An r-combination of n elements: is a selection of r of the objects where the order of the objects in the selection is immaterial.

An r- permutation of n elements is an ordered selection of r of the objects.

1.Permutation with no repetition: The number of r-permutations of n objects, $^nP_r = \frac{n!}{(n-r)!}$.

2.Permutation with unlimited repetition: The no of r– permutations with unlimited repetition n^r.

3.Permutation with restricted repetition: If there n objects of which m1 are of the first kind, m2 are of the second kind, ..., mk are of the kth kind. The number of permutations of all the objects in this case is $\frac{n!}{m1!\,m2!\cdots mk!}$

4.Combinations without repetition: The number of r-combinations of n objects without repetition is $^nC_r = \frac{n!}{(n-r)!\,r!}$

Distributing r different objects to n distinct cells:

Such that each cell has at most one object: ^nP_r

If we allow each cell to hold any number of objects: n^r

Distributing r identical objects to n distinct cells:

Such that each cell has at most one object: ^nC_r

How many combination of 1, 2, 3, 4 are possible if we select 3 at a time with repetition?

123	111	112	221	331	441
124	222	113	223	332	442
134	333	114	224	334	443
234	444				

$\underline{1\,2\,\cdots\,6}$: $^6C_3$

123

124 ⋯

$\underline{\underline{20}}$

$\boxed{1, 2, \cdots 6 \text{ select } 3 \text{ without repetition}}$

$c_1c_2c_3$

444

(456)

$c_1\,c_2\,c_3$		$d_1d_2d_3$
224	$\longrightarrow$	236
334	$\longrightarrow$	346
114	$\longrightarrow$	126

Consider a combination of $\underline{n}$ objects in which we select $\underline{r}$ of them $\underline{\text{with}}$ repetition.

Let the n objects be $1, 2, 3, \ldots, n$

Let $c_1\ c_2 \ldots c_r$ be any one combination of the n objects with repetition s.t

$$c_1 \le c_2 \le \cdots \le c_r$$

Define $d_1\ d_2 \ldots d_r$ as follows.

$$d_1 = c_1 + 0, \quad d_2 = c_2 + 1 \quad \ldots \quad d_r = c_r + (r-1)$$

Observe that whatever be $c_1\ c_2 \ldots c_r$, the $d_1\ d_2 \ldots d_r$ are all distinct

Every distinct r- combination of $c_1\ c_2 \ldots c_r$ produces a distinct set of $d_1\ d_2 \ldots d_r$

The combination of $(n+r-1)$ objects in which we are selecting r of them $\underline{\text{without}}$ repetition is ${}^{n+r-1}C_r$

Note : Dist^n of r- identical objects into n distinct cells s.t each cell can hold any number of objects is ${}^{n+r-1}C_r$

Eg: $\underline{4}$ diff flavors of icecream. If we want 6 icecreams if they are not necessarily of different flavors.

$n = 4, \ r = 6, \quad {}^{4+6-1}C_6 = 84$

1st
2nd

n : 1 2 3 4

Q1. If 5 men A,B,C,D,E intend to speak at a meeting, (i) in how many orders can they do so without B speaking before A?
(ii) how many orders are there in which A speaks immediately before B?

(i) $\underline{A}\ \underbrace{\_\ \_\ \_\ \_}_{4!}$

$\_\ \underline{A}\ \underbrace{\_\ \_\ \_}$

C, D, E: 3 B + 2 rem: $3!$

$\_\ \_\ \underline{A}\ \underbrace{\_\ \_}$

C, D, E: 3×2 B one rem: $2!$

$\_\ \_\ \_\ \underline{A}\ \underline{B}$

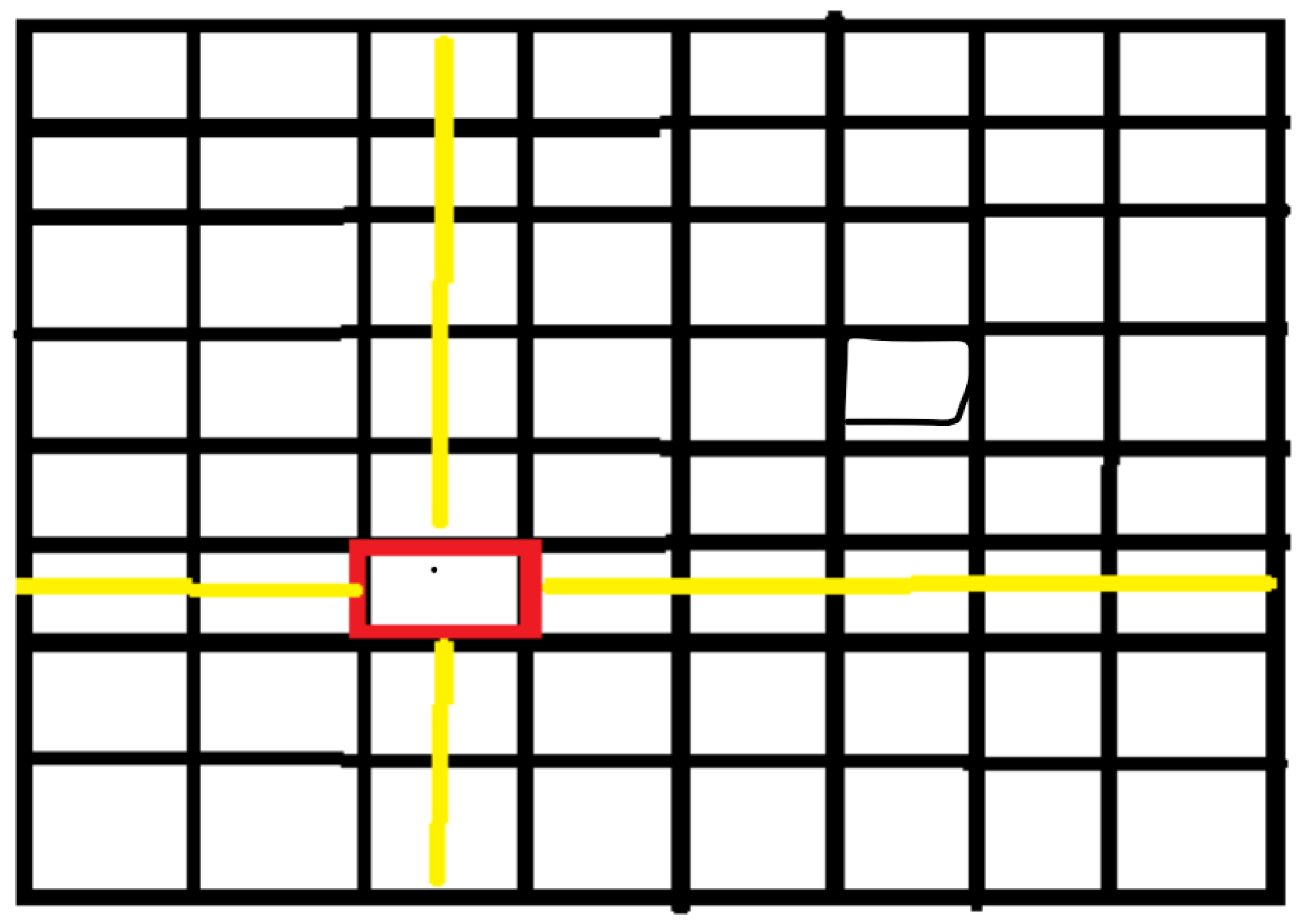
$\_\ \_\ \_\ \_\ \underline{A}$ → 0

$$4! + 3\times 3! + 3\times 2\times 2! + 3!\times 1 + 0 = 60$$

(ii) Consider AB as one unit $(AB)\ \_\ \_\ \_$

So $4!$ ways.

Q2. In how many ways can 2 squares be selected one by one from 8 × 8 chess board such that they are not in the same row and same columns?



Ans: $64 \times (7 \times 7)$

Q3. Three identical dice are rolled. How many different outcomes can be recorded?

1 2 . . 6

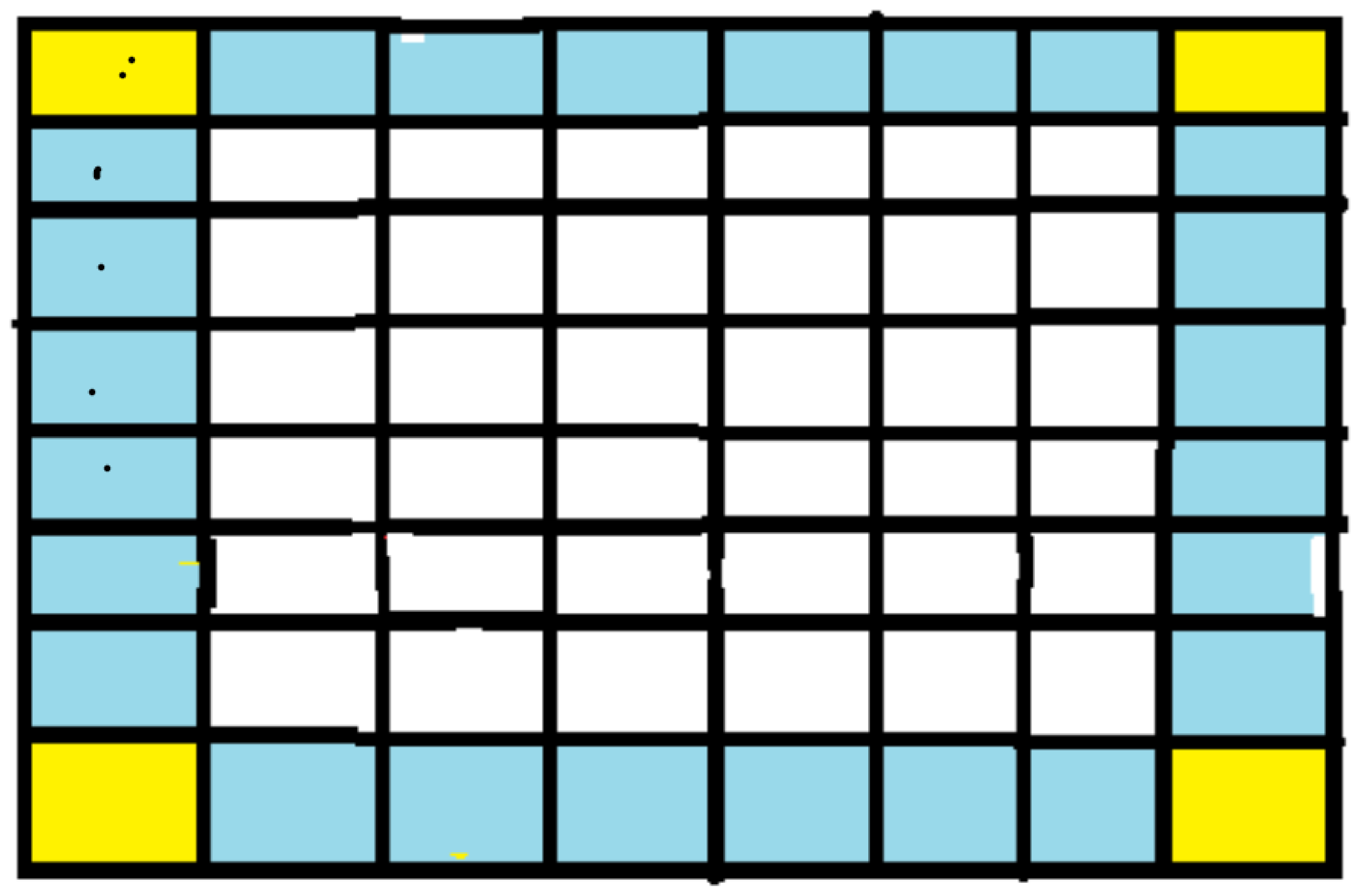
$n = 6$
$r = 3$

_ _ _

selecting 3 out of 1, 2, 3, 4, 5, 6 with repetition.

$^{6+3-1}C_3 = {}^8C_3 = 56$

Q4. In how many ways can two adjacent squares can be selected from an 8 × 8 chess board?



Yellow Blue

$$4 \times 2 + 24 \times 3 + 36 \times 4 = 224$$

Q5. Among all 7 digits numbers, how many of them contain exactly three 9s?

Begining with 9 or Begining with 1, 2, ... 8

Begin with 9 **(6C2)*9*9*9*9**

Begin with 1,2,...,8 **8*(6C3)*9*9*9**

$$6C_2 \cdot 9^4 + 8 \cdot 6C_3\, 9^3 = 215055$$

Q6. A bit is either 0 or 1, a byte is a sequence of 8 bits. (i) Find the number of bytes that can be formed.
Also find number of bytes (ii) that begin with 1,1 and end with 1,1.
(iii) begin with 1,1 and do not end with 1,1.
(iv) begin with 1,1 or end with 1,1.

(i) $\underset{0,1}{2} \times \underset{0,1}{2} \times \ldots \underset{0,1}{2}$: 2^8

(ii) $1\ 1\ \underset{0,1}{\_}\ \underset{0,1}{\_}\ \underset{0,1}{\_}\ \underset{0,1}{\_}\ 1\ 1$: 2^4

(iii) $1\ 1\ \_\ \_\ \_\ \_\ \_\ \_$
$-(1\ 1\ \_\ \_\ \_\ \_\ 1\ 1)$
$= 2^6 - 2^4 = 48$ ✓

(iv) $1\ 1\ \_\ \_\ \_\ \_\ \_\ \_$
$+(\_\ \_\ \_\ \_\ \_\ \_\ 1\ 1)$
$-(1\ 1\ \_\ \_\ \_\ \_\ 1\ 1)$
$2^6 + 2^6 - 2^4 = 112$

OR

$\left.\begin{matrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{matrix}\right\}$ last 2

$1\ 1\ \underset{0,1}{\_}\ \underset{0,1}{\_}\ \underset{0,1}{\_}\ \underset{0,1}{\_}\ \_\ \_$

$2^4 \times 3 = 48$

Q7. In how many ways 3 integers can be selected from 3n consecutive integers such that the sum is a multiple of 3?

$S1$: 1 4 7 . . . $3n-2$
$S2$: 2 5 8 . . . $3n-1$
$S3$: 3 6 9 . . . $3n$

sum is a multiple of 3 if we select 3 number from the same set or one from each set

Any 3 from S_1 or from S_2 or S_3 or one from S_1, S_2, S_3 each.

$$nC_3 + nC_3 + nC_3 + nC_1 \times nC_1 \times nC_1 = 3\,nC_3 + n^3$$

EXTRA Questions:

Q1. The number of squares of all possible sizes in an 8×8 chess board is-------------------.
ANS: 204
Q2. The number of ways to choose 3 days out of 7 days (With repetition) is--------------------.
ANS: 9C3=84
Q3. A shop sells 6 different flavors of ice-cream. In how many ways a customer can choose 4 ice-cream cones if (i) they are all of different flavors?
(ii) they are not necessarily of different flavors?
ANS: (i): 6C4 (ii) 9C4
Q4. There are 6 different French books, 8 different Russian boos and 5 different Spanish boos. How many ways are there to arrange the books in a row on a shelf with all books of the same language grouped together?
ANS:6!8!5!3!
Q5. How many odd integers between 100 and 999 have distinct digits?
ANS:320
Q6: A student is to answer 12 of the 15 questions in an exam. How many choices does the student have
(i) In all
(ii) If he must answer the first 2 questions
(iii) If he must answer the first or the second but not both
(iv) If he must answer exactly 3 of the first 5
(v) If he must answer at least 3 out of first five
ANS: (i) 455
(ii) 286
(iii) 156
(iv) 100
(v) 445