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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER B.E DEGREE END SEMESTER EXAMINATION- 2009

SUB: ENGG. MATHEMATICS I (MAT – 101)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All questions carry equal marks

1A. Find the n^{th} derivatives of

i) $\dfrac{x^2+4x+1}{x^3+2x^2-x-2}$ (ii) $e^{2x}\,.\cos x.\sin^2 2x$

1B. Trace the following curve with explanations

$y^2x^2 = x^2 - a^2$

1C. A variable plane at a constant distance p from the origin meets the coordinate axes at A, B, C. Through A, B, C planes are drawn parallel to coordinate planes. Show that the locus of their point of intersection is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

(4 + 3 + 3)

2A. If $y = \sin\ m\sin^{-1}x$, show that $(1-x^2)y_{n+2} = (2n+1)\,x\,y_{n+1} + (n^2 - m^2)\,y_n$

2B. Evaluate :

(i) $\displaystyle\int_0^{\pi} \frac{\sin^2\theta\sqrt{1-\cos\theta}}{1+\cos\theta}\,d\theta$ (ii) $\displaystyle\int_0^{2a} x^{9/2}\,(2ax-x^2)^{-1/2}\,dx$

2C. A line with direction cosines proportional to 2, 7, – 5 is drawn to intersect the lines $\dfrac{x-8}{3}=\dfrac{y-6}{-1}=\dfrac{z+1}{1}$ and $\dfrac{x+3}{-3}=\dfrac{y-3}{2}=\dfrac{z-6}{4}$.

Find the coordinates of the point of intersection and the length intercepted.

(3 + 4+ 3)

3A. Find the nature of

(i) $1+\dfrac{3}{7}+\dfrac{3.6}{7.10}+\dfrac{3.6.9}{7.10.13}+.....$

(ii) $1+\dfrac{2^2}{3^2}x+\dfrac{2^2.4^2}{3^2.5^2}x^2+.....$

3B. Sketch and find perimeter of the curve $r = a(1 - \cos\theta)$.

3C. Find the evolute of $x = a\cos t + \log\tan t/2$, $y = a\sin t$.

(4 + 3+ 3)

4A. Evaluate :

(i) $\lim_{x\to 0} \dfrac{e^x \sin x - x - x^2}{x^2 + x\log(1-x)}$

(ii) $\lim_{x\to 0} \dfrac{1+x^{1/x} - e}{x}$

4B. Find the angle between the curves
$r^2 \sin 2\theta = 4$, $r^2 = 16\sin 2\theta$

4C. Find the centre and the radius of the circle of intersection by the plane $x+4y+z = 4$ and the sphere $x^2 + y^2 + z^2 - x - z - 2 = 0$.

(4 + 3+ 3)

5A. Find the first three nonzero terms in the Maclaurin's series expansion
$f(x) = (\sin^{-1} x)^2$.

5B. The tangents at two points P, Q on the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ are at right angles. Show that if ρ_1 and ρ_2 be the radii of curvature at these points, then show that $\rho_1^2 + \rho_2^2 = 16a^2$.

5C. Find the volume of the solid obtained by revolution of the curve $y^2(2a - x) = x^3$ about its asymptote.

(3 + 4+ 3)

6A. Transform the equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ into polar coordinates.

6B. State Cauchy's mean value theorem and verify it for
$f(x) = \sqrt{x}$ $g(x) = \dfrac{1}{\sqrt{x}}$ in [a,b]

6C. Find the maximum possible error in calculating g if $T = 2\pi\sqrt{l/g}$, given 1% and 0.5% errors are possible in l and T respectively.

(4 + 3+ 3)

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