LECTURE 5 & 6

KARNAUGH MAP (K – MAP)

EXAMPLE 4:

$$F(x,y,z)=\prod_M(0,2,5,7) = \sum_m(1,3,4,6)$$

x \ yz	00	01	11	10
0	0	1	1	0
1	1	0	0	1

POS

$F = (x+z)(\bar{x}+\bar{z})$

SOP

$F = \bar{x}z + x\bar{z}$

NAND

$\overline{\overline{F}} = \overline{\overline{\bar{x}z + x\bar{z}}} = F$

$= \overline{\overline{\bar{x}z} \cdot \overline{x\bar{z}}}$

Draw ckt here

NAND ONLY

NOR

$F = \overline{\overline{F}} = \overline{\overline{(x+z)(\bar{x}+\bar{z})}}$

$= \overline{\overline{(x+z)}+\overline{(\bar{x}+\bar{z})}}$

NOR ONLY

FOUR VARIABLE K – MAP

4 Variables $2^4 = 16$ combinations

SOP using minterms

A. SOP: -

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	m_0 $\bar{A}\bar{B}\bar{C}\bar{D}$ 0	m_1 $\bar{A}\bar{B}\bar{C}D$ 1	m_3 $\bar{A}\bar{B}CD$ 3	m_2 $\bar{A}\bar{B}C\bar{D}$ 2
$\bar{A}B$ 01	m_4 $\bar{A}B\bar{C}\bar{D}$ 4	m_5 $\bar{A}B\bar{C}D$ 5	m_7 $\bar{A}BCD$ 7	m_6 $\bar{A}BC\bar{D}$ 6
AB 11	m_{12} $AB\bar{C}\bar{D}$ 12	m_{13} $AB\bar{C}D$ 13	m_{15} $ABCD$ 15	m_{14} $ABC\bar{D}$ 14
$A\bar{B}$ 10	m_8 $A\bar{B}\bar{C}\bar{D}$ 8	m_9 $A\bar{B}\bar{C}D$ 9	m_{11} $A\bar{B}CD$ 11	m_{10} $A\bar{B}C\bar{D}$ 10

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	3	2
$\bar{A}B$ 01	4	5	7	6
AB 11	12	13	15	14
$A\bar{B}$ 10	8	9	11	10

ABCD
0000
0001
0010
0011
0100
0101
0110
0111
1000 m_8
1001
1010
1011
1100
1101
1110
1111

FOUR VARIABLE K – MAP

POS: -

$\overline{A}\overline{B}$ sop

A+B \ C+D	C+D 0 0	$C+\overline{D}$ 0 1	$\overline{C}+\overline{D}$ 1 1	$\overline{C}+D$ 1 0
$\boxed{A+B}$ 0 0	$A+B+C+D$ 0	$A+B+C+\overline{D}$ 1	$A+B+\overline{C}+\overline{D}$ 3	$A+B+\overline{C}+D$ 2
$A+\overline{B}$ 0 1	$A+\overline{B}+C+D$ 4	$A+\overline{B}+C+\overline{D}$ 5	$A+\overline{B}+\overline{C}+\overline{D}$ 7	$A+\overline{B}+\overline{C}+D$ 6
$\overline{A}+\overline{B}$ 1 1	$\overline{A}+\overline{B}+C+D$ 12	$\overline{A}+\overline{B}+C+\overline{D}$ 13	$\overline{A}+\overline{B}+\overline{C}+\overline{D}$ 15	$\overline{A}+\overline{B}+\overline{C}+D$ 14
$\overline{A}+B$ 1 0	$\overline{A}+B+C+D$ 8	$\overline{A}+B+C+\overline{D}$ 9	$\overline{A}+B+\overline{C}+\overline{D}$ 11	$\overline{A}+B+\overline{C}+D$ 10

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0 $\bar{A}\bar{B}\bar{C}\bar{D}$	1	3	2
$\bar{A}B$ 01	4	5	7	6
AB 11	12	13	15	14
$A\bar{B}$ 10	8	9	11	10

for SOP

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	M_0 $A+B+C+D$	M_1 $A+B+C+\bar{D}$		
$A+\bar{B}$ 01				
$\bar{A}+\bar{B}$ 11				
$\bar{A}+B$ 10				

for POS

EXAMPLE 1:

- Simplify the following expression into
  - SOP & give NAND realization
  - POS & give NOR realization

$$F(A,B,C,D) = \sum_m (1,5,6,7,8,9,10,14) = \prod_M (0,2,3,4,11,12,13,15)$$

AB \ CD	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	0	0	0	1
10	1	1	0	1

SOP Group 1's

$$F = \bar{A}\bar{C}D + \bar{A}BC + AC\bar{D} + A\bar{B}\bar{C}$$

$$F = \bar{A}BD + BC\bar{D} + A\bar{B}\bar{D} + \bar{B}\bar{C}D$$

$m_5 + m_7$

$$\bar{A}B\bar{C}D + \bar{A}BCD \Rightarrow \bar{A}B(\bar{C}+C)D = \bar{A}BD$$

CONTINUED…

$$F(A,B,C,D) = \sum(1,5,6,7,8,9,10,14) = \prod_{M}(0,2,3,4,11,12,13,15)$$

- POS

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	0	1	0	0
$A+\bar{B}$ 01	0	1	1	1
$\bar{A}+\bar{B}$ 11	0	0	0	1
$\bar{A}+B$ 10	1	1	0	1

POS

1 2 3 4

$$F = (A+C+D)\cdot(A+B+\bar{C})\cdot(\bar{A}+\bar{B}+C)\cdot(\bar{A}+\bar{C}+\bar{D})$$

OR

1 2 3 4

$$F = (\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{D})(B+\bar{C}+\bar{D})(A+B+\bar{D})$$

EXAMPLE 2:

$$F(A,B,C,D)=\sum_m(0,1,2,4,5,6,8,9,10,12,13)$$

MSB LSB

$\bar{C}+\bar{D}$

F: AB (MSB) \ CD (LSB)	00 $\bar{C}\bar{D}$	01 $\bar{C}D$	11 CD	10 $C\bar{D}$
00 $\bar{A}\bar{B}$	1	1	0	1
01 $\bar{A}B$	1	1	0	1
11 AB	1	1	0	0
10 $A\bar{B}$	1	1	0	1

SOP: club 1's

$$\boxed{F=\overset{1}{\bar{C}}+\overset{2}{\bar{A}\bar{D}}+\overset{3}{\bar{B}\bar{D}}}$$

POS

$$\boxed{F=(\bar{C}+\bar{D})(\bar{A}+\bar{B}+\bar{C})}$$

$F=m_0+m_1+m_2+m_4+m_5+m_6+m_8+m_9+m_{10}+m_{12}+m_{13}$

$x+x+x+x+\ldots\ldots\Rightarrow x$

Note: A pair of minterm's combined $\rightarrow$ 1 Variable Vanishes

For Ex:

$m_{10}+m_2$

$A\bar{B}C\bar{D}+\bar{A}\bar{B}C\bar{D}$ — 2 4 I/P AND gate

$\rightarrow (A+\bar{A})\bar{B}C\bar{D}=\bar{B}C\bar{D}$ — ONE 3 I/P AND

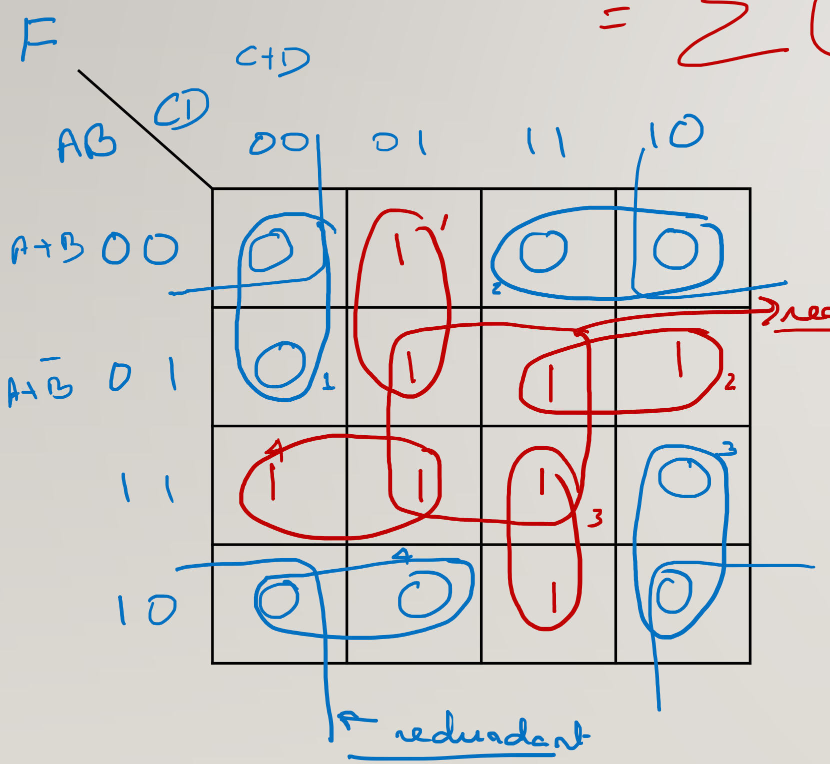
Possible

$m_0+m_2+m_8+m_{10}$

EXAMPLE 3:

$$F(A,B,C,D) = \prod_{M}(0,2,3,4,8,9,10,14)$$

$$= \sum (1,5,6,7,11,12,13,15)$$



POS

$$F = (A+C+D)\,(A+B+\bar{C})\,(\bar{A}+\bar{C}+D)\,.\,(\bar{A}+B+C)$$

SOP

$$F = \bar{A}\bar{C}D + \bar{A}BC + ACD + AB\bar{C} + (\quad)$$

redundant

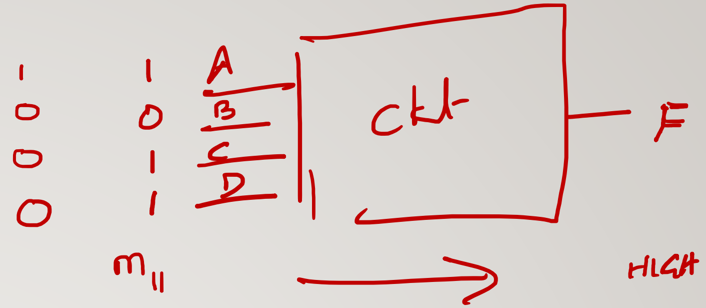
(1, 5) (6, 7) (15, 11) (12, 13) (5, 7, 13, 15) Quad

EXAMPLE 4:

$$F(A,B,C,D) = \overline{C}(\overline{A}\overline{B}\overline{D} + D) + A\overline{B}C + \overline{D}$$

MSB LSB

$= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{C}D + A\bar{B}C + \bar{D}$



m_0

$\bar{C}+\bar{D}$

$(\bar{A}+A)(\bar{B}+B)\bar{C}D$

$1 \cdot 1 \cdot \bar{C}D$

$\bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + A\bar{B}\bar{C}D + AB\bar{C}D$

$m_1 \quad m_5 \quad m_9 \quad m_{13}$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	1 (1)	0 (3)	1 (2)
$\bar{A}B$	1 (4)	1 (5)	0 (7)	1 (6)
AB	1 (12)	1 (13)	0 (15)	1 (14)
$A\bar{B}$	1 (8)	1 (9)	1 (11)	1 (10)

$F = \sum_m (0, 1, 2, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14)$

$\bar{C}D \rightarrow$ A & B are missing
Quad group

$A\bar{B}C \rightarrow$ 1 variable missing D
pair group m_{10} & m_{11}

$A\bar{B}C(\bar{D}+D)$

$= A\bar{B}C\bar{D} + A\bar{B}CD$

$m_{10} \quad m_{11}$

$\bar{D} \rightarrow$ 3 variable missing
group 8 minterms

Simplified Expression

$$\boxed{F = \bar{C} + \bar{D} + A\bar{B}}$$ SOP

$\underbrace{\bar{C}+\bar{D}}_{x} + yz$

POS

$$\boxed{F = (A + \bar{C} + \bar{D})(\bar{B} + \bar{C} + \bar{D})}$$

Distributive law

$x + y \cdot z = (x+y)(x+z)$

$x(y+z) = xy + xz$

$F = (\bar{C} + \bar{D} + A)(\bar{C} + \bar{D} + \bar{B})$

EXAMPLE 5:

$$F(A,B,C,D) = D(\overline{A}+B) + \overline{B}(C+AD) = \sum_m (1, 2, 3, 5, 7, 9, 10, 11, 13, 15)$$

$$= \bar{A}D + BD + \bar{B}C + A\bar{B}D$$

$$= \prod_M (0, 4, 6, 8, 12, 14)$$

F

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	1	1
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	1	1	1

$A+B$

$POS = (C+D)(\bar{B}+D)$

distributive law

$SOP = D + (\bar{B}C) \rightarrow (\bar{B}+D)(C+D)$

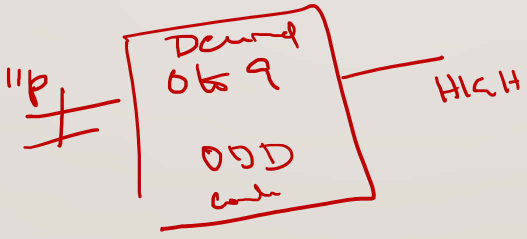
distributive law

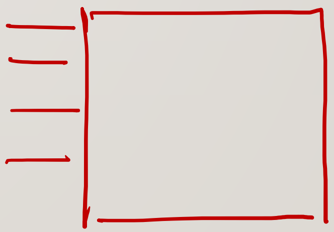
DON'T CARE CONDITION

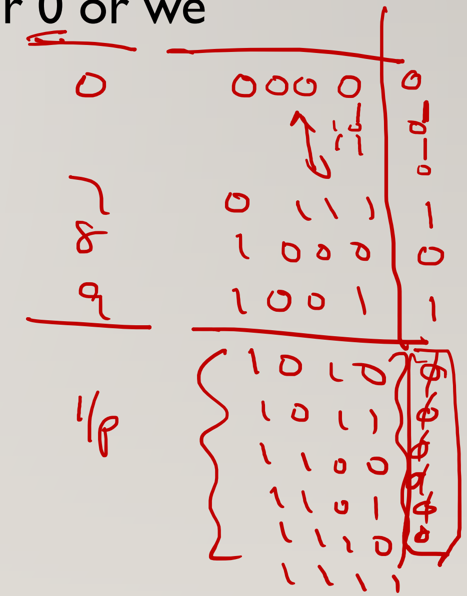
- The "Don't Care" conditions indicate the input combinations which are invalid for a particular circuit.

ϕ d

- While forming groups of cells, we can consider a "Don't Care" cell as either 1 or 0 or we can simply ignore that cell.
- Therefore, "Don't Care" condition are used to form a larger group of cells.



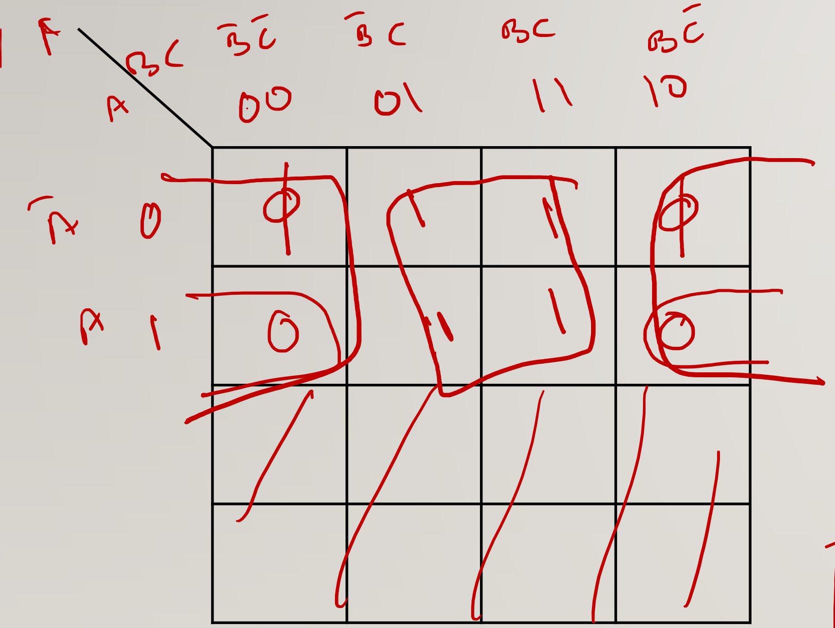




EXAMPLE 1:

$$F(A,B,C) = \sum_m (1,3,5,7) + \sum_d (0,2)$$

$$= \prod_M (4,6) \cdot \prod_d (0,2)$$



$\boxed{F = C}$ SOP

only 0's

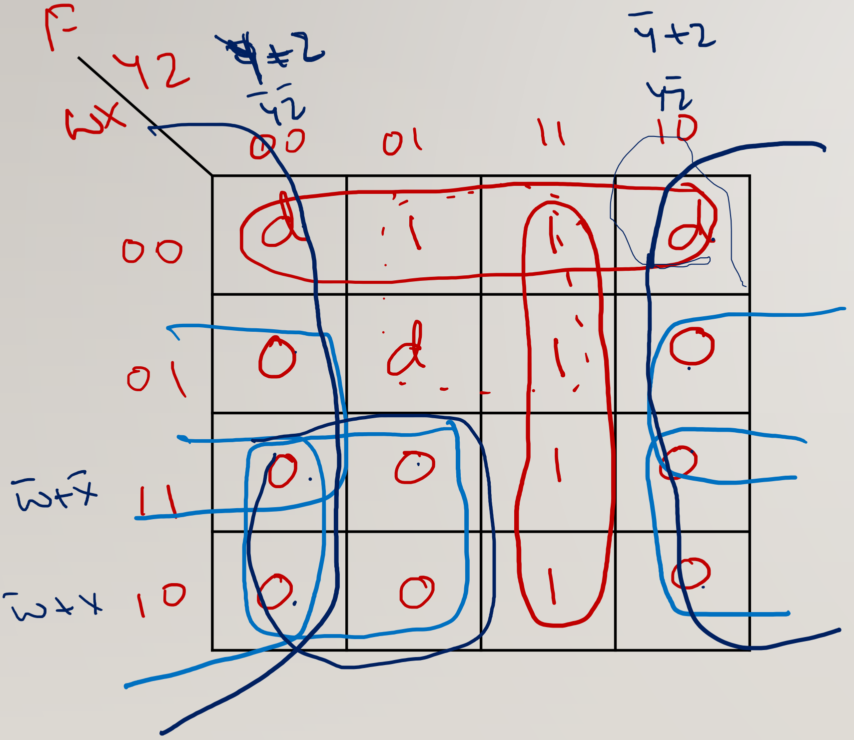
POS $= (\bar{A} + C)$

with don't care

$\boxed{F = C}$ POS

EXAMPLE 2:

$$F(W,X,Y,Z)=\sum_m(1,3,7,11,15)+\sum_d(0,2,5) \;=\; \prod_m(4,6,8,9,10,12,13,14)\cdot\prod_d(0,2,5)$$



SOP

$F = YZ + \bar{W}\bar{X}$

OR

$F = YZ + \bar{W}Z$

POS $F = (\bar{W}+Y)(\bar{W}+Z)(\bar{X}+Z)$

Consider clubbing with don't cares

$F = Z + (\bar{W}+Y)$

$YZ + \bar{W}Z$

EXAMPLE 3:

$$F(W,X,Y,Z) = \prod_{M}(0,1,3,5,8,9,14) \cdot \prod_{D}(2,6,10) = \sum_{m} 4,7,11,12,13,15 + \sum_{d} 2,6,10$$

F: WX \ YZ	00	01	11	10
00	0 [0]	0 [1]	0 [3]	ϕ [2]
01	1 [4]	0 [5]	1 [7]	ϕ [6]
11	1 [12]	1 [13]	1 [15]	0 [14]
10	0 [8]	0 [9]	1 [11]	ϕ [10]

$SOP = x\bar{y}\bar{z} + wx\bar{y} + xyz + wyz$

$POS = (w+x)(\bar{y}+z)(x+y)(w+y+\bar{z})$

P. Note:

SOP / POS

Clubbing rule

1. Club max Number of 1's / 0's along with 'ϕ'
   to get max combinations
2. Check for uncovered 1's / 0's
3. if Any uncovered 1's | 0's → along with ϕ's go for max clubbing

EXAMPLE 4:

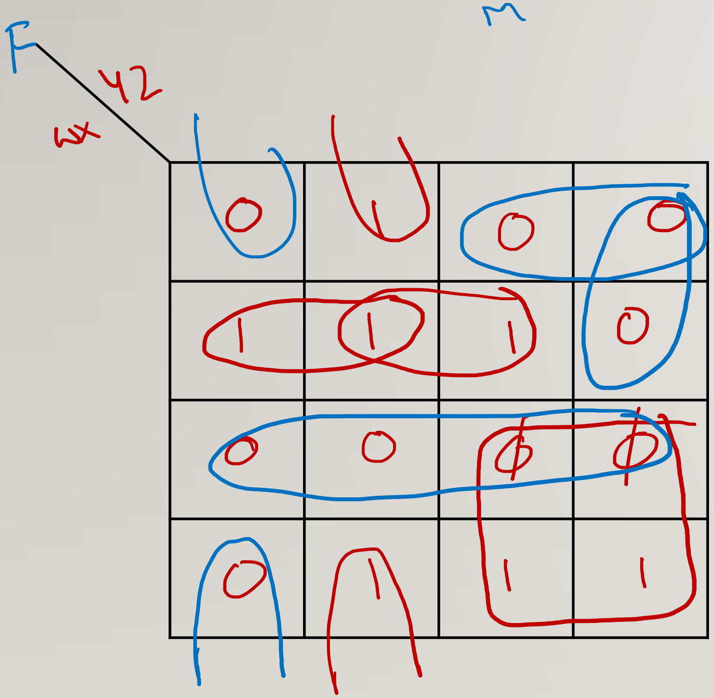
$$F(W, X, Y, Z) = \sum_m (1,4,5,7,9,10,11) + D(14,15)$$

$$= \prod_m (0,2,3,6,8,12,13) \cdot D(14,15)$$

$\phi(\quad)$

$d(\quad)$

$D(\quad)$

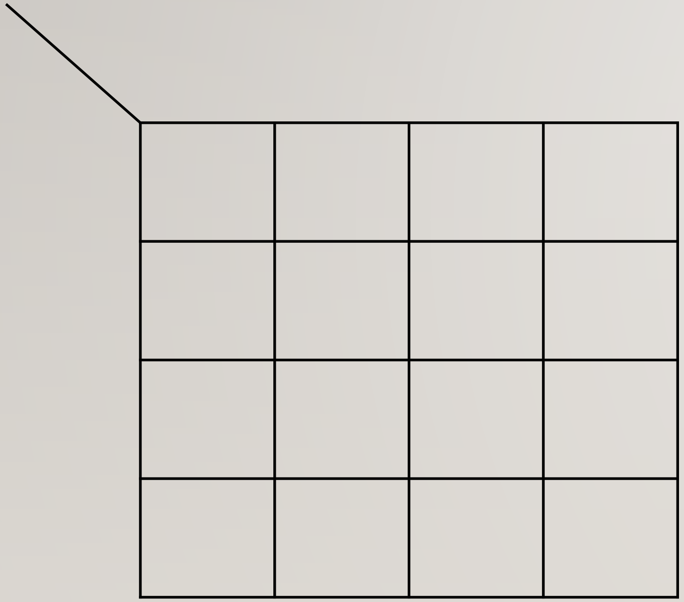


SOP ⇒ F = pls. write

POS ⇒ F = pls. write

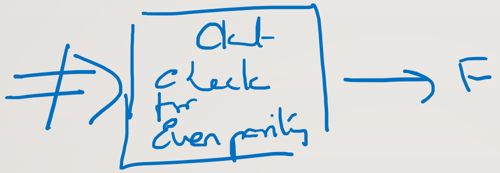
EXAMPLE 5:

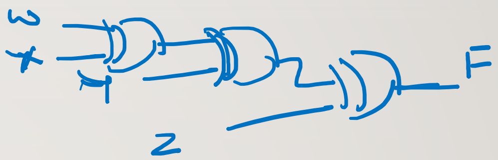
Design a combinational circuit with 4- input lines that represents a decimal digit in BCD and 4- output lines that generates 2's complement of input digit.



EXAMPLE 6:

Design a combinational circuit to check for even parity of 4 bits. A logic '1' output is required when the 4 bits constitute an even parity.





Even parity

i/p → for Even number of 1's in i/p → if Yes → 1, No → 0

4-bit

w x y z	F	
0 0 0 0	ϕ	m_0
0 0 0 1	0	m_1
0 0 1 0	0	m_2
0 0 1 1	1	m_3
0 1 0 0	0	m_4
0 1 0 1	1	
0 1 1 0	1	
0 1 1 1	0	
1 0 0 0	0	
1 0 0 1	1	
1 0 1 0	1	
1 0 1 1	0	
1 1 0 0	1	
1 1 0 1	0	
1 1 1 0	0	
1 1 1 1	1	

F

wx \ yz	00	01	11	10
00	ϕ	0	1	0
01	0	1	0	1
11	1	0	1	0
10	0	1	0	1

$$= \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}yz + \bar{w}x\bar{y}z + \bar{w}xy\bar{z} + wx\bar{y}\bar{z} + wxyz + w\bar{x}\bar{y}z + w\bar{x}y\bar{z}$$

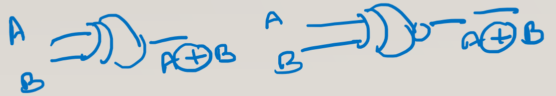
$$= \bar{w}\bar{x}(\bar{y}\bar{z}+yz) + \bar{w}x(\bar{y}z+y\bar{z}) + wx(\bar{y}\bar{z}+yz) + w\bar{x}(\bar{y}z+y\bar{z})$$

$$= \bar{w}\bar{x}(\overline{y\oplus z}) + wx(\overline{y\oplus z}) + \bar{w}x(y\oplus z) + w\bar{x}(y\oplus z)$$

$$= (\bar{w}\bar{x}+wx)\,\overline{y\oplus z} + (\bar{w}x+w\bar{x})(y\oplus z)$$

$$= \underbrace{(\overline{w\oplus x})}_{\bar{A}}\underbrace{(\overline{y\oplus z})}_{\bar{B}} + \underbrace{(w\oplus x)}_{A}\underbrace{(y\oplus z)}_{B} = \underline{w\oplus x\oplus y\oplus z}$$

EX-OR Gates   EX-NOR



EXAMPLE 7:

Design a combinational circuit that multiplies by ‘5’ an input decimal digit represented in BCD. The output is also in BCD.