

4A. Solve : $(D^2 - 2D + 4)y = e^x \cos x$

4B. Using double integration, find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$

4C. (i) Find the Laplace transform of $f(t) = \begin{cases} e^{2t} & 0 < t < 1 \\ 2 & t > 1 \end{cases}$

(ii) Find $L^{-1} \left\{ \frac{s+2}{s^2 + 4s + 5} \right\}$

(3 + 3+4)

5A. Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cos e^{-x}$ by the method of variation of parameters.

5B. Investigate the value of λ and μ so that the equations

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + \lambda z = \mu$$

have

(i) no solution (ii) a unique solution (iii) an infinite number of solutions

5C. A spring is such that it would be stretched 6 inches by a 12 pound weight. Let the weight be attached to the spring and pulled down 4 inches below the equilibrium point. If the weight is started with an upward velocity of 2ft/sec describe the motion. No damping or impressed force is present.

(3 + 3+ 4)

6A. Solve : $(1+x)^2 \frac{d^2y}{dx^2} + 1+x \frac{dy}{dx} + y = 2\sin \log(1+x)$.

6B. Obtain the relation between Beta and Gamma functions.

6C. Find the Laplace transform of

$$f(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}, \quad f\left(t + \frac{2\pi}{\omega}\right) = f(t) \quad \forall t$$

(3 + 3+ 4)
