Q1. Find the no of permutations of the word INSTITUTION?
(i) How many of them begin with I?
(ii) How many of them begin with I and end with N?
(iii) How many permutations are with 3 Ts not together?

Soln :

$$\frac{11!}{\underset{3I's}{\underset{\uparrow}{3!}}\ \underset{2N's}{\underset{\uparrow}{2!}}\ \underset{3T's}{\underset{\uparrow}{3!}}} =$$

(i) I - - - - - - - - - -

$$\frac{10!}{\underset{I's}{2!}\ \underset{2N's}{2!}\ \underset{3T's}{3!}}$$

(ii) I - - - - - - - - - N

$$\frac{9!}{\underset{I's}{2!}\ \underset{T's}{3!}}$$

(iii) Number of permutations with 3 T's are together.

$$\frac{9!}{2!\,3!}$$

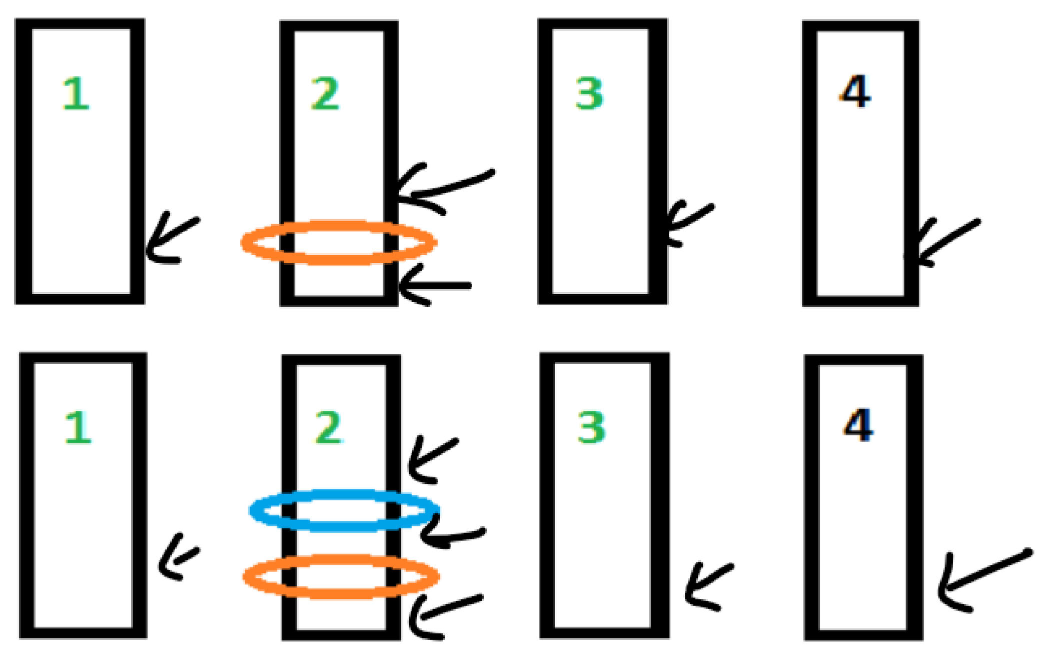
Req Ans : Total - 3 T's together

$$\frac{11!}{3!\,2!\,3!} - \frac{9!}{2!\ 3!} =$$

Q2. In how many ways can a lady wear five rings on the fingers (not the thumb) of her right hand?

First ring has 4 positions.

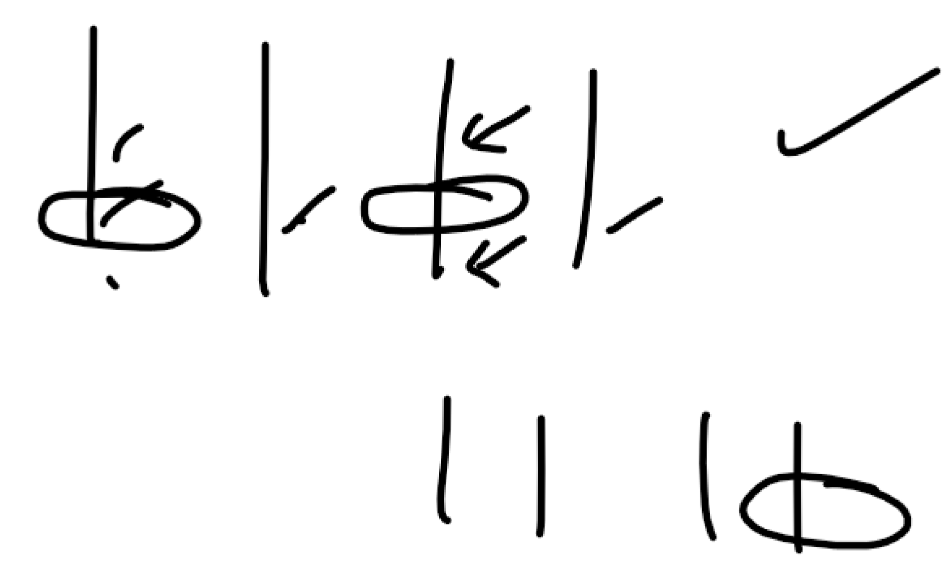
Second ring has 5 " .



3rd ring has 6 positions.

4th " " 7 "

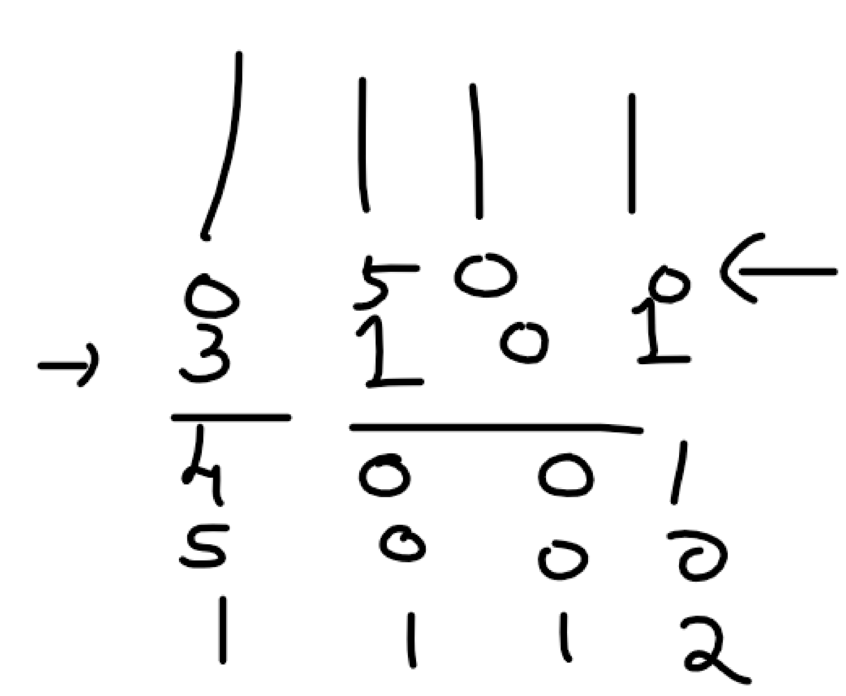
5th " " 8 "



Ans : $4 \times 5 \times 6 \times 7 \times 8 = 6720$

Or Consider all 5 rings are identical.

Distn of 5 identical rings into 4 distinct fingers s.t each finger can hold any no. of rings can be done is ${}^{4+5-1}C_5 = {}^{8}C_5$

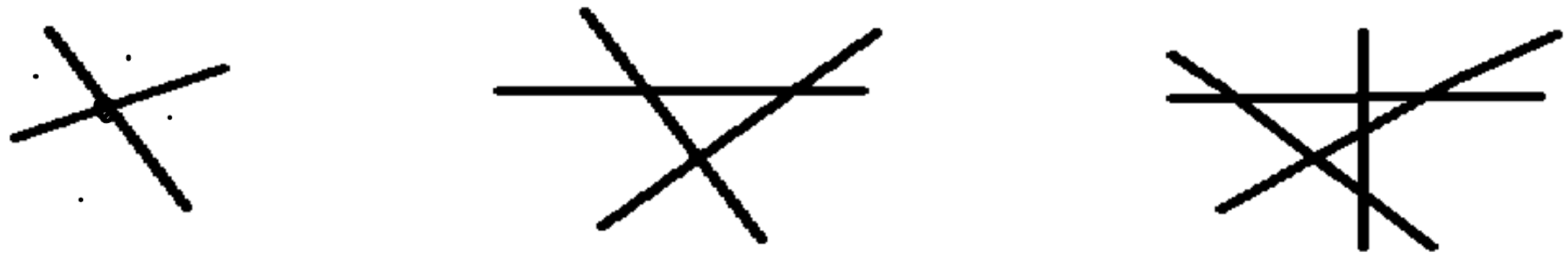


But as 5 rings can be arranged in $5!$ ways, So

Ans is ${}^{8}C_5 \times 5! = 6720$

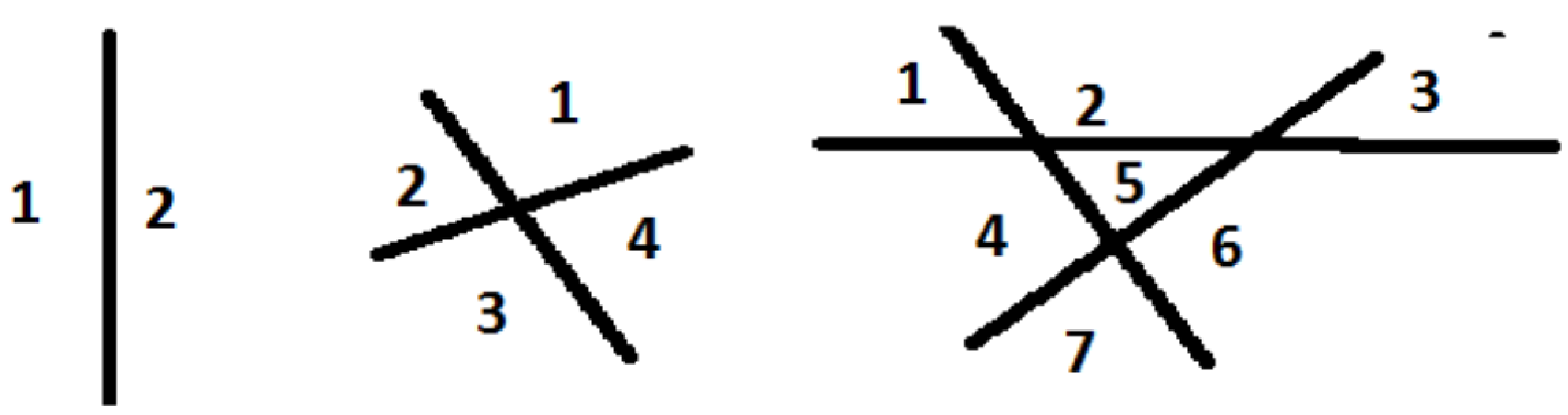
Q3. How many points of intersection are formed by n lines drawn in a plane if no two are parallel and no three concurrent? Into how many regions is the plane divided?

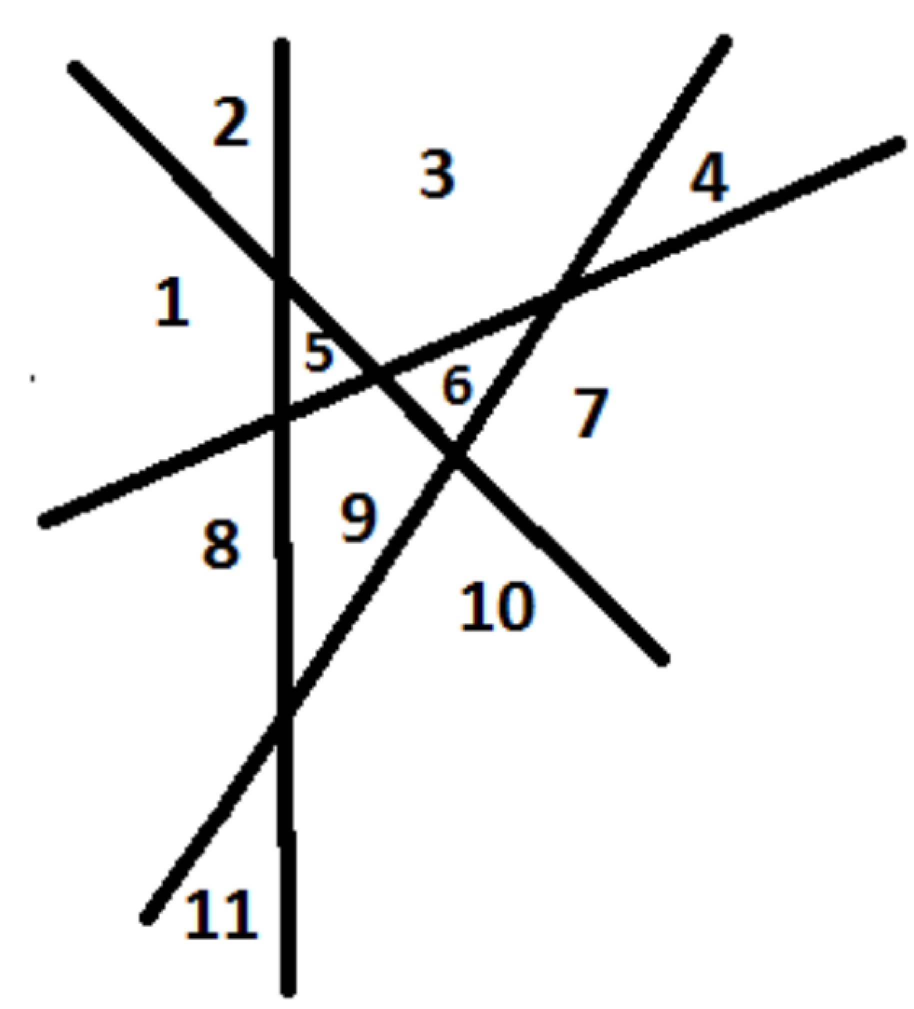
Soln :



Two lines 1 intersecting point
3 lines (1+2) intersecting points
4 lines (1+2+3) intersecting points
⋮
n lines $(1+2+3+\cdots+(n-1))$ intersecting points

Ans : $\frac{n(n-1)}{2} = {}^{n}C_2$





1 line 2 regions =intersecting points of 2 lines +1
2 line 4 regions =intersecting points of 3 lines +1
3 line 7 regions =intersecting points of 4 lines +1
4 line 11 regions =intersecting points of 5 lines+1

n line : intersecting points of (n+1) lines +1

for n line $= {}^{n+1}C_2 + 1 = \frac{n^2+n+2}{2}$ regions

Q4. 6 distinct symbols are transmitted through a communication channel. A total of 12 blanks are to be inserted between the symbols with at least two blank spaces between every pair of symbols. In how many ways can we arrange symbols and blanks?

11 12 23
22 13
: :
55 15 25

1 2 3 4 5

* _ _ * _ _ * _ _ * _ _ * _ _ * ←

2 blanks (identical) needs to be inserted between the 6 symbols (5 positions) allowing repetition

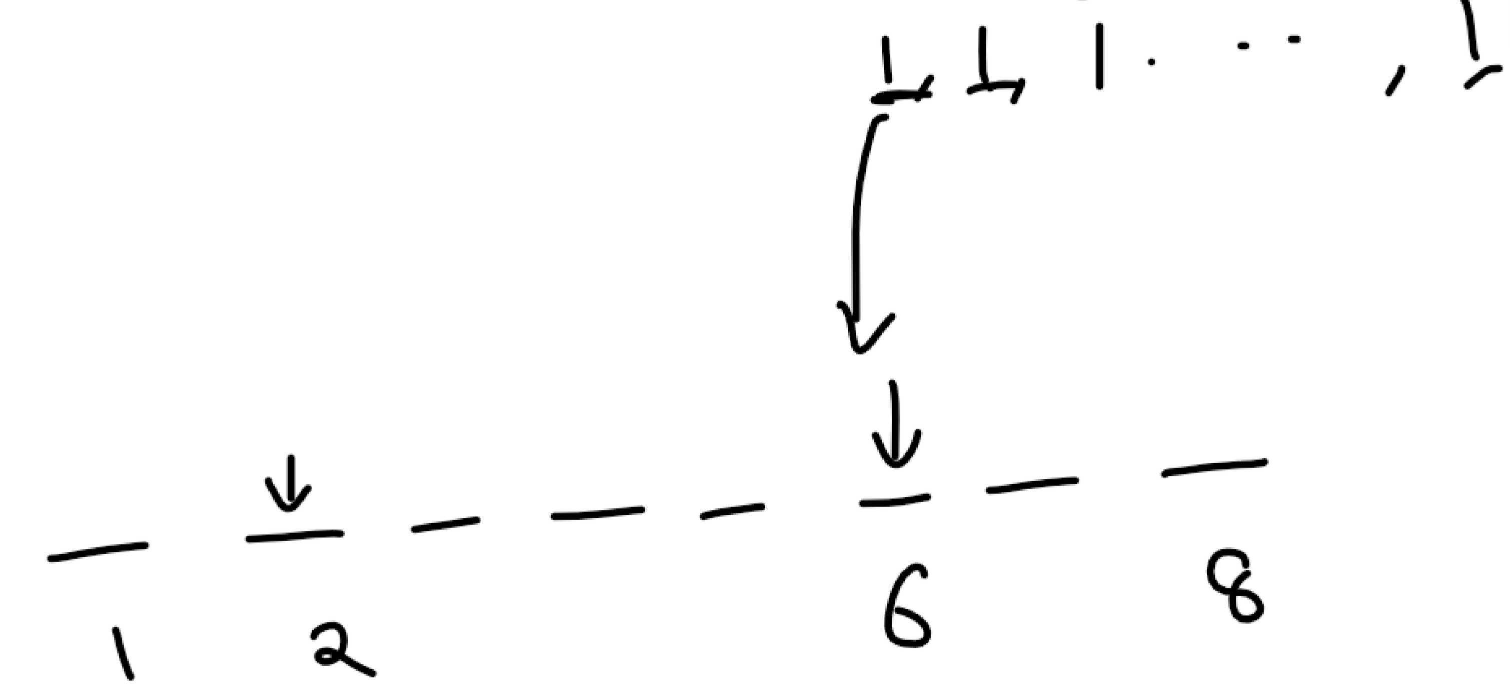
This can be done is $^{5+2-1}C_2$ ways.

As all 6 symbols are distinct, this can be done is $6C_2 \times 6!$ ways.

Q5. In how many ways can an examiner assign 30 Marks to 8 questions such that no question recieves less than 2 marks?

Soln : Initially we give 2 marks each to all 8 questions.

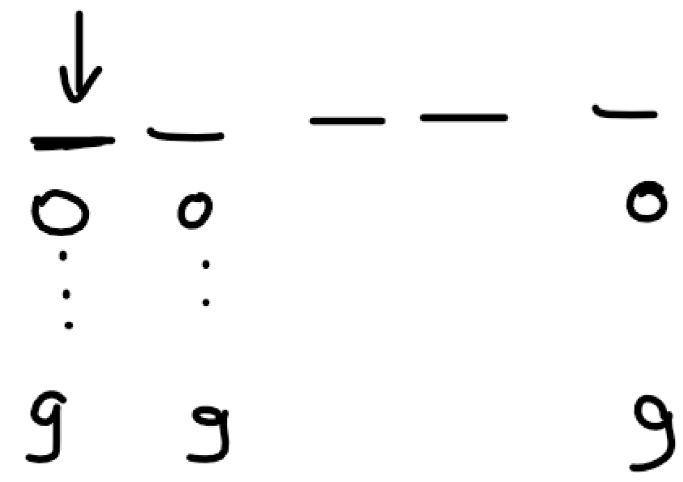
Now $\underline{14}$ remaining marks (identical) to be given to 8 question allowing repetition.



$r = 14$
$n = 8$

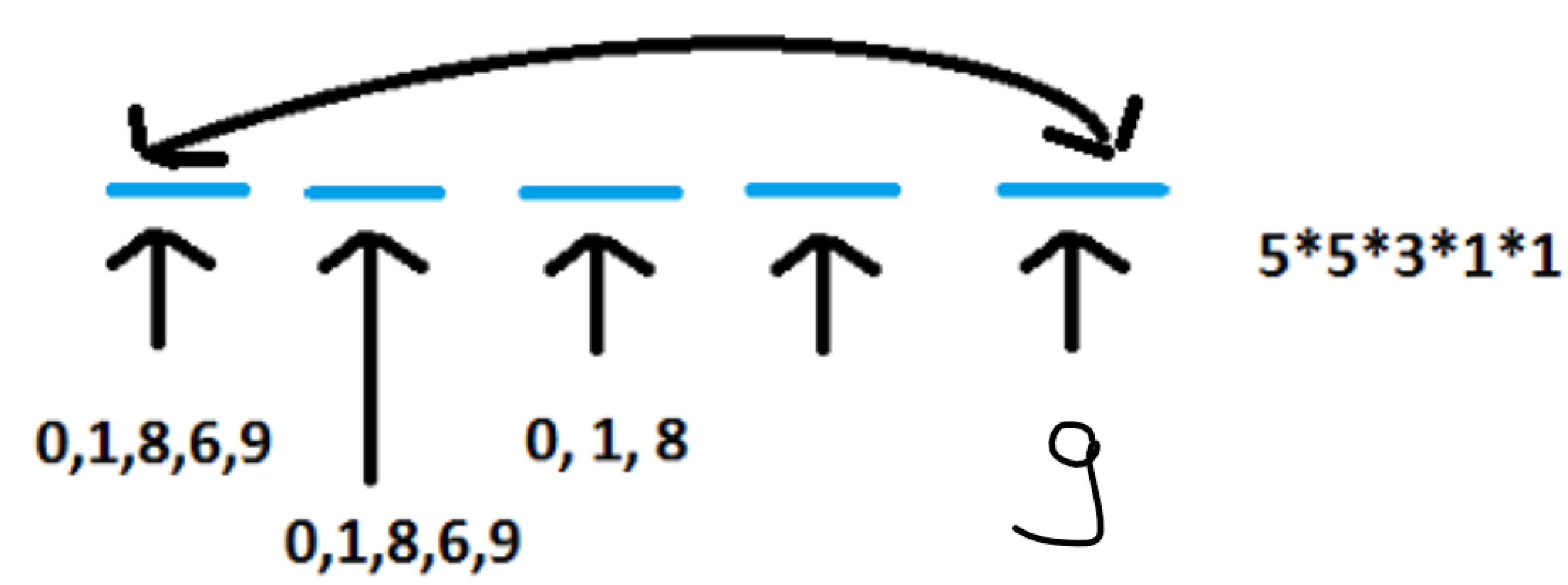
Req Ans is $^{8+14-1}C_{14} = \underline{\underline{^{21}C_{14}}}$

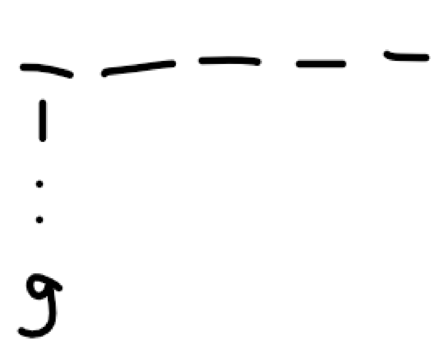
Q5. Suppose we print all FIVE-digit numbers on slips of paper with one number on each slip. However, since the digits 0, 1, 6, 8, and 9 become 0, 1, 9, 8, and 6 when they are read right side up or upside down, there are pairs of numbers that can share the same slip if the slips are read right side up or upside down. For example, we can make up one slip for the numbers 89166 and 99168. The question is then how many distinct slips will we have to make up for all five-digit numbers?



There are 10^5 distinct 5 digit numbers.
Among these numbers, 5^5 of them can be read either right side up or upside down

Eg: 16091, 80108, 61819
There are some numbers made up of digits 0, 1, 6, 8, 9 can read same either right side up or upside down.





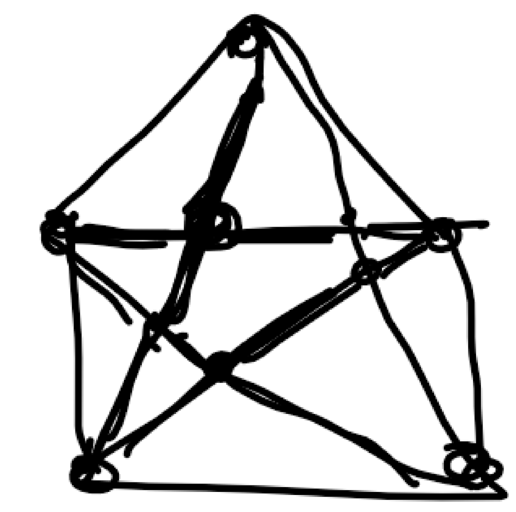
There are 3×5^2 numbers (center digit must be 0, 1, 8, and 5^{th} digit must be on 1^{st} digit turned upside down & so on)

$\Rightarrow$ $5^5 - 3\times5^2$ numbers that can be read either right side up or upside down but will read differently. $\therefore$ Ans $10^5 - \left(\frac{5^5-3\times5^2}{2}\right) =$

Q6. If no three diagonals of a convex decagon meet at the same point inside the ~~pentagon (or~~ decagon), into how many line segments are the diagonals divided by their intersections?

There are $^{10}C_2 - 10 = 35$ diagonals

There are $\underline{^{10}C_4} = 210$ intersecting points.

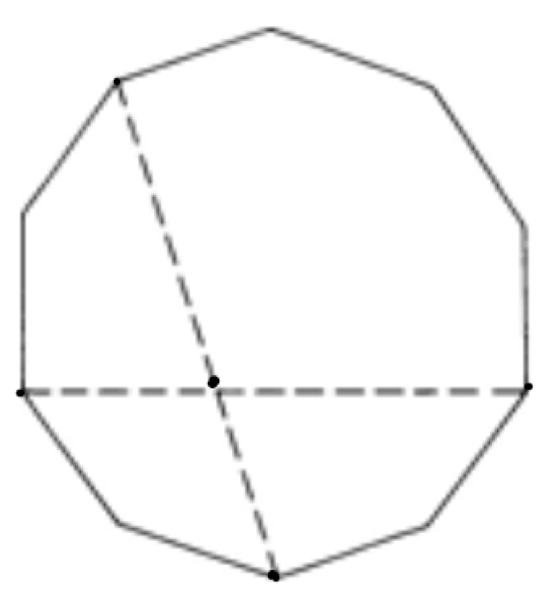


$^5C_2 - 5$

$^{10}C_2 - 10 = 5$

Ans : 15

$2 \times 5 + 5 = \underline{\underline{15}}$



One intersecting point lies on 2 diagonals

suppose in a diagonal if we have k intersecting points, then it will have (k+1) line segments.

Total number of linesegments are

$2 \times 210 + 35 = \underline{\underline{\qquad}}$

In Hexagon ______ line segments?

Q7. In how many ways can 5 different messages be delivered by 3 messengers (A,B,C) if no messenger is left unemployed. The order in which a messenger delivers his messages is immaterial?

Soln : A can deliver 3 msges , B - 1 and C-1

or A can deliver 1 msges, B-3 , C-1 or . . . .

$\Rightarrow (3,1,1)\ (1,3,1)\ (1,1,3)\ (2\ 2\ 1)\ (2\ 1\ 2)\ (1\ 2\ 2)$

$$\Rightarrow 3\left(\frac{5!}{3!} + \frac{5!}{2!\,2!}\right) = 150 \text{ ways.}$$

Extra Questions:

Q1. Find the sum of all 4 digit numbers which are formed by the digits 1,2,5,6? Ans: 93324

Q2. Find the sum of all 4 digit numbers which are formed by the digits 0,1,2,3 with and without repetition of digits?

Ans:6(1+2+3)1000+4[(1+2+3)100+(1+2+3)10+(1+2+3)]=38664.

64((1+2+3)1000+48[(1+2+3)100+(1+2+3)10+(1+2+3)]

Q3. Out of 5 mathematicians and 7 physicists a committee consisting of 2 mathematician and 3 physicists has to be formed. In how many ways it could be done of

(i) There is no restriction

(ii) 1 particular physicists must be in the committee

(iii) 2 particular mathematicians cannot be in a committee.

ANS: (i) $(5C2)(7C3)$ (ii) $(5C2)(6C2)$ (iii) $(3C2)(7C3)$

Q4. In how many ways can the letters a,b,c,d,e,f be arranged so that b is always to the immediate left of the letter e. Ans: 5!

Q5. How many odd numbers between 100 and 999 have distinct digits? Ans: 320

Q6. If repetition is not allowed, how many 4 digit numbers can be formed from the 6 digits 1,2,3,5,7,8,

i. How many of the numbers are lesser than 4000

ii. How many are even

iii. How many are odd

iv. How many are multiple of 5

v. How many contain both the digits 5 and 3

ANS: 360 (i) 180 ii. 120 iii. 240 iv. 60 v. 288

Q7. How many 7 letter palindromes can be made out of the English alphabets? Ans: 26^4=