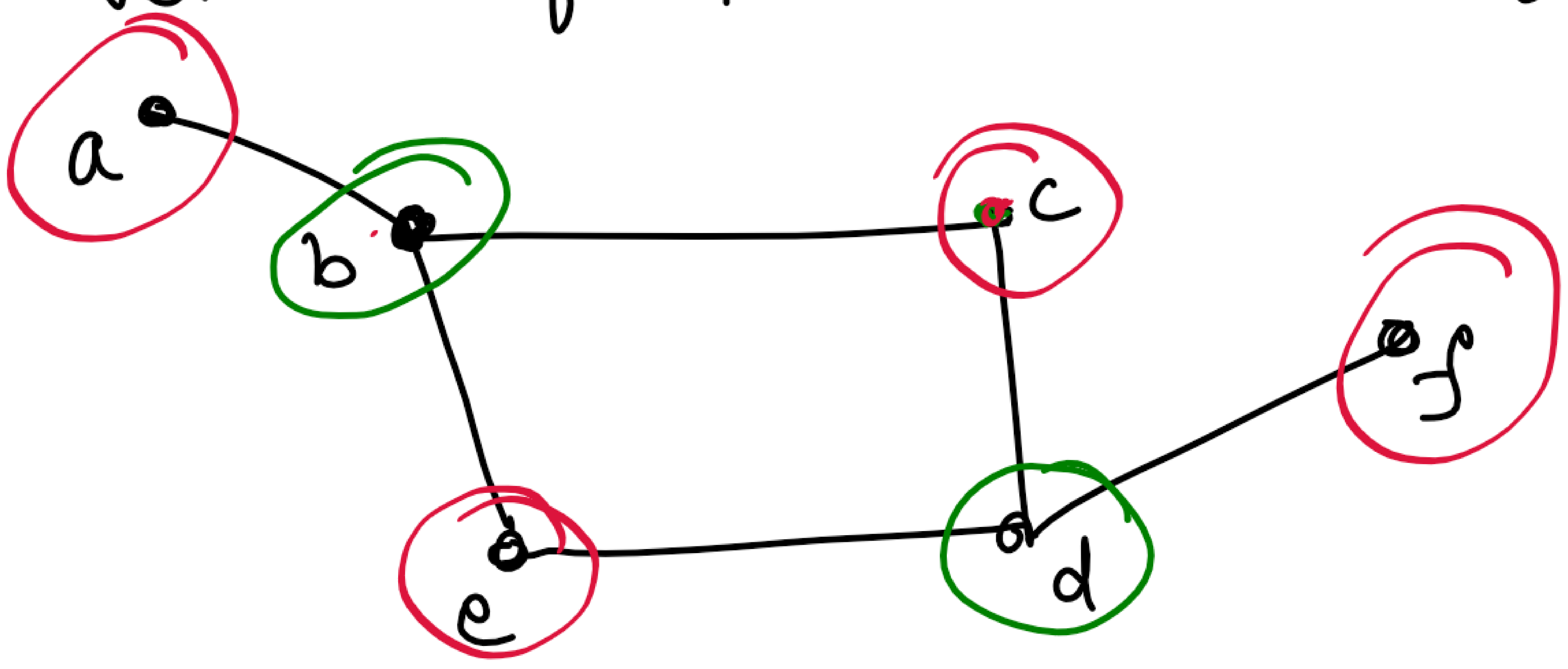
$$= \underbrace{(p-1) + (p-1) + \cdots + (p-1)}_{p}$$

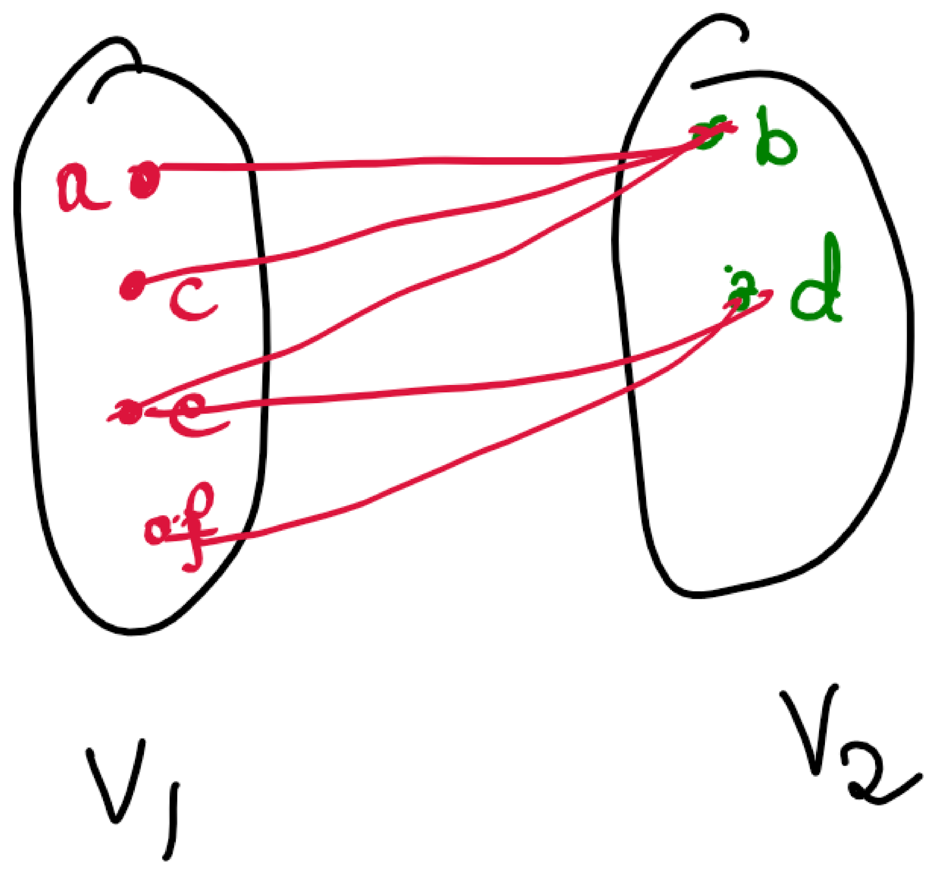
$$2q = p(p-1)$$

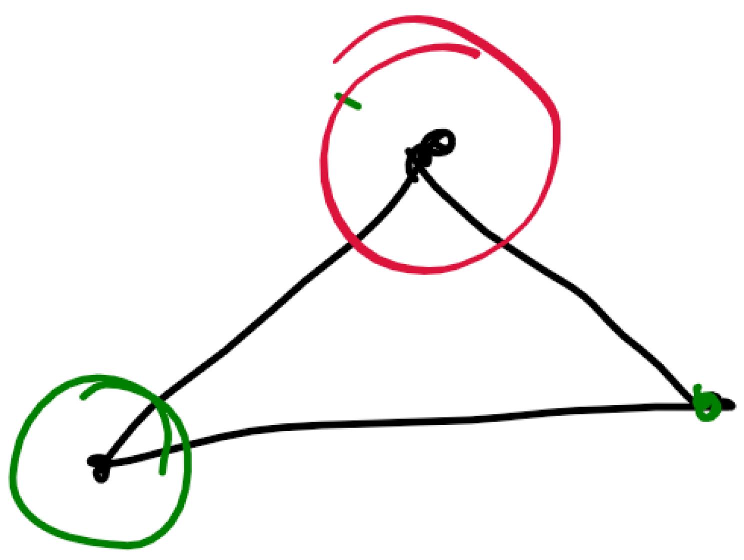
$$\therefore \boxed{q = \frac{p(p-1)}{2}}$$

(4) Bipartite graph:

A bigraph or a bipartite graph is the one whose vertex set $V(G)$ can be partitioned into two disjoint subsets V_1 & V_2 such that every edge of the graph joins the vertices of V_1 with vertices of V_2





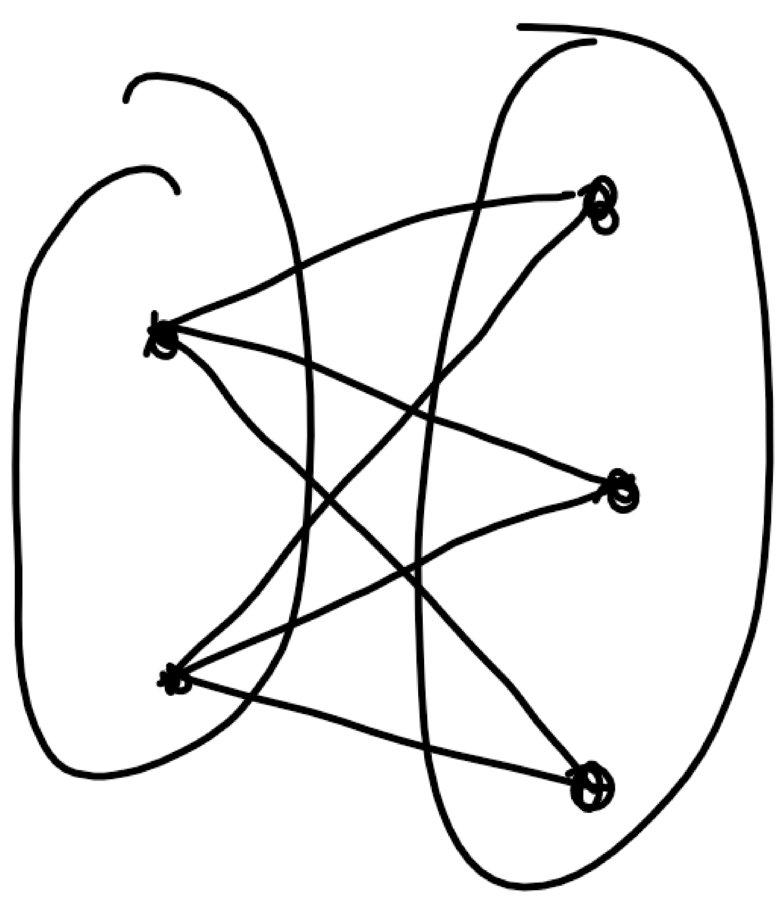


Not a bipartite graph

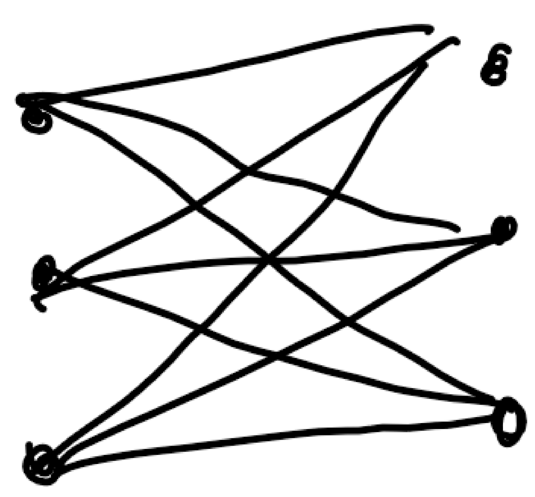
Complete bipartite graph :- If the graph contains every possible edges joining vertices of V_1 with vertices of V_2

$$|V_1| = p \;,\; |V_2| = q, \longrightarrow K_{p,q}$$

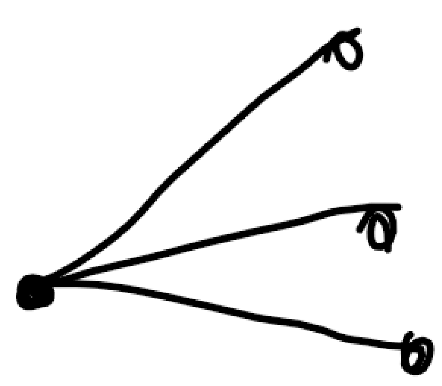
$K_{2,3}$



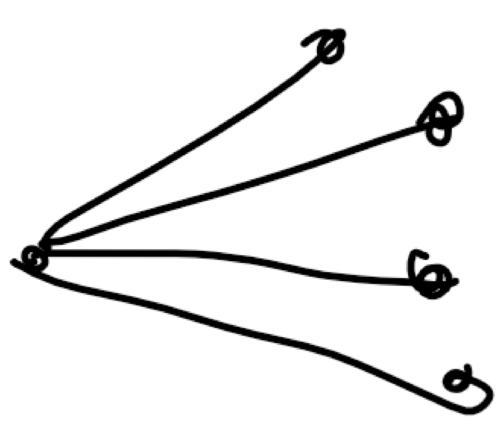
$K_{3,3}$



Star graph: $K_{1,p}$

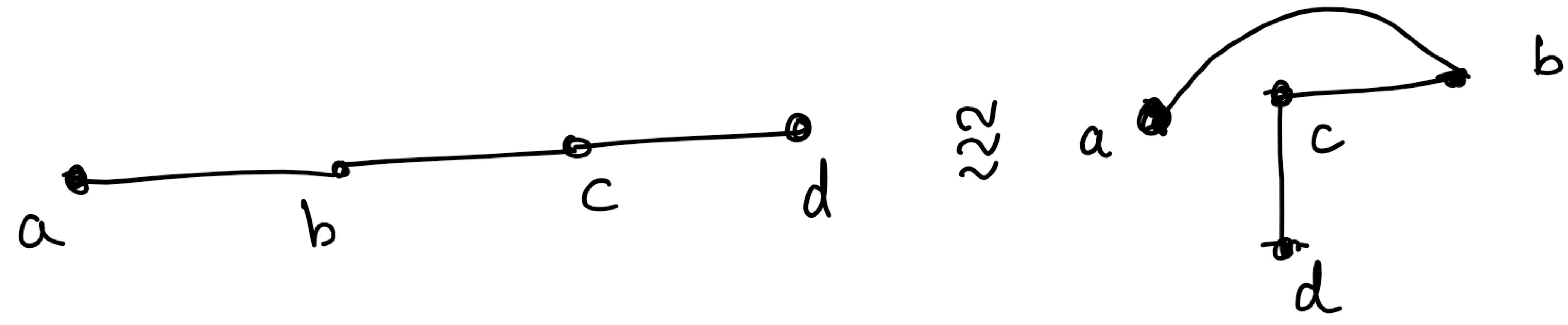


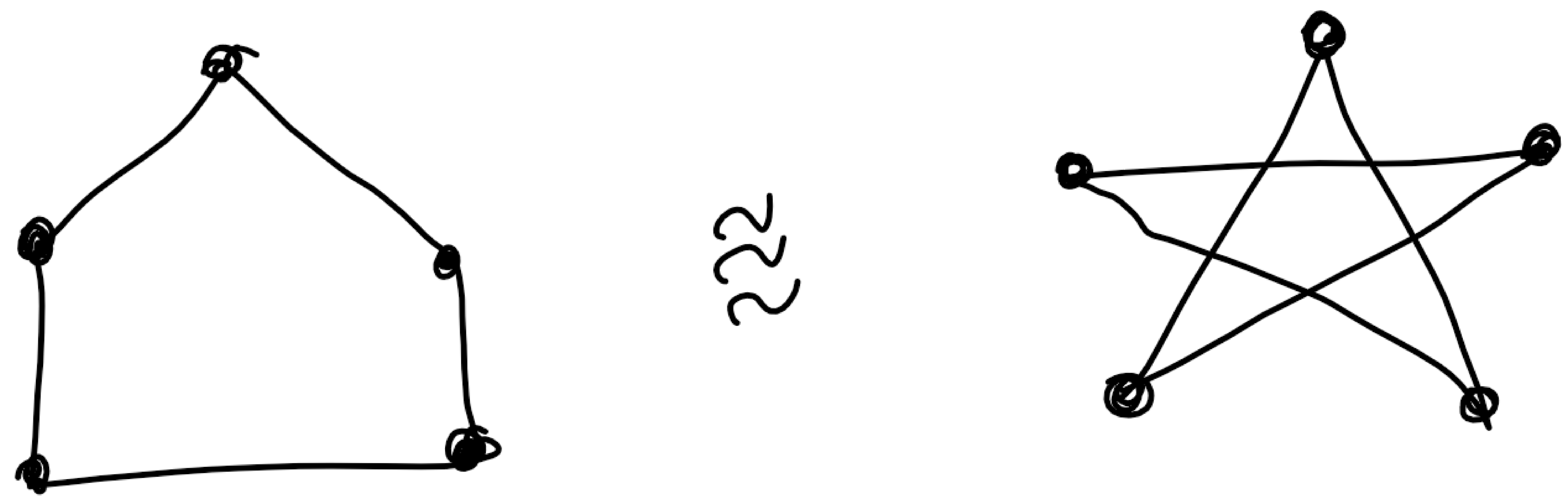
$K_{1,3}$



$K_{1,4}$

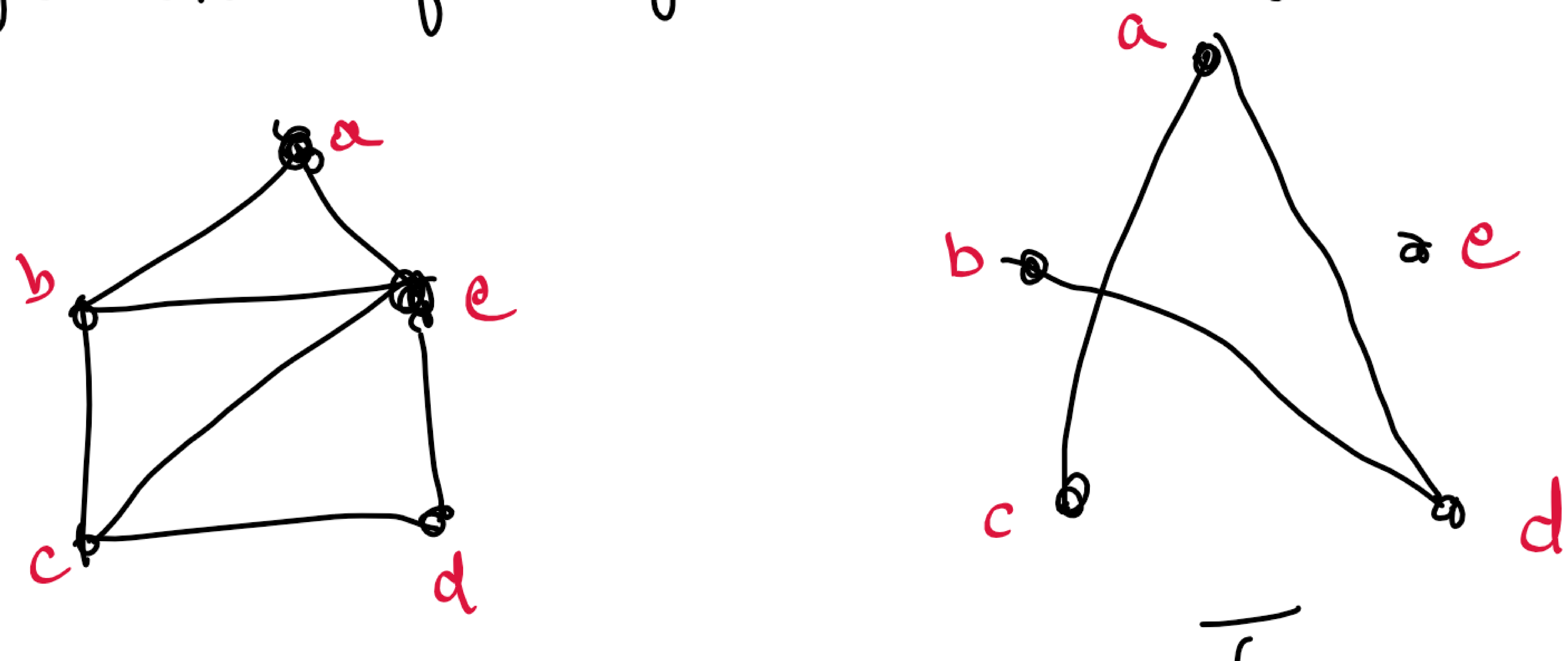
Two graphs are isomorphic : Two graphs G_1 & G_2 are isomorphic if there is one-to-one correspondence b/wn the vertices & b/wn the edges preserving the incidence.





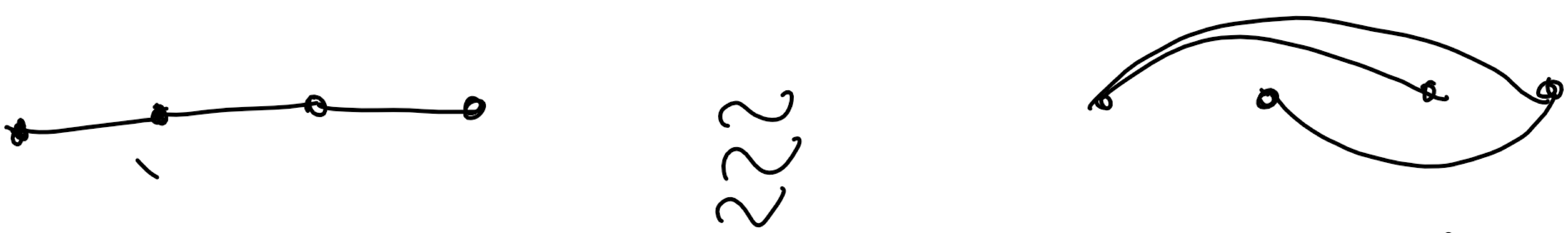
Complement of a graph

The complement of G, denoted by $\bar{G}$ is a graph with $V(\bar{G}) = V(G)$ and any 2 vertices in $\bar{G}$ are adjacent if they are not adjacent in G

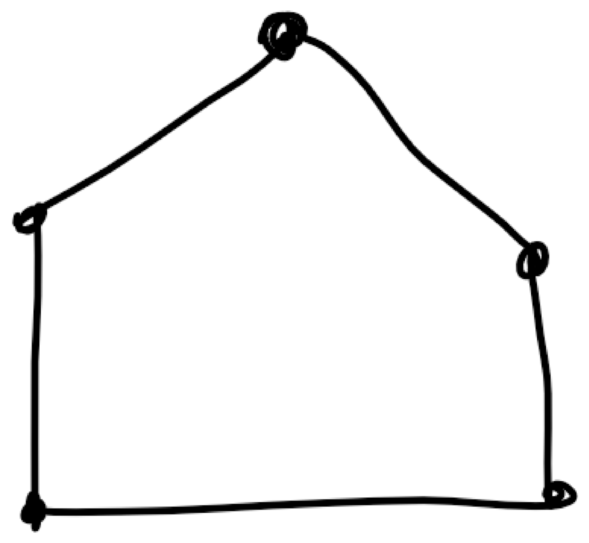


G $\qquad$ $\bar{G}$

Self complementary graph :- if $\bar{G}$ is isomorphic to G

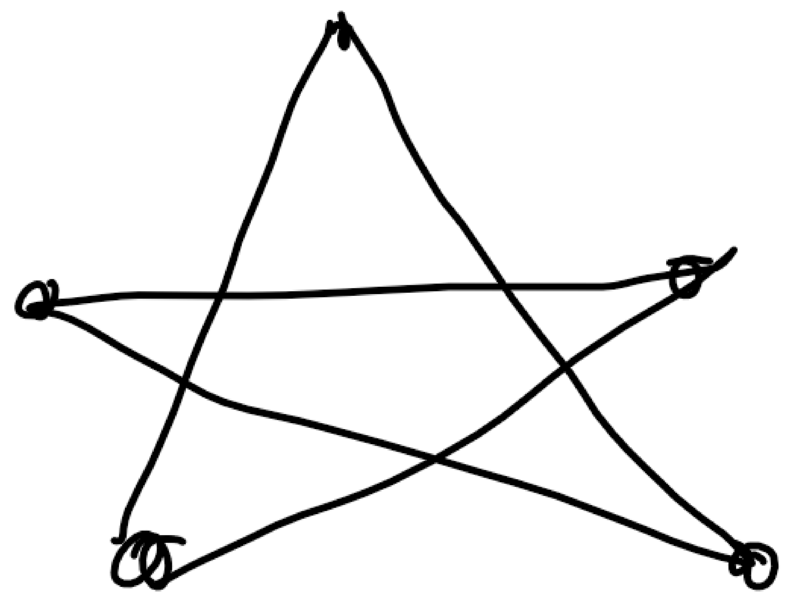


$G = P_4$ $\qquad$ $\bar{G} = P_4$



C_5

≋



$\overline{C_5} = C_5$

$P_4 \rightarrow$ smallest self comp graph