

PERMUTATIONS: (Arrangements, order is considered)

No of ways of arranging r objects out of total of n objects:

Case 1: With no repetition: $\longrightarrow n P_r$

Case 2: With unlimited repetition: $\longrightarrow n^r$

No of ways of arrangements of n objects where

n_1 objects are of kind 1, n_2 objects are of kind 2, ..., n_k objects are of kind k , such that $n_1 + n_2 + \dots + n_k = n$

$$\left. \begin{array}{l} \text{No of ways of arrangements of } n \text{ objects where} \\ n_1 \text{ objects are of kind 1, } n_2 \text{ objects are of kind 2, ..., } n_k \\ \text{objects are of kind } k, \text{ such that } n_1 + n_2 + \dots + n_k = n \end{array} \right\} \longrightarrow \frac{n!}{n_1! n_2! n_3! \dots n_k!}$$

COMBINATIONS:

(selection order is immaterial)

No of ways of selecting r objects out of total of n objects:

Case 1: With no repetition: $\longrightarrow n C_r$

Case 2: With unlimited repetition: $\longrightarrow {}^{n+r-1}C_r$

(Assignment of 'r' obj^s into 'n' boxes)CASE 1: When the objects are distinct

Number of distributions of r distinct objects to n different cells

Subcase 1: If each box can hold at most one object

(selecting r boxes out of tot of n boxes s.t there is no rep of boxes, (order ✓))

$${}^n P_r$$

Subcase 2: If each box can hold any number of objects

$$n^r$$

select 'r' boxes out of 'n' boxes with order in mind

CASE 2: When the objects are identical

Number of distributions of r identical objects to n different cells

Subcase 1: If each box can hold at most one object

$${}^n C_r$$

Subcase 2: If each box can hold any number of objects

$$\frac{n+r-1}{r} C_r$$

for 'r' objects, I've to select r boxes out of total of n boxes, but order doesn't matter.

Note:

Number of non-negative solutions of the equation

$$x_1 + x_2 + \dots + x_n = r$$

n x_i 's summing up to r"No of ways of selecting r x_i 's out of total of n x_i 's with repetition allowed"

$$\frac{n+r-1}{r} C_r$$

(Distⁿ of r 1's to n boxes with rep allowed)

$$x_1 + x_2 + x_3 = 6$$

$$(n=3, r=6)$$

select 6 x_i 's out of x_1, x_2, x_3 with rep allowed

$$\frac{3+6-1}{6} C_6 = {}^8 C_6 =$$

$$x_1 + x_2 + x_3 = 6$$

nonneg solns

$$(114) \\ (222)$$

$$x_1, x_1, x_2, x_2, x_2, x_2 = (2, 40)$$

selecting any 6 x_i 's out of x_1, x_2, x_3 with reptⁿ

Q2: Find the no of permutations of the word INSTITUTION?

(i) How many of them begin with I and end with N?

(ii) How many permutations are with 3 T's not together?

Soln 11 letters $\xrightarrow{\substack{I(3) \\ T(3) \\ N(2)}} \frac{11!}{3! \cdot 3! \cdot 2!}$

i) $\boxed{I \quad \quad \quad \dots \quad \quad N} \xrightarrow{\quad} \frac{9!}{2! \cdot 3!}$

iii) 3 T's not together = Total - 3 T's together = $\frac{11!}{3! \cdot 3! \cdot 2!} - \frac{9!}{3! \cdot 2!}$

3 T's together: $\boxed{\text{TTT} \quad \quad \quad}$

Q3: In how many ways 3 integers can be selected from $3n$ consecutive integers such that the sum is a multiple of 3?

Soln $1, 2, \dots, 3n$
ex:- $3 + 6 + 9 = 18 \checkmark$

So (Rem 0) $\rightarrow 3 \quad 6 \quad 9 \quad \dots \quad 3n$

S_1 (Rem 2) $\rightarrow 2 \quad 5 \quad 8 \quad \dots \quad 3n-1$

S_2 (Rem 1) $\rightarrow 1 \quad 4 \quad 7 \quad \dots \quad 3n-2$

$\begin{matrix} 2 & 2 & 2 \\ \uparrow & \uparrow & \uparrow \\ 2 & 2 & 2 \end{matrix} = 6$
 $\begin{matrix} S_1 & S_2 & S_2 \\ \downarrow & \downarrow & \downarrow \\ 2 & 3 & 3 \end{matrix} = 8$

$\left(\begin{matrix} \text{All 3 elts} \\ \text{from } S_0 \end{matrix} \right) + \left(\begin{matrix} \text{All 3 elts} \\ \text{from } S_1 \end{matrix} \right) + \left(\begin{matrix} \text{All from} \\ S_3 \end{matrix} \right) + \left(\begin{matrix} \text{one from } S_1 \\ \text{one from } S_2 \\ \text{One from } S_2 \end{matrix} \right)$
 ${}^nC_3 + {}^nC_3 + {}^nC_3 + {}^nC_1 \cdot {}^nC_1 \cdot {}^nC_1$

Ans = $3({}^nC_3) + ({}^nC_1)^3$

Q4: In how many ways one right and one left shoe can be selected from 6 pairs of shoes without obtaining a pair?

soln

$${}^6C_1 \cdot {}^5C_1 = \boxed{30}$$

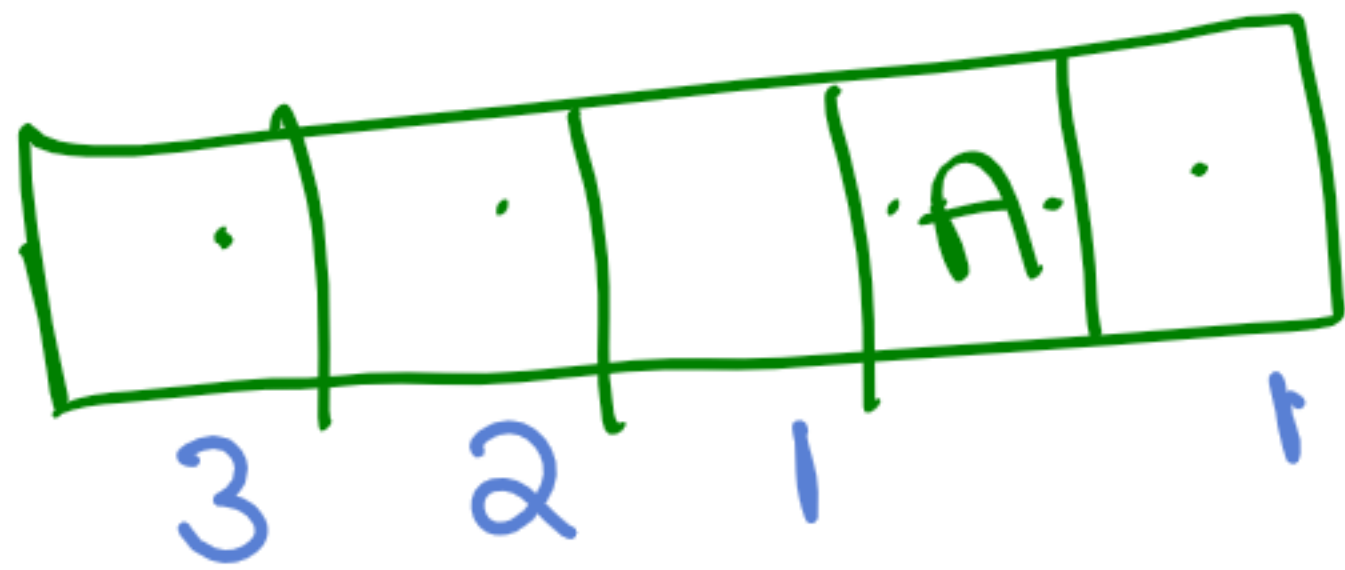
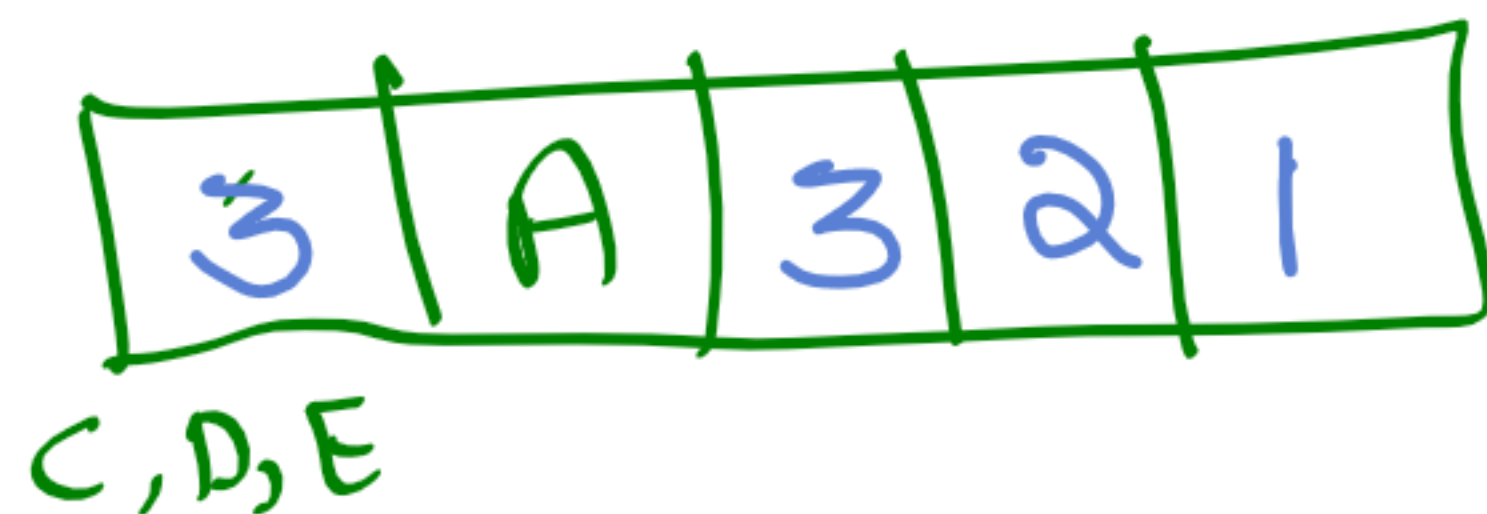
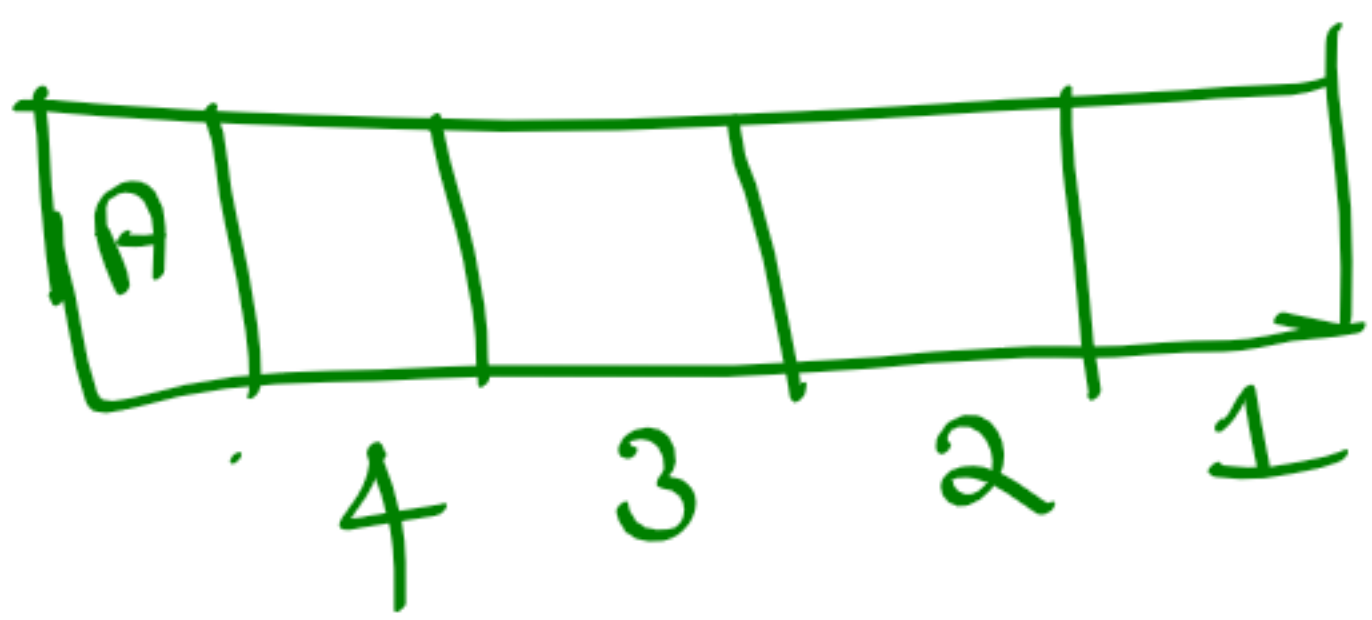
6 pairs \Rightarrow $\underbrace{6}_{A} \text{ left} + \underbrace{6}_{A} \text{ right}$

Q5: If 5 men A, B, C, D, E intend to speak at a meeting,

- in how many orders can they do so without B speaking before A?
- How many orders are there in which A speaks immediately before B?

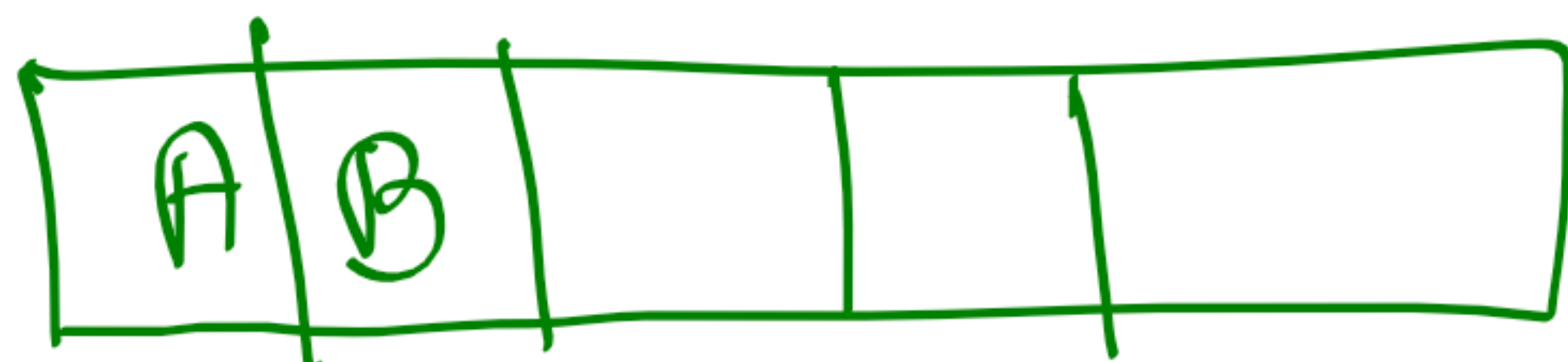
soln

i) B shouldn't speak before A :



$$4! + 18 + 12 + 6$$

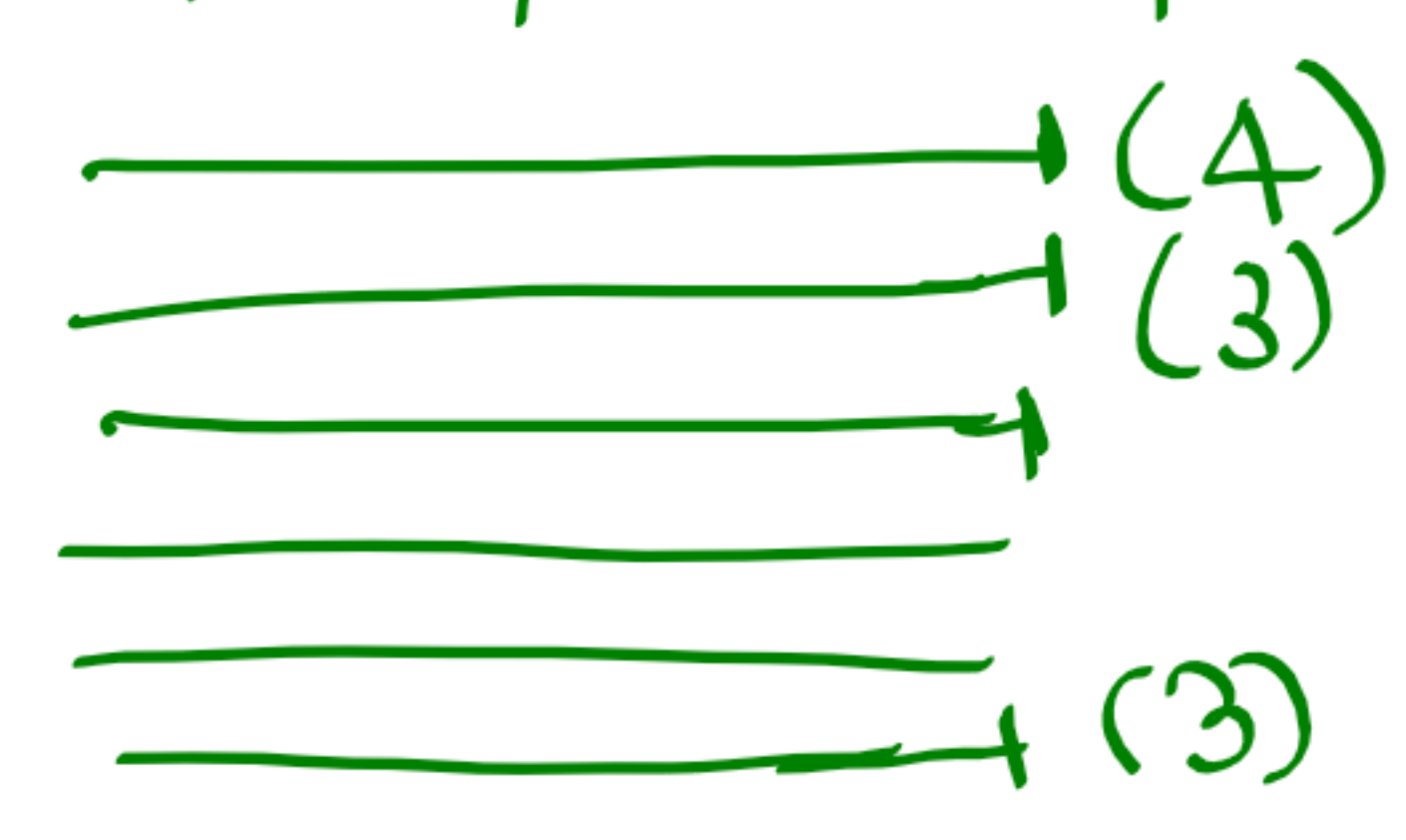
ii) A speaks immediately before B



$$\text{Ans} = \underline{\underline{4!}}$$

Q6: A new national flag has to be designed with 6 vertical strips in yellow, green, blue and red. In how many ways can this be done s.t no two adjacent strips have same color?

Soln Y, G, B, R / 6 strips



Ans = 4×3^5

Q7: In how many ways can 2 squares be selected one by one from 8×8 chess board such that they are not in the same row and same columns?

Soln 8×8 board



$\boxed{64 \quad 35} \rightarrow$

${}^{64}C_1 \times {}^{49}C_1 = \underline{\underline{64 \times 49}} \quad (\because 64-8-7)$

Q8: In how many ways can 5 different msgs be delivered by 3 messengers if no messenger is left unemployed. The order in which a messenger delivers his msg is immaterial?

Soln 5 msgs \rightarrow 3 ppl s.t nobody is left unemployed

i) One person 3 msgs & other two del one each

$(3!1, 13!1, 11!3)$
 $\downarrow \quad \downarrow \quad \downarrow$
 $\frac{5!}{3!} + \frac{5!}{3!} + \frac{5!}{3!} = 3 \left(\frac{5!}{3!} \right)$

$P_1 P_2 P_3 \equiv (3, 1, 1)_{m_1 m_2 m_3}$
 $\neq (3, 2, 0) \times$
 $(3, 1, 1) \neq (1, 3, 1)$
 $P_1 \rightarrow 3(m_1, m_2, m_3)$
 $P_2 \rightarrow 1(m_4)$
 $P_3 \rightarrow 1(m_5)$
 $P_1 \rightarrow 1 \text{ msg}$
 $P_2 \rightarrow 3$
 $P_3 \rightarrow 1$

ii) 2 ppl deliver 2 msgs and one person del one msg

$(221, 122, 212)$

$\boxed{\frac{5! \times 3}{2! 2!}}$

$P_1 \rightarrow (P_1, m_2) \rightarrow (2)$
 $P_2 \rightarrow m_3 m_4 \rightarrow (2!)$
 $P_3 \rightarrow m_5$

$\text{Ans} = 3 \left(\frac{5!}{3!} \right) + 3 \left(\frac{5!}{2! 2!} \right)$

if the second was'nt there, then $3(5!) + 3(5!)$

Q9: Find the sum of all 4 digit numbers that can be obtained using the digits 1,2,3,4 once in each?

soln
Total no of 4 dig nos $\Rightarrow 4! = 24$

in every col, each no occurs exactly 6 times

unit : $1(6) + 2(6) + 3(6) + 4(6) = 60$

10th place : $[1(6) + 2(6) + 3(6) + 4(6)] \times 10 = 600$

100th place : $[1(6) + 2(6) + 3(6) + 4(6)] \times 100 = 6000$

1000th place : $[1(6) + 2(6) + 3(6) + 4(6)] \times 1000 = 60000$

$\therefore \text{Sum} = (60 + 600 + 6000 + 60000)$
 $= \underline{\underline{66660}}$

sum of all 4 dig nos formed by 1,2,3,4

$$\begin{array}{r} 1234 \\ 2134 \\ 1342 \\ \vdots \\ 4321 \\ \hline \text{Ans} = ? \end{array}$$

Q10: In how many ways can an examiner assign 30M to 8 questions such that no question receives less than 2 marks?

soln

Dist 30M to 8 Q^s s.t each Qⁿ receives ≥ 2 marks

$$\begin{array}{ccccccc} Q_1 & Q_2 & & & & & Q_8 \\ \downarrow & \downarrow & & & & & \downarrow \\ 2 & 2 & & & & & 2 \end{array}$$

Give 2M each to all 8 questⁿ first. (30-16) are left
 \therefore Rem 14 marks should be dist^d to 8 questⁿ
s.t each questⁿ receives any no of marks

$n=8$ $r=14$ $\therefore n+r-1C_r = {}^{14+8-1}C_{14}$

Q11: 3 identical dice are rolled. How many outcomes can be recorded?

soln

$n=6$ $r=3$

$${}^{6+3-1}C_3$$

selecting 3 nos out of tot of 6 nos (1,2,3,4,5,6) with repetition

Dist of r obj to n boxes s.t
i) every box can get any no of obj^s
ii) every box can get atmost one obj

or
selecting r obj^s out of n obj^s s.t
i) with rep
ii) no rep

if order is consid^d
 6^3

① How many first thousand +ve integers have distinct digits?

$$\text{Ans} = 738$$

② In how many ways can 12 white and 12 black pawns can be placed on black squares of 8×8 chessboard?

$$\text{Ans} = {}^{32}C_{12} {}^{20}C_{12}$$

③ How many 7 letters palindromes are there?

$$\text{Ans} = 26 \times 25 \times 24 \times 23 \rightarrow \text{w/d sep}$$

$$26^4 \rightarrow \text{w/d "}$$

④ A shop sells 6 diff flavours of icecream. In how many ways, a customer can choose 4 icecream cones if

i) they are all of diff flavours

ii) they are not necessarily of diff flavours

iii) they contain only 2 or 3 flavours

soln

$$\text{i) } {}^6C_4 \quad \text{ii) } {}^{6+4-1}C_4 \quad \text{iii) } 105$$

⑤ How many integer solns are there to the eqn
 $x_1 + x_2 + x_3 + x_4 = 12$ with $x_i \geq 0$

i) How many of them are s.t $x_i \geq 1$

ii) How many of them with $x_1 \geq 2, x_2 \geq 2, x_3 \geq 4$ & $x_4 \geq 0$

soln

$$455$$

$$\text{i) } {}^{11}C_8 \quad \text{ii) } {}^7C_4$$

⑥ In how many ways can 2 adjacent squares be selected from 8×8 chess board?

soln

$$112$$