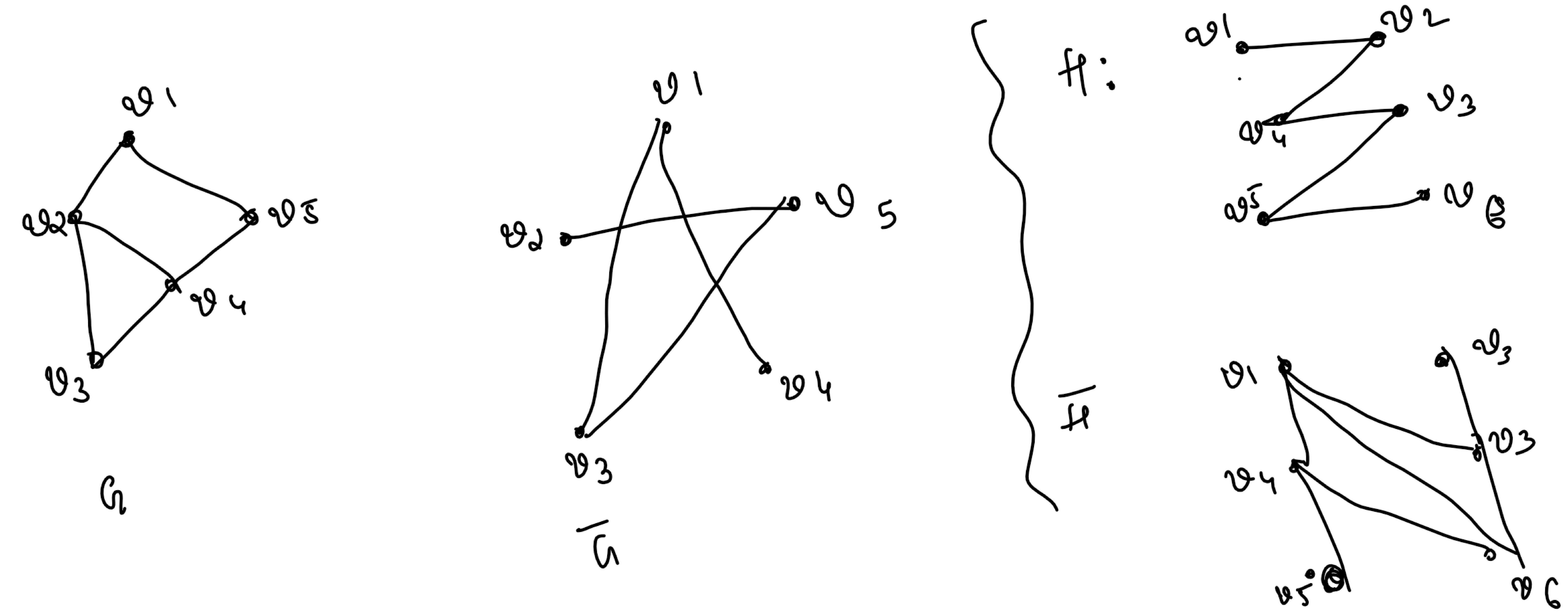


THEOREM 1:

For any graph G on six vertices, either G or $\bar{G}$ contains a triangle.

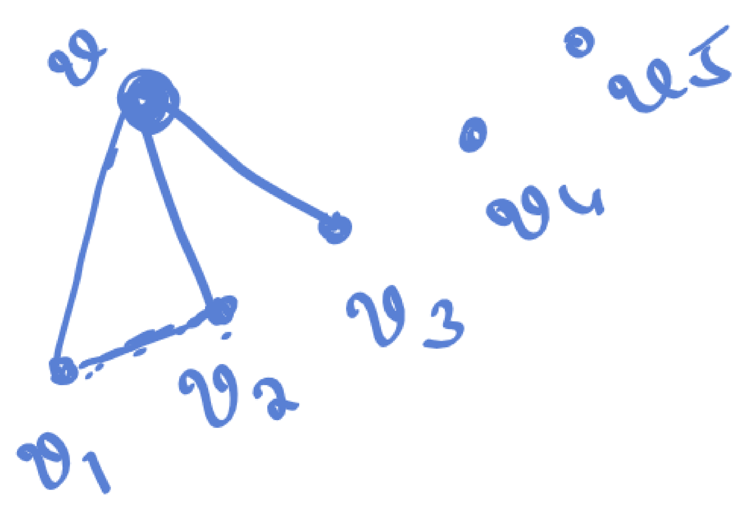
Proof:

Let G be a graph on 6 vertices. Let v be any vertex in G. Note that v is adjacent with other 5 vertices either in G or in $\bar{G}$. We assume that, let v is adjacent with v_1, v_2, v_3 in G. If any two of these vertices say, v_1 and v_2 are adjacent, then v, v_1, v_2 forms a triangle. If no two of them are adjacent, then v_1, v_2, v_3 forms a triangle in $\bar{G}$.

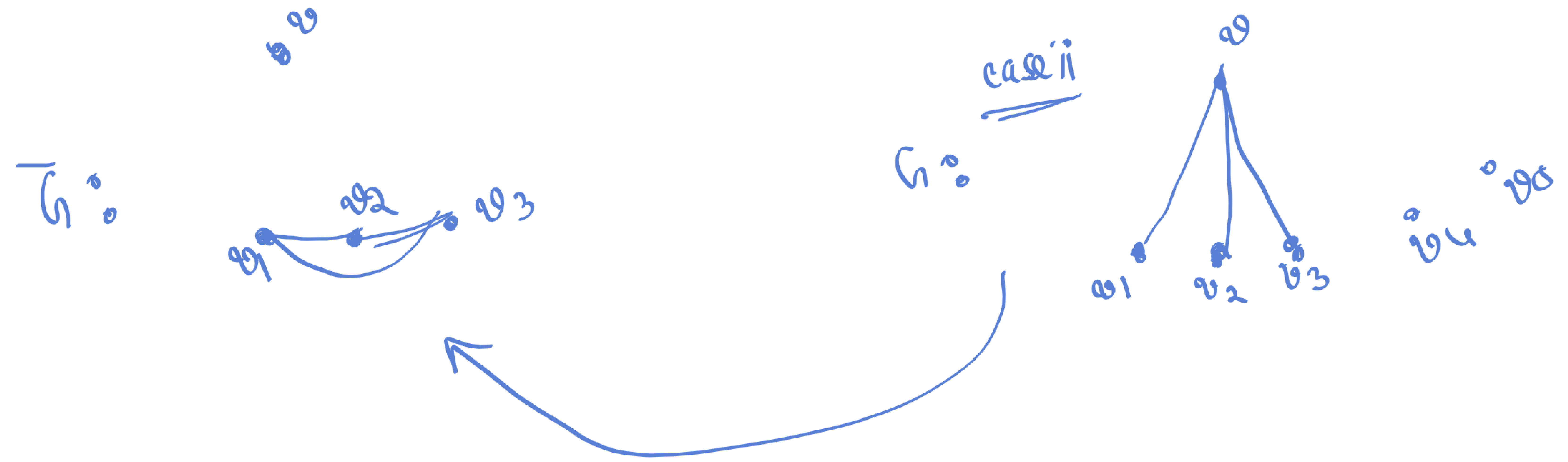
'G'

$v \in V(G)$

case i



case ii



Ramsey

In a party of 6 ppl, there are 3 mutual acquitances & 3 mutual non acquitances

THEOREM 2:

Ex: C_5

P_4

"There are no self comp graph on 6 vertices".

Let G be a self-complementary graph. Then the number of vertices in G is of the form $4n$ or $4n+1$.

Proof:

vertices

edges

Let G be a (p, q) graph. which is self comp

We know that the number of edges in $k_p = \frac{p(p-1)}{2}$

Thus, No of edges in G+ No of edges in $\bar{G} = \frac{p(p-1)}{2}$ -----(1)

Since G is self-complementary,

No of edges in G= No of edges in $\bar{G}$

From (1), No of edges in $\bar{G} = \frac{p(p-1)}{2}$ −No of edges in G

$$q = \frac{p(p-1)}{2} - q$$

That is, $q = \frac{p(p-1)}{4}$.

Thus either $4|p$ or $4|p-1$, which implies, either $p = 4n$ or $p = 4n + 1$.

$\underbrace{\text{No of edges in } G}_{q}$ + No of edges in G $= \frac{p(p-1)}{2}$

$$q + q = \frac{p(p-1)}{2}$$

$$2q = \frac{p(p-1)}{2}$$

$$q = \frac{p(p-1)}{4}$$

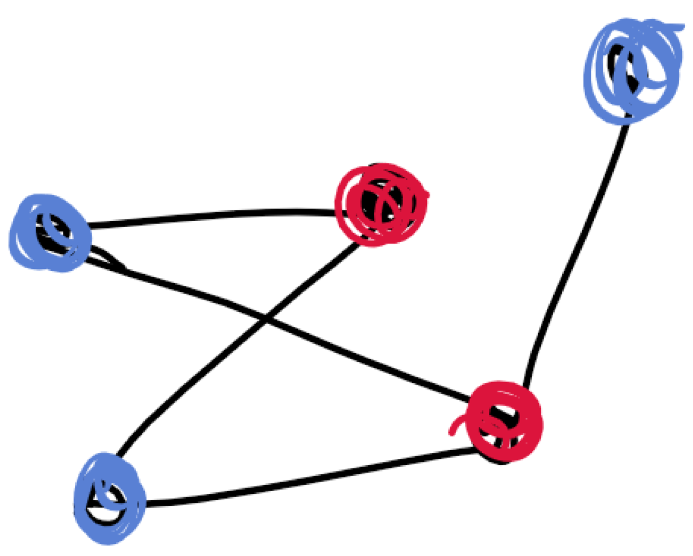
q has to be an integer

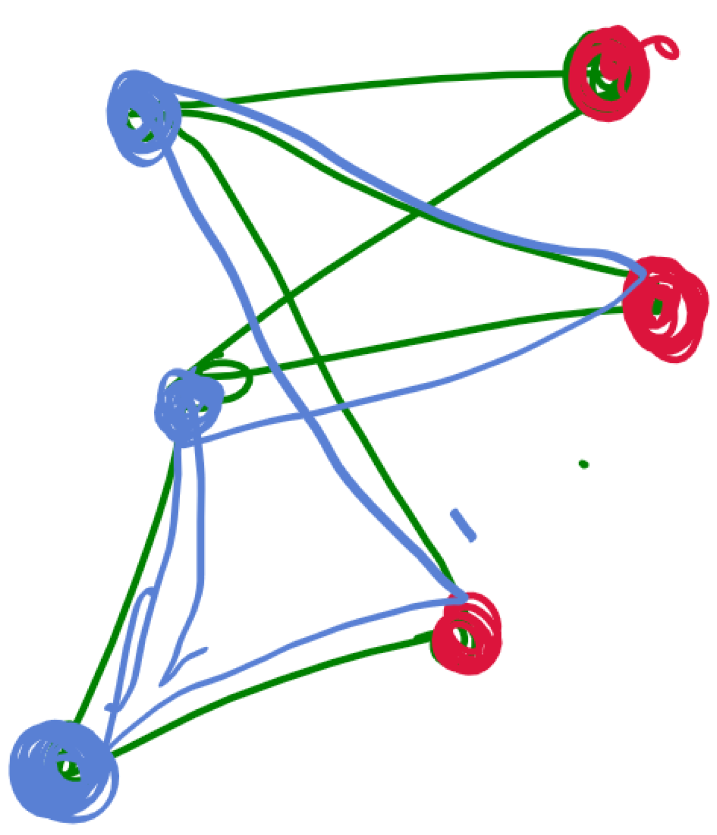
$4|p$ or $4|(p-1)$

No of vertices in a self complementary graph is either $4n$ or $4n+1$ for $n \in \mathbb{Z}$.

V_1

V_2





Not bipartite

THEOREM:

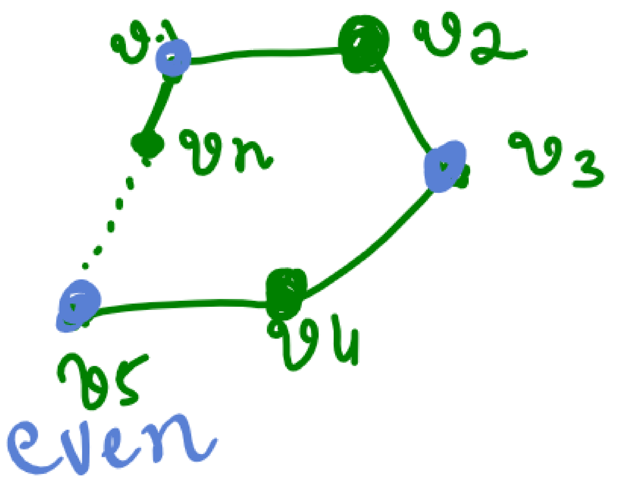
A graph is bipartite if and only if all the cycles are of even length.

Proof   Bipartite, P.T all cycles are of even length

$G \rightarrow V(G) = V_1 \cup V_2$

Let G be a connected bipartite graph. Then its vertex set V can be partitioned into two sets V_1 and V_2 such that every edge of G joins a vertex of V_1 with a vertex of V_2. Thus, every cycle $v_1, v_2, \ldots v_n, v_1$ in G necessarily has its oddly subscripted vertices in V_1(say). i.e, $v_1, v_3, \ldots \in V_1$ and other vertices $v_2, v_4, \ldots \in V_2$. In a cycle $v_1, v_2, \ldots v_n, v_1$: v_n, v_1 is an edge in G. Since, $v_1 \in V_1$ we must have $v_n \in V_2$. This implies n is even. Hence, the length of the cycle is even.

$v_1 v_2 v_3 v_4 \cdots v_n v_1$



$v_1 v_3 v_5 \cdots \in V_1$

$v_2 v_4 v_6 \cdots \in V_2$

$v_n \in V_2$, $n \rightarrow$ even

all cycles are of even length ⇒ Bip

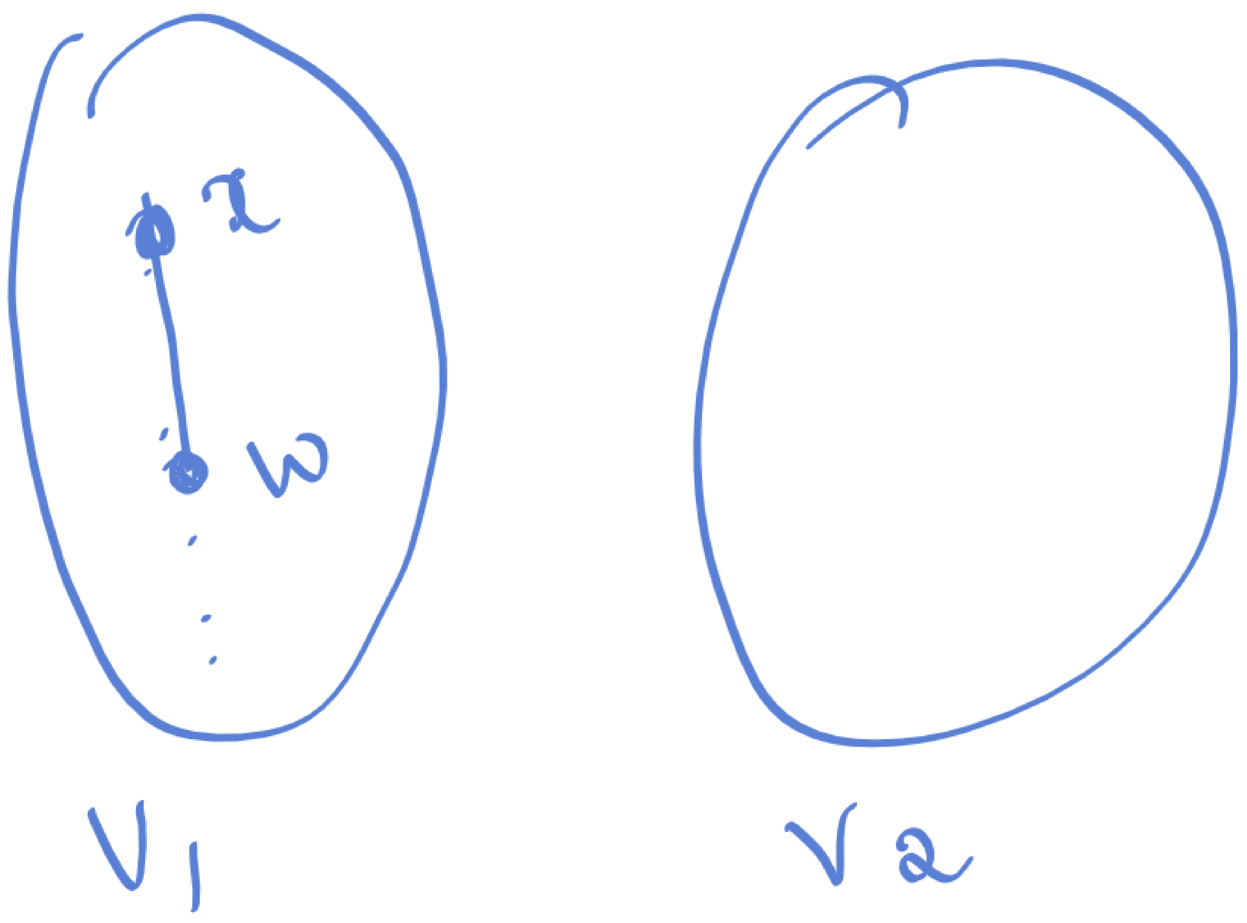
Conversly, suppose that G is a connected graph with no odd cycles. Let $u \in G$ be any vertex. Let $V_1 = \{v \in V/d(u, v) = even\}$, $V_2 = \{v \in V/d(u, v) = odd\}$. Then, $V = V_1 \cup V_2$, $V_1 \cap V_2 = \Phi$. We must prove that no two vertices in V_1 and V_2 are adjacent. Suppose that $x, w \in V_1$ be adjacent. $w \in V_1 \Rightarrow d(u, w) = 2k$ and $x \in V_1 \Rightarrow d(u, x) = 2l$. Thus, the path $u - w - x - u$ forms a cycle of length $2k + 2l + 1$, odd a contradiction. Therefore, x and w cannot be adjacent. That is no two vertices in V_1 are adjacent. Similarly we can prove no two vertices in V_2 are adjacent. Hence, the graph is bipartite. □

'G'

$u \in V(G)$

$V_1 = \{v \mid d(u,v) = \text{even}\}$

$V_2 = \{w \mid d(u,w) = \text{odd}\}$

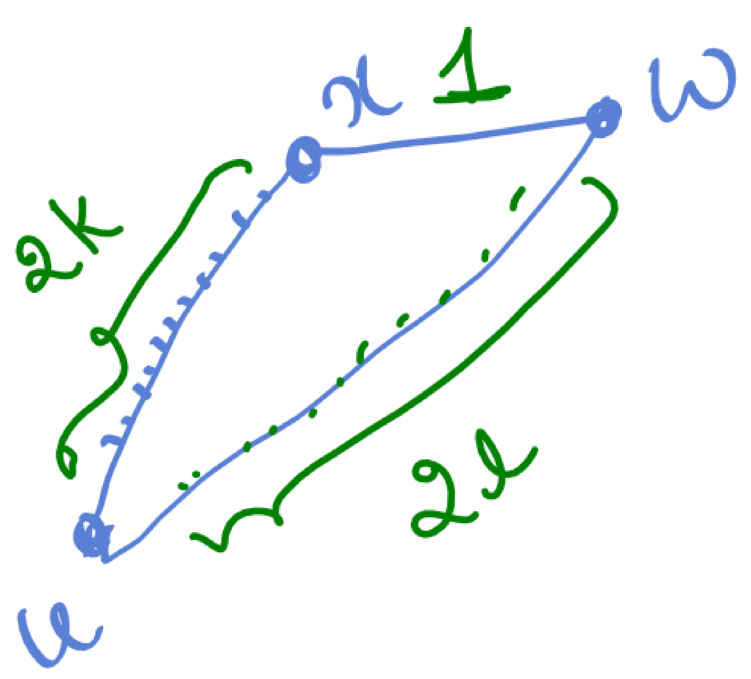


consider $x, w \in V_1$. P.T x & w are nonadjacent

Suppose x & w are adjacent

$d(u, w) = 2k$

$d(u, x) = 2l$



length $= 2k + 2l + 1$

$=$ odd no

contradictn

∴ x & w are nonadjacent

THEOREM:

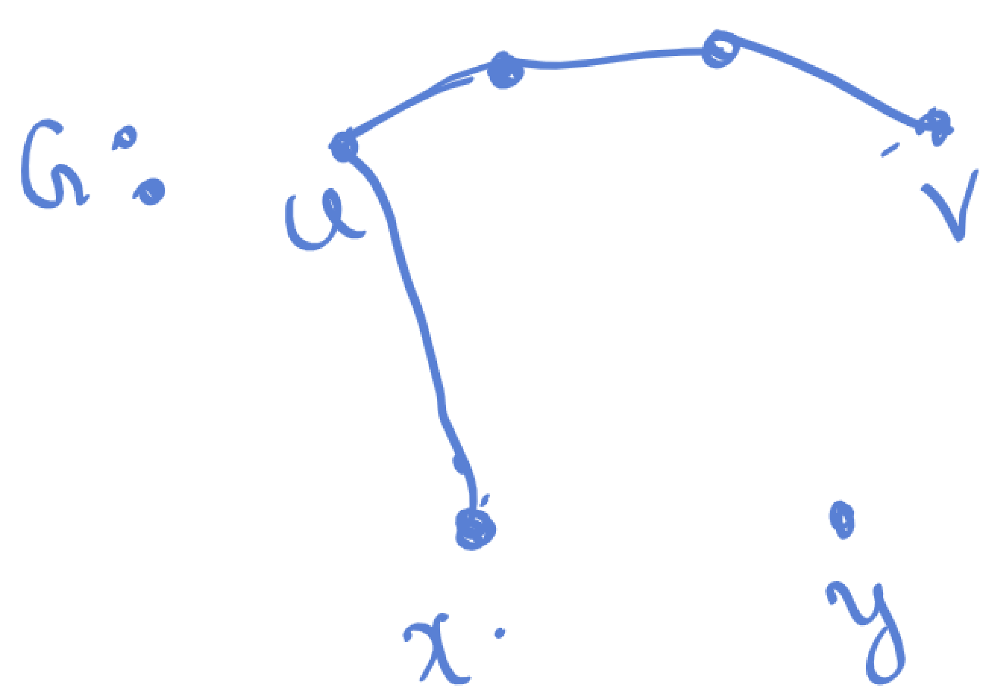
If $diam(G) \geq 3$, then $diam(\bar{G}) \leq 3$

Proof

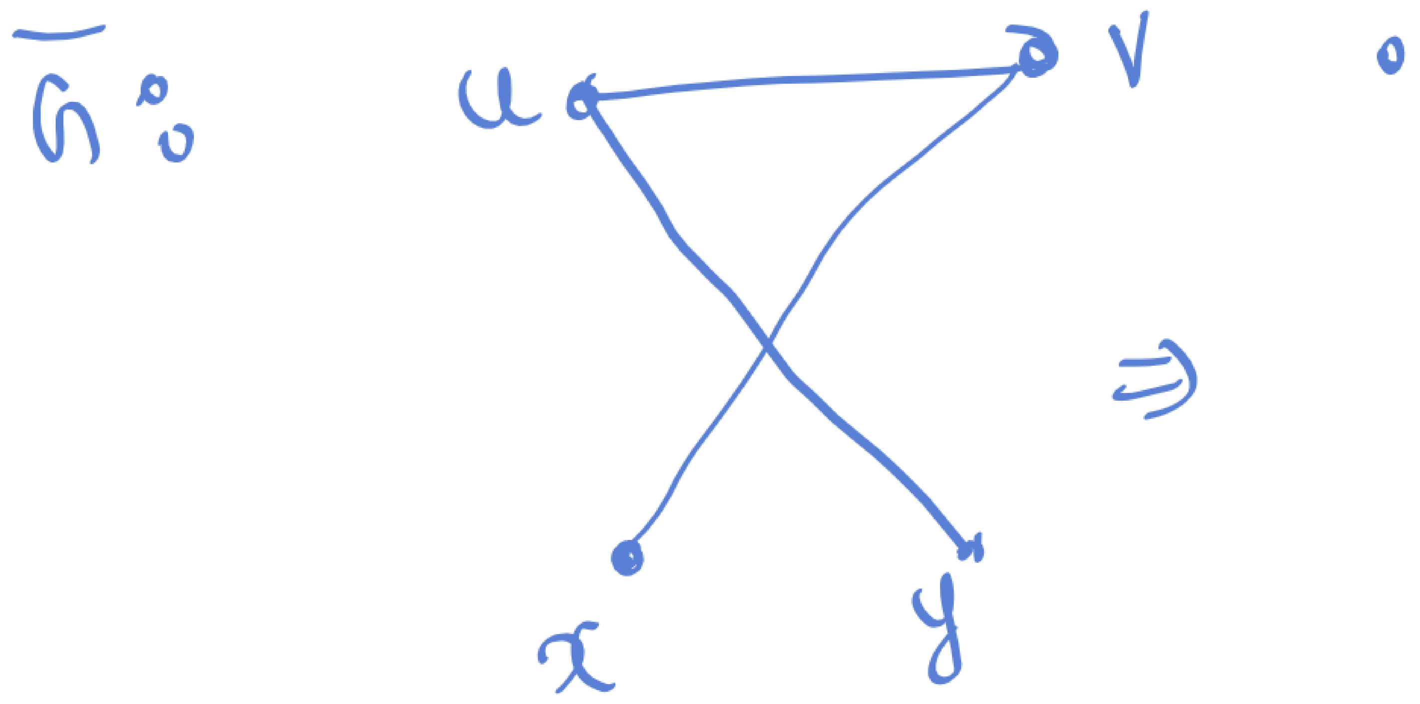
Let x and y be any two vertices in G. Since $diam(G) \geq 3$, there exist vertices u and v at distance 3 in G. Hence, uv is an edge in $\overline{G}$. Since u and v have no common neighbour in G, both x and y are each adjacent to u or v in $\overline{G}$. It follows that $d(x, y) \leq 3$ in $\overline{G}$ and hence $d(\overline{G}) \leq 3$) □

Let G s.t $diam(G) \geq 3$

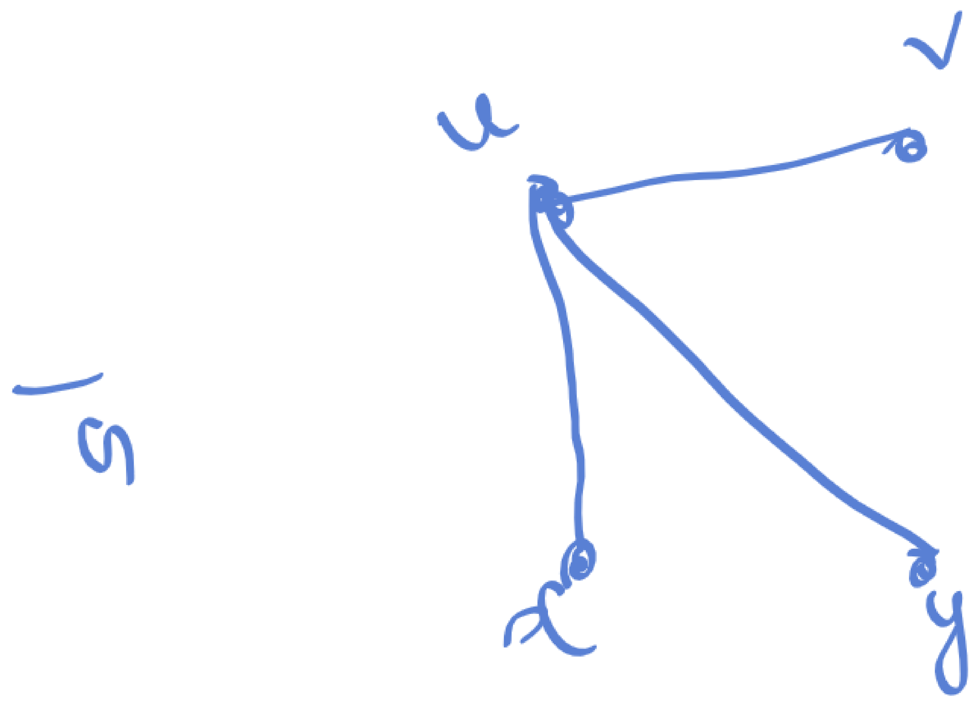
$\exists$ u & v s.t $d(u,v) = 3$



No vertex in G is commonly adjacent to both u & v



$\Rightarrow$



THEOREM:

Every nontrivial self-complementary graph has diameter 2 or 3

Proof

Let G be a self complementary graph. Clearly, G cannot have diameter 1.Since $G \cong K_n$ which is not self complementary graph. Hence, self complementary graphs have diameter atleast 2. Suppose that $diam(G) \geqslant 3$. By the above theorem, $\text{diam}(\overline{G}) \leq 3$. Hence, diameter of every self complementary graph is either 2 or 3. □

self comp, $G \cong \overline{G}$

$diam(G) \geqslant 3$, then

$diam(\overline{G}) \leq 3$

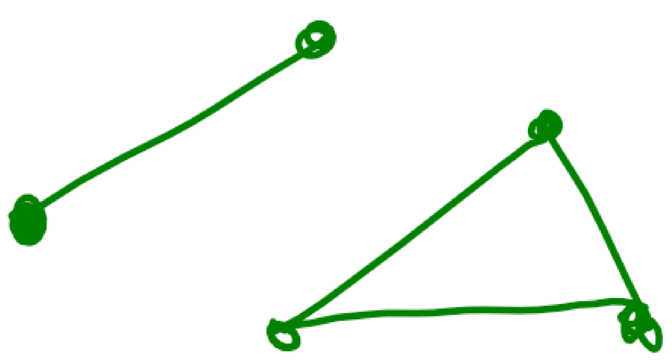
$= 3$

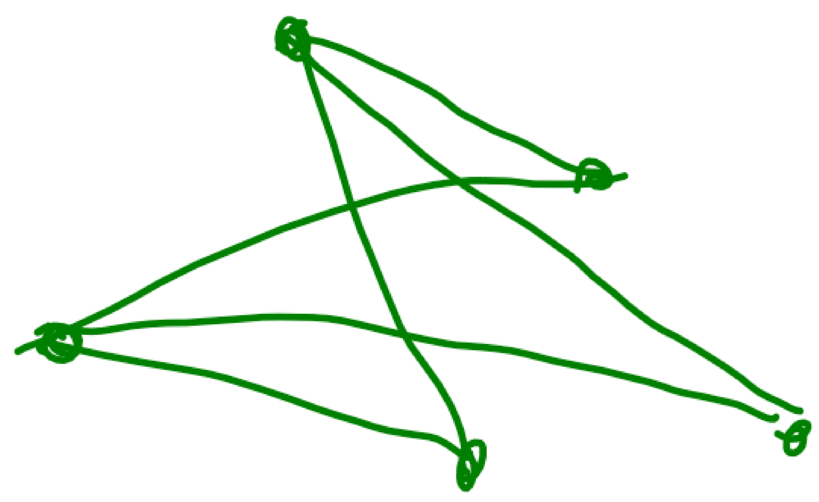
2

G $\overline{G}$

G:

$\overline{G}$:





Theorem

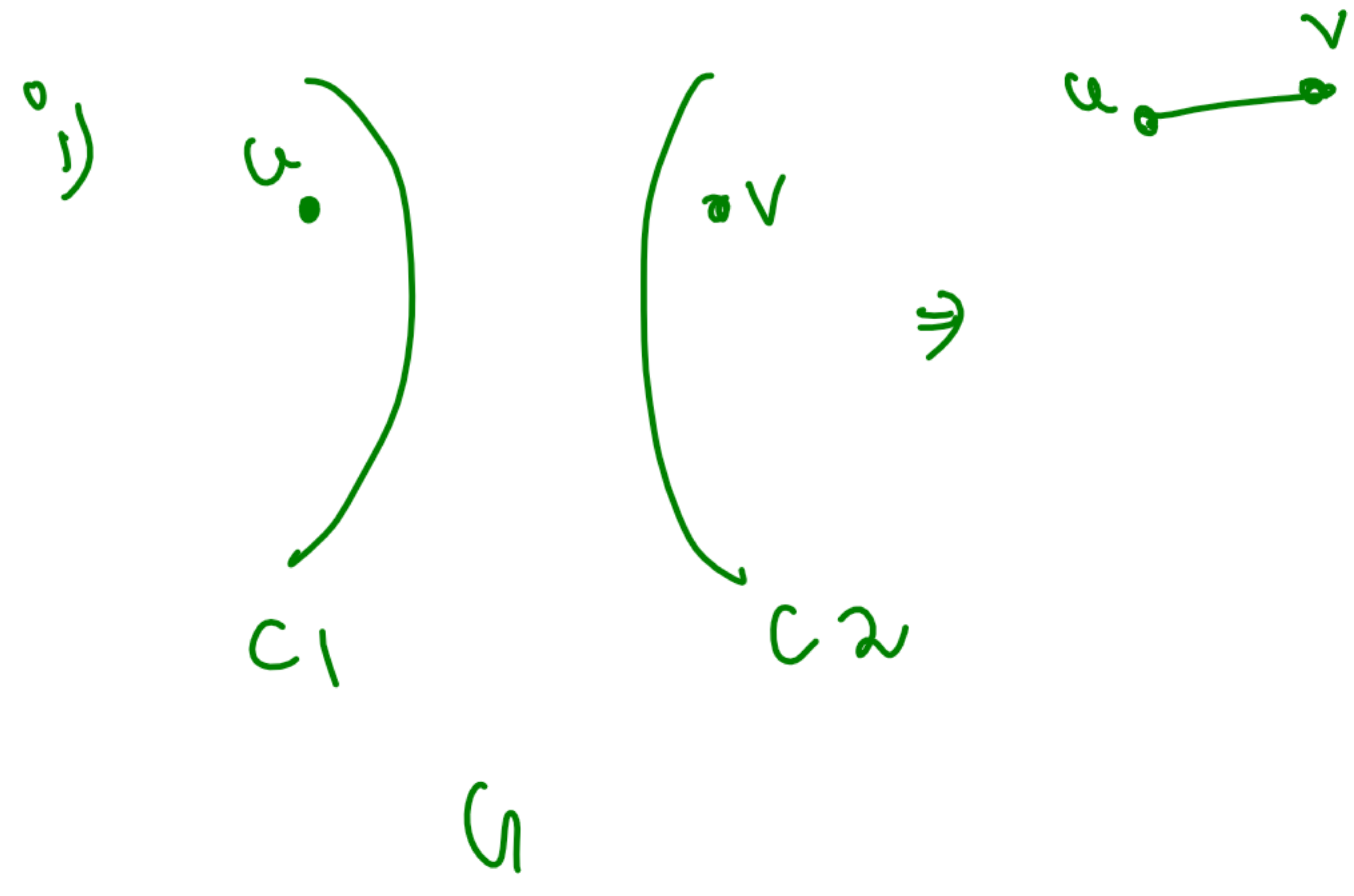
For any graph G, ~~show that~~ either G or $\overline{G}$ is connected.

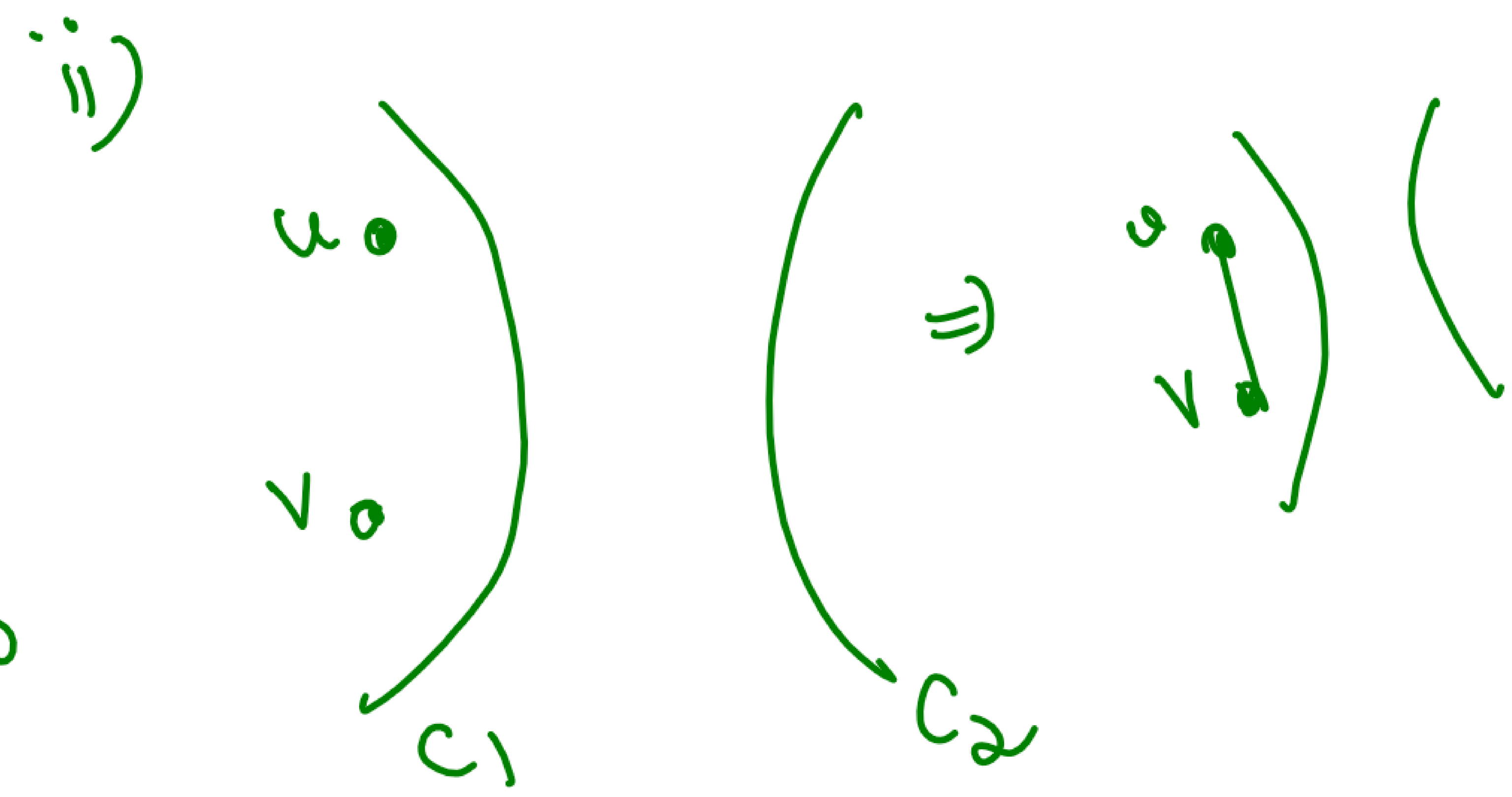
If G itself is connected, there is nothing to prove. Suppose that the graph G is disconnected and has two components C_1 and C_2. Let u and v be any two vertices, we have the following cases.

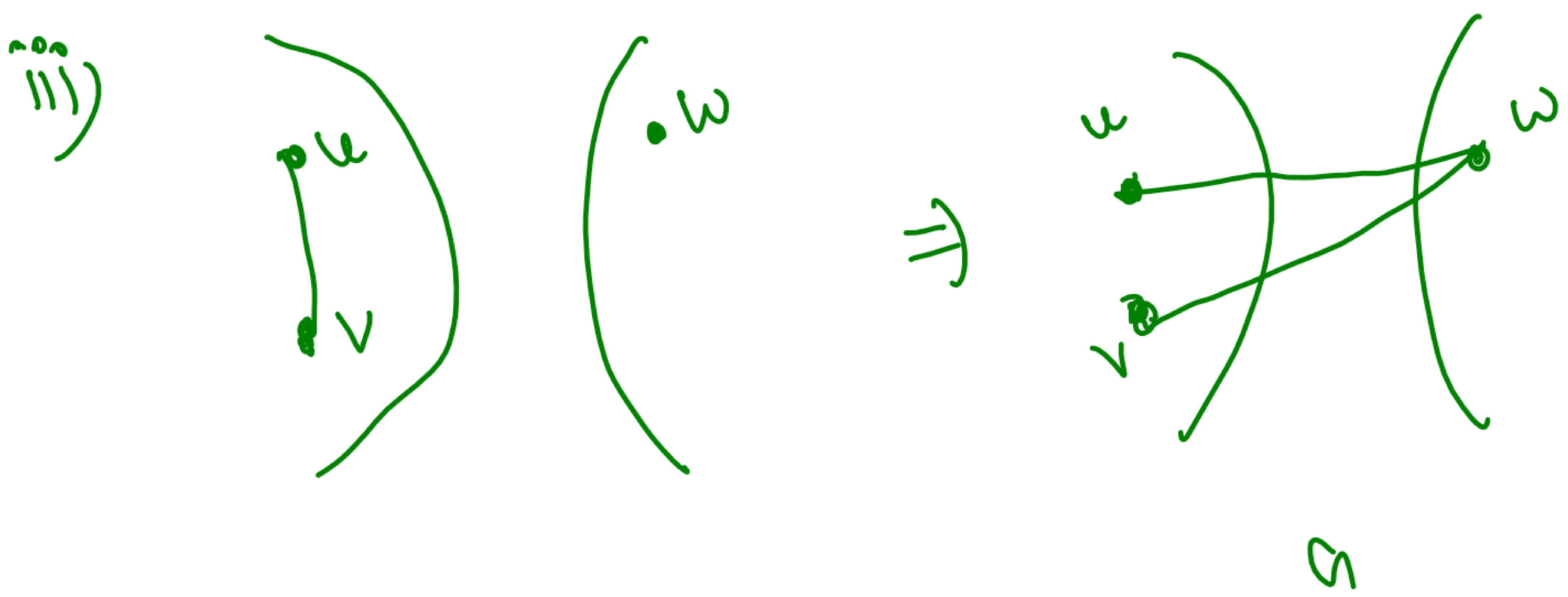
(i) If u and v are in different components and are not adjacent in G.Then u and v are adjacent in $\overline{G}$. We have, uv path, hence $\overline{G}$ is connected.

(ii) If u and v belong to the same component but they are not adjacent in G. Hence, they are adjacent in $\overline{G}$. Hence, we have uv path.

(iii) Suppose that u and v are adjacent in G(Obviously, they belong to the same component).Then we can find w in another component (which does not contain u and v). We have a uv path via w in $\overline{G}$. That is, $u \sim w$ and $v \sim w$.

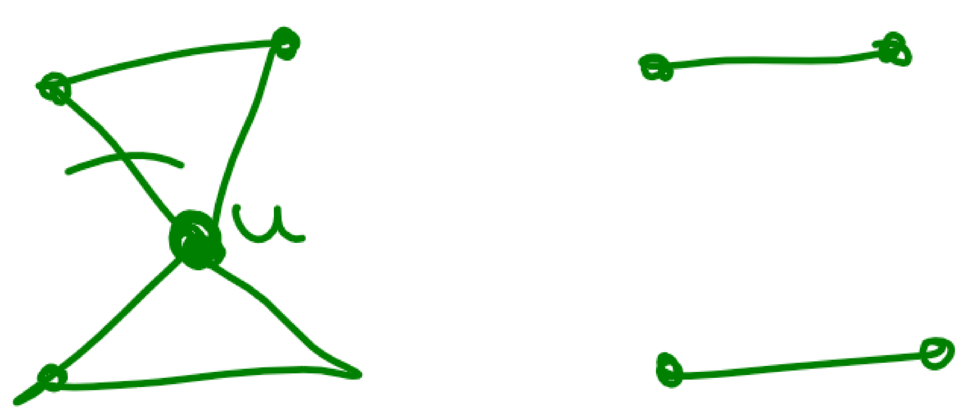




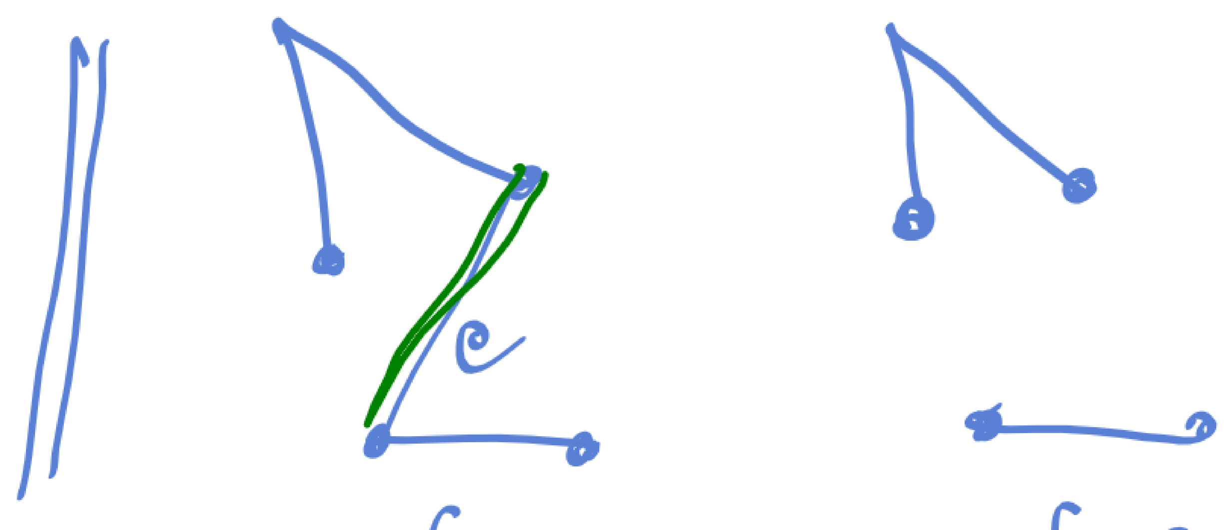


CUT VERTEEX, BRIDGE

cut vertex : vertex whose removal increases the no of components



G G-u



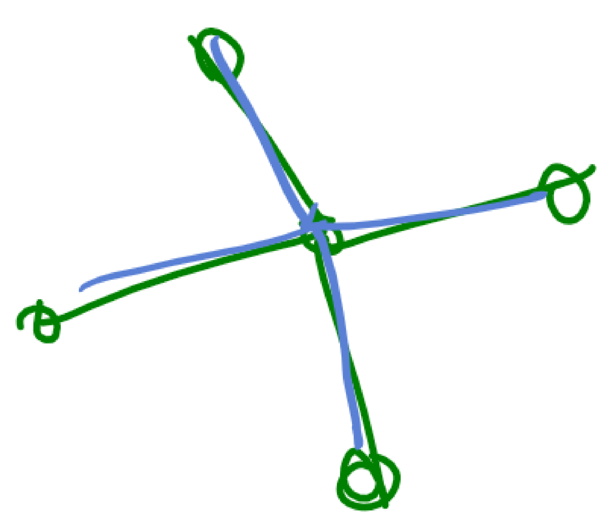
G G-e

Bridge : An edge whose removal increases the no of components

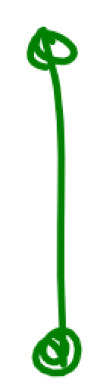
TREES

→ A tree is a connected acyclic graph

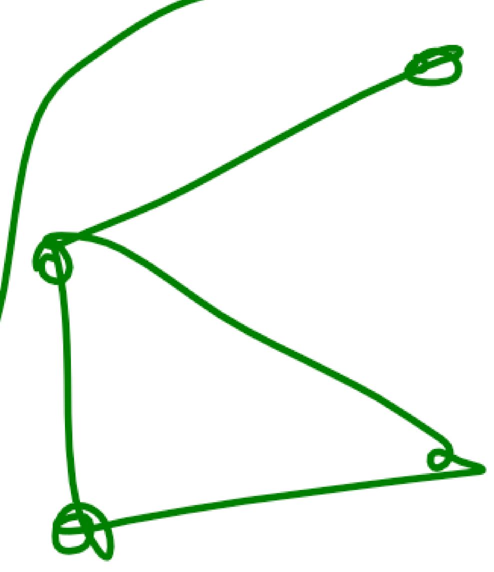
Forest → acyclic graph



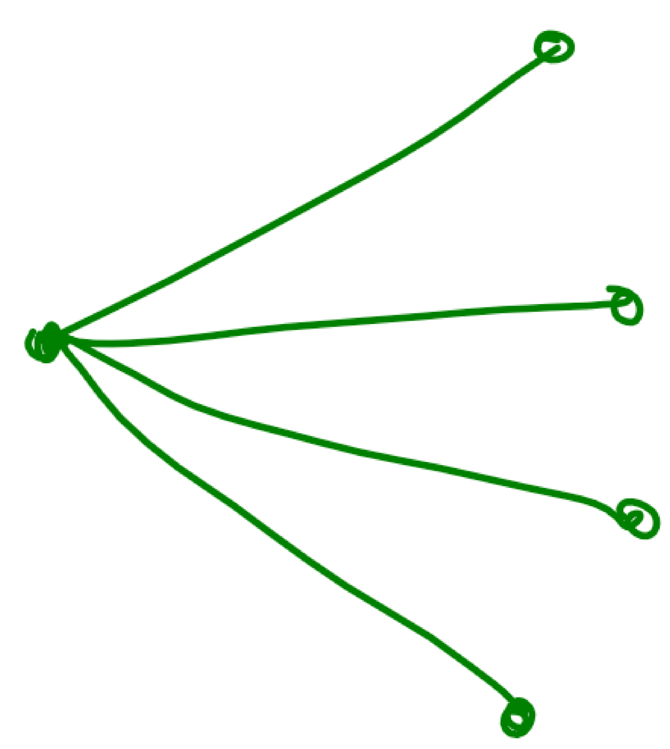
$p=5$
$q=4$



$p=2$
$q=1$

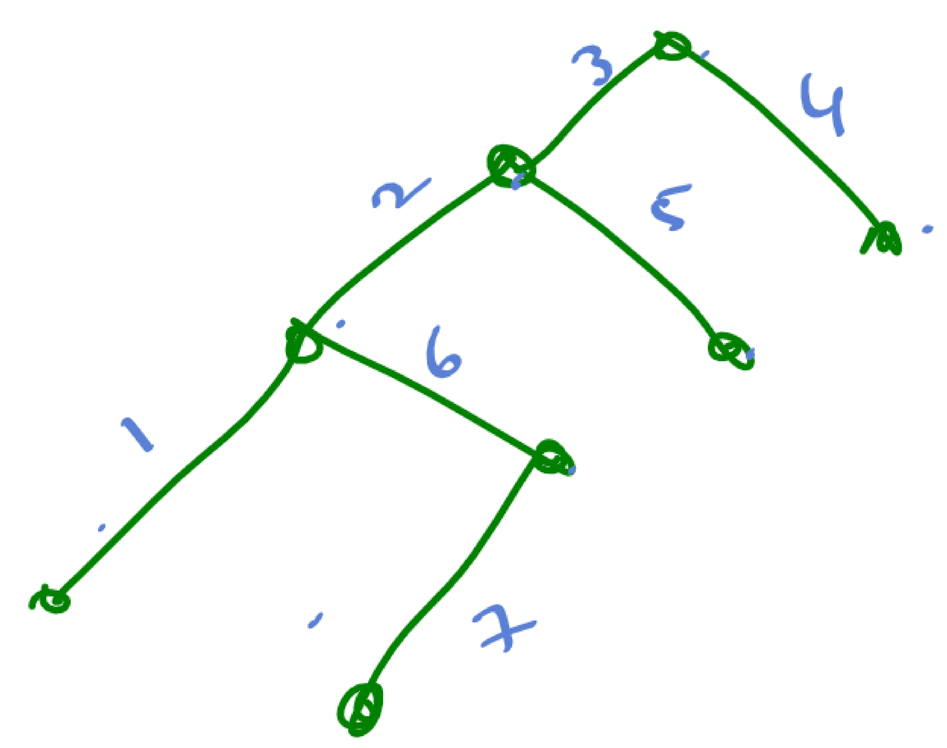


Not a tree



$K_{1,4}$ (star graph)

$p=5$
$q=4$



$p=8$
$q=7$

A tree on p vertices have $(p-1)$ edges

Theorem

A graph G is a tree if and only if between every pair of vertices there exist a unique path.