Basic Electrical Technology

Introduction

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Course Outline: BET [ELE 1051] $\langle [L\ T\ P\ C] = [2\ 1\ 0\ 3] \rangle$

- **DC Circuit Analysis:**
  - Circuit elements, Sources, Resistance, Inductance, Capacitance, Mesh Current and Node Voltage Analysis, Superposition, Thevenin's and Max Power Transfer Theorems

- **Magnetic Circuit Analysis and Electromagnetism:**
  - Magnetism, Laws of magnetism, series and parallel magnetic circuits, Electromagnetic induction, Magnetic coupling, induced emfs, mesh current equations

- **Single phase AC Circuit Analysis:**
  - Generation, Representation, AC through R, L and C, Series and parallel circuits, Power, Power factor, Resonance in series and parallel AC circuits

Course Outline ELE 1051 [2 1 0 3]

- **3 phase AC Circuit Analysis:**
  - Generation, Representation, Types of connection – Star & Delta, Analysis of balanced and unbalanced loads, Measurement of Power

- **Overview of Power System Components:**
  - Electrical Power System – An overview, Generation, Transmission, Distribution, Utilization of Electric Power; Overview of Electrical Machines, Types, working principle & applications; Measurement of Energy: Energy meters

Course Plan

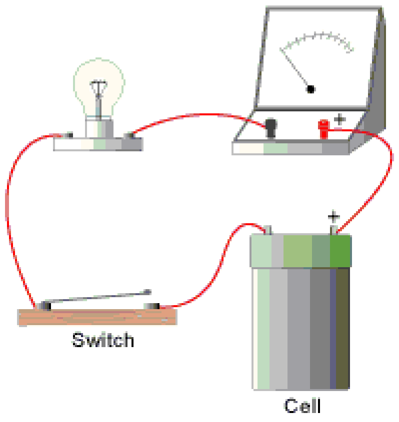
Assessment

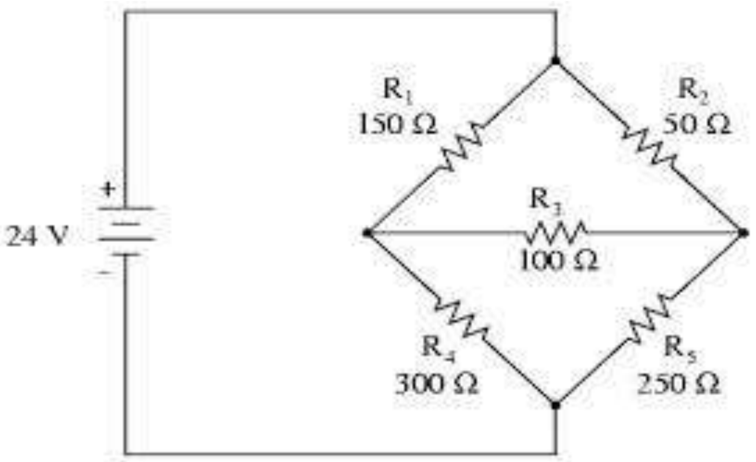
- **In-Semester Assessment - 50%**
  - **2 Sessionals: 15 marks each – 1 hour duration**
  - **4 Quizzes : 5 marks each – 20 minutes**
- **End-Semester Examination – 50%**
  - **Written Examination : 50 marks – 3 hours duration**
  - **Minimum Pass Marks for End Semester Exam: 18 marks**
- ***In order to clear the course a student must secure minimum pass marks (which could be between 35 to 50 marks) which is calculated by adding the marks obtained in In-Semester and End-Semester Exams.***
- **All questions are to be answered**
- Attendance requirement : 75 % (which is regularly updated in SLcM)
- NO use of Mobiles in the classroom.
- Maintain Lecture Notes
- Bring Calculators

What is an Electric Circuit?

Definition:

“An interconnection of simple electrical devices with at least one closed path in which current may flow”





Circuit Elements

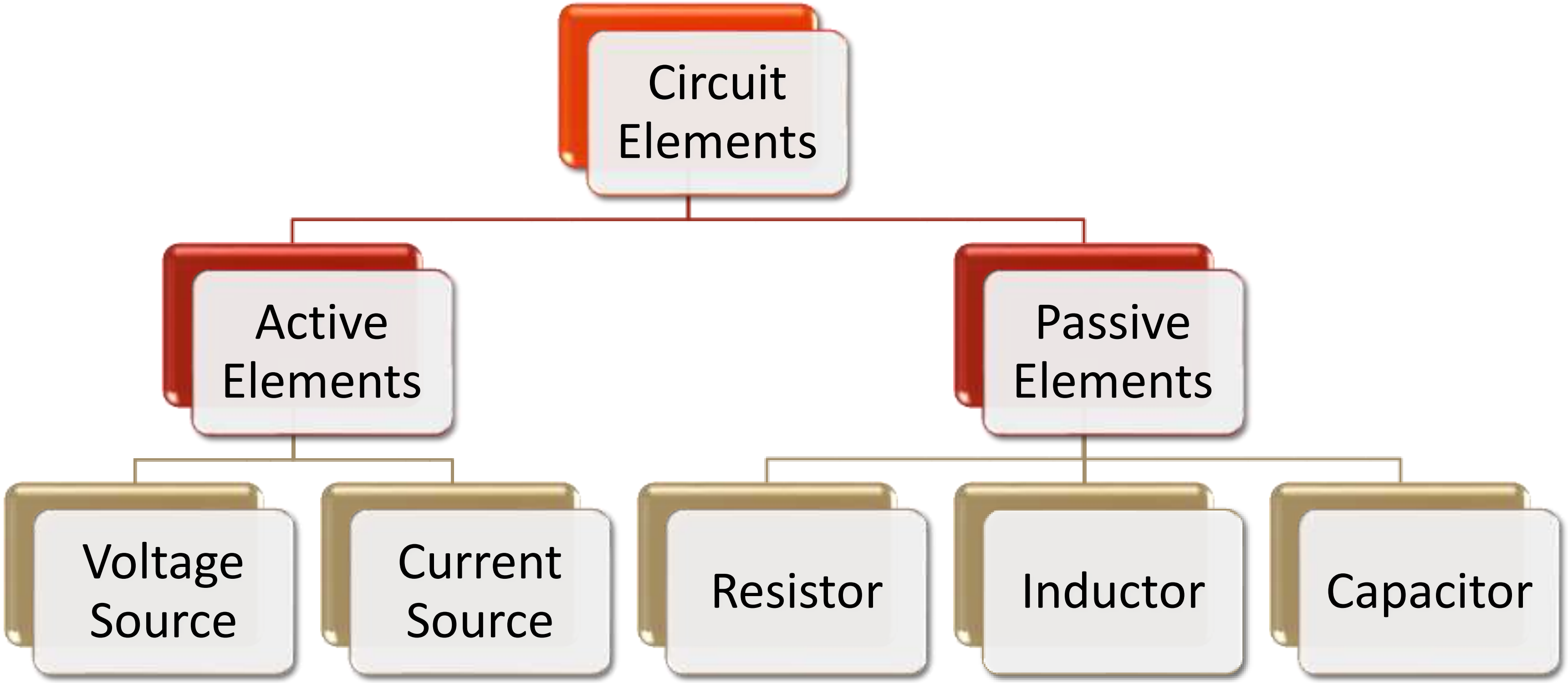
- **Active & Passive**
  - Active Elements: Voltage & Current Source
  - Passive Elements: Resistor, Inductor & Capacitor
- **Linear & Non-linear Elements**
  - Linear: Resistor, Inductor, Capacitor
  - Nonlinear: Diode, LDR (Light Dependent Resistor), Thermistor, transistor
- **Unilateral & Bilateral Elements**
  - Unilateral (Current Flow in one direction): Diode, Transistor
  - Bilateral: Resistor, Inductor, Capacitor*
- **Lumped & Distributed**

Discuss only ***lumped linear bilateral*** circuit elements

Basic Electrical Technology

Circuit Elements

Classification of Circuit Elements

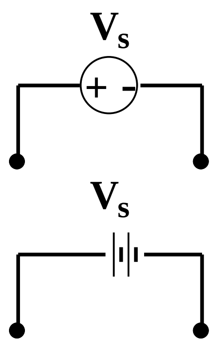


Active Elements - Sources

Voltage Source:

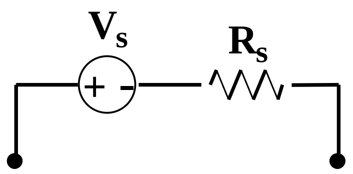
- **Ideal:**
  - Maintains constant voltage irrespective of connected load
  - Internal resistance $R_s = 0$

Ideal Voltage Source (DC)



- **Practical:**
  - Terminal voltage changes based on the connected load
  - Internal resistance $R_s \neq 0$

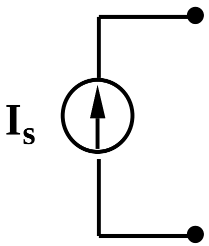
Practical Voltage Source



Active Elements - Sources

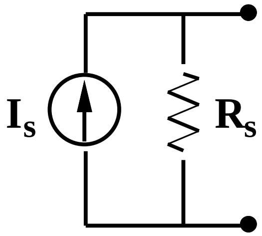
Current Source:

Ideal Current Source (DC)



- **Ideal:**
  - Maintains constant current irrespective of the load connected
  - Internal resistance $\boldsymbol{R_s} = \infty$

Practical Current Source



- **Practical:**
  - Output current changes based on the connected load
  - Internal resistance $\boldsymbol{R_s} < \infty$

Resistor

Energy Consuming Element

Resistor

- **Passive electric device** that **dissipates energy**
- **Resistance:** property which opposes flow of current
  - Symbol: **R**
  - Unit: Ohms (Ω)
  - Power Consumed $= \boldsymbol{I^2 R}$

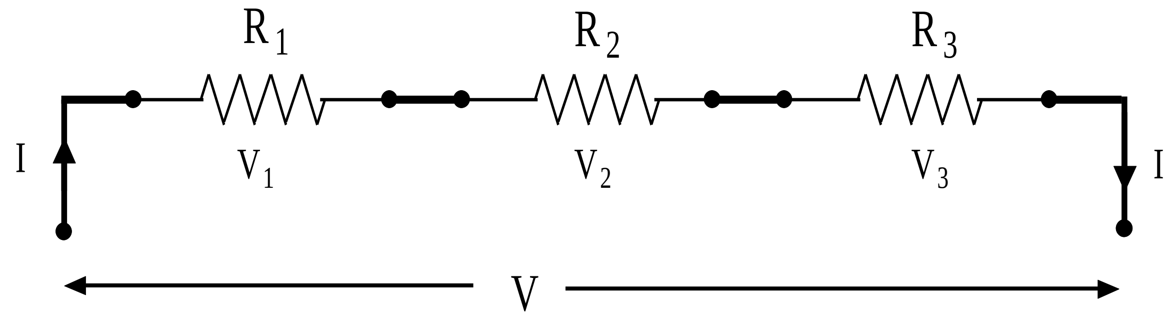
- **Conductance**
  - Reciprocal of resistance
  - Symbol: G
  - Unit – Siemens (S)

Resistors in Series

Current (I) is same

$$\boldsymbol{V = V_1 + V_2 + V_3}$$

$$\boldsymbol{R_{eq} = R_1 + R_2 + R_3}$$

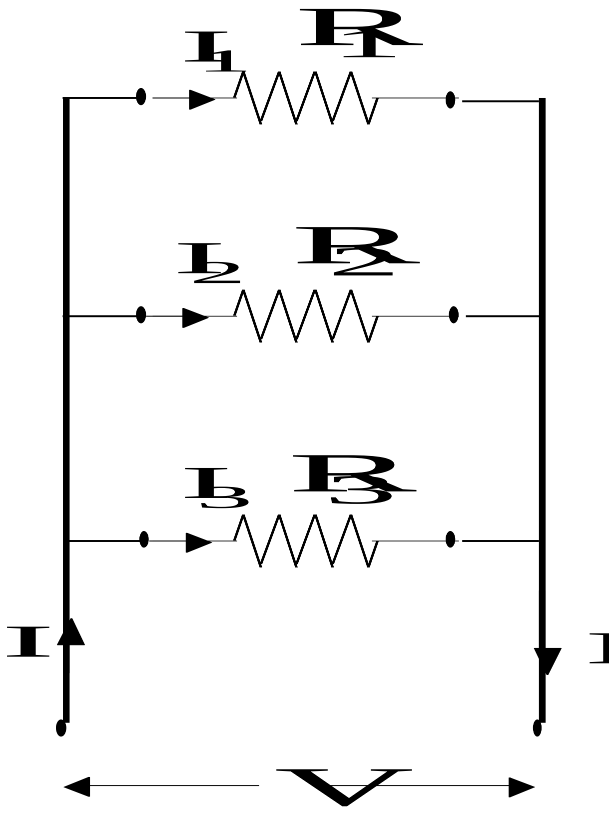


Resistors in Parallel

Voltage (V) is same

$$I = I_1 + I_2 + I_3$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



Inductor

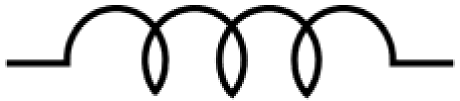
Energy Storing Element

Inductor

- **Passive** electric device that **stores energy in its magnetic field** when current flows through it

- A coil of wire wound on a core
  - Eg.: Air core Inductor, iron core inductor

- **Inductance**: property which opposes rate of change of current
  - Symbol: **L**
  - Unit: Henry (H)



- The voltage across inductor is proportional to the rate of change of current through it

$$v_L = L\frac{di}{dt}$$

Inductive Circuit

For a coil uniformly wound on **a non-magnetic core** of uniform cross section, self inductance is given by

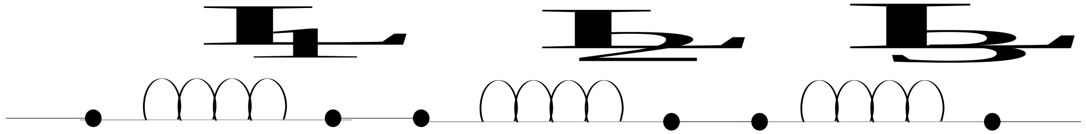
$$L = \frac{\mu_0 A N^2}{l}$$

Where,
$l =$ length of the magnetic circuit in meters
$A =$ cross sectional area in square meters
$\mu_o =$ Permeability of air $= 4 \times 10^{-7}$
$N = No.\, of\ turns\ in\ the\ coil$

Equivalent Inductance

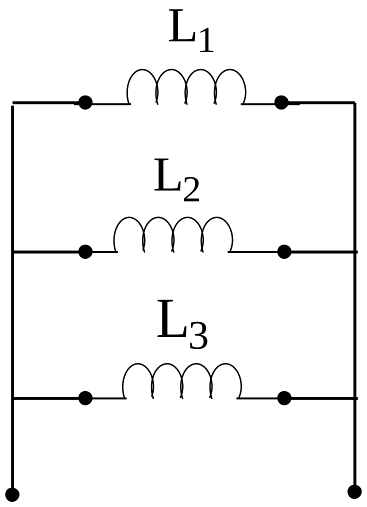
Inductors in series

$$L_{eq} = L_1 + L_2 + \ldots\ldots + L_n$$



Inductors in Parallel

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots\ldots. + \frac{1}{L_n}$$



Energy Stored in an Inductor

- Instantaneous power,

$$p = v_L . i = L\, i \frac{di}{dt}$$

- Energy absorbed in 'dt' time is

$$dw = L\, i\, di$$

- Energy absorbed by the magnetic field when current increases from $\mathbf{0}$ to I amperes, is

$$W = \int_0^I L\, i\, di = \frac{1}{2} L\, I^2$$

Capacitor

Energy Storing Element

Capacitors

- **Passive electric device** that **stores energy in the electric field** between a pair of closely spaced conductors

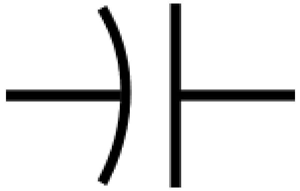
- **Capacitance**: Property which opposes the rate of change of voltage
  - Symbol: **C**
  - Unit: Farad (F)

- The capacitive current is proportional to the rate of change of voltage across it

$$\boldsymbol{i_c = C\frac{dv_c}{dt}}$$

- Charge stored in a capacitor whose plates are maintained at constant voltage:

$$\boldsymbol{Q = CV}$$



Terminologies

- Electric field strength,

$$E = \frac{V}{d}\ volts/m$$

- Electric flux density,

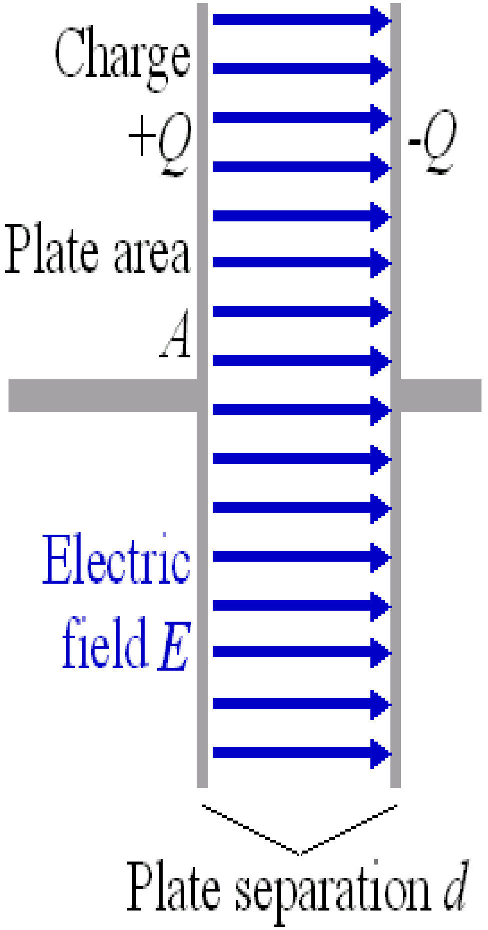
$$D = \frac{Q}{A}\ C/m^2$$

- Permittivity of free space,

$$\varepsilon_0 = 8.854 \times 10^{-12}\ F/m$$

- Relative permittivity, ε_r
- Capacitance of parallel plate capacitor

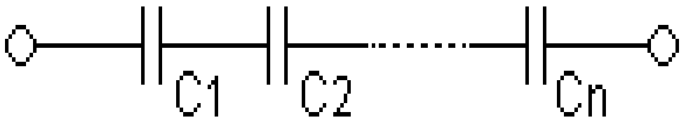
$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

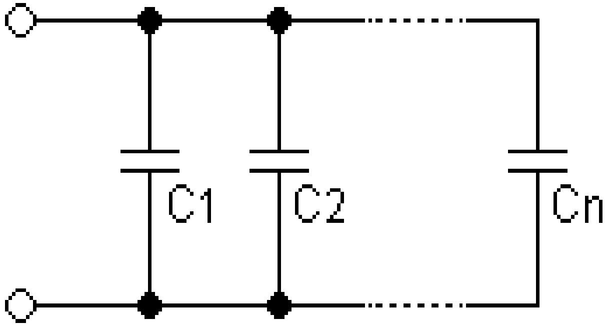


Equivalent Capacitance

Capacitors in Series

$$\frac{1}{C_{eq}} = \frac{1}{C1} + \frac{1}{C2} + \ \ldots \ldots + \frac{1}{Cn}$$





Capacitors in Parallel

$$C_{eq} = C_1 + C_2 + \ \ldots .. + C_n$$

Energy stored in a Capacitor

- Instantaneous power

$$\boldsymbol{p = v_c \times i = C\, v_c \frac{dv_c}{dt}}$$

- Energy supplied during '$\boldsymbol{dt}$' time is:

$$\boldsymbol{dw = C\, v_c\, dv_c}$$

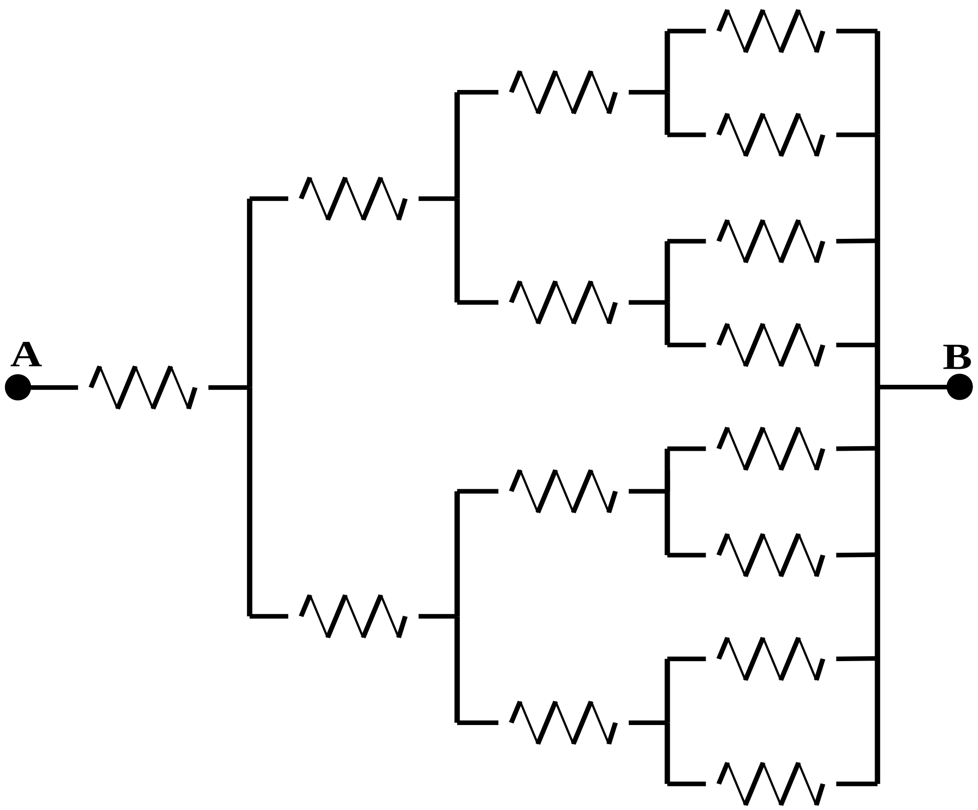
- Energy stored in the electric field when potential rises from $\boldsymbol{0}$ to $\boldsymbol{V}$ volts is,

$$\boldsymbol{W = \int_0^V C\, v_c dv_c = \frac{1}{2} CV^2 \ \text{Joules}}$$

Illustration 1

a) 15 resistors are connected as shown in the diagram. Each of the resistors has resistance 1 Ω. Find the equivalent resistance of the network between A & B.

b) What will be the equivalent resistance of this network if the resistors arranged in the sequence extends to infinity?



Ans:

a) 1.875 Ω

b) 2 Ω

Illustration 2

Two incandescent bulbs have the following ratings:

Bulb-1: 120 V, 60 W; **Bulb-2:** 240 V, 480 W

a) Both of them are connected in series with a voltage source.
   i. Which bulb will glow brighter and why?
   ii. What is the maximum voltage that can be applied so that non of the bulbs fuse?

b) Now both of them are connected in parallel with a voltage source.
   i. Which bulb will glow brighter and why?
   ii. What is the maximum voltage that can be applied so that non of the bulbs fuse?

Assume that the incandescent bulbs are purely resistive.

Ans:

a) i) Bulb-1 since it consumes more power, ii) 180 V

b) i) Bulb-2 since it consumes more power, ii) 120 V

Illustration 3

Two incandescent bulbs of 40 W and 60 W ratings are connected in series across the mains. Then which of the following statement(s) is(are) correct?

a) The bulbs together will consume 100 W

b) The bulbs together will consume 50 W

c) The 60 W bulb glows brighter

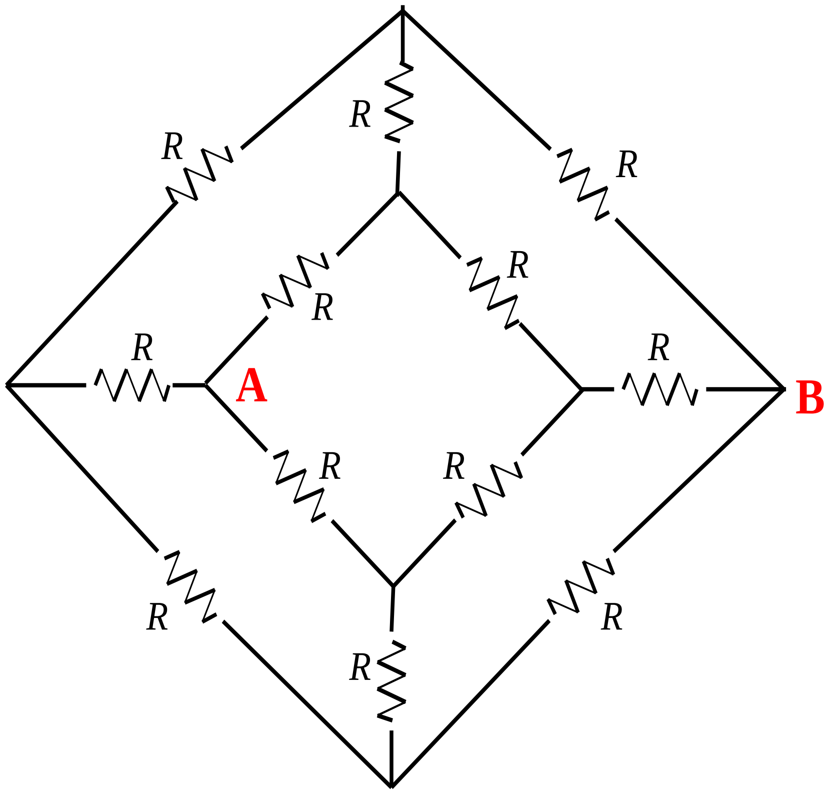
d) The 40 W bulb glows brighter

Assume the voltage rating of both the bulbs to be same.

Ans: d) The 40 W bulb glows brighter

Homework 1

Reduce the network to its equivalent resistance between terminals A and B



Ans: $\frac{5}{6}R$

Basic Electrical Technology

Source Transformation & Star-Delta Transformation

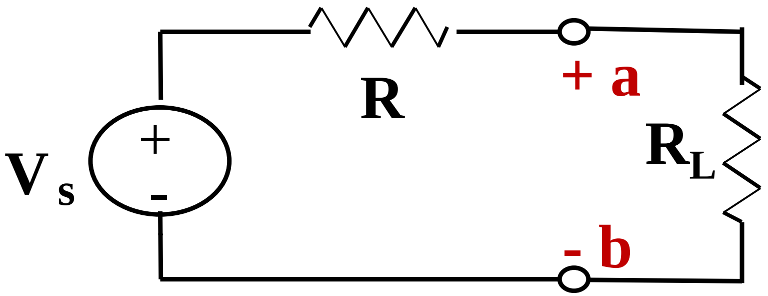
Source Transformation

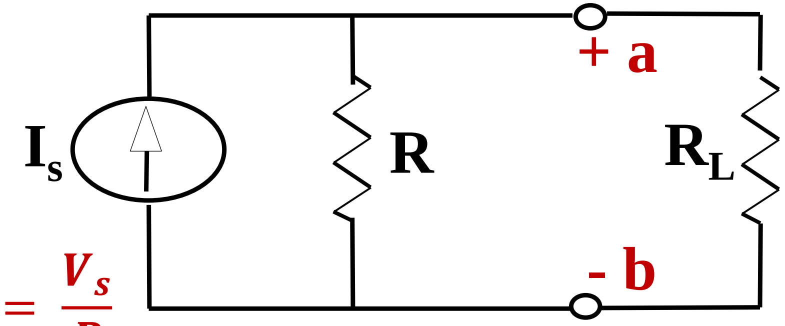
Practical Voltage Source ⟷ **converted to** ⟷ **Practical Current Source**

Source Transformation

Practical Voltage source

Practical Current source





$$I_s = \frac{V_s}{R}$$

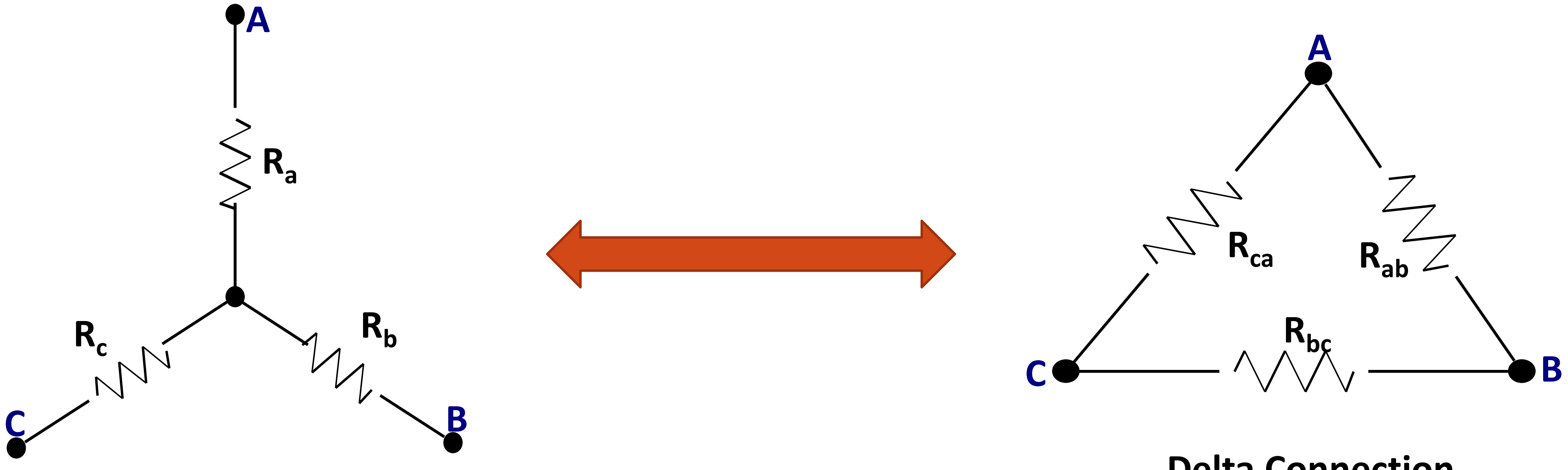
Source Transformation

Practical Voltage source

Practical Current source

$V_s = R \times I_s$

Star & Delta Connections

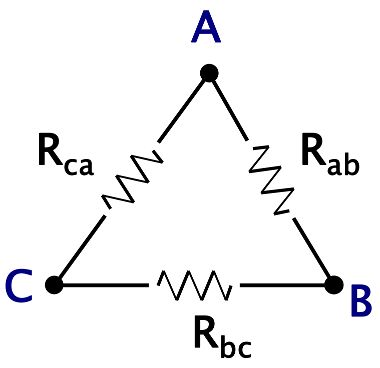


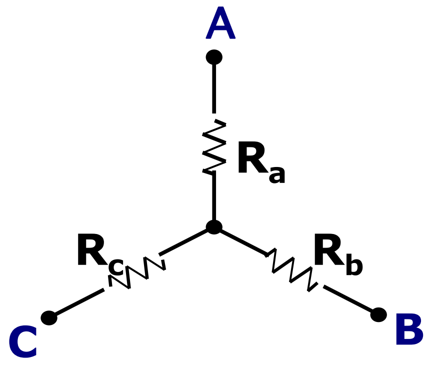
Star Connection

Delta Connection

Star-Delta Transformation

Delta to Star Transformation





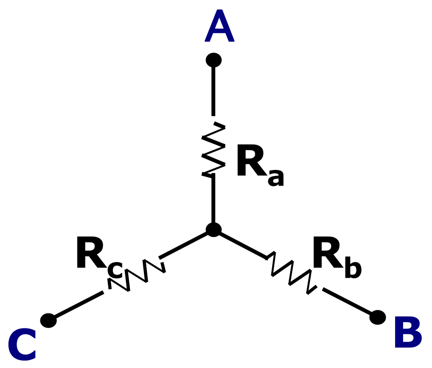
$$R_a = \frac{R_{ab}\,R_{ca}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ab}\,R_{ca}}{\sum R_{ab}}$$

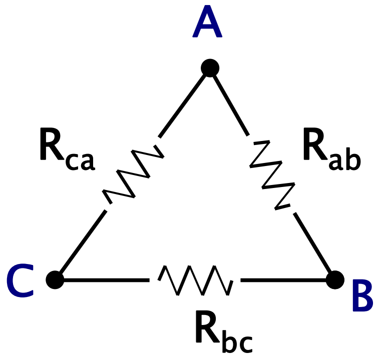
$$R_b = \frac{R_{bc}\,R_{ab}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{bc}\,R_{ab}}{\sum R_{ab}}$$

$$R_c = \frac{R_{ca}\,R_{bc}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ca}\,R_{bc}}{\sum R_{ab}}$$

Star-Delta Transformation

Star to Delta Transformation





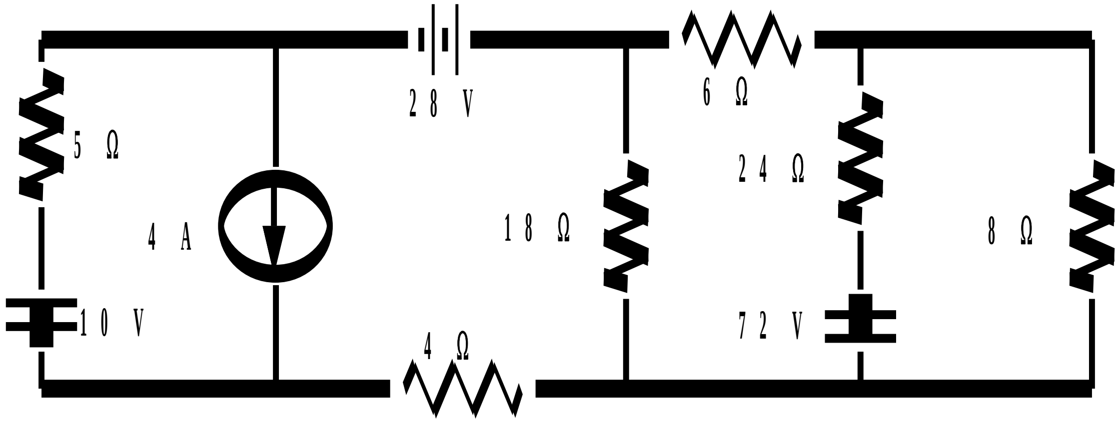
$$R_{ab} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_c} = \frac{\sum R_aR_b}{R_c}$$

$$R_{bc} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_a} = \frac{\sum R_bR_c}{R_a}$$

$$R_{ca} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_b} = \frac{\sum R_cR_a}{R_b}$$

Illustration 1

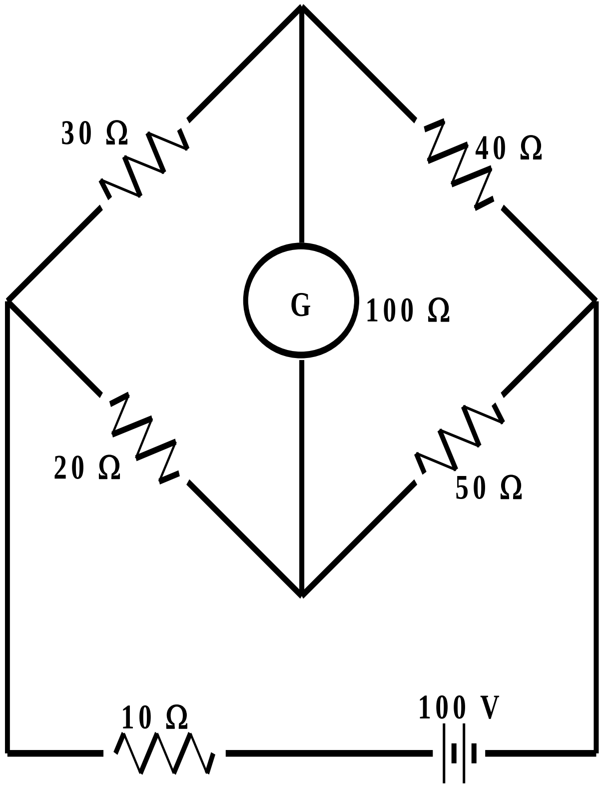
Find the current through 8 Ω resistor by source transformation method.



Ans: 1 A

Illustration 2

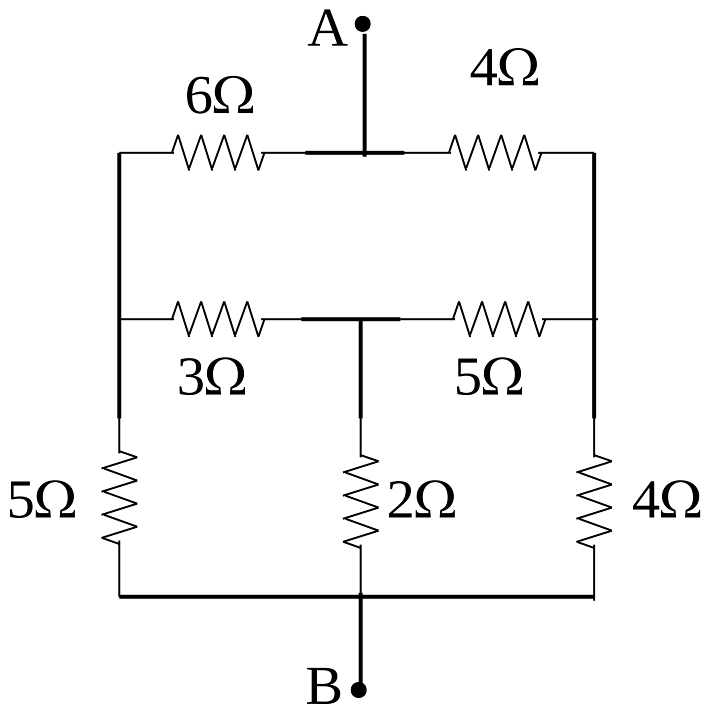
For the circuit shown, determine the total power supplied by the source using star-delta transformation



Ans: $P_{supplied}$ = 223.12 W

Illustration 3

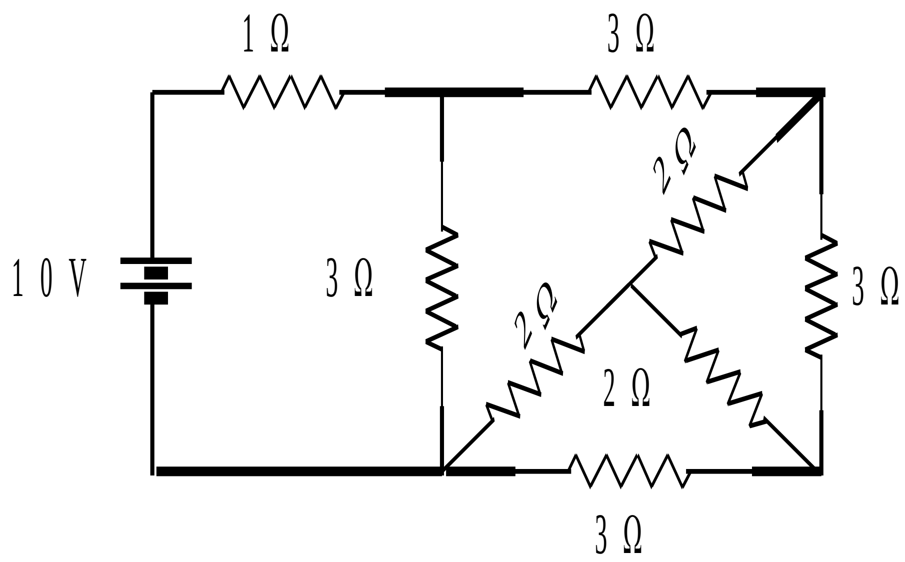
Determine the equivalent resistance between the terminals A and B.



Ans: 3.85 Ω

Homework 1

Find the power dissipated in 1Ω resistor and the power delivered by 10V source using network reduction technique



Ans: $P_{1\Omega} = 11.65$ W, $P_{10V} = 34.2$ W

Basic Electrical Technology

Mesh Current Analysis

Objective

Application of KVL for the analysis of DC circuits

Introduction

Mesh

- A closed path for the flow of current

Kirchhoff's Voltage Law (KVL)

- The algebraic sum of voltages in a mesh is zero

Mesh Current Analysis Method

- Transform all the current sources present in the circuit to voltage sources

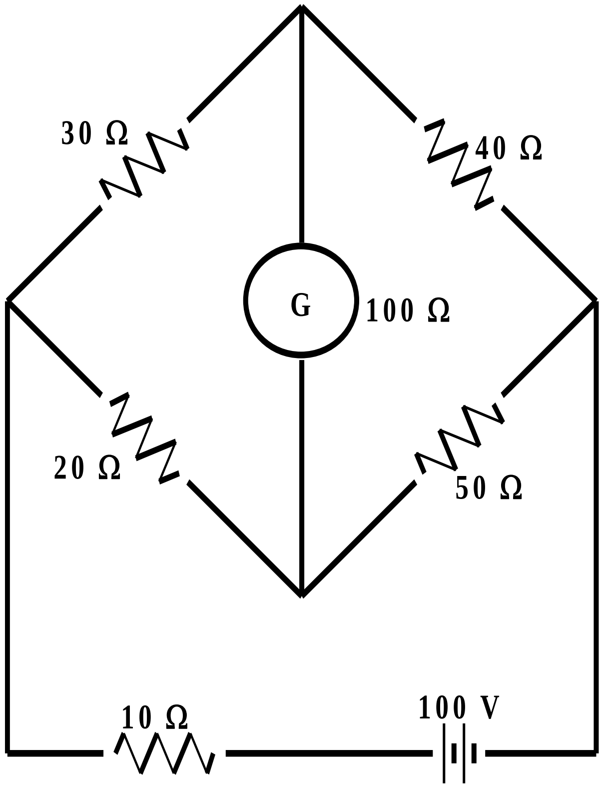
- Mark different currents in all the independent meshes of the given network

- Write KVL equations for these independent meshes

- Solve for the currents

Illustration 1

Determine the current through the galvanometer “**G**”



Answer : 84 mA

Illustration 1 contd...

How to write the network equations by inspection?

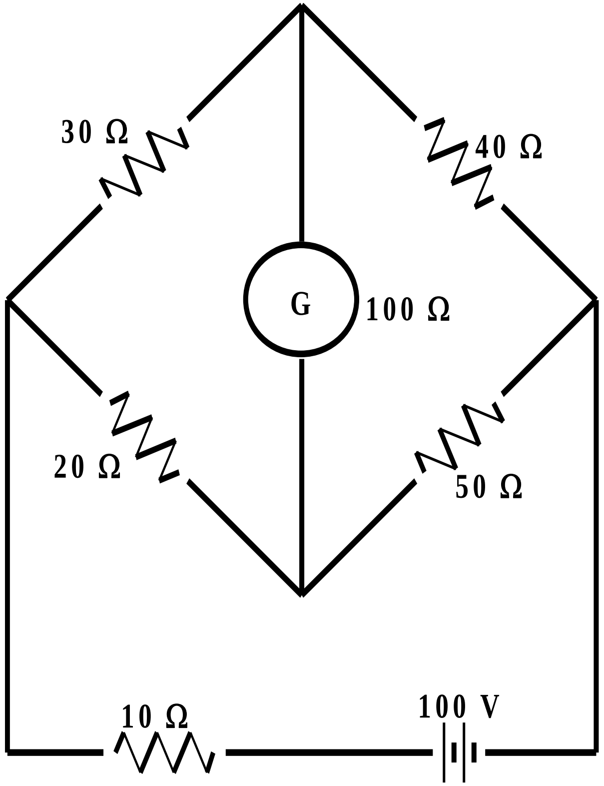
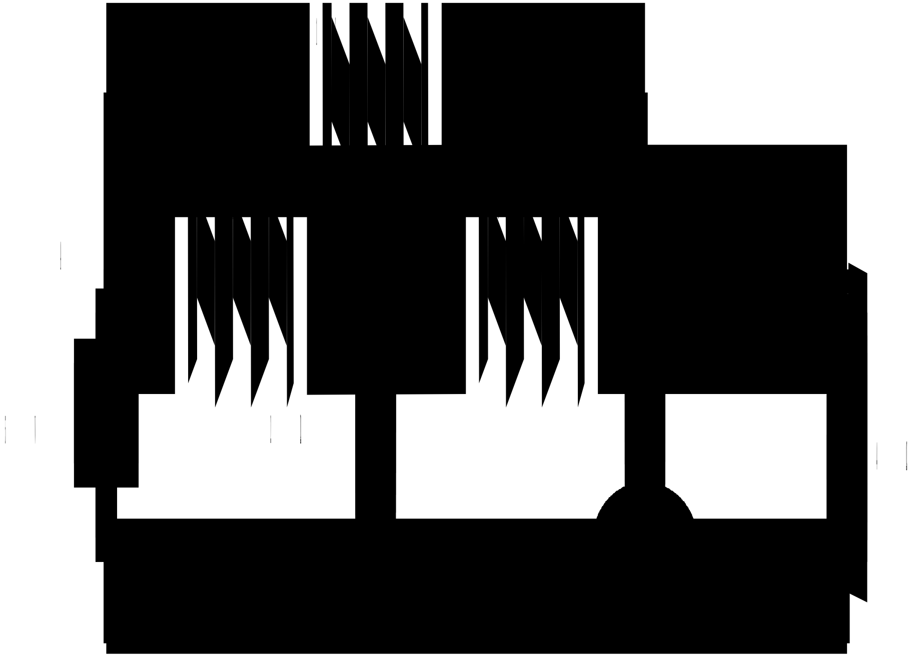


Illustration 2

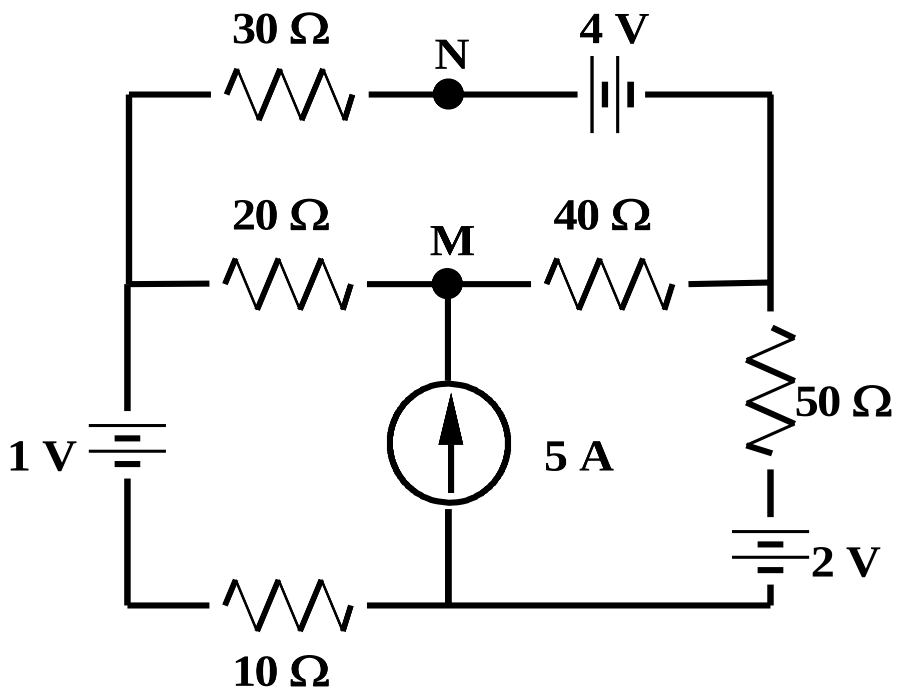
Determine the current and its direction through the 2 Ω resistor. Also, determine the potential difference between A & B



Answer : $I_{2\Omega}$=0.575 A (downwards)
V_A-V_B = +2.7 V

Illustration 3

Find the power supplied by the 5 A current source. Also, determine the voltage between the points M & N.



Answer : P_{5A} = 556.5 W
V_M -V_N = 55.8 V

Summary

- Mesh currents are determined
- Other operating conditions can be determined using the mesh currents
- Concept of super-mesh: If there is a current source between two meshes

Basic Electrical Technology

Node Voltage Analysis

Objective

- Application of KCL for analysis of DC circuits

Introduction

Node

- A point in the circuit where three or more than three elements are joined

Kirchhoff's Current Law (KCL)

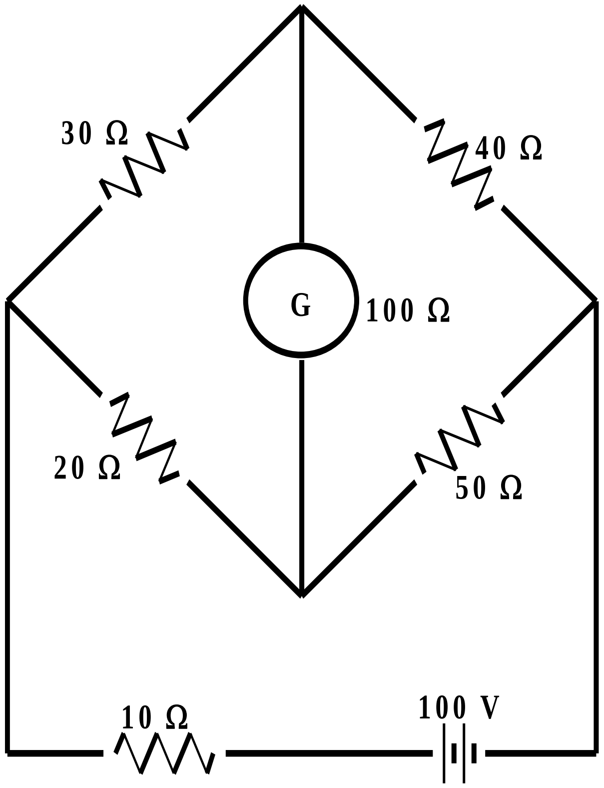
- The algebraic sum of currents at a node is zero

Node Voltage Analysis Method

- Convert all the voltage sources in the circuit to current sources
- Identify nodes in the circuit and assign a voltage for each node
- One of the nodes is the reference node
- Write KCL equations for all the nodes
- Solve for voltages

Illustration 1

Determine the current through the galvanometer “**G**”



Ans: 84 mA

Illustration 1 contd…

How to write the network equations by inspection?

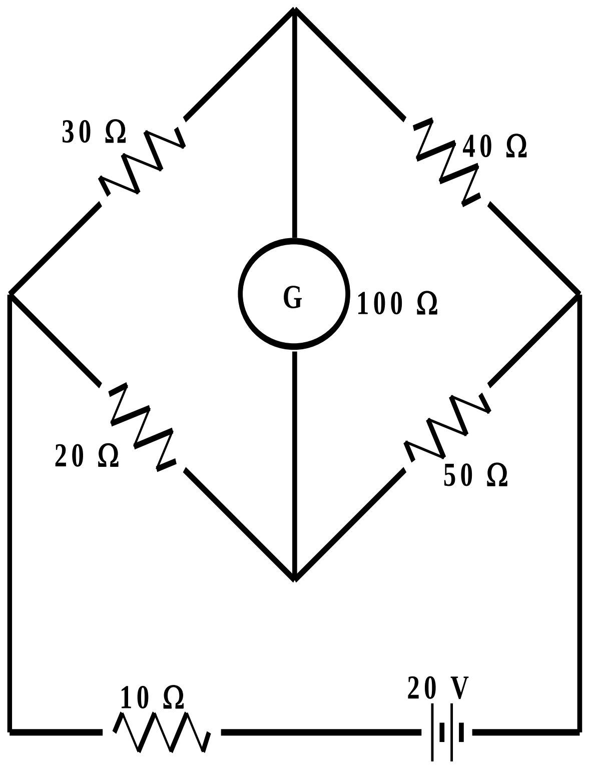
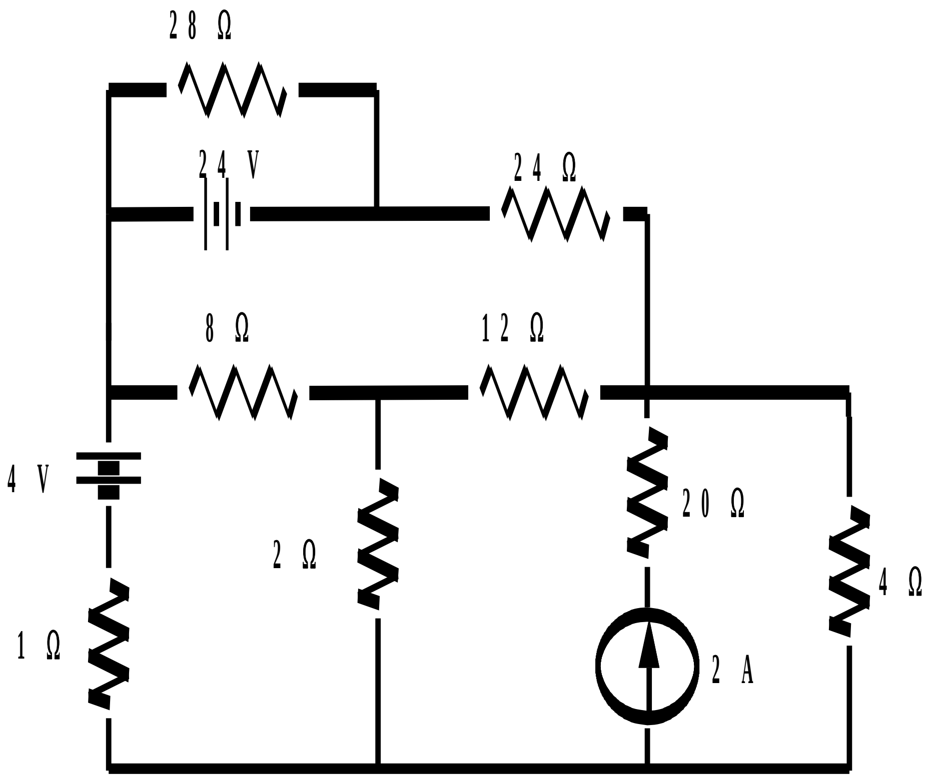


Illustration 2

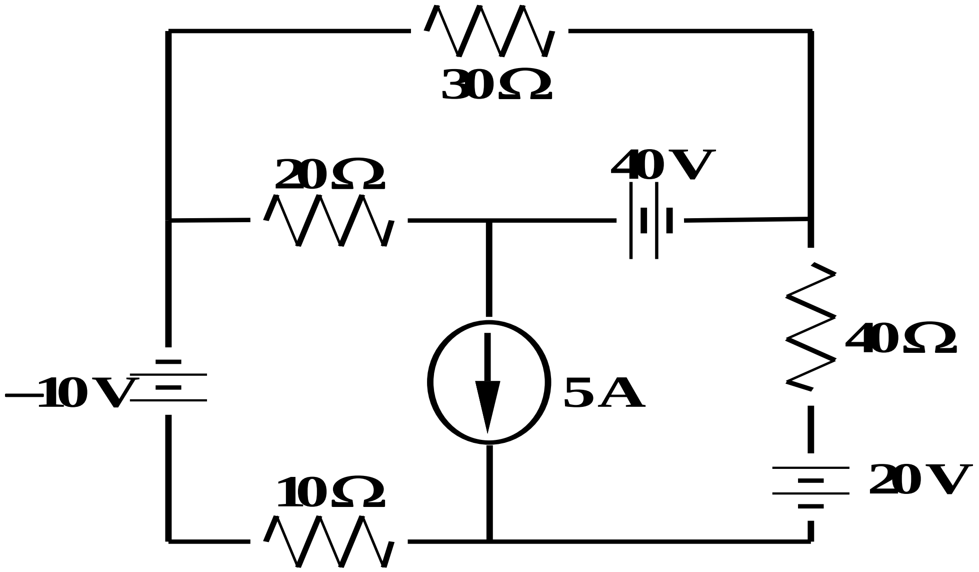
Determine the power dissipated in 8 Ω resistor. Is the 4 V source charging or discharging?



Ans: $P_{8\Omega}$ = 1.386 W
Charging

Illustration 3

Find the current through 40 V battery. Is the battery charging or discharging?



Ans: 4.19 A, Discharging

Summary

- Node voltages are determined

- Other operating conditions can be determined using the node voltages

- Concept of super-node:- If there is a voltage source between two nodes

Basic Electrical Technology

Thevenin's Theorem

Definition

- Any **linear, bilateral** network may be replaced by a single voltage source (called Thevenin's equivalent voltage, $\mathbf{V_{Th}}$) in series with one resistance (called Thevenin's equivalent resistance, $\mathbf{R_{Th}}$) across the load terminals.

- Thevenin's equivalent voltage, $\mathbf{V_{Th}}$, is the open circuit voltage at the load terminals.

- Thevenin's equivalent resistance, $\mathbf{R_{Th}}$, is the equivalent resistance at the load terminals, after replacing the sources by their internal resistances.

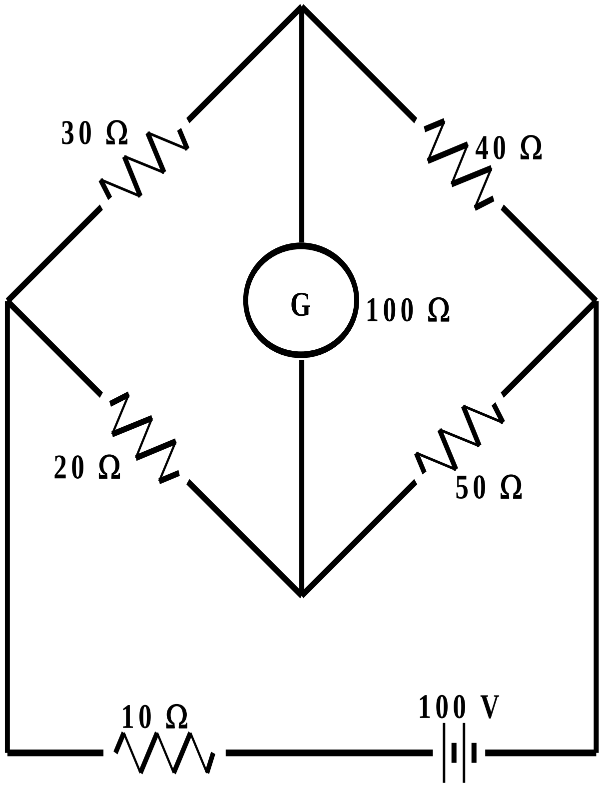
Procedure

- **To find V_{Th} :**
  - Remove the load and keep the terminals open circuited.
  - Apply mesh current / node voltage method
  - Find the voltage across the open circuited terminals.

- **To find R_{Th} :**
  - Keep the load terminals open.
  - Replace all the sources by their internal resistances.
  - Find the equivalent resistance with respect to open circuited load terminals.

Illustration 1

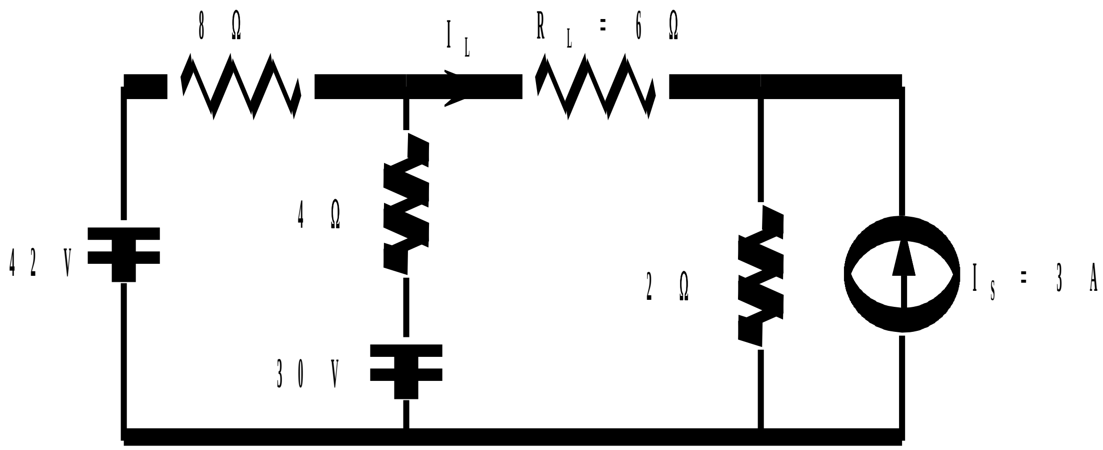
Determine the current through the galvanometer using Thevenin's Theorem



Answer : 84 mA

Illustration 2

For the circuit shown find the current $\mathbf{I_L}$ through 6 Ω resistor using Thevenin's theorem



Ans: I_L = 2.625 A

Illustration 3

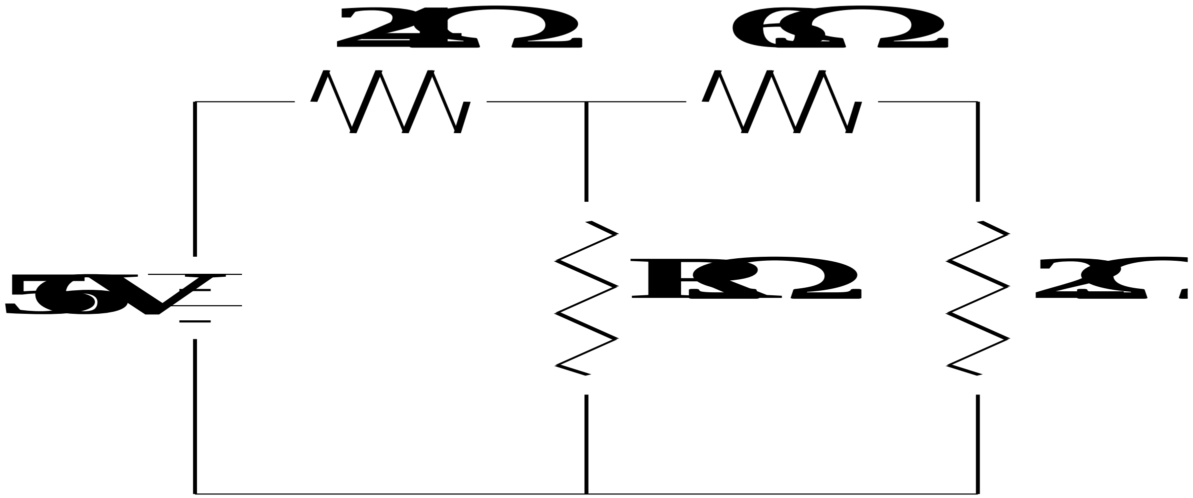
The box shown in the adjacent figure consists of independent dc sources and resistances. Measurements are taken by connecting an ammeter in series with the resistor R_L and the results are shown in the table below. Find the value of R_L for which the current is 0.6 A

R_L	I
10 Ω	2.0 A
20 Ω	1.5 A
?	0.6 A

Ans: $R_L = 80\ \Omega$

Homework 1

Using Thevenin’s theorem, find the value of **R** such that the current through 2 Ω resistor is 1 A



Ans: 8 Ω

Basic Electrical Technology

Superposition & Maximum Power Transfer Theorems

Terminologies Used

- **Linear element:** V-I characteristics is linear. E.g: R, L, C
- **Non-linear element:** V-I characteristics is non-linear. E.g: Diode
- **Unilateral element:** Property changes with the direction of operation. E.g: Diode
- **Bi-lateral element:** Property does not change with direction of operation. E.g: R, L, C
- **Linear Circuit:** Circuit with linear elements only
- **Bi-lateral circuit:** Circuit with bi-lateral elements only.
- **Response:** The output of the network. E.g: current, voltage

Superposition Theorem

Definition

- In **any linear, bi-lateral** network, **total response** is the **sum** of partial responses.

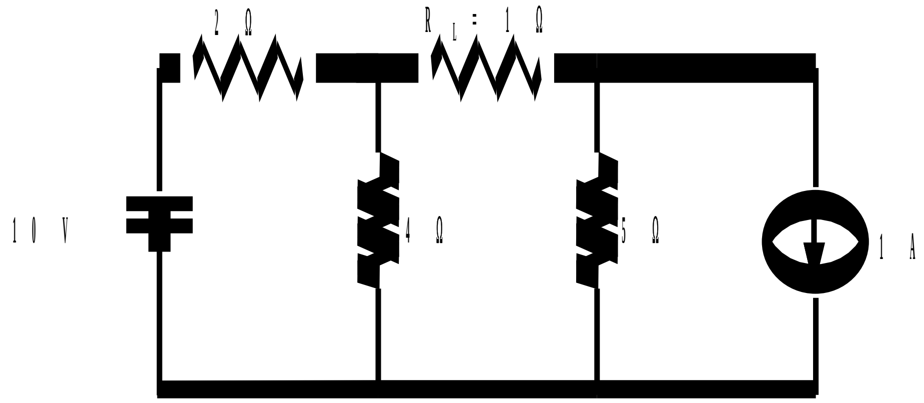
- In any linear, bilateral network, the total response may be determined by adding the responses due to individual sources, considering one source at a time and replacing the other sources by their internal resistances.

Procedure

1. Draw the circuit with passive elements only.
2. Place one of the sources in its position.
3. Replace the other sources by their internal resistances. Ex: Ideal voltage source by short circuit, ideal current source by open circuit.
4. Find the response using one of the methods, i.e., network reduction, mesh current, node voltage methods.
5. Repeat the procedure for all the sources.
6. Add the responses due to individual sources.

Illustration 1

Find the current through 1 Ω resistor using Superposition theorem



Ans: $I_{1\Omega}$ = 1.59 A

Illustration 2

Find the current I_0 using Superposition theorem

Ans: I_0 = -0.105 A

Maximum Power Transfer Theorem

Definition

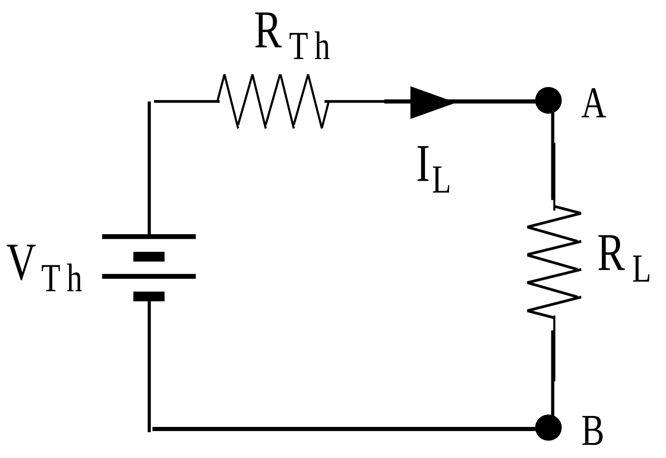
In **any linear, bi-lateral network**, maximum power will be transferred to the load from the network when the load resistance is equal to the internal resistance of the network.

Proof

Consider the Thevenin's equivalent circuit of a network

$I_L = \frac{V_{Th}}{R_{Th}+R_L}$

$P_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$



$For P_L \ to\ be\ maximum, \frac{dP_L}{dR_L} = 0$

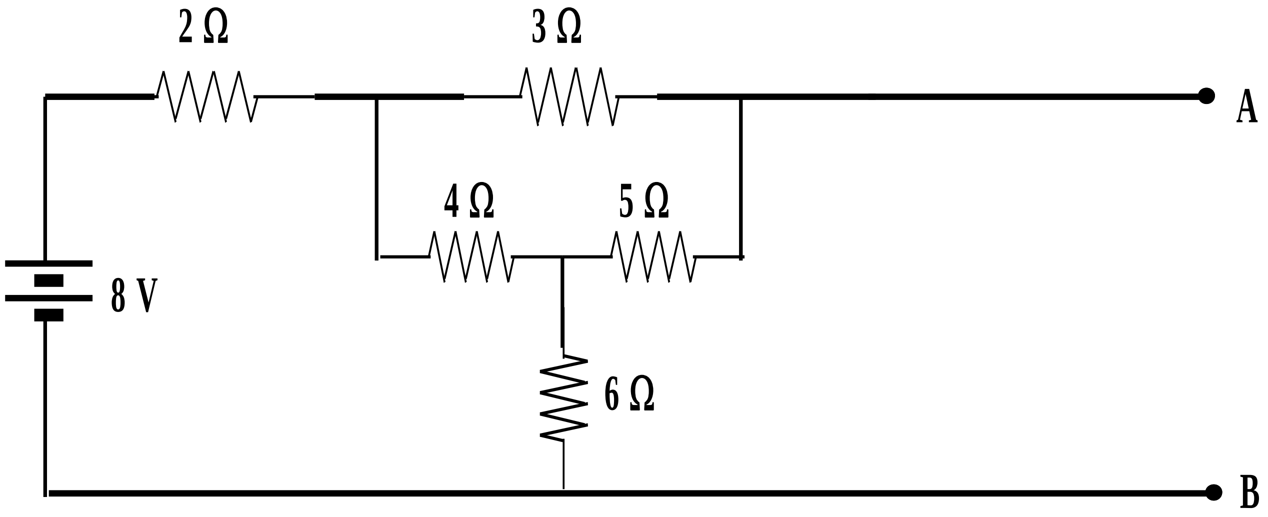
$which\ yields, \boldsymbol{R_L = R_{Th}}$

$Maximum\ Power,$

$P_{L-max} = \left(\frac{V_{Th}}{R_{Th}+R_{Th}}\right)^2 R_{Th} = \frac{V_{Th}^2}{4R_{Th}}$

Illustration 3

Determine the value of resistor to be connected across the terminals A & B such that maximum power is transferred to the that resistor. Also, find the value of maximum power.



Ans: 3.41 Ω, 2.43 W

Basic Electrical Technology

RL Transient

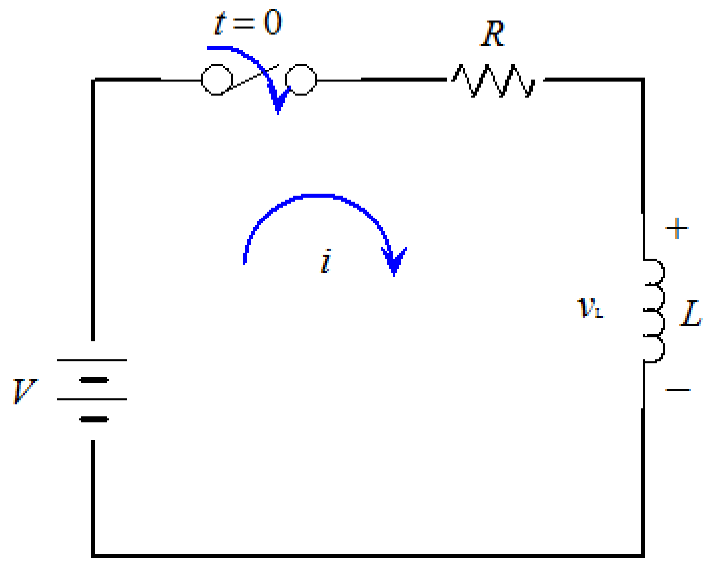
Growth of Current in an Inductive Circuit

Applying KVL,

$$V - R\,i - L\frac{di}{dt} = 0$$

Initial Conditions,

$$At\ t = 0\ sec, i = 0\ A$$



Final current & voltage equation,

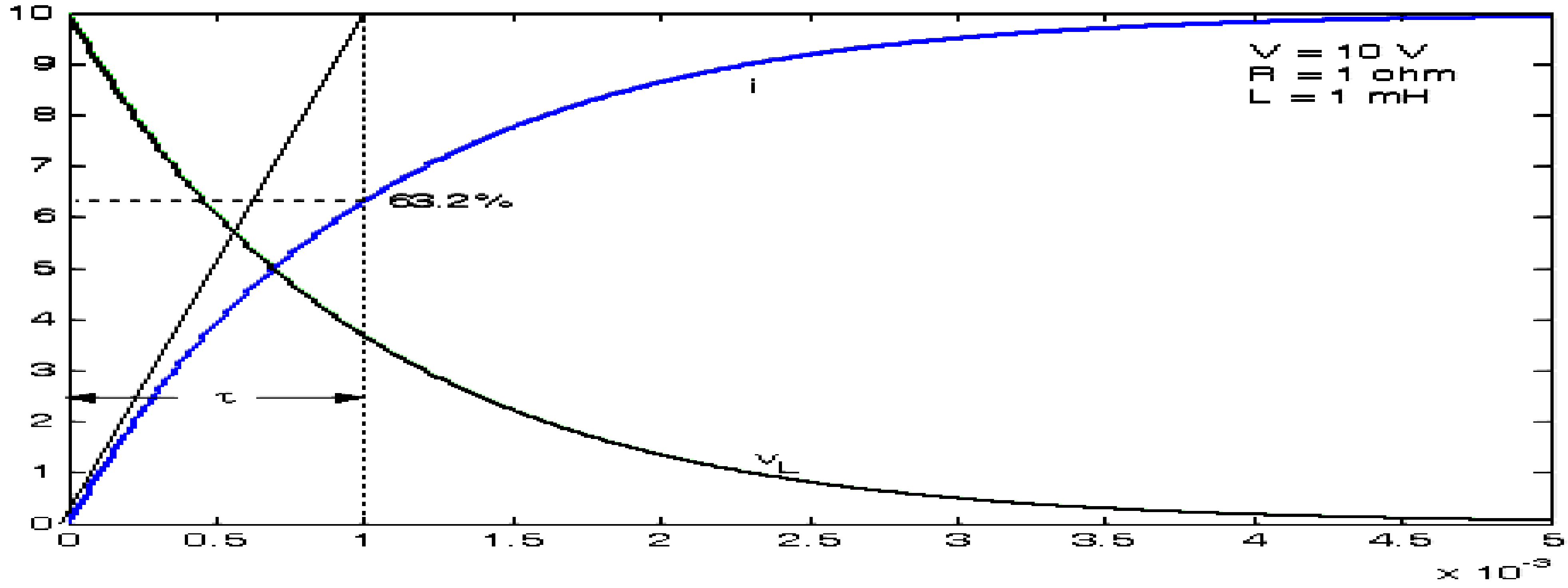
$$i = \frac{V}{R}\left(1 - \ e^{-\frac{Rt}{L}}\right)$$

$$v_L = V\,e^{-\left(\frac{R}{L}\right)t}$$

Growth of current in an inductive circuit

Time Constant (τ): Time taken by the current through the inductor to reach its final steady state value, had the initial rate of rise been maintained constant

$$\tau = \frac{L}{R}$$

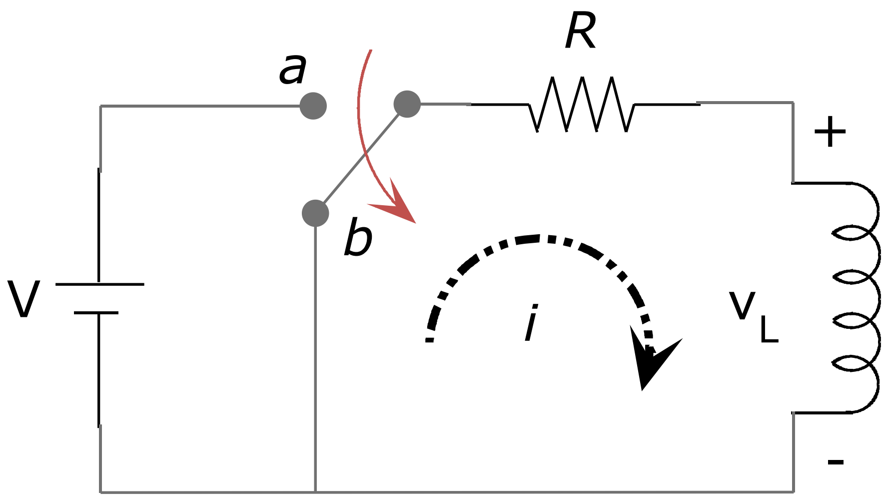


Decay of current in an Inductive Circuit

- Initial current is through inductor is

$$I_0 = V/R$$

- At $t = 0$, switch is moved from position ***a*** to ***b***



Applying KVL,

$$L\frac{di}{dt} + R\,i = 0$$

Using initial conditions and then solving

$$i = I_0 e^{\left(\frac{-Rt}{L}\right)}$$

$$v_L = -V\,e^{-\left(\frac{Rt}{L}\right)}$$

Decay of current in an Inductive Circuit

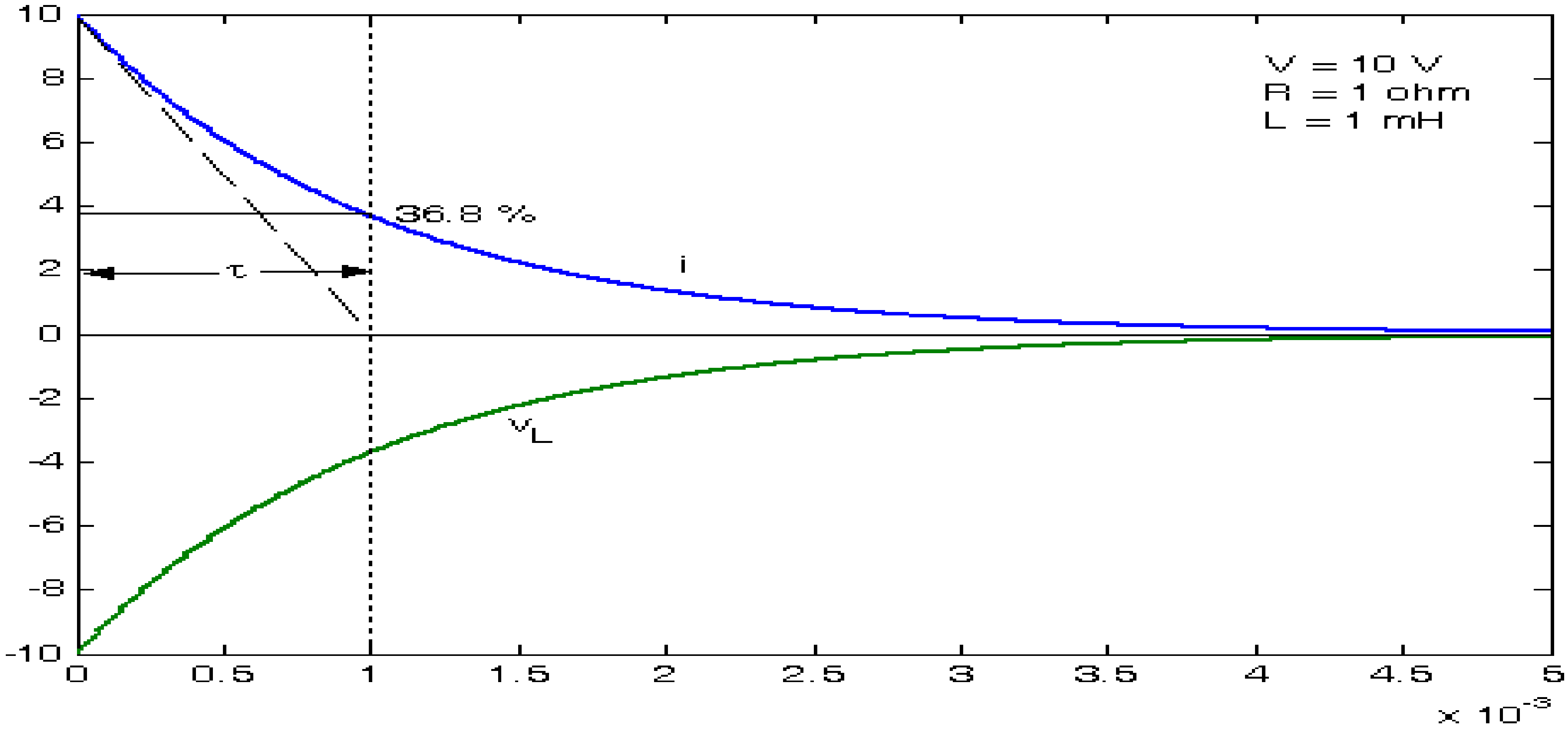
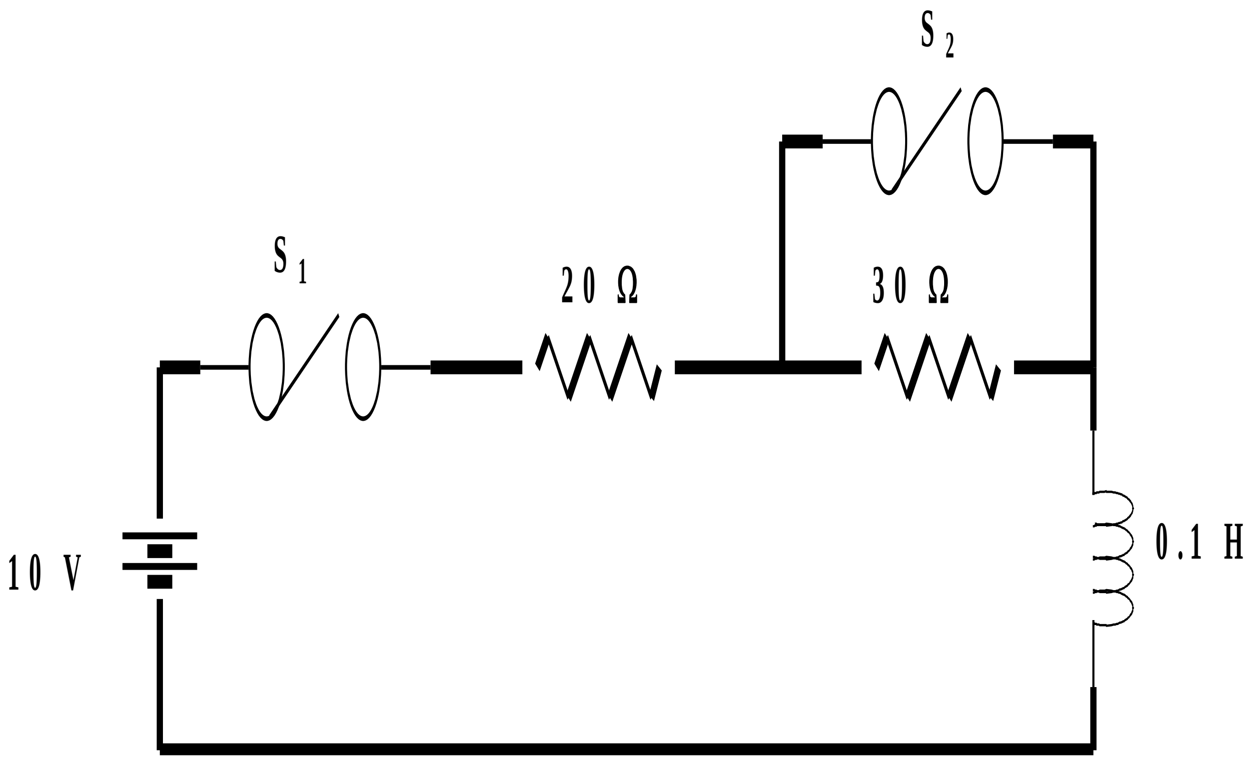
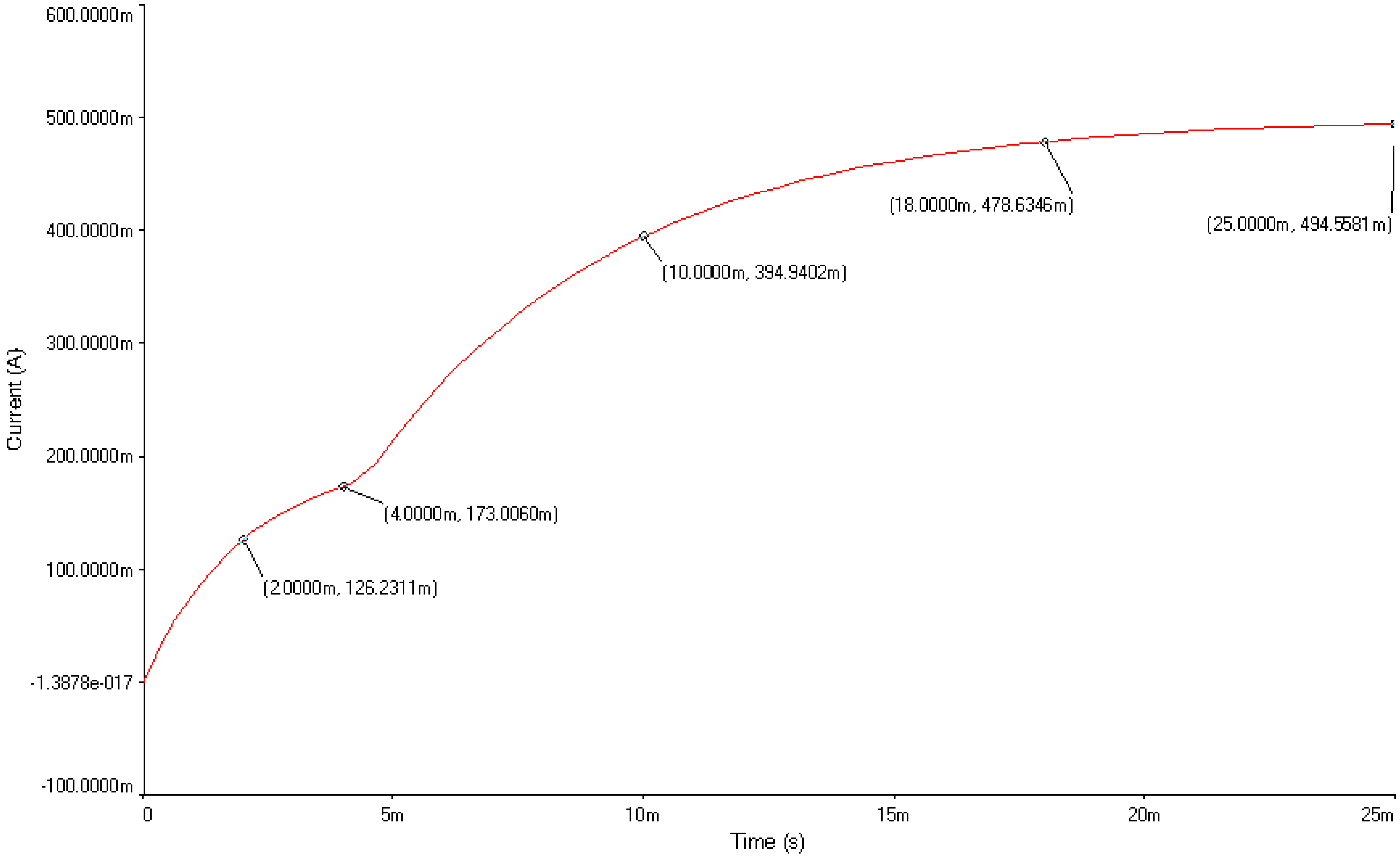


Illustration 1

In the circuit shown below, both the switches, S_1 & S_2 , are open initially. At t = 0 sec, S_1 is closed (& S_2 remains open). At t = 4 ms S_2 is closed. Sketch the inductor current $\boldsymbol{i(t)}$ for $0 \leq t \leq 25$ ms.



Solution



600.0000m
500.0000m
400.0000m
300.0000m
200.0000m
100.0000m
-1.3878e-017
-100.0000m
Current (A)
0
5m
10m
15m
20m
25m
Time (s)
(2.0000m, 126.2311m)
(4.0000m, 173.0060m)
(10.0000m, 394.9402m)
(18.0000m, 478.6346m)
(25.0000m, 494.5581m)

Illustration 2

A coil of resistance 5Ω and inductance of 0.02H is connected to a battery of voltage 12V for a long time. At t = 0, the coil is short circuited. Find the time taken for the current to reach the value 1.2A.

Ans: 2.77 m-sec

Illustration 3

An R-L series circuit is designed for a steady current of 250mA. A current of 120 mA flows in the circuit at an instant 0.1 sec after connecting the supply voltage. Calculate i) time constant of the circuit ii) the time from closing the circuit at which the circuit current has reached 200 mA.

Ans:

i. $\zeta = 0.1529$ s

ii. $t = 0.2461$ s

Basic Electrical Technology

RC Transient

Charging of a Capacitor through a Resistor

Applying KVL,

$$V - Ri - v_c = 0$$

where, $i = C\frac{dv_c}{dt}$

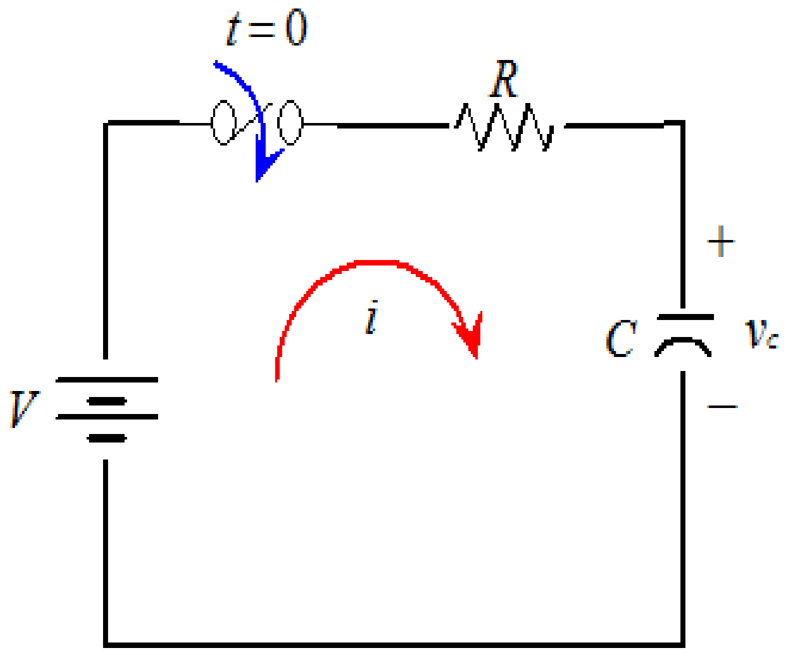
Initial Conditions,

$$At\ t = 0\ sec, V_c = 0\ V$$

Final current & voltage equation,

$$v_c = V\left(1 - e^{-\frac{1}{RC}t}\right)$$

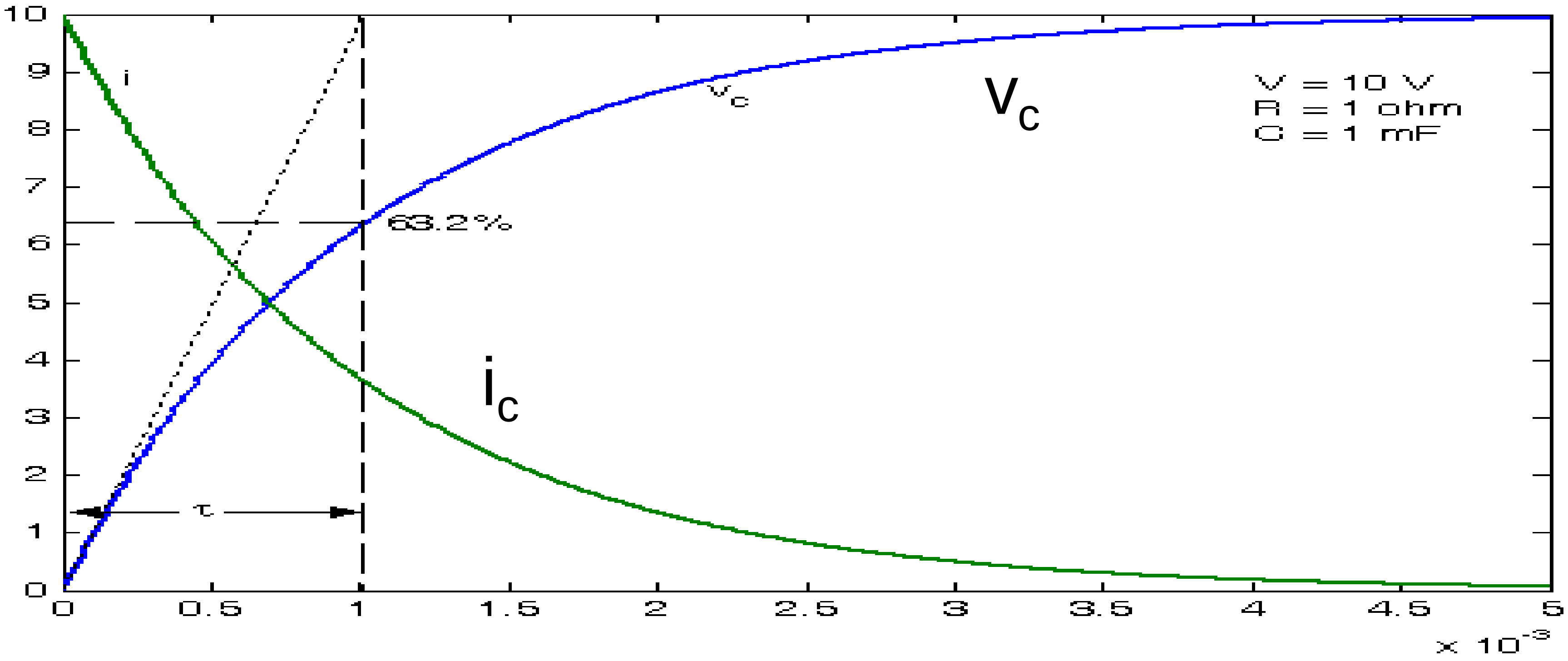
$$i_c = \left(\frac{V}{R}\right) e^{-\left(\frac{1}{RC}\right)t}$$



Growth of current in an inductive circuit

Time Constant (τ): Time taken by the voltage of the capacitor to reach its final steady state value, had the initial rate of rise been maintained constant

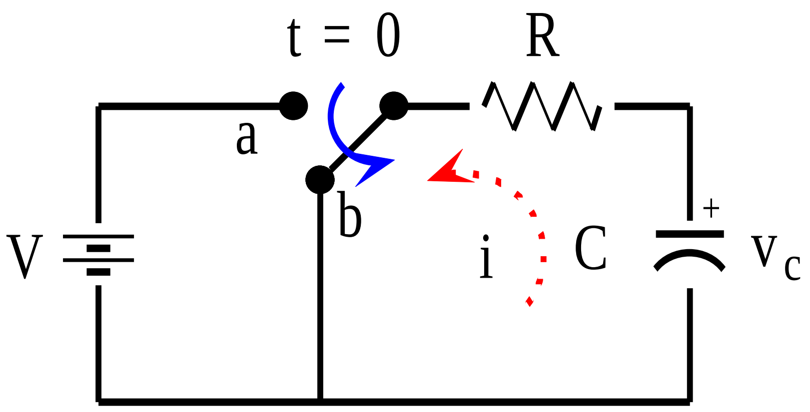
$$\tau = RC$$



Discharging of a Capacitor through a Resistor

- Capacitor is initially charged to a voltage $\boldsymbol{V}$

- At $\boldsymbol{t = 0}$, switch is moved from position $\boldsymbol{a}$ to $\boldsymbol{b}$



Applying KVL,

$$\boldsymbol{v_c - Ri = 0}$$

Where, $\boldsymbol{i = -C\frac{dv_c}{dt}}$

Using initial conditions and then solving

$$\boldsymbol{v_c = V\, e^{-\left(\frac{1}{RC}\right)t}}$$

$$\boldsymbol{i_c = -I\, e^{-\left(\frac{1}{RC}\right)t}}$$

Discharging of a Capacitor through a Resistor

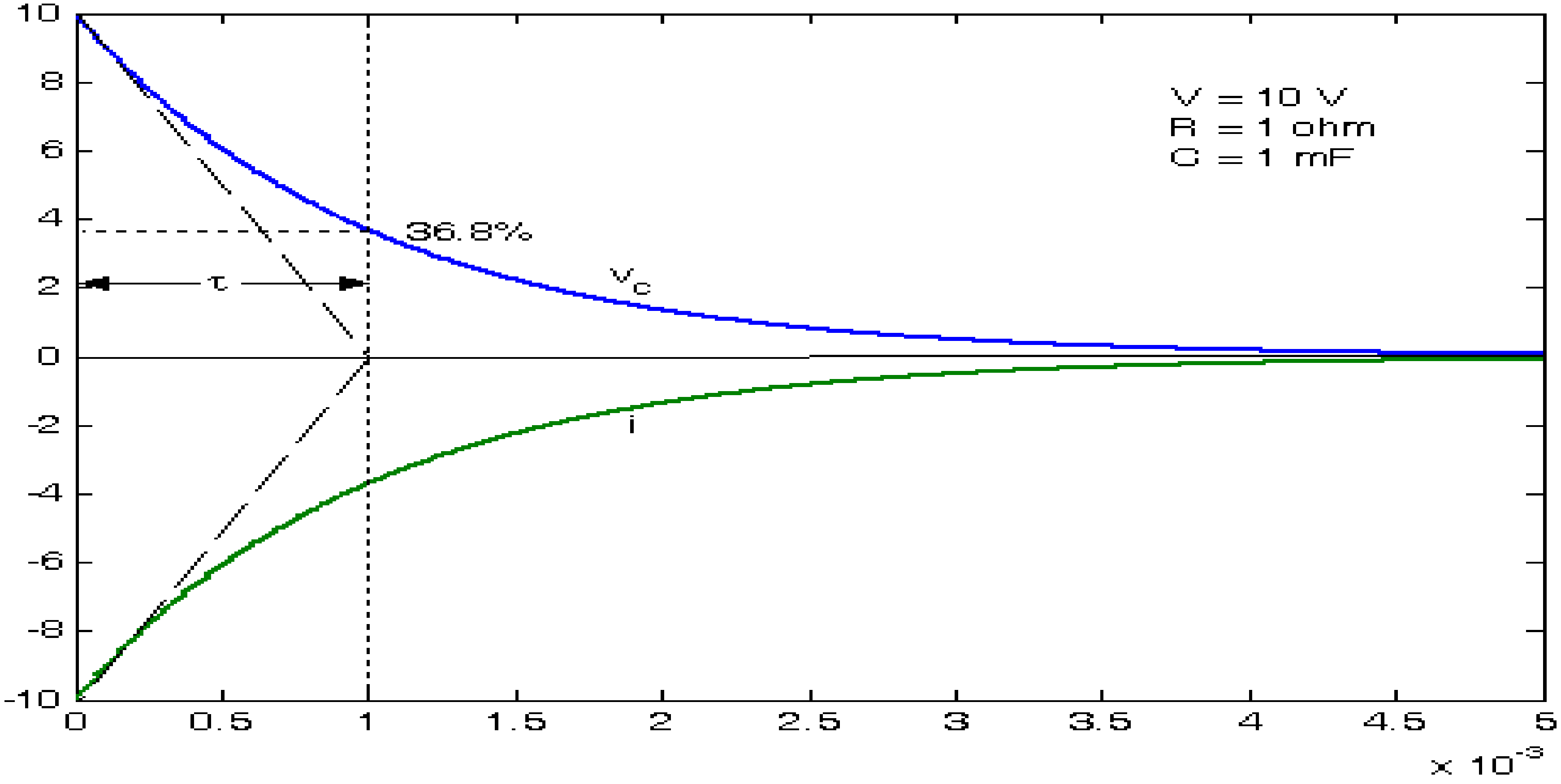
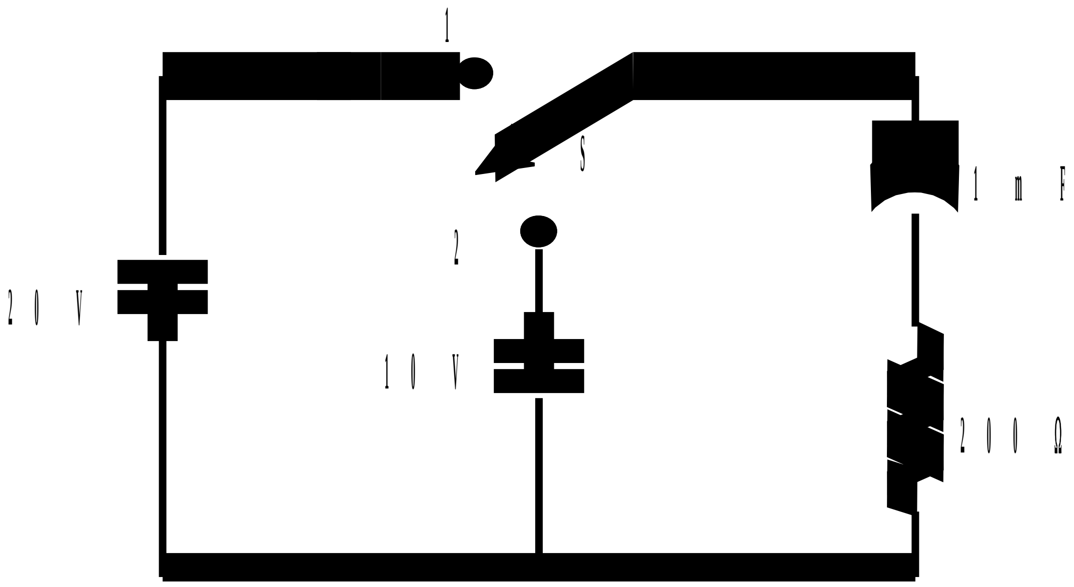


Illustration 1

In the network shown in the figure, the switch is closed to position 1 at t = 0 & is moved to position 2 at t = 0.4 sec. Determine the voltage across the capacitor vc(t) & sketch it for $0 \leq t \leq 1$ second

Also find the value of 't' for which vc(t) = 0



Solution

$v_c = 20(1 - e^{-t/0.2})$
$At\ t = 0.4\ sec, v_c = 17.29\ V$

$After\ 0.4\ second, the\ switch\ is\ in\ postion\ 2$
$v_c = -10 + 27.29e^{-(t-0.4)/0.2}$
$At\ t = 1\ sec, v_c = -8.64\ V$
$\boldsymbol{Ans:\ At\ t = 0.6\ sec, v_c = 0\ V}$

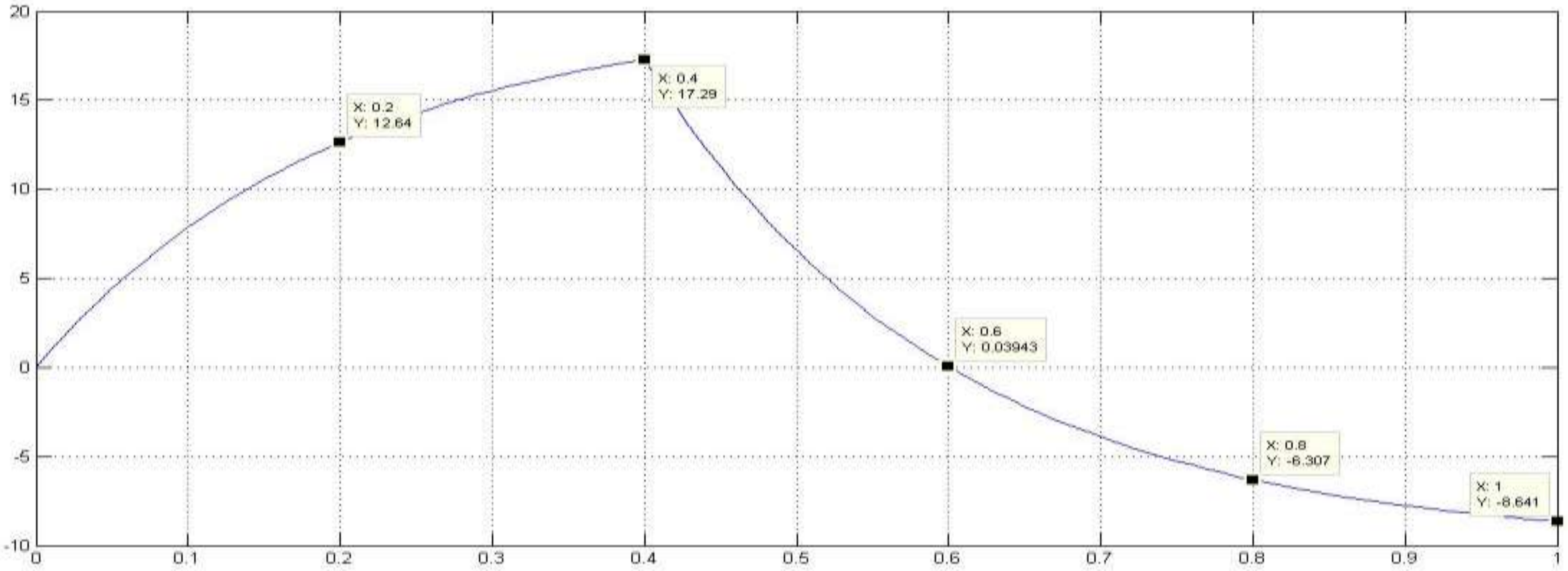


Illustration 2

An 8uF capacitor is connected in series with a 0.5MΩ resistor, across a 200 V d.c. supply through a switch. At t=0 seconds, the switched is turned on, Calculate

i. Time constant of the circuit

ii. Initial charging current.

iii. Time taken for the potential difference across the capacitor to grow to 160V.

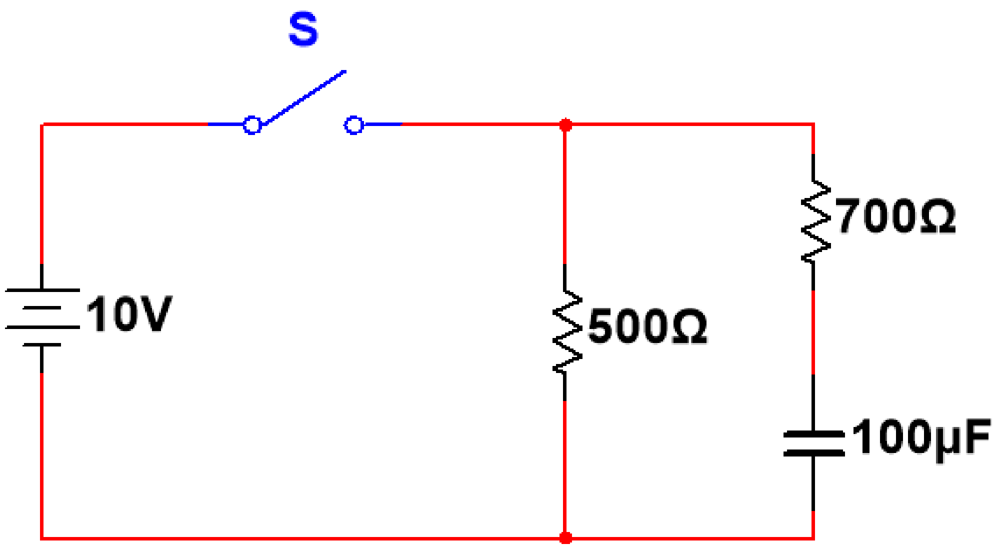
iv. Current & potential difference across the capacitor 4.0 seconds after the switch is turned on.

v. Derive the expressions used

Ans:
(i) 4 seconds, (ii) 400 μA , (iii) 6.44 seconds
(iv) 126.424 V & (v) 147.15 μA

Illustration 3

For the circuit shown in the figure, the switch 'S' is closed at t = 0 seconds. Determine how long it takes after the switch is closed for the total current drawn from the supply reaches 25mA



Ans:

t = 0.0735 s

Basic Electrical Technology

Introduction to Magnetism

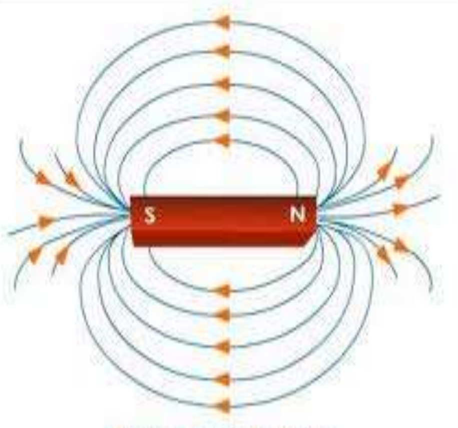
Magnetism

- A physical phenomena by which materials exert attractive or repulsive force on other materials.

- **Magnetic Materials**
  - Properties:
    - Points in the direction of geometric north and south pole when suspended freely and attracts iron fillings.
  - Classification:
    - Natural Magnets: Lodestone
    - Temporary magnets (exhibits these properties when subjected to external force)
    - Non-magnetic materials.

Definitions

- **Magnetic Line of Force**
  - Closed path radiating from north pole, passes through the surrounding, terminates at south pole and is from south to north pole within the body of the magnet.



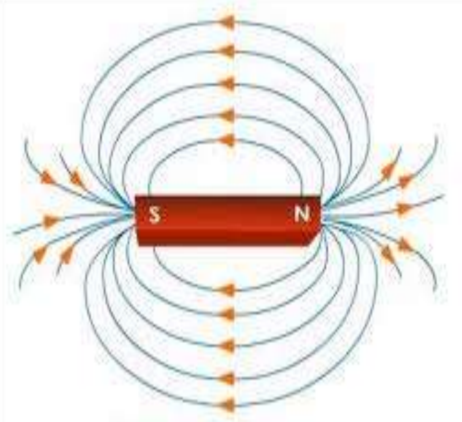
Field Lines Around a Bar Magnet

- **Magnetic Field**
  - The space around which magnetic lines of force act.
  - Strong near the magnet and weakens at points away from the magnet.

- **Magnetic Flux (ϕ)**
  - Analogous to Electric Current
  - Number of magnetic lines of force created in a magnetic circuit.
  - Unit : Weber (Wb)

Definitions

- **Magnetic Flux Density (B)**
  - Analogous to Current Density
  - No. of magnetic lines of force created in a magnetic circuit per unit area normal to the direction of flux lines
  - $\boldsymbol{B} = \boldsymbol{\Phi}/\boldsymbol{A}$
  - Unit : Wb/m^2 (Tesla)



Field Lines Around a Bar Magnet

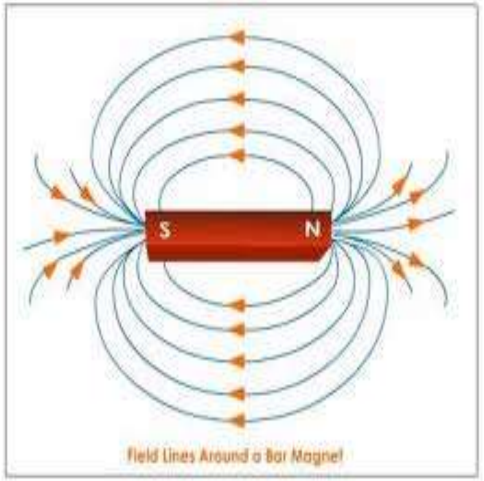
- **Magneto Motive Force (F)**
  - Analogous to EMF
  - Force which drives the magnetic lines of force through a magnetic circuit
  - $\boldsymbol{F} = \boldsymbol{\Phi} \times \boldsymbol{S} = \boldsymbol{N} \times \boldsymbol{I}$,

    Where, $\boldsymbol{\Phi}$ = Magnetic flux, $\boldsymbol{S}$ = Reluctance of the magnetic path,

    $\boldsymbol{N}$ = No. of turns of the coil, $\boldsymbol{I}$ = Current flowing through the coil
  - Unit: AT (Ampere-Turns)

Definitions

- **Magnetic Field Strength (H)**
  - Analogous to Electric Field Strength
  - The magneto motive force per meter length of the magnetic circuit
  - $\boldsymbol{H = (N \times I)/l}$
  - Unit: AT/m

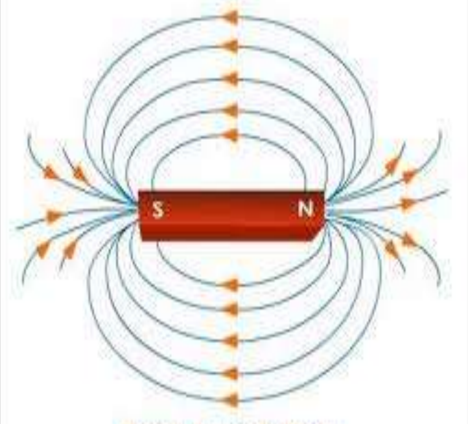


- **Permeability (μ)**
  - Analogous to Conductivity
  - A property of a magnetic material which indicates the ability of magnetic circuit to carry magnetic flux.
  - $\boldsymbol{\mu = B\ /\ H}$
  - $\mu_0 = 4\pi \times 10^{-7} \Longrightarrow$ Permeability of free space or air or non magnetic material
  - Unit: H/m

- **Relative Permeability (μ_r)**
  - Permeability of the material with reference to air / vacuum
  - $\boldsymbol{\mu_r = \mu/\mu_0}$

Definitions

- **Reluctance (S)**
  - Analogous to Resistance
  - Opposition of a magnetic circuit to the setting up of magnetic flux in it.
  - Unit: AT/Wb



Field Lines Around a Bar Magnet

- **Derivation of an expression for reluctance**

$$H = (N \times I)/l$$

$$\mu = B \,/\, H$$

$$B \;= \Phi /\, A$$

$$F \;= N \;\times\; I = H \;\times l = (B/\mu) \times l = ((\Phi /\, A\,)/\mu) \times l = \frac{\Phi}{\mu A} \times l$$

$$F = \frac{\Phi}{\mu_0 \mu_r A} \times l = \Phi \times S$$

$$S = \frac{l}{\mu_0 \mu_r A}$$

Illustration

A ring made of ferromagnetic material has 500 mm^2 as cross-sectional area and 400 mm as mean circumference. A coil of 600 turns is wound uniformly around it. Calculate:

a) The reluctance of the ring,

b) The current required to produce a flux density of 1.6 T in the ring.

Take μ_r of the ferromagnetic material as 800 for flux density of 1.6 T

Ans:

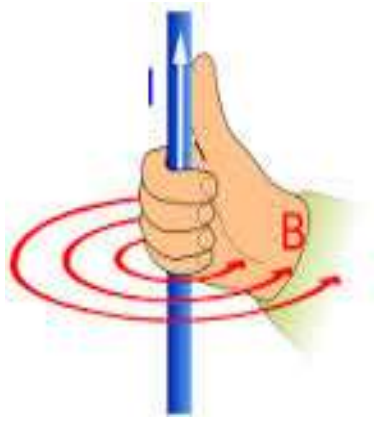
a) 795774.72 AT/Wb

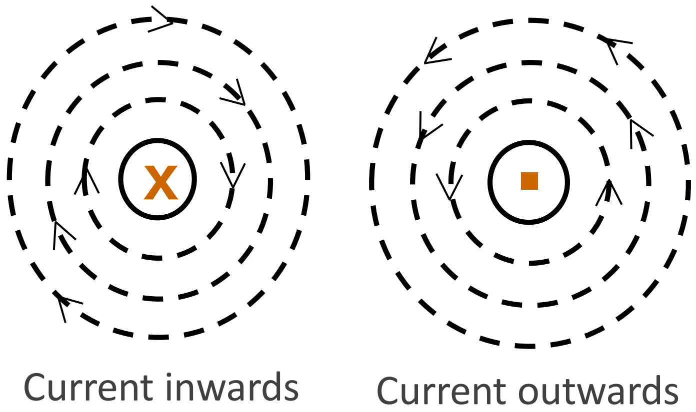
b) 1.06 A

Magnetic Field (in a Current-Carrying Conductor)

- An electric current flowing in a conductor creates a magnetic field around it.

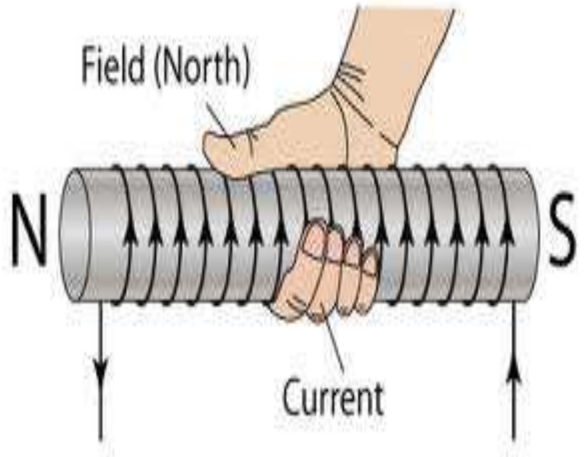
- **Direction of magnetic field**
  - **By Maxwell's Right Hand Grip Rule:**
  - Assume that the current carrying conductor is held in right hand so that the fingers wrap around the conductor and the thumb is stretched along the direction of current. Wrapped fingers will show the direction of circular magnetic field lines.

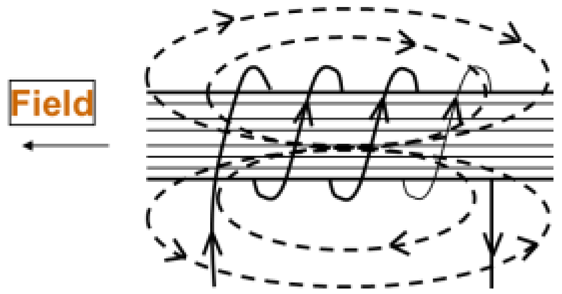




Magnetic Field (in a Solenoid)

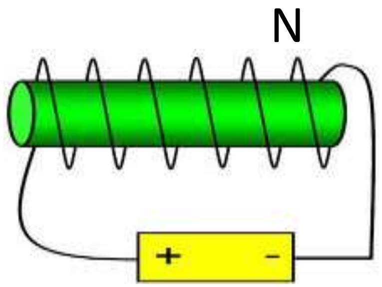
- **Direction of magnetic field**
  - **By Right Hand Grip Rule:**
  - If the coil is gripped with the right hand, with the fingers pointing in the direction of the current, then the thumb, outstretched parallel to the axis of the solenoid, points in the direction of the magnetic field inside the solenoid

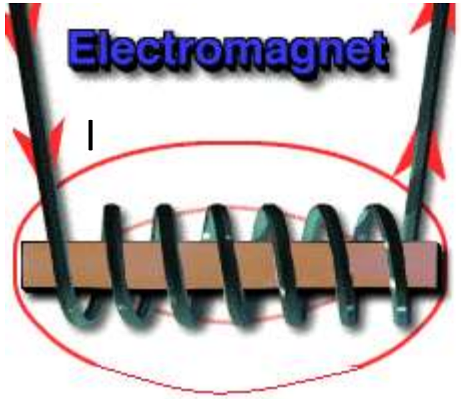




Electromagnets

- **Principle:** An electric current flowing in a conductor creates a magnetic field around it
- Strength of the field is proportional to the amount of current in the coil.
- The field disappears when the current is turned off.
- A simple electromagnet consists of a coil of insulated wire wrapped around an iron core.
- Widely used as components of motors, generators, relays etc.



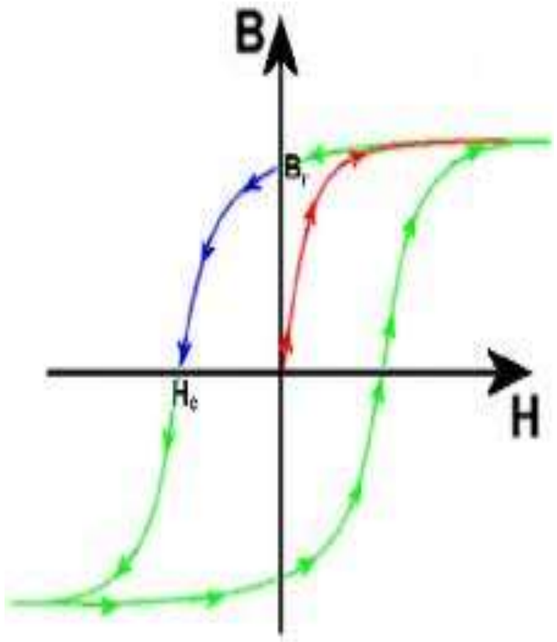


Losses in Magnetic Circuit

Hysteresis Loss

- Lagging of magnetization or flux density behind the magnetizing force is called **Magnetic Hysteresis**

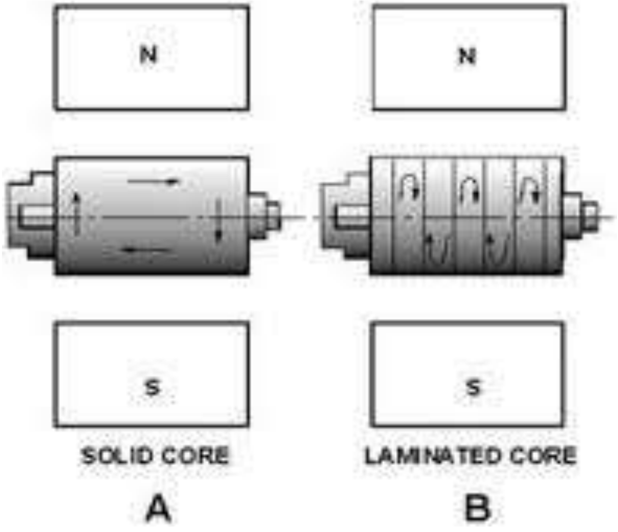
- The energy dissipated as heat in the process of magnetization and demagnetization which is proportional to the area of hysteresis loop is the **Hysteresis Loss**



Hysteresis loop

Losses in Magnetic Circuit

- **Eddy Current Loss**
  - The varying flux in the magnetic core induces emf and hence eddy current within the material.
  - Flow in closed loops in planes perpendicular to the magnetic field
  - Results in loss of power and heating of the material.
  - Cores of electric machines are laminated to reduce eddy current loss



Comparison of Electric and Magnetic Circuits

Analogy:

Electric Circuits	Magnetic Circuits
Current	Flux
Current Density	Flux Density
EMF	MMF
Conductivity	Permeability
Resistance	Reluctance

Differences:

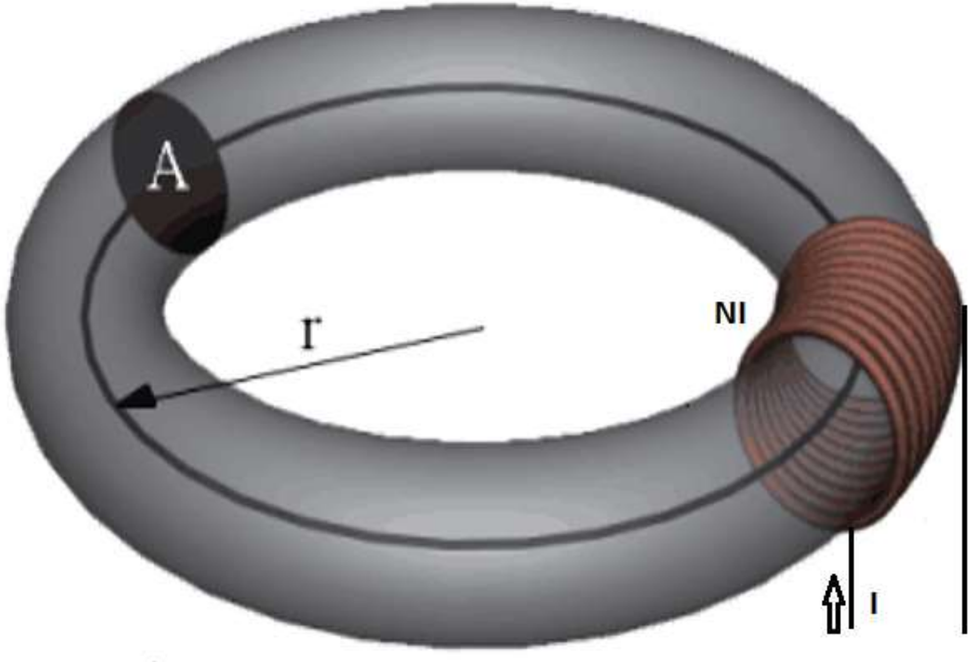
Electric Circuits	Magnetic Circuits
Current actually flows	Flux does not flow
Current can not flow in air / vacuum	Flux can be created in air / vacuum

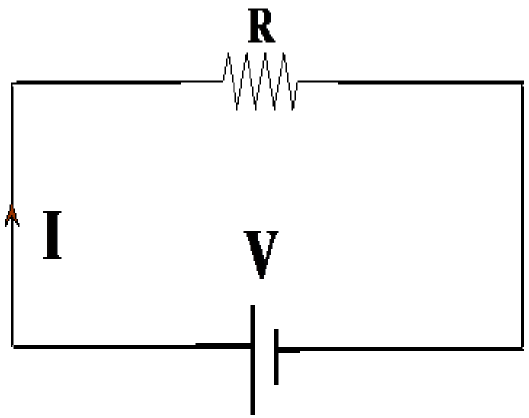
Basic Electrical Technology

Series Magnetic Circuits

Magnetic Circuits

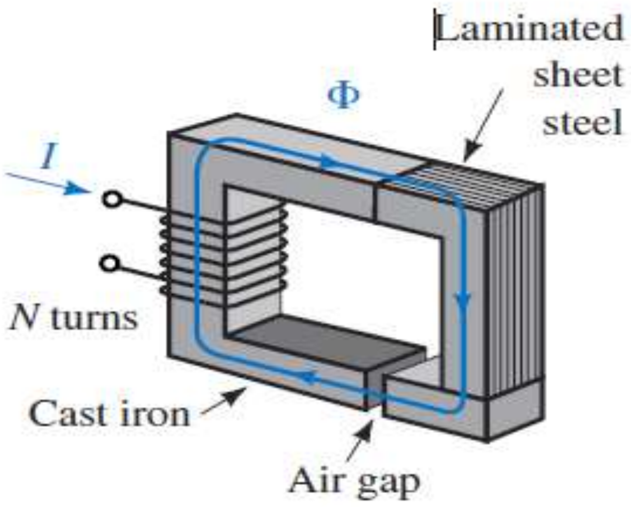
The complete closed path followed by any group of magnetic lines of flux





Series Magnetic Circuit

- Flux ϕ is the same in all sections if leakage flux is neglected.
- Flux density and reluctance in each section may vary, depending on its effective cross-sectional area and material.
- Equivalent reluctance is the sum of reluctance of different parts/elements.
- The resultant MMF is the sum of MMFs in each individual parts/elements



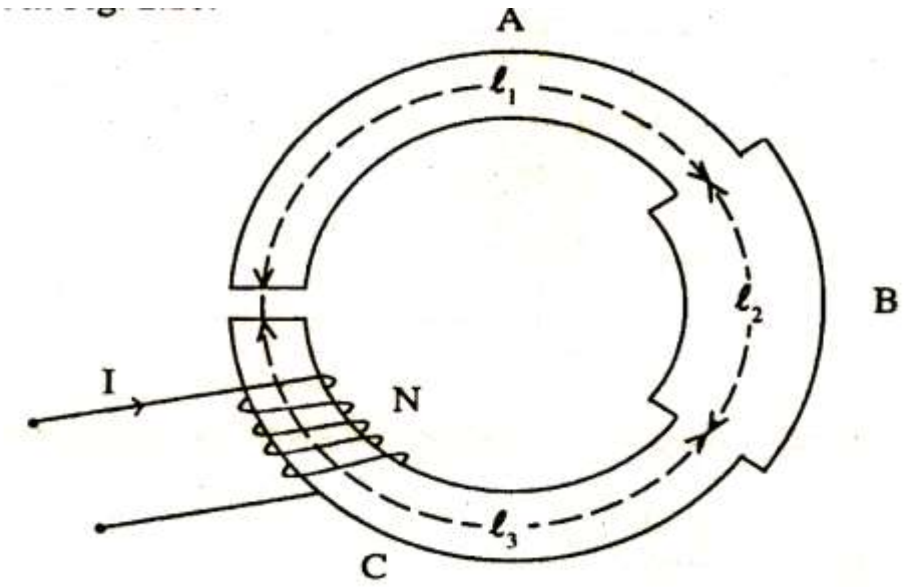
Rectangular shaped series magnetic circuit with air gap.

Series Magnetic Circuit

$$\boldsymbol{S_1 = \frac{l_1}{\mu_0\,\mu_{r1}\,A_1},\ S_2 = \frac{l_2}{\mu_0\,\mu_{r2}\,A_2}}$$

$$\boldsymbol{S_3 = \frac{l_3}{\mu_0\,\mu_{r3}\,A_1},\ S_g = \frac{l_g}{\mu_0\,A_1}}$$

$$\boldsymbol{S_T = S_1 + S_2 + S_3 + S_g}$$



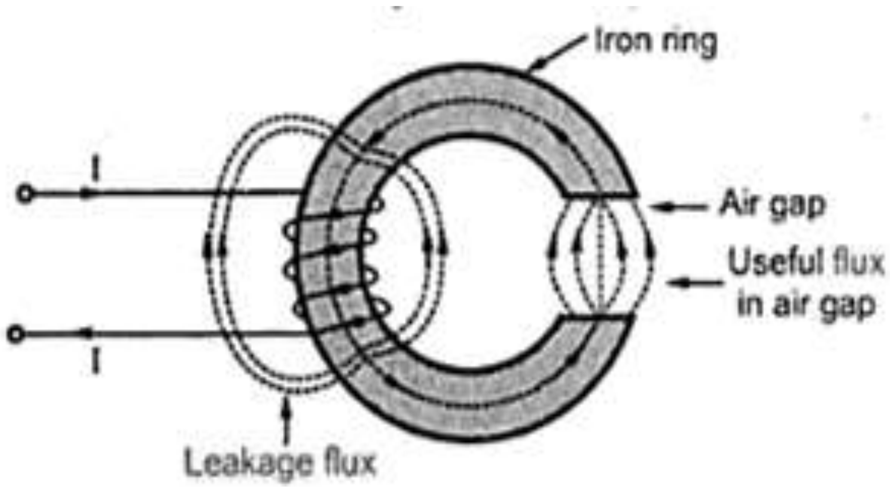
$$Total\ mmf\ = \phi_1 S_1 + \phi_2 S_2 + \phi_3 S_3 + \phi_g S_g$$

$$= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

$$= \left(\frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_3 l_3}{\mu_0 \mu_{r3}} + \frac{B_g l_g}{\mu_0}\right)$$

Useful & Leakage Flux

- **Magnetic leakage:**
  - The passage of magnetic flux outside the path along which it can do useful work.



- Total flux of coil = Useful flux + Leakage flux

- **Leakage Coefficient**:

$$\lambda = \frac{\text{Total Flux of the Coil}}{\text{Useful Flux}}$$

Illustration 1

An iron ring has a circular cross- sectional area of 5 cm^2 and a mean circumference of 100 cm. The ring is uniformly wound with a coil of 1000 turns. Relative permeability of iron is 800.

a) Find the current required to produce a flux of 1 mWb in the ring.
b) If a saw cut of 2 mm wide is made in the ring, find the flux produced, if the current is same as that found in **part a**.
c) Find the current required to produce the same flux as in **part a** for the cut made in the ring in **part b**.

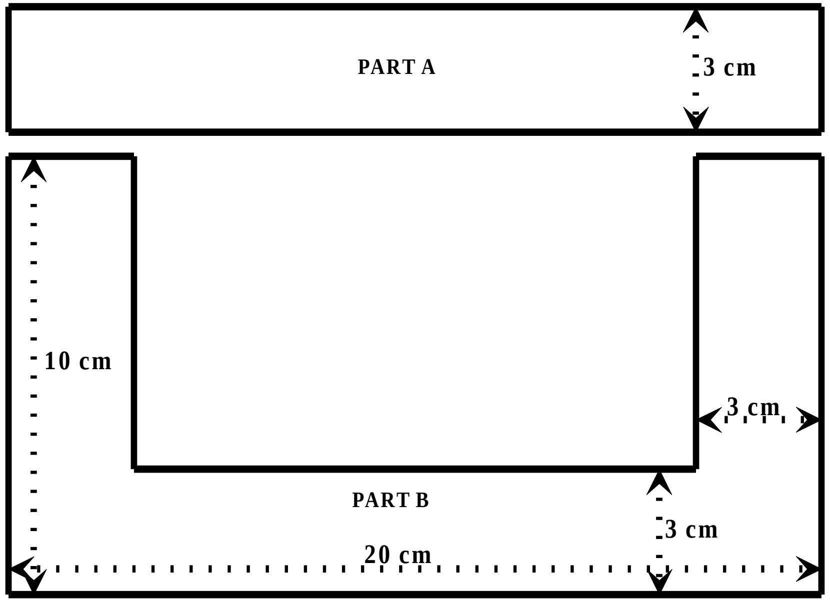
Ans:

a) 1.99 A
b) 0.385 mWb
c) 5.17 A

Illustration 2

The magnetic circuit shown in the figure is made of iron having a square cross-section of 3 cm side. It has two parts A and B, with relative permeabilities of 1000 and 1200 respectively, separated by two air gaps, each 2 mm wide. The part B is wound with a total of 1000 turns of wire on the two side limbs carrying a current of 2.5 A. Calculate

a) The reluctances of Part-A, Part-B & air gaps,
b) the total reluctance
c) the mmf
d) the flux and the flux density.



Hint:

Length of Part A = 1.5 + (20-1.5-1.5)+1.5 = 20cm
Length of Part B = (10-1.5)+(20-1.5-1.5)+(10-1.5) = 34 cm

Ans:

S_A = 176838.83AT/Wb,
S_B = 250521.67AT/Wb
S_g = 3536776.51AT/Wb
S_T = 3964137AT/Wb
mmf = 2500 AT
Φ = 0.63 mWb, B = 0.7 T

Illustration 3

A ring of cross sectional area 12 cm^2 has 3 parts made of following materials:

Part	Material	Length	Relative Permeability
A	Iron	25 cm	800
B	Steel	18 cm	1100
C	Air	2 mm	---

A coil of 660 turns carrying a current of 2.1 A is wound uniformly on the ring. Determine the flux density in the air gap. Assume no leakage and fringing effect.

Ans: 0.703 Wb/m^2

Basic Electrical Technology

Parallel Magnetic Circuits

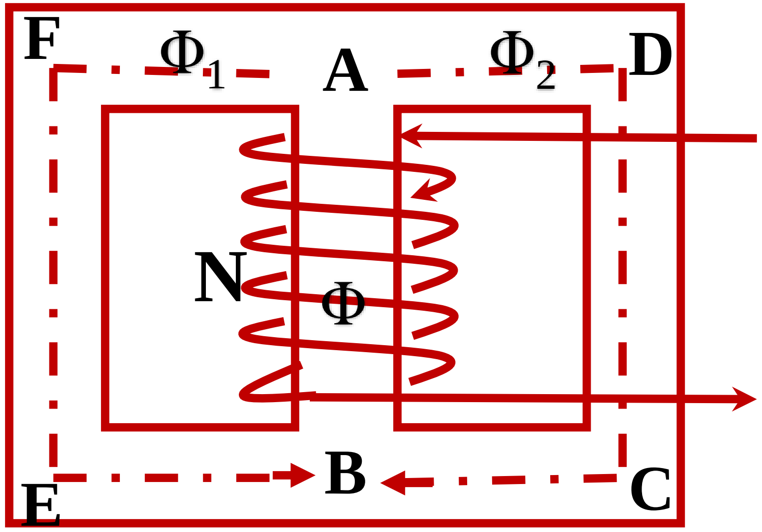
Parallel Magnetic Circuit

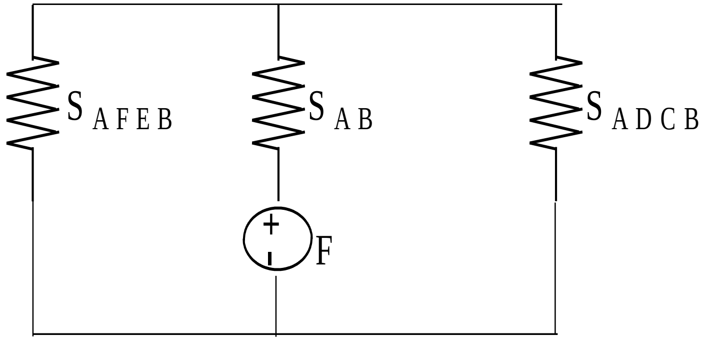
- More than one path for flux
- $\Phi = \Phi_1 + \Phi_2$

$$S_{AB} = \frac{l_{AB}}{\mu_0 \, \mu_{rAB} \, A_{AB}}$$

$$S_{ADCB} = \frac{l_{ADCB}}{\mu_0 \, \mu_{rADCB} \, A_{ADCB}}$$

$$S_{AFEB} = \frac{l_{AFEB}}{\mu_0 \, \mu_{rAFEB} \, A_{AFEB}}$$





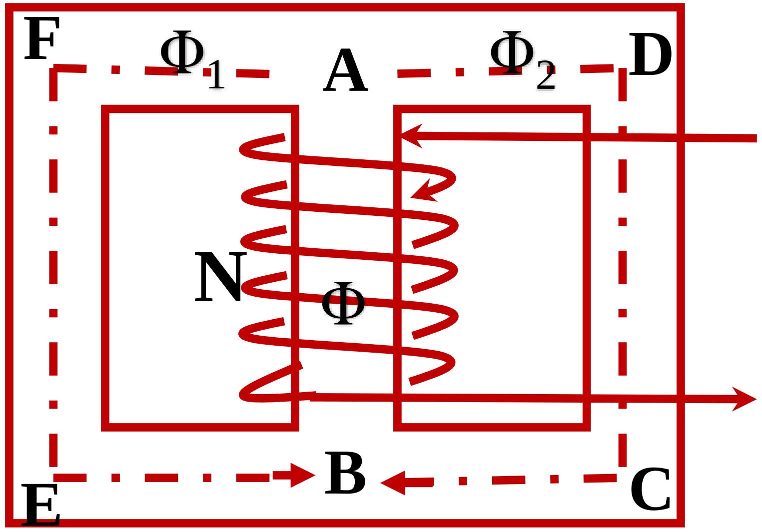
Analogous Electrical Circuit

Parallel Magnetic Circuit

- $(Mmf)_{Total} = (Mmf)_{AB} + (Mmf)_{ADCB}$

  OR

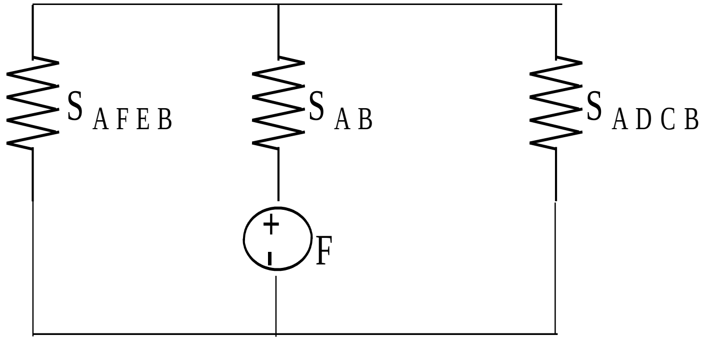
  $(Mmf)_{Total} = (Mmf)_{AB} + (Mmf)_{AFEB}$



- $(Mmf)_{Total} = \Phi\, S_{AB} + \Phi_1\, S_{ADCB}$

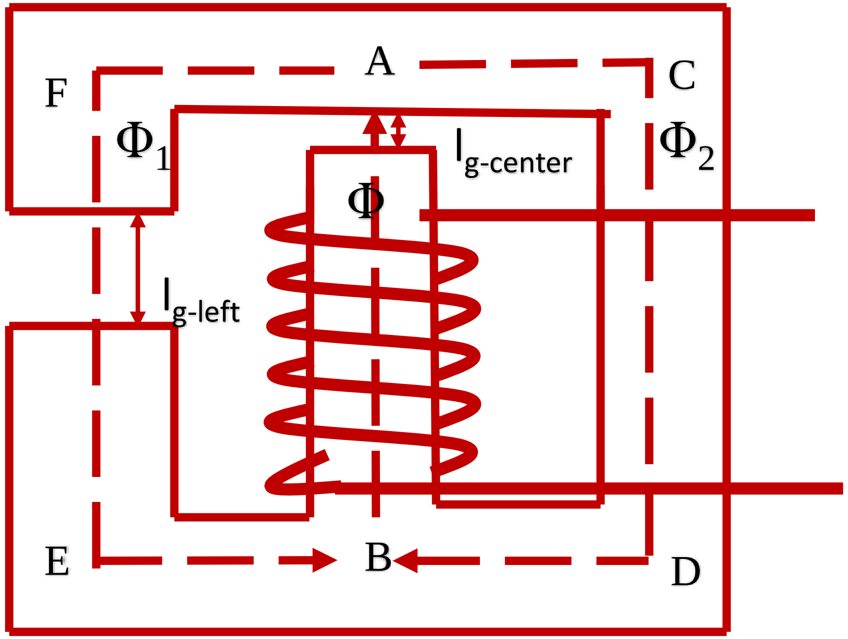
  OR

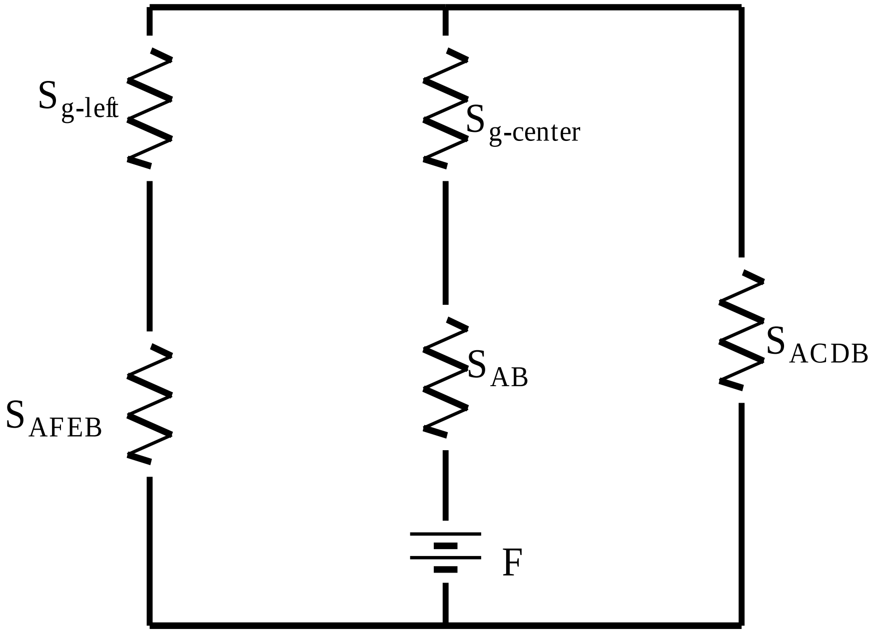
  $(Mmf)_{Total} = \Phi\, S_{AB} + \Phi_2\, S_{AFEB}$



Analogous Electrical Circuit

Parallel Magnetic Circuit with Air Gap

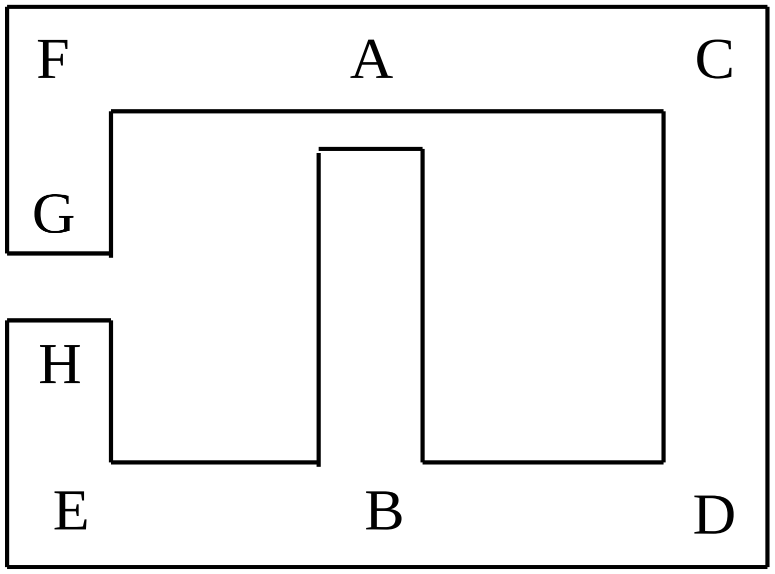




$$S_{AFEB} = \frac{(l_{AFEB} - l_{gleft})}{\mu_0\, \mu_{rAFEB}\, A_{AFEB}}; \qquad S_{AB} = \frac{(l_{AB} - l_{gcenter})}{\mu_0\, \mu_{rAB}\, A_{AB}}$$

Illustration 1

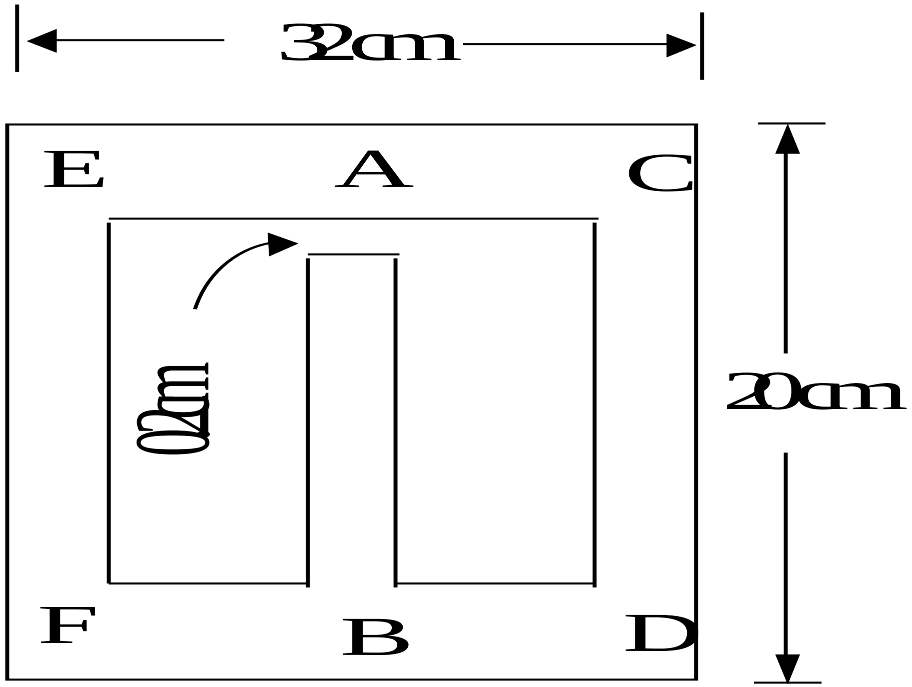
The magnetic circuit shown in Fig. is made of a material having relative permeability of 2000. The central limb is wound with 1000 turns and has an airgap of length of 2mm. The side limb airgap is 8 mm. Calculate the current required to set up a flux of 2.6 mWb in the central limb. Mean lengths of various sections are as follows: AB = 24 cm, ACDB = AFGHEB = 60 cm. Cross sectional area of the structure is 10 cm^2.



Ans: 4.98 A

Illustration 2

A coil carrying a current of 2.8 A is wound on the left limb of the cast steel symmetrical frame of uniform square cross section 16 cm^2 as shown in Fig. Calculate the number of turns in the coil to produce a flux of 1.8 mWb in the air gap of 0.2 cm length. The relative permeability of cast steel is 1200.



Ans: 1480

Basic Electrical Technology

[ELE 1051]

Electromagnetic induction

Faraday's Laws of Electromagnetic Induction

First Law:

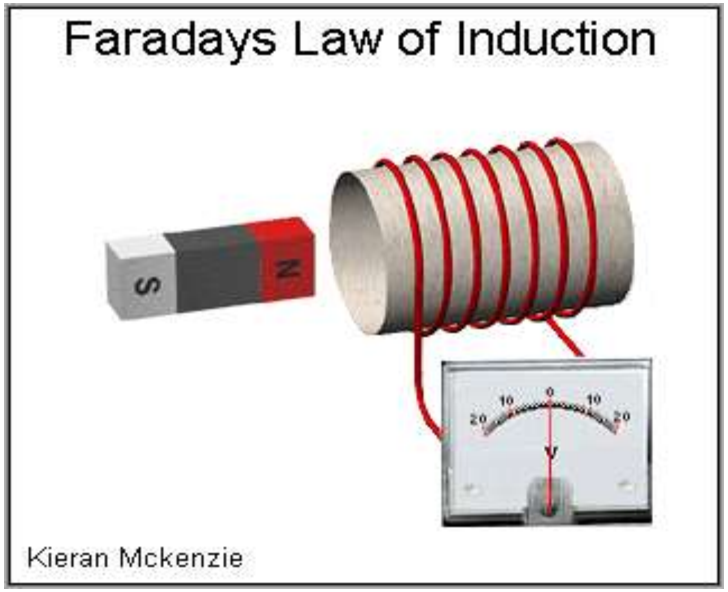
Whenever the magnetic field linking with a conductor changes, an EMF will be induced in that conductor

Second Law:

The magnitude of the induced EMF is proportional to the rate of change of the magnetic flux linking the conductor

$$e = N\frac{d\phi}{dt}$$

Where N = number of turns in the coil



Lenz's Law

The electro-magnetically induced emf always acts in such a direction to set up a current opposing the motion or change of flux responsible for inducing the emf.

$$e = -N\frac{d\phi}{dt}$$

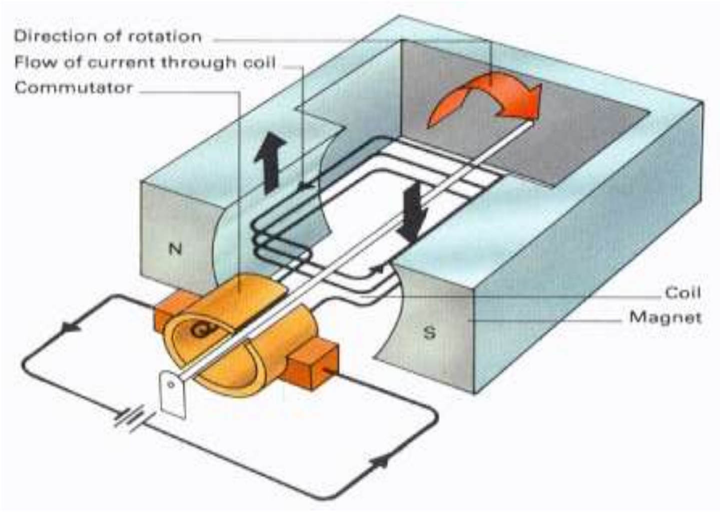
Fleming's Right Hand Rule

If the first, second and the thumb of the right hand are held at right angles to each other,
first finger indicates the direction of the magnetic flux
and
thumb finger indicates the direction of motion of the conductor relative to the magnetic field,
then
the second finger represents the direction of induced EMF.

Types of induced EMF

Dynamically induced EMF:

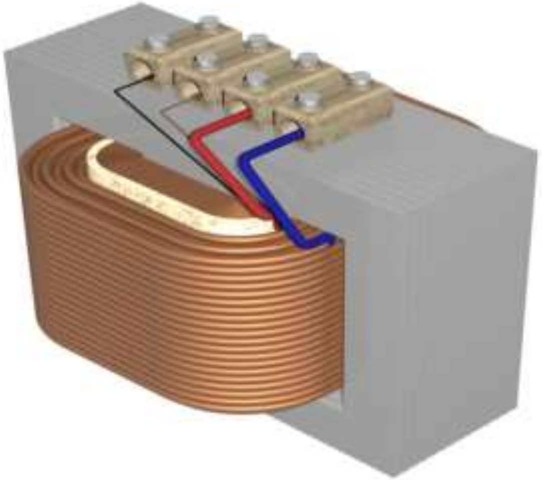
- The voltage induced in the conductor due to relative motion of conductor and magnetic field
- $e = B\, l\, v\, Sin\theta$
- Either conductor or magnetic field is moving
- Principle of **Electric generator**



Types of induced EMF

Statically Induced EMF:

- The voltage induced in the conductor due to change in the magnetic field
- Conductor is stationary
- Magnetic Field is changing in a stationary magnetic system
- Eg: Transformer

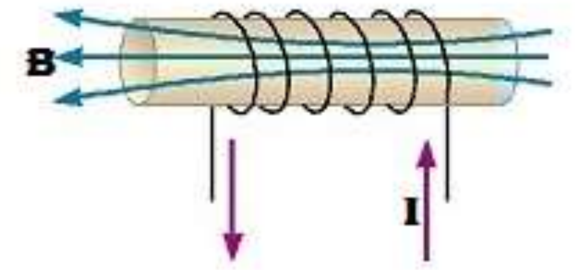


Types of Statically induced EMF

Self Induced Emf:

The induced emf in a coil proportional to the rate of the change of the magnetic flux passing through it due to its own current.

$$\boldsymbol{e = -L\frac{di}{dt}}$$



Self Inductance L:

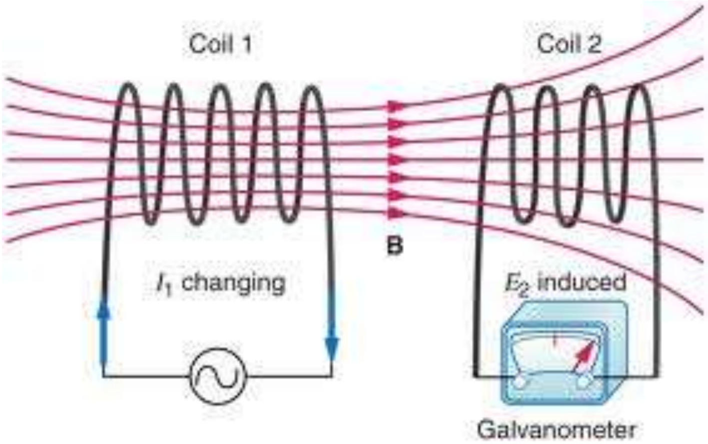
The proportionality constant is called the self inductance, L.
Unit is Henry

$$e = -N\frac{d\phi}{dt} = -L\frac{di}{dt}$$

$$L = N\frac{d\phi}{di}$$

Mutually Induced Emf:

The induced emf in a coil due to the change of flux produced by the change of current in the nearby coil



Mutual Inductance M:

This proportionality constant is called the mutual inductance, M

If Coil 1 is excited:

Mutually induced emf e_2 in Coil 2, $$e_2 = N_2 \frac{d\phi_{12}}{dt} = M \frac{di_1}{dt}$$

Mutual Inductance, $$M = N_2 \frac{d\phi_{12}}{di_1}$$

If coil 2 is excited: $M = N_1 \frac{d\phi_{21}}{di_2}$

Coupling Coefficient (k)

Gives an idea about the degree of magnetic coupling between two coils.

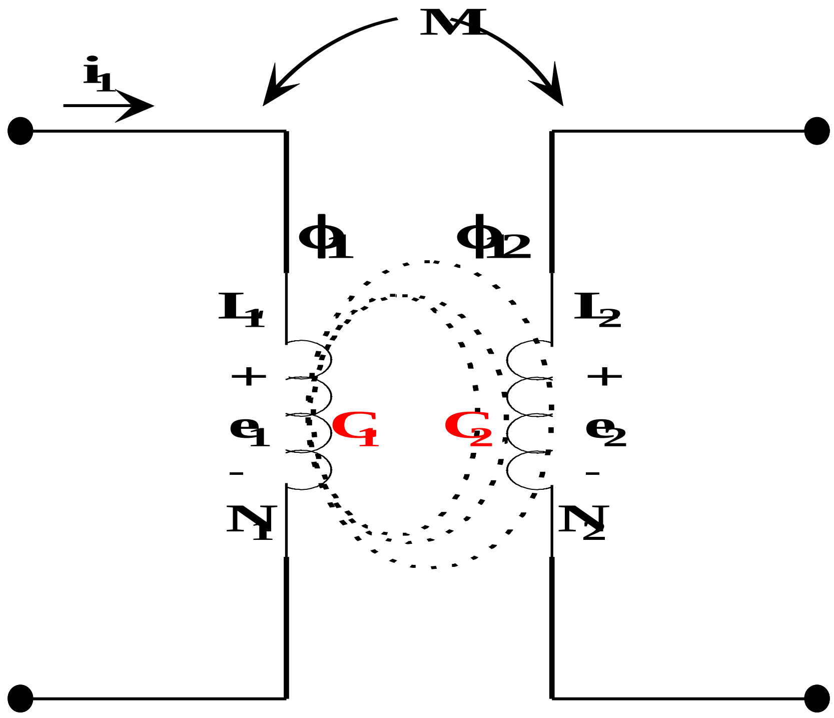
$$M = N_2 \frac{d\phi_{12}}{di_1} = N_1 \frac{d\phi_{21}}{di_2}$$

$$where, \phi_{12} = k\,\phi_1; \;\; \phi_{21} = k\,\phi_2$$

$$M^2 = \left(N_2 k \frac{d\phi_1}{di_1}\right)\left(N_1 k \frac{d\phi_2}{di_2}\right)$$

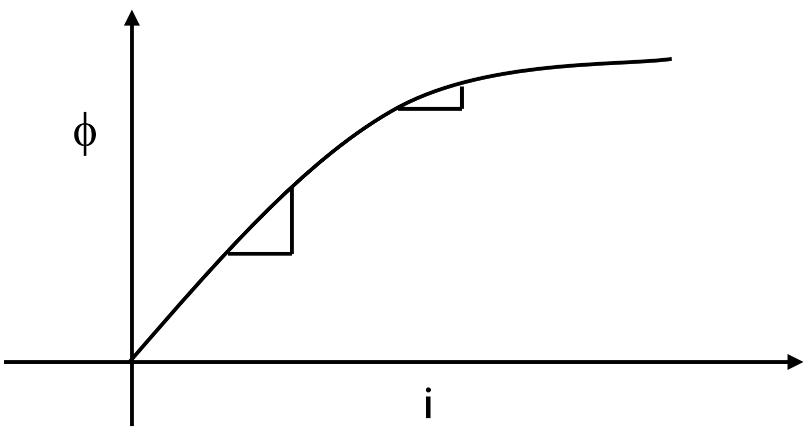
$$= k^2 L_1 L_2$$

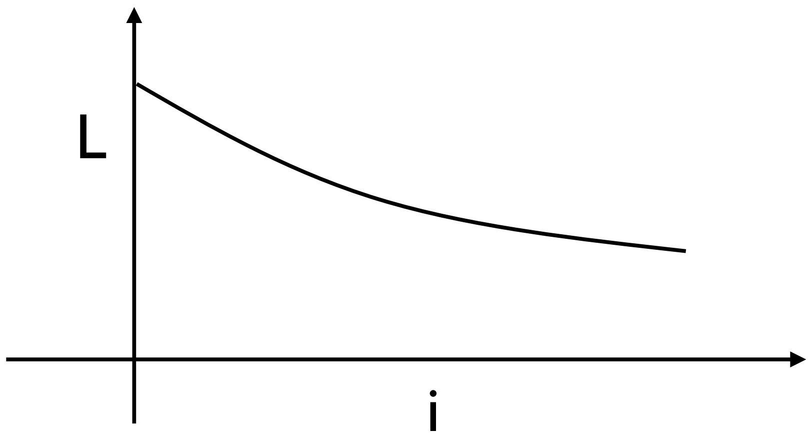
$$\boldsymbol{k = \frac{M}{\sqrt{L_1 L_2}}}$$



Ferrite core inductor

- Ferrite core – used to reduce the size of the inductor
- $L = N\frac{d\phi}{di}$

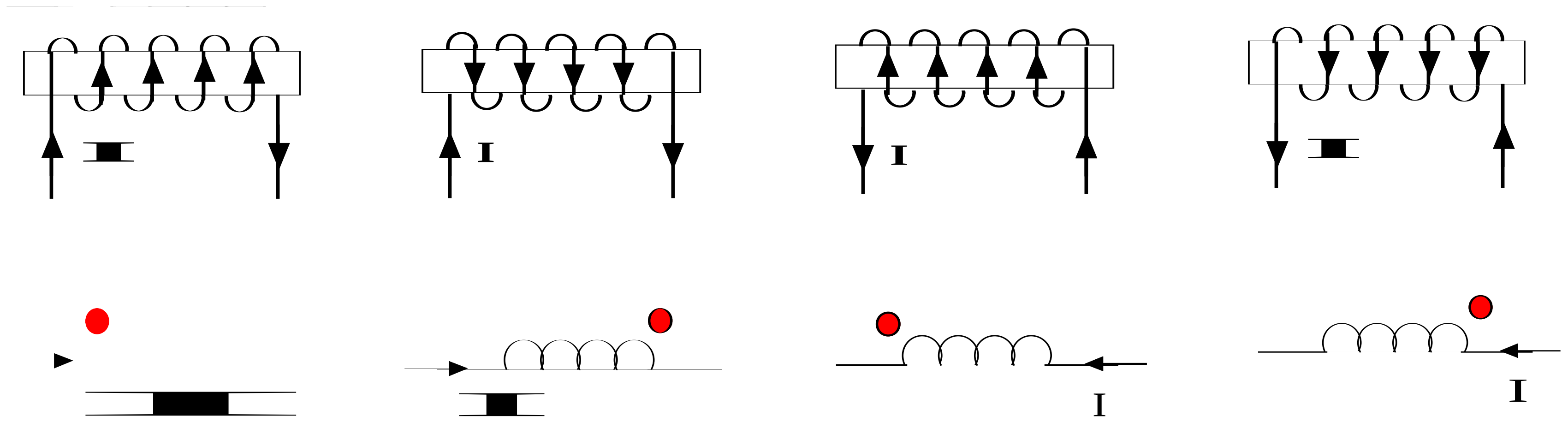




Coupled Circuits

- Polarity of mutually induced emf depends on
  - current direction
  - physical construction of the coils
- Obtaining the dotted equivalent: Right Hand Grip Rule

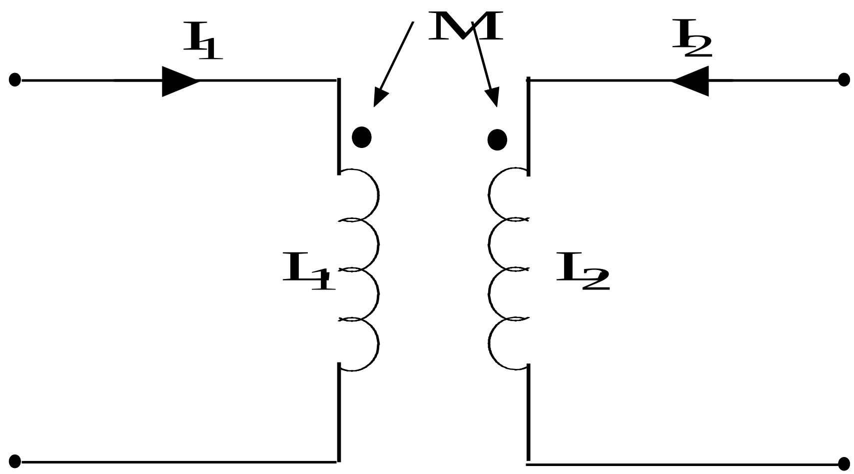
  Place the dot at the terminal directed by the thumb



Dot Rule for coupled coils

- Dot Rule helps in determining the sign of mutually induced emf without going into the details of physical construction
- **Dot Rule:**
  - ✓ If currents enter (or leave) the dotted terminals in both the coils, the sign of mutually induced emf is same as that of sign of self induced emf. (Additive coupling)
  - ✓ If the current enters the dotted terminal in one coil and leaves the dotted terminal in the other coil, the sign of mutually induced emf is opposite to that of sign of self induced emf. (Subtractive coupling)

Additive Coupling: (Fluxes are aiding)

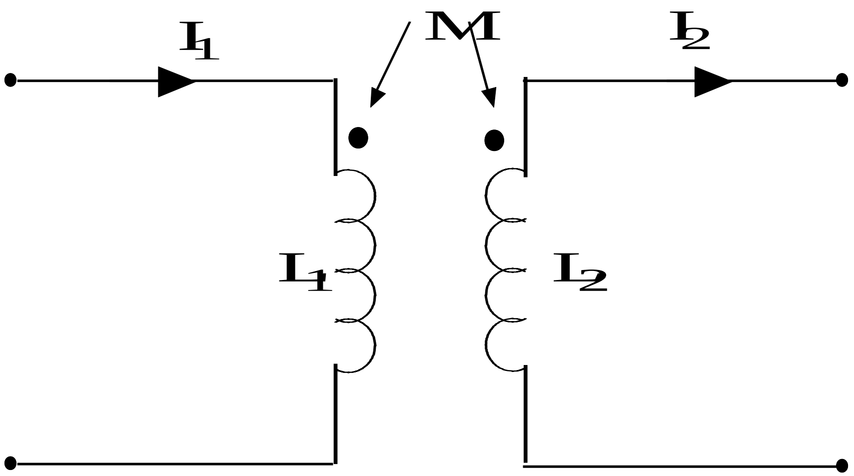


Self induced emf in L₁ = $-L_1 \dfrac{di_1}{dt}$

Mutually induced emf in L₁ = $-M \dfrac{di_2}{dt}$

Total induced emf in L₁ = $-\left(L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt} \right)$

Subtractive Coupling: (Fluxes are opposing)

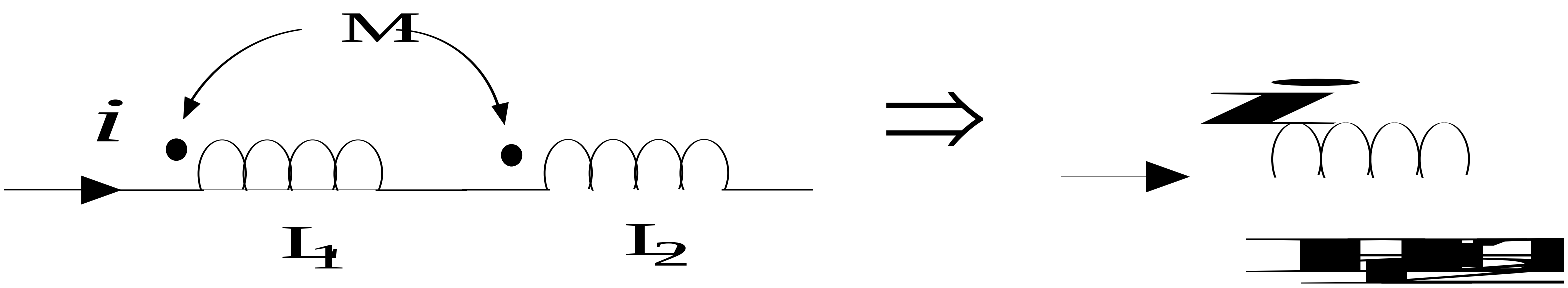


Self induced emf in L$_1$ = $-L_1 \dfrac{di_1}{dt}$

Mutually induced emf in L$_1$ = $+M \dfrac{di_2}{dt}$

Total induced emf in L$_1$ = $\left(-L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt}\right)$

Coupled coils in Series - Aiding



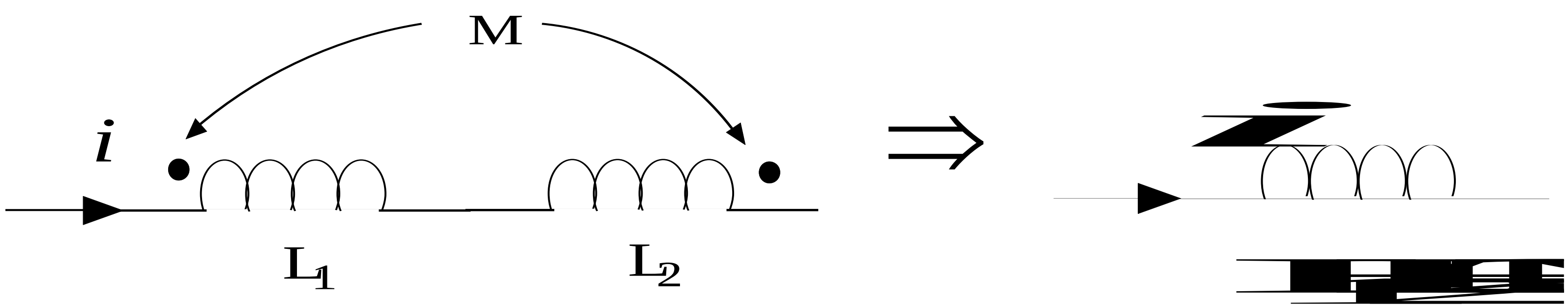
$$e_1 = L_1 \frac{di}{dt} + M \frac{di}{dt}$$

$$e_2 = L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$e = L_{eq} \frac{di}{dt} = e_1 + e_2 = (L_1 + L_2 + 2M) \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + 2M$$

Coupled coils in Series - Opposing



$$e_1 = L_1\frac{di}{dt} - M\frac{di}{dt}$$

$$e_2 = L_2\frac{di}{dt} - M\frac{di}{dt}$$

$$e = L_{eq}\frac{di}{dt} = e_1 + e_2 = (L_1 + L_2 - 2M)\frac{di}{dt}$$

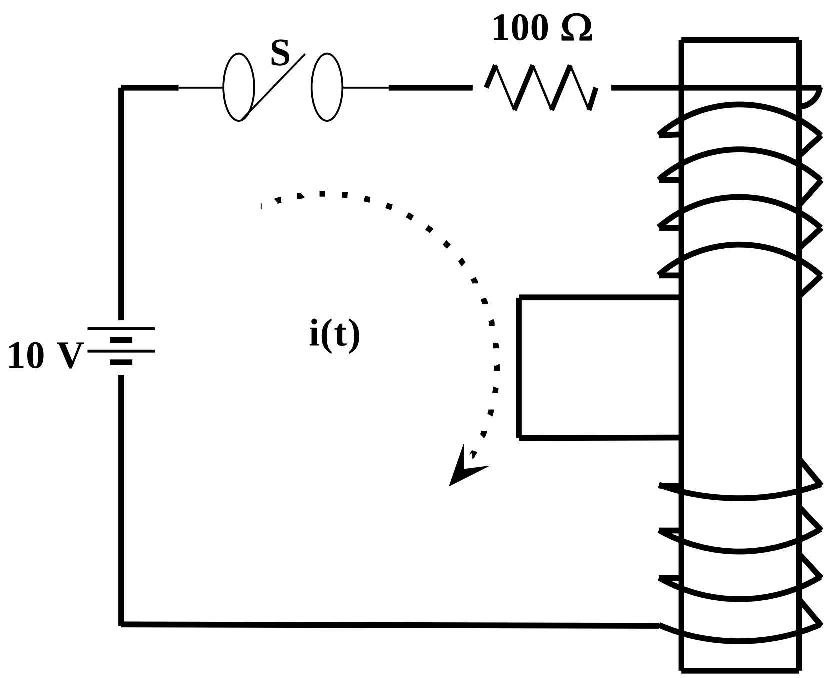
$$L_{eq} = L_1 + L_2 - 2M$$

Example 1

For the circuit shown in the figure: L1 = 0.3 H, L2 = 0.2 H & k = 0.8

If the switch is closed at t = 0, find

a) Initial value of current in the circuit
b) Final value of current in the circuit
c) Time constant of the circuit



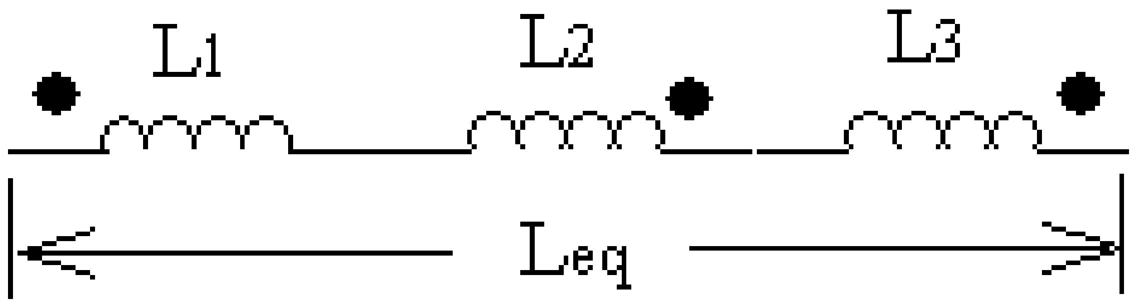
Example 1

Three magnetically coupled inductive coils having the following data are connected in series as shown in Figure.

$L_1 = 0.12$ H; $L_2 = 0.14$ H; $L_3 = 0.16$ H

$k_{12} = 0.3$; $k_{23} = 0.6$; $k_{31} = 0.9$

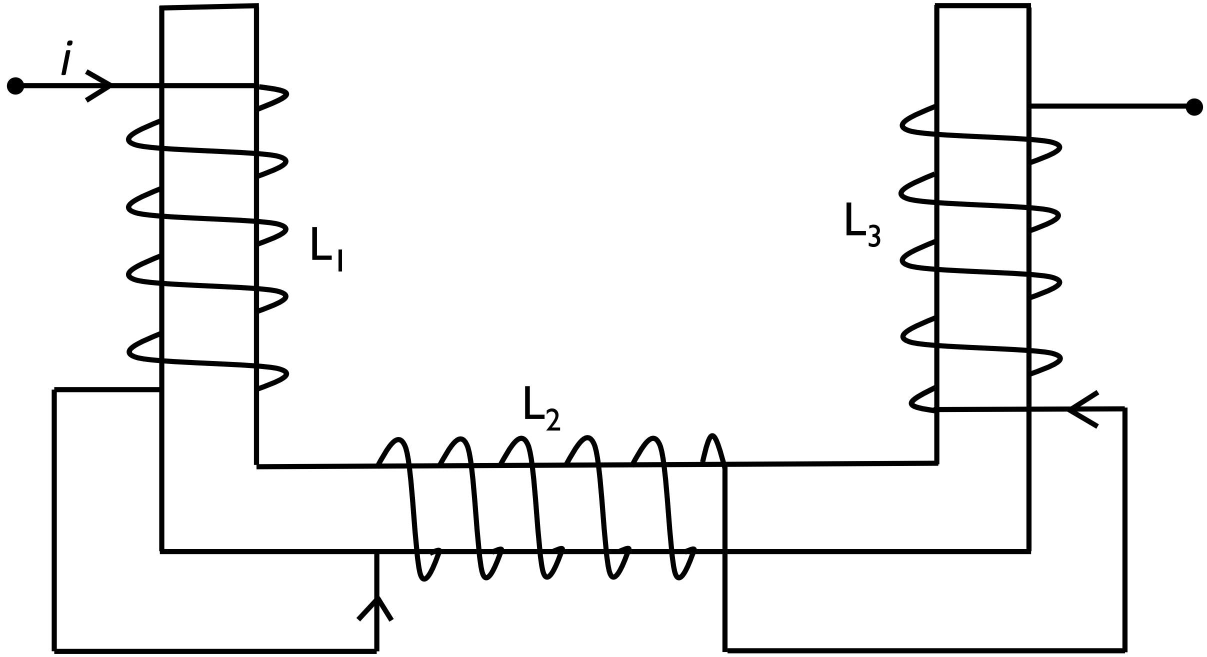
Find the equivalent inductance of the circuit.



Ans: 0.272 H

Example 1

Three magnetically coupled inductive coils having the following data are connected in series as shown in Figure. L_1 = 0.3 H; L_2 = 0.6 H; L_3 = 0.8H and the coefficients of coupling are ,k_{12} = 0.8; k_{23} = 0.75; k_{31} = 0.5
Draw the dotted equivalent circuit of the figure, also find the equivalent inductance of the circuit.



Ans : 0.472 H

Example 3

Two similar coils have a coupling coefficient of 0.4. When they are connected in series aiding, the equivalent inductance is 560mH. Calculate: i) self-inductance of both the coils. i) Total inductance when the coils are connected in series opposition. iii) total energy stored due to a current of 3A when the coils are connected in series opposition.

Ans: 0.2 H, 0.24 H, 1.08 J

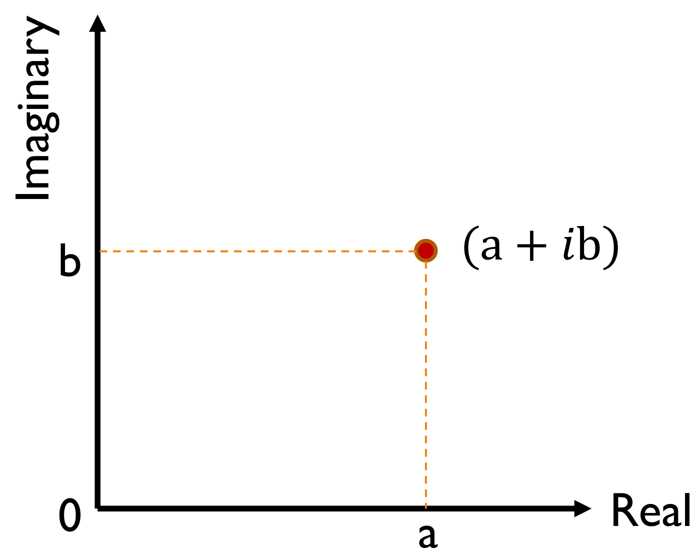
Review of complex algebra

- Representation of complex numbers
- Conversion of complex forms
- Arithmetic operation on complex numbers

Complex Number

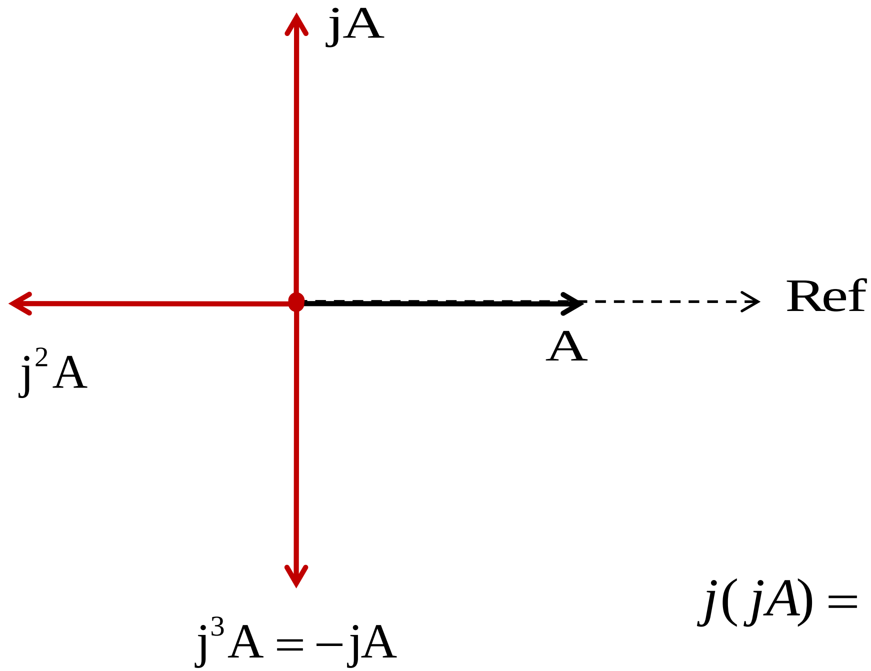
- A **complex number** is of the form **a + *i* b**

- Represented on complex plane as:



The operator 'j'

$j = 1\angle 90°$



$$j(jA) = j^2A = -A$$

$$\boxed{\text{Therefore, } \quad j^2 = -1; \quad j = \sqrt{-1}}$$

The operator 'j' rotates the given vector by 90 degrees in anti-clockwise direction

Representation of a complex number

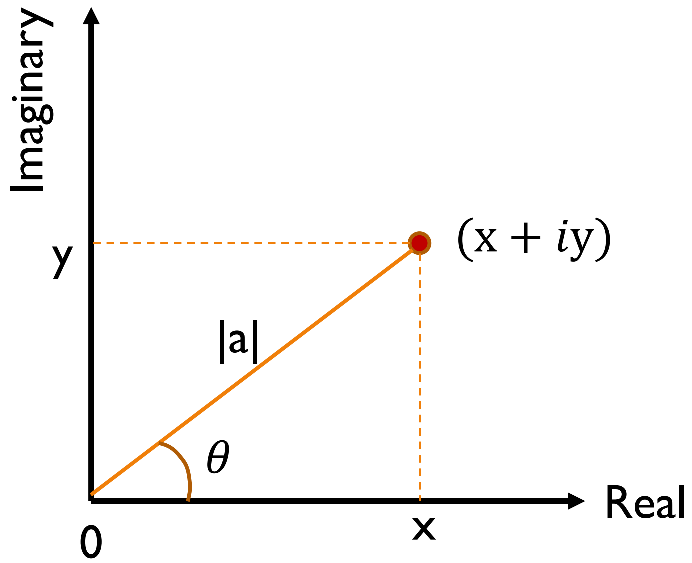
- **Rectangular form**: $\mathbf{a = x \pm \mathit{j}y}$
- **Polar form**: $\mathbf{a = |\mathit{a}|\angle \pm \theta}$
- **Exponential form**: $\mathbf{a = |\mathit{a}|e^{\pm \mathit{j}\theta}}$
- **Trigonometric form:** $\mathbf{a = |a|(cos\theta \pm \mathit{j}sin\theta)}$

Rectangular $\leftrightarrow$ Polar conversion

- **Rectangular to polar:**

$$|a| = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$



- **Polar to Rectangular:**

$$x = |a| \cos\theta$$

$$y = |a| \sin\theta$$

Rectangular ↔ Polar conversion

- Convert the following into polar form

  1) $3 + j\,4 \quad = 5\,\angle 53.13°$

  2) $8 + j\,6 \quad = 10\,\angle 36.87°$

  3) $8 - j\,6 \quad = 10\,\angle\text{-}36.87°$

- Convert the following into rectangular form

  1) $5\,\angle 30° \quad = 4.33 + j\,2.5$

  2) $3\,\angle\text{-}60° \quad = 1.5 - j\,2.59$

  3) $-(10\,\angle 45°) \quad = -\,7.07 - j\,7.07$

Arithmetic operation

Let $a_1 = x_1 + jy_1 = r_1\angle\theta_1$ $a_2 = x_2 + jy_2 = r_2\angle\theta_2$

$a_1 = 4 + j6 = 7.21\angle 56.3°$ $a_2 = 2 - j4 = 4.47\angle - 63.43°$

Addition: $$\boxed{\mathbf{a}_1 + \mathbf{a}_2 = (\mathbf{x}_1 + \mathbf{x}_2) + j(\mathbf{y}_1 + \mathbf{y}_2)}$$

$$a_1 + a_2 = (4 + 2) + j(6 - 4) = 6 + j2$$

Subtraction: $$\boxed{\mathbf{a}_1 - \mathbf{a}_2 = (\mathbf{x}_1 - \mathbf{x}_2) + j(\mathbf{y}_1 - \mathbf{y}_2)}$$

$$a_1 - a_2 = (4 - 2) + j(6 + 4) = 2 + j10$$

'Rectangular form is used for addition and subtraction of complex numbers'

Arithmetic operation

Let $a_1 = x_1 + jy_1 = r_1\angle\theta_1$ $\qquad a_2 = x_2 + jy_2 = r_2\angle\theta_2$

$a_1 = 4 + j6 = 7.21\angle 56.3°$ $\qquad a_2 = 2 - j4 = 4.47\angle -63.43°$

Multiplication:

$$\boxed{a_1\, a_2 = r_1\, r_2\angle(\theta_1 + \theta_2)}$$

$$a_1\, a_2 = (7.21)(4.47)\angle(56.3° - 63.43°) = 32.22\angle -7.13°$$

Division:

$$\boxed{\frac{a_1}{a_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)}$$

$$\frac{a_1}{a_2} = \frac{7.21}{4.47}\angle(56.3° - (-63.43°)) = 1.61\angle 119.73°$$

'Polar form is used to for multiplication and division of complex numbers'

Exercise

Ex. 1: $a_1 = 3 + j5 = 5.83\angle 59.03°$

$a_2 = 5 - j4 = 6.40\angle - 38.65°$

Compute $a = \dfrac{a_1 a_2}{a_1 + a_2}$

Ans:
$a = 4.63\angle 13.26°$

Ex. 2: $a_1 = 4 + j4$ $\quad a_2 = 5 - j4$ $\quad a_3 = 8 + j2$

Compute $a_{12} = a_1 + a_2 + \dfrac{a_1 a_2}{a_3}$, similarly a_{23} & a_{31}

Ans: $a_{12} = 13.36\angle - 2.52°$ $\quad a_{23} = 18.90\angle - 47.52°$ $\quad a_{31} = 17.21\angle 50.17°$

Ex. 3: $a_{12} = 7 + j4$ $\quad a_{23} = 9 + j11$ $\quad a_{31} = 35 - j3$

Compute $a_1 = \dfrac{a_{12} a_{31}}{\sum a_{12}}$, similarly a_2 & a_3

Ans: $a_1 = 5.40\angle 11.60°$ $\quad a_2 = 2.18\angle 67.21°$ $\quad a_3 = 9.52\angle 32.57°$

Summary

- **Review of complex algebra**
  - Rectangular form is used for addition and subtraction of complex numbers
  - Polar form is used to for multiplication and division of complex numbers

- **'j' operator**
  - $j = 1\angle 90°$
  - Rotates a vector by 90 degree in the anti-clockwise direction

Basic Electrical Technology

[ELE 1051]

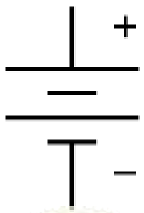
SINGLE PHASE AC CIRCUITS

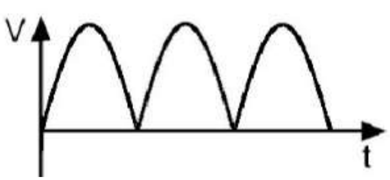
L14 - Introduction to AC, Generation of AC, Average & RMS value

Topics covered…

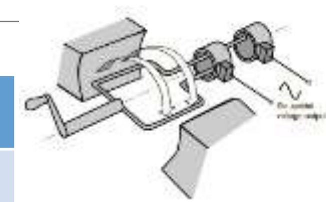
- DC vs AC
- Generation of AC
- Terminologies
- Average value and Root Mean Square (RMS) value

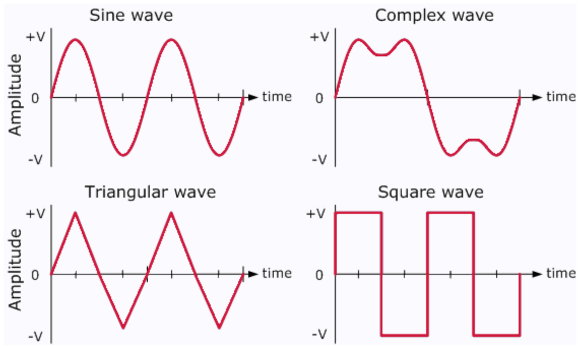
DC vs. AC





	DC	AC
Obtained from	Battery / cell / derived from AC	AC Generator
Polarity	Positive and Negative	Oscillatory
Frequency	Zero	50Hz or 60Hz
Types	Constant or pulsating	**Sinusoidal**, Trapezoidal, Triangular, Square





Terminologies

Period(T) :

- one cycle time

Frequency (f):

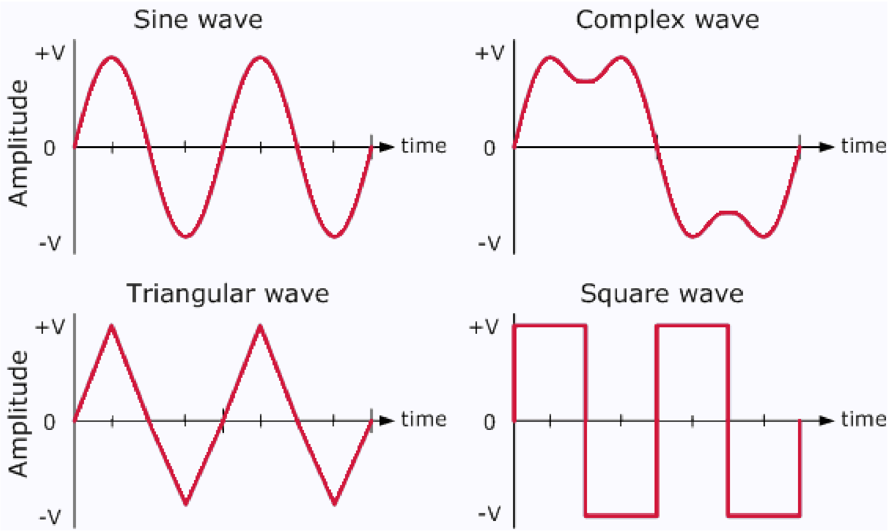
- $f = \frac{1}{T}$

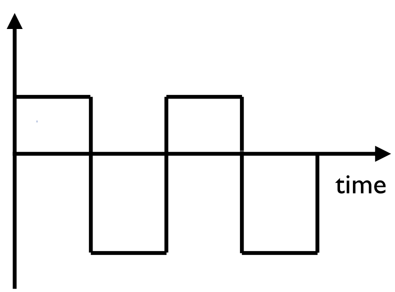
Average value :

- $\boldsymbol{F_{avg} = \frac{1}{T}\int_0^T f(t)dt}$

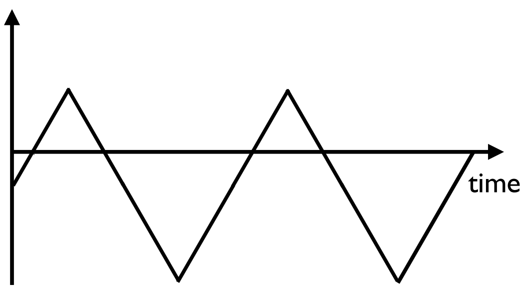
RMS value :

- $\boldsymbol{F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}}$





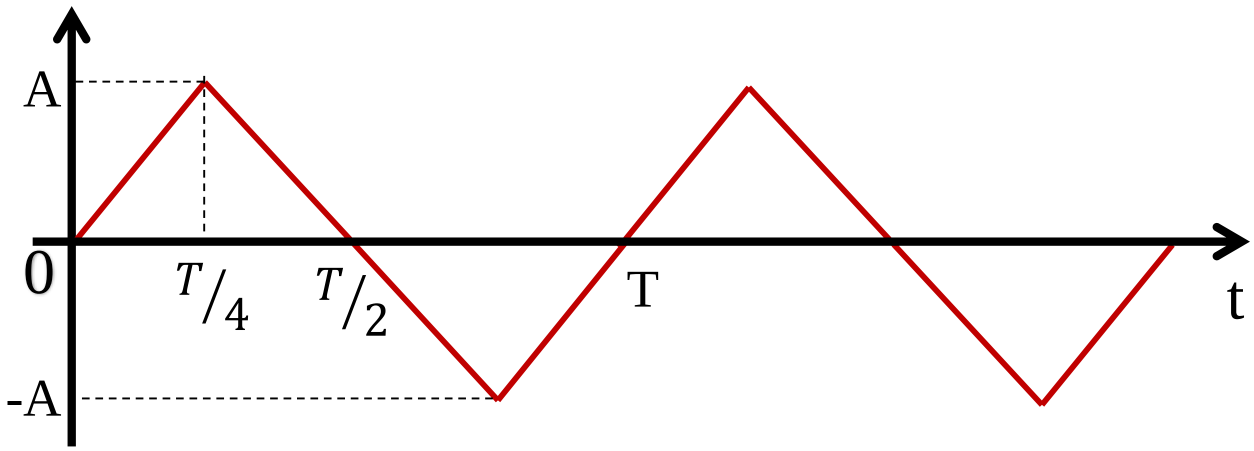
Asymmetric square wave



Asymmetric triangular waveform,

Exercise 1

Find the Average value and RMS value of the given non-sinusoidal waveform



Answer:

$$Average\ Value = \frac{A}{2} \qquad RMS\ Value = \frac{A}{\sqrt{3}}$$

Generation of Alternating EMF

[Generation of Alternating EMF]

EMF Equation

EMF induced per conductor is

$$e = B\,l\,v\,\sin\theta$$

EMF Induced in one Coil is

$$e = 2\,B\,l\,v\,\sin\theta$$

If, b = width of the coil,

$v = \pi\,b\,n$ *'n' is the speed in revolutions per sec.*

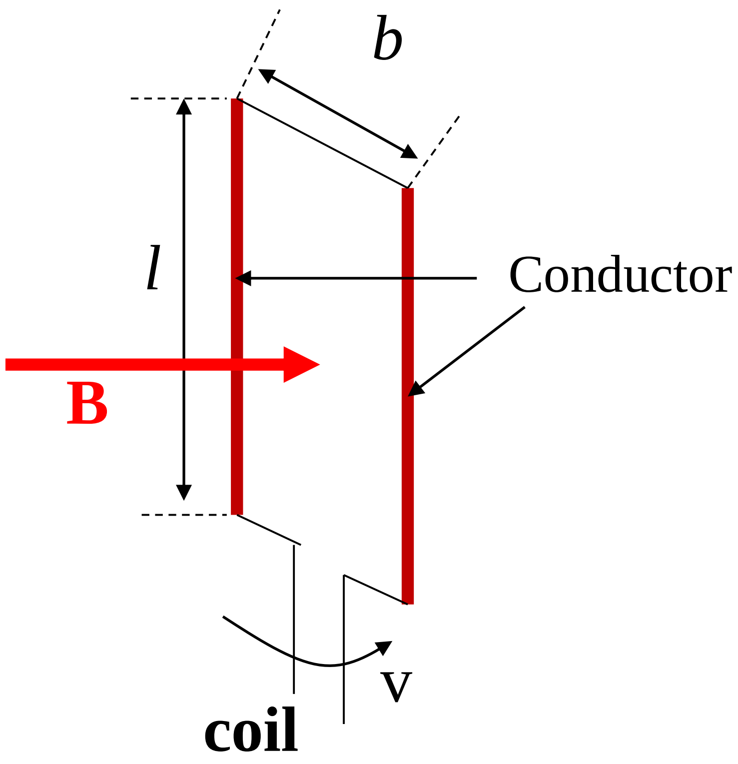
$$e = 2\,B\,l\,b\,\pi\,n\,\sin\theta$$

$$= 2\,B\,A\,\pi\,n\,\sin\theta$$

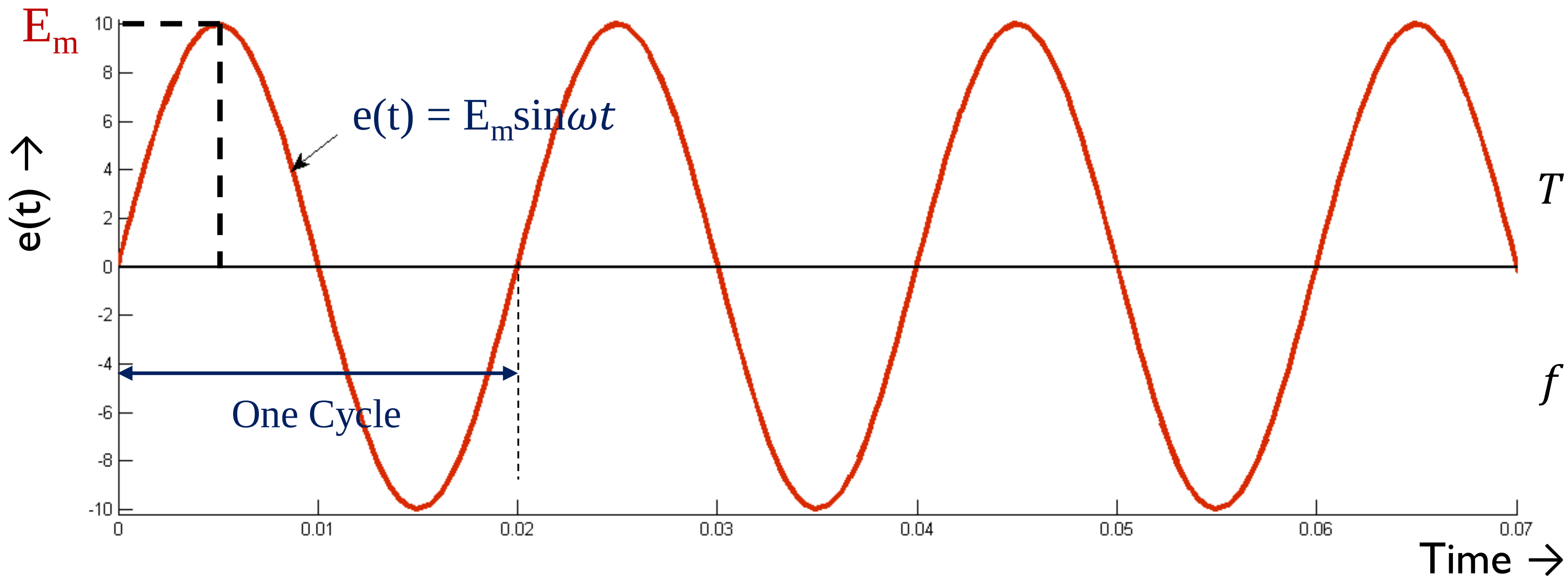
If there are N turns in the coil, the emf induced is,

$$e = 2\,\pi\,n\,B\,A\,N\,\sin\theta$$

$$\boxed{e = E_m \sin\theta}$$



Sinusoidal AC waveform



$T = 0.02s$

$f = 50Hz$

Cycle: Each repetition of the alternating quantity, recurring at equal intervals

Period (T): Duration of one cycle

Instantaneous Value (e(t)): The magnitude of a waveform at any instant in time

Peak Amplitude: Maximum value or peak value of alternating quantity

Frequency (f): Number of cycles in one second (Hz) $f = \frac{1}{T}$

Average value of Sinusoidal Alternating Current

Definition: *"It is that steady current which transfers the same amount of charge to any circuit during the given interval of time, as is transferred by the alternating current to the same circuit during the same time"*

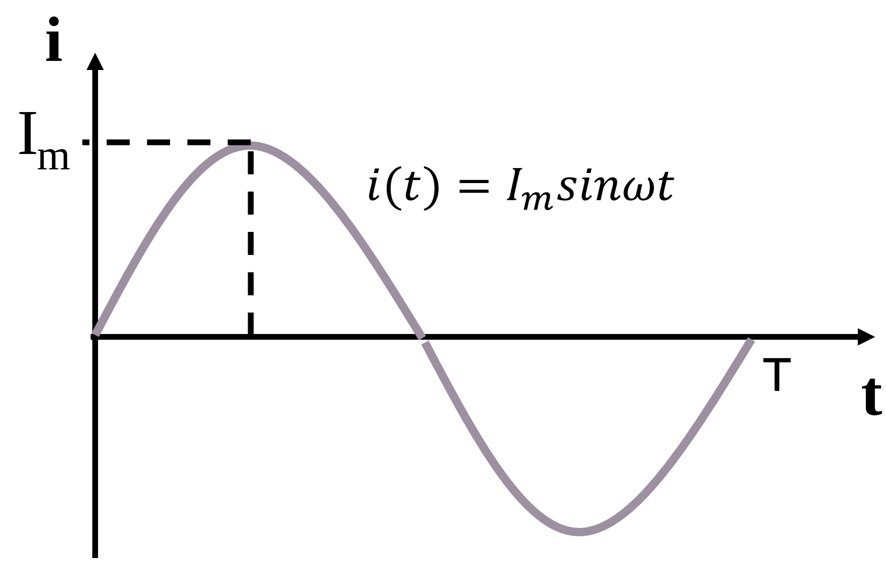
For a periodic function f(t) with period T,

$$F_{avg} = \frac{1}{T}\int_0^T f(t)dt$$

For sinusoidal signal,

$$I_{avg} = \frac{1}{T/2}\int_0^{T/2} I_m \sin\omega t dt$$

$$\boxed{I_{av} = \frac{2I_m}{\pi}}$$



RMS value of Sinusoidal Alternating Current

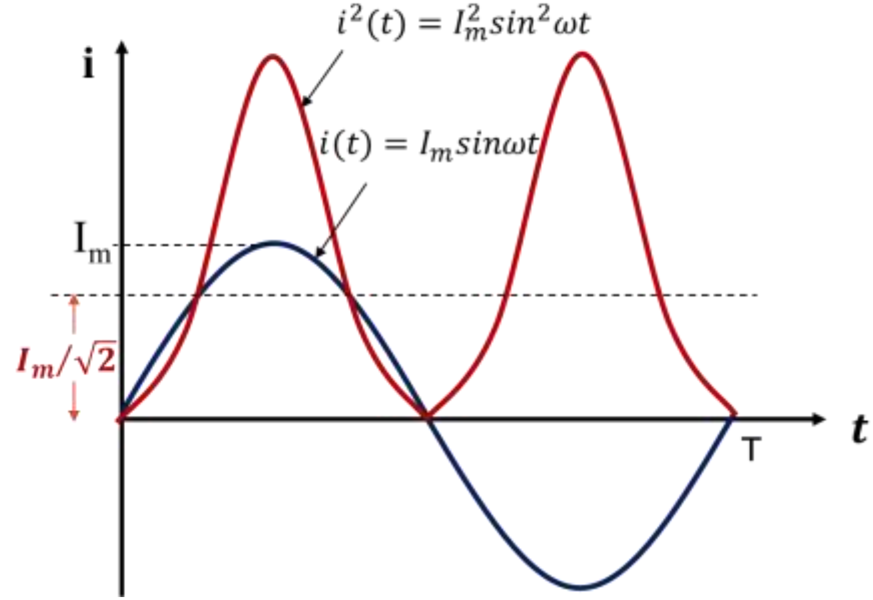
Definition: *"It is that value of direct current which when flowing through a circuit produces the same amount of heat for a given interval of time as that of the alternating current flowing through the same circuit during the same time"*

For a periodic function f(t) with period T,

$$F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 sin^2\omega t\ dt}$$

$$\boxed{I_{RMS} = \frac{I_m}{\sqrt{2}}}$$



Form Factor & Peak Factor

$$\boldsymbol{Form\ Factor = \frac{RMS\ Value}{Average\ Value} = 1.11\ for\ sinusoidal}$$

$$\boldsymbol{Peak\ Factor = \frac{Maximum\ Value}{RMS\ Value} = \sqrt{2}\ for\ sinusoidal}$$

Exercise 1

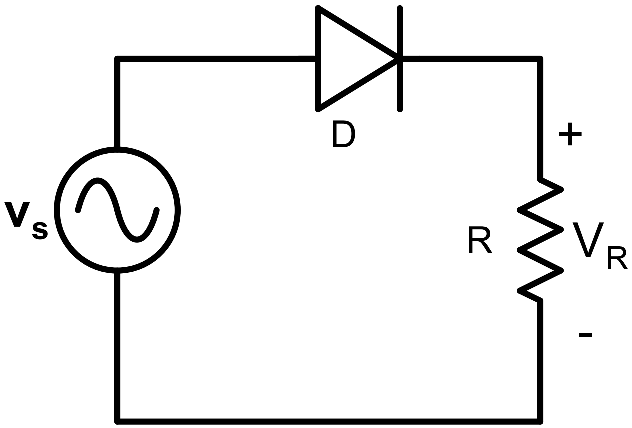
If an alternating voltage has the equation **$v(t) = 141.4 \sin 377t$**, calculate

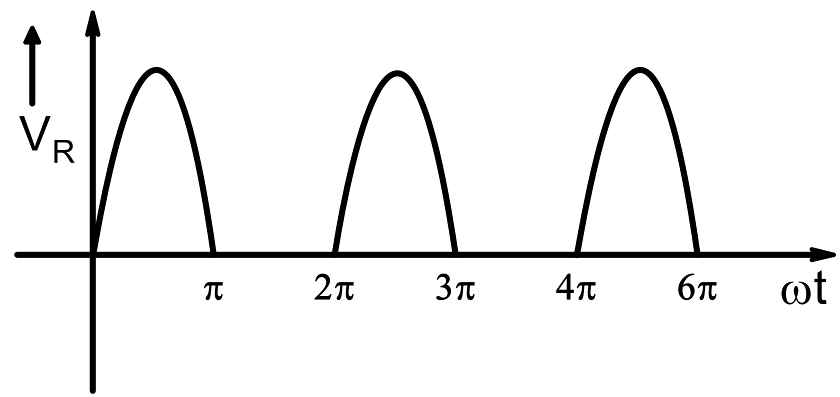
a. Maximum voltage value
b. RMS value of the voltage
c. Frequency
d. The instantaneous voltage when t = 3ms

Answers:

a) Maximum Value = 141.4 V
b) RMS Value = 100 V
c) Frequency = 60 Hz
d) Instantaneous voltage = 127.8V

For the circuit shown below, sketch the the voltage across the resistance, & then find the Average value and RMS value of the same.





Ans: $V_{R\,avg} = \frac{V_m}{\pi}$; $V_{R\,RMS} = \frac{V_m}{2}$

Summary

- Alternating quantity
  - time dependent
  - takes positive & negative values in every cycle
  - Types: **Sinusoidal**, triangular, square
- Generation of AC
- RMS value / Effective value for Sinusoidal AC

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

- Average value of a Sinusoidal AC

$$I_{av} = \frac{2I_m}{\pi}$$

Basic Electrical Technology

[ELE 1051]

SINGLE PHASE AC CIRCUITS

L15 – AC Representation & Response

Topics covered…

- Representation of AC
  - Mathematical form
  - Graphical form
  - Phasors
- AC response of
  - Pure Resistor
  - Pure Inductor
  - Pure Capacitor

Representing AC

- Consider three sinusoidal signals $x(t), y(t)$ & $z(t)$ with same frequency

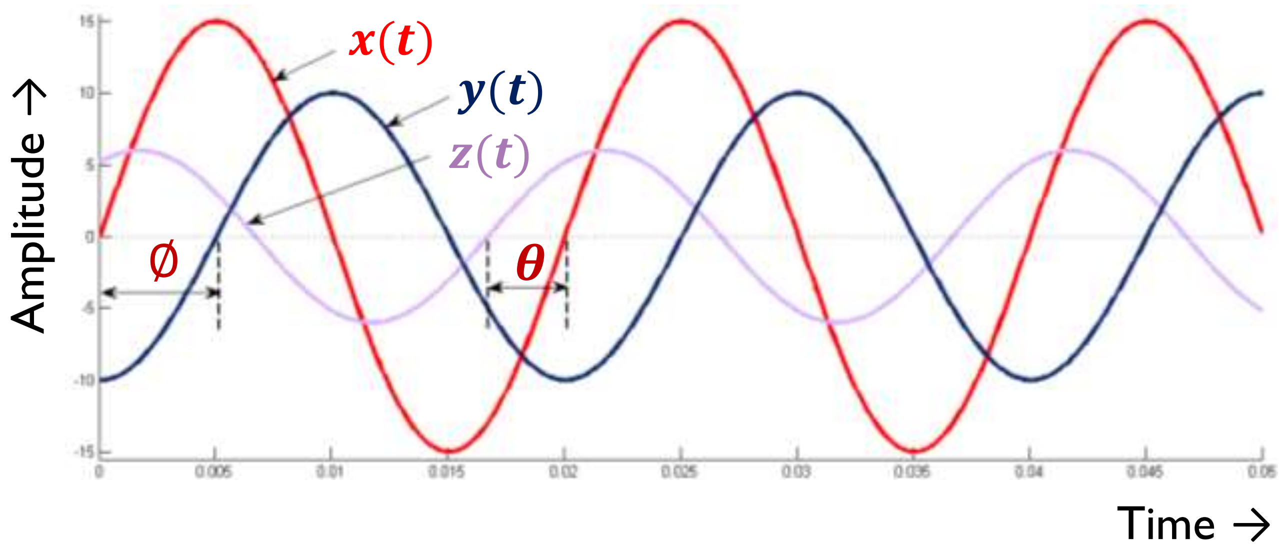
Mathematical Representation

$$x(t) = X_m sin(\omega t)$$

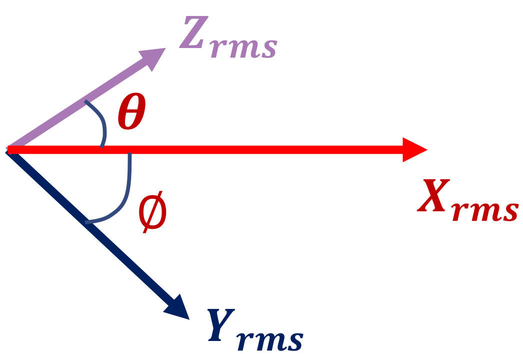
$$y(t) = Y_m \sin(\omega t - \emptyset)$$

$$z(t) = Z_m \sin(\omega t + \theta)$$

Graphical Representation

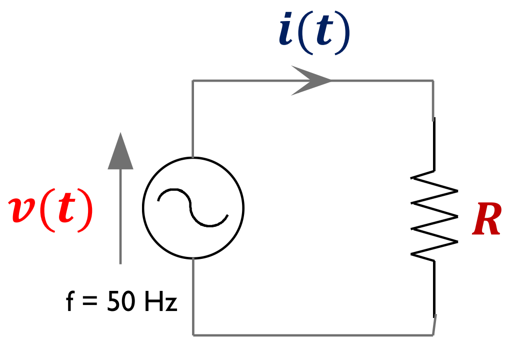


Phasor Representation



- Representing the relationship between sinusoidal signals with same frequency in graphical or mathematical form is tedious
- Phasor representation is often used

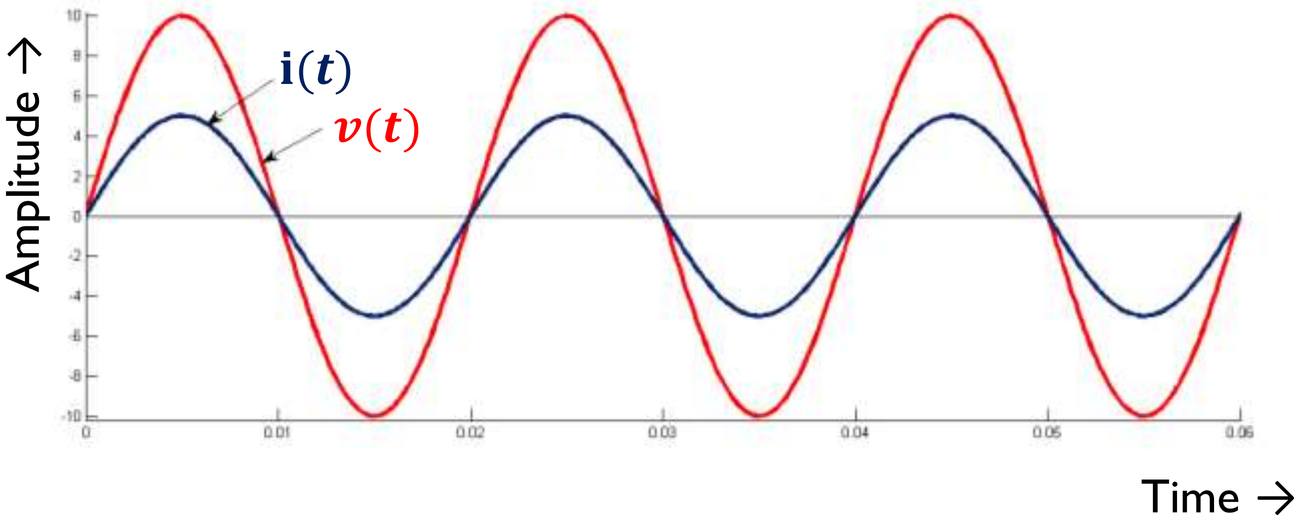
R circuit response with AC supply



$$i(t) = \frac{v(t)}{R}$$

'Current through the resistor is in phase with the voltage across it'

<u>*Graphical Representation*</u>

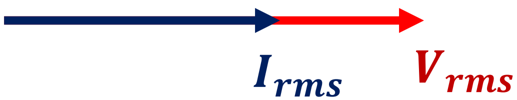


<u>*Mathematical Representation*</u>

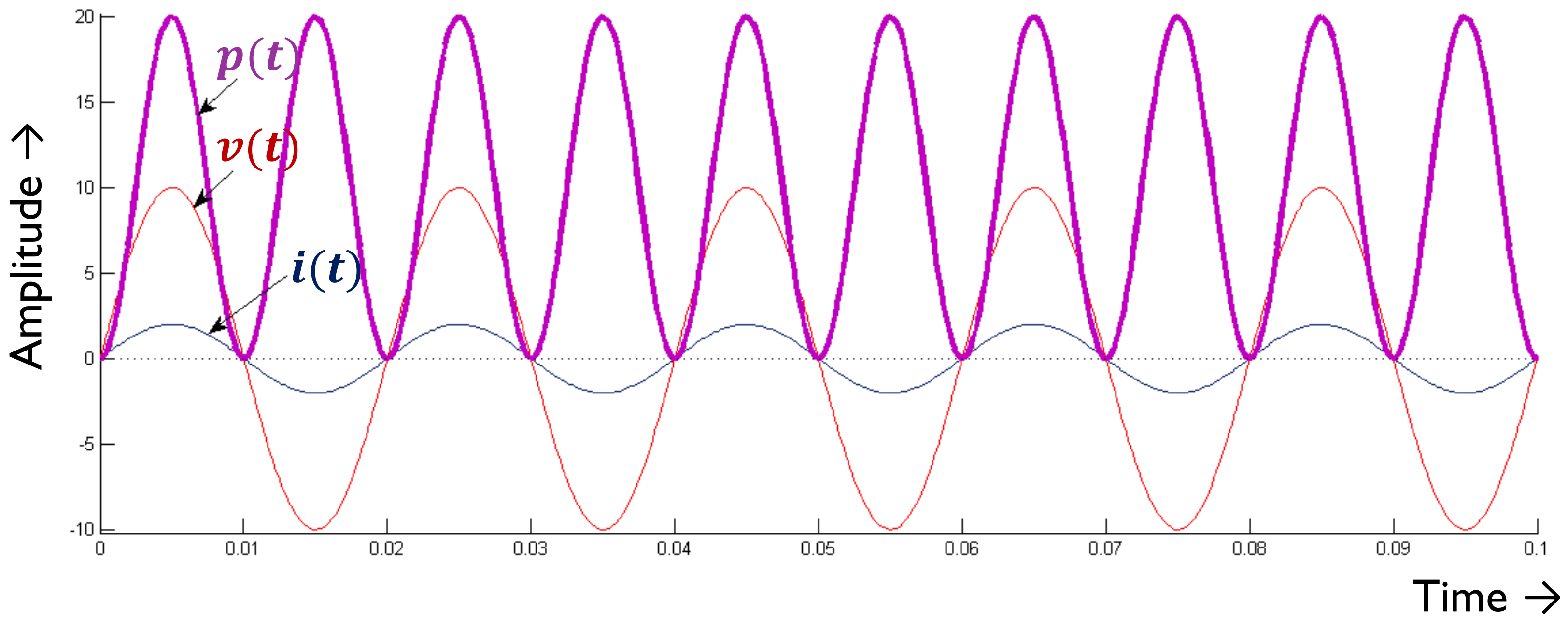
$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t)$$

<u>*Phasor Representation*</u>



Power Consumed - Pure Resistive Circuit



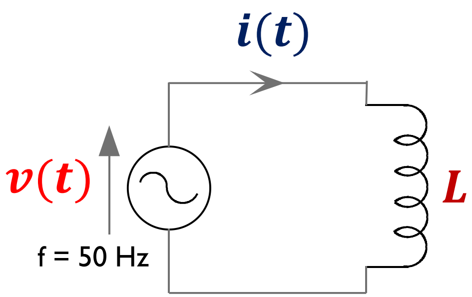
Instantaneous power,

$p(t) = v(t).i(t) = V_m I_m sin^2 \omega t$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = \frac{V_m I_m}{2} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R}}$$

L circuit response with AC supply



$$i(t) = \frac{1}{L}\int v(t)dt$$

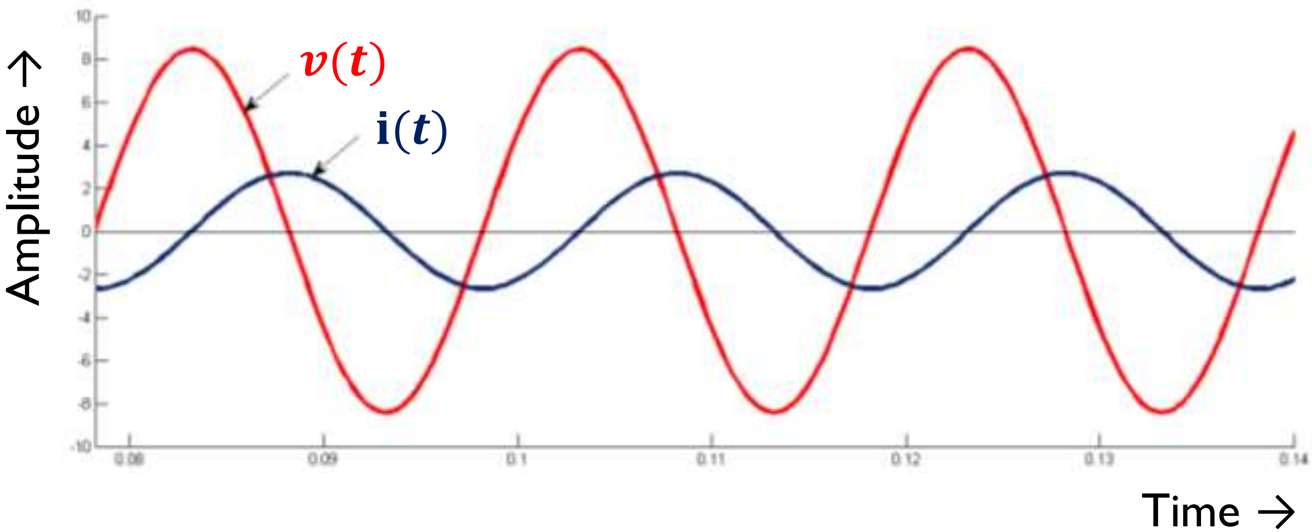
'Current through the inductor lags the voltage across it by 90°'

$\bar{V} = V\angle 0°$ $\bar{I} = I\angle -90°$

$$\frac{\bar{\boldsymbol{V}}}{\bar{\boldsymbol{I}}} = \frac{\boldsymbol{V\angle 0°}}{\boldsymbol{I\angle -90°}} = \boldsymbol{jX_L} \quad where\ \frac{V}{I} = X_L$$

$\boldsymbol{X_L}$ *is called* ***Inductive Reactance***

<u>*Graphical Representation*</u>



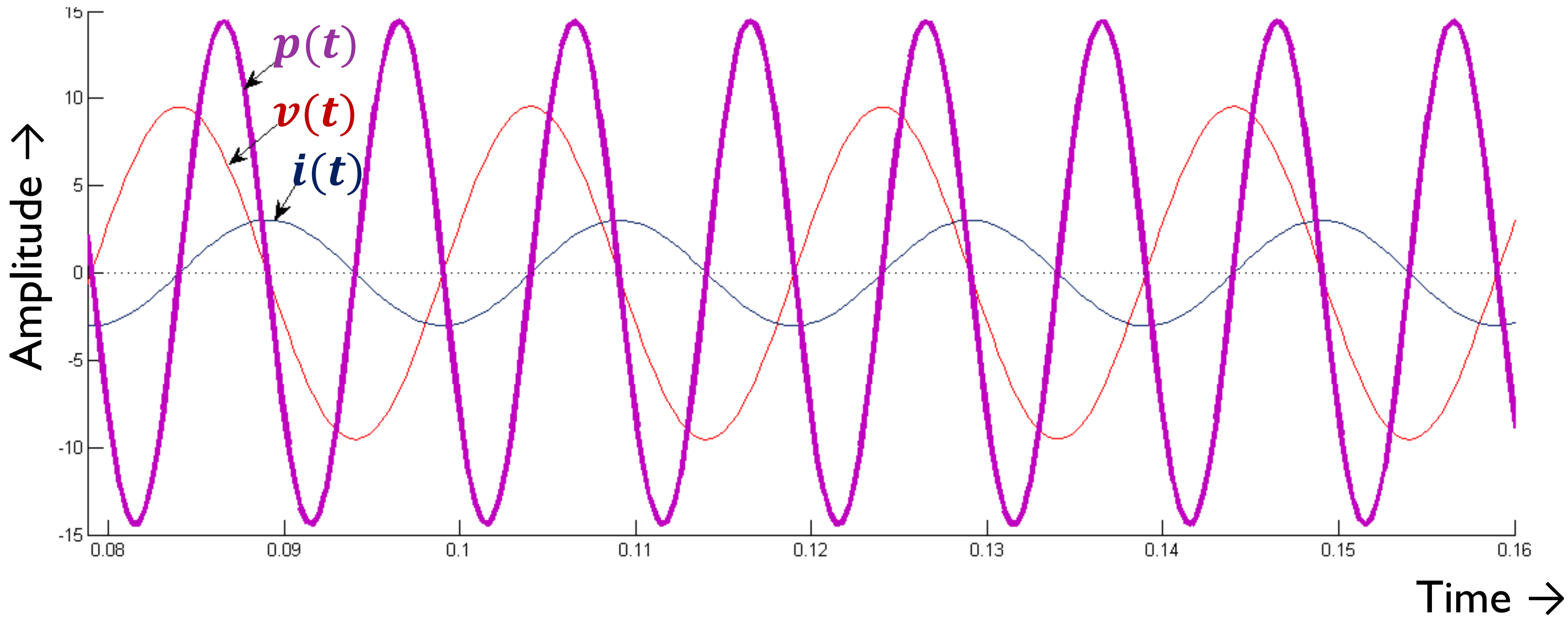
<u>*Mathematical Representation*</u>

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t - 90°)$$

<u>*Phasor Representation*</u>

Power Consumed - Pure Inductive Circuit



Instantaneous power,

$$p(t) = v(t).\,i(t)$$

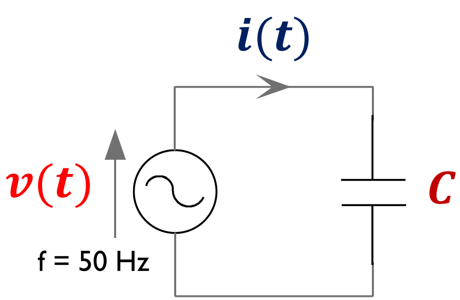
$$= V_m I_m \sin\omega t\,.\,\sin(\omega t - 90\,°)$$

$$= -\frac{V_m I_m}{2}\sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = 0}}$$

C circuit response with AC supply



$$i(t) = C\frac{dv(t)}{dt}$$

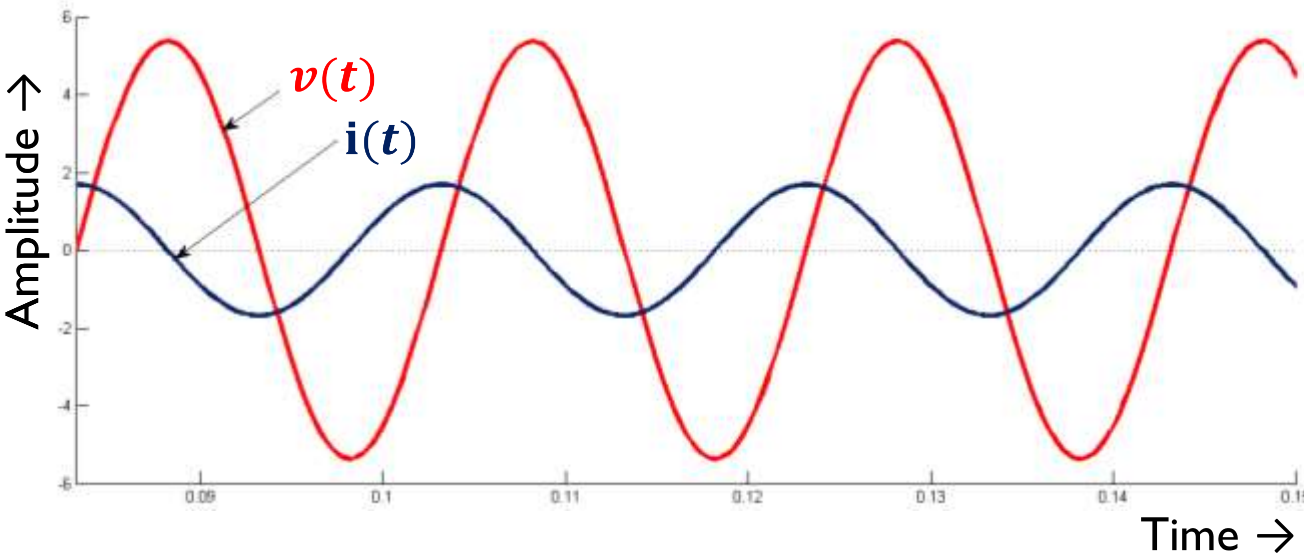
'Current through the capacitor leads the voltage across it by 90°'

$\bar{V} = V\angle 0°$ $\quad\quad$ $\bar{I} = I\angle 90°$

$$\frac{\bar{\boldsymbol{V}}}{\bar{\boldsymbol{I}}} = \frac{\boldsymbol{V\angle 0°}}{\boldsymbol{I\angle 90°}} = \boldsymbol{-jX_C} \quad where\ \frac{V}{I} = X_C$$

$\boldsymbol{X_C}$ *is called* ***Capacitive Reactance***

Graphical Representation

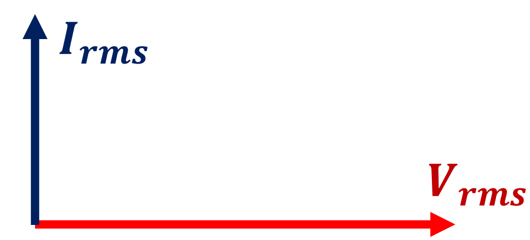


Mathematical Representation

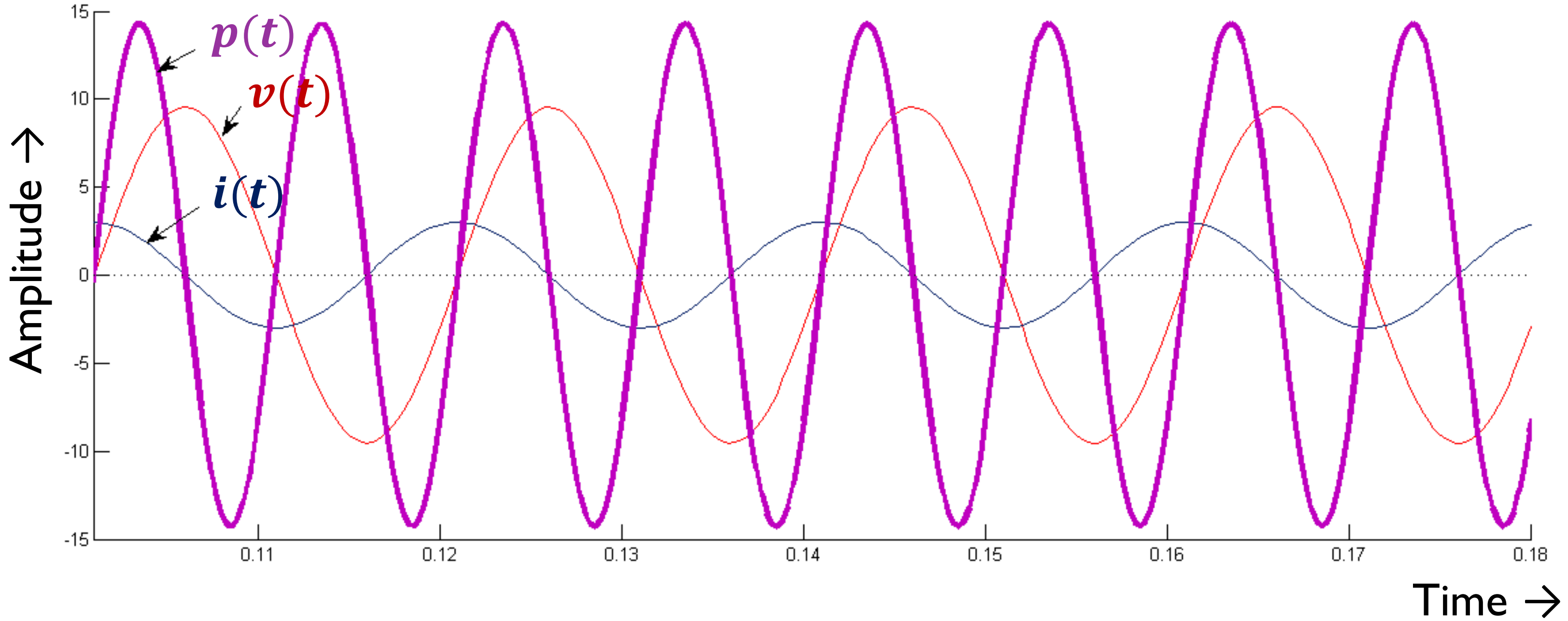
$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t + 90°)$$

Phasor Representation



Power Consumed - Pure Capacitive Circuit



Instantaneous power,

$$p(t) = v(t).\,i(t)$$

$$= V_m I_m \sin \omega t \,.\, \sin(\omega t + 90\,°)$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{P_{avg} = 0}$$

Summary

- Sinusoidal alternating signals of same frequency can be represented graphically by ***Phasors***
- **Define:** Inductive and capacitive Reactances

	R	L	C
Voltage, current relationship	v(t) in phase with I(t)	i(t) lags v(t) by 90°	i(t) leads v(t) by 90°
Power associated	$I^2R = \frac{V^2}{R}$ (Active Power)	I^2X_L (Reactive Power)	I^2X_C (Reactive Power)

Basic Electrical Technology

[ELE 1051]

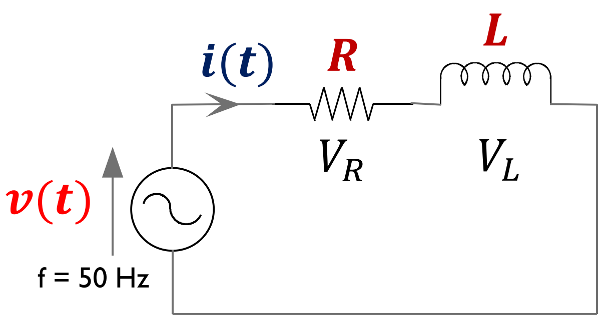
SINGLE PHASE AC CIRCUITS

L16,L17, RL,RC,RLC series circuit

Topics covered…

- AC response of
  - Series RL
  - Series RC
  - Series RLC

RL circuit analysis



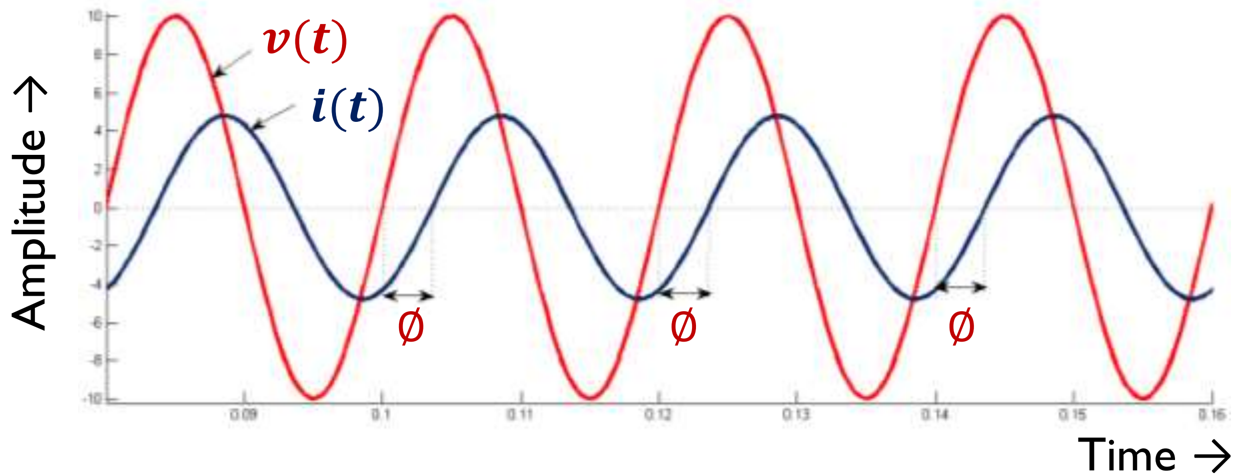
Let $\bar{I}$ *be along the reference*

$$\overline{V_R} = \bar{I}R$$

$$\overline{V_L} = j\bar{I}X_L$$

$$\bar{V} = \overline{V_R} + \overline{V_L} = |V|\angle\emptyset$$

Graphical Representation



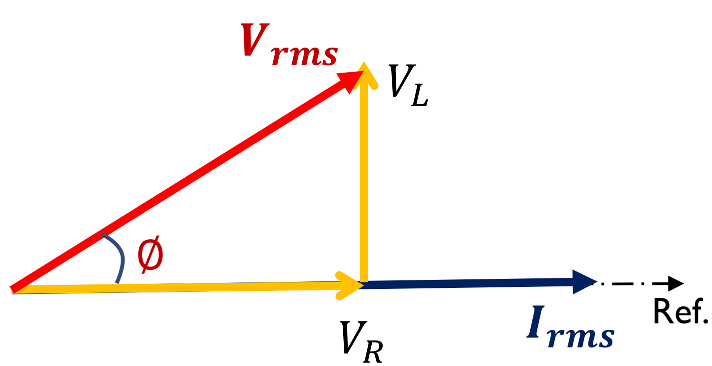
Mathematical Representation

$$i(t) = I_m \sin(\omega t)$$

$$v(t) = V_m \sin(\omega t + \emptyset)$$

$\emptyset$ – *Phase Angle*

Phasor Representation



Impedance

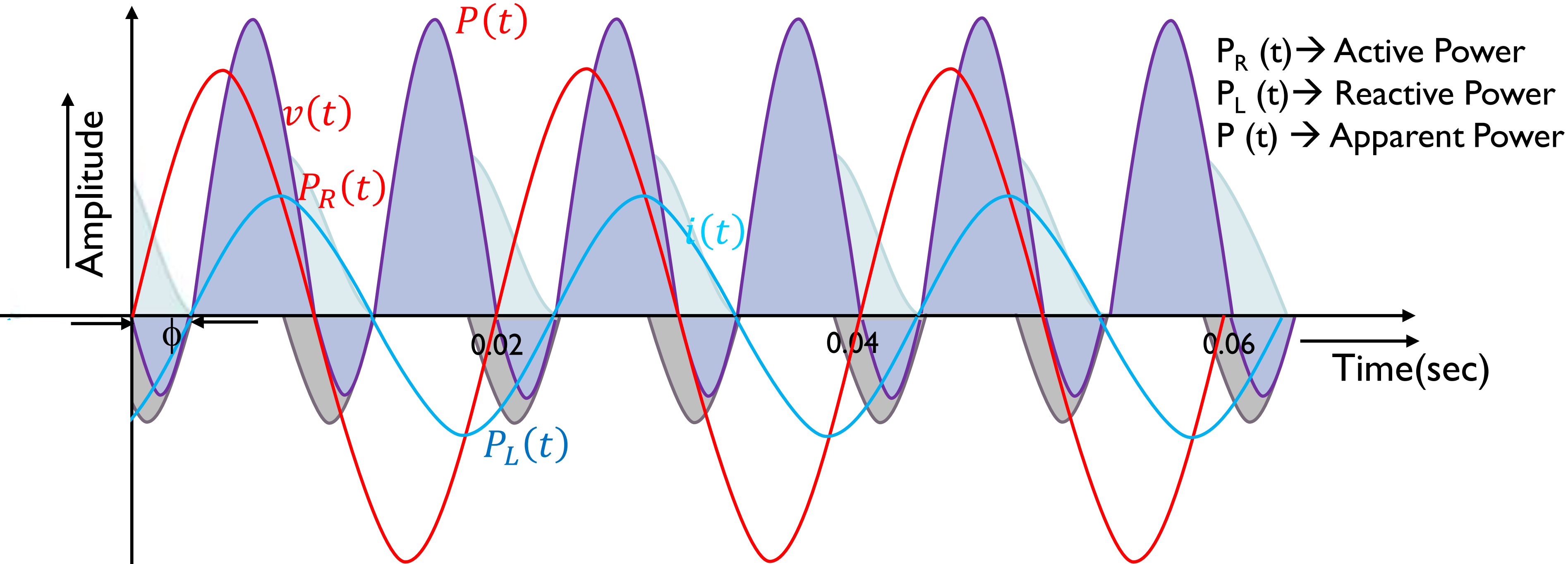
$$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R + jX_L)}{\bar{I}} = R + jX_L = |Z|\angle\emptyset$$

Z – *Impedance of the circuit*

$$\therefore R = |Z|\cos\emptyset \qquad X_L = |Z|\sin\emptyset$$

$$|Z| = \sqrt{R^2 + X_L^2} \qquad \emptyset = \tan^{-1}\frac{X_L}{R}$$

Power associated - RL circuit



Instantaneous power,

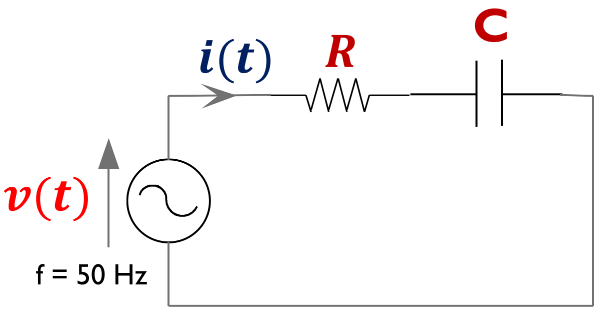
$$p(t) = v(t).\,i(t)$$
$$= V_m I_m \sin \omega t\,.\,\sin(\omega t + \emptyset)$$
$$= V_{rms} I_{rms} [\cos \emptyset - \cos(2\omega t + \emptyset)]$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2}\cos\emptyset}$$

$$\boxed{\mathbf{P_{avg} = V_{rms} I_{rms} \cos \emptyset}}$$

$\cos\emptyset$ *is called the* ***Power Factor***

RC circuit analysis



Let $\bar{I}$ *be along the reference*

$$\overline{V_R} = \bar{I}R$$

$$\overline{V_C} = -j\bar{I}X_C$$

$$\overline{V} = \overline{V_R} + \overline{V_C} = |V|\angle - \emptyset$$

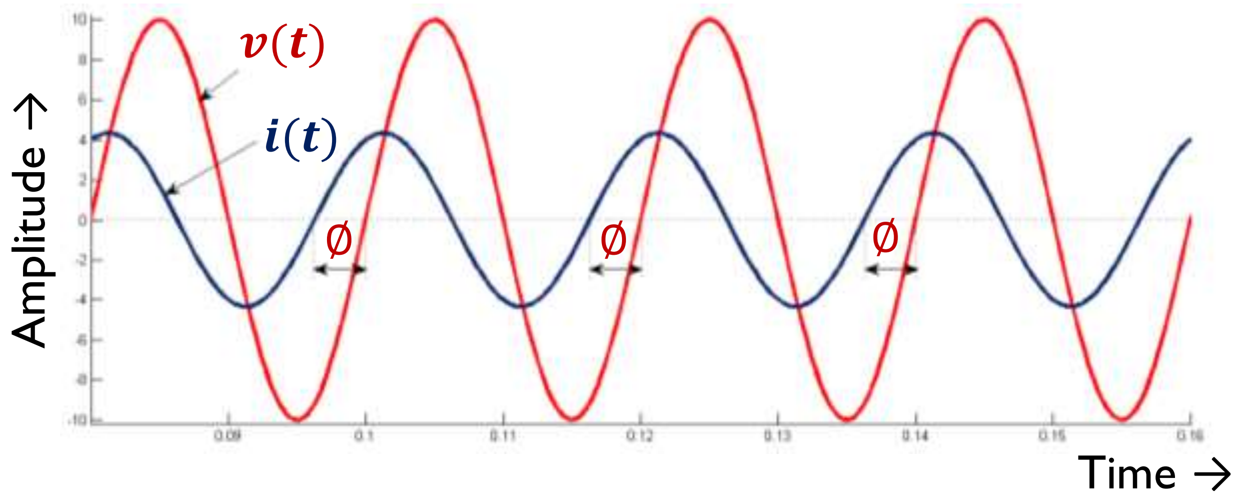
Mathematical Representation

$$i(t) = I_m \sin(\omega t)$$

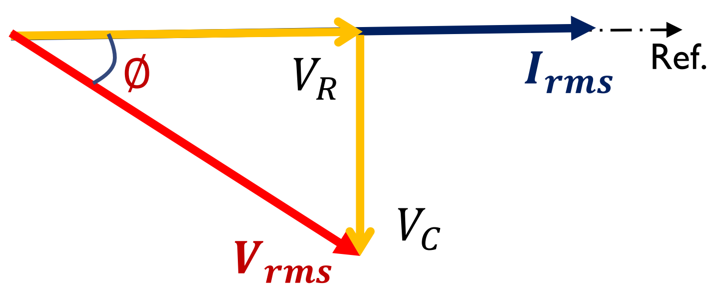
$$v(t) = V_m \sin(\omega t - \emptyset)$$

$\emptyset$ *– Phase Angle*

Graphical Representation



Phasor Representation



Impedance

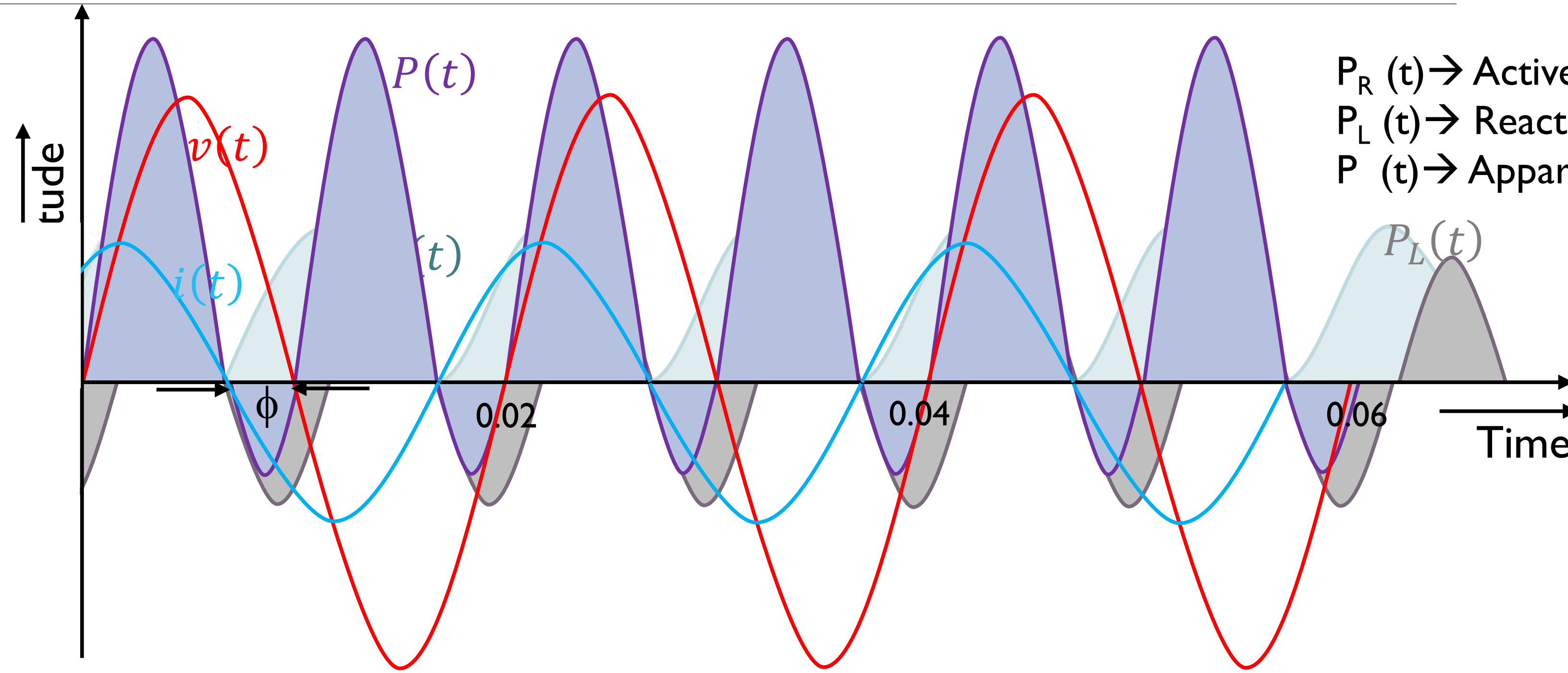
$$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R - jX_L)}{\bar{I}} = R - jX_L = |Z|\angle - \emptyset$$

Z – Impedance of the circuit

$$\therefore \boldsymbol{R = |Z|\cos\emptyset} \qquad \boldsymbol{X_C = |Z|\sin\emptyset}$$

$$\boldsymbol{|Z| = \sqrt{R^2 + X_C^2}} \qquad \boldsymbol{\emptyset = \tan^{-1}\frac{X_C}{R}}$$

Power associated - RC circuit



P_R (t)→ Active Power
P_L (t)→ Reactive Power
P (t)→ Apparent Power

Instantaneous power,

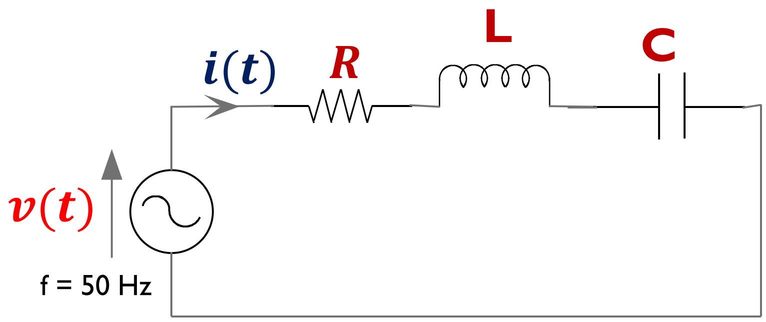
$$p(t) = v(t).i(t)$$
$$= V_m I_m \sin \omega t \,.\, \sin(\omega t - \emptyset)$$
$$= V_{rms} I_{rms}[\cos\emptyset - \cos(2\omega t - \emptyset)]$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2}\cos\emptyset}$$

$$\boxed{\mathbf{P_{avg} = V_{rms} I_{rms} \cos\emptyset}}$$

Ratio of Active power to apparent power is called the ***Power Factor***

RLC circuit



$Let\ i(t)\ be\ the\ reference$

$\boldsymbol{Impedance, Z = R + j(X_L \sim X_c)}$

$if\ X_L = X_C$	$\Longrightarrow$	$Resistive\ circuit\ (Resonance\ condition)$
$if\ X_L > X_C$	$\Longrightarrow$	$RL\ series\ circuit$
$if\ X_L < X_C$	$\Longrightarrow$	$RC\ series\ circuit$

Illustration 1

A resistance of 50Ω is connected in series with an inductance of 200mH and capacitance of 101.321μF across a 230V, 50 Hz, single phase AC supply. Obtain,

a) *Impedance of the circuit*

b) *Current drawn*

c) *Power factor*

d) *Power consumed*

e) *Phasor diagram*

Ans:

$59.050\angle 32.14°\ \Omega$

$3.898\angle -32.14°\ A$

$0.846\ lag$

$759.15\ W$

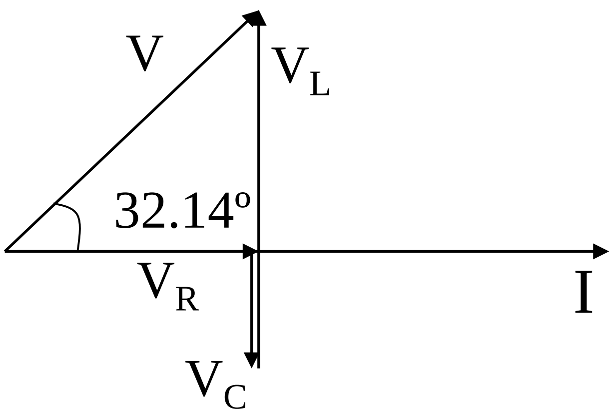
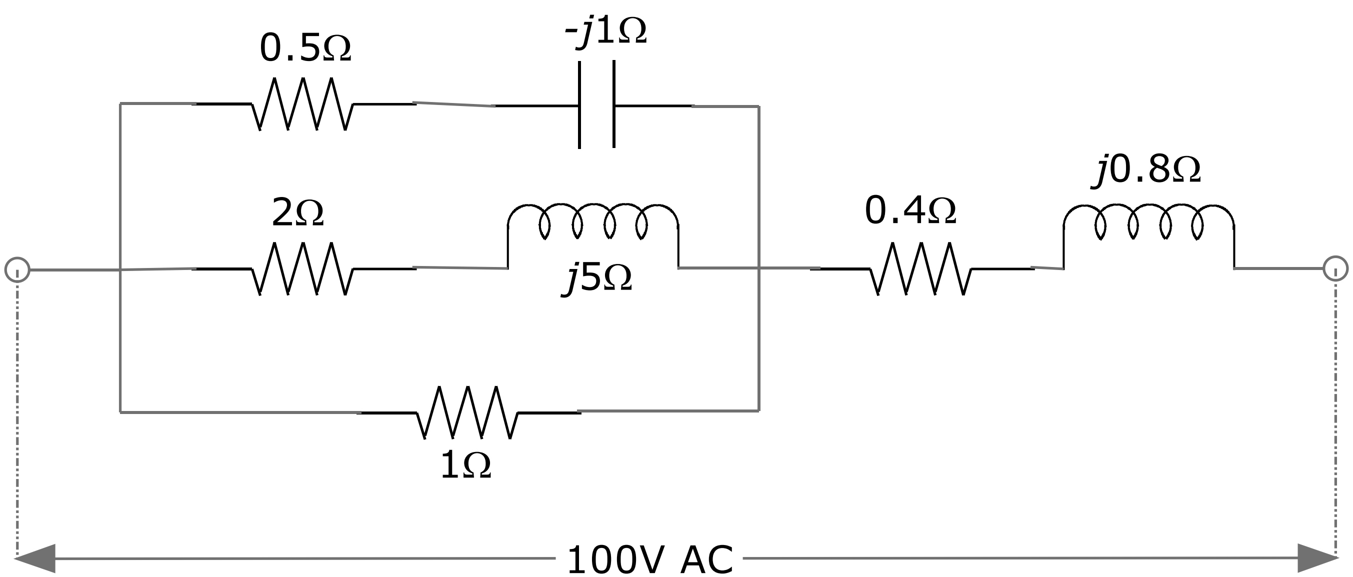


Illustration 2

Determine the impedance of the circuit shown and the power consumed in each branch



Ans:

$Z = 1.12\angle 29.5°\Omega$

$1.25\ kW;\ 0.216\ kW; 3.12\ kW; 3.19\ kW$

Summary

- **Define:** Impedance
- **Define:** Active Power; Reactive Power; Apparent Power; Power Factor

	RL	**RC**
Voltage, current relationship	i(t) lags v(t) by angle θ	i(t) leads v(t) by angle θ
Power associated	$\mathrm{S} = VI$ $\mathrm{P} = VI\cos\theta$ $\mathrm{Q} = VI\sin\theta$	$\mathrm{S} = VI$ $\mathrm{P} = VI\cos\theta$ $\mathrm{Q} = -VI\sin\theta$

Basic Electrical Technology

[ELE 1051]

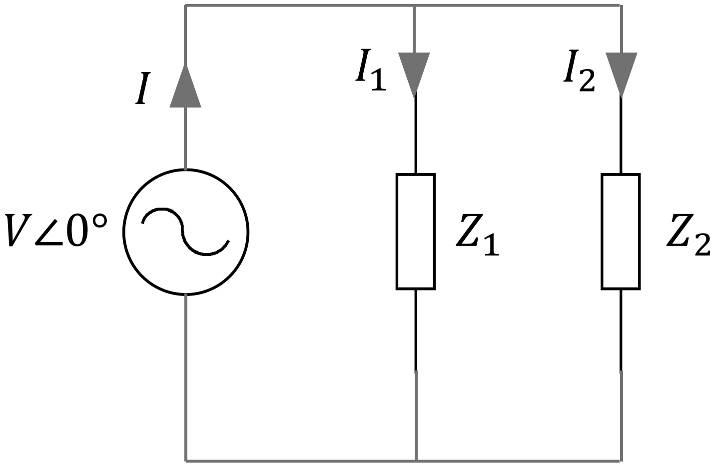
SINGLE PHASE AC CIRCUITS

L18 –Parallel circuits

Topics covered…

- Parallel circuit
  - Admittance
  - Conductance
  - Susceptance

Impedance in parallel



Let $Z_1 = R_1 + jX_1 \qquad Z_2 = R_2 + jX_2$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad I_1 = I * \frac{Z_2}{Z_1 + Z_2} \qquad I_2 = I * \frac{Z_1}{Z_1 + Z_2}$$

$$Y_{eq} = Y_1 + Y_2 \qquad Y:\ Admittance$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_1} = \frac{1}{(R_1 + jX_1)} * \frac{(R_1 - jX_1)}{(R_1 - jX_1)} = \frac{R_1}{(R_1^2 + X_1^2)} - j\frac{X_1}{(R_1^2 + X_1^2)} = G_1 - jB_1$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 + jX_2} = \frac{1}{(R_2 + jX_2)} * \frac{(R_2 - jX_2)}{(R_2 - jX_2)} = \frac{R_2}{(R_2^2 + X_2^2)} - j\frac{X_2}{(R_2^2 + X_2^2)} = G_2 - jB_2$$

$G:\ Conductance \qquad B:\ Susceptance$

$$B_{eq} = \frac{X_1 + X_2}{(R_1^2 + X_1^2)\,(R_2^2 + X_2^2)}$$

$$\mathbf{Y_{eq} = (G_1 + G_2) - j(B_1 + B_2) = G_{eq} - jB_{eq}}$$

$$G_{eq} = \frac{R_1 + R_2}{(R_1^2 + X_1^2)(R_2^2 + X_2^2)}$$

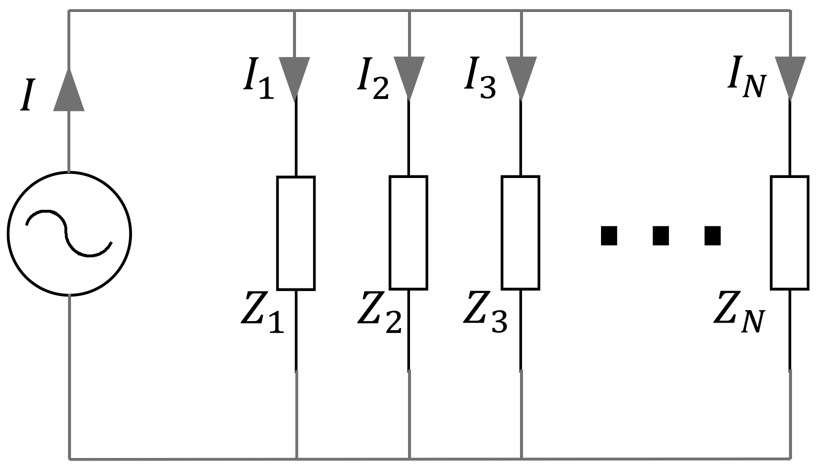
Impedance in parallel

For 'N' impedances connected in parallel,

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}$$

$$Y_{eq} = Y_1 + Y_2 + Y_3 + \cdots + Y_N$$

$$\mathbf{Y_{eq}} = \mathbf{G_{eq}} \pm j\mathbf{B_{eq}}$$



$$I_1 = VY_1; I_2 = VY_2; I_3 = VY_3; \ldots\ldots I_N = VY_N$$

$$I = I_1 + I_2 + I_3 + \cdots + I_N = VY_{eq}$$

Network equations for AC circuits

KVL Equation

(Matrix form)

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

$$[V] = [Z][I]$$

KCL Equation

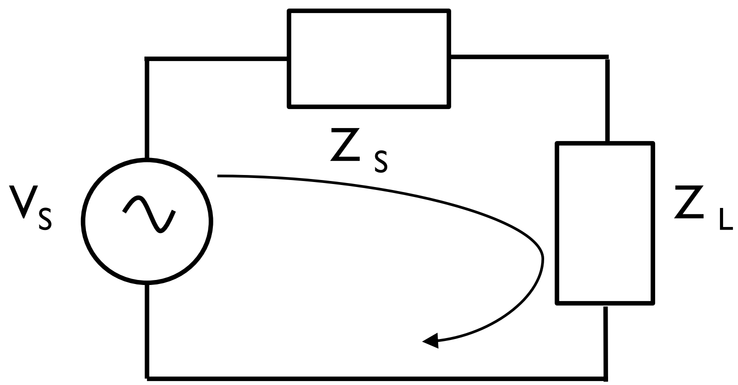
(Matrix form)

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

$$[I] = [Y][V]$$

All the other theorems are applicable to the AC circuits

Maximum power transfer theorem



	Type of load	Condition of maximum power transfer
Case 1	Load is purely resistive	$R_L = \sqrt{R_S^2 + X_S^2}$
Case 2	Both R_L & X_L are variable	$Z_L = Z_{TH}^*$
Case 3	X_L is fixed & R_L is variable	$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$

Crammers rule

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

Solution for the linear simultaneous equations above is as follows

__Step 1__: finding the determinant

$$\Delta = \begin{vmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{vmatrix}$$

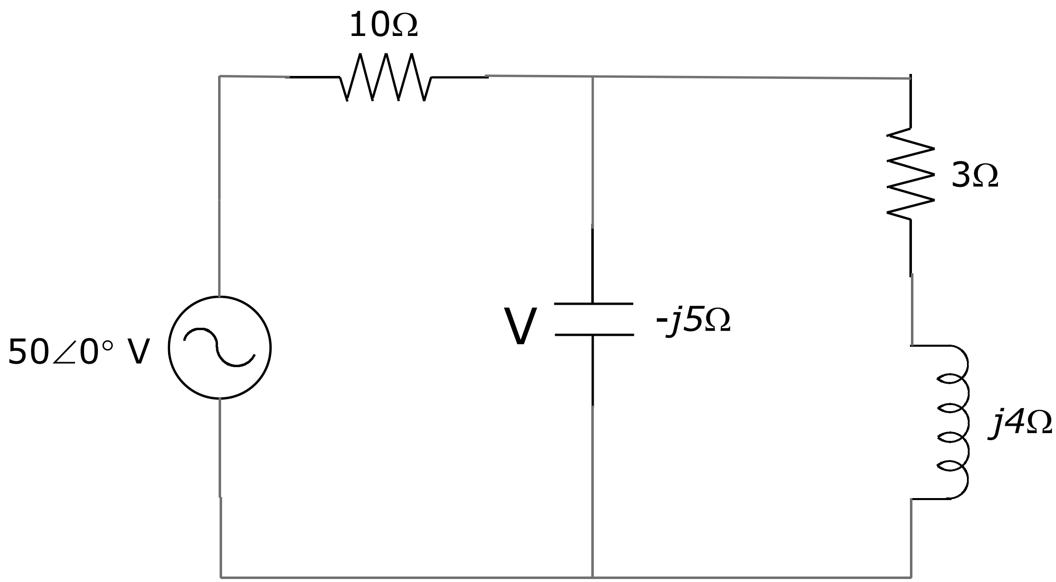
__Step 2__: finding the determinant after substituting first column with RHS column matrix

$$\Delta_1 = \begin{vmatrix} \boldsymbol{V_1} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ \boldsymbol{V_N} & \cdots & Z_{NN} \end{vmatrix}$$

__Step 3__ :Solution for I_1 $\quad I_1 = \dfrac{\Delta_1}{\Delta}$

Illustration 1

Assigning two mesh currents, find the voltage V across the capacitor in the following circuit

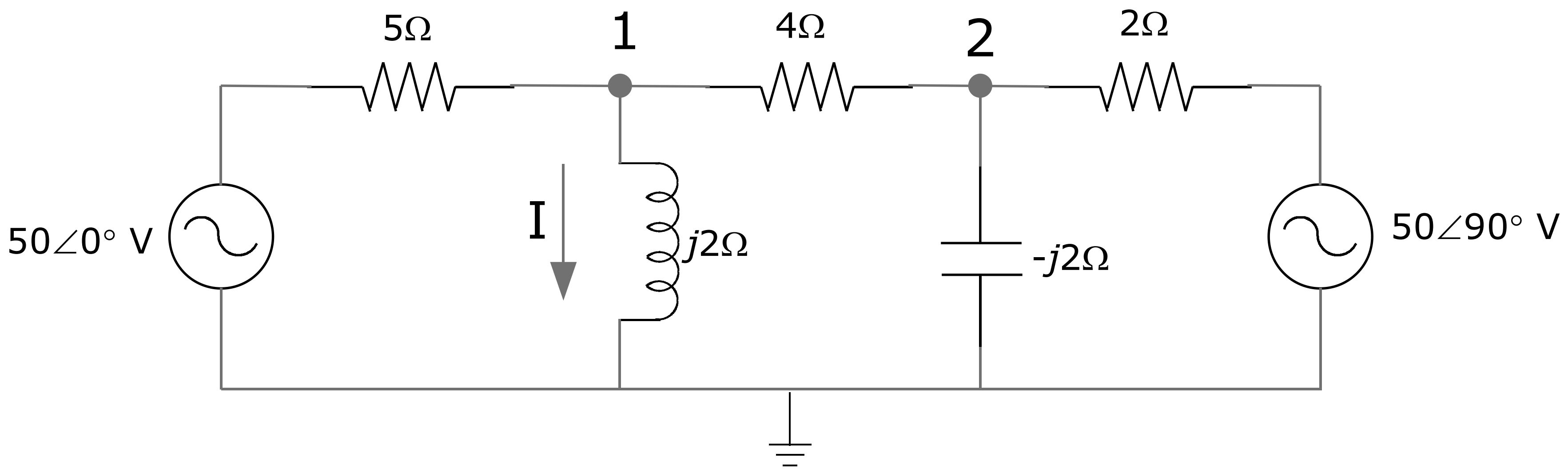


Ans:

$V = 22.36\angle -10.30° V$

Illustration 2

Use node voltage method to obtain the current I in the network



Ans:

$$I = 12.38\angle -17.75^{\circ}\ A$$

Summary

- **Define:** Conductance; Susceptance; Admittance
- All network equations & theorem are applicable to AC circuits

Basic Electrical Technology

[ELE 1051]

SINGLE PHASE AC CIRCUITS

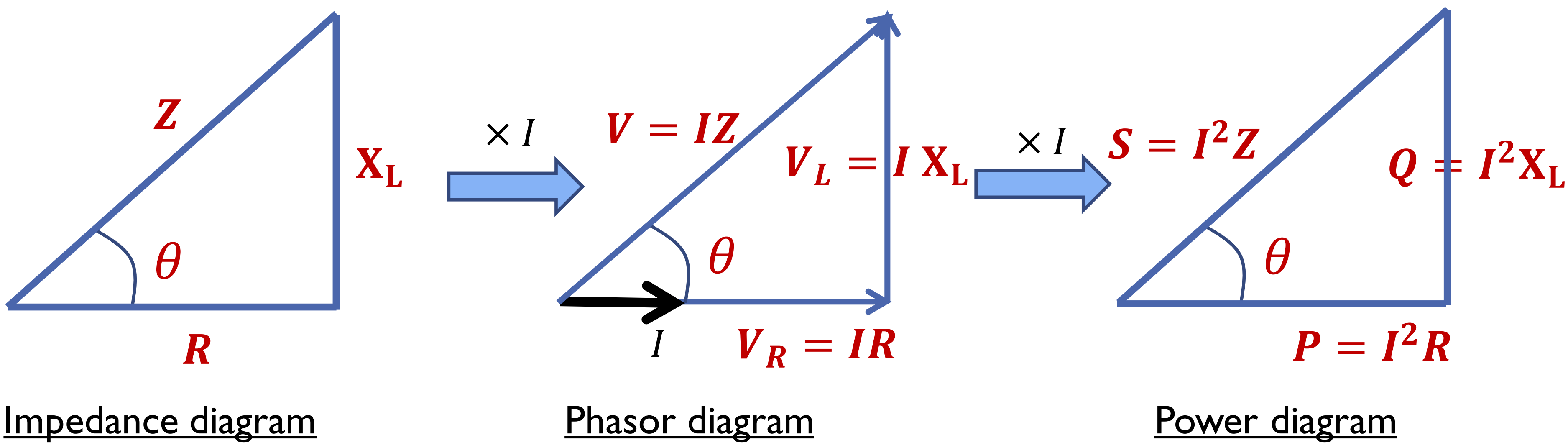
L19 – Power in AC circuits

Topics covered…

- Impedance, phasor & Power Diagram
- Concept of power factor and its significance
- Need for power factor improvement

Power associated in RL load

For RL load:



$$S = P + jQ$$

Where,
S = Apparent Power (VA)
P = Active Power (W)
Q = Reactive Power (var)

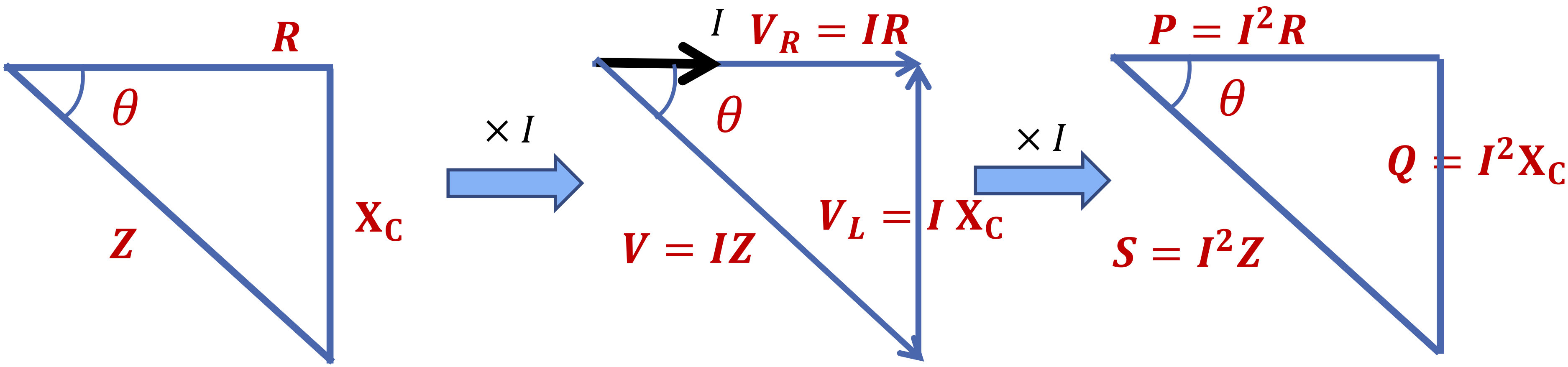
$$S = VI$$

$$P = VI \cos \emptyset$$

$$Q = VI \sin \emptyset$$

Power associated in RL load

For RC load:



Impedance diagram

Phasor diagram

Power diagram

$$S = P - jQ$$

Where,
S = Apparent Power (VA)
P = Active Power (W)
Q = Reactive Power (var)

$$S = VI$$

$$P = VI\cos\emptyset$$

$$Q = VI\sin\emptyset$$

Power Factor

$$\textbf{Power Factor} = \frac{\textbf{Active Power P in watts}}{\textbf{Apparent Power S in voltamperes}}$$

$$\cos\theta = \frac{P}{S} = \frac{P}{VI}$$

- For an impedance Z,

$$\cos\theta = \frac{IR}{V} = \frac{IR}{IZ} = \frac{resistance}{impedance}$$

- Power factor is *lagging* when the *current lags the supply voltage*
- Power factor is *leading* when the *current leads the supply voltage*
- For a resistive load, power factor is Unity

Disadvantages of Low Power Factor

1) Draws more reactive power
2) For supplying a certain amount of active power (consumed), the apparent power to be supplied increased
3) Cost of supply increases
4) Voltage drop occurs in the vicinity of consumer

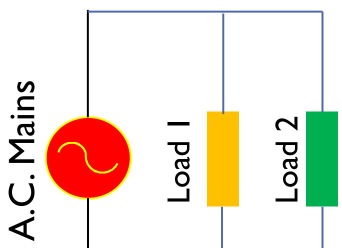
- Increased transmission losses
- Hence bulk consumers are advised to maintain the power factor close to unity by power utilities.

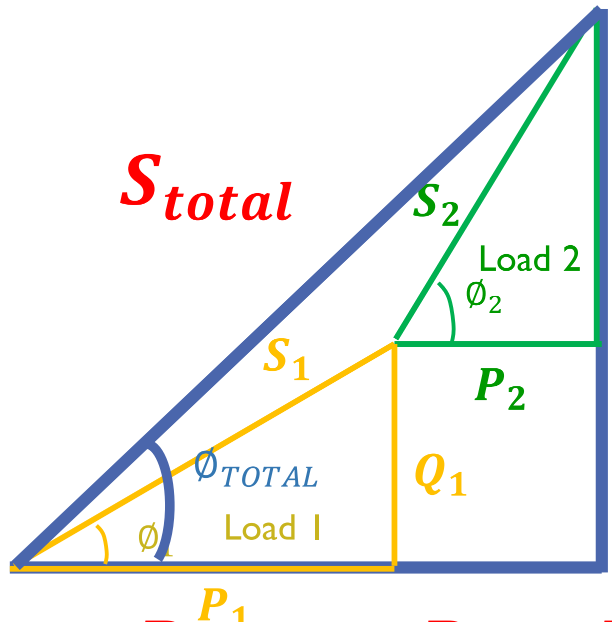
Remedial Measures

- Most of the industrial loads are inductive in nature
- Reactive power demand of Inductive loads can be compensated with capacitive loads
- Hence it is possible to localise reactive power requirement by connecting parallel capacitors across the load

Power Triangle

- Practically, loads are in connected parallel
- Majority of the loads are inductive in nature





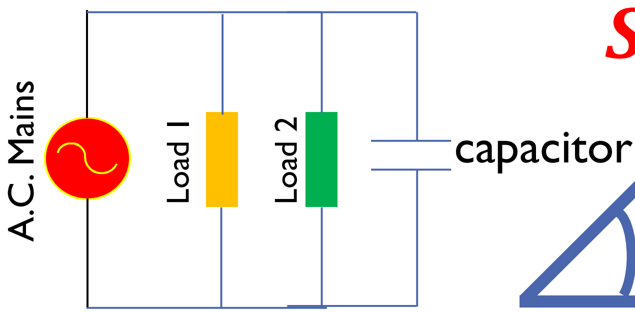
$$Q_{total} = Q_1 + Q_2$$

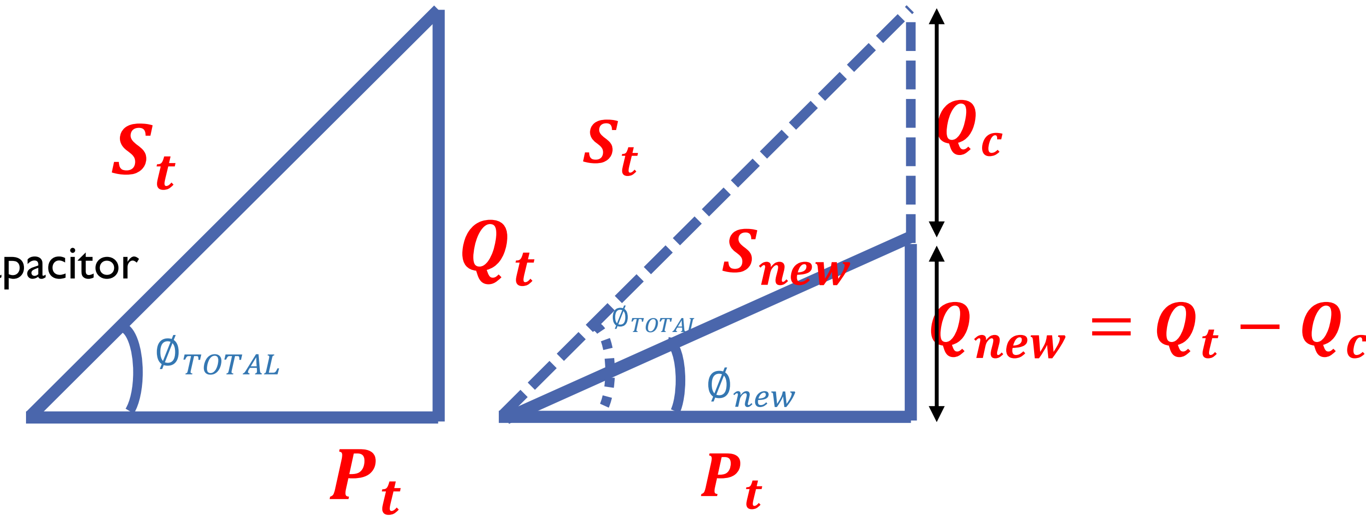
$$P_{total} = P_1 + P_2$$

$$S_{total} = P_{total} + jQ_{total}$$

Power Factor Improvement

- Connect capacitor parallel to the load
- Energy stored by the capacitor provides the required reactive power by the load



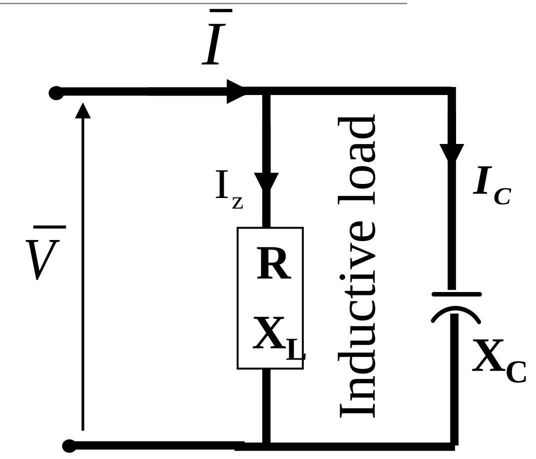


<u>Calculation of capacitor value</u>

- Calculate Q_c needed to improve power factor to $cos\emptyset_{new}$
- Calculate $X_c = \frac{V^2}{Q_C}$ & $c = \frac{X_c}{2\pi f}$

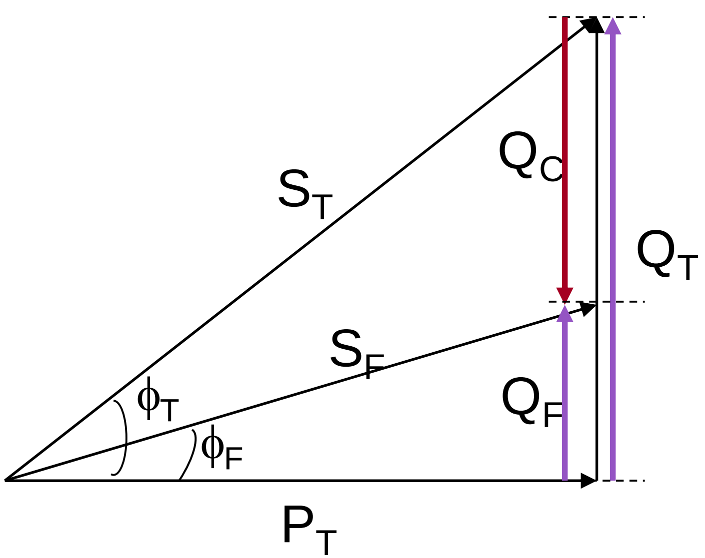
Power Factor Improvement

Since most of the loads are inductive in nature, a capacitor connected in parallel to the inductive load improves the power factor



Determination of Capacitor Value:

- ϕ_T is the over all power angle
- ϕ_F is the final power angle after connecting the capacitor across the load



- $S_T = P_T/\text{Cos}\phi_T$ & $S_F = P_T/\text{Cos}\phi_F$
- $Q_T = (P_T/\text{Cos}\phi_T) * \text{Sin}\ \phi_T$
- $Q_F = (P_T/\text{Cos}\phi_F) * \text{Sin}\ \phi_F$
- $Q_C = (Q_T - Q_F) = P_T(\text{Tan}\ \phi_T - \text{Tan}\ \phi_F)$
- $X_C = V^2/\ Q_C$

Illustration 1

A single-phase motor takes 8.3 A at a power factor of 0.866 lagging when connected to a 230 V, 50 Hz supply. Capacitance bank is now connected in parallel with the motor to raise the power factor to unity. Determine the capacitance value

Ans: $57.4\ \mu F$

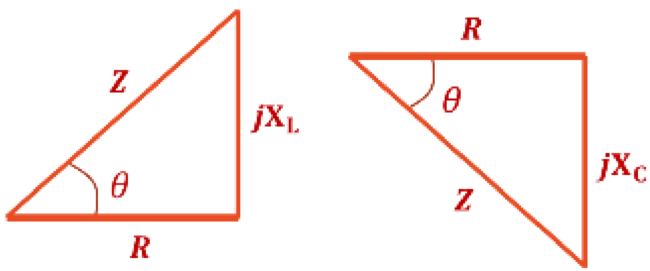
Illustration 2

A single-phase load of 5 kW operates at a power factor of 0.6 lagging. It is proposed to improve this power factor to 0.95 lagging by connecting a capacitor across the load. Calculate the kvar rating of the capacitor

Ans: 5.02 kvar

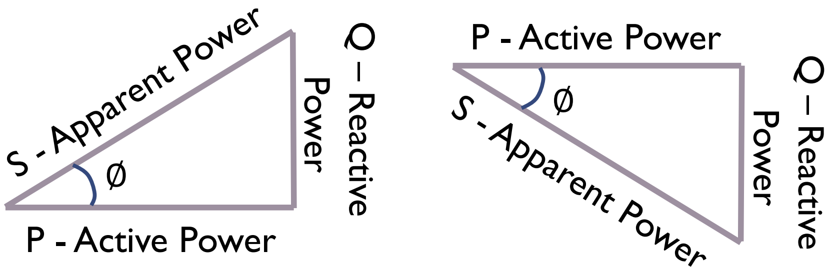
Summary

Impedance diagram



RL Load RC Load

Power diagram



RL Load RC Load

- Power in AC circuit is given by, $\boldsymbol{S = VI}$
- For any load with impedance Z,
  - Active Power (watt), $\boldsymbol{P = VI\cos\theta = I^2Z\cos\theta = I^2R}$
  - Reactive Power (var), $\boldsymbol{Q = VI\sin\theta = I^2Z\sin\theta = I^2X}$
  - Apparent Power (VA), $\boldsymbol{S = VI = I^2Z}$
- $\boldsymbol{Power\ Factor} = \dfrac{\boldsymbol{Active\ Power}}{\boldsymbol{Apparent\ Power}} = \dfrac{P}{S} = \dfrac{R}{Z} = \cos\theta$

Summary

- Low power factor loads must be avoided

- Capacitor bank connected parallel to the load serves as the source of reactive power for the load
  - Improves load power factor
  - Reduces transmission and distribution losses

Basic Electrical Technology

[ELE 1051]

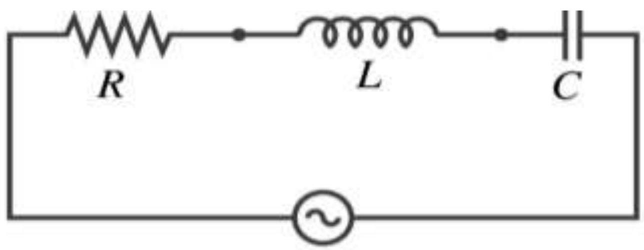
SINGLE PHASE AC CIRCUITS

L20 -Resonance

Topics covered…

- Resonance in series RLC circuit
- What is half power frequency, bandwidth and quality factor in series RLC circuit?
- Resonance in parallel circuits

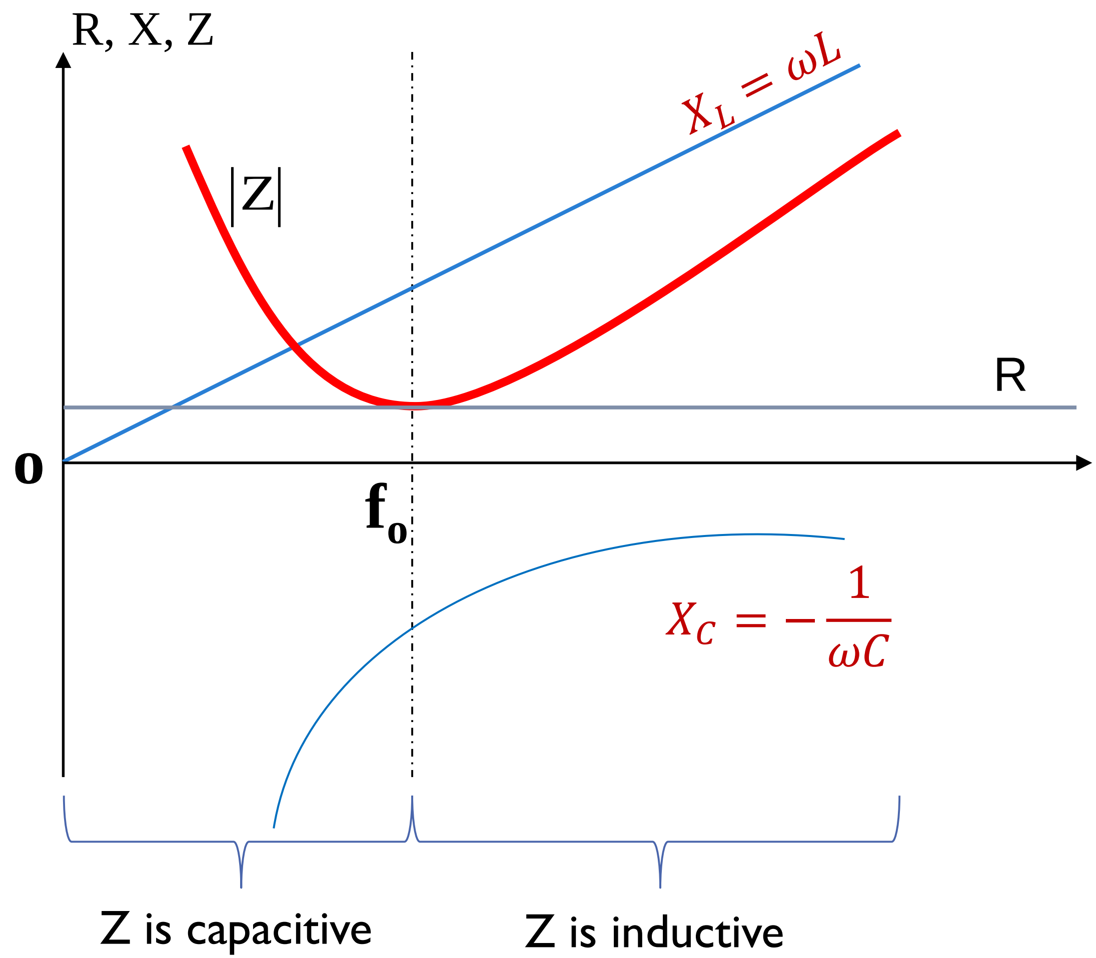
Series Resonance



v(t), variable frequency

$$Z = R + j(X_L \sim X_C)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



'f_0 is called the resonant frequency'

Series Resonance

- **When series RLC circuit is at resonance,**
  - Current is in phase with voltage
  - Circuit power factor is unity
  - $X_L = X_C$
  - $Z = R$

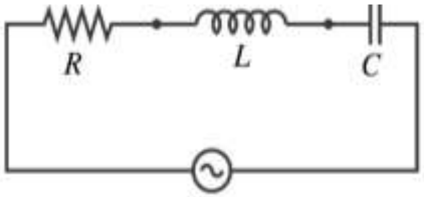
- **Resonant frequency for a series RLC circuit is obtained as follows:**

$$\text{Imaginary part of } Z_{eq} = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\boxed{\boldsymbol{f_0 = \frac{1}{2\pi\sqrt{LC}}\ hertz}}$$

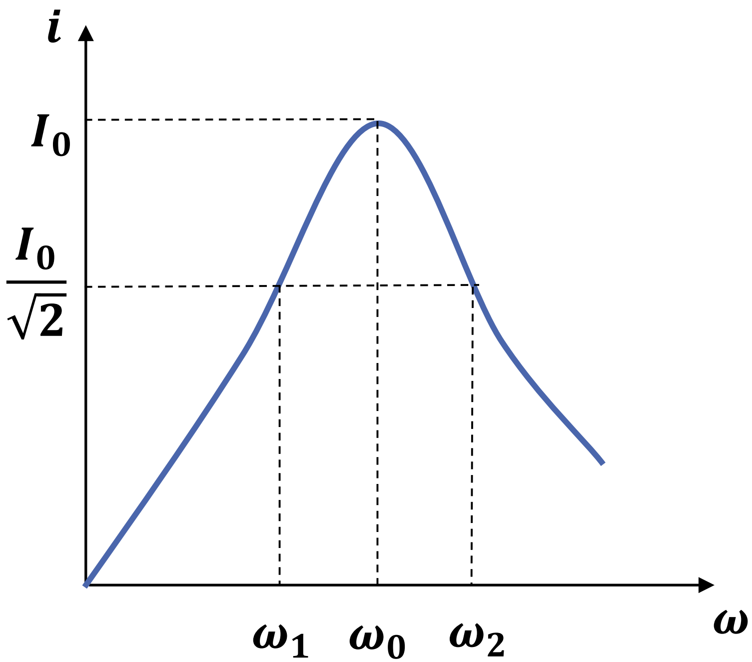


v(t), variable frequency

$$Z = R + j(X_L \sim X_C)$$

Current vs. Frequency in RLC Series Circuit

Variation of current with frequency



$$I_0 = I_{max} = \frac{V_{rms}}{R}$$

- **Half Power Frequency**

  'Frequency at which the power is half of the power at resonant frequency'

$$Power = \frac{1}{2} I_0^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R$$

$$At\ \omega_1\ and\ \omega_2, I = \frac{I_0}{\sqrt{2}}$$

$\omega_1 = Lower\ half\ power\ frequency$

$\omega_2 = Upper\ half\ power\ frequency$

$$\mathbf{Bandwidth = \omega_2 - \omega_1}$$

In practice the curve of $|I|$ against ω is not symmetrical about the resonant frequency

Half Power Frequency

$$Impedance\ at\ \omega_1 and\ \omega_2, |Z| = {V_0}\Big/{\frac{I_0}{\sqrt{2}}} = \sqrt{2}R$$

Below Resonant frequency $\omega_0, |X_C| > |X_L|$

At ω_1,

$$\sqrt{R^2 + (X_C - X_L)^2} = \sqrt{2}R$$

$$X_C - X_L = R$$

$$\frac{1}{\omega_1 C} - \omega_1 L = \text{R}$$

$$\boxed{\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

Above Resonant frequency $\omega_0, |X_L| > |X_C|$

At ω_2,

$$\sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2}R$$

$$X_L - X_C = R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = \text{R}$$

$$\boxed{\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

$$\omega_2\omega_1 = \frac{1}{LC} = \omega_0^2 \qquad \omega_2 - \omega_2 = \frac{R}{L}$$

Quality Factor for series circuit

- At resonance, V_C and V_L can be very much greater than applied voltage

$$|V_C| = |I|X_C = \frac{V.X_C}{\sqrt{R^2 + (X_L - X_C)^2}}$$

At resonance, $X_L = X_C$

$$V_C = \frac{V}{R}X_C$$

$$V_C = \frac{V}{\omega_0 CR} = \boldsymbol{Q}V$$

$$\boldsymbol{Q = \frac{Resonant\ frequency}{Bandwidth}}$$

$$\boxed{\boldsymbol{Q = \frac{1}{\omega_0 CR} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}}}$$

Q is termed the Q factor or voltage magnification

- High value of Q can lead to component damage
- Careful design necessary
- Larger the value of Q, more symmetrical the curve appears about the resonant frequency

Illustration 1

A circuit having a resistance of 4Ω and inductance of 0.5H and a variable capacitance in series, is connected across a 100V, 50Hz supply. Calculate:

a) The capacitance to give resonance
b) The voltages across the inductor and the capacitor
c) The Q factor of the circuit

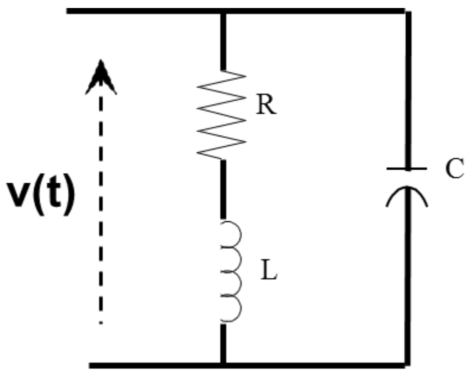
Ans:
C = 20.3μF
$V_C = V_L = 3930V$
Q = 39.3

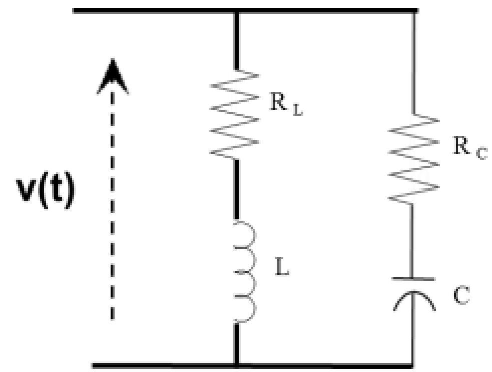
Illustration 2

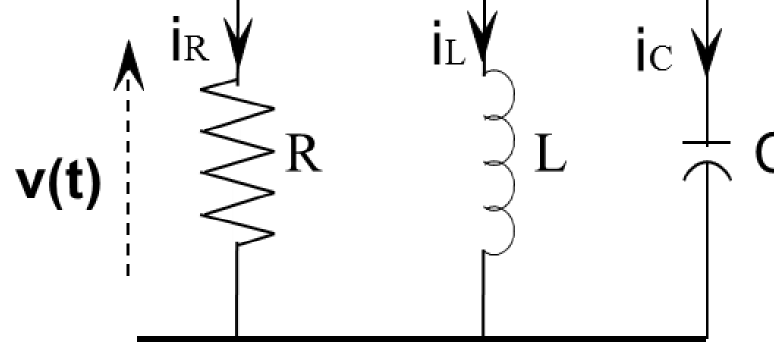
The bandwidth of a series resonant circuit is 500 Hz. If the resonant frequency is 6000 Hz, what is the value of Q? If R =10 Ω, what is the value of the inductive reactance at resonance? Calculate the inductance and capacitance of the circuit

Ans:
Q = 12
$X_L = 120\Omega$
L = 3.18mH; C = 0.22μF

Resonance in parallel circuits





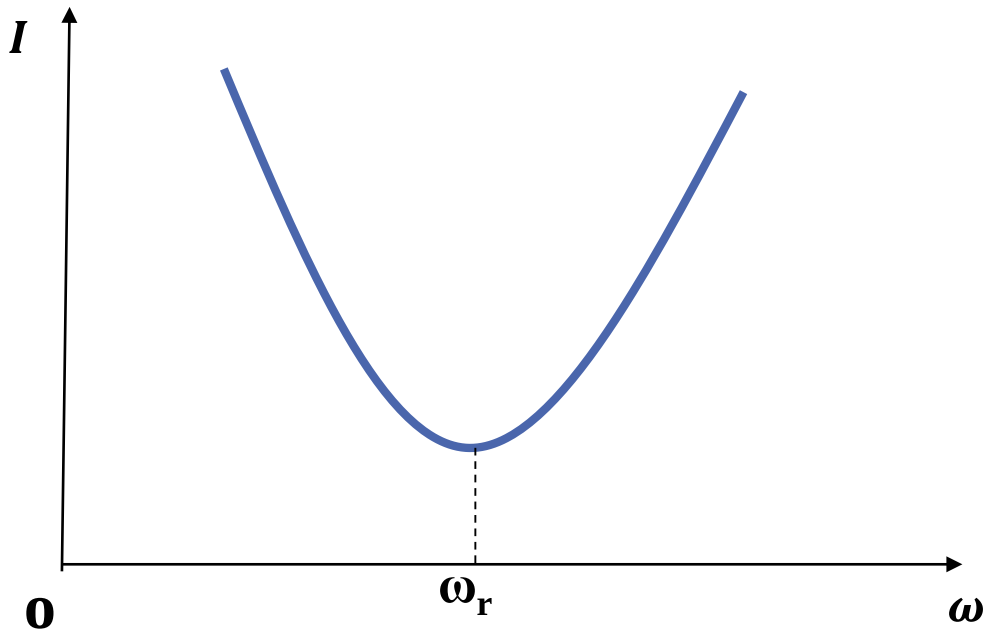


<u>Steps to obtain the expression of resonant frequency in parallel circuits</u>

- Obtain the net admittance of the circuit
- Equate the imaginary part (susceptance) to zero and obtain the expression of ω_r

The expression for resonant frequency depends on circuit configuration

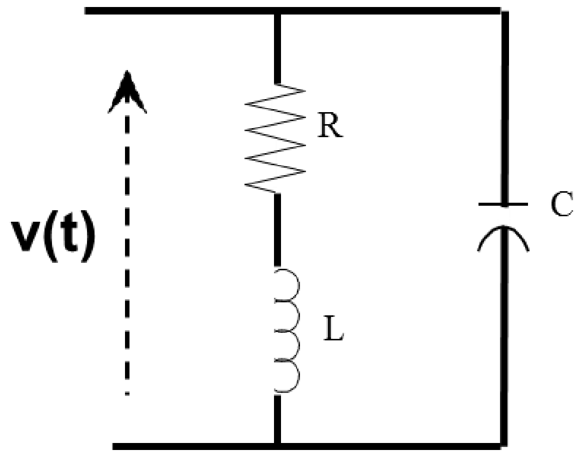
Current vs. Frequency in parallel Circuits



- At resonance
  - Impedance is maximum
  - Resultant current minimum

Illustration 1

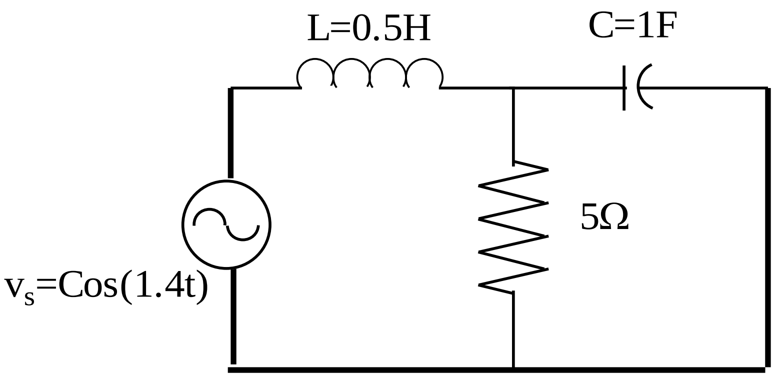
Obtain the expression for resonant frequency for the given parallel circuit



$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

Illustration 2

Show that circuit given in figure will be at resonance at supply frequency



Z = 0.099 at $\omega = 1.4\ rad/sec$

Basic Electrical Technology

[ELE 1051]

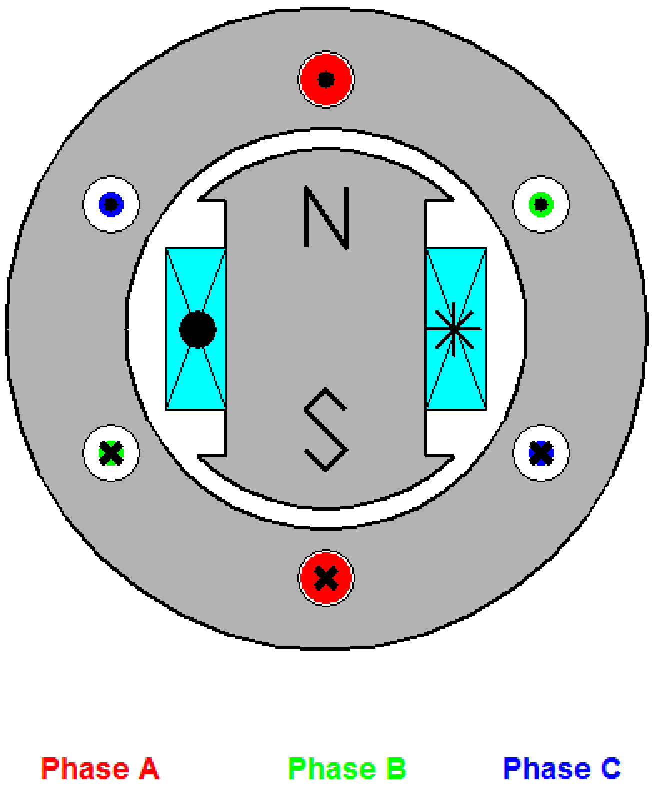
Three Phase AC Circuits

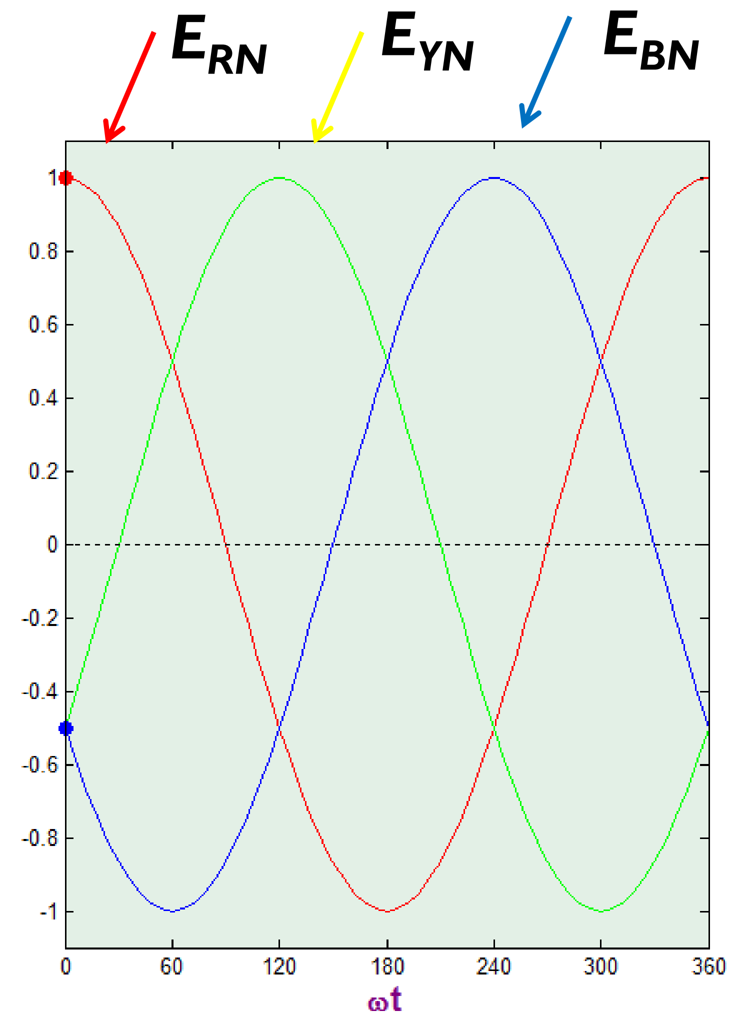
L21 – Generation & Representation of three phase supply

Topics Covered

- *Generation of Three Phase Supply*
- *Representation of Three Phase Excitation*
- *Relationship between Phase and Line Voltages*

Generation of Three Phase





Courtesy : www.ece.umn.edu

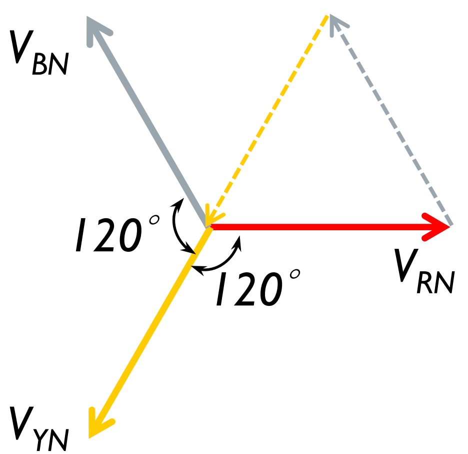
3 Phase Excitation (Phase Voltages)

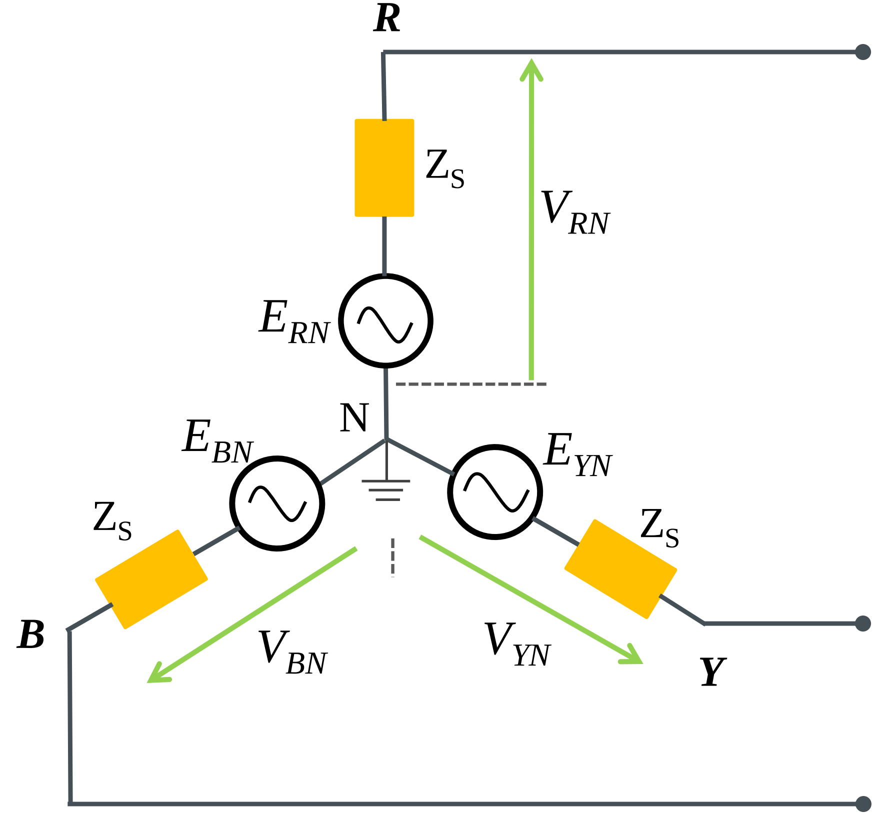
Phase Voltages,

$$\hat{V}_{RN} = V_m\ Sin(\omega t)$$

$$\hat{V}_{YN} = V_m\ Sin(\omega t - 120°)$$

$$\hat{V}_{BN} = V_m\ Sin(\omega t - 240°)$$



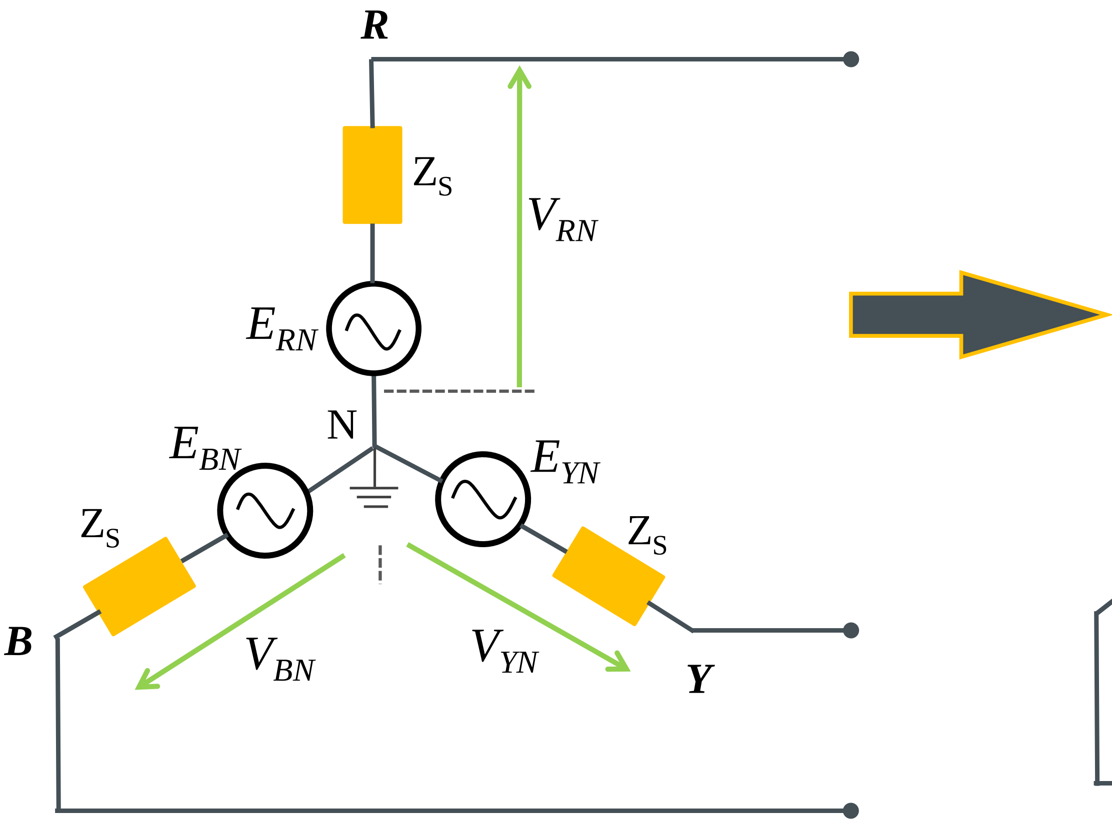


Three Phase Source

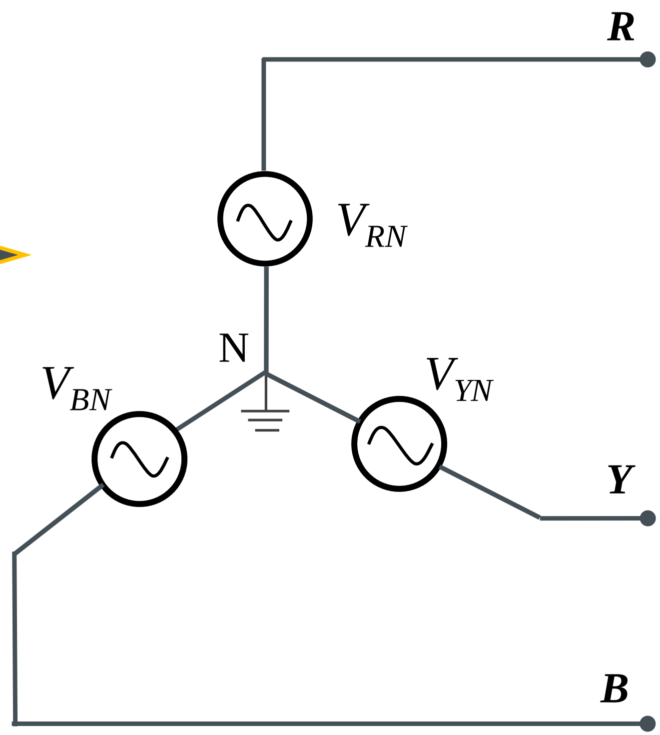
Summing up the phase voltages,

$$\hat{V}_{RN} + \hat{V}_{YN} + \hat{V}_{BN} = 0$$

3 Phase Excitation (Phase Voltages)..

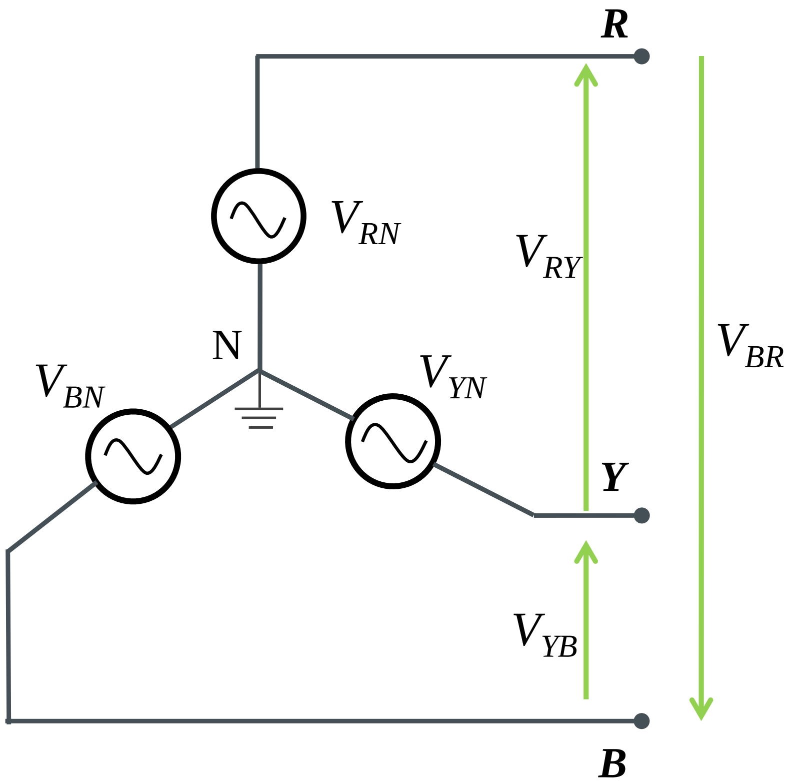


Three Phase Source

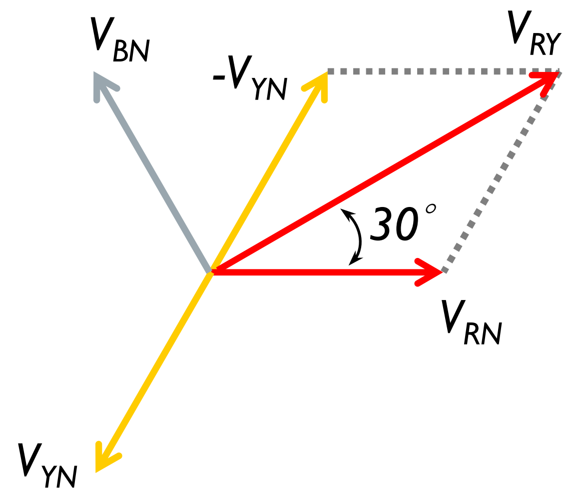


Three Phase Source

3 Phase Excitation (Line Voltages)



Three Phase Source



Line Voltages,

$$\hat{V}_{RY} = \hat{V}_{RN} - \hat{V}_{YN}$$

$$= V_m\,Sin(\omega t) - V_m\,Sin(\omega t - 120°)$$

$$= \sqrt{3} \times V_m\,Sin(\omega t + 30)$$

3 Phase Excitation (Line Voltages)…

Similarly,

$$\hat{V}_{YB} = \hat{V}_{YN} - \hat{V}_{BN}$$

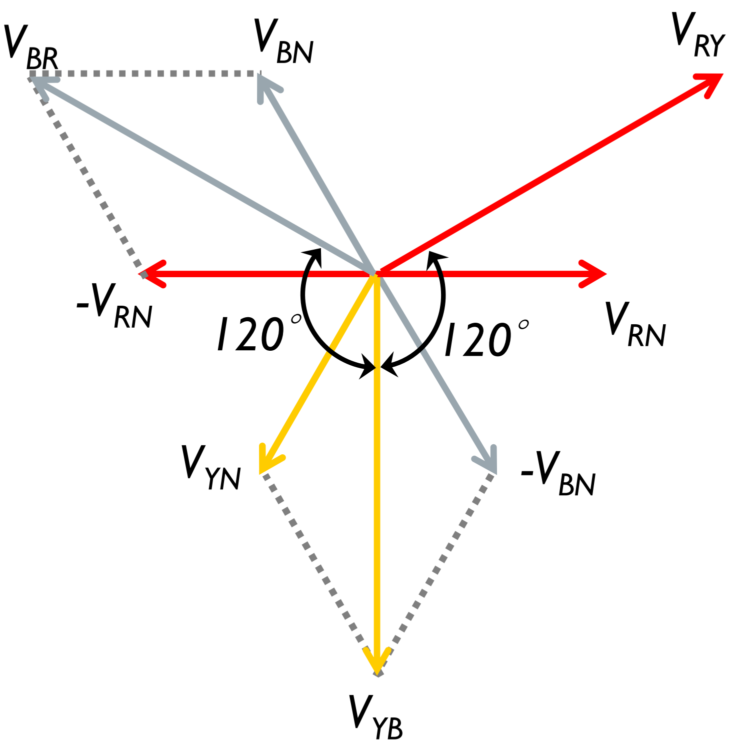
$$= V_m\ Sin(\omega t - 120) - \ V_m\ Sin(\omega t - 240°)$$

$$= \sqrt{3} \times V_m\ Sin(\omega t - 90)$$

$$= V_{RY}\ Sin(\omega t - 120)$$

$$\hat{V}_{BR} = \hat{V}_{BN} - \hat{V}_{RN}$$

$$= V_{RY}\ Sin(\omega t + 120)$$



Summing up the Line voltages,

$$\hat{V}_{RY} + \hat{V}_{YB} + \hat{V}_{BR} = 0$$

In a Three Phase balanced Supply, the summation of Phase voltages and summation of Line Voltages is zero.

Relation b/w Phase & Line Voltages

Phase Voltages

$$\hat{V}_{RN} = V_m\, Sin(\omega t)$$

$$\hat{V}_{YN} = V_m\, Sin(\omega t - 120°)$$

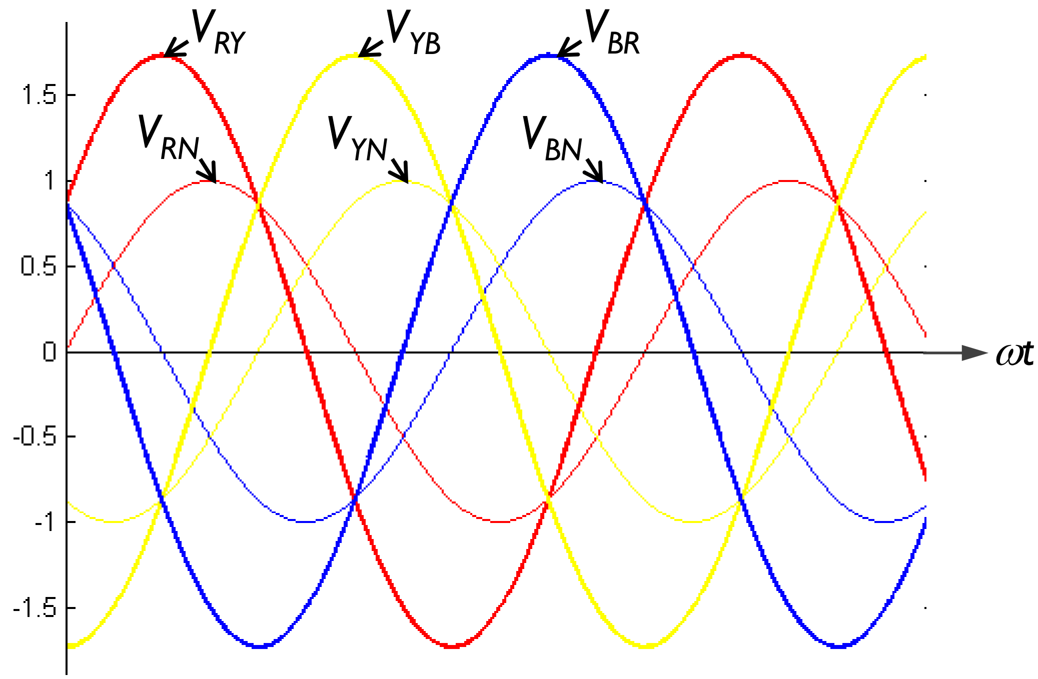
$$\hat{V}_{BN} = V_m\, Sin(\omega t - 240°)$$

Line Voltages

$$\hat{V}_{RY} = \sqrt{3} \times V_m\, Sin(\omega t + 30)$$

$$\hat{V}_{YB} = \sqrt{3} \times V_m\, Sin(\omega t - 90)$$

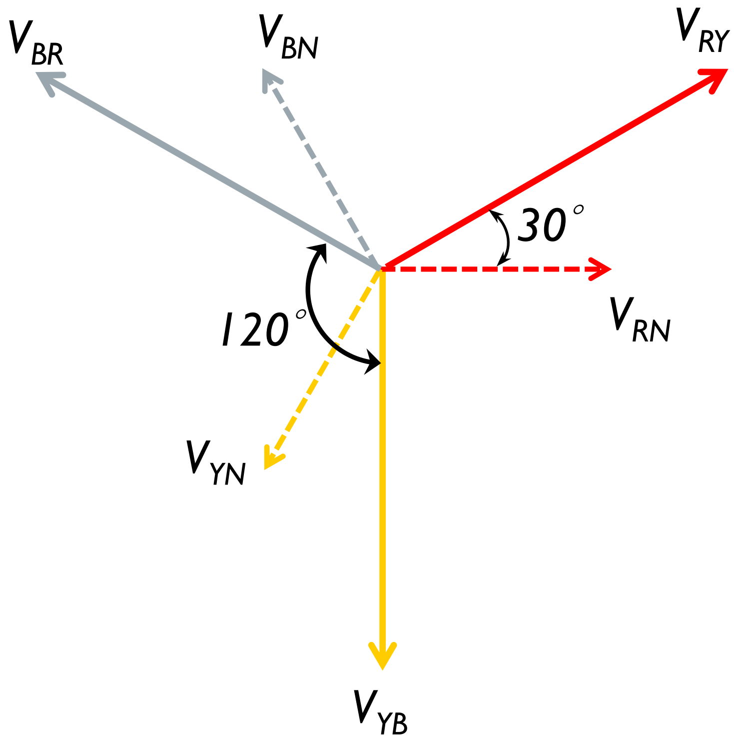
$$\hat{V}_{BR} = \sqrt{3} \times V_m\, Sin(\omega t + 150)$$



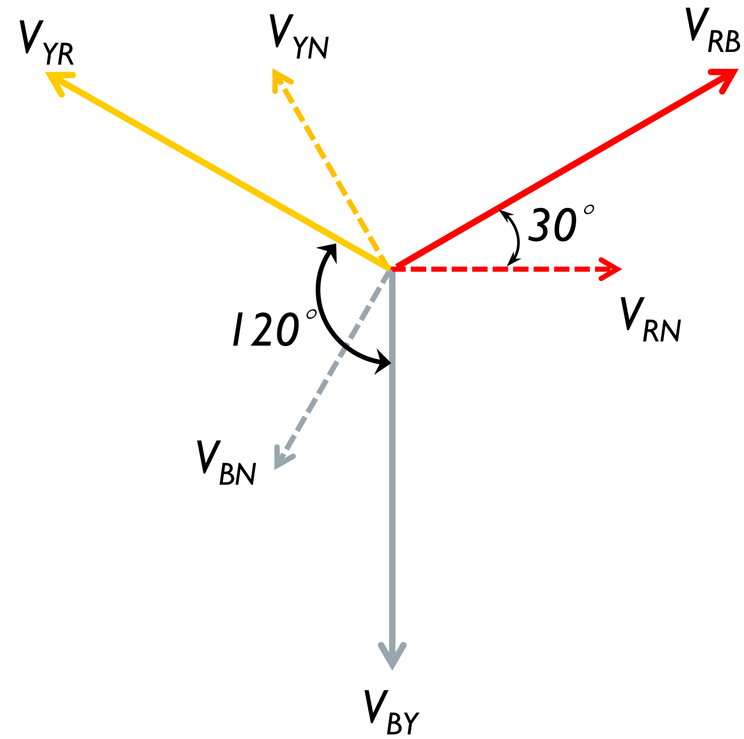
$$|V_{Line}| = \sqrt{3}\ |V_{Phase}|$$

Phase Sequence

1. RYB



2. RBY



Exercise- I

Given the phase voltage V_{RN} of a 3 phase balanced RYB system as 240V, express the phase and line voltages mathematically. Also sketch the phasor diagram.

Solution:

Phase Voltages:

$$\hat{V}_{RN} = 240 \times \sqrt{2} \times \ Sin(\omega t)$$

$$\hat{V}_{YN} = 240 \times \sqrt{2} \times Sin(\omega t - 120°)$$

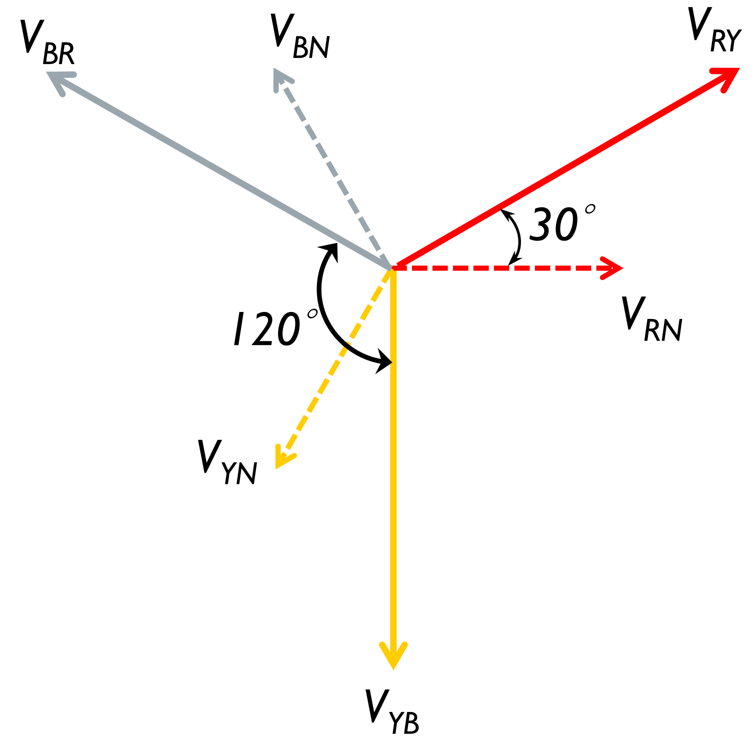
$$\hat{V}_{BN} = 240 \times \sqrt{2} \times Sin(\omega t - 240°)$$

Line Voltages:

$$\hat{V}_{RY} = \sqrt{3} \times 240 \times \sqrt{2} \times \ Sin(\omega t + 30°)$$

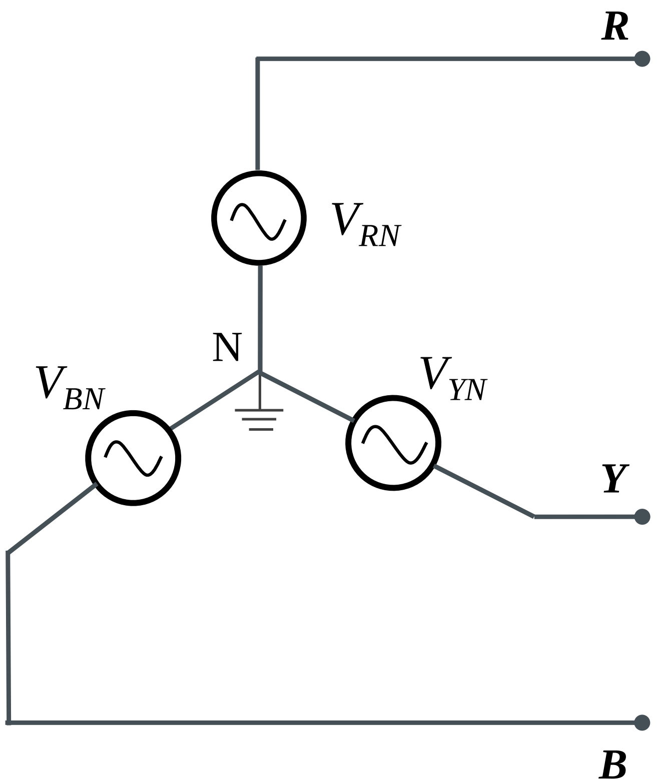
$$\hat{V}_{YB} = \sqrt{3} \times 240 \times \sqrt{2} \times Sin(\omega t - 90°)$$

$$\hat{V}_{BR} = \sqrt{3} \times 240 \times \sqrt{2} \times Sin(\omega t + 150°)$$

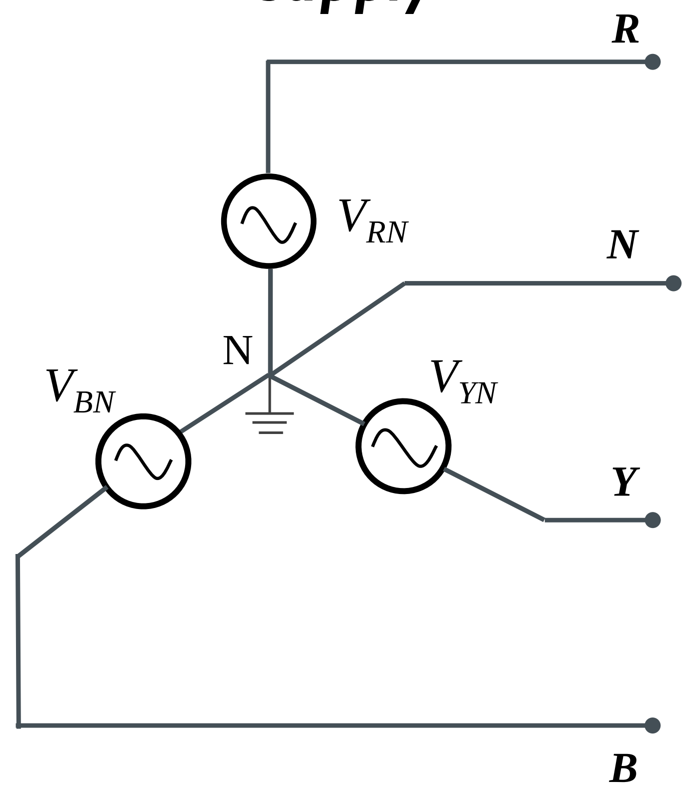


3 Phase 3 Wire & 4 Wire Supply

3 Phase 3 Wire Supply



3 Phase 4 Wire Supply



Summary

In a three phase balanced supply,

- ✓ *Summation of phase voltages = zero*
- ✓ *Summation of Line voltages = zero*
- ✓ *Line voltage is √3 x Phase Voltage*
- ✓ *In an RYB sequence, V_{RY} leads V_{RN} by 30°*
- ✓ *Power transmission is generally through 3 phase 3 wire network and distribution is through 3 phase 4 wire network.*

Basic Electrical Technology

[ELE 1051]

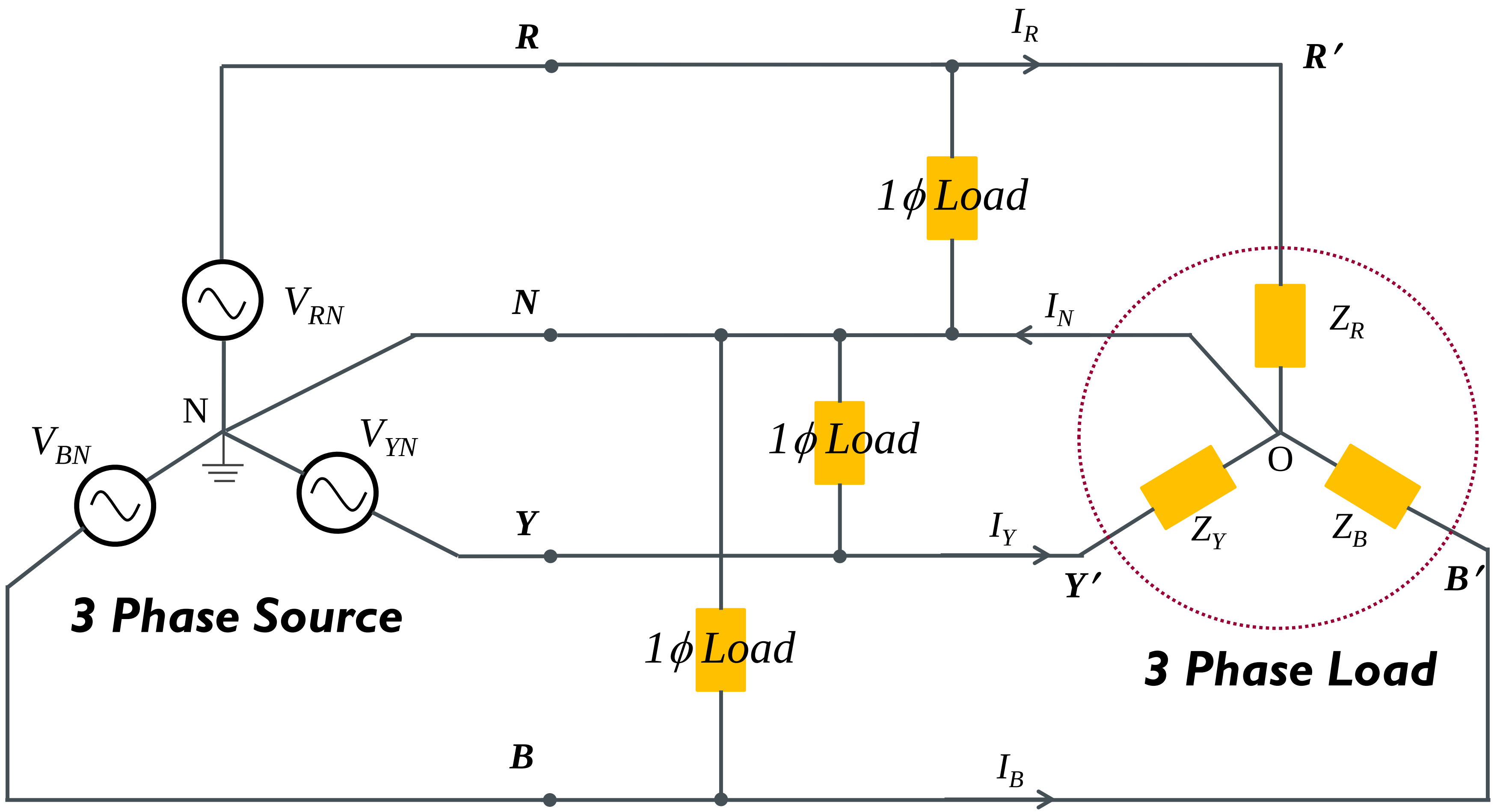
Three Phase AC Circuits

L22 ,L23– Star & Delta Connected Balanced Loads & Unbalanced loads

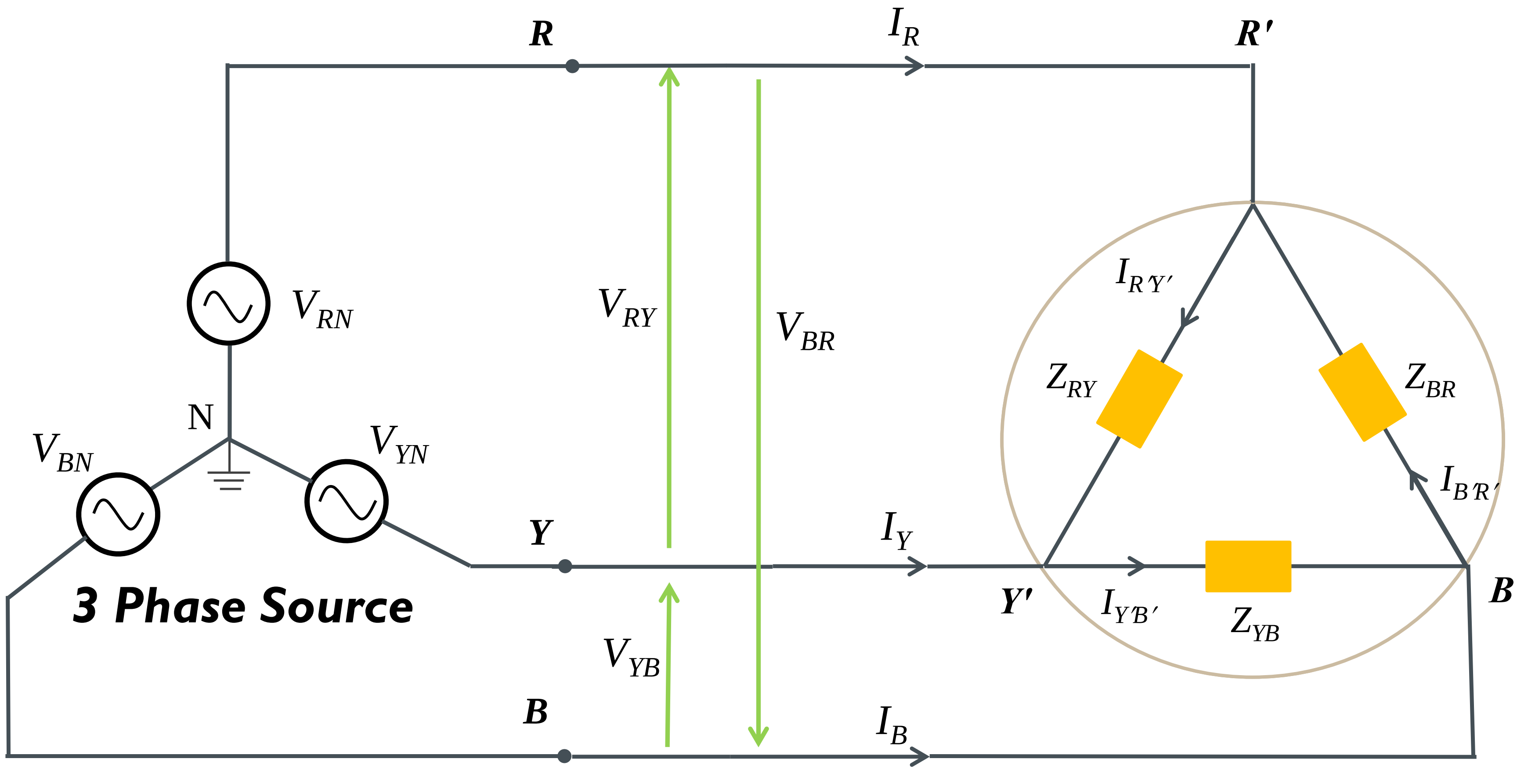
Topics Covered

- *Three Phase loads*
- *Analysis of balanced/unbalanced star/delta connected loads with 3 phase excitation.*
- *Phase and line voltage/current relations.*
- *Neutral shift and circulating currents with unbalanced loads.*

3 Phase 4 Wire Supply

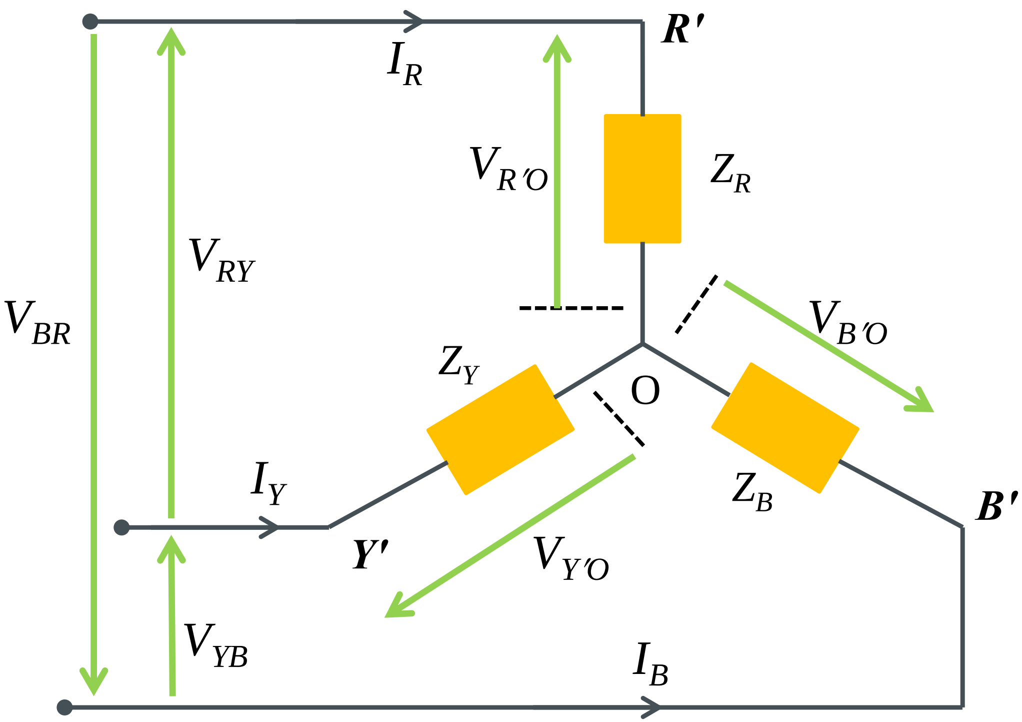


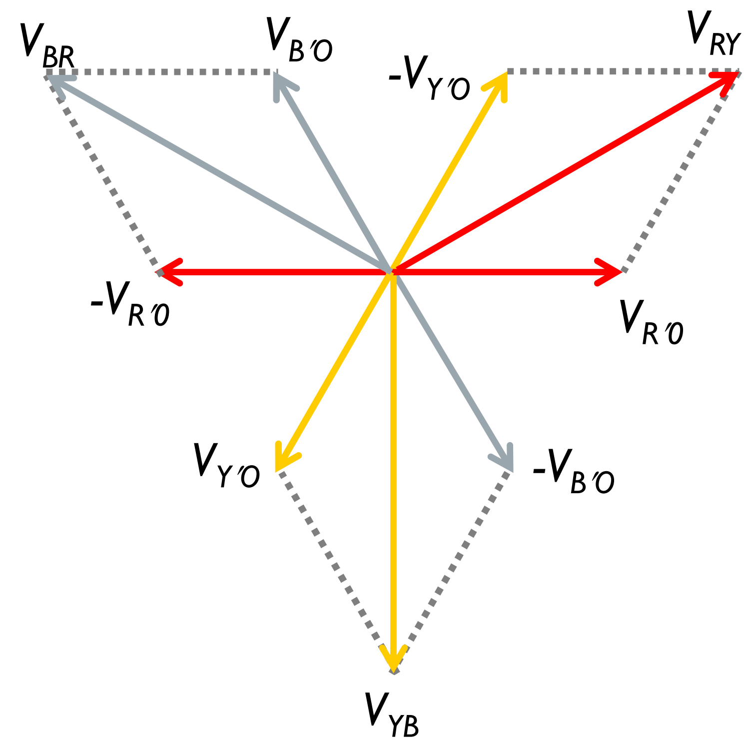
Type of 3 Phase Loads



Delta Connected Load

Star Connected Load





Phase Voltages: $V_{R'O}, V_{Y'O}, V_{B'O}$

Line Voltages:

$$\hat{V}_{RY} = \hat{V}_{R'O} - \hat{V}_{Y'O}$$

$$\hat{V}_{YB} = \hat{V}_{Y'O} - \hat{V}_{B'O}$$

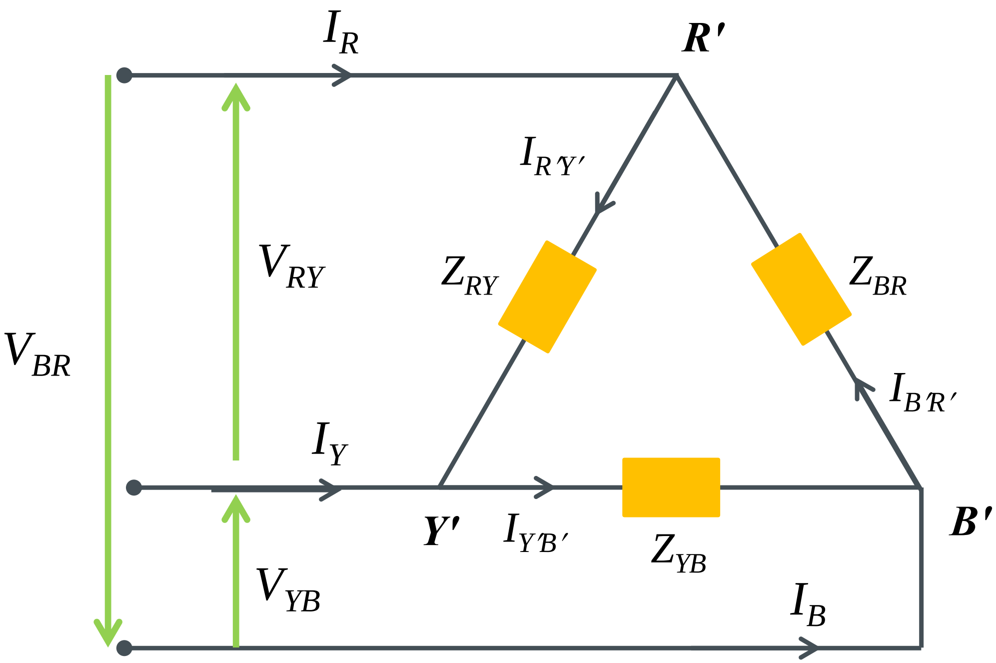
$$\hat{V}_{BR} = \hat{V}_{B'O} - \hat{V}_{R'O}$$

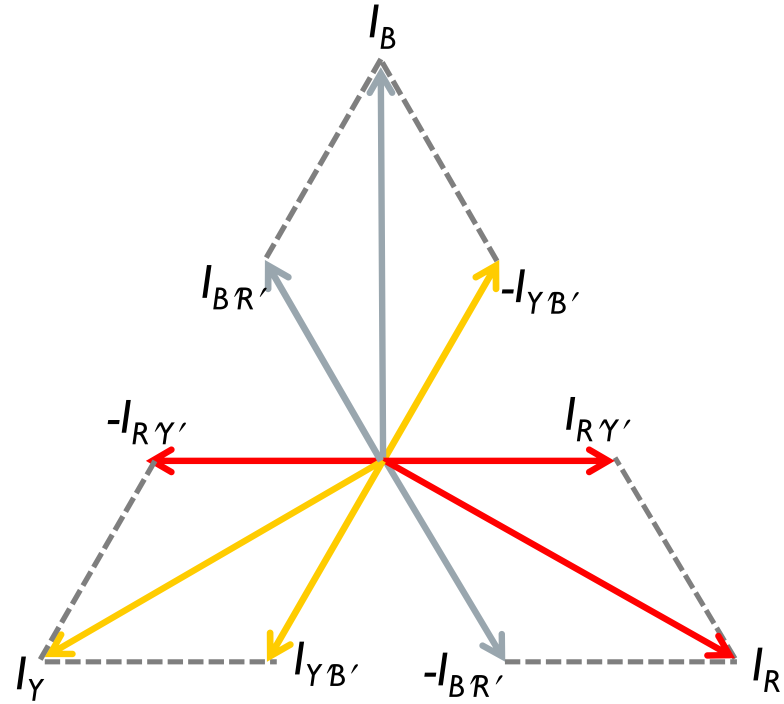
Line Currents = Phase Currents

For Balanced load,

Line Voltage = √3 X Phase Voltage

Delta Connected Load





Phase Currents: $I_{R'Y'}, I_{Y'B'}, I_{B'R'}$

Line Currents:

$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$

$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'}$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'}$$

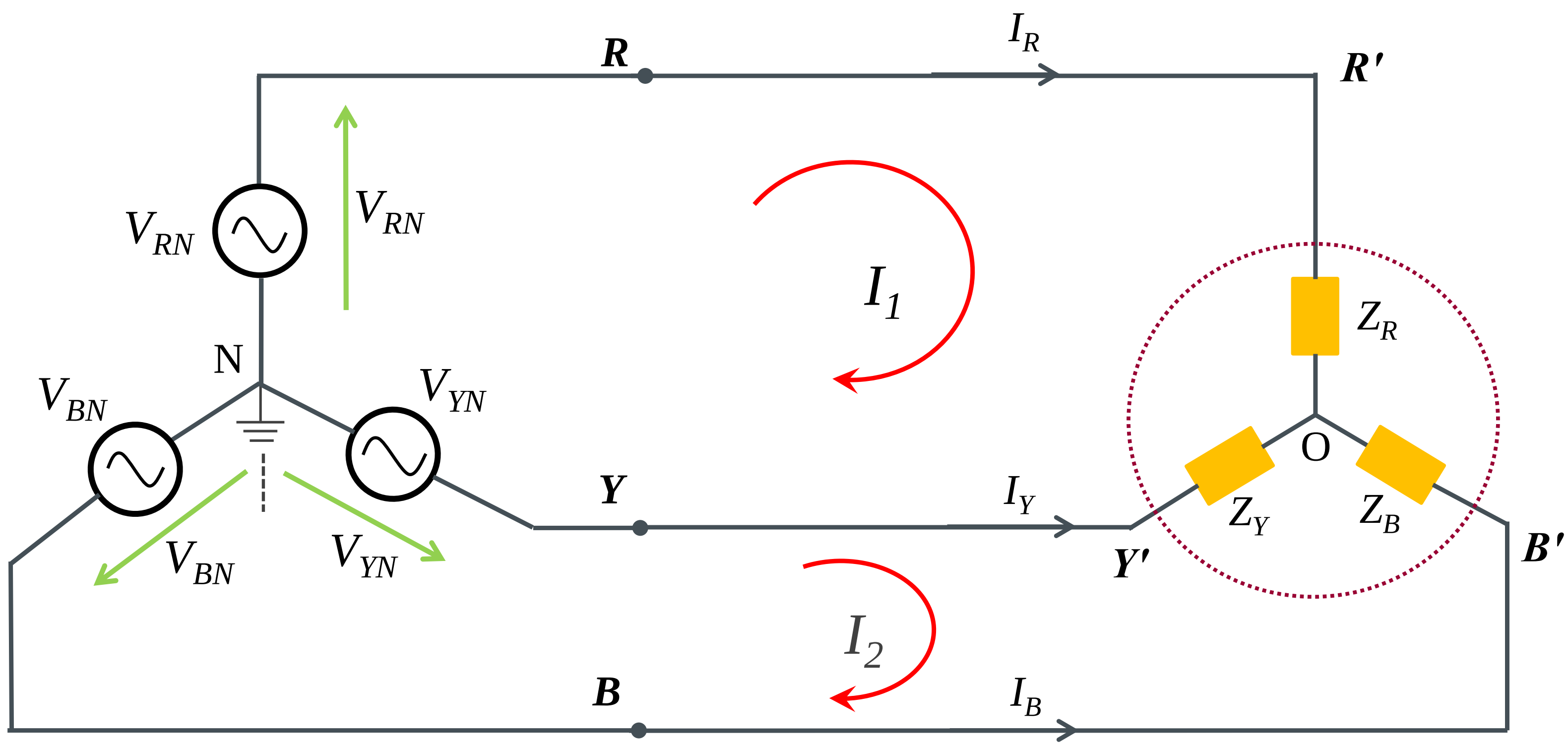
Phase Voltages = Line Voltages

For Balanced load,

Line Current = √3 X Phase Current

3φ System with Y Connected Load

Consider the 3 phase star load connected to a 3 wire balanced source. Two mesh currents $\hat{I}_1$ and $\hat{I}_2$ are assumed to flow as shown in Fig.



Writing mesh equations,

$$\begin{bmatrix} \bar{Z}_R + \bar{Z}_Y & -\bar{Z}_Y \\ -\bar{Z}_Y & \bar{Z}_Y + \bar{Z}_B \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} \hat{V}_{RN} - \hat{V}_{YN} \\ \hat{V}_{YN} - \hat{V}_{BN} \end{bmatrix}$$

Loop Current Analysis…

Using Cramer's Rule,

$$\hat{I}_1 = \frac{\begin{vmatrix} \hat{V}_{RN} - \hat{V}_{YN} & -\bar{Z}_Y \\ \hat{V}_{YN} - \hat{V}_{BN} & \bar{Z}_Y + \bar{Z}_B \end{vmatrix}}{\begin{vmatrix} \bar{Z}_R + \bar{Z}_Y & -\bar{Z}_Y \\ -\bar{Z}_Y & \bar{Z}_Y + \bar{Z}_B \end{vmatrix}} \qquad \hat{I}_2 = \frac{\begin{vmatrix} \bar{Z}_R + \bar{Z}_Y & \hat{V}_{RN} - \hat{V}_{YN} \\ -\bar{Z}_Y & \hat{V}_{YN} - \hat{V}_{BN} \end{vmatrix}}{\begin{vmatrix} \bar{Z}_R + \bar{Z}_Y & -\bar{Z}_Y \\ -\bar{Z}_Y & \bar{Z}_Y + \bar{Z}_B \end{vmatrix}}$$

The line currents are determined using the following equations:

$$\hat{I}_R = \hat{I}_1$$

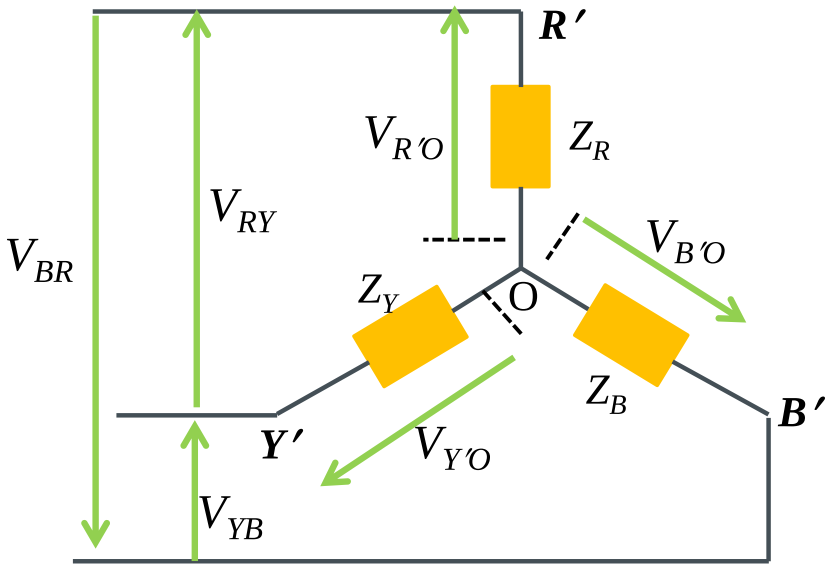
$$\hat{I}_Y = \hat{I}_2 - \hat{I}_1$$

$$\hat{I}_B = -\hat{I}_2$$

Balanced Star Connected Load

If $Z_R = Z_Y = Z_B = Z\angle\theta$

Then, $|V_{R'O}| = |V_{Y'O}| = |V_{B'O}| = V_{Ph}$



Phase Voltages:

$$\hat{V}_{R'O} = V_{Ph}\angle 0°$$

$$\hat{V}_{Y'O} = V_{Ph}\angle -120°$$

$$\hat{V}_{B'O} = V_{Ph}\angle +120°$$

Line Voltages:

$$\hat{V}_{RY} = \hat{V}_{R'O} - \hat{V}_{Y'O}$$

$$= V_{Ph}\angle 0° - V_{Ph}\angle -120° = \sqrt{3} \times V_{Ph}\angle 30°$$

$$\hat{V}_{YB} = \hat{V}_{Y'O} - \hat{V}_{B'O} = \sqrt{3} \times V_{Ph}\angle -90°$$

$$\hat{V}_{BR} = \hat{V}_{B'O} - \hat{V}_{R'O} = \sqrt{3} \times V_{Ph}\angle 150°$$

Illustration- I

Three loads $Z_A = 10\angle 0^0\ \Omega$; $Z_B = 15\angle -30^0\ \Omega$ and $Z_C = 20\angle 45^0$ are connected in star across a balanced, 3 phase, 400 V, RYB supply. Determine (a) line currents (b) Phase Voltages (c) Neutral shift voltage, V_{ON}.

Solution:

The three phase load is supplied with a balanced supply of 400V, hence the line voltages appearing across the load are:

$$\hat{V}_{RY} = 400\angle 0° \text{ (Reference Voltage)}$$

$$\hat{V}_{YB} = 400\angle -120°$$

$$\hat{V}_{BR} = 400\angle +120°$$

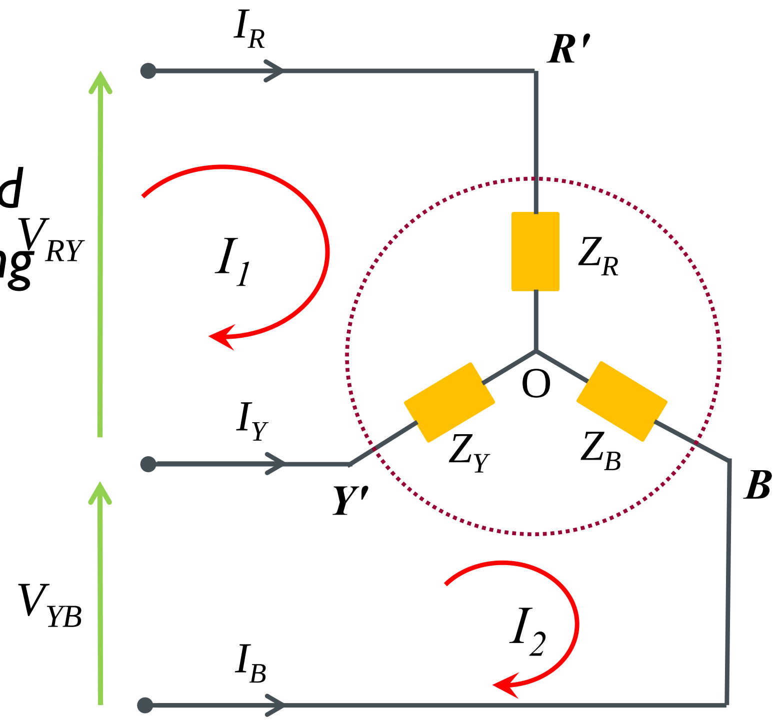


Illustration- I …

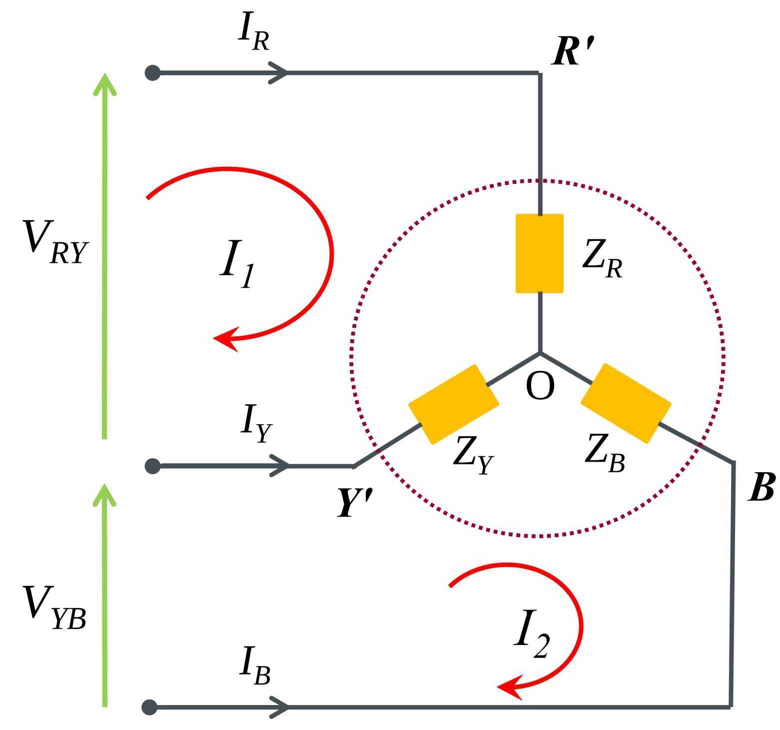
Writing Mesh Equation in Matrix form,

$$\begin{bmatrix} 10\angle 0 + 15\angle -30 & -15\angle -30 \\ -15\angle -30 & 15\angle -30 + 20\angle 45 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 400\angle 0° \\ 400\angle -120° \end{bmatrix}$$

Using Cramer's rule,

$$\hat{I}_1 = \mathbf{9.783\angle -17.87}\ A$$

$$\hat{I}_2 = \mathbf{16.69\angle -116.63}\ A$$



(i) The line currents are

$$\hat{I}_R = \hat{I}_1 = \mathbf{9.783\angle -17.87}\ A$$

$$\hat{I}_Y = \hat{I}_2 - \hat{I}_1 = \mathbf{20.59\angle -144.63}\ A$$

$$\hat{I}_B = -\hat{I}_2 = \mathbf{16.69\angle 63.37}\ A$$

Illustration-1 …

(ii) Phase Voltages are determined using the following equations.

$$\hat{V}_{R'O} = \hat{I}_R \times \bar{Z}_A = \mathbf{97.83\angle -7.87\,V}$$

$$\hat{V}_{Y'O} = \hat{I}_Y \times \bar{Z}_B = \mathbf{308.85\angle -174.63\,V}$$

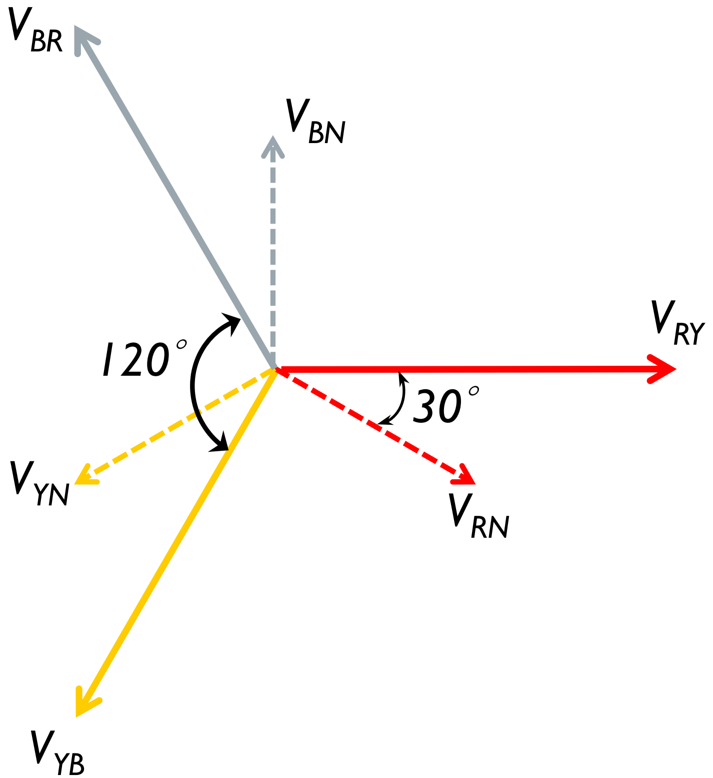
$$\hat{V}_{B'O} = \hat{I}_B \times \bar{Z}_C = \mathbf{338\angle 108.37\,V}$$

(c)Neutral Shift Voltage (Von)

$$\hat{V}_{RY} = 400\angle 0^\circ \textit{ (Reference Voltage)}$$

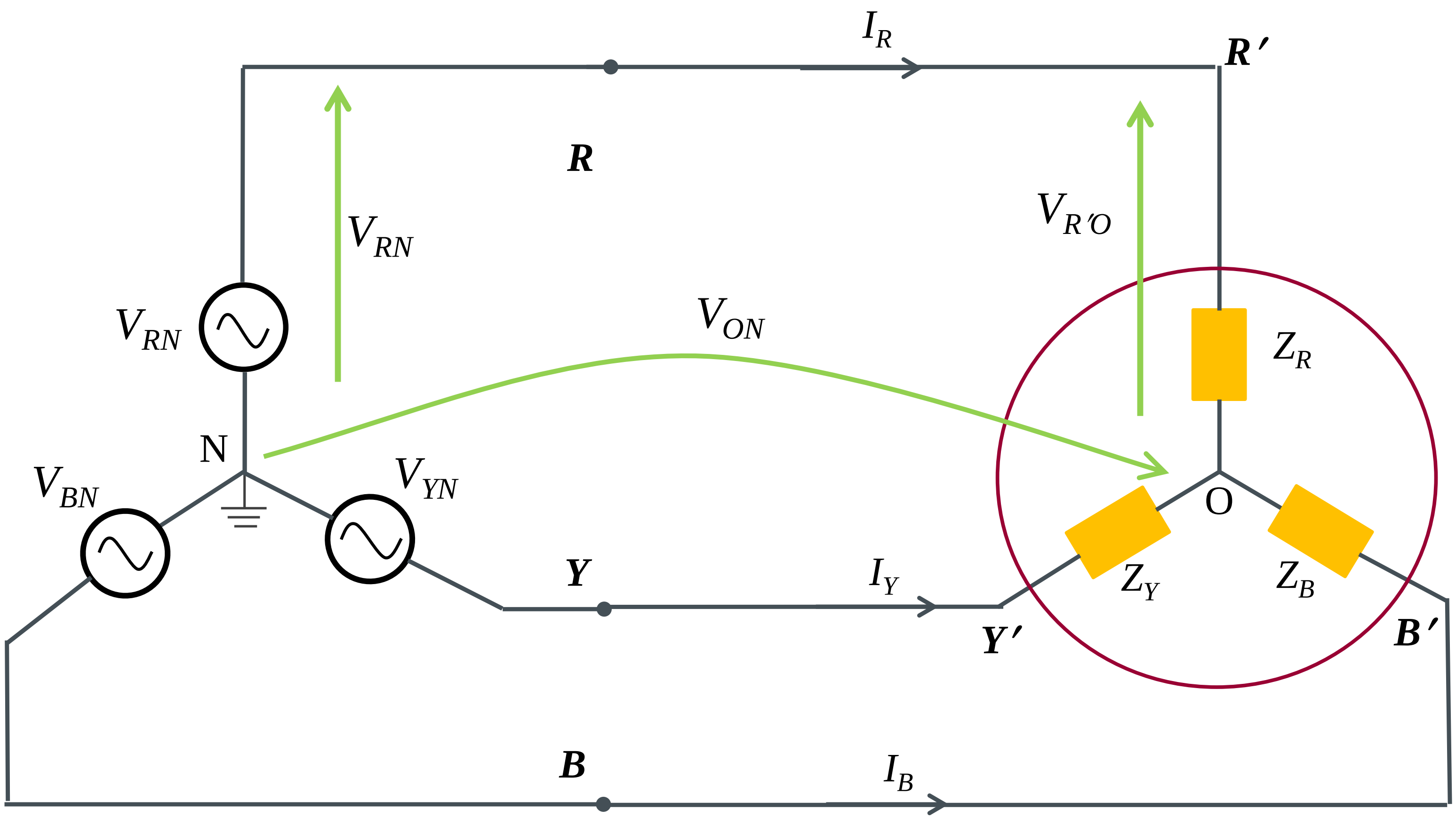
$$\hat{V}_{RN} = \frac{400}{\sqrt{3}}\angle -30^\circ$$

$$\hat{V}_{YN} = \frac{400}{\sqrt{3}}\angle(-30-120)$$



$$\hat{V}_{BN} = \frac{400}{\sqrt{3}}\angle(-30^\circ - 240)$$

Illustration-I …

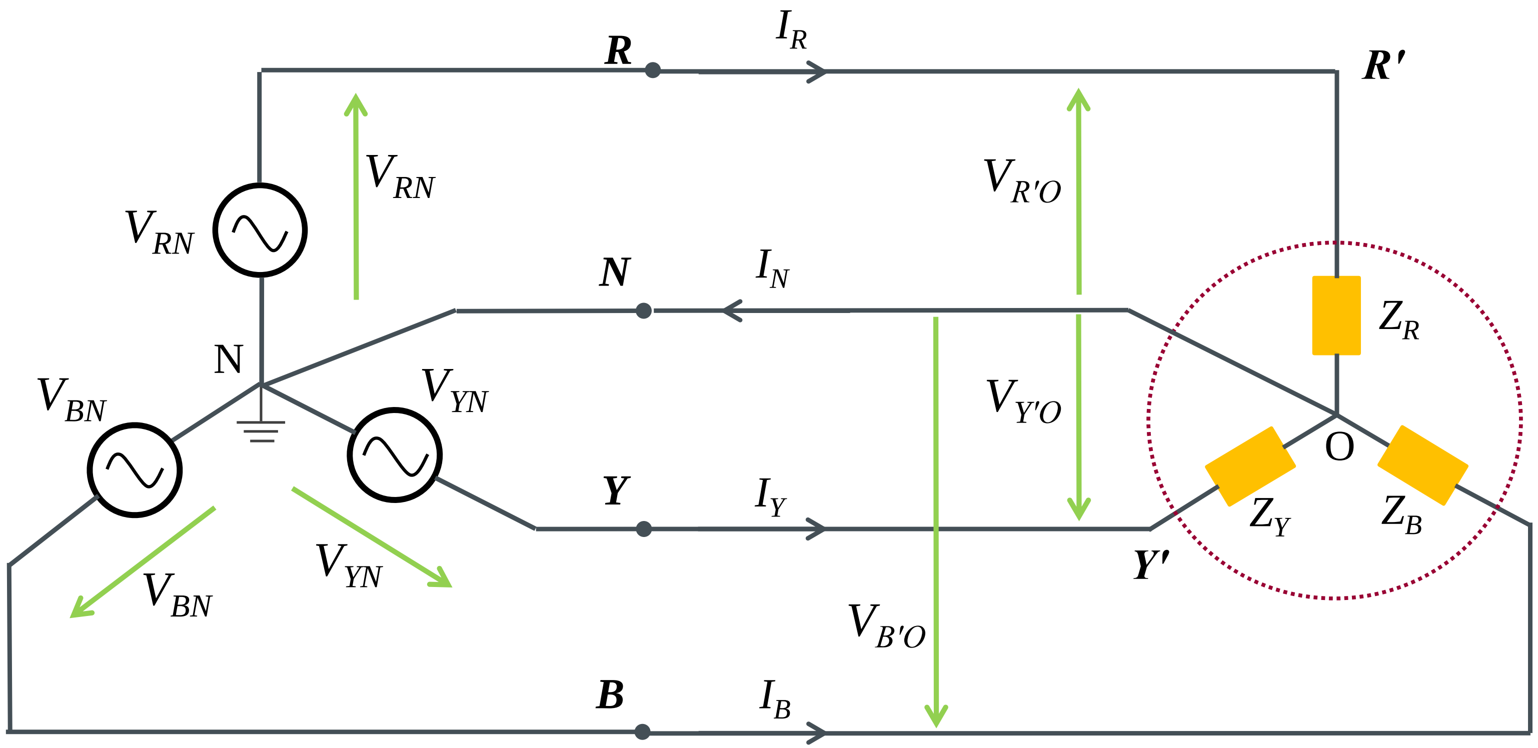


Applying KVL, $\hat{V}_{RN} - \hat{V}_{R'O} - \hat{V}_{ON} = 0$

$$\hat{V}_{ON} = \hat{V}_{RN} - \hat{V}_{R'O} = 145.07\angle - 44.7°\ V$$

3φ , 4 Wire System with Y Load

Consider the 3 phase star load connected to a 4 wire balanced source.



Phase Voltages of Load,

$$\hat{V}_{R'O} = \hat{V}_{RN}$$

$$\hat{V}_{Y'O} = \hat{V}_{YN}$$

$$\hat{V}_{B'O} = \hat{V}_{BN}$$

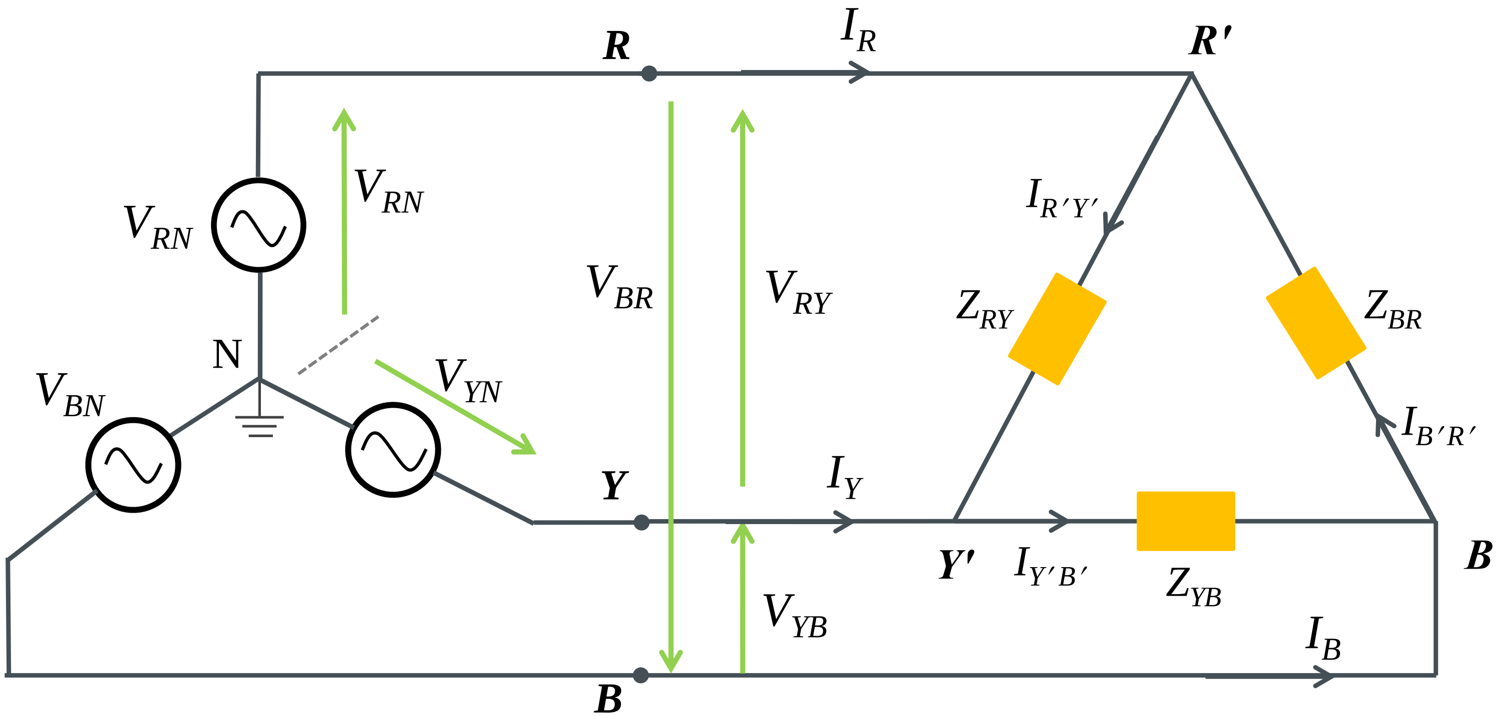
Neutral Current:

$$\hat{I}_N = \hat{I}_R + \hat{I}_Y + \hat{I}_B$$

$$\hat{I}_N = 0; \text{(If } Z_R = Z_Y = Z_B = Z\angle\theta° \text{)}$$

3φ , 3 Wire System with Δ Load

Consider the 3 phase Delta load connected to a 3 wire balanced source.



Phase Currents,

$$\hat{I}_{R'Y'} = \frac{\hat{V}_{RY}}{\bar{Z}_{RY}} \qquad \hat{I}_{Y'B'} = \frac{\hat{V}_{YB}}{\bar{Z}_{YB}} \qquad \hat{I}_{B'R'} = \frac{\hat{V}_{B'R'}}{\bar{Z}_{BR}}$$

Line Currents,

$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$

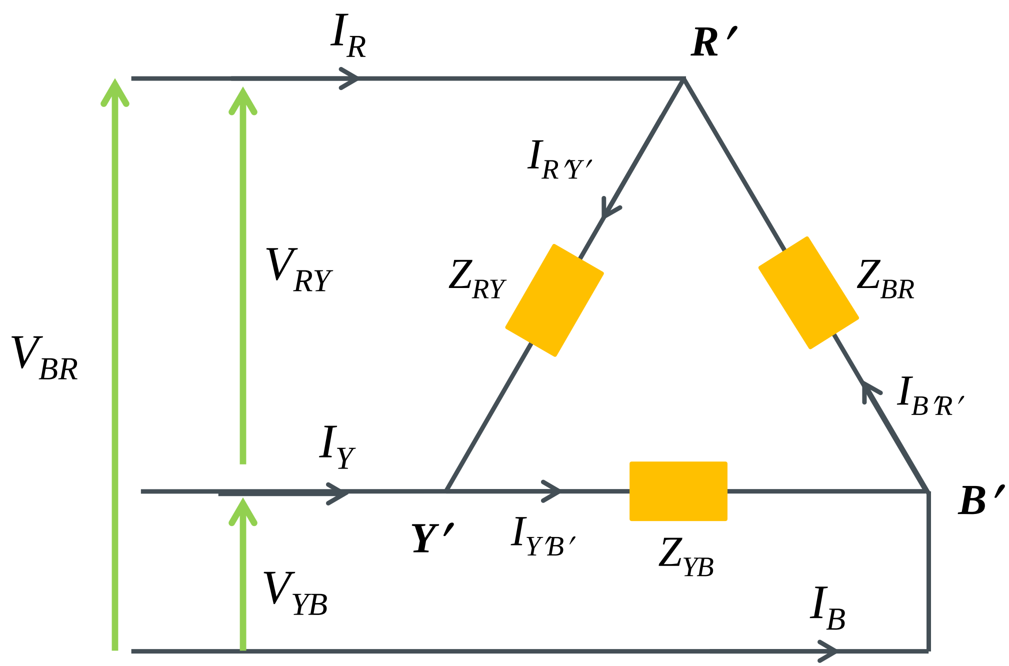
$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'}$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'}$$

Balanced Δ Load

If $Z_R = Z_Y = Z_B = Z\angle\theta$

Then, $|I_{R'Y'}| = |I_{Y'B'}| = |I_{B'R'}| = I_{Ph}$



Phase Currents:

$$\hat{I}_{R'Y'} = I_{Ph}\angle 0°$$

$$\hat{I}_{Y'B'} = I_{Ph}\angle -120°$$

$$\hat{I}_{B'R'} = I_{Ph}\angle +120°$$

Line Currents:

$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$

$$= I_{Ph}\angle 0° - I_{Ph}\angle +120° = \sqrt{3} \times I\angle -30°$$

$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'} = \sqrt{3} \times I_{Ph}\angle -150°$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'} = \sqrt{3} \times I_{Ph}\angle 90°$$

Illustration-2

Three loads, Z_R=50+j40 Ω, Z_Y=100 Ω and Z_B=80-j60 Ω are connected in Delta across a balanced 3 phase, 415V, 50 Hz supply. Determine

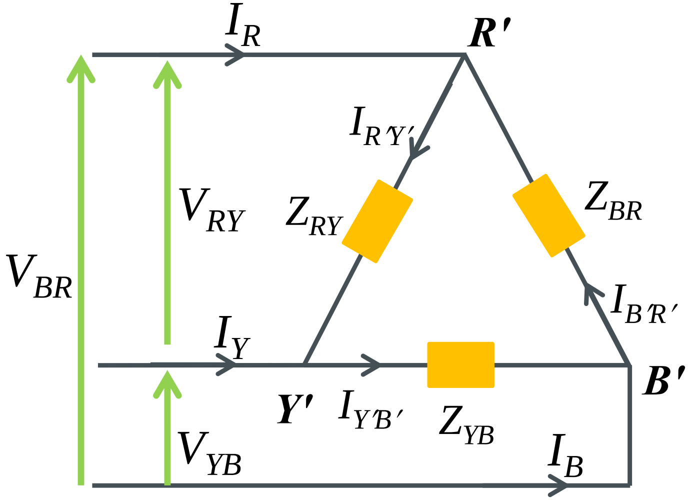
(i) Phase Currents

(ii) Line Currents and hence draw the complete phasor diagram.

Assume a phase sequence of RYB.

Solution:

The three phase load is supplied with a balanced supply of 415V, hence the line voltages appearing across the load are:



$$\hat{V}_{RY} = 415\angle 0° \text{ (Reference Voltage)}$$

$$\hat{V}_{YB} = 415\angle -120°$$

$$\hat{V}_{BR} = 415\angle +120°$$

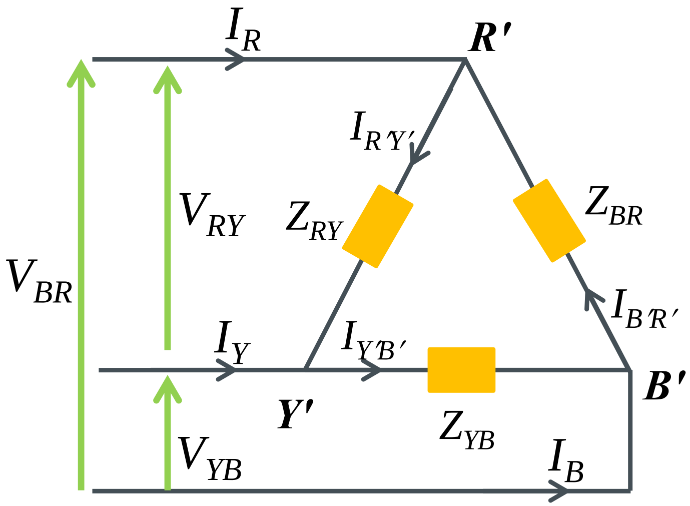
Illustration-2…

(i) Calculating the phase currents,

$$\hat{I}_{R'Y'} = \frac{\hat{V}_{RY}}{\bar{Z}_{RY}} = \mathbf{6.48\angle -38.66°}\ A$$

$$\hat{I}_{Y'B'} = \frac{\hat{V}_{YB}}{\bar{Z}_{YB}} = \mathbf{4.15\angle -120°}\ A$$

$$\hat{I}_{B'R'} = \frac{\hat{V}_{BR}}{\bar{Z}_{BR}} = \mathbf{4.15\angle 156.87°}\ A$$



(il) Calculating the Line currents,

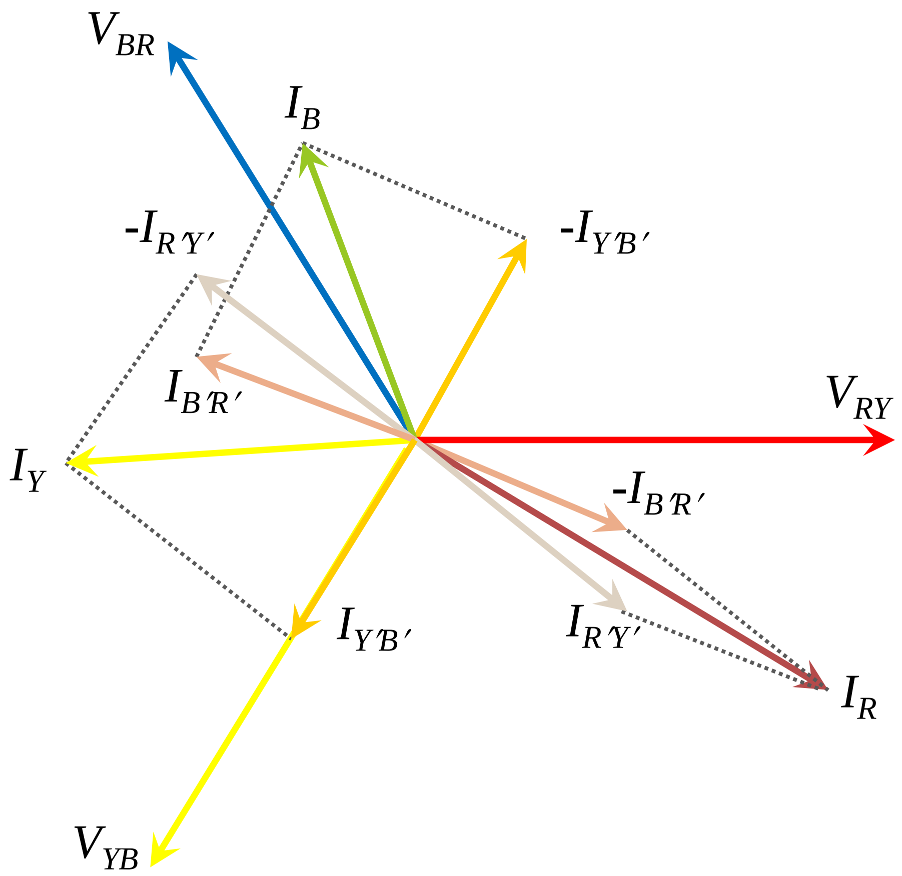
$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'} = \mathbf{10.537\angle -32.61°}\ A$$

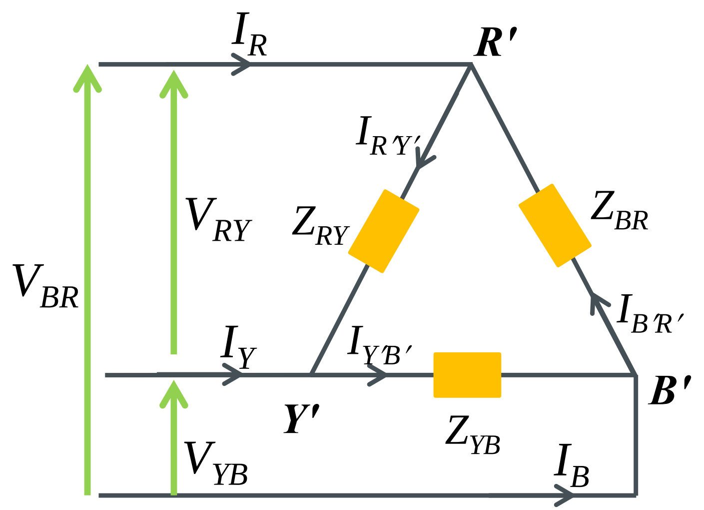
$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'} = \mathbf{7.149\angle 176.35°}\ A$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'} = \mathbf{5.506\angle 108.44°}\ A$$

Illustration-2…

(ii) Phasor Diagram,





$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$

$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'}$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'}$$

Summary

Analysis of balanced/unbalanced three phase star/delta connected load with 3 phase balanced excitation is performed.

- *For Balanced Star connected load, the line voltage = $\sqrt{3}$ x phase voltage.*
- *For Balanced Delta connected load, the line current = $\sqrt{3}$ x phase current.*

Basic Electrical Technology

[ELE 1051]

Three Phase AC Circuits

L24 – Power Associated with Three Phase System

Topics Covered

Power in 3 phase system: active, reactive and apparent.

Power measurement

Three Phase Power

1. Star Connected Load

Complex Power,

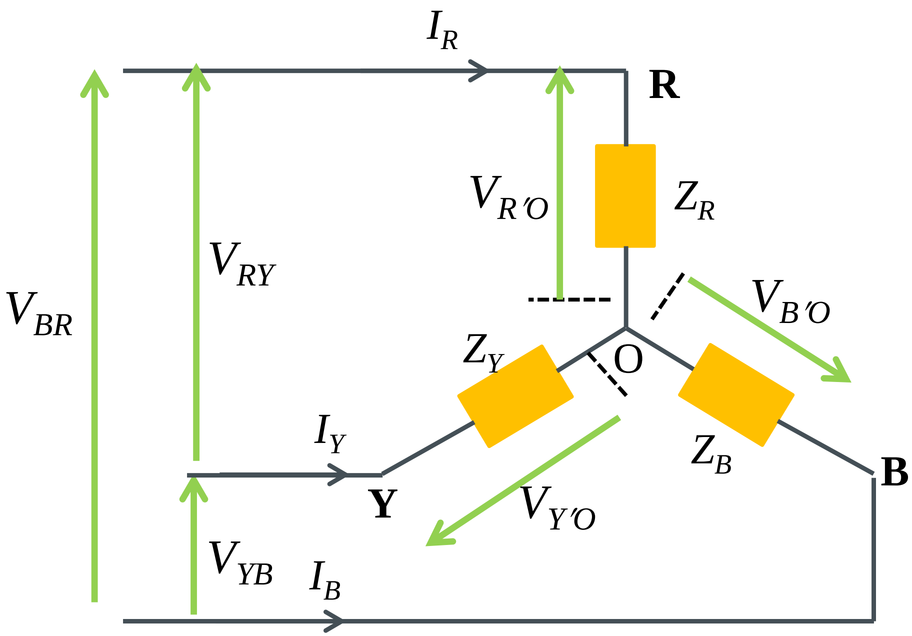
$$S = V_{R'O} I_R^{\,*} + V_{Y'O} I_Y^{\,*} + V_{B'O} I_B^{\,*}$$

Active Power,

$$P = V_{R'O}\, I_R\, Cos\angle(V_{R'O}\, \&\, I_R) + V_{Y'O}\, I_Y\, Cos\angle(V_{Y'O}\, \&\, I_Y) + V_{B'O}\, I_B\, Cos\angle(V_{B'O}\, \&\, I_B)$$

Reactive Power,

$$Q = V_{R'O}\, I_R\, Sin\angle(V_{R'O}\, \&\, I_R) + V_{Y'O}\, I_Y\, Sin\angle(V_{Y'O}\, \&\, I_Y) + V_{B'O}\, I_B\, Sin\angle(V_{B'O}\, \&\, I_B)$$



For Balanced Load,

Complex Power, $S = \sqrt{3}\, V_L\, I_L^{\,*}$

Active Power, $P = \sqrt{3}\, V_L\, I_L\, Cos\angle \pm \theta$

Reactive Power, $Q = \sqrt{3}\, V_L\, I_L\, Sin\angle \pm \theta$

Apparent Power, $S = \sqrt{3}\, V_L\, I_L$

Three Phase Power…

2. Delta Connected Load

Complex Power,

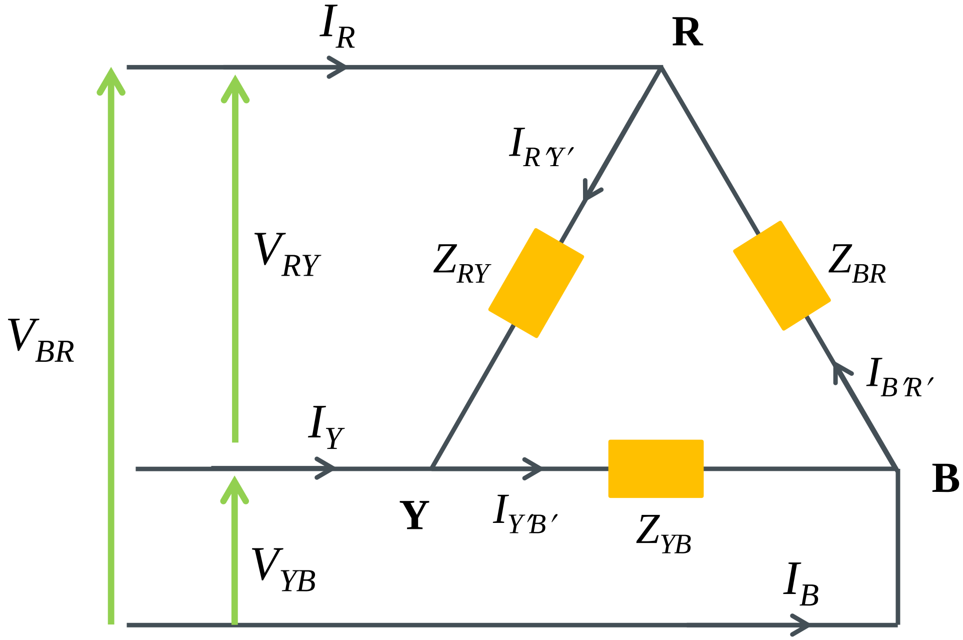
$$S = V_{RY} I_{R'Y'}{}^* + V_{YB} I_{Y'B'}{}^* + V_{BR} I_{B'R'}{}^*$$

Active Power,

$$P = V_{RY}\, I_{R'Y'}\, Cos\angle(\, V_{RY}\, \&\, I_{RY})$$
$$+ V_{YB}\, I_{Y'B'}\, Cos\angle(\, V_{YB}\, \&\, I_{YB})$$
$$+ V_{BR}\, I_{B'R'}\, Cos\angle(\, V_{BR}\, \&\, I_{BR})$$

Reactive Power,

$$Q = V_{RY}\, I_{R'Y'}\, Sin\angle(\, V_{RY}\, \&\, I_Y)$$
$$+ V_{YB}\, I_{Y'B'}\, Sin\angle(\, V_{YB}\, \&\, I_{YB})$$
$$+ V_{BR}\, I_{B'R'}\, Sin\angle(\, V_{BR}\, \&\, I_{BR})$$



For Balanced Load,

Complex Power, $S = 3\, V_{ph}\, I_{ph}{}^*$

Active Power, $P = \sqrt{3}\, V_L\, I_L\, Cos\angle \pm \theta$

Reactive Power, $Q = \sqrt{3}\, V_L\, I_L\, Sin\angle \pm \theta$

Apparent Power, $S = \sqrt{3}\, V_L\, I_L$

Exercise-1

A balanced star connected load of 8+j6 Ω per phase is connected to a 3 phase, 415V supply. Find the line currents, power factor, power, reactive volt amperes and total volt amperes.

Exercise-2

A star connected load is supplied from a symmetrical three phase, 440V system. The branch impedances of the load are ,$Z_R = 5\angle 30^\circ\ \Omega$, $Z_Y = 10\angle 45^\circ\ \Omega$, $Z_B = 10\angle 60^\circ\ \Omega$*. Find the active power supplied by the source.*

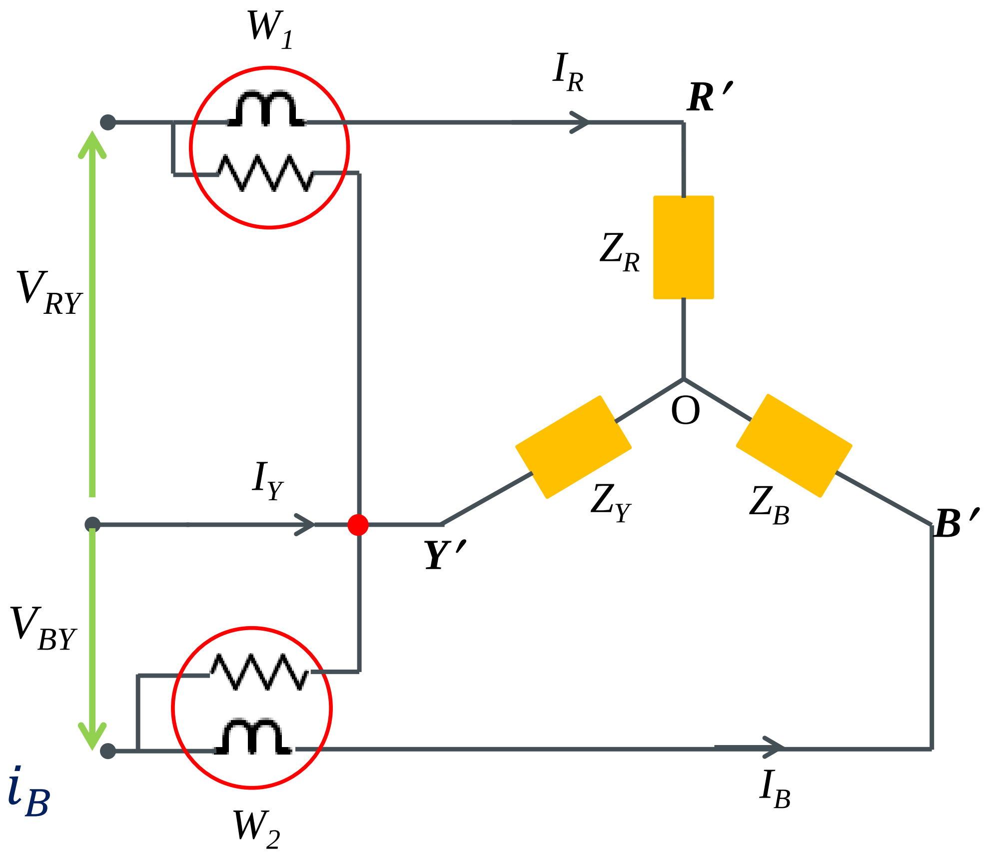
Measurement of 3 Ph. Active Power

1. Star Connected Load using 2 Wattmeter's

Wattmeter Reading,

$$W_1 = v_{RY}\ i_R = (v_{R'O} - v_{Y'O})\ i_R$$

$$W_2 = v_{BY}\ i_B = (v_{B'O} - v_{Y'O})\ i_B$$



Total Active Power,

$$= W_1 + W_2$$

$$= (v_{R'O} - v_{Y'O})\ i_R + \ (v_{B'O} - v_{Y'O})\ i_B$$

$$= v_{R'O}\ i_R - v_{Y'O}(\ i_R + i_B) + \ v_{B'O}\ i_B$$

$$= v_{R'O}\ i_R + v_{Y'O}\ i_Y + \ v_{B'O}\ i_B \quad Since,\ i_R + i_Y + i_B = 0$$

Measurement of 3 Ph. Active Power

2. Balanced Load (Star Connected) using 2 Wattmeter's

Wattmeter Reading,

$$W_1 = V_{RY}\, I_R\, Cos\angle(\, V_{RY}\, \&\, I_R)$$

$$= V_L\, I_L\, Cos(30° + \theta)$$

$$W_2 = V_{BY}\, I_B\, Cos\angle(\, V_{BY}\, \&\, I_B)$$

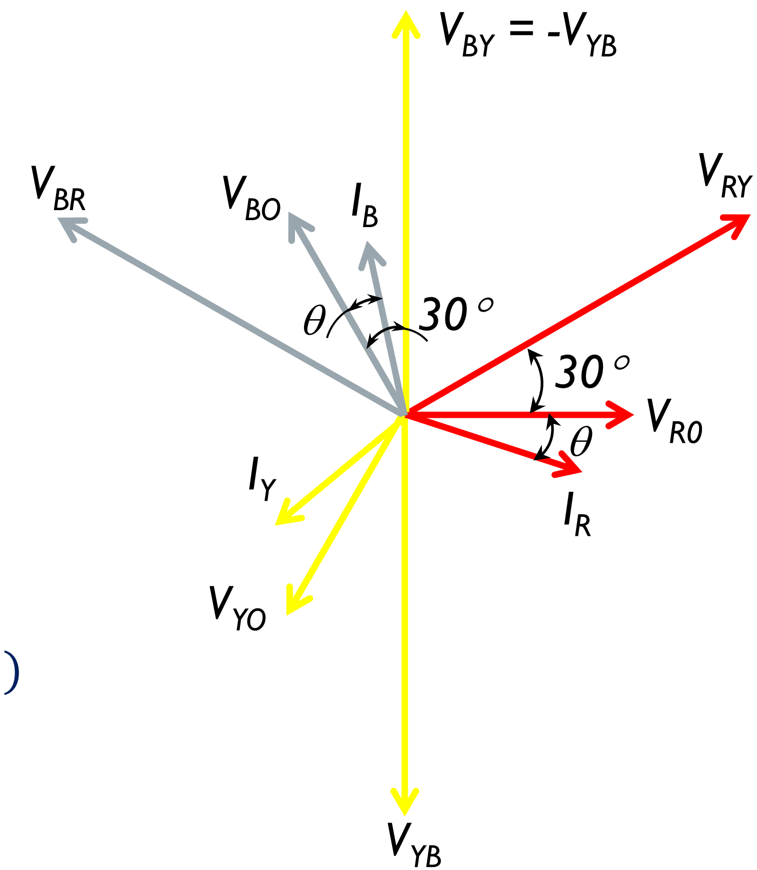
$$= V_L\, I_L\, Cos(30° - \theta)$$

Total active power consumed,

$$P = W_1 + W_2$$

$$= V_L\, I_L\, Cos(30° + \theta) + V_L\, I_L\, Cos(30° - \theta)$$

$$= \sqrt{3} \times V_L\, I_L\, Cos\theta$$



Meas. of 3 Ph. Active Power…

Summation of two wattmeters,

$$W_1 + W_2 = \sqrt{3} \times V_L \times I_L \times Cos\ \theta$$

Difference in the reading of two wattmeters,

$$W_2 - W_1 = V_L \times I_L \times Sin\ \theta$$

Hence,

$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{Sin\ \theta}{\sqrt{3} \times Cos\ \theta}$$

$$\theta = \tan^{-1}\left[\sqrt{3} \times \frac{W_2 - W_1}{W_2 + W_1}\right]$$

Power factor of the Balanced Load = $Cos\theta = Cos\left\{\tan^{-1}\left[\sqrt{3} \times \frac{W_2 - W_1}{W_2 + W_1}\right]\right\}$

Measurement of 3 Ph. Active Power

3. Balanced Load (Star Connected) using 1 Wattmeter

Wattmeter Reading,

$$W_1 = V_{R'O}\, I_R\, Cos\angle(\, V_{R'O}\ \&\ I_R)$$

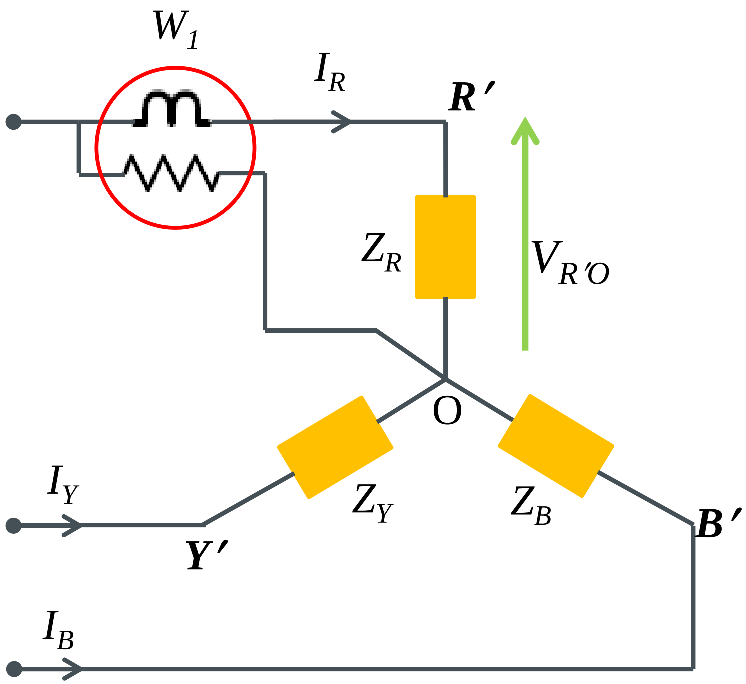
$$= V_{Ph}\, I_{Ph}\, Cos\theta$$

Total active power consumed,

$$= 3 \times W_1$$

$$= 3 \times V_{Ph}\, I_{Ph}\, Cos\theta$$

$$= \sqrt{3} \times V_L I_L Cos\theta$$



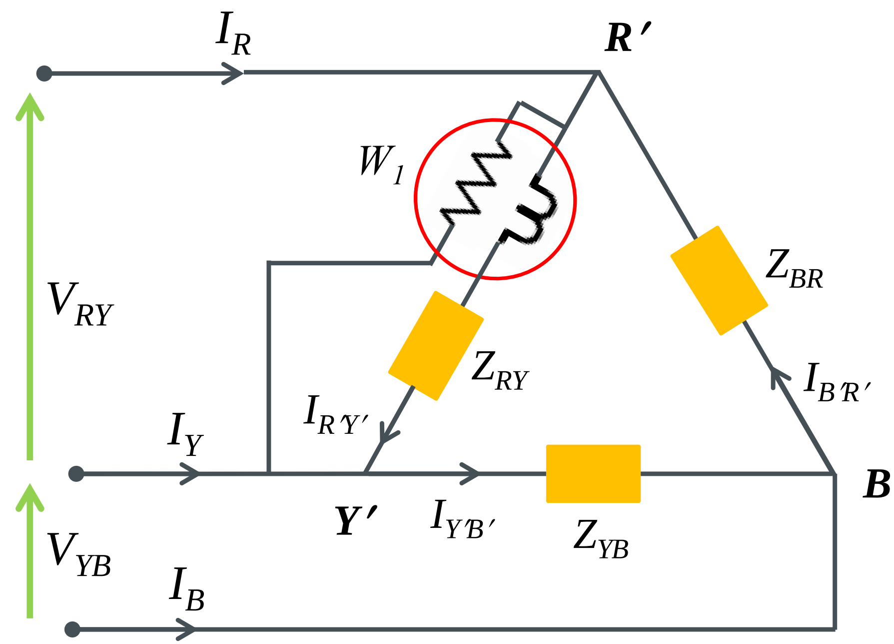
Measurement of 3 Ph. Active Power

4. Balanced Load (Delta Connected) using 1 Wattmeter

Wattmeter Reading,

$$W_1 = V_{R'Y'}\, I_{R'Y'}\, Cos\angle(\, V_{R'Y'} \,\&\, I_{R'Y'})$$

$$= V_L\, I_{Ph}\, Cos\theta$$



Total active power consumed,

$$= 3 \times W_1$$

$$= 3 \times V_L\, I_{Ph}\, Cos\theta$$

$$= \sqrt{3} \times V_L\, I_L\, Cos\theta$$

Exercise-3

Three identical impedances of (8+j6) Ω are connected in delta across a symmetrical 3 phase, 3 wire 400 V system. Calculate the power factor using wattmeter readings.

Exercise-4

Three loads $Z_R = 5\angle 30^\circ\ \Omega, Z_Y = 10\angle 45^\circ\ \Omega, Z_B = 10\angle 60^\circ\ \Omega$ *are connected in Star to R,Y and B Phase respectively. The current coils of the two wattmeters are connected in R & Y lines. If the supply voltage is 415V, 50 Hz, determine the reading of the two wattmeters. Assume the phase sequence is RBY.*

Summary

Measurement of Active Power for a three phase Star/Delta connected balanced/unbalanced load can be performed by using two wattmeters.

For a balanced Load, the Load Power factor can be measured by using one or two wattmeter method.

Measurement of power for a balanced Star/Delta load can be performed using one wattmeter.

Basic Electrical Technology
[ELE 1051]

L25 - Electrical Power system components

Outline

Power System Components

- Generation
- Transmission, Distribution
- Protection & Control

Types of Loads

Power System Background

Branch of Electrical Sciences dealing with *Generation, Transmission* & *Distribution* of electrical energy.

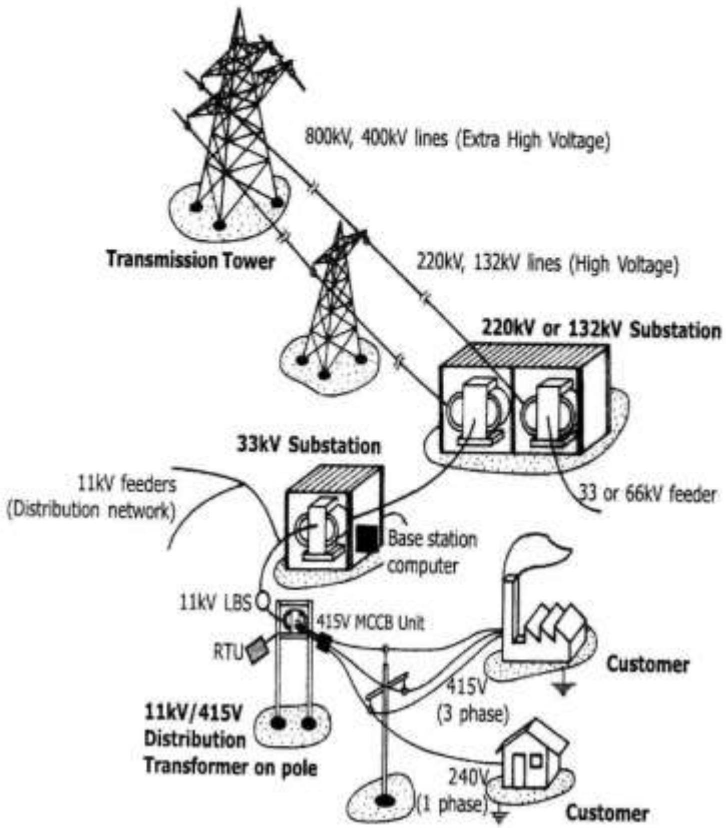
Pearl Street Station in New York City, 1882

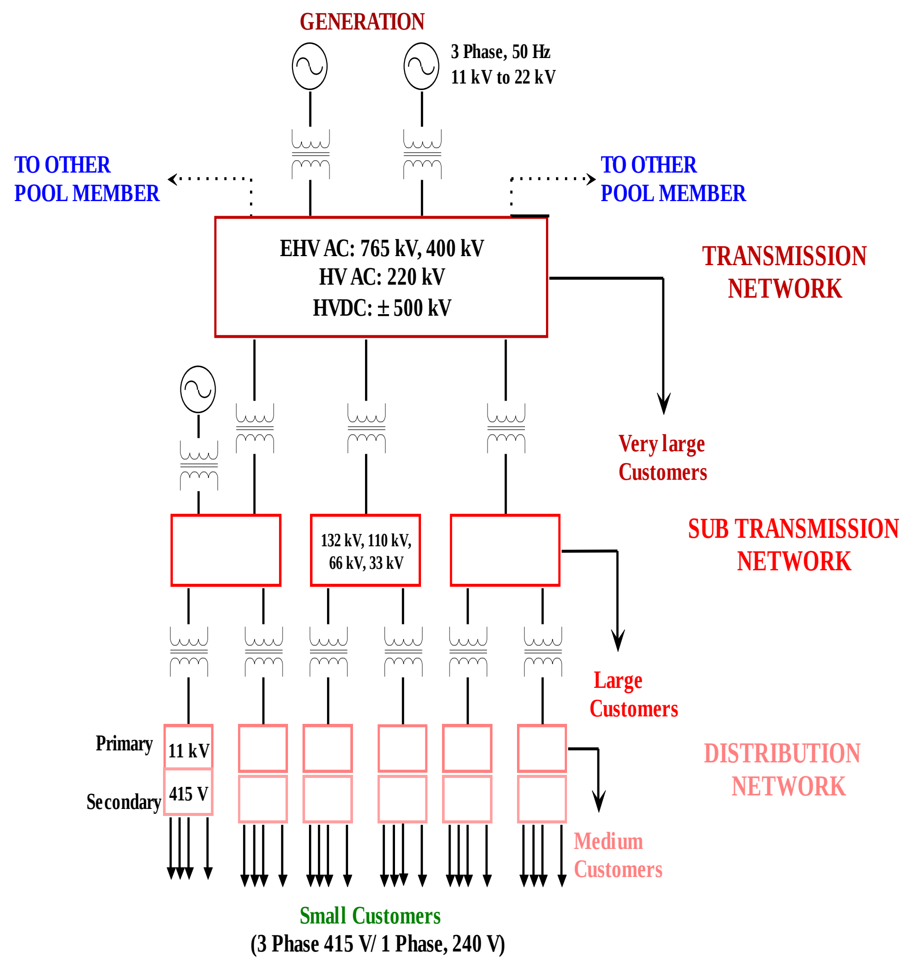
- “Illuminating Companies” by Thomas A Edison
- Concept of DC power generation

Three phase AC power system, 1896

- 2 generators and a transmission line @ 25 Hz.

Power System Structure





Courtesy: Olle I Elgerd

Power System Components

Generation subsystem

Transmission subsystem

Sub-transmission subsystem

Distribution subsystem

Protection and Control subsystem

Generation Subsystem

Primary Sources of Energy

- Fossil Fuel
  - Coal, Oil, Natural Gas
- Renewable Energy
  - Water, Solar, Wind, Tidal, Geo-thermal etc.
- Nuclear Energy

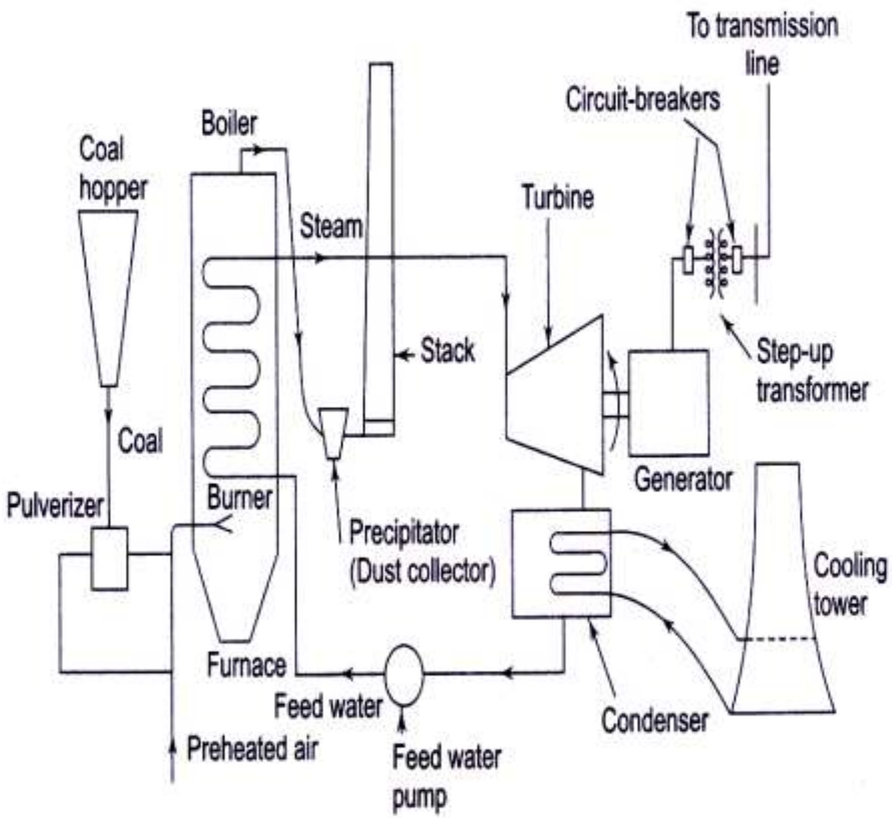
Generation Subsystem

Thermal Power Stations

- Coal Fired
  - Turbo alternators driven by steam turbine
- Oil Fired
  - Crude oil or Residual oil
- Gas Fired
  - Combined cycle- First stage: Gas turbine, Second stage: Steam Turbine
- Diesel Fired
  - IC engines as prime mover
  - Standby power plants

Generation Subsystem

Coal Fired Power plant



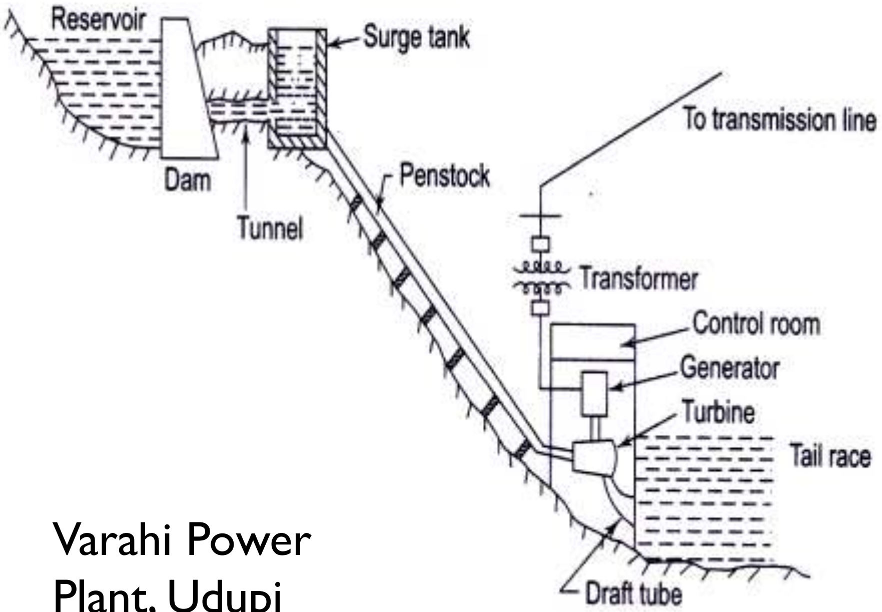
UPCL, Padubidri, Mangalore

Generation Subsystem

Hydroelectric Power Station

- Salient Pole alternators driven by turbines.
- Turbines: Impulse Turbine & Reaction Turbine
- Pumped storage plants

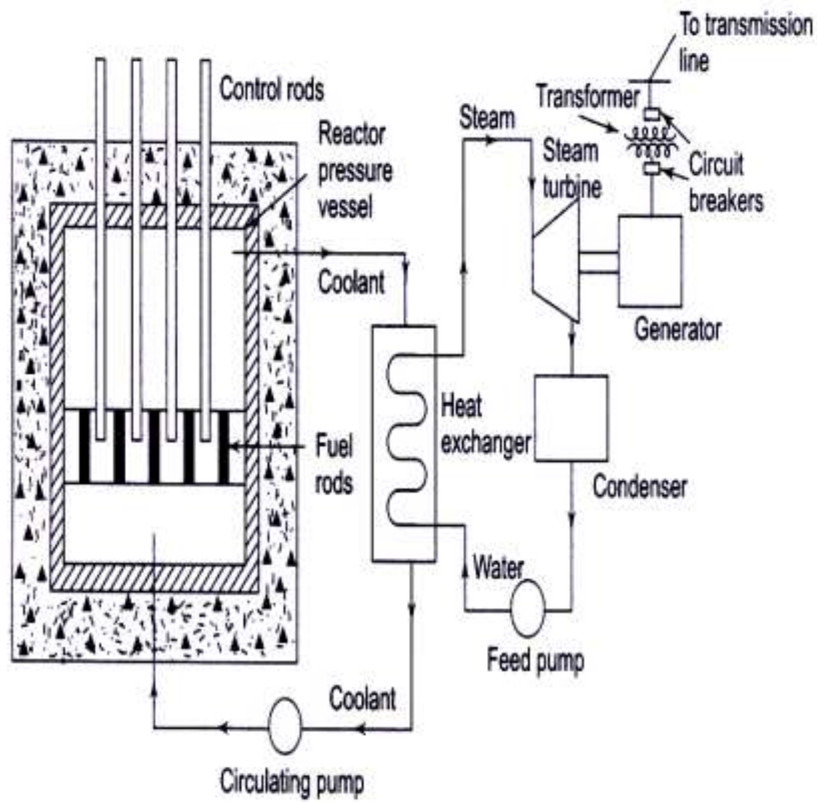
Varahi Power Plant, Udupi Dist.



Generation Subsystem

Nuclear Power Plant

- Fissile Material $^{235}_{92}U,\ ^{239}_{94}Pu$
- Moderator
  - D2O, Graphite
- Control rods
  - Boron OR Cadmium
- Fast Breeder Reactors
  - Liquid metal (alloy of Na & K) is coolant



$$^{238}_{92}U + \,^{1}_{0}n \rightarrow \,^{239}_{92}U - \beta \rightarrow \,^{239}_{93}Np - \beta \rightarrow \,^{239}_{94}Pu$$

Generation Subsystem

Non Conventional Power Stations

- Wind Power Stations
- Solar Power Stations
- Micro-Hydel Power Stations
- Bio-Mass Power Stations
- Geothermal Power Stations

Wind Farm in Karnataka

Solar Park, Charanka Village, Gujarat

1MW hydro plant, HP

Bio-mass Plant, Chattisgarh

Share of Renewable resources in India

Resource	Potential (MW)	Upto 9th Plan	Upto 10th Plan	11th Plan Target	Upto 30.09.10	Cumulative Achievement	12th Plan Projection (2017)	13th Plan Projection (2022)
Wind Power	48,500	1,667	5,427	9,000	4,714	12,809	27,300	38,500
Small Hydro Power	15,000	1,438	538	1,400	759	2,823	5,000	6,600
Bio Power	23,700	390	795	1,780	1,079	2,505	5,100	7,300
Solar Power	20-30 MW/sq km	2	1	50	8	18	4,000	20,000
Total		3,497	6,761	12,230	6,560	18,155	41,400	72,400

Source: Ministry of New & Renewable Energy, Govt. of India

Transmission, Sub-transmission & Distribution Subsystems

Transmission networks- EHV AC or HVDC

- Operates @765 kV/400 kV/ 220 kV AC or ± 500 kV DC.

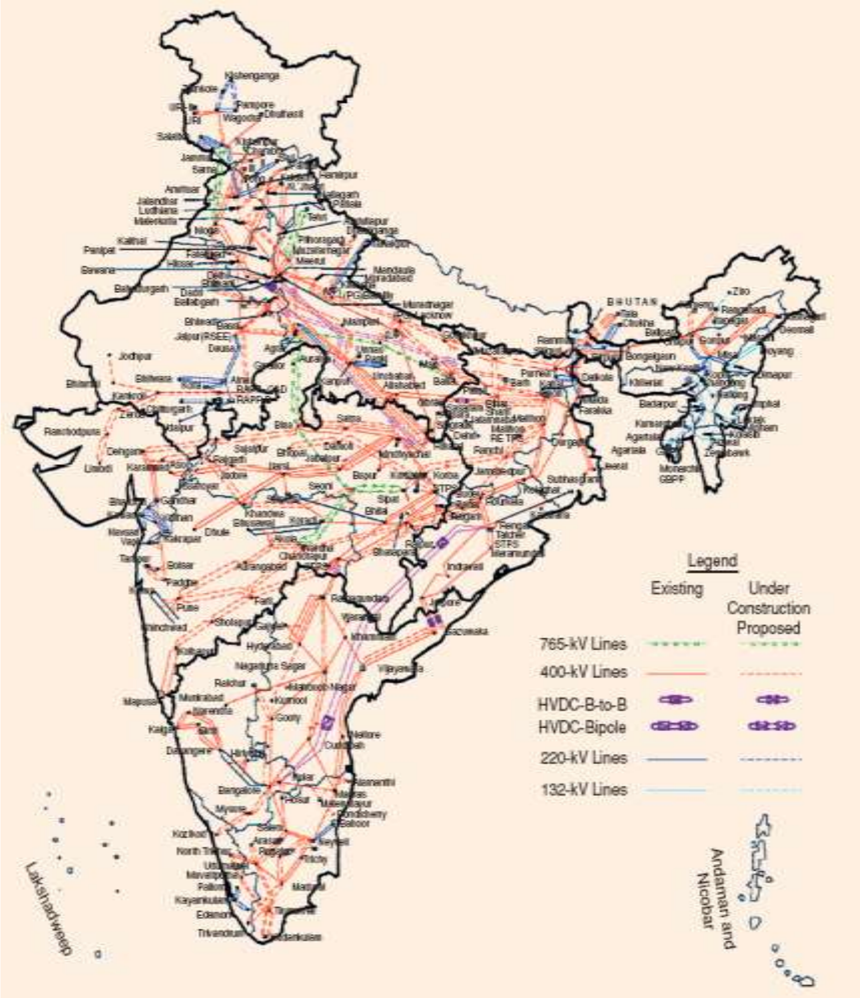
AC Sub-Transmission networks

- Operates @ 132 kV/ 110kV/ 66 kV/ 33 kV

AC Distribution Network

- Primary side: 11 kV
- Secondary side: 415 V, 4 Wire

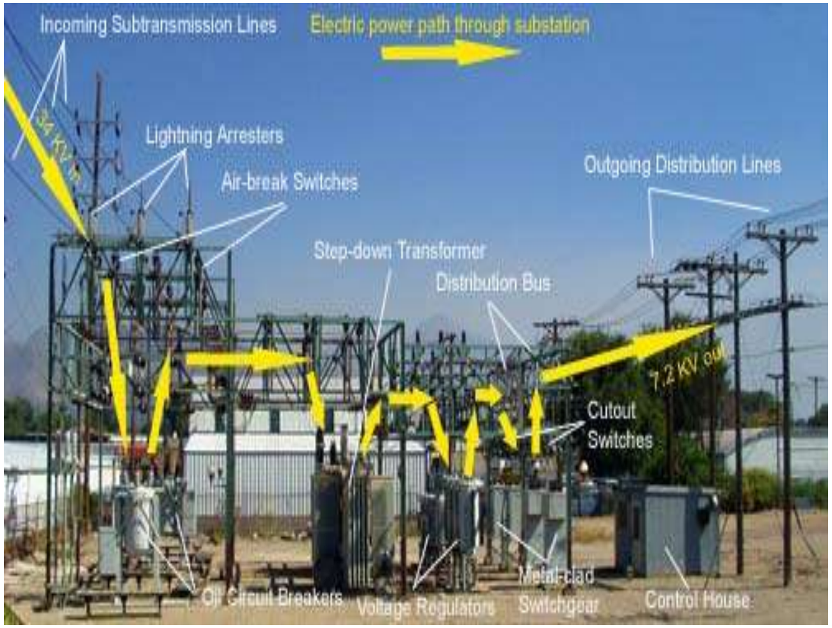
Transmission Network – A Glance



Substation

Substation Components

- Lightning Arrester
- Carrier line communication equipment (Wave Trap)
- Instrument Transformers (CT, PT)
- Circuit Breakers
- Isolators
- Bus Bars
- Power Transformers
- Control Room



Protection & Control Subsystem

Fail free power is *Hypothetical*.

Faults: Open Circuit & Short Circuit

Faults detection : *Relays*. Fault Isolation: *Circuit Breakers*

Modern Trend: Supervisory Control And Data Acquisition (SCADA) systems.

Types of Loads

Industrial Loads

- 3 Phase
- Complex Tariff Structure

Domestic Loads/Commercial Loads

- 1 Phase
- Tariff based on energy consumed- kWH

Domestic Loads and Power Ratings

Incandescent lamps - (5 W to 100 W)

Fluorescent lamps - (20 W & 40 W); CFL - (5 W to 25 W)

Air Conditioner (1.5 T) - 1800 W

Electric Iron - 750 W

Heaters/ Geysers – 2000 W

Ceiling Fan – 60 W

Washing Machine (with heater) – 2.5 kW

Refrigerator – 160 W

PC – 200 W, Laptop – 40 W

Reduce Electricity bill by minimizing the use of heating / environmental conditioning gadgets

Indian Power Sector – A Glance

Sector	MW	Percentage
State	93,540.70	37.4
Central	68,393.30	27.3
Private	88,322.96	35.3
Total	2,50,256.95	100.0

As on
31/07/2014

Source: Ministry of Power,
Govt. of India

Indian Power Sector - A Glance

Fuel	MW	Percentage
Total Thermal	1,72,986.09	69.1
Coal	149,178.39	59.6
Gas	22,607.95	9.0
Oil	1,199.75	0.52
Hydro (Renewable)	40,798.76	16.3
Nuclear	4,780	1.9
RES*(MNRE)	31,692.11	12.7
Total	2,50,256.95	100

*RES include small hydro, bio-mass, urban and industrial waste power and wind energy

As on 31/07/2014

Source: Ministry of Power, Govt. of India

Summary

Detailed discussion of various power generating sources.

Different levels of voltages at transmission, sub-transmission and distribution stage.

Types of loads.

Indian Power Sector

Basic Electrical Technology

[ELE 1051]

L26 – Transformers & DC Motors

Contents

Introduction

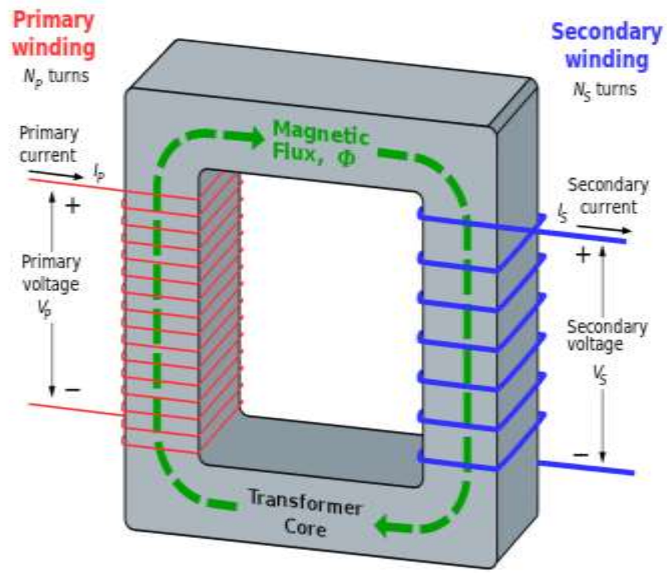
Static device with AC excitation

Transfers energy between two or more magnetically coupled circuits without change in frequency.

Principle of operation: *Electromagnetic Induction*

Electric circuits are linked by a common ferromagnetic core

Ferromagnetic core ensures *maximum* magnetic flux linkage



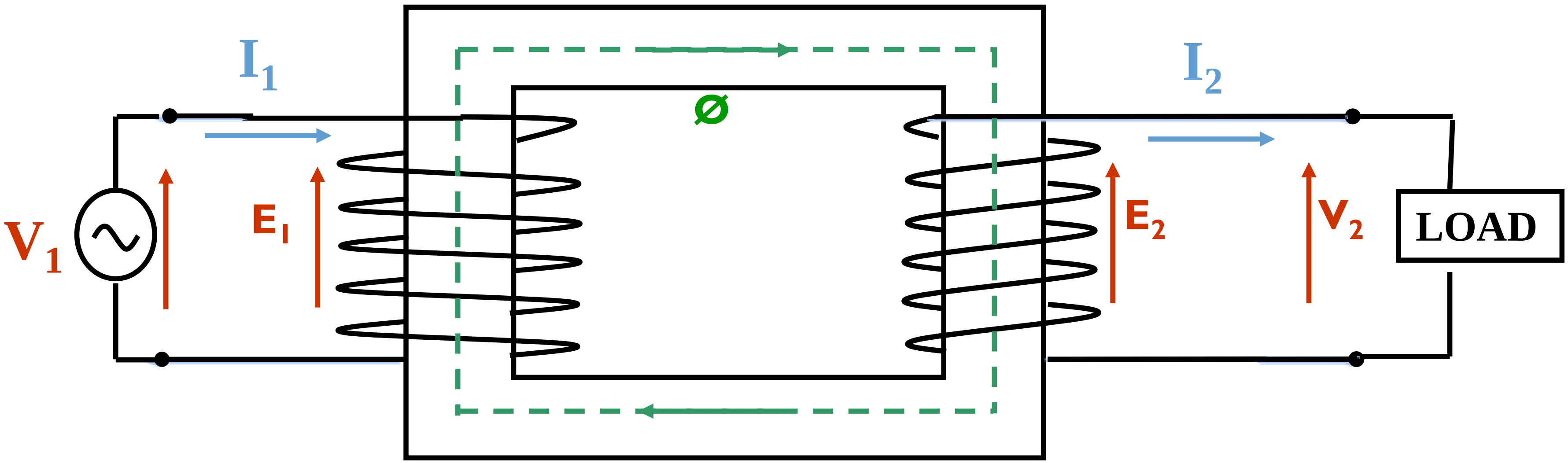
Applications:

- Electric power systems
- Power transmission, distribution networks
- Electronic circuits
- Electric traction

Types

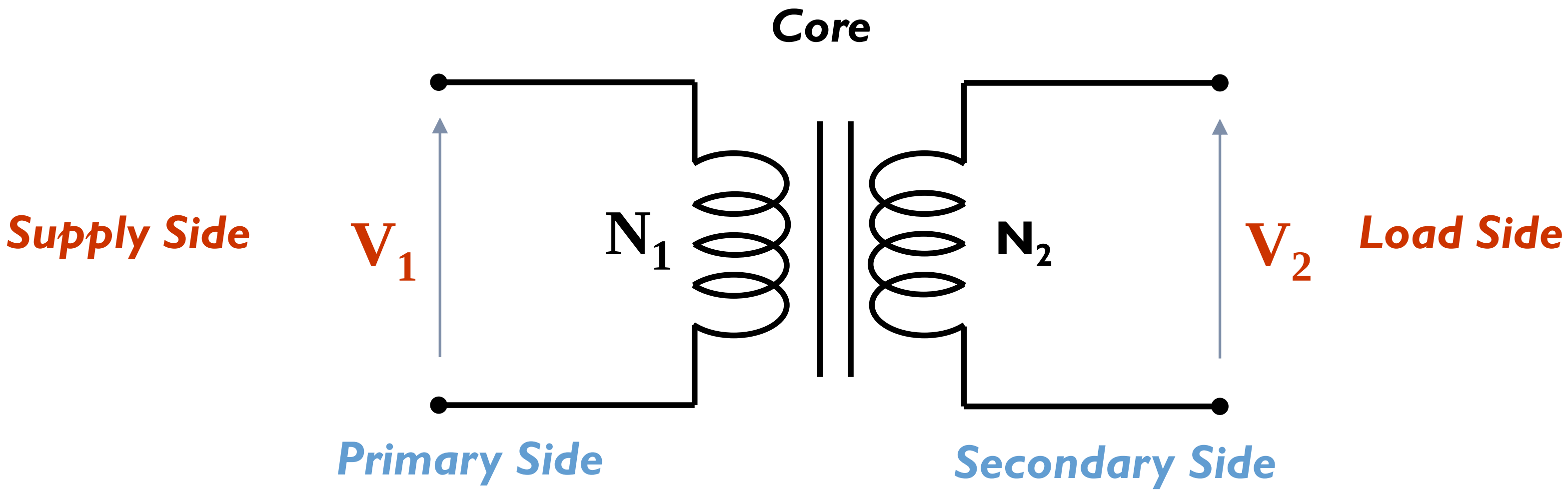
Based on Construction	***Based on Function***	***Based on Windings***
Core Type	Step Up	Single Winding
Shell Type	Step Down	2 or 3 Windings

Operation of Transformer



- Magnetic Core : Flux path
- Flux Linkages : Primary & Secondary
- Induced Emf :
  - Primary – Self Induced Emf
  - Secondary – Mutually Induced Emf

Representation



N_1 = Number of turns on primary
N_2 = Number of turns on secondary

Emf Equation of Transformer

$$\boldsymbol{Core\ flux}, \emptyset = \emptyset_m Sin\omega t$$

$$\boldsymbol{Induced\ Emf}, e = -N\frac{d\emptyset}{dt} = N\omega\emptyset_m Sin(\omega t - 90°)$$

$$e = E_m Sin(\omega t - 90°)$$

$$\boldsymbol{where}, E_m = N\omega\emptyset_m \rightarrow Maximum\ value\ of\ self\ induced\ emf$$

$$\boldsymbol{RMS\ value\ of\ self\ induced\ emf}, E = \frac{E_m}{\sqrt{2}} = \frac{N\omega\emptyset_m}{\sqrt{2}} = \frac{2\pi f N\emptyset_m}{\sqrt{2}}$$

$$\boldsymbol{Primary\ Induced\ Emf}, E_1 = 4.44\ N_1 f\emptyset_m$$
$$\boldsymbol{Secondary\ Induced\ Emf}, E_2 = 4.44\ N_2 f\emptyset_m$$

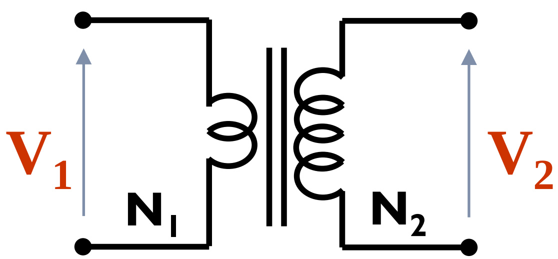
Emf Equation of Ideal Transformer…

$$\frac{V_1}{V_2} \cong \frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a = Turns\ Ratio$$

$$where, V_1 \& V_2\ are\ the\ terminal\ voltages,$$

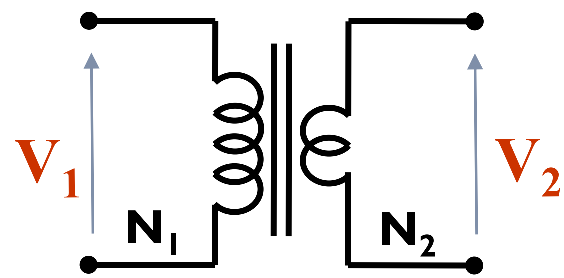
$$E_1 \&\ E_1 are\ the\ induced\ RMS\ voltages,$$

$N_2 > N_1$: Step Up



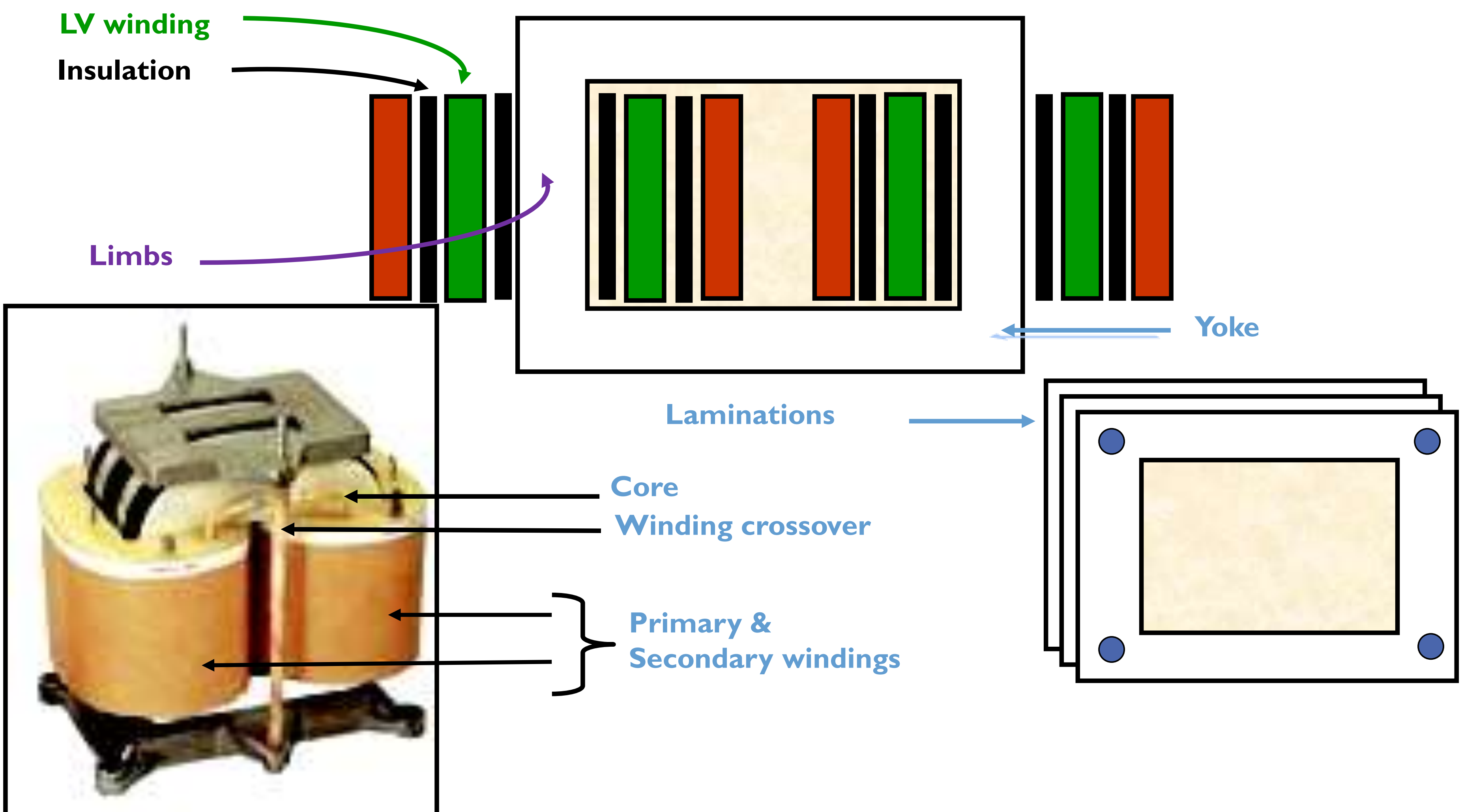
$E_2 > E_1$
$V_2 > V_1$
$I_1 > I_2$

$N_2 < N_1$: Step down

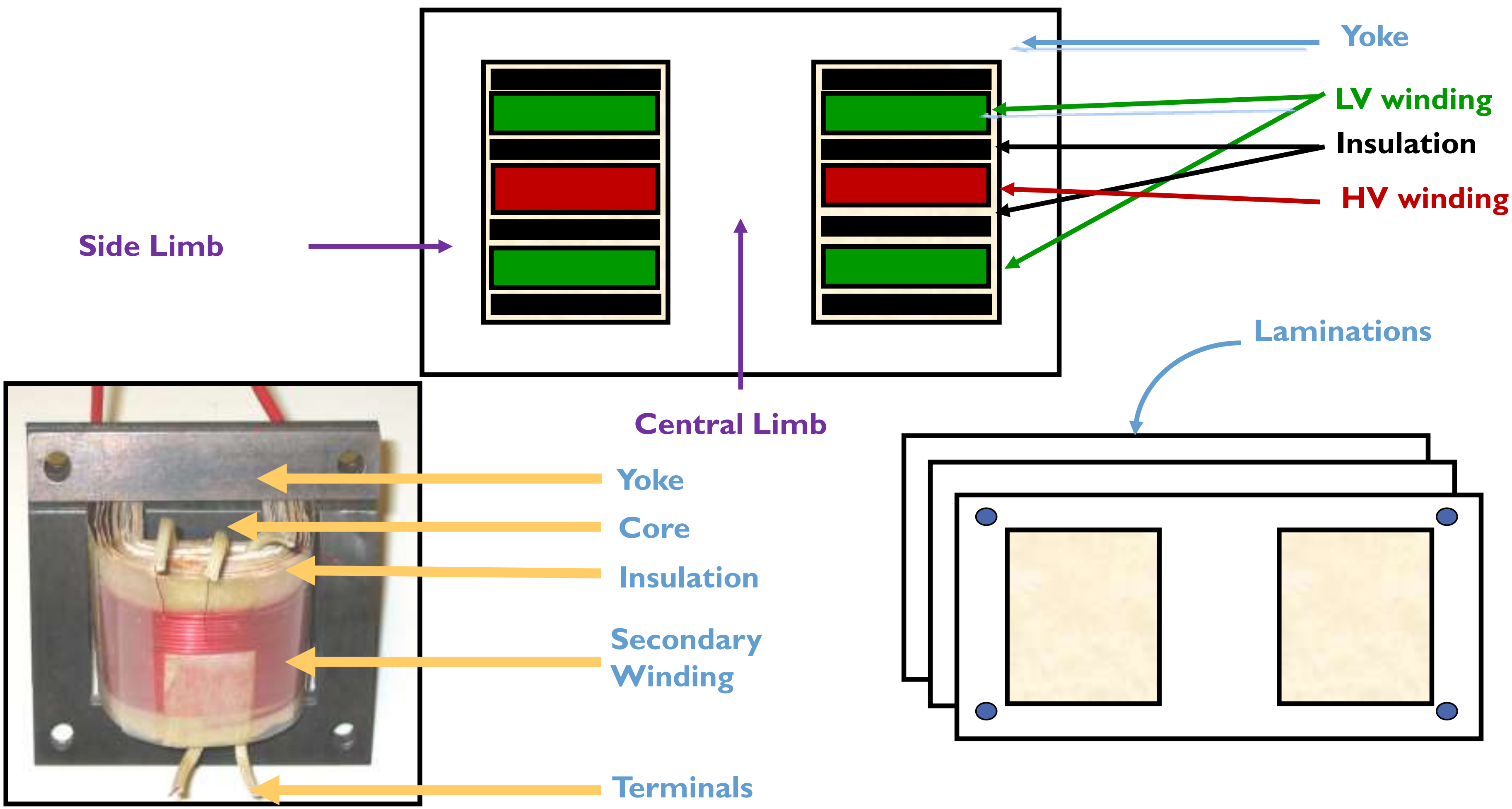


$E_2 < E_1$
$V_2 < V_1$
$I_1 < I_2$

Construction- *Core Type*



Construction- *Shell Type*



Losses & Efficiency

Core Loss

- Hysteresis Loss
- Eddy Current Loss

- Depends on flux which is constant hence the loss is constant

- Minimized using high graded core material and lamination

Copper Loss

- Winding Resistance (in primary and secondary)

- Current (or Load) dependent, hence variable loss

Total Loss = Core Loss + Copper Loss

Efficiency: Very high 97% to 99% (since it is a static device)

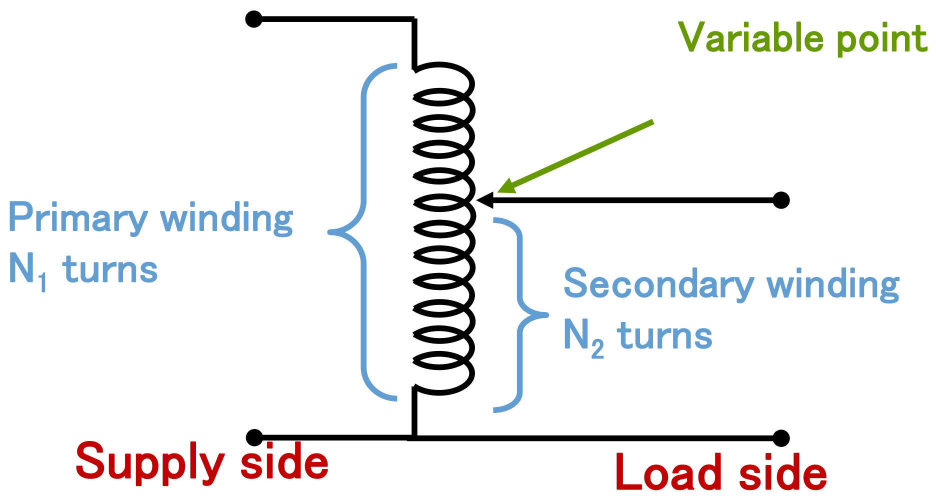
Auto Transformer

One winding transformer
- Part of winding common to primary & secondary circuits

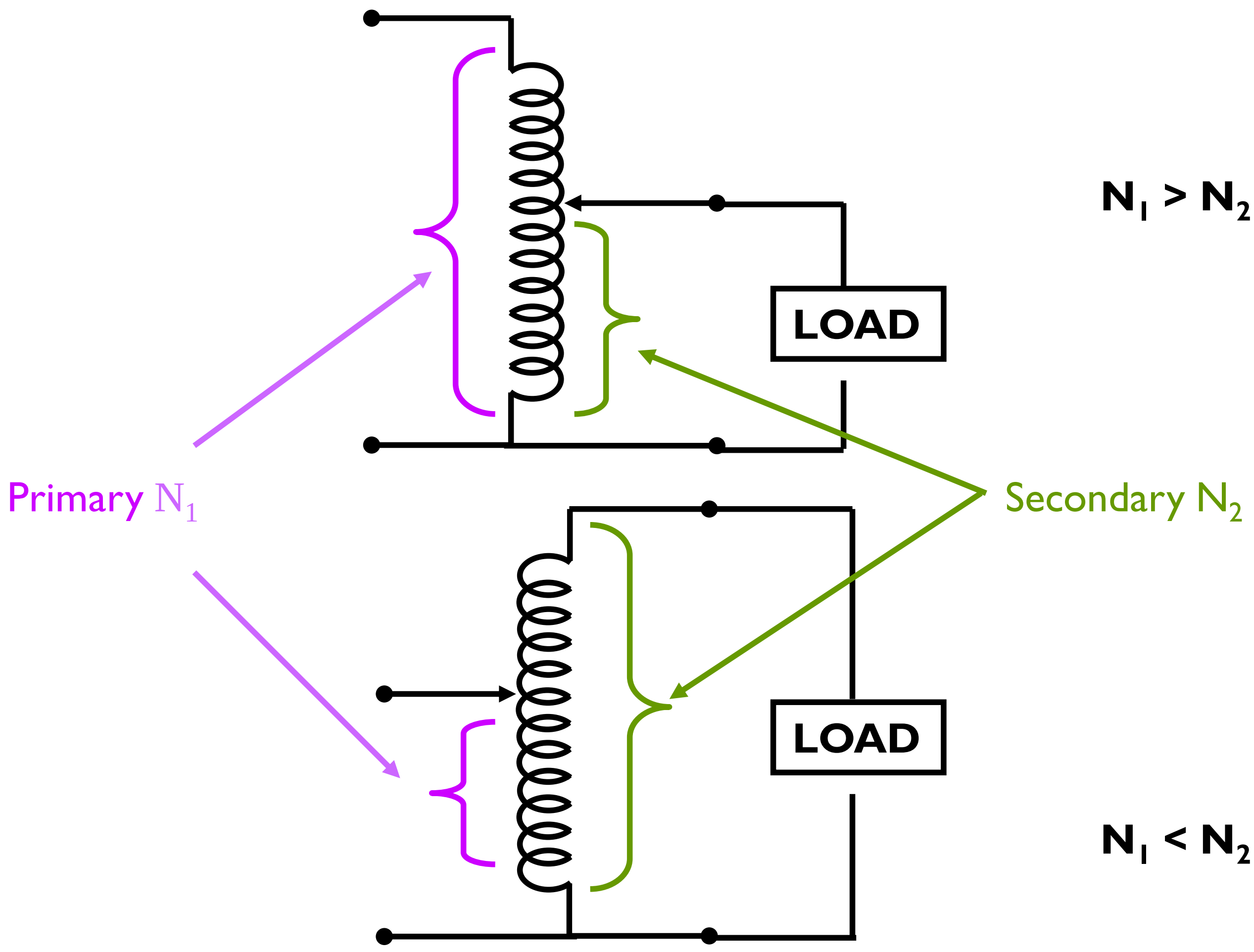
One winding wound over the entire core

Secondary winding can be varied using variable point

Used in power applications to interconnect systems operating at different voltage classes, for example 138 kV to 66 kV for transmission



Auto Transformer- Types



$N_1 > N_2$: Step Down type

$N_1 < N_2$: Step Up type

3 Phase Transformer

3 primary coils & 3 secondary coils.

Possible connections of primary & secondary windings
- *star/star*
- *star/delta*
- *delta/delta*
- *delta/star*

3 single-phase transformers of similar ratings can be connected to form a 3 phase transformer

Application

Power Transformer: Used in electric transmission network

Distribution Transformer: Used in electric distribution networks

Instrument Transformers (PT & CT): Used for high voltage & current measurement

Isolation Transformer: 1:1 transformers used in circuits to provide electrical isolation.

Constant Voltage Transformer: Used as voltage regulators

High frequency Transformer: Transformers designed for operating with high frequency – ferrite core

Introduction to Electric machines

Electric motors may be broadly classified as

- D.C motors
- Induction motors
- Synchronous motors

The last two categories of motors operate on alternating current.

Both ac and dc motors work with the Law of electro magnetic induction and Law of electromagnetic interaction such as Faraday's law, Lenz's law, Ampere's law, Biot-Savart's law etc…

DC Motors

Introduction

- DC motors have excellent dynamic performance & speed control characteristics when compared with its AC counter parts.

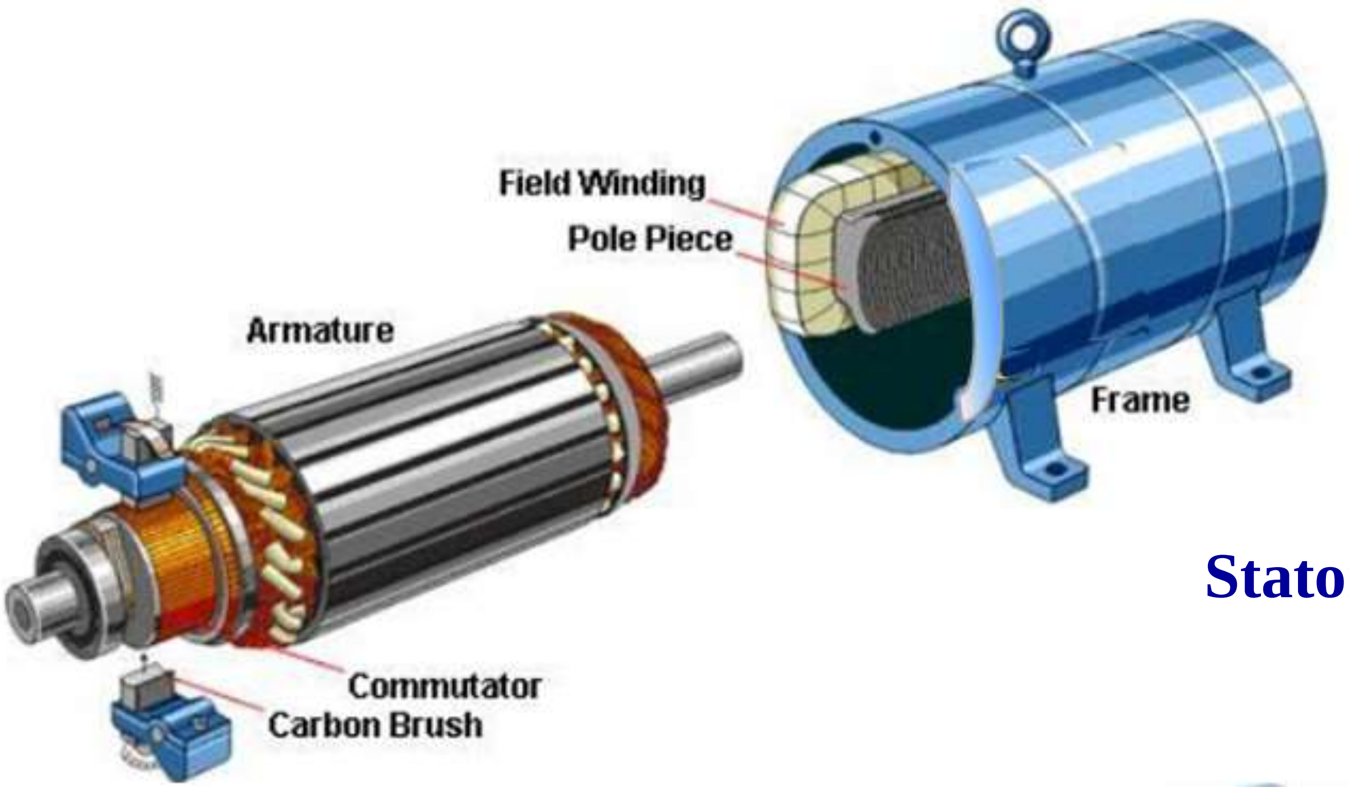
- DC Motors are classified as Shunt, Series, Compound, Separately excited etc… based on how the field system is connected with armature system.

- The most preferred drive motors for Traction and sophisticated control requirement.

- With the advancement of Power Electronics, AC motor performance also matches with that of DC motors

- Many DC motors are getting replaced by AC motors with enhanced control features because of the ease in maintenance and power r supply requirements.

Construction



Stator

Rotor

Construction

Major parts are the stator and the rotor with an air gap in between.

- **Stator** : houses the field winding
- **Rotor** : carries the armature winding

Stator consists of

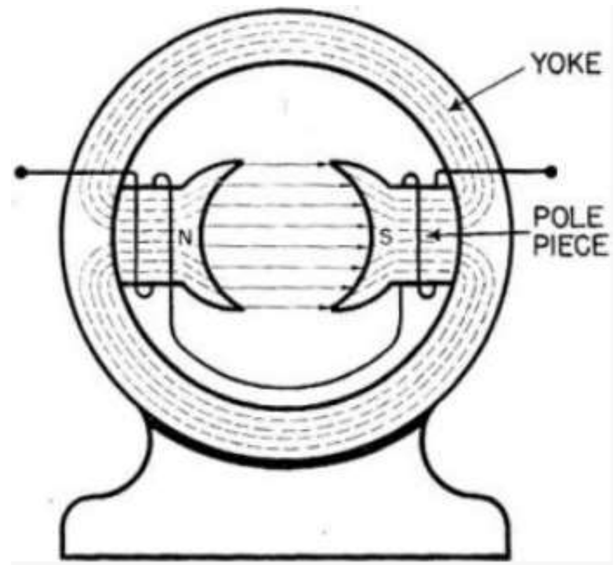
- Yoke(or frame)
- Poles
- Brushes, brush holders and end covers

Rotor consists of

- Armature
- Commutator

Construction

Yoke



- Outer shell housing all the parts
- Two end covers supporting the bearings
- Cast steel is used as a material

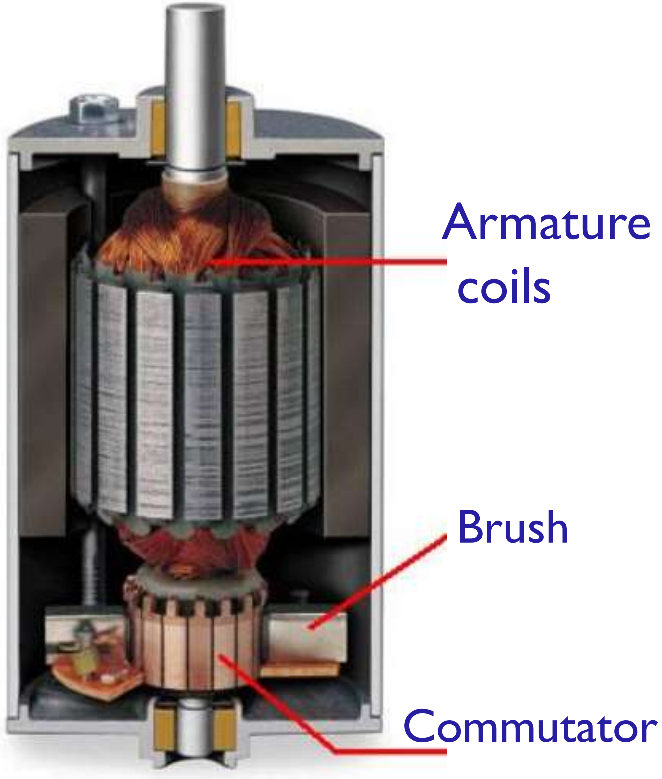
Main Poles

- The field coils are wound
- When excited with dc , produce alternate North and South poles

Construction

Armature

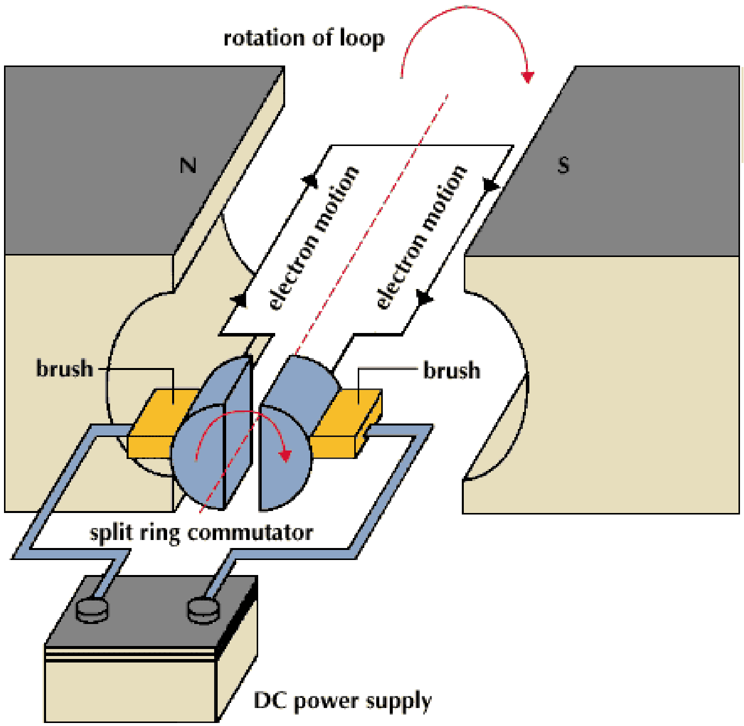
- Rotating part of the DC machine.
- Core is slotted to receive the armature winding.

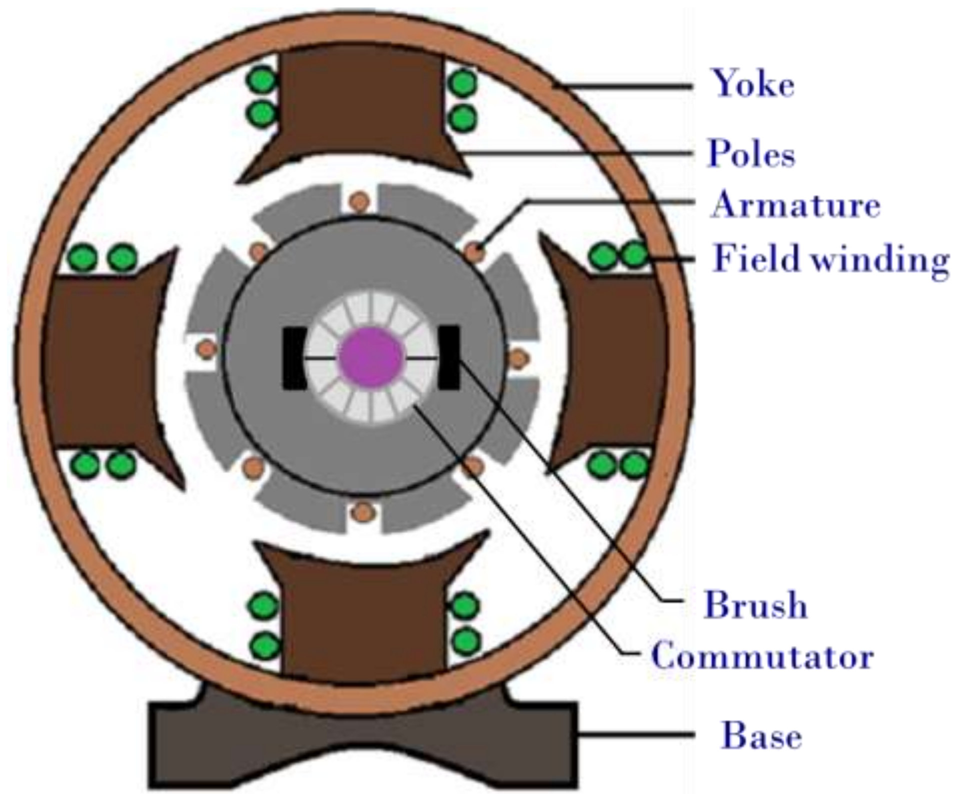


Commutator

- Mechanical Rectifier which rectifies AC to DC
- Carbon Brushes rest on the surface of the commutator.
- The brush holders provide slots for the brushes to be placed

Working Principle





Working Principle

- Current carrying armature conductors placed in the magnetic field experience a force which rotates the armature.
- Induced emf in armature conductor regulates the current drawn to match with the connected load.
- The motor continues to operate in an equilibrium with motor torque balancing load torque and the tractive effort.

$$V = E_b + I_a R_a$$

$$N \propto \frac{E_b}{\emptyset}$$

$$T \propto I_a \times \emptyset$$

V = Voltage applied(Volts)
Eb = Induced Back e.m.f(Volts)
Ia = Armature current(Amperes)
Ra = Armature resistance(ohms)
N = Speed of the motor(r.p.m)
T = Torque developed(Nm)
Ø = Flux (Webers)

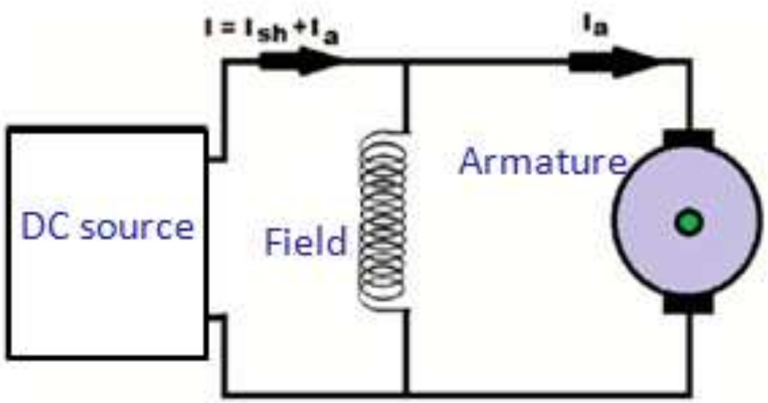
Types of DC Motors

- **Shunt-Wound.**
- **Series-Wound.**
- **Compound-Wound.**
- **Separately excited DC motor.**

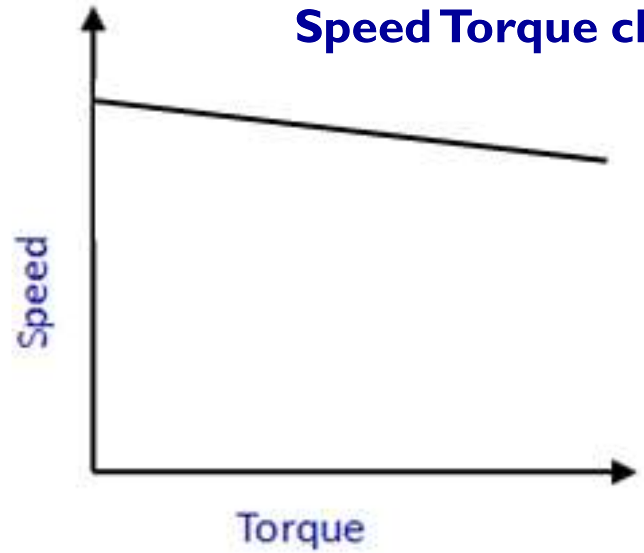
Types of DC Motors

DC Shunt Motor

- Torque proportional to armature current.
- Currents in the field and armature are independent of one another.
- As a result, these motors have excellent speed control.



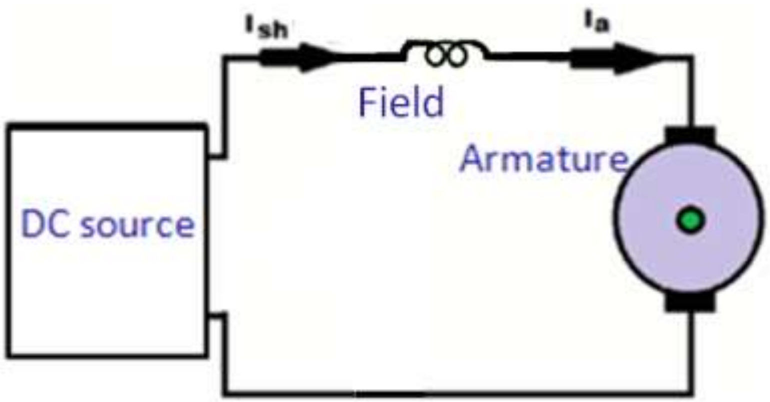
Speed Torque characteristics

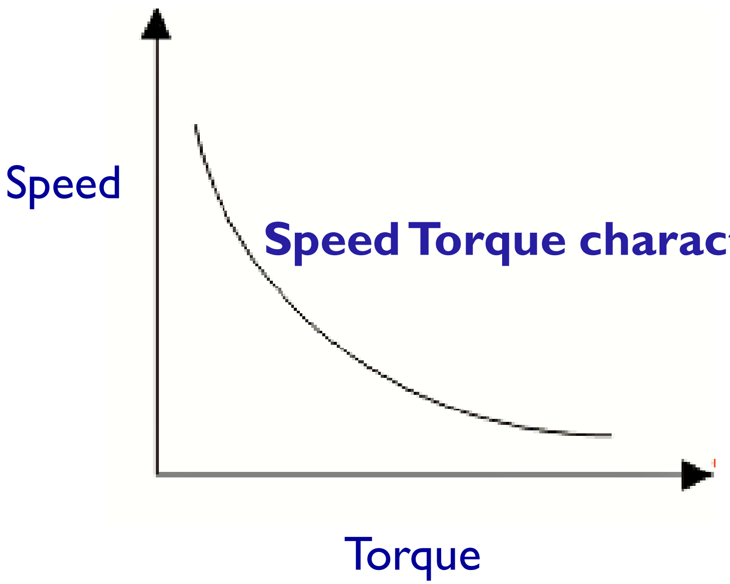


Types of DC Motors

DC Series Motor

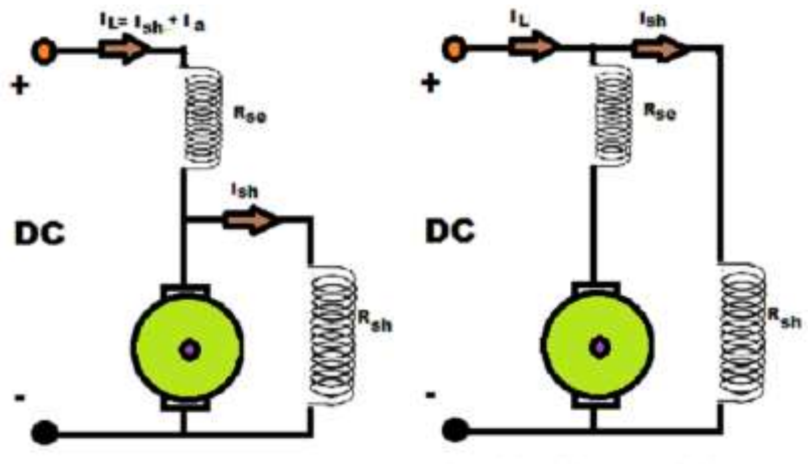
- Field and armature currents are equal
- Torque is proportional to the square of the armature current
- Starting torque quite high and its gets regulated automatically as speed increases.
- Most preferred for Traction.



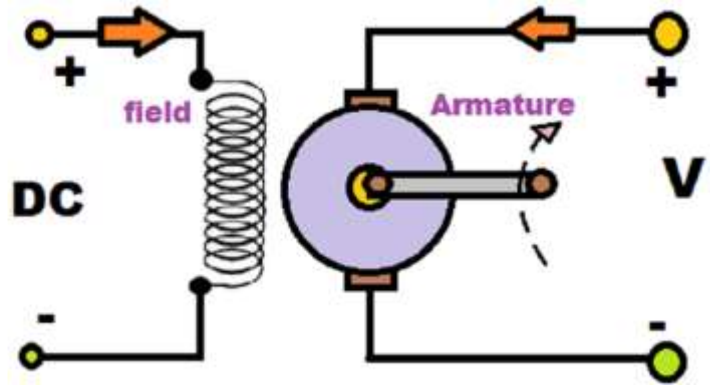


Types of DC Motors

DC compound motor



Seperately excited DC motor



Applications

- D.C. shunt motor is used for drive requiring moderate torques.

- D.C. series motor: Electric traction, high speed tools

- D.C. compound motor :Rolling mills and other loads requiring large momentary torques

Summary

- D.C Motors can be shunt wound , series wound , compound wound or separately excited .
  - For a d.c motor

$$V = E_b + I_a R_a$$

$$N \propto \frac{E_b}{\emptyset} \qquad T \propto I \times \emptyset$$

Basic Electrical Technology

[ELE 1051]

L27- AC motors

Induction Motors

Introduction

Nearly 80% of the world's ac motors are poly-phase induction motors.

It has simple and rugged construction.

It is available from fractional horsepower ratings to megawatt levels.

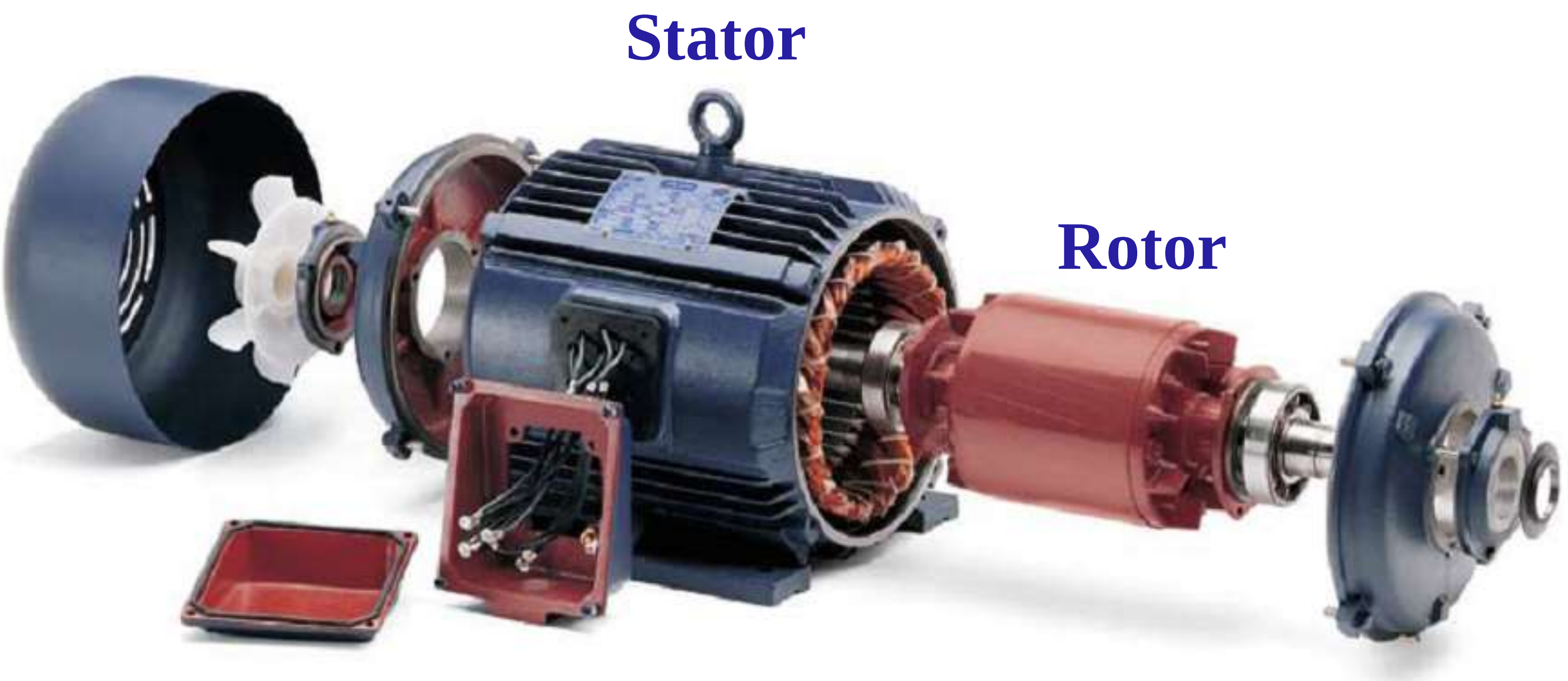
There are machines available to operate from 3 phase or single phase electrical input

Single phase induction motors are restricted to small power levels (say less than 2 hp)

3 phase induction motors are widely used as pump & fan drives

For all practical purpose, it may be considered as a constant speed drive with full load slip around 3 %.

Construction



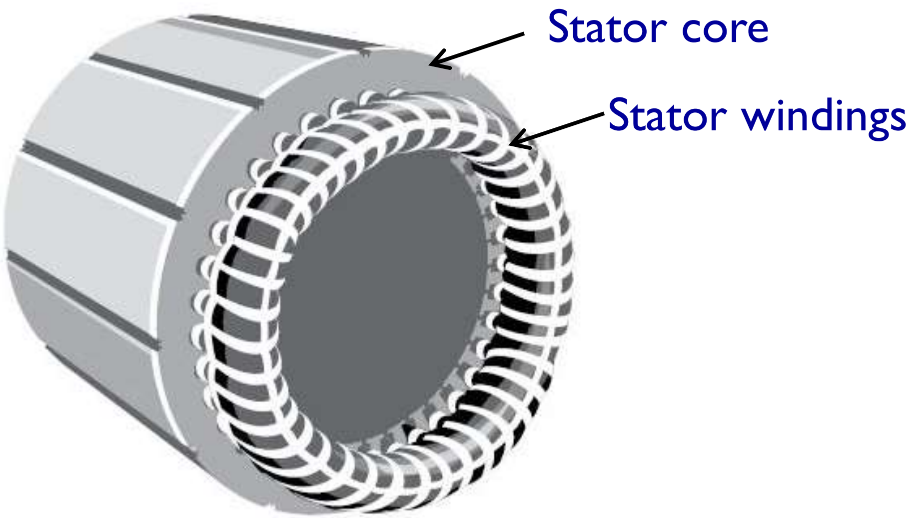
Construction

Stator

- **Stator frame** (cast iron) provides mechanical support to the stator core

- **Stator core** laminated and slotted to carry the 3 phase windings

- The balanced windings are displaced in space by 120 degrees electrical



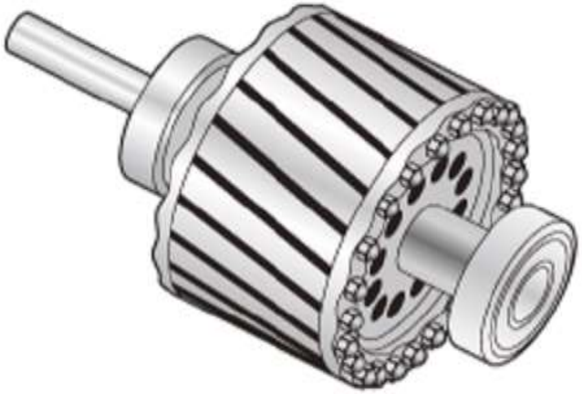
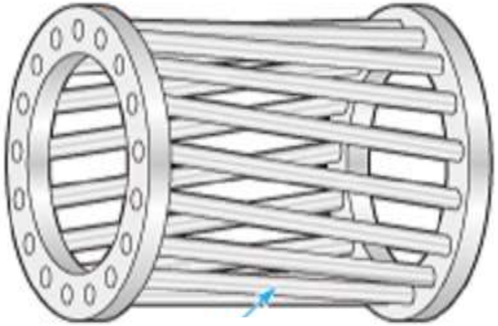
Construction

- Types on the basis of rotor construction
  - **Squirrel Cage Rotor**
  - **Slip Ring Rotor**
- Cylindrical Laminated core
- Slots cutout on outer periphery
- Conductors placed in slots

Construction

Squirrel Cage Rotor

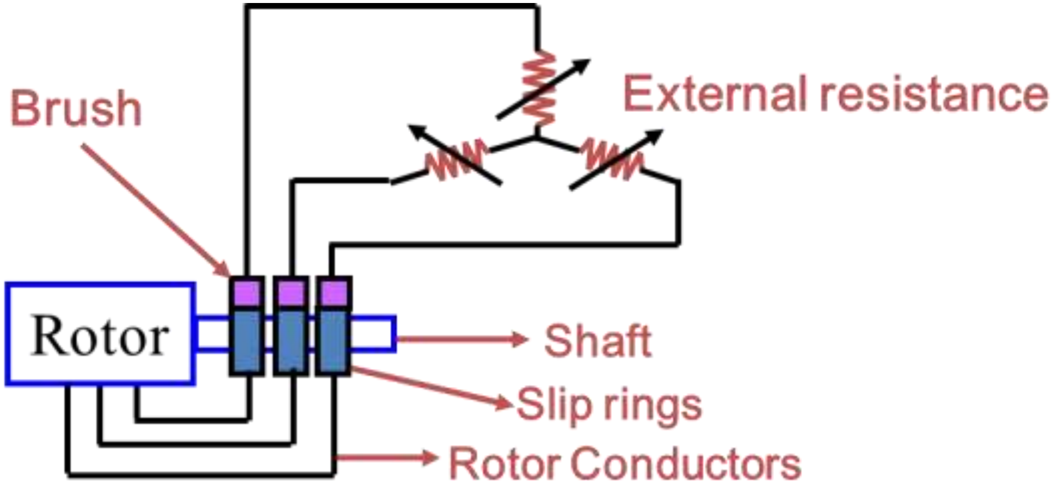
- Skewed arrangement
- Copper or Aluminum Bars
- Conductors shorted by end rings
- Closed rotor circuit
- Cheap, rugged and needs little or no maintenance

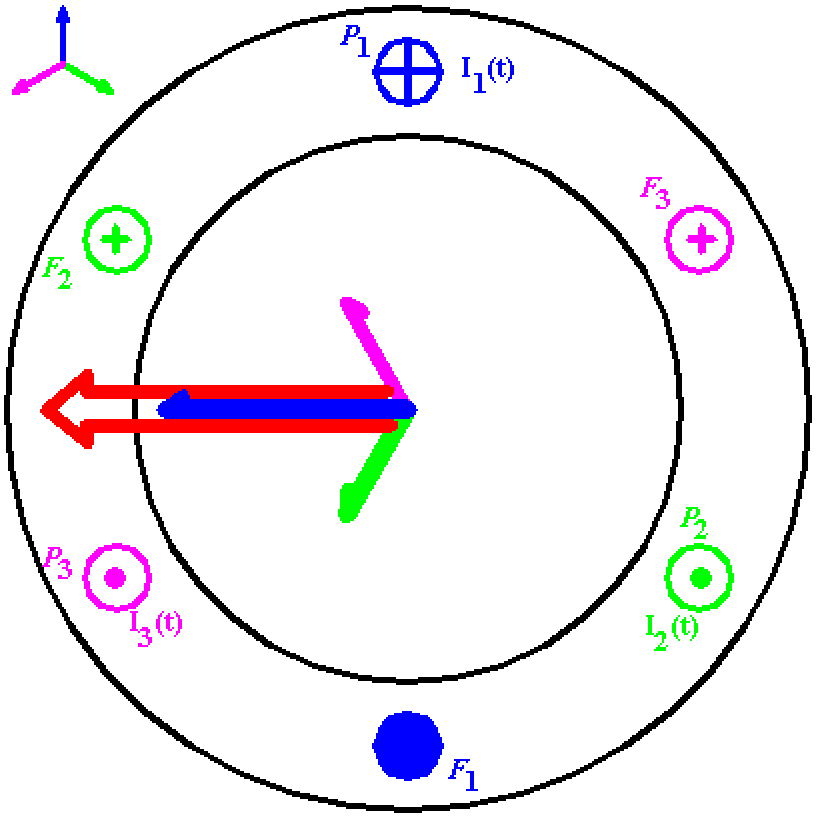

Construction

Wound Rotor

- rotor winding is uniformly distributed and is usually connected in star.
- The terminals of the winding are brought out to three slip rings
- Slip rings in contact with brushes
- Brushes connected to external resistance for higher starting torque

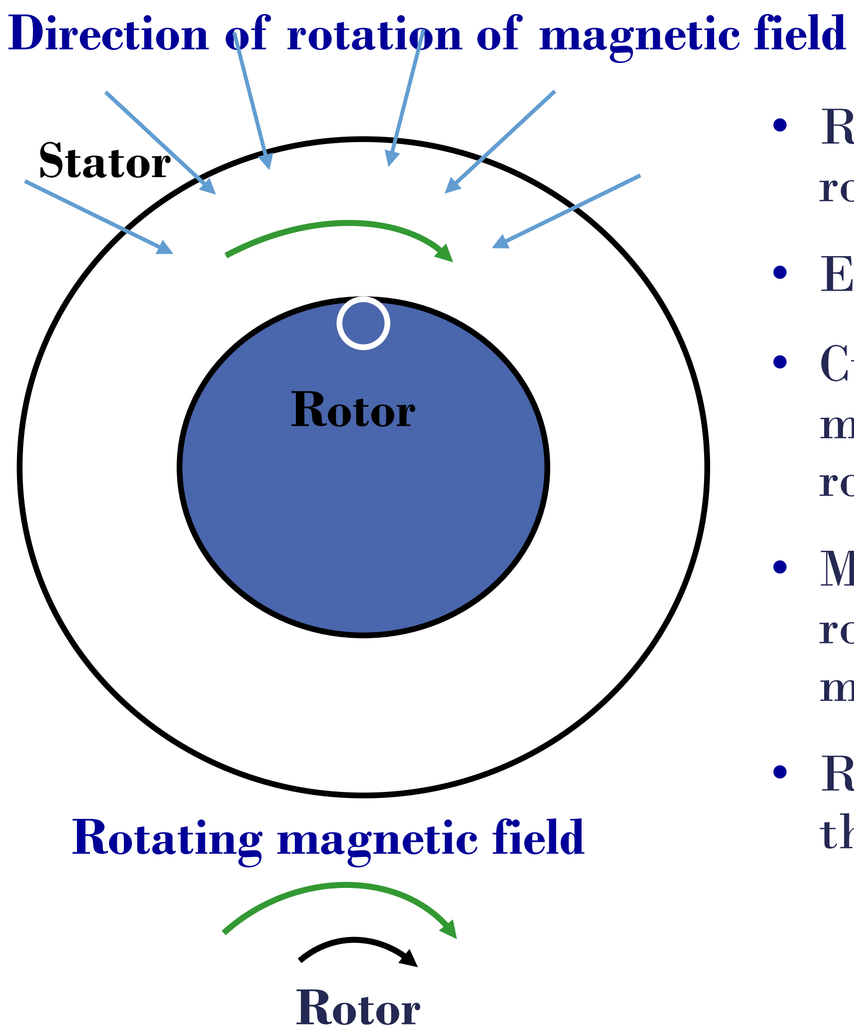


Working Principle



3-phase currents flowing in the stator winding produce a rotating magnetic field rotating at synchronous speed.

Working Principle



- Rotating magnetic field is cut by the rotor conductor
- EMF is induced in rotor conductor
- Current in the rotor conductor sets up a magnetic field which opposes the rotation of main field
- Main field is independent and hence rotor field tries to catch up the speed of main field to reduce the relative speed
- Rotor rotates in the same direction as that of rotating magnetic field

Working Principle

The axis of the magnetic field rotates at a synchronous speed

$$\boxed{N_S = \frac{120\,f}{P}}$$

N_S = Speed of RMF, rpm
f = Frequency of ac supply, Hz
P = No. of poles

N_S = Synchronous Speed,rpm
N = rotor speed, rpm

- If $N = N_S$,
  - ✓ No flux cut by rotor conductors
  - ✓ No emf induced across rotor conductors
  - ✓ No current flow, no torque

Hence N < Ns must for rotor rotation

Working Principle

Slip speed = $(N_S - N)$, rpm

$$\boxed{\% s = \frac{N_S - N}{N_S} \text{ x } 100\ \%}$$

- For rotor speed N, relative speed = $\mathbf{N_S - N}$

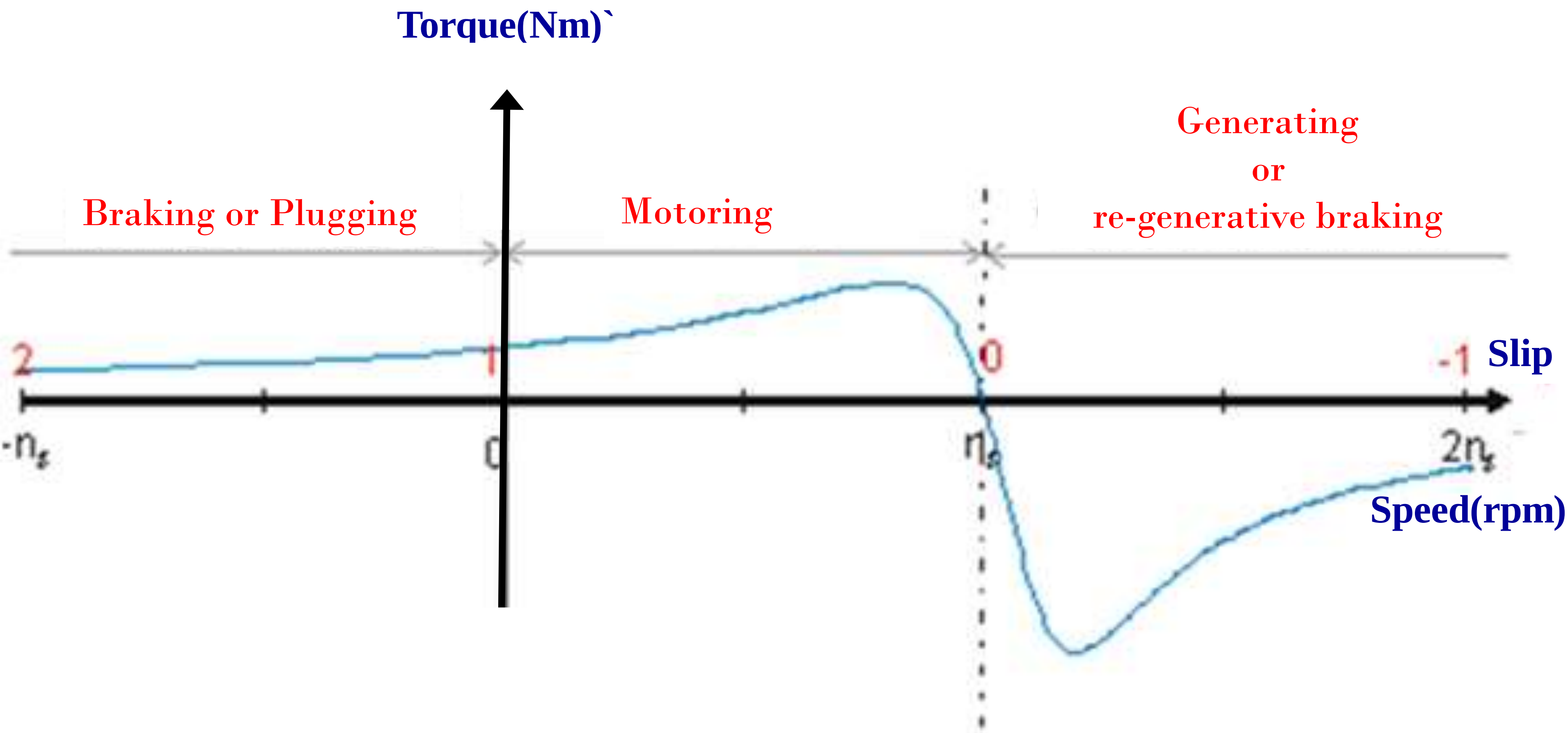
$$f_r = \frac{P\,(N_S - N)}{120}$$

$$\boxed{\mathbf{f_r = s\,f}}$$

Note

At instant of starting, N = 0, s =1 , $f_r = f$

Speed-Torque Characteristics



Applications

- **Industrial & Commercial Applications**
  - Pumping Systems
  - Refrigeration Systems
  - Compressors
  - Fans & Blowers
  - Industrial Drives

Single Phase Induction Motor

Introduction

- Construction is similar to 3 phase induction motor except the stator has TWO winding - starting & running.

- The rotor is of squirrel cage type

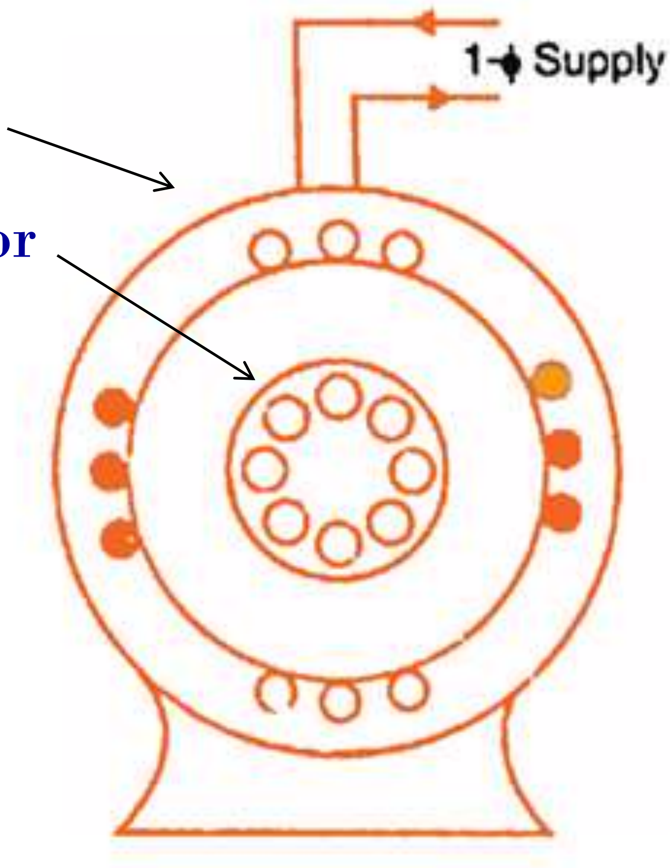
- A capacitor is connected in series with the starting winding to achieve phase split.

- The motor is started as two phase machine.

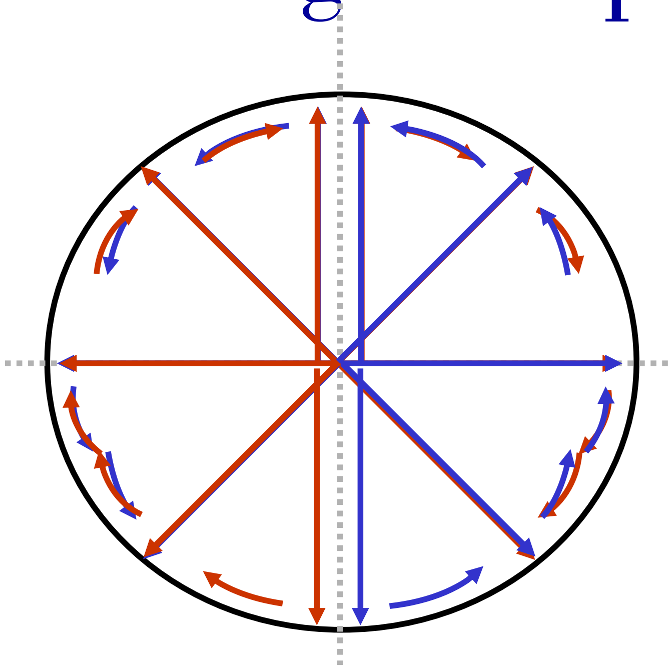
Construction

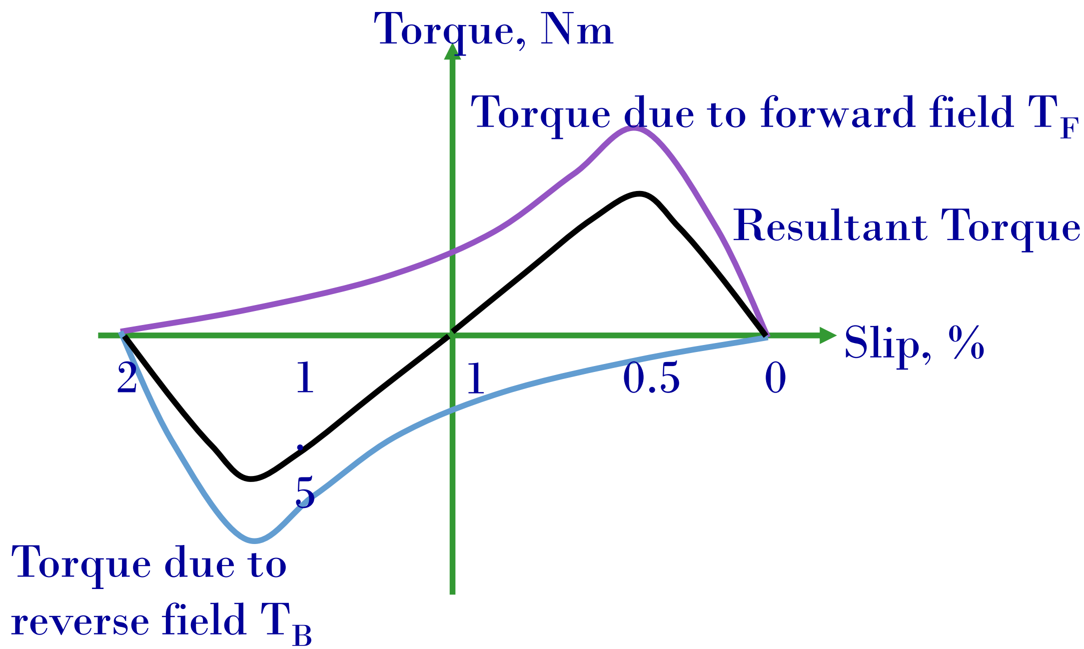
Stator

Rotor



Working Principle –Double field revolving theory





Resultant field is alternating. Hence,

Single Phase induction motors are **Not Self Starting** in nature

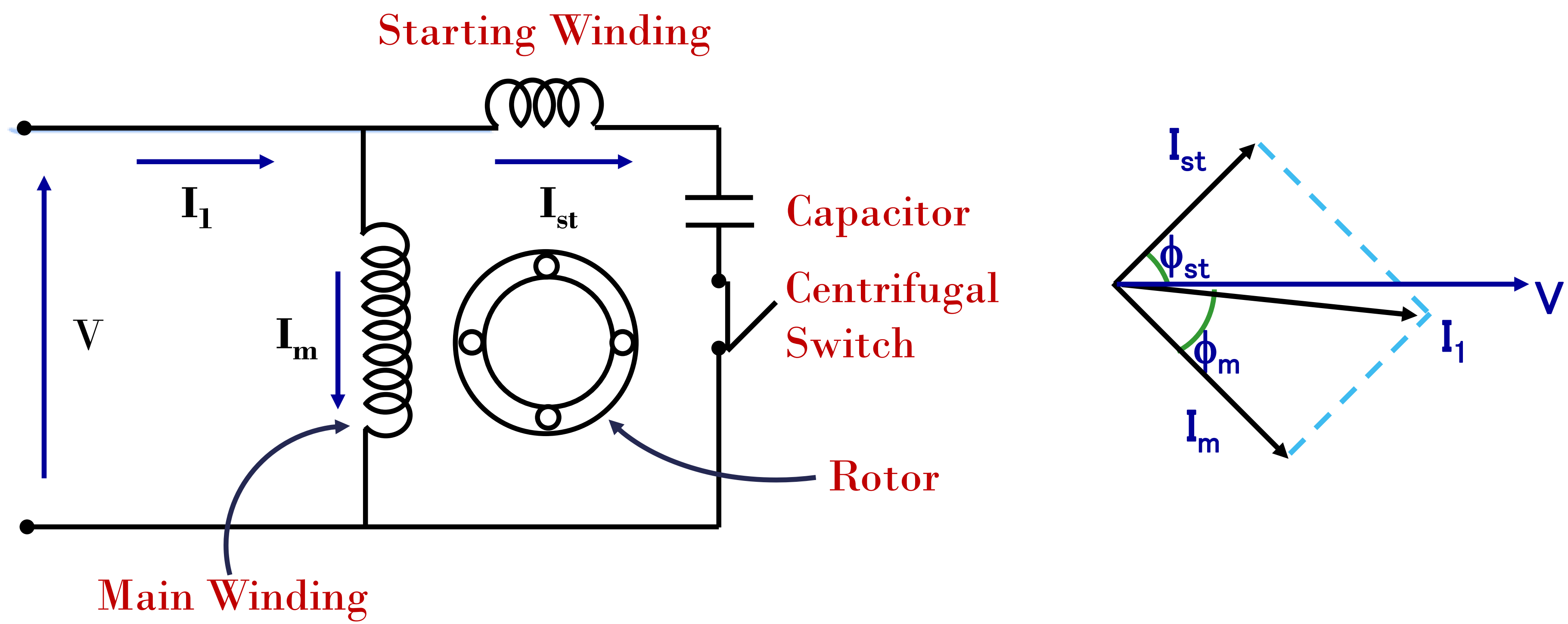
Starting

- The motor is started as two phase induction motor
- Phase split is achieved by connecting a series capacitor with starting winding

Types

- Capacitor Start Motor – pump / compressor motors
- Capacitor Run Motor – ceiling fan motor

Capacitor Start Motor



- Centrifugal Switch opens the circuit when speed is near about rated speed
- Capacitor present in circuit only at starting

Applications

- Low power applications such as Air conditioners, Pumps, Fans.

- Refrigerators, Washing Machines etc…

Summary

- Three phase Induction motors have inherent self- starting torque . It can never run at synchronous speed.

$$N_S = \frac{120f}{P} \qquad \%s = \frac{N_S - N}{N_S} \times 100$$

- Single phase Induction motors are used for small power applications The common forms are capacitor start and capacitor run motor.

Basic Electrical Technology

[ELE 1051]

SINGLE PHASE AC CIRCUITS

L19 – Power in AC circuits

Topics Covered

- **Synchronous Motors**
  - Introduction
  - Construction
  - Working Principle
  - Applications

Synchronous Motors

Introduction

- Synchronous motors are constant speed AC motors which always run at synchronous speed irrespective of connected load.
- Its power factor of operation is controllable.
- Field system is DC excited or made up of permanent magnets
- Synchronous condensers are used for power factor improvements.

Construction

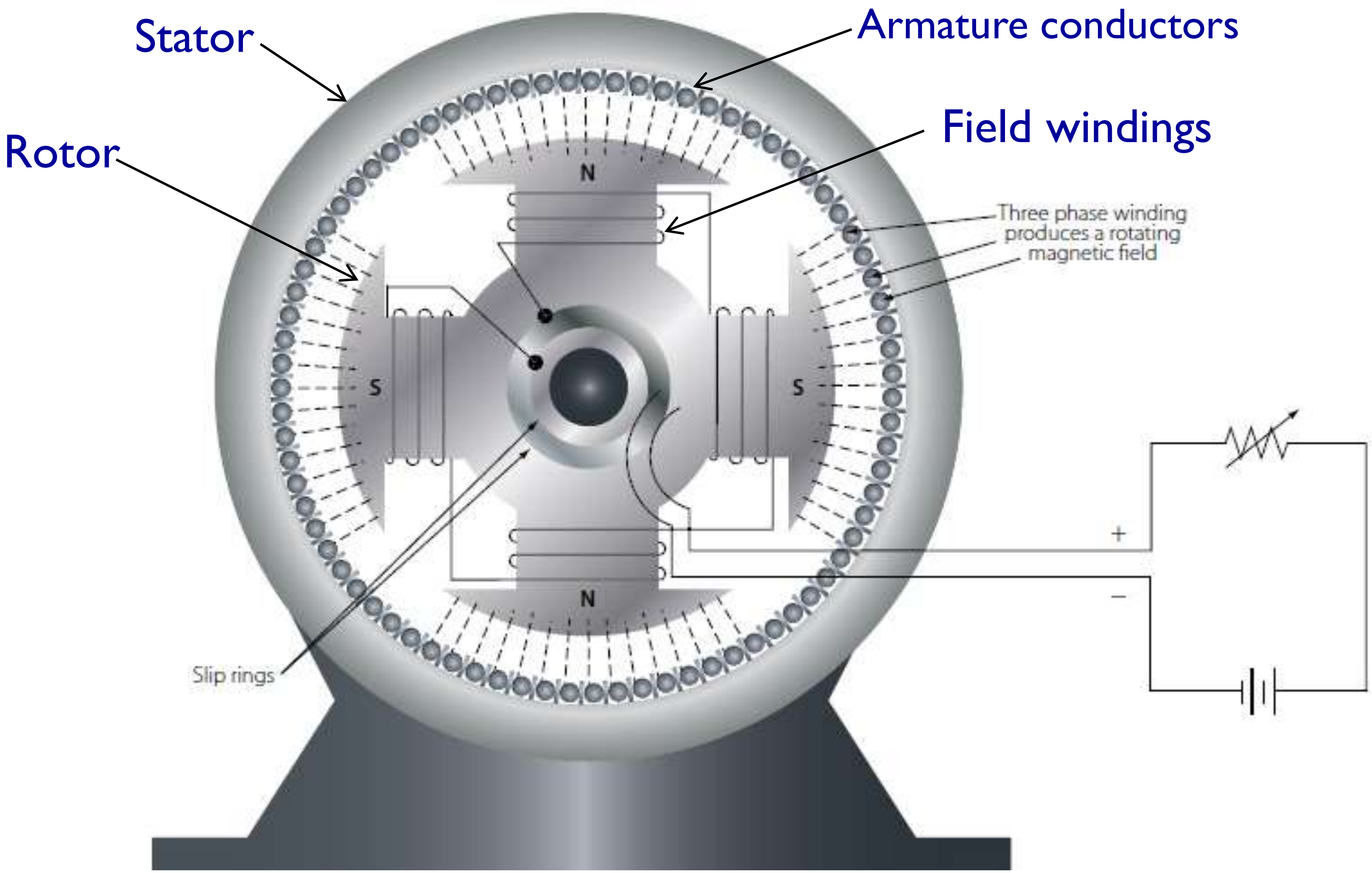
Stator

- Has slots on the inner periphery to accommodate the armature windings

Rotor

- Carries the field windings that produces the required flux.

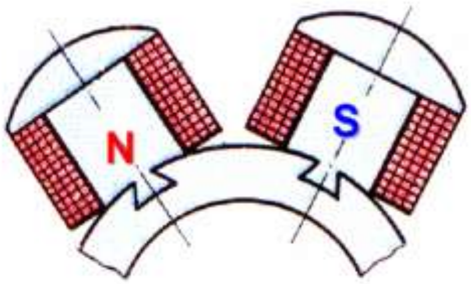
Construction



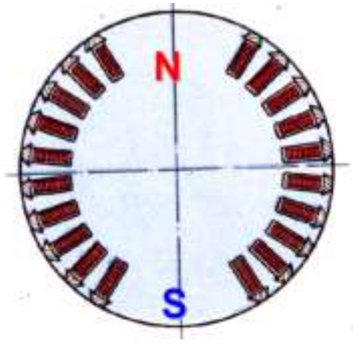
Construction

Synchronous machines are of two types based on the rotor structure

- **Salient pole type**
  - Low speed applications
- **Cylindrical Rotor Type**
  - Robust and used for high speed applications

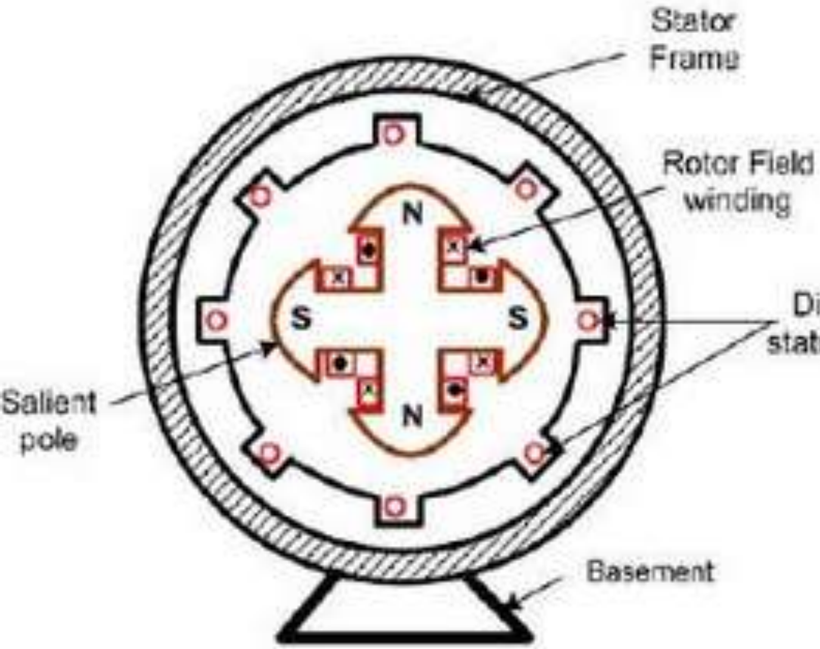


Salient-Pole

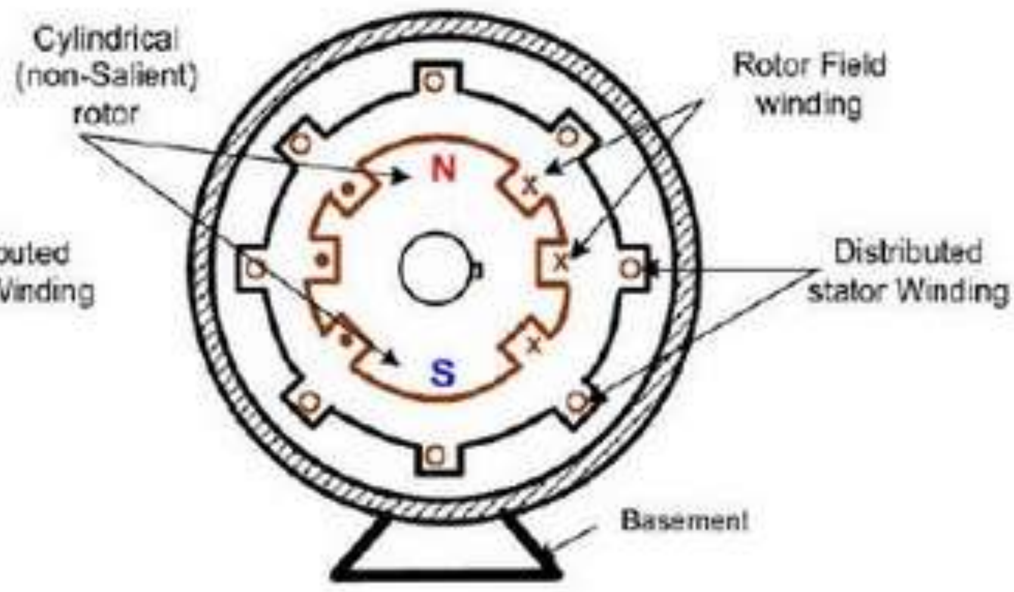


Non-Salient-Pole

Construction



Salient Pole



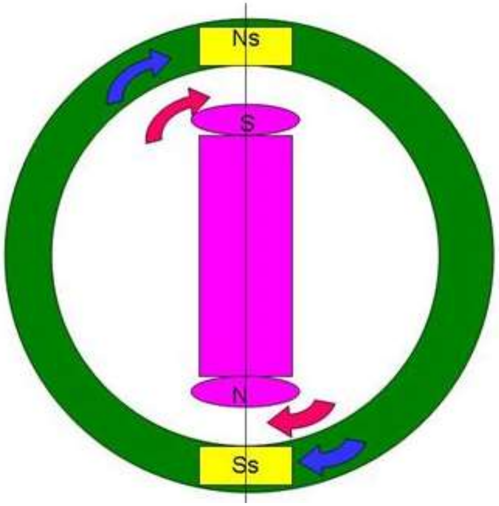
Non Salient Pole

Principle of Operation

- Armature energized from a 3 phase ac source , and the machine is started as induction motor with damper windings.

- After achieving the full speed, field winding is excited and the stator and rotor field gets magnetically locked.

Rotor with damper windings



Rotor poles get locked to the unlike poles in the stator .Torque is now unidirectional

Power Factor Control

- By varying excitation current one can change the operating power factor.
- With normal excitation unity power factor can be achieved.
- Under excited synchronous motor operates with lagging power factor.
- Over excited synchronous motor operates with leading power factor - Synchronous Condenser

Applications

- Constant speed applications

- Used for power factor improvement

Energy Meters

Topics Covered

- *Working Principle of Energy Meters*
- *Introduction to Digital Energy Measurement*
- *Electricity Tariff*

Working Principle

Energy is the total power delivered or consumed over a time interval,

Energy = Power x Time

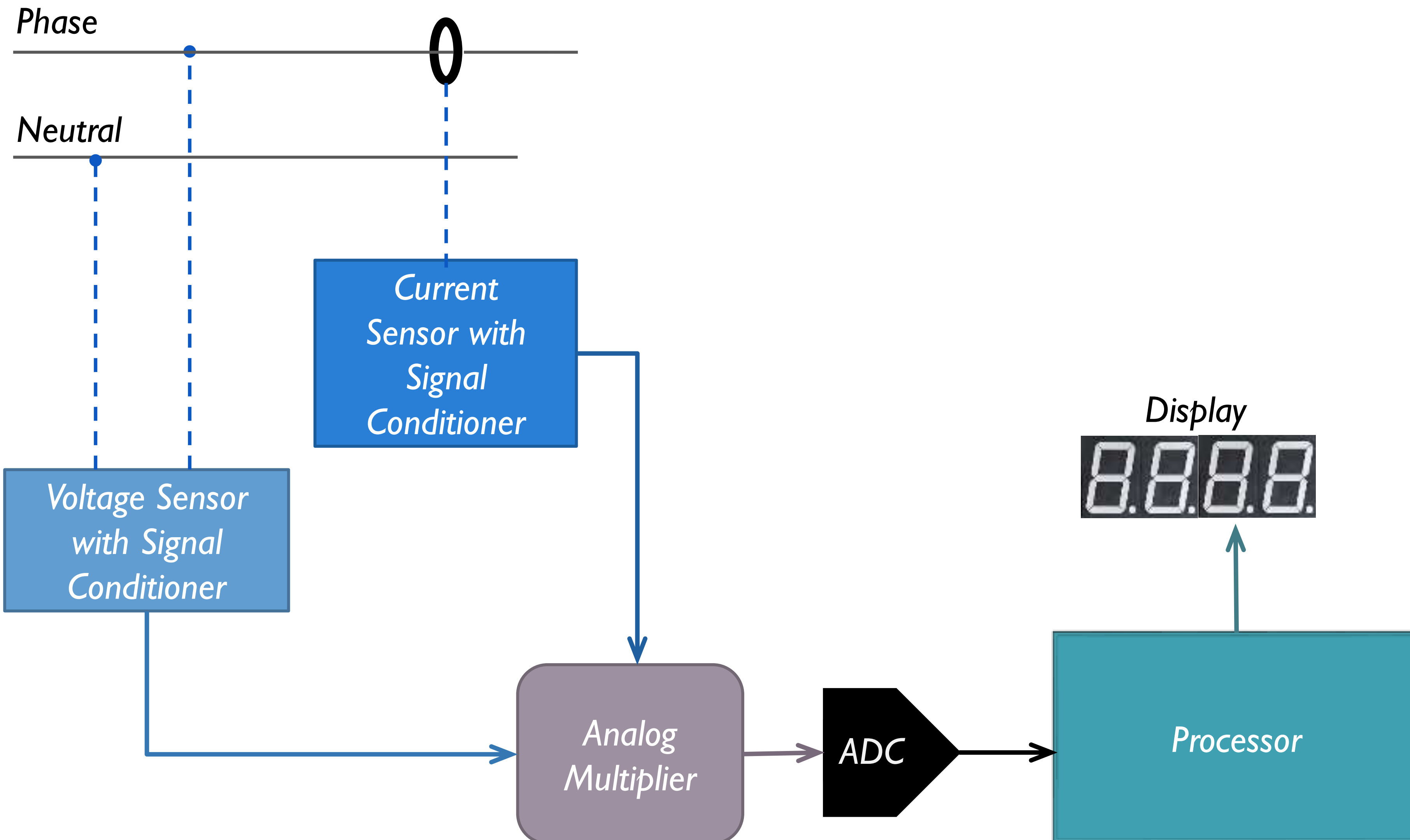
Electrical energy developed as work or dissipated as heat over an interval of time 't' may be expressed as:

$$Energy = \int_0^t v\, i\ dt$$

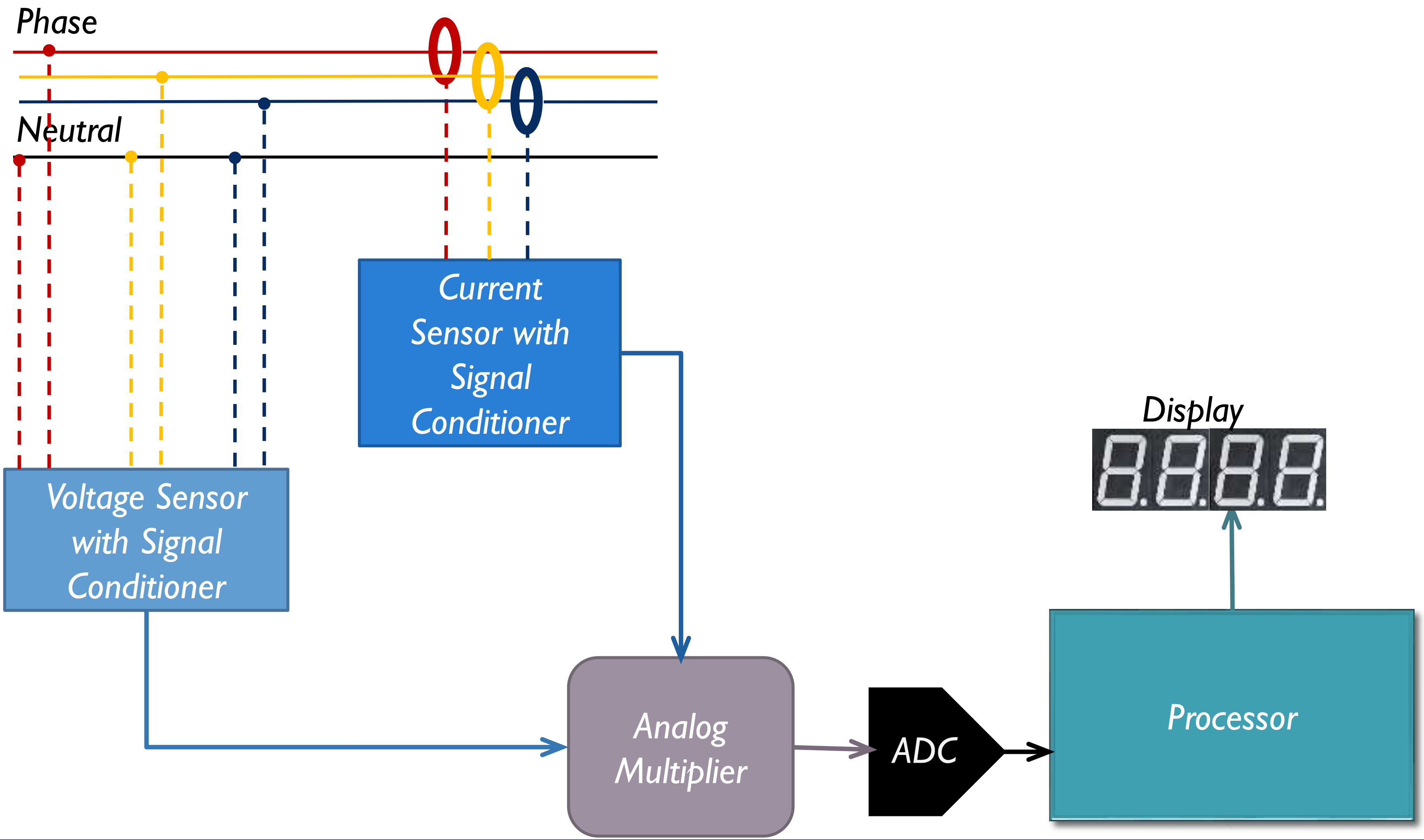
v – Applied voltage in (volts)
i – current (A)
t – time (hr)

Unit of Energy: ***kWh***

Digital Energy Meter



Three Phase Digital Energy Meter



Electricity Tariff

Installation	Power Supply	Tariff
Industries	11kV and above	Demand Charges (per kVA) Power Factor Surcharge (per unit) Energy Charges (per kWh)
Hotels/Restaurant/ Cinemas/Petrol Bunks/Banks/ Commercial Complexes	400V Three Phase 230V Single Phase	Sanctioned Load (per kW) Power Factor Surcharge (per unit) Energy Charges (per kWh)
Residential	400V Three Phase 230V Single Phase	Sanctioned Load (per kW) Energy Charges (per kWh) Rebate for Solar Installations

Reference: MESCOM 'Electricity Tariff-2014' dated 06/05/2014

Summary

- Synchronous motors are constant speed AC motors which always run at synchronous speed. Synchronous condensers are used for power factor improvements.

- Energy can be measured by integrating power over an interval of time.

- Electricity Tariff depend on the type of consumer.