DEPARTMENT OF CIVIL ENGINEERING

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL

LEARNING MATERIAL

Mechanics

of

Solids

LECTURE- 1

- **Concept of Rigid body**
- **Force and its characteristics**
- **Principle of transmissibility**
- **Classification of Force System**
- **Resultant of Coplanar concurrent forces**
- **Composition of forces**
- **Resolution of a force**
- **Rectangular Components of a force**
- **Sign convention**

INTRODUCTION

- **Basic principles and concepts**

Mechanics may be defined as the branch of physical science which describes and predicts the state of rest or motion of bodies under the action of forces.

Mechanics of solids is classified into

a) Mechanics of Rigid Bodies and b) Mechanics of Deformable Bodies

Rigid Body:

The body, which does not, undergoes any change in its dimensions or shape, even after application of force is called as rigid body

Actually solid bodies are never rigid; they deform under the action of applied forces. In those cases where this deformation is negligible compared to the size of the body, the body may be considered to be rigid.

Force :

It is that agent which causes or tends to cause, changes or tends to change the state of rest or of motion of a mass.

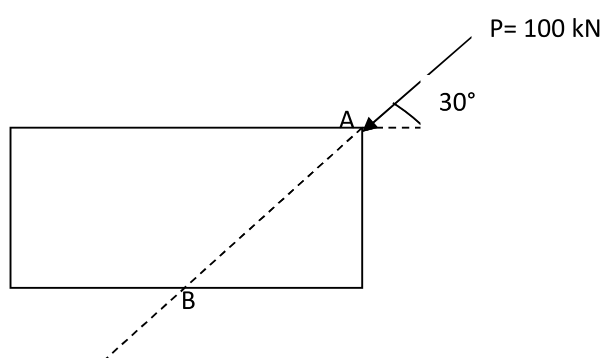
A force is fully defined only when the following four characteristics are known:

Magnitude

Direction

Point of application and

Line of action.

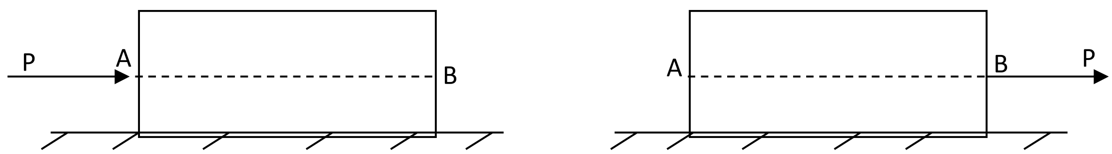


The force P applied externally has the following characteristics:

Magnitude of the force is 100 kN. Line of action is represented by AB which is inclined at an angle of 30° with the horizontal. An arrow drawn as shown in figure indicates the direction. The point on the body where the arrow makes contact is the point of application (i.e., point A)

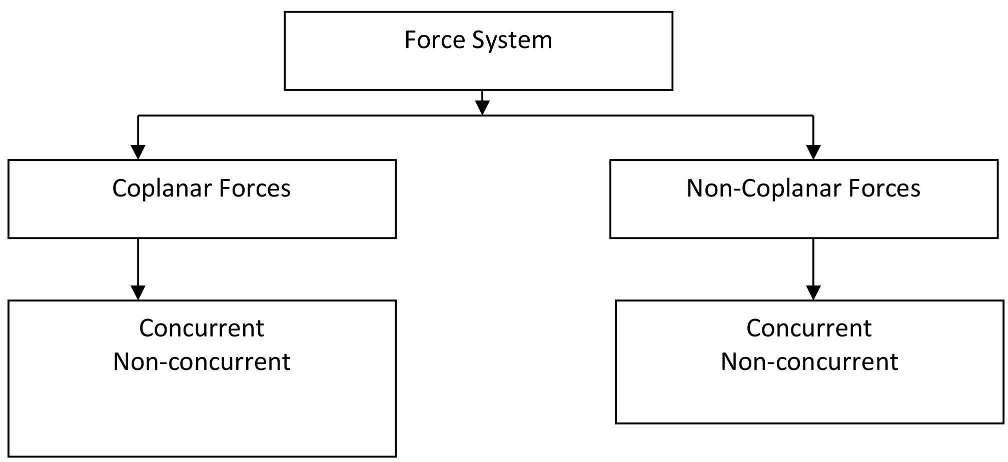
Principle of transmissibility:

The external effect of a force on a rigid body is the same for all points of application along its line of action



In the figure , P is the force acting on a rigid body at point A. As per the law of transmissibility of forces, if the point of application of this force is shifted to B, which is another point on the line of action of the force P, the original state of rest or motion of the body remains unchanged.

Classification of force system :



Coplanar Concurrent forces : Line of action of all the forces pass through a single point and the forces lie in the same plane. Example: A sphere resting on a smooth horizontal floor

Coplanar non-concurrent forces: all the forces do not meet at a point, but lie in a single plane. Example: The force on a ladder resting against a wall when a person stands on a rung which is not at its center of gravity.

Non- coplanar concurrent forces: All the forces do not lie in the same plane but their lines of action pass through a single point. Example: A tripod carrying a camera.

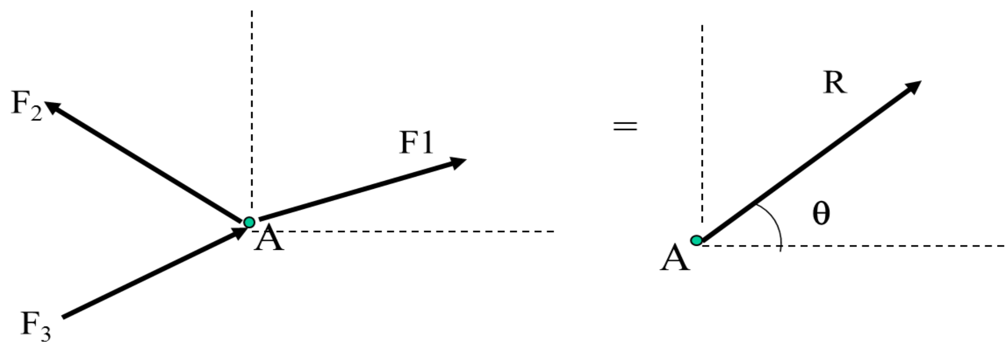
Non- coplanar non-concurrent forces: All the forces do not lie in the same plane and their lines of action do not pass through a single point. Example: The force acting on a moving bus.

Collinear forces: Line of action of all forces act along the same line. Example: Forces on a rope in a tug of war.

- ## <u>Resultant of coplanar concurrent and non concurrent force system</u>

<u>Resultant</u> : It is defined as that single force which can replace a set of forces, in a force system, and cause the same external effect.

The process of obtaining the resultant of a given force system is called '**Composition of forces'**.



<u>Component of a force:</u>

Component of a force, in simple terms, is the effect of a force in a certain direction. A force can be split into infinite number of components along infinite directions. Usually, a force is split into two mutually perpendicular components, one along the x-direction and the other along y-direction (generally horizontal and vertical, respectively). Such components that are mutually perpendicular are called 'Rectangular Components'.

The process of obtaining the components of a force is called '**Resolution of a force**'.

<u>Rectangular components of a force:</u>

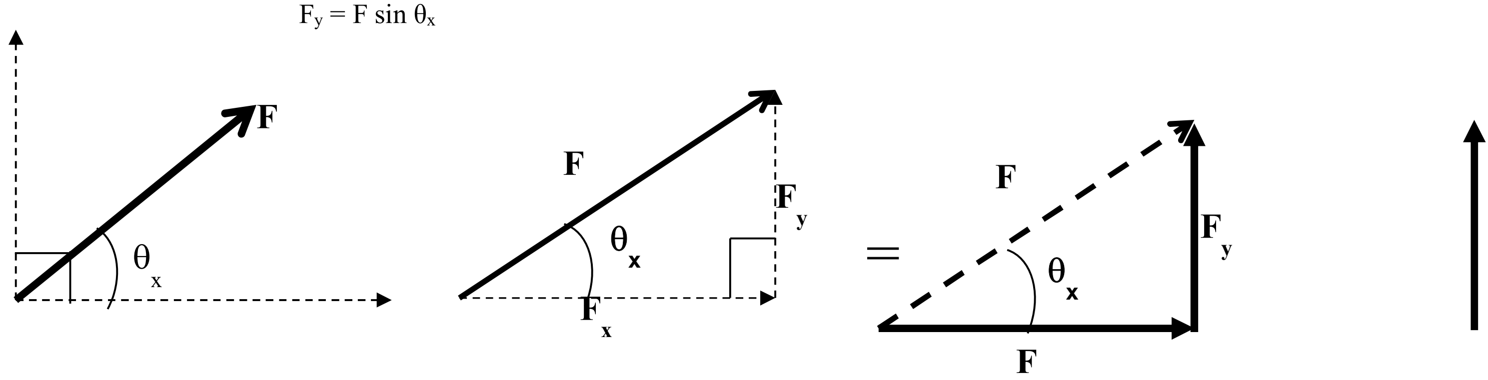
Consider a force F making an angle θ_x with x-axis.

The resolved part of the force F along x-axis is given by

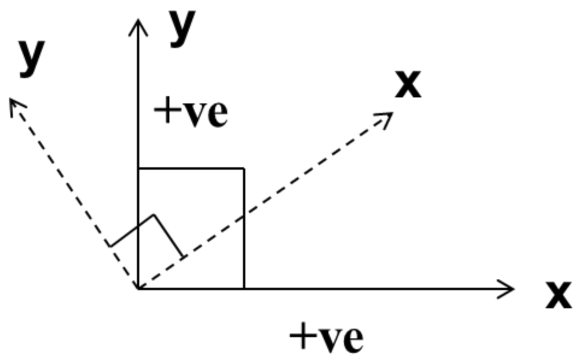
$$F_x = F \cos \theta_x$$

The resolved part of the force F along y-axis is given by

$$F_y = F \sin \theta_x$$



Sign Convention for force components:



The above diagram gives the sign convention for force components, i.e., force components that are directed along positive x-direction are taken +ve for summation along the x-direction.

LECTURE-2

- **Resultant of Coplanar non-concurrent force systems**
- **Moment of a force**
- **Varignon's theorem**
- **Couple**
- **Moment of a couple**
- **Resolution of a force into a force and couple**
- **Properties of a couple**

RESULTANT OF COPLANAR NON -CONCURRENT FORCE SYSTEM

If all the forces in a system lie in the same plane and the lines of action of these forces do not pass through a single point, the system is said to be coplanar non concurrent force system

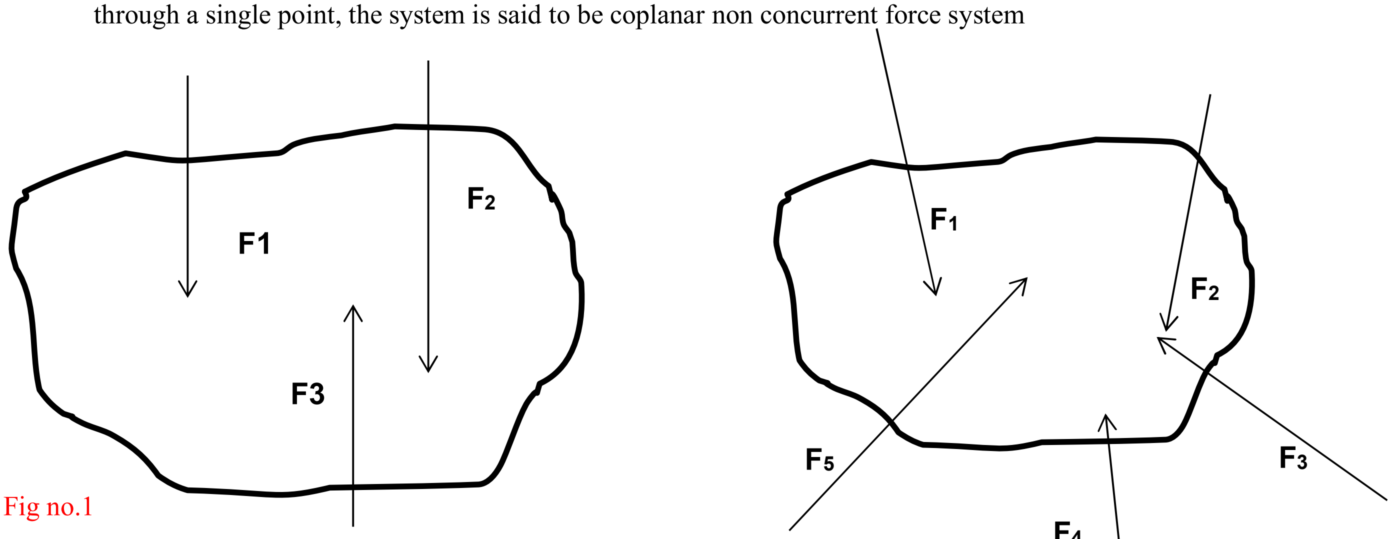


Fig no.1

Parallel Force System – Lines of action of individual forces are parallel to each other.

Non-Parallel Force System – Lines of action of the forces are not parallel to each other

Moment of a force about an axis:

The applied force can also tend to rotate the body about an axis in addition to motion. This rotational tendency is known as moment.

Definition: Moment is the tendency of a force to make a rigid body to rotate about an axis.

This is a vector quantity having both magnitude and direction

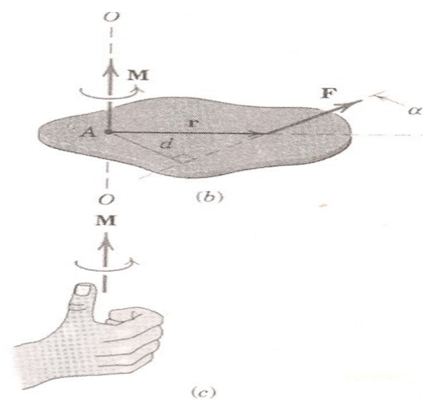


Fig no.2

Moment Axis: This is the axis about which rotational tendency is determined. It is perpendicular to the plane comprising moment arm and line of action of the force (axis 0-0 in the figure no. 2)

Moment Center: This is the position of axis on a co-planar system. (A).

Moment Arm: Perpendicular distance from the line of action of the force to moment center. Distance AB = d.

Magnitude of moment: It is computed as the product of magnitude of the force and the perpendicular distance from the line of action to the point about which moment is computed, (Moment center). MA = F×d.

Sense of moment:

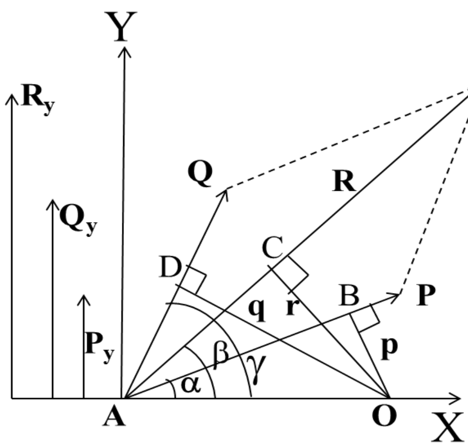
The sense is obtained by 'Right Hand Thumb' rule.

'If the fingers of the right hand are curled in the direction of rotational tendency of the body, the extended thumb represents the sense of moment vector'.

For the purpose of additions, the moment direction may be considered by using a suitable sign convention such as +ve for counterclockwise and –ve for clockwise rotations or vice-versa.

Varignon's theorem (Principle of moments) – [Varignon – French Mathematician(1654-1722)]

The moment of a force about a moment center or axis is equal to the algebraic sum of the moments of its component forces about the same moment center (axis).



Let ‘R’ be the given force.‘P’ & ‘Q’ are component forces of ‘R’.‘O’ is the moment center. p, r and q are moment arms from ‘O’ of P, R and Q respectively.α, β and γ are the inclinations of ‘P’, ‘R’ and ‘Q’ respectively w.r.to X – axis.

We have,

$R_y = P_y + Q_y$

R Sinβ = P Sinα + Q Sin γ ----(1)

From Δle AOB, p/AO = Sin α

From Δle AOC, r/AO = Sin β

From Δle AOD, q/AO = Sin γ

From (1),

$\therefore$ R ×(r/AO) = P ×(p/AO) + Q ×(q/AO)

i.e., R × r = P × p + Q × q

Moment of resultant R about O = algebraic sum of moments of component forces P & Q about same moment center ‘O’.

Couple

Two parallel, non collinear (separated by certain distance) forces that are equal in magnitude and opposite

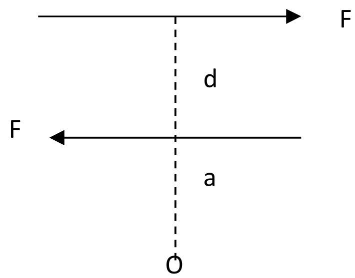
in direction form ‘couple’. The algebraic summation of the two forces forming couple is zero. Hence, couple does not produce any translation and produces only rotation.

Moment of a Couple:

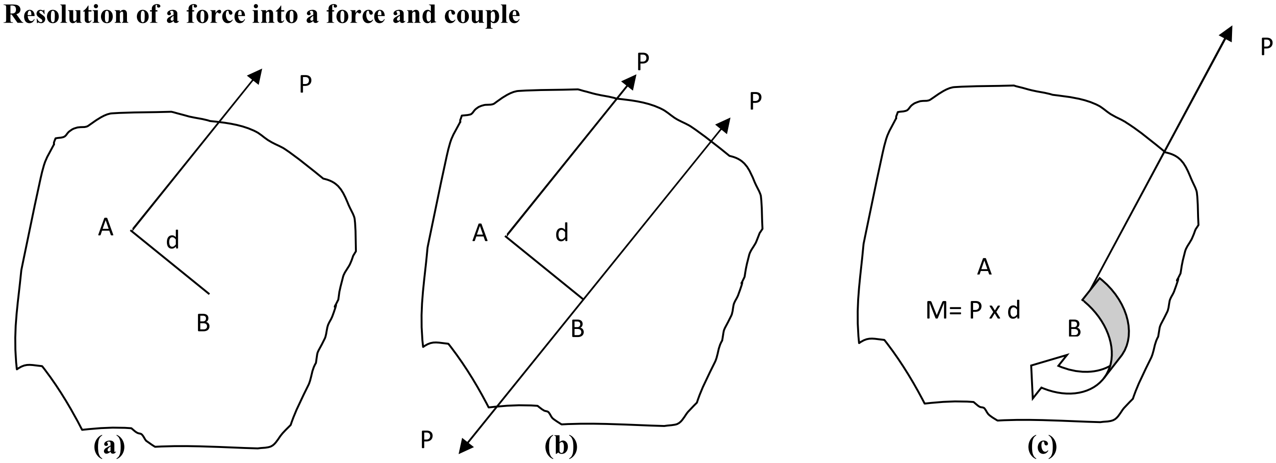
Consider two equal and opposite forces separated by a distance ‘d’. Let ‘O’ be the moment center at a distance ‘a’ from one of the forces. The sum of moments of two forces about the point ‘O’ is, $\sum$ Mo = F × (a + d) – F × a = F× d

Thus, the moment of the couple about ‘O’ is independent of the location as it is independent of ‘a’.

The moment of a couple about any point is constant and is equal to the product of one of the forces and the perpendicular distance between them.



Resolution of a force into a force and couple



In figure (a), P is a force acting on a body at A. Now it can be shown that P at A may be resolved into a force P at B and a couple of magnitude M = Pd, where d is the perpendicular distance of B from the line of action of P through A

By applying equal and opposite forces P at B the system of forces is not disturbed. Hence the system of forces in the fig (b) is the same as the system given in fig (a). Now the original force P at A and the opposite force P at B form a couple of magnitude Pd. The system in fig (b) can be replaced by the system shown in fig (c). Thus, the given force P at A is replaced by a force P at B and a moment Pd.

LECTURE-3

Application Problems 1-3

LECTURE-4

Application Problems 4-5

TUTORIAL - 1

Tutorial Problems

LECTURE- 5

Equilibrium of Coplanar concurrent and non-current force system

- **Equilibrium of a particle,**
- **Conditions for Equilibrium,**
- **Space Diagram& Free Body Diagram (FBD)**
- **Types of Support and Support Reactions**

Equilibrium of a particle

Consider a particle 'A' acted upon by two equal and opposite forces.

If the net effect of the given force is zero and the particle is said to be in equilibrium.

For example, consider the following fig 2.1, the net force applied on the point 'O' to is zero.

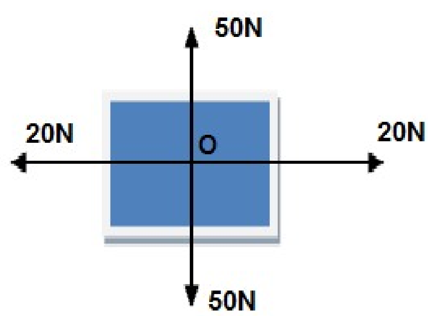


Fig. 1 Example for equilibrium

When the resultant of all the forces acting on a particle is zero, the particle is in equilibrium

A body is said to be in equilibrium when it does not have any translator or rotatory motion in any direction. This means that when the body is in equilibrium, the following two simultaneous conditions are to be satisfied

Equilibrium conditions

The algebraic sum of the components of the forces along each of three mutually perpendicular axes is zero. In case of coplanar forces, $\Sigma F_X = 0$; $\Sigma F_Y = 0$; $\Sigma M = 0$

Graphical conditions for equilibrium

Triangle Law:

If three forces are in equilibrium, then, they form a closed triangle when represented in a Tip to Tail arrangement, as shown in Figure 2.2.

Fig. 2.2 Triangle Law of Forces

Polygonal Law:

If more than three forces are in equilibrium, then, they form a closed polygon when represented in a Tip to Tail arrangement, as shown in Figure 2.3.

Fig. 2.3 Polygonal Law of Forces

Lami's Theorem:

If a system of three "concurrent" forces is in equilibrium, then, each force of the system is proportional to sine of the angle between the other two forces (and constant of proportionality is the same for all the forces).

Note: While using Lami's theorem, all the three forces should be either directed away or all directed towards the point of concurrence.

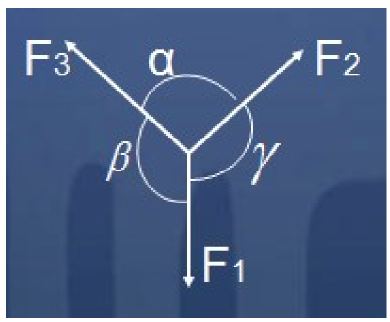


Fig. 2.4 Lami's theorem for concurrent co-planar forces under equilibrium

So the unknown force can be computed using the Lami's theorem by the following equation.

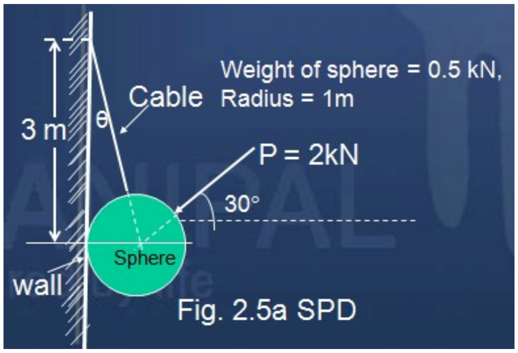
$$\frac{F_1}{\text{Sin}\,\alpha} = \frac{F_2}{\text{Sin}\,\beta} = \frac{F_3}{\text{Sin}\,\gamma}$$

Space diagram and free body diagram

Problems in Engineering Mechanics always involve the interaction of bodies upon one another, and are derived from an actual physical situation.

A sketch showing the physical conditions of the problem is known as **space diagram**. Eg, the nature of supports provided; size, shape and location of various bodies; forces applied on the bodies, etc., is known as **space diagram**.(Refer Fig.2.5a)

An isolated view of a body which shows only the external forces acting on the body is called **Free Body Diagram** [FBD], also it is an isolated diagram of the body being analyzed (called free body), in which, the body is shown freed from all its supports and contacting bodies/surfaces. Instead of the supports and contacting bodies/surfaces, the reactive forces exerted by them on the free body are shown, along with all other applied forces. (Refer Fig.2.5b)



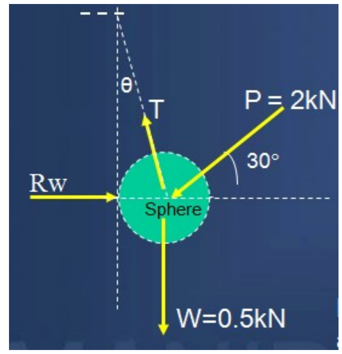


Fig.2.5a Space diagram   Fig.2.5b Free Body Diagram

Guidelines for drawing FBD:

1. Tensile Force: It is a force trying to pull or extend the body. It is represented by a vector directed away from the body.
2. Compressive Force: It is force trying to push or contract the body. It is represented by a vector directed towards the body.
3. Reactions at smooth surfaces: The reactions of smooth surfaces, like walls, floors, Inclined planes, etc. will be normal to the surface and pointing towards the body.
4. Forces in Link rods/connecting rods: These forces will be acting along the axis of the rod, either towards or away from the body. (They are either compressive or tensile in nature).
5. Forces in Cables (Strings or Chords): These can only be tensile forces. Thus, these forces will be along the cable and directed away from the body.

Support Reactions:

Beam: beam is a structural member which supports the load perpendicular to its centre line.

Supports: A structure is subjected to external forces and transfers these forces through the supports on to the foundation. Therefore the support reactions and the external forces together keep the structure in equilibrium.

Types of supports: Roller support, Hinged Support, Fixed support (Built-in)

Roller Support: Contact force is normal to the surface on which the roller moves. The reaction will always be perpendicular to the plane of the roller. Roller support will offer only one independent reaction component as shown in Fig. 2.6. (Direction of which is known.)

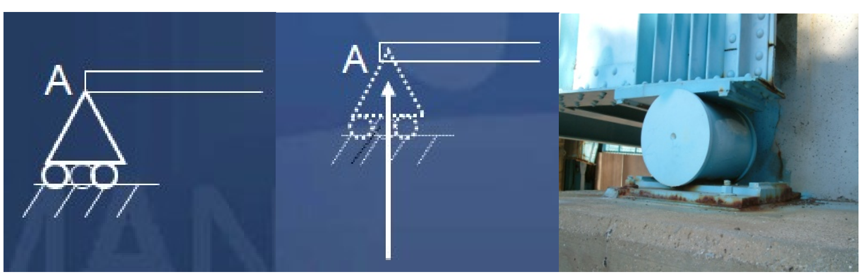


Fig. 2.6 Roller Support representation, reaction exerted by roller, and example of roller

Hinged/pinned Support: This support does not allow any translatory movement of the rigid body. There will be two independent reaction components at the support. The resultant reaction can be resolved into two mutually perpendicular components. Or it can be shown as resultant reaction inclined at an angle with respect to a reference direction.

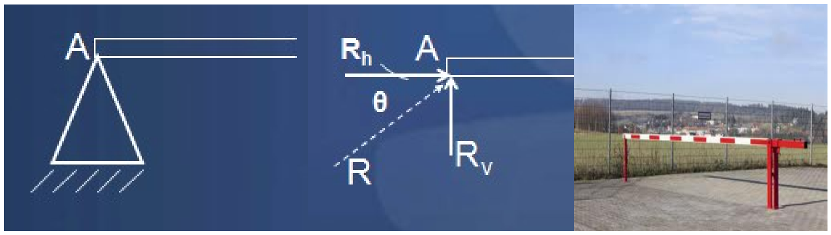


Fig. 2.7 Hinged/pinned Support representation, reaction exerted by hinge, and example of hinge

Fixed Support: This type of support not only prevents the translatory movement of the rigid body, but also the rotation of the rigid body. Hence there will be 3 independent reaction components of forces. Hence there will be 3 unknown components of forces, two mutually perpendicular reactive force components and a reactive moment as shown in the figure 2.8.

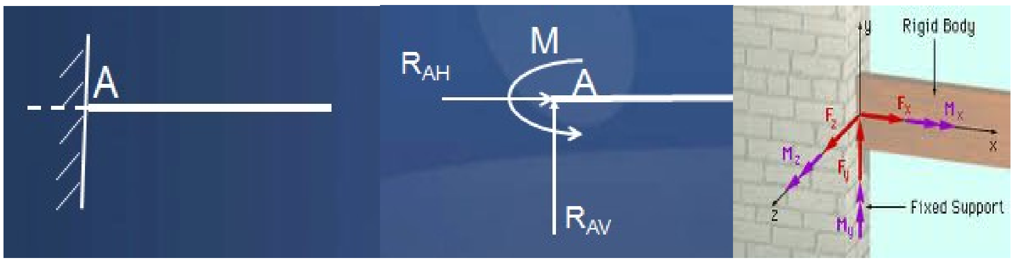


Fig. 2.7 Fixed Support representation, reaction exerted by fixed support, and an example

LECTURE- 6

- **Types of Beams**
- **Types on Loads on beam**
- **Application Problems**

Types of Beams: A member which is subjected to predominantly transverse loads and supported in such a way that rigid body motion is prevented is known as beam. It is classified based on the support conditions. A beam generally supported by a hinge or roller at the ends having one span (distance between the supports) is called as **simply supported beam**. A beam which is fixed at one end and free at another end is called as a **cantilever beam**. If one end or both ends of the beam project beyond the support it is known as **overhanging beam.**

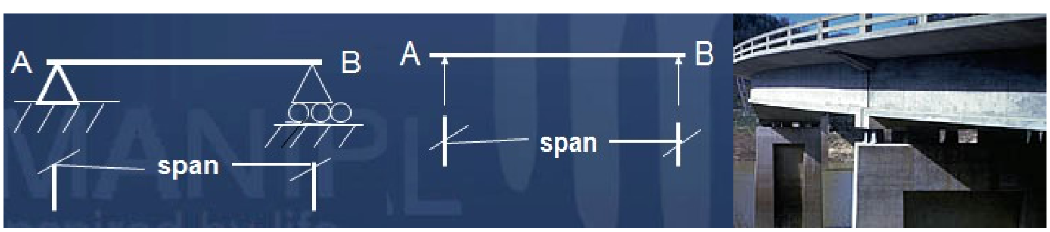


Fig. 2.8 Simply Supported Beam representation, reaction exerted by fixed support, an example

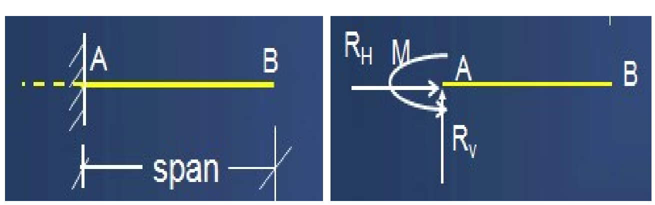


Fig. 2.9 Cantilever Beam representation, reaction exerted by fixed support.

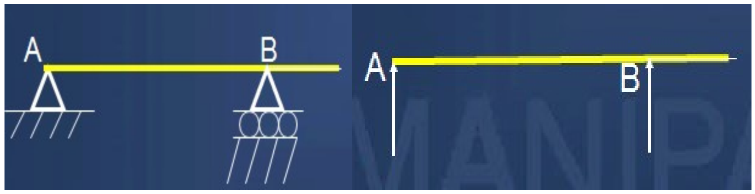


Fig. 2.10 Overhanging Beam representation, reaction exerted by fixed support.

<u>Types of loads on structures</u>:

1. Concentrated Loads – This is the load acting for very small length of the beam. (Also known as point load, Total load W is acting at one point) Fig.2.11a
2. Uniformly distributed load – This is the load acting for a considerable length of the beam with same intensity of w kN/m throughout its spread. Total intensity, $W = w \times L$ (acts at L/2 from one end of the spread) Fig.2.11b
3. Uniformly varying load – This load acts for a considerable length of the beam with intensity varying linearly from '0' at one end to w kN/m to the other representing a triangular distribution. Total intensity of load = area of triangular spread of the load

   $W = 1/2\times w \times L$. (acts at $2\times L/3$ from 'Zero' load end) Fig. 2.11c

4. Trapezoidal load - A combination of UDL and UVL. (Fig.2.11d)

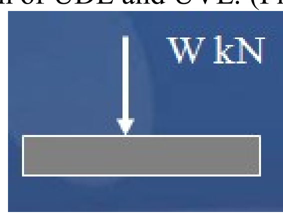


Fig.2.11a Point Load

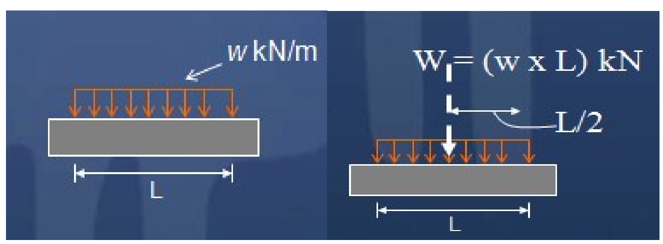
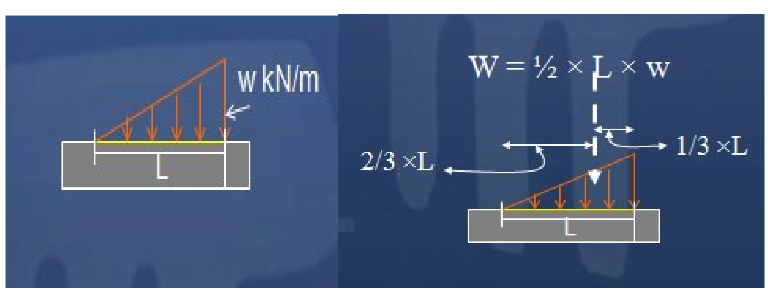
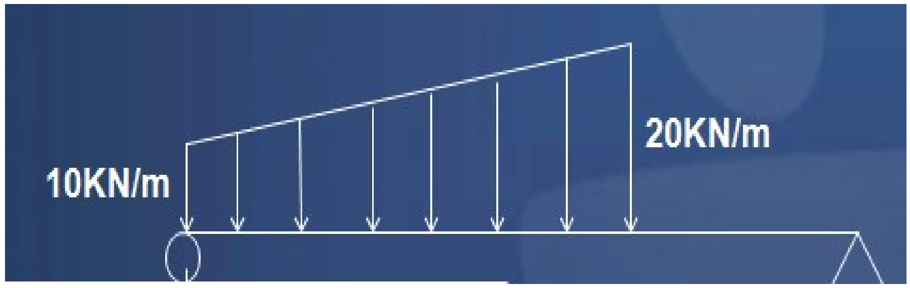


Fig.2.11b Uniformly Distributed Load (UDL)



2.11c Uniformly Varying Load (UVL)



2.11d Trapezoidal Load

Statically determinate beam and statically indeterminate beam:

Using the equations of equilibrium,if all the reaction components can be found out, then the beam is a statically determinate beam ,and if all the reaction components cannot be found out using equations of equilibrium, then the beam is a statically indeterminate beam.

Application Problems 1-2

LECTURE-7

Application Problems 3-5

TUTORIAL-2

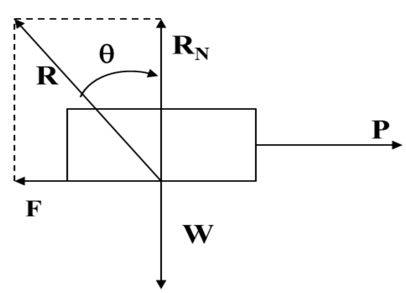
Application Problems

LECTURE-8

Friction

- **Introduction**
- **Statical, Limiting and Kinetic Fricion**
- **Terms and Definitions**
- **Laws of dry friction**
- **Application Problem**

Introduction:

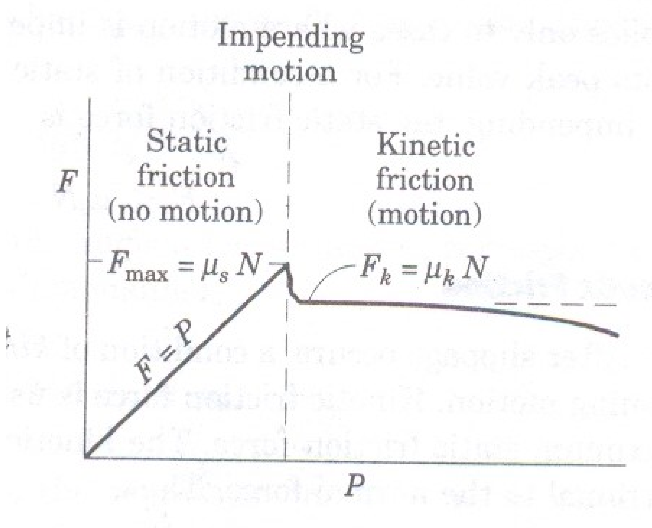


Friction may be defined broadly as the resistance that occurs between two bodies in contact when they tend to slide or roll relative to one another. Bellow discussed are the three approximate rules of friction that were established by Charles A. Coulomb (1736-1806), but are still useful today.

Frictional resistance is due in large part to the roughness of various materials, such as metals, concrete, wood, plastic, and composites. It is seen that surface roughness determines, in part, the amount of friction that exists between bodies. One can analyze the performance of several mechanical devices in which friction plays a central role. These devices include the wedge, screw and jackscrew, as well as belt-pulley assemblies, clutches and brakes.

Friction can be simultaneously beneficial and harmful. For example, friction enables to walk, but it also causes the soles of shoes to wear out. Friction can be used to produce desirable wear. For example, it may be used to grind and polish metals and to "break in" a new car engine.

Statical, Limiting and Kinetic Fricion:



Frictional resistance has the remarkable property of adjusting itself in magnitude of force producing or tending to produce the motion so that the motion is prevented.

When **P** = 0, **F** = 0 → block under equilibrium.When **P** increases, **F** also increases proportionately to maintain equilibrium. However there is a limit beyond which the magnitude of this friction cannot increase.

When the block is on the verge of motion(motion of the block is impending) **F** attains maximum possible value, which is termed as Limiting Friction. When the applied force is less than the limiting friction, the body remains at rest and such frictional resistance is called the static friction.

Further if **P** is increased, the value of **F** decreases rapidly and then remains fairly a constant thereafter. However at high speeds it tends to decrease. This frictional resistance experienced by the body while in motion is known as Dynamic friction OR Kinetic Friction.

Terms and Definitions:

1) **Coefficient of friction (μ)** : When F has reached Fmax, it bears a constant ratio with the normal reaction. This constant is called Coefficient of friction .

$$\mu = F_{max} / R_N$$

2) Total reaction (R): Frictional force and the normal reaction could be replaced by a single force called Total reaction.

$$R=\sqrt{F^2 + R_N^2}$$

3) Angle of Friction (φ): It is the angle made by the total reaction with the normal reaction when F has reached F_{max}.

$$\tan\varphi=F_{max} / R_N$$

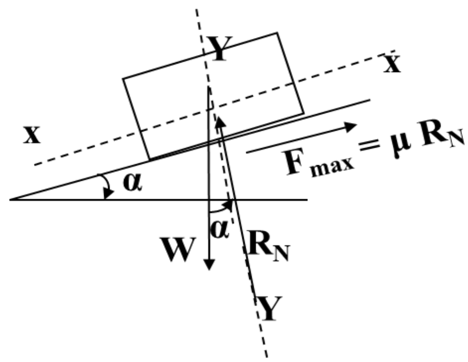
4) Angle of repose

When granular material is heaped, there exists a limit for the inclination of the surface. Beyond that angle, the grains start rolling down. This limiting angle upto which the grains repose (sleep) is called the angle of repose of the granular material.

Significance of Angle of repose:

The angle that an inclined plane makes with the horizontal, when the body supported on the plane is on the verge of motion due to its self -weight is equal to the angle of repose.

Angle of repose is numerically equal to ***Angle of limiting friction***



$$\Sigma F_X = 0$$

$$F_{max} = W \sin\alpha \text{ - - - - - - } (1)$$

$$\Sigma F_Y = 0$$

$$R_N = W \cos\alpha \text{ - - - - - - - } (2)$$

$$\frac{(1)}{(2)}$$

$$\tan\alpha = \frac{F_{max}}{R_N} = \mu = \tan\phi$$

$$\alpha = \phi$$

Laws of dry friction

1. The magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces.(Experimentally proved)

2. The force of friction is independent of the area of contact between the two surfaces.

3. For low velocities the total amount of friction that can be developed is practically independent of velocity. It is less than the frictional force corresponding to impending motion.

Application Problem 1

LECTURE-9

Application Problems 2-3

TUTORIAL-3

Tutorial Class problems

CENTROID AND MOMENT OF INERTIA

LECTURE-10

CENTROID:-

- **Centre of gravity**
- **Centroid of simple figures using method of moment**
- **Axis of symmetry**
- **To locate the centroid of following plane areas –**

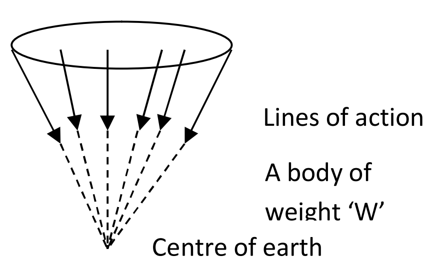
  rectangle, triangle, semicircle, quarter circle from first principles

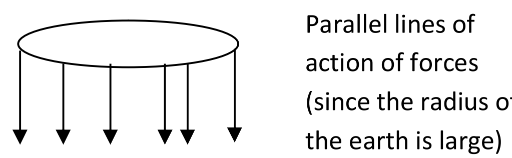
Introduction

In engineering mechanics applications, it is often required to consider line elements, areas, volumes, thin plates and solid bodies. In all such cases, a body can be considered as a collection of large number of particles each of which is adhering to its adjacent particles. These particles may have different or uniform size and density. But it is necessary to define a point where the distributed quantities are assumed to be concentrated.

Consider a body of weight W. Each of the particles in a body is subjected to gravitational force directed towards the center of the earth. The magnitude of this force depends on the mass of the body. Since the size of the bodies are small when compared to the size of the earth, gravitational forces can be assumed to be parallel. Hence the total weight of the body can be determined by finding the sum of all such gravitational forces acting vertically downwards, towards the centre of the earth.

It has been assumed so far that the attraction exerted by the earth on a rigid body could be represented by a single force W. This force, called the force of gravity or the weight of the body, is to be applied at the center of gravity of the body. Actually the earth exerts a force on each of the particles forming the body. The action of the earth on a rigid body should thus be represented by a large number of small forces distributed over the entire body. All of these small forces can be replaced by a single equivalent force W.





Centre of gravity of a body is the point at which the whole weight of the body may be assumed to be concentrated. A body has only one center of gravity for all positions of the body. It is represented by CG. or simply G or C.

Thus *Centre of gravity can be defined as the point through which the resultant of force of gravity of the body acts.*

When speaking of an actual physical body, we use the term "centre of mass". The term centroid is used when the calculation concerns a geometrical shape only. In case of plane figures such as rectangle, triangle, circle, semicircle, etc if the total area is concentrated at one and only one point it is defined as centroid.

There are two major differences between "center of gravity" and "center of mass": 1) The term "center of gravity" aplies to the bodies with mass and weight, while the term "centroid" applies to plane areas.2) Center of gravity of a body is the point through which the resultant gravitational force (weight) of the body acts for any orientation of the body while centroid is the point in a plane area such that the moment of the area, about any axis, through that points zero. $\overline{X}$ A=∫xdA

$\overline{Y}$ A= ∫ydA

These equations define coordinates $\overline{X}$ and $\overline{Y}$ of centre of gravity of a homogeneous plate.

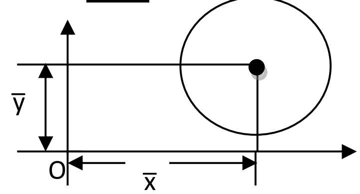
The integral ∫xdA is known as the first moment of area A with respect to the y-axis denoted by Q_y. Similarly

The integral ∫ydA defines the first moment of area with respect the x-axis and is denoted by Q_x.

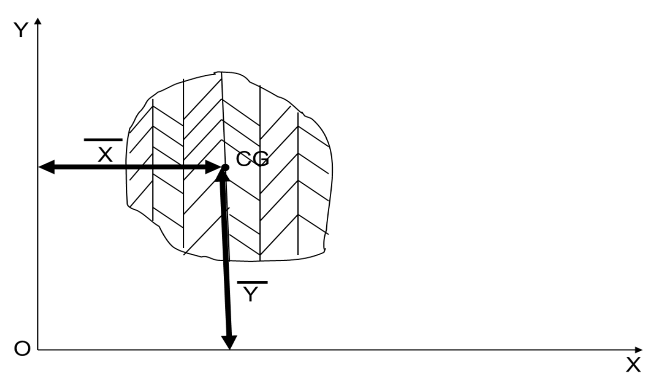
Q_y=∫xdA Q_x=∫ydA

i.e. $Q_y=\overline{x}\ dA$ $Q_x=\overline{y}dA$

Note:



Centroid of Simple figures: using method of moment (First moment of area)



Moment of Total area 'A' about y-axis = Algebraic Sum of moment of elemental '*d*A' about the same axis

$A\overline{x} = (a1)\ x1 + (a2)\ x2 + (a3)\ x3 + \ldots\ldots\ldots+(an)\ xn$

$$\overline{x} = \frac{(a_1)\,x_1 + (a_2)\,x_2 + (a_3)\,x_3 + \ldots\ldots\ldots + (a_n)\,x_n}{A}$$

Similarly

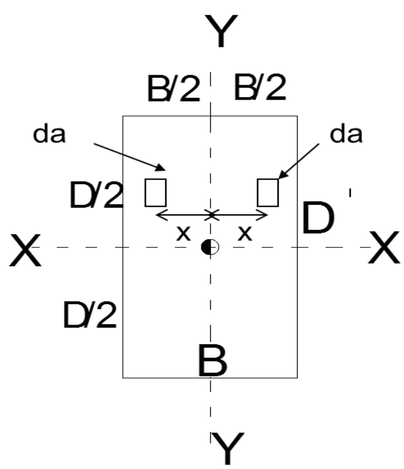
$$\overline{Y} = \frac{(a_1)\, y_1 + (a_2)\, y_2 + (a_3)\, y_3 + \ldots\ldots\ldots + (a_n)\, y_n}{A}$$

where ($A = a_1 + a_2 + a_3 + a_4 + \ldots\ldots.. + a_n$)

Axis of symmetry

It is an axis w.r.t. which for an elementary area on one side of the axis , there is a corresponding elementary area on the other side of the axis (the first moment of these elementary areas about the axis balance each other)

If an area has an axis of symmetry, then the centroid must lie on that axis. If an area has two axes of symmetry, then the centroid must lie at the point of intersection of these axes.



CENTROID OF SIMPLE FIGURES

First moment of area for

Rectangle:-

Derivation using first principles

Triangle:-

Derivation using first principles

LECTURE-11

CENTROID OF SIMPLE FIGURES

First moment of area for

Semi-circle:-

Derivation using first principles

Illustrative Example

LECTURE-12

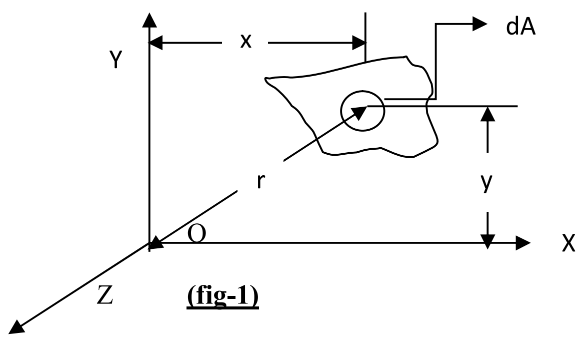
- **Second Moment of Area.**
- **Radius of gyration.**
- **Perpendicular Axis Theorem.**
- **Parallel Axis Theorem.**

MOMENT OF INERTIA (SECOND MOMENT OF AREA):-

Second moment of Area: The product of the elemental area and square of the perpendicular distance between the centroid of area and the axis of reference is the "Moment of Inertia" about the reference axis.Consider a plane area'A' and let OX, OY be its reference axes with 'dA' as the area of element, as shown in figure.

The product of area and square of the distance from reference axis is called second moment of area about the reference axis.

Second moment of area is also known as moment of inertia and is represented by the letter 'I'.



(fig-1)

The particular axis about which the second moment is considered is denoted by the subscripts. For eg. the second moment of area about XX and YY axes of area 'A' can be expressed as

$I_{ox}=\int y^2 dA$

$I_{oy}=\int x^2 dA$

And 'y' and 'x' are the distance of the elemental area dA from XX and YY axes respectively.

Note:

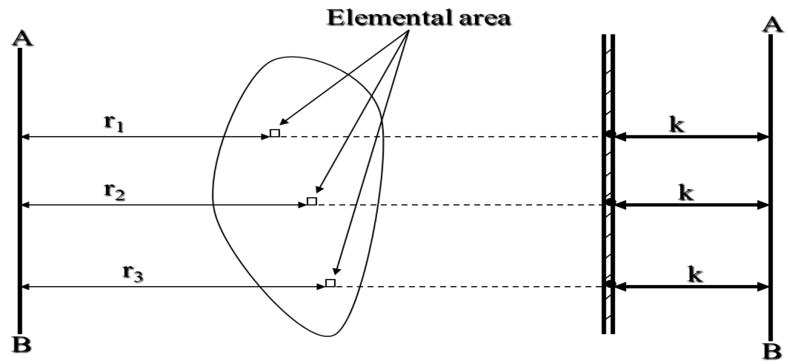
Second moment of area can be considered as the sum of a number of elements each consisting of an area multiplied by distance squared

unit is m^4.

- Second moment of area can be considered as the sum of a number of elements each consisting of an area multiplied by distance squared.
- The dimension of the second moment of area is given by L^4 and its unit is m^4.
- Sign of each term is +ve since the distance is squared.
- The first moment of area about the centroidal axis is zero where as the second moment of area about the centroidal axis is non zero.

- Moment of inertia plays a major role in design of beams, columns, machine and also helps in selection of members in structural design.

Radius of gyration



The term radius of gyration is defined as the distance from the axis to a point where the concentrated area of the same size could be placed to have the same second moment of area with respect to the given axis.

$I_{AB}= dak^2+dak^2+$--------------

$I_{AB}= \sum dak^2$

$I_{AB}= A\ k^2$

$k=\sqrt{I_{AB}/A}$

<u>Perpendicular Axis theorem: Ref (fig-1)</u>

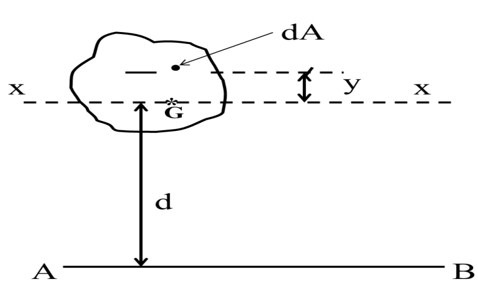
The moment of inertia of an area about an axis perpendicular to the plane of the area is called “Polar Moment of Inertia” and it is denoted by symbol I_{zz} or J or I_p. The moment of inertia of an area in xy plane w.r.to z. axis is $I_{zz} = I_p = J = \int r^2 dA = \int (x^2 + y^2)\ dA = \int x^2 dA + \int y^2 dA = I_{xx} + I_{yy}$

Hence polar M.I. for an area w.r.t. an axis perpendicular to its plane of area is equal to the sum of the M.I. about any two mutually perpendicular axes in its plane, passing through the point of intersection of the polar axis and the area.

Parallel Axis theorem

Moment of inertia of any area about any axis AB is equal to the M.I. about parallel centroidal axis plus the product of the total area and square of the distance between the two axes.

$I_{AB}= I_{xx}+Ad^2$



$I_{AB} = \int(y+d)^2 dA$

$I_{AB} = \int y^2 dA + d^2\int dA + 2d\int y dA$

Since $\int y dA$ about centroidal axis is zero,

$I_{AB} = Ixx + A.d^2$

Therefore

Moment of inertia of an area about an axis is equal to the sum of moment of inertia about an axis passing through the centroid parallel to the given axis and the product of area and square of the distance between the two parallel axis.

In parallel axis theorem, two important points to be noted are

1. the axes between which the transfer is made must be parallel
2. one of the axes must pass through the centroid of the area.

Application of moment of inertia

Moment of inertia (MI) is merely a mathematical term. However when used in combination with other terms (given below) it has significance

Bending Formula

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Torsion Formula

$$\frac{T}{I_p} = \frac{\tau}{R} = \frac{G\theta}{L}$$

Deflection formula

$$M = EI\frac{d^2y}{dx^2}$$

Column formula

$$P_{CR} = \frac{\pi^2 EI}{L^2}$$

Moment of inertia plays a major role in design of beams, columns, machine and also helps in selection of members in structural design.

LECTURE-13

MOMENT OF INERTIA BY DIRECT INTEGRATION

Second moment of area for

Rectangle:-

Derivation

Triangle:-

Derivation

Circle:-

Derivation

Centroid and moment of inertia of plane figures :

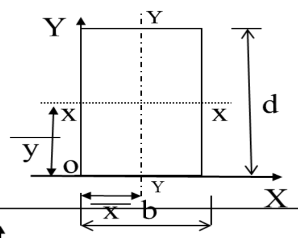
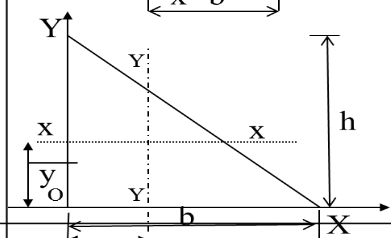
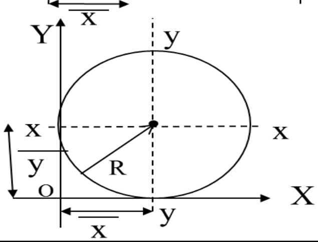
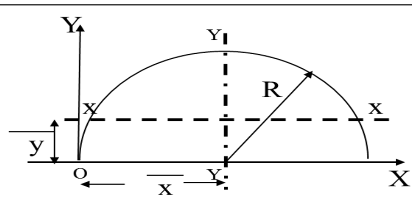
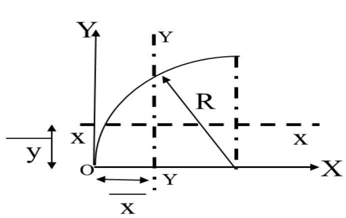
	FIGURE	$\bar{x}$	$\bar{y}$	I_{x-x}	I_{y-y}
1		b/2	d/2	$bd^3/12$	$db^3/12$
2		b/3	h/3	$bh^3/36$	$hb^3/36$
3		R	R	$\pi R^4/4$	$\pi R^4/4$

	FIGURE	$\bar{x}$	$\bar{y}$	I_{x-x}	I_{y-y}
4		R	$4R/3\pi$	$0.11R^4$	$\pi R^4/8$
5		$R - 4R/3\pi$	$4R/3\pi$	$0.055R^4$	$0.055R^4$

TUTORIAL-04

Application Problems of L10-L13

TUTORIAL-05

Application Problems of L10-L13

TUTORIAL-06

Application Problems of L10-L13

LECTURE-14

- ♦ **Introduction**
- ♦ **Mechanical properties of materials**
- ♦ **Normal stress and strain**
- ♦ **Hooke's law**
- ♦ **Modulus of elasticity**

♦ Introduction:

There are different types of structures built by man-buildings, bridges, water tanks, roads, transmission towers, machines etc. Such structures are built up of a number of structural elements joined together suitably. In the case of any structure, two types of designs are involved-functional or architectural and structural. Functional or architectural design deals with aspects other than strengths of the structure, like aesthetics, utility, orientation, general layout etc. Once this important aspect of design is taken care of, structural designers take over. Their work involves analysis of the structure and its elements to find the forces and moments that they have to withstand and then design the dimensions of the elements and their interconnections. Structures are designed to withstand loads, that is, forces and moments due to different causes. The subject strength of materials deals with the relations between externally applied loads and their internal effects on bodies. The bodies are no longer assumed to be rigid and the deformations, however small, are of major interest.

Strength of materials is an inter-disciplinary subject. Mechanical and chemical engineers need this subject to know the strength of various materials for the design of machineries and pressure vessels. Civil engineers need this subject for the design of trusses, slabs, beams, columns etc., of buildings and bridges. Aeronautical engineers need this subject for the design of aircraft. Similarly mining, electrical, electronics and computer engineers should also know this subject for one or the other reason.

An important aspect of the analysis and design of structures relates to the deformations caused by the loads applied to a structure. Clearly, it is important to avoid deformations so large that they may prevent the structure from fulfilling the purpose for which it was intended. By considering engineering structures as deformable and analyzing the deformations in their various members, it will be possible for us to compute forces that are statically indeterminate i.e. indeterminate within the framework of statics. To determine actual distribution of stresses within a member, it is necessary to analyze the deformations that take place in that member.

Strength of materials or mechanics of materials involves analytical methods for determining the **strength**, **stiffness** (deformation characteristics), and **stability** of various load carrying members.

♦ Mechanical properties of materials:

The following are the most important mechanical properties of engineering materials.

- Elasticity
- Plasticity
- Ductility
- Brittleness
- Malleability

- **Elasticity:**

Elasticity is the property by virtue of which a material deformed under the load is enabled to return to its original dimension when the load is removed. If a body regains completely its original shape, it is said to be perfectly elastic.

- **Plasticity:**

Plasticity is the property by virtue of which a material is permanently deformed by the application of load and it has no tendency to recover. Every elastic material possesses the property of plasticity. Under the action of large forces, most engineering materials become plastic and behave in a manner similar to a viscous liquid.

- **Ductility:**

Ductility is the property of a material which permits it to be drawn out in to slender rod or wire. Wrought metals such as steel, wrought iron, alloys of aluminium and alloys of copper are ductile. Ductile metals have high degree of plasticity beyond the elastic limit. Ductility is measured by the percentage of elongation occurring in a given distance between two points marked on the specimen along its length.

- **Brittleness:**

Brittleness in a material is indicated by its ability to undergo only slight deformation before rupturing. Brittle materials are much stronger in compression than in tension. Most cast metals are brittle immediately after casting.

- **Malleability:**

If the shape of a metal body can be altered by hammering or rolling the body without cracking or breaking the metal, the metal is said to be Malleable.

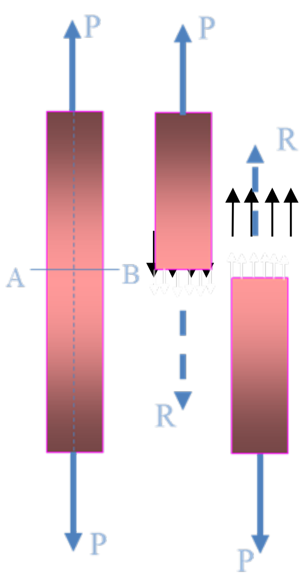
♦ <u>**Normal stress and strain:**</u>

Whenever a body is subjected to forces, the body deforms. The bodies in practice are not rigid. The body resists the deformation by developing stresses. The body reaches an equilibrium position when the internal stress resultant equals the external force. The deformation of the body and the stress can be related for any body depending upon its material property.

No engineering material is perfectly rigid and hence, when a material is subjected to external load, it undergoes deformation. While undergoing deformation, the particles of the material offer a resisting force (internal force). When this resisting force equals applied load the equilibrium condition exists and hence the deformation stops. These internal forces maintain the externally applied forces in equilibrium.

The internal force resisting the deformation per unit area is called as stress or intensity of **stress.**

$$\boxed{\text{Stress} = \frac{\text{Internal resisting force}}{\text{Resisting cross sectional area area}} = \frac{R}{A}}$$



Consider a uniform bar of cross sectional area *A*, subjected to a tensile force P. Consider a section *AB* normal to the direction of force P. Let R is the total resisting force acting on the cross section *AB*.

Then for equilibrium condition, R = P

Then from the definition of stress, normal stress = $\sigma = \frac{R}{P} = \frac{P}{A}$

Symbol: σ = Normal Stress

Intensity of resisting force perpendicular or normal to the section is called the **normal stress**.

Normal stress may be ***tensile*** or ***compressive.***

Tensile stress: Stresses that cause pulling on the surface of the section, (particles of the materials tend to pull apart causing extension in the direction of force).

Compressive stress: Stresses that cause pushing on the surface of the section, (particles of the materials tend to push together causing shortening in the direction of force).

Units:

SI unit for stress is N/m^2 also designated as a ***pascal*** (Pa)

Pascal, $Pa = N/m^2$

Kilopascal, $1kPa = 1000\ N/m^2$

Megapascal, $1\ MPa = 1\times10^6\ N/m^2 = 1\times10^6\ N/(10^6 mm^2) = 1N/mm^2$

Gigapascal, $1GPa = 1\times10^9\ N/m^2 = 1\times10^3\ MPa = 1\times10^3\ N/mm^2$

Strain:

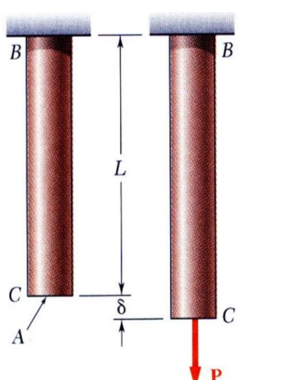
When a load acts on the material it will undergo deformation. is a measure of deformation produced by the application of external forces.

If a bar is subjected to a direct load, and hence a stress, the bar will changes in length. If the bar has an original length L and change in length by an amount δL, the linear strain produced is defined as,

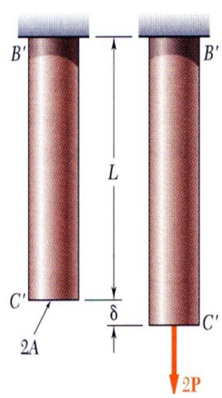
Linear strain,

$$\varepsilon = \frac{\delta L}{L} = \frac{\text{Change in the length}}{\text{Original length}}$$

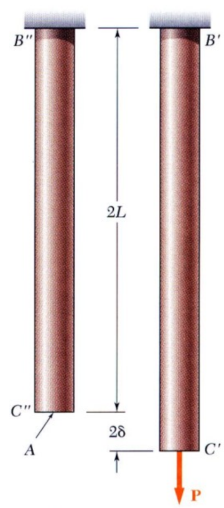
Strain is a dimensionless quantity



$$\sigma = \frac{P}{A} = \text{stress}$$
$$\varepsilon = \frac{\delta}{L} = \text{normal strain}$$



$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$
$$\varepsilon = \frac{\delta}{L}$$



$$\sigma = \frac{P}{A}$$
$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

Hooke's Law:

For all practical purposes, up to certain limit the relationship between normal stress and linear strain may be said to be linear for all materials

stress (σ) α strain (ε)

$$\frac{Stress(\sigma)}{Strain(\varepsilon)} = \text{constant}$$

Thomas Young introduced a constant of proportionality that came to be known as Young's modulus.

$$\frac{Stress(\sigma)}{Strain(\varepsilon)} = \text{Young's Modulus or Modulus of elasticity}$$

Young's Modulus is defined as the ratio of normal stress to linear strain within the proportionality limit.

$$E=\frac{Stress(\sigma)}{Strain(\varepsilon)} = \frac{\text{PL}}{\text{Adl}}$$

The value of the Young's modulus is a definite property of a material. From the experiments, it is known that strain is always a very small quantity, hence E must be large.

The following table shows modulus of elasticity of important materials:

Material	Modulus of elasticity
Steel	210 GPa
Aluminium	73Gpa
Brass	96 – 110 GPa
Cast Iron	83 – 170 GPa
Concrete	17 – 31 GPa
Rubber	0.0007 – 0.004 GPa
Tungsten	340 – 380 GPa

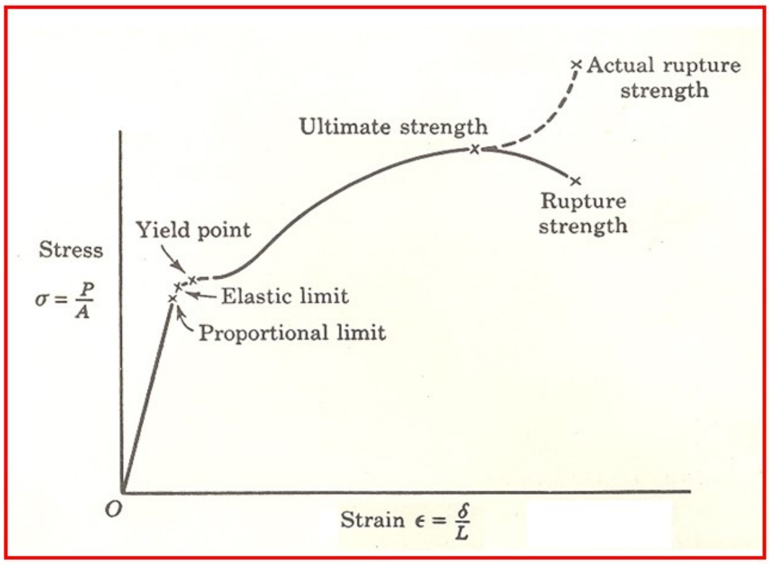
LECTURE -15

- **Tension test on ductile and brittle material**
- **Factor of safety**
- **Allowable stress**

- **Tension test on ductile and brittle material:**

  In order to compare the strength of various materials it is necessary to carry out some standard form of test to establish their relative properties. One such test is the standard tensile test in which a circular bar of uniform cross section is subjected to a gradually increasing tensile load until failure occurs. The test is conducted using Universal testing machine. Measurement of change in length over a selected gauge length of the bar is recorded throughout the loading operation by means of extensometers.

<u>Stress –strain diagram:</u> A graph of load verses extension or stress against strain is drawn as shown in the figure.



<u>Typical tensile test curve for mild steel</u>

Limit of Proportionality:

From the origin O to a point called proportionality limit the stress strain diagram is a straight line. That is stress is proportional to strain. Hence proportional limit is the maximum stress up to which the stress – strain relationship is a straight line and material behaves elastically.

From this we deduce the well known relation, first postulated by Robert Hooke, that stress is proportional to strain. Beyond this point, the stress is no longer proportional to strain

$$\sigma_P = \frac{\text{Load at proportionality limit}}{\text{Original crosssectional area}} = \frac{P_P}{A}$$

Elastic limit:

It is the stress beyond which the material will not return to its original shape when unloaded but will retain a permanent deformation called permanent set. For most practical purposes it can often be assumed that points corresponding proportional limit and elastic limit coincide.

Beyond the elastic limit plastic deformation occurs and strains are not totally recoverable. There will be thus some permanent deformation when load is removed.

$$\sigma_E = \frac{\text{Load at elatic limit}}{\text{Original cross sectional area}} = \frac{P_E}{A}$$

Yield point:

It is the point at which there is an appreciable elongation or yielding of the material without any corresponding increase of load.

$$\sigma_Y = \frac{\text{Load at yield point}}{\text{Original cross sectional area}} = \frac{P_Y}{A}$$

Ultimate strength:

It is the stress corresponding to maximum load recorded during the test. It is stress corresponding to maximum ordinate in the stress-strain graph.

$$\sigma_U = \frac{\text{Maximum load taken by the material}}{\text{Original cross sectional area}} = \frac{P_U}{A}$$

Rupture strength (Nominal Breaking stress):

It is the stress at failure. For most ductile material including structural steel breaking stress is somewhat lower than ultimate strength because the rupture strength is computed by dividing the rupture load (Breaking load) by the original cross sectional area.

$$\sigma_B = \frac{\text{Load at failure}}{\text{Original cross sectional area}} = \frac{P_B}{A}$$

True breaking stress:

$$\sigma_B = \frac{\text{Load at failure}}{\text{Actual cross sectional area at rupture}} = \frac{P_B}{A_{Act}}$$

Ductile Materials:

The capacity of a material to allow these large plastic deformations is a measure of ductility of the material. The capacity of a material to allow large extension i.e. the ability to be drawn out plastically is termed as its ductility. Material with high ductility are termed ductile material.

Example: Low carbon steel, mild steel, gold, silver, aluminum

A measure of ductility is obtained by measurements of the percentage elongation or percentage reduction in area, defined as,

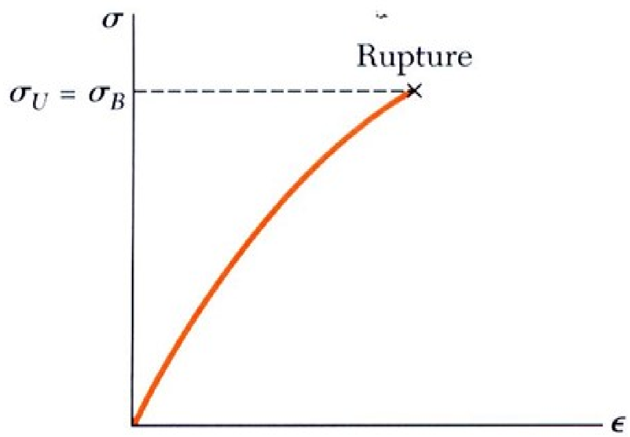
$$\text{Percentage elongation} = \frac{\text{Increase in the gauge length (upto fracture)}}{\text{Original gauge length}} \text{x} 100$$

$$\text{Percentage reduction in area} = \frac{\text{Reduction in cross sectional area of neck portion (at fracture)}}{\text{Original cross sectional area}} \text{x} 100$$

<u>Brittle Materials</u>: (Stress-strain Diagram)

A brittle material is one which exhibits relatively small extensions before fracture so that plastic region of the tensile test graph is much reduced.

Example: steel with higher carbon content, cast iron, concrete, brick



Stress-strain diagram for a typical brittle material

<u>Working stress</u>: It is obvious that one cannot take risk of loading a member to its ultimate strength, in practice. The maximum stress to which the material of a member is subjected to in practice is called working stress.

This value should be well within the elastic limit in elastic design method.

<u>Factor of safety</u>: Because of uncertainty of loading conditions, design procedure, production methods, etc., designers generally introduce a *factor of safety* into their design, defined as follows

$$\text{Factor of safety} = \frac{\text{Maximum stress}}{\text{Allowable working stress}} \; OR \; \frac{\text{Yield stress (or proof stress)}}{\text{Allowable working stress}}$$

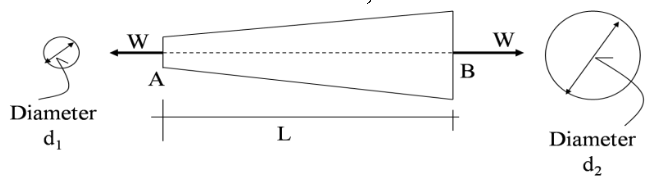
<u>LECTURE 16:</u>

Application Problems of L19 – L20

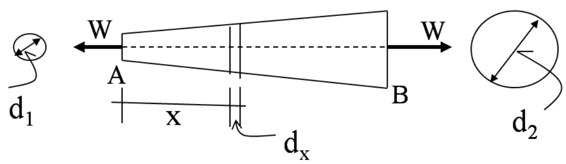
<u>LECTURE 17:</u>

- **Expression for deformation of a tapered Circular bar**
- **Expression for deformation of a Trapeziodal Plate**
- **Numerical problems 6-7**

Derive an expression for the total extension of the tapered bar of circular cross section shown in the figure, when subjected to an axial tensile load , W.



Consider an element of length, dx at a distance x from A.



Diameter at x, $= d_1 + \frac{(d_2 - d_1)}{L} \times x$, c/s area at x, $= \pi d_1^2 / 4 = \frac{\pi}{4}(d_1 + kx)^2$

Change in length over a length dx is, $= \left(\frac{PL}{AE}\right)_{dx} = \left(\frac{Wdx}{\frac{\pi}{4}(d_1 + kx)^2 \times E}\right)$

Change in length over a length L is, $= \int_0^L \left(\frac{Wdx}{\frac{\pi}{4}(d_1 + kx)^2 \times E}\right)$

$$= \int_{d_1}^{d_2} \left(\frac{W\frac{dt}{k}}{\frac{\pi}{4}(t)^2 \times E}\right)$$

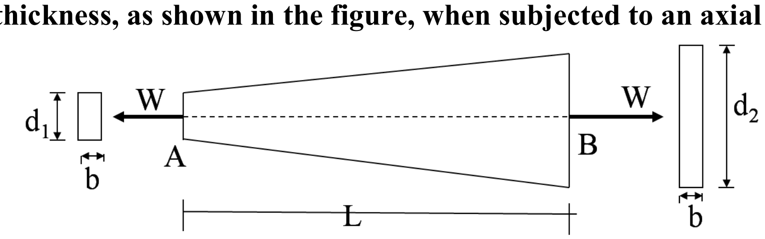
Put $d_1 + kx = t$,

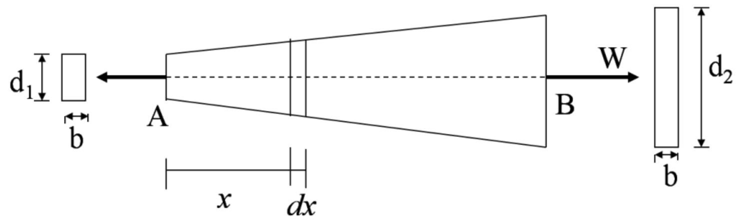
Then, $k\,dx = dt$

$$= \frac{4W}{\pi E k}\left[\frac{t^{-2+1}}{-1}\right]_{d_1}^{d_2} = \frac{4W}{\pi E k}\left[\frac{-1}{t}\right]_{d_1}^{d_2} = \frac{-4W}{\pi E k}\left[\frac{1}{(d_1 + kx)}\right]_{d_1}^{d_2}$$

$$= \frac{4WL}{\pi E d_1 d_2} = \frac{WL}{\frac{\pi d_1 d_2}{4} \times E}$$

Derive an expression for the total extension of the tapered bar AB of rectangular cross section and uniform thickness, as shown in the figure, when subjected to an axial tensile load ,W.





Consider an element of length, dx at a distance x from A,

depth at x, $= d_1 + \frac{(d_2 - d_1)}{L} \times x \quad = d_1 + k \times x$ c/s area at x, $= (d_1 + kx)b$

Change in length over a length dx is, $= \left(\frac{PL}{AE}\right)_{dx} = \left(\frac{Wdx}{(d_1 + kx)b \times E}\right)$

Change in length over a length L is,

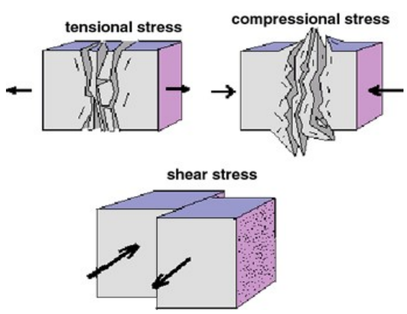
$$= \int_0^L \left(\frac{Wdx}{(d_1 + kx)b \times E}\right) \qquad = \frac{P}{b \times E \times k}\left(\log_e d_2 - \log_e d_1\right)$$

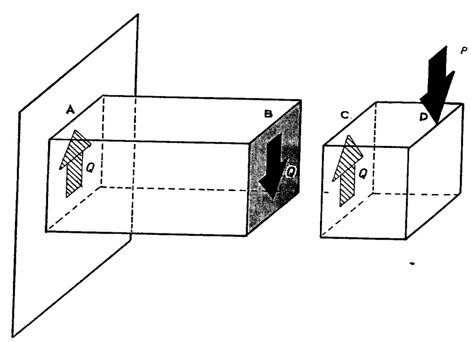
Application Problems of L22

LECTURE-18

- **Shear stress**
- **Shear strain**
- **Modulus of rigidity**
- **State of simple shear & Complementary shear**
- **Direct stress due to pure shear**

- **Shear stress:**





Consider a block or portion of a material shown in Fig.(a) subjected to a set of equal and opposite forces P. then there is a tendency for one layer of the material to slide over another to produce the form of failure as shown in Fig.(b)

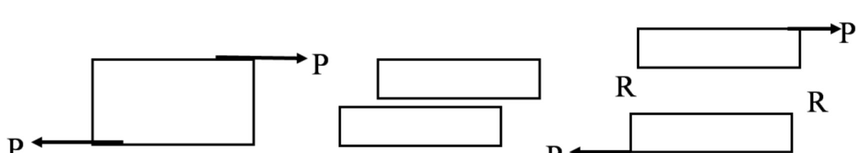


Fig. a Fig. b Fig. c

The resisting force developed by any plane (or section) of the block will be parallel to the surface as shown in Fig.(c).

The resisting forces acting parallel to the surface per unit area is called as shear stress.

$$\text{Shear stress}(\tau) = \frac{\text{Shear resistance}}{\text{Area resisting shear}} = \frac{R}{A} = \frac{P}{A}$$

This shear stress will always be tangential to the area on which it acts.

If block ABCD subjected to shearing stress as shown in Fig.(d), then it undergoes deformation. The shape will not remain rectangular, it changes into the form shown in Fig.(e), as AB′C′D.

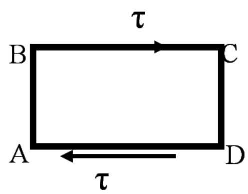


Fig. d

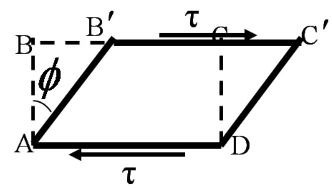


Fig. e

Shear strain is defined as the change in angle between two line element which are originally right angles to one another.

$$\text{shear strain} = \frac{BB'}{AB} = \tan\phi \approx \phi$$

The angle of deformation   is then termed as shear strain

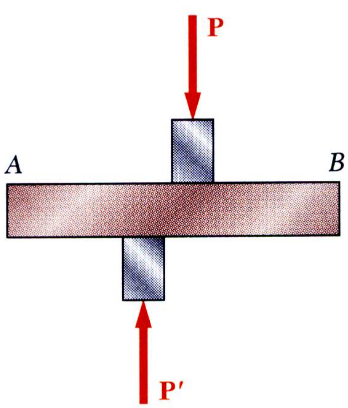
The angle of deformation is measured in radians and hence is non-dimensional.

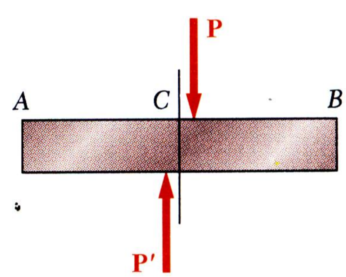
- **Shear modulus:**

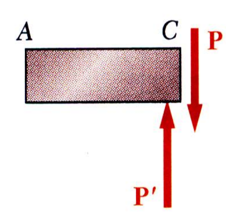
For materials within the proportionality limit the shear strain is proportional to the shear stress. Hence the ratio of shear stress to shear strain is a constant within the proportionality limit.

$$\frac{\text{Shear stress } (\tau)}{\text{Shear strain } (\phi)} = \text{constant} = \text{G} = \text{Shear Modulus or Modulus of Rigidity}$$

For Mild steel, G= 80GPa = 80,000MPa = 80,000N/mm^2







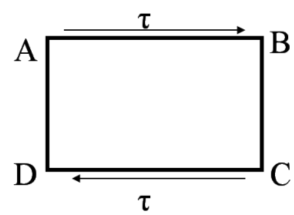
- Forces P and P' are applied transversely to the member AB.
- Corresponding internal forces act in the plane of section C and are called *shearing* forces.
- The resultant of the internal shear force distribution is defined as the *shear* of the section and is equal to the load P.

$$\tau_{\text{ave}} = \frac{P}{A}$$

- The corresponding average shear stress is,
- The shear stress distribution cannot be assumed to be uniform.

♦ State of simple shear:

Consider an element ABCD in a strained material subjected to shear stress, τ as shown in the figure .



Force on the face AB = P = $\tau \times AB \times t$

Where, t is the thickness of the element.

Force on the face DC is also equal to P

Now consider the equilibrium of the element.

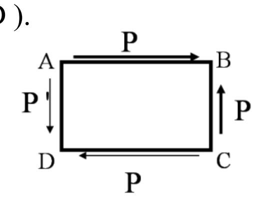
(i.e., $\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma M = 0$.)

For the force diagram shown,

$\Sigma Fx = 0$, & $\Sigma Fy = 0$, But ΣM not equal to 0

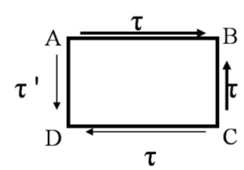
The element is subjected to a clockwise moment $= P \times AD = (\tau \times AB \times t) \times AD$

But, as the element is actually in equilibrium, there must be another pair of forces say P' acting on faces AD and BC, such that they produce a anticlockwise moment equal to (P × AD).



$P' \times AB = P \times AD = (\tau \times AB \times t) \times AD$ ----- (1)

If τ^1 is the intensity of the shear stress on the faces AD and BC, then P ' can be written as, $P' = \tau' \times AD \times t$

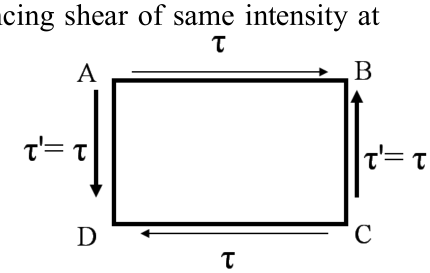


Eqn.(1) can be written as

$(\tau' \times AD \times t) \times AB = (\tau \times AB \times t) \times AD$ ----- (1)

$\tau' = \tau$

Thus in a strained material a shear stress is always accompanied by a balancing shear of same intensity at right



angles to itself. This balancing shear is called "complementary shear"

.

The shear and the complementary shear together constitute a state of simple shear.

- **Direct stress due to pure shear:**

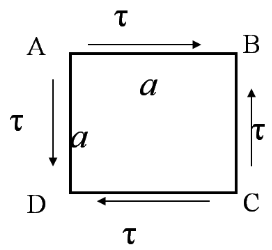


Fig.(a).

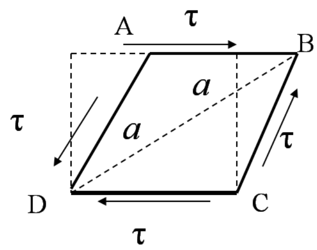
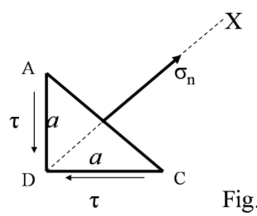


Fig.(b).

Fig.(b) shows the deformed shape of the element. The length of diagonal DB increases, indicating that it is subjected to tensile stress. Similarly the length of diagonal AC decreases indicating that compressive stress. Now consider the section, ADC of the element, Fig.(c).



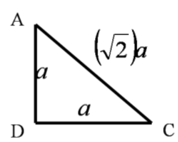


Fig.(c).

Resolving the forces in σ_n direction, i.e., in the X-direction shown. For equilibrium,

$$\sum Fx = 0$$
$$= \sigma_n \times \left(\sqrt{2} \times a \times 1\right) - 2\left(\tau \times a \times \cos 45\right)$$
$$\sigma_n = \tau$$

Therefore the intensity of normal tensile stress developed on plane BD is numerically equal to the intensity of shear stress. Similarly it can be proved that the intensity of compressive stress developed on plane AC is numerically equal to the intensity of shear stress.

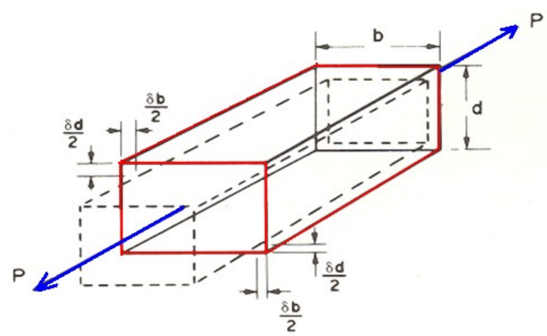
LECTURE-19

- **Poissons ratio**
- **Volumetric strain**
- **Bulk modulus**
- **Relationship between volumetric strain and linear strain**
- **Numerical problems 8-10**

- **Poissons ratio:**

Consider the rectangular bar shown in Fig.(a) subjected to a tensile load. Under the action of this load the bar will increase in length by an amount δL giving a longitudinal strain in the bar of

$$\varepsilon_l = \frac{\delta l}{l}$$



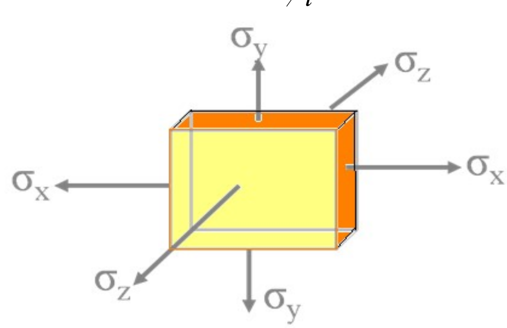
The bar will also exhibit, reduction in dimension laterally, i.e. its breadth and depth will both reduce. These change in lateral dimension is measured as strains in the lateral direction as given below.

$$\varepsilon_{lat} = -\frac{\delta b}{b} = -\frac{\delta d}{d}$$

The associated lateral strains will be equal and are of opposite sense to the longitudinal strain provided the load on the material is retained within the elastic range. The ratio of the lateral and longitudinal strains will always be constant. This ratio is termed ***Poisson's ratio*** **(μ).**

$$\text{Poissons ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{(-\delta d/d)}{\delta l/l} \quad \text{OR} \quad \frac{(-\delta b/b)}{\delta l/l}$$

General case:



Strain in X-direction = $\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$

Strain in Y-direction = $\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$

Strain in Z-direction = $\varepsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$

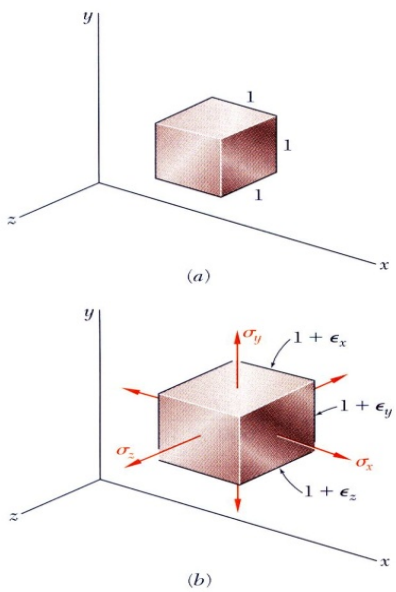
- **Bulk Modulus:**

A body subjected to three mutually perpendicular equal direct stresses undergoes volumetric change without distortion of shape.

If V is the original volume and dV is the change in volume, then dV/V is called volumetric strain.

A body subjected to three mutually perpendicular equal direct stresses then the ratio of stress to volumetric strain is called Bulk Modulus. K $= \frac{\sigma}{\left(\frac{dV}{V}\right)}$

- **Relationship between volumetric strain and linear:**



Consider a cube of side 1unit, subjected to three mutually perpendicular direct stresses as shown in the figure.

Relative to the unstressed state, the change in volume per unit volume is

$$\frac{dV}{1} = \left[(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)\right] - 1 = \left[1+\varepsilon_x+\varepsilon_y+\varepsilon_z\right] - 1$$

$= \varepsilon_x + \varepsilon_y + \varepsilon_z$

= change in volume per unit volume

Volumetric strain= $\frac{dV}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$$= \left(\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}\right) + \left(\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}\right) + \left(\frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}\right)$$

$$\frac{dV}{V} = \frac{1-2\mu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

$$\frac{dV}{V} = \frac{1-2\mu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

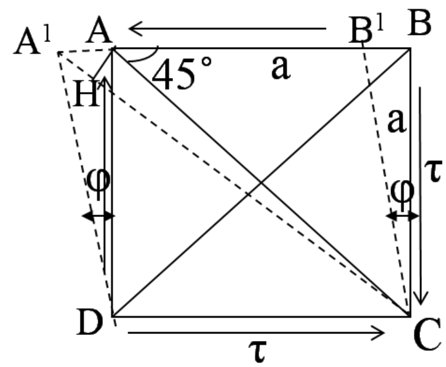
$$\frac{dV}{V} = \frac{1-2\mu}{E}(3\sigma)$$

$$But\ K = \frac{\sigma}{\left(\frac{dV}{V}\right)}$$

LECTURE-20

- **Relationship between modulus of elasticity and modulus of rigidity**
- **Relationship between E, G and K**
- **Numerical problems- 11-13**

- **Relationship between young's modulus of elasticity (E) and modulus of rigidity (G):**



Consider a square element ABCD of side 'a' subjected to pure shear 'τ'. DA'B'C is the deformed shape due to shear τ. Drop a perpendicular AH to diagonal A'C.

Strain in the diagonal AC = $\tau/E - \mu(-\tau/E)$ [$\sigma_n = \tau$]

$= \tau/E\,[1 + \mu]$ -----------(1)

Strain along the diagonal AC=(A'C–AC)/AC=(A'C–CH)/AC=A'H/AC

In Δle AA'H

Cos 45° = A'H/AA'

A'H= AA' × 1/√2

AC = √2 × AD ($AC = \sqrt{AD^2 + AD^2}$)

Strain along the diagonal AC = $\frac{A'H}{AC} = \frac{AC}{\sqrt{2}\sqrt{2}AD} = \frac{AA'}{2AD} = \frac{\phi}{2}$ ---(2)

Modulus of rigidity = G = τ /φ

φ = τ /G

Substituting in (2)

Strain along the diagonal AC = τ /2G ------(3)

Equating (1) & (3)

$\tau/2G = \tau/E[1+\mu]$

E=2G(1+ μ)

- **Relationship between E, G and K:**

We have

$E = 2G(1+ \mu)$ -----------(1)

$E = 3K(1-2\mu)$ -----------(2)

Equating (1) & (2)

$2G(1+ \mu) =3K(1- 2\mu)$

$2G + 2G\mu=3K- 6K\mu$

$\mu= (3K- 2G) /(2G +6K)$

Substituting in (1)

$E = 2G[1+(3K – 2G)/ (2G+6K)]$

$E = 18GK/(2G+6K)$

$\mathbf{E = 9GK/(G+3K)}$

TUTORIAL-07

Application Problems T1 – T4

TUTORIAL - 08

Application Problems T5 – T8

TUTORIAL - 09

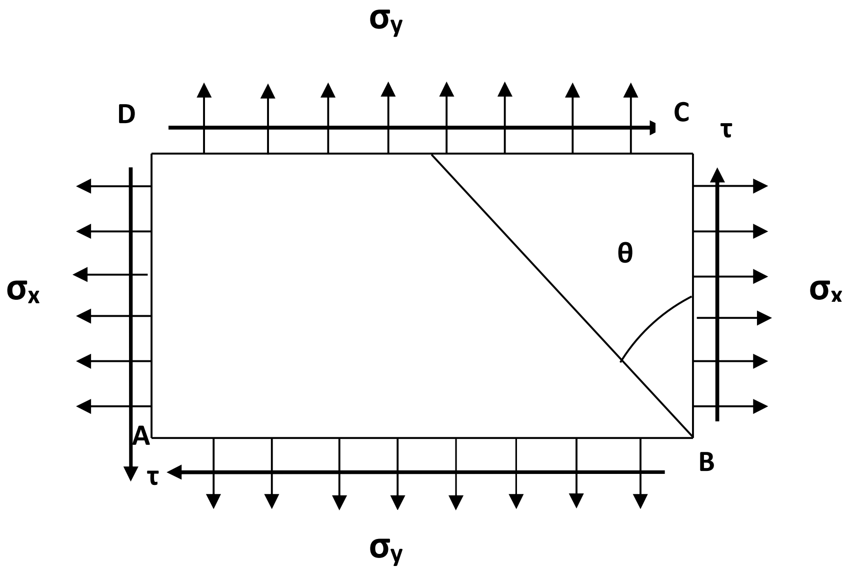
Application Problems T9 – T11

LECTURE-21

Stresses due to fluid pressure in Thin Cylinders

- **Introduction**
- **Circumferential Stress or Hoop Stress.**
- **Longitudinal Stress**
- **Maximum Shear Stress**
- **Evaluation of Strain**
- **Joint Efficiency**
- **Application Problems N 1-2**

Introduction

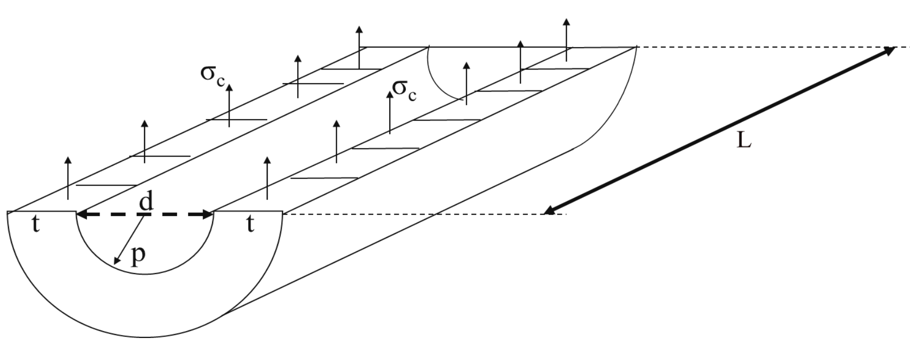


A cylinder is considered to be thin when the metal thickness is small compared to internal diameter. i. e., when the wall thickness, ‘t’ is equal to or less than ‘d/20’, where ‘d’ is the internal diameter of the cylinder or shell, we consider the cylinder or shell to be thin, otherwise thick.

Cylindrical or spherical vessels are commonly used in industies to serve as boilers or tanks. When under pressure, the material of which they are made is subjected to a loading from all directions. Although this is the case, the vessel can be analyzed in a simpler manner provided it has a thin wall. In general, “thin walled cylinder” refers to a vessel having an inner radius to wall thickness ratio of 10 or more ($r/t >= 10$). Specifically, when $r/t = 0$ the results of a thin-wall analysis will predict a stress that is approximately 4% less than the actual maximum stress in the vessel. For larger r/t ratios this error will be even smaller.

When the vessel wall is thin, the stress distribution throughout its thickness will not vary significantly, and so we will assume that it is uniform or constant. Using this assumption, one can analyse the state of stress in thin-walled cylindrical and spherical pressure vessels. In both cases, the pressure in the vessel is understood to be the gauge pressure, since it measures the pressure above atmospheric pressure, which is assumed to exist both inside and outside the vessel’s wall.

CIRCUMFERENTIAL or HOOP STRESS (σ_C):



Consider a thin cylinder of internal diameter d, thickness 't' subjected to internal pressure 'p' as shown in the figure.

Bursting force = Pressure intensity × projected area of curved surface with horizontal

= p × d × L------------------(1)

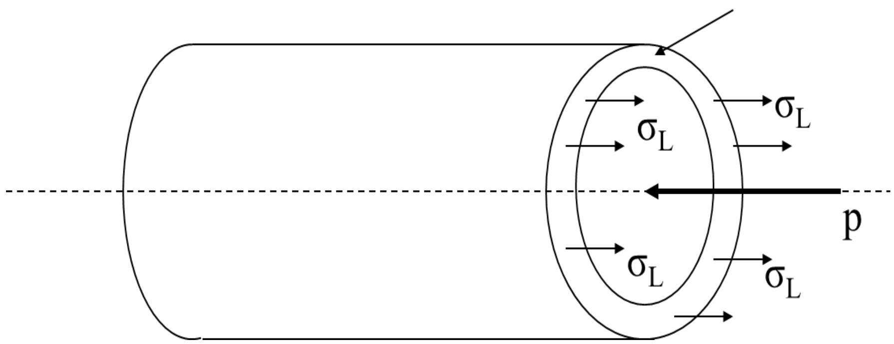
Force due to hoop stress = 2×σ_c×t×L--------------------(2)

For equilibrium, equating (1) & (2) we get

$2\times\sigma_c\times t\times L = p \times d \times L$

$\sigma_c = Pd/2t$

LONGITUDINAL STRESS (σ_L):



$$\text{Longitudinal bursting force (on the end plates)} = p\times\frac{\pi}{4}\times d^2$$

Area of cross section resisting this force $= \pi \times d \times t$

Let σ_L = Longitudinal stress of the material of the cylinder.

$\therefore$ Resisting force $= \sigma_L \times \pi \times d \times t$

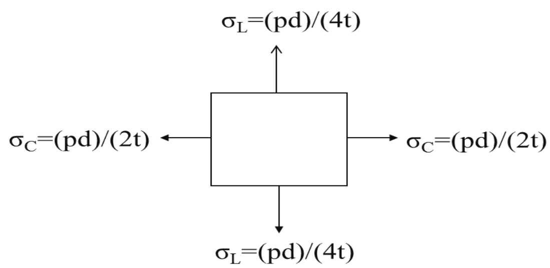
Under equillibrium, bursting force = resisting force

$$\text{i.e., } p\times\frac{\pi}{4}\times d^2 = \sigma_L \times \pi \times d \times t$$

$$\therefore \quad \text{Longitudinal stress, } \sigma_L = \frac{p\times d}{4\times t}$$

$$\sigma_C = 2\times\sigma_L$$

EVALUATION OF STRAINS:



A point on the surface of thin cylinder is subjected to biaxial stress system, (Hoop stress and Longitudinal stress) mutually perpendicular to each other, as shown in the figure.

Circumferential strain, ε_c:

$$\underline{\text{Circumfere ntial strain, } \varepsilon_C :}$$

$$\varepsilon_C = \frac{\sigma_C}{E} - \mu \times \frac{\sigma_L}{E} = 2 \times \frac{\sigma_L}{E} - \mu \times \frac{\sigma_L}{E}$$

$$= \frac{\sigma_L}{E} \times (2 - \mu)$$

$$\text{i.e.,} \quad \underline{\underline{\varepsilon_C = \frac{\delta d}{d} = \frac{p \times d}{4 \times t \times E} \times (2 - \mu)}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\underline{\text{Longitudin al strain, } \varepsilon_L :}$$

$$\varepsilon_L = \frac{\sigma_L}{E} - \mu \times \frac{\sigma_C}{E} = \frac{\sigma_L}{E} - \mu \times \frac{(2 \times \sigma_L)}{E} = \frac{\sigma_L}{E} \times (1 - 2 \times \mu)$$

$$\text{i.e.,} \quad \underline{\underline{\varepsilon_L = \frac{\delta l}{L} = \frac{p \times d}{4 \times t \times E} \times (1 - 2 \times \mu)}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

VOLUMETRIC STRAIN :

$$\text{We have volume, } V = \frac{\pi}{4} \times d^2 \times L.$$

$$\frac{dv}{v} = \frac{\frac{\pi}{4}(d + \delta d)^2 \times (L + \delta L) - \frac{\pi}{4} \times d^2 \times L}{\frac{\pi}{4} \times d^2 \times L}$$

$$\frac{dv}{v} = \frac{(d^2 + \delta d^2 + 2d \times \delta d)(L + \delta L) - d^2 L}{d^2 L}$$

$$\frac{\mathbf{dV}}{\mathbf{V}} = \frac{(d^2 \times L) + (\delta d^2 \times L) + (2d \times \delta d \times L) + (d^2 \times \delta L) + (\delta d^2 \times \delta L) + (2d \times \delta d \times \delta L) - d^2 \times L)}{d^2 L}$$

Neglecting smaller quantities we have

$$\frac{\mathbf{dV}}{\mathbf{V}} = \frac{(2d \times \delta d \times L) + (d^2 \times \delta L)}{d^2 L}$$

$$\frac{dV}{V} = \frac{2\delta d}{d} + \frac{\delta L}{L}$$

$$\frac{dV}{V} = 2\varepsilon_c + \varepsilon_L$$

$$\frac{dV}{V} = \frac{2Pd}{2tE}(1 - \frac{\mu}{2}) + \frac{Pd}{2tE}(\frac{1}{2} - \mu)$$

$$\frac{dV}{V} = \frac{Pd}{2tE}(2 - \mu) + \frac{Pd}{2tE}(\frac{1}{2} - \mu)$$

$$dV = \frac{Pd}{2tE}(\frac{5}{2} - 2\mu)V$$

Maximum Shear stress :

$$\text{Maximum Shear stress, } \tau_{max} = \frac{\sigma_C - \sigma_L}{2}$$

$$= \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2}$$

$$\text{i.e., } \tau_{max} = \frac{pd}{8t}$$

- **JOINT EFFICIENCY**

The cylindrical shells like boilers are having two types of joints namely Longitudinal and Circumferential joints.

Let η_L=Efficiency of Longitudinal joint

and η_C=Efficiency of Circumferential joint.

Circumferential stress is given by,

$$\sigma_C = \frac{p \times d}{2 \times t \times \eta_L} \quad \text{............(1)}$$

Longitudinal stress is given by,

$$\sigma_L = \frac{p \times d}{4 \times t \times \eta_C} \quad \text{............(2)}$$

Application problems N 1-2

LECTURE-22

Compound bars

- **Introduction to statically indeterminate problems**
- **Compound bars subjected to external loads**
- **Illustrative Example**

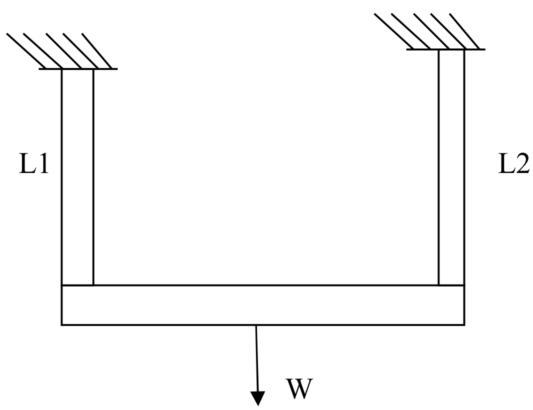
Structure for which equilibrium equations are sufficient to obtain the solution are classified as statically determinate. But for some combination of members subjected to axial loads, the solution cannot be obtained by merely using equilibrium equations.The structural problems with number of unknowns greater than the number independent equilibrium equations are called statically indeterminate.

A compound bar is a form of statically indeterminate structure. A compound bar is one which is made of two or more than two materials rigidly fixed, so that they sustain together an externally applied load sharing the load in correspondence with their modulus of Elasticity and cross sectional area.

In such cases

(i) Applied load is equal to sum of the loads carried by each bar (Equilibrium condition).

(ii) Change in length in all the materials are same or proportional, depending on the condition (compatibility condition).



The following equations are required to solve the problems on statically indeterminate structure.

1) Equilibrium equations based on free body diagram of the structure or part of the structure.

2) Equations based on geometric relations regarding elastic deformations, produced by the loads (Compatibility Equation).

structural member composed of two or more elements of different materials rigidly connected together at their ends to form a parallel arrangement and subjected to axial loading is termed a compound bar. The different materials of the member may have same length or different length.

For a compound/composite bar,

i. Total external load on the composite bar is equal to the sum of the loads carried by them independently i.e. $P_1+P_2 = W$

$\mathbf{W = \sigma_1 A_1 + \sigma_2 A_2}$

ii. Extension or compression in each bar is equal (strain in each bar is equal if equal length)

i.e $\delta_1 = \delta_2$

$(\sigma_1/ E_1)L_1 = (\sigma_2 /E_2)L_2$

$\sigma_1 = \sigma_2 \times (E_1/E_2)(L_2/L_1)$

E1/E2 is called modular ratio

Application Problems N 1-2

LECTURE- 23

Application Problems N 3-4

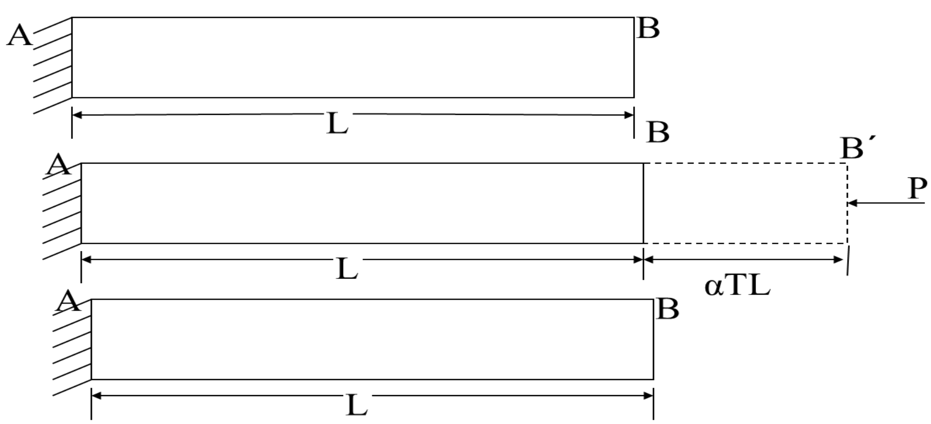
TUTORIAL - 10

Tutorial Class T1 –T2

LECTURE- 24

- **Temperature stress**
- **Compound bars subjected to temperature stresses**
- **Illustrative problems**

Any material is capable of expanding or contracting freely due to rise or fall in temperature. If it is subjected to rise in temperature of T˚C, it expands freely by an amount 'αTL' as shown in figure. Where α is the coefficient of linear expansion, T˚C = rise in temperature and L = original length.



However, if the material is constrained and is not fully or partly free to change its length, then stresses are developed in the material, called temperature (or thermal) stresses. If the material is constrained fully, then there is no change in length (and no corresponding strain). The material is however stressed because it is not free to expand or contract to its normal length.

From the above figure it is seen that 'B' shifts to B' by an amount 'αTL'. If this expansion is to be prevented a compressive force is required at B'.

Temperature strain = $\alpha TL/(L + \alpha TL) \approx \alpha TL/L = \alpha T$

Temperature stress = αTE

Hence the temperature strain is the ratio of expansion or contraction prevented to its original length.

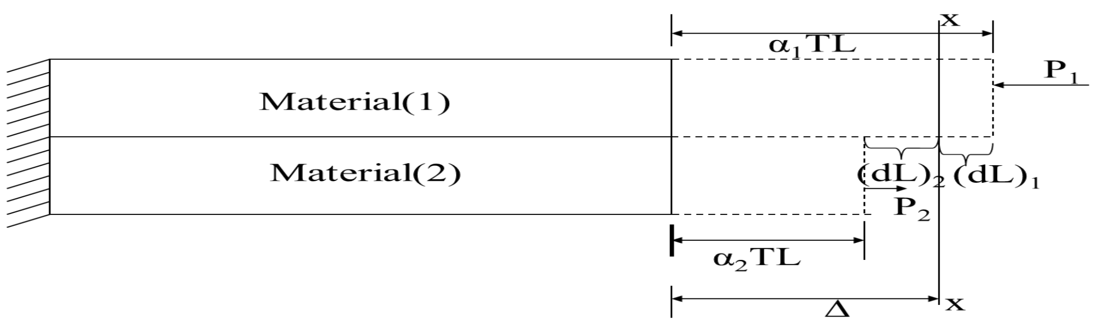
If a gap δ is provided for expansion then

Temperature strain = $(\alpha TL - \delta) / L$

Temperature stress = $[(\alpha TL - \delta)/L]\, E$

Thermal stresses in composite bars

The above discussion applies to bars of a single material only. When a bar consists of two materials rigidly joined together, thermal stresses are induced even without the bar being constrained. This is due to the difference in the thermal expansion characteristics of the materials.



When a compound bar is subjected to change in temperature, both the materials will experience stresses of opposite nature.

Compressive force on material (1) = tensile force on material (2)

$$\sigma_1 A_1 = \sigma_2 A_2 \text{ (there is no external load)}$$

$\sigma_1 = (\sigma_2 A_2)/A_1$ ------------------ (1)

As the two bars are connected together, the actual position of the bars will be at XX.

Actual expansion in material (1) = actual expansion in material (2)

$$\alpha_1 TL - (dL)_1 = \alpha_2 TL + (dL)_2$$

$$\alpha_1 TL - (\sigma_1 / E_1)\, L = \alpha_2 TL + (\sigma_2 / E_2)\, L$$

$\alpha T - (\sigma_1 / E_1) = \alpha_2 T + \sigma_2 / E_2$ --------------------------(2)

From (1) and (2) magnitude of σ_1 and σ_2 can be found out.

Application problem N-5

TUTORIAL - 11

Tutorial Class T3 –T4

TUTORIAL - 12

Tutorial Class T5 –T7

Shear Force and Bending Moment Diagrams
[SFD & BMD]

- **Introduction**
- **Shear force and Bending moment at a section.**
- **Sign Convention**

Introduction:

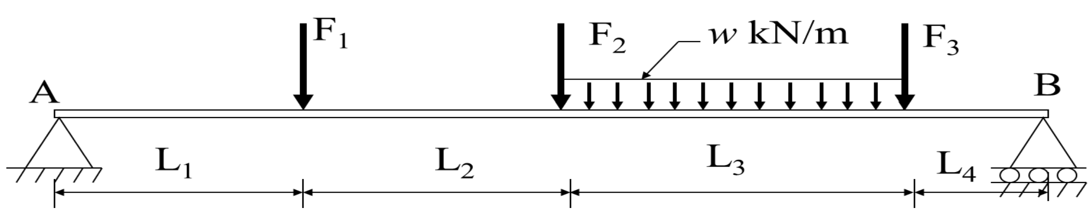
A beam is said to be statically determinate, if its reaction components can be determined by using equations of static equilibriums only. Commonly encountered statically determinate beams are :

a) Cantilever beams
b) Simply supported beams, and
c) Overhanging beams ..

These beams are usually subjected to the following types of transverse loads : *i)* Concentrated loads
ii) Uniformly distributed loads *iii)* Uniformly varying loads,
and *iv)* Externally applied moments.

The beams transfer the applied loads to the support. In the process of transferring the applied loads to the supports the beams develop resistance to moments and transverse shear forces at all its cross sections.

In this chapter the moment and shear force caused at all sections of beams will be determined and shown diagrammatically.



The above Fig. shows that a simply supported beam is subjected to typical transverse loading. When the beam, in general, subjected to transverse loads, the shear force and bending moments are varying from section to section of the beam. It is possible to calculate these shear force and bending moments at various sections of the beam. If we plot the calculated shear force and bending moments along length of beam the diagrams what we get are called shear force diagram and bending moment diagram.

Shear Force and Bending Moments at a section

Shear force at a section: The algebraic sum of the vertical forces acting on the beam either to the left or right of the section is known as the *shear force at a section.*

Sign convention for shear forces:

LEFT side portion of the section is considered

Positive (+) Sign → For the vertical forces acting upward

Negative (-) Sign → For the vertical forces acting downward

<u>RIGHT side portion of the section is considered</u>

Positive (+) Sign → For the vertical forces acting downward

Negative (-) Sign → For the vertical forces acting upward.

<u>Bending moment (BM) at section:</u> The algebraic sum of the moments of all forces acting on the beam either to the left or right of the section is known as the *bending moment at a section*

<u>Sign convention for bending moments:</u>

The bending moment is considered as <u>Sagging Bending Moment</u> if it tends to bend the beam to a curvature having convexity at the bottom as shown in the Fig. given below. Sagging Bending Moment is considered as positive bending moment.

Similarly the bending moment is considered as hogging bending moment if it tends to bend the beam to a curvature having convexity at the top as shown in the Fig. given below. Hogging Bending Moment is considered as Negative Bending Moment.

<u>Shear Force and Bending Moment Diagrams</u>
(SFD & BMD)

<u>Shear Force Diagram (SFD):</u>

The diagram which shows the variation of shear force along the length of the beam is called *Shear Force Diagram (SFD).*

<u>Bending Moment Diagram (BMD):</u>

The diagram which shows the variation of bending moment along the length of the beam is called *Bending Moment Diagram (BMD).*

<u>Point of Contra flexure [Inflection point]:</u>

It is the point on the bending moment diagram where bending moment changes the sign from positive to negative or vice versa. It is also called Inflection point. At the point of contra flexure or inflection point the bending moment is zero.

Variation of Shear force and bending moments

Type of load / SFD/BMD	**Between point loads OR for no load region**	**Uniformly distributed load**	**Uniformly varying load**
Shear Force Diagram	Horizontal line	Inclined line	Two-degree curve (Parabola)
Bending Moment Diagram	Inclined line	Two-degree curve (Parabola)	Three-degree curve (Cubic-parabola)