11/11/2021

Q- Solve by gauss elimination method

$$x_1 + 2x_2 - 3x_3 - 4x_4 = 6$$
$$x_1 + 3x_2 + x_3 - 2x_4 = 4$$
$$2x_1 + 5x_2 - 2x_3 - 5x_4 = 10$$

$$[A:B] = \left[\begin{array}{cccc:c} 1 & 2 & -3 & -4 & 6 \\ 1 & 3 & 1 & -2 & 4 \\ 2 & 5 & -2 & -5 & 10 \end{array}\right]$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

$$A \sim \left[\begin{array}{cccc:c} 1 & 2 & -3 & -4 & 6 \\ 0 & 1 & 4 & 2 & -2 \\ 0 & 1 & 4 & 3 & -2 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\sim \left[\begin{array}{cccc:c} 1 & 2 & -3 & -4 & 6 \\ 0 & 1 & 4 & 2 & -2 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \qquad \therefore \rho[A:B] = 3$$

Also, echelon form of A : $A \sim \begin{bmatrix} 1 & 2 & -3 & -4 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\therefore \rho(A) = 3$

$\therefore \rho[A:B] = \rho(A) = 3 < 4$ (No. of unknown)

$\Rightarrow$ system is consistent and has infinitely many solutions.

The equivalent matrix equation is

$$\begin{bmatrix} 0 & 2 & -3 & -4 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 + 2x_2 - 3x_3 - 4x_4 = 6$

$\Rightarrow x_2 + 4x_3 + 2x_4 = -2$

$\Rightarrow x_4 = 0$

when $x_4 = 0$, $x_1 + 2x_2 - 3x_3 = 6$ and $x_2 + 4x_3 = -2$

Let $x_3 = k$, be any real no.

Then, $x_2 = -2 - 4k$

$\therefore x_1 = 11k + 10$

$\therefore$ solution $x = \begin{bmatrix} 11k+10 \\ -2-4k \\ k \\ 0 \end{bmatrix}$ where, k is any real number

* Inverse of a matrix using row reduced elementary transformat

" Let A be a square matrix

working rule: Let B be the matrix of the given matrix A inverse

$$AB = I$$

make this an identity matrix $\leftarrow$ $(A)B = I$

Apply same row elementary transformations to the matrices A and I simultaneously to make A an Identity.

Gauss Jordan Method: [NOTE: only row transformations are allowed]

Let A be a square matrix

$[A|I]$ $\rightarrow$ apply row elementary transformation.

will become the inverse.

to make A as an identity matrix, simultaneously apply same to I

eg. using gauss Jordan method find the inverse of

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

Ans $[A|I] = \left[\begin{array}{cc:cc} 2 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right]$

$R_1 \rightarrow R_1/2$

$[A|I] \sim \left[\begin{array}{cc:cc} 1 & 1/2 & 1/2 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right]$

$R_2 \to R_2 - 3R_1 \quad \sim \left[\begin{array}{cc:cc} 1 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & -3/2 & 1 \end{array}\right]$

$R_1 \to R_1 - R_2 \quad \sim \left[\begin{array}{cc:cc} 1 & 0 & 2 & -1 \\ 0 & 1/2 & -3/2 & 1 \end{array}\right]$

$R_2 \to 2R_2 \quad \sim \left[\begin{array}{cc:cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -3 & 2 \end{array}\right]$

$\therefore \quad A^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ Ans

Q2 $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$. Find the inverse of A using gauss jordan method.

$[A \mid I] = \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$

$\sim \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & -1 & 1 & 0 \\ 0 & -2 & 2 & -1 & 0 & 1 \end{array}\right]$

$R_3 \to R_3 - R_2$

$\sim \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$

$R_2 \to R_2 - R_3$

$\sim \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$

$R_2 \rightarrow R_2/2 \quad \sim \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & -1 & 1/2 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$

$R_1 \rightarrow R_1 - R_2 \quad \sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1/2 & 1 & -1/2 \\ 0 & 1 & 0 & 1/2 & -1 & 1/2 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$

$$\therefore \quad A^{-1} = \begin{bmatrix} 1/2 & 1 & -1/2 \\ 1/2 & -1 & 1/2 \\ 0 & -1 & 1 \end{bmatrix}$$

Q Using gauss-Jordan method, solve the system of eqs.

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 2y + 5z = 40$$

$[A X = B]$

make Identity

Apply transformation to B too

Ans the matrix equation is

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 40 \end{bmatrix}$$

make this an identity matrix

$R_2 \rightarrow R_2 - 2R_1, \quad R_3 \rightarrow R_3 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 13 \end{bmatrix}$$

$R_3 \rightarrow R_3 + \frac{1}{5} R_2 \quad \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & 0 & 12/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 13 \end{bmatrix}$

$R_3 \rightarrow R_3 \times 5/12 \quad \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_3 \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -15 \\ 5 \end{bmatrix}$

$R_2 \to R_2/-5 \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -15 \\ 5 \end{bmatrix}$

$R_1 \to R_1 - R_2,$
$R_1 \to R_1 - R_3 \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \Rightarrow x = 1,\ y = 3,\ z = 5$ Ans

Gauss-Jacobi's method and Gauss-Seidel method.

5. Iterative method to solve the system of linear equation.

3.1
$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

step 1: diagonal dominance condition :

for 1st equation $|a_1| \geq |b_1| + |c_1|$

for 2nd eq $|b_2| \geq |a_2| + |c_2|$

for 3rd eq $|c_3| \geq |a_3| + |b_3|$

rearrangement can be done

Q. Solve the system of equation by gauss's-jacobi method.

$$3x + 20y - z = -18 \quad \text{—— (ii)}$$
$$2x - 3y + 20z = 25 \quad \text{—— (iii)}$$
$$20x + y - 2z = 17 \quad \text{—— (i)}$$

example of rearrangement

step 2:

$$\left.\begin{aligned} \text{Then, } x &= \frac{1}{a_1}(d_1 - b_1 y - c_1 z) \quad \text{from eq (1)} \\ y &= \frac{1}{b_2}(d_2 - a_2 x - c_2 z) \quad \text{from eq (2)} \\ z &= \frac{1}{c_3}[d_3 - a_3 x - b_3 y) \quad \text{from eq (3)} \end{aligned}\right\} \text{—— (1)}$$

Gauss-Jacobi Method

Let $x_0 = y_0 = z_0 = 0$ [initial approx value of $x = y = z = 0$]

iteration 1: put $x = x_0, y = y_0 = z = z_0$ in eq (1)

$$\Rightarrow \quad x' = \frac{1}{a_1}(d_1 - b_1 y_0 - c_1 z_0) = \frac{d_1}{a_1}$$

$$\Rightarrow \quad y' = \frac{1}{b_2}(d_2) = \frac{d_2}{b_2}$$

$$\Rightarrow \quad z' = \frac{d_3}{c_3}$$

Iteration 2:

$$x^2 = \frac{1}{a_1}(d_1 - b_1 y^{(1)} - c_1 z^{(1)})$$

$$y^2 = \frac{1}{b_2}(d_2 - a_2 x^{(1)} - c_2 z^{(1)})$$

$$z^2 = \frac{1}{c_3}(d_3 - a_3 x^{(1)} - b_3 y^{(1)})$$

Gauss-Seidal (faster method because we use improved values)

Let $y_0 = z_0 = 0$

iteration 1:

$$x^{(1)} = \frac{1}{a_1}(d_1 - b_1 y_0 - c_1 z_0) = ?$$

$$y^{(1)} = \frac{1}{b_2}(d_2 - a_2 x^{(1)} - 0) = ?$$

$$z^{(1)} = \frac{1}{c_3}(d_3 - a_3 x^{(1)} - b_3 y^{(1)}) = ?$$

iteration II:

$$x^{(2)} = \frac{1}{a_1}(d_1 - b_1 y^{(1)} - c_1 z^{(1)})$$

$$y^{(2)} = \frac{1}{b_2}(d_2 - a_2 x^{(2)} - c_2 z^{(1)})$$

$$z^{(2)} = \frac{1}{c_3}(d_3 - a_3 x^{(3)} - b_3 y^{(2)})$$