Theorem: A tree with p vertices has p − 1 edges.

Proof: Proof is by induction on the number of vertices.

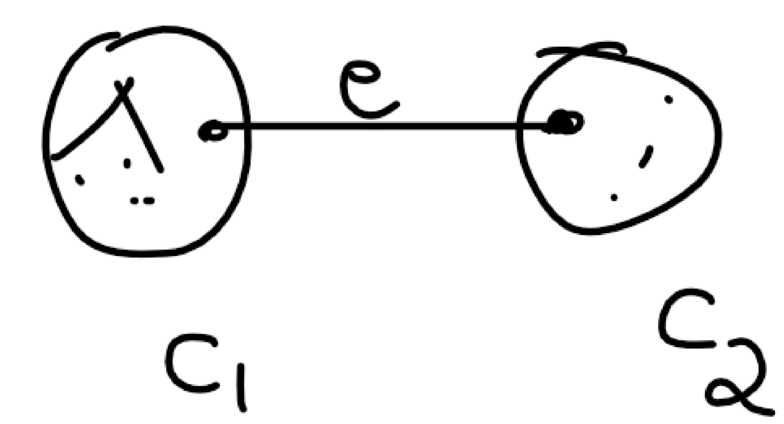
If $p=1$, we get a tree with zero edge.

If $p=2$, we get a tree with 1 edge

If $p=3$, we get a tree with 2 edges

Assume that the statement is true with all tree with k vertices ($k<p$).

Let G be a tree with p vertices. Since G is a tree there exist a unique path between every pair of vertices. Thus, removal of an edge 'e' from G will disconnect the graph G. Further $G-e$ consists of exactly 2 components with number of vertices say m and n with $m+n=p$



Each component is a tree.

By induction, the component C_1 with m vertices has $m-1$ edges and the component C_2 with n vertices has $n-1$ edges. ($m<p$, & $n<p$)

Thus the no. of edges in $G = (m-1)+(n-1)+1$

$$= p-1$$

Every tree with p vertices has $p-1$ edges

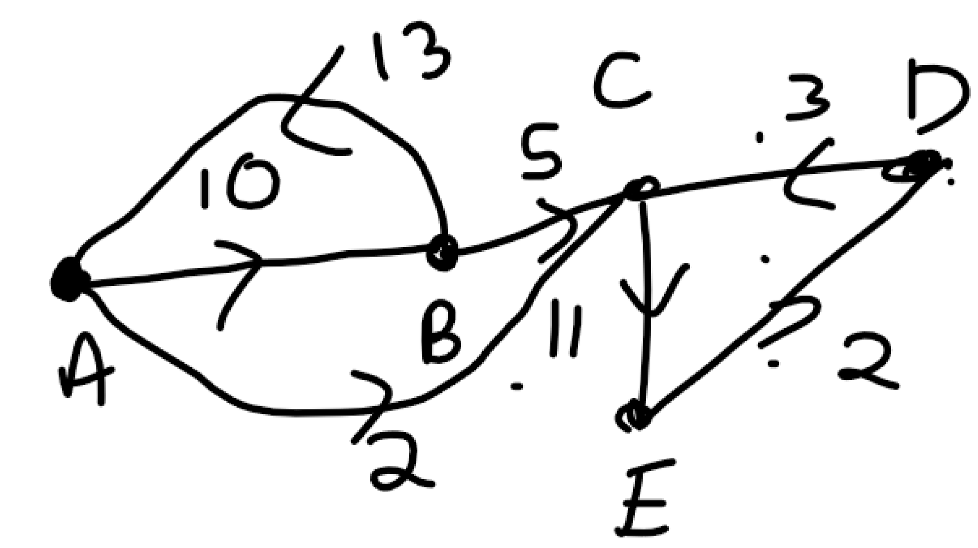
Algorithm to find Shortest paths in graphs:

Shortest paths in graphs: The graph G has n vertices and a distance (weight) associated with each edge of the graph G. We represent the graph as a distance matrix D.

The distance matrix D = (dij) where, dij = 0 , if i = j.
dij = ∞, if i is not joined to j by an edge.
dij = distance associated with an edge from i to j, if i is joined to j by an edge.

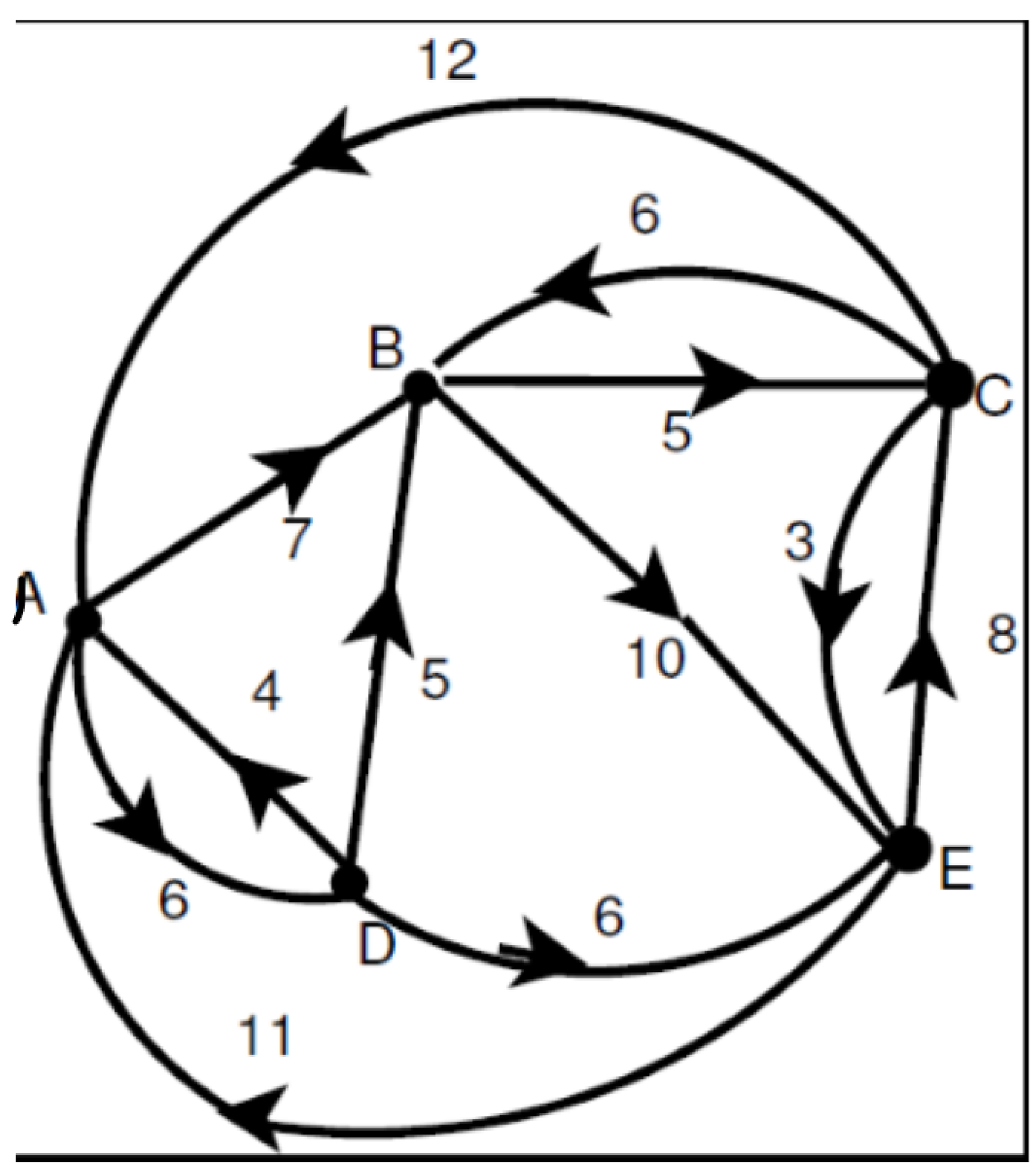
Dijkstra's algorithm to ind the shortest distance between the vertices of a graph G from a fixed vertex:

Define two sets K and U, where K consists of those vertices which have been fully investigated and between which the best path is known, and U of those vertices which have not yet been processed. Clearly, every vertex belongs to either K or U but not both. Let a vertex r be selected from which we shall find the shortest paths to all the other vertices of the network.



$$\begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ A & \begin{pmatrix} 0 & 10 & 2 & \infty \\ & & & \end{pmatrix} \end{array}$$

Example: Implement Dijkstra's algorithm to find shortest path from the vertex B to all other vertices of following graph G.



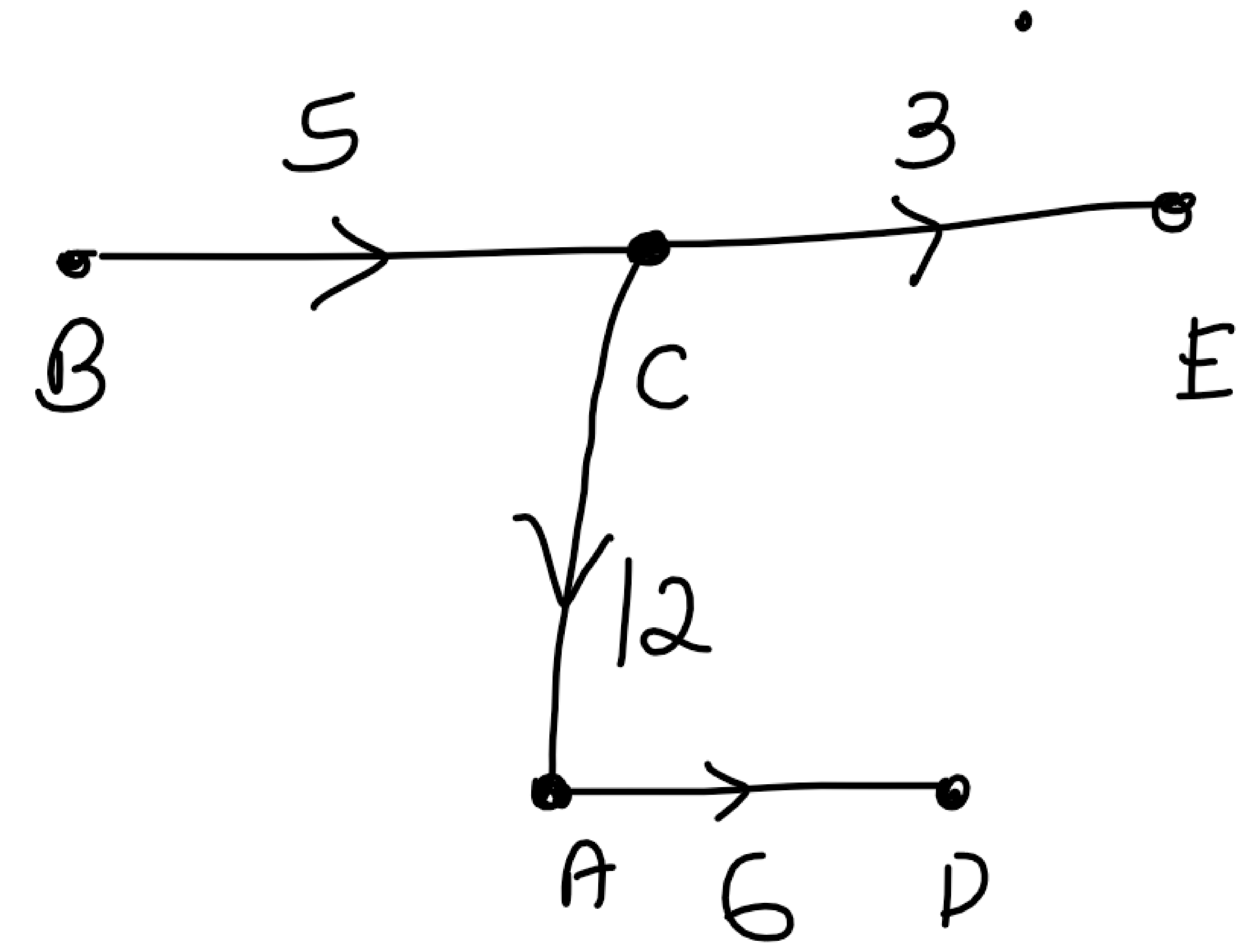
$$D = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 7 & \infty & 6 & \infty \\ \infty & 0 & 5 & \infty & 10 \\ 12 & 6 & 0 & \infty & 3 \\ 4 & 5 & \infty & 0 & 6 \\ 11 & \infty & 8 & \infty & 0 \end{bmatrix} \end{array}$$

$K = \{B\}$ $U = \{A, C, D, E\}$

	A	C	D	E
best-d	∞	5	∞	10
tree	B	B	B	B

set $C = 5$

$K = \{B, C\}$ $U = \{A, D, E\}$



	A	D	E
best d			
Through B	∞	∞	10
Through C	$12+5$	∞	$5+3$

$K = \{B, C, E\}$ set $E = 8$ ✓

$U = \{A, D\}$

	A	D
Through B	∞	∞
Through C	$\boxed{17}$	∞
Through E	$11+8$	∞

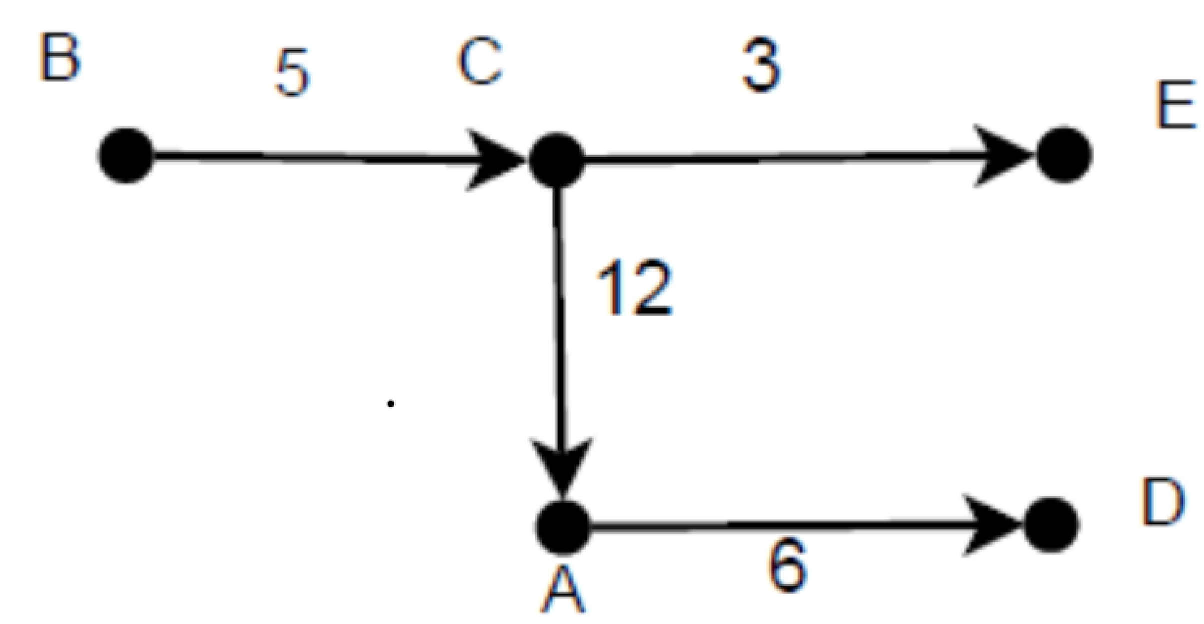
set $E = 8$

set $C = 5$

set $A = 17$

$K = \{B, C, E, A\}$ $U = \{D\}$

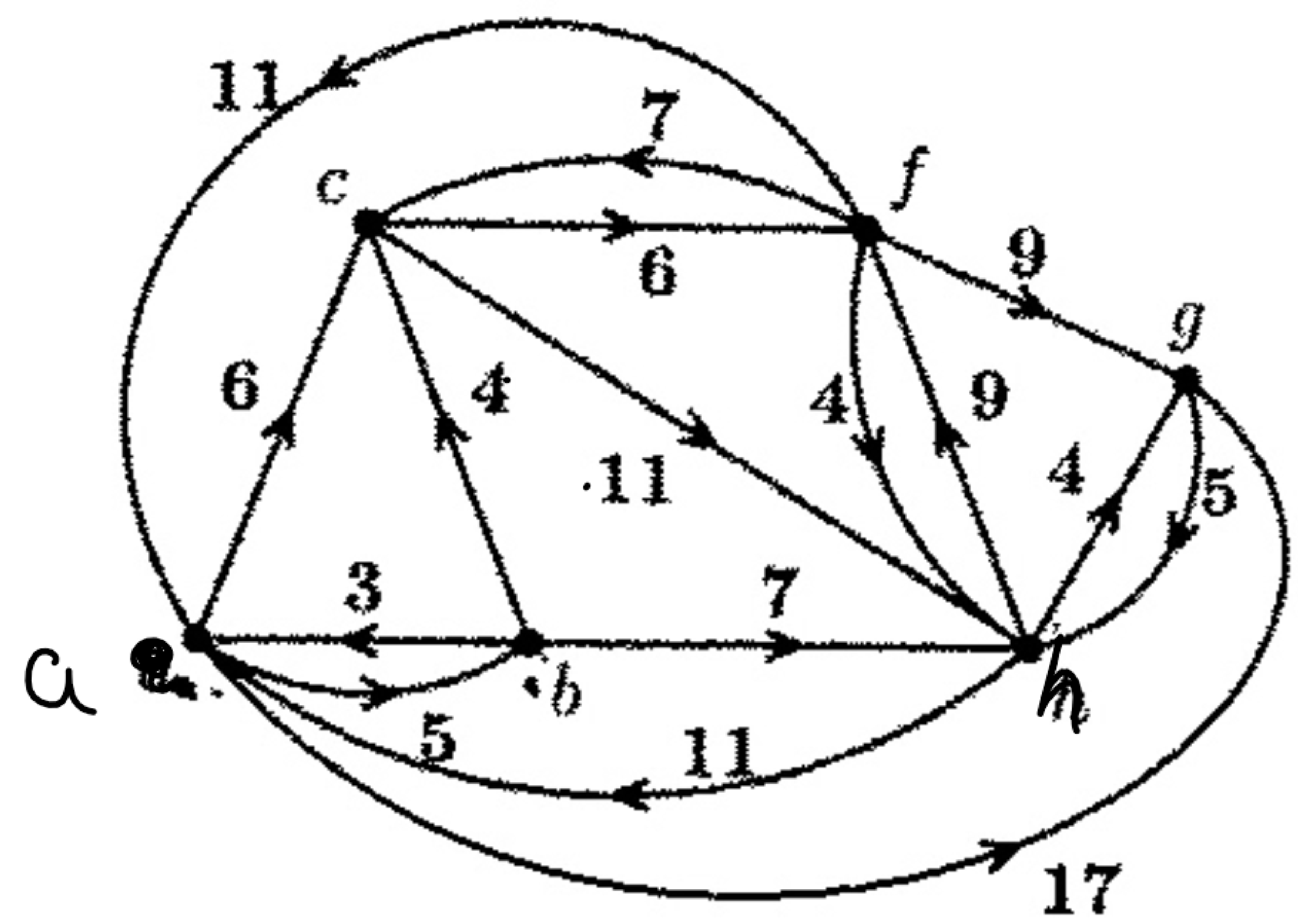
		D
Through	B	∞
	C	∞
	E	∞
	A	$6+17=23$



Tree

	A	C	D	E
best d	17	5	23	8
tree	C	B	A	C

Example: Implement Dijkstra's algorithm to find shortest path from $\underline{c}$ to all other vertices of the following network.



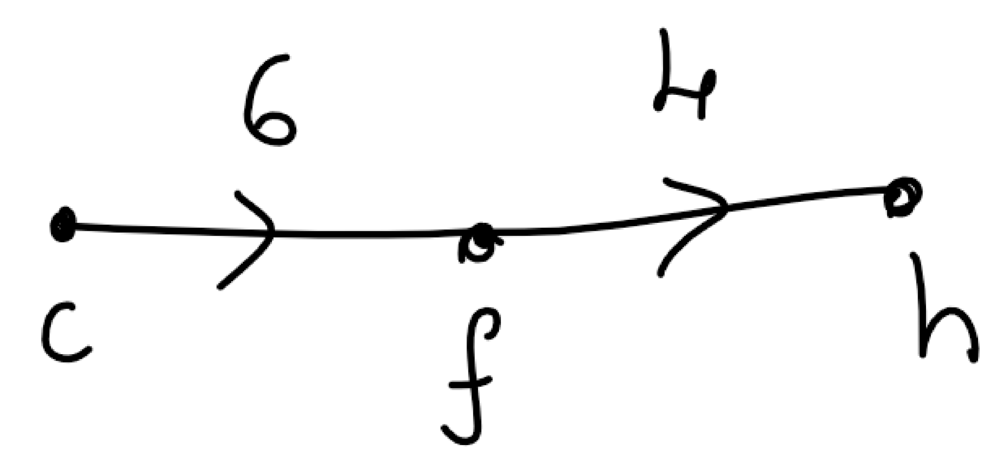
	a	b	c	f	g	h
a	0	5	6	∞	17	∞
b	3	0	4	∞	∞	7
c	∞	∞	0	$\underline{\underline{6}}$	∞	11
f	11	∞	7	0	9	4
g	∞	∞	∞	∞	0	5
h	11	∞	∞	9	4	0

$K = \{c\}$ $\qquad U = \{a \ b \ f \ g \ h\}$

set $f = 6$

$K = \{c, f\}$ $\qquad U = \{a, b, g \ h\}$

	a	b	g	h
c	∞	∞	∞	11
f	11+6	∞	9+6	$\underline{\underline{4+6}}$



set $h = 10$

$K = \{c, f, h\}$ $\qquad U = \{a, b, g\}$

	a	b	g
c	∞	∞	∞
f	11+6	∞	9+6
h	11+10	∞	$\underline{\underline{4+10}}$

set $g = 14$

$K = \{ c \ f \ \ h \ g \}$

$U = \{ a \ \ b \}$

	a	b
c	∞	∞
f	$\underline{\underline{17}}$	∞
h	21	∞
g	∞	∞

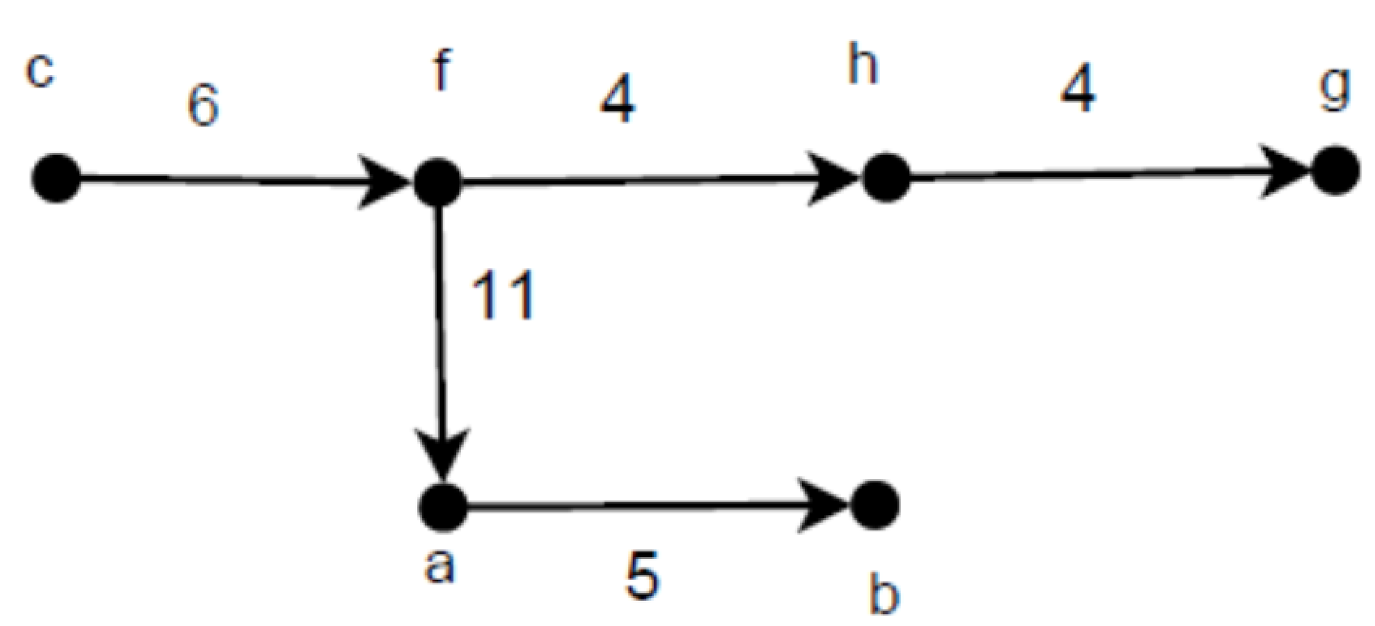
set $a = 17$

$K = \{ c \ f \ \ h \ \ g \ \ a \}$

$U = \{ b \}$

	b
c	∞
f	∞
h	∞
g	∞
a	$17+5$

$b = 22$



Tree

	a	b	f	g	h
bestd	17	22	6	14	10
tree	f	a	c	h	f

Example: Implement Dijkstra's algorithm to find shortest path from c to all other vertices of the following network.