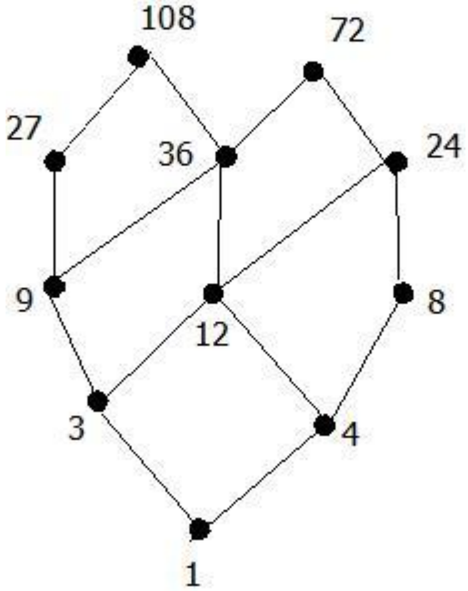
Scheme Set 3 (IN Sem Exam Mathematics CCE/ICT/CSE)

1. Hasse diagram:



2M

Length of the longest chain is 5. 1M

2. Let a_i be the property that $(i, i+1)$ occurs consecutively, $1 \leq i \leq n-1$.
Total number of permutations: $n!$
We have $N(a_i) = (n-1)!$
$N(a_i a_j) = (n-2)!$
$N(a_i a_j a_k) = (n-3)!$
…..
$N(a_1 a_2 \ldots . a_{n-1}) = 1$ 1M
Using the principle of inclusion and exclusion
$N(a_1' a_2' \ldots . a_{n-1}') = n! - \binom{n-1}{1}(n-1)! + \binom{n-1}{2}(n-2)! + \cdots + (-1)^n \binom{n-1}{n-1}$ 1M
$= (n-1)! \left\{ \left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \cdots\right) + n\left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \cdots\right) \right\}$
$= (n-1)! \left\{ \frac{1}{e} + \frac{n}{e} \right\}$
$= \frac{(n-1)!(n+1)}{e}$
Thus, the proportion of the permutation is $\frac{\frac{(n-1)!(n+1)}{e}}{n!} = \frac{n+1}{ne}$ 1M

3. Consider the Ferrers diagram of a partition of n into even parts. Every row in this diagram has an even number of dots. Therefore, in the conjugate diagram, every column has an even number of dots. Observe that the size of the last column, say t_1, is the number of occurrences of the largest part. As t_1 is even, the largest part occurs an even number of times. Now remove all rows corresponding to the largest part, and let t_2 be the size of the last column in the resulting diagram. Then t_2 is even, since t_1 as well as $t_1 + t_2$ are even. But t_2 is the number of occurrences of the second-largest part. Proceeding similarly, we find that all parts occur an even number of times, in the conjugate partition. Since conjugation is a bijection, we get the required result. 3M

4.

x_1	x_2	x_3	$E(x_1, x_2, x_3)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

CNF: $(x_1 \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3}) \wedge (x_1 \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3})$

DNF: $(x_1 \wedge \overline{x_2} \wedge \overline{x_3}) \vee (x_1 \wedge x_2 \wedge \overline{x_3}) \vee (\overline{x_1} \wedge \overline{x_2} \wedge \overline{x_3})$ 4M

5. Lexicographical: 68th : 35142
108th : 52431 2M
Fike's: 68th : seq; 0222 ,
permutation is 21534
108th : seq ; 0022 ,

permutation is 31524 2M