MECHANICS OF SOLIDS

CIE 1051

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

LECTURE 0

Mechanics of Solids

PART- I

Mechanics of Rigid Bodies

PART- II

Mechanics of Deformable Bodies

COURSE CONTENT IN BRIEF

PART I Mechanics of Rigid Bodies

1. Resultant of concurrent and non-concurrent coplanar forces
2. Equilibrium of concurrent and non-concurrent coplanar forces
3. Centroid of plane areas
4. Moment of Inertia of plane areas

PART II Mechanics of Deformable Bodies

6. Simple stresses and strains

7. Statically indeterminate problems and thermal stresses

8. Stresses due to fluid pressure in thin cylinders

9. Concepts of SFD and BMD

REFERENCE BOOKS

1. Engineering Mechanics - Meriam & Craige, John Wiley & Sons.
2. Engineering Mechanics - Irwing Shames, Prentice Hall of India.
3. Mechanics for Engineers - Beer and Johnston, McGraw Hills Edition
4. Mechanics of Materials - E.P.Popov
5. Mechanics of Materials - E J Hearn
6. Strength of materials - Beer and Johnston
7. Strength of materials - F L Singer & Andrew Pytel
8. Strength of Materials - B.S. Basavarajaiah & P. Mahadevappa
9. Strength of Materials - S S Bhavikatti

LECTURE 1

- Concept of Rigid body
- Force and its characteristics
- Principle of transmissibility
- Classification of Force System
- Resultant of Coplanar concurrent forces
- Composition of forces
- Resolution of a force
- Rectangular Components of a force
- Sign convention

INTRODUCTION TO RIGID BODY MECHANICS

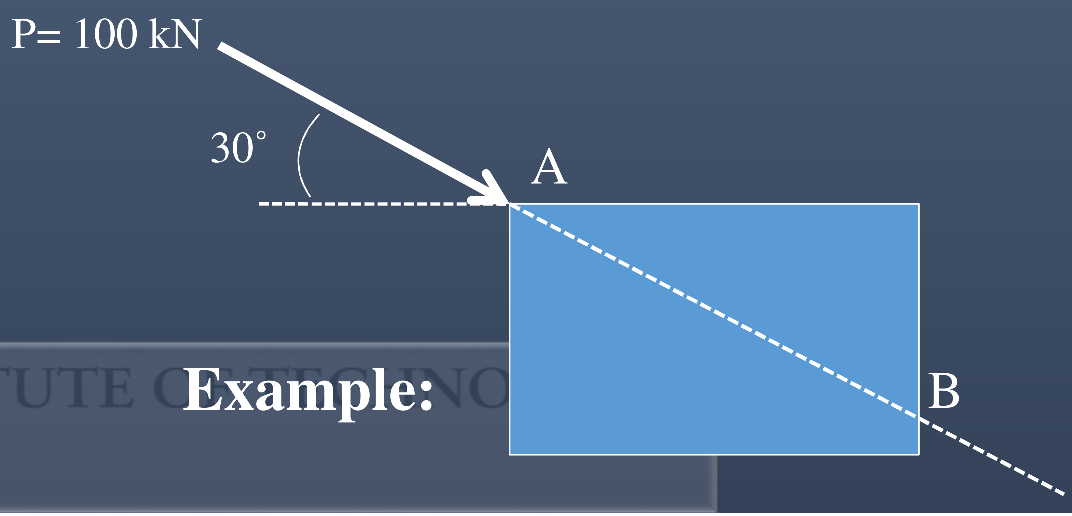
Rigid Body

- A body is said to be rigid if it does not get deformed under the action of external load.
- It is defined as a definite amount of matter, the parts of which are fixed in position relative to one another.
- Actually solid bodies are never rigid; they deform under the action of applied forces. In those cases where this deformation is negligible compared to the size of the body, the body may be considered to be rigid.

Force

- It is defined as the action of one body on another. (OR) It is that agent which causes or tends to cause changes or tends to change the state of rest or motion of a mass.
- Characteristics of a force:
  1. Magnitude
  2. Direction
  3. Point of application and
  4. Line of action

Example:



Principle of Transmissibility

- The state of rest or motion of the block will be the same if a force of magnitude P is applied as a push at A or as a pull at B.

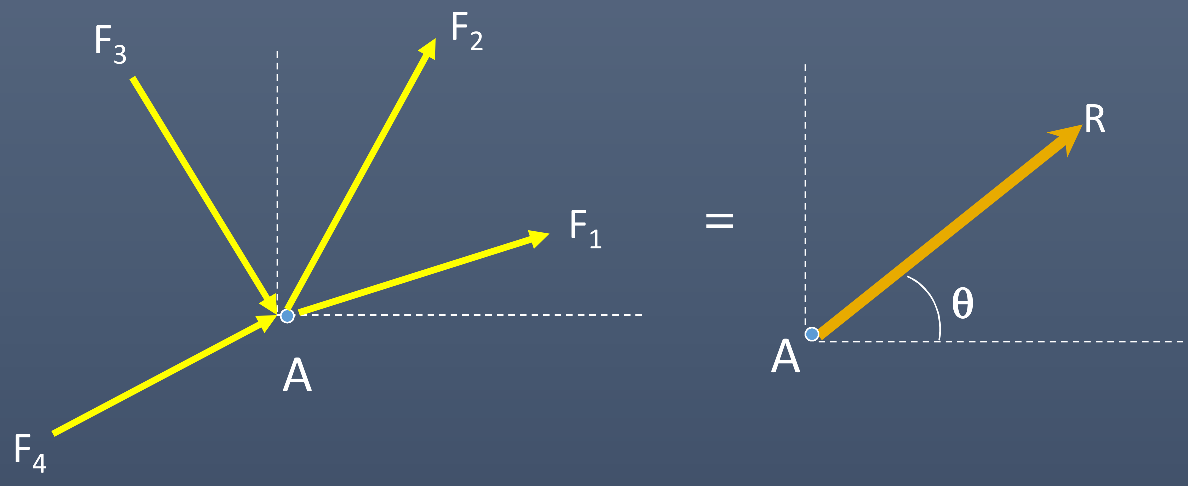
OR

- The state of rest or of uniform motion of a rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.

Classification of Force System

- System of forces
  - Collinear forces
  - Coplanar forces
    - Concurrent forces
    - Non concurrent forces
      - Like parallel forces
      - Unlike parallel forces
  - Non coplanar forces
    - Concurrent forces
    - Non concurrent forces
      - Like parallel forces
      - Unlike parallel forces

RESULTANT OF COPLANAR CONCURRENT FORCE SYSTEM



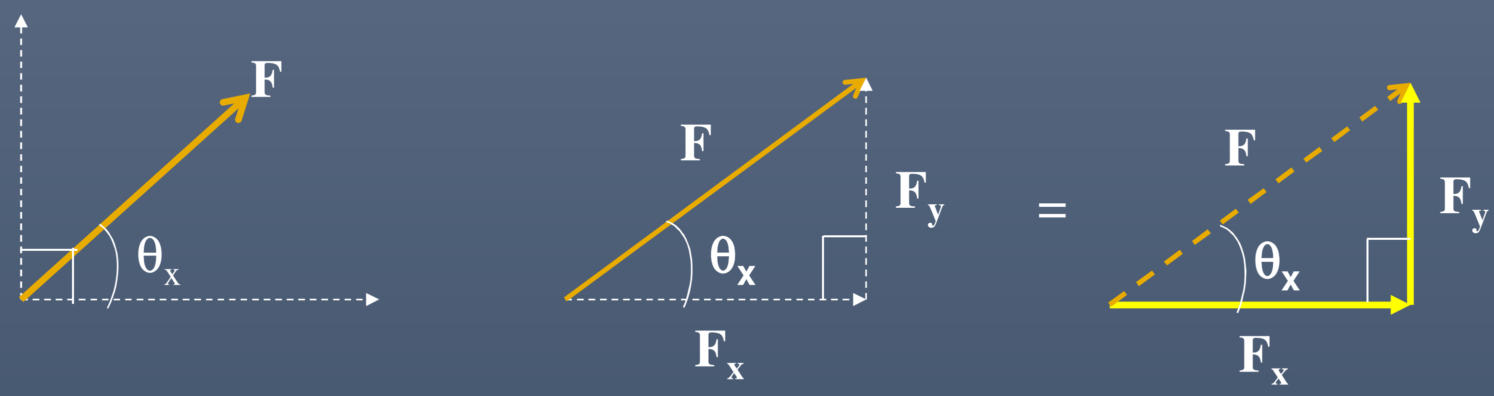
It is defined as that single force which can replace a set of forces, in a force system, and cause the same external effect.

- **Composition of forces:** It is the process of obtaining the resultant of a given force system.

- **Component of a force :** In simple terms, it is the effect of a force in a certain direction.

- **Resolution of a force:** The process of obtaining the components of a force.

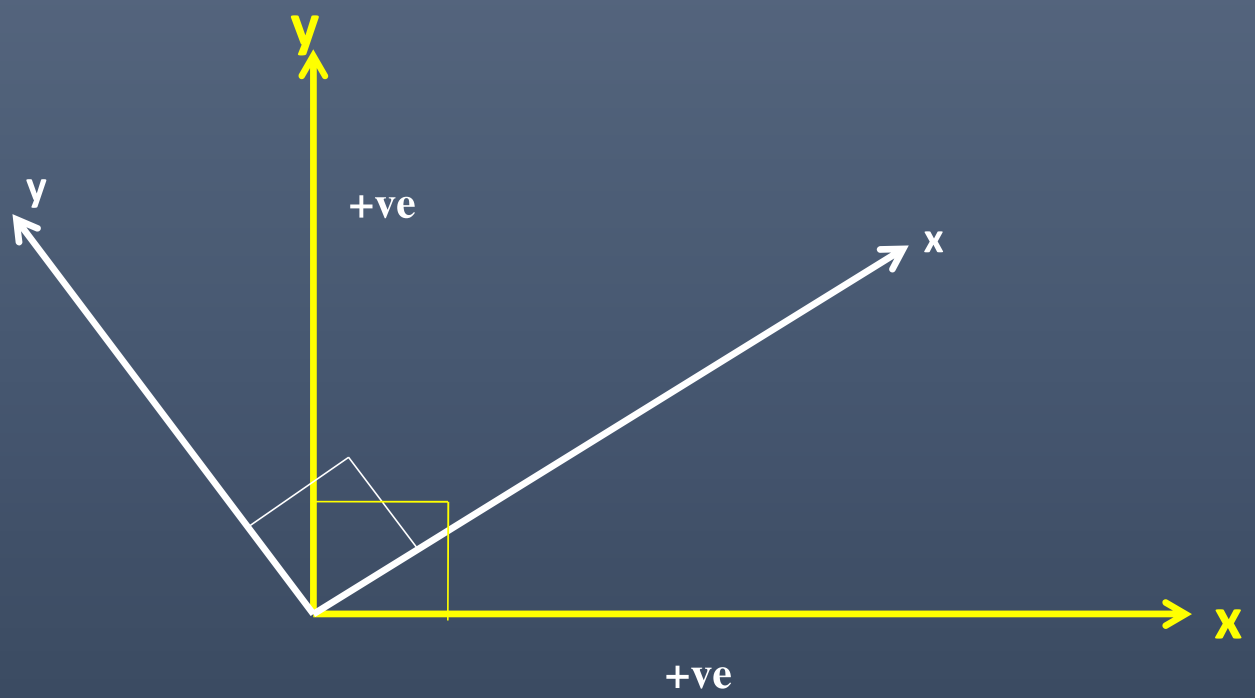
Rectangular Components of a force



- The components that are mutually perpendicular are called ‘**Rectangular Components**’.

- Consider a force F making an angle θ_x with x-axis.
  Resolved part of the force F along x-axis is given by $F_x = F \cos \theta_x$
  Resolved part of the force F along y-axis is given by $F_y = F \sin \theta_x$

Sign convention



LECTURE 2

- Resultant of Coplanar non-concurrent force systems
- Moment of a force
- Varignon's theorem
- Couple
- Moment of a couple
- Resolution of a force into a force and couple
- Properties of a couple

RESULTANT OF COPLANAR NON-CONCURRENT FORCE SYSTEM

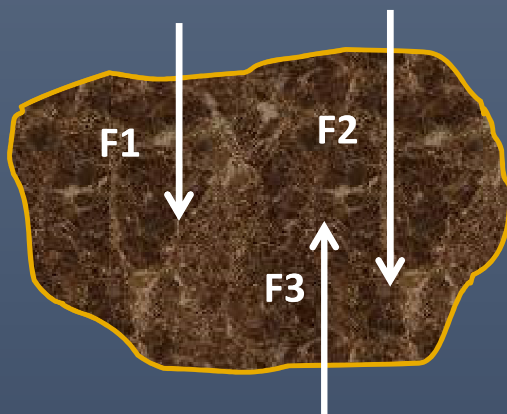


Fig. 1

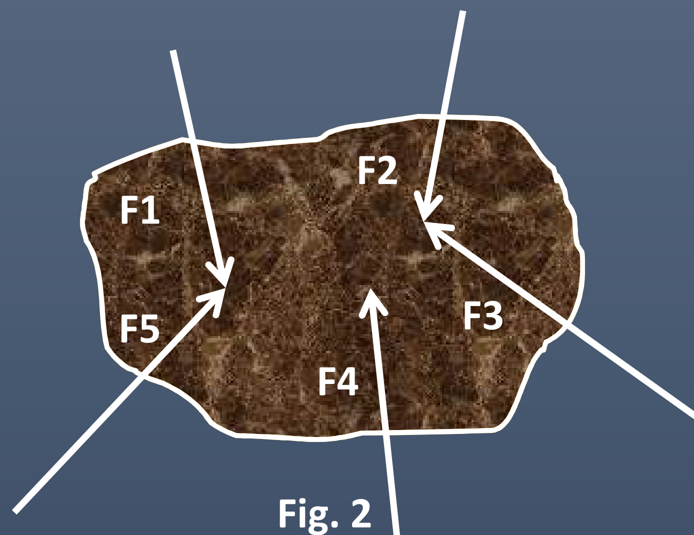
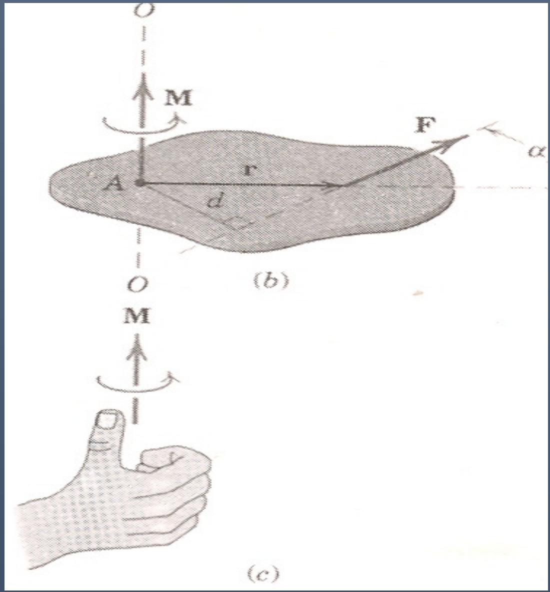


Fig. 2

- **Parallel Force System –** Lines of action of individual forces are parallel to each other.
- **Non-Parallel Force System –** Lines of action of the forces are not parallel to each other.

Moment

- Moment of a force about an axis:
  - Moment Axis: the axis about which rotational tendency is determined.
  - Moment Center: This is the position of axis on co-planar system.
  - Moment Arm: Perpendicular distance from the line of action of the force to moment center.



- Magnitude of moment:

$M_A = F \times d$

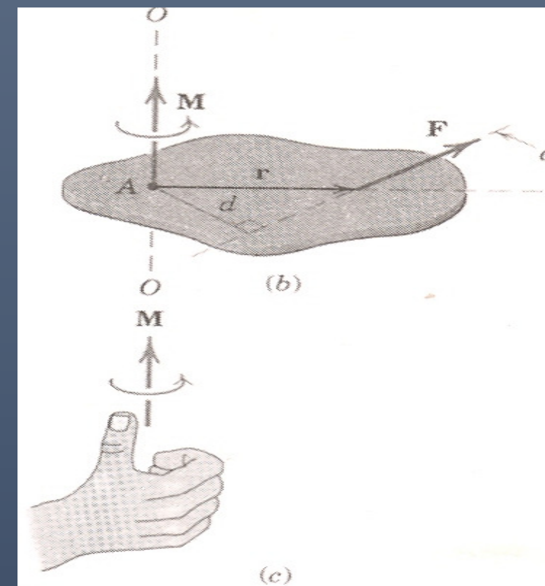
= Rotation effect because of the force F, about the point A (about an axis *0-0*)

Unit – kN-m, N-mm etc.

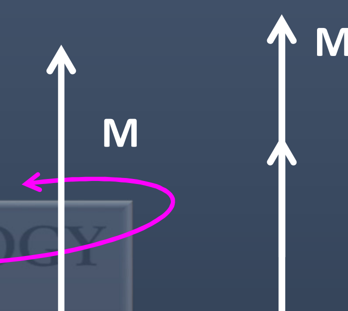
Sense of a Moment

‘Right Hand Thumb’ rule:

‘If the fingers of the right hand are curled in the direction of rotational tendency of the body, the extended thumb represents the sense of moment vector’.

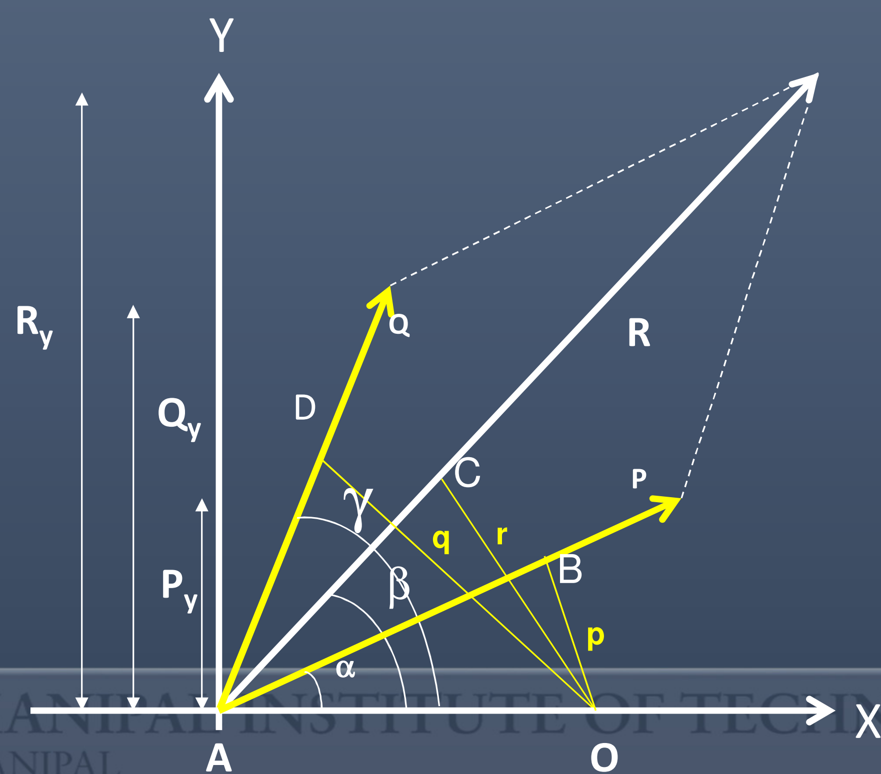


For the purpose of additions, the moment direction may be considered by using a suitable sign convention such as +ve for counterclockwise and –ve for clockwise rotations or vice-versa.



Varignon's Theorem (Principle of moments)

Proof (by Scalar Formulation):



Let ‘R’ be the given force. ‘P’ & ‘Q’ are component forces of ‘R’.‘O’ is the moment center. p, r and q are moment arms from ‘O’ of P, R and Q respectively.α, β and γ are the inclinations of ‘P’, ‘R’ and ‘Q’ respectively w.r.to X – axis.

We have,

$R_y = P_y + Q_y$

$R \, Sin\beta = P \, Sin\alpha + Q \, Sin \, \gamma$ ----(1)

From Δle AOB, $p/AO = Sin \, \alpha$

From Δle AOC, $r/AO = Sin \, \beta$

From Δle AOD, $q/AO = Sin \, \gamma$

From (1),

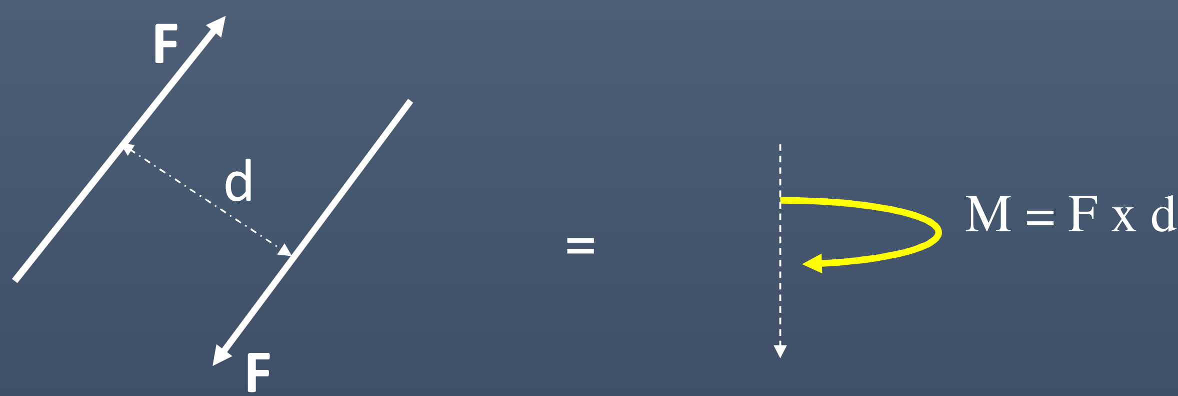
$$R \times (r/AO) = P \times (p/AO) + Q \times (q/AO)$$

i.e., $R \times r = P \times p + Q \times q$

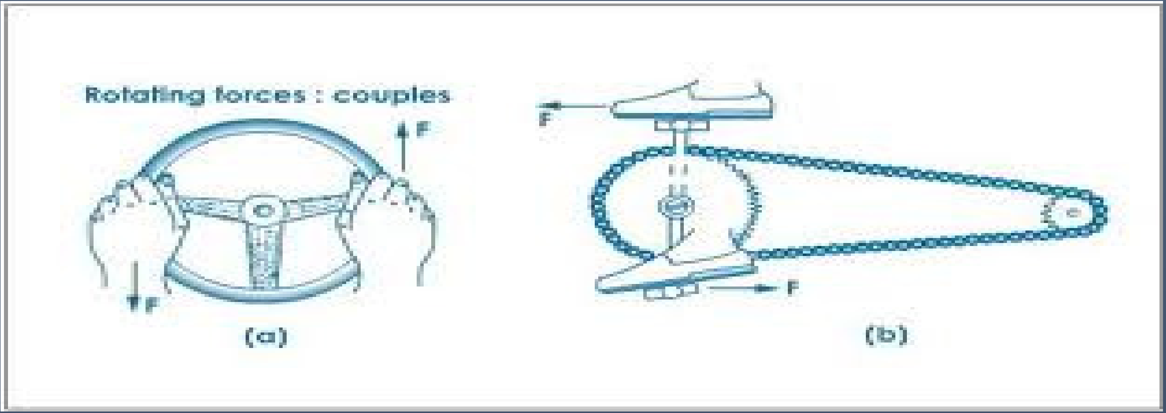
Moment of resultant R about O = algebraic sum of moments of component forces P & Q about same moment center 'O'.

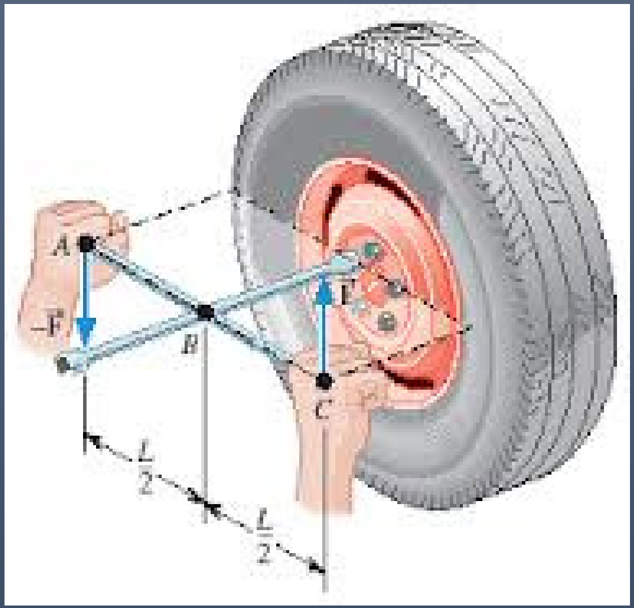
Couple

Two parallel, non collinear (separated by certain distance) forces that are equal in magnitude and opposite in direction form 'couple'.

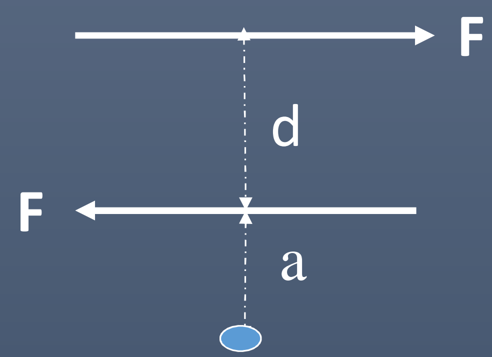


<u>Couple does not produce any translation but produces only rotation</u>.


Rotating forces : couples
F
F
(a)
F
F
(b)


A
B
C
$-\vec{F}$
$\vec{F}$
$\frac{L}{2}$
$\frac{L}{2}$

Moment of a Couple



- Consider two equal and opposite forces separated by a distance ‘d’. Let ‘O’ be the moment center at a distance ‘a’ from one of the forces.
- The sum of moments of two forces about the point ‘O’ is,

$$\sum Mo = F \times (a + d) - F \times a = F \times d$$

Thus, the moment of the couple about ‘O’ is independent of the location as it is independent of ‘a’.

Properties of a Couple

- The algebraic sum of the components of the two forces is zero. i.e. the resultant force of a couple is zero
- The moment of a couple is constant for any point chosen in the plane of the couple
- A couple can be balanced by an equal and opposite couple in the same plane.
- Two or more couples can be reduced to a single couple of moment equal to the algebraic sum of the moments of the given couples
- The moment of a couple is independent of the choice of the axis of moments (moment centre). The moment of a couple is the same with respect to any axis perpendicular to the plane of the couple.

Properties of a Couple cont…

- The effect of a couple is unchanged if the couple is rotated through any angle.
- Since the only effect of a couple is a moment and this moment is the same about any point, the effect of a couple is unchanged if :

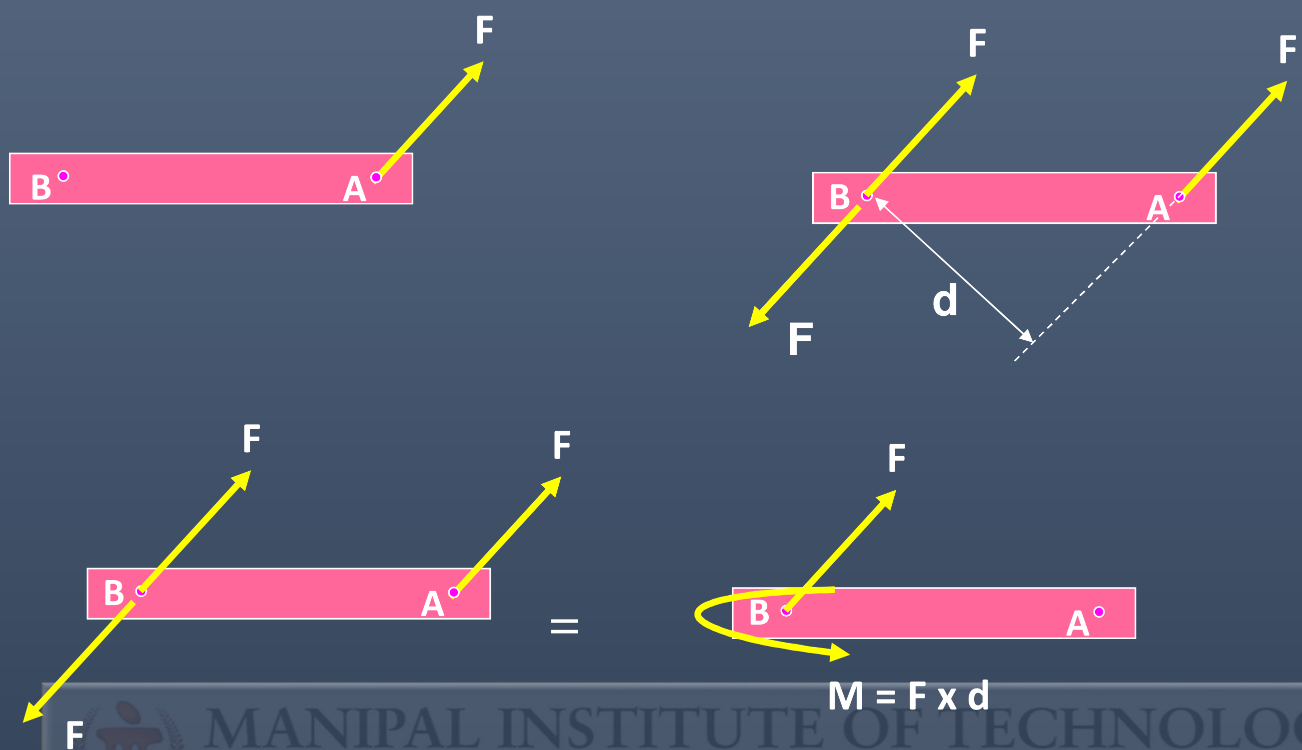
  The couple is rotated though any angle.

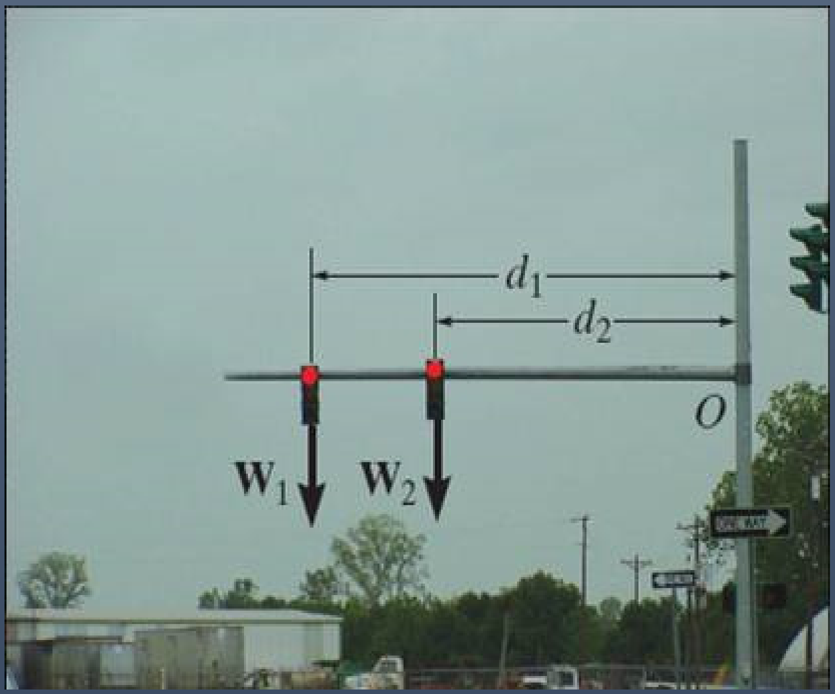
  The couple is shifted to any other position.

  The couple is replaced by another pair of forces whose rotational effect is the same

Resolution of a force into a force-couple system

To replace the force F acting at the point A to the point B



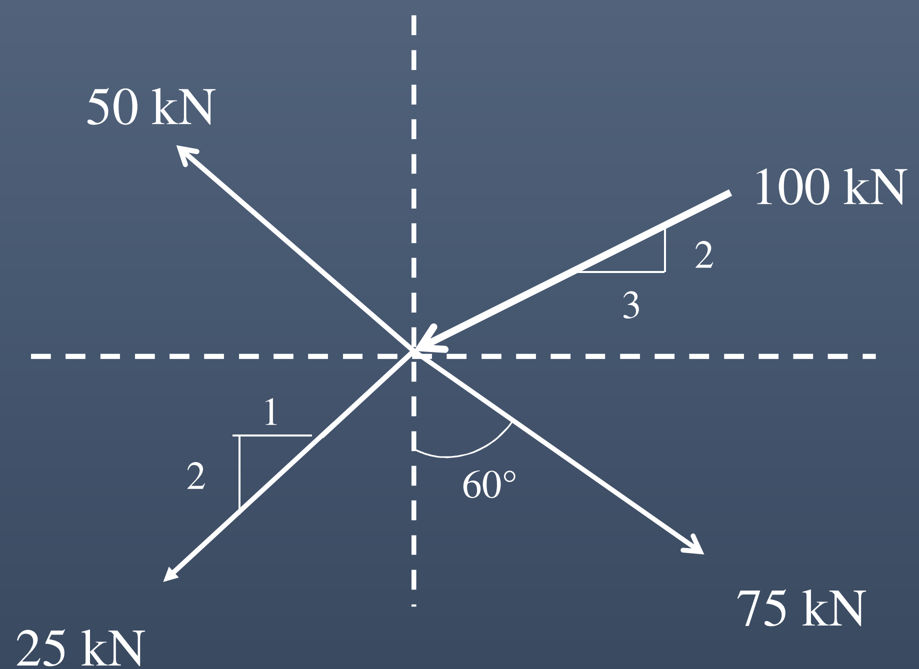

d_1
d_2
O
$\mathbf{W}_1$
$\mathbf{W}_2$
ONE WAY


$(\mathbf{M}_R)_O$
O
$\mathbf{W}_R$
ONE WAY

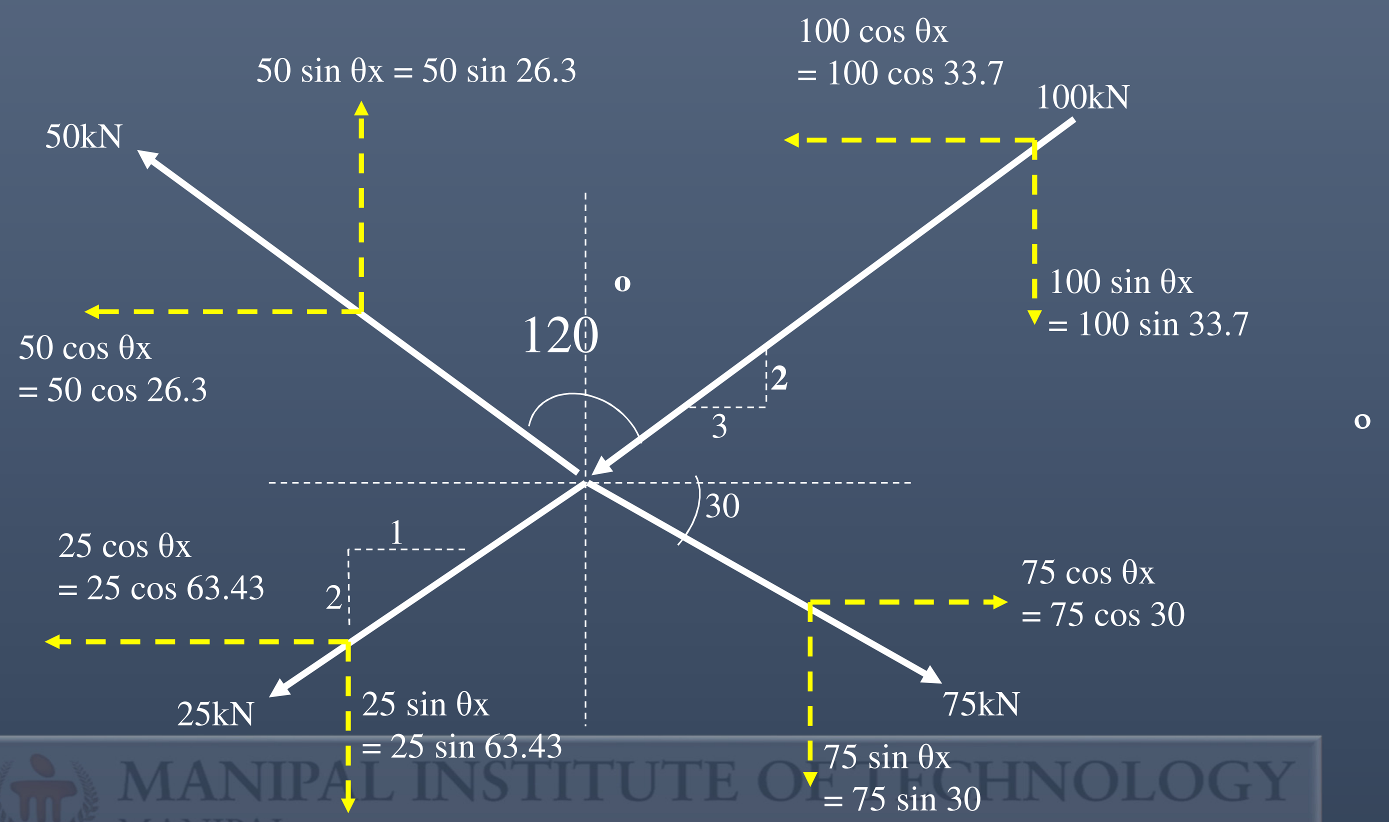
LECTURE 3

- Application Problems

1 . Obtain the resultant of the concurrent coplanar forces acting as shown in figure



Solution:


50 sin θx = 50 sin 26.3
100 cos θx
= 100 cos 33.7
100kN
50kN
o
120
100 sin θx
= 100 sin 33.7
50 cos θx
= 50 cos 26.3
2
3
o
30
1
25 cos θx
= 25 cos 63.43
2
75 cos θx
= 75 cos 30
25kN
25 sin θx
= 25 sin 63.43
75kN
75 sin θx
= 75 sin 30

Solution:

Force	X-comp.	Y-comp
100	-100 cos33.7	-100 sin33.7
50	-50 cos26.3	+50 sin26.3
25	-25cos 63.43	-25 sin63.43
75	+75 cos30	-75 sin30
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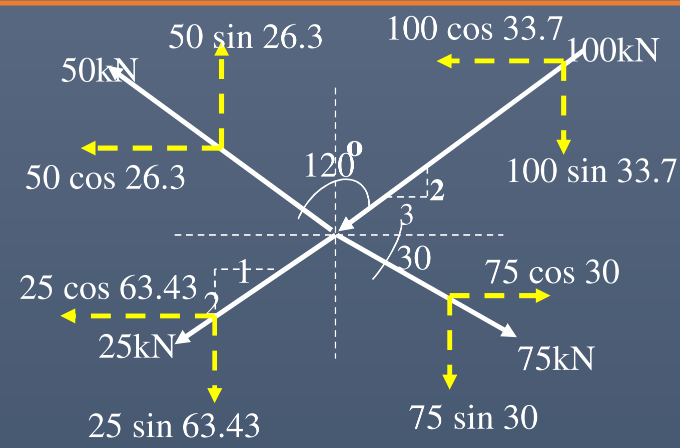
R	ΣFx =	ΣFy =
	- 74.26 kN	-93.17 kN



$$\sum F_x = -50 \cos 26.31 - 100 \cos 33.69 - 25 \cos 63.43 + 75 \cos 30$$

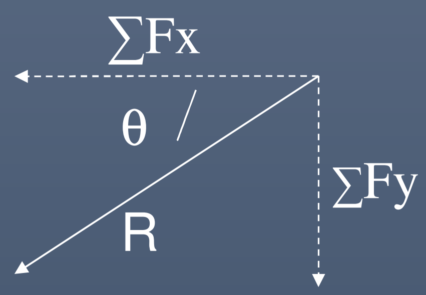
$$= -74.26\text{kN} = 74.26\text{kN} \leftarrow$$

$$\sum F_Y = 50 \sin 26.31 - 100 \sin 33.69 - 75 \sin 30 - 25 \sin 63.43$$

$$= -93.17\text{kN} = 93.17\text{kN} \downarrow$$

Contd..

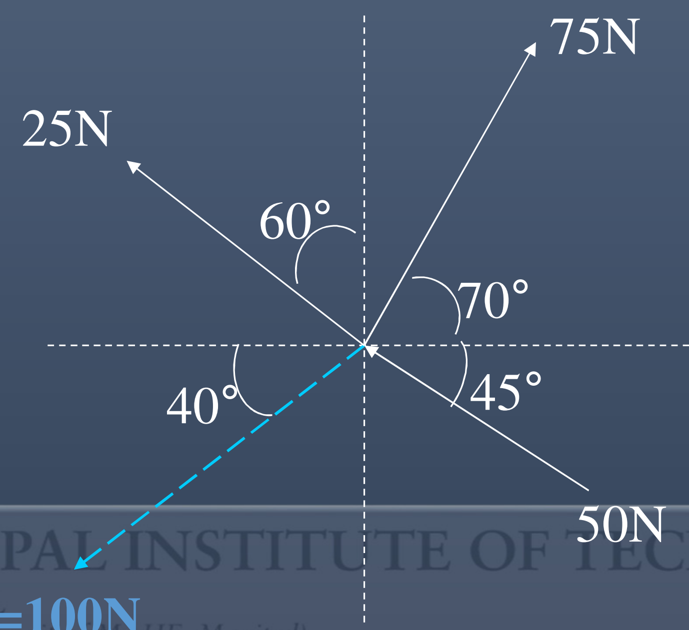
Answers:



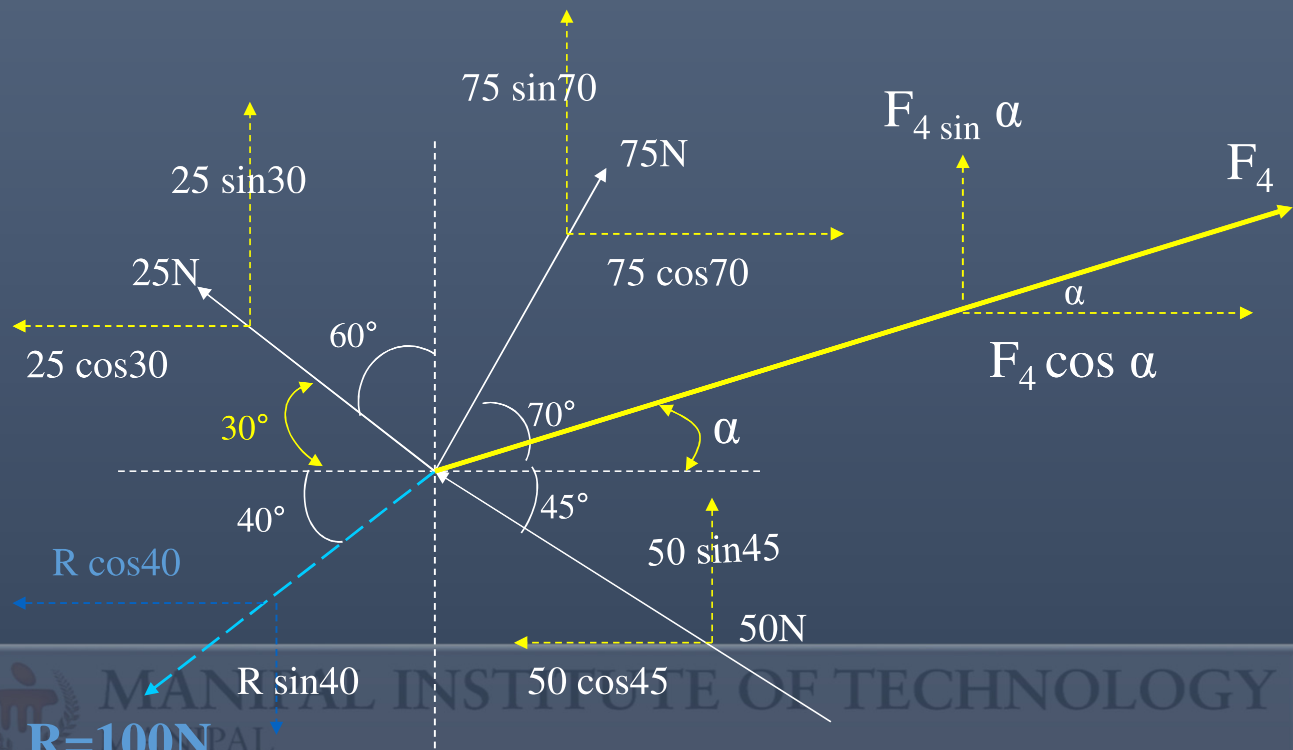
$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = 119.14 \text{ kN}$$

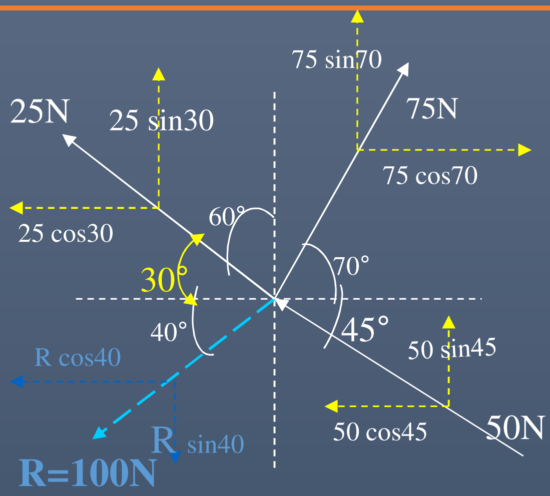
$$\Theta = \tan^{-1}(\sum F_y / \sum F_x) = 51.44^{\circ}$$

2. A system of concurrent coplanar forces has <u>four</u> forces of which only three are shown in figure. If the resultant is a force R = 100N acting as indicated, obtain the unknown fourth force.



- Assume the fourth force (F_4) in the first quadrant, at an angle α, as shown.
- The 25 N force makes an angle of 30° w.r.t. horizontal
- R is the resultant of Four forces including F_4





Force	X-comp.	Y-comp
F_4	$+F_4 \cos\alpha$	$+F_4 \sin\alpha$
50	- 50 cos45	+50 sin45
25	- 25 cos30	+25 sin30
75	+75 cos70	+75 sin70
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R	ΣFx = - R cos40	ΣFy = -R sin40
	= -100 cos40N	= - 100 sin40N

$\xrightarrow[+ve]{}$ ΣFx = -Rcos40

$$\Sigma F_x = -R\cos 40 = F_4\cos\alpha + 75\cos 70 - 50\cos 45 - 25\sin 60$$

$$F_4\cos\alpha = -45.25\text{N}$$

$\uparrow^{+ve}$ $\Sigma Fy = -R\sin 40$

$\Sigma Fy = -R\sin 40 = F_4\sin\alpha + 75\sin 70 + 25\cos 60 + 50\sin 45$

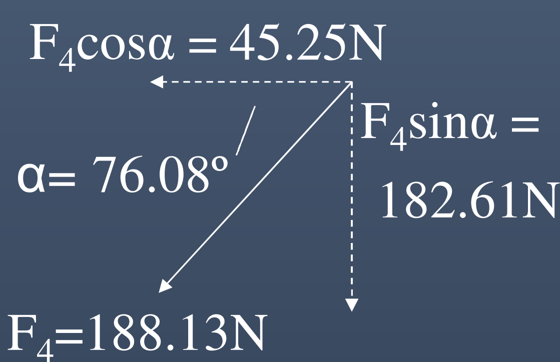
$\therefore\ F_4\sin\alpha = -182.61N$;

Answers:

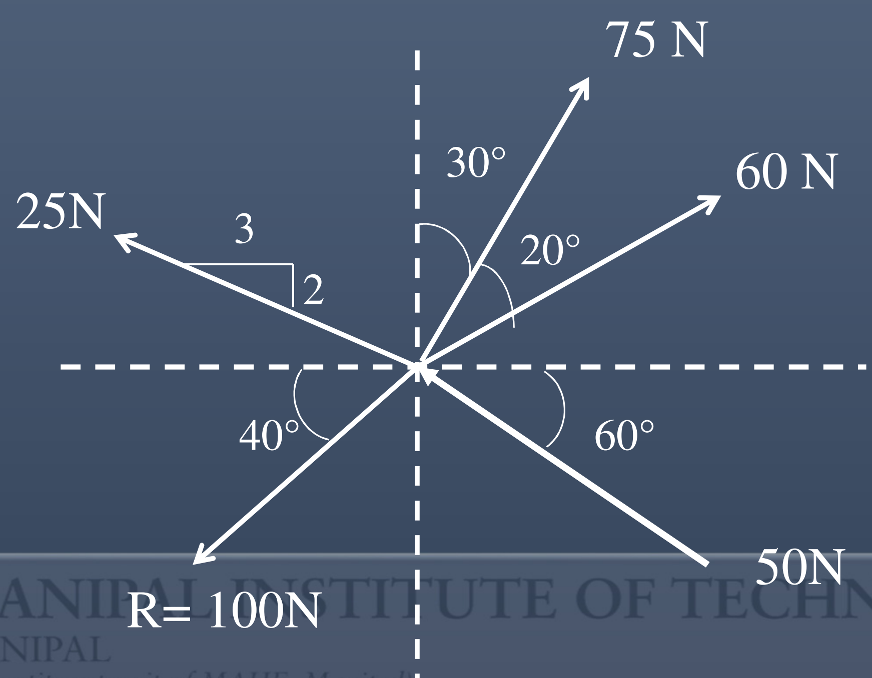
$\tan\alpha = (F_4\sin\alpha / F_4\cos\alpha)$

$\alpha = 76.08°$

& $F_4 = 188.13N$



3. A system of concurrent coplanar forces has five forces of which only four forces are shown in the Fig. If the resultant is a force R = 100N acting as indicated, obtain the unknown fifth force.



Ans:

$F_4 \cos\alpha =$ 114.265N

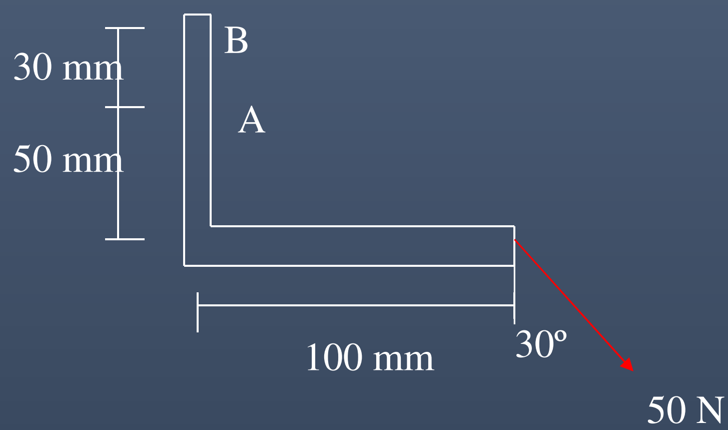
$F_4 \sin\alpha =$ 224.97N

α= 63.073º

F_4= 252.33N

LECTURE 4

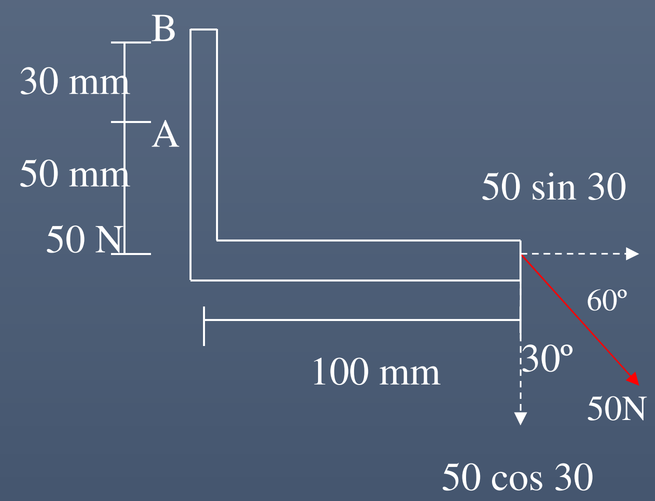
3. A 50 N force is applied to the corner of a plate as Shown in the fig. Determine an equivalent force-couple system at A.



Force – Couple System at A:

$F_x = 50 \times \sin 30 = 25$ N.

$F_y = 50 \times \cos 30 = 43.3$ N



a) Force – Couple System at A:

$F_x = 50 \times \sin 30 = 25$ N.

$F_y = 50 \times \cos 30 = 43.3$ N

These forces can be moved to **A** by adding the couple.

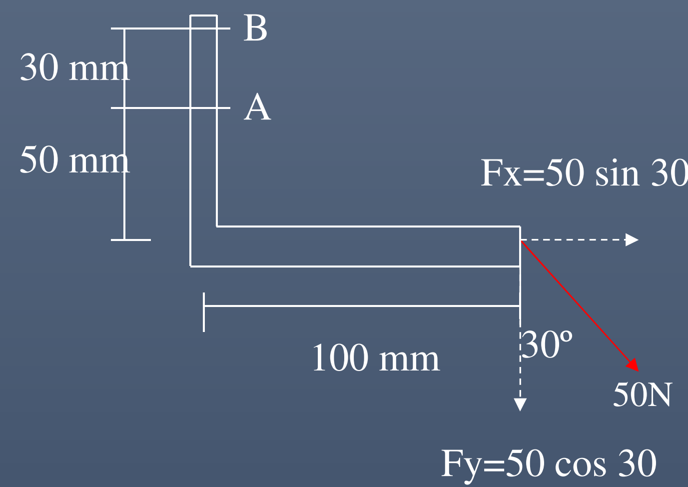
Moment of the couple about A

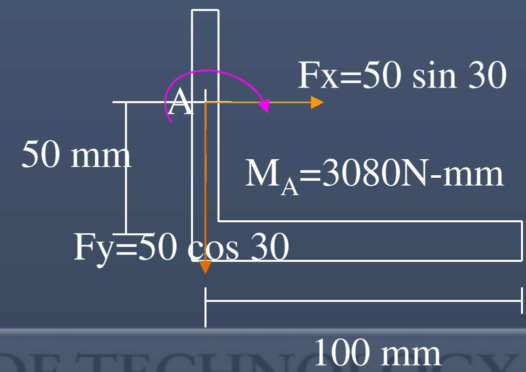
$+\ \sum M_A = F_x \times 50 - F_y \times 100$

$= 25 \times 50 - 43.3 \times 100$

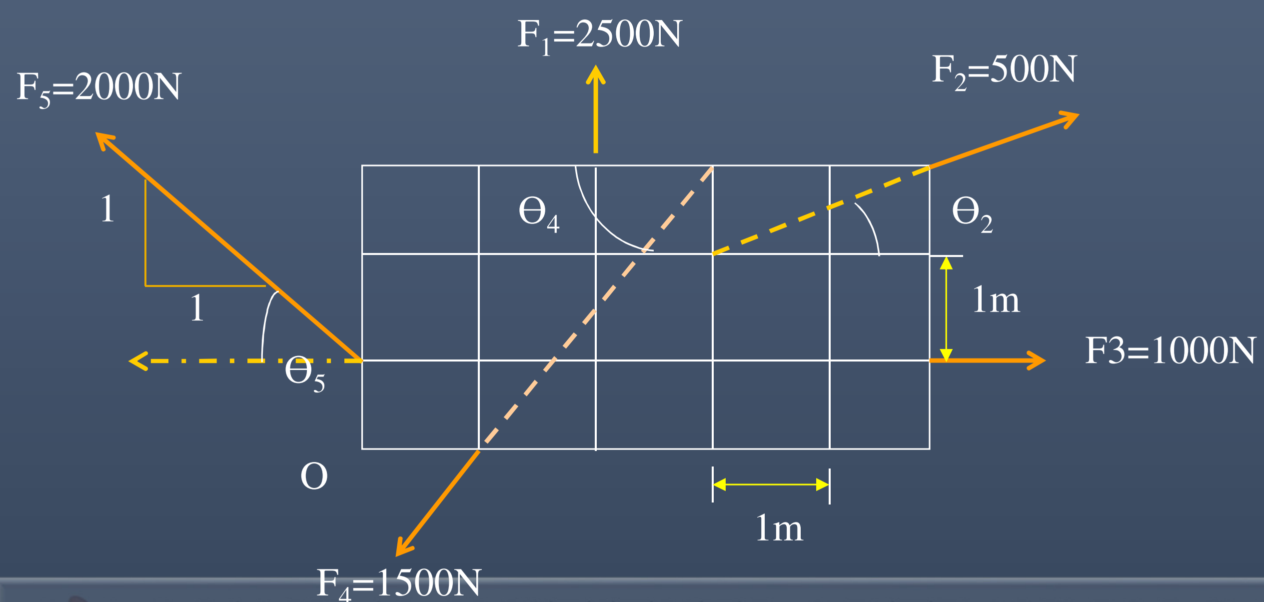
$= -3080$ N-mm.

$= 3080$ N-mm





4. Find the resultant and its position w.r.to ‘O’ of the non-concurrent system of forces shown in the figure.



A) To find the resultant –

$\Theta_2 = \tan^{-1}(1/2) = 26.56°$

$\Theta_4 = \tan-1(3/2) = 56.31°$

$\Theta_5 = \tan^{-1}(1/1) = 45°$

$+ \Sigma F_x = R_x = F_2 \cos\Theta_2 + F_3$

$- F_4 \cos\Theta_4 - F_5 \cos \Theta_5$

$= 500 \times \cos 26.56 + 1000 –$

$1500 \times \cos 56.31 - 2000 \times \cos 45$

$= -799.03N = \underline{799.03N\leftarrow}$

$+\uparrow\Sigma F_y = R_y = F_1 + F_2 \sin \Theta_2 - F_4 \sin \Theta_4 + F_5 \sin \Theta_5$

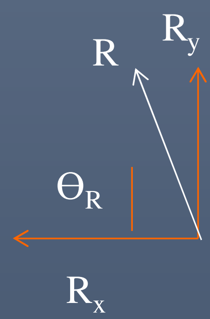
$= 2500+500 \sin 26.56-1500 \sin 56.31+2000 \sin 45$

$=\underline{2889.70N}\uparrow$

$\therefore$ Resultant R = $\sqrt{R_x^{\ 2} + R_y^{\ 2}} = \sqrt{799.03^2 + 2889.7^2}$

= $\underline{2998.14N}$

$\Theta_R = \tan^{-1}\left\{\dfrac{R_y}{R_x}\right\}$ = $\tan^{-1}(2889.7/799.03)$ = $\underline{74.54°}$



B) Position of Resultant w.r.to 'O':

By Varignon's theorem, Moment of the resultant about 'O'

= Algebraic sum of the moments of its components about 'O'.

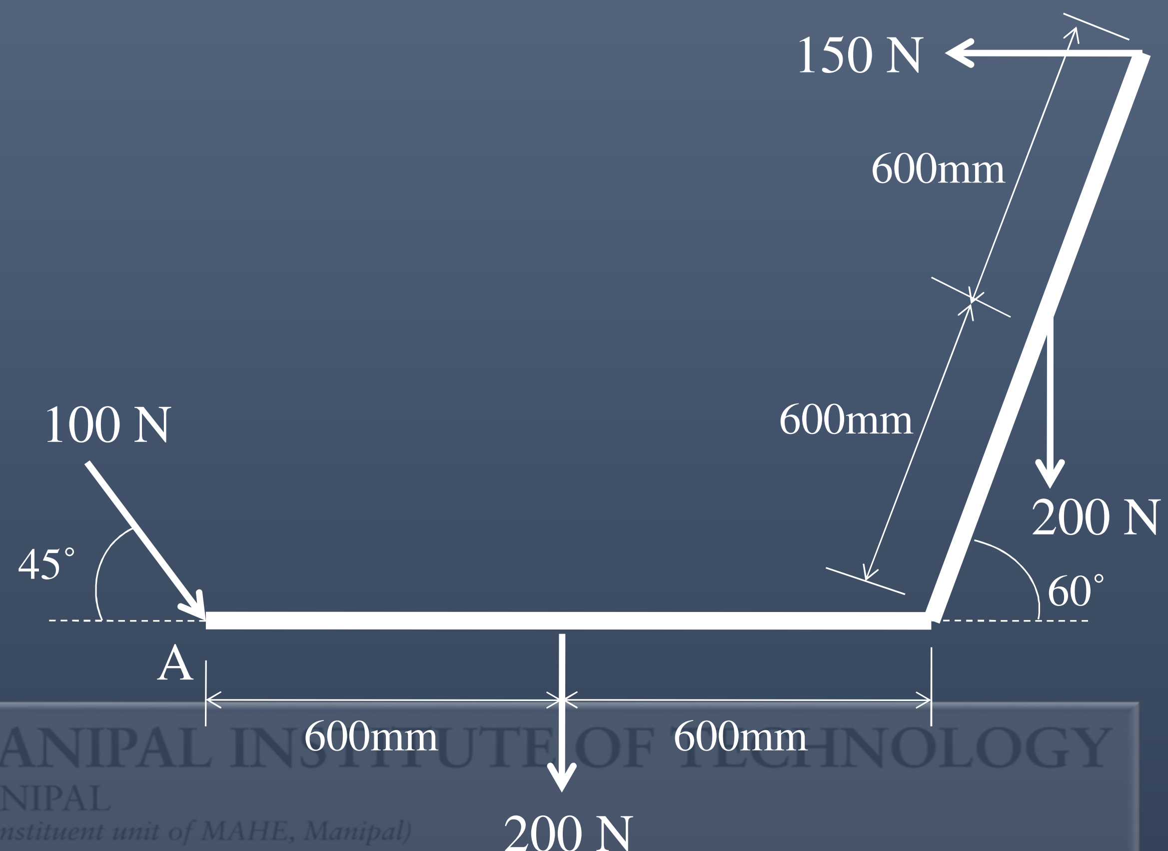
+ M_o =R×d = +2500×2 + 500×sin26.56×5 – 500× cos26.56×3 - 1000×1- 1500× cos56.31×0 –1500×sin56.3×1+2000× cos45×1-2000×sin45×0

= 2998.14 × d

d = 1.43 m from O.

TUTORIAL 1

1. Locate the resultant with respect to point A.



Soln:

$$\xrightarrow{+ve} \Sigma F_x = 100 \cos 45 - 150 = -79.289 \text{ N}$$

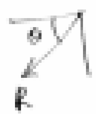
$$\uparrow^{+ve} \Sigma F_y = -100 \sin 45 - 200 - 200 = -470.710 \text{ N}$$

$$\curvearrowright^{+ve} \Sigma M_A = (200 \times 600) + 200 \times [1200 + 600 \cos 60] - 150 \times 1200 \sin 60$$

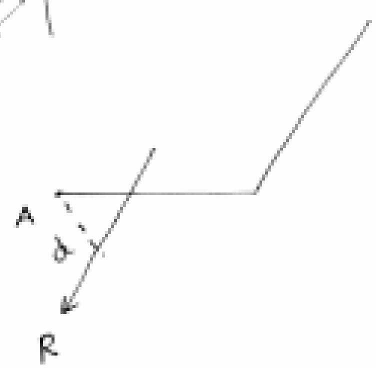
$$= 264115.42 \text{ N-mm}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \underline{\underline{477.341 \text{ N}}}$$

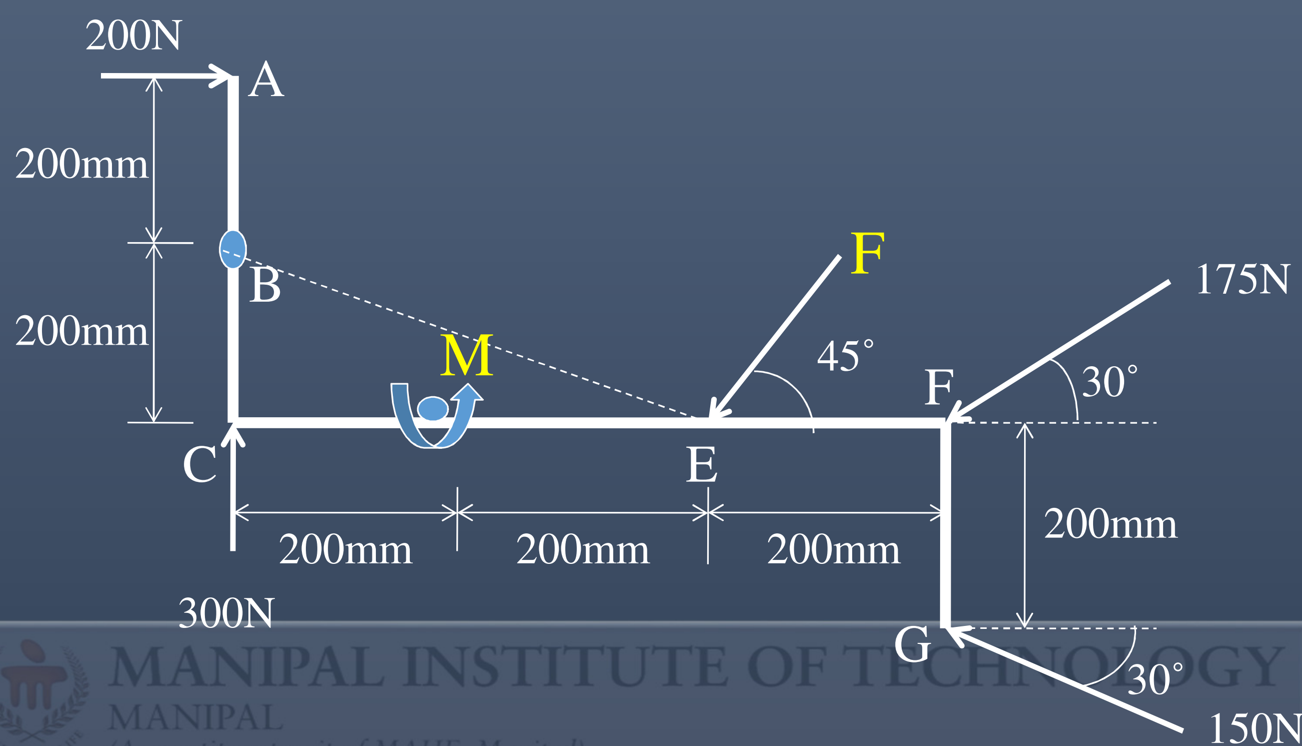
$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \underline{\underline{80.44^\circ}}$$



$$d = \frac{\Sigma M_A}{R} = \underline{\underline{553.305 \text{ mm}}}$$



2. For the configuration shown in the figure, resultant of the force system acting on the frame passes through the point B and E as shown in the figure. Find the values of F and M.



Soln: Hence resultant passes through B & E

Net moment about B & E is zero

$\Sigma M_B = 0 \qquad \Sigma M_E = 0$

+ve

$\Sigma M_E = 0$

$$200 \times 400 + 300 \times 400 - M + 175 \times \sin 30 \times 200 - 150 \sin 30 \times 200 + 150 \cos 30 \times 200 = 0$$

$$\underline{M = 228480.76 \text{ N-mm}}$$

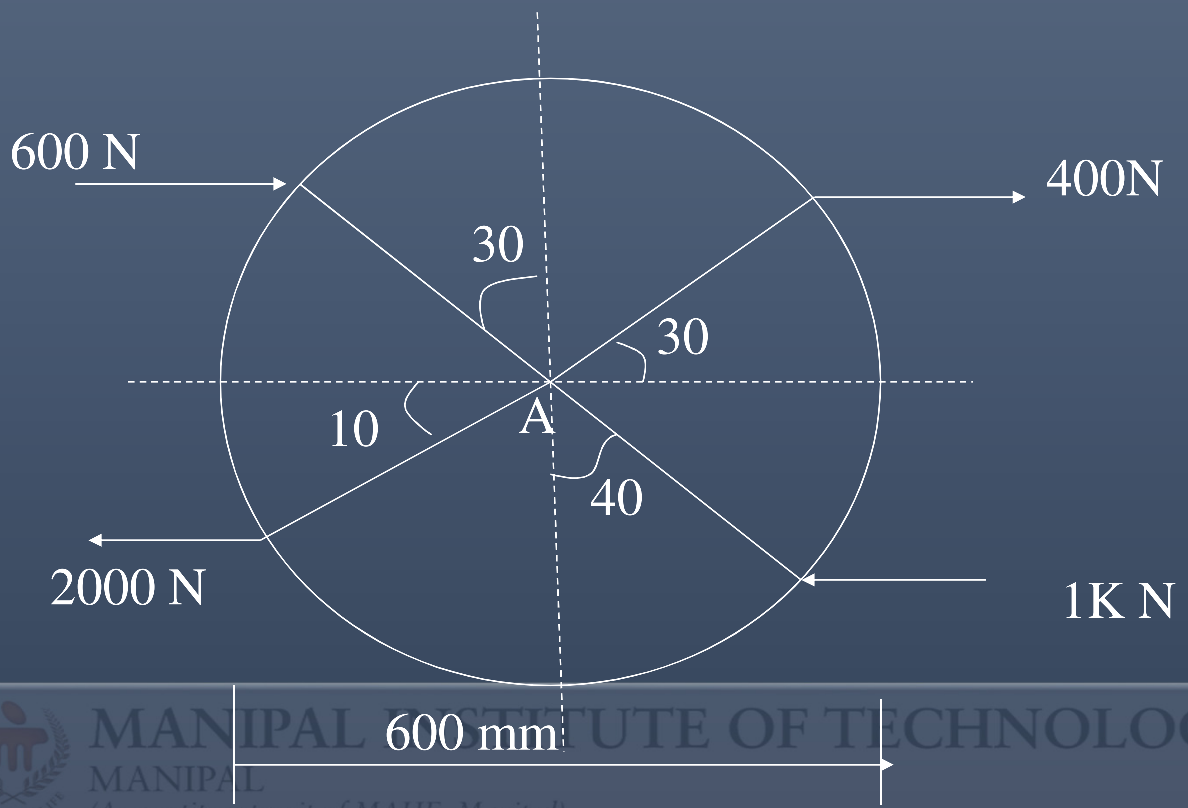
+ve

$\Sigma M_B = 0$

$$200 \times 200 - 228480.16 + F \sin 45 \times 400 + F \cos 45 \times 200 + 175 \sin 30 \times 600 + 175 \cos 30 \times 200 - 150 \sin 30 \times 600 + 150 \cos 30 \times 400 = 0$$

$$\underline{F = 232.656 \text{ N}}$$

3. Determine the resultant of the parallel coplanar force system shown in fig. Locate the resultant with respect to A .



Soln:

$$\Sigma F_x = 600 + 400 - 2000 - 1000 = -2000\,N$$

$\circlearrowright$ +ve

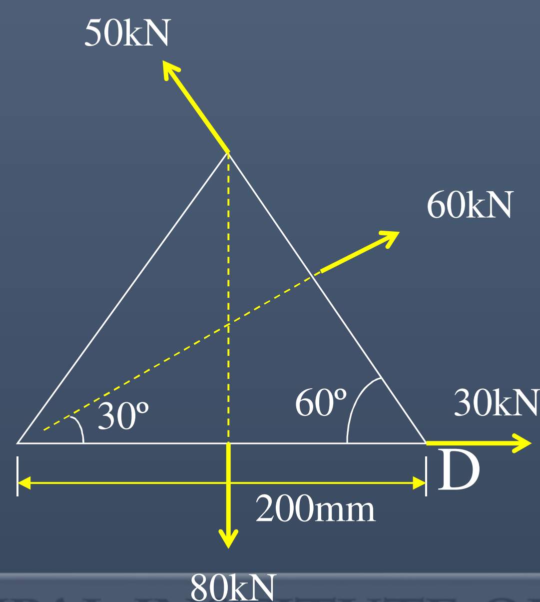
$$\Sigma M_A = 600 \times 300 \cos 30 + 400 \times 300 \times \sin 30 + 1000 \times \cos 40 \times 300 + 2000 \times 300 \times \sin 10$$

$$= 549886.81\ N\text{-}mm$$

$R = 2000\,N$

$$d = \frac{\Sigma M_A}{R} = 274.943\,mm$$

4. An equilateral triangle of sides 200mm is acted upon by 4 forces as shown in the figure. Determine magnitude and direction of the resultant and its position from point ‘D’.



Resultant & its inclination:

Resolving forces

$\xrightarrow{+} \Sigma F_x = R_x = +30 + 60\cos 30^\circ - 50\cos 60^\circ$

$= +56.96\text{kN.}$

$+\uparrow \Sigma F_y = R_y = -80 + 60\sin 30^\circ + 50\sin 60^\circ$

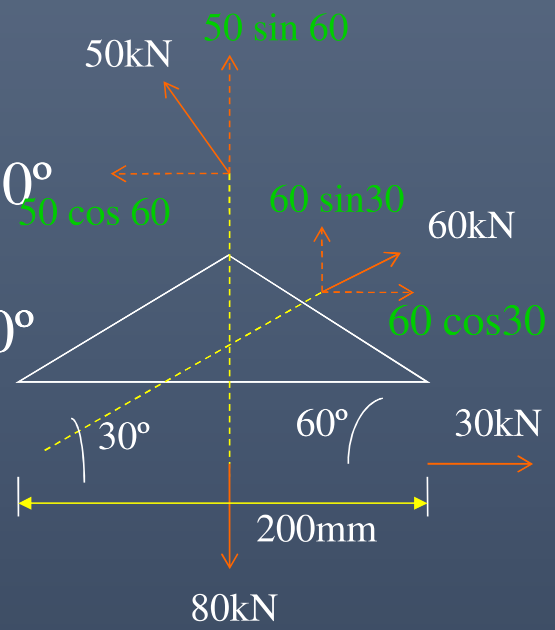
$= -6.69\text{kN.}$

$R = \sqrt{Rx^2 + Ry^2} = \sqrt{56.96^2 + 6.69^2} = 57.35kN$

Inclination w.r.to horizontal $= \theta_R$

$= \tan^{-1}(R_y/R_x)$

$= \tan^{-1}(6.69/56.96) = \underline{6.7^\circ}$



b) Position w.r.to D:

Moment of the component forces about D:

+ M_D = - 60 × 100 + 80 × 100 = 2000kNmm.

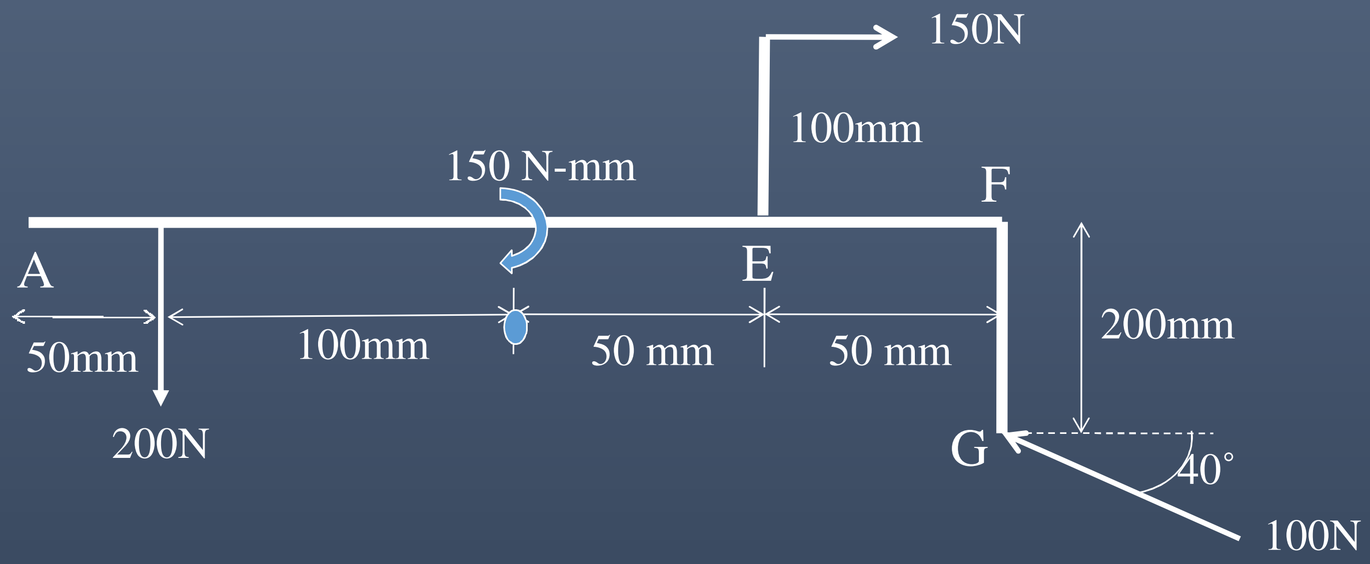
= R × d

where 'd' = perpendicular distance from point D to the line of action of R.

or, 2000= Rxd.

∴ d =2000/57.35 = 34.87mm

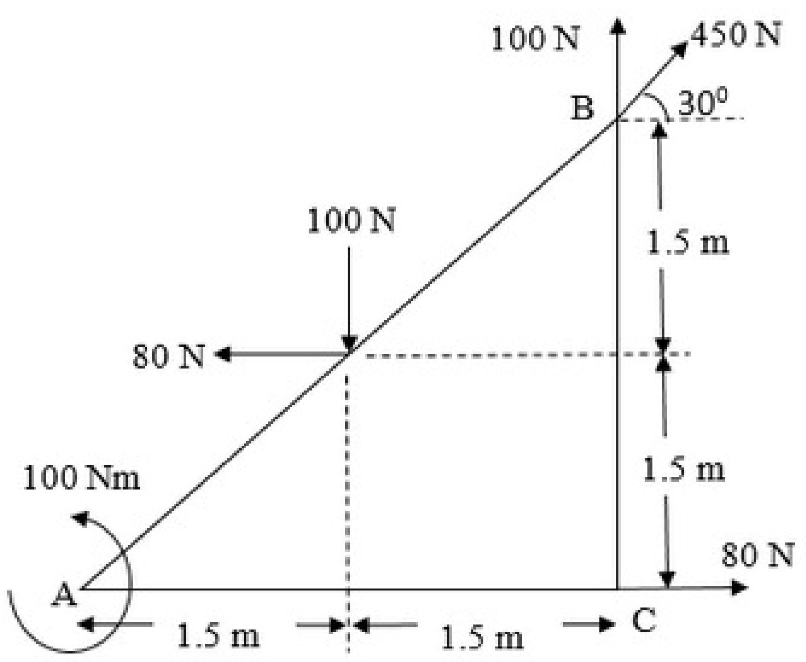
5. Determine the resultant of the coplanar force system shown in fig. Locate the resultant with respect to A



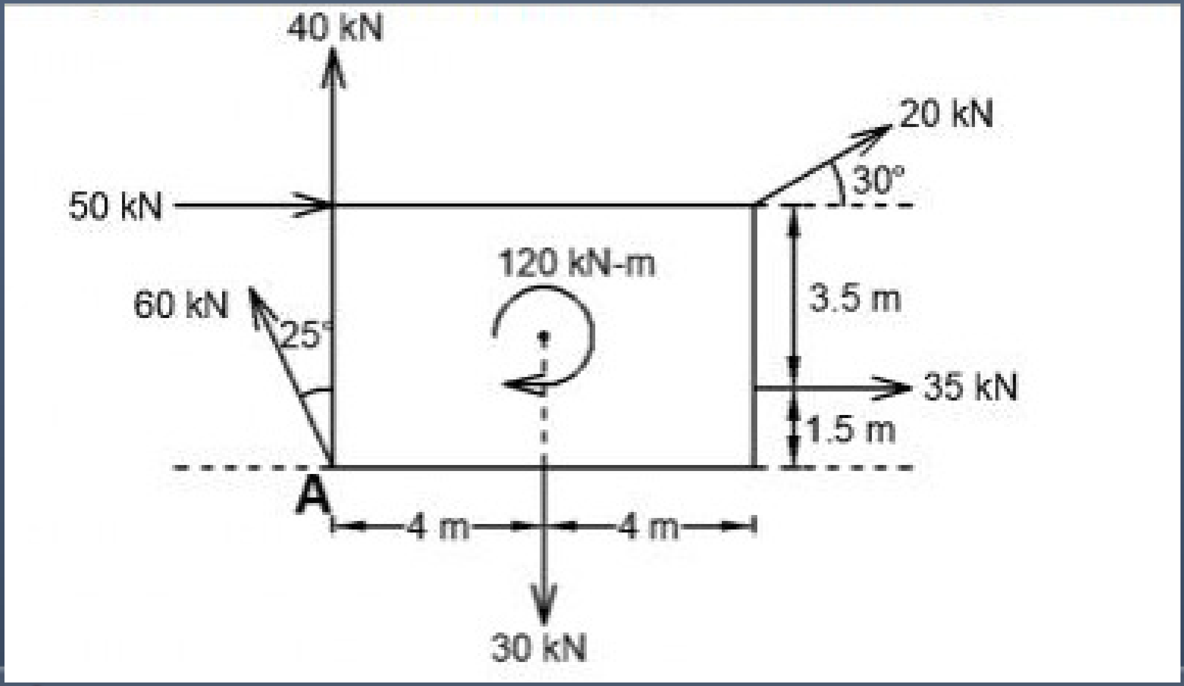
TUTORIAL (Additional)

1.

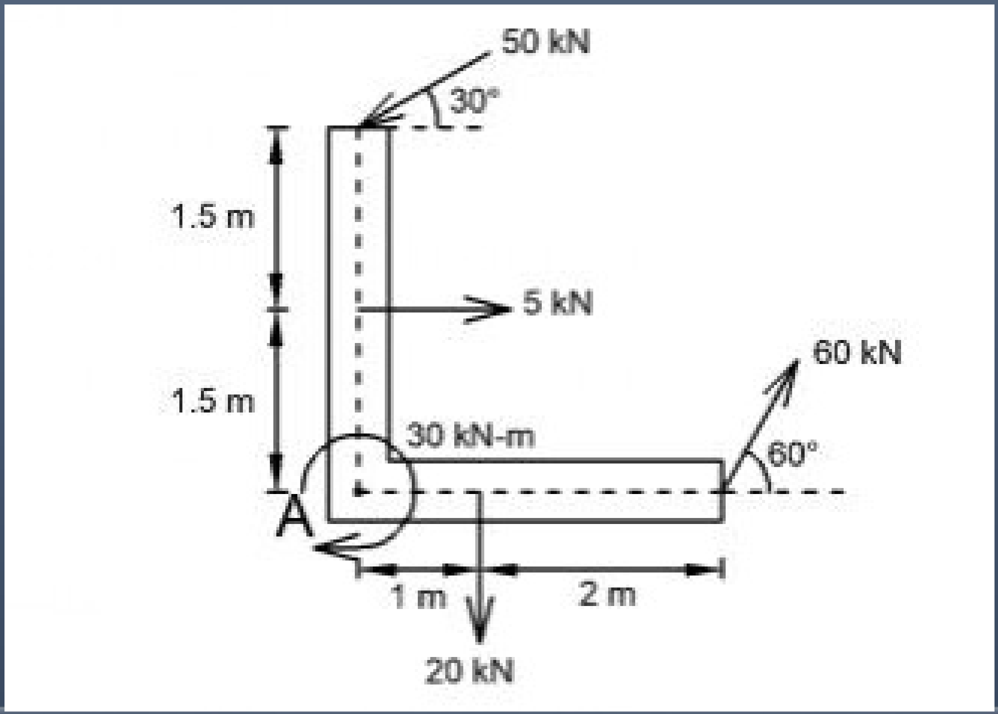
Locate the resultant of a force system shown in the figure with respect to C.



2. Locate the resultant of coplanar non-concurrent force system shown in figure with respect to ‘A’.



3. Locate the resultant of coplanar non-concurrent force system shown in figure with respect to ‘A’.



4.

Find magnitude, direction and position of a resultant force for a system of forces shown in the figure with respect to 'O'.