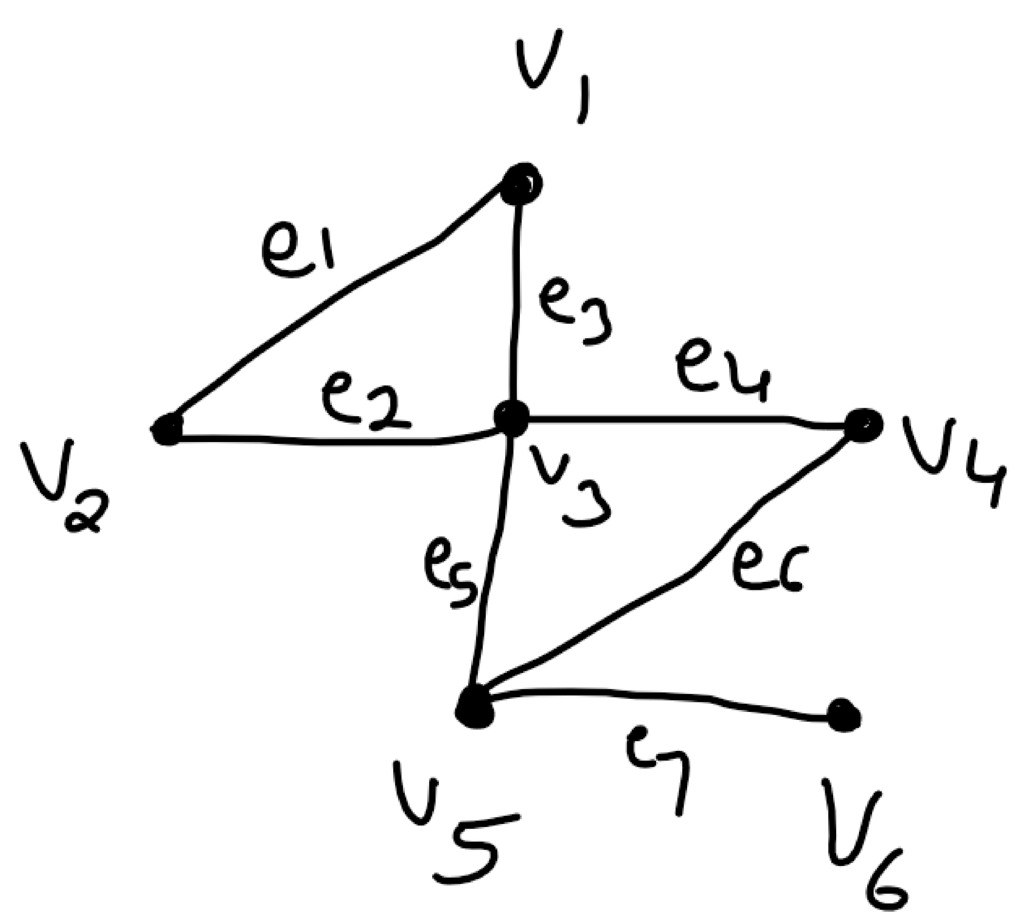
A labelled graph is a graph in which every vertex and every edge is labelled.
Most of the times, a graph means a labelled graph.

Question 1: Draw a labelled graph G having (4, 3, 2, 2, 2, 1) as degree sequence.



$\deg(v_3) = 4$

$dg(v_5) = 3$

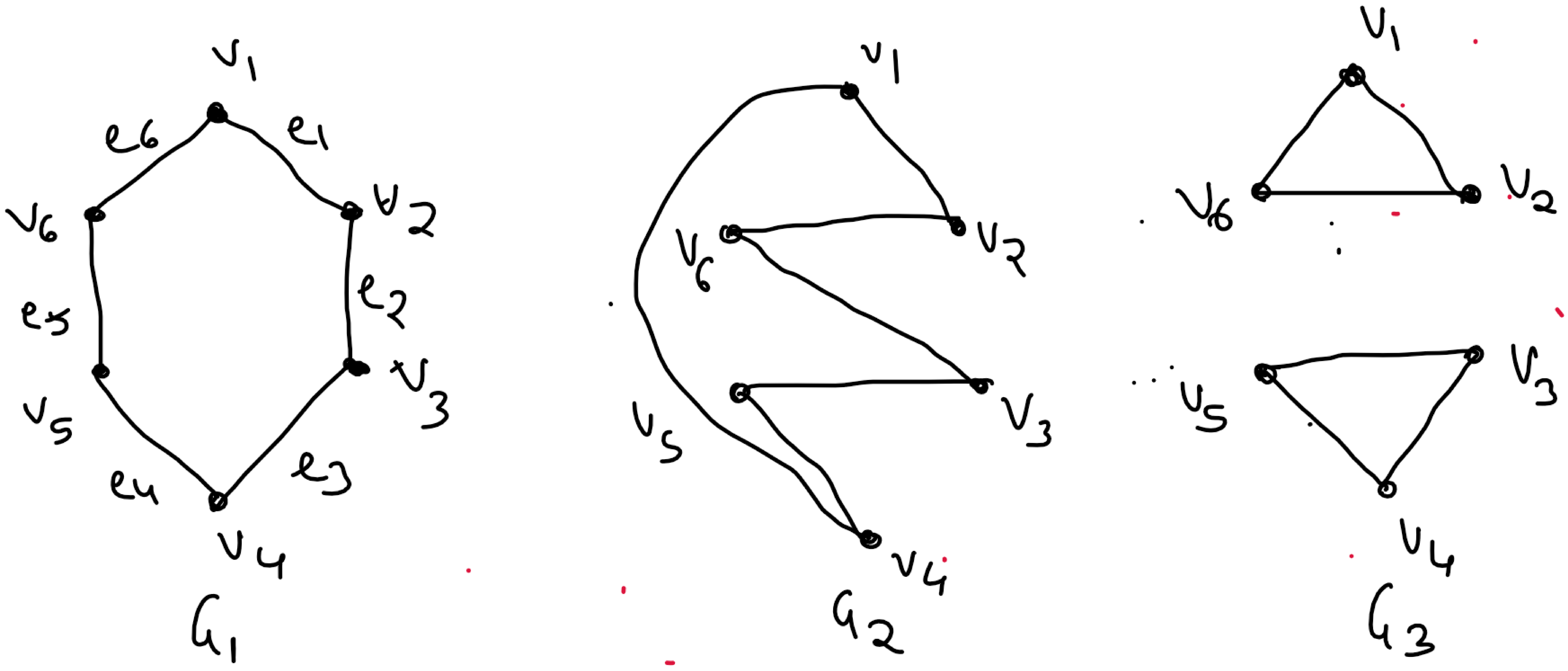
$dg(v_4) = dg(v_1) = dg(v_2) = 2$

$dg(v_6) = 1$

$\delta(G) = 1$ $\Delta(G) = 4$ $|E(G)| = 7$

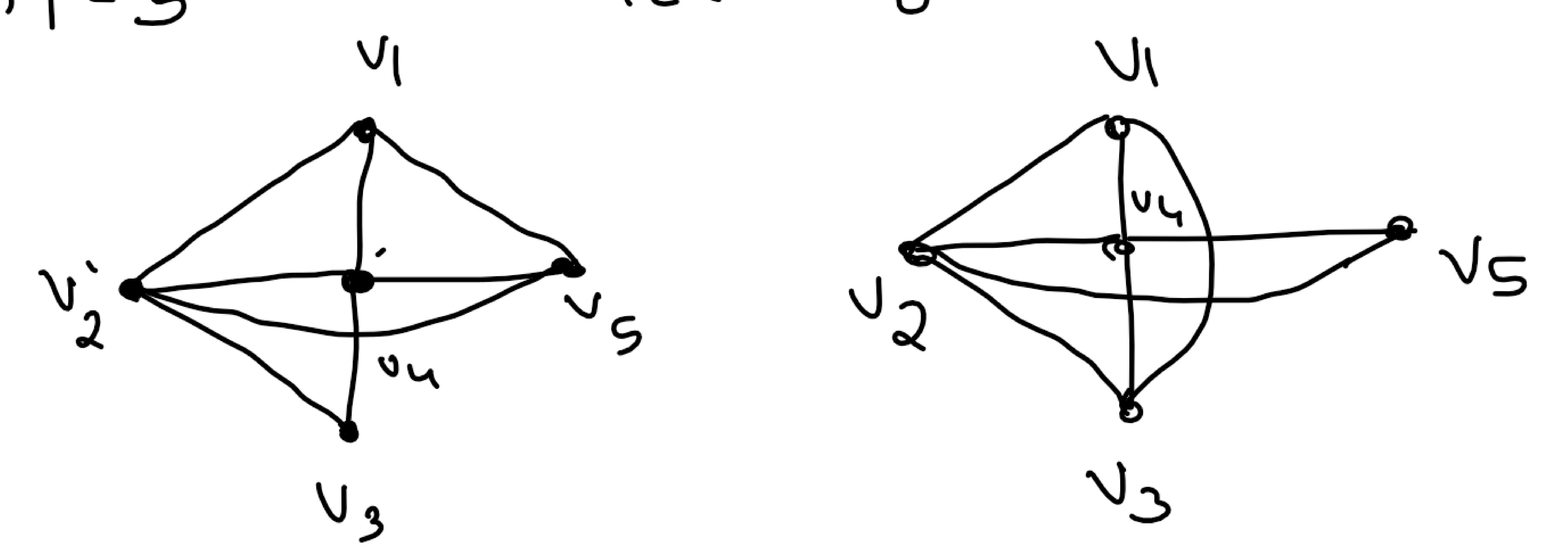
Question 2: Draw a graph G having (2, 2, 2, 2, 2, 2) as degree sequence. Is there exists more than one graph on a given degree sequence? Yes it is possible. [G_1 & G_3 are different]

$|V(G)| = p = 6$ $\qquad$ $|E(G)| = q = 6$



Question 3: If possible draw a graph G having (4, 4, 3, 3, 2) as degree sequence. How many edges can this graph contain?

$|V(G)| = 5$ $\qquad$ $|E(G)| = 8$



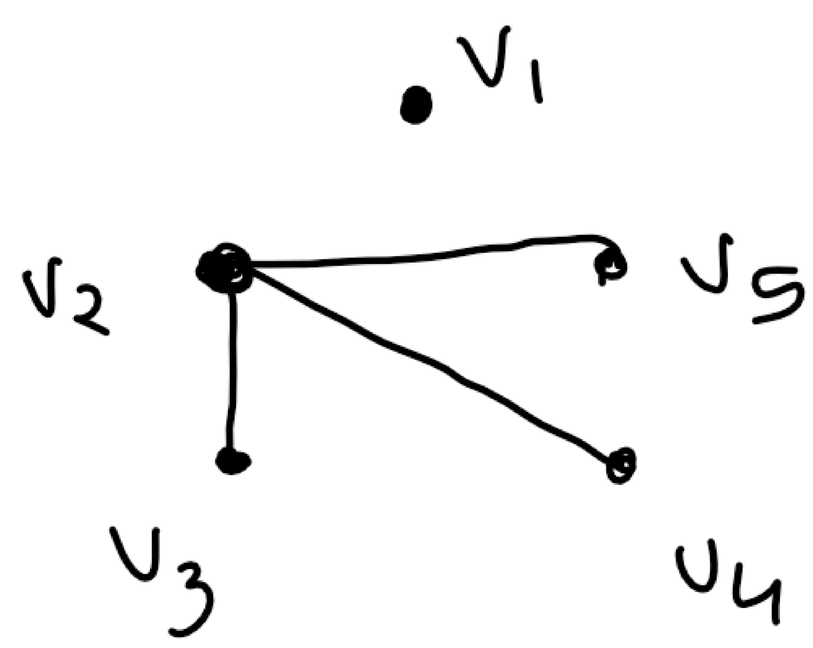
Theorem 1: Let G be a (p, q) graph. The sum of degrees of a graph G is twice the number of edges i.e.

$$\sum_{v \in V(G)} \deg(v) = 2q$$

Proof: Since every edge is incident with 2 vertices, each edge contributes 2 to the sum of degrees of the vertices.

Question 4: If possible draw a graph G having (4, 3, 2, 1, 0) as degree sequence. If not possible, then explain why.

$|V(G)| = 5 = p$



$0 \leq \deg(v) \leq p-1$

Not possible to draw.

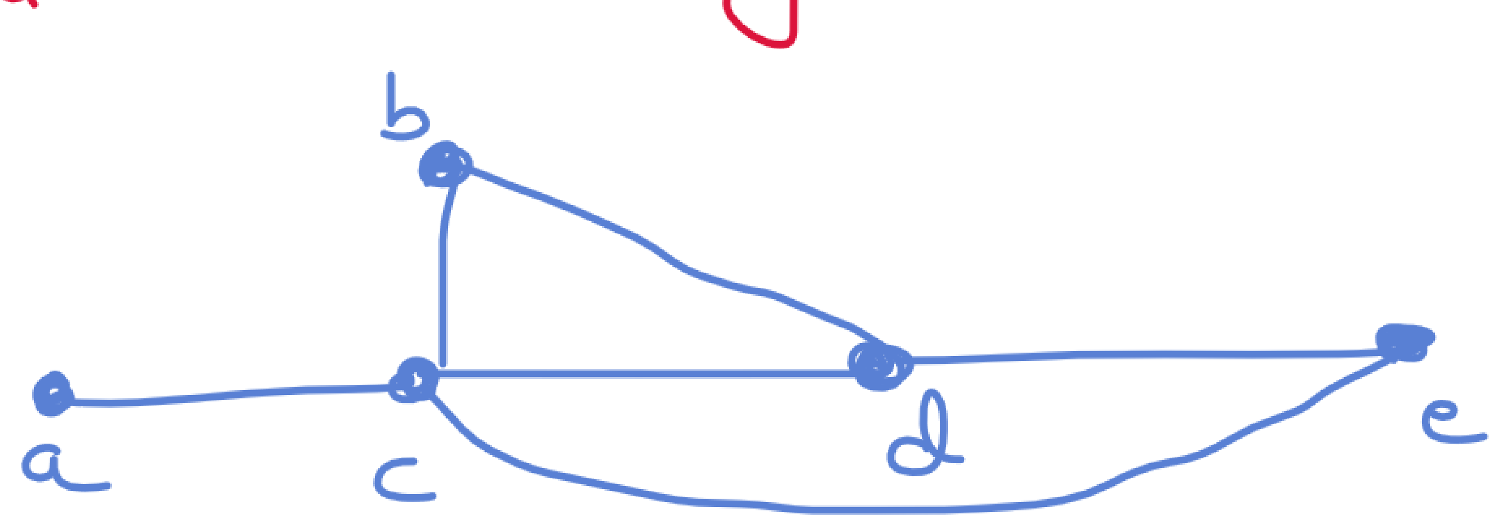
A graph is perfect if no two vertices are of same degree.

Theorem 2: No graph is perfect. OR In a (p, q) graph G, there exists at least 2 vertices with same degree.

Proof: We know that any vertex $v \in V(G)$, satisfies $0 \leq \deg(v) \leq p-1$.
Suppose G has a vertex with degree 0, then G cannot have a vertex with degree $p-1$. Therefore, for p vertices we have $p-1$ choices.

Hence, there exists at least 2 vertices with same degree.

Example: A graph with exactly 2 vertices of same degre and remaining all vertices are of distinct degrees.



deg seq: (1, 2, 2, 3, 4)

Question 5: Draw a graph with degree sequence as (3, 3, 2, 2, 1).

$p = 5$, Not possible.

Theorem 3 : In any graph, the number of vertices of odd degree is even.

Proof: Let S_e : sum of all degrees of vertices which have even degrees

S_o : Sum of all degrees of vertices which have odd degrees.

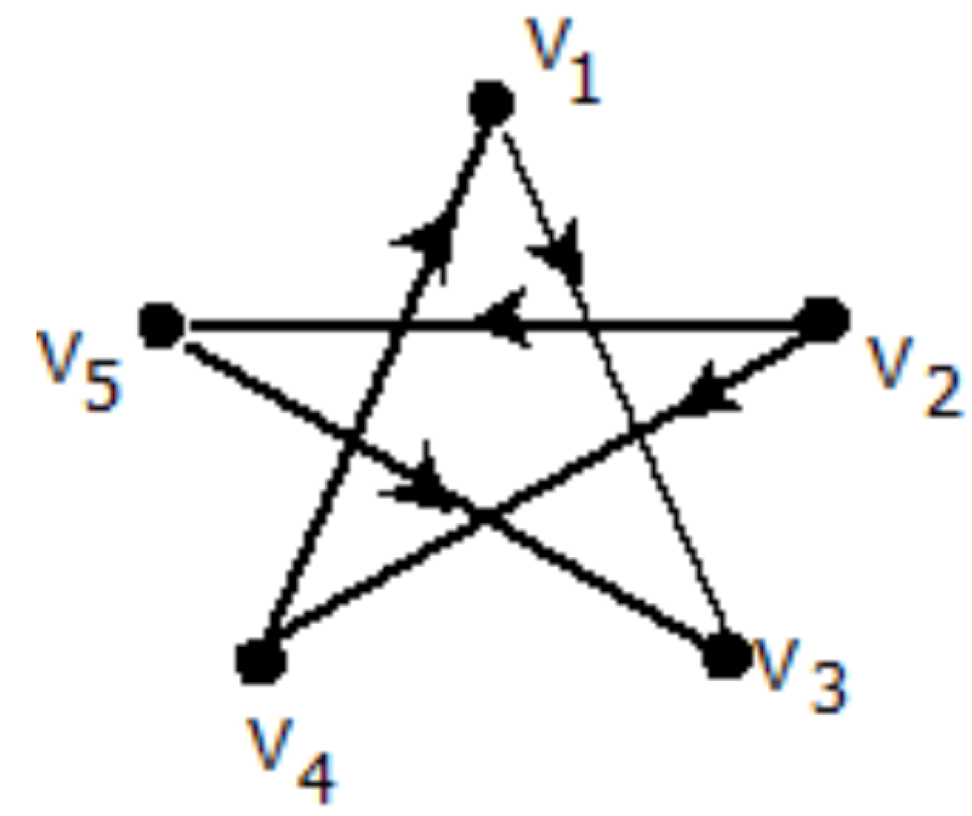
By definition, $S_e + S_o = 2q$

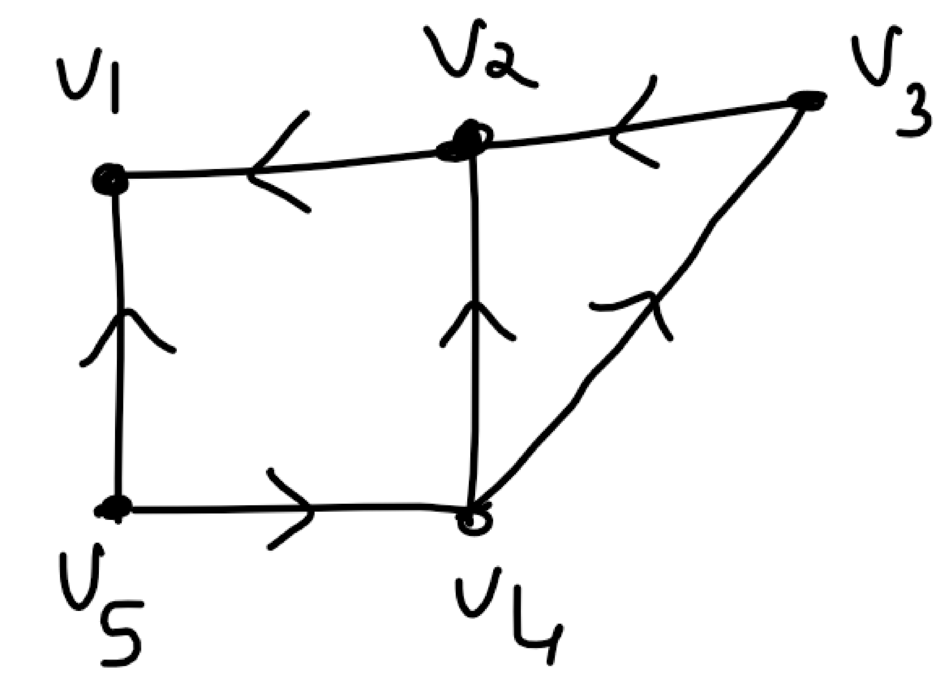
$\Rightarrow S_o = 2q - S_e$ an even number

Each term in S_o is odd

$\therefore S_o$ is even only if there are even number of terms in S_o.

A directed graph or digraph D consists of a finite nonempty set V of vertices together with a prescribed collection X of ordered pairs of distinct vertices. The elements of X are directed edges or arcs. By definition, a digraph has no loops or multiple arcs.

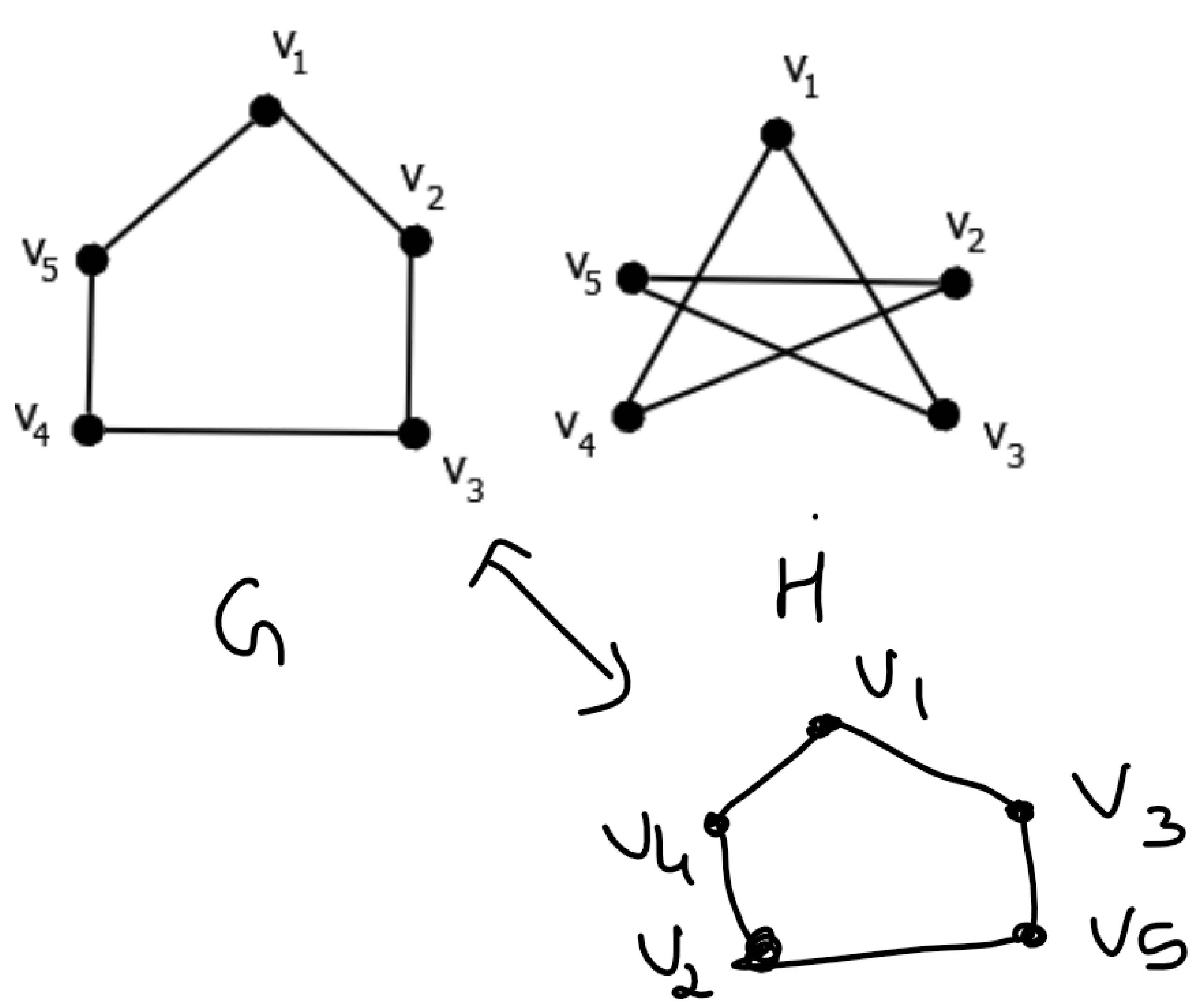


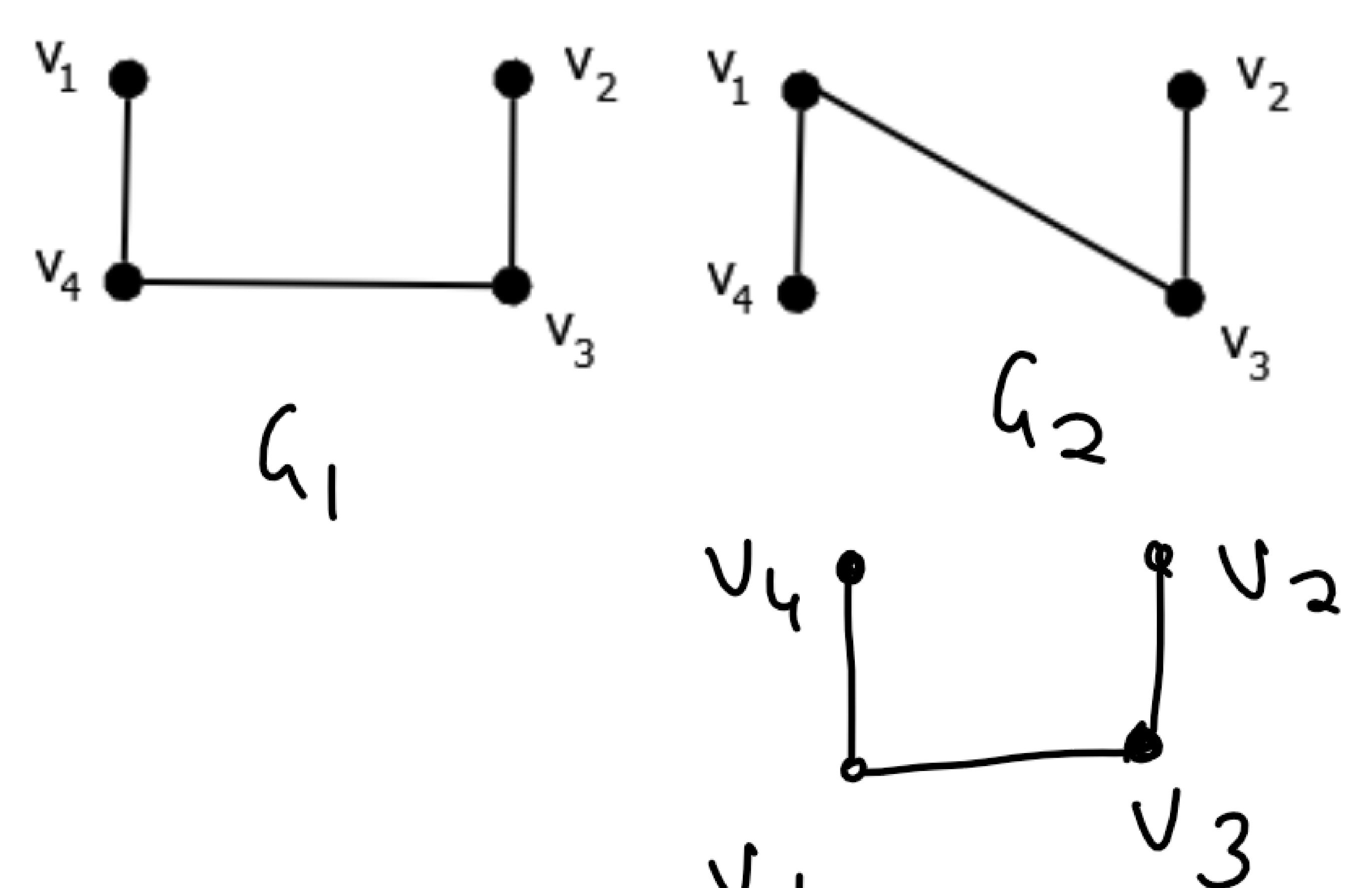


Two graphs G and H are isomorphic (written $G \cong H$ or sometimes G = H) if there exists one-to-one correspondence between their vertices and between their edges such that structure is preserved.

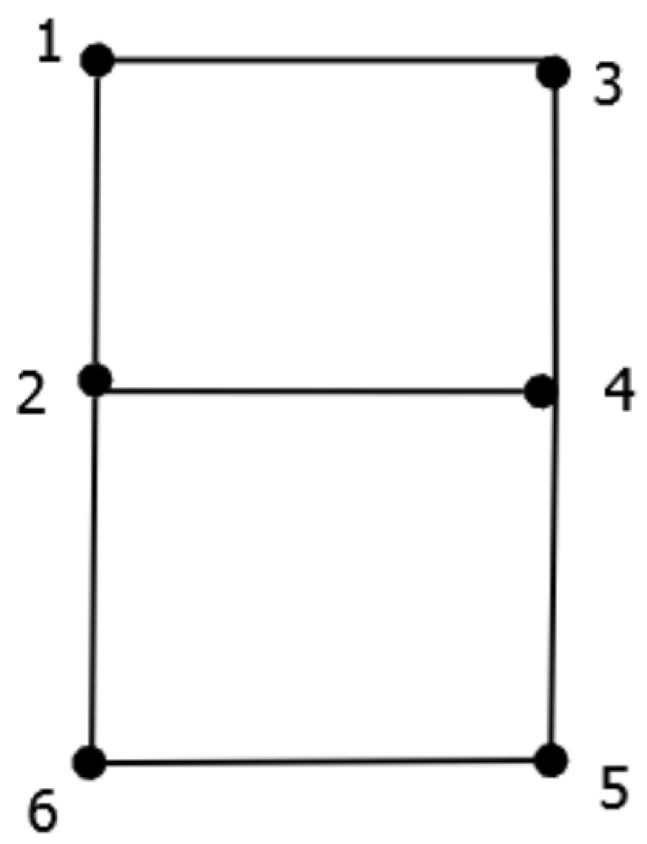
$G \cong H$

$G_1 \cong G_2$

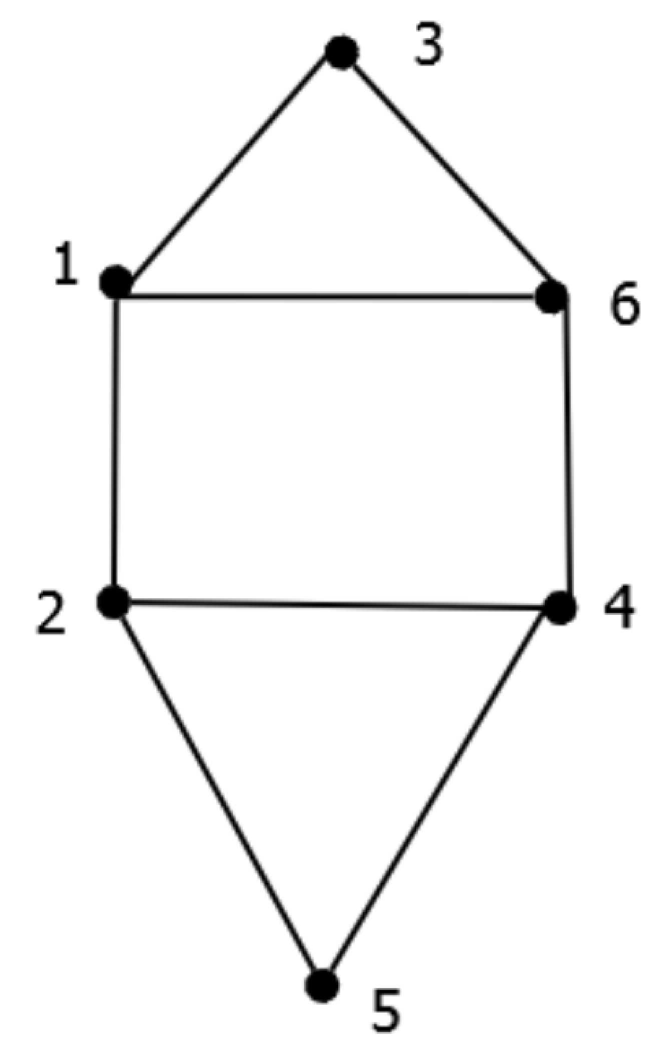




Question 6: Check whether the following graphs are isomorphic or not?



G

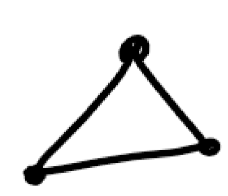


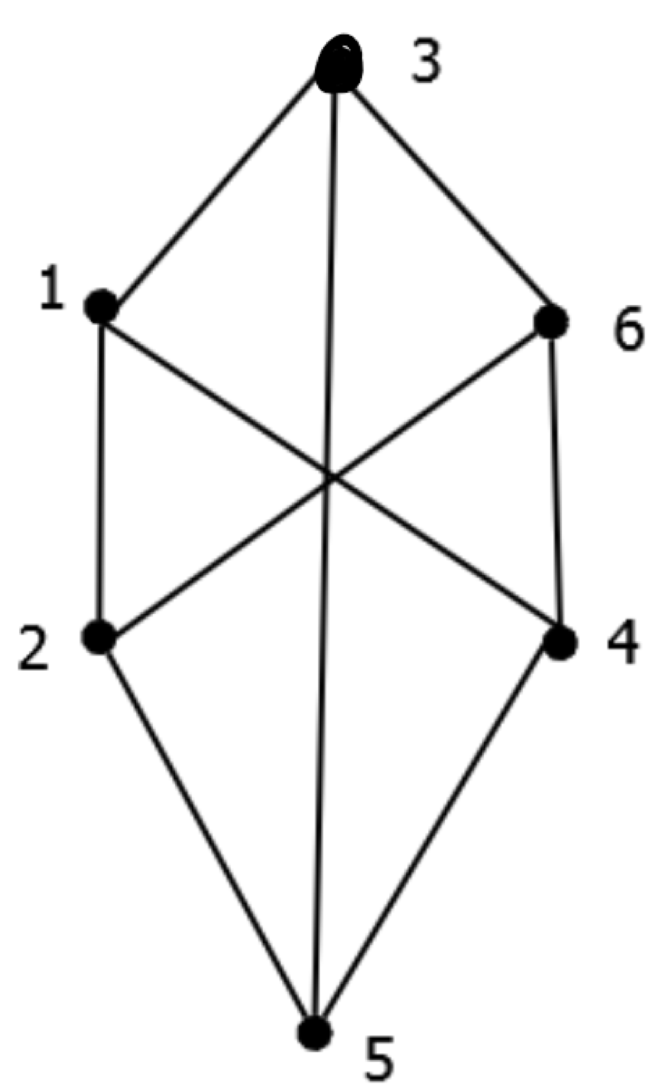
H

$G \rightarrow p = 6,\ q = 7$
$(3\ 3\ 2\ 2\ 2\ 2)$

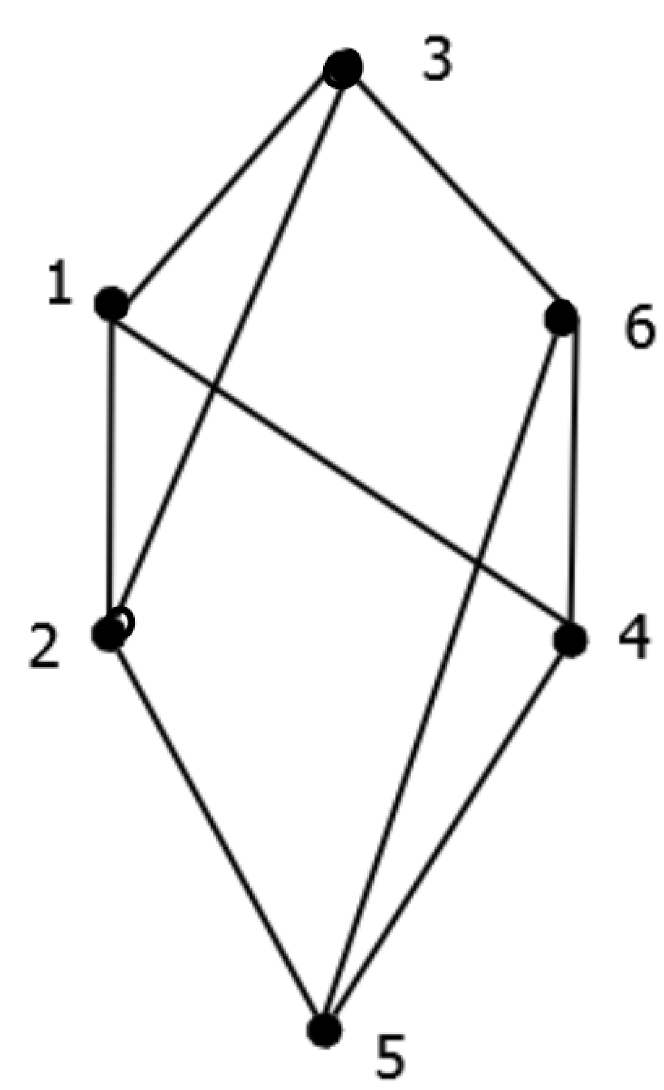
$H \rightarrow p = 6,\ q = 8$
$(3\ 3\ 3\ 3\ 2\ 2)$

$G \neq H$
G is not isomorphic since dg seq is not same.

 → triangle



G



H

$G: p = 6,\ q = 9$
$(3\ 3\ 3\ 3\ 3\ 3)$

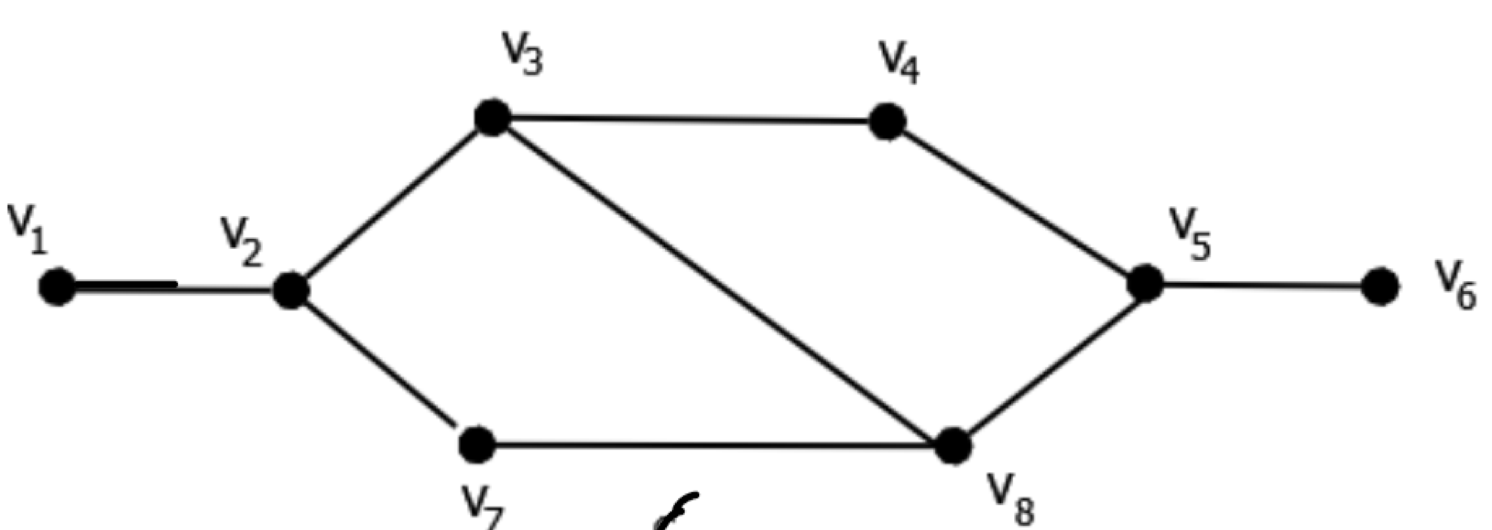
$H: p = 6,\ q = 9$
$(3\ 3\ 3\ 3\ 3\ 3)$

Not isomorphic. G has no triangle but H has.

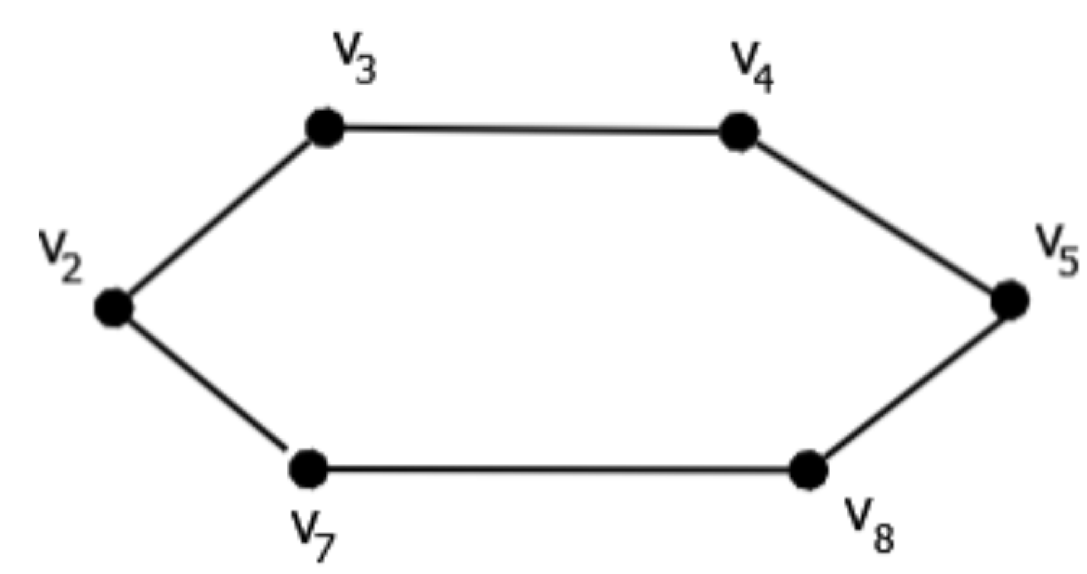
Subgraph of a graph :

A graph $\underline{H}$ is called a subgraph of G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.

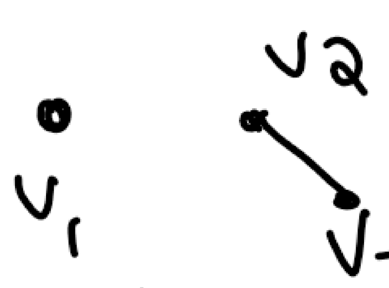
A subgraph H of G is called a spanning subgraph if $V(H) = V(G)$.



G

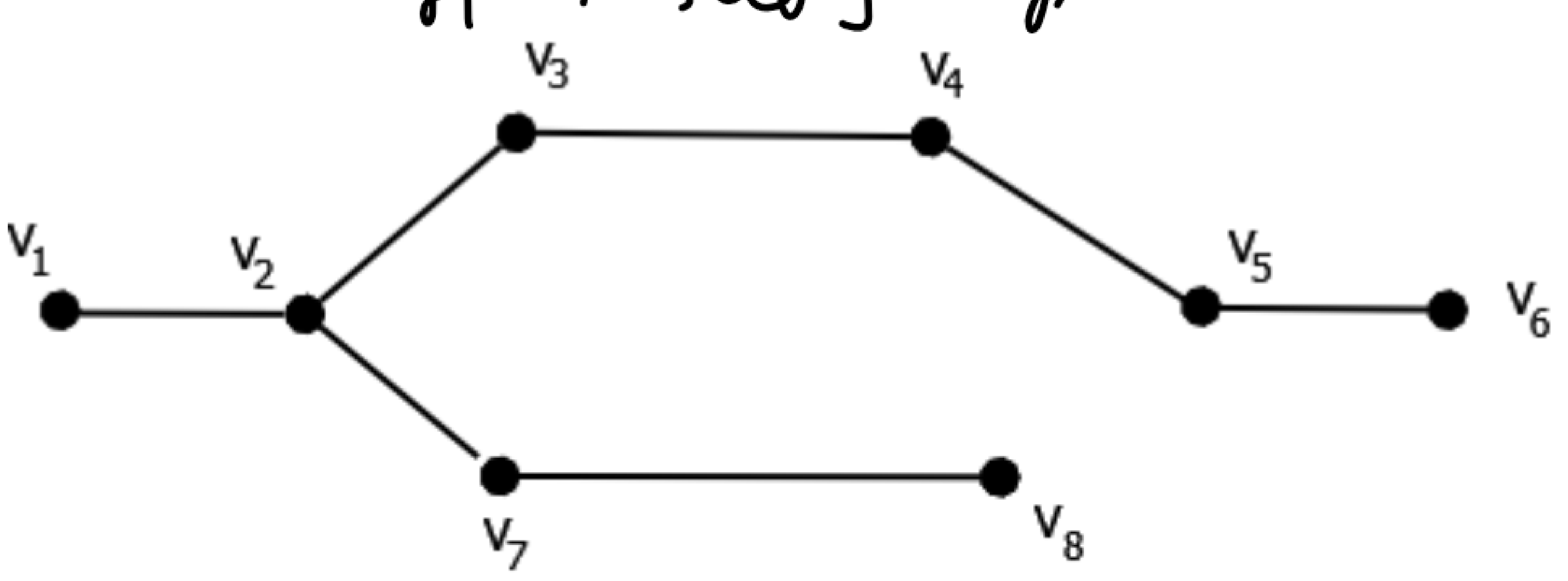


H_1 subgraph

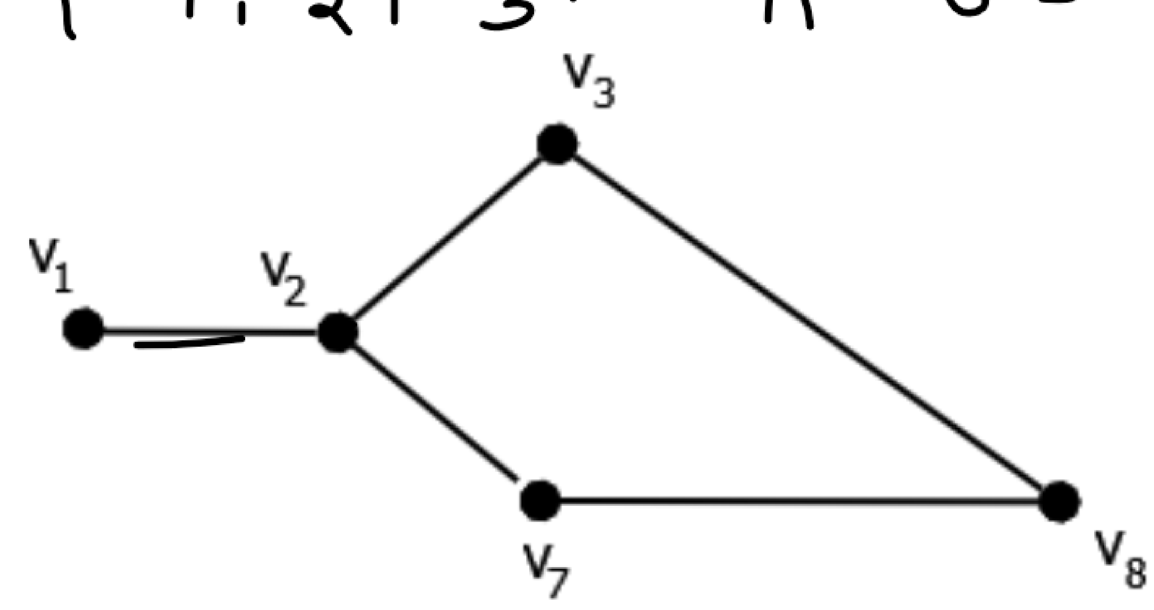


$H \rightarrow$ subgraph

$S = \{v_1, v_2, v_3, v_7, v_8\}$ ✓



$H_3 \rightarrow$ spanning



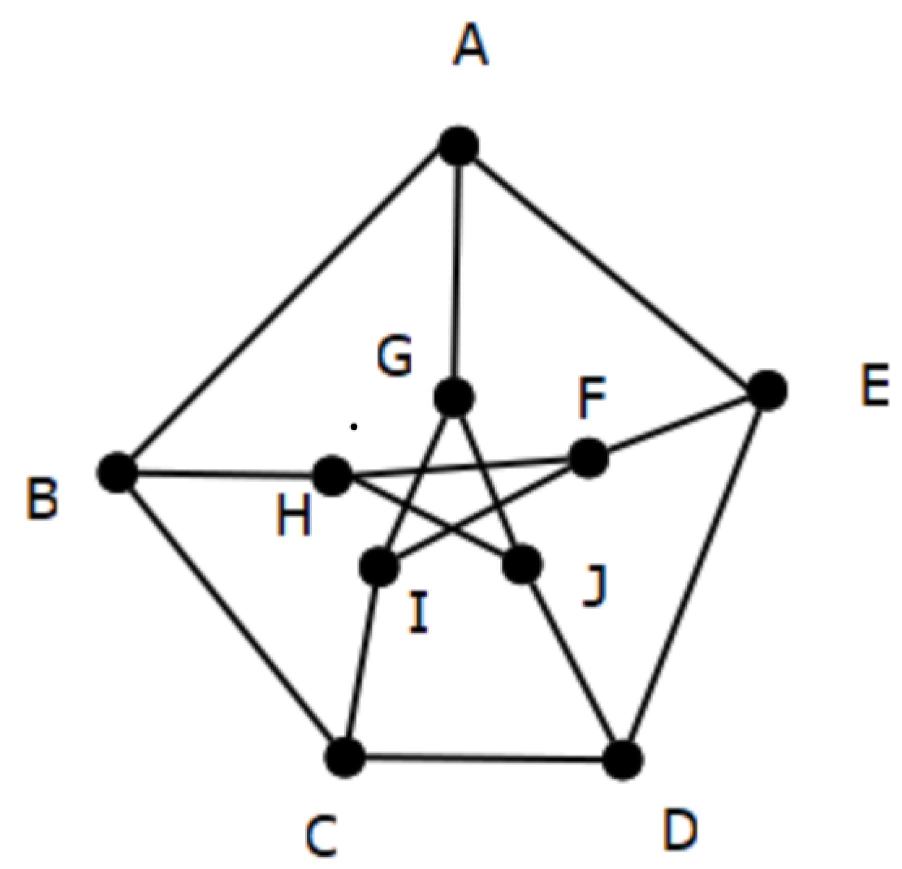
$H_2 \rightarrow$ graph induced by S.

Let $\underline{S}$ be a subset of the vertex set $V(G)$ of G.

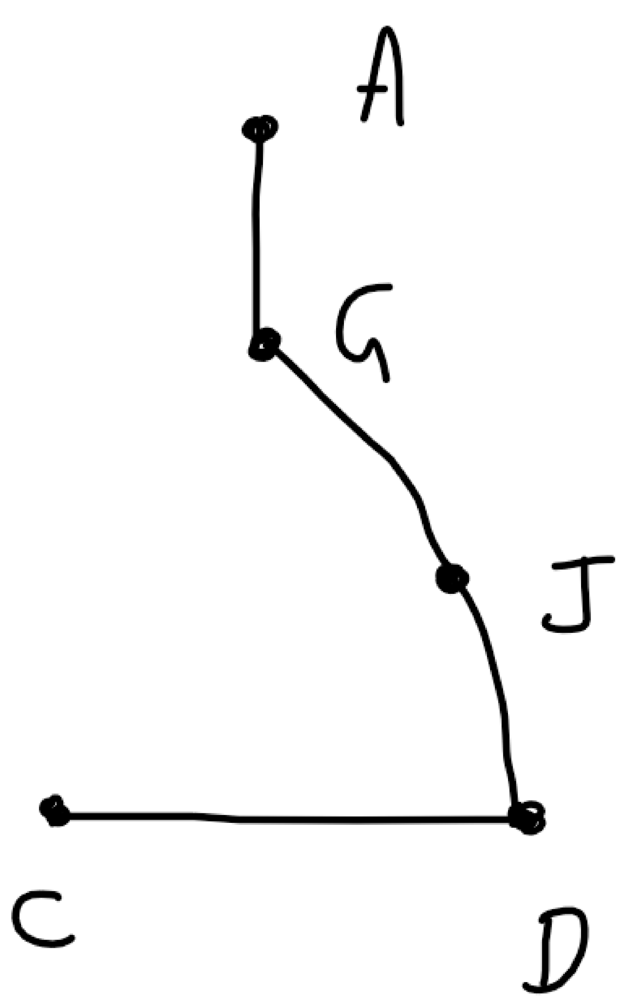
Then ; the subgraph induced by S, denoted by $\langle S \rangle$ is the maximal subgraph of G with S as the vertex set.

Thus, 2 vertices of S are adjacent in $\langle S \rangle$ if and only if they are adjacent in $\underline{G}$.

Find the induced subgraph of the graph G spanned by the set S = {A, C,D, G, J}. Also find one spanning subgraph of the graph G.

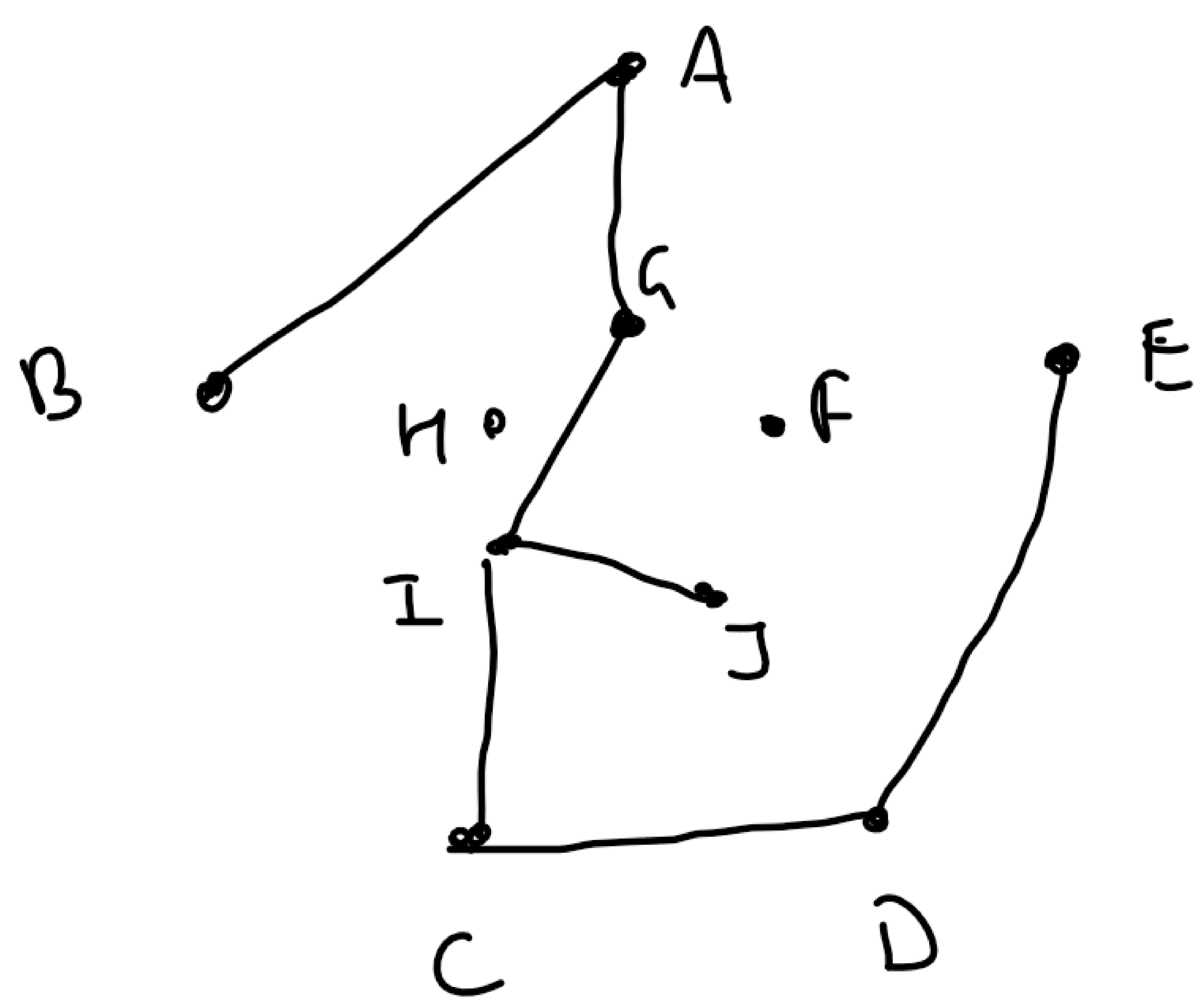


G



Graph induced by S

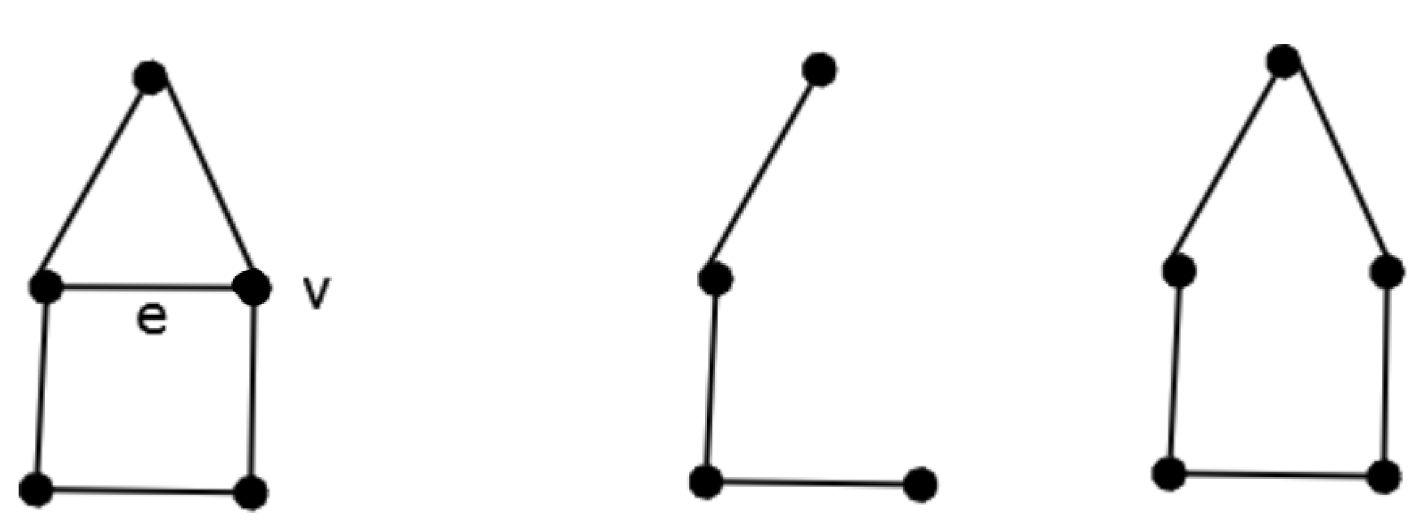
$\langle S \rangle$



← spanning subgraph

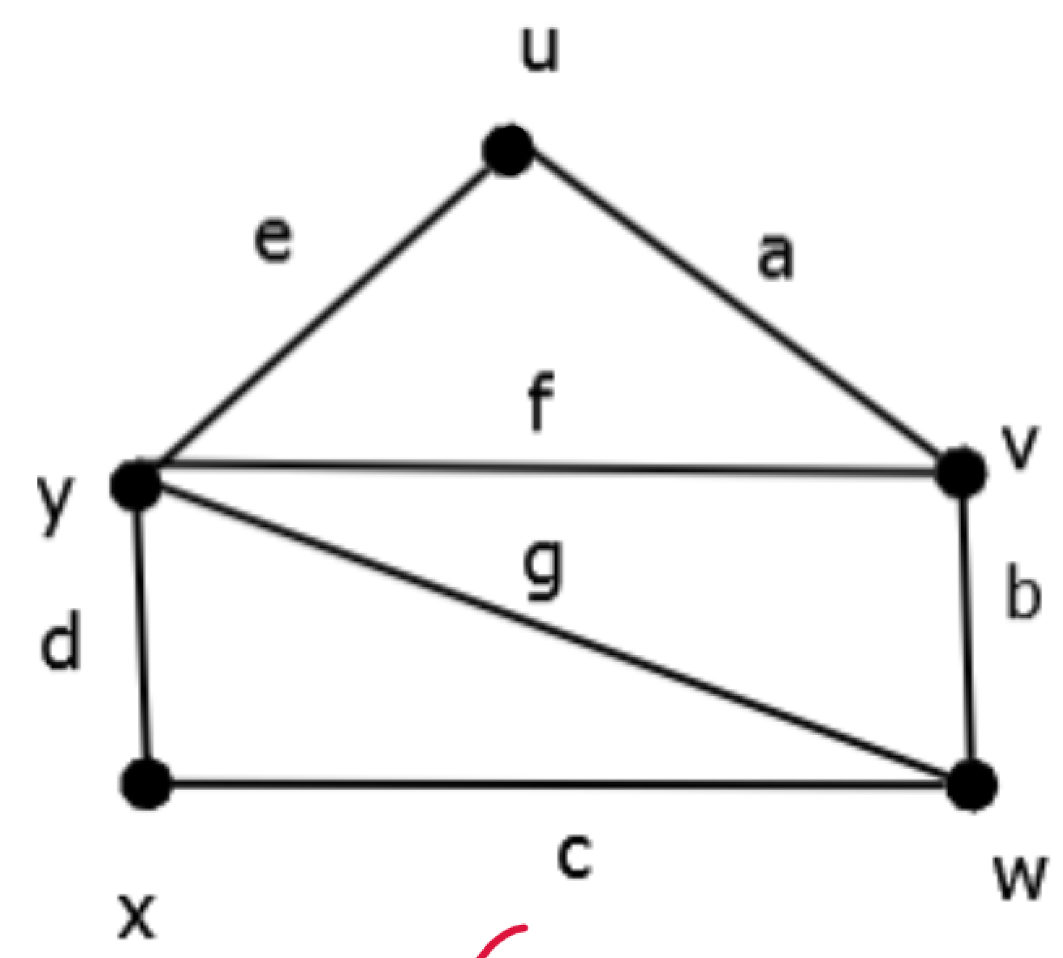
The removal of a vertex v from a graph G results in that subgraph G − v of G consisting of all vertices of G except v and all edges not incident with v.

The removal of an edge e from G yields the spanning subgraph G − e containing all edges of G except e.



G G-v G-e

A walk in G is an alternating sequence of vertices and edges, begining and ending with vertices, in which each each edge is incident with the 2 vertices immediately preceeding and following it.



G

Walk : u a v b w b v f y

Walk : y f v b w g y d x

Walk → (y v w y x)

Length : 4

In a simple graph, a walk $v_0 e_1 v_1 e_2 v_2 e_3 \cdots e_k v_k$ is determined by the sequence of vertices

$v_0 \; v_1 \; v_2 \cdots v_k$.

A walk is closed if the starting and end vertex is same.

The length of a walk is the number of edges in a walk.

A walk is a trail if all the edges are distinct, and a path if all the vertices and edges are distinct. A closed path is a cycle.

Trail example : u v w y v (u a v b w g y f v)

Path example : v w y x (v b w g y d x)

Cycle example : y v u y , y v w x y

A cycle of length 3 is also known as triangle. (u y v u) is a triangle.

The girth of a graph G is denoted by g(G) is the length of the shortest cycle (if any) in G.

The circumference c(G) is the length of the longest cycle in G.

g(G) = 3 and c(G) = 5