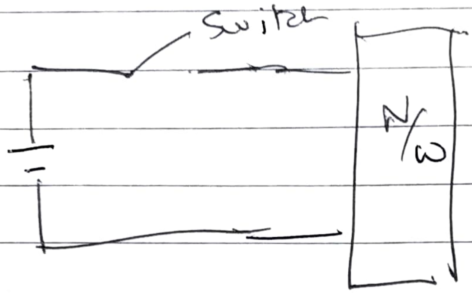
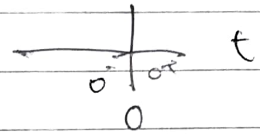
Transient Analysis





$t = 0^-$ → Just before switch is opened / closed
$t = 0^+$ → Just after " " " "

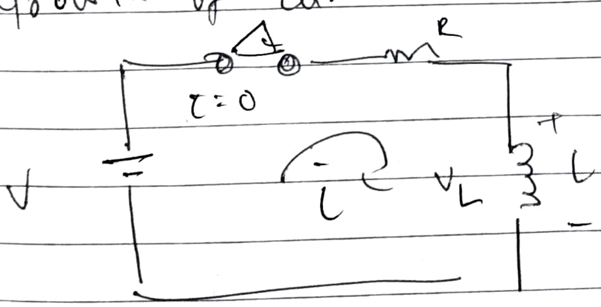
* The current in the inductor cannot change instantaneously

$$i_L(0^-) = i_L(0^+)$$

* The voltage in the capacitor cannot change instantaneously

$$V_c(0^-) = V_c(0^+)$$

① Growth of current in an Inductive circuit.



$t = 0^-$ → switch was open

* $i_L(0^-) = i_L(0^+) = 0$ → initial condition

KVL → closed switch

$V - V_R - V_L = 0$

$V - iR - L\frac{di}{dt} = 0$ $\Rightarrow V = iR + L\frac{di}{dt}$

(First order LDE)

$i(t) = i_{(h)}(t) + i_{(p)}(t)$

↳ homogeneous sol^n ↳ particular sol^n

$\frac{di}{dt} = D$

$\frac{d^2i}{dt^2} = D^2$

→ so on

→ Homogeneous sol^n

Characteristic eq^n

$DL + R = 0$ $\Rightarrow D = -R/L$

↳ Roots of char. eq^n

$\boxed{i_h = Ae^{-R/L\, t}}$ → homo sol^n

→ Particular sol^n → same form as the input

$i_p(t) = c$ → const $\Rightarrow \frac{dc}{dt} = 0$

$V = iR + L\frac{di}{dt}$ $\Rightarrow V = (cR + L/0)$

$\Rightarrow c = V/R$

$\Rightarrow i_{(p)}(t) = V/R$

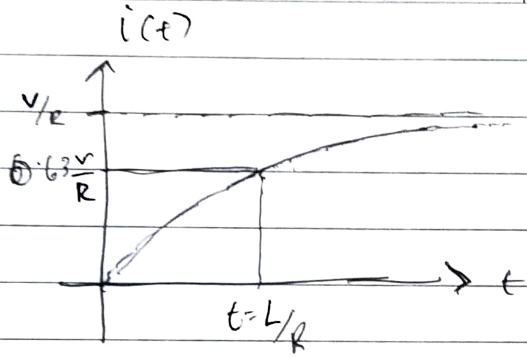
$\Rightarrow i(t) = Ae^{-R/L\, t} + V/R$

Using $i=0$ @ $t=0$

$$A + \frac{V}{R} = 0 \Rightarrow A = -V/R$$

$$\Rightarrow i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow i(t=\infty) = V/R$$



$t = L/R$

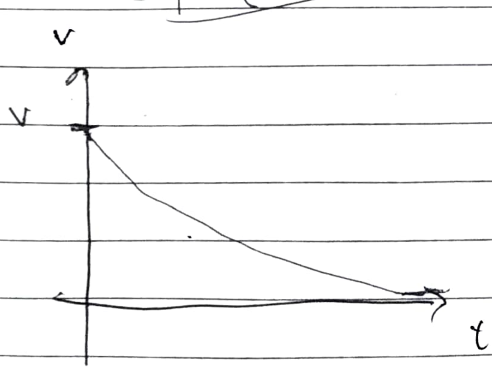
$$\Rightarrow i = \frac{V}{R}\left(1 - \frac{1}{e}\right)$$

$$i\left(\frac{L}{R}\right) = 0.63\,\frac{V}{R}$$

$L/R \rightarrow$ time const.

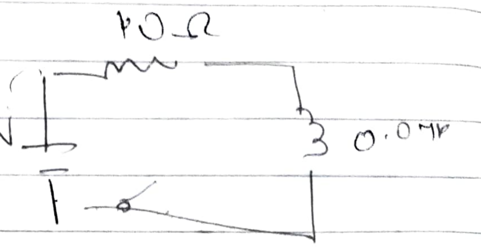
$$V_L(t) = L\frac{di_L}{dt} = \frac{V}{\cancel{R}} \times \frac{\cancel{R}}{\cancel{L}} \times \cancel{L}\, e^{-R/L\, t}$$

$$= V e^{-R/L\, t}$$



$$\Rightarrow \boxed{V_R = i_L(t) R}$$

Q A coil of inductance 0.04H & R = 10 is connected to a 120V dc. Determine



a → final value of current
b → time const
c → i @ t = time const.
d → expected time for the current to rise within 1% of its final value

a $I(\infty) = V/R = 12$ A

b ~~time~~ ~~of~~ $\tau = L/R = 0.04/10 = 0.004$ sec

c $I(\tau) = 0.63 \times 12 = 7.56$

d 1% of final value $= \frac{12}{100} = 0.12$

$$\Rightarrow \frac{V}{R \times 100} = \frac{V}{R}\left(1 - e^{-t/0.004}\right)$$

$$\frac{1}{100} = 1 - e^{-t/0.004}$$

$$\frac{99}{100} = e^{-t/0.004}$$

$$\frac{t}{0.004} = \ln\left(\frac{99}{100}\right) \quad \ln\left(\frac{100}{99}\right)$$

$$\boxed{t = 18.421 \text{ msec}}$$

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Q An RL series circuit is designed for a steady current of 250 mA. A current of 120 mA flows in the circuit at an instant 0.1 sec after connecting from supply voltage. Calc.

i) Time const

ii) Time from closing the circuit at which the current has reached 200 mA.

$$i_0 = \frac{V}{R} = 250 \text{ mA}$$

$$i(0.1s) = 120 \text{ mA}$$

$$\frac{120}{1000} = \frac{250}{1000}\left(1 - e^{-t/0.1\tau}\right)$$

$$\frac{12}{25} = 1 - e^{-t/0.1\tau}$$

$$e^{-t/0.1\tau} = \frac{13}{25} \qquad \frac{t}{0.1\tau} = \ln\left(\frac{25}{13}\right)$$

$$\Rightarrow \boxed{\tau = 0.1529 \text{ sec.}}$$

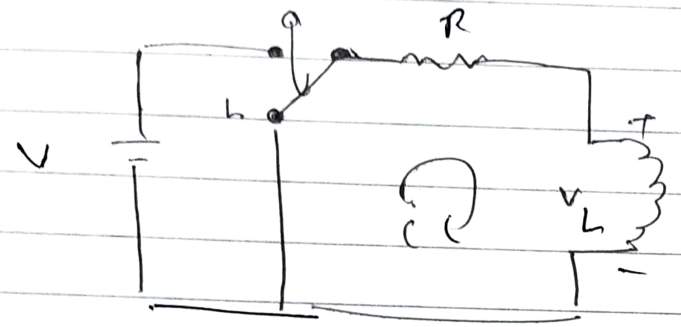
$$200 \text{ mA} = 250 \text{ mA}\left(1 - e^{-t/0.1529}\right)$$

$$\frac{1}{5} = e^{-t/0.1529}$$

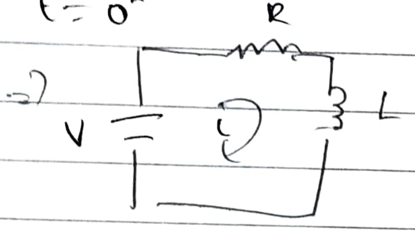
$$\Rightarrow t = 0.1529 \ln 5$$

$$= \boxed{0.2461 \text{ sec}}$$

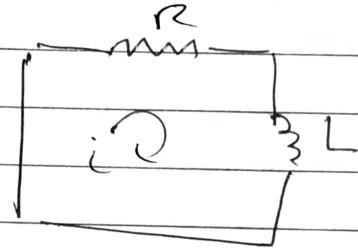
Decay of current in an Inductive circuit



$t = 0^-$ $\qquad\qquad\qquad\qquad\qquad$ $t = 0^+$



$\Rightarrow$



$i(0^-) = V/R \qquad\qquad i(0^+) = V/R$

$$-iR - L\frac{di}{dt} = 0$$

$$iR + L\frac{di}{dt} = 0 \rightarrow \text{First order LDE}$$

$\hookrightarrow i_{particular} = 0 \quad (V = 0)$

$$i(t) = i_h(t)$$

$$\Rightarrow PL + R = 0 \Rightarrow P = -R/L$$

$$\Rightarrow i_h(t) = A e^{-\frac{R}{L}t}$$

$$\Rightarrow \boxed{i(t) = A e^{-R/L\, t}}$$

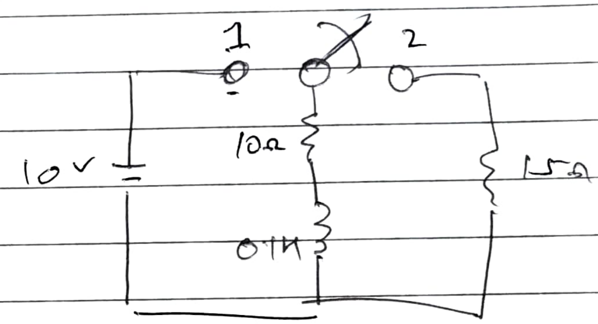
$$A = V/R \leftarrow t = 0^+ \quad i(0^+) = V/R$$

$$\Rightarrow \boxed{i(t) = \frac{V}{R} e^{-R/L\, t}}$$

$$V_L(t) = L\frac{di_L}{dt} \Rightarrow L\frac{V}{R}\times\frac{-R}{L}\times e^{-R/L\,t}$$

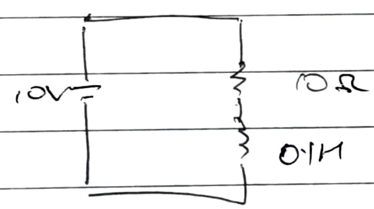
$$\boxed{V_L(t) = -V e^{-R/L\,t}}$$

Q In network, switch is closed to position 1 @ $t=0$ & is moved to position 2 @ 10ms. Determine
(a) $i_L(t)$ → sketch i_L



$0 < t < 10ms$

$$i_L(0^-) = 0 = i_L(0^+)$$



$$i = \frac{V}{R}\left(1-e^{-R/L\,t}\right) = \frac{10}{10}\left(1-e^{-\frac{10}{0.1}\times t}\right)$$

$$= \boxed{1-e^{-100t}}$$ ~~at~~ t=

when $t \geq 10ms$

$$i(t_{10ms}) = 1-e^{-\left(100\times\frac{10}{1000}\right)} = 1-1/e = \boxed{0.63}$$

$R = 25\Omega \Rightarrow i = 0.63\,e^{-(25/0.1)(t-10ms)}$
$= 0.63\,e^{-250(t-10ms)}$

$$i(t) = \left.\begin{matrix} 1-e^{-100t} \\ 0.63\,e^{-250(t-10ms)} \end{matrix}\right\}\begin{matrix} 0 \leq t \leq 10ms \\ t \geq 10ms \end{matrix}$$