Basic Electrical Technology

[ELE 1051]

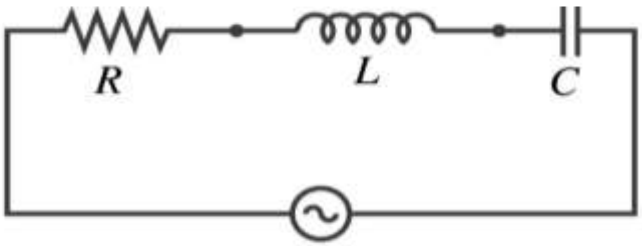
SINGLE PHASE AC CIRCUITS

L20 -Resonance

Topics covered…

- Resonance in series RLC circuit
- What is half power frequency, bandwidth and quality factor in series RLC circuit?
- Resonance in parallel circuits

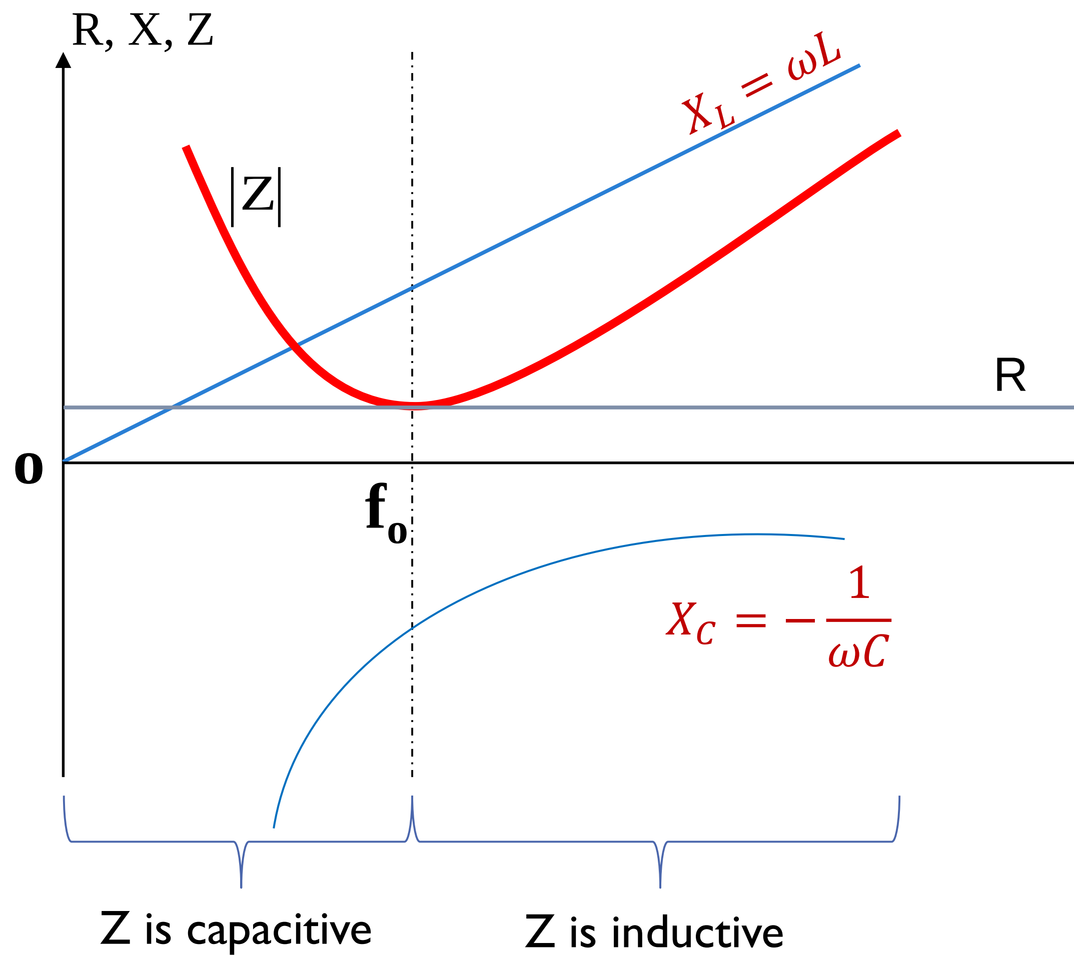
Series Resonance



v(t), variable frequency

$$Z = R + j(X_L \sim X_C)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



'f_0 is called the resonant frequency'

Series Resonance

- **When series RLC circuit is at resonance,**
  - Current is in phase with voltage
  - Circuit power factor is unity
  - $X_L = X_C$
  - $Z = R$

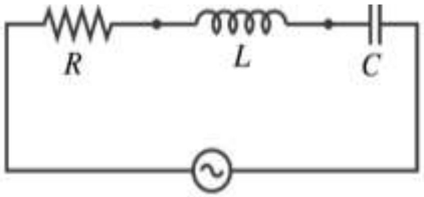
- **Resonant frequency for a series RLC circuit is obtained as follows:**

$$\textit{Imaginary part of } Z_{eq} = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \ \textit{hertz}}$$

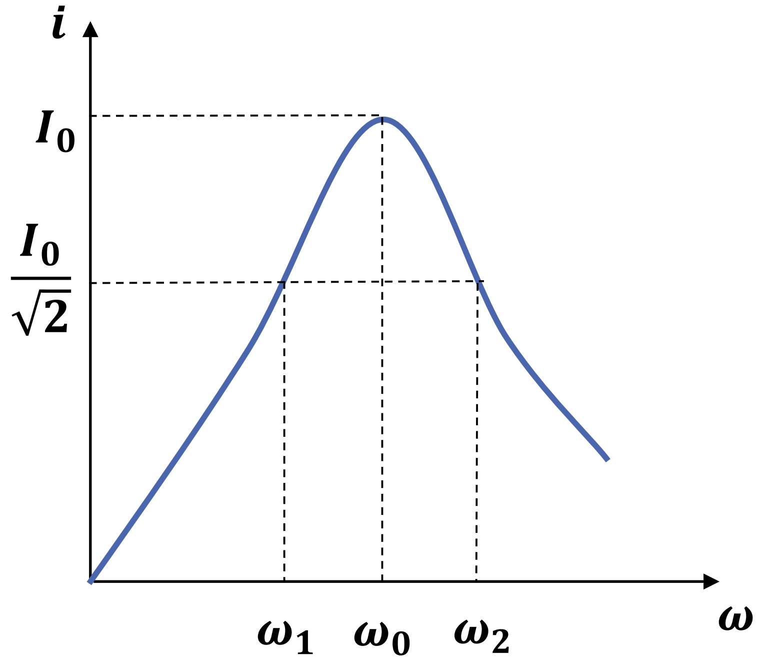


v(t), variable frequency

$$Z = R + j(X_L \sim X_C)$$

Current vs. Frequency in RLC Series Circuit

Variation of current with frequency



$$I_0 = I_{max} = \frac{V_{rms}}{R}$$

- **Half Power Frequency**

  'Frequency at which the power is half of the power at resonant frequency'

$$Power = \frac{1}{2} I_0^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R$$

$$At\ \omega_1\ and\ \omega_2, I = \frac{I_0}{\sqrt{2}}$$

$$\omega_1 = Lower\ half\ power\ frequency$$

$$\omega_2 = Upper\ half\ power\ frequency$$

$$\mathbf{Bandwidth = \omega_2 - \omega_1}$$

In practice the curve of $|I|$ against ω is not symmetrical about the resonant frequency

Half Power Frequency

$$Impedance\ at\ \omega_1 and\ \omega_2, |Z| = {V_0}\Big/{\frac{I_0}{\sqrt{2}}} = \sqrt{2}R$$

Below Resonant frequency $\omega_0, |X_C| > |X_L|$

At ω_1,

$$\sqrt{R^2 + (X_C - X_L)^2} = \sqrt{2}R$$

$$X_C - X_L = R$$

$$\frac{1}{\omega_1 C} - \omega_1 L = \mathrm{R}$$

$$\boxed{\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

Above Resonant frequency $\omega_0, |X_L| > |X_C|$

At ω_2,

$$\sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2}R$$

$$X_L - X_C = R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = \mathrm{R}$$

$$\boxed{\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

$$\omega_2 \omega_1 = \frac{1}{LC} = \omega_0^2 \qquad \omega_2 - \omega_2 = \frac{R}{L}$$

Quality Factor for series circuit

- At resonance, V_C and V_L can be very much greater than applied voltage

$$|V_C| = |I|X_C = \frac{V.X_C}{\sqrt{R^2 + (X_L - X_C)^2}}$$

At resonance, $X_L = X_C$

$$V_C = \frac{V}{R}X_C$$

$$V_C = \frac{V}{\omega_0 CR} = \boldsymbol{Q}V$$

$$\boldsymbol{Q = \frac{Resonant\ frequency}{Bandwidth}}$$

$$\boxed{\boldsymbol{Q = \frac{1}{\omega_0 CR} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}}}$$

Q is termed the Q factor or voltage magnification

- High value of Q can lead to component damage
- Careful design necessary
- Larger the value of Q, more symmetrical the curve appears about the resonant frequency

Illustration 1

A circuit having a resistance of 4Ω and inductance of 0.5H and a variable capacitance in series, is connected across a 100V, 50Hz supply. Calculate:

a) The capacitance to give resonance
b) The voltages across the inductor and the capacitor
c) The Q factor of the circuit

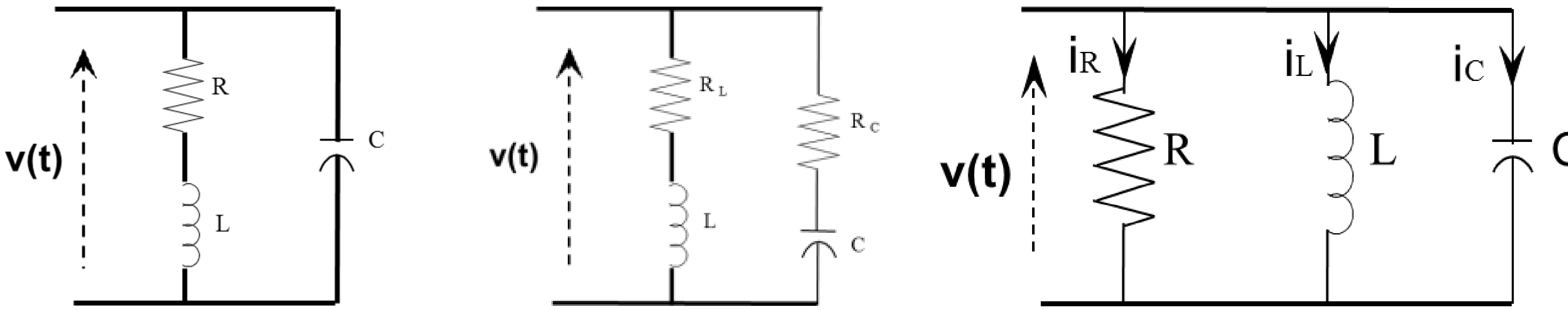
Ans:
C = 20.3μF
$V_C = V_L = 3930V$
Q = 39.3

Illustration 2

The bandwidth of a series resonant circuit is 500 Hz. If the resonant frequency is 6000 Hz, what is the value of Q? If R =10 Ω, what is the value of the inductive reactance at resonance? Calculate the inductance and capacitance of the circuit

Ans:
Q = 12
$X_L = 120\Omega$
L = 3.18mH; C = 0.22μF

Resonance in parallel circuits

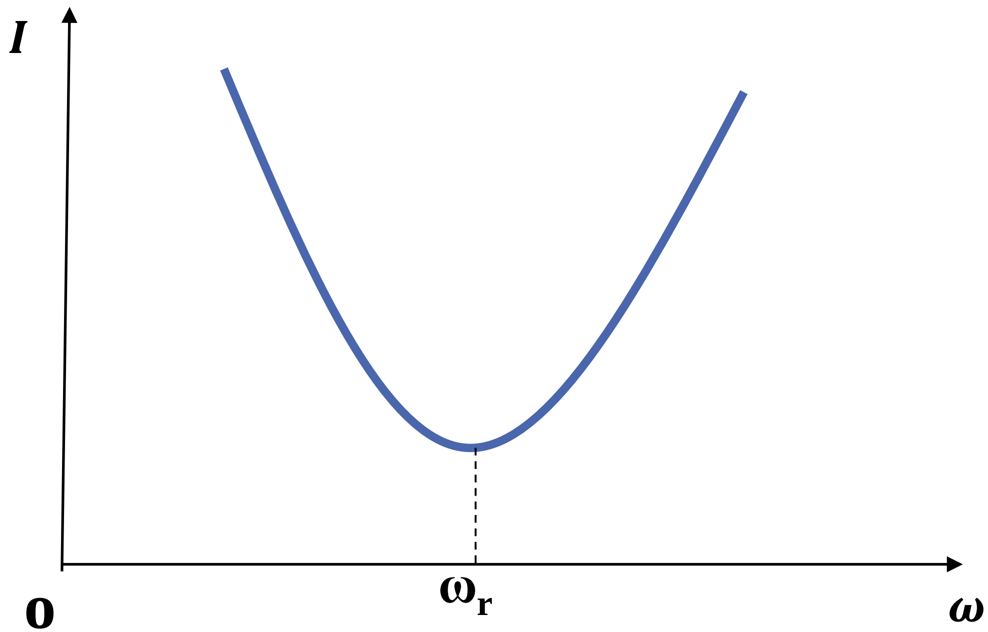


Steps to obtain the expression of resonant frequency in parallel circuits

- Obtain the net admittance of the circuit
- Equate the imaginary part (susceptance) to zero and obtain the expression of ω_r

The expression for resonant frequency depends on circuit configuration

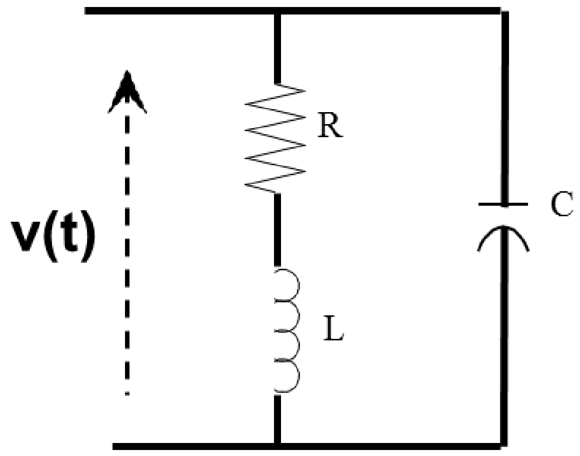
Current vs. Frequency in parallel Circuits



- At resonance
  - Impedance is maximum
  - Resultant current minimum

Illustration 1

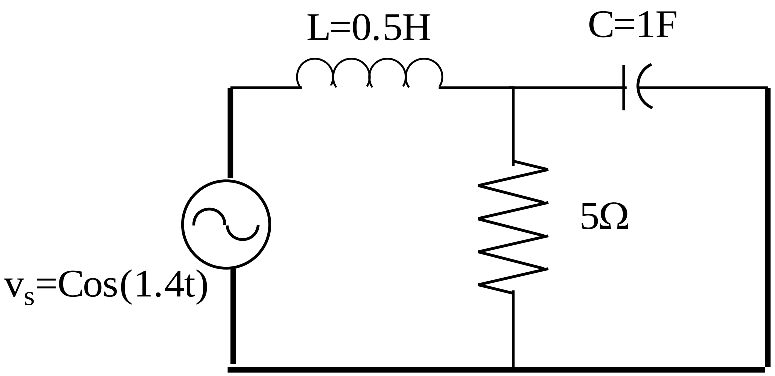
Obtain the expression for resonant frequency for the given parallel circuit



$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

Illustration 2

Show that circuit given in figure will be at resonance at supply frequency



Z = 0.099 at $\omega = 1.4\, rad/sec$