MECHANICS OF DEFORMABLE BODIES

LECTURE 14

Contents:

Introduction

Mechanical properties of materials

Normal stress and strain

Hooke's law

Modulus of elasticity

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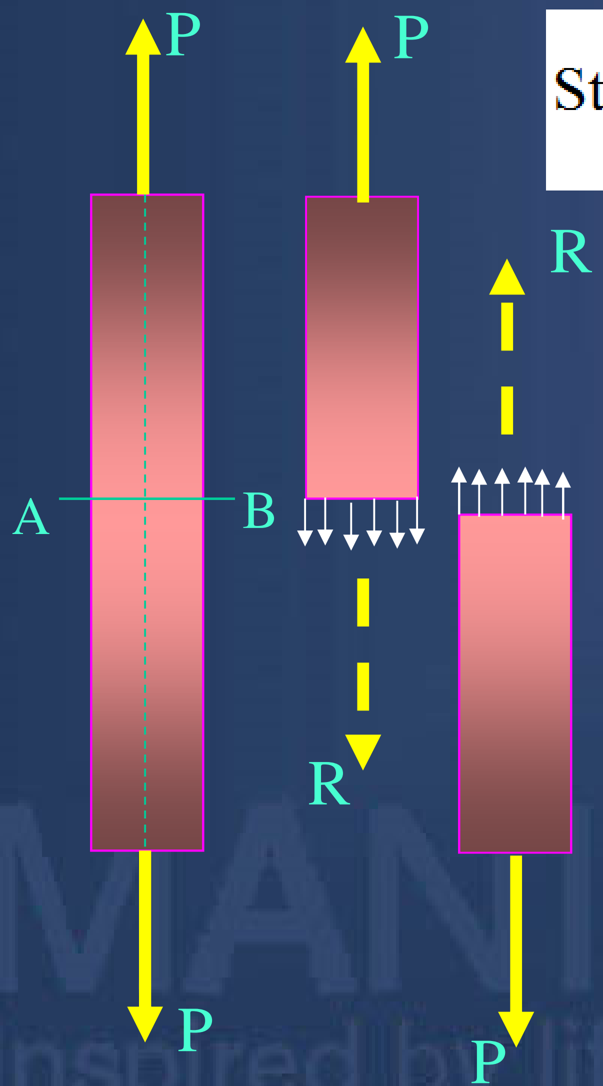
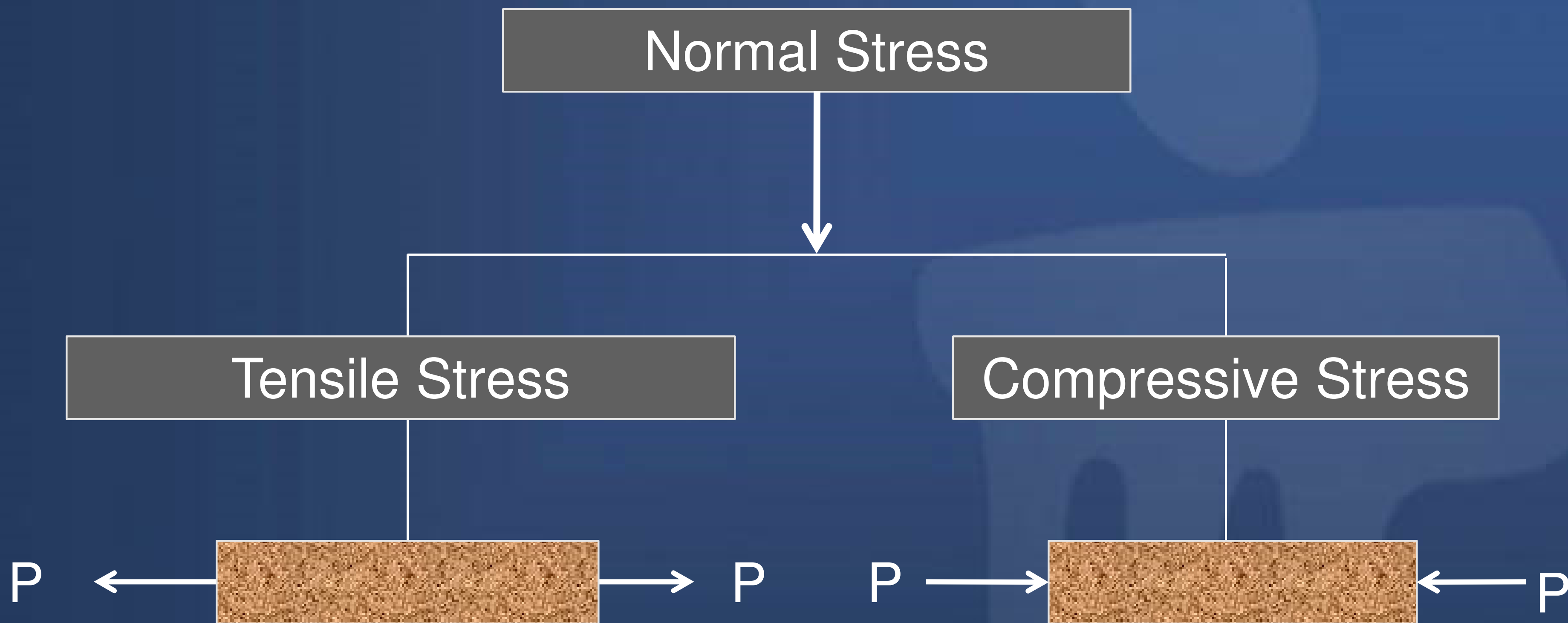
Normal stress



$$\text{Stress} = \frac{\text{Internal resisting force}}{\text{Resisting cross sectional area}} = \frac{R}{A}$$

The body resists the deformation by developing stresses.



Normal Stress
Tensile Stress
Compressive Stress
P
P
P
P

Units:

SI unit for stress is Pascal (Pa)

Pa= N/m^2

	N/m^2	N/mm^2
1kPa	10^3	10^{-3}
1MPa	10^6	1
1GPa	10^9	10^3

Kilopascal, 1kPa= 1000 N/m^2

Megapascal , 1MPa = $1 \times 10^6\ N/m^2$

$= 1 \times 10^6\ N/(10^6\ mm^2) = 1 N/mm^2$

1MPa = 1 N/mm^2

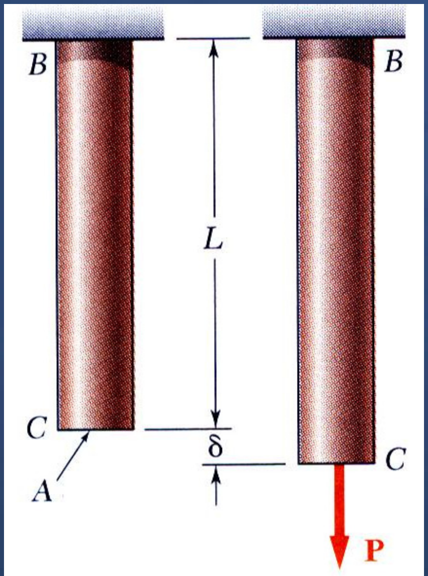
Gigapascal, 1GPa = $1 \times 10^9\ N/m^2$

$= 1 \times 10^3$ MPa

$= 1 \times 10^3\ N/mm^2$

STRAIN

$$\varepsilon = \frac{\delta L}{L} = \frac{\text{Change in the length}}{\text{Original length}}$$



$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{Linear strain}$$

Hooks law and Modulus of elasticity

Hooks law:

$$\frac{Stress(\sigma)}{Strain(\varepsilon)} = \text{constant}$$

Modulus of elasticity:

$$\frac{Stress(\sigma)}{Strain(\varepsilon)} = \frac{\text{PL}}{\text{Adl}}$$

The following table shows modulus of elasticity of important materials:

Material	Modulus of elasticity
Steel	210 GPa
Aluminium	73Gpa
Brass	96 – 110 GPa
Cast Iron	83 – 170 GPa
Concrete	17 – 31 GPa
Rubber	0.0007 – 0.004 GPa
Tungsten	340 – 380 GPa

Tension test on ductile and brittle material

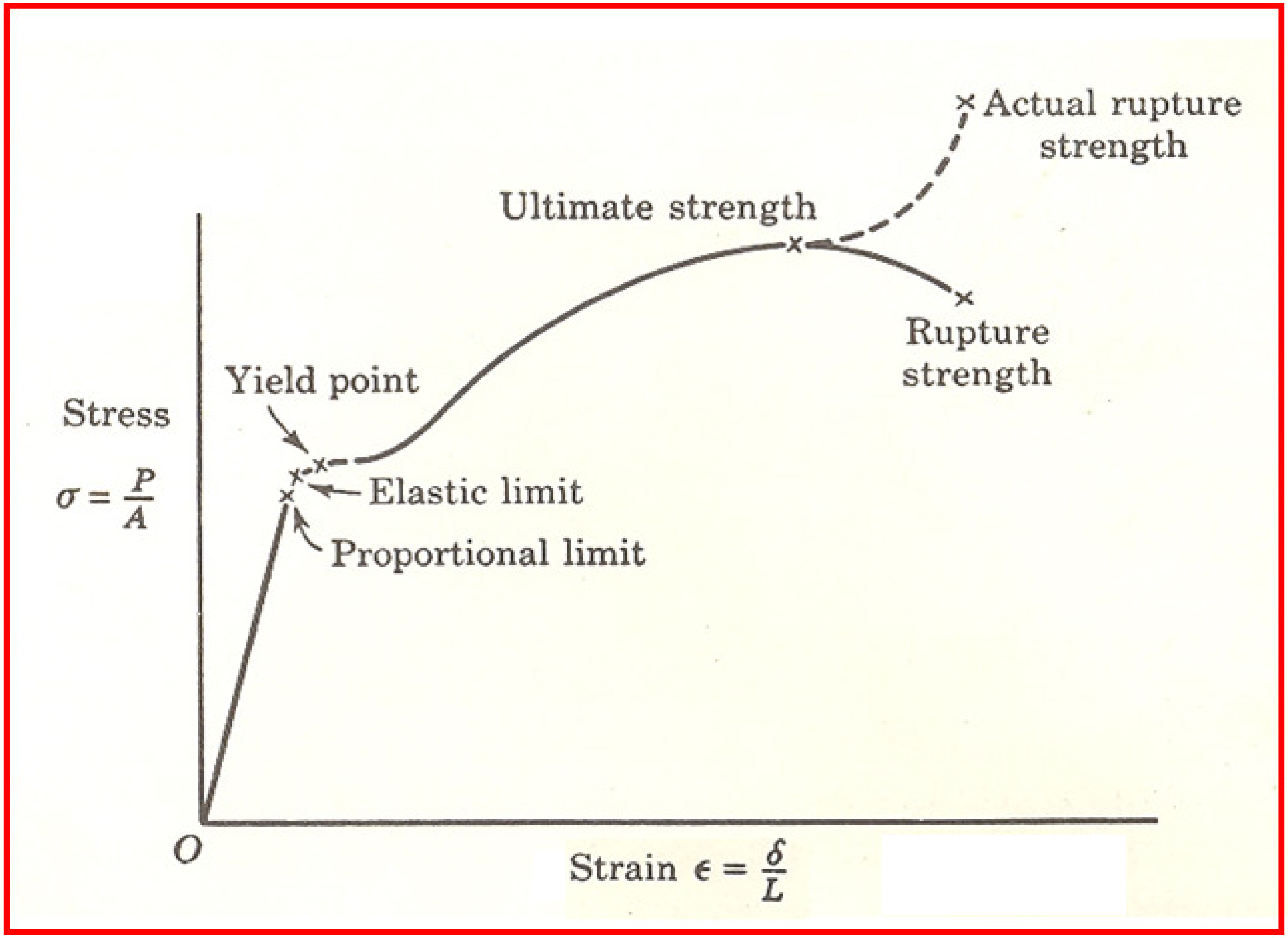
Factor of safety

Allowable stress

HOME

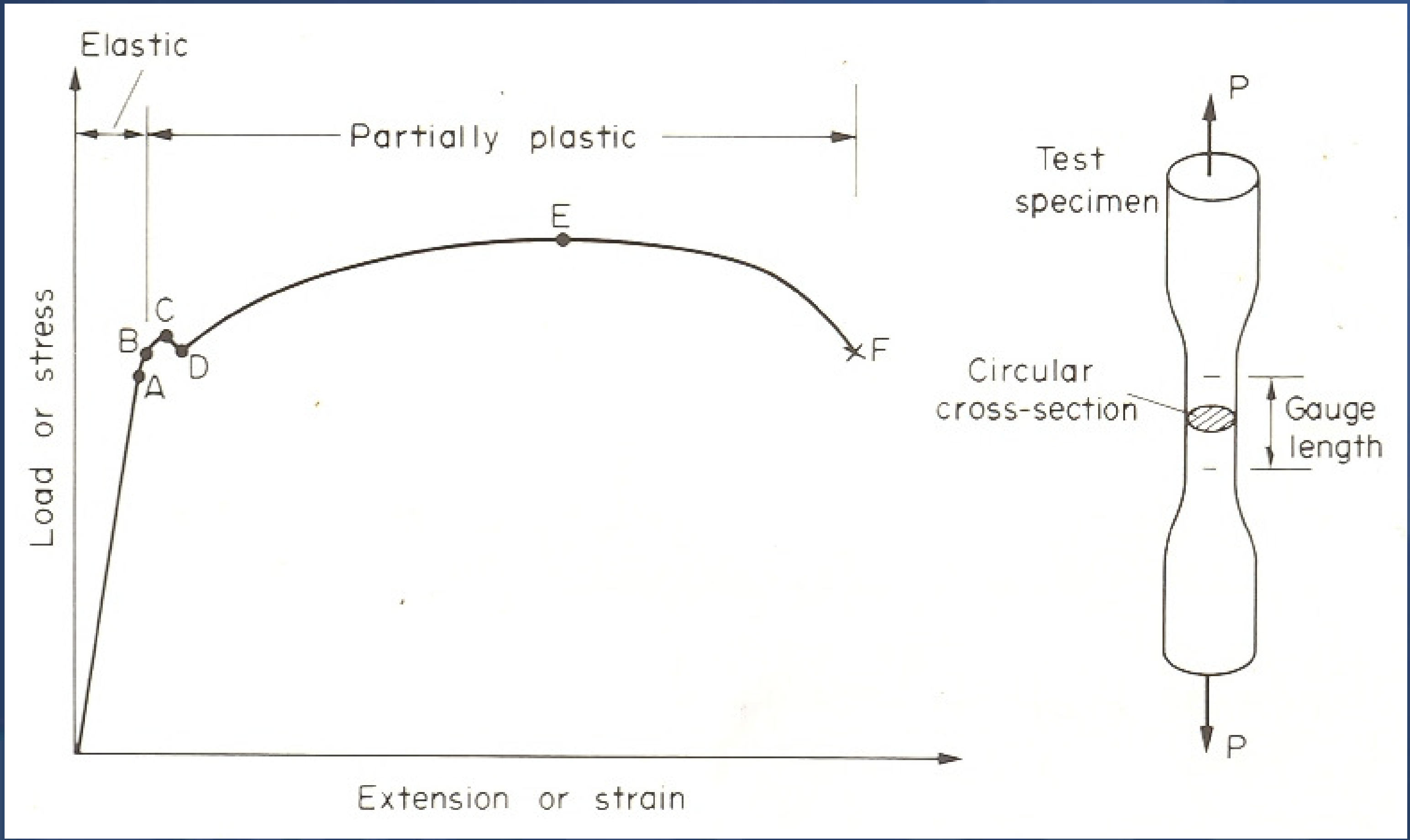
Tension test on ductile and brittle material

STRESS-STRAIN DIAGRAM



Typical tensile test curve for mild steel

STRESS-STRAIN DIAGRAM



Typical tensile test curve for mild steel showing upper yield point and lower yield point and also the elastic range and plastic range

Limit of Proportionality: $\sigma_P = \dfrac{\text{Load at proportionality limit}}{\text{Original crosssectional area}} = \dfrac{P_P}{A}$

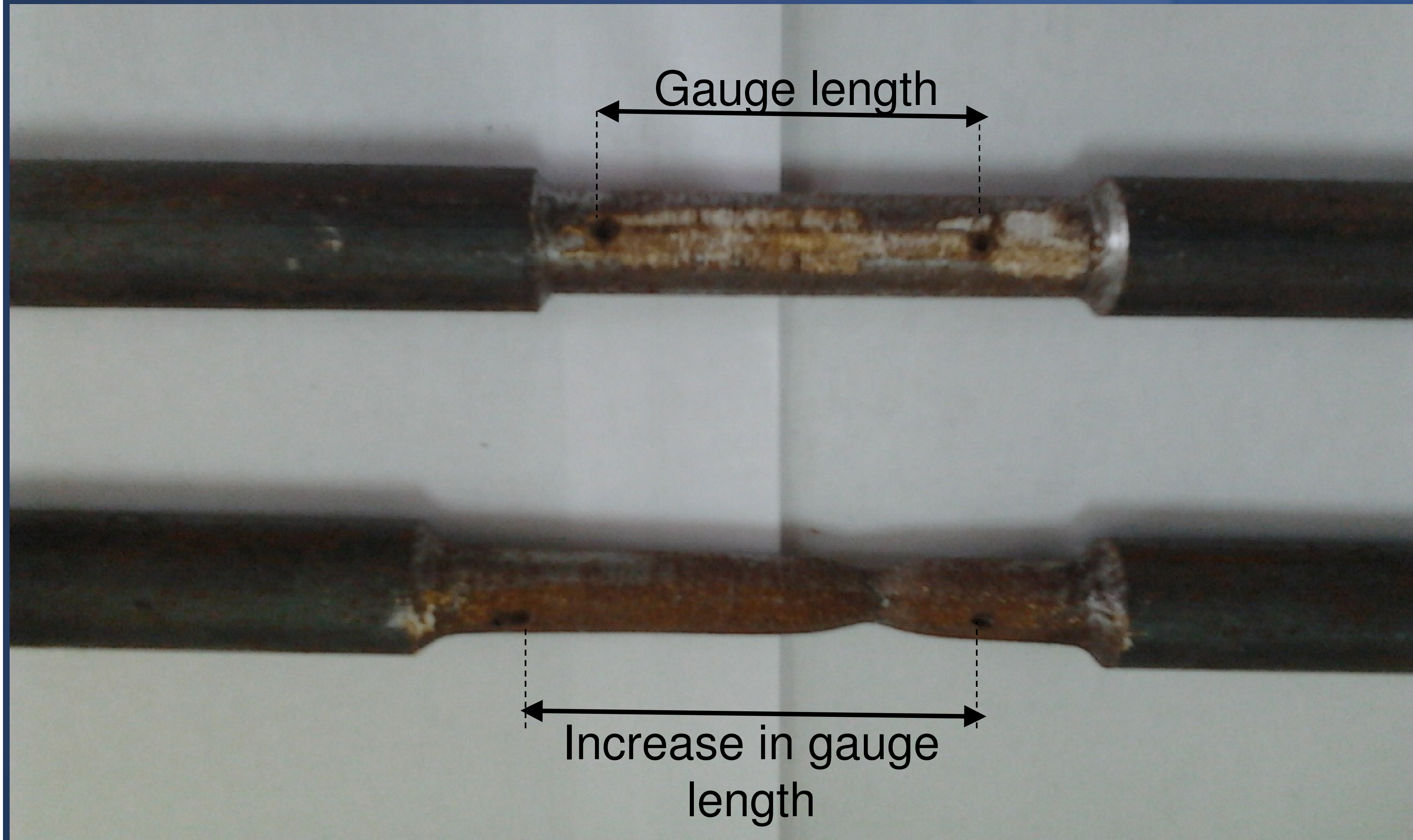
Elastic limit: $\sigma_E = \dfrac{\text{Load at elatic limit}}{\text{Original cross sectional area}} = \dfrac{P_E}{A}$

Yield point: $\sigma_Y = \dfrac{\text{Load at yield point}}{\text{Original cross sectional area}} = \dfrac{P_Y}{A}$

Ultimate strength: $\sigma_U = \dfrac{\text{Maximum load taken by the material}}{\text{Original cross sectional area}} = \dfrac{P_U}{A}$

Rupture strength (Nominal Breaking stress): $\sigma_B = \dfrac{\text{Load at failure}}{\text{Original cross sectional area}} = \dfrac{P_B}{A}$

True breaking stress: $\sigma_B = \dfrac{\text{Load at failure}}{\text{Actual cross sectional area}} = \dfrac{P_B}{A}$


Gauge length
Increase in gauge length

Ductile Materials

Percentage elongation

Percentage reduction in area

Measures of ductility

Cup and cone fracture for a Ductile Material ►

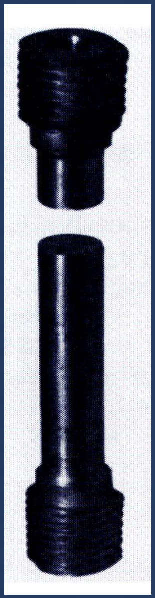
$$\text{Percentage elongation} = \frac{\text{Increase in the gauge length (upto fracture)}}{\text{Original gauge length}} \text{x}100$$

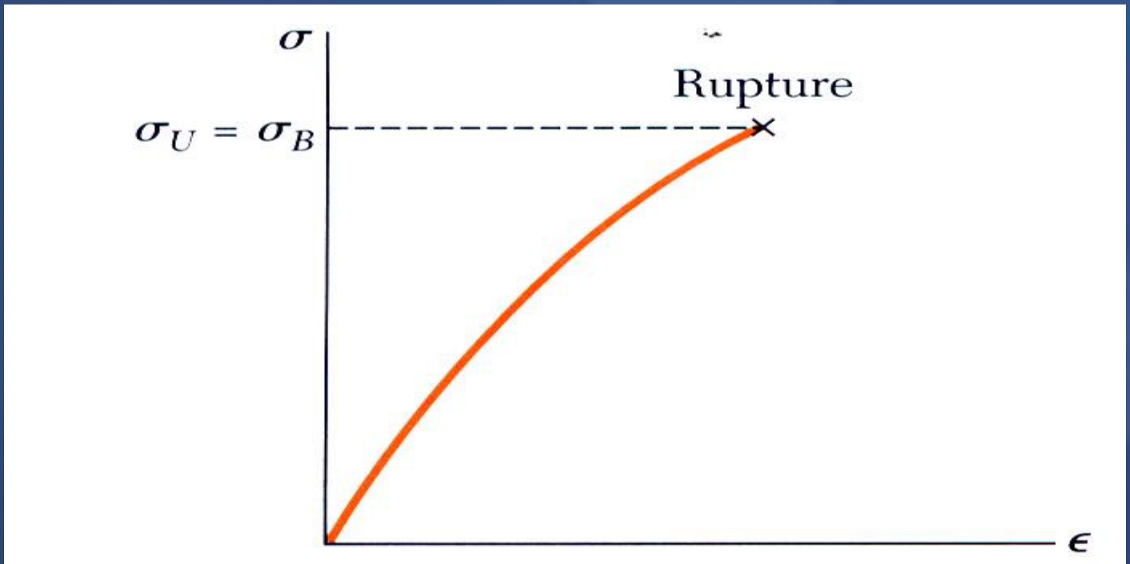
$$\text{Percentage reduction in area} = \frac{\text{Reduction in cross sectional area of neck portion (at fracture)}}{\text{Original cross sectional area}} \text{x}100$$

Example: Low carbon steel, mild steel, gold, silver, aluminum

Stress-strain Diagram

Brittle Materials :





Stress-strain diagram for a typical brittle material

Working stress & Factor of safety

Ductile Material:

Working stress = Yield Stress / Factor of Safety

Brittle Material:

Working stress = Ultimate Stress / Factor of Safety

Factor of Safety = Maximum stress / Allowable working stress

LECTURE 16

Contents:

Numerical problems

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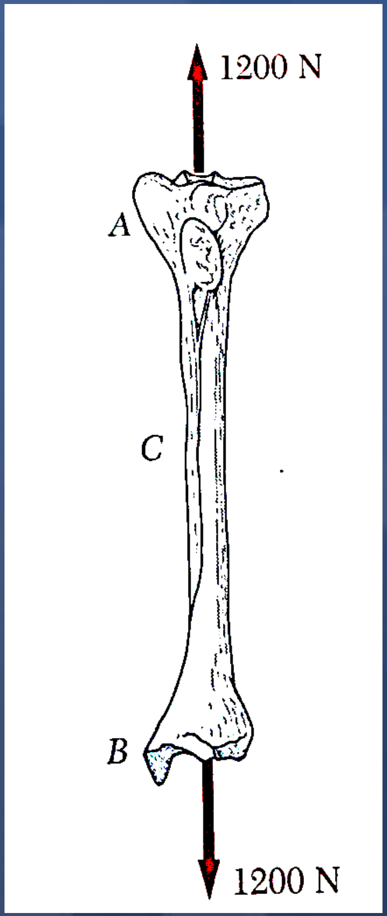
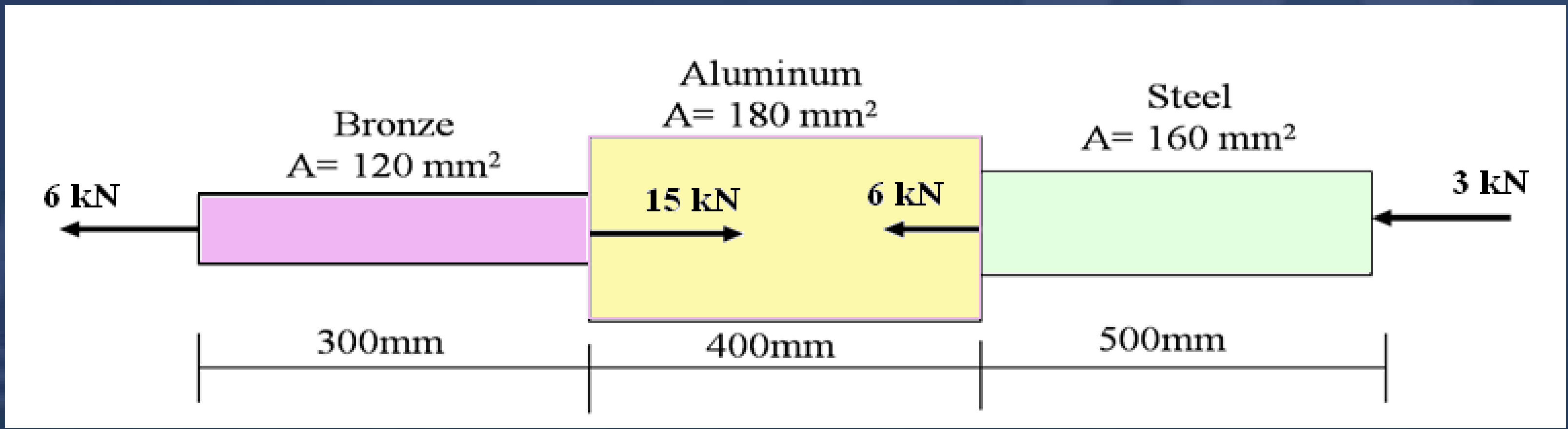
N1. A strain gauge located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200 N forces as shown. Assuming the cross section of the bone at C to be annular and Knowing that its outer diameter is 25mm, determine the inner diameter of the bones cross section at C.



N2. A composite bar consists of an aluminum section rigidly fastened between a bronze section and a steel section as shown in figure. Axial loads are applied at the positions indicated. Determine the stress in each section. Also determine the change in each section and the change in total length.

Given E_b = 100 GPa, E_a = 70 GPa, E_s = 200 GPa



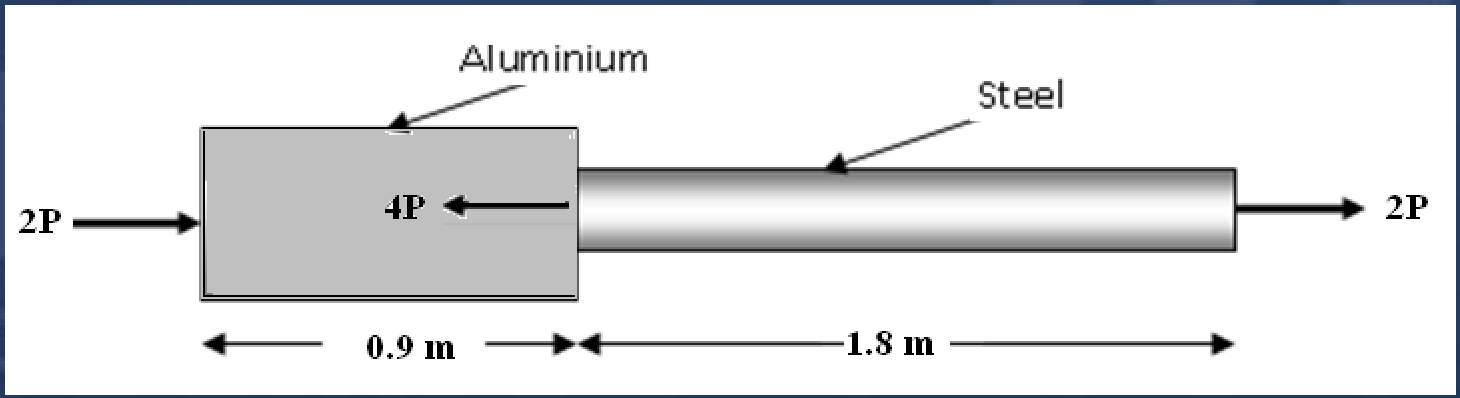
N3. An aluminum rod is fastened to a steel rod as shown. Axial loads are applied at the positions shown. The area of cross section of aluminum and steel rods are 400 mm^2 and 200mm^2 respectively. Find maximum value of P that will satisfy the following conditions.

a) $\sigma_s \leq 140$ MPa

b) $\sigma_a \leq 80$ MPa

c) Total elongation ≤ 0.5 mm,

Take $E_a = 70$ GPa and $E_s = 210$ GPa



N4. A member ABCD is subjected to point loads P1, P2,P3 and P4 as shown in figure below.

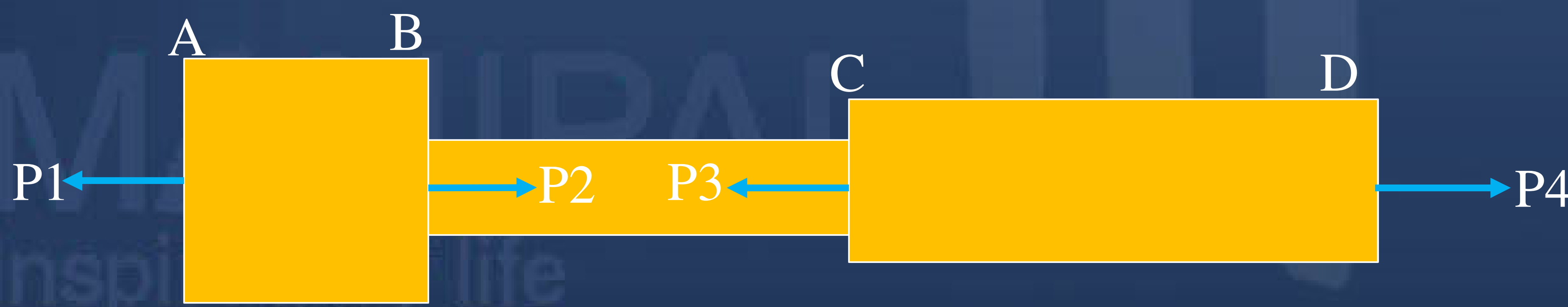
Calculate the force P3 necessary for equilibrium if P1 = 120 kN, P2 = 220 kN and P4 = 160 kN.

Determine the net change in the length of the member. Take $E = 200\ GN/m^2$.

Given: area and length of AB: $1600\ mm^2$, 0.75 m ;

area and length of BC: $625\ mm^2$, 1.0 m;

area and length of CD: $900 mm^2$, 1.2 m.



LECTURE 17

Contents:

Expression for deformation of a tapered bar

Expression for deformation of a tapered flat

Application problems

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N5. Find the modulus of elasticity of the material of a tapering bar from the following data: The bar has 20 mm diameter at one end, 40 mm diameter at the other, length 1.0 m and axial load of 10 kN. The elongation observed was 0.1 mm.

N6. A tapered bar of rectangular cross section is 20 mm wide at one end and 40 mm wide at the other, 8 mm thick and 800 mm long. The elongation of 0.08 mm was observed under load P. find the load P, if the modulus of elasticity of the material of the bar is 100 GPa.

N7. A uniform steel rod of diameter 20 mm is connected to an aluminium rod of diameter 60 mm at one end. The aluminium rod tapers to a diameter of 20 mm at the other end. The steel rod is 0.6 m long and is connected rigidly to 60 mm diameter end of the aluminium rod which is 0.4 m long. If E = 200 GPa for steel and 70 GPa for aluminium, find the total extension under an axial load of 30 kN.

LECTURE 18

Contents:

Shear stress

Shear strain

Modulus of rigidity

State of simple shear & Complementary shear

Direct stress due to pure shear

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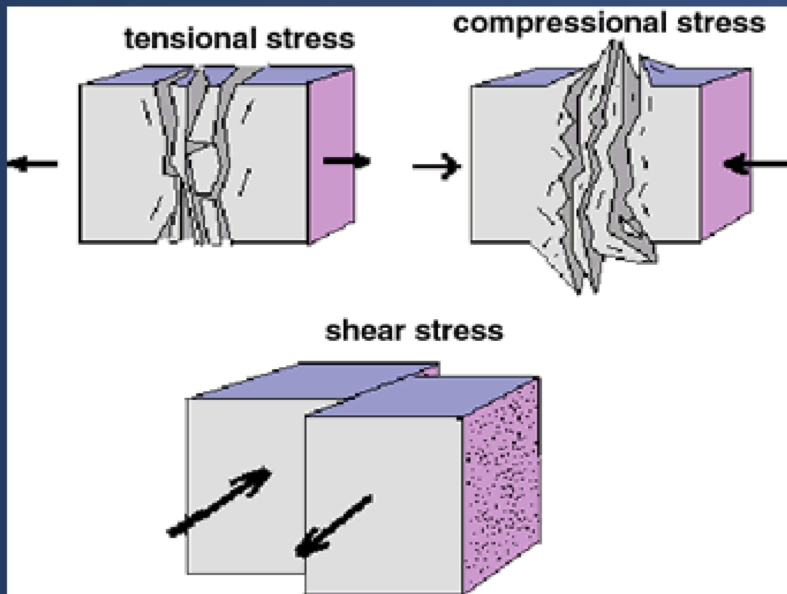
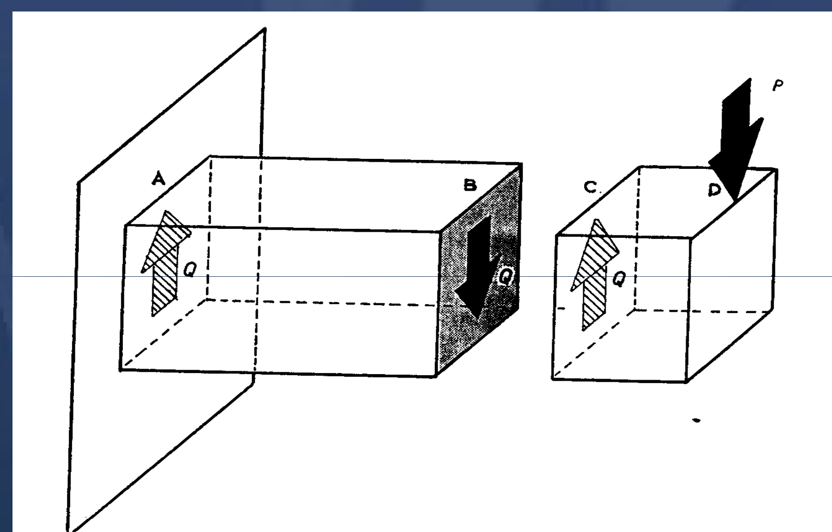
SHEAR STRESS





SHEAR STRESS

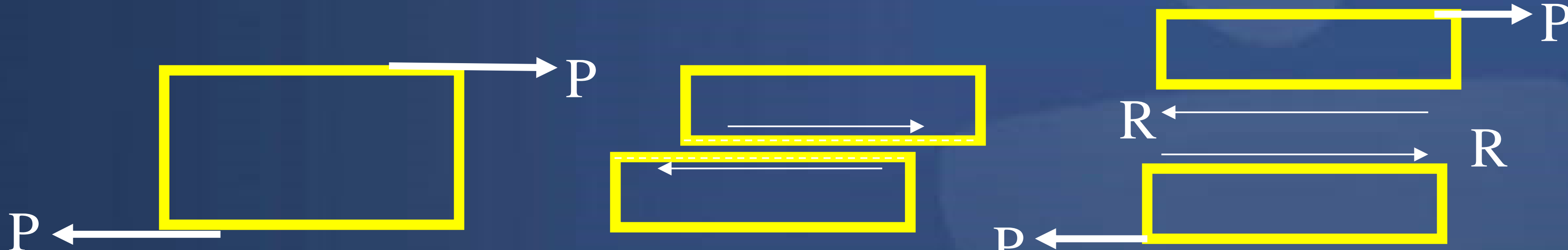


Fig. a Fig. b Fig. c

$$\text{Shear stress}(\tau) = \frac{\text{Shear resistance}}{\text{Area resisting shear}} = \frac{R}{A} = \frac{P}{A}$$

This shear stress will always be tangential to the area on which it acts

SHEAR STRAIN

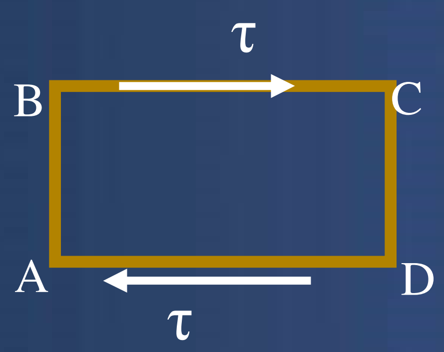


Fig. d

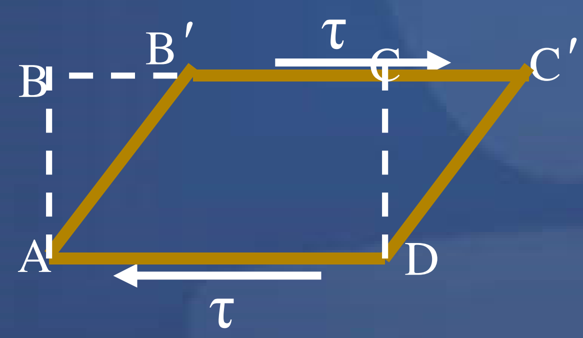
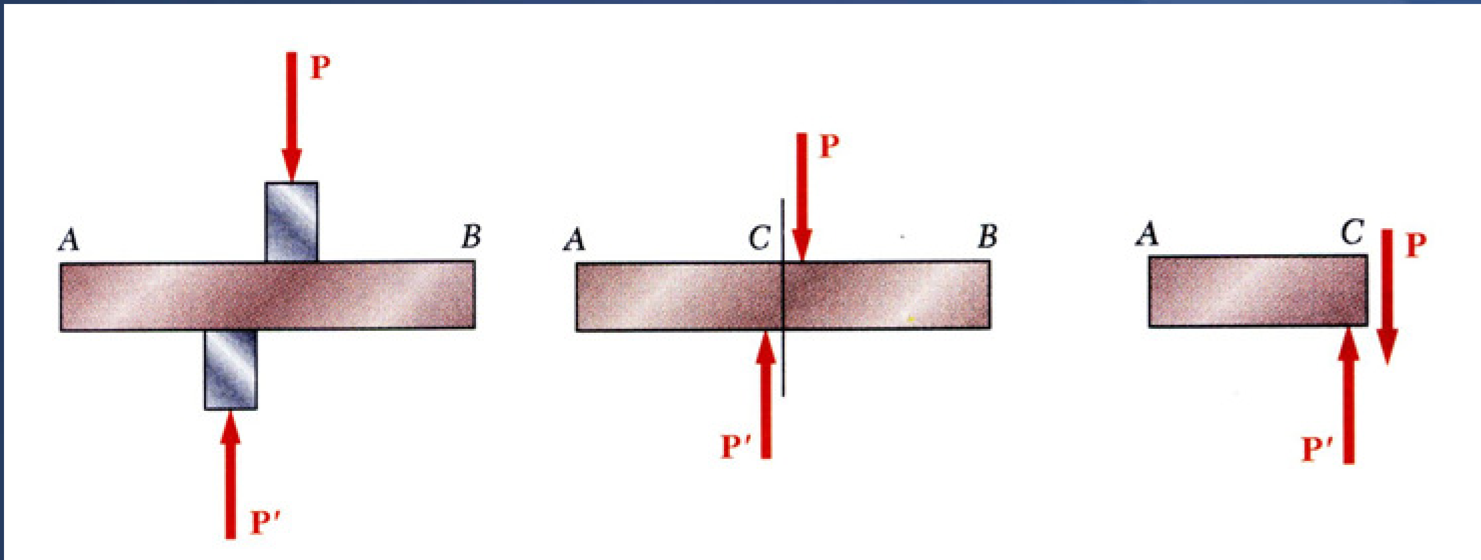


Fig. e

$$\text{shear strain} = \frac{BB'}{AB} = \tan\phi \approx \phi$$

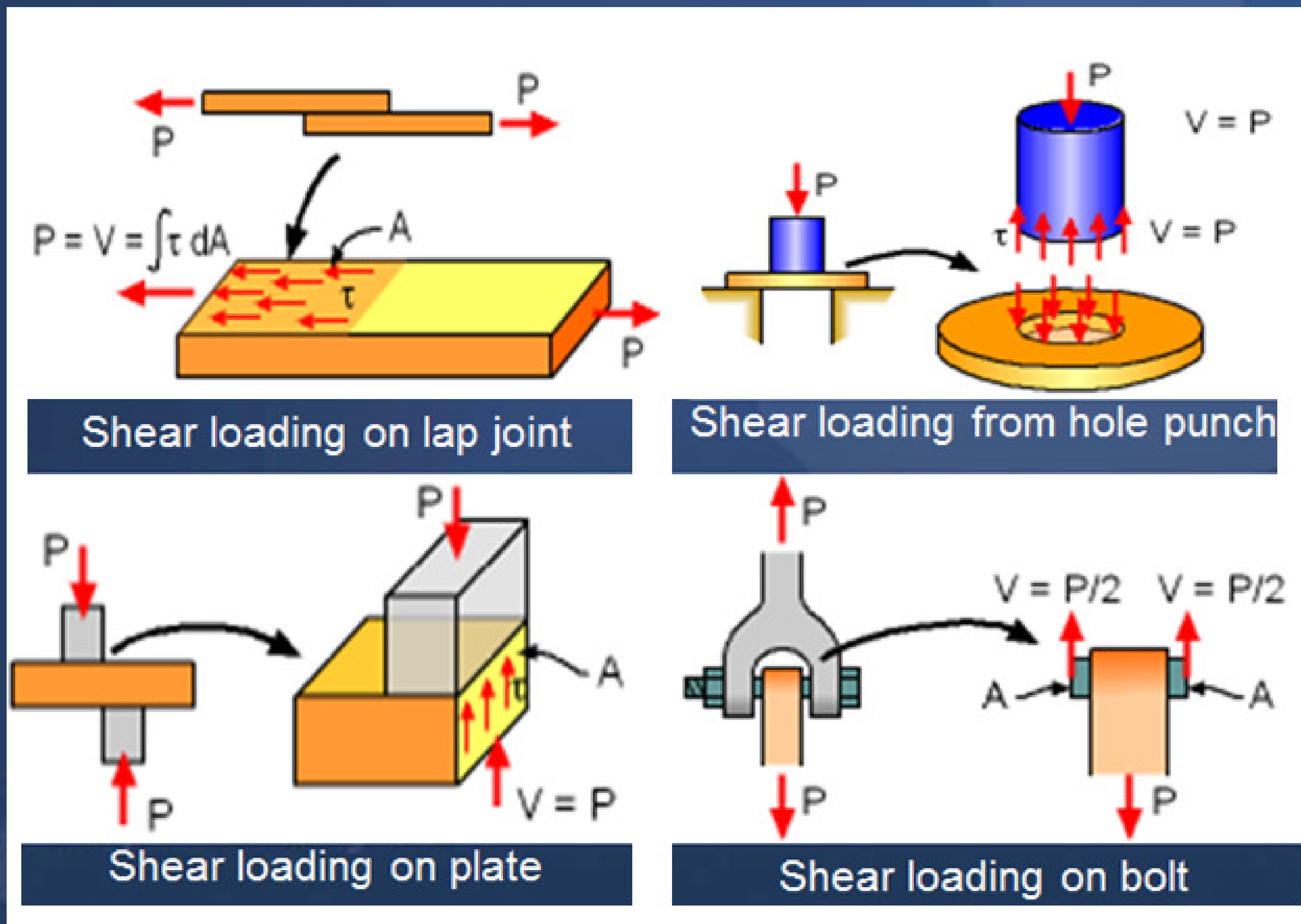
Shear modulus:

$$\frac{\text{Shear stress } (\tau)}{\text{Shear strain } (\phi)} = \text{constant} = \text{G} = \text{Shear Modulus or Modulus of Rigidity}$$

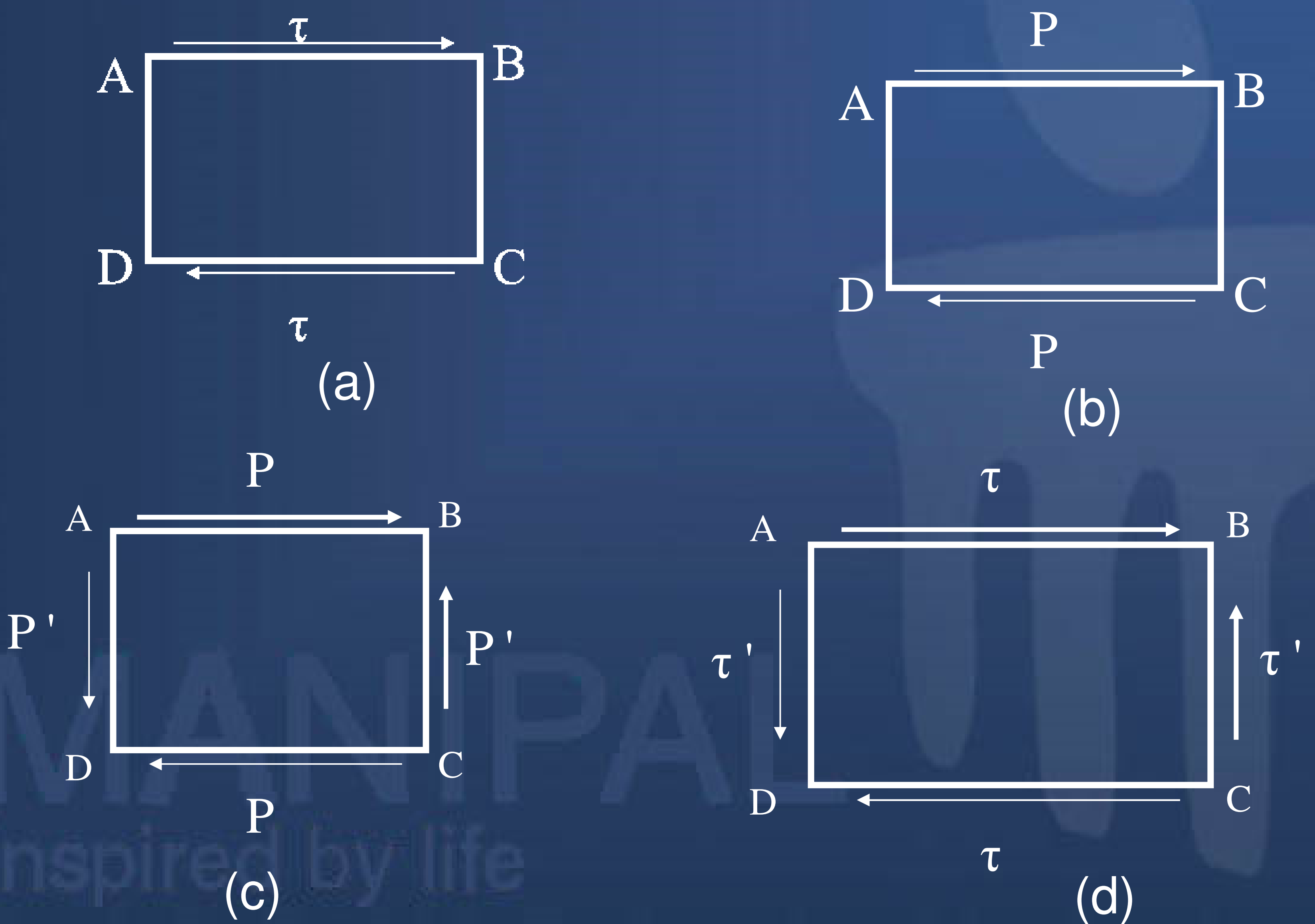


$$\tau_{ave} = \frac{P}{A}$$

Examples of Shear



State of simple shear



Direct stress due to pure shear

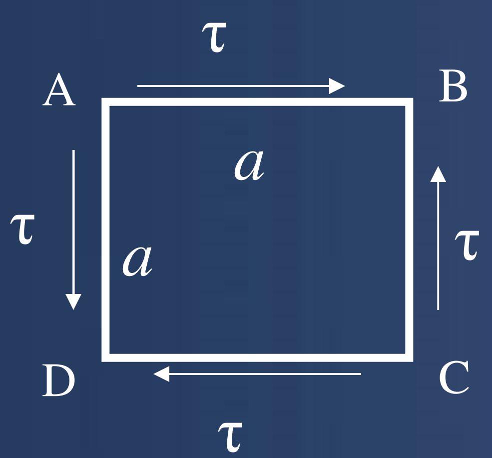


Fig.(a).

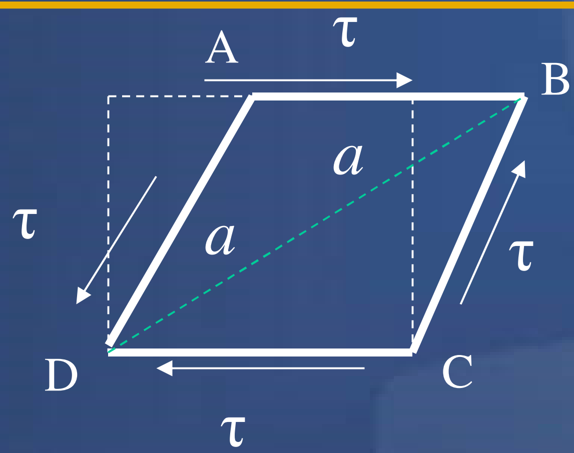


Fig.(b).

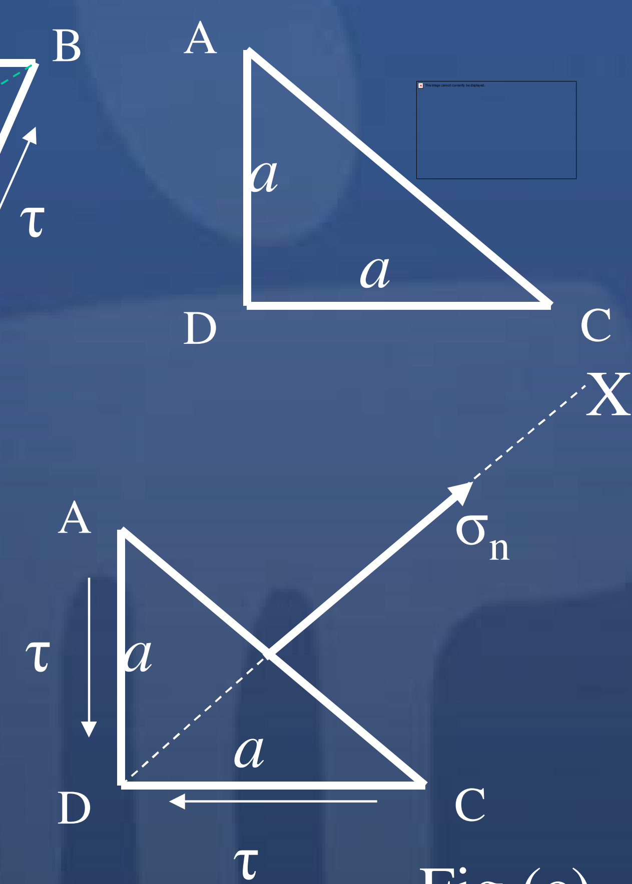


Fig.(c).

$$\Sigma F_x = 0$$

$$= \sigma_n \times \left(\sqrt{2} \times a \times 1\right) - 2 \times (\tau \times a \times \cos 45)$$

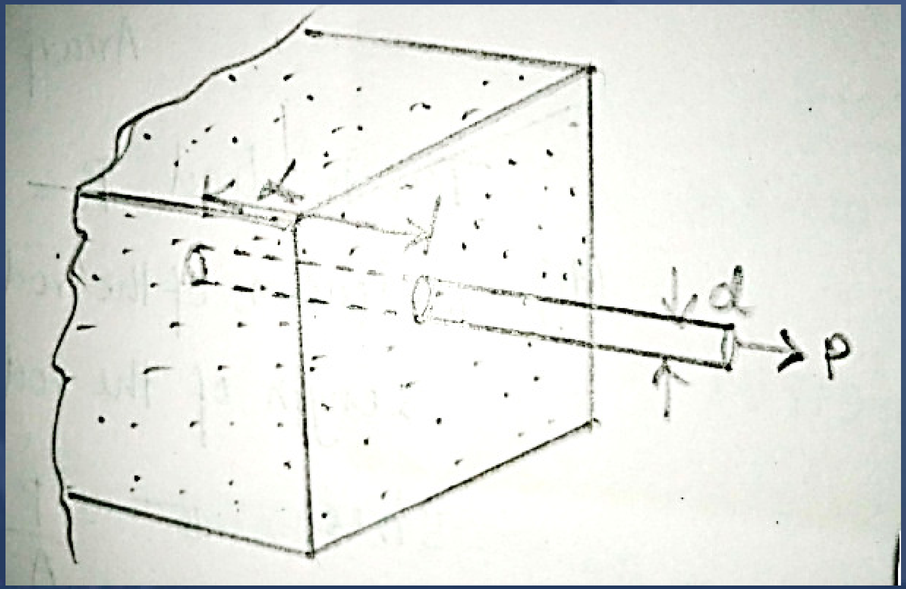
For equilibrium, $\sigma_n = \tau$

Direct stress due to pure shear

Therefore the intensity of normal tensile stress developed on plane BD is numerically equal to the intensity of shear stress.

Similarly it can be proved that the intensity of compressive stress developed on plane AC is numerically equal to the intensity of shear stress.

N8. To check the bond strength between reinforcing bars and concrete, a tensile force of P=30 kN is applied to the end of the bar of diameter d=12 mm and length L= 100 mm. Calculate the average shear stress developed between steel and concrete.

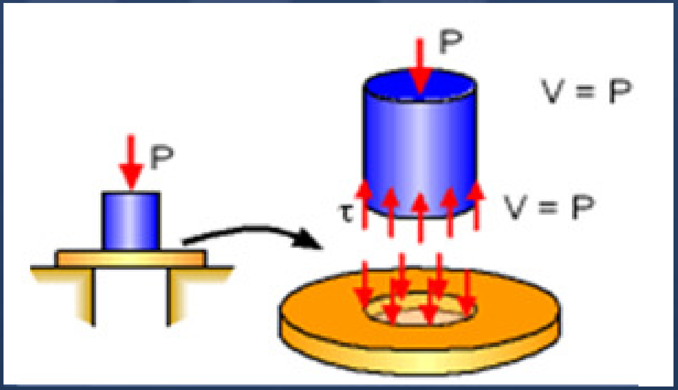


N9. A hole is to be punched out of a plate having an ultimate shear stress of 300 MPa. If the compressive stress in the punch is limited to 400 MPa,

determine:

(a) Maximum thickness of the plate for which a 100 mm dia hole can be punched.

(b) If the plate is 10mm thick, smallest diameter hole that can be punched.



Ans:
t=33.33 mm
d=30 mm

LECTURE 19

Contents:

Poisson’s ratio
Volumetric strain
Bulk modulus
Relationship between volumetric strain and linear strain

HOME

POISSON'S RATIO

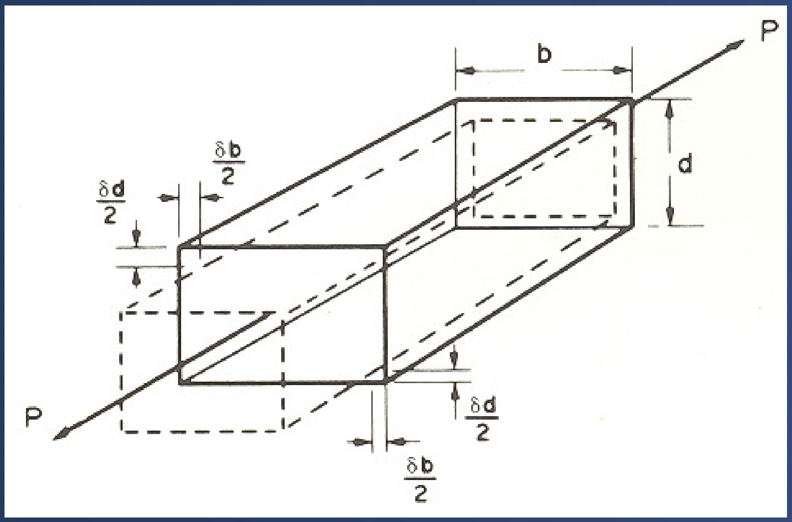


Fig.(a)

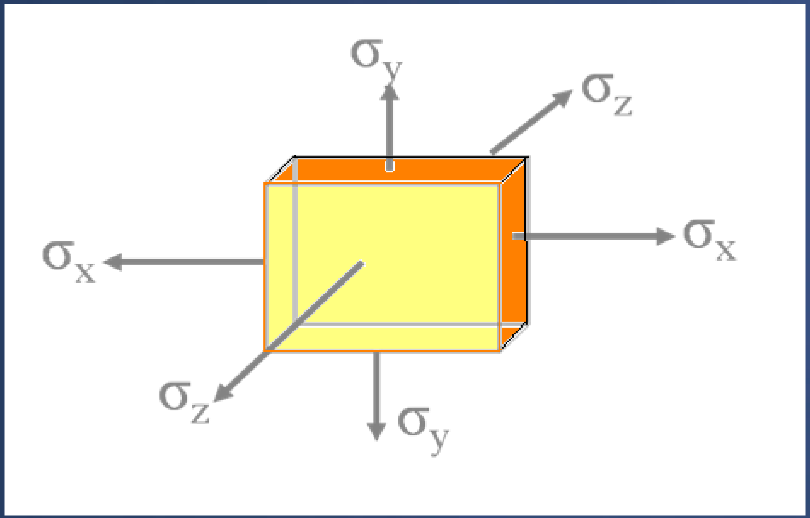
$$\varepsilon_{lat} = -\frac{\delta b}{b} = -\frac{\delta d}{d}$$

$$\epsilon_l = \frac{\delta l}{l}$$

Poisson's ratio = $\dfrac{\textit{Lateral Strain}}{\textit{Longitudinal Strain}}$

$$= \frac{\left(-\frac{\delta b}{b}\right)}{\frac{\delta l}{l}} \quad \text{or} \quad \frac{\left(-\frac{\delta d}{d}\right)}{\frac{\delta l}{l}}$$

General case:



Strain in X-direction = $$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

Strain in Y-direction = $$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

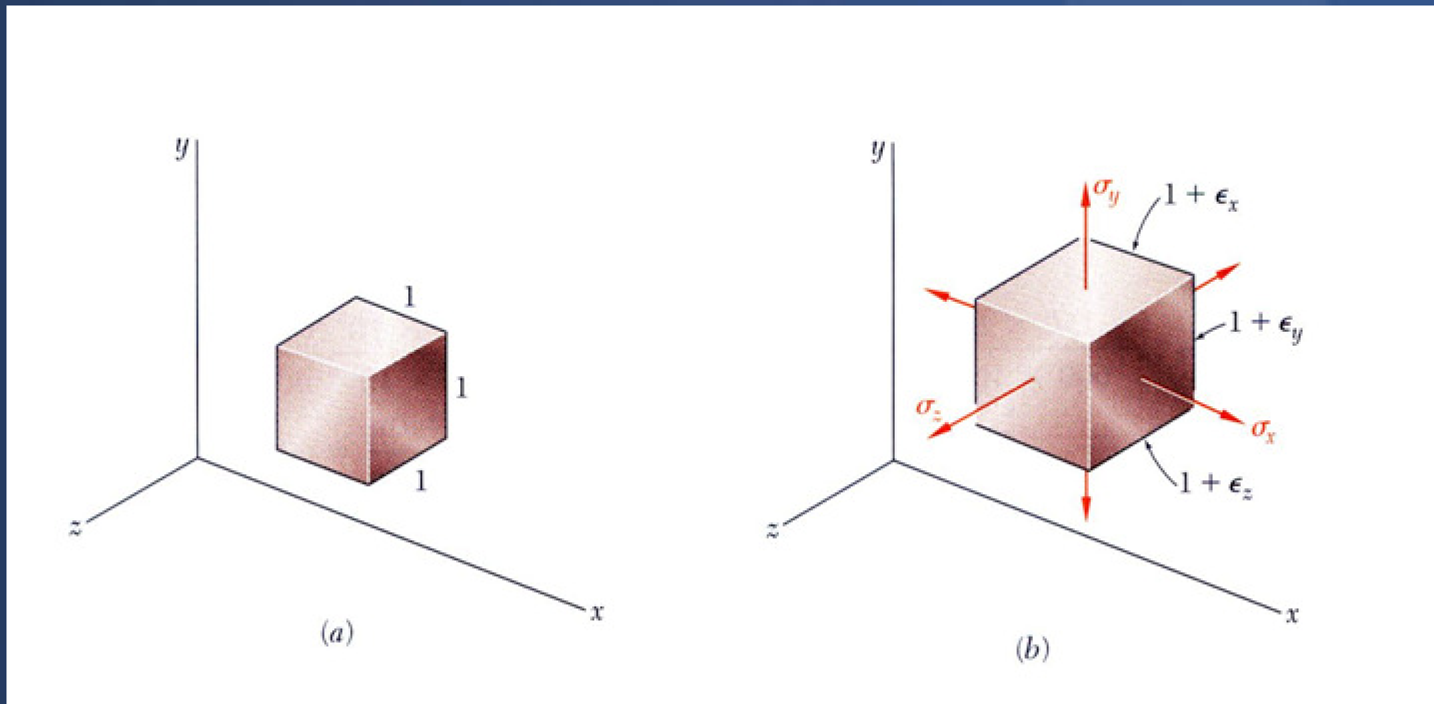
Strain in Z-direction = $$\varepsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$$

Bulk Modulus

Bulk modulus, $K = \dfrac{\sigma}{\left(\dfrac{dV}{V}\right)}$

A body subjected to three mutually perpendicular equal direct stresses then the ratio of stress to volumetric strain is called Bulk Modulus.

Relationship between volumetric strain and linear strain



$$\frac{dV}{1} = \left[(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)\right] - 1 = \left[1+\varepsilon_x+\varepsilon_y+\varepsilon_z\right] - 1$$

$$= \varepsilon_x + \varepsilon_y + \varepsilon_z$$

= change in volume per unit volume

Relationship between volumetric strain and linear strain

Volumetric Strain

$$\frac{dV}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \left(\frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} \right) + \left(\frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} \right) + \left(\frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} \right)$$

$$= \frac{1-2\mu}{E} \left(\sigma_x + \sigma_y + \sigma_z \right)$$

For element subjected to uniform hydrostatic pressure,

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

$$\frac{dV}{V} = \frac{1-2\mu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\frac{dV}{V} = \frac{1-2\mu}{E}(3\sigma)$$

$$K = \frac{\sigma}{\left(\frac{dV}{V}\right)}$$

$$E = 3K(1-2\mu)$$

N10. A bar of metal 100x50 mm in cross section is 250 mm long. It carries a tensile load of 400 kN in the direction of its length, a compressive load of 4000 kN on its 100 mm x 250 mm faces and a tensile load of 2000 kN on its 50 mm x 250 mm faces. If $E=2x10^5$ N/mm^2 and poisson's ratio is 0.25, find the change in volume of the bar.

What change must be made in the 4000 kN load in order that there shall be no change in volume of the bar.

N11. A bar of steel 40 mm x 40 mm cross section and 150 mm long is subjected to a tensile load of 200 kN along its longitudinal axis and tensile load of 600 kN and 400 kN along lateral axis.

Find,

(a) Change in each dimension and change in volume

(b) What longitudinal force alone can produce same longitudinal strain as in case (a).

Given E= 200 GPa μ=0.3

LECTURE 20

Contents:

Relationship between modulus of elasticity and modulus of rigidity

Relationship between E, G and K

Application problems

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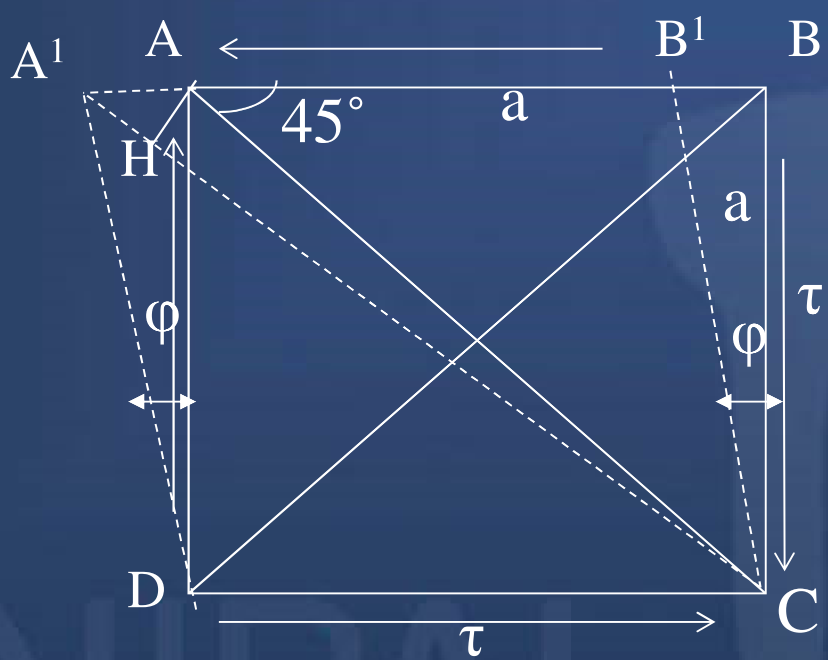
Relationship between young's modulus of elasticity (E) and modulus of rigidity (G) :-



Consider a square element ABCD of side ‘a’ subjected to pure shear‘τ’. DA'B'C is the deformed shape due to shear τ.

Drop a perpendicular AH to diagonal A'C.

Strain in the diagonal AC = $\tau /E - \mu (- \tau /E)$ [$\sigma_n = \tau$]

$= \tau /E [1 + \mu]$ -----------(1)

Strain along the diagonal AC=(A'C–AC)/AC=(A'C–CH)/AC=A'H/AC

In ∆le AA'H

$\text{Cos } 45^\circ = A'H/AA'$

$A'H = AA' \times 1/\sqrt{2}$

$AC = \sqrt{2} \times AD \qquad (AC = \sqrt{AD^2 + AD^2})$

Strain along the diagonal AC = $AA'/ (\sqrt{2} \times \sqrt{2} \times AD) = \varphi/2$ ----(2)

Modulus of rigidity = $G = \tau/\varphi$

$$\varphi = \tau/G$$

Substituting in (2)

Strain along the diagonal AC = $\tau/2G$ -----------(3)

Equating (1) & (3)

$\tau/2G = \tau/E[1+\mu]$

$$\boxed{E=2G(1+\mu)}$$

Relationship between E, G, and K:-

We have

$E = 2G(1+\mu)$ -----------(1)

$E = 3K(1-2\mu)$ -----------(2)

Equating (1) & (2)

$2G(1+\mu) = 3K(1-2\mu)$

$2G + 2G\mu = 3K - 6K\mu$

$\mu = (3K - 2G)/(2G + 6K)$

Substituting in (1)

$E = 2G[1 + (3K - 2G)/(2G+6K)]$

$E = 18GK/(2G+6K)$

$$\boxed{E = 9GK/(G+3K)}$$

N12. A circular rod of 100 mm dia and 500 mm length is subjected to a tensile force of 2000 kN. Determine the modulus of rigidity, bulk modulus and the change in volume, if the poisson's ratio=0.3 and $E=2\times10^5$ N/mm^2.

Ans:
$G=0.77\times10^5$ N/mm^2
$K=1.67\times10^5$ N/mm^2
$dv=1994.9$ mm^3

N13. The modulus of rigidity of a material is 0.8×10^5 N/mm^2 . When a 6 mm x 6 mm bar of this material is subjected to an axial pull of 3600 N, it was found that the lateral dimension of bar is changed to 5.9991 mm x 5.9991 mm.

Find μ and E.

Ans:
$\mu = 0.32$
$E = 2.105 \times 10^5$ N/mm^2

TUTORIAL 8

Contents:

Tutorial problems

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T1. Find the Young's modulus of a brass rod of diameter 25 mm and of length 250 mm which is subjected to a tensile load of 75 kN when the extension of the rod is equal to 0.3 mm.

T2. The ultimate stress, for a hollow steel column which carries an axial load of 2.0 MN is 480 N/mm^2. If the external diameter of the column is 200 mm, determine the internal diameter. Take factor of safety as 3.

T3. A brass bar having cross section area of 900 mm^2, is subjected to axial forces as shown below. The lengths of portion AB, BC and CD are 0.6 m, 0.8 m and 1.0 m respectively. Determine the total elongation of the bar. Take E = 100 GPa.

T4. A tensile load of 80 kN is acting on a rod of diameter 80 mm and of length 8 m. A bore of diameter 60 mm is made centrally on the rod. To what length the rod should be bored so that the total extension will increase 25% under the same tensile load. Take E = 200 GPa.

TUTORIAL 9

Contents:

Tutorial problems

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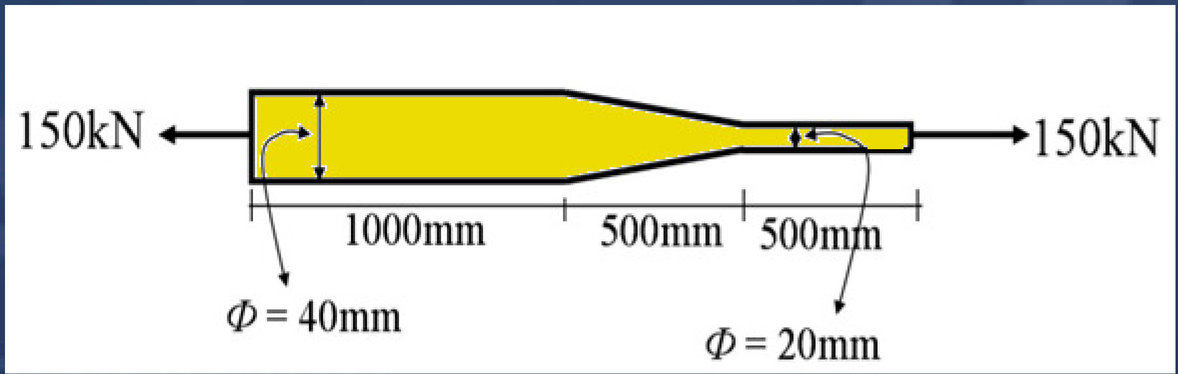
T5. A steel flat of thickness 16 mm tapers uniformly from 80 mm at one end to 50 mm at the other end in a length of 800 mm, If the flat is subjected to a load of 120 kN, find the extension of the flat. Also calculate the percentage error if average area is used for calculating its extension. Take $E=2x10^5$ Mpa

T6. A two meter long steel bar is having uniform diameter of 40 mm for a length of 1 m, in the next 0.5 m its diameter gradually reduces to 20 mm and for remaining 0.5 m length diameter remains 20 mm uniform as shown in the figure. If a load of 150 kN is applied at the ends, find the stresses in each section of the bar and total extension of the bar. Take E = 200 GPa.



T7. A tensile load of 50 kN is acting on rod of 50 mm diameter and length of 5m. Determine the length of a bore of 25mm that can be made central in the rod, if the total extension is not to exceed by 25 percent under the same tensile load. Take $E = 2.05 \times 10^5$ N/mm^2

TUTORIAL 10

Contents:

Tutorial problems

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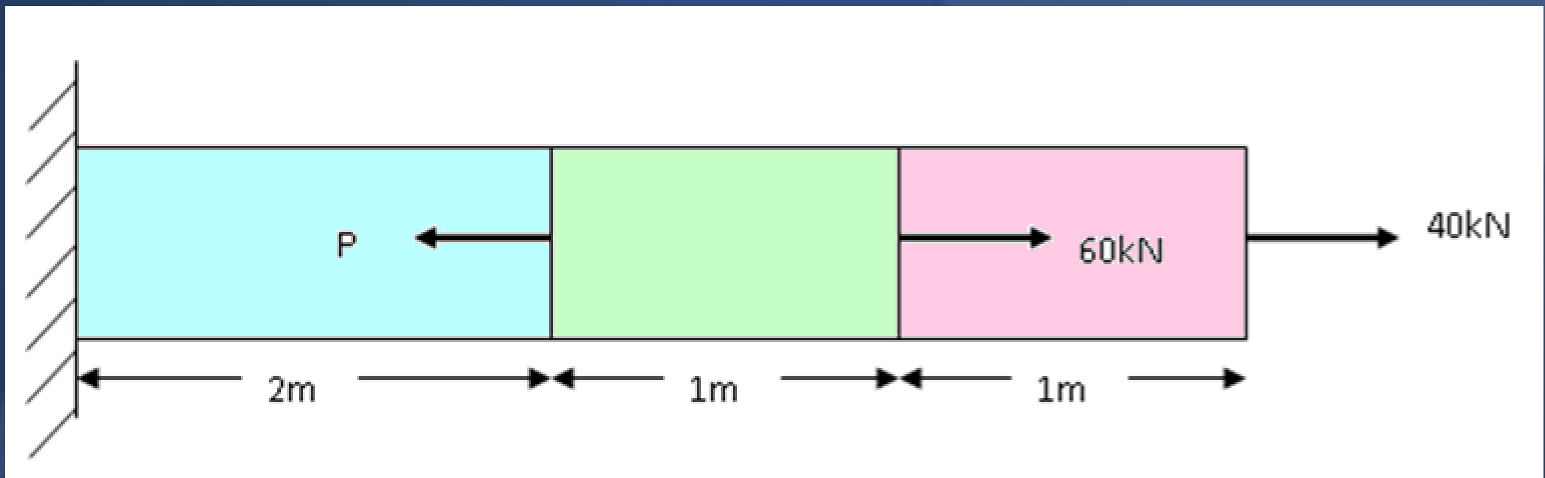
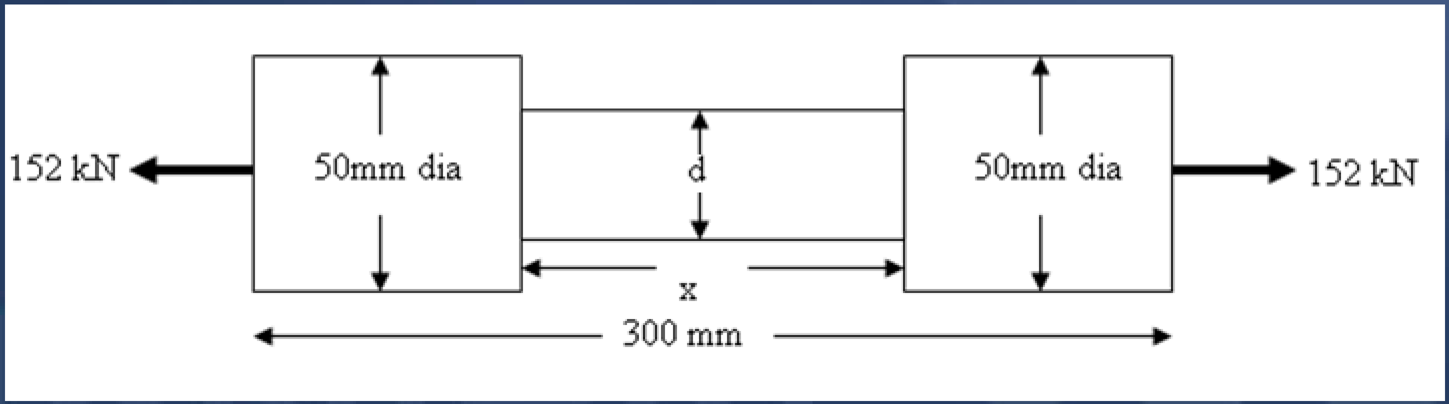
T8. Determine the magnitude of the load P necessary to produce zero net change in the length of the bar shown in the figure below. Take A=400 mm^2.



Ans:
P=170 kN

T9. For the bar shown below, determine diameter of the central portion and its length, if the total extension of the bar is 0.16 mm. Take E=200 GPa. Stress at central portion is limited to 140 N/mm^2



Ans:
d=37.18mm
x=140.23mm

HOME

T10. A tension test is carried out subjected on a mild steel tube of external diameter 18 mm and internal diameter 12 mm. An an axial load of 2 kN produces an extension of 3.36 x 10^{-3} mm on a length of 50 mm and a lateral contraction of 3.62 x 10^{-4} mm of outer diameter.

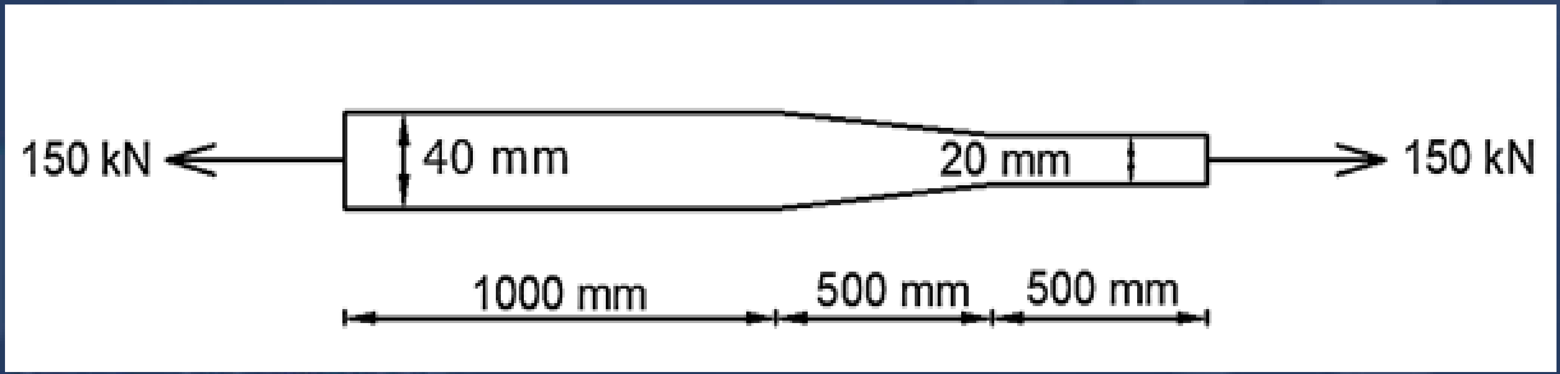
Determine E, μ,G and K.

Ans:
$E=2.11x10^{5}$ N/mm^{2}
μ=0.3
$G=81.15x10^{3}$ N/mm^{2}
$K=175.42x10^{3}$ N/mm^{2}

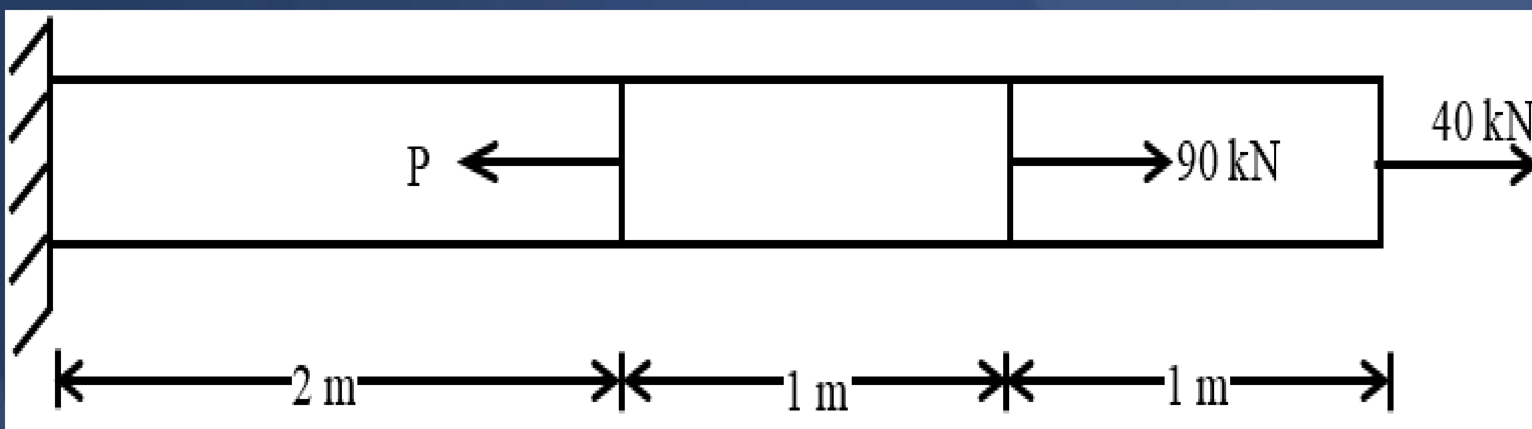
ADDITIONAL TUTORIAL PROBLEMS

HOME

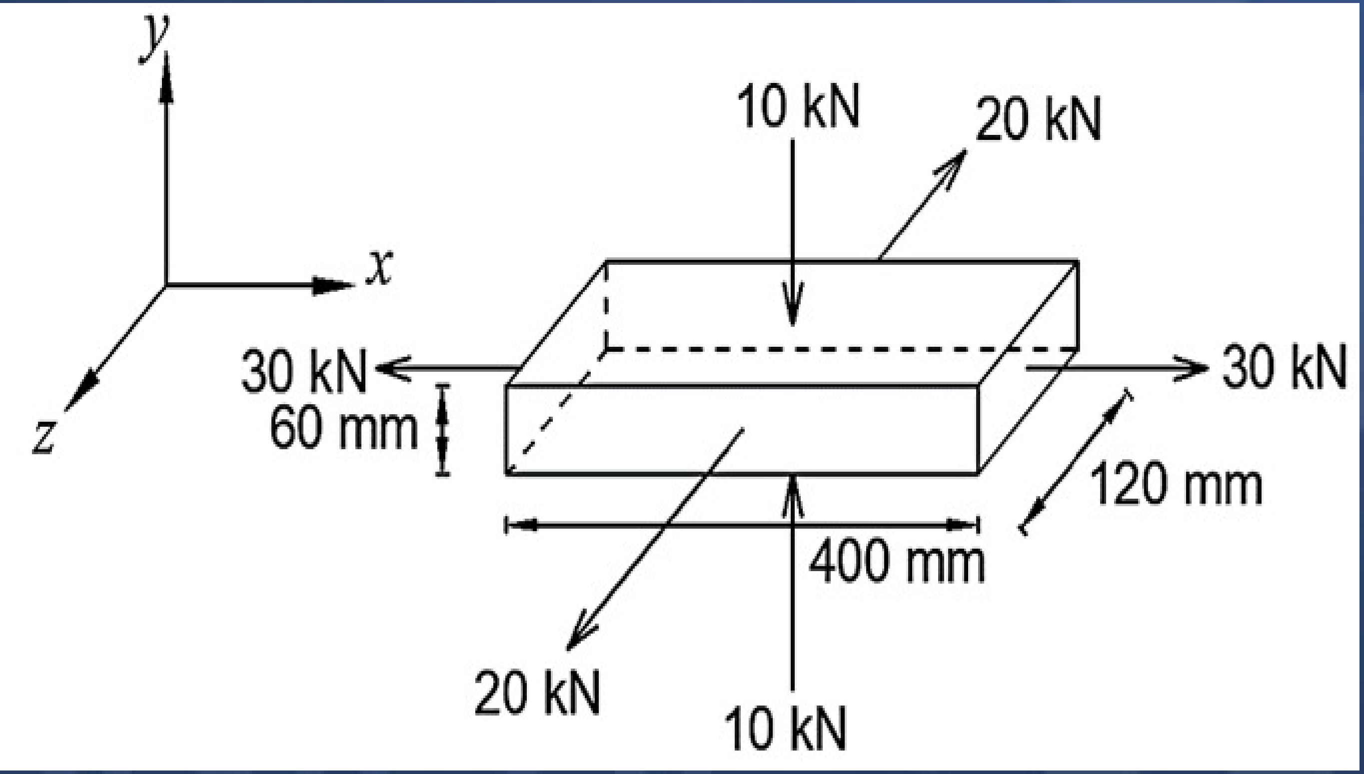
AT1. A two meter long steel bar is having uniform diameter of 40 mm for a length of 1 m, in the next 0.5 m its diameter gradually reduces to 20 mm and for remaining 0.5 m length diameter remains 20 mm uniform as shown in the figure. If a load of 150 kN is applied at the ends, find the stress in each section and total extension of the bar. Take E = 200 Gpa



AT2. Determine the magnitude of the load P necessary to produce zero net change in the length of the bar shown in the figure below. Take A=400 mm^2.



AT3. A steel bar of 400 mm x 120 mm x 60 mm is subjected to forces as shown in the figure. Find the change in dimension. Taking E = 200 GPa and μ = 0.25.



AT4. A circular concrete pillar consists of six steel rods of total area 2280 mm^2. Determine the area of concrete required when it has to carry a load of 1000 kN. Take allowable stresses for steel & concrete as 140 MPa & 8 MPa respectively. Take $E_s = 15\ E_c$.