Reg.No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER B.Tech DEGREE END SEMESTER EXAMINATION – DECEMBER, 2012

Sub: MAT 101 - ENGG. MATHEMATICS I

(REVISED CREDIT SYSTEM - 2011)

Time: 3 Hrs. **Max. Marks: 50**

Note: a). Answer any FIVE full questions b). All questions carry equal marks

1A. Find the nth derivative of i) $e^{3x}\sin^2 x\cos^2 x$ *ii*) $\dfrac{x^4}{(x+1)(x+2)}$

1B. If $y = e^{m\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$

1C. Obtain the reduction formula for $\int \cos^n x\,dx$ and hence evaluate $\int_0^{\frac{\pi}{2}} \mathrm{Sin}^n x\,dx$

(3+3+4)

2A. Evaluate

i) $\int_0^{2a} x^2\sqrt{(2ax-x^2)}\,dx$ ii) $\int_0^{\infty} \frac{x^2}{(1+x^2)^{7/2}}\,dx$

2B. Find the equation of the right circular cone generated when the straight line 2x +3y= 6, z =0 revolves about y – axis.

2C. Trace the following curve with explanation

y $(1 - x^2)=x^2$ **(3+3+4)**

3A. Find the region of convergence of the following power series.

(i) $1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+....$

(ii) $\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+...$

3B. Find the equation of the plane containing the line 2x –y + z -3 = 0, 3x + y + z =5 and at a distance of $1/\sqrt{6}$

3C. Find first three non zero terms in Maclurin's series expansion of f(x) = tanx

(3+3+4)

4A. Test the nature of the series $\sum_{n=1}^{\infty} \frac{n!2^n}{n^n}$

4B. If $H = f\left(y-z,\ z-x,\ x-y\right)$ then prove that $\frac{\partial H}{\partial x}+\frac{\partial H}{\partial y}+\frac{\partial H}{\partial z}=0$

4C. Find the evolute of the rectangular hyperbola $2xy = a^2$. **(3+3+4)**

5A. Sketch and find the area common to the circle $r = a\sqrt{2}$ and $r = 2a\cos\theta$.

5B. In the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that the radius of curvature at an end of the major axis is equal to semi-latus rectum of the ellipse

5C. Find the length of the cardiode $r = a(1+\cos\theta)$ lying outside the circle $r = a\cos\theta$. **(3+3+4)**

6A. Find magnitude and the equations of the shortest distance between the lines

$\frac{x}{2}=\frac{y-6}{2}=\frac{z}{-1}$, $\frac{x+3}{2}=\frac{y-6}{3}=\frac{z-3}{-2}$.

6B. Find the volume of the solid formed by revolving the curve $x = a(\theta+\sin\theta)$, $y = a(1-\cos\theta)$ a>0 about x-axis.

6C. If the sides of a triangle ABC vary in such a way that its circum radius remains constant, prove that $\frac{da}{\cos A}+\frac{db}{\cos B}+\frac{dc}{\cos C}=0$

(3+3+4)

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