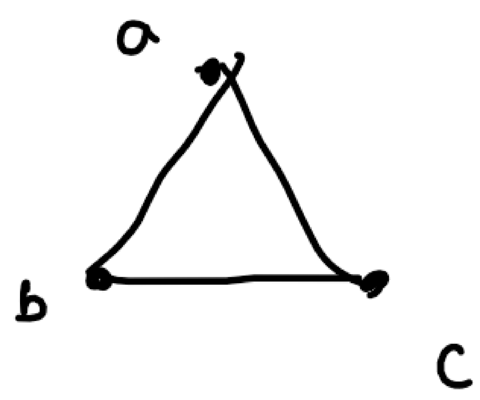
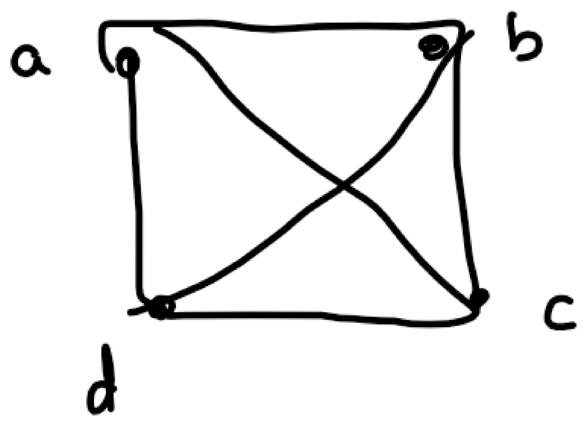
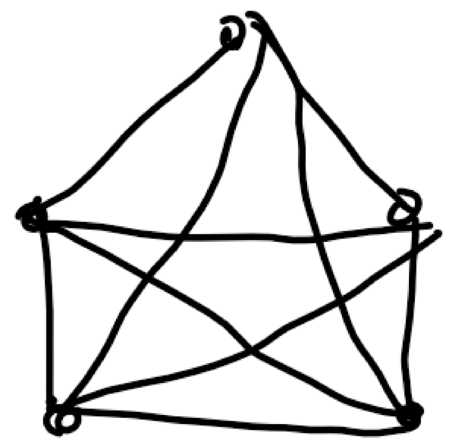
complete graph (K_n) :



K_3



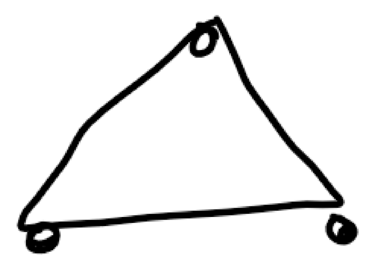
K_4



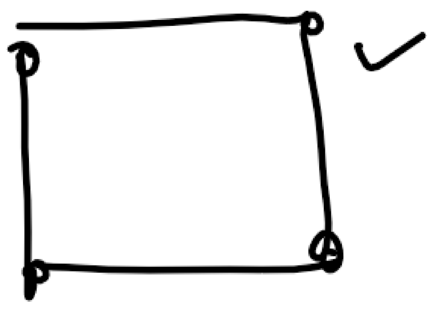
K_5

No of edges in $K_n \Rightarrow \frac{n(n-1)}{2}$

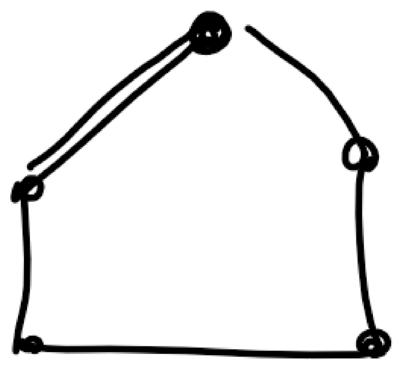
cycle graph (C_n)



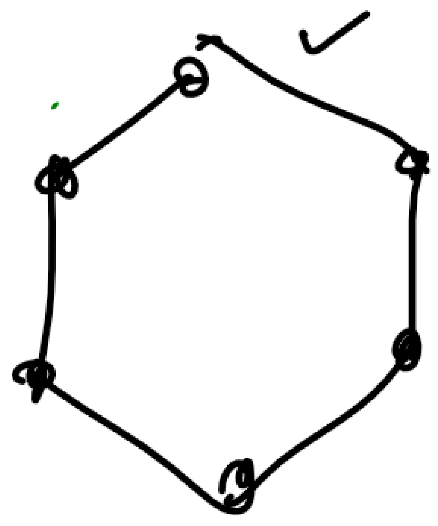
C_3



C_4



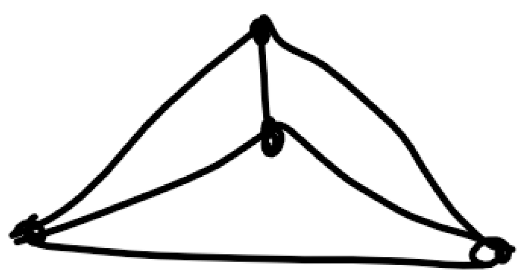
C_5



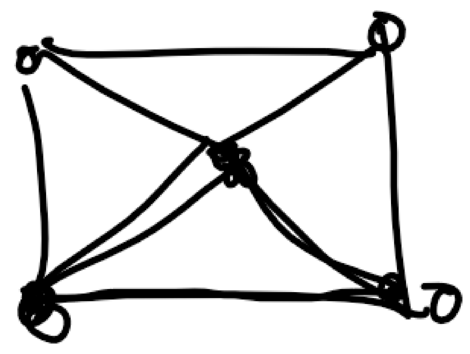
C_6

No of edges in $C_n = n$

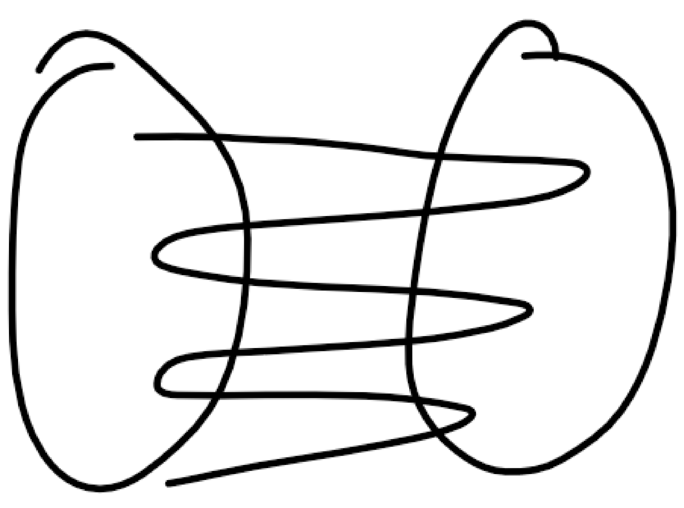
wheel graph ($W_{1,n}$)



$W_{1,3}$



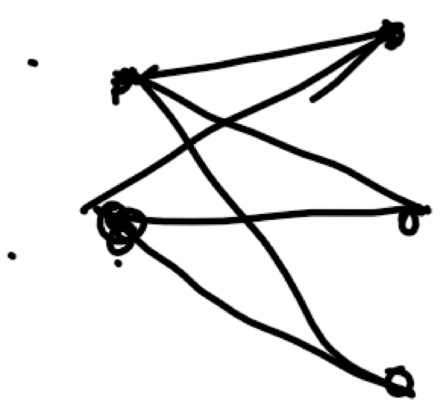
Bipartite graph



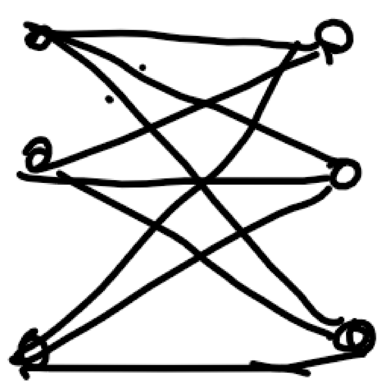
V_1 V_2

Bipartite $\Longleftrightarrow$ all cycles present in it are of even length

complete bipartite graph ($K_{m,n}$)

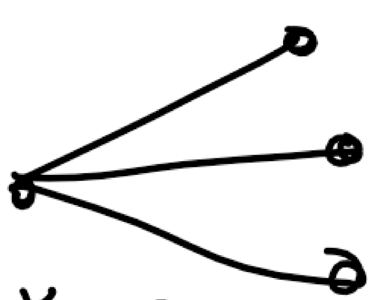


$K_{2,3}$



$K_{3,3}$

star graph ($K_{1,n}$)



$K_{1,3}$

Regular graph

all vertices have same deg

Tree

Acyclic, connected

* Every tree on n vertices has $(n-1)$ edges
* Bipartite

Theorem

A graph G is a tree if and only if between every pair of vertices there exist a unique path.

Proof.

Let G be a tree then G is connected. Hence, there exist atleast one path between every pair of vertices. Suppose that between two vertices say u and v, there are two distinct paths then union of these two paths will contain a cycle; a contradiction. Thus, if G is a tree, there is atmost one path joining any two vertices. Conversly, suppose that there is a unique path between every pair of vertices in G.Then G is connected. A cycle in the graph implies that there is atleast one pair of vertices u and v such that there are two distinct paths between u and v. Which is not possible because of our hypothesis. Hence, G is acyclic and therefore it is a tree. □

Let G be a tree ⇒ I've to S.T there exists unique path b/wn every pair of vertices

Theorem

A tree with p vertices has $p-1$ edges.

Proof.

The theorem will be proved by induction on the number of vertices.

If $p = 1$, we get a tree with one vertex and no edge. If $p = 2$, we get a tree with two vertices and one edge. If $p = 3$, we get a tree with three vertices and two edges. Assume that, the statement is true with all tree with k vertices ($k < p$). Let G be a tree with p vertices. Since G is a tree there exist a unique path between every pair of vertices in G. Thus, removal of an edge e from G will disconnect the graph G. Further, $G-e$ consists of exactly two components with number of vertices say m and n with $m+n = p$. Each component is again a tree. By induction, the component with m vertices has $m-1$ edges and the component with n vertices has $n-1$ edges. Thus, the number of edges in $G = m-1+n-1+1 = m+n-1 = p-1$.

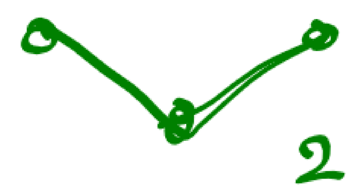
$p=1$

0 edges

$p=2$

1 edge

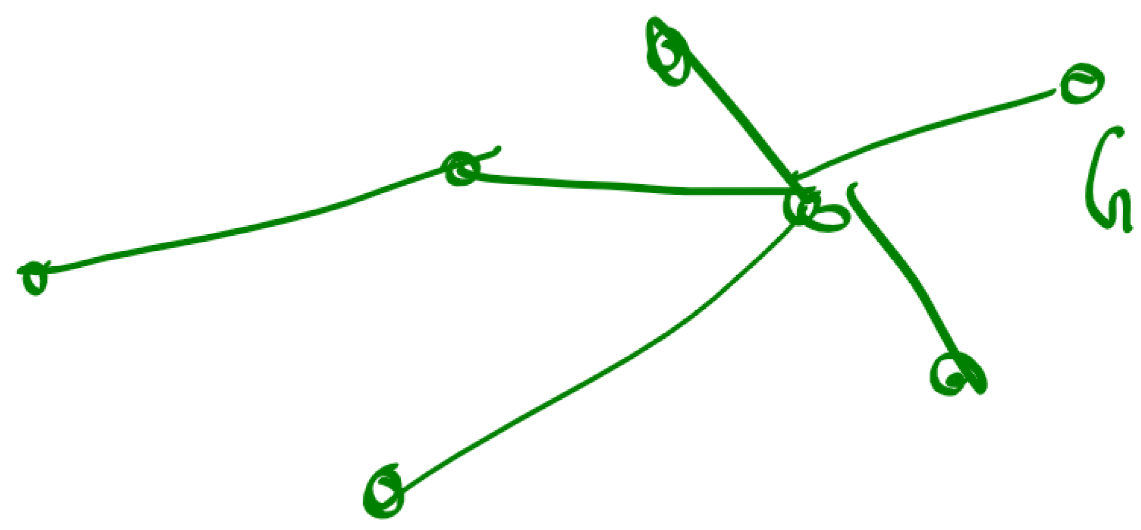
$p=3$



2 edges

Assume that statement is true for $K<p$

P.T for p vertices



$$G-e: \begin{cases} C_1 \to m \text{ vertices} \\ C_2 \to n \text{ vertices} \end{cases}$$

$m+n=p$

C_1 has $(m-1)$ edges

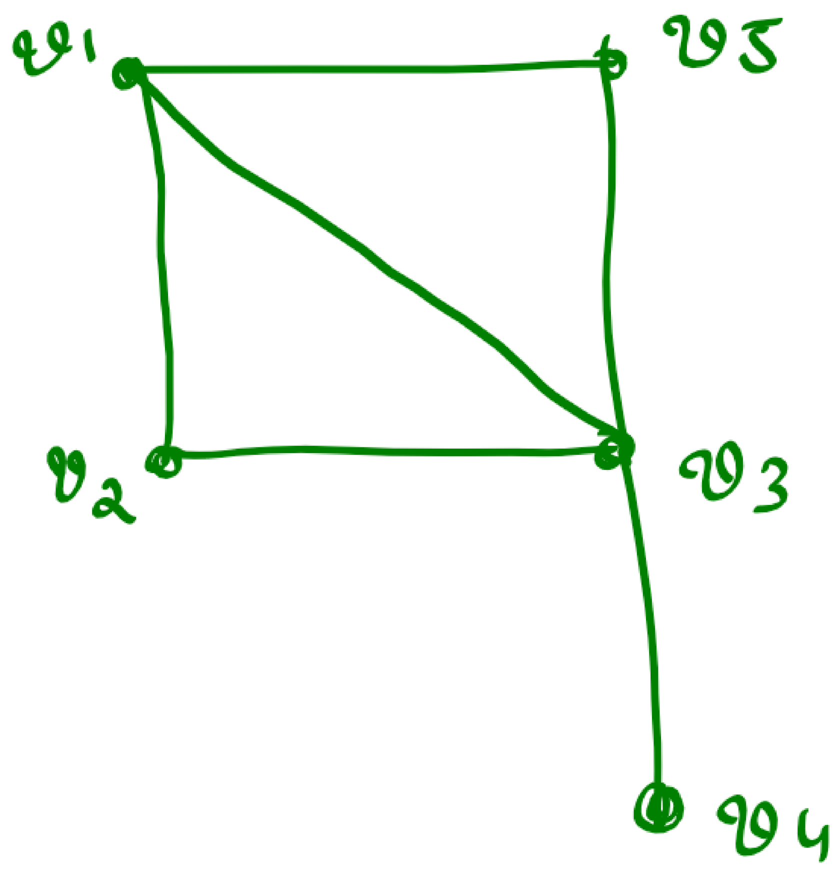
C_2 has $(n-1)$ edges

$$\therefore \quad m-\cancel{1} + n-1 + \cancel{1}$$

$$p-1$$

Matrices

① Adjacency matrix :-

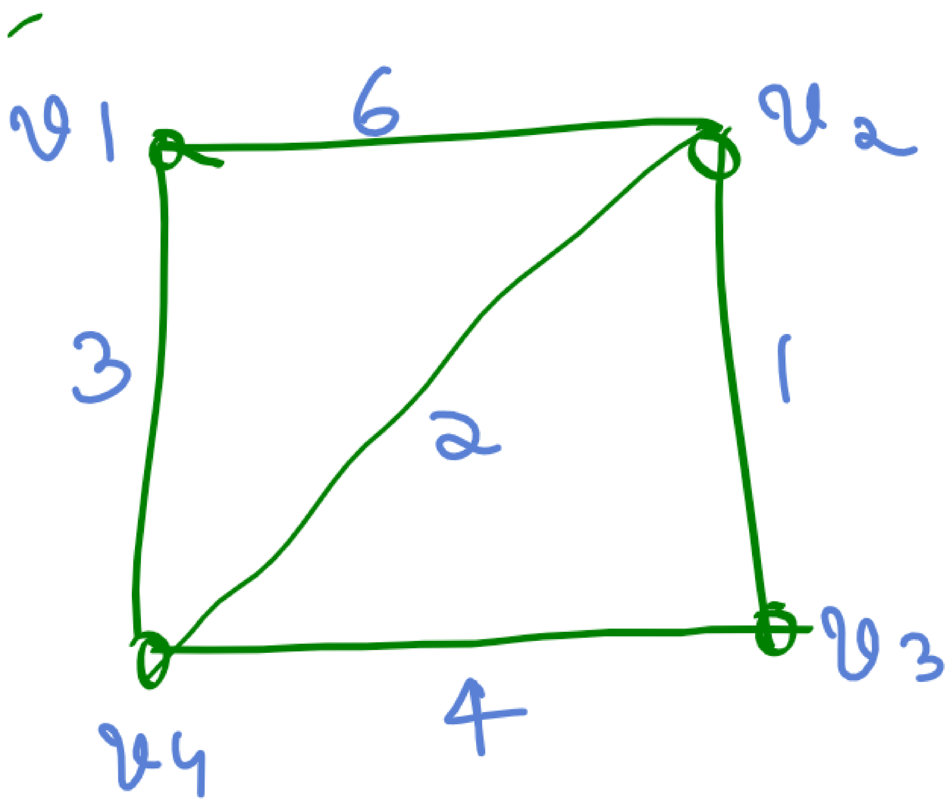


$$
\begin{array}{c|ccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 \\
\hline
v_1 & 0 & 1 & 1 & 0 & 1 \\
v_2 & 1 & 0 & 1 & 0 & 0 \\
v_3 & 1 & 1 & 0 & 1 & 1 \\
v_4 & 0 & 0 & 1 & 0 & 0 \\
v_5 & 1 & 0 & 1 & 0 & 0
\end{array}
$$

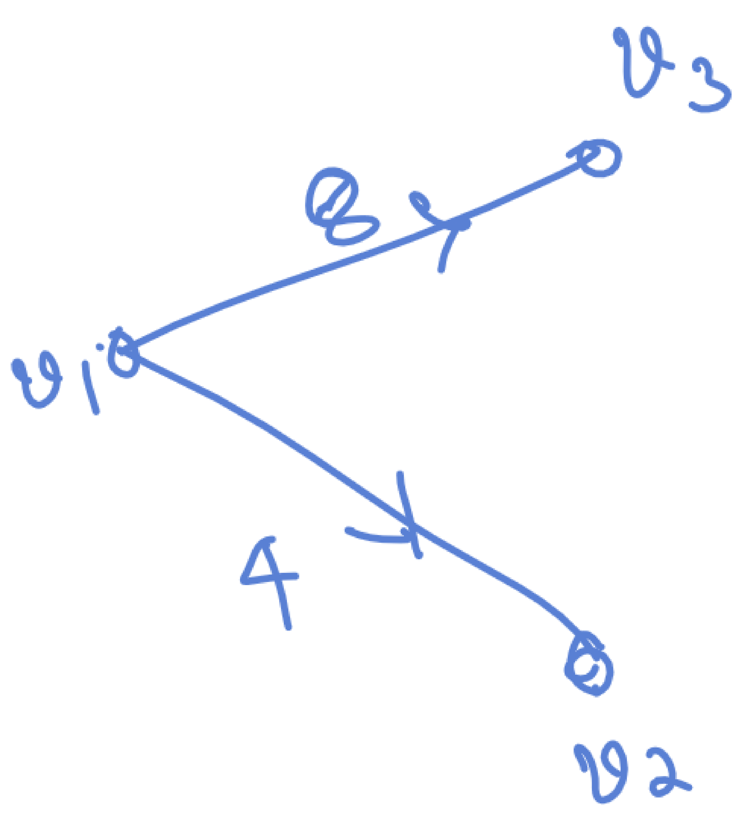
symmetric

$$(a_{ij}) = \begin{cases} 1 & v_i \text{ is adj to } v_j \\ 0 & \text{else} \end{cases}$$

Distance matrix :-



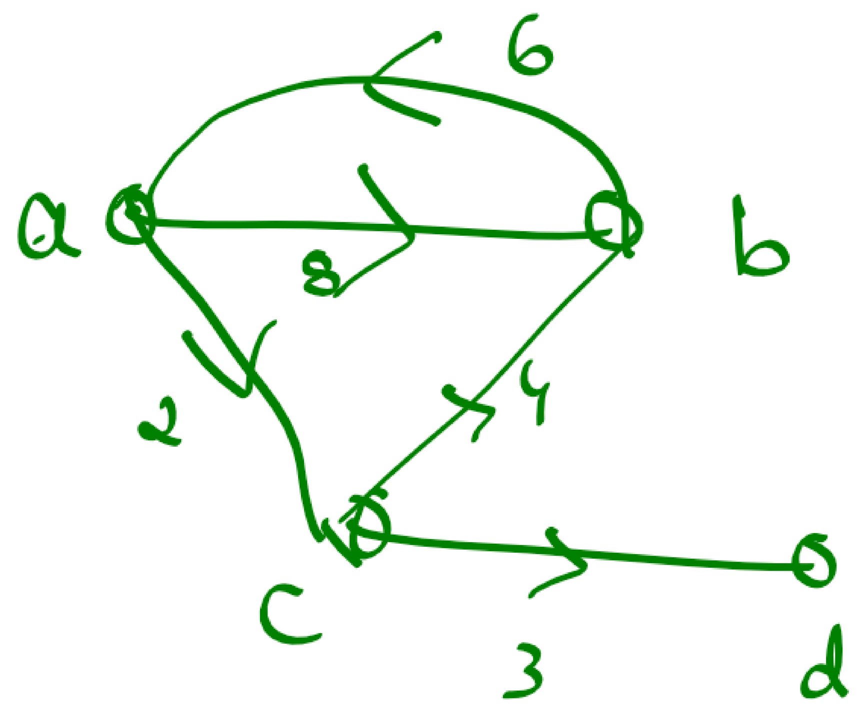
$$d_{ij} = \begin{cases} 0 & \text{if } i=j \\ \infty & v_i \,\&\, v_j \text{ are not adj} \\ w & v_i \,\&\, v_j \text{ are adj} \end{cases}$$

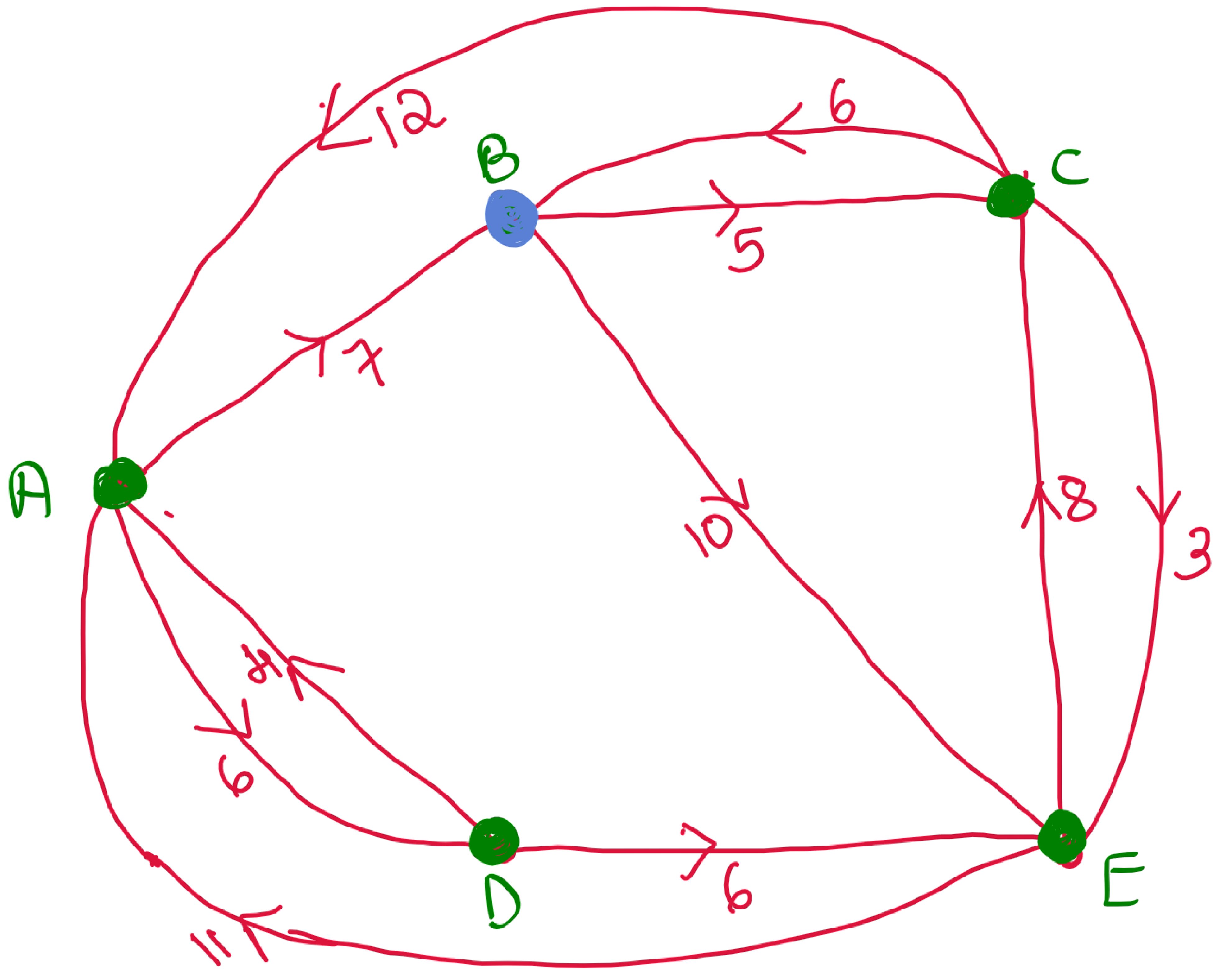


$$
\begin{array}{c|ccc}
 & v_1 & v_2 & v_3 \\
\hline
v_1 & 0 & 4 & 8 \\
v_2 & \infty & 0 & \infty \\
v_3 & \infty & \infty & 0
\end{array}
$$

Dijkstra's Algorithm

to find shortest path b/w the vertices in a weighted graph





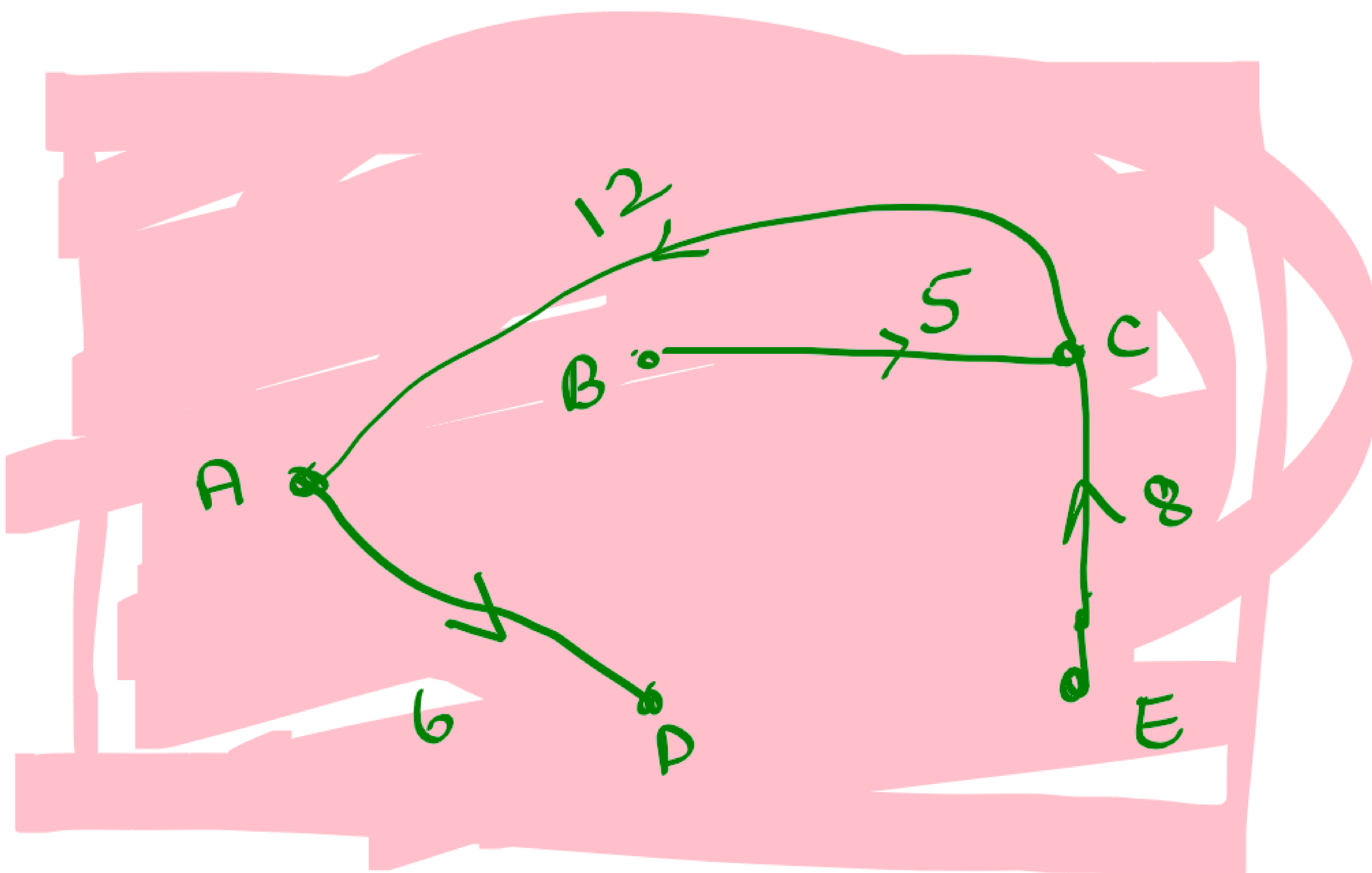
shortest path from B to all Other vertice

$$D = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 7 & \infty & 6 & \infty \\ B & \infty & 0 & 5 & \infty & 10 \\ C & 12 & 6 & 0 & \infty & 3 \\ D & 4 & 5 & \infty & 0 & 6 \\ E & 11 & \infty & 8 & \infty & 0 \end{array}$$

$K = \{B\}$

$U = \{A, C, D, E\}$

	A	C	D	E
Best dist	∞	5	∞	10
tree	B	B	B	B



1st Iteration:- $U = \{B, C\}$

$K = \{A, D, E\}$

$d(C) = 5$

	A	D	E
Best dist	17	∞	8
tree	C	B	C

A: $\infty > 12+5 = 17$

D: $\infty < \infty + 5$

E: $10 > 3+5$

2nd Iteration:-

$K = \{B, C, E\}$

$U = \{A, D\}$

$d(E) = 8$

A: $17 < 11+8$

D: $\infty < \infty + 8$

	A	D
Best dist	17	∞
tree	C	B

3rd iteration:-

$K = \{B, C, E, A\}$

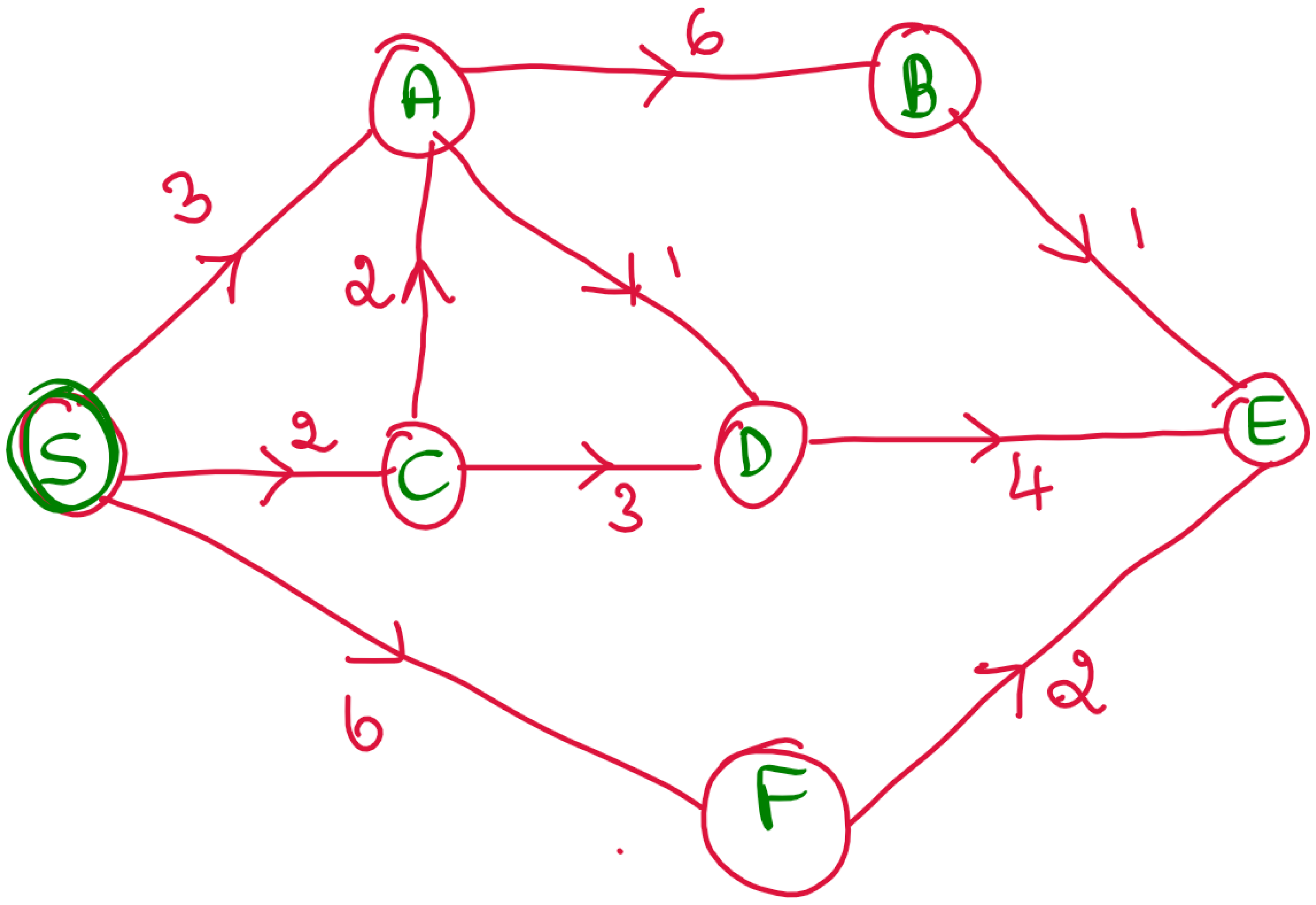
$d(A) = 17$

$U = \{D\}$

	D
	23
	A

$D: \infty > 6+17$

②



$$
\begin{array}{c|ccccccc}
 & A & B & C & D & E & F & S \\
\hline
A & 0 & 6 & \infty & 1 & \infty & \infty & \infty \\
B & \infty & 0 & \infty & \infty & 1 & \infty & \infty \\
C & 2 & \infty & 0 & 3 & \infty & \infty & \infty \\
D & \infty & \infty & \infty & 0 & 4 & \infty & \infty \\
E & \infty & \infty & \infty & \infty & 0 & \infty & \infty \\
F & \infty & \infty & \infty & \infty & 2 & 0 & \infty \\
S & 3 & \infty & 2 & \infty & \infty & 6 & 0
\end{array}
$$

$K = \{S\}$

$U = \{A, B, C, D, E, F\}$

	A	B	C	D	E	F
Best dist	3	∞	2	∞	∞	6
tree	S	S	S	S	S	S

Ist iteration :- $K = \{S, C\}$ $d(C) = 2$

$U = \{A, B, D, E, F\}$

	A	B	D	E	F
Best dist	3	∞	5	∞	6
tree	S	S	C	S	S

A: $3 < 2+2$
B: $\infty < \infty+2$
D: $\infty > 3+2$
E: $\infty < \infty+2$
F: $6 < \infty+2$

2nd iteration:-

$K = \{S, C, A\}$ $d(A) = 3$

$U = \{B, D, E, F\}$

	B	D	E	F
Best dist	9	4	∞	6
tree	A	A	S	S

B: $\infty > 6+3$
D: $5 > 1+3$
E: $\infty < \infty+3$
F: $6 < \infty+3$

3rd iteration:

$K = \{S, C, A, D\}$ $d(D) = 4$

$U = \{B, E, F\}$

	B	E	F
Best di	9	8	6
tree	A	D	S

Fourth iteration:-

$K = \{S, A, C, D, F\}$ $d(F) = 6$

$U = \{B, E\}$

	B	E
	9	8
	A	F

Fifth iteration

$K = \{S, C, A, D, E, F\}$ $d(E) = 8$

$U = \{B\}$

	B
	9
	A