1. 12 people are made to sit around a round table. Find the number of ways in which they can sit such that John and Mary are not sitting together?

   Ans: $11! - 2 \times 10!$

2. Find the number of ways in which 3 exams can be schedules in a 5 days period such that
   i. No two exams are scheduled on the same day
   ii. There are no restrictions on the number of exams on one day

   Ans: $60, 125$

3. How many odd numbers between 100 and 999 have distinct digits?

   Ans: $320$

4. How many ways can 12 white and 12 black pawns can be placed on black squares of $8 \times 8$
   Chess board?

   Ans:$\binom{32}{12}\binom{20}{12}$

5. Determine the number of ways to seat 5 boys in a row of 12 chairs?

   Ans: $\binom{12}{5} \times 5!$

6. A shop sells 6 flavors of ice-creams. How many ways, a customer can choose 4 ice-cream cones if
   i. If they all are of different flavors
   ii. If they are not necessarily of different flavors
   iii. If they contain only three or four flavors
   iv. If they contain 3 different flavors

   i. Ans. $\binom{6}{4}$
   ii. Ans. $\binom{9}{4}$
   iii. Ans. 75
   iv. Ans. 60

7. Out of 5 mathematicians and 7 engineers, a committee consisting of 2 mathemacians and 3 engineers has to be formed. In how many ways it can be done such that
   i. There is no restrictiom
   ii. One particular engineer must be in the committee

iii. 2 particular mathematicians should not be in the committee

i. Ans . 350
ii. Ans. 150
iii. Ans. 105
iv.

8. A student is to answer 12 of the 15 questions in an exam. How many choices does the student have if
   i. In all
   ii. If he must answer first two questions
   iii. If he must answer first or second but not both questions
   iv. If he must answer exactly three of the first five questions
   v. If he must answer Atleast three of the first five questions

i. $\binom{15}{12}$
ii. $\binom{15}{13}$
iii. $\binom{13}{11}\binom{2}{1}$
iv. $\binom{5}{3}\binom{10}{9}$

9. A person has to visit one of the 12 temples on each evening of a given week. In how many ways can we plan his week if he will not visit a temple more than once

Ans: 3991680

10. Given integers 1,2,…11. Tow groups are made, first group contains 5 integers and second group contains 2 integers. In how many ways can the selection be made with unrestricted repetition if
   i. There are no further restriction
   ii. A group has either all odd integers or all even integers

i. $\binom{15}{5}\binom{12}{2}$

$$ii. \binom{10}{5}\binom{6}{2} + \binom{9}{5}\binom{7}{2} + \binom{9}{5}\binom{6}{2} + \binom{10}{5}\binom{7}{2}$$

11. How many ways are there distribute 27 identical jelly beans among 3 kids
   i. Without restriction
   ii. Each kid getting exactly 9 beans
   iii. Each kid has at least one

i. $\binom{29}{27}$

ii. 1

iii. $\binom{26}{24}$

12. How many are there to assign 100 different diplomats to 5 different continents?

Ans: $5^{100}$

13. How many ways are there to distribute 20 identical sticks of red candy and 15 identical sticks of black candy among 5 kids?

Ans: $\binom{24}{20}\binom{19}{15}$

14. A message is made up of 12 different symbols and is to be transmitted through a communication channel in addition to the 12 symbols with at least 3 spaces between each pair of consecutive symbols. In how many ways can we transmit such a message with 45 blank spaces?

Ans: $\binom{22}{12}12!$

15. A bakery sells 6 different kind of pastry. Is the bakery has a dozen of each kind. How many different options for a dozen of pastry are there? What if a box is to contain at least one of each kind of pastry?

i. $\binom{17}{12}$

ii. $\binom{11}{6}$

16. In how many ways can we distribute 8 identical balls to 4 distinct boxes such that
    i. No container is left empty
    ii. $4^{th}$ box has odd number of balls

i. $\binom{7}{4}$

ii. 70

17. In how many ways can 10 identical marbles are distributed among 5 kids
    i. Without repetition
    ii. Each kid gets at least one marble
    iii. Oldest kid gets at least two marbles

i. $\binom{14}{10}$
ii. $\binom{9}{5}$
iii. $\binom{12}{8}$

18. A variable name in programming language must be either a letter or a letter followed by a decimal digit. How many different variable names are there in this language?

Ans: 26+260

19. In a row of 20 seats, in how many ways can three blocks of consecutive seats with 5 seats each in each block can be arranged?

Ans: $\frac{8!}{3!5!}$

20. In how many ways can 10 boys and 5 gilrs stand in a line such that no two gilrs are next to each other?

Ans: $10!\binom{11}{5}5!$

21. If repetition is not allowed, how many 4 digit numbers can be formed from the 6 digits 1,2,3,5,7,8

i. How many of the numbers are lesser than 4000
ii. How many are even
iii. How many are odd
iv. How many are multiple of 5
v. How many contain both the digits 5 and 3

360

i. 180
ii. 120
iii. 240
iv. 60
v. 288

22. Find the no of ways to paint 12 offices so that 3 of them will be green, 2 of them pink, 2 of them yellow and remaining ones white

Ans: $\frac{12!}{2!2!3!5!}$

23. Among all 7 digit decimal numbers, how many of them contain exactly three 9s?

Ans: $\binom{6}{2} 9^4 + \binom{6}{3} 8 \times 9^4$

24. In how many ways can two numbers be sleceted from the integers 1,2,…,100 so that the
    i. Sum is even
    ii. Sum is odd

i. $\binom{50}{2} + \binom{50}{2}$
ii. 2500

25. Three integers are selected from the integers 1,2,…,1000. In how many ways can these integers be selected such that their sum is divisible by 4?

Ans: $\binom{250}{3} + 3\binom{250}{2}\binom{250}{1} + \binom{250}{1}\binom{250}{1}\binom{250}{1}$

26. A computer password consists of a letter of the alphabet followed by 3 or 4 digits. Find total number of passwords
    i. That can be formed
    ii. In which no digit repeats

a. $26(10)^4 + 26(10)^3$
b. 1497

27. Out of 5 mathematicians and 7 physicists a committee consisting of 2 mathematicians and 3 physicists has to be formed. In how many ways it could be done of
    i. There is no restriction
    ii. 1 particular physicist must be in the committee
    iii. 2 particular mathematicians cannot be in a committee.

i. $\binom{5}{2}\binom{7}{2}$
ii. $\binom{5}{2}\binom{6}{2}$
iii. $\binom{3}{2}\binom{7}{3}$

28. There are 15 true false questions in an exam. In how many ways can a student do the exam, if he can also choose not to answer some of the questions?

Ans: $3^{15}$

29.How many 7 letter palindromes can be made out of the English alphabets?

Ans: $26^4$

30.In how many ways can the letters $a, b, c, d, e, f$ be arranged so that b is always to the immediate left of the letter e.

Ans: 5!

31.In how many ways the letters $a, a, a, a, a, b, c, d, e$ are permuted such that no $b, c, d, e$ are adjacent? In how many ways the letters $a, a, a, a, a, b, c, d, e$ are permuted such that no two $a'$s are together?

Ans: $\frac{9!}{5!} - \frac{6!4!}{5!}$

4!

**On generating functions and principles of inclusion and exclusion**

32.How many ways can an examiner assign 30 marks to 8 questions so that no question receives less than 2 marks using generating function?

Sol: $(x^2 + x^3 + \cdots + x^{16})^8$

Co-eff of $x^{30}$ is $\binom{21}{4}$.

33.In how many ways can 4 letters of the word EAGLE be arranged using Generating function?

GF: $\left(1 + x + \frac{x^2}{2}\right)(1 + x)^3$

Co-eff of $\frac{x^4}{4!}$ is 60.

34. How many ways are there to split 6 copies of one book, 7 copies of a second book and 11 copies of a third book between 2 teachers, if each teacher gets 12 books and each teacher gets at least 2 copies of each book?

GF: $(x^2 + x^3 + x^4)(x^2 + \cdots + x^5)(x^2 + \cdots + x^9)$

Co-eff of $x^{12}$is $\binom{8}{6} - \binom{5}{3} - \binom{4}{2} = 12$

35. How many ways are there to select 300 chocolate candies from 7 types if each type cones in boxes of 20 and if at least one but not more than 5 boxes of each type are chosen?(Hint : Solve in terms of boxes of chocolates)

GF:$[(x^{20} + (x^{20})^2 + \cdots + (x^{20})^5]^7$

Co-eff of $x^{300}$ $\binom{14}{8} - 7\left(\binom{9}{3}\right) = 2415.$

36. How many of the first 1000 integers are not divisible by 2,3,5 or 7?

Ans: 228

37. How many n-digit ternary (0,1,2) sequences are there with at least one 0, at least one 1 and at least one 2? (Using Inclusion Exclusion)(U can do it with generating function)

Soln:

$A_0$ =No. of n-digit ternary sequence with no 0's.

$A_1$ = No. of n-digit ternary sequence with no 1's.

$A_2$ = No. of n-digit ternary sequence with no 2's.

$N = 3^n, N(A_0) = N(A_1) = N(A_2) = 2^n$

$N(A_0A_1) = N(A_0A_2) = N(A_1A_2) = 1.$

Required Answer is $N(A_0'A_1'A_2') = 3^n - 3.2^n + 3.$

38. If a number n has only 2 distinct prime factors $p_1$and $p_2$, show that $f(n)$ the number of positive integer less than n and relatively prime to n is $f(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})$. Hence find $f(12) and\ f(135)$.
(Hint Use inclusion exclusion).

39. How many r-digit quaternary sequences are there that have even number of 0's and an even number of 1's.

GF: $\left(1 + \frac{x^2}{2!} + \cdots\right)^2 \left(1 + x + \frac{x^2}{2!} \ldots\right)^2$

Co-eff of $\frac{x^r}{r!}$ is $\frac{4^r}{4} + \frac{2^r}{2}$.

40.Find the number of ways in which 25 distinct objects can be placed in 3 distinct boxes such that no box is empty using generating function.

GF: $\left(x + \frac{x^2}{2!} + \cdots\right)^3 = (e^x - 1)^3$

Co-eff of $\frac{x^{25}}{25!}$ Is $3^{25} - 3(2^{25}) + 3$