Engineering Mathematics-IV
MAT 2256

- **Introduction to Probability**
- **One dimentional random variable**
- **Two dimentional random variables**
- **Probability Distributions**
- **Functions of 1D and 2D random variables**
- **Moment generating function**
- **Sampling Theory**

References:

- Meyer P.L. - Introduction to probability and statistical applications, 2nd edition, 1980, Oxford and IBH Publishing, Delhi.
- Miller, Freund and Johnson - Probability and Statistics for Engineers, 8th edn, PHI, 2011.
- Hogg and Craig, Introduction to Mathematical Statistics, 6th edition, Pearson education, New Dehli, 2012.
- Ross Sheldon M, Introduction to Probability and Statistics for Engineers and Scientists, Elseveir, 2010.

Probability

SET THEORY

A set is a collection of well-defined objects. The objects comprising the set are called elements.

If x is an element of a set A, then we write $x \in A$. If x is not an element, then we write $x \notin A$.

Subset: Let A & B be two sets. A is said to be a subset of B,

if $x \in A \Rightarrow x \in B$ & is denoted by $A \subseteq B$.

Equality of sets: Two sets A and B are said to be equal if $A \subseteq B$ & $B \subseteq A$ and we write $A = B$.

Universal set: All sets under consideration are taken to be subsets of a fixed set. This set is called universal set & is denoted by U.

Null set: A set containing no elements is called a null set and is denoted by ϕ.

Singleton set: A set containing a single element is called a singleton set.

SET OPERATIONS:

Let A and B be two sets.

Union: Union of A and B is denoted by **$A \cup B$** and is defined as

$A \cup B = \{x \mid x \in A$ or $x \in B\}$.

Intersection: Intersection of A and B is defined as

$A \cap B = \{x \mid x \in A$ and $x \in B\}$. If **$A \cap B = \phi$**, then A & B are disjoint sets.

Difference: The difference A\B is defined as A\B = **$\{x \mid x \in A$ & $x \notin B\} = A \cap B'$**

Complement: Complement of the set A is denoted by A' **or** A^c **or** $\bar{A}$ and is defined as

$A' = \{x \mid x \in U$ but $x \notin A\}$.

Introduction to Probability:

Probability is a numerical measure which indicates the chance of occurrence.

Random experiment: An experiment in which the outcome cannot be predicted.

It is an experiment which does not have a unique outcome.

Examples: Tossing of a coin, rolling of a die.

Sample Space: The set of all possible outcomes of a random experiment is called the sample space and is denoted by S.

Examples:

In tossing of a coin: S = {H, T}

In tossing of two coins: S={HH, HT, TH, TT}

In rolling of a die: S ={1,2,3,4,5,6}

Elements of sample space is called sample points

If a sample space has finite number of elements then it is called a finite sample space. Otherwise the sample space is said to be an infinite sample space.

Examples:

While tossing two coins simultaneously, the sample space is

$S = \{HH, HT, TH, TT\}$ - finite sample space

Consider rolling of a die till a 5 appears

$S = \{5, 15, 25 \dots 65, 115, 125, \dots, 215, \dots\}$ - infinite sample space

Event: An event is a subset of the sample space.

Null Event: An event, which does not contain any element, is called a null event or an impossible event, denoted by ϕ.

Certain Event: If the event contains all the elements of the sample space, then it is called a certain event.

<u>Mutually Exclusive Events:</u> Two events A and B are said to be mutually exclusive if both of them cannot occur simultaneously.

i.e., if occurrence of one event prevents the occurrence of the other, then the events are said to be mutually exclusive.

A and B are mutually exclusive if $A \cap B = \phi$.

Examples:

While tossing a coin, either a head turns up or a tail but not both.

In rolling of a die all six faces are mutually exclusive

Equally likely outcomes: If all outcomes of a random experiment have equal chances of occurrence, then the outcomes are said to be equally likely.

Examples:

In tossing of an unbiased coin, head and tail are equally likely.

In rolling of an honest die, all six faces are equally likely.

Exhaustive cases: The total number of possible outcomes of a random experiment is called exhaustive cases for that experiment.

In tossing of a coin, exhaustive cases = 2

In tossing of 2 coins, exhaustive cases = 4

In tossing n coins, exhaustive cases = 2^n

In rolling of two dice, exhaustive cases = 36

Favourable cases: An outcome x is said to be favourable to an event A, if x belongs to A. The total number of outcomes favourable to A is called favourable cases to A.

Examples:

In tossing of two coins, favourable cases for getting two heads is 1, for getting exactly one head is 2 and for getting at least 1 heads is 3.

In drawing a card from a pack, there are 4 cases favouring a king, 2 cases favouring a red queen and 26 cases favouring a black card.

Probability is a quantitative measure of chances of occurrence.

There are 3 approaches to the study of probability.

1. Classical approach

2. Statistical or empirical approach

3. Axiomatic approach

Classical Definition of Probability: If an event A can occur in m different ways out of a total of n ways all of which are equally likely and mutually exclusive, then the probability of the event A is given by,

$$P(A) = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes} = \frac{m}{n}$$

- For a null set, m = 0. Hence P(ϕ) = 0
- For the sample space $m = n$. Hence $\boldsymbol{P(S) = 1}$
- $0 \leq m \leq n$. Hence $0 \leq \frac{m}{n} \leq 1$ i.e., $0 \leq P(A) \leq 1$
- m outcomes are favourable to A $\Longrightarrow$ remaining $n - m$ are favourable to A'.

Hence $P(A') = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$, **i.e., P(A) + P(A′) = 1**

Statistical Definition of Probability:

If an experiment is repeated a number of times under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event occurs to the number of trials, as the number of trials become indefinitely large, is called the probability of that event.

i.e. if an event A occurs m times in n trials then P(A) = $\lim_{n \to \infty} \frac{m}{n}$

Axiomatic Definition of Probability:

Consider a random experiment with sample space S. Associated with this random experiment, many events can be defined.

Let for every event A, a real number $P(A) = p$ be assigned. Then $P(A)$ is the probability of event A, if the following axioms are satisfied.

(i) $0 \leq P(A) \leq 1$

(ii) $P(S) = 1$

(iii) For two mutually exclusive events A and B, $P(A \cup B) = P(A) + P(B)$ $(\because P(A \cap B) = 0)$

(iv) If $A_1, A_2, \ldots, A_n$ are pairwise mutually exclusive events of S, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

Addition Theorem:

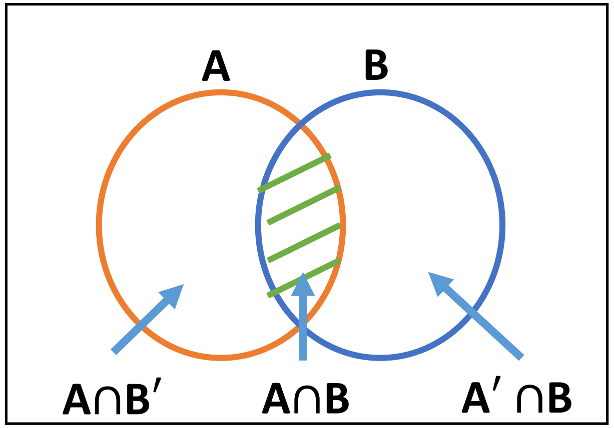
If A and B are any two events of S then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

From venn diagram,

$A \cup B = A \cup (A' \cap B)$

$P(A \cup B) = P(A) + P(A' \cap B)$ ---- (i)



(Note that **A∩B′, A∩B and A′ ∩B** are mutually exclusive**)**

Also, $B = (A \cap B) \cup (A' \cap B)$

$P(B) = P(A \cap B) + P(A' \cap B)$ --- (ii)

From (i) and (ii),

$P(A \cup B) - P(B) = P(A) + P(A' \cap B) - P(A \cap B) - P(A' \cap B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Note: If A and B are mutually exclusive, then $P(A \cap B) = 0$, then

$$P(A \cup B) = P(A) + P(B)$$

Theorem: If A, B and C are any three events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + \mathrm{P}(P(A \cap B \cap C)$$

Proof: Let $B \cup C = D$

$$P(A \cup B \cup C) = P(A \cup D)$$

$$= P(A) + P(D) - P(A \cap D)$$

$$= P(A) + P(B \cup C) - P\big(A \cap (B \cup C)\big)$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)]$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - \{P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)\}$$

In general,

$$P(A_1 \cup A_2 \cup \cdots \cup A_k) = \sum_{i=1}^{k} P(A_i) - \sum_{i<j=2}^{k} P\left(A_i \cap A_j\right) + \sum_{i<j<r=3}^{k} P\left(A_i \cap A_j \cap A_r\right) - \ldots +$$

$$(-1)^{k-1} P(A_1 \cap A_2 \cap \cdots \cap A_k)$$