To get 100^{th} permutation of $n=5$ in lexicographic (using Algorithm)

$$99 = c_4 \times 4! + c_3 \times 3! + c_2 \times 2! + c_1 \times 1!$$

$0 \leq c_i \leq i$

$$99 = \underline{4} \times 4! + \underline{0} \times 6 + \underline{1} \times 2 + \underline{1} \times 1$$

$c_4 \quad c_3 \quad c_2 \quad c_1$

$\underline{4} \quad 0 \quad 1 \quad 1 \longleftrightarrow 1\ 2\ 3\ 4\ 5 \longrightarrow 5$

$c_4 + 1 = 4+1 = 5$ remove the element in the 5^{th} position.

$\underline{0} \quad 1 \quad 1 \longleftrightarrow \underline{1}\ 2\ 3\ 4 \longrightarrow 1$

$c_3 + 1 = 0+1 = 1$ remove the element in 1^{st} position

$\underline{1} \quad 1 \longleftrightarrow 2\ 3\ 4 \longrightarrow 3$

$c_2 + 1 = 2$

$1 \longleftrightarrow 2\ 4 \longrightarrow 4$

$c_1 + 1 = 2$

2

100^{th} permutation is $\underline{\underline{5\ 1\ 3\ 4\ 2}}$

$\underline{35}^{th}$ permutation of $n = 6$.

$$34 = c_5 \times 5! + c_4 \times 4! + c_3 \times 3! + c_2 \times 2! + c_1 \times 1!$$
$$= 0 \times 5! + 1 \times 24 + 1 \times 6 + 2 \times 2 + 0 \times 1$$

$c_5\ c_4\ c_3\ c_2\ c_1$
$0\ 1\ 1\ 2\ 0$

$\underline{0}1120 \longleftrightarrow \underline{1}23456 \longrightarrow 1$
$\underline{1}120 \longleftrightarrow 2\underline{3}456 \longrightarrow 3$
$\underline{1}20 \longleftrightarrow 2\underline{4}56 \longrightarrow 4$
$\underline{2}0 \longleftrightarrow 25\underline{6} \longrightarrow 6$
$0 \longleftrightarrow \underline{2}5 \longrightarrow 2$
5

$\boxed{1\ 3\ 4\ 6\ 2\ 5}$

Reverse Lexicographic Ordering

In this we fix the largest element in the last position as far as possible & permute the remaining.

Example:

1234	1243	1342	2341
2134	2143	3142	3241
1324	1423	1432	2431
3124	4123	4132	4231
2314	2413	3412	3421
3214	4213	4312	4321

50th & 35th permutation of n=5 in R.L.O ordering

12345	12435
21345	21435
13245	14235
31245	41235
23145	24135
32145	42135

24th

54	12534
54	21534
54	15234
54	51234
54	$\boxed{25134}$ (35th)
54	134

48th

12453
$\boxed{21453}$ (50th)
253
253
153
153

72

96

120th

Algorithm to find k^{th} permutation of n in Reverse lexicographic ordering.

Step 1: $K-1 = c_{n-1}(n-1)! + c_{n-2}(n-2)! + \dots + c_1 1!$

where $0 \le c_i \le i$

Step 2: Consider

$$c_{n-1} \quad c_{n-2} \quad \dots \quad c_2 \ c_1 \leftrightarrow n \quad n-1 \quad \dots \quad 3 \ 2 \ 1$$

Step 3: Pick the element in the position $c_{n-1}+1$ as last element of the permutation.

Step 4: Remove c_{n-1} and the selected last element

Step 5: Repeat step 2 onwards.

Example:

To get 35^{th} permutation when $n=5$.

$$34 = c_4 \times 4! + c_3 \times 3! + c_2 \times 2! + c_1 \times 1!$$

$$(c_4\ c_3\ c_2\ c_1) = (1\ 1\ 2\ 0)$$

$$1120 \longleftrightarrow 54321 \longrightarrow 4$$

$$120 \longleftrightarrow 5321 \longrightarrow 3$$

$$20 \longleftrightarrow 521 \longrightarrow 1$$

$$0 \longleftrightarrow 52 \longrightarrow 5$$

$$2$$

$$2\ 5\ 1\ 3\ 4$$

100^{th} permutation of 1, 2, 3, 4, 5 in Reverse Lexicographic:

$$99 = c_4 \times 4! + c_3 \times 3! + c_2 \times 2! + c_1 \times 1! \qquad 0 \le c_i \le i$$

$$= 4 \times 4! + 0 \times 6 + 1 \times 2 + 1 \times 1$$

$$c_4\, c_3\, c_2\, c_1 = 4011$$

$\underline{4}011 \longleftrightarrow 5432\underline{1} \longrightarrow 1$

$\underline{0}11 \longleftrightarrow \underline{5}432 \longrightarrow 5$

$\underline{1}1 \longleftrightarrow 4\underline{3}2 \longrightarrow 3$

$1 \longleftrightarrow 4\underline{2} \longrightarrow 2$

4

$$\boxed{42351}$$

Fike's order

To find k^{th} permutation of n, first we obtain Fike's sequence.

$$k-1 = c_1 \times \frac{n!}{2!} + c_2 \times \frac{n!}{3!} + \cdots + c_{n-2} \times \frac{n!}{(n-1)!} + c_{n-1}$$ ✓

$0 \le c_i \le i$

To get Fike's sequence,

$$\begin{array}{rccccc} & 1 & 2 & 3 & \cdots & n-1 \\ - & c_1 & c_2 & c_3 & \cdots & c_{n-1} \\ \hline & d_2 & d_3 & d_4 & & d_n \end{array}$$

Fike's sequence is $(d_2\, d_3 \cdots d_n)$.

Next, to find k^{th} permutation,

Let $d_2 + 1 = r$ then interchange 2^{nd} and r^{th} position in $1\ 2\ 3 \cdots r \cdots n$

$1\ r\ 3 \cdots 2 \cdots n$

Next,

let $d_3 + 1 = s$, then interchange 3^{rd} and s^{th} position in $1\ r\ 3 \cdots s \cdots 2 \cdots n$

$1\ r\ s \cdots 3 \cdots 2 \cdots n$

$\vdots$

continue till $d_n + 1 = t$, then interchange n and t^{th} position to get k^{th} permutation

Q1. Find 50^{th} and 100^{th} permutation for $n=5$ with initial permutation $\underline{01234}$ using Fike's ordering.

Soln:
$$49 = c_1 \times \frac{5!}{2!} + c_2 \times \frac{5!}{3!} + c_3 \times \frac{5!}{4!} + c_4 \times 1$$

$$0 \le c_i \le i$$

$$\underline{\underline{49}} = c_1 \times 60 + c_2 \times 20 + c_3 \times 5 + c_4 \times 1$$
$$= 0 \times 60 + 2 \times 20 + 1 \times 5 + 4 \times 1$$
$$(c_1\ c_2\ c_3\ c_4) = (0\ 2\ 1\ 4)$$

$$\begin{array}{rcccc} & 1 & 2 & 3 & 4 \\ - & 0 & 2 & 1 & 4 \\ \hline & 1 & 0 & 2 & 0 \\ & d_2 & d_3 & d_4 & d_5 \end{array}$$

$1\ 0\ 2\ 0 \longleftrightarrow 50^{th}$ Fike's sequence

$1\ \ 0\ \ 2\ \ 0 \longleftrightarrow 0\ 1\ 2\ 3\ 4$

$d_{\underline{2}}+1 = 1+1 = \underline{\underline{2}}$

Interchange 2^{nd} & 2^{nd} element $\Rightarrow$ No change

$0\ 1\ 2\ 3\ 4$

$d_3+1 = 0+1 = 1$

Interchange 3^{rd} & 1^{st}

$2\ 1\ 0\ 3\ 4$

$d_{\underline{4}}+1 = 2+1 = \underline{\underline{3}}$

Interchange 4^{th} & 3^{rd} element.

$2\ 1\ 3\ 0\ 4$

$d_{\underline{5}}+1 = 0+1 = \underline{1}$

Interchange 1^{st} & 5^{th}

$\boxed{4\ 1\ 3\ 0\ 2}$ ✓

Q2. Find 79^{th} and 111^{th} permutation for $n=5$ with initial permutation 1, 2, 3, 4, 5 using Fike's ordering.

Soln :

Q1 : 100^{th} permutation of 0 1 2 3 4

$$99 = c_1 \times 60 + c_2 \times 20 + c_3 \times 5 + c_4 \times 1$$

$$= \underline{1} \times 60 + \underline{1} \times 20 + \underline{3} \times 5 + \underline{4} \times \underline{1}$$

$$c_1 c_2 c_3 c_4 = 1134$$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 1 & 1 & 3 & 4 \\ \hline 0 & 1 & 0 & 0 \end{array} \leftarrow d_2 d_3 d_4 d_5 \quad (\text{Fike's seq})$$

0 1 0 0 $\longleftrightarrow$ 0 1 2 3 4

$d_2 + 1 = \underline{1}$ 1 0 2 3 4

$d_3 + 1 = 1 + 1 = \underline{2}$ 1 2 0 3 4

$d_4 + 1 = 0 + 1 = \underline{1}$ 3 2 0 1 4

$d_5 + 1 = 0 + 1 = \underline{1}$ 4 2 0 1 3

$78 =$

$c_1 c_2 c_3 c_4 = 1\,0\,3\,3$

$d_2 d_3 d_4 d_5 = 0\,2\,0\,1$ ✓

$0\,2\,0\,1 \longleftrightarrow 1\,2\,3\,4\,5$

$d_2 + 1 = 1$ 　　 $2\,1\,3\,4\,5$

$d_3 + 1 = 3$ 　　 $2\,1\,3\,4\,5$

$d_4 + 1 = 1$ 　　 $4\,1\,3\,2\,5$

$d_5 + 1 = 2$ 　　 $\boxed{4\,5\,3\,2\,1}$

$110 =$

$c_1 c_2 c_3 c_4 = 1\,2\,2\,0$

$d_2 d_3 d_4 d_5 = 0\,0\,1\,4$

$0\,0\,1\,4$ 　　 $1\,2\,3\,4\,5$

$d_2 + 1 = 1$ 　　 $2\,1\,3\,4\,5$

$d_3 + 1 = 1$ 　　 $3\,1\,2\,4\,5$

$d_4 + 1 = 2$ 　　 $3\,4\,2\,1\,5$

$d_5 + 1 = 5$ 　　 $\boxed{3\,4\,2\,1\,5}$ ✓

Immediate next permutation of 43215

in Lex ico ordering is $\underline{43251}$ ✓

in Reverse Lexicographic ordering is $\underline{12354}$

Immediate next permutation of 431250

in Reverse Lexico graphic ordering is $\underline{234150}$

341250

431250

431250

431250

$\underline{234150}$

Please Note:

Quiz 1 is on 10.11.2021 (Mathematics)

Time: 6.15PM – 6.45PM