Propositional and Predicate Calculus

Propositional calculus:

A proposition is a declarative sentence that is either true or false.

Eg: It rained yesterday

'True' or 'False' are called truth values of the propositions and are usually denoted by T and F respectively.

A proposition that is true under all circumstances is called a "Tautology".

Eg: 15 is divisible by 3

A proposition that is false under all circumstances is called "Contradiction"

Eg: -3 is a natural number

Two or more propositions can be combined using words like 'and', 'or', "if, then", "iff" etc. These are called "logical connectivities"

A proposition having one or more logical connectivities is called a Compound Proposition. Otherwise is called Simple / Atom.

Two propositions p and q are said to be Equivalent if when p is T, q is also T and when p is F, q is also F & conversely.

① Negation:

Let p be a proposition. We define Negation of p denoted by $\neg p$ to be a proposition which is true when p is false & false when p is true.

$\neg$	p	$\neg p$
	T	F
	F	T

② Disjunction:

Let p and q be two propositions. The Disjunction of two propositions is denoted by $p \vee q$ (read as p or q) & is defined as follows:

$\vee$	p	q	$p \vee q$
	T	T	T
	T	F	T
	F	T	T
	F	F	F

$\vee \rightarrow$ or

③ Conjunction:

The Conjunction of two propositions denoted by $p \wedge q$ (read as p and q) & is ~~denoted~~ defined ~~by~~ as follows.

$\wedge$	p	q	$p \wedge q$
	T	T	T
	T	F	F
	F	T	F
	F	F	F

$\wedge \rightarrow$ and

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(4) Conditional: The conditional statement is denoted by $p \to q$ (read as if p then q) & is defined as follows:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

* p is called the first component or ANTECEDENT
q is called the second component or CONSEQUENT

For the conditional $p \Rightarrow q$,

(i) $q \to p$ is called the converse

(ii) $\neg p \to \neg q$ is called the inverse

iii) $\neg q \to \neg p$ is called the contrapositive

p	q	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$	$\neg p \vee q$
T	T	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	T	T
			✓	✓	*	*

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From the truth table, we obserbe that

* $p \to q$ and $\neg q \to \neg p$ are logically equivalent.

* $q \to p$ and $\neg p \to \neg q$ are logically equivalent
  i.e., Inverse & converse are logically equivalent.

* $p \to q$ and $\neg p \vee q$ are logically equivalent.

Eg: There are two restaurants next to each other. One has a sign that says "Good food is not cheap" and the other has a sign that says, "Cheap food is not good". Are the signs saying the same thing?

Let G : Food is good
C : Food is cheap.

G	C	$G \to \neg C$	$C \to \neg G$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

$p \to q$
T F $\to$ F

$\therefore$ Both are saying the same thing.

5 **Biconditional**:

Let p & q be two propositions.
The biconditional is denoted by $p \leftrightarrow q$ (read as p if and only if q OR p iff q) and is defined as follows:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$\leftrightarrow$

Well formed formulas – (WFF)

A WFF is a formula generated using the following groups:

(i) A statement variable is a WFF

(ii) If 'A' is WFF, then $\neg A$ is also WFF

(iii) If 'A' and 'B' are WFF's then $A \vee B$, $A \wedge B$, $A \rightarrow B$ and $A \leftrightarrow B$ are also WFF.

(iv) A string of symbols consisting of statements variable, connectivities & parantheses is a WFF iff it can be obtained by finitely many applications of the rules (i), (ii) & (iii).

Eg: $p \wedge q$, $\neg(p \wedge q)$, $(\neg(p \rightarrow q)) \vee r$, $((p \rightarrow q) \rightarrow r)$ are WFF.

$p \wedge q \rightarrow r$, $\neg p \rightarrow \wedge r$, $\neg p \wedge q$ are not WFF

this either

$\neg p \wedge q$ or $\neg(p \wedge q)$

$(p \wedge q) \rightarrow r$

$p \wedge (q \rightarrow r)$

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Equivalence of formulas –

Let A & B be two statements formulas and $p_1, p_2, \ldots, p_n$ denote all variables occuring in A and B. If the truth value of A is same as that of B for each of the 2^n possible sets of assignments to the variables $p_1, p_2, \ldots, p_n$. Then A and B are said to be equivalent and we write $A \Leftrightarrow B$

Remark:

Two statements formulas A & B are equivalent iff $A \leftrightarrow B$ is a tautology.
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Table of Equivalences:

① $\neg\neg p \Leftrightarrow p$

② $p \vee q \Leftrightarrow q \vee p$, $p \wedge q \Leftrightarrow q \wedge p$ commutative

③ $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$
$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$ associative

④ $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ distributive
$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

⑤ $p \wedge (p \vee q) \Leftrightarrow p$, $p \vee (p \wedge q) \Leftrightarrow p$ absorption

⑥ $p \wedge p \Leftrightarrow p$, $p \vee p \Leftrightarrow p$ Idempotent

⑦ $p \wedge (\neg p) \Leftrightarrow F$, $p \vee (\neg p) \Leftrightarrow T$

⑧ $p \wedge F \Leftrightarrow F$, $p \vee F \Leftrightarrow p$
$p \wedge T \Leftrightarrow p$, $p \vee T \Leftrightarrow T$

⑨ $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

⑩ $(p \rightarrow q) \Leftrightarrow \neg p \vee q$, $\neg(p \rightarrow q) \Leftrightarrow p \wedge \neg q$

⑪ $(p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$, $(q \rightarrow p) \Leftrightarrow (\neg p \rightarrow \neg q)$