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Q Solve using Gauss Jacobi & Seidel.

$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$
$$20x + y - 2z = 17$$

diagonal dominance → ✗

~~3 20~~

⇒ Rearrange

$$\left.\begin{aligned} 20x + y - 2z &= 17 \\ 3x + 20y - z &= -18 \\ 2x - 3y + 20z &= 25 \end{aligned}\right\}$$ New System

diagonal dom ✓

$x = x_0,\ y = y_0,\ z = z_0$ (0 initially).

$$\left.\begin{aligned} x^0 &= \frac{1}{20}(17 - y + 2z) \\ y^0 &= \frac{1}{20}(-18 - 3x + z) \\ z^0 &= \frac{1}{20}(25 - 2x + 3y) \end{aligned}\right\}$$ $x^0 = y^0 = z^0 = 0$ (initial approx solⁿ.)

→ Gauss Jacobi solⁿ.

⇒ $x' = 17/20 = 0.85$

$y' = -18/20 = -0.9$

$z' = 25/20 = 1.25$

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$x^{(2)} = \frac{1}{20}(17 - y' + 2z')$

$= \frac{1}{20}(17 + 0.9 + 2(1.25))$

$= \frac{1}{20}(17 + 0.9 + 2.5) = \frac{\cancel{18.6}\ 20.4}{20}$ $= \cancel{0.93}\ 1.020$

$y^{(2)} = \frac{1}{20}(-18 - 3x' + z')$

$= \frac{1}{20}(-18 - 2.55 + 1.25) = \frac{-19.3}{20} = -0.965$

$z^2 = \frac{1}{20}(25 - 2x' + 3y') =$

$= \frac{1}{20}(25 - 1.7 + 2.7) = \frac{20.6}{20} = 1.03$

$x^3 = \frac{1}{20}(17 - y^2 + 2z^2)$

$= \frac{1}{20}(17 + 0.965 + 2.06) = \boxed{1.00125}$

$y^3 = \frac{1}{20}(-18 - 3x^2 + z^2)$

$= \frac{1}{20}(-18 - 3.06 + 1.03) = \boxed{-1.0015}$

$z^3 = \frac{1}{20}(25 - 2x^2 + 3y^2)$

$= \frac{1}{20}(25 - \underbrace{2(1.020)}_{-2.04} + \underbrace{3(\cancel{1.03})}_{3.09}) = \boxed{\cancel{1.3025}}$ $3(-0.965)$

$\boxed{1.00325}$

$x^4 = \frac{1}{20}(17 - y^3 + 2z^3)$

$= \frac{1}{20}[17 + 1.0015 + 2(1.00325)]$

$= 1.0004$

$y^4 = \frac{1}{20}(-18 - 3(1.00125) + 1.00325)$

$= -1.000025$

$z^4 = \frac{1}{20}(25 - \underbrace{2(1.00125)}_{2.0025} + 3(-1.0015))$

$= \boxed{0.99965}$

$x^5 = \frac{1}{20}(17 - (-1.000025) + 2(0.99965))$

$= \boxed{0.99996625}$

$y^5 = \frac{1}{20}(-18 - 3(1.0004) + 0.99965)$

$= -1.0000775$

$z^5 = \frac{1}{20}(25 - \underbrace{2(1.0004)}_{2.0008} + \underbrace{3(-1.000025)}_{-3.000075})$

$= 0.99995625$

$x^6 = \frac{1}{20}(17 - (-1.0000775) + 2(0.99995625))$

$= 0.9999995$

$y^6 = \frac{1}{20}(-18 - 3(0.99996625) + 0.99995625)$

$= 0.99999 7125$

$z^6 = \frac{1}{20}(25 - 2(0.99996625) + 3(-1.0000775)) = 1.000022125$

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From iterations 5 & 6, values of x, y & z are same upto 3 decimal pts.

$\Rightarrow$ approx sol^n is $x = 0.9999995 \approx 1$

$y = -0.99999997125 \approx -1$

$z = 1.0000022125 \approx 1$

Gauss Seidal method

Let $y^0 = z^0 = 0$ be the initial approx sol^n

Iteration 1

$$x^1 = \frac{1}{20}(17 - y^0 + 2z^0) = 17/20 = 0.85$$

$$y^1 = \frac{1}{10}(-18 - 3x^1 + z^0)$$

$$= \frac{1}{\cancel{20}}(-18 - \quad = -1.0875$$

$$z^1 = \frac{1}{20}(25 - 2x^1 + 3y^1) = \cancel{1.465} \ \boxed{1.010875}$$

I2

$$x^2 = \frac{1}{20}(17 - y^1 + 2z^1) = 1.0024625$$

$$y^2 = \frac{1}{10}(-18 - 3(x^2) + z^1) = -0.999825625$$

$$z^2 = \frac{1}{20}(25 - 2x^2 + 3y^2) = 0.99977794$$

2.004295

$x^3 = \frac{1}{20}(17 - y^2 + 2z^2) = 0.99996927$

$y^3 = \frac{1}{20}(-18 - 3x^3 + z^2) = -1.0000062$

$z^3 = \frac{1}{20}(25 - 2x^3 + 3y^3) = \cancel{0.00}$

-3.0000192 $\quad 0.99991007$

$x^4 = \frac{1}{20}(17 + 1.0000062 + 2(0.99)) = \boxed{0.99999\,1315}$

$y^4 = \frac{1}{20}(-18 - 3x^4 + z^3) = \boxed{-1.0000032}$

$= \frac{1}{20} \cancel{(-18}$

$z^4 = \frac{1}{20}(25 - 2x^2 + 3y^2) = \boxed{1.00000039}$

3.0000096

from I3, I4, values of $x, y, z = 1, -1, 1$

Q $\cancel{5x - y + z = 10}$
$2x + 4y = 12$
$x + y + 5z = -1$

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Q $5x - y + z = 10$
$2x + 4y + 0z = 12$
$x + y + 5z = -1$

} Diagonal Dom ✓

Jacobi → 2 DP   Seidal → 3DP.

⇒ $x = \frac{1}{5}(10 + y - z)$
$y = \frac{1}{4}(12 - 2x)$
$z = \frac{1}{5}(-1 - x - y)$

Gauss Jacobi

$x^0 = y^0 = z^0 = 0$

⇒ $x_1 = 10/5 = 2.$
$y_1 = 12/4 = ~~2.5~~\ 3$
$z^1 = -1/5 = -0.2$

⇒ $x_2 = \frac{1}{5}(10 + 3 - (-0.2)) = \frac{10 + 2.5 + 0.2}{5} = \frac{12.7}{5}$ $\boxed{2.64}$

$y_2 = 1/4(12 - 2\times 2) = 6/4 = \boxed{}$ $\boxed{2}$

$z_2 = 1/5(-1 - 2 + 3) = \boxed{}$ $\boxed{-1.2}$

$x_3 = \frac{1}{5}(10 + 2 + 1.2) = 13.2/5 = \boxed{2.64}$

$y_3 = \frac{1}{4}(12 - 2\times 2.64) = \boxed{1.68}$

$z_3 = \frac{1}{5}(-1 - 2.64 - 2) = \boxed{-1.128}$

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$x_4 = \frac{1}{5}(10 + 1.68 + 1.128) = 2.5616$

$y_4 = \frac{1}{4}(12 - 2\times 2.64) = 1.68$

$z_4 = \frac{(-1 - 2.64 - 1.68)}{5} = -1.064$

After 4 iterations, approx values of x, y, z : (to 2 DP)

$$\boxed{x = 2.56, \quad y = 1.68, \quad z = -1.06}$$

Gauss Seidal method.

Let $y^0 = z^0 = 0$

$\Rightarrow x_1 = 10/5 = 2$

$y_1 = \cancel{10}\ \frac{1}{4}(12 - 2\times 2) = 2$

$z_1 = \frac{1}{5}(-1 - 2 - 2) = -1$

$x_2 = \frac{1}{5}(10 + 2 + 1) = 13/5 = \cancel{6.5}\ 2.6$

$y_2 = \frac{1}{4}(12 - 2\times 2.6) = 1.7$

$z_2 = \frac{1}{4}(-1 - 2.6 - 1.7) = -1.06$

$x_3 = \frac{1}{10}(10 + 1.7 + 1.06) = 2.552$

$y_3 = \frac{1}{4}(12 - 2\times 2.552) = 1.724$

$z_3 = \frac{1}{5}(-1 - 2.552 - 1.724) = \cancel{-1.00}\ -1.0552$

$x_4 = \frac{1}{5}(10 + 1.724 + 1.0552) = 2.55584$

$y_4 = \frac{1}{4}(12 - 2 \times 2.55584) = 1.72208$

$z_4 = \frac{1}{5}(-1 - 2.55584 - 1.72208) = -1.055584$

After 4 iterations, approx values (upto 3DP)

$x = 2.556$, $y = 1.722$, $z = -1.056$

Eigen Values & Eigen vectors.

Consider a matrix $A = \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix}$

Let $e_1 = (1 \quad 0)$, $e_2 = (0 \quad 1)$ be two 2-D vectors.

$Ae_1 = \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = (1, 2) \neq$ any multiple of e_1

$Ae_2 = \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} = (0, 4)$

$= 4e_2$

Let A be a square matrix, then the eigen vectors of A are those vectors who when multiplied, still be parallel.

$\Rightarrow AX = \lambda X$

vector, Scalar (eigen value)

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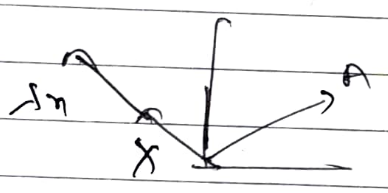
Consider $A = \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix}$ for $x = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$$Ax = \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} = 2x$$

$\Rightarrow$ $x \rightarrow$ eigen vector, $2 \rightarrow$ eigen value of x

Graphical explanation



Consider $B = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$ for $v = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$$Bv = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} = -2v$$

$v \rightarrow$ eigen vector, $-2 \rightarrow$ eigen value of v

Let A be sq. matrix of order N. Let λ be a scalar.

$\Rightarrow$ $(A - \lambda I)_{n \times n} \rightarrow$ <u>characteristic matrix</u>

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Determinant

→ $|A - \lambda I| = 0$

↳ characteristic eqn of the given matrix
eq of degree 'n' in λ.

→ Roots of eqn $|A - \lambda I| = 0$ is called the eigen values of the given matrix A

⇒ $\lambda_1, \lambda_2 \, --- \, \lambda_n$ → eigen values

then find respective x_i for λ_i

⇒ $AX = \lambda_i (IX)$

$AX - (\lambda_i I)X = 0$

$(A - \lambda_i I)X = 0$

Properties of Eigen Values

→ Let A be $n \times n$. Assuming A has n distinct eigen values $\lambda_1, \lambda_2 \, -- \, \lambda_n$, then eigen values are:

→ $A^T \rightarrow \lambda_1, \lambda_2 \, --- \, \lambda_n$

→ A^{-1} (If exists) → $1/\lambda_1, 1/\lambda_2 \, --- \, 1/\lambda_n$

→ $A - \alpha I \rightarrow \lambda_1 - \alpha, \lambda_2 - \alpha \, ---- \, \lambda_n - \alpha$

→ A^k, $k \geq 0$, $k \in W$ → $\lambda_1^k, \lambda_2^k, \, ---- \, \lambda_n^k$