25 November 2021 03:14 PM

$$I_{AB} = \int da \cdot (d+y)^2$$

$$= \int da \cdot d^2 + \int da \cdot y^2 + \int da \cdot 2 d \cdot y$$

$$= A \cdot d^2 + I_{xx} + 2d \cancel{\int da \cdot y}^{\,0}$$

$$\boxed{I_{AB} = I_{xx} + A \cdot d^2}$$

29 November 2021 07:55 AM

$$A\bar{y} = \int_0^H da \cdot y$$

$$da = b \cdot dy$$

$$da = \left(\frac{B}{H}(H-y)\right) \cdot dy$$

$$\left(\tfrac{1}{2} \times B \times H\right) \times \bar{y} = \int_0^H \left(\frac{B}{H}(H-y) \cdot dy\right) \cdot y$$

$$\boxed{\bar{y} = H/3}$$

$$\boxed{\bar{x} = B/3} \rightarrow da = \left[\frac{H}{B}(B-x)\right] \cdot dx$$

29 November 2021 07:55 AM

Semi circle

$\bar{x} = R$

$$\frac{\pi R^2}{2} \times \bar{y} = \int_0^R \int_0^\pi da \cdot y$$

$$\bar{y} = \int_0^R \int_0^\pi [d\theta \cdot r \cdot dr][r \sin\theta]$$

$$\boxed{\bar{y} = \frac{4R}{3\pi}}$$

$da = d\theta \cdot r \cdot dr$

$y = r \cdot \sin\theta$

$$I_{AB} = \int_0^\pi \int_0^R da \cdot y^2$$

$$\boxed{I_{AB} = \frac{\pi R^4}{8}}$$

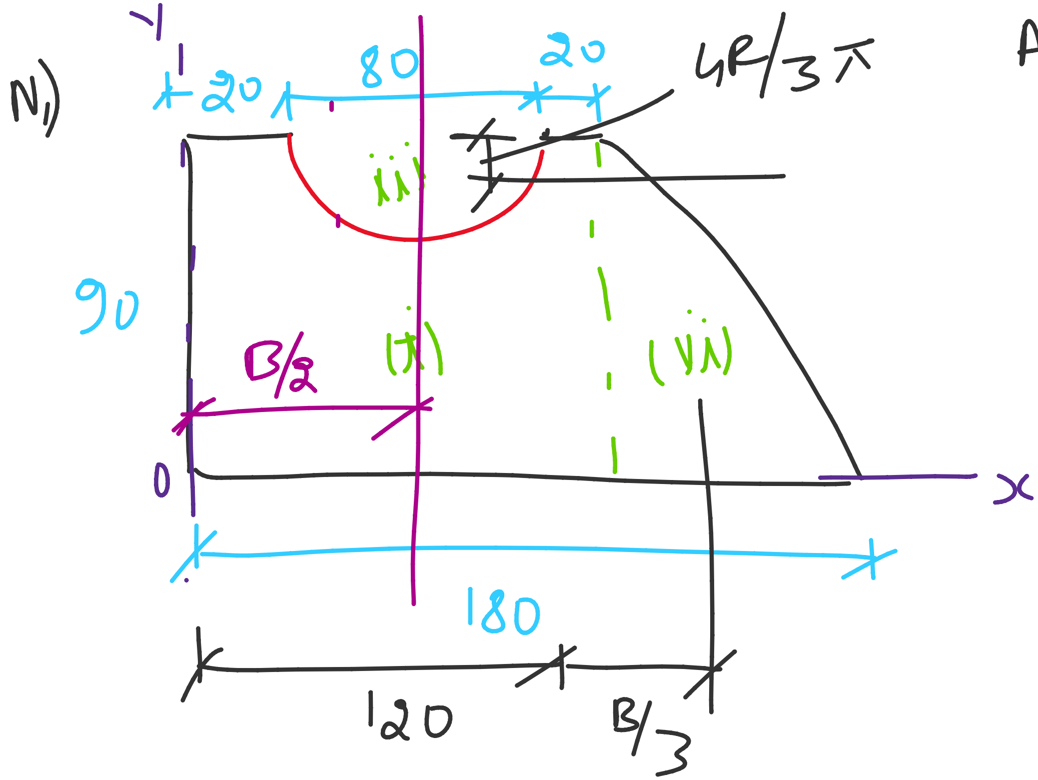
$$\boxed{I_{xx} = 0.11 R^4}$$

$I_{AB} = I_{xx} + Ad^2$

$$\frac{\pi R^4}{8} = I_{xx} + \left(\frac{\pi R^2}{2}\right) \times \left(\frac{4R}{3\pi}\right)^2$$

29 November 2021 07:55 AM

N1)



$A_i = 120 \times 90$, $A_{ii} = \frac{1}{2} \times 60 \times 90$

$A_{iii} = \frac{\pi 40^2}{2}$. $\boxed{A = 10986.73}$ mm^2

$$\bar{x} = \frac{\Sigma a_i x_i}{A}$$

$$= \frac{\left[120 \times 90 \times 120/2\right]_i + \left[\frac{1}{2} \times 60 \times 90 \times (120 + 60/3)\right]_{ii} - \left[\frac{\pi 40^2}{2} \times 2r + 40\right]}{\left[120 \times 90\right] + \left[\frac{1}{2} \times 60 \times 90\right] - \left[\frac{\pi 40^2}{2}\right]}$$

$$\boxed{\bar{x} = 79.66 \text{ mm}}$$

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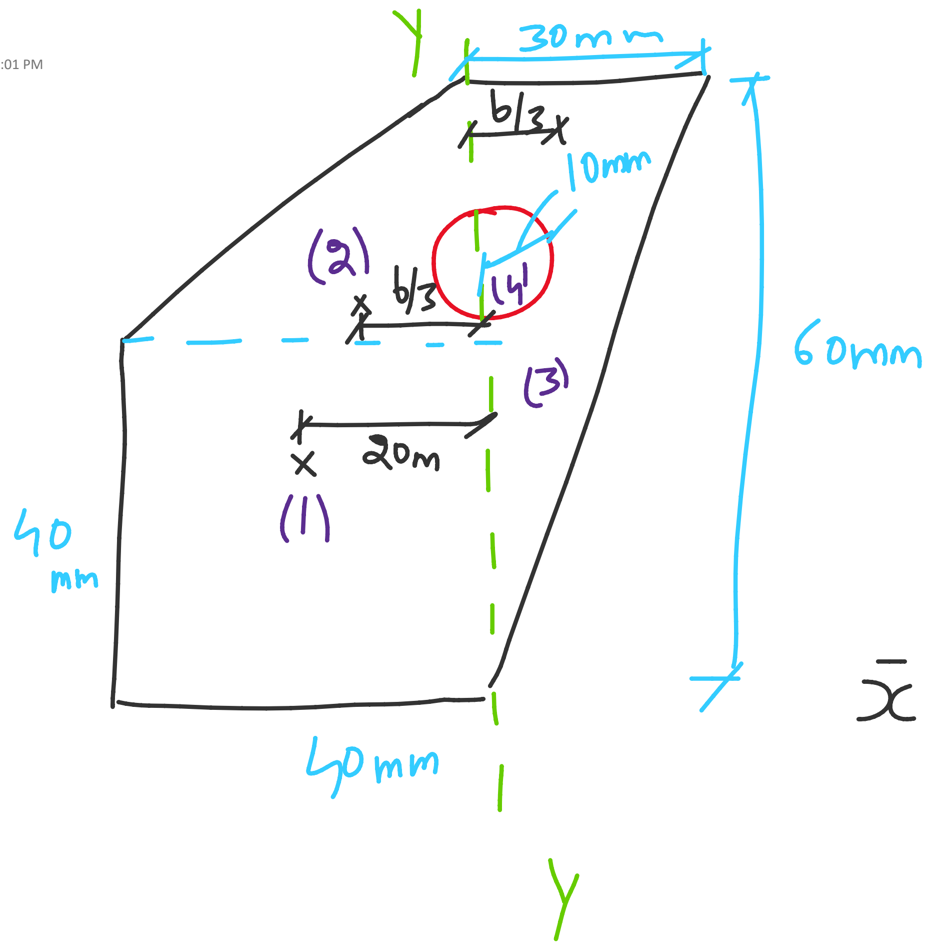
$$\bar{y} = \frac{\Sigma a_i y_i}{A}$$

$$= \frac{\left[120 \times 90 \times \left(90/2\right)\right] + \left[1/2 \times 60 \times 90 \times \left(90/3\right)\right] - \left[\frac{\pi 40^2}{2} \times \left(90 - \frac{4 \times 40}{3\pi}\right)\right]}{10986.73}$$

$$\boxed{\bar{y} = \underline{34.90} \text{ mm}}$$

02 December 2021 02:01 PM

T1)



$$a_1 = 40 \times 40 = 1600\ mm^2$$

$$a_2 = \tfrac{1}{2} \times 40 \times 20 = 400\ mm^2$$

$$a_3 = \tfrac{1}{2} \times 30 \times 60 = 900\ mm^2$$

$$a_4 = -\pi \times 10^2 = -314.16\ mm^2$$

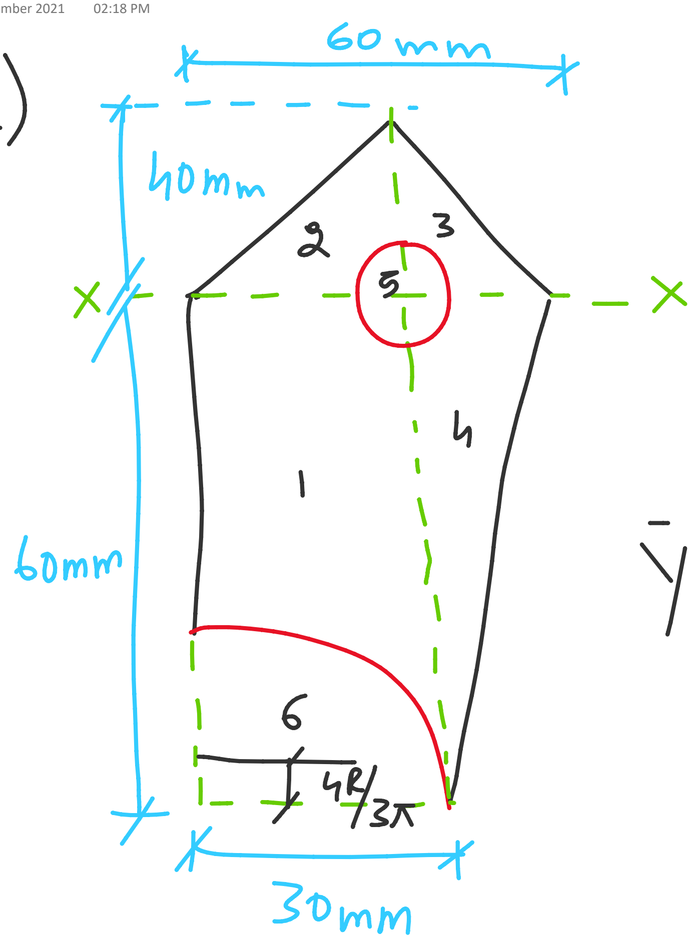
$$\Sigma_A = 2585.84\ mm^2$$

$$\bar{x} = \frac{\Sigma a_i x_i}{A} = \frac{\left[1600 \times -40/2\right]_1 + \left[400 \times -40/3\right]_2 + \left[900 \times 30/3\right]_3 - \left[314.16 \times 0\right]_4}{2585.84}$$

$$\boxed{\bar{x} = -10.95mm}$$

02 December 2021 02:18 PM

T2)



$a_1 = 60 \times 30 = 1800\,mm^2$

$a_2 = \frac{1}{2} \times 30 \times 40 = 600\,mm^2$

$a_3 = \frac{1}{2} \times 30 \times 40 = 600\,mm^2$

$a_4 = \frac{1}{2} \times 30 \times 60 = 900\,mm^2$

$a_5 = -\pi \times 10^2 = -314.16\,mm^2$

$a_6 = -\frac{\pi \times 30^2}{4}$

$= -706.86\,mm^2$

$\Sigma A = 2878.98\,mm^2$

$$\bar{Y} = \frac{\Sigma a_i y_i}{\Sigma A} = \frac{[1800 \times -60/2]_1 + [600 \times 40/3]_2 + [600 \times 40/3]_3 + [900 \times -60/3]_4 - [\cancel{314.16 \times 0}]_5 - \left[706.86 \times -\left(60 - \frac{4 \times 30}{3\pi}\right)\right]_6}{2878.98}$$

$$\boxed{\bar{Y} = -7.84\ mm}$$

02 December 2021 02:39 PM

T3)

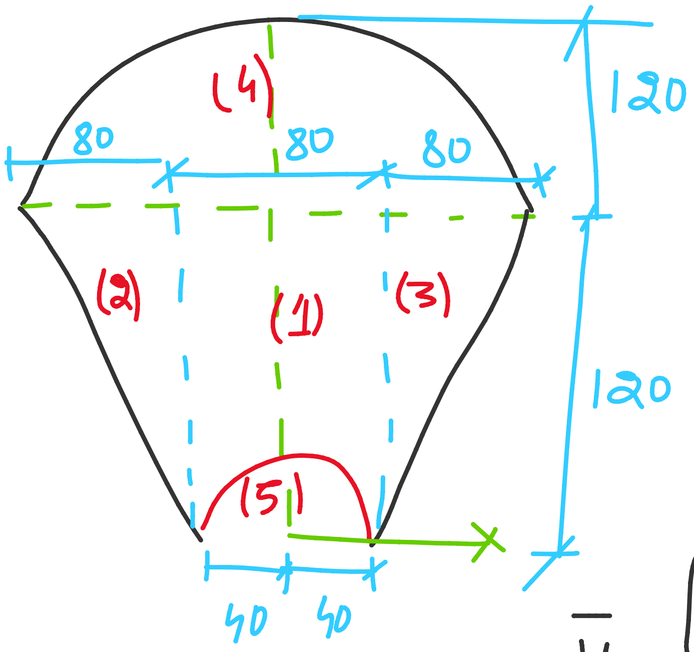
$$\bar{x} = 0$$

$$\bar{y} = \frac{\left[223000 \times (60+10)\right] + \left[(120 \times 10) \times 10/2\right]}{223000 + (120 \times 10)}$$

$$\boxed{\bar{y} = \underline{69.65} \text{ mm}}$$

02 December 2021 02:55 PM

N2)



$$a_1 = 80 \times 120 = 9600\,mm^2$$

$$a_2 = \tfrac{1}{2} \times 80 \times 120 = 4800\,mm^2$$

$$a_3 = \tfrac{1}{2} \times 80 \times 120 = 4800\,mm^2$$

$$a_4 = \frac{\pi \times 120^2}{2} = 22619.46\,mm^2$$

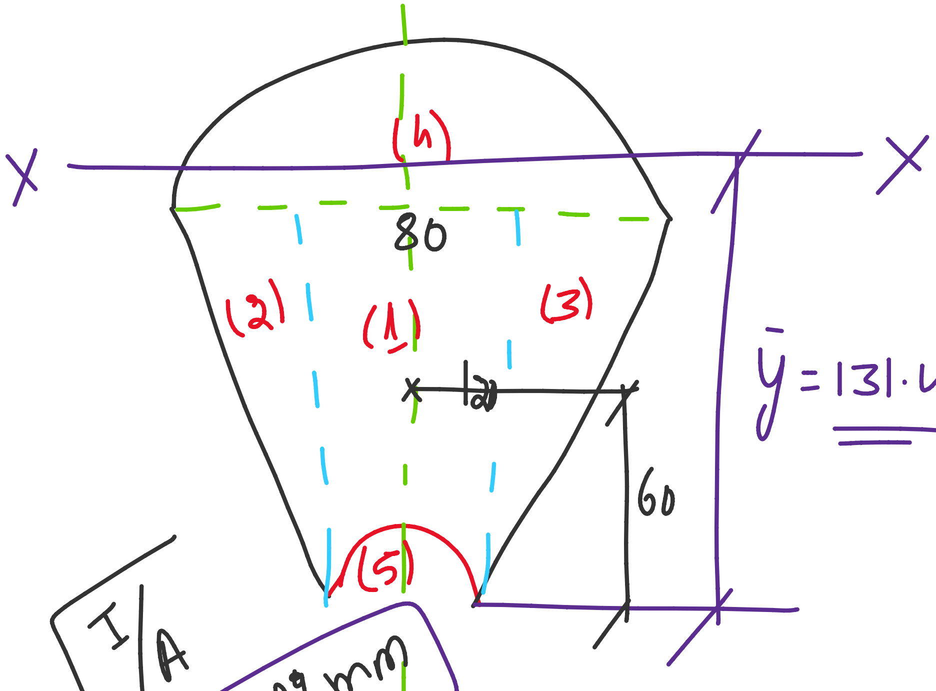
$$a_5 = -\frac{\pi \times 40^2}{2} = -2513.27\,mm^2$$

$$\Sigma A = 39306.19\,mm^2$$

$$\bar{y} = \frac{\left[9600 \times 120/2\right] + 2\left[4800 \times 2/3 \times 120\right] + \left[22619.46 \times \left(120 + \frac{4 \times 120}{3\pi}\right)\right] - \left[2513.27 \times \frac{4 \times 40}{3\pi}\right]}{39306.19}$$

$$\boxed{\bar{y} = 131.47\ mm}$$

02 December 2021 03:13 PM



$\bar{y} = 131.47 mm$

$K = \sqrt{I/A}$

$\boxed{K = 55.09 \ mm}$

$$\boxed{I_{xx} = 11.93 \times 10^7 \ mm^4}$$

$$I_{xx} = \left[\frac{bd^3}{12} + Ad^2\right]_1 + 2\left[\frac{bh^3}{36} \cancel{+ Ad^2}\right]_{2,3}$$

$$+ \left[0.11R^4 + Ad^2\right]_4 - \left[0.11R^4 \cancel{+ Ad^2}\right]_5$$

$$= \left[\frac{80 \times 120^3}{12} + (80 \times 120) \times (131.47 - 60)^2\right]_1$$

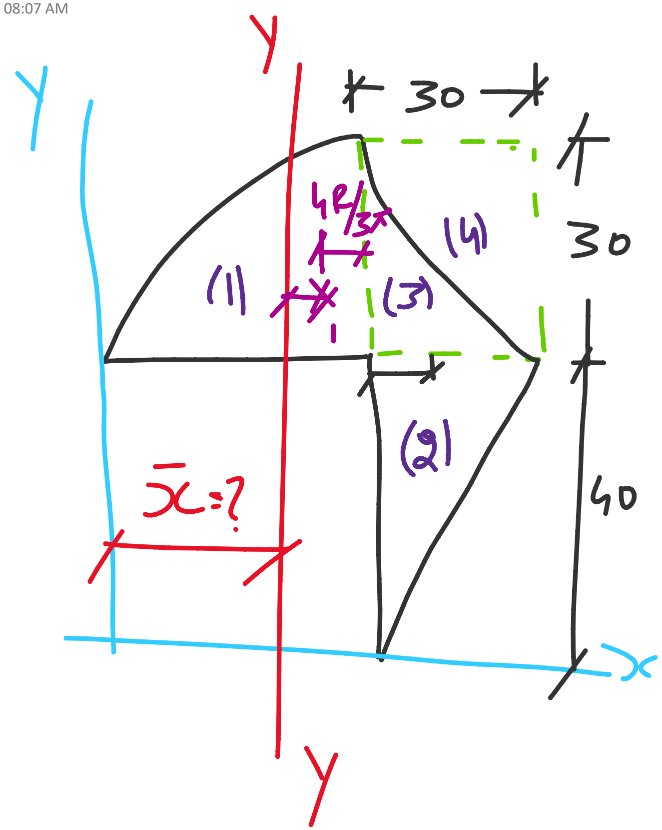
$$+ 2\left[\frac{80 \times 120^3}{30} + \left(\tfrac{1}{2} \times 80 \times 120\right) \times \left(131.47 - \tfrac{2}{3} \times 120\right)^2\right]_{2,3}$$

$$+ \left[0.11 \times 120^4 + \left(\frac{\pi \times 120^2}{2} \times \left(\frac{4 \times 120}{3\pi} - 11.47\right)^2\right)\right]_4$$

$$- \left[0.11 \times 40^4 + \left(\frac{\pi \times 40^2}{2} \times \left(131.47 - \frac{4 \times 40}{3\pi}\right)^2\right)\right]_5$$

06 December 2021 08:07 AM

N3)



$$\bar{x} = \frac{\left[\frac{\pi \times 30^2}{4} \times \left(30 - \frac{4 \times 30}{3\pi}\right)\right]_1 + \left[\frac{1}{2} \times 30 \times 40 \times \left(30 + \frac{1}{3} \times 30\right)\right]_2 + \left[30 \times 30 \times \left(30 + \frac{30}{2}\right)\right]_3 - \left[\frac{\pi \times 30^2}{4} \times \left(60 - \frac{4 \times 30}{3\pi}\right)\right]_4}{\left(\frac{\pi \times 30^2}{\cancel{4}}\right) + \left(\frac{1}{2} \times 30 \times 40\right) + \left(30 \times 30\right) - \frac{\pi \times 30^2}{\cancel{4}}}$$

$$\boxed{\bar{x} = \underline{28.86}\ \text{mm}}$$

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$$I_{yy} = \left[0.055(30)^4 + \frac{\pi \times 30^2}{4} \times \left(28.86 - \left(30 - \frac{4\times 30}{3\pi}\right)\right)^2\right]_1 +$$

$$\left[\frac{40\times 30^3}{36} + \left(\tfrac{1}{2}\times 40\times 30\right)\times\left(\bar{x} - 30 + \left(\tfrac{1}{3}\times 30\right)\right)^2\right]_2 +$$

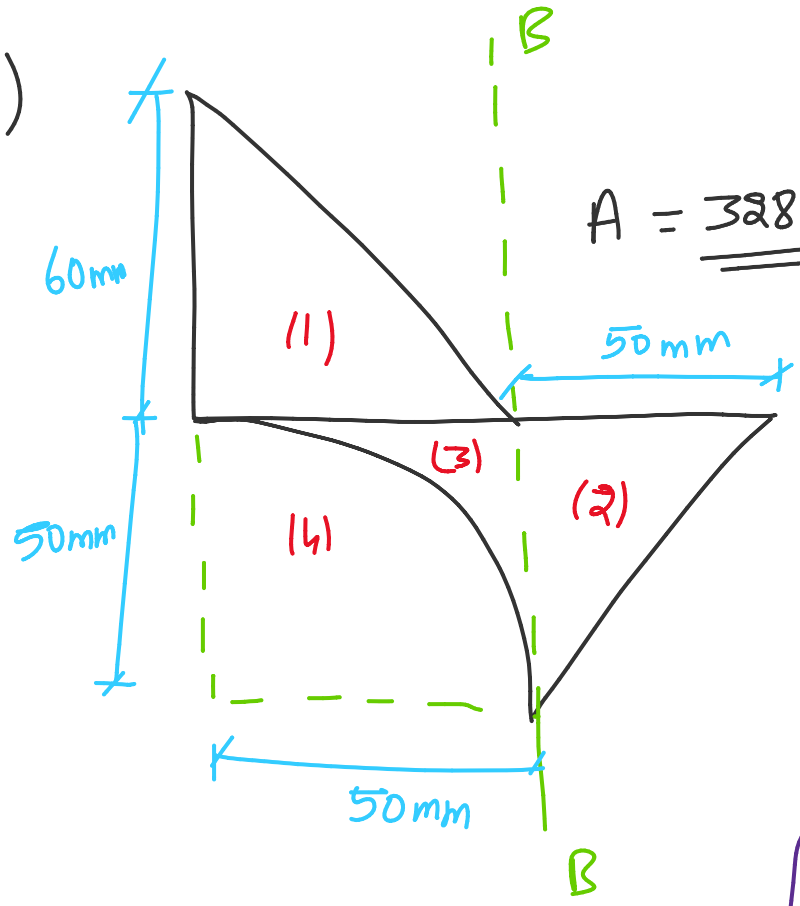
$$\left[\frac{30\times 30^3}{12} + (30\times 30)\times\left(\bar{x} - 30 + \left(\tfrac{30}{2}\right)\right)^2\right]_3 -$$

$$\left[0.055\times(30^4) + \left(\frac{\pi\times 30^2}{4}\right)\times\left(28.86 - \left(60 - \frac{4\times 30}{3\pi}\right)\right)^2\right]_4$$

$$\boxed{I_{yy} = \underline{2.62\times 10^5}\ mm^4}$$

06 December 2021 08:42 AM

T4)



$$A = \underline{\underline{3286.50\,mm^2}}$$

$$I_{BB} = \left[\frac{60\times50^3}{36} + \left(\tfrac{1}{2}\times60\times50\right)\times\left(50-\tfrac{1}{3}\times50\right)^2\right]_1$$

$$+\left[\frac{50\times50^3}{36} + \left(\tfrac{1}{2}\times50\times50\right)\times\left(\tfrac{1}{3}\times50\right)^2\right]_2$$

$$+\left[\frac{50\times50^3}{12} + (50\times50)\times\left(\tfrac{50}{2}\right)^2\right]_3$$

$$-\left[0.055\times50^4 + \left(\frac{\pi\times50^2}{4}\right)\times\left(50-\frac{4\times50}{3\pi}\right)^2\right]_4$$

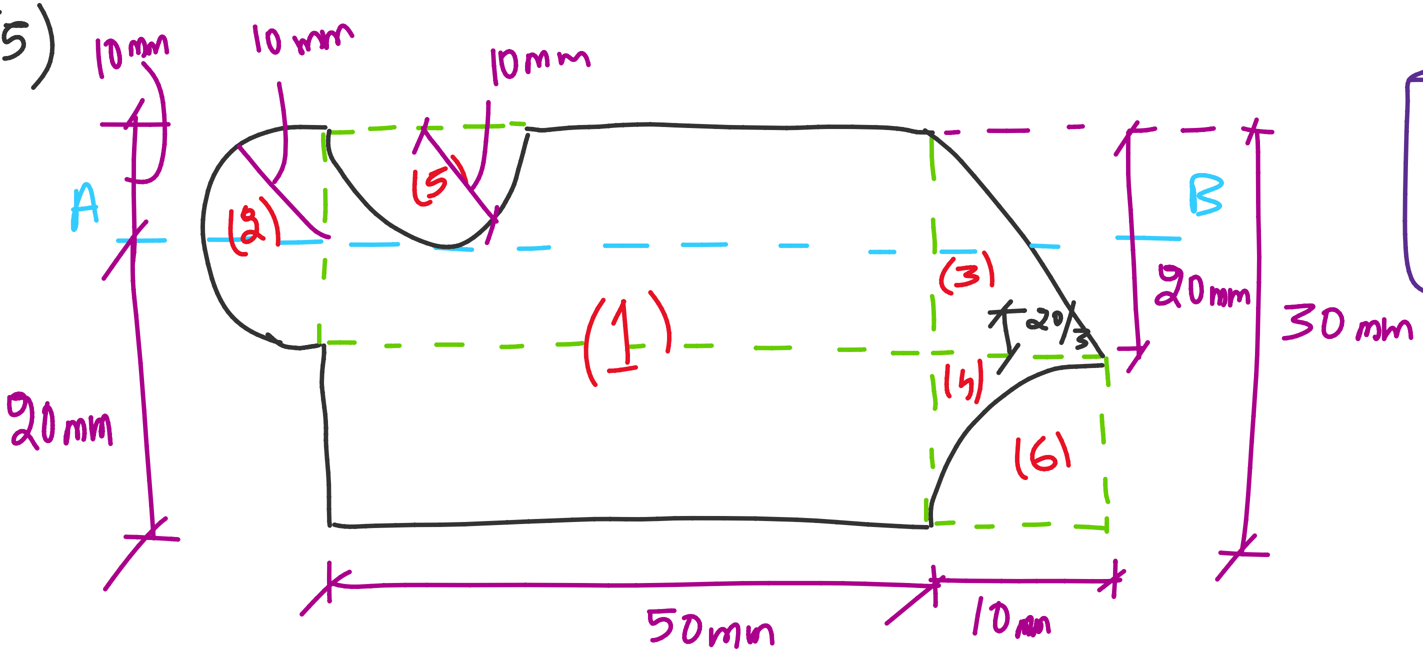
$$\boxed{I_{BB} = 25\times10^5 \quad mm^4}$$

$$K_{BB} = \sqrt{\frac{I_{BB}}{A}} =$$

$$\boxed{K_{BB} = \underline{27.57}\,mm}$$

06 December 2021 09:07 AM

T5)



$$I_{AB} = \boxed{13.09 \times 10^4}\ mm^4$$

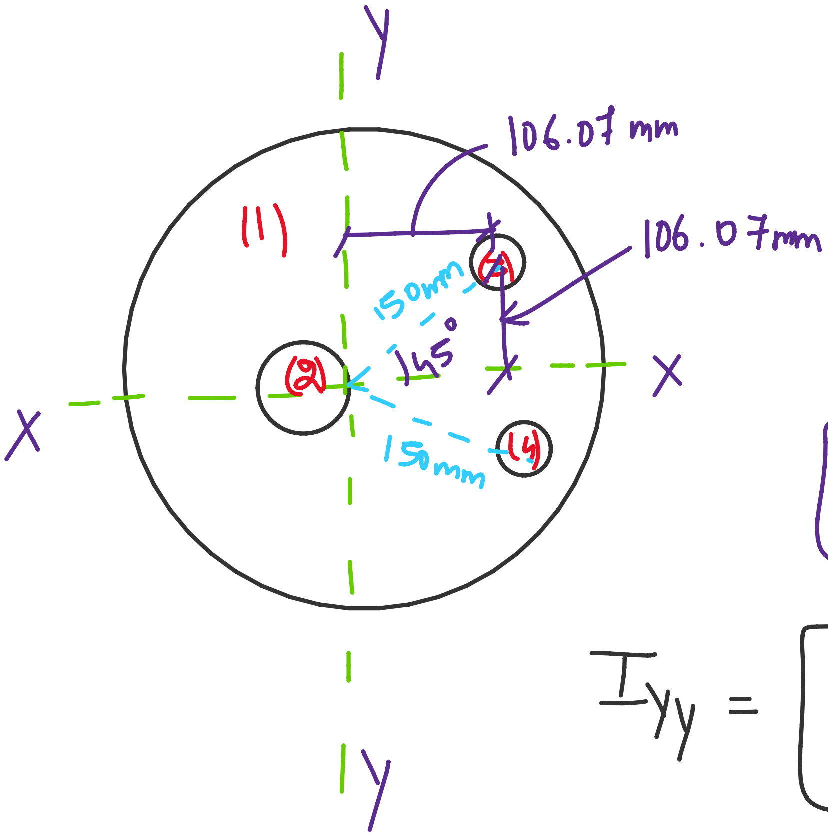
$$I_{AB} = \left[\frac{50\times30^3}{12} + (50\times30)\times(20-15)^2\right]_1 + \left[\frac{\pi\times10^4}{8} + \cancelto{0}{\left(\frac{\pi\times10^2}{2}\right)\times(0)^2}\right]_2$$

$$+ \left[\frac{10\times20^3}{36} + \left(\tfrac{1}{2}\times10\times20\right)\times\left(20-\left(10+\tfrac{20}{3}\right)\right)^2\right]_3 + \left[\frac{10\times10^3}{12} + (10\times10)\times\left(20-\tfrac{10}{2}\right)^2\right]_4$$

$$- \left[0.11\times10^4 + \left(\frac{\pi\times10^2}{2}\right)\times\left(10-\frac{4\times10}{3\pi}\right)^2\right]_5 - \left[0.055\times10^4 + \left(\frac{\pi\times10^2}{4}\right)\times\left(20-\frac{4\times10}{3\pi}\right)^2\right]_6$$

09 December 2021 02:05 PM

T6)



$$I_{xx} = \left[\frac{\pi \times 200^4}{4}\right]_1 - \left[\frac{\pi \times 50^4}{4}\right]_2$$

$$-2\left[\frac{\pi \times 25^4}{4} + (\pi \times 25^2) \times (106.07)^2\right]_{3,4}$$

$$\boxed{I_{xx} = 1.24 \times 10^9 \ mm^4}$$

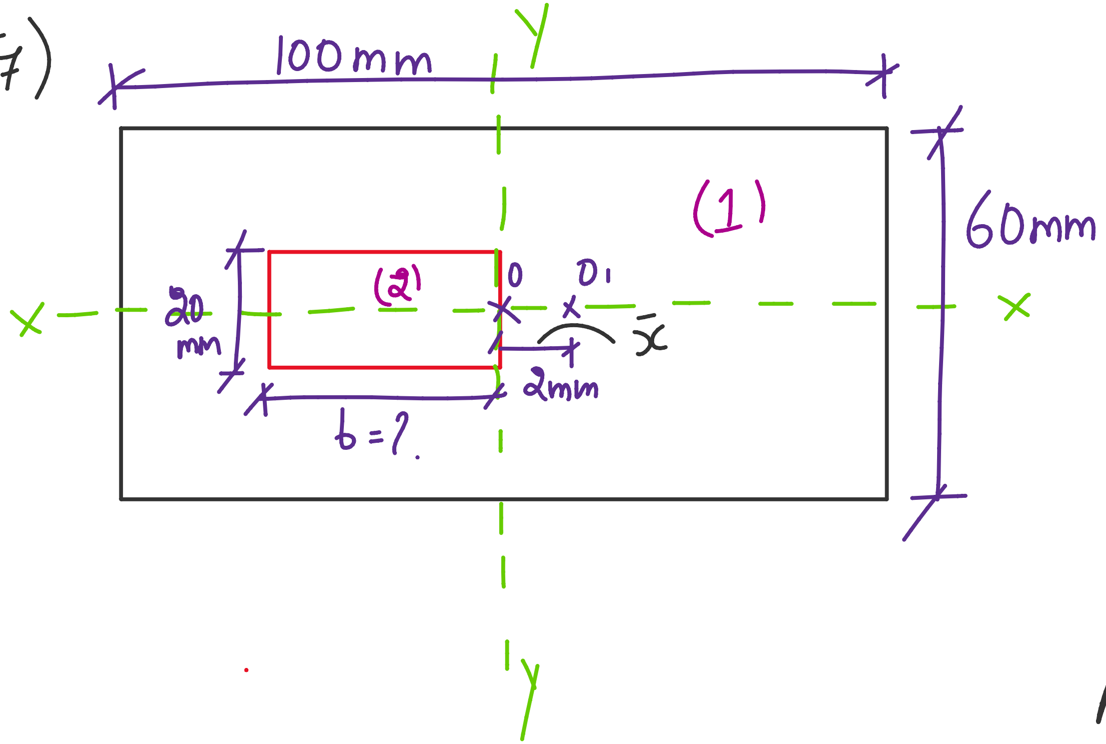
$$I_{yy} = \left[\frac{\pi \times 200^4}{4}\right]_1 - \left[\frac{\pi \times 50^4}{4} + (\pi \times 50^2) \times (50^2)\right]_2$$

$$-2\left[\frac{\pi \times 25^4}{4} + (\pi \times 25^2) \times (106.07)^2\right]_{3,4}$$

$$\boxed{I_{yy} = 1.18 \times 10^9 \ mm^4}$$

09 December 2021 02:28 PM

T7)



$\bar{x} = 2\,mm$

$$2mm = \frac{[60\times\cancel{100\times 0}]_1 - [20\times b\times -b/2]}{[60\times100] - [20\times b]}$$

$$\boxed{b = 32.69mm}$$

Area to be removed $= 20 \times 32.69$

$= 653.97 mm^2$

Area of hatched portion $= [60\times100] - [653.97]$

$$\boxed{= 5346.03 mm^2}$$