III sem B.Tech (IT/CCE)

ICT 2154 Digital Systems
&
ICT 2171 Digital systems and computer organisation

- **Topics covered**
- MINTERM, MAXTERM
- Writing Boolean expressions for the given truth table
- Design of a combinational circuit for the given problem
- Realization of a logic circuit using Universal gates

MINTERMS & MAXTERMS

- MIN and MAX are means to represent the inputs through logical AND /OR operations
- Consider a 3-variable Boolean function, F(a,b,c)
- Prepare a table with 6 columns
- In first column, write all possible binary combinations, possible with three input variables, one below the other. Ex:000,001…., .. . 111
- In 2^{nd} column, write decimal equivalent of all the corresponding input combinations.

Rough

	INPUTS A B C	MINTERMS product	Decimal notation of minterms	Maxterms SUM	Decimal notation of max~~min~~terms
0	0 0 0	$\rightarrow \bar{A}\bar{B}\bar{C} = 1.1.1 = 1$	m_0	$A+B+C$	M_0
1	0 0 1	$\bar{A}\bar{B}C$	m_1	$A+B+\bar{C}$ $0+0+0=0$	M_1
2	0 1 0	$\bar{A}B\bar{C}$	m_2	$A+\bar{B}+C$	M_2
3	0 1 1	$\bar{A}BC$	m_3	$A+\bar{B}+\bar{C}$	M_3
4	1 0 0	$A\bar{B}\bar{C}$	m_4	$\bar{A}+B+C$	M_4
5	1 0 1	$A\bar{B}C$	m_5	$\bar{A}+B+\bar{C}$	M_5
6	1 1 0	$AB\bar{C}$	m_6	$\bar{A}+\bar{B}+C$	M_6
7	1 1 1	ABC	m_7	$\bar{A}+\bar{B}+\bar{C}$	M_7

Minterms (Standard products) and Maxterms (standard sums)

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

Write the Boolean function for f(x, y, z) given below

	x	y	z	f
	0	0	0	1
	0	0	1	1
2	0	1	0	0
3	0	1	1	0
	1	0	0	1
	1	0	1	1
	1	1	0	1
7	1	1	1	0

$$f(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + x\bar{y}\bar{z} + \overset{101}{x\bar{y}z} + xy\bar{z}$$

$$= m_0 + m_1 + m_4 + m_5 + m_6$$

$$= \sum_m (0, 1, 4, 5, 6)$$

$$f(x,y,z) = (\underset{0}{x} + \bar{y} + z) \cdot (x + \bar{y} + \bar{z}) \cdot (\bar{x} + \bar{y} + \bar{z})$$

$x=0 \quad y=0 \quad z=0 \qquad 1 \cdot 1 \cdot 1 = 1$

$x=1 \quad y=1 \quad z=1 \Rightarrow f = 1 \cdot 1 \cdot 0 = 0$

$$= M_2 \cdot M_3 \cdot M_7 = \prod M(2, 3, 7)$$

Boolean function can be represented as (i) sum of minterms and (ii) product of maxterms.

Sum of Minterms and product of maxterms expressions

x	y	z	f	$\bar{f}$
0	0	0	1	0
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

$A + \bar{A}B = A + B$

Sum of minterms : Canonical form

$F(x, y, z) = x'y'z' + x'y'z + xy'z' + xy'z + xyz'$

$= m_0 + m_1 + m_4 + m_5 + m_6$

$= \Sigma(0, 1,4,5,6)$

Sum of product(simplified) : Standard forms

$F(x, y, z) = \boxed{\bar{y} + xy\bar{z}} = \bar{y} + x\bar{z}$

Product of maxterms : Canonical form

$F(x, y, z) = (x+y'+z).(x+y'+z').(x'+y'+z')$

$= M_2 . M_3 . M_7$

$= \Pi(2,3,7)$

Product of sum : Standard forms

$F(x, y, z) = (x+y')(y'+z')$

Rough

Relationship between sum of minterms and product of maxterms

- Sum of minterms : Canonical form
- F(x , y, z)= x'y'z' + x'y' z +xy'z'+xy'z+ xyz'

$= m_0 + m_1 + m_4 + m_5 + m_6$

$= \underset{m}{\Sigma}(0, 1,4,5,6)$

$f(x,y,z) = \Sigma m(0,1,4,5,6) = \pi M(2,3,7)$

$F'(x , y, z) = \underset{m}{\Sigma}(2, 3, 7) = m2+m3+m7$

Taking complement on both sides and applying DeMorgan's theorem

$\bar{\bar{F}} =$ $F(x , y, z) = (m2+m3+m7)' = m2'.m3'.m7'$

$= (x'yz')' .(x'yz)'. (xyz)' = (x+y'+z).(x+y'+z') .(x'+y'+z')$

$= M2.M3. M7 = \Pi (2,3,7)$ ⇒ Product of Maxterm for f

$\mathbf{m_j' = M_j}$

Express the Boolean function F(a,b,c)= $\overline{ab'+c'}$ using Sum of minterms and Product of maxterms

$c+\bar{c}=1$

- Two methods
    1. Identify the missing term and include them in the expression using the postulates : x+0 =x , x.1 =x , x+x'=1, x.x'=0

    2. Write the truth table from the given expression and then write sum of minterms and product of maxterms

F(a,b,c)= ab'+c' using method 1

- $= a\bar{b}\cdot 1 + \bar{c}\cdot 1$

  $= a\bar{b}\cdot(c+\bar{c}) + \bar{c}\cdot(a+\bar{a})$

  $= \underbrace{a\bar{b}c + a\bar{b}\bar{c}}_{} + a\bar{c} + \bar{a}\bar{c}$

  $\quad ,, \quad + a\bar{c}(b+\bar{b}) + \bar{a}\bar{c}(b+\bar{b})$

$= m_4 + m_5 + m_1 + m_0$

$= \sum_m (0,2,4,5,\cancel{6})$ $\Rightarrow \bar{f}(a,b,c) = \sum_m (1,3,7)$

$F(a,b,c) = \pi M(1,3,7)$ $\Rightarrow \bar{f}(a,b,c) = \pi M(0,2,4,5,6)$

$F(a,b,c) = ab' + c'$ using method 2

$c = 0$

$a = 1 \quad b = 0$

a	b	c	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

$$f(a,b,c) = \Sigma m(0,2,4,5,6)$$

$$= \Pi M(1,3,7)$$

Design a combinational circuit that takes 3-bit input and generates an output high whenever the input is a prime number.
Draw the circuit using basic logic gates

$F(A, B, C)$

1. Truth table
2. $F = \Sigma m(\ ) \text{ or } \Pi M(\ )$
3. Simplified expression for F_2
4. Draw the ckt

1

	A	B	C	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

$\bar{A}\bar{B}\bar{C} + \bar{A}BC$

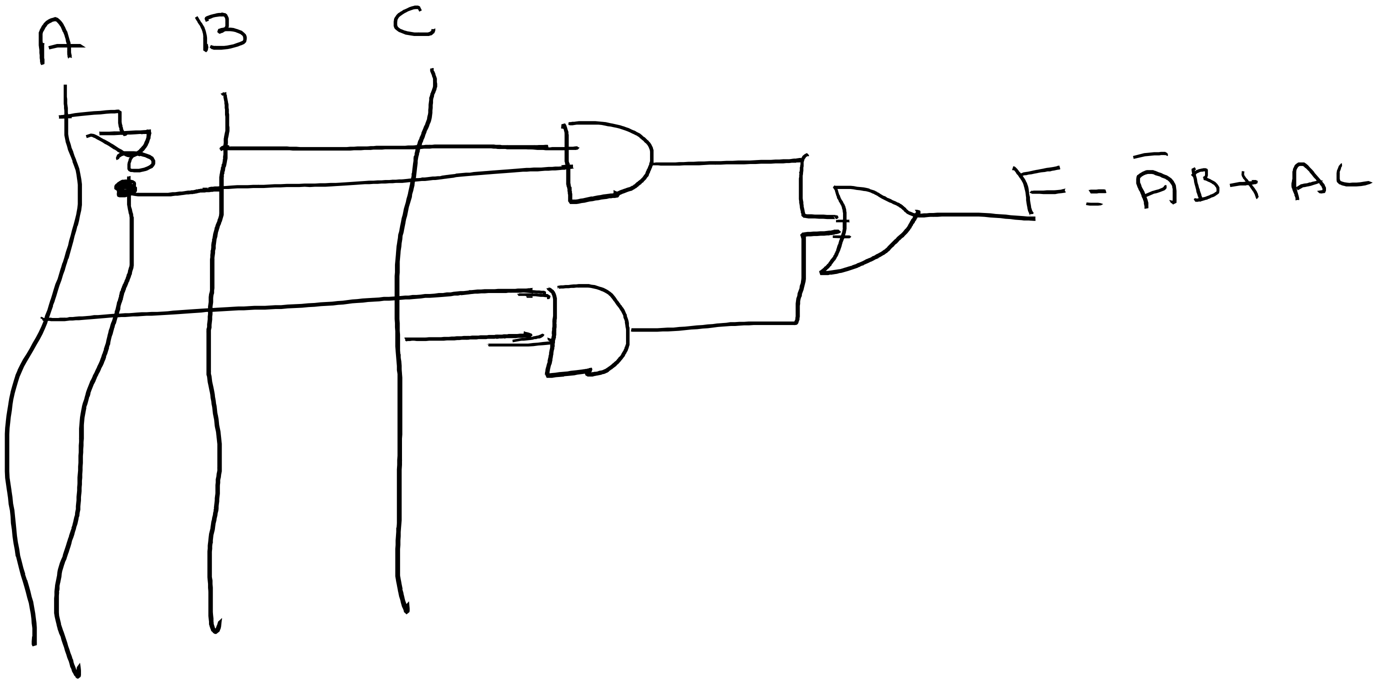
$(\bar{A}B\cancel{C})$

$F(A, B, C) = \Sigma m(2, 3, 5, 7)$

$= \bar{A}B + AC$

Rough

$$F(A,B,C) = \bar{A}B + AC$$



Design a combinational circuit that takes 2-bit input and outputs the square of the input

(1)

	A_1	A_0	Y_3 (MSB)	Y_2	Y_1	Y_0 (LSB)	
0	0	0	0	0	0	0	0
1	0	1	0	0	0	1	1
2	1	0	0	1	0	0	4
3	1	1	1	0	0	1	9

(2)

$Y_3 = A_1 \cdot A_0 = \Sigma m(3)$

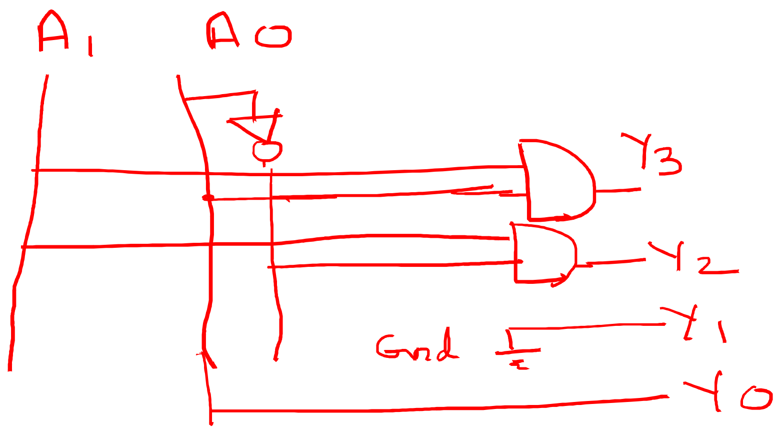
$Y_2 = A_1 \overline{A_0} = \Sigma m(2)$

$Y_1 = 0$

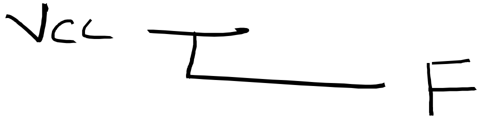
$Y_0 = \cancel{\overline{A_1}} A_0 + \cancel{A_1} A_0 = \Sigma m(1, 3)$

$= A_0$

(3)



if o/p F = 1



Rough

Drawing the circuit using only universal gates

- 1. F(A,B,C,D) = AB + CD using only NAND Gates

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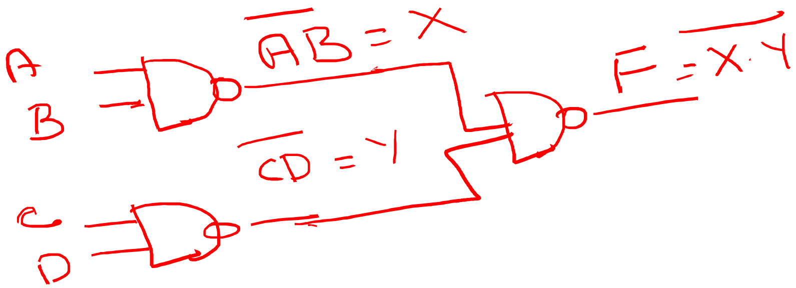
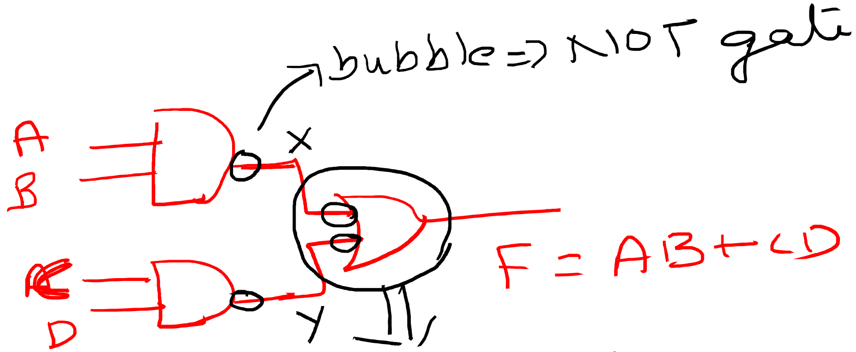
$$\bar{F} = \overline{AB+CD} = \overline{AB}\cdot\overline{CD}$$

$$\bar{\bar{F}} = \overline{(\overline{AB})(\overline{CD})} = F$$

X Y





$F = AB + CD$

$\bar{X} + \bar{Y} = \overline{X \cdot Y}$

$A + \bar{A}B = A + B$

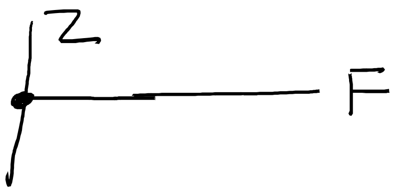
2. F(x,y,z) = xz + y'z+x'yz using only NAND gates

- $= xz + z(\underline{\bar{y} + \bar{x}y}) = xz + z(\bar{y} + \bar{x})$

$F = z$

$\cancel{= xz +}$

$= z(x + \bar{y} + \bar{x}) = z(1 + \bar{y}) = z$

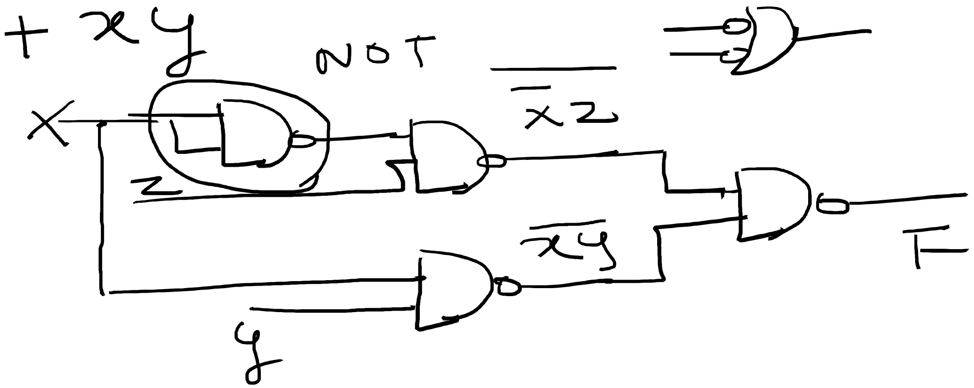


Q $F(x,y,z) = \bar{x}z + xy$

$\bar{F} = \overline{\bar{x}z} \cdot \overline{xy}$

$F = \bar{\bar{F}} = \overline{\overline{\bar{x}z} \cdot \overline{xy}}$

NOT



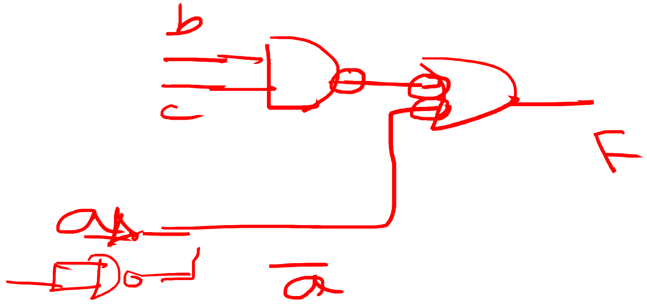
3. Realize the simplified $F(x,y,z) = \Sigma$ (1,2,3,4,5,7) using only NAND gates

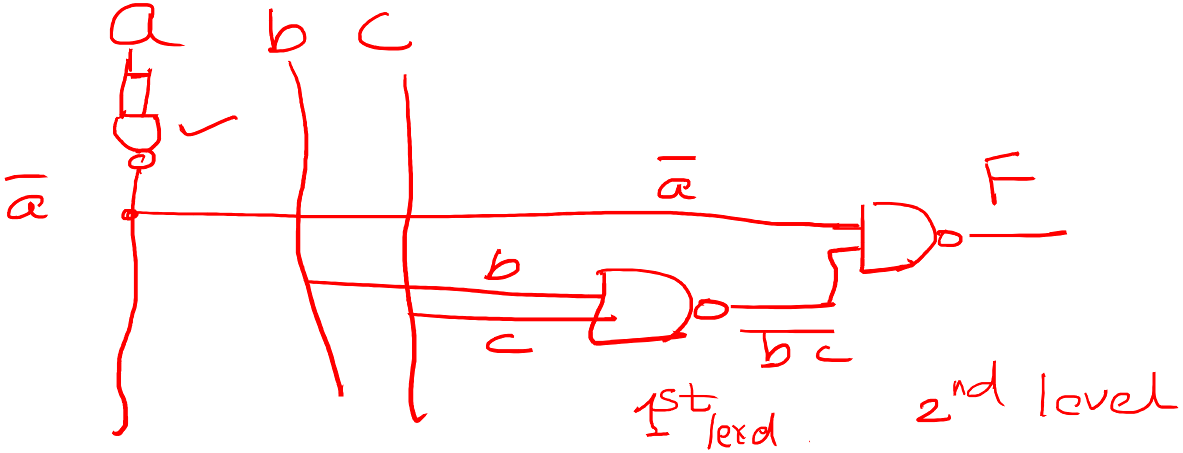
•

4. F(a,b,c)= a+bc using only NAND gates

- $\overline{F} = \overline{a} \cdot \overline{bc}$

  $\overline{\overline{F}} = \overline{\overline{a} \cdot \overline{bc}}$





Drawing the circuits using only NOR gates

- 1. F(x,y,z) = (x+y) (y+z)

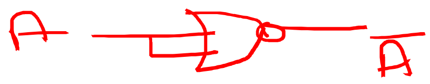
$$\overline{F} = \overline{(x+y)} + \overline{(y+z)}$$

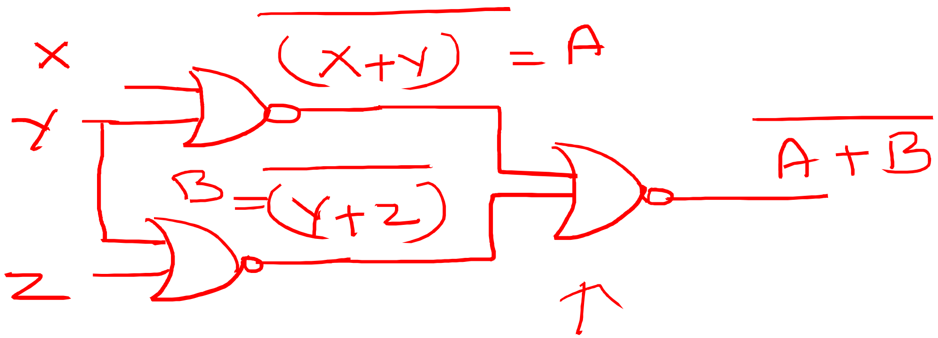
$$\overline{\overline{F}} = \overline{\overline{(x+y)} + \overline{(y+z)}}$$

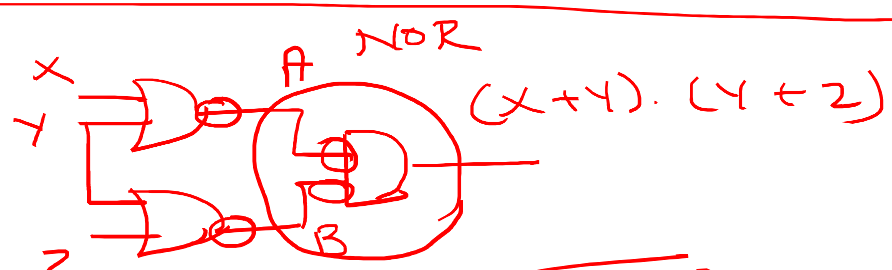
NOT using NOR

$$F = \overline{A+B}$$

$$= \overline{A+A} = \overline{A}$$



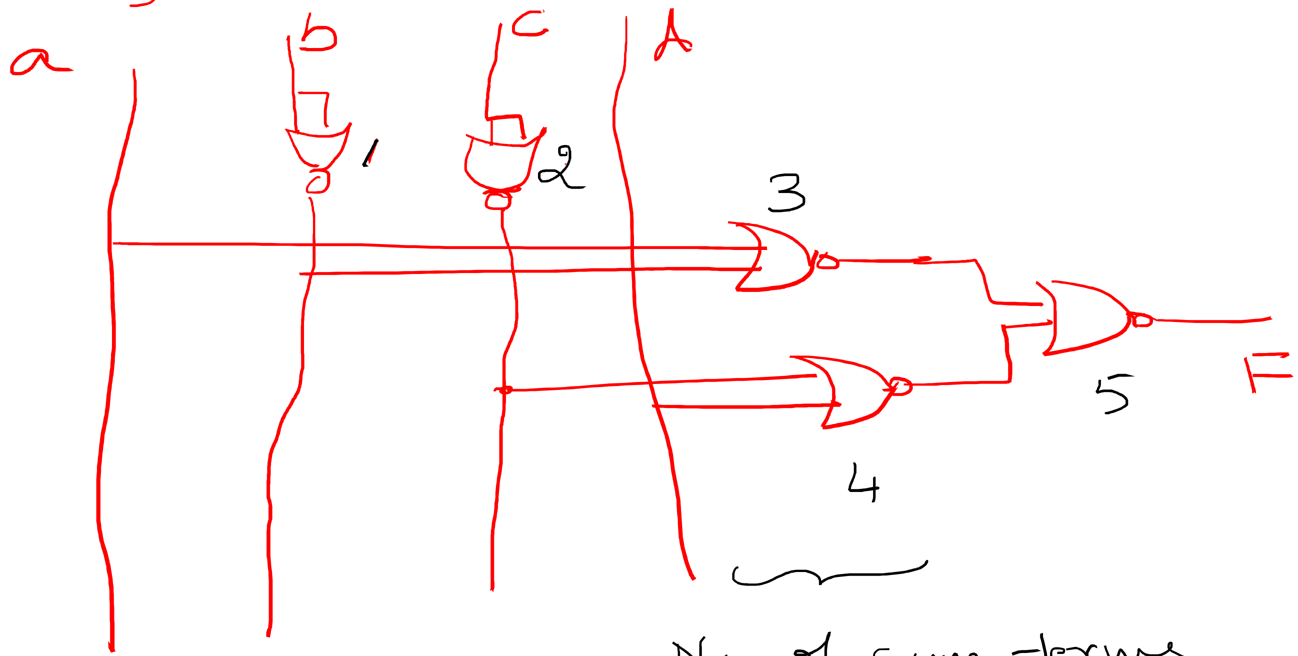




$$\overline{A} \cdot \overline{B} = \overline{(A+B)} \Rightarrow \text{NOR}$$

2. $F(a, b, c, d) = \underbrace{\overbrace{(a+\underbrace{b'}_{1})}^{3}\overbrace{(\underbrace{c'}_{2}+d)}^{4}}_{5}$

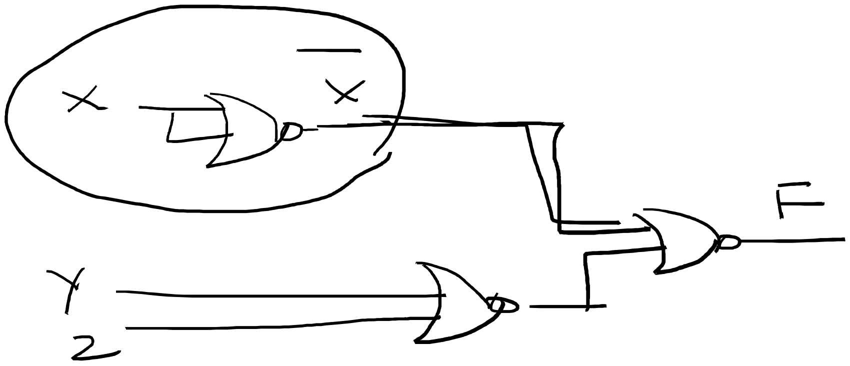
=



No. of sum terms

3. f(x,y,z) = x. (y+z)

$$\Rightarrow f = \overline{\bar{x} + \overline{(y+z)}}$$



4. f(a, b, c, d)= a.b.(c+d)

$$= \overline{\underset{1}{\overline{a}} + \underset{2}{\overline{b}} + \underset{3}{\overline{(c+d)}}}^{\,4}$$

Any questions