Poset : $(A, \leq)$ s.t $\leq$ satisfying i) ref
ii) transitive
iii) antisym

Hasse Diagram :

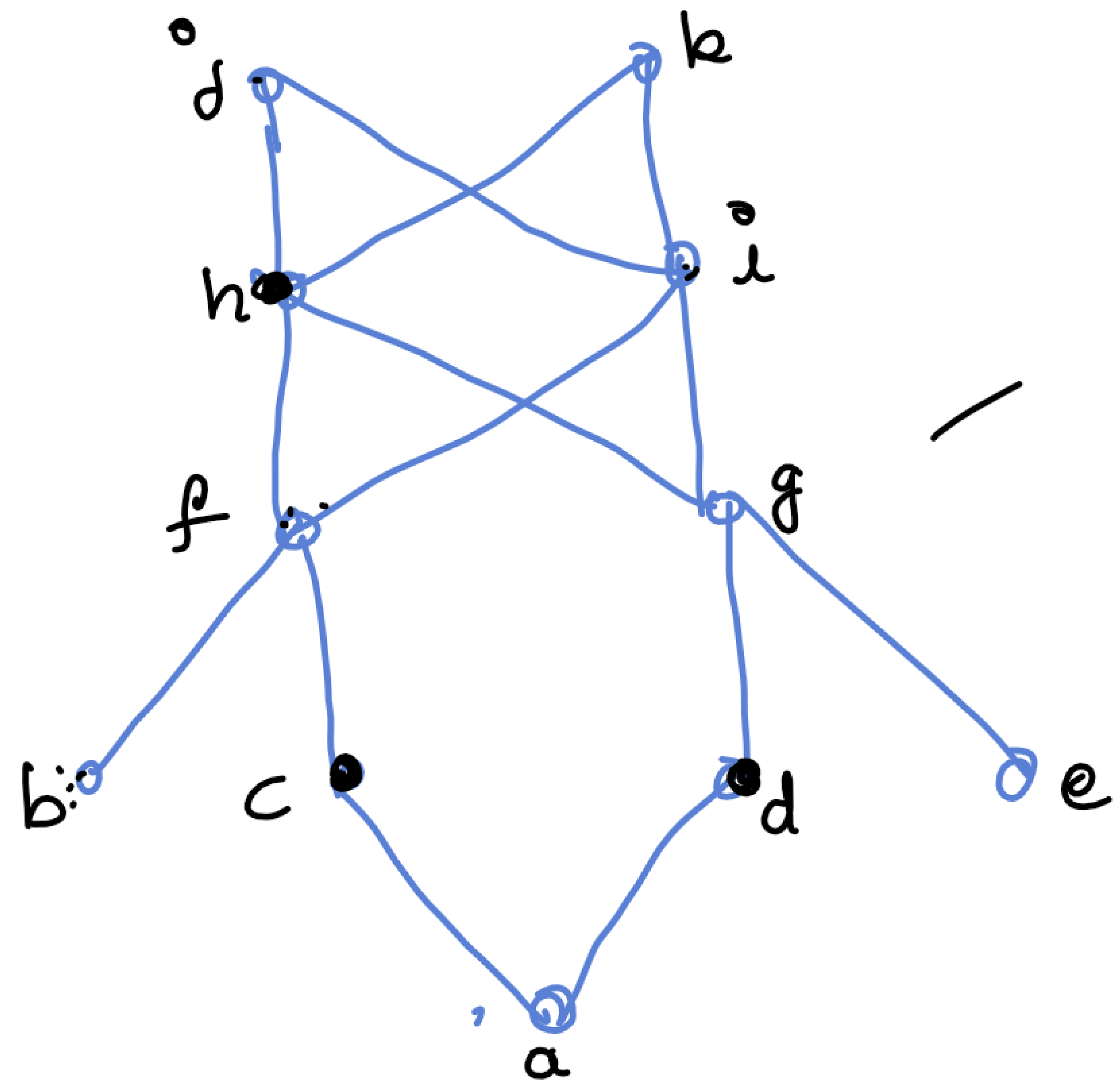
Upper bound : In a poset $(A, \leq)$, an element c is said to be an upper bound of a & b if $a \leq c$ and $b \leq c$

Least upper bound (lub) : An elt 'c' is said to be a least upper bound (supremum) of a & b if
i) c is an upper bound of a & b
and
ii) there is no other other upper bound 'd' of a & b s.t $d \leq c$

Lower bound : An elt 'c' is said to be a lower bound of a & b if $c \leq a$ & $c \leq b$

Greatest lower bound (glb) : An elt c is said to be glb of a & b if
i) c is a lower bound of a & b
and
ii) there no other lower bound 'd' of a & b s.t $c \leq d$

example 1:



consider the elts f & g

① h, i, j, k are all the upper bounds of f & g

② least upper bound for f & g

$lub(f,g) = h$

$lub(f,g) = i$

③ $lub(b,f) = f$

$b \le f$ & $f \le f$

f is the least one

④ $lub(c,d) = h$

$c \le h$, $d \le h$

$lub(c,d) = i$

⑤ a, b, c, d, e, f, g are the lower bounds of h & i

⑥ $glb(h,i) = f$

$glb(h,i) = g$

⑦ $glb(b,f) = b$

⑧ $glb(h,g) = g$

$lub(h,g) = h$

* Consider a poset $(N, |)$

① For any 2 elts a & b, the upper bounds are : all elements which are common multiples of a & b

② $lub(a,b) = lcm(a,b)$

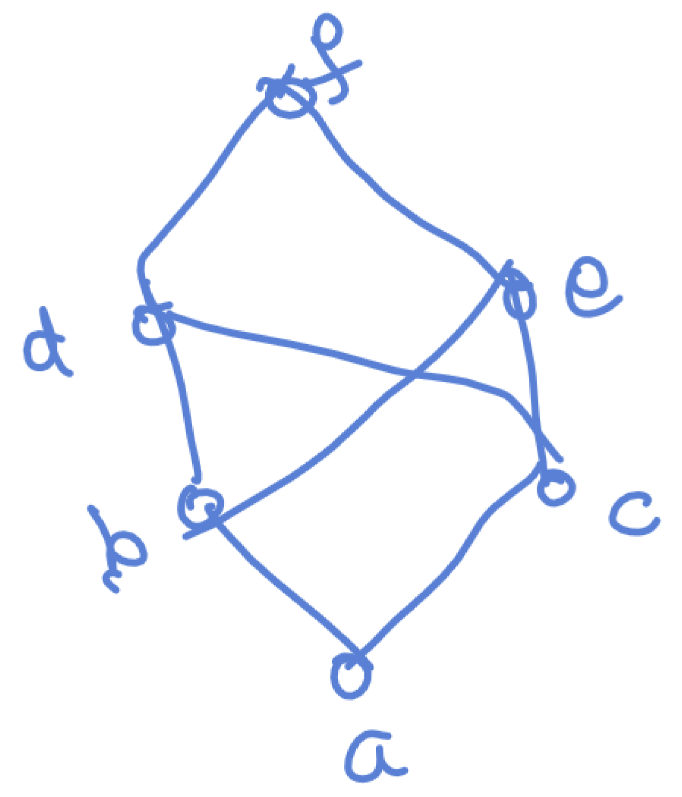
③ For any 2 elts a & b, the lower bounds are the common divisors of a & b

④ $glb(a,b) = gcd(a,b)$

Lattice (George Boole)

A partially orderd set is said to be a lattice if every two elements in the set have unique least upper bound & unique greatest lower bound.

① ex 1 (in prev page) is not a lattice

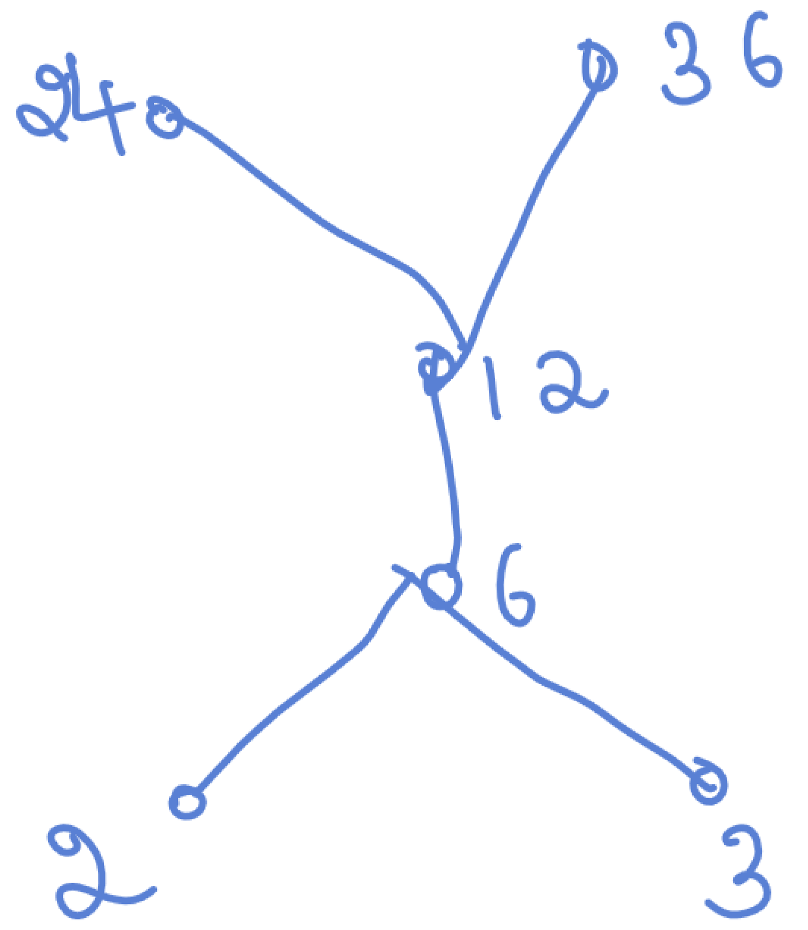
② 

is not a lattice

Bcz, the elts d & e have two glb's

ie glb(d, e) = b

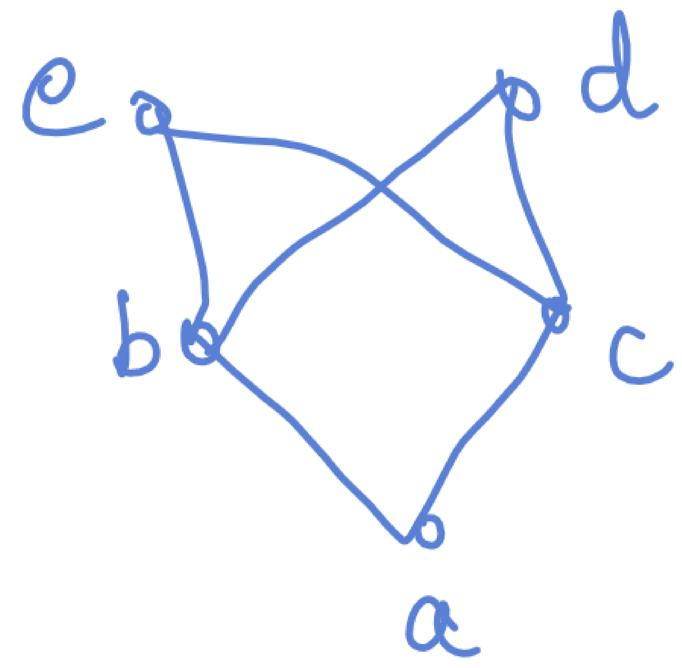
glb(d, e) = c

③ 

lub (24, 36) is not present

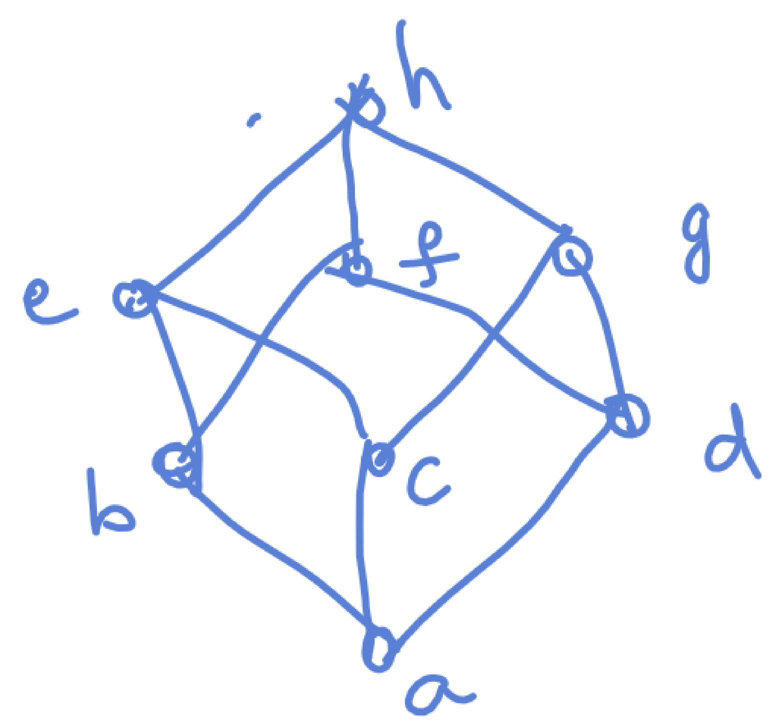
glb (2, 3) is not present

∴ Not a lattice

③ 

Not a lattice

lub(e, g) is not present

④ 

⑤