MANIPAL INSTITUTE OF TECHNOLOGY, MANIPAL

Semester III – CCE/CSE/ICT

Sessional I

Duration: 1 Hour

Subject: Engineering Mathematics III

Max. Marks: 15

Part A

$5 \times 1 = 5$ marks

1. The number of squares of all possible sizes in an 8×8 chessboard is _____.
2. "Every positive integer has a self-conjugate partition". Say whether the statement is true or false, with justification.
3. Draw the Hasse diagram of the poset $(P, \mid)$, where P is the set of all positive divisors of 36 and $\mid$ is the "divides" relation ($m \mid n$, if n is a multiple of m).
4. The 60^{th} permutation of $1,2,3,4,5$ in lexicographical order is _____.
5. The 90^{th} permutation in Fike's ordering starting with 12345 is _____.

Part B

$5 \times 2 = 10$ marks

6. Find the number of parts into which a plane is divided by n straight lines, no two of which are parallel and no three of which are concurrent.
7. Show that the number of partitions of n with at most k parts is equal to the number of partitions of n with no part larger than k. Hence define a formula for the number of partitions of n with exactly k parts.
8. Using the principle of inclusion and exclusion, derive a formula for the number of permutations of $\{1,2,\ldots,n\}$ in which i is not in the i^{th} position for any i, $1 \le i \le n$.
9. How many subsets of five integers chosen (without repetition) from $1,2,\ldots,20$ are there with no consecutive integers (e.g., if 5 is in the set, then 4 and 6 cannot be in it)?
10. A point (x, y) in the first quadrant of the xy-plane defines a rectangle with points $(0,0)$, $(x,0)$, $(0,y)$, and (x,y) as its vertices. Consider n such rectangles defined by the points (x_i, y_i), $1 \le i \le n$. Let $P = \{(x_i, y_i) \mid 1 \le i \le n\}$. Define a relation $\le$ on P as follows. $(x_i, y_i) \le (x_j, y_j)$ if and only if $x_i \le x_j$ and $y_i \le y_j$. Show that $\le$ defined above is a partial ordering relation on P.