Scheme Set 3 (IN Sem Exam Mathematics CCE/ICT/CSE)

1. Let a, b, c be elements in a lattice (A, ≤). Show that, a ≤ b if and only if $a\vee(b\wedge c) \leq b \wedge (a\vee c)$.
   Solution: Suppose $a \leq b$.
   As $a \leq b$ and $a \leq (a\vee c)$, we get $a \leq b\wedge(a\vee c)$-----(1) 1M
   Now, we have $(b\wedge c) \leq b$ and $(b\wedge c) \leq c \leq (a\vee c)$.
   Thus $(b\wedge c) \leq b\wedge(a\vee c)$--------------------------------(2)
   From (1) and (2), we get $a\vee(b\wedge c) \leq b\wedge(a\vee c)$. 1M
   Conversely, suppose $a\vee(b\wedge c) \leq b\wedge(a\vee c)$.
   Thus $a \leq a\vee(b\wedge c) \leq b\wedge(a\vee c) \leq b$. 1M

2. Show that the number of derangements of n distinct objects is approximately $\frac{n!}{e}$.

   Solution: Let a_k be the property that the element k is in the kth position, $1 \leq k \leq n$.
   N=$n!$, $N(a_i) = (n-1)!$, $N(a_i a_j) = (n-2)!, \ldots . N(a_1 \ldots a_n) = 1$. 1M
   $N(a_1' \ldots . a_n') = N - \sum N(a_i) + \cdots . +(-1)^n N(a_1 \ldots a_n)$
   $= n!\,(\frac{1}{2} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!})$
   $= n!\left(1 - \frac{1}{1!} + \frac{1}{2!} \ldots . +(-1)^n \frac{1}{n!} + \cdots .\right) = \frac{n!}{e}$ for n large. 2M

3. How many different strings can be formed using 2 A's, 3 B's, 2 C's, and 1 E, once each? In how many of these strings are all the vowels non-adjacent?
   Solution: Total = $\frac{8!}{2!\,3!\,2!} = 1680$ 1M
   We can arrange the 3 B's and 2Cs in $\frac{5!}{3!2!}$ways. Then we have to place the 2A's and 1E in different locations between and around the already arranged consonants. There are 6 locations available, so select any two for the A's in $^6C_2$ ways, and then select one place for E out of the 4 remaining places.
   Thus (5! / 3! 2!) × $^6C_2$ ×4 2M

4. How that the number of partitions of n in which odd parts are not repeated but even parts can occur any number times is equal to the number of partitions of n in which every part is either odd or a multiple of 4.
   Solution: GF of number of partitions of n in which odd parts are not repeated but even parts can occur any number times is
   $G_1(x) = (1+x)(1-x^2)^{-1}(1+x^3)(1-x^4)^{-1} \ldots \ldots$ 0.5M
   GF of number of partitions of n in which every part is either odd or a multiple of 4 is $G_2(x) = (1-x)^{-1}(1-x^3)^{-1}(1-x^4)^{-1} \ldots.$ 0.5M
   Consider $G_2(x) = (1-x)^{-1}(1-x^3)^{-1}(1-x^4)^{-1} \ldots.$
   $= \frac{(1+x)}{(1-x)(1+x)} \frac{(1+x^3)}{(1-x^3)(1+x^3)} (1-x^4)^{-1} \frac{(1+x^5)}{(1-x^5)(1+x^5)} \ldots$
   $= (1+x)(1-x^2)^{-1}(1+x^3)(1-x^4)^{-1}(1+x^5)(1-x^6)^{-1} \ldots$
   $= G_1(x)$ 2M

5. Compute the CNF and DNF of the Boolean expression $E(x_1, x_2, x_3) = \overline{a \wedge (\overline{b} \vee (\overline{c} \wedge a))}$

Solution: DNF: $(\overline{x_1} \wedge \overline{x_2} \wedge \overline{x_3}) \vee (\overline{x_1} \wedge \overline{x_2} \wedge x_3) \vee (\overline{x_1} \wedge x_2 \wedge \overline{x_3}) \vee (\overline{x_1} \wedge x_2 \wedge x_3)$
$\vee (x_1 \wedge x_2 \wedge x_3)$

CNF: $(\overline{x_1} \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3)$ 4M

6. Find both the 78th and 112th permutations of 1, 2, 3, 4, 5 in each of (i) lexicographical order (ii) Fike's order.

Solution: Lexico: 78^{th}: 41532

112^{th} : 53241 2M

Fikes: 78^{th}: seq; 0202, permutation is 41523

112^{th}: seq; 0013, permutation is 34251 2M