Q12: 6 distinct symbols are transmitted through a communication channel. A total of 12 blanks are to be inserted between the symbols with at least two blank spaces between every pair of symbols. In how many ways can we arrange symbols and blanks?

12 spaces

soln

$^{2+5-1}C_2 \times 6!$ → No of ways of arranging 6 symbols as they are distinct

$S_1 \{\{ S_2 \{ \{ S_3 \{ \{ S_4 \{ \{ S_5 \{ \{ S_6$

Remaining 2 blank spaces must be distributed to 5 places wd reptn

Ans = $^6C_2 \cdot 6!$

Q13: Suppose we print all 5 digits numbers on slips of paper with one number on each slip. However, since the digits 0,1,6,8 and 9 become 0,1,9,8 and 6 when they are read upside down, there are pairs of numbers which can share the same slip if the slips are read right side up or upside down (89166=99168). How many distinct slips will we have to make up for all the 5 digits numbers?

soln

0, 1, 6, 8, 9

↓

0, 1, 9, 8, 6

ex :- 89166 ⇔ 99168 (same slip)

How many slips are needed to print all 5 dig nos?

Keeping in mind that some slips are shared by 2 nos

81018 ⇔ 81018

Total no of 5 dig no^{s} $\Longrightarrow 10^5$

0,1,2,3,4,5,6,7,8,9.

Now I wanna subtract those no^{s} which can share the same slip.

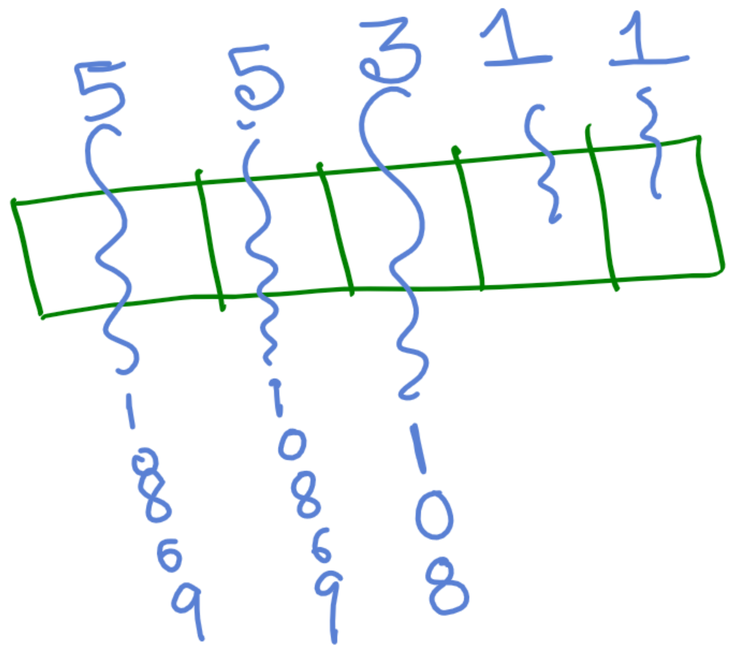
Counting the no^{s} of 5 dig no^{s} made of 1,8,6,9,0

$\Rightarrow 5^5$

Among the no^{s} made of 1,8,69,0, there are some no^{s}, even when u flip, they are read same

ex:- 81018

16091 $\longleftrightarrow$ 16091



∴ The no^{s} which are read same even when u flip $\Rightarrow$

$5^2 \times 3$ -

∴ Total no of 5 dig no^{s} which can share the same slip $= 5^5 - (5^2 \times 3)$

∴ Total no of slips :-

$$10^5 - \frac{1}{2}\left[5^5 - (5^2 \times 3)\right]$$

* Every 2 no^{s} from that set shares the same slip

if 0 is not allowed in Ist place

what is the ans?

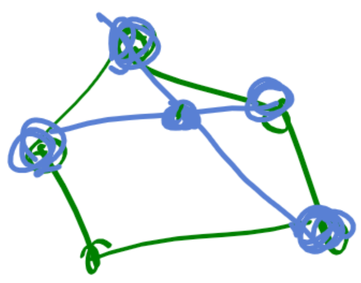
Q14: If no three diagonals of a convex decagon meet at the same point inside the decagon, into how many line segments are the diagonals divided by their intersection?

Soln

No of diagonals in a decagon : $^{10}C_2 - 10 = 35$

Total no of intersection bt̄n the diagonals :- $^{10}C_4$

$= 210$



Bt every intersectn pt lies on 2 diagonals (xy by 2)

Suppose a line has 'k' intersectn points, the no of line seg fᵭmed is $\rightarrow (k+1)$

∴ Total no of line segments

$$35 + 2(210)$$

Fᵭ a polygon of n-sides :-

$$\left({}^{n}C_2 - n\right) + 2\left({}^{n}C_4\right)$$

↳ Total no of intersectn pts

↳ Every intersectn pt lie on 2 lines

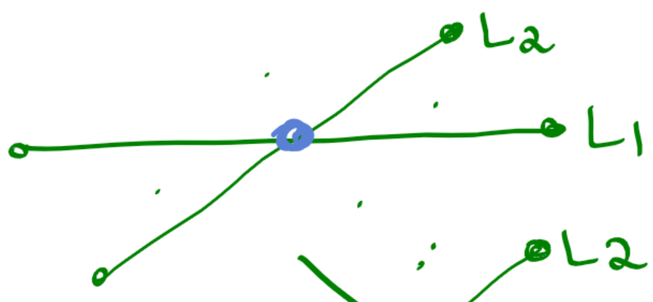
Q16: How many points of intersections are formed by n lines drawn in a plane if no two lines are parallel and no three are concurrent? In to how many regions is the plane divided?

Soln

$n=1$:

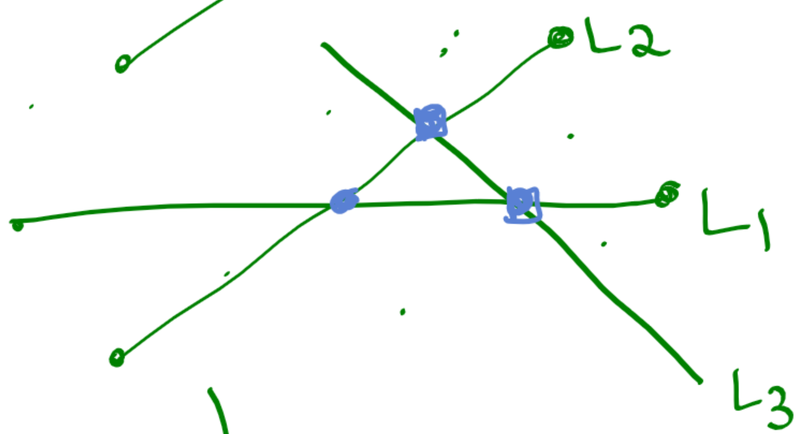
No point of intersectn

$n=2$:



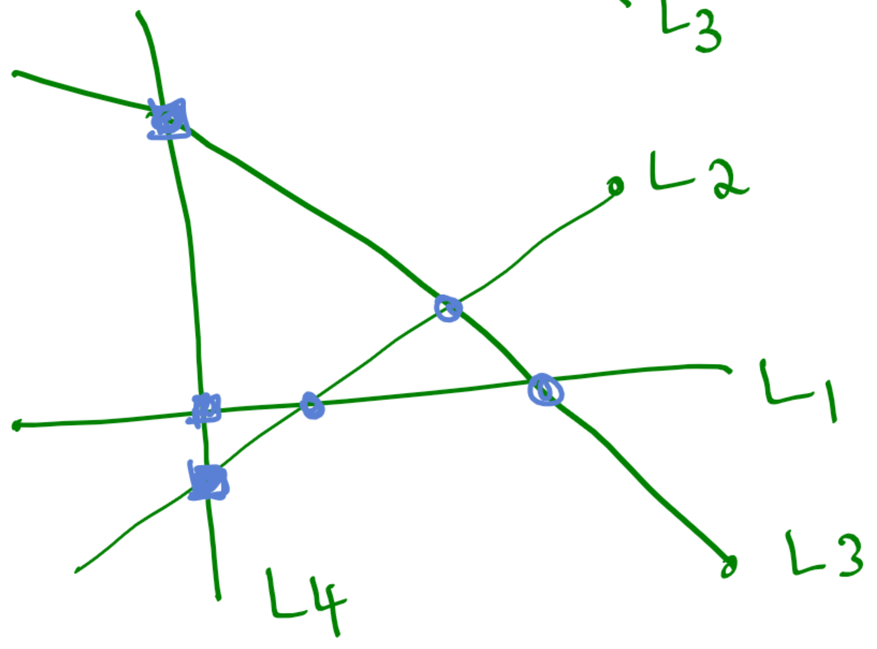
1

$n=3$:-



1old one + 2 new
= (3)

$n=4$



3old + 3new
= (6)

$n=5$: 4new + 6old = (10)

$n=6$: 5new + 10old = (15)

when there n lines drawn ;

$$= (n-1) + [1+2+3+\ldots+(n-2)]$$

$$= 1+2+3+\ldots(n-1)$$

$$= \frac{n(n-1)}{2}.$$

∴ Total no of intersectn pts $\Rightarrow \frac{n(n-1)}{2}$.

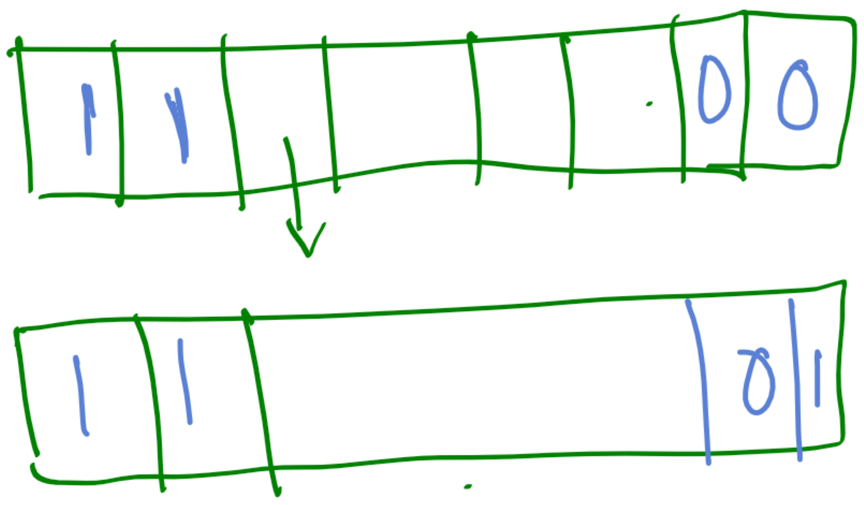
No of regions $\Longrightarrow$? $\frac{1}{2}[n^2+n+2]$

Q19: A bit is either 0 or 1, a byte is a seq of 8 bits. Find the number of bytes which begin with 11 and do not end with 11?

Soln

No of bytes starting with 11, but not ending with 11

It can end with 00, 10, 01



$\Rightarrow 2^4$

$\Rightarrow 2^4$

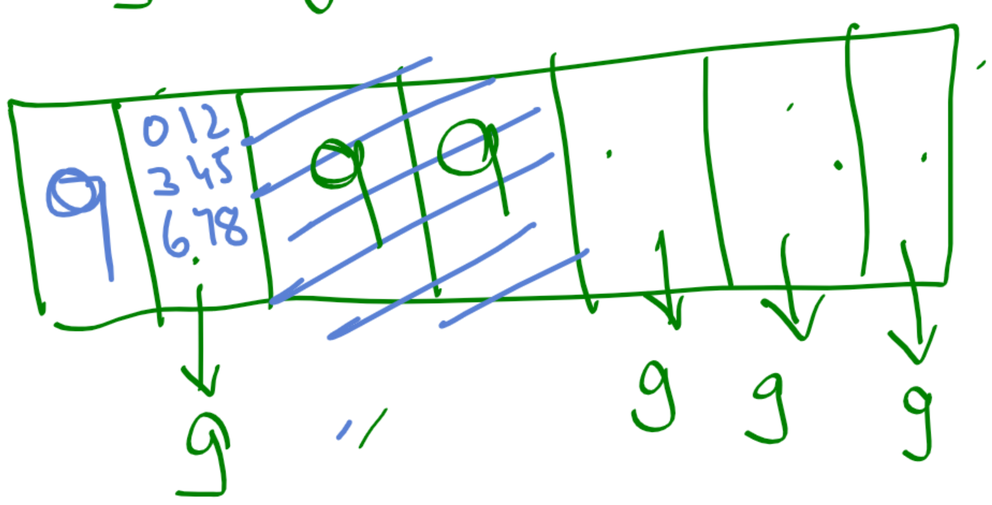
$\Rightarrow 2^4$

$$\text{Ans} = 3 \times 2^4$$

Q15: Among all 7 digits decimal numbers, how many of them contain exactly three 9s?

Soln

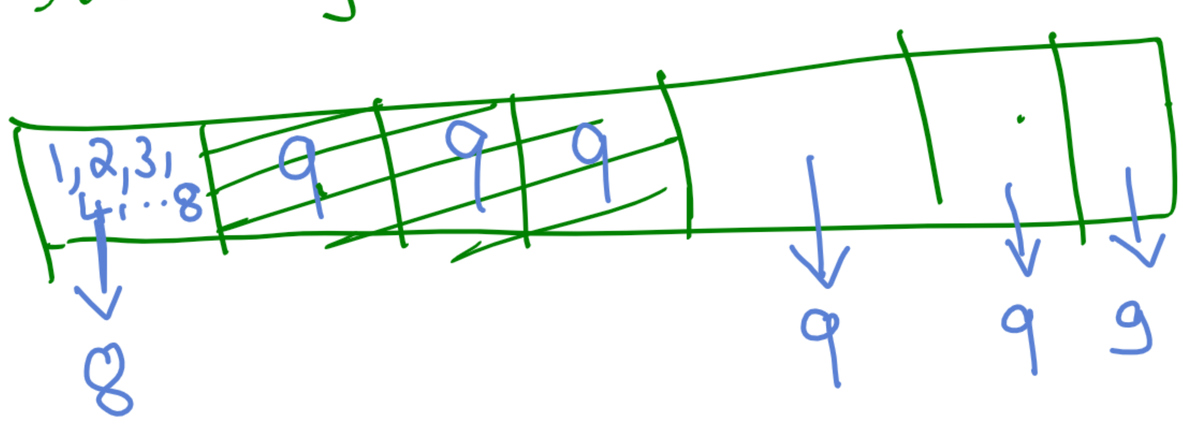
i) starting dig is 9 :-



9992340.
9990000
9992333
9209009

$9^4 \times {}^6C_2$

ii) Not starting with 9 :



$(8 \times 9^3)\,{}^6C_3$

$\therefore$ Ans $\Rightarrow$ $(8 \times 9^3)\,{}^6C_3 + (9^4)\,{}^6C_2$

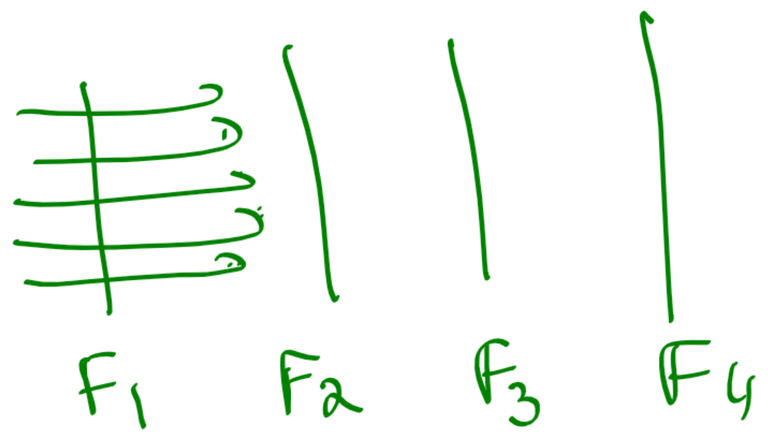
Q17:In how many ways can a lady wear 5 rings on her fingers(Not on thumb) on her right hand?

Solm

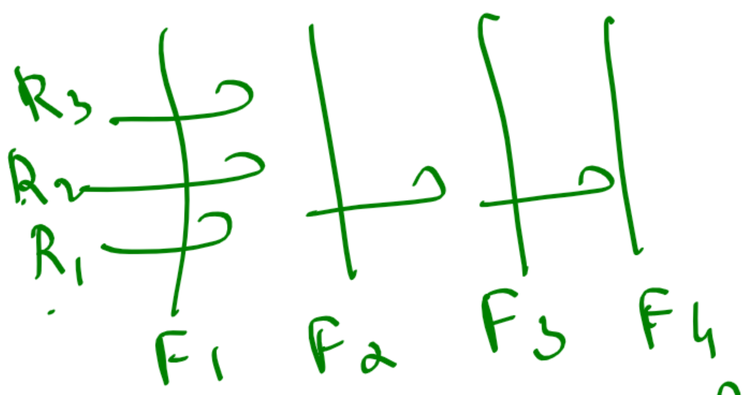
5 rings on 4 fingers

Dist of 5 rings on 4 fingers. Sit each finger can get any no$ rings

$$n+r-1C_r = 5+4-1C_5$$



$$= {}^8C_5$$



since the rings are distinct, to enclude the order

$$5! \times {}^8C_5$$

Distⁿ of r objects to n boxes s.t each box ....

$\approx$

Selectⁿ of r boxes out of total of n boxes

{ with order (distinct) without order (identical) }

{ with reptⁿ (each box can have more than one obj) wdt rep (each box has atmost one obj