Mechanics of Deformable Bodies

08 December 2021 11:56 AM

Example problems on Simple Stresses and Strains

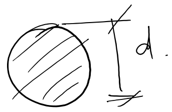
N1. A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m2.

$$120\ MN/m^2 = 120\ N/mm^2$$

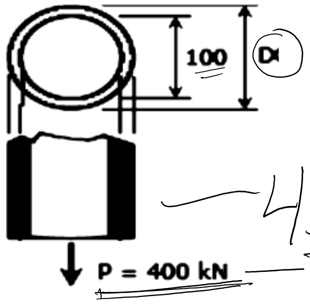
$$\sigma = \frac{P}{A} = \frac{400\times10^3}{\frac{\pi}{4}(d_o^2 - d_i^2)}$$

$$120 = \frac{400\times10^3}{\frac{\pi}{4}(d_o^2 - 100^2)}$$

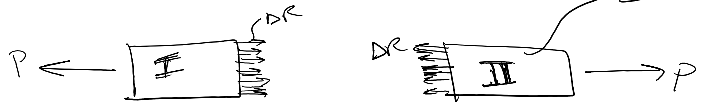
$$\boxed{d_o = 119.7\ mm}$$



$$A = \frac{\pi d^2}{4} = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$$



$\Sigma F_H = 0 \quad P = \Sigma \Delta R = R$

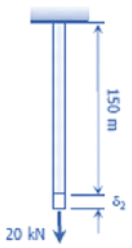


$\Sigma F_H = 0 \quad +P - \Sigma \Delta R = 0$

$P = \Sigma \Delta R = R$

N2. A steel rod having a cross-sectional area of 300 mm2 and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. Find the total elongation of the rod if E = 200 x 103 MN/m2. Neglect the self-weight of the rod.

$200\times10^3\ MN/m^2$



Data :

$P = 20\ kN = 20\times10^3\ N.$

$E = 200\times10^9\ N/m^2 = 2\times10^5\ N/mm^2$

$L = 150\times10^3\ mm, \quad A = 300\ mm^2$

$$\delta = \frac{PL}{AE} = \frac{20\times10^3\times150\times10^3}{300\times2\times10^5} = 50\ mm$$

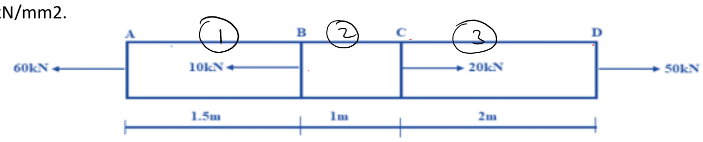
$\sigma = \epsilon E$

$$\sigma = \frac{P}{A} = 66.67\ N/mm^2$$

$$\epsilon = \frac{\sigma}{E} = \frac{66.67}{2\times10^5} = 3.333\times10^{-4}$$

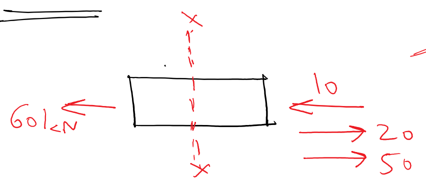
$$\frac{\delta}{L} = \epsilon \Rightarrow \delta = 3.333\times10^{-4}\times150\times10^3 = 50\ mm$$

N3. A steel bar of 25 mm diameter is acted upon by force as shown in figure. Determine the stress in each section and total elongation of the bar. Take E= 200 kN/mm2.



$$A = \frac{\pi\times25^2}{4} = 490.87\ mm^2$$

FBD Part 1



of Equilibrium $\Sigma F_x = 0$

$P_1 = 60\ kN.$

$60 \leftarrow\ T \rightarrow 60$

$$\sigma_1 = \frac{P_1}{A} = \frac{60\times10^3}{} = 122.2\ N/mm^2$$

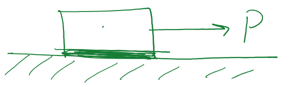
60 kN ⟶ 20, ⟶ 50

60 ⟵ T ⟶ 60

$P_1 = 60\ kN$.

$\sigma_1 = \frac{P_1}{A} = \frac{60\times10^3}{490.87} = \underline{122.2\ N/mm^2}$

$\delta_1 = \frac{P_1 L_1}{AE} = \frac{60\times10^3\times1500}{490.87\times2\times10^5} = \underline{0.917 mm}$



sliding ⟹

$\epsilon = 0$

0

Part (2)

kN 60, 10 ⟵ ⟶ 20 kN, ⟶ 50

70 kN ⟵ T ⟶ 70 kN

$P_2 = 70\times10^3\ N$ (T)

$\sigma_2 = P_2/A_2$

$\sigma_2 = \underline{142.6\ N/mm^2}$

$\delta_2 = \frac{70\times10^3\times1000}{490.87\times2\times10^5} = \underline{0.713 mm}$

Part (3)

⟵ 10, ⟵ 60, ⟶ 20 ; ⟶ 50

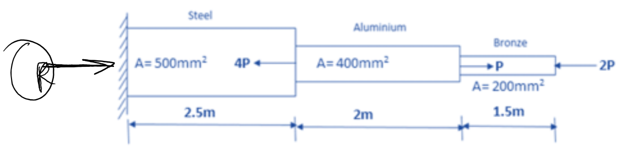
$\sigma_3 = \frac{50\times10^3}{490.87} = \underline{101.86\ N/mm^2}$

50 ⟵ T ⟶ 50.

$\delta_3 = \frac{50\times10^3\times2000}{490.87\times2\times10^5} = \underline{1.01\ mm}$

$\delta_{Total} = \sum \delta_i = 1.01 + 0.713 + 0.917 = \underline{2.649\ mm}$

N4. An aluminum rod is rigidly attached between a steel and bronze rod as shown in the figure. Axial loads are applied at the positions indicated. Find the maximum value of P that will not exceed stress in steel = 140 MPa, stress in aluminum = 90 MPa, stress in bronze = 100 MPa.



Bronze

$R + P - 4P -$

$\Sigma F_x = 0$.

⟶ P ; ⟵ 4P ; ⟶ 5P ; Br. ⟵ 2P

$2P \longrightarrow$ Br. $\longleftarrow 2P$

Compression.

$\sigma_{Br} = \frac{2P}{A_{Br}} = \frac{2P}{200} = 100\ N/mm^2$

$\boxed{P = 10\ kN}$ —— (1)

4P ⟵ Al ⟶ P ; 5P ⟶ ⟵ 2P

Compression

$\sigma_{Al} = \frac{P}{400} = 90$

$\boxed{P = 36\ kN}$ —— (2)

5P ⟶ Steel ⟵ 4P, ⟶ P, ⟵ 2P

$\sigma_{St} = \frac{5P}{500} = 140$.

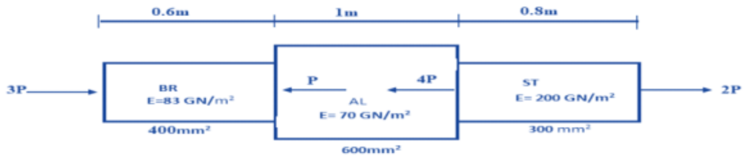
$\boxed{P = 14\ kN}$ —— (3)

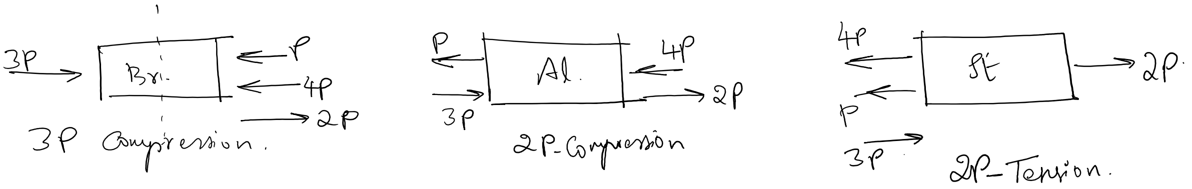
... so that all three members

$\boxed{P = 14\,kN}$ ——— (3)

Answer is $\boxed{P = 10\,kN}$ such that all three members will be below stress limit.

N5. Determine the value of force 'P' if the elongation of the bar is restricted to 2 mm.



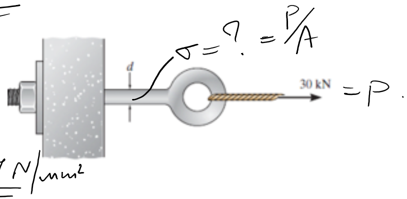


$$\delta_{Br} + \delta_{Al} + \delta_{st} = 2\,mm$$

$$\frac{-3P\times600}{400\times83000} + \frac{(-2P\times1000)}{600\times70000} + \frac{2P\times800}{300\times2\times10^5} = 2\,mm$$

$$\boxed{P = -26.60\,kN}$$

N6. If the eyebolt is made of a material having a yield stress σ_y = 250 MPa (N/mm^2), determine the minimum required diameter *'d'* of its shank. Apply a factor of safety F.S. = 1.5 against yielding.



$$\text{Allowable stress} = \frac{\sigma_y}{FS} = \frac{250}{1.5}\,N/mm^2 = 166.67\,N/mm^2$$

$$\sigma_{allow} = P/A$$

$$A = P/\sigma_{allowable} = \frac{30\times10^3}{166.67} = 180\,mm^2$$

$$\frac{\pi d^2}{4} = A \Rightarrow d = \sqrt{\frac{180\times4}{\pi}} = 15.138\,mm$$

N7. Tension test was conducted on a specimen and the following observations were made:

Diameter= 25 mm, Gauge length = 200 mm, least count of extensometer = 0.001 mm

At load of 24 kN and extensometer reading = 60

At load of 38 kN and extensometer reading = 94

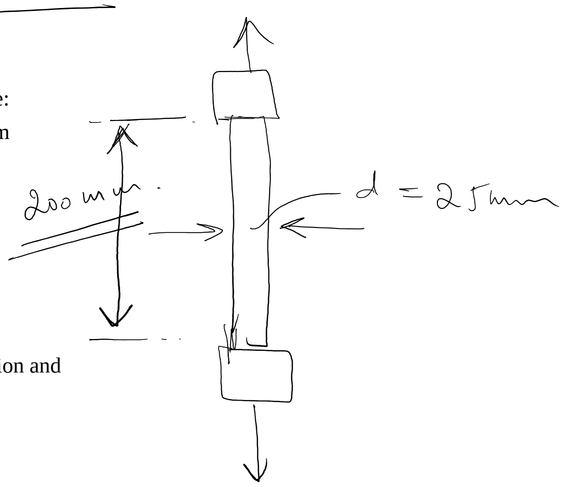
Yield load = 99 kN

Maximum load = 157 kN

Diameter of neck = 17mm

Final extension over 110 mm, original length = 132 mm.

Find a) Modulus of elasticity b) Yield stress c) ultimate stress d) percentage elongation and percentage reduction in area.



P	δL (mm)
@ 24 kN	$60\times0.001 = 0.06$
@ 38 kN	$94\times0.001 = 0.094$

@ 24 kN | $60 \times 0.001 = 0.06$

@ 38 kN | $94 \times 0.001 = 0.094$

$E = \frac{\sigma}{\epsilon}$ } with proportionality limit

$$E_1 = \frac{24\times10^3/(\pi\times25^2/4)}{(0.06)/200} = 1.6297\times10^5\ N/mm^2$$

$$E_2 = \frac{38\times10^3/(\pi\times25^2/4)}{(0.094/200)} = 1.647\times10^5\ N/mm^2$$

$$E = \frac{E_1+E_2}{2}$$

(1) $E = 1.638\times10^5\ N/mm^2$

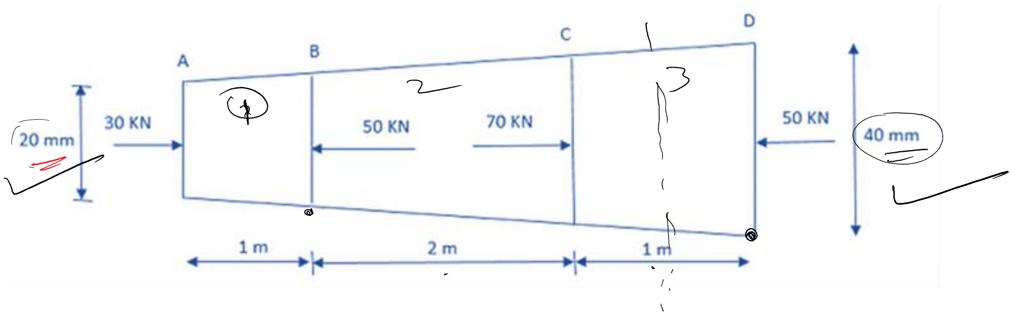
(2) $\sigma_y = \frac{\text{yield load}}{\text{c/s Area}} = \frac{99\times10^3}{\pi\times25^2/4} = 201.68\ N/mm^2$

(3) $\sigma_u = \frac{\text{ultimate load}}{\text{c/s Area}} = \frac{157\times10^3}{\left(\frac{\pi\times25^2}{4}\right)} = 319.83\ N/mm^2$

(4) % Elongation $= \frac{\text{final length} - \text{initial length}}{\text{initial length}}\times100 = \frac{132-110}{110}\times100 = 20\ \%$

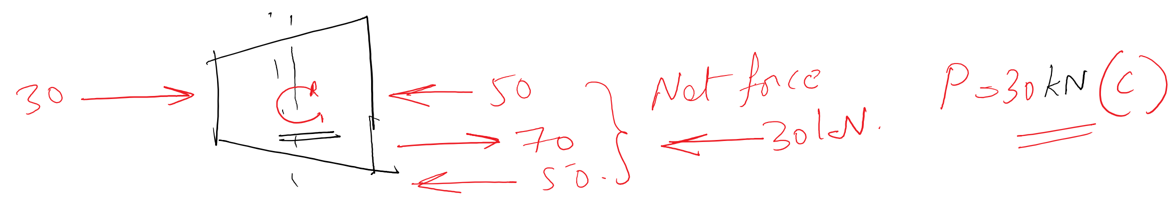
(5) % reduction in area $= \frac{A_0 - A_f}{A_0}\times100 = 53.75\ \%$

N8. A tapered bar of rectangular cross-section 4 m long and 15 mm thick is subjected to load as shown in Figure. Determine maximum and minimum stress at each section. Also, find the change in length of the bar. Take E for the bar material as 120 GPa.



$L = 4m$ $\quad t = 15mm$ $\quad E = 1.2\times10^5\ N/mm^2 = 120\times10^9\ N/m^2$

Section (1)



$d_{@A} = 20$ mm

$$d@B = 40 - \frac{(40-20)}{4000} \times 3000 = \underline{\underline{25\ mm}}$$

$$\underline{\underline{A@A}} = 20 \times t = 20 \times 15 = \underline{\underline{300\ mm^2}} \quad A@B = 25 \times t = 25 \times 15 = \underline{\underline{375\ N/mm^2}}$$

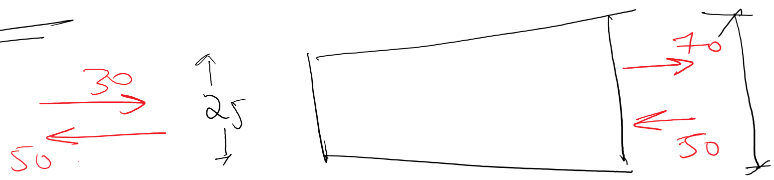
$$(\sigma_{max})_1 = \frac{30 \times 10^3}{300} = \underline{\underline{100}}\ N/mm^2$$

$$(\sigma_{min})_r = \frac{30 \times 10^3}{375} = \underline{\underline{80}}\ N/mm^2$$

$$(\delta)_{AB} = \frac{(-30 \times 10^3) \times 1000 \times 2.303 \times \log_{10}\left(\frac{25}{20}\right)}{15 \times 1.2 \times 10^5\ (25-20)}$$

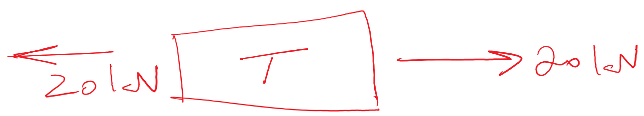
$$= \underline{\underline{-0.744\ mm}}$$

Part B–C



$$40 - \frac{(40-20)}{4000} \times 1000 = \underline{\underline{35\ mm}}$$

Net force.



$$P = +\underline{\underline{20 \times 10^3\ N}}$$

(T)

$$(\sigma_{max})_{BC} = \frac{20 \times 10^3}{t \times b_{min}} = \frac{20 \times 10^3}{15 \times 25} = \underline{\underline{53.33}}\ N/mm^2$$

$$(\sigma_{min})_{BC} = \frac{20 \times 10^3}{t \times b_{max}} = \frac{20 \times 10^3}{15 \times 35} = \underline{\underline{38.09\ N/mm^2}}$$

$$(\delta L)_{BC} = \frac{+20 \times 10^3 \times 2000 \times 2.303 \log_{10}\left(\frac{35}{25}\right)}{15 \times 1.2 \times 10^5 \times (35-25)}$$

$$= \underline{\underline{+0.747\ mm}}$$

$$(\delta L)_{CD} = \underline{\underline{-0.74\ mm}}$$

$$\left(\delta L = \underline{\underline{-0.738\ mm}}\right)$$

$$(\delta L)_{CD} = -0.74 \text{ mm}$$

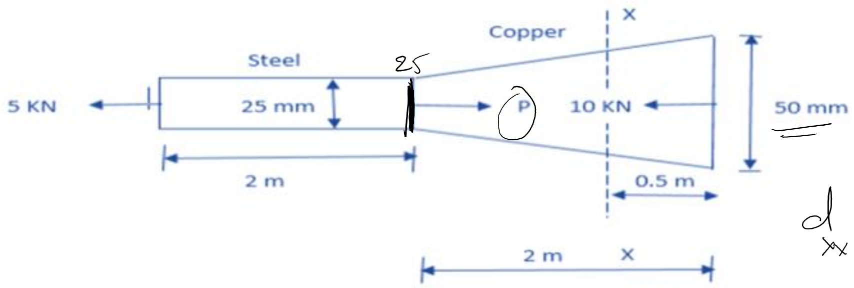
Part C-D

$$b_c = 40 - \left(\frac{40-20}{4000}\right) \times 1000 = 35$$

$$(\delta L)_{CD} = \frac{2.303 \times (-50\times 1000) \times \log_{10}\left(\frac{40}{35}\right)}{15 \times 1.2\times 10^5 \times (40-35)}$$

$$= -0.74 \text{ mm}$$

N9. A composite bar made of steel and copper bars of diameters 25 mm and 50 mm respectively. Calculate the stress at section X-X for the copper bar and stress in steel bar for the setup shown in Figure. E_s = 200 GPa, E_{Cu} = 100 GPa.

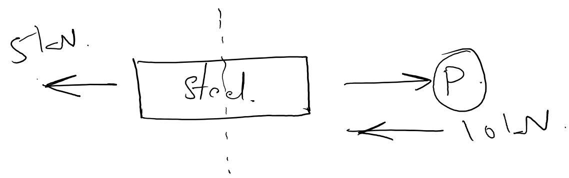


$$d_{xx} = 50 - \frac{(50-25)\,500}{2000}$$

$$d_{xx} = 43.75 \text{ mm}$$

Solution:

To find P.



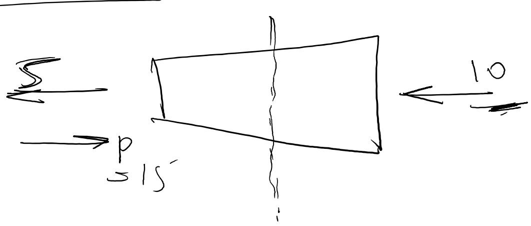
$$\Sigma F_x = 0.$$

$$-5 + P - 10 = 0 \implies \boxed{P = 15 \text{ kN}}$$

To find stress in steel $\quad W_s = 5 \text{ kN (T)}$

$$\sigma_s = \frac{5\times 10^3}{\left(\frac{\pi\, 25^2}{4}\right)} = 10.18 \text{ N/mm}^2$$

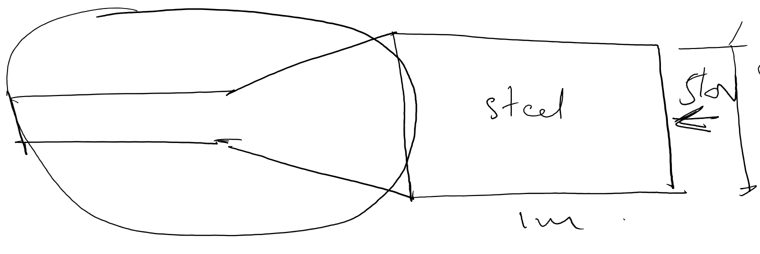
To find stress @x-x in tapered bar.



$$W_c = 10 \text{ kN (C)}$$

$$\sigma_{xx} = \frac{10\times 10^3}{\ldots} \qquad \frac{10\times 10^3 \times 4}{\ldots} \qquad 6.65 \text{ N/mm}^2$$

$$\sigma_{xx} = \frac{10 \times 10^3}{\frac{\pi \times d_{xy}^2}{4}} = \frac{10 \times 10^3 \times 4}{\pi \times 43.75^2} = \underline{6.65 \ N/mm^2}$$



50 uniform.. $200 \times 10^9 \ N/m^2$

$E_s = 200 \ GPa$

$E_{al} = 100 \ GPa$

Deformation of tapered bar with circular cross-section

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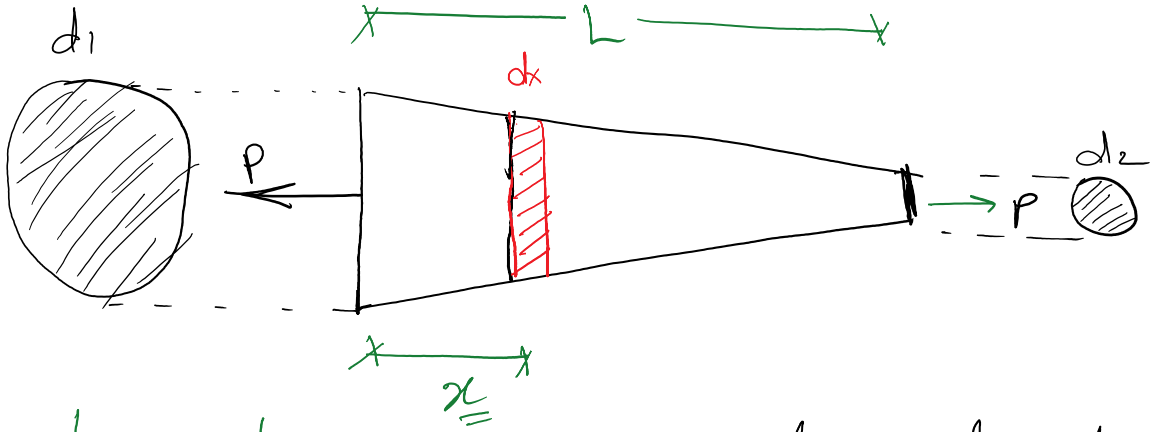

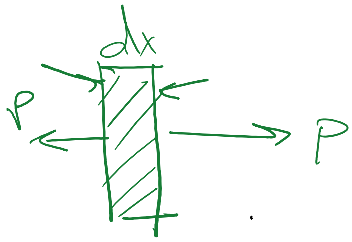

$$\int_0^L \delta_{dx} = \int_0^L \frac{P\,dx}{(A)_x E}$$

Consider diameter @ $x = d$

$$d = d_1 - \left(\frac{d_1-d_2}{L}\right)x$$

$$d = d_1 - kx \quad \text{where} \quad k = \left(\frac{d_1-d_2}{L}\right)$$

$$A_x = \frac{\pi}{4}(d_1-kx)^2$$

$$\delta_x = \frac{P\,dx}{\frac{\pi}{4}(d_1-kx)^2 E} \qquad \therefore \quad \delta_L = \int_0^L \delta x$$

$$\delta_L = \int_0^L \frac{P\,dx}{\frac{\pi}{4}E\,(d_1-kx)^2} = \frac{4P}{\pi E}\int_0^L \frac{dx}{(d_1-kx)^2}$$

$$= \frac{4P}{\pi E}\left[\frac{+1}{(d_1-kx)}\right]_0^L \frac{1}{+k} = \frac{4P}{\pi E}\left[\frac{1}{d_1-kL} - \frac{1}{d_1}\right]$$

$$= \frac{4P}{\pi E}\left\{\frac{1}{\left(\frac{d_1-d_2}{L}\right)}\right\}\left[\frac{1}{\cancel{d_1} - \left(\frac{\cancel{d_1}-d_2}{\cancel{L}}\right)\cancel{L}} - \frac{1}{d_1}\right]$$

$$= \frac{4PL}{\pi E\,(\cancel{d_1-d_2})}\,\frac{(\cancel{d_1-d_2})}{(d_1 d_2)} = \frac{4PL}{\pi E\, d_1 d_2}$$

$$-\frac{4PL}{\pi E\,\cancel{(d_1-d_2)}}\,\frac{\cancel{(d_1-d_2)}}{(d_1 d_2)} = \frac{4PL}{\pi E\, d_1 d_2}$$

$$\boxed{\delta_L = \frac{4PL}{\pi E\, d_1 d_2}}$$

Deformation of tapered rectangular bar

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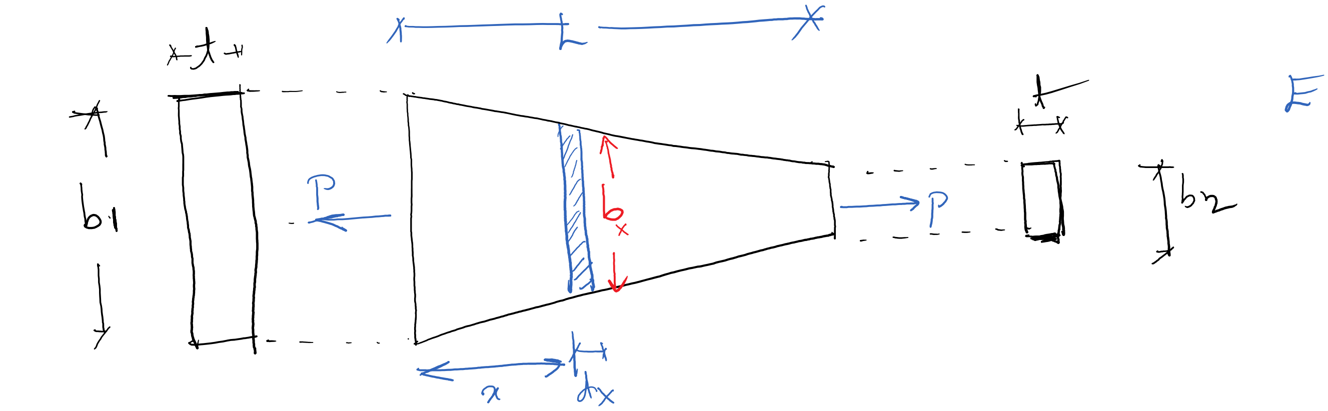

change in width over a length of $L = b_1 - b_2$

rate of change in width $= \left(\frac{b_1 - b_2}{L}\right)$

length change over a length of $x = \frac{(b_1 - b_2)}{L} x$

$$b_x = b_1 - \left(\frac{b_1 - b_2}{L}\right) x = b_1 - kx \quad \text{where } k = \left(\frac{b_1 - b_2}{L}\right)$$

$$\delta_x = \frac{P\,dx}{t\,(b_1 - kx)\,E} \qquad \delta = \int_0^L \delta x$$

$$\delta = \int_0^L \frac{P\,dx}{tE\,(b_1 - kx)} = \frac{P}{tE}\int_0^L \frac{dx}{(b_1 - kx)}$$

Hint : $\left[\int \frac{1}{(a+bx)} = \frac{\log (a+bx)}{b}\right]$

$$\delta_L = \frac{P}{tE}\left[\frac{\log (b_1 - kx)}{-k}\right]_0^L = \frac{-P}{tEk}\left[\log (b_1 - kL) - \log (b_1)\right]$$

$$\delta = \frac{P}{tEk}\left[\log b_1 - \log (b_1 - kL)\right]$$

$$\frac{\quad}{tEK}\left[\quad\right]$$

$$= \frac{P}{tE\left(\frac{b_1-b_2}{L}\right)}\left[\log b_1 - \log\left\{\cancel{b_1} - \frac{\cancel{b_1}-b_2}{\cancel{L}}\cdot \cancel{L}\right\}\right]$$

$$= \frac{PL}{tE(b_1-b_2)}(\log b_1 - \log b_2)$$

$$\boxed{\delta_L = \frac{PL}{tE(b_1-b_2)}\log_e(b_1/b_2)}$$

$$\boxed{\delta_L = \frac{PL\,2.303}{tE(b_1-b_2)}\log_{10}\left(\frac{b_1}{b_2}\right)}$$

Shear Stress and strains numericals

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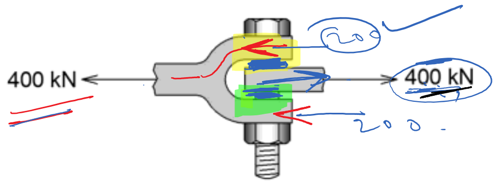
N10. Find the smallest diameter of bolt that can be used in the shackle shown in Figure. The shearing strength of the bolt is 300 MPa.

$\Sigma F_x = 0$



$$\tau = \frac{P}{2A} = \frac{400 \times 1000}{2 \times \left(\frac{\pi \times d^2}{4}\right)}$$

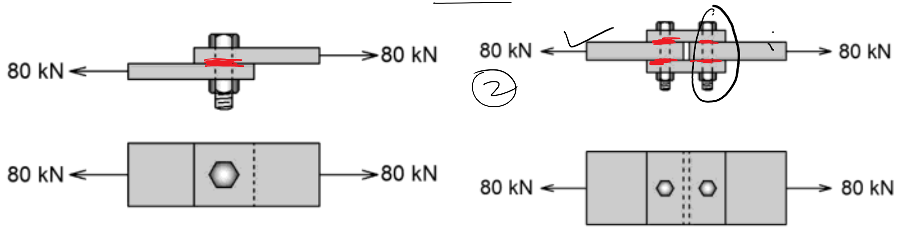
$1 \text{ MPa} = 1 \text{ N/mm}^2$

$10^{-6} \times 1 \times 10^{6} \text{ N/mm}^2$

$$300 = \frac{4 \times 10^5 \times 4}{2 \times \pi \times d^2}$$

$$d = \sqrt{\frac{4 \times 10^5 \times 4}{2\pi \times 300}} \Rightarrow \boxed{d = 29.13 \text{ mm}}$$

N11.Check the safety of bolt against shear failure when it is used in following configurations. (a) Lap joint, (b) Double cover butt joint. Take bolt diameter as 15 mm, shearing strength of bolt material = 280 MPa.



$\tau_{max} = 280 \text{ N/mm}^2$

(a) Lap joint: $\tau = \frac{P}{A} \Rightarrow \tau = \frac{80 \times 10^3}{\left(\frac{\pi \times 15^2}{4}\right)} = 452.92 \text{ N/mm}^2$

$\tau >> \tau_{max}$ ✗ unsafe against shear

Approach 1 d increase.

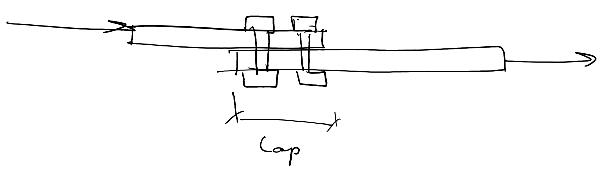
$$\tau_{max} = \frac{P}{\left(\frac{\pi \times d^2}{4}\right)} = \qquad 280 = \frac{80 \times 10^3}{\left(\frac{\pi \times d^2}{4}\right)}.$$

$d = 19.07$ mm

≈ 20 mm

Approach 2 Add additional bolt

provide 2 bolts



(b) Double cover butt joint

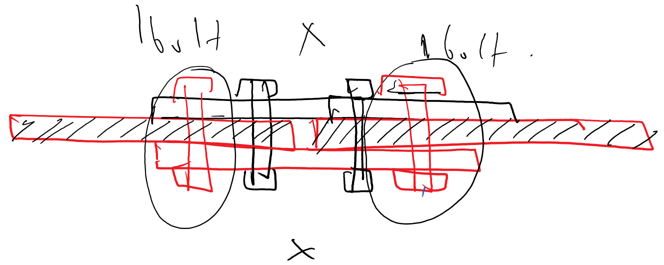
Cap

(b) Double cover butt joint

$$\tau = \frac{P}{2A}.$$

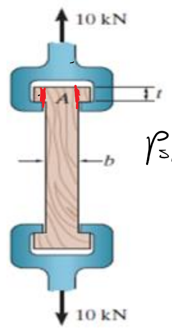
$$\tau = \frac{80\times10^3}{\left\{2\times\left(\frac{\pi\times15^2}{4}\right)\right\}} = 226.35\ N/mm^2.$$

$\tau_{max} > \tau$ Hence safe in shear.



$$\tau = \frac{P}{2A\times2}$$

N12. The wood specimen is subjected to the pull of 10 kN in a tension testing machine. If the allowable normal stress for the wood is $\sigma_{t\ allowable} = 12$ MPa and the allowable shear stress is $\tau_{allowable} = 1.2$ MPa, determine the required dimensions *b* and *t* so that the specimen reaches these stresses simultaneously. The specimen has a width of 25 mm.



P=10.

(1) $\sigma = \frac{P}{A} = \frac{10\times10^3}{b\times25}$

$12 = \frac{10\times10^3}{b\times25} \Rightarrow \boxed{b = 33.33\ mm}$

(2) $\tau = \frac{P}{2A} = \frac{10\times10^3}{2\times t\times25}$

$1.2 = \frac{10\times10^3}{2\times t\times25} \Rightarrow \boxed{t = 166.66\ mm}$

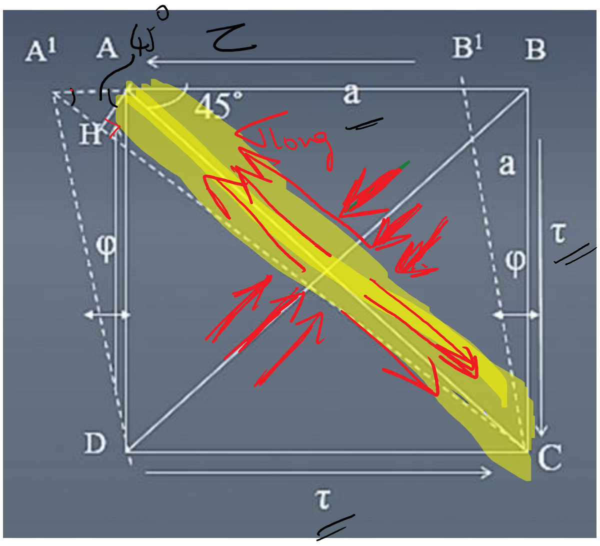
E and G relation

29 December 2021 04:07 PM

ϕ = shear strain

drop a $\perp^r$ AH onto A'C

$\angle AA'H = \angle A'AH = 45^\circ$



we know $G = \frac{\tau}{\phi}$

$E = \frac{\sigma}{\epsilon}$

$\epsilon_{long} = \frac{\sigma_{long}}{E} - \mu \frac{\sigma_{lat}}{E}$

$\sigma_{long} = \tau$ $\quad \sigma_{lat} = -\tau$

Strain in diagonal AC.

$$\epsilon_{AC} = \frac{\tau}{E} - \mu\frac{(-\tau)}{E} = \frac{\tau}{E}(1+\mu) \quad \text{—— (1)}$$

Also change in length of AC = A'H

strain in AC = $\frac{A'H}{AC}$

$\underline{\underline{A'H}}$ in $\Delta AA'H$ = $AA' \cos 45^\circ = AA'\frac{1}{\sqrt{2}}$ —— (2)

in $\Delta A'AD$, $\tan\phi = \frac{A'A}{AD}$

$\phi = \frac{AA'}{AD}$ $\quad A'A = AD\,\phi$ —— (3)

(3) in (2) $\quad A'H = \frac{AD\,\phi}{\sqrt{2}} = \frac{a\phi}{\sqrt{2}}$ —— (4)

ϵ_{AC} Strain in AC = $\frac{A'H}{AC} = \frac{\cancel{a}\phi}{\sqrt{2}\times\sqrt{2}\cancel{a}} = \underline{\underline{\phi/2}}$ —— (5)

we know that $G = \frac{\tau}{\phi}$ $\quad \phi = \tau/G$

Eqn (5) becomes $\quad \epsilon_{AC} = \frac{\tau}{\ldots}$ —— (6)

Eqn (5) becomes $= \epsilon_{AC} = \dfrac{\tau}{G \times 2}$ — (6)

(6) = (1)

$$\cancel{\frac{\tau}{E}}(1+\mu) = \cancel{\frac{\tau}{2G}}.$$

$$\boxed{E = 2G(1+\mu)}$$

Also.

$$\boxed{E = 3K(1-2\mu)}$$

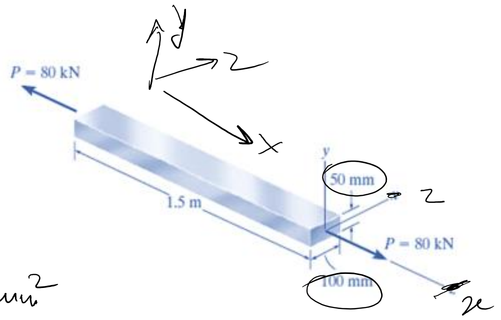
E = 9GK/(G+3K)

Problems on Volumetric stress and EGK

29 December 2021 04:14 PM

N13. A bar made of steel has the dimensions shown in Fig. If an axial force of is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. Take E = 200 GPa and μ = 0.3.

80 kN



$P = 80\ kN \qquad E = 200\ GPa$

$= 2\times10^5\ N/mm^2$

$\sigma_x = \frac{P}{A} = \frac{80\times10^3}{50\times100} = \underline{\underline{16\ N/mm^2}}$

$\sigma_y = 0$

$\sigma_z = 0$

$\epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\cancel{\sigma_y}}{E} - \mu\frac{\cancel{\sigma_z}}{E} = \frac{16}{2\times10^5} = \underline{\underline{8\times10^{-5}}}$

$\epsilon_y = \frac{\cancel{\sigma_y}}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\cancel{\sigma_z}}{E} = -0.3\times\frac{16}{2\times10^5} = \underline{\underline{-2.4\times10^{-5}}}$

$\epsilon_z = \frac{\cancel{\sigma_z}}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\cancel{\sigma_y}}{E} = -0.3\times\frac{16}{2\times10^5} = -2.4\times10^{-5}$

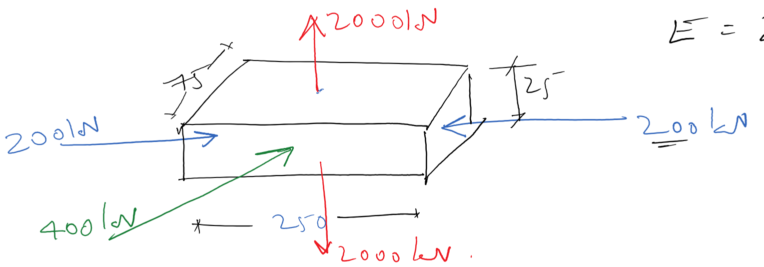
$\delta_x = 8\times10^{-5}\times1500 = \underline{\underline{0.12}}\ mm \qquad L' = \underline{\underline{1500.12}}\ mm$

$\delta_y = -2.4\times10^{-5}\times50 = \underline{\underline{-0.0012\,mm}} \qquad B' = \underline{\underline{49.9988}}\ mm$

$\delta_z = -2.4\times10^{-5}\times100 = \underline{\underline{-0.0024}}\ mm \qquad D' = \underline{\underline{99.9976}}\ mm$

N14. A rectangular bar 250 mm long is 75 mm wide and 25 mm thick. It is loaded with an axial compressive load of 200 kN on 75 mm × 25 mm face, a normal tensile force of 2000 kN on face 75 mm × 250 mm and a compressive force 400 kN on face 25 mm × 250 mm. Calculate the changes in length, breadth, thickness and volume of the bar. Take E = 200 GPa and μ = 0.3.

What change must be made in the 2000 kN load in order that there shall be no change in volume of the bar.



$E = 200\ GPa = 2\times10^5\ MPa$

$= 2\times10^5\ N/mm^2$

$\delta_x = \epsilon_x \times L_x \qquad \epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$

$\sigma_x = \frac{-200\times10^3}{75\times25} = \underline{\underline{-106.66\ N/mm^2}} \qquad \sigma_z = \frac{-400\times10^3}{25\times250} = \underline{\underline{-64\ N/mm^2}}$

$\sigma_y = \frac{+2000\times10^3}{75\times250} = \underline{\underline{+106.66\ N/mm^2}}$

$\epsilon_x = \underline{-106.66} - 0.3\times106.66 - 0.3\times(-64) = -5.97\times10^{-4}$

$\overline{75 \times 250}$

$$\epsilon_x = \frac{-106.66}{2\times10^5} - \frac{0.3\times106.66}{2\times10^5} - \frac{0.3\times(-64)}{2\times10^5} = \underline{\underline{-5.97\times10^{-4}}}$$

$$\delta L_x = L_x \times \epsilon_x = -5.97\times10^{-4}\times250 = \underline{\underline{-0.149\ mm}}$$

$$\epsilon_y = \frac{1}{2\times10^5}\left(106.66 - 0.3(-106.66) - (-64)\times0.3\right) = \underline{\underline{7.893\times10^{-4}}}$$

$$\delta L_y = \epsilon_y \times L_y = 7.893\times10^{-4}\times25 = \underline{\underline{0.0197\ mm}}$$

$$\epsilon_z = \underline{-3.2\times10^{-4}} \qquad \delta L_z = \underline{\underline{-0.024\ mm}}$$

$$\frac{dv}{V} = \epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z = 10^{-4}(-3.2+7.893-5.97)$$

$$= \underline{\underline{-1.277\times10^{-4}}} \Rightarrow dv = \epsilon_v \times (250\times75\times25)$$

$$dv = \underline{\underline{-600\times10^3\ mm^3}}$$

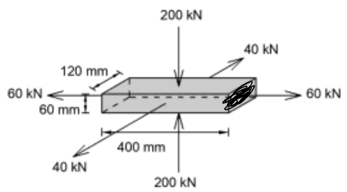
Part (2) $\quad dv = \underline{\underline{0}}$ (No change in vol.)

$$\frac{dv}{V} = \epsilon_v = 0$$

$$\epsilon_v = \left(\frac{1-2\mu}{E}\right)\underbrace{(\sigma_x + \sigma_y + \sigma_z)}_{0} = 0 \Rightarrow \sigma_x + \sigma_y + \sigma_z = 0$$

$$-106.66 + \sigma_y - 64 = 0 \Rightarrow \boxed{\sigma_y = 170.66}$$

$$\frac{P_y}{A_{Ay}} = \sigma_y \qquad 170.66 = \frac{P_y}{(250\times75)} \Rightarrow \boxed{P_y = 3200\ kN}$$



T1. A steel bar of 400 mm x 120 mm x 60 mm is subjected to forces as shown in the figure. Find,
(a) Change in each dimension and change in volume.
(b) What longitudinal force alone can produce same longitudinal strain as in case (a).
Take E = 200 GPa and μ = 0.25.

$$\sigma_x = \frac{60\times10^3}{60\times120} = \underline{\underline{8.33\ N/mm^2}} \quad (T)$$

$$\sigma_y = \frac{-200\times10^3}{400\times120} = \underline{\underline{-4.167\ N/mm^2}} \quad (C)$$

$$\sigma_z = \frac{+40\times10^3}{60\times400} = \underline{+1.667\ N/mm^2} \quad (T)$$

$$\epsilon_x = \frac{8.33}{200\times10^3} - 0.25\left(\frac{-4.167}{200\times10^3} + \frac{1.667}{200\times10^3}\right) = \underline{\underline{4.479\times10^{-5}}}$$

$$\delta l_x = \underline{\underline{17.917\times10^{-3}\ mm}}$$

$$\epsilon_y = -3.333\times10^{-5} \qquad \delta L_y = -\underline{\underline{2\times10^{-3}}}\ mm$$

$$\epsilon_z = 3.125\times10^{-5} \qquad \delta L_z = \underline{\underline{0.375\times10^{-3}\ mm}}$$

$$\epsilon_v = 1.458\times10^{-5} \qquad dv = \underline{\underline{42\ mm^3}}$$

$$\epsilon_x = \frac{\delta L_x}{l} = \frac{Pl/AE}{l} = \frac{P}{AE} = \frac{P}{120\times80\times2\times10^5}$$

$$\epsilon_x = \frac{\delta L_x}{L_x} = \frac{PL/AE}{L} = \frac{P}{AE} = \frac{P}{120\times 80\times 2\times 10^5}$$

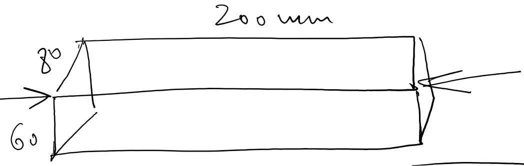
$$4479\times 10^{-5} \times 120\times 80\times 2\times 10^5 = P$$

$$\boxed{P = 86.\ kN}$$

T2. A mild steel bar 200 mm long and 80 mm × 60 mm in cross–section is subjected to a longitudinal axial compressive load of 700 kN. Determine the value of the lateral forces necessary to prevent any transverse strain. Evaluate the resultant change in length.

Take E = 200 kN/m^2 and μ = 0.25.

$\epsilon_z = \epsilon_y = 0$



$$\sigma_x = \frac{700\times 10^3}{80\times 60}$$

$$= 145.83\ N/mm^2$$

To find ? σ_z & σ_y in turn

P_z, P_y

$$\epsilon_z = 0 = \frac{\sigma_z}{2\times 10^5} - \frac{0.25(-145.83)}{2\times 10^5} - 0.25\left(\frac{\sigma_y}{2\times 10^5}\right) \quad (1)$$

$$\sigma_y - 0.25\,\sigma_z = -36.458 \quad (1)$$

$$\epsilon_y = 0 = \frac{\sigma_y}{2\times 10^5} - \frac{0.25(-145.83)}{2\times 10^5} - \frac{0.25(\sigma_z)}{2\times 10^5} \quad (2)$$

$$\sigma_z - 0.25\,\sigma_y = -36.458 \quad (2)$$

solve (1) & (2)

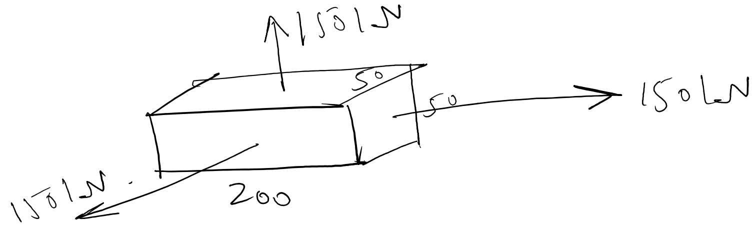
$\sigma_y = -48.61$

$\sigma_z = -48.61$

$P_z = \sigma_z \times 60\times 400 = -1152\ kN\ (C)$

$P_y = \sigma_y \times 80 \times 400 = -1555.52\ (C)$

T3. A 200 mm x 50 mm x 50 mm steel block is subjected to a tensile force of 150 kN on all three sides. The modulus of elasticity and Poisson's ratio of the material are 200 GPa and 0.3 respectively. Find the change in the volume of the block.



N15. A bar of certain material 50 mm square is subjected to an axial pull of 150 kN. The extension over a length of 100 mm is 0.05 mm and the decrease in each side is 0.00625 mm. Calculate, (a) Modulus of Elasticity, (b) Poisson's ratio, (c) Shear Modulus and (d) Bulk Modulus

$$E = \frac{\sigma}{\epsilon_{long}} = \frac{150\times 10^3/(50\times 50)}{(0.05/100)}$$

$$E = \frac{P/A}{\ } \quad = \frac{60}{\ } = 120\times 10^3\ MPa$$

$$\mu = \frac{\epsilon_{lat}}{\epsilon_{long}}$$

$$= (0.00625/50)$$

ϵ_{long} $(0.05/100)$ | ϵ_{long}

$E = \frac{P/A}{(\delta L/L)} = \frac{60}{5\times10^{-4}} = 120\times10^{3}\,MPa$ | $= \frac{(0.00625/50)}{5\times10^{-4}}$

$\mu = \frac{1.25\times10^{-4}}{5\times10^{-4}} = 0.25$

$E = 2G(1+\mu)$

$G = \frac{E}{2(1+\mu)} = 48\times10^{3}\,MPa$ & $E = 3K(1-2\mu)$

$K = \frac{E}{3(1-2\mu)} = 80\times10^{3}\,N/mm^2$

N16. A round bar of diameter 40 mm is subjected to an axial pull of 80 kN and the reduction in diameter is found to be 0.00775 mm. Find Poisson's ratio and modulus of elasticity for the material of the bar, if the shear modulus is 40 kN/mm ^{2}.

$G = 40\times10^{3}\,N/mm^2$ $\quad P = 80\times10^{3}\,N$ $\quad d = 40\,mm$

$\delta d = 0.00775\,mm$ $\quad E = ?$ $\quad \mu = ?$

$\mu = \frac{\epsilon_{lat}}{\epsilon_{long}}$ $\quad \epsilon_{lat} = \frac{0.00775}{40} = 1.9375\times10^{-4}$

$\epsilon_{long} = \frac{\delta L}{L} = \frac{\sigma}{E} = \frac{P/A}{E} = \frac{\frac{80\times10^{3}}{(\pi\times40^2/4)}}{E}$

Now $\mu = \frac{1.9375\times10^{-4}\,E}{63.66}$ $\quad \boxed{E\times3.0434\times10^{-6} = \mu}$ — (1)

$E = 2G(1+\mu)$ $\quad E = 2\times40\times10^{3}(1+E\times3.0434\times10^{-6})$

$\boxed{E = 1.05\times10^{5}\,N/mm^2}$ $\quad \boxed{\mu = 0.322}$

T4.A circular rod of 100 mm diameter and 500 mm length is subjected to a tensile force of 2000kN. Determine the modulus of rigidity, bulk modulus and the change in volume, if the

Poisson's ratio = 0.3 and E = 2x10^5 N/mm^2.

$L = 500\,mm$ $\quad d = 100\,mm$ $\quad P = 2000\times10^{3}\,N$ | $G = ?$

$\mu = 0.3$ $\quad E = 2\times10^{5}\,N/mm^2$ | $K = ?$

$dV = ?$

$E = 2G(1+\mu)$ $\quad G = \frac{E}{2(1+\mu)} = 7.692\times10^{4}\,N/mm^2$

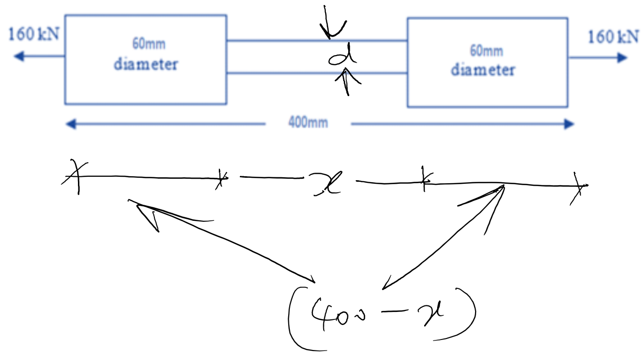
$E = 3K(1-2\mu)$ $\quad K = \frac{E}{3(1-2\mu)} = 1.667\times10^{5}\,N/mm^2$

$\frac{dV}{V} = \left(\frac{1-2\mu}{E}\right)(\sigma_x + \cancel{\sigma_y} + \cancel{\sigma_z})$ 0 0 $\quad$ also $\sigma = \frac{2000\times10^{3}}{\left(\frac{\pi\times100^2}{4}\right)}$

$dV = \left\{\left(\frac{\pi\times100^2}{4}\right)\times500\right\}\times\left(\frac{1-2\times0.3}{2\times10^{5}}\right)\times254.64 = 2000\,mm^3$

$$dv = \left\{\left(\frac{\pi\times 60^2}{4}\right)\times 500\right\}\times\left(\frac{1-2\times 0.3}{2\times 10^5}\right)\times 254.64 = \underline{\underline{2000}}\ mm^3$$

T5. The bar shown in the figure is subjected to a tensile load of 160 kN, If stress in the middle portion is limited to 150 MPa, determine the diameter and length of the middle portion if the total elongation of the bar is to be 0.2 mm. Take E = 210 GPa.



① $P = 160\ kN\ (T)$

$\sigma = 150\ MPa$

$\sigma = P/A \Rightarrow 150 = \dfrac{160\times 10^3}{(\pi\times d^2/4)}$

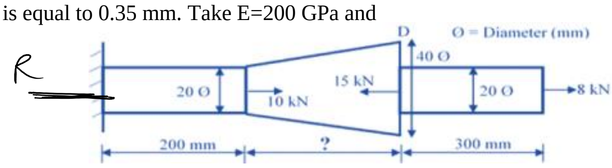
$d = \underline{\underline{36.85\ mm}}$

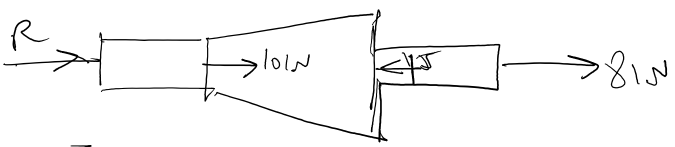
$$\delta L = \underbrace{\delta l_1}_{\text{left \& right}} + \underbrace{\delta l_2}_{(x)} = \frac{160\times 10^3\times(400-x)}{\left(\frac{\pi\times 60^2}{4}\right)\times 2.1\times 10^5} + \frac{160\times 10^3\times x}{\left(\frac{\pi\times 36.85^2}{4}\right)\times 2.1\times 10^5}$$

$\delta L = 0.2\ mm$

$\boxed{x = 208.05\ mm}$

T6. Determine the length of the tapered portion if the total deformation of the stepped bar is equal to 0.35 mm. Take E=200 GPa and find the stress in each section.

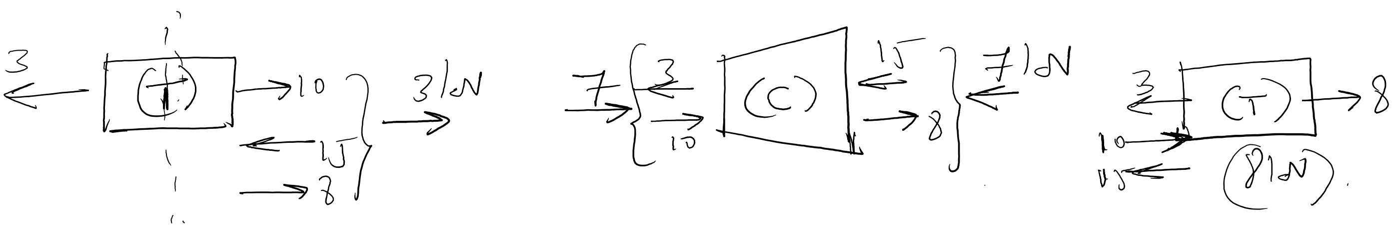




$\Sigma F_x = 0.$

$R + 10 - 15 + 8 = 0.$

$R = \underline{\underline{-3\ kN}}$ (leftward).



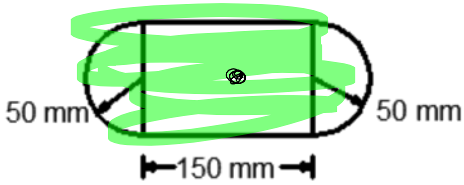
$$\delta L = \frac{3\times 10^3\times 200}{\left(\frac{\pi\times 20^2}{4}\right)\times 2\times 10^5} + \frac{(-7\times 10^3\times l)\times 4}{\pi\times 2\times 10^5\times 20\times 40} + \frac{8\times 10^3\times 300}{\left(\frac{\pi\times 20^2}{4}\right)\times 2\times 10^5}$$

Ans.

Also, $\delta l = 0.35$ mm find l.

$$\boxed{l = 225.31\text{ mm}}$$

T7. A 6 mm thick plate is to be punched of a shape shown in figure. Determine the minimum punching force to be applied on punch. The ultimate shear strength of the plate is 320 MPa. What is the corresponding compressive stress in the punch?



Force P in punch $= \tau_{max} \times A$

$$= 320 \times \underbrace{\left(150 + 150 + \left(\frac{\pi \times 100}{2}\right) \times 2\right) \times 6}_{\text{Area resisting shear.}}$$

$$= 1179.18 \text{ kN}$$

Compressive stress in punch $= \dfrac{P}{A} = \dfrac{1179.18 \text{ kN}}{(150 \times 100) + \left(\frac{\pi \times 100^2}{4}\right)}$

$$\sigma = \frac{1179.18 \times 10^3}{22853.98} = 51.6 \text{ N/mm}^2$$

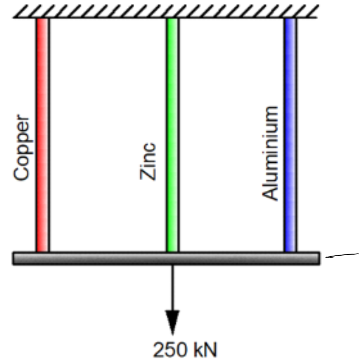
STRESSES DUE TO FLUID PRESSURE IN THIN CYLINDERS

03 January 2022 01:34 PM

Compound bars

24 January 2022 12:18 PM

N1. Three bars made of copper, zinc and aluminium each of 2.5 m length have cross section 500 mm^2, 750 mm^2 and 1000 mm^2 respectively. These bars are rigidly connected through a rigid horizontal bar. This compound member is subjected to a longitudinal pull of 250 kN such that the rigid bar is horizontal even after the loading. Calculate the stresses in each bars and determine the total deformation of this compound bar. Take E for copper = 1.3×10^5 N/mm^2, for zinc = 1.0×10^5 N/mm^2 and aluminium = 0.8×10^5 N/mm^2



$$W = W_{Cu} + W_{Zn} + W_{Al}$$

$$250 \times 10^3 \, N = W_{Cu} + W_{Zn} + W_{Al} \quad \text{Eq. (1)}$$

Compatibility

$$dL_{Cu} = dL_{Zn} = dL_{Al}$$

$$\frac{\sigma_{Cu} \times 2.5 \times 1000}{500 \times 1.3 \times 10^5} = \frac{\sigma_{Zn} \times 2.5 \times 10^3}{750 \times 1 \times 10^5} = \frac{\sigma_{Al} \times 2.5 \times 10^3}{1000 \times 0.8 \times 10^5}$$

$$\boxed{\sigma_{Cu} = 0.86 \, \sigma_{Zn}} \qquad \boxed{\sigma_{Cu} = 0.812 \, \sigma_{Al}}$$

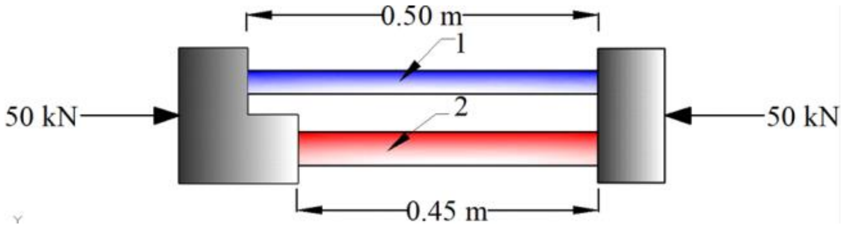
$$250 \times 10^3 = \sigma_{Cu} A_{Cu} + \sigma_{Zn} A_{Zn} + \sigma_{Al} A_{Al}$$

$$250 \times 10^3 = \sigma_{Cu} \times 500 + \frac{1}{0.86} \sigma_{Cu} \times 750 + \frac{\sigma_{Cu}}{0.812} \times 1000$$

$$\boxed{\sigma_{Cu} = 96.243 \, N/mm^2} \quad \therefore \quad \boxed{\begin{aligned} \sigma_{Al} &= 118.52 \, N/mm^2 \\ \sigma_{Zn} &= 111.135 \, N/mm^2 \end{aligned}}$$

N2.Two bars, one of 0.5 m length and the other 0.45 m length are rigidly connected at ends as shown in fig. Find the stresses in each bar when a compressive force of 50 kN acts on the compound bar. Given, A_{st1} = 25000 mm^2, A_{st2} = 50000 mm^2, $E_1 = 2 \times 10^5$ N/mm^2 and $E_2 = 1 \times 10^5$ N/mm^2

N2.Two bars, one of 0.5 m length and the other 0.45 m length are rigidly connected at ends as shown in fig. Find the stresses in each bar when a compressive force of 50 kN acts on the compound bar. Given, A_{st1} = 25000 mm², A_{st2} = 50000 mm². $E_1 = 2\times10^5$ N/mm² and $E_2 = 1\times10^5$ N/mm².



$$dL_1 = dL_2$$

$$\frac{W_1 L_1}{E_1 A_1} = \frac{W_2 L_2}{A_2 E_2}$$

$$\frac{W_1 \times 500}{2\times10^5 \times 25000} = \frac{W_2 \times 450}{50000 \times 1\times10^5}$$

$$W_1 = 0.9\, W_2$$

$$W = W_1 + W_2.$$

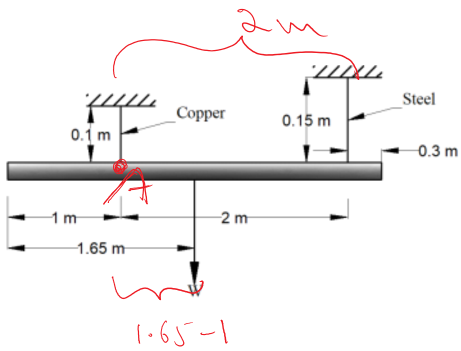
$$50\times10^3 = W_2\times0.9 + W_2$$

$$\boxed{W_2 = 26.315\,kN} \quad \boxed{W_1 = 23.68\,kN}$$

$$\sigma_1 = \frac{W_1}{A_1} = \underline{0.947\ N/mm^2}$$

$$\sigma_2 = \frac{W_2}{A_2} = \underline{0.526\ N/mm^2}$$

N3. A uniform concrete slab of total weight 'W' is to be attached to two wires whose lower ends are at the same level as shown in fig. Determine the ratio of areas of rods so that slab remains horizontal. $E_s = 2\times10^5$ N/mm² and $E_{cu} = 1\times10^5$ N/mm².



Compatibility

$$dL_{cu} = dL_s.$$

$$\frac{\sigma_{cu}\times100}{1\times10^5} = \frac{\sigma_s\times150}{2\times10^5}$$

$$\sigma_{cu} = 0.75\,\sigma_{st}$$

$$\frac{W_{cu}}{A_{cu}} = 0.75\,\frac{W_s}{A_s} \Rightarrow \boxed{\frac{A_s}{A_{cu}} = 0.75\,\frac{W_s}{W_{cu}}} \text{ — (1)}$$

Equilibrium

$$W = W_{cu} + W_s$$

$$\Sigma F_y = 0.$$

$$\Sigma M_A$$

$\sum M_{@A}$

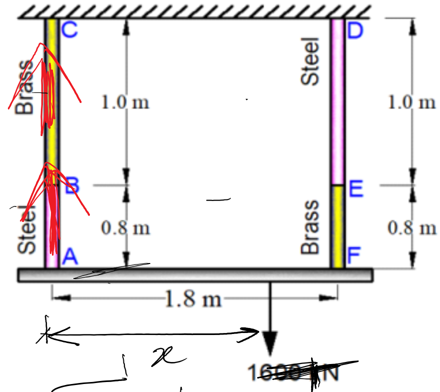
$W \times 0.65 = W_s \times 2$

$\boxed{W_s = 0.325\,W} \quad \therefore \quad \boxed{W_{cu} = 0.675\,W}$ — (2)

Substitute (2) in (1).

$$\frac{A_s}{A_{cu}} = 0.75 \times \frac{0.325\,\cancel{W}}{0.675\,\cancel{W}} = \boxed{0.36}$$

N4. Two rigidly connected parallel composite bars ABC and DEF are suspended at the upper ends and support a rigid horizontal bar of weight 500 N as shown in fig. Each bar is of 10 mm diameter and 1.8 m long. Segments AB and DE are of steel and segments BC and EF are of Brass. Determine the position of additional weight 1600 N to be attached so that the rigid bar remains horizontal after loading. Also calculate stresses in each rod. Take Es = 210 GPa and Ebr = 100 GPa



(1) 500 N.

Compatibility condition

$$(dL_s + dL_B)_{ABC} = (dL_s + dL_B)_{DEF}$$

$$\frac{\sigma'_{ABC} L_s}{E_s} + \frac{\sigma'_{ABC} L_B}{E_B} = \frac{\sigma'_{DEF} L_s}{E_S} + \frac{\sigma'_{DEF} L_B}{E_B}$$

$$\sigma'_{ABC}\left(\frac{800}{2.1\times10^5} + \frac{1000}{1\times10^5}\right) = \sigma'_{DEF}\left(\frac{800}{1\times10^5} + \frac{1000}{2.1\times10^5}\right).$$

$$\boxed{\sigma'_{ABC} = 0.92\,\sigma'_{DEF}} \text{ — (1)}$$

Equilibrium

$$500 = \sigma'_{ABC} \times A + \sigma'_{DEF} \times A$$

$$\frac{500}{(\pi \times 10^2/4)} = \sigma'_{ABC} + \sigma'_{DEF} \text{ — (2)}$$

sub (1) → (2) $\quad \sigma'_{ABC} = 3.05\ N/mm^2$

sub ① → ②

$$\boxed{\begin{aligned}\sigma'_{ABC} &= 3.05\ N/mm^2\\ \sigma'_{DEF} &= 3.32\ N/mm^2\end{aligned}}$$

Case ② (additional load 1600 N)

Compatibility condition

$$\boxed{\sigma''_{ABC} = 0.92\,\sigma''_{DEF}} \quad \text{—— ③}$$

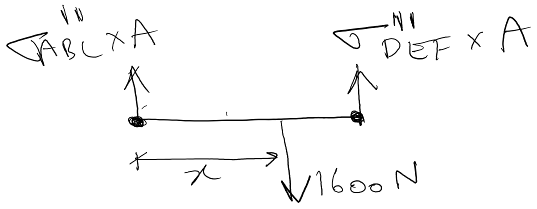
Equilibrium conditions

$$1600 = \sigma''_{ABC}\,A + \sigma''_{DEF}\,A$$

$$\frac{1600}{(\pi\times10^2/4)} = \sigma''_{ABC} + \sigma''_{DEF} \Longrightarrow \boxed{\begin{aligned}\sigma''_{ABC} &= 9.76\ N/mm^2\\ \sigma''_{DEF} &= 10.61\ N/mm^2\end{aligned}}$$

total stress $\sigma_{ABC} = \sigma'_{ABC} + \sigma''_{ABC} = 12.81\ N/mm^2$

$\sigma_{DEF} = \sigma'_{DEF} + \sigma''_{DEF} = 13.93\ N/mm^2$



$$\Sigma M@A = 0$$

$$+\sigma''_{DEF}\times A\times1.8 - 1600\,x = 0$$

$$10.61\times\pi\times10^2/4\times1.8 - 1600\,x = 0$$

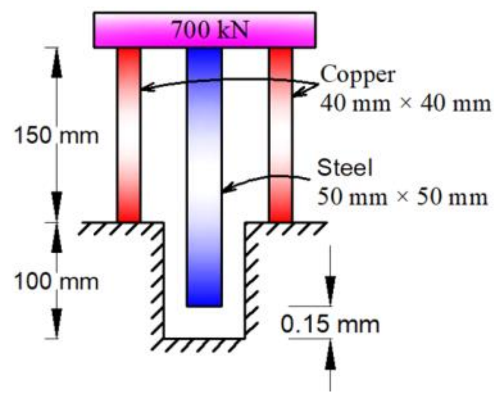
$$\boxed{x = 0.93\ m}$$

N5. A compound bar made of steel and copper bars is loaded with 700 kN load as shown. If a gap of 0.15 mm is left between ste el bar and the floor, determine the forces in copper and steel. Consider $E_s = 2.1\times10^5\ N/mm^2$, $E_{cu} = 1\times10^5\ N/mm^2$

Compatibility

$$dL_c = dL_s + 0.15$$

$$\frac{P_{cu}\times L_{cu}}{E_{cu}\times A_{cu}} = \frac{P_{st}\times L_{st}}{E_{st}\times A_{st}} + 0.15$$



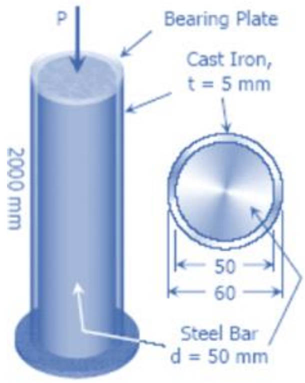
$$\frac{P_{cu}\times150}{1\times10^5\times2\times40\times40} = \frac{P_{st}\times249.85}{2.1\times10^5\times50\times50} + 0.15 \quad \text{—— ②}$$

$$\frac{\cdots}{1\times10^5 \times 2\times40\times40} = \frac{P_{st}\times 249.85}{2\times10^5\times50\times50} + 0.15 \quad \text{——— (2)}$$

$$P_{cu} = P_{st} + 320\times10^3 \quad \text{——— (3)} \qquad \text{Equilibrium} \Rightarrow 700\times10^3 = 2P_{cu} + P_{st}$$

$$\boxed{P_{st} = 20\ kN} \quad \boxed{P_{cu} = 340\ kN}$$

T1. A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of a cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, E = 200 GPa, and for cast iron, E = 100 GPa.



Compatibility

$$(dL)_{st} = (dL)_i = 0.8\,mm$$

$$\frac{P_{st}\times L_{st}}{A_{st}\times E_{st}} = \frac{P_i\times L_i}{A_i\times E_i} = 0.8\,mm$$

$$\frac{P_{st}\times 2000}{\left(\frac{\pi\times50^2}{4}\right)\times2\times10^5} = 0.8\,mm.$$

$$\boxed{P_{st} = 157.08\ kN}$$

$$\frac{P_i\times2000}{\left(\frac{\pi\times(60^2-50^2)}{4}\right)\times1\times10^5} = 0.8$$

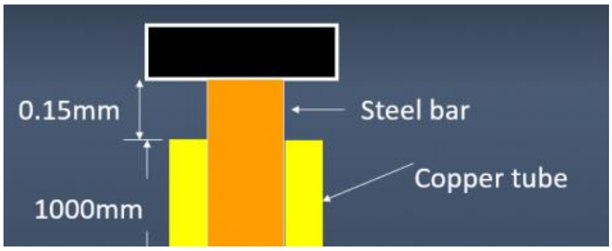
$$\boxed{P_i = 345.58\ kN}$$

Total load required $= P_i + P_{st}$.

$$P = \underline{502.66\ kN}.$$

T2. A mild steel bar of c/s 490 mm^2 is surrounded by a copper tube of c/s 210 mm^2 as shown. When they are placed centrally over a rigid bar, it is found that steel bar is 0.15 mm longer. Over this unit a rigid plate carrying a load of 80 kN is placed. Find the stress in each material.

Take E_s = 200 GPa, E_c = 100 GPa.



$$80\,kN = \sigma_{cu}\times A_{cu} + \sigma_{st}\,A_{st}$$

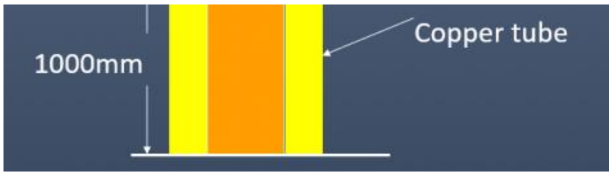
Compatibility

$$0.15 + (dL \cdots = (dL \cdots$$

$$0.154\,(dL_{cu}) = (dL)_{st}$$

$$\sigma_{cu} = 54.87\ N/mm^2$$

$$\sigma_{st} = 139.84\ N/mm^2$$



T3. A reinforced concrete column 200mm diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of reinforcing steel if the allowable stress are 6 MPa and 120 MPa for the concrete and steel respectively. Take E_{co}= 14 GPa, E_{st}= 200 GPa.

T4. A reinforced concrete column 250 mm × 250 mm in cross section is reinforced with 8 steel rods of total cross sectional area 2500 mm². The column carries a compressive load of 390 kN. If the modulus of elasticity for steel is 15 times that of concrete,

find (i) the stresses in concrete and steel. (ii) Area of steel required so that column may support a load of 480 kN if maximum stress in concrete is 4.5 N/mm².

T3 Compatibility

$$dL_{st} = dL_c$$

$$\frac{\sigma_{st} \times \cancel{l_{st}}}{E_{st}} = \frac{\sigma_c \times \cancel{L_c}}{E_c}$$

$$\sigma_{st} = \frac{E_{st}}{E_c}\,\sigma_c \qquad \boxed{\sigma_{st} = 14.28\,\sigma_c} \text{ — (1)}$$

case (1) if $\sigma_c = 6\ MPa$

what is $\sigma_{st} = ?$ $\quad \sigma_{st} = 14.28 \times 6 = 85.68\ N/mm^2$

$\sigma_{st} = 85.68 << 120\ N/mm^2$

Case (1) is valid

case (2) if $\sigma_{st} = 120\ N/mm^2$

— 120 ... 8.4 N/ ... $>> 6\ N/mm^2$

case (2) if $\sigma_{st} = 120\ N/mm^2$

$$\sigma_c = \frac{120}{14.28} = 8.4\ N/mm^2 >> 6\ N/mm^2$$

$\therefore$ Not valid

Equilibrium

$$W = \sigma_{st} A_{st} + \sigma_c A_c.$$

$$300 \times 10^3 = 85.68 \times A_{st} + 6 \times \left(\frac{\pi \times 200^2}{4} - A_{st}\right)$$

$$\boxed{A_{st} = 1398.35\ mm^2}$$

dia 8 mm, 10, 12, 16, 20, 25, 32

$$\text{No. of 16 mm bars} = \frac{1398.35}{\left(\pi \times \frac{16^2}{4}\right)} \simeq 6.9 \simeq \boxed{7}$$