Extra questions on permutations and combinations - 1

1. 12 people are made to sit around a round table. Find the number of ways in which they can sit such that John and Mary are not sitting together?

   Ans: $11! - 2 \times 10!$

2. Determine the number of ways to seat 5 boys in a row of 12 chairs?

   Ans: $12P_5$

3. Out of 5 mathematicians and 7 engineers, a committee consisting of 2 mathematicians and 3 engineers has to be formed. In how many ways it can be done such that
   i. There is no restriction
   ii. One particular engineer must be in the committee
   iii. 2 particular mathematicians should not be in the committee

   i. Ans . 350
   ii. Ans. 150
   iii. Ans. 105

4. A student is to answer 12 of the 15 questions in an exam. How many choices does the student have if
   i. In all
   ii. If he must answer first two questions
   iii. If he must answer first or second but not both questions
   iv. If he must answer exactly three of the first five questions
   v. If he must answer At least three of the first five questions

   i. $\binom{15}{12}$
   ii. $\binom{15}{13}$
   iii. $\binom{13}{11}\binom{2}{1}$
   iv. $\binom{5}{3}\binom{10}{9}$

5. A person has to visit one of the 12 temples on each evening of a given week. In how many ways can we plan his week if he will not visit a temple more than once. Ans: $12P_7 = 3991680$

6. Given integers 1,2,…11. Two groups are made, first group contains 5 integers and second group contains 2 integers. In how many ways can the selection be made with unrestricted repetition if
   i. There are no further restriction
   ii. A group has either all odd integers or all even integers

(i) Ans: $\binom{15}{5}\binom{12}{2}$

(ii) $\binom{10}{5}\binom{6}{2}+\binom{9}{5}\binom{7}{2}+\binom{9}{5}\binom{6}{2}+\binom{10}{5}\binom{7}{2}$

7. How many ways are there distribute 27 identical jelly beans among 3 kids
   i. Without restriction
   ii. Each kid getting exactly 9 beans
   iii. Each kid has at least one

   i. $\binom{29}{27}$
   ii. 1
   iii. $\binom{26}{24}$

8. How many are there to assign 100 different diplomats to 5 different continents? Ans: 5^{100}

9. How many ways are there to distribute 20 identical sticks of red candy and 15 identical sticks of black candy among 5 kids? Ans: $\binom{24}{20}\binom{19}{15}$

10. A message is made up of 12 different symbols and is to be transmitted through a communication channel in addition to the 12 symbols with at least 3 spaces between each pair of consecutive symbols. In how many ways can we transmit such a message with 45 blank spaces? Ans: $\binom{23}{12}12!$

11. A bakery sells 6 different kind of pastry. Is the bakery has a dozen of each kind. How many different options for a dozen of pastry are there? What if a box is to contain at least one of each kind of pastry?

    i. $\binom{17}{12}$
    ii. $\binom{11}{6}$

12. In how many ways can we distribute 8 identical balls to 4 distinct boxes such that
    i. No container is left empty
    ii. 4th box has odd number of balls

    Ans i. $\binom{7}{4}$ ii. 70

13. In how many ways can 10 identical marbles are distributed among 5 kids

i. Without repetition
ii. Each kid gets at least one marble
iii. Oldest kid gets at least two marbles

Ans: i. $\binom{14}{10}$ ii. $\binom{9}{5}$ iii. $\binom{12}{8}$

14. A variable name in programming language must be either a letter or a letter followed by a decimal digit. How many different variable names are there in this language? Ans: 26+260

15. In a row of 20 seats, in how many ways can three blocks of consecutive seats with 5 seats each in each block can be arranged?

Ans: $\frac{8!}{3!5!}$

16. In how many ways can 10 boys and 5 girls stand in a line such that no two girls are next to each other? Ans: $10!\binom{11}{5}5!$

17. If repetition is not allowed, how many 4 digit numbers can be formed from the 6 digits 1,2,3,5,7,8
i. How many of the numbers are lesser than 4000
ii. How many are even
iii. How many are odd
iv. How many are multiple of 5
v. How many contain both the digits 5 and 3

ANS: 360 (i) 180 ii. 120 iii. 240 iv. 60 v. 288

18. Find the no of ways to paint 12 offices so that 3 of them will be green, 2 of them pink, 2 of them yellow and remaining ones white Ans: $\frac{12!}{2!2!3!5!}$

19. In how many ways can two numbers be selected from the integers 1,2,…,100 so that the
i. Sum is even
ii. Sum is odd

$ans: i.\binom{50}{2} + \binom{50}{2}$ ii. 2500

20. Three integers are selected from the integers 1,2,…,1000. In how many ways can these integers be selected such that their sum is divisible by 4?

Ans: $\binom{250}{3} + 3\binom{250}{2}\binom{250}{1} + \binom{250}{1}\binom{250}{1}\binom{250}{1}$

21. In how many ways can 5 men and 5 women be seated in a round table if no two women may be seated side by side?

Ans: In a Circular arrangement, Total Permutation is (n-1)!

The total number of people we have are 10. Now, since we don't want two women to sit together, we can't have two men sitting together either.

Step1: Fix all the boys first around the table. This can be done in (5-1)!

Step2: Now we have 5 places in between these men where we can fit available 5 Women. This can be done in 5! Ways.

Total number of ways = 4! * 5!

22. Bob is about to hang his 8 shirts in the wardrobe. He has four different styles of shirt, two identical ones of each particular style. How many different arrangements are possible if no two identical shirts are next to one another?

Soln: Total= $\frac{8!}{(2!)^4}$ = 2520

Ans=Total-1656=864

23. How many distinct results can be recorded from one throw of five dice? Ans: $10C_5$

24. Find the sum of all 4 digit numbers which are formed by the digits 1,2,5,6? Ans: 93324

25. Find the sum of all 4 digit numbers which are formed by the digits 0,1,2,3 with and without repetition of digits?

Ans: $6(1 + 2 + 3 + 4)1000 + 4[(1 + 2 + 3)100 + (1 + 2 + 3)10 + (1 + 2 = 3)]=38664.$
$64((1 + 2 + 3 + 4)1000 + 48[(1 + 2 + 3)100 + (1 + 2 + 3)10 + (1 + 2 + 3)]$

26. A computer password consists of a letter of the alphabet followed by 3 or 4 digits. Find total number of passwords

(i) That can be formed

(ii) In which no digit repeats

Ans: (i) $26(10)^4 + 26(10)^3$

(ii)149760

27. Out of 5 mathematicians and 7 physicists a committee consisting of 2 mathematician and 3 physicists has to be formed. In how many ways it could be done of
    (i) There is no restriction
    (ii) 1 particular physicists must be in the committee
    (iii) 2 particular mathematicians cannot be in a committee.

    Ans: (i) $5C_2 7C_3$ (ii) $5C_2 6C_2$ (iii) $3C_2 7C_3$

28. There are 15 true false questions in an exam. In how many ways can a student do the exam, if he can also choose not to answer some of the questions? Ans: 3^{15}

29. How many 7 letter palindromes can be made out of the English alphabets? Ans: 26^4

30. In how many ways can the letters a, b, c, d, e, f be arranged so that b is always to the immediate left of the letter e. Ans: 5!

31. In how many ways the letters $a, a, a, a, a, b, c, d, e$ are permuted such that no b, c, d, e are adjacent? In how many ways the letters $a, a, a, a, a, b, c, d, e$ are permuted such that no two a's are together?
    Ans: $\frac{9!}{5!} - \frac{6!4!}{5!}$
    $\frac{9!}{5!} - 5!$.

32. How many odd numbers between 100 and 999 have distinct digits? Ans: 320

33. A shop sells 6 flavors of ice-creams. How many ways, a customer can choose 4 ice-cream cones if
    i. If they all are of different flavors
    ii. If they are not necessarily of different flavors
    iii. If they contain only three or four flavors

iv. If they contain 3 different flavors

i. Ans. $\binom{6}{4}$
ii. Ans. $\binom{9}{4}$
iii. Ans. 105
iv. Ans. 60