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MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER B.E DEGREE END SEMESTER EXAMINATION- 2009

SUB: ENGG. MATHEMATICS I (MAT – 101)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All questions carry equal marks

1A. Find the n^{th} derivative of

(i) $\dfrac{3x^2-3x-5}{(x-1)^2(2x+3)}$ (ii) $\sinh 2x \cos^2 x \sin 2x$

1B. Trace the following curve with explanations
$y^2a^2 - x^4 = x^2(a^2 + y^2)$, $a > 0$.

1C. Find the reflection of the point (1, 3, 4) through the plane $2x - y + z + 3 = 0$.

(4 + 3+ 3)

2A. If $\cos^{-1}\left(\dfrac{y}{b}\right) = \log\left(\dfrac{x}{n}\right)^n$, then prove that $x^2 y_{n+2} + (2n + 1) xy_{n+1} + 2n^2 y_n = 0$

2B. Evaluate :

(i) $\displaystyle\int_0^\infty \frac{x^2}{\left(\sqrt{1+x^6}\right)^7}dx$ (ii) $\displaystyle\int_0^2 x^{5/2}\sqrt{2-x}\,dx$

2C. Find the magnitude and shortest distance between the lines $\dfrac{x-3}{1} = \dfrac{y-5}{-5} = \dfrac{z-7}{1}$ and $\dfrac{x+1}{7} = \dfrac{y+1}{-6} = \dfrac{z+1}{1}$.

(3 + 4+ 3)

3A. Find the nature of the following series

(i) $\displaystyle\sum\left(\left(\frac{n+1}{n}\right)^{n+1} - \frac{n+1}{n}\right)^{-n}$ (ii) $\displaystyle\sum\frac{(n+1)^n x^n}{n^{n+1}}$

3B. Sketch and find the area bounded by the curve $r^2 = a^2\cos 2\theta$, $a>0$.

3C. Find the evolute of $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. (4 + 3+ 3)

4A. Evaluate :

(i) $\underset{x\to 0}{\mathrm{Lt}} \dfrac{1+\sin x-\cos x+\log(1-x)}{x\ \tan^2 x}$

(ii) $\underset{x\to 0}{\mathrm{Lt}} \left(\dfrac{\tan x}{x}\right)^{1/x}$

4B. Find the angle between the curves
$r^m = a^m \cos m\theta$, $r^m = a^m \sin m\theta$

4C. The radius of a normal section of a right circular cylinder is 2 units; the axis lies along the straight line $\dfrac{x-1}{2}=\dfrac{y+3}{(-1)}=\dfrac{z-2}{5}$, find its equation.

(4 + 3+ 3)

5A. Find the first three nonzero terms in the expansion of $f(x) = \log \sec x$.

5B. Show that the radius of curvature at any point of the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ is equal to three times the length of the perpendicular from the origin to the tangent.

5C. Find the volume of the solid generated by revolving the curve $x = a(\theta + \sin\theta)$, y a $(1 - \cos\theta)$ about its base.

(4 + 3+ 3)

6A. If $u = F(x-y, y-z, z-x)$, prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$.

6B. State and prove Lagrange's mean value theorem.

6C. If the sides of a plane triangle ABC vary in such a way that its circum – radius remains a constant, then prove that

$$\frac{\delta a}{\cos A}+\frac{\delta b}{\cos B}+\frac{\delta c}{\cos C}=0$$

(3 + 3+ 4)

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