THEORY OF INFERENCE

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Let A and B be two statement formula. We say that " B logically follows from A" or "B is a valid conclusion of the premise A" iff $A \rightarrow B$ is a Tautology i.e., $A \implies B$

To demonstrate that a particular formula is valid consequence of a given set of premises, we use the follow rules of inference.

Rule P : *A premise may be introduced at any point in the derivation*

Rule T : *A formula S may be introduced in a derivation if S is tautologically implied by any one or more of the preceding formulas in the derivation*

Question : Demonstrate that r is a valid inference from the premises $p \to q, \;\; q \to r \;\; and \;\; p$

Solution :

$$\begin{aligned} p \to q & \qquad (Rule\ P) \\ p & \qquad (Rule\ P) \\ q & \qquad (Rule\ T, \; p \wedge (p \to q) \implies q) \\ q \to r & \qquad (Rule\ P) \\ r & \qquad (Rule\ T, q \wedge (q \to r) \implies r) \end{aligned}$$

Question : RVS follows logically from the premises $C \wedge D$, $C \vee D \to \neg H$, $\neg H \to (A \wedge \neg B)$, $(A \wedge \neg B) \to (R \vee S)$.
Solution :

$$
\begin{aligned}
C \vee D \to \neg H & \qquad (Rule\ P) \\
\neg H \to A \wedge \neg B & \qquad (Rule\ P) \\
C \vee D \to A \wedge \neg B & \qquad (Rule\ T) \\
A \wedge \neg B \to R \vee S & \qquad (Rule\ P) \\
C \vee D \to R \vee S & \qquad (Rule\ T) \\
C \vee D & \qquad (Rule\ P) \\
R \vee S & \qquad (Rule\ T)
\end{aligned}
$$

Question : Show that $S \vee r$ is tautologically implied by $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow s)$

Solution :

$$
\begin{array}{rl}
p \vee q & (Rule\ P) \\
\neg p \rightarrow q & (Rule\ T\ i.e.,\ p \rightarrow q \Leftrightarrow \neg p \vee q) \\
q \rightarrow S & (Rule\ P) \\
\neg p \rightarrow S & (Rule\ T) \\
\neg S \rightarrow p & (Rule\ T, p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p) \\
p \rightarrow r & (Rule\ P) \\
\neg S \rightarrow r & (Rule\ T) \\
S \vee r & (Rule\ T, p \rightarrow q \Leftrightarrow \neg(\neg p \vee q))
\end{array}
$$

Exercise Q1 : $R \wedge (p \vee q)$ is a valid conclusion from the premises $p \vee q$, $q \rightarrow r$, $p \rightarrow M$ and $\neg M$.

Exercise Q2 : If A works hard, then either B or C will enjoy themselves. If B enjoys himself, then A will not work hard. If D enjoys himself, then C will not, therefore prove that if A works hard, D will not enjoy himself.

Solution hint :

$$\begin{aligned} A &: \quad A \textit{ works hard} \\ B &: \quad B \textit{ will enjoy himself} \\ C &: \quad C \textit{ will enjoy himself} \\ D &: \quad D \textit{ will enjoy himself} \end{aligned}$$

To prove $A \rightarrow \neg D$ follows from $A \rightarrow B \vee C$, $B \rightarrow \neg A$ and $D \rightarrow \neg C$