Reg. No.										

MANIPAL ACADEMY OF HIGHER EDUCATION

(Deemed University)

SECOND SEMESTER B.E. DEGREE EXAMINATION – NOV/DEC 2006

SUBJECT: ENGINEERING MATHEMATICS – II (MAT 102)

(CREDIT SYSTEM)

Saturday, December 16, 2006

Time: 3 Hrs. Max. Marks: 100

✍ ***Answer any FIVE full questions.***

1A. Solve: $\dfrac{dy}{dx} = (4x + y + 1)^2;\ y(0) = 1$.

1B. Solve: $(1+x)^2 \dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 2\sin[\log(1+x)]$.

1C. Solve: $\dfrac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$.

(6+7+7 = 20 marks)

2A. Solve: $y(1+xy+x^2y^2)dx+x(1-xy+x^2y^2)dy=0$.

2B. Solve: $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^x \sin x$.

2C. Solve: $\dfrac{dx}{dt} + y = 1 + \sin t$

$\dfrac{dy}{dt} + x = \cos t$

(7+7+6 = 20 marks)

3A. Find the Laplace transform of

$$f(t) = \begin{cases} t^2 & 0 < t < 2 \\ t-1 & 2 < t < 3 \\ 7 & t > 3 \end{cases}$$

Using unit step functions.

3B. Find i) $L\left[\dfrac{e^{at} - e^{bt}}{t}\right]$ ii) $L^{-1}\left[\dfrac{2s+1}{s^2+3s+2}\right]$.

3C. Solve the differential equation using the Laplace transform:

$\dfrac{d^2y}{dt^2} - 3\dfrac{dy}{dt} + 2y = 4e^{2t};\quad y(0) = -3,\quad y'(0) = 5$.

(7+7+6 = 20 marks)

4A. Change the order of integration and evaluate: $\displaystyle\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx$.

4B. Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ using triple integrals.

4C. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

(6+7+7 = 20 marks)

5A. Find the volume bounded by the cylinder $x^2+y^2 = 4$ and the planes $y+z = 4$ and $z = 0$.

5B. Evaluate: $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dx\,dy$.

5C. Find the maximum and minimum distances of the point (3, 4,12) from the sphere $x^2+y^2+z^2=1$.

(7+6+7 = 20 marks)

6A. Test for consistency and solve:

$x + 2y + 3z = 14$

$4x + 5y + 7z = 35$

$3x + 3y + 4z = 21$

6B. Find the rank for the following matrices:

i) $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$

ii) $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

6C. Show that the representation of any vector in terms of a set of basis vectors is unique.

(7+7+6 = 20 marks)

7A. Using Gram-schmidt orthogonalisation process, construct an orthonormal set of vectors from {(1, 1, 1), (0, 1, 1), (1, 2, 3)}

7B. Give an example each for the following with proper justification.

i) Basis for E^3.

ii) Linearly dependent set in E^3.

7C. Find the inverse of the matrix

$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ using elementary row transformations.

(7+6+7 = 20 marks)