

t-distribution

Let W denote a random variable that is $N(0, 1)$; let V denote a random variable that is $\chi^2(r)$; and let W and V be independent. Then the joint pdf of W and V , say $h(w, v)$, is the product of the pdf of W and that of V or

$$h(w, v) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-w^2/2} \frac{1}{\Gamma(r/2)2^{r/2}} v^{r/2-1} e^{-v/2} & -\infty < w < \infty, \quad 0 < v < \infty \\ 0 & \text{elsewhere.} \end{cases}$$

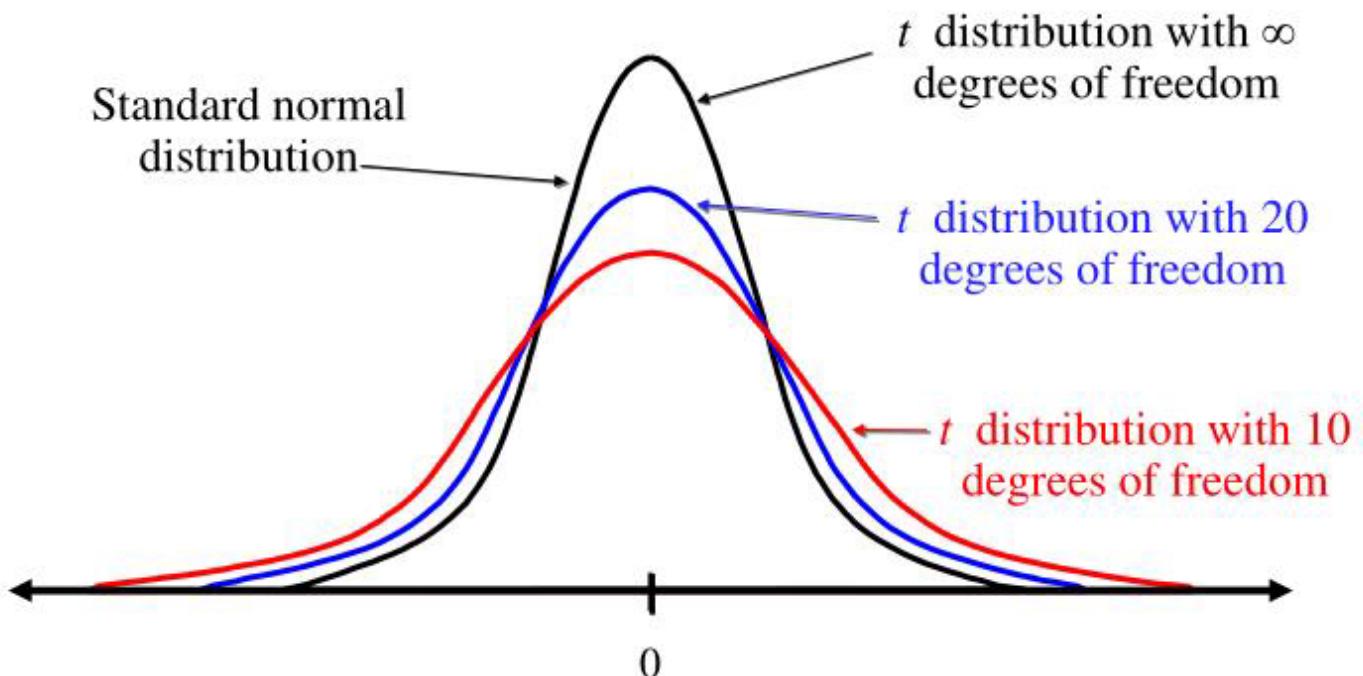
Define a new random variable T by writing

$$T = \frac{W}{\sqrt{V/r}}.$$

The distribution followed by random variable T is called the t-distribution (see Table of t-distribution and for its pdf). The probability density function is symmetric, and its overall shape resembles the bell shape of a normally distributed variable with mean 0 and variance 1, except that it is a bit lower and wider. As the number of degrees of freedom grows, the t-distribution approaches the normal distribution with mean 0 and variance 1.

t Distribution

The t-distribution is used when n is **small** and σ is **unknown**.



However in practice, a t -distribution is created by the expression $\frac{\bar{x} - \mu}{S / \sqrt{n}}$, where \bar{x} and S are the sample mean and sample deviation from all possible random samples of size n taken from a normally distributed measurement. This usage was first discovered by W.S. Gosset while employing statistical methods as a brewer at Guinness Brewery in Dublin, Ireland. After studying under Karl Pearson in London in 1907, Gosset published two papers in 1908, under the pseudonym "Student," that introduced the "Student t -distribution." In 1912, Ronald Fisher graduated from Cambridge University. He spent the next 13 years developing the mathematical theory and further statistical applications of Student's t -distribution.

F- distribution

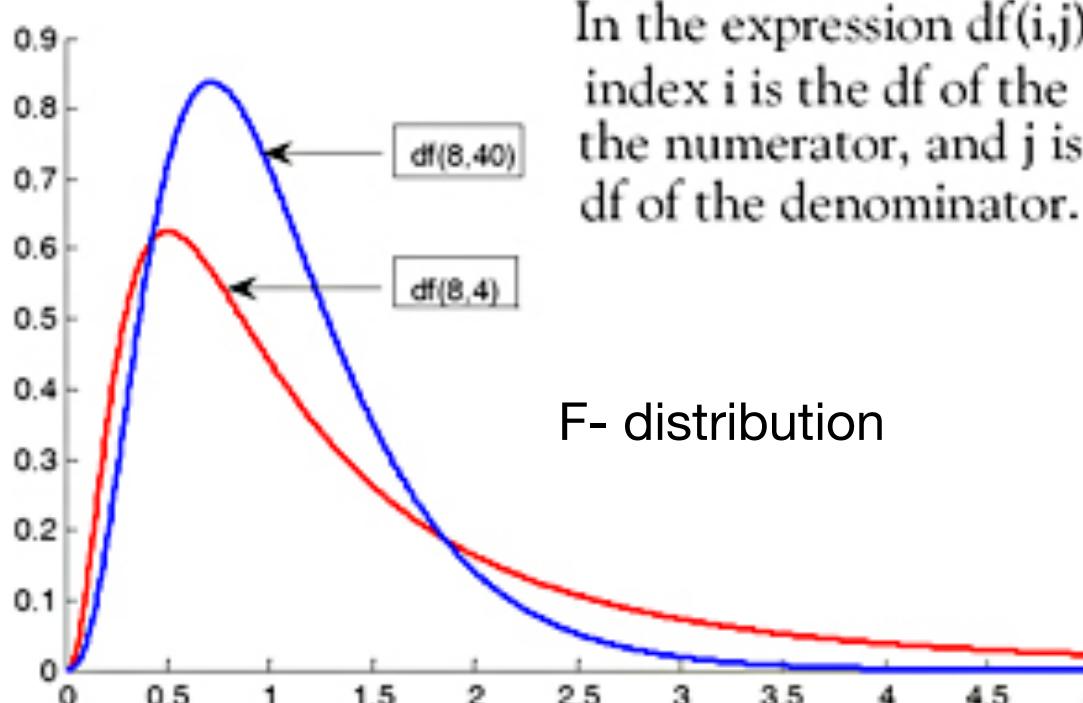
Next consider two independent chi-square random variables U and V having r_1 and r_2 degrees of freedom, respectively. The joint pdf $h(u, v)$ of U and V is then

$$h(u, v) = \begin{cases} \frac{1}{\Gamma(r_1/2)\Gamma(r_2/2)2^{(r_1+r_2)/2}} u^{r_1/2-1} v^{r_2/2-1} e^{-(u+v)/2} & 0 < u, v < \infty \\ 0 & \text{elsewhere.} \end{cases}$$

We define the new random variable

$$W = \frac{U/r_1}{V/r_2}$$

The distribution followed by random variable W is called the F-distribution (see Table of F-distribution for its pdf)



Tables

Chi-square Distribution

The following table presents selected quantiles of chi-square distribution; i.e., the values x such that

$$P(X \leq x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2} dw,$$

for selected degrees of freedom r .

r	$P(X \leq x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892

Normal Distribution

The following table presents the standard normal distribution. The probabilities tabled are

$$P(X \leq x) = \Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dw.$$

Note that only the probabilities for $x \geq 0$ are tabled. To obtain the probabilities for $x < 0$, use the identity $\Phi(-x) = 1 - \Phi(x)$.

t-Distribution

The following table presents selected quantiles of the *t*-distribution; i.e., the values x such that

$$P(X \leq x) = \int_{-\infty}^x \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r} \Gamma(r/2)(1+w^2/r)^{(r+1)/2}} dw$$

for selected degrees of freedom r . The last row gives the standard normal quantiles.

r	$P(X \leq x)$					
	0.900	0.950	0.975	0.990	0.995	0.999
1	3.078	6.314	12.706	31.821	63.657	318.309
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
∞	1.282	1.645	1.960	2.326	2.576	3.090

F-Distribution

The following table presents selected quantiles of the F -distribution; i.e, the values x such that

$$P(X \leq x) = \int_0^x \frac{\Gamma[(r_1 + r_2)/2](r_1/r_2)^{r_1/2} w^{r_1/2-1}}{\Gamma(r_1/2)\Gamma(r_2/2)(1 + r_1 w/r_2)^{(r_1+r_2)/2}} dw,$$

for selected numerator and denominator degrees of freedom r_1 and r_2 , respectively.

$P(X \leq x)$	r_2	r_1							
		1	2	3	4	5	6	7	8
0.950	1	161.450	199.500	215.710	224.580	230.160	233.990	236.770	238.880
0.975	1	647.790	799.500	864.160	899.580	921.850	937.110	948.220	956.660
0.990	1	4052.180	4999.500	5403.350	5624.580	5763.650	5858.990	5928.360	5981.070
0.950	2	18.510	19.000	19.160	19.250	19.300	19.330	19.350	19.370
0.975	2	38.510	39.000	39.170	39.250	39.300	39.330	39.360	39.370
0.990	2	98.500	99.000	99.170	99.250	99.300	99.330	99.360	99.370
0.950	3	10.130	9.550	9.280	9.120	9.010	8.940	8.890	8.850
0.975	3	17.440	16.040	15.440	15.100	14.880	14.730	14.620	14.540
0.990	3	34.120	30.820	29.460	28.710	28.240	27.910	27.670	27.490
0.950	4	7.710	6.940	6.590	6.390	6.260	6.160	6.090	6.040
0.975	4	12.220	10.650	9.980	9.600	9.360	9.200	9.070	8.980
0.990	4	21.200	18.000	16.690	15.980	15.520	15.210	14.980	14.800
0.950	5	6.610	5.790	5.410	5.190	5.050	4.950	4.880	4.820
0.975	5	10.010	8.430	7.760	7.390	7.150	6.980	6.850	6.760
0.990	5	16.260	13.270	12.060	11.390	10.970	10.670	10.460	10.290
0.950	6	5.990	5.140	4.760	4.530	4.390	4.280	4.210	4.150
0.975	6	8.810	7.260	6.600	6.230	5.990	5.820	5.700	5.600
0.990	6	13.750	10.920	9.780	9.150	8.750	8.470	8.260	8.100
0.950	7	5.590	4.740	4.350	4.120	3.970	3.870	3.790	3.730
0.975	7	8.070	6.540	5.890	5.520	5.290	5.120	4.990	4.900
0.990	7	12.250	9.550	8.450	7.850	7.460	7.190	6.990	6.840
0.950	8	5.320	4.460	4.070	3.840	3.690	3.580	3.500	3.440
0.975	8	7.570	6.060	5.420	5.050	4.820	4.650	4.530	4.430
0.990	8	11.260	8.650	7.590	7.010	6.630	6.370	6.180	6.030
0.950	9	5.120	4.260	3.860	3.630	3.480	3.370	3.290	3.230
0.975	9	7.210	5.710	5.080	4.720	4.480	4.320	4.200	4.100
0.990	9	10.560	8.020	6.990	6.420	6.060	5.800	5.610	5.470
0.950	10	4.960	4.100	3.710	3.480	3.330	3.220	3.140	3.070
0.975	10	6.940	5.460	4.830	4.470	4.240	4.070	3.950	3.850
0.990	10	10.040	7.560	6.550	5.990	5.640	5.390	5.200	5.060
0.950	11	4.840	3.980	3.590	3.360	3.200	3.090	3.010	2.950
0.975	11	6.720	5.260	4.630	4.280	4.040	3.880	3.760	3.660
0.990	11	9.650	7.210	6.220	5.670	5.320	5.070	4.890	4.740
0.950	12	4.750	3.890	3.490	3.260	3.110	3.000	2.910	2.850
0.975	12	6.550	5.100	4.470	4.120	3.890	3.730	3.610	3.510
0.990	12	9.330	6.930	5.950	5.410	5.060	4.820	4.640	4.500
0.950	13	4.670	3.810	3.410	3.180	3.030	2.920	2.830	2.770
0.975	13	6.410	4.970	4.350	4.000	3.770	3.600	3.480	3.390
0.990	13	9.070	6.700	5.740	5.210	4.860	4.620	4.440	4.300
0.950	14	4.600	3.740	3.340	3.110	2.960	2.850	2.760	2.700
0.975	14	6.300	4.860	4.240	3.890	3.660	3.500	3.380	3.290
0.990	14	8.860	6.510	5.560	5.040	4.690	4.460	4.280	4.140
0.950	15	4.540	3.680	3.290	3.060	2.900	2.790	2.710	2.640
0.975	15	6.200	4.770	4.150	3.800	3.580	3.410	3.290	3.200
0.990	15	8.680	6.360	5.420	4.890	4.560	4.320	4.140	4.000
0.950	16	4.490	3.630	3.240	3.010	2.850	2.740	2.660	2.590
0.975	16	6.120	4.690	4.080	3.730	3.500	3.340	3.220	3.120
0.990	16	8.530	6.230	5.290	4.770	4.440	4.200	4.030	3.890

F-Distribution

$P(X \leq x)$	r_2	r_1							
		9	10	11	12	13	14	15	16
0.950	1	240.540	241.880	242.980	243.910	244.690	245.360	245.950	246.460
0.975	1	963.280	968.630	973.030	976.710	979.840	982.530	984.870	986.920
0.990	1	6022.470	6055.850	6083.320	6106.320	6125.860	6142.670	6157.280	6170.100
0.950	2	19.380	19.400	19.400	19.410	19.420	19.420	19.430	19.430
0.975	2	39.390	39.400	39.410	39.410	39.420	39.430	39.430	39.440
0.990	2	99.390	99.400	99.410	99.420	99.420	99.430	99.430	99.440
0.950	3	8.810	8.790	8.760	8.740	8.730	8.710	8.700	8.690
0.975	3	14.470	14.420	14.370	14.340	14.300	14.280	14.250	14.230
0.990	3	27.350	27.230	27.130	27.050	26.980	26.920	26.870	26.830
0.950	4	6.000	5.960	5.940	5.910	5.890	5.870	5.860	5.840
0.975	4	8.900	8.840	8.790	8.750	8.710	8.680	8.660	8.630
0.990	4	14.660	14.550	14.450	14.370	14.310	14.250	14.200	14.150
0.950	5	4.770	4.740	4.700	4.680	4.660	4.640	4.620	4.600
0.975	5	6.680	6.620	6.570	6.520	6.490	6.460	6.430	6.400
0.990	5	10.160	10.050	9.960	9.890	9.820	9.770	9.720	9.680
0.950	6	4.100	4.060	4.030	4.000	3.980	3.960	3.940	3.920
0.975	6	5.520	5.460	5.410	5.370	5.330	5.300	5.270	5.240
0.990	6	7.980	7.870	7.790	7.720	7.660	7.600	7.560	7.520
0.950	7	3.680	3.640	3.600	3.570	3.550	3.530	3.510	3.490
0.975	7	4.820	4.760	4.710	4.670	4.630	4.600	4.570	4.540
0.990	7	6.720	6.620	6.540	6.470	6.410	6.360	6.310	6.280
0.950	8	3.390	3.350	3.310	3.280	3.260	3.240	3.220	3.200
0.975	8	4.360	4.300	4.240	4.200	4.160	4.130	4.100	4.080
0.990	8	5.910	5.810	5.730	5.670	5.610	5.560	5.520	5.480
0.950	9	3.180	3.140	3.100	3.070	3.050	3.030	3.010	2.990
0.975	9	4.030	3.960	3.910	3.870	3.830	3.800	3.770	3.740
0.990	9	5.350	5.260	5.180	5.110	5.050	5.010	4.960	4.920
0.950	10	3.020	2.980	2.940	2.910	2.890	2.860	2.850	2.830
0.975	10	3.780	3.720	3.660	3.620	3.580	3.550	3.520	3.500
0.990	10	4.940	4.850	4.770	4.710	4.650	4.600	4.560	4.520
0.950	11	2.900	2.850	2.820	2.790	2.760	2.740	2.720	2.700
0.975	11	3.590	3.530	3.470	3.430	3.390	3.360	3.330	3.300
0.990	11	4.630	4.540	4.460	4.400	4.340	4.290	4.250	4.210
0.950	12	2.800	2.750	2.720	2.690	2.660	2.640	2.620	2.600
0.975	12	3.440	3.370	3.320	3.280	3.240	3.210	3.180	3.150
0.990	12	4.390	4.300	4.220	4.160	4.100	4.050	4.010	3.970
0.950	13	2.710	2.670	2.630	2.600	2.580	2.550	2.530	2.510
0.975	13	3.310	3.250	3.200	3.150	3.120	3.080	3.050	3.030
0.990	13	4.190	4.100	4.020	3.960	3.910	3.860	3.820	3.780
0.950	14	2.650	2.600	2.570	2.530	2.510	2.480	2.460	2.440
0.975	14	3.210	3.150	3.090	3.050	3.010	2.980	2.950	2.920
0.990	14	4.030	3.940	3.860	3.800	3.750	3.700	3.660	3.620
0.950	15	2.590	2.540	2.510	2.480	2.450	2.420	2.400	2.380
0.975	15	3.120	3.060	3.010	2.960	2.920	2.890	2.860	2.840
0.990	15	3.890	3.800	3.730	3.670	3.610	3.560	3.520	3.490
0.950	16	2.540	2.490	2.460	2.420	2.400	2.370	2.350	2.330
0.975	16	3.050	2.990	2.930	2.890	2.850	2.820	2.790	2.760
0.990	16	3.780	3.690	3.620	3.550	3.500	3.450	3.410	3.370