CHAPTER - 1

① Basic permutations & combinations

② Generating fns — Permutation (distinct obj)
exponential g f
— Combinations (identical obj)
enumerators

③ Principle of inclusion & exclusion : Derrangements
" No of integers $\leq a$, which are rel prime to a"

④ Partition & compositions

⑤ Ferrers graph representation

⑥ Ordering — Lexico
Reverse lexico
Fike's

QUIZ

* Basic counting principle, general p&C probs
* Principle of inclusion & exclusn
* gf for general probs
* gf for partitions
* Ferrers graphs
* ordering → Immediate next permutatn
→ Fike's sequence

ex: what is the next imediate permutation of 4657 in lexico, where intial permutatn is 4567

4657
$\Downarrow$
4675

ex:- Next permutation TMHA where intial $\Rightarrow$ MATH
i) lexico
ii) Reverse lexico

soltn

MATH
1 2 3 4

k^{th} :- TMHA
3 1 4 2

$(k+1)^{th}$:- ?

lexico :- TAMH
3 2 1 4

Reverse :-

* What is 50th permutation in Fike's ordering when the intial permutation is PQRST

What is the Fike's sequence corresp to 50th permutatatn of PQRST.

n = 5

soltn

$$49 = \_\left(\frac{5!}{2!}\right) + \_\left(\frac{5!}{3!}\right) + \_\left(\frac{5!}{4!}\right) + \_\left(\frac{5!}{5!}\right)$$

$$49 = 0(60) + 2(20) + 1(5) + 4(1)$$

$49 = 0(60) + 2(20) + 1(5) + 4(1)$

0214 is to be subtracted from 12,...(n-1)

$$\begin{array}{r} 1234 \\ -\ 0214 \\ \hline 1020 \end{array}$$

$\therefore$ Fike's seq :- 1020

LATTICE THEORY

Binary Relation:

A binary relation R from a set A to a set B is a subset of $A \times B$

ie $R \subseteq A \times B$

$aRb \ni$ a is Related to b by a relation R.

$$R = \left\{ (a,b) \;\middle|\; \begin{matrix} a \in A \\ b \in B \end{matrix} \right\} \subseteq A \times B$$

* A binary relation on a set A is a binary relation from A to itself

* Identity relation :- A relation which relates an elt to itself.

* $n(A) = m$

  $n(B) = n$

  $n(A \times B) = mn$

  No of relations which can be defined from A to B

  $= 2^{mn}$ (?)

Types of relations: "A relation 'R' on set 'A'"

① Reflexive relation:

A relation R is said to be reflexive if $(a,a) \in R$ for all $a \in A$. In other words, every elt is related to itself

ex:- $A \to$ set of all +ve integers

$R: \{(a,b) \in R \text{ iff } a|b\}$

$(2,4) \in R$

$(6,8) \notin R$

Take any elt $a \in A$, since $a|a$, $(a,a) \in R$

$\therefore$ R is reflexive

② $A \leftarrow$ set of straight lines

$R_1 = (L_1, L_2) \in R_1$ iff L_1 ||el to L_2

$R_2 = (L_1, L_2) \in R_2$ iff $L_1 \perp^r$ to L_2

i) R_1 is reflexive ($\because$ Every line is ||el to itself)

ii) R_2 is not reflexive

③ A: set of all real nos

$R: (a,b) \in R$ iff $a < b$

Clearly, it is not reflexive.

$R': (a,b) \in R$ iff $a \leq b$, R' is reflexive

(2) Symmetric relation:

A relation R is symmetric

if $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$

ex:- $A = \{$set of all straight lines$\}$

R_1: $(L_1, L_2) \in R_1$ iff $L_1 \parallel$el to L_2

R_2: $(L_1, L_2) \in R_2$ iff $L_1 \parallel$el to L_2

If $(L_1, L_2) \in R_1$, L_1 is $\parallel$le to L_2, which means

L_2 is $\parallel$lel to L_1, $\Rightarrow (L_2, L_1) \in R_1$

$\therefore$ Both R_1 & R_2 are symmetric.

ex:- A: set of all +ve integers

R: $(a, b) \in R$ iff $a | b$

$(2, 4) \in R \Rightarrow 2 | 4$

But $4 \nmid 2$

$\therefore (4, 2) \notin R$

$\therefore$ Not symmetric

* A: ppl in a colony

R: $(P_1, P_2) \in R$ if P_1 is a sibling of P_2

Manoj

Ria

Boy ——— Girls

③ Antisymmetric :

$(a,b) \in R$ implies $(b,a) \notin R$ unless $a=b$

In other words, if both (a,b) and (b,a) are in R, then $a=b$

ex:- A = Natural no's

$R : (a,b) \in R$ if $a \leq b$

* For any 2 distinct elts a,b, if $(a,b) \in R \Rightarrow (b,a) \notin R$

$(a,b) \in R$ & $(b,a) \in R$ iff $a=b$

ex:- A : +ve integers

$R : (a,b) \in R$ iff $a \mid b$

It is antisymmetric ($2 \mid 4$ $(2,4) \in R$ $4 \nmid 2$ $(4,2) \notin R$)

* $R = \{(a,b)\ (a,c)\ (c,a)\}$
    * Not reflexive
    * Not sym ($\because (b,a) \notin R$
    * Not antisy ($\because (a,c) \in R$ & $(c,a) \in R$)

* $R = \{(a,a)\ (b,b)\}$ $A = \{a,b\}$
    * Reflexive
    * sym
    * Antisymmetric

④ Transitive Relation:

If $(a,b) \in R$, $(b,c) \in R \Longrightarrow (a,c) \in R$
for all $a, b, c \in A$

i) A: set of all st. lines
R_1: ||el to
R_2: ⊥r to

R_1 is transitive
R_2 is not transitive

② A: set of all +ve integers
R: '|'

Transitive

③ A: set of all natural nos
R: $\leq$

Transitive

④ A: set of all natural nos
R: $\geq$ $(a,b) \in R$ if $a \geq b$

$(x,y) \in R \Rightarrow x \geq y$
$(y,z) \in R \Rightarrow y \geq z$
$\therefore x \geq z \quad \therefore (x,z) \in R$

(5) Irreflexive :

For all $a \in A$, $(a,a) \nmid R$.

(6) Equivalence Relation :

→ Reflexive

→ symmetric

→ Transitive

(7) Partial Ordering Relation :

→ Reflexive

→ antisymmetric

→ Transitive.

POSET (Partial Ordering set)

A nonempty set with a partial ordering relation is called a poset. It is often denoted as (A, R)

ex:- A : set of all +ve integers

$R : (a,b) \in R$ iff $a \mid b$

Is 'R' a partial ordering relation ?

(ref, antisym, transit)

∴ R is a partial ordering relation

(A, R) is poset. , $(A, \mid)$ is a poset ex :- $U = \{a, b, c\}$

A : set of all subsets of U

R : $\subseteq$

soln

$A = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$

R : $(S_1, S_2) \in R$ if $S_1 \subseteq S_2$

Reflexive : ✓

antisym : $\{b\} \subseteq \{b,c\}$.

$\{b,c\} \nsubseteq \{b\}$

✓

Transitive : ✓

$\subseteq$ is a partial ordering relation.

$\therefore$ $(A, \subseteq)$ is a poset

* $(A, |)$ is a poset

$(A, \leq)$ "

$(A, \geq)$ "

* $(A$, "$\parallel$el to"$)$ is not a poset
* $(A$, "$\perp^r$ to"$)$ is not a poset