Propositional Calculus

Proposition:

A declarative sentence that is either true or false.

Ex: It rained yesterday.

'True' or 'false' are called the truth values of the proposition and are denoted by T and F resp

Tautology:

A proposition that is true under all circumstances ex:- 15 is divisible by 3

contradiction:

A proposition which is false under all circumstances

logical connectivities:

Two or more propositions can be combined using words like "and", "or", "iff", "if, then" etc. These are called logical connectivities

Compound proposition:

A proposition having one or more logical connectivities is called a compound proposition. Otherwise the proposition is called "simple / atom"

Equivalent

Two propositions p and q are said to be equivalent if when p is T, q is also T and when p is F, q is also F and conversely

ex:- P: He was born in 1934

q: He'll be 60 years old in 1994

p and q are equivalent

ex2: p: x is a prime no
q: x is not divisible by 2
∴ p and q are not equivalent, as x not ÷ble by 2 doesnt mean its prime

① Negation ($\neg p$)

Let p be a proposition, we define negation of p denoted by $\neg p$ to be a proposition which is true when p is false and is false when p is true

$\neg$ p	$\neg$ p
T	F
F	T

p: monthly vol of the sales is < 20K
$\neg p$: monthly vol of the sales exceeds & equal to 20K

② Disjunction ($p \vee q$)

Let p and q be 2 propositions. The disjunction of 2 proposition is denoted by $p \vee q$ (read as p & q)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

③ Conjunction ($p \wedge q$)

The conjunction of 2 propositions p & q denoted by $p \wedge q$ (read as p and q) & is defined as

$\wedge$	p	q	$p \wedge q$
	T	T	T
	T	F	F
	F	T	F
	F	F	F

④ Conditional ($p \rightarrow q$)

The conditional statement is denoted by '$p \rightarrow q$'

read as "if p, then q".

$\rightarrow$	p	q	$p \rightarrow q$
	T	T	T
	T	F	F
	F	T	T
	F	F	T

* p is called the 'Ist component' or ANTECEDENT

q is called the '2nd component' or consequent

* For the conditional $p \rightarrow q$,

i) $q \rightarrow p$ is called 'converse'

ii) $\neg p \rightarrow \neg q$ is called 'inverse'

iii) $\neg q \rightarrow \neg p$ is called 'contrapositive'

p	q	$p \rightarrow q$	$q \rightarrow p$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Scanned by CamScanner

we observe that

i) $p \to q$ and ~~converse~~ $\neg q \to \neg p$ are logically equivalent

ii) Inverse and converse are logically equiv

$q \to p$ and $\neg p \to \neg q$ are "

* Also consider $\neg p \vee q$

p	q	$p \to q$	$\neg p \vee q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

we can say that $p \to q$ & $\neg p \vee q$ are logically equivalent

Example

There are 2 restaurants next to each other. One has a sign that says "Good food is not cheap". The other has a sign that says "Cheap food is not good". Are the signs saying the same thing?

Soln :

A: Food is good

B : Food is cheap

we have to examine $A \to \neg B$ and $B \to \neg A$

A	B	$A \to \neg B$	$B \to \neg A$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

Both are saying the same thing

(8) John made 2 statements

i) I love Lucy

ii) If I love Lucy, then I also love Vivian

Given than John either told the truth & lied in both the cases. Determine whether John really loves Lucy

solm

P: John loves Lucy

Q: John loves Vivian

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

If John lies (p & p→q is false), not possible

ie Both p and p→q cant be false

But both p and p→q can be true

Since both p & p→q can't be false, both p & p→q must be true which confirms that John loves Lucy

⑤ Biconditional

Let P and q be two propositⁿs. The biconditional is denoted by $P \leftrightarrow q$, read as "p iff q"

$\leftrightarrow$	p	q	$p \leftrightarrow q$
	T	T	T
	T	F	F
	F	T	F
	F	F	T

Example:-

An island has 2 tribes of natives. Any native from the first tribe always tells the truth, while any native from the other tribe alws lies. You arrive at the island and ask a native if there is a gold at the island. He answers "there is gold on the island iff I always tell the truth". Which tribe is he from? Is there gold on the island?

Sol^n

P: There is gold on the island

q: I always tell the truth

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Consider the follow cases:

case i: If the person belongs to I^{st} tribe. Then q is true and the statement $p \leftrightarrow q$ is true. From the truth table p is also true

∴ There is gold

case ii: If the person belongs to the 2nd tribe. Then q is false and $p \leftrightarrow q$ ~~is true~~. ~~[illegible]~~ must be false. From the truth table p is true ie There is gold

Thus there is gold on the island, the native could have been from either tribe

Well formed formulas (WFF)

A WFF is a formula generated using following groups

i) A statement variable is a WFF

ii) If A is WFF, then $\neg A$ is also a WFF

iii) If A and B are WFFs, $A \vee B$, $A \wedge B$; $A \rightarrow B$, $A \leftrightarrow B$ are also WFFs

iv) A string of symbols consisting of statement variables, connectives & paranthesis is a WFF iff it can be obtained by finite many applications of the rules (i), (ii) & iii)

ex:- $p \wedge q$, $\neg(p \wedge q)$, $(\neg(p \rightarrow q)) \vee r)$
$((p \rightarrow q) \rightarrow r)$ are WFFs

ex:- $p \wedge q \rightarrow r$ is not a WFF as it can be
$(p \wedge q) \rightarrow r$ & $p \wedge (q \rightarrow r)$

$\neg p \rightarrow \wedge r$, $\neg p \wedge q$ are not WFFs

Equivalence of formulas

Let A and B be two statement formulas. and $P_1, P_2, .. P_n$ denote all the variables occuring in A and B. If the truth value of A is same as that of B for each of 2^n possible set of assignments to the variables $P_1, P_2, .. P_n$. Then A and B are said to be equivalent.

We write $A \Longleftrightarrow B$

Two statement formulas A and B are equivalent iff $A \leftrightarrow B$ is a tautology

~~Table of tool~~

Table of equivalence

① $\neg\neg P \Leftrightarrow P$

② $P \vee q \Leftrightarrow q \vee P$
$P \wedge q \Leftrightarrow q \wedge P$ } commutative

③ $P \vee (q \vee r) \Leftrightarrow (P \vee q) \vee r$
$P \wedge (q \wedge r) \Leftrightarrow (P \wedge q) \wedge r$ } Associative

④ $P \vee (q \wedge r) \Leftrightarrow (P \vee q) \wedge (P \vee r)$
$P \wedge (q \vee r) \Leftrightarrow (P \wedge q) \vee (P \wedge r)$ } Distributive

⑤ $P \vee (P \wedge q) \Leftrightarrow P$
$P \wedge (P \vee q) \Leftrightarrow P$ } Absorption

⑥ $P \wedge P \Leftrightarrow P$
$P \vee P \Leftrightarrow P$ } Idempotent

⑦ $P \wedge (\neg P) \Leftrightarrow F$
$P \vee (\neg P) \Leftrightarrow T$

(8) $P \wedge F \Leftrightarrow F$

$P \vee F \Leftrightarrow P$

(9) $P \wedge T \Leftrightarrow P$

$P \vee T \Leftrightarrow T$

(10) $\neg(P \vee q) = \neg P \wedge \neg q$

$\neg(P \wedge q) = \neg P \vee \neg q$

(11) $(P \rightarrow q) \Leftrightarrow \neg P \vee q$

$\neg(P \rightarrow q) \Leftrightarrow P \wedge \neg q$

(12) $(P \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg P)$

$q \rightarrow P \Leftrightarrow (\neg P \rightarrow \neg q)$

Scanned by CamScanner

Problems

① S.T $p \to (q \to r) \Leftrightarrow p \to (\neg q \vee r) \Leftrightarrow (p \wedge q) \to r$

sol$\underline{\text{m}}$

$p \to (q \to r)$

$\Leftrightarrow p \to (\neg q \vee r)$ $\quad$ ($\because p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow \neg p \vee (\neg q \vee r)$ $\quad$ ($\because$ same rule

$\Leftrightarrow (\neg p \vee \neg q) \vee r$ $\quad$ ($\because$ associative

$\Leftrightarrow \neg(p \wedge q) \vee r$

$\Leftrightarrow p \wedge q \to r$

② S.T $(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

sol$\underline{\text{m}}$

$(\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r)$

$\Leftrightarrow (\neg p \wedge (\neg q \wedge r)) \vee (r \wedge (q \vee p))$ $\quad$ ($\because$ dist

$\Leftrightarrow ((\neg p \wedge \neg q) \wedge r) \vee (r \wedge (p \vee q))$ $\quad$ ($\because$ assoc

$\Leftrightarrow (\neg(p \vee q) \wedge r) \vee (r \wedge (p \vee q))$

$\Leftrightarrow r \wedge [(p \vee q) \vee \neg(p \vee q)]$ $\quad$ ($\because$ dist

$\Leftrightarrow r \wedge T$

$\Leftrightarrow r$

③ S.T $\left((p \vee q) \wedge \neg[(\neg p) \wedge (\neg q \vee \neg r)]\right) \vee (\neg p \wedge \neg q)$
$\vee (\neg p \wedge \neg r)$ is a tautology

sol$\underline{\text{m}}$

$[(p \vee q) \wedge \neg((\neg p) \wedge (\neg q \vee \neg r))] \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$

$[(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))] \vee [\neg p \wedge (\neg q \vee \neg r)]$

$[(p \vee q) \wedge \neg\neg p \vee (q \wedge r)] \vee [\neg p \wedge \neg(\cancel{p \wedge r})]$ $q \wedge r$

$[(p \vee q) \wedge (p \vee (q \wedge r))] \vee \neg[p \vee (q \wedge r)]$

$[(p \vee q) \wedge (p \vee q) \wedge (p \vee r)] \vee \neg[p \vee (q \wedge r)]$

$[(p \vee q) \wedge (p \vee r)] \wedge \neg[p \vee (q \wedge r)]$

$[p \vee (q \wedge r)] \wedge \neg[p \vee (q \wedge r)]$

T

$\therefore$ its tautology

④ SoT $q \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$ is tautogy

soln

$q \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

$q \vee [(p \wedge \neg q) \vee (\neg p \wedge \neg q)]$

$q \vee [\neg q \wedge (p \vee \neg p)]$

$q \vee [\neg q \wedge T]$

$q \vee T$

T

Scanned by CamScanner

Tautological Implications

A is said to tautologically imply to statement B if $A \to B$ is a tautology. In this case, we write $A \Rightarrow B$ (read as A implies B)

① S.T $\neg q \wedge (p \to q) \Rightarrow \neg p$

$p \to q$ is true means P.T if p is T, q is T & q is F, p is F

Soln

Suppose $\neg q \wedge (p \to q)$ is true

$\neg q$ is true and $p \to q$ is true

q is false and $p \to q$ is true

$\Rightarrow p$ is false

$\Rightarrow \neg p$ is true

$\therefore \neg p \wedge (p \to q) \Rightarrow \neg p$

Remark: To S.T $A \Rightarrow B$, we can assume B is false and S.T A is false

ntd

consider again $\neg p$ is false,

$\Rightarrow p$ is true

If q is true, $\neg q$ is false and its understood that $\neg q \wedge (p \to q)$ is false

If q is ~~true~~ false, $\neg q$ is true and $p \to q$ is false

Agn $\neg q \wedge (p \to q)$ is false

② S.T $\neg(p \to q) \Rightarrow \neg q$

soln We say $A \Rightarrow B$ if $A \to B$ is true in all condn

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p\to q) \Rightarrow \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

② $(p \vee q) \wedge (p \to r) \wedge (q \to r) \Rightarrow r$

soln

Suppose $(p \vee q) \wedge (p \to r) \wedge (q \to r)$ is true

$(p \vee q)$ is true, $(p \to r)$ is true, $(q \to r)$ is true

Now $(p \vee q)$ is true when either both p & q are true or either one of them is true

If p is true,

Now if $p \to r$ is true $\Rightarrow$ r is true

If q is true,

Now if $q \to r$ is true $\Rightarrow$ r is true.

$p \to q$ is false only when TF $\Rightarrow$ F

If $p \to q$ is true, then q is true

T T T

T F F

F T T

F F T

③ Prove that i) $\neg p \Rightarrow p \to q$

ii) $p \wedge (p \to q) \Rightarrow q$

iii) $p \wedge q \Rightarrow p$

Soln

i) Assume $\neg p$ is true

$\Rightarrow p$ is false

Now q is either true or false.

when q is true, $p \to q$ is true

when q is false, $p \to q$ is true

$\therefore p \to q$ is true

$\therefore \neg p \to (p \to q)$ is true

$\therefore \neg p \Rightarrow (p \to q)$

ii) Assume $p \wedge (p \to q)$ is true

$\Rightarrow p$ is true and $p \to q$ is true

$\Rightarrow q$ is true

$\therefore p \wedge (p \to q) \to q$ is true

$\therefore p \wedge (p \to q) \Rightarrow q$

iii) Assume $p \wedge q$ is true

Both p & q are true

p is true

$\therefore p \wedge q \to p$ is true

$p \wedge q \Rightarrow p$

Table of Tautological Implications

1. $p \wedge q \Longrightarrow p$
   $p \wedge q \Longrightarrow q$

2. $p \Rightarrow p \vee q$
   $q \Rightarrow p \vee q$

3. $\neg p \Longrightarrow p \to q$

4. $q \Longrightarrow p \to q$

5. $\neg(p \to q) \Longrightarrow p$

6. $\neg(p \to q) \Longrightarrow \neg q$

7. * $p \wedge (p \to q) \Longrightarrow q$

8. * $\neg q \wedge (p \to q) \Longrightarrow \neg p$

9. $\neg p \wedge (p \vee q) \Longrightarrow q$

10. * $(p \to q) \wedge (q \to r) \Longrightarrow p \to r$

11. $(p \vee q) \wedge (p \to r) \wedge (q \to r) \Longrightarrow r$

Scanned by CamScanner

Theory of Inference

Let A and B be two statement formulas. We say that "B logically follows from A" & "B is a valid conclusion of the premise A" iff $A \to B$ is a tautology. ie $A \Rightarrow B$

To demonstrate that a particular formula is valid consequence of a given set of premises, we use the follow rules of inference

1. Rule P: A premise may be introduced at any point in the derivation
2. Rule T: A formula S may be introduced in a derivation if S is tautologically implied by any one or more of the preceding formulas in the derivation

1. Demonstrate that r is a valid inference from the premises $p \to q$, $q \to r$ & p

Soln

$p \to q$ (Rule P)

p (Rule P)

q (Rule T) $p \wedge (p \to q) \Rightarrow q$

$q \to r$ (Rule P)

r (Rule T, $q \wedge (q \to r) \Rightarrow r$

② S.T $R \vee S$ follows logically from the premises $C \vee D$, $C \vee D \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$, $(A \wedge \neg B) \rightarrow (R \vee S)$

Soln

$C \vee D \longrightarrow \neg H$ (Rule P

$\neg H \longrightarrow A \wedge \neg B$ (Rule P

$C \vee D \longrightarrow A \wedge \neg B$ (Rule T, by (1) & (2)

$A \wedge \neg B \longrightarrow R \vee S$ (Rule P

$C \vee D \longrightarrow R \vee S$ (Rule T

$C \vee D$ (Rule P

$R \vee S$ (Rule T $P \vee (P \rightarrow q) \Leftrightarrow q$

③ S.T $S \vee R$ is tautologically implied by $(P \vee q) \wedge (P \rightarrow R) \wedge (q \rightarrow S)$

Soln

$P \vee q$ (Rule P)

$\neg P \rightarrow q$ (Rule T ie $P \rightarrow q \Leftrightarrow \neg P \vee q$

$q \rightarrow S$ (Rule P

$\neg P \rightarrow S$ (Rule T (2) & (3)

$\neg S \rightarrow P$ (Rule T $P \rightarrow q \Leftrightarrow \neg q \rightarrow \neg P$

$P \rightarrow R$ (Rule P

$\neg S \rightarrow R$ (Rule T prev 2 steps

$S \vee R$ (Rule T $P \rightarrow q \Leftrightarrow (\neg P \vee q)$

(4) S.T. $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \to R$, $P \to M$ and $\neg M$.

Soln

(i)	$P \to M$	(Rule P)	
(ii)	$\neg M$	(Rule P	
(iii)	$\neg P$	(Rule T	$\neg Q \wedge (P \to Q) \Rightarrow \neg P$ $\neg M (P \to M) \Rightarrow \neg P$
(iv)	$P \vee Q$	(Rule P)	
(v)	Q	(Rule T by (iii) (iv)	$\boxed{\neg P \wedge (P \vee Q) \Rightarrow Q}$
vi)	$Q \to R$	(Rule P)	
vii	R	Rule T By vi, v	$\boxed{Q \vee (Q \to R) = R}$
viii)	$R \wedge (P \vee Q)$	Rule T, by iv & vii	

(5) If A works hard, then either B or C will enjoy themselves. If B enjoys himself, then A will not work hard. If D enjoys himself, then C will not. Therefore P.T. if A works hard, D will not enjoy himself.

Soln:
A: A works hard
B: B will enjoy himself
C: C will enjoy himself
D: D will enjoy himself

To prove $A \to \neg D$ follows from $A \to B \vee C$, $B \to \neg A$ and $D \to \neg C$

1)	$A \to B \vee C$	(Rule P	
2)	$\neg(B \vee C) \to \neg A$	(Rule T	$P \to Q \Leftrightarrow \neg Q \to \neg P$
3)	$\neg B \wedge \neg C \to \neg A$	(Rule T)	

&

4) $\neg B \rightarrow (\neg C \rightarrow \neg A)$ (Rule T) $A \rightarrow B \Leftrightarrow \neg B \rightarrow \neg A$

5) $\neg B \rightarrow (A \rightarrow C)$ (Rule T ,,

6) $\neg(A \rightarrow C) \rightarrow B$ (Rule T ,,

7) $B \rightarrow \neg A$ (Rule P

8) $\neg(A \rightarrow C) \rightarrow \neg A$ (Rule T) $(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow P \rightarrow R$

9) $A \rightarrow A \rightarrow C$ (Rule T

10) $A \wedge A \rightarrow C$

11) $A \rightarrow C$

12 $D \rightarrow \neg C$

13 $C \rightarrow \neg D$

14) $A \rightarrow \neg D$

Predicate Calculus

A part of declarative sentence describing the properties of an object & relation among obj's" is called a 'predicate'

Ex: consider two propositions,

Ram is a bachelor

Shyam is a bachelor

1 Both Ram & Shyam have the same property of having bachelor. The part "is a bachelor" is called a predicate.

In propositional calculus, there is no symbolic representation "is a bochelor"

The two propositions can be replaced by a single proposition "x is a bachelor". By replacing x by Rame & shyam & any other names we get many propositions

The predicate is denoted by capital letters and names of individulas or objects by small letters.

Let "B" denote the predicate "is bachelor", then the sentence "x is a bachelor" can be writen as $B(x)$, where x is a predicate variable

$B(x)$ is also called a propositional fn, which becomes a statement when values are submitted in place of x.

$B(x) \rightarrow$ 1 place predicate

A predicate requiring m(>0) names is called m-place predicate

ex :- x is taller than y ; $T(x,y)$ } Both are
x sits b/w A and B } 2 place predicate

universal and extential quantifiers

Quantifiers are words that refer to quantities such as 'some' & 'all' and indicate how frequently certain statement is true. [Quantifiers are the words, expressions & phrases that indicate the no of elts that a particular statement pertains to]

The phrase "for all" ($\forall$) is called the 'universal quantifier'.

ex :- All human beings are mortal
For all natural nos" n, 2n is an even no

The phrase "there exists" ($\exists$) is called the 'extential quantifier'

ex :- There exists x s.t $x^2 = 5$

This can be written as $(\exists x)\, P(x)$ where

$P(x) : x^2 = 5$

Scanned by CamScanner

- $\exists x$ represents the following
  - There exists an x
  - There is an x
  - For some x
  - There is atleast one x

① $(\forall x)P(x)$ is true iff $P(x)$ is true $\forall$ x in U
$(\forall x)P(x)$ is false iff $P(x)$ is false for atleast one x in U

② $(\exists x)(P(x))$ is true if $P(x)$ is true for atleast one x in U
$(\exists x)(P(x))$ is false if $P(x)$ is false for every x in U

Example

Let $D = \{1, 2, 3, \ldots, 9\}$ Determine the truth value of each of the follow statements

1) $(\forall x)\ x+4 < 15$ T
2) $(\exists x)\ x+4 = 10$ T
3) $(\exists x)\ x+4 > 15$ F
4) $(\forall x)\ (x+4) \leq 10$ F

Symbolise the Statement

1) All men are mortal

Let $M(x)$: x is a man
$H(x)$: x is mortal

$(\forall x)(M(x) \longrightarrow H(x))$

2 An integer is either +ve & -ve

$(\forall x)(M(x) \longrightarrow N(x))$

where $M(x)$: x is an integer

$N(x)$: x is either +ve & -ve

Negation of quantified statements

① $\neg(\forall x)\, P(x) \Longleftrightarrow (\exists x)\neg P(x)$

② $\neg(\exists x)\, P(x)$: $\Longleftrightarrow (\forall x)\neg P(x)$

Negate the follow statements :

① For all real nos x, if $x>3$ then $x^2>9$

soln

Let $P(x)$: $x>3$

$Q(x)$: $x^2>9$

$\therefore (\forall x)(P(x) \longrightarrow Q(x))$

Negation is : $\neg(\forall x)(P(x) \longrightarrow Q(x))$

$\Leftrightarrow (\exists x)\neg(P(x) \rightarrow Q(x))$

$\neg(P\rightarrow Q) \Leftrightarrow P \wedge \neg q$ $\qquad \Leftrightarrow (\exists x)(P(x) \wedge \neg(Q(x)))$

ie there exists a real no x s/t $x>3$ & $x^2 \leq 9$

② ⓐ Ottawa is a small town

The negation is :

* It is not the case that Ottawa is a small town

* Ottawa is not a small town

(b) Every city in Canada is clean

* It's not the case that every city in Canada is clean
* Not every city in Canada is clean
* Some city in Canada is not clean

It's incorrect if we write;

"Every city in Canada is not clean".

Rules of inference

The method of derivation involving predicate formulas uses the rules of inference given for the statement calculus and also certain additional rules which are required to deal with the formulas involving quantifiers. (In addn to Rule P & T)

① Rule US (Universal Specification)

From $(\forall x) A(x)$, we can conclude $A(y)$

$$(\forall x) A(x) \Rightarrow A(y)$$

② Rule ES (Existential Specification)

From $(\exists x) A(x)$ one can conclude $A(y)$ provided that y is not free in any given premise & also not free in any prior step of the derivation

$$(\exists x) A(x) \Rightarrow A(y)$$

③ Rule EG (Existential Generalization)

From $A(x)$ one can conclude $(\exists y) A(y)$

$$A(x) \Rightarrow (\exists y) A(y)$$

④ Rule UG (Universal Generalization)

From $A(x)$ one can conclude $(\forall y) A(y)$

① S.T $(\forall x)[H(x) \to M(x)] \wedge H(s) \Rightarrow M(s)$

Note that this prob is a symbolic representn & translation of a well known argument known as "Socrates argument" which is given by

All men are mortal

Socrates is a man

Therefore Socrates is a mortal

Denote $H(x)$: x is a man

$M(x)$: x is mortal

s : Socrates

$H(s)$: Socrates is a man

$M(s)$: Socrates is mortal

Soln

(1)	$(\forall x)\ [H(x) \longrightarrow M(x)]$	Rule P
(2)	$H(s) \longrightarrow M(s)$	Rule US
(3)	$H(s)$	Rule P
(4)	$M(s)$	Rule T

$(P \wedge (P \to q) \Rightarrow P$

(2) S.T $(\forall x)[P(x) \rightarrow Q(x)] \wedge \forall x[Q(x) \rightarrow R(x)]$
$\Rightarrow (\forall x)[P(x) \rightarrow R(x)]$

Soln

(1)	$(\forall x)[P(x) \longrightarrow Q(x)]$	Rule P
(2)	$P(y) \longrightarrow Q(y)$	Rule US
(3)	$\forall(x)[Q(x) \longrightarrow R(x)]$	Rule P
(4)	$Q(y) \longrightarrow R(y)$	Rule US
(5)	$P(y) \longrightarrow R(y)$	Rule T, (2) & (4) $P \rightarrow q,\ q \rightarrow r \Rightarrow P \rightarrow q$
(6)	$(\forall x)[P(x) \longrightarrow R(x)]$	Rule UG

(3) S.T $(\exists x) M(x)$ follows logically from the premises $(\forall x)[H(x) \rightarrow M(x)]$ and $(\exists x) H(x)$

Soln

(1)	$(\exists x)\ H(x)$	Rule P
(2)	$H(y)$	Rule ES
(3)	$(\forall x)[H(x) \longrightarrow M(x)]$	Rule P
(4)	$H(y) \longrightarrow M(y)$	Rule US by (3)
(5)	$M(y)$	Rule T by (2) & (4)
(6)	$(\exists x)\ M(x)$	Rule EG