# Transient Analysis

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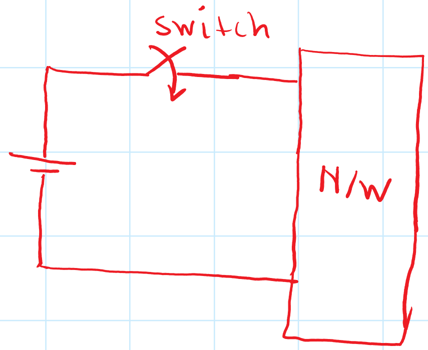


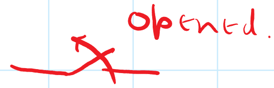


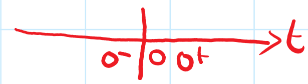


* $t = 0^-$ Just before the switch is opened or closed.

$t = 0^+$ Just after the switch is opened or closed.

The current in the inductor cannot change instantaneously

$$i_L(0^-) = i_L(0^+)$$

The voltage in the capacitor cannot change instantaneously

$$V_c(0^-) = V_c(0^+)$$

# Growth of Current in an Inductive Circuit

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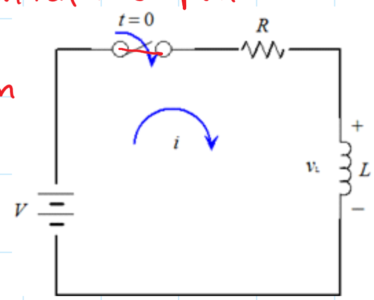
$t = 0^-$ switch was open.




* $i_L(0^-) = i_L(0^+) = 0$ initial condition

$$V - V_R - V_L = 0$$

$$V - iR - L\frac{di}{dt} = 0$$

$$0 = iR + LD(i(t))$$

$$V = iR + L\frac{di}{dt}$$ → First order linear differential equation

$$i(t) = i_h(t) + i_p(t)$$

homogeneous solution

Particular solution

$$V = iR + L\frac{di}{dt} + C\frac{d^2 i}{dt}$$

$$CD^2 + LD + R = 0$$

Homogeneous solution

characteristics equation ✓

$$DL + R = 0$$

$$D = -\frac{R}{L} = a$$ Roots of the characteristics equation

$$i_h(t) = A e^{-\frac{R}{L}t} = A e^{at}$$ ✓ → homogeneous solution

* Particular solution → same form as the input  $V \to dc$

$V = \sin(t + \phi)$

$i_p(t) = C_1 \sin(t) + C_2 \cos(t)$ ✓

$$i_p(t) = C$$

$$V = iR + L\frac{di}{dt}$$

$$V = CR + L0$$

$$C = \frac{V}{R} \qquad i_p(t) = \frac{V}{R}$$ ✓

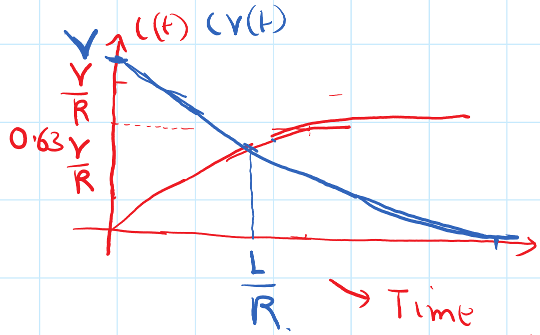
$$i(t) = i_h(t) + i_p(t)$$

$$= A e^{-\frac{R}{L}t} + \frac{V}{R} \qquad i(0^+) = 0$$

$$i(0^+) = A + \frac{V}{R} = 0$$

$$A = -\frac{V}{R}$$

$$\boxed{i(t) = -\frac{V}{R} e^{-\frac{R}{L}t} + \frac{V}{R} = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$




→ Time constant

$t = 0 \quad i = 0$

$t = \infty \quad i = \frac{V}{R}$

$t = \frac{L}{R} \quad \frac{V}{R}(1 - e^{-1})$

$i\left(\frac{L}{R}\right) = 0.63\frac{V}{R}$

$$V_L(t) = L\frac{di_L}{dt}$$

$$= \frac{V}{\cancel{R}} \times \frac{\cancel{R}}{\cancel{L}} \times \cancel{L}\, e^{-\frac{R}{L}t}$$

$$\boxed{V_L(t) = V e^{-\frac{R}{L}t}}$$

$$\boxed{V_R = i_L(t)\, R}$$

# Decay of current in an Inductive Circuit

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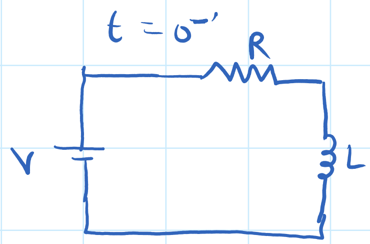
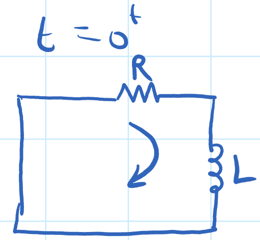
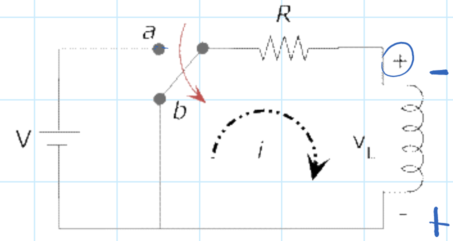
$$i(t) = \frac{V}{R}\left(1 - e^{-t/z}\right)$$




$$i(0^-) = \frac{V}{R} = i(0^+)$$

$$-iR - L\frac{di}{dt} = 0$$

$$iR + L\frac{di}{dt} = 0$$

$$i_h(t) + i_p(t) \qquad y_p(t) = 0$$

$$DL + R = 0$$

$$D = -\frac{R}{L}$$

$$i_h(t) = A e^{-\frac{R}{L}t}$$

$$i(t) = i_h(t) = A e^{-\frac{R}{L}t}$$

$$t = 0 \quad i(0^+) = \frac{V}{R}$$

$$\frac{V}{R} = A$$

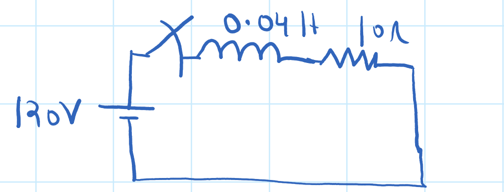
$$\boxed{i_L(t) = \frac{V}{R} e^{-\frac{R}{L}t}}$$

$$V_L(t) = L\frac{di_L}{dt} = \cancel{L}\frac{V}{\cancel{R}} \times -\frac{\cancel{R}}{\cancel{L}} e^{-\frac{R}{L}t}$$

$$\boxed{V_L(t) = -V e^{-\frac{R}{L}t}}$$

$$\boxed{v_L(t) = -V e^{-\frac{R}{L}t}}$$

- A coil of inductance 0.04 H and resistance 10  is connected to a 120 V, d.c. supply. Determine
- (a) the final value of current.
- (b) the time constant of the circuit.
- (c) the value of current after a time equal to the time constant from the instant the supply voltage is connected.
- (d) the expected time for the current to rise to within 1% of its final value.




$$I(\infty) = \frac{V}{R}\left(1-\cancel{e^{-\frac{R}{L}t}}\right)$$
$0$

$$I(\infty) = \frac{V}{R} = \frac{120}{10} = 12A$$

$$\tau = \frac{L}{R} = \frac{0.04}{10} = 0.004 \text{ sec}$$

$$I(\tau) = 0.63\frac{V}{R}$$
$$= 0.63 \times 12$$
$$= 7.56A.$$

1% of final value $\quad \frac{V}{R100}$

$$i_L(t) = \frac{V}{R}\left(1-e^{-\frac{R}{L}t}\right)$$

$$\frac{\cancel{V}}{\cancel{R}\times 100} = \frac{\cancel{V}}{\cancel{R}}\left(1-e^{-\frac{t}{0.004}}\right)$$

$t = 18.421\text{ ms}.$

- An R-L series circuit is designed for a steady current of 250mA. A current of 120 mA flows in the circuit at an instant 0.1 sec after connecting the supply voltage. Calculate i) time constant of the circuit ii) the time from closing the circuit at which the circuit current has reached 200 mA.

$$\frac{V}{R} = 250\,mA$$

$$I_L(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

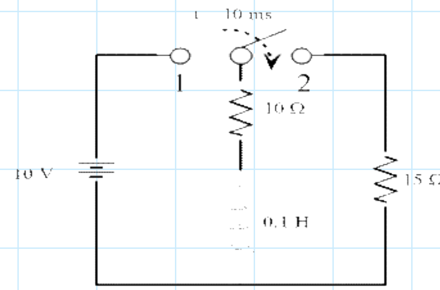
$$120\,mA = 250\,mA\left(1 - e^{-\frac{0.1}{\tau}}\right)$$

$$\frac{120}{250} = 1 - e^{\frac{-0.1}{\tau}}$$

$$\tau = 0.1529\ \text{sec.}$$

$$200\,mA = 250\,mA\left(1 - e^{-\frac{t}{0.1529}}\right)$$

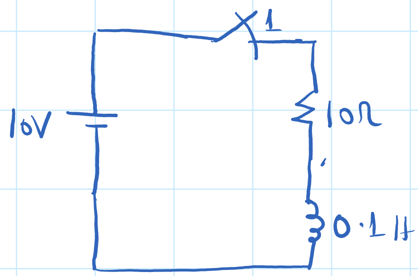
$$t = 0.2461\ \text{sec.}$$




- In the network shown in figure, the switch is closed to position 1 at t = 0 and is moved to position 2 at 10 ms. Determine iL(t) & sketch it.

$0 < t < 10ms$

$i_L(0^-) = 0 \quad i_L(0^+)$




$$i_L(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

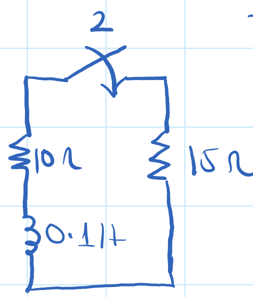
$$= \frac{10}{10}\left(1 - e^{\frac{}{0.1}t}\right)$$

$$i_L(t) = \left(1 - e^{-100t}\right)$$

2) $t \geq 10ms.$

$i_L(0^-) = i_L(0^+)$

$$= 1 - e^{-100 \times 10 \times 10^{-3}} = 0.63\ A$$




$$i_L(t) = 0.63\, e^{-\frac{25}{0.1}(t - 10ms)}$$

$$i(t) = \begin{cases} 1 - e^{-100t} & 0 \leq t < 10ms \\ 0.63\, e^{-\frac{25}{0.1}(t - 10ms)} & t \geq 10ms. \end{cases}$$

Ans: (a) 12A ; (b) 4ms (c) 7.58A (d) 18.421ms

Ans: i)Time constant = 0.1529 s ii) t= 0.2461 s