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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

Second Semester B.Tech Degree End Semester Examination – May 2013

MAT 102: Engineering Mathematics II

(Revised Credit System - 2011)

Time: 3 Hrs. **Max. Marks: 50**

Note : a) Answer any FIVE full questions. **b) All questions carry equal marks.**

1a. Expand $f(x, y) = e^x \cos(y)$ in powers of x and y up to third degree terms.

1b. Using Gram-Schmidt's process construct an orthonormal basis for the vectors {(0, 1, 0), (2, 3, 0), (0, 2, 4)}.

1c. Solve the differential equations:

(i) $\frac{dy}{dx} = \frac{x+y-2}{y-x-4}$ (ii) $y\,dx + (x + x^3y^2)\,dy = 0$

(3 + 3 + 4)

2a. Solve: $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by the method of variation of parameters.

2b. Evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{x}{\sqrt{x^2+y^2}}\,dxdy$ by changing the order of integration.

2c. Evaluate the following using Beta and Gamma functions

(i) $\int_{-\infty}^{\infty} e^{-x^2}\,dx$ (ii) $\int_0^{\pi/2} \sqrt{\cot\theta}\,d\theta$ **(3 + 3 + 4)**

3a. Using double integration, find the area lying inside $r = a(1+\cos\theta)$ and outside $r = a$.

3b. A rectangular box open at the top has a volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction.

3c. Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8x^2e^{2x}\sin 2x$. **(3 + 3 + 4)**

4a. Using triple integrals, find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

4b. Solve: $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = \log x.$

4c. Express the following function f(t) in terms of unit step function and hence find its Laplace transform

$$f(t) = \begin{cases} t^2 & 0 \le t < 2 \\ t-1 & 2 \le t < 3 \\ 7 & t \ge 3 \end{cases}$$

(3 + 3 + 4)

5a. Solve the differential equation using Laplace transform,

$\dfrac{d^2y}{dt^2} - 3\dfrac{dy}{dt} + 2y = e^{3t}$, given $y(0) = 1$ and $y'(0) = 0$.

5b. Find the inverse of the matrix $A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ using elementary row transformations.

5c. Solve the differential equations:

(i) $(xy + x)\,dx - (x^2y^2 + x^2 + y^2 + 1)\,dy = 0$

(ii) $(2y + x^3e^x)\,dx - x\,dy = 0$

(3 + 3 + 4)

6a. Solve by Gauss elimination method:

$$4y + 2z + t = 3, \quad x + y + z + 3t = 5,$$
$$3x + y + z = 4, \quad x - y + 2z + 4t = 7$$

6b. A simple electrical circuit consists of resistance **R** and inductance **L** in a series with constant e.m.f. **E**. Find the current at any time **t** if initially there is no current in the circuit.

6c. Evaluate:

(i) $L\left\{\dfrac{1-\cos t}{t}\right\}$ (ii) $L^{-1}\left\{\dfrac{s}{s^2 + 3s + 2}\right\}$ **(3 + 3 + 4)**

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