INSTITUTION of EMINENCE

Basic Electrical Technology

[ELE 1051]

SINGLE PHASE AC CIRCUITS

L15 – AC Representation & Response

Topics covered…

- Representation of AC
  - Mathematical form
  - Graphical form
  - Phasors
- AC response of
  - Pure Resistor
  - Pure Inductor
  - Pure Capacitor

Representing AC

- Consider three sinusoidal signals $x(t), y(t)$ & $z(t)$ with same frequency

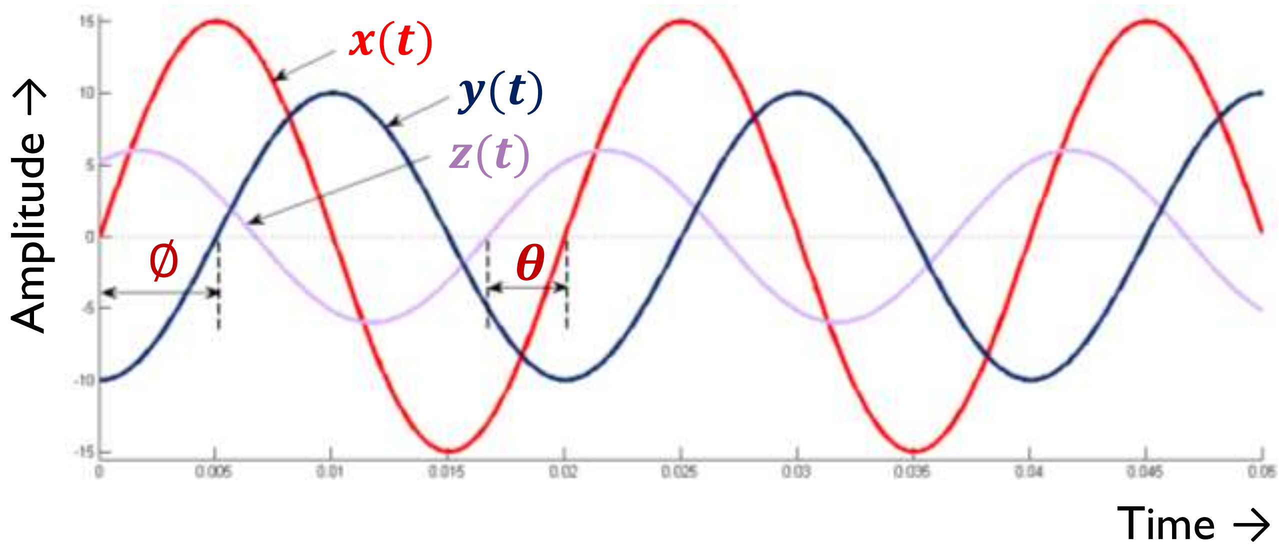
Mathematical Representation

$$x(t) = X_m sin(\omega t)$$

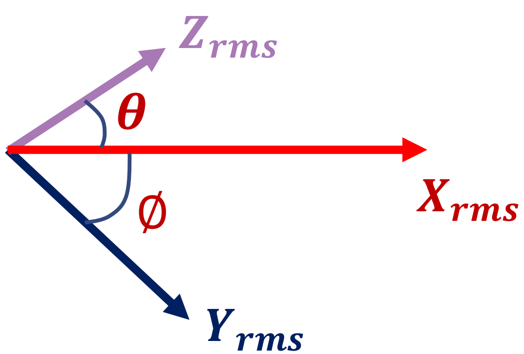
$$y(t) = Y_m \sin(\omega t - \emptyset)$$

$$z(t) = Z_m \sin(\omega t + \theta)$$

Graphical Representation

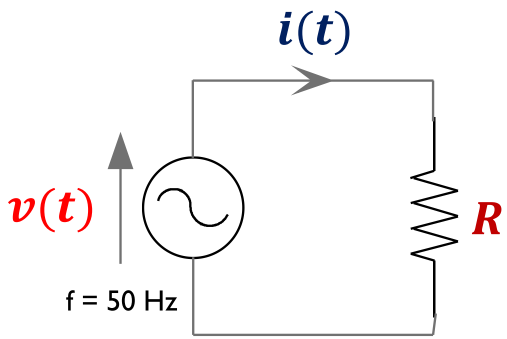


Phasor Representation



- Representing the relationship between sinusoidal signals with same frequency in graphical or mathematical form is tedious
- Phasor representation is often used

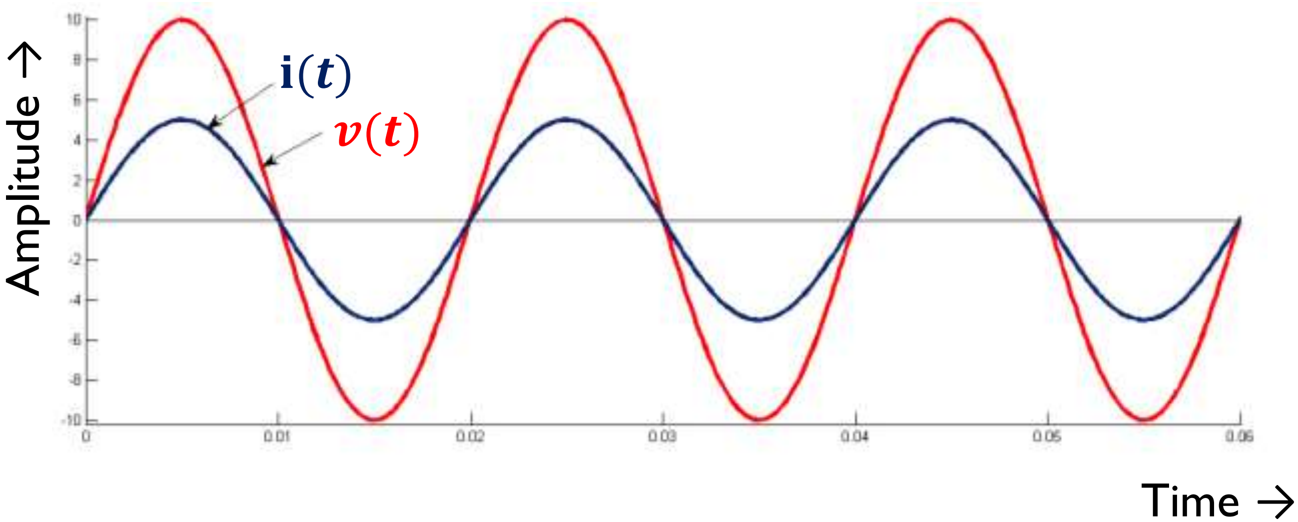
R circuit response with AC supply



$$i(t) = \frac{v(t)}{R}$$

'Current through the resistor is in phase with the voltage across it'

<u>*Graphical Representation*</u>

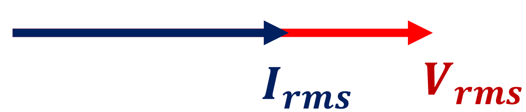


<u>*Mathematical Representation*</u>

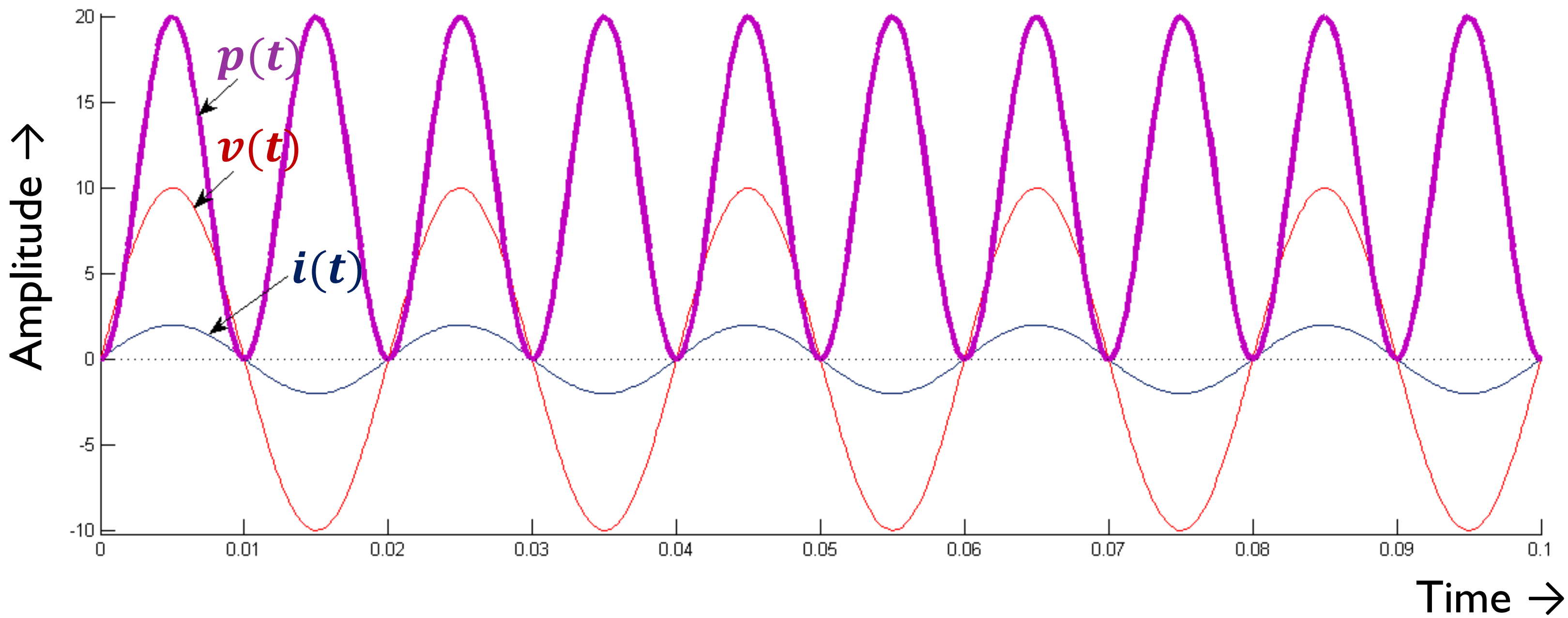
$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t)$$

<u>*Phasor Representation*</u>



Power Consumed - Pure Resistive Circuit



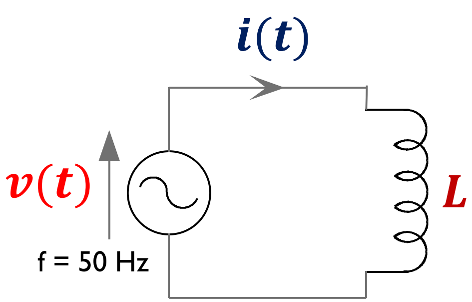
Instantaneous power,

$$p(t) = v(t).i(t) = V_m I_m sin^2 \omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = \frac{V_m I_m}{2} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R}}$$

L circuit response with AC supply



$$i(t) = \frac{1}{L}\int v(t)dt$$

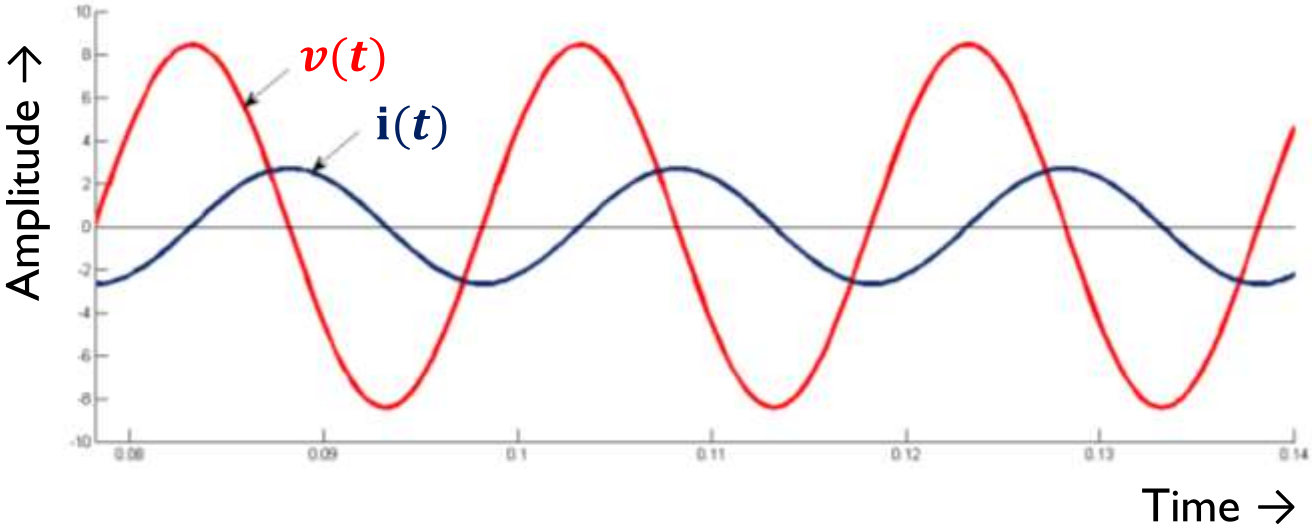
'Current through the inductor lags the voltage across it by 90°'

$\bar{V} = V\angle 0°$ $\quad\quad$ $\bar{I} = I\angle -90°$

$$\frac{\bar{\boldsymbol{V}}}{\bar{\boldsymbol{I}}} = \frac{\boldsymbol{V\angle 0°}}{\boldsymbol{I\angle -90°}} = \boldsymbol{jX_L} \quad where\ \frac{V}{I} = X_L$$

$\boldsymbol{X_L}$ *is called* ***Inductive Reactance***

<u>*Graphical Representation*</u>



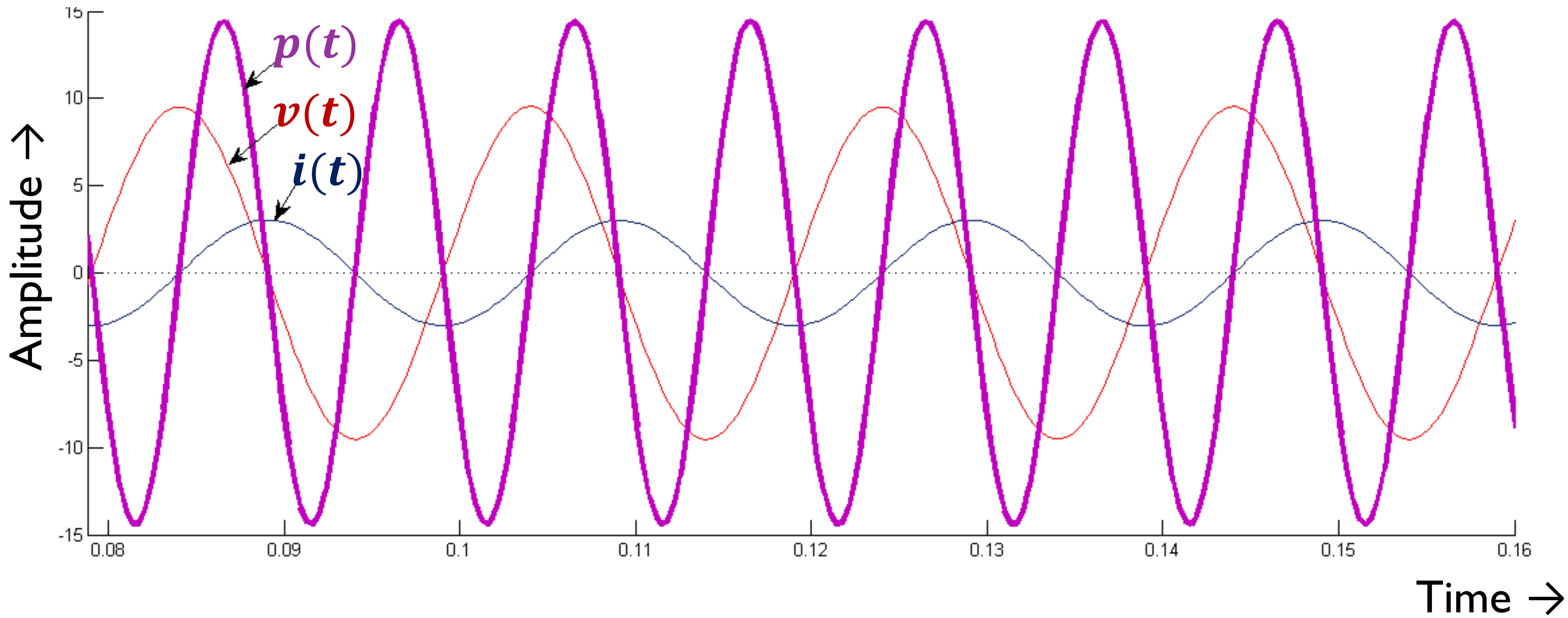
<u>*Mathematical Representation*</u>

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t - 90°)$$

<u>*Phasor Representation*</u>

Power Consumed - Pure Inductive Circuit



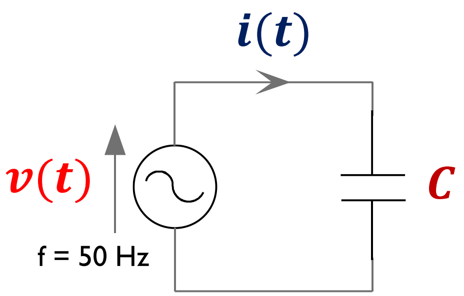
Instantaneous power,

$$p(t) = v(t).\,i(t)$$
$$= V_m I_m \sin \omega t \,.\, \sin(\omega t - 90\,°)$$
$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = 0}}$$

C circuit response with AC supply



$$i(t) = C\frac{dv(t)}{dt}$$

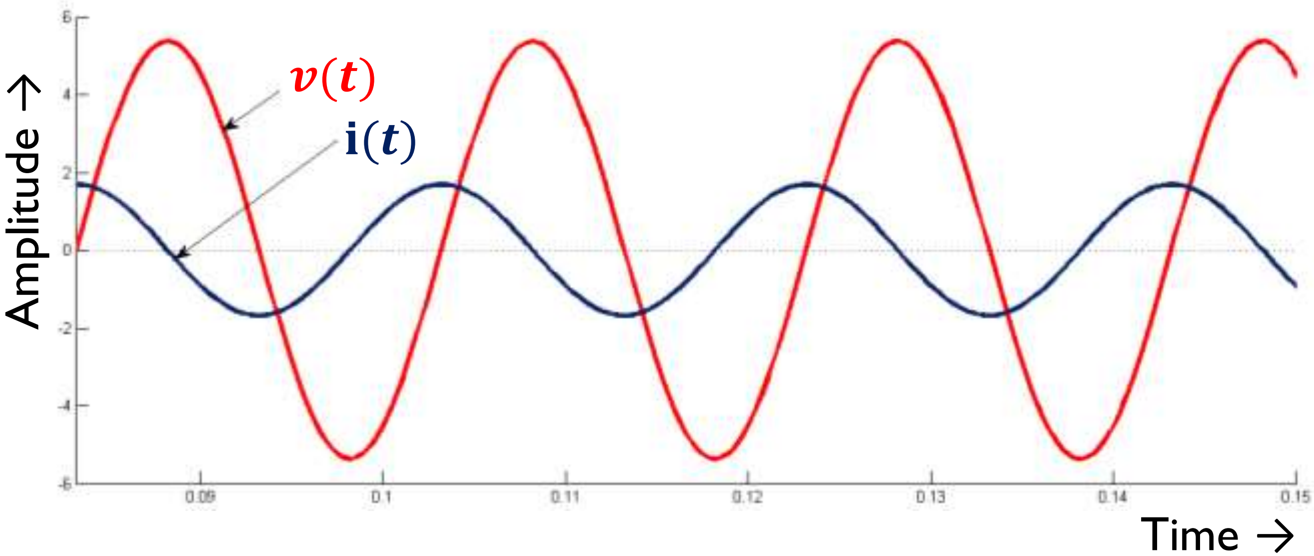
'Current through the capacitor leads the voltage across it by 90°'

$\bar{V} = V\angle 0°$ $\qquad \bar{I} = I\angle 90°$

$$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0°}{I\angle 90°} = -jX_C \qquad where\ \frac{V}{I} = X_C$$

X_C *is called* ***Capacitive Reactance***

Graphical Representation

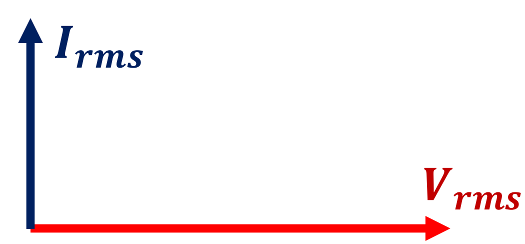


Mathematical Representation

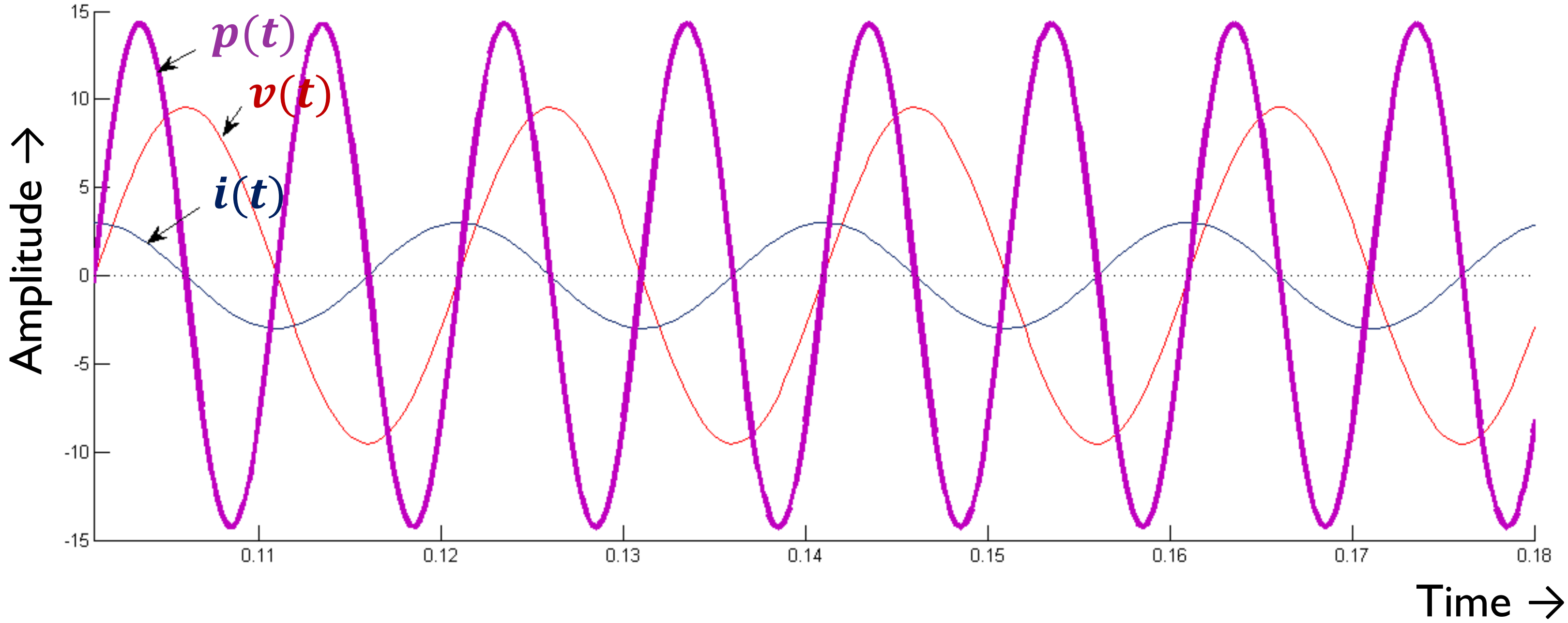
$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t + 90°)$$

Phasor Representation



Power Consumed - Pure Capacitive Circuit



Instantaneous power,

$$p(t) = v(t).\, i(t)$$
$$= V_m I_m \sin \omega t \, . \sin(\omega t + 90\,°)$$
$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = 0}}$$

Summary

- Sinusoidal alternating signals of same frequency can be represented graphically by ***Phasors***
- **Define:** Inductive and capacitive Reactances

	R	L	C
Voltage, current relationship	v(t) in phase with I(t)	i(t) lags v(t) by 90°	i(t) leads v(t) by 90°
Power associated	$I^2R = \frac{V^2}{R}$ (Active Power)	I^2X_L (Reactive Power)	I^2X_C (Reactive Power)