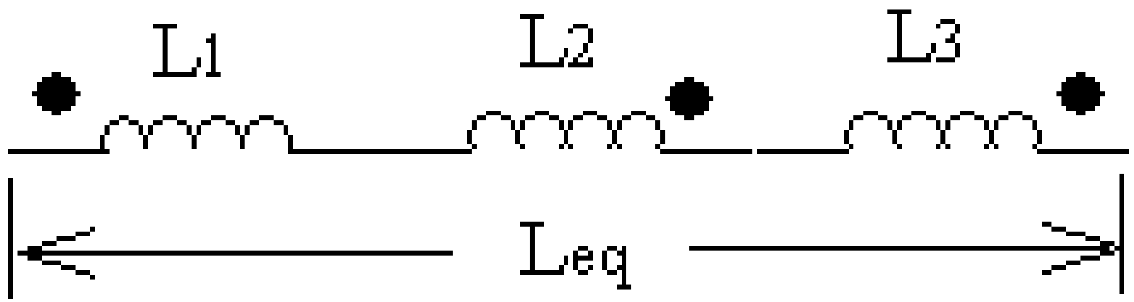
Example 1

Three magnetically coupled inductive coils having the following data are connected in series as shown in Figure.

$L_1 = 0.12$ H; $L_2 = 0.14$ H; $L_3 = 0.16$ H

$k_{12} = 0.3$; $k_{23} = 0.6$; $k_{31} = 0.9$

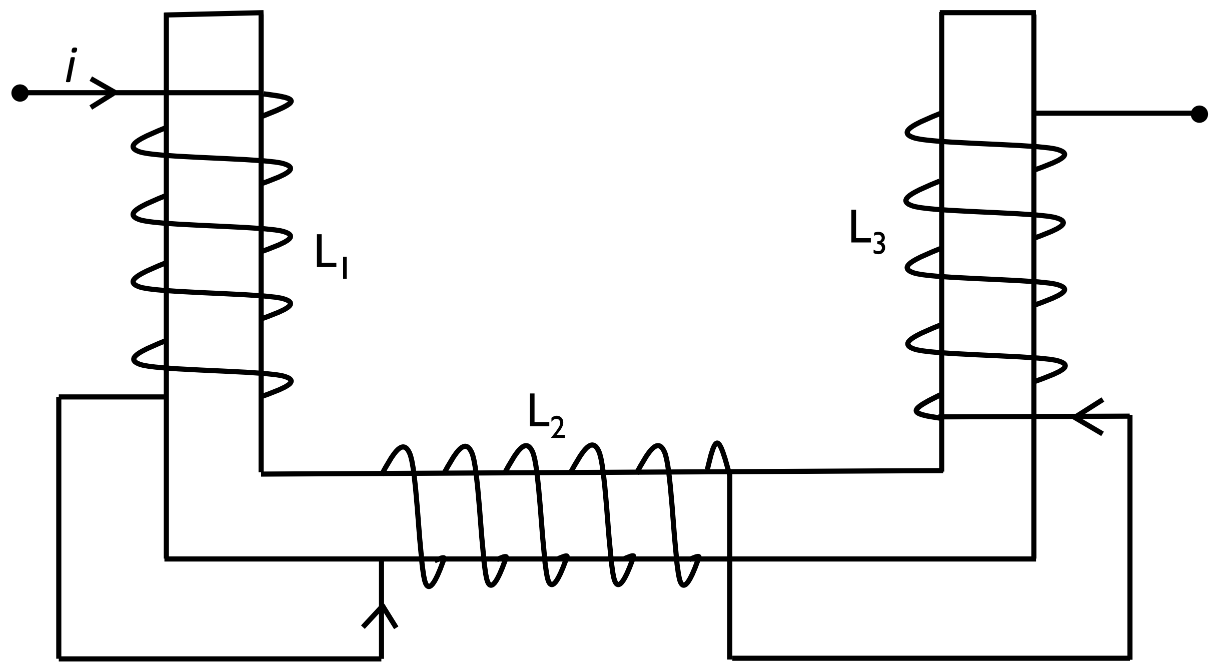
Find the equivalent inductance of the circuit.



Ans: 0.272 H

Example 1

Three magnetically coupled inductive coils having the following data are connected in series as shown in Figure. $L_1 = 0.3$ H; $L_2 = 0.6$ H; $L_3 = 0.8$H and the coefficients of coupling are ,$k_{12} = 0.8$; $k_{23} = 0.75$; $k_{31} = 0.5$
Draw the dotted equivalent circuit of the figure, also find the equivalent inductance of the circuit.



Ans : 0.472 H

Example 3

Two similar coils have a coupling coefficient of 0.4. When they are connected in series aiding, the equivalent inductance is 560mH. Calculate: i) self-inductance of both the coils. i) Total inductance when the coils are connected in series opposition. iii) total energy stored due to a current of 3A when the coils are connected in series opposition.

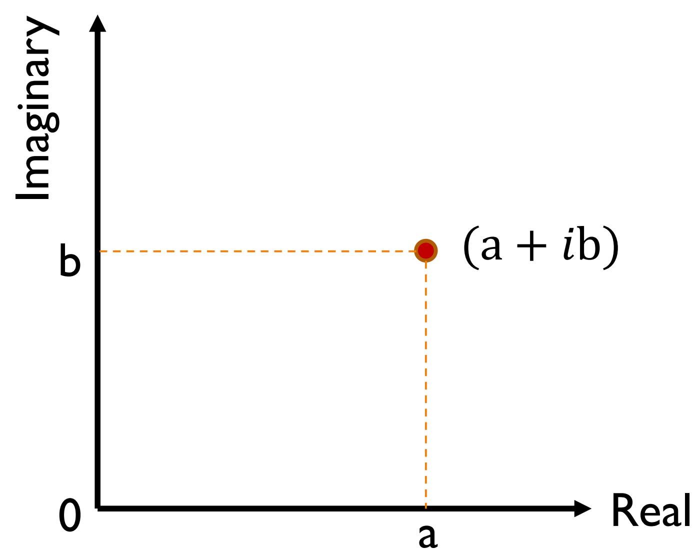
Ans: 0.2 H, 0.24 H, 1.08 J

Review of complex algebra

- Representation of complex numbers
- Conversion of complex forms
- Arithmetic operation on complex numbers

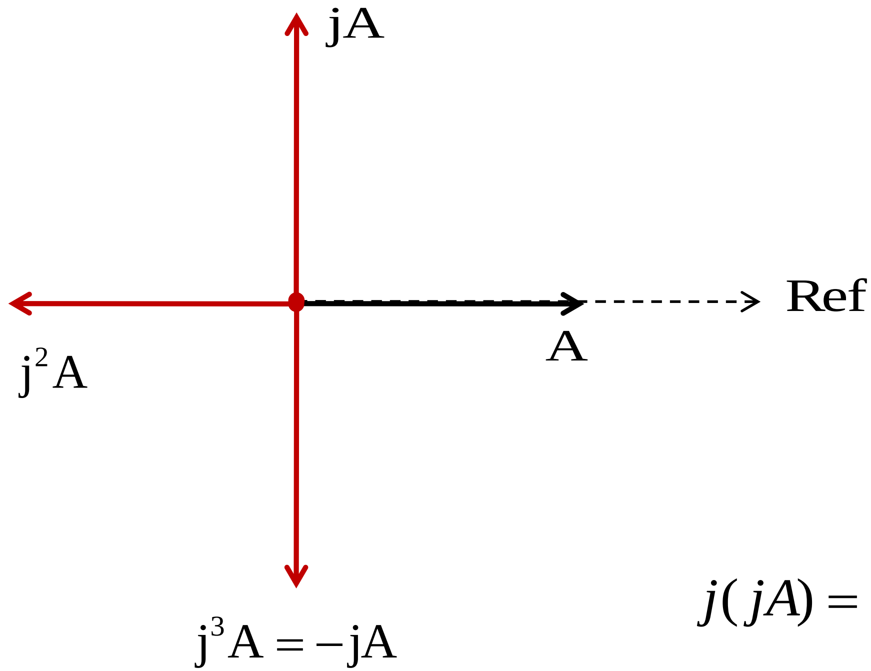
Complex Number

- A **complex number** is of the form **a + *i* b**
- Represented on complex plane as:



The operator ‘j’

$$j = 1\angle 90°$$



$$j(jA) = j^2A = -A$$

$$\boxed{\text{Therefore,} \quad j^2 = -1; \quad j = \sqrt{-1}}$$

The operator ‘j’ rotates the given vector by 90 degrees in anti-clockwise direction

Representation of a complex number

- **Rectangular form**: $\mathbf{a = x \pm \mathit{j}y}$
- **Polar form**: $\mathbf{a} = |\boldsymbol{a}|\angle \pm \boldsymbol{\theta}$
- **Exponential form**: $\mathbf{a} = |\boldsymbol{a}|\mathbf{e}^{\pm \boldsymbol{j\theta}}$
- **Trigonometric form:** $\mathbf{a = |a|(cos\theta \pm \mathit{j}sin\theta)}$

Rectangular ↔ Polar conversion

- **Rectangular to polar:**

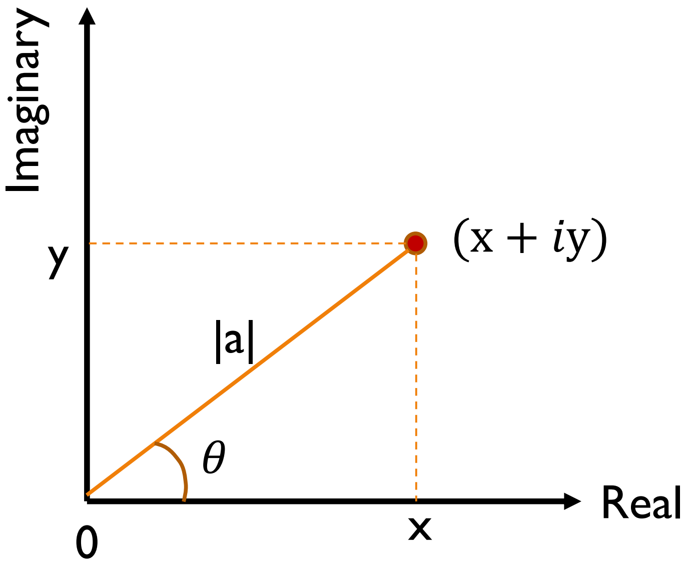
$$|a| = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

- **Polar to Rectangular:**

$$x = |a| \cos\theta$$

$$y = |a| \sin\theta$$



Rectangular ↔ Polar conversion

- Convert the following into polar form

  1) $3 + j\,4 \qquad = 5\,\angle 53.13°$

  2) $8 + j\,6 \qquad = 10\,\angle 36.87°$

  3) $8 - j\,6 \qquad = 10\,\angle -36.87°$

- Convert the following into rectangular form

  1) $5\,\angle 30° \qquad = 4.33 + j\,2.5$

  2) $3\,\angle -60° \qquad = 1.5 - j\,2.59$

  3) $-(10\,\angle 45°) \qquad = -7.07 - j\,7.07$

Arithmetic operation

Let $a_1 = x_1 + jy_1 = r_1\angle\theta_1$ $a_2 = x_2 + jy_2 = r_2\angle\theta_2$

$a_1 = 4 + j6 = 7.21\angle 56.3°$ $a_2 = 2 - j4 = 4.47\angle - 63.43°$

Addition: $\boxed{\mathbf{a}_1 + \mathbf{a}_2 = (\mathbf{x}_1 + \mathbf{x}_2) + j(\mathbf{y}_1 + \mathbf{y}_2)}$

$$a_1 + a_2 = (4 + 2) + j(6 - 4) = 6 + j2$$

Subtraction: $\boxed{\mathbf{a}_1 - \mathbf{a}_2 = (\mathbf{x}_1 - \mathbf{x}_2) + j(\mathbf{y}_1 - \mathbf{y}_2)}$

$$a_1 - a_2 = (4 - 2) + j(6 + 4) = 2 + j10$$

'Rectangular form is used for addition and subtraction of complex numbers'

Arithmetic operation

Let $\boldsymbol{a_1 = \mathrm{x}_1 + jy_1 = r_1\angle\theta_1}$ $\boldsymbol{a_2 = \mathrm{x}_2 + jy_2 = r_2\angle\theta_2}$

$a_1 = 4 + j6 = 7.21\angle 56.3°$ $a_2 = 2 - j4 = 4.47\angle - 63.43°$

Multiplication:

$$\boxed{\mathbf{a_1\, a_2 = r_1\, r_2\angle(\theta_1 + \theta_2)}}$$

$$a_1\, a_2 = (7.21)(4.47)\angle(56.3° - 63.43°) = 32.22\angle - 7.13°$$

Division:

$$\boxed{\boldsymbol{\frac{a_1}{a_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)}}$$

$$\frac{a_1}{a_2} = \frac{7.21}{4.47}\angle(56.3° - (-63.43°)) = 1.61\angle 119.73°$$

'Polar form is used to for multiplication and division of complex numbers'

Exercise

Ex. 1: $a_1 = 3 + j5 = 5.83\angle 59.03°$

$a_2 = 5 - j4 = 6.40\angle - 38.65°$

Compute $a = \dfrac{a_1 a_2}{a_1 + a_2}$

Ans:
$a = 4.63\angle 13.26°$

Ex. 2: $a_1 = 4 + j4$ $\quad a_2 = 5 - j4$ $\quad a_3 = 8 + j2$

Compute $a_{12} = a_1 + a_2 + \dfrac{a_1 a_2}{a_3}$, similarly a_{23} & a_{31}

Ans: $a_{12} = 13.36\angle - 2.52°$ $\quad a_{23} = 18.90\angle - 47.52°$ $\quad a_{31} = 17.21\angle 50.17°$

Ex. 3: $a_{12} = 7 + j4$ $\quad a_{23} = 9 + j11$ $\quad a_{31} = 35 - j3$

Compute $a_1 = \dfrac{a_{12} a_{31}}{\sum a_{12}}$, similarly a_2 & a_3

Ans: $a_1 = 5.40\angle 11.60°$ $\quad a_2 = 2.18\angle 67.21°$ $\quad a_3 = 9.52\angle 32.57°$

Summary

- **Review of complex algebra**
  - Rectangular form is used for addition and subtraction of complex numbers
  - Polar form is used to for multiplication and division of complex numbers

- **'j' operator**
  - $j = 1\angle 90^\circ$
  - Rotates a vector by 90 degree in the anti-clockwise direction