Chapter 2

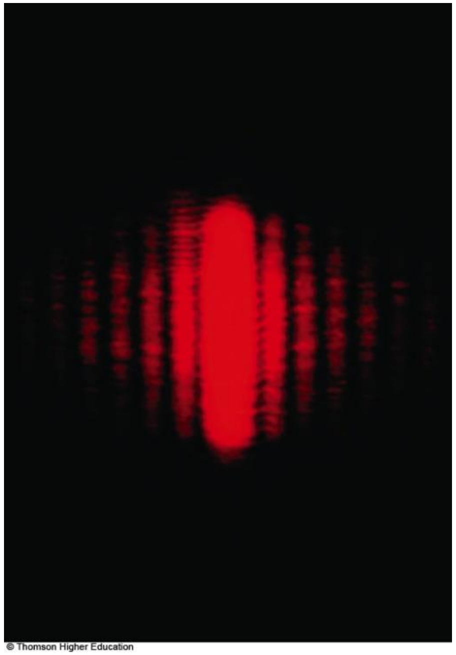
DIFFRACTION PATTERNS AND POLARIZATION

OBJECTIVES

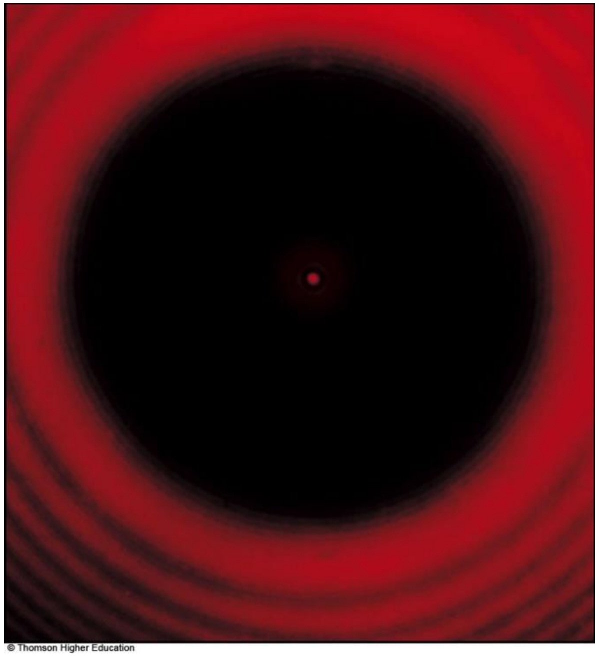
- To understand the principles of diffraction.
- To explain the intensity distribution in diffraction under various conditions.
- To explain the diffraction of light waves at single, multiple slits and circular apertures.
- To understand polarization phenomena and various techniques used to produce polarized light.

Introduction to Diffraction Patterns

Light of wavelength comparable to or larger than the width of a slit spreads out in all forward directions upon passing through the slit. This phenomenon is called *diffraction*



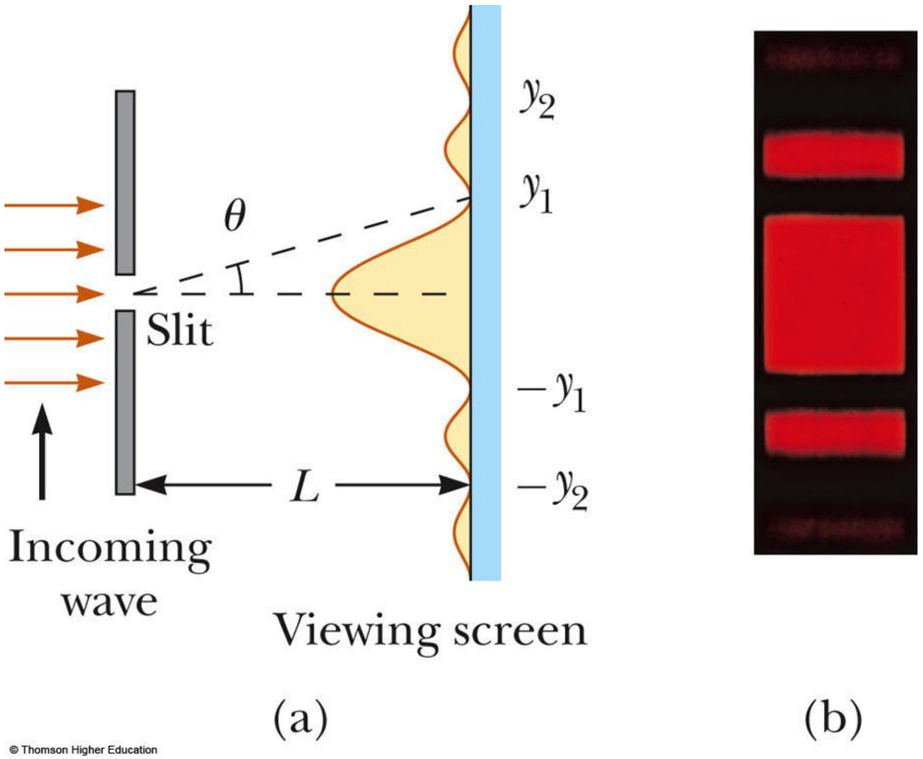
(a) The diffraction pattern that appears on a screen when light passes through a narrow vertical slit. The pattern consists of a broad central fringe and a series of less intense and narrower side fringes



(b) Diffraction pattern created by the illumination of a penny, with the penny positioned midway between the screen and light source.

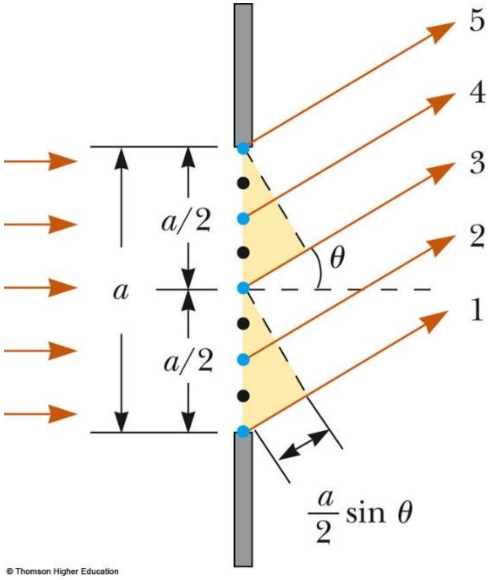
Diffraction Patterns from Narrow Slits

- Here we assume the observing screen is far from the slit and the rays reaching the screen are approximately parallel.
- In laboratory, this situation can also be achieved experimentally by using a converging lens to focus the parallel rays on a nearby screen.
- In this model, the pattern on the screen is called a **Fraunhofer diffraction pattern.**



Qualitative analysis of single slit diffraction

- To analyze the diffraction pattern, let's divide the slit into two halves as shown in Figure.
- If this path difference is exactly half a wavelength (corresponding to a phase difference of 180°), the pairs of waves cancel each other and destructive interference results.
- Ray 1 and Ray 3 are out of phase by 180°
- Similarly Ray 2 and Ray 4 are out of phase by 180°



- Hence we can write $\frac{a}{2}\sin\theta = \pm\frac{\lambda}{2}$
- Dividing the slit into four equal parts and using similar reasoning, we find that the viewing screen is also dark when $\sin\theta = \pm 2\frac{\lambda}{a}$
- Therefore, the general condition for destructive interference (**Diffraction minima**) is

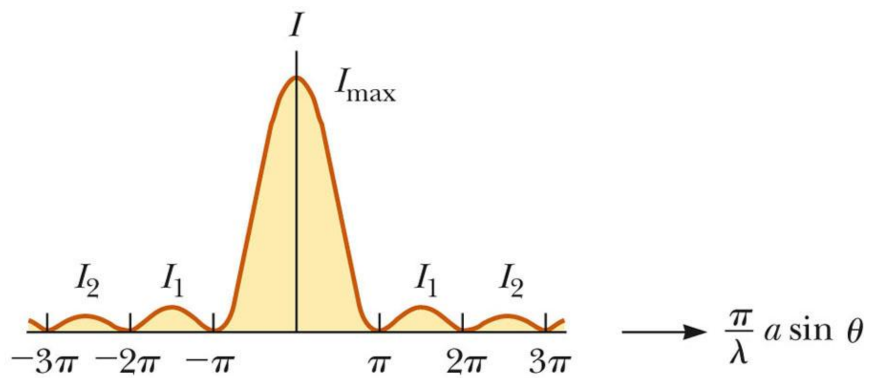
$$\boxed{\sin\theta_{\text{dark}} = m\frac{\lambda}{a}} \quad m = \pm 1, \pm 2, \pm 3, \ldots$$

Intensity of Single-Slit Diffraction Patterns

Analysis of the intensity variation in a diffraction pattern from a single slit of width '*a*' shows that the intensity is given by

$$I = I_{\max}\left[\frac{\sin\left(\pi a \sin\theta/\lambda\right)}{\pi a \sin\theta/\lambda}\right]^2$$

where $I_{\max}$ is the intensity at $\theta = 0$ (the central maximum) and λ is the wavelength of light used to illuminate the slit.



(a)

(b)

Light of wavelength 540 nm passes through a slit of width 0.200 mm. (a) The width of the central maximum on a screen is 8.10 mm. How far is the screen from the slit? (b) Determine the width of the first bright fringe to the side of the central maximum.

a) 1.5 m b) 4.05 mm

A screen is placed 50.0 cm from a single slit, which is illuminated with light of wavelength 690 nm. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?

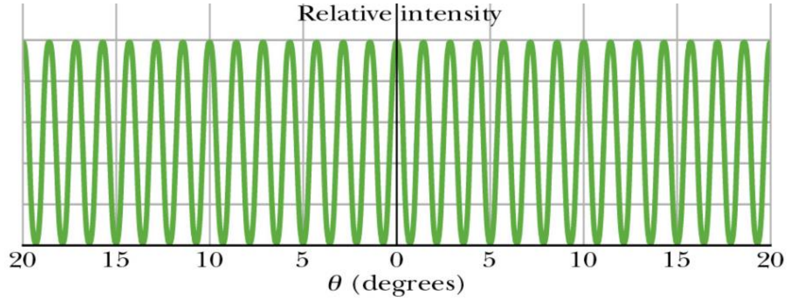
2.3×10^{-4} m

A diffraction pattern is formed on a screen 120 cm away from a 0.400-mm-wide slit. Monochromatic 546.1-nm light is used. Calculate the fractional intensity I/I_{max} at a point on the screen 4.10 mm from the center of the principal maximum. (Use the calculator in radian mode)

1.62×10^{-2}

Intensity of Two-Slit Diffraction Patterns

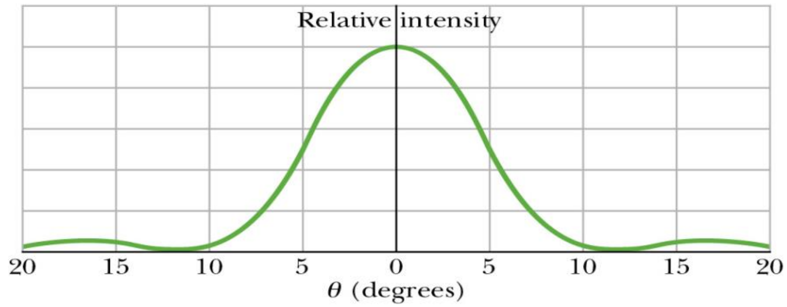
a<< λ



(a)

Pure two slit interference

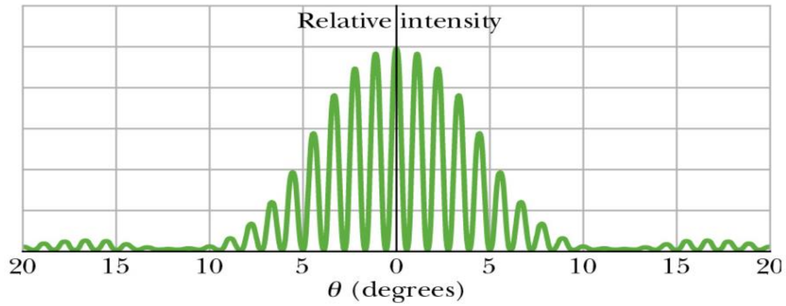
a ~ λ



(b)

Single slit diffraction

a ~ λ



(c)

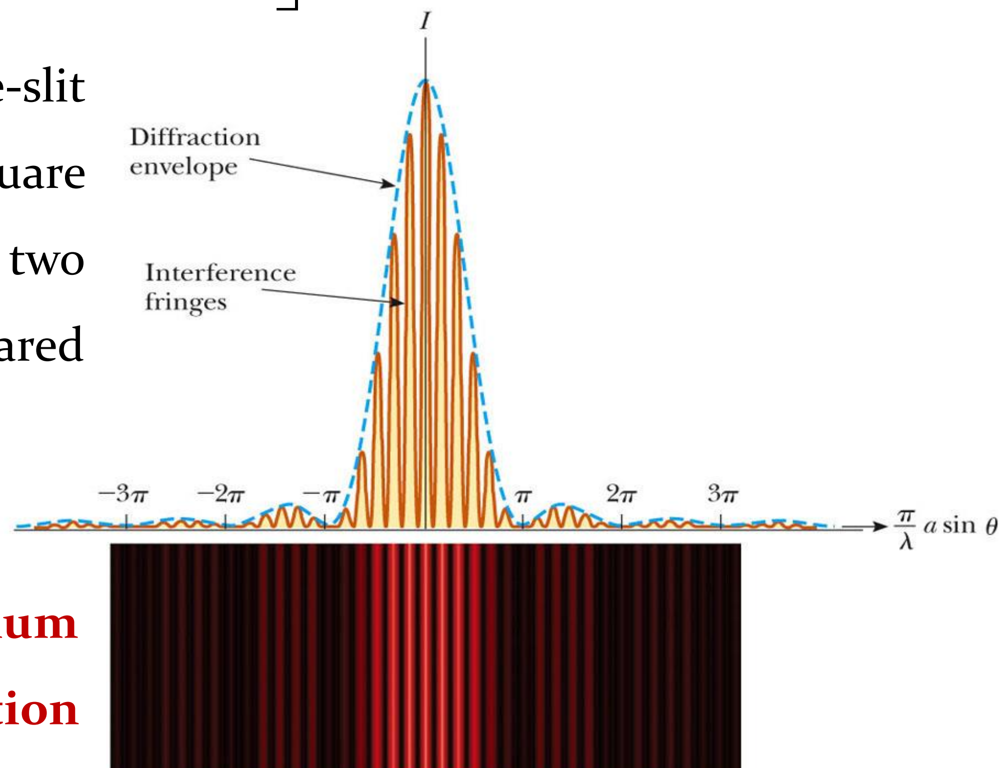
Double slit diffraction

Intensity of Two-Slit Diffraction Patterns

When more than one slit is present, we must consider not only diffraction patterns due to the individual slits but also the interference patterns due to the waves coming from different slits. Intensity due to combined effect is given by

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left[\frac{\sin\left(\pi a \sin\theta / \lambda\right)}{\pi a \sin\theta / \lambda}\right]^2$$

Above equation represents the single-slit diffraction pattern (the factor in square brackets) acting as an “envelope” for a two slit interference pattern (the cosine-squared factor).



If $\frac{d}{a} = m$ where m is an interger

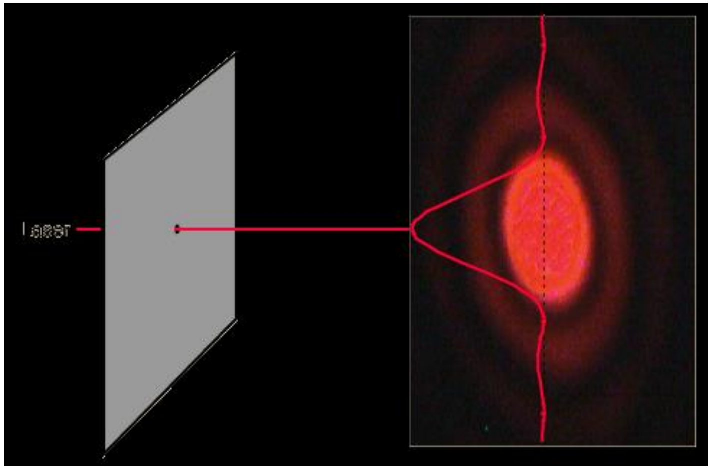
In this case, mth interference maximum coincides with first diffraction minimum.

Intensity of Two-Slit Diffraction Patterns

If $\frac{d}{a} = m + \frac{1}{2}$ where m is an interger

In this case, m^{th} interference minimum coincides with first diffraction minimum.

Resolution of Single-Slit and Circular Apertures



When light from a point source passes through a small circular aperture, it does not produce a bright dot as an image, but rather a diffuse circular disc known as Airy's disc surrounded by much fainter concentric circular rings. This example of diffraction is of great importance because the eye and many optical instruments have circular apertures.

Resolution of Single-Slit and Circular Apertures

The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light.

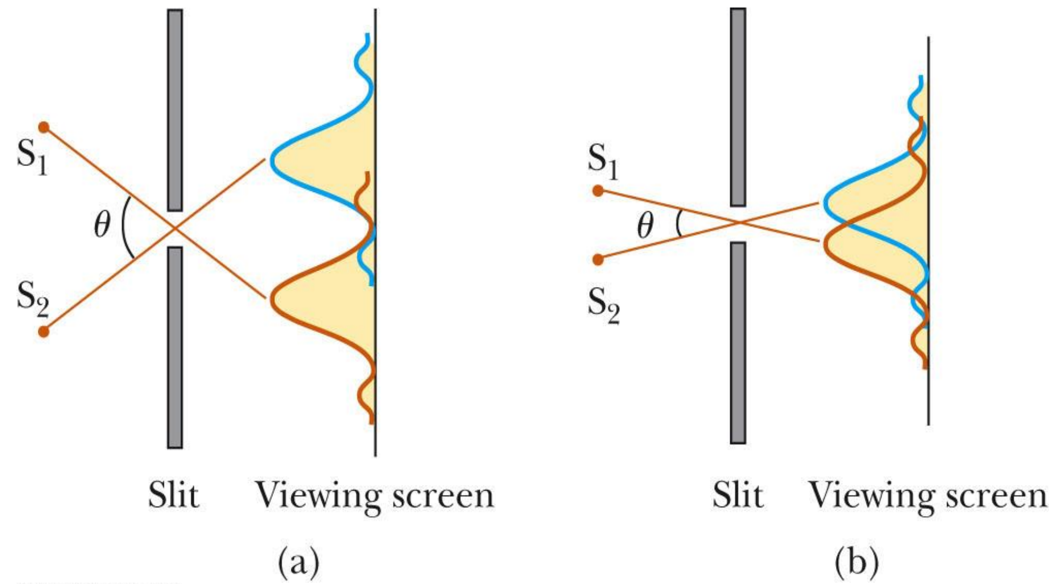
When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved. This limiting condition of resolution is known as **Rayleigh's criterion.**

In case of single-slit diffraction pattern, this occurs at the angle for which

$$\sin\theta = \frac{\lambda}{a}$$

Since $\lambda << a$

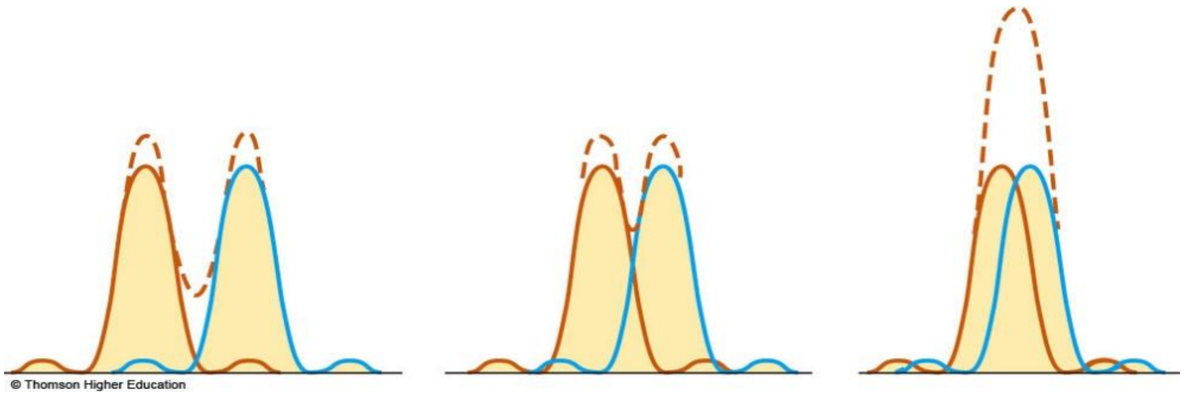
$$\theta_{min} = \frac{\lambda}{a}$$

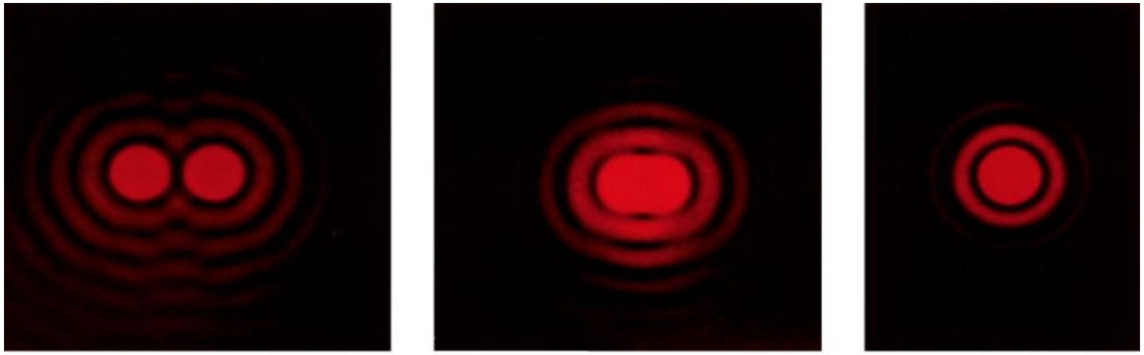


Circular Aperture: Analysis shows that the limiting angle of resolution of the circular aperture is

$$\theta_{min} = 1.22\,\frac{\lambda}{D}$$

where D is the diameter of the aperture. θ_{min} is the smallest angular separation for which we can resolve the images of two objects.





(a) (b) (c)

a. Well resolved

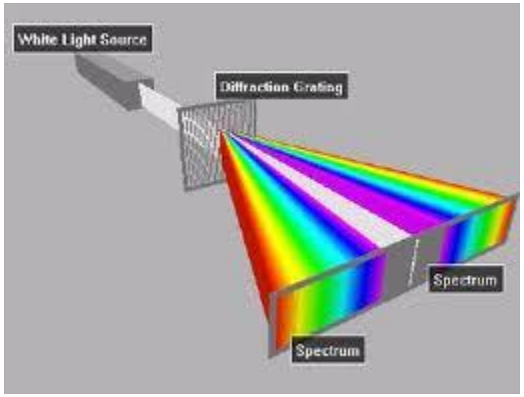
b. Just resolved

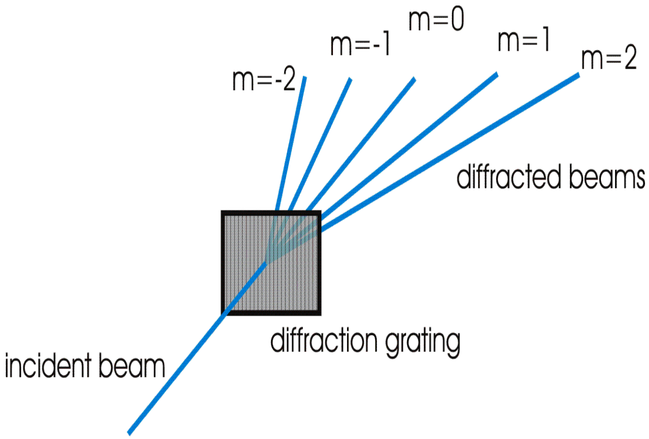
c. Not resolved

The angular resolution of a radio telescope is to be 0.100° when the incident waves have a wavelength of 3.00 mm. What minimum diameter is required for the telescope's receiving dish?

2.1 m

Diffraction Grating





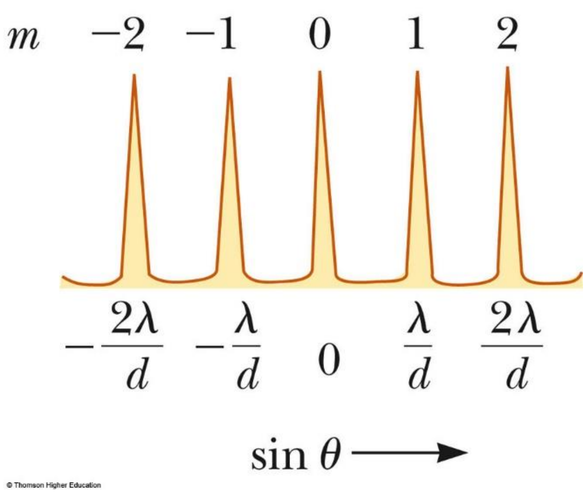
m = 0 m = 1 m = 2 m = 3

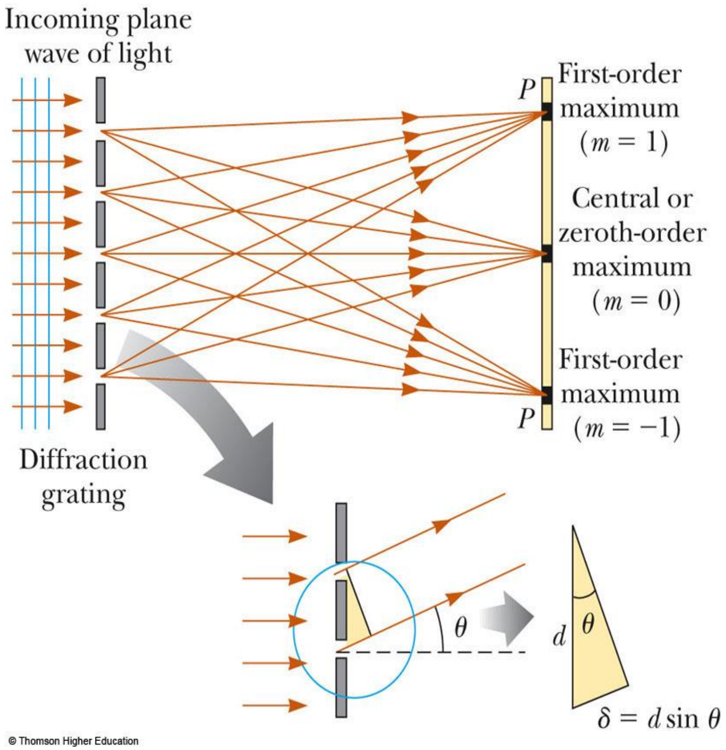
Sample spectra of visible light emitted by a gaseous source

Diffraction Grating

- A *transmission grating* can be made by cutting parallel grooves on a glass plate with a precision ruling machine. The spaces between the grooves are transparent to the light and hence act as separate slits.
- A *reflection grating* can be made by cutting parallel grooves on the surface of a reflective material.

$$d \sin \theta_{\text{bright}} = m\lambda \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$





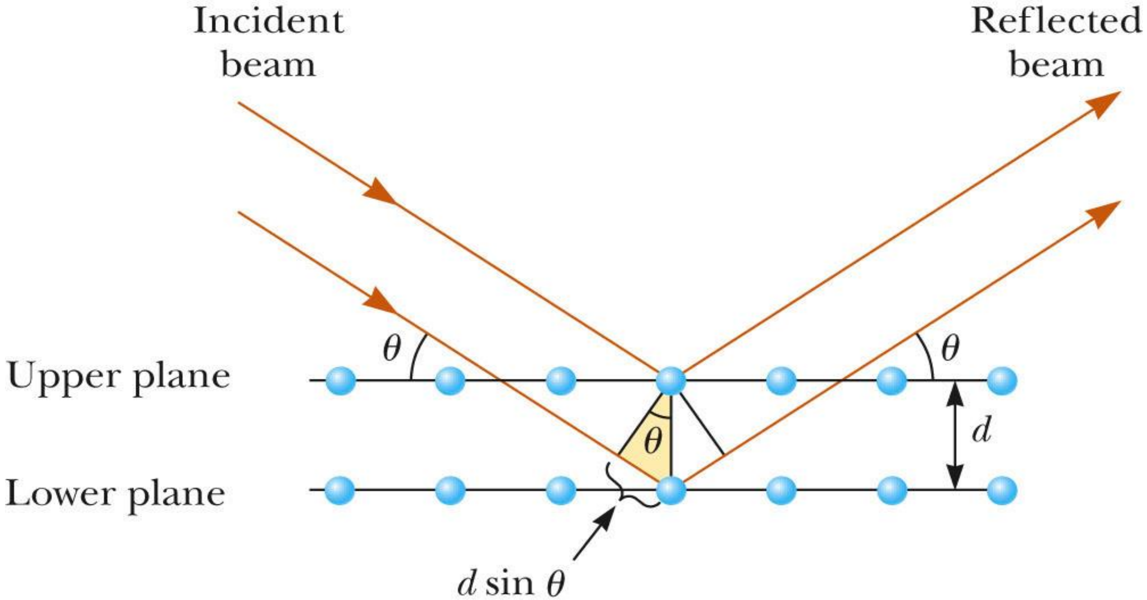
Light of wavelength 500 nm is incident normally on a diffraction grating. If the third-order maximum of the diffraction pattern is observed at 32.0°, (a) what is the number of rulings per centimeter for the grating? (b) Determine the total number of primary maxima that can be observed in this situation.

(a) 3.53×10^3 rulings per cm

(b) Total number of primary maxima = 2m+1 = 11

Diffraction of X-Rays by Crystals

X-rays are **used (Instead of visible radiation)** to produce the diffraction pattern because their wavelength λ is typically the same order of magnitude (1–10 nm) as the d spacing between planes in the crystal.

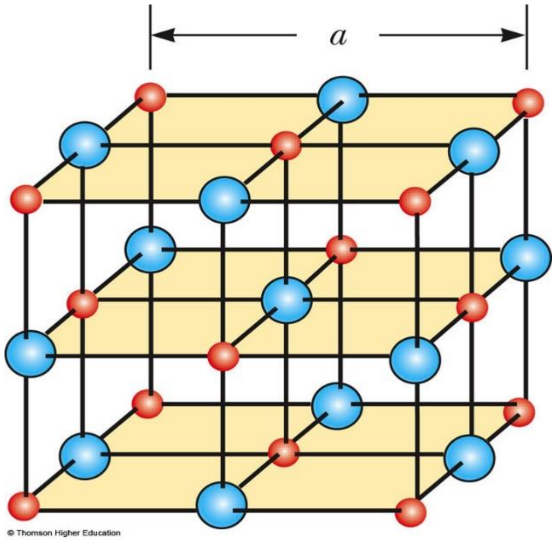


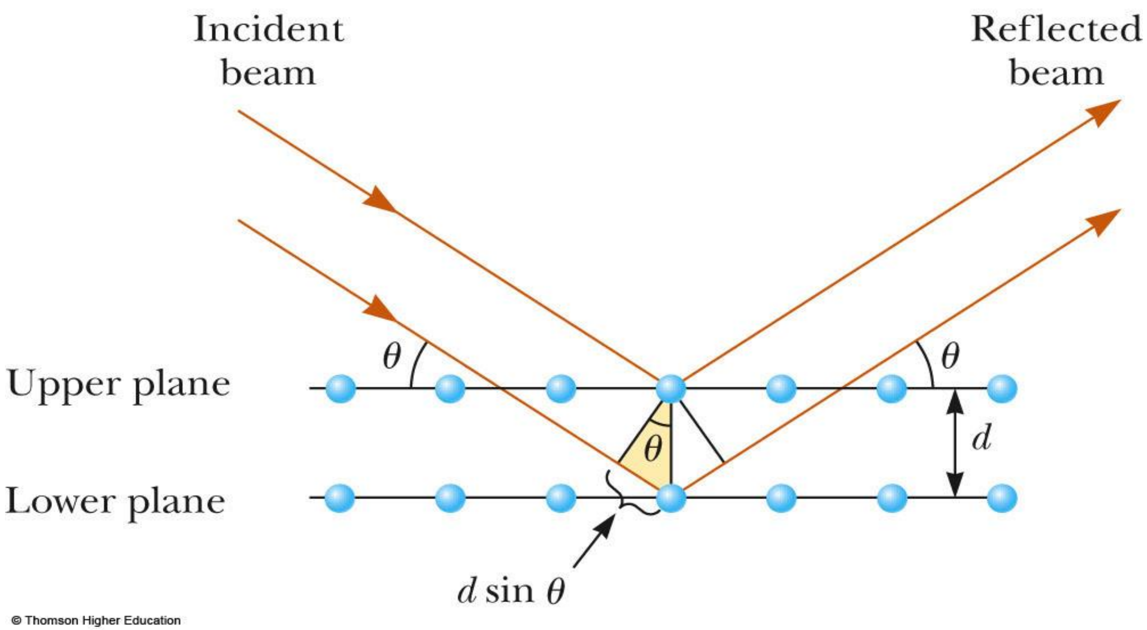
Diffraction of X-Rays by Crystals

- Crystal acts as 3D grating for X-rays.
- Condition for constructive interference (maxima in the reflected beam) is

$$2d \sin\theta = m\lambda \quad m = 1, 2, 3, \ldots$$

This condition is known as **Bragg's law**





The first-order diffraction maximum is observed at 12.6° for a crystal having a spacing between planes of atoms of 0.250 nm. (a) What wavelength x-ray is used to observe this first-order pattern? (b) How many orders can be observed for this crystal at this wavelength?

(a) Wavelength = 0.109 nm

(b) 4 orders

If the spacing between planes of atoms in a NaCl crystal is 0.281 nm, what is the predicted angle at which 0.140-nm x-rays are diffracted in a first-order maximum? Calculate the angle of incident of the x-ray beam.

$$\theta = 14.4°$$

Angle of incidence = 90^{o}-14.4^{o} = 75.6^{o}

Secondary maxima in multiples slits

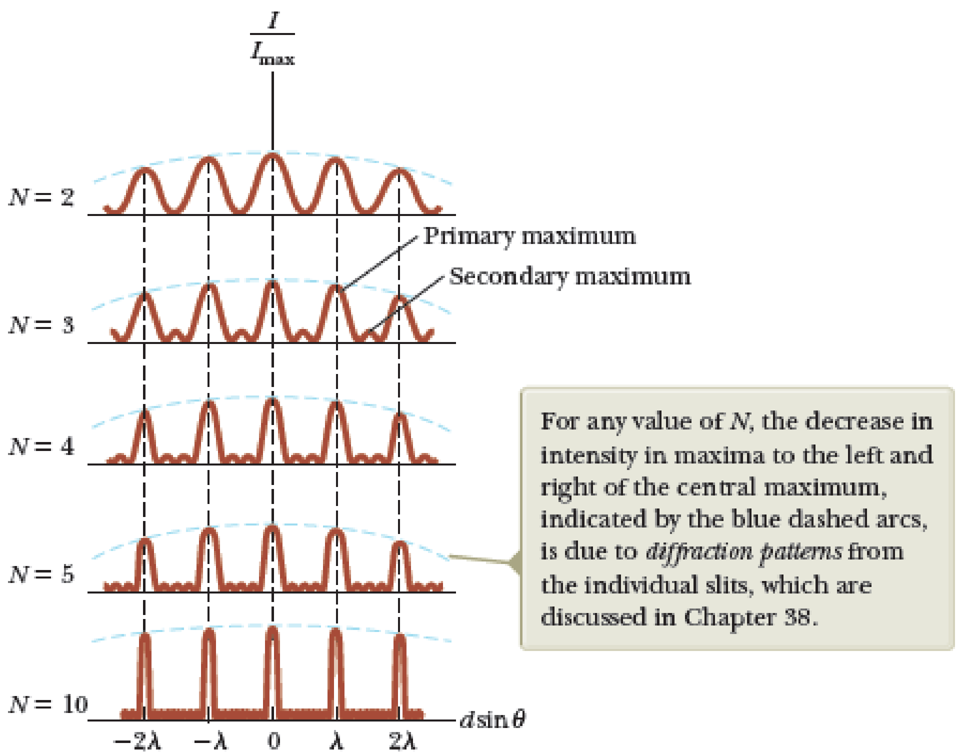
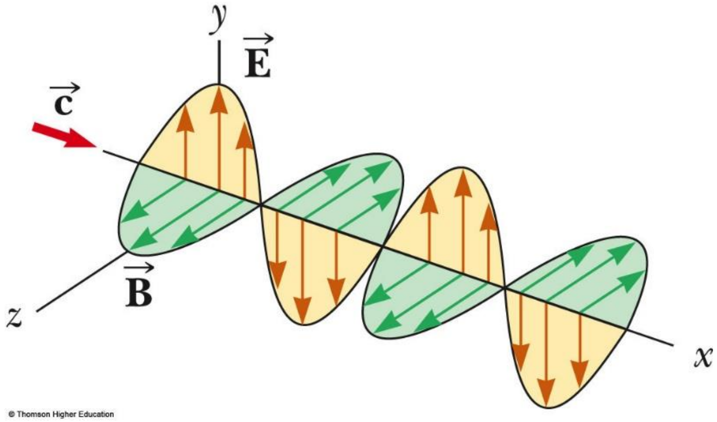


Figure 37.7 Multiple-slit interference patterns. As N, the number of slits, is increased, the primary maxima (the tallest peaks in each graph) become narrower but remain fixed in position and the number of secondary maxima increases.

Number of secondary maxima is always N-2, where N is the number of slits

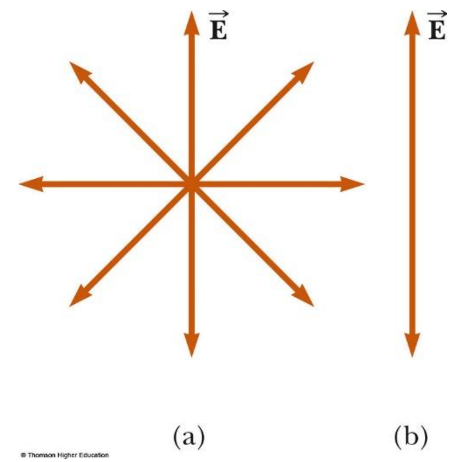
Polarization of Light Waves

Polarization of Light Waves

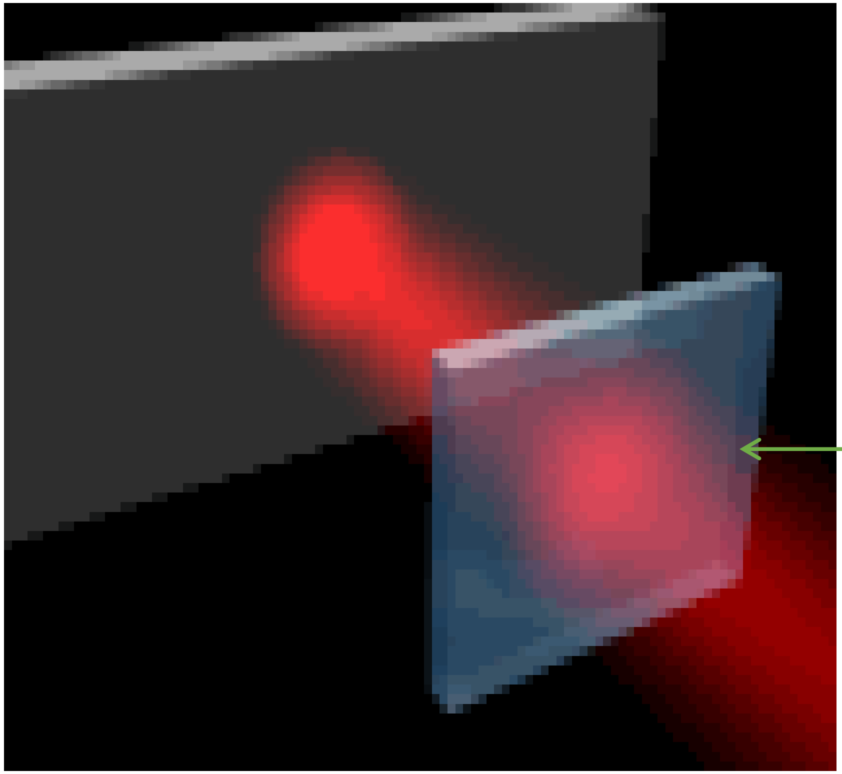


Schematic diagram of an electromagnetic wave propagating at velocity c in the x direction. The electric field vibrates in the xy plane, and the magnetic field vibrates in the xz plane.

Unpolarized light is an uniform mixture of linear polarizations at all possible angles



(a) A representation of an unpolarized light beam viewed along the direction of propagation. The transverse electric field can vibrate in any direction in the plane of the page with equal probability. (b) A linearly polarized light beam with the electric field vibrating in the vertical direction.



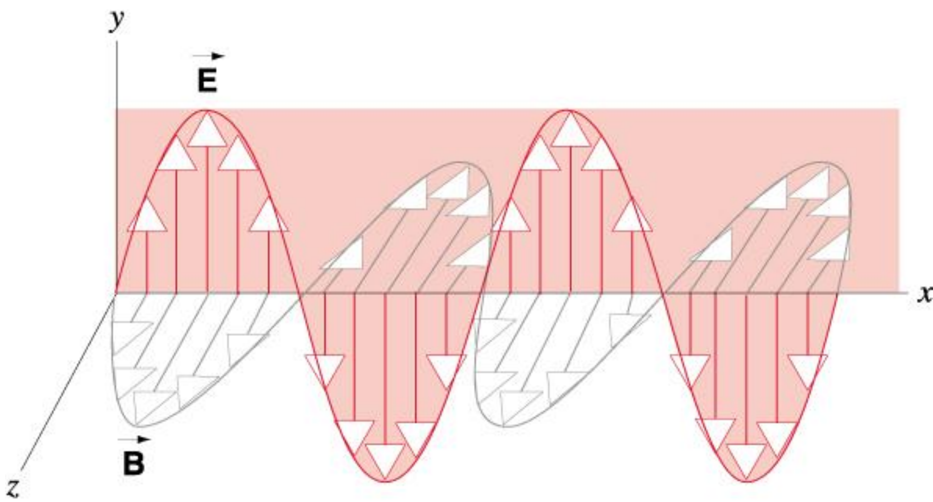
Polaroid (Polarizing material)

A polarizer is an optical filter that lets light waves of a specific polarization pass through while blocking light waves of other polarizations. It can filter a beam of light of undefined or mixed polarization into a beam of well-defined polarization, that is polarized light.

POLARIZATION OF ELECTROMAGNETIC WAVES

- By convention, we define the direction of polarization of the wave to be the direction of the electric vector.

- The plane determined by electric vector E and direction of propagation of wave is called plane of polarization (xy plane in figure)



Plane electromagnetic wave

A wave is said to be linearly polarized if the resultant electric field vibrates in the same direction at all times at a particular point

How do we polarize an upolarized light?

Polarization by Selective Absorption

Polarization by Reflection

Polarization by Double Refraction

Polarization by Scattering

Polarization by Selective Absorption

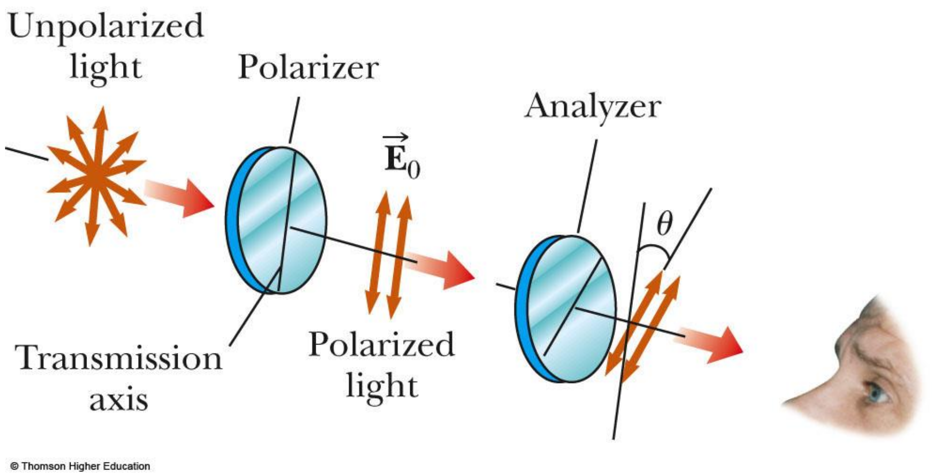
Polaroid is a device which polarizes unpolarized light when light passes through it.

Malus's law

The component of $\vec{E}_0$ parallel to the analyzer axis, which is transmitted through the analyzer, is $E_o \cos\theta$.

Intensity is square of magnitude of field vector. Hence,

$$I = I_{\max} \cos^2 \theta$$



*Two polarizing sheets whose transmission axes make an angle **θ** with each other. Only a fraction of the polarized light incident on the analyzer is transmitted through it.*

I_{max} is the intensity of the polarized beam incident on the analyzer.

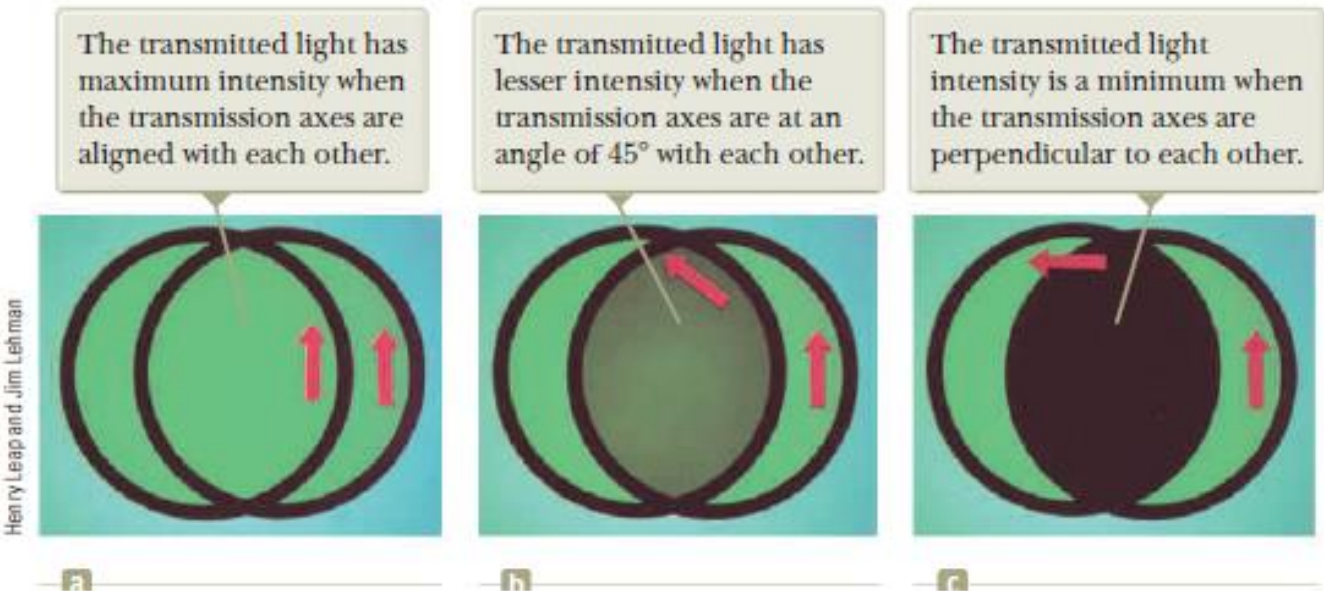
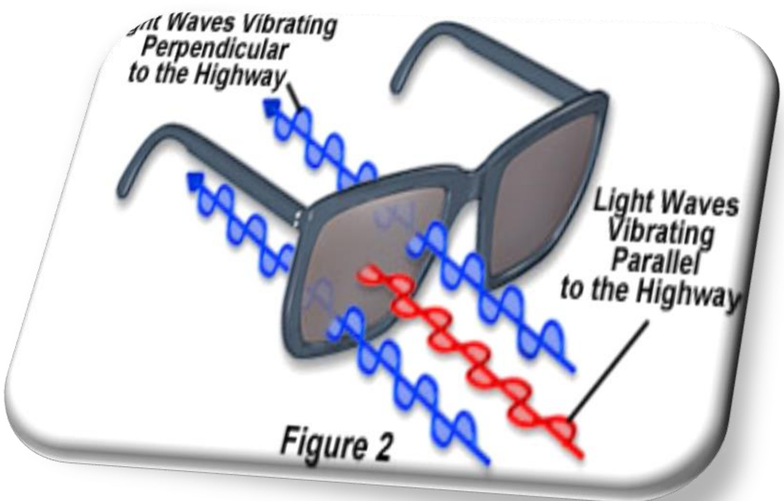


Figure 38.27 The intensity of light transmitted through two polarizers depends on the relative orientation of their transmission axes. The red arrows indicate the transmission axes of the polarizers.

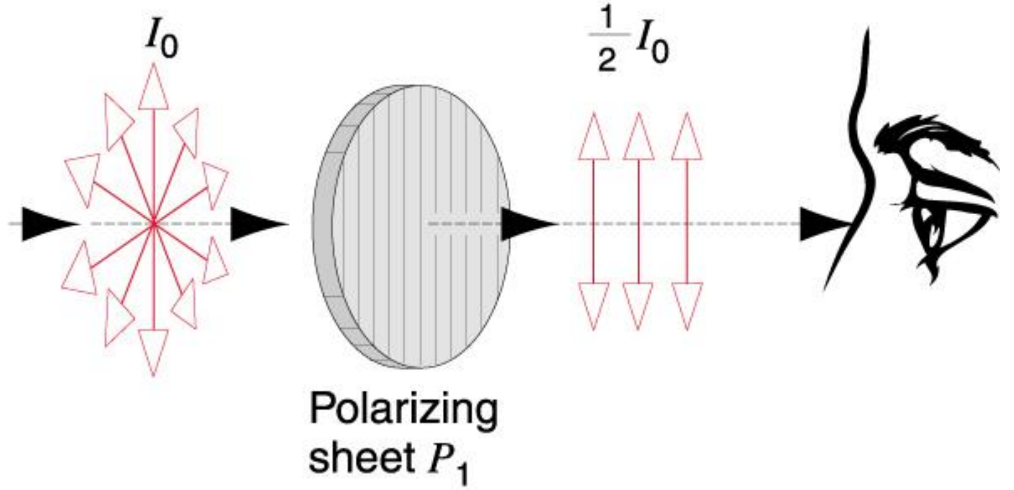
SUN GLASS



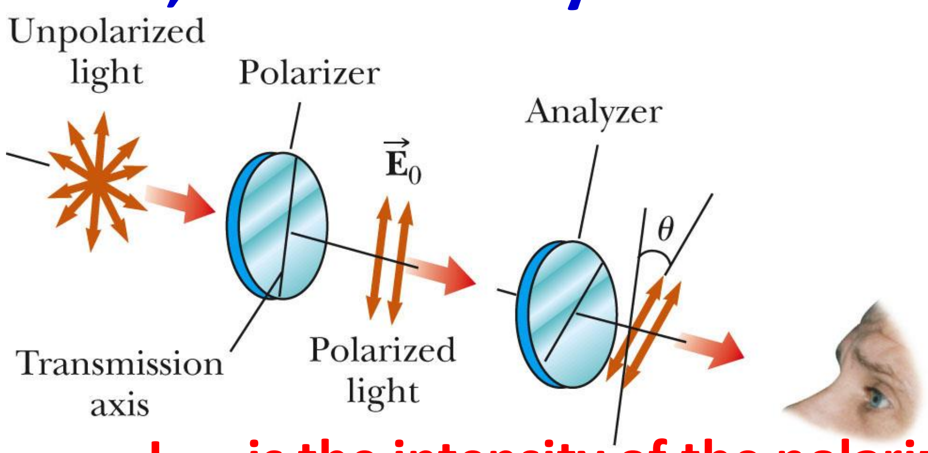
If you rotate a pair of polarizing sunglasses, you will find that they cut road glare much better in some positions than in others.

All reflections from objects above water are partially polarized. With polarized sunglasses the sea appears more transparent! See all the glare on the image to the left. The rocks below the surface are washed out. Rotate the polarizer to align its transmission axis vertical and . . . : the glare disappears!

If an unpolarized light passes through a polarizing sheet, the intensity reduces to half its original intensity



If a linearly polarized light passes through a polarizing sheet, the intensity will be



$$I = I_{max} \cos^2\theta$$

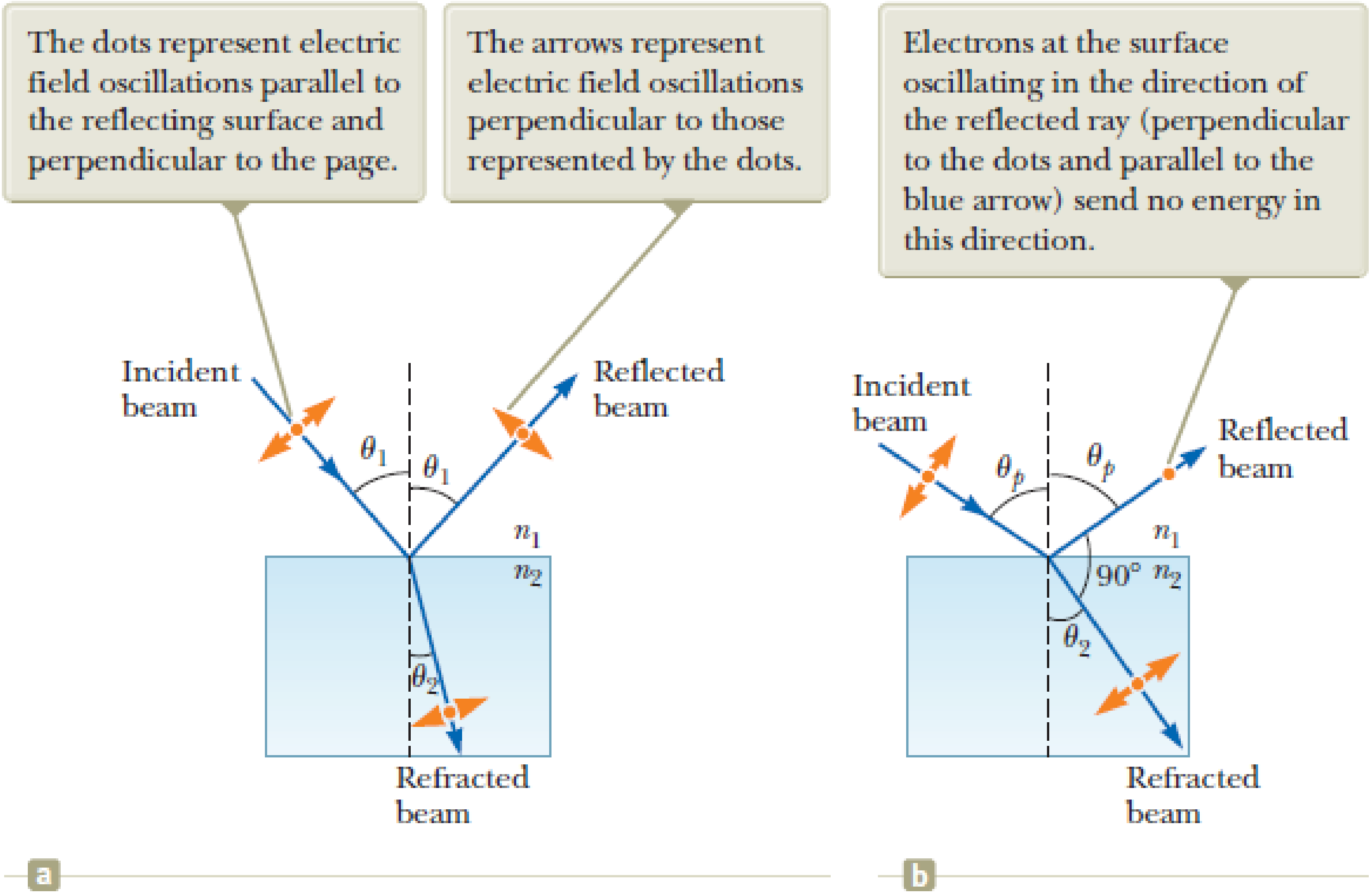
(Law of Malus)

I_{max} is the intensity of the polarized beam incident on the analyzer.

Plane-polarized light is incident on a single polarizing disk with the direction of E parallel to the direction of the transmission axis. Through what angle should the disk be rotated so that the intensity in the transmitted beam is reduced by a factor of (a) 3.00, (b) 5.00, and (c) 10.0?

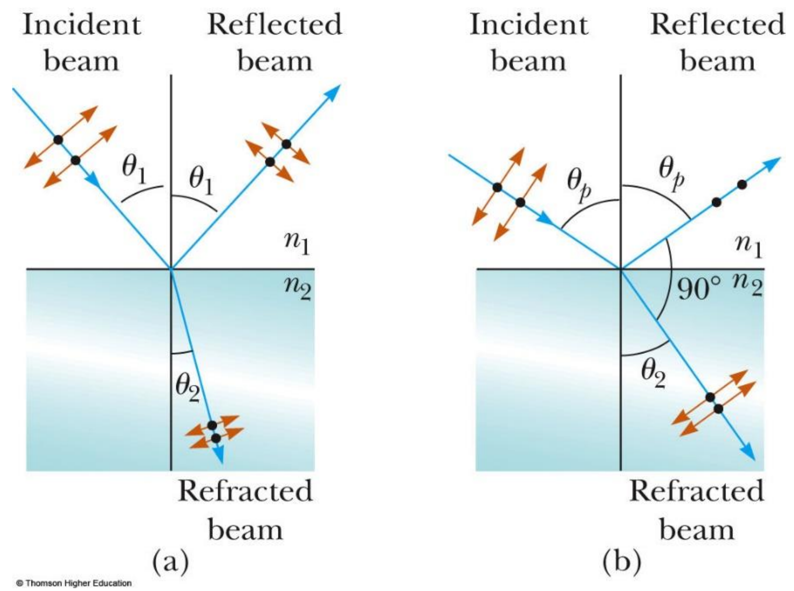
(a) 54.7^0 (b) 63.4^0 (c) 71.6^0

Polarization by Reflection



Polarization by Reflection

For one particular angle of incidence (θ_p), the reflected light is completely polarized **with its electric field vector parallel to the surface.**



Using Snell's law of refraction

$$\frac{n_2}{n_1} = \frac{\sin \theta_p}{\sin \theta_2}$$

But, $\theta_2 = 90 - \theta_p$. So, we can write,

$$\tan \theta_p = \frac{n_2}{n_1}$$

*This expression is called **Brewster's law**, and the polarizing angle $\boldsymbol{\theta_p}$ is sometimes called **Brewster's angle***

Polarization by Double Refraction

Polarization by Double Refraction

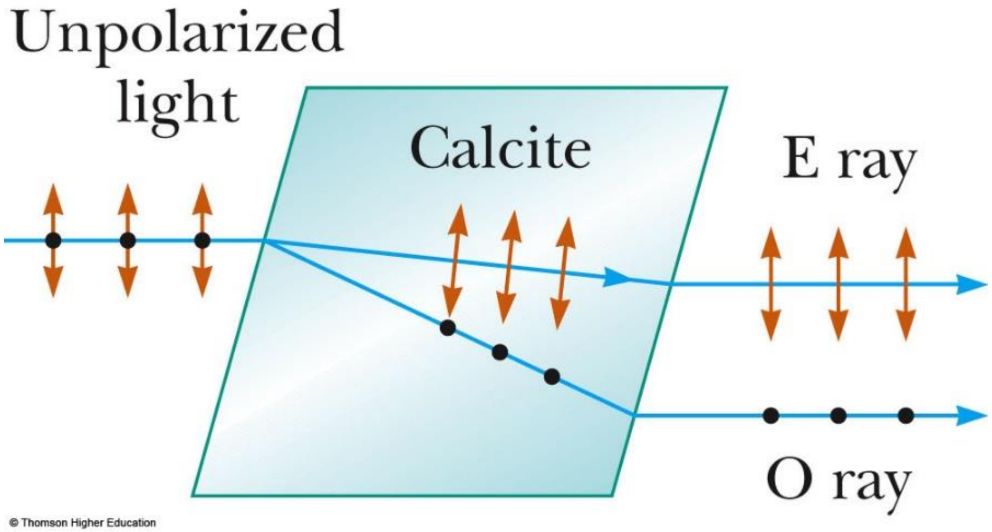


Figure 38.31 A calcite crystal produces a double image because it is a birefringent (double-refracting) material.

Unpolarized light incident at an angle to the optic axis in a calcite crystal splits into an ordinary (O) ray and an extraordinary (E) ray.

They have mutually perpendicular polarizations.

DOUBLE REFRACTION

Principal indices of refraction (n_o, n_e) of some doubly refracting crystals for sodium light

Crystal	Formula	n_o	n_e	n_e-n_o
Ice	H_2O	1.309	1.313	+0.004
Quartz	SiO_2	1.544	1.553	+0.009
Wurzite	ZnS	2.356	2.378	+0.022
Calcite	$CaCO_3$	1.658	1.486	-0.172

$n_e>n_o$

$n_e<n_o$

Polarization by Scattering

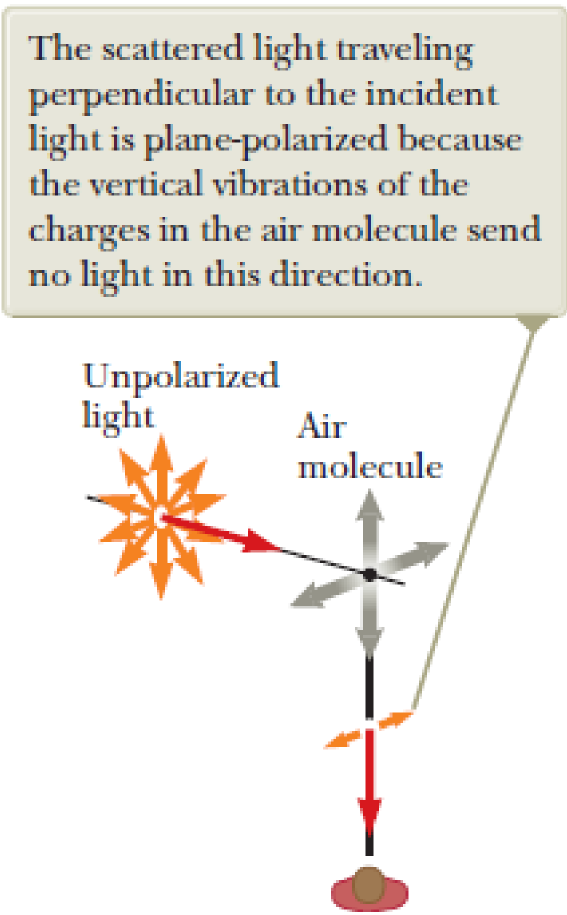
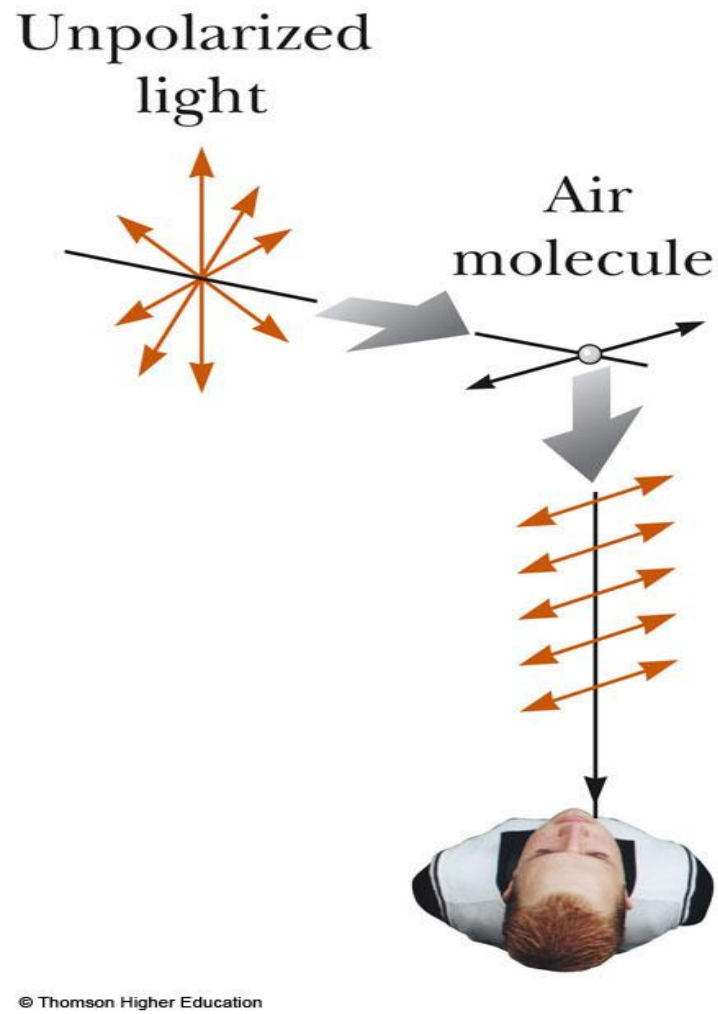


Figure 38.33 The scattering of unpolarized sunlight by air molecules.

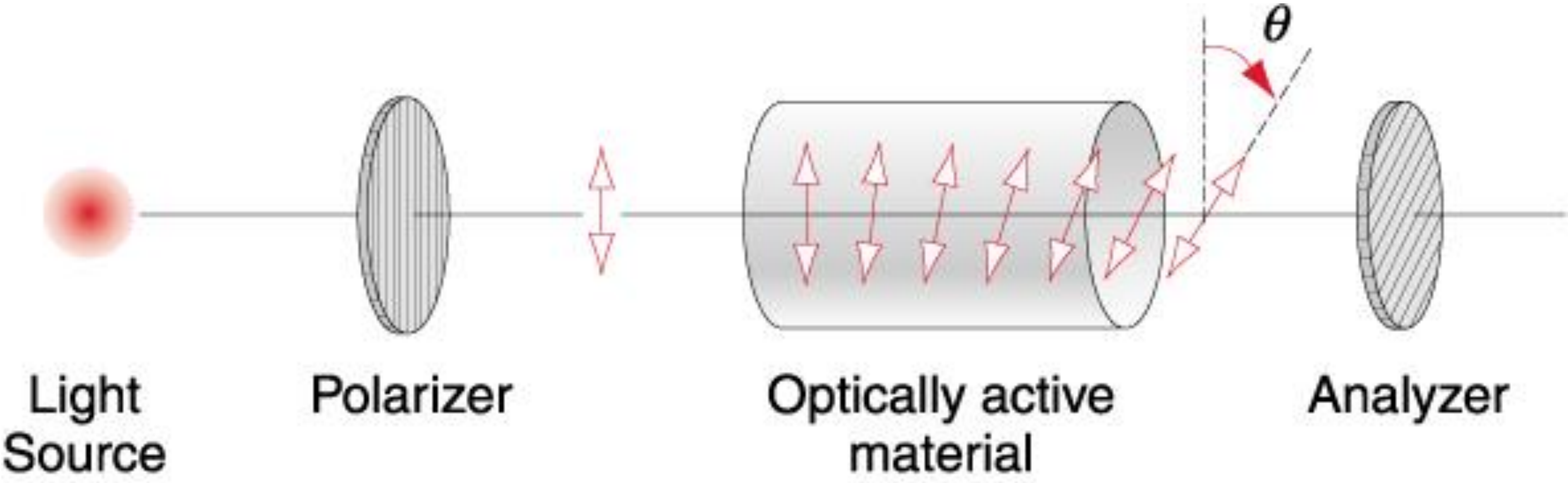


The scattering of unpolarized sunlight by air molecules.

Optical Activity

Optical Activity

- A material is said to be optically active if it rotates the plane of polarization of any light transmitted through the material.
- The angle through which the light is rotated by a specific material depends on the length of the path through the material and on concentration if the material is in solution.



Unpolarized light passes through two ideal Polaroid sheets. The axis of the first is vertical, and the axis of the second is at 30.0° to the vertical. What fraction of the incident light is transmitted?

$$I/I_{max} = 0.375$$

1. Explain the term diffraction of light.
2. Discuss qualitatively, the Fraunhofer diffraction at a single-slit.
3. Draw a schematic plot of the intensity of light in single slit diffraction against phase difference.
4. Explain briefly diffraction at a circular aperture.
5. State and explain Rayleigh's criterion for optical resolution.
6. Effect of diffraction is ignored in the case of Young's double slit interference. Give reason.
7. Discuss qualitatively, the diffraction due to multiple slits.
8. What is diffraction grating? Write the grating equation.
9. Briefly explain x-ray diffraction and Bragg's law.
10. Distinguish between unpolarized and linearly polarized light.
11. Explain Malus's law.
12. How to produce linearly polarized light by (a) selective absorption, (b) reflection, (c) double refraction, (d) scattering ? Explain.