

Trees (Non-linear DS).

Tree consists of :

- 1) Distinguished node r : root node.
 - 2) and zero or more non-empty (sub)trees. T_1, \dots, T_k each of whose roots are connected by a directed edge from r .
- Every node is root of some sub tree.
 - A tree can also be empty.
 - In a tree with n nodes; the number of edges are $(n-1)$. Each node other than the root node is associated with an edge.
 - Normally root is drawn at the top.

Level of a Node:

- Defined by letting the root be at level one.
If a node is at level L , its child nodes are at level $L+1$.

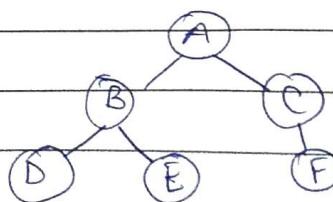
Degree of a node:

$\text{degree}(A) = \text{no. of sub trees of that node.}$

- Degree can also be 0

$\text{degree}(E) = 0$

$\text{degree}(A) = 2$



Degree of a tree:

- max { degree(node) }.
- maximum degree of any node.

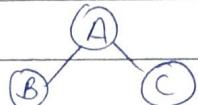
Tree is a Graph which is:

↳ connected

↳ acyclic (cycle not possible).

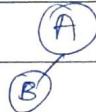
Parent = Node of a subtree.

A is parent of B and C.



Children = Root of the subtree.

B is a child of A.



Sibling: children of same parent.

B & C.

Leaf (Terminal node):

Degree = 0 (no children).

Deg > 0 => non-leaf (non-terminal) node.

Path from n_i to n_k :

- Unique sequence of nodes: n_i is the parent of n_{i+1} for $1 \leq i \leq k$.

Length of path:

- Number of edges on the path.

Depth of n_i :

- Length of unique path from the root to n_i . (Same as level).

$$\text{Depth}(\text{root}) = 1.$$

Height of n_i :

- Length of longest path from n_i to leaf.

Height (Depth) of a tree:

height (root).

depth (deepest root).

Ancestors of a node:

- all the nodes along the path from the node up to the root.

Descendants of a node:

- all nodes in the subtrees.

LL

rows	columns		
r	c	v	→ no. of non-zero elements.
a[0]	4	2	* [2] → [for zeroth value].
a[1]	1	1	10 } non-zero elements
a[2]	3	0	20 }

This and
★ are
always
same value.

0	0	1	
1	0	0	
2	0	0	
3	20	0	

→ r = 3 c = 0 v = 20

- we read from left to right in each row.

class sparse.

{

int row, col, val;

public:

sparse() { row = col = val = 0; }

sparse(int r, int c, int v)

{

row = r; col = c; val = v;

}

array of objects as parameter.

void showTranspose(sparse *);

void fastTranspose(sparse *);

void display(sparse *);

{

↳ (sparse a[]);

void main()

{

→ array of objects.

sparse a[10];

cout << "Enter no. of non-zero elements";

cin >> n;

cin >> r >> c;

a[0] = sparse(r, c, n);

rows & columns.

11

```

for (i=1; i<=n; i++)
{
    cout << "Enter row, col, val";
    cin >> r >> c >> v;
    s[i] = sparse(r, c, v);
}

```

Sparse Matrix:

	0	1	2	3	
0	0	2	0	0	No. of zeroes = 12.
1	1	0	0	0	(3)
2	0	0	0	4	No. of non-zeroes
3	0	3	0	0	(nz) = 4

	Row	Col	Val
a[0]	4	4	4
a[1]	0	1	2
a[2]	1	0	1
a[3]	2	2	4
a[4]	3	1	3

Slow Transpose:

	Row	Col	Val
b[0]	4	4	4
b[1]	0	1	1
b[2]	1	0	1
b[3]	1	3	3
b[4]	2	2	4
b[5]			

void SM::slowTranspose (SM a[])

{

 SM b[10]; int ctr = 1; pass;
 for (pass = 0; pass < a[0].col; pass++)

{
 for (obj = 1; obj <= a[0].val; obj++)

{
 if (a[obj].col == pass)

 b[ctr].col = a[obj].row;

 b[ctr].row = a[obj].col;

 b[ctr].val = a[obj].val;

 ctr = ctr + 1;

}

{

 b[0].row = a[0].col;

 b[0].col = a[0].row;

 b[0].val = a[0].val;

}

void SM::display (SM a[])

{ int obj = 0;

 cout << "\t" << a[0].rows << "\t" << a[0].cols << "\t" <<

 val;

 while (obj <= a[0].val)

{

 cout << "\t" << a[obj].row;

 cout << "\t" << a[obj].col << "\t" <<

 a[obj].val;

 obj = obj + 1;

}

{

Slow:

for pass = 0 to $a[0].c \rightarrow$ Exclusive
 for obj = 1 to $a[0].v \rightarrow$ Inclusive.
 if ($a[obj].cat == pass$)
 we're checking column.
 $b[ctr].r = a[obj].c;$
 $b[ctr].c = a[obj].r;$
 $b[ctr].val = a[obj].val;$

exchange \leftarrow row & column. $b[0].r = a[0].c ; b[0] = a[0].r;$
 $b[0].v = a[0].val$

- Columns \rightarrow rows.
- Rows \rightarrow column.
- Values remain same.
- Rows of new matrix are in ascending order.
- columns of each row are in ascending order.
- Time-space $\& = c * e$ (much more).

Fast Transpose:

- we compare each object.
- input matrices, start position.
- Time-space : $c + e$ (lesser)
 \Rightarrow better & faster.

16/9/22

	c_0	c_1	c_2	c_3
r_0	0	1	4	0
r_1	0	0	3	0
r_2	0	5	0	6

3×4 .

no. of non-zero
terms in that
column.

	0	1	2	3
rowterm	0	2	2	1
Start posn	1	1	3	5

↳ we start with these for rowterm
no. of rows.

	r	c	v
$a[0]$	3	4	5
$a[1]$	0	1	1
$a[2]$	0	2	4
$a[3]$	1	2	3
$a[4]$	2	1	5
$a[5]$	2	3	6

Transpose = r c v

$b[0]$	4	3	5
$b[1]$	1	0	1
$b[2]$	1	2	5
2nd $\rightarrow b[3]$	2	0	4
$b[4]$	2	1	3
3rd $\rightarrow b[5]$	3	2	6

$$\begin{bmatrix} 0 & 10 & 5 & 0 \\ 1 & 0 & 6 & 0 \\ 2 & 4 & 0 & 8 \end{bmatrix}$$

	r	c	v
a[0]	3	4	7
a[1]	0	1	10
a[2]	0	2	5
a[3]	1	0	1
a[4]	1	2	6
a[5]	2	0	2
a[6]	2	1	4
a[7]	2	3	8

↳ companion.

	0	1	2	3
newterm	2	2	2	1
startpos	1	3	5	7

	r	c	v
b[0]	4	3	7
b[1]	0	1	1
b[2]	0	2	2
b[3]	1	0	10
b[4]	1	2	4
b[5]	2	0	5
b[6]	2	1	6
b[7]	3	2	8

19/9/22.

→ Code for fast transpose:

	1	0	0	0
	2	3	0	5
	0	4	0	0
	↓	↓	↓	↓
Rowterm	0	1	2	3
Start posn	2	2	0	1

$$\text{start posn}[0] = 1. \quad \text{start posn}[i] = \text{start posn}[i-1] + \text{rowterm}[i-1].$$

	r	c	v
a[0]	3	4	5

a[1]	0	0	1
a[2]	1	0	2
a[3]	1	1	3
a[4]	1	3	5
a[5]	2	1	4

Code :

```
void SM::fastTranspose(SM a[])
{

```

```
    int rowterm[20], startpos[20];
    for (i=0; i<a[0].col; i++)
        rowterm[i]=0;

```

```
    for (i=1; i<=a[0].val; i++)
        rowterm[a[i].col]++;
    startpos[0]=1;
}
```

```

for (i=1, i<a[0].col; i++)
    startpos[i] = startpos[i-1] +
        rowterm[i-1];
    b[0].col = a[0].row;
    b[0].row = a[0].col;
    b[0].val = a[0].val;
    int b_i;
    for (i=1; i<=a[0].val; i++)
    {
        b_i = startpos[a[i].col];
        b[b_i].col = a[i].row;
        b[b_i].row = a[i].col;
        b[b_i].val = a[i].val;
        startpos[a[i].col]++;
    }
    cout << "Transpose array of objects = ";
    display(b);
}

```

	r	c	v
b[0]	4	3	5
b[1]	0	0	1
b[2]	0	1	2
b[3]	1	1	3
b[4]	1	2	4
b[5]	3	1	5

11

	0	1	2
0	0	0	0
1	1	0	5
2	2	3	0
3	0	4	0

4x3.

	r	c	v	→ We take 1st element all of column.
a[0]	4	3	5	- We check start posn of that number.
a[1]	1	0	1	
a[2]	1	2	5	- We place that no.
a[3]	2	0	2	in the startposn no.
a[4]	2	1	3	of row 2 increment
a[5]	3	1	4	its start posn by 1.
				- We swap values of r & c while writing b.

rowterm	0	1	2	- value of v
	2	2	1	remains same.
startposn	1	3	5	+ This is the sum of the two.

This startposn is always 1.

b[0]	3	4	5	0 0 0 X
b[1]	0	1	1	{ +
b[2]	0	2	2	{ 2
b[3]	1	2	3	{ r
b[4]	1	3	4	{ u → 0 0 0
b[5]	2	1	5	{ r ₂

Pointers:

→ memory location or a variable which stores the memory address of another variable or memory.

Uses:

- Accessing array elements.
- call by reference (arrays & strings to fns).
- dynamic allocation & deallocation of memory.
- linked lists.
- increase execution speed.
- reduces complexity.

→ address of - (operator).

$p = \& op$.

↳ used to get address.

→ $\&(n+y)$: invalid since it does not have a memory location.

`int *p;`

↳ declares variable p as pointer.

↳ this points only to variable that is of integer type.

`float *f;` → only to variable of float type.

$\&$ = reference operator. (address of).

$*$ = dereference operator. (value pointed to).

`n = *p;`

↳ it gives the value stored in address p.
(value pointed by p).

`int * p = &a, d;`

↳ Invalid. Cannot declare anything after pointer.

Scale Factor:

- increment or decrement given to a pointer.
 - changes the address of pointer to next memory address.
 - when we increment a pointer, its value is increased by the length of the data type that it points to.
- This length is called scale factor.

`p1++;` pointing to initial value = 2800
 $\Rightarrow p1 = p1 + 1$
 \Rightarrow value of `p1` = 2802.

7/10/22.

→ Array name is a pointer which represents base address (location of zeroth element).

$n[10] = \{ 11, 21, \dots \}$

↳ value of `n` is memory location of 11.

$x = \&n[0].$

→ When array is declared, compiler allocates base address & amount of memory required.

`int n[5] = { 1, 2, 3, 4, 5 };`

`int * p;`

`p = n;` OR `p = &n[0];`

`p = &n` : Invalid. (It won't point at base address; it will copy address of `n`).

Successive array elements can be accessed:

`cout << *p;`

\downarrow
`p++;`

OR

`cout << *(p+i);`

$i++;$

increments by size
of the pointer's
datatype.

(for `int` : 2 bytes).

(memory is increment so 1 increment = 2 byte increment
for integer).

$p[0] \equiv * (p+0)$

$p[1] \equiv * (p+1)$

$p[2] \equiv * (p+2)$

and so on.

`cout << *(p+i)`

(in for loop, will
also print the
array).

- for float value; increment by 1 =
increment by 4 bytes.

`if p = 1000`

$p+1 = 1002$

$p+2 = 1004$

`content = * (&a[i])`

$\hookrightarrow (p+i)$: address.

`for (j=0; j<5; j++)` { prints elements of
`cout << *p++;` } array p.

→ We can write diff. programs using this.
(Exchange values, etc).

Pointers and Character Strings:

```
char name[] = "Example";
```

size of char = 1.

```
char * cptr = name;
```

E	1000
---	------

```
while (*cptr != '\0')
```

X	1001
---	------

```
    cptr++;
```

A	1002
---	------

length = cptr - name;

address of

base address

final char.

(does not change).

M	1003
---	------

P	1004
---	------

L	1005
---	------

E	1006
---	------

R	1007
--------------	------

$$a[i][j] = *(*(\downarrow a + i) + j)$$

row

column

Array of Pointers:

name[0] → "String 1"

name[1] → "String 2"

name[2] → "String 3"

Instead of 2D array like name[3][25], we can also use array of pointers.

⇒ char * name[3] = {"---", " ---", " ---"};

↳ we can have variable length of each where as in char[3][25], 25 is fixed.

- 25 memory locations are allocated whether we use or not (garbage value).
- 25 is the maximum length.
- In pointers we can have varying no. of length (no maxm or garbage value).

```
for (i=0; i<2; i++)
    cout << name[i];
```

Pointers to Structures :

`struct name`

```
{  
    data members;  
}
```

→ There are no fns inside struct.

`struct inventory`

```
{  
    char name[30];  
    int number;  
    float price;
```

`}; → we can also initialize = {product[2], *ptr;`
`inventory product[2], *ptr;`

`ptr = product; (ptr = &product[0]);`

`ptr → name : product[0].name`

`ptr → number : product[0].number`

`ptr → price : product[0].price`

`ptr = &product[1];`

`ptr → name : same as product[1].name.`

`member`

`→ = selection operator. (arrow operator).`

`ptr → name OR (*ptr).name`

`ptr is holding address.`

`*ptr.name : () is req. because . has higher precedence than *.`

Returning Pointers:

```
int * larger (int * n, int * y)
{
    if (*n > *y)
        return (n);
    else
        return (y);
}
```

Dynamic Memory Allocation:

→ constructor is called.
new is used.

```
int * p; float * q;
p = new int;
q = new float;
```

OR

```
int * p = new int; (we can write in
float * q = new float; one line).
```

```
int * p = new int(25); (memory of 1)
int * p = new int[25]; (array is allocated
& base address is returned).
```

```
p = new int[3][4][5]; } multidimensional
p = new int[m][3]; } allocation.
```

Dynamic Memory De-allocation:

→ destructor is called.

delete is used.

```
delete p;
```

Dangling pointer problem:

$\text{delete } p;$

$p = 0; \quad (p = \text{NULL})$

→ Pointer pointing to deallocated memory.

Memory leak problem:

Memory which is no longer needed
is not released.

$p \boxed{1111}$

$p = 0;$

$p \boxed{0} \quad \boxed{1111}$

↳ we do not know how to
fetch this memory.

→ pointer got diff. value ; previous memory
is still there.

8/10/22.

Linked List:

- Linear DS.

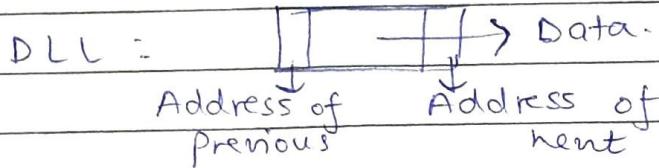
Types:

SLL: Singly.

DLL: Doubly.

CSLL: Circular SLL.

CDLL: Circular DLL.



→ Dynamic in nature.

Code:

```
class SLL {
```

```
    int data;
```

```
    SLL *next; // pointer to class object.
```

```
public:
```

```
    // set of operations on SLL.
```

global scope
SLL *first = NULL; or 3; SLL *first = NULL;
↳ Address of 1st node. (Scope should be global).

```
void main()
```

```
{
```

```
    SLL obj;
```

Operations on SLL:

- i) Inserting a new node (At the end of list).

ASCII value of NULL = 0.

-/-

- 2) Inserting new node in the beginning.
- 3) Count no. of nodes.
- 4) Display

3

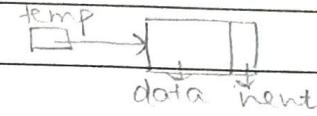
insert → begin.
→ end.
→ before value.
→ after value.

del → deleting a
node with
specific value.

Insert at end:

void SLL::ins_end()

{
 new object is created
 → which has two parts → node
 SLL *temp = new SLL;
 → temp holds address of
 newly created node.



cout << "Enter value";
cin >> temp → data;
 → value will be
 saved here.
 → next is set to null.

temp → next = NULL;

if (first == NULL) //Empty? 0 +
 ↓ condition NULL -

This can also be
written as if (!first).

first = temp;

else → no body.

{ → independent / stand alone for loop

for (SLL *curr = first; curr → next != NULL;
 curr = curr → next);

curr holds address of last node

curr → next = temp;

{ // else.

{ // ins-end.

10/10/22

Insert front:

void SLL::ins_front()

{

SLL * temp = new SLL;

cout << "Enter a value";

cin >> temp → data;

if (first == NULL)

 first = temp;

else

{

 temp → next = first;

 first = temp;

{ // else.

{ // ins-front.

Display:

void SLL::display()

{

if (first == NULL)

 cout << "List is empty";

else

{

 SLL * curr = first;

 while (curr != NULL)

```

}
cout << curr->data << " ";
curr = curr->next;
} // while.      → visiting each node until
} // else.          current becomes null
} // display.       and displaying it.

```

Count no. of elements =

int → returning no. of elements.

~~void SLL::count()~~

{

int c=0;

if (first == NULL)

cout << "Empty";

else

{

SLL * curr = first

while (curr != NULL)

{

c = c + 1;

curr = curr->next;

{

return(c);

{

→ if is not required.

We are returning no. of elements.

Delete :

Delete first :

```
void SLL :: del-first()
{
    SLL * temp;
    if (first == NULL)
        cout << "List is empty";
    else
    {
        curr temp = first; → We are deleting
        first = first -> next;
        delete (temp);
    }
}
```

Delete last :

```
void SLL :: del-last()
```

{

```
    SLL * temp; SLL * prev;
    if (first == NULL)
        cout << "Empty, deletion not possible";
    else
```

{

①

②

```
    for (SLL * curr = first; curr -> next != NULL;
         curr = curr -> next) {  
        prev = curr; ③  
    }
```

- We take prev; it is equal to second last element. (We need to put that to NULL).
- It is not an independent for now.

```
prev->next = NULL;
delete(curr);
```

{

}

→ We also need to check if only 1 node is present. We add an else if:

```
else if(first->next == NULL)
```

{

```
temp = first; } OR delete(first)
first = first->next; } first = NULL;
delete(temp); }
```

{

prev: previous to current.

Delete a node having specific value:

```
void SLL::del-specific()
```

{

```
SLL *temp, *prev; int n; int flag=0;
cout << "Enter node value to be deleted";
cin >> n;
```

```
if(first == NULL)
```

```
cout << "Empty";
```

```
else if(first->data == n)
```

{

```
temp = first;
```

```
first = first->next;
```

```
delete(temp);
```

{

```
flag=1;
```

```
else
```

{

SLL *curr = first;
while (curr != NULL)

{

if (curr->data == n)

{

prev->next = curr->next;

delete (curr);

flag = 1;

break;

{

prev = curr;

curr = curr->next;

} //while.

} //else.

if (flag)

cout << n << "Found" & deleted";

else

cout << n << "Not found";

{

11/10/22

Reversing SLL:

- We will put it in an array and make a new list.



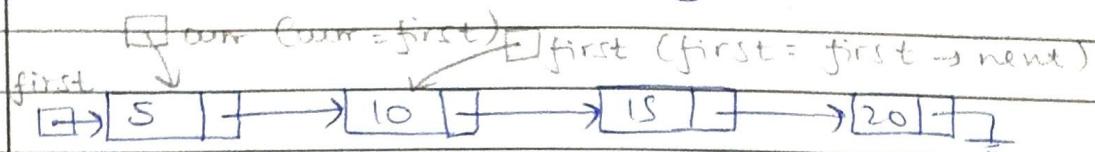
and so on.

- We put each element at start one by one.

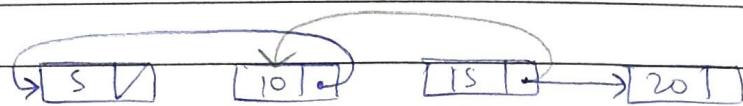
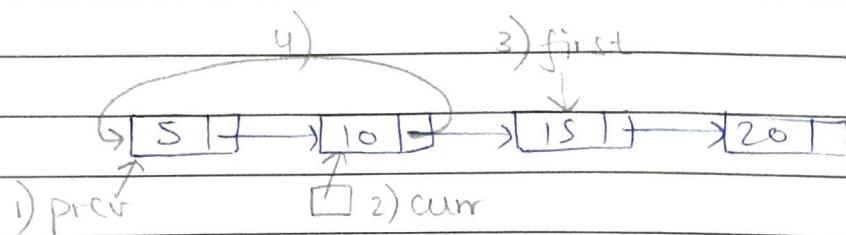
Pseudo code :

- 1) prev = curr
- 2) curr = first
- 3) first = first → next
- 4) curr → next = prev

} while (first != NULL)



Initial: prev = NULL.
curr = NULL.



At last: first = ~~curr~~; curr;

Code:

void SLL :: reverse()

{

SLL * prev, * curr; prev = curr = NULL;
while (first) // while (first != NULL)
{

 prev = curr;

 curr = first;

 first = first → next;

 curr → next = prev;

}

 first = curr;

 display();

}

Sorting SLL:

void SLL::sort() //Ascending Order.

{

 SLL *i, *j;

 for (i = first; i->next != NULL;
 i = i->next)

{

 for (j = i->next; j != NULL;
 j = j->next)

{

 if (i->data > j->data)

{

 int t = i->data;

 i->data = j->data;

 j->data = t;

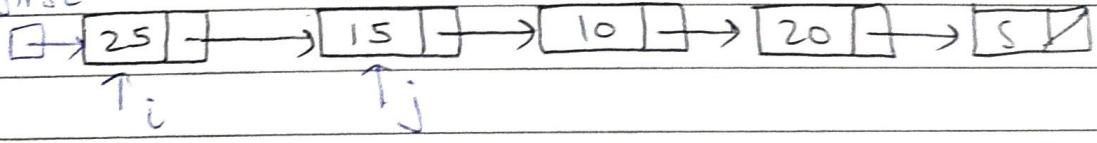
}

{

{

{

first

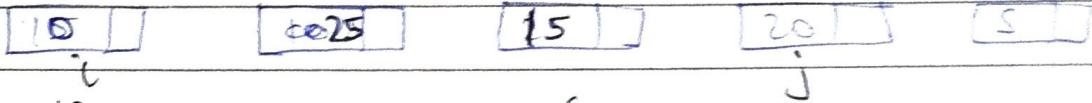


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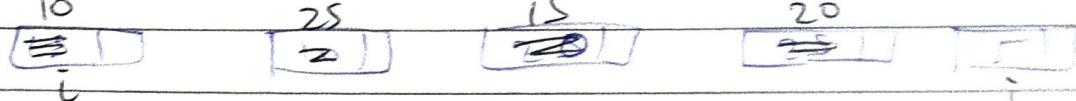


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=>



=>



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Iterations:

4 n-1

3 n-2

2 n-3

1 n-4

13/10/22.

Main function:

Menu: SLLobj:

- 1) Insert a node to first list
obj.end-ins(f1);
- 2) Insert a node into second list.
first2 = obj.ins-end(f2);
- 3) Display 1st list obj.display(f1);
- 4) Display 2nd list obj.display(f2);
- 5) Concatenate list 1 and 2.
- 6) Merge list 1 and 2.

Display:

```
void SLL::display(SLL *first)  
{
```

```
    SLL *curr = first;
```

```
    while (curr) // while curr != NULL).
```

```
{
```

```
    cout << curr->data;
```

```
    curr = curr->next;
```

```
}
```

```
{
```

List Concatenation:



SLL * SLL := concat (SLL * first1, SLL * first2)

{

SLL * first3;

f3 is not a copy. To make duplicate we need to call insert again.

if (first1 == NULL)

 first3 = first2;

else if (first2 == NULL)

 first3 = first1;

else

{

 SLL * curr = first1;

 while (curr → next != NULL) // holds

 the address of last node of first1.

 curr = curr → next;

 curr → next = first2;

 first3 = curr;

{

 return (first3);

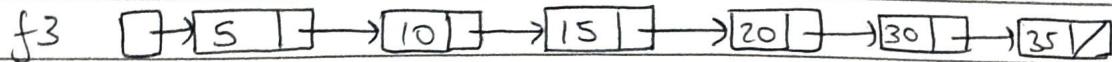
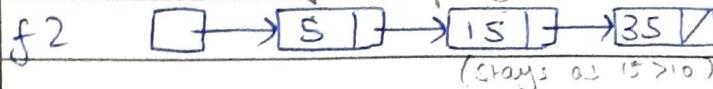
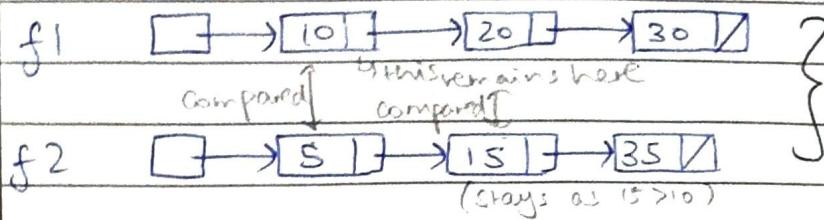
{

Comparisons:

- / -

List Merge: 10 > 5 15 > 10 20 > 15 35 > 20

35 > 30



It is in ascending order.

17/10/22.

Code =

```
SLL * merge (SLL *a, SLL *b)  
{
```

```
    if (!a) // (a == NULL).  
        return (b);
```

```
    if (!b)  
        return (a);
```

```
    SLL * res; = NULL;  
    while (a && b) // while (a != NULL &&  
    {  
        b = NULL).  
            if (a->data < b->data)
```

```
                res = insert_end (res, a->data)  
                a = a->next;
```

(we write res as the
smaller one and increment
only that). (Then we check
the next!).

```
            else if (a->data > b->data)  
                {
```

```
                    res = insert_end (res, b->data);
```

```
                    b = b->next;
```

```
}
```

```
        else // a->data == b->data.
```

{

res = insert_end (res, a->data);

a = a->next;

b = b->next;

{

{

while (a) If there are remaining nodes in a
{ if b = null then they are added.

{

res = insert_end (res, a->data);

a = a->next;

{ // remaining nodes in a are added
to res list.

while (b)

{

res = insert_end (res, b->data);

b = b->next;

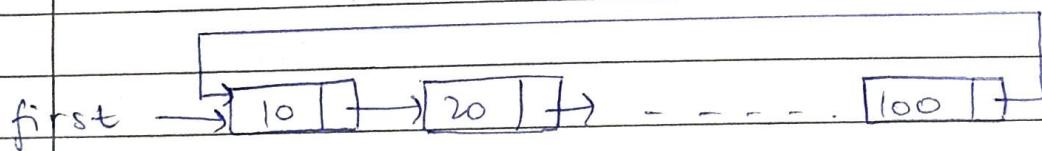
{

display (res);

{ return (res);

{

Circular SLL :



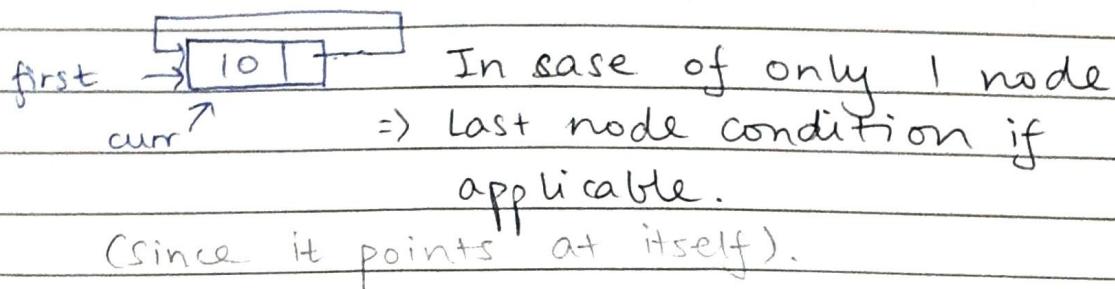
Empty = if (first == NULL).

Last node = curr = first;

while (curr->next != first)

curr = curr->next;

//current holds address of last node.



Code:

```
class CSU
{
    int data;
    CSU * next;
public:
    CSU * insert_end(CSU *, int data);
    void display(CSU *);
};

CSU * first = NULL;
```

Insert end :

```
CSU * CSU :: insert_end(CSU * f, int data)
{
    CSU * temp = new CSU;
    temp->data = data;
    if (f == NULL) (Empty case).
}

f = temp;
f->next = f; (make it point to itself).
```

_ / _

```
else
{
    csu * ctemp = f;
    while (c->next != f)
        c = c->next;
    c->next = temp;
    temp->next = f;
}
// else.
display(f);
return f;
```

Display =

```
void csu::display(csu *f)
{
    csu *c = f;
    while (c->next != NULL)
    {
        cout << c->data;
        c = c->next;
    }
    cout << c->data; // last node value.
```

OR do

```
{}
cout << c->data;
c = c->next;
} while (c != f);
```

Linked List Representation of Stack: (LIFO)

ins-end() : push() } end to first
del-end() : pop() } (display)
OR

ins-first(): push() } first to end
del-end(); pop() } (display)

Empty: first == NULL;

Represenh of Queue: (FIFO)

ins-end() : insert } first to last.
del-first() : del }

OR

ins-first(): insert } last to first.
del-end(): del }

Empty: first == NULL;

- Circular Q we can represent using circular DLL.
- It is not required since in LL space is utilized properly and therefore we do not require circular Q.