**IT-Section B**

**Assignment-3** **MAT 2155** **Submission Date: 26 - 29th October**

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1. Let $a, b, c$ be elements in a Lattice $(A, \leq)$. Show that, $a \leq b$ if and only if $a \vee (b \wedge c) \leq b \wedge (a \vee c)$
2. Show that a lattice is distributive if and only if for any elements $a, b, c$ in the lattice $(a \vee b) \wedge c \leq a \vee (b \wedge c)$
3. Show that a lattice is modular if and only if the following condition holds
$$a \vee \big(b \wedge (a \vee c)\big) = (a \vee b) \wedge (a \vee c)$$
4. In any lattice L, prove that
$$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$
for all $a, b, c \in L$.

5. Prove that, in any lattice L, the distributive inequalities
   (i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$
   (ii) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$
6. Let $(A, \vee, \wedge)$ be an algebraic system, where $\vee$ and $\wedge$ are binary operations satisfying the commutative and absorption laws.
   (i) Define a binary relation $\leq$ on A as follows:
   For all $a, b \in A$, $a \leq b$ if and only if $a \wedge b = a$. Show that '$\leq$' is a partial ordering relation.
   (ii) Show that $a \vee b$ is least upper bound of $a$ and $b$ in $(A, \leq)$
   (iii) Show that $a \wedge b$ is greatest lower bound of $a$ and $b$ in $(A, \leq)$
7. Let $(P, \leq)$ be a poset. Suppose the length of the longest chains in P is $n$, then prove that, the elements in P can be partitioned into $n$ disjoint antichains.
8. For elements $a$ and $b$ in a Boolean algebra, show that $a \leq b$ if and only if $\overline{a} \vee b = 1$.
9. For a fixed integer $n \geq 1$, let $B$ be the set of all binary strings of length $n$. Define a relation $\leq$ on $B$ as follows. For two strings $x = x_1x_2 \dots x_n$ and $y = y_1y_2 \dots y_n$ in $B$, $x \leq y$ if and only if for each $i$, whenever the bit $y_i = 0$, the bit $x_i = 0$ as well (Example: $0101 \leq 1101$). Show that $(B, \leq)$ is a lattice. Describe the join and meet operations of two strings in terms of the bits of the string.
10. Let $E(x_1, x_2, x_3) = \overline{x}_1 \wedge (\overline{x}_2 \vee x_3) \wedge (x_2 \vee \overline{x}_3)$ be a Boolean Expression over the two-valued Boolean algebra $(\{0,1\}, \vee, \wedge, {}^{-})$. Write $E(x_1, x_2, x_3)$ in CNF and DNF respectively.