STATICALLY INDETERMINATE MEMBERS

LECTURE 22

- Introduction
- Compound bars subjected to external loads
- Illustrative Example

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STATICALLY INDETERMINATE MEMBERS

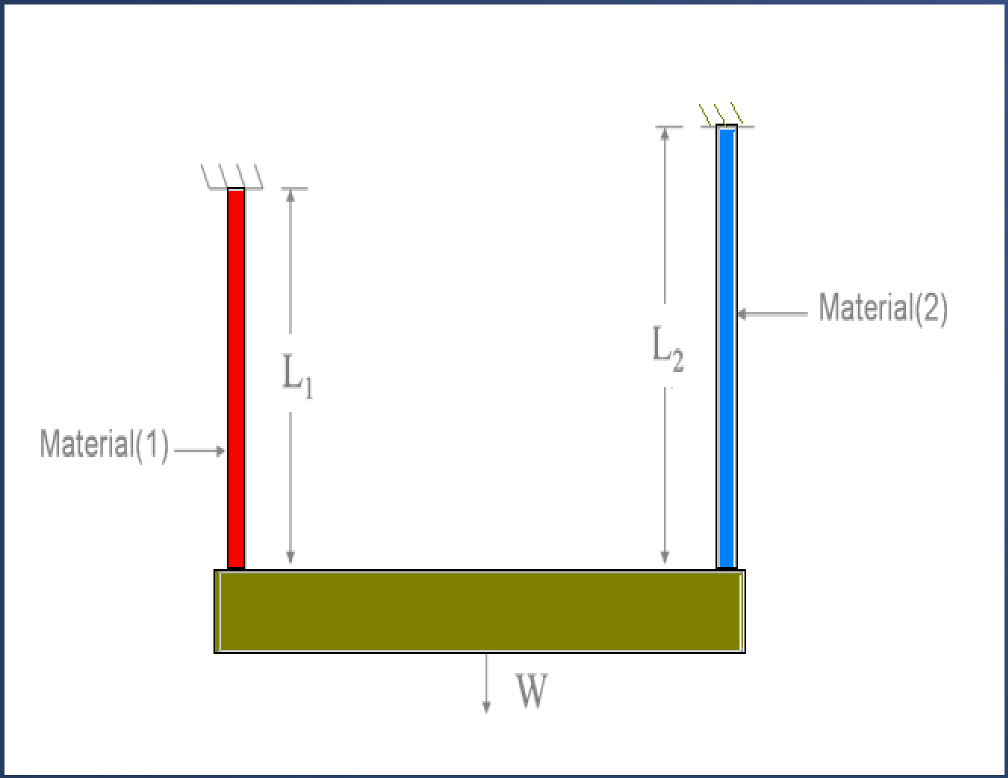
Introduction

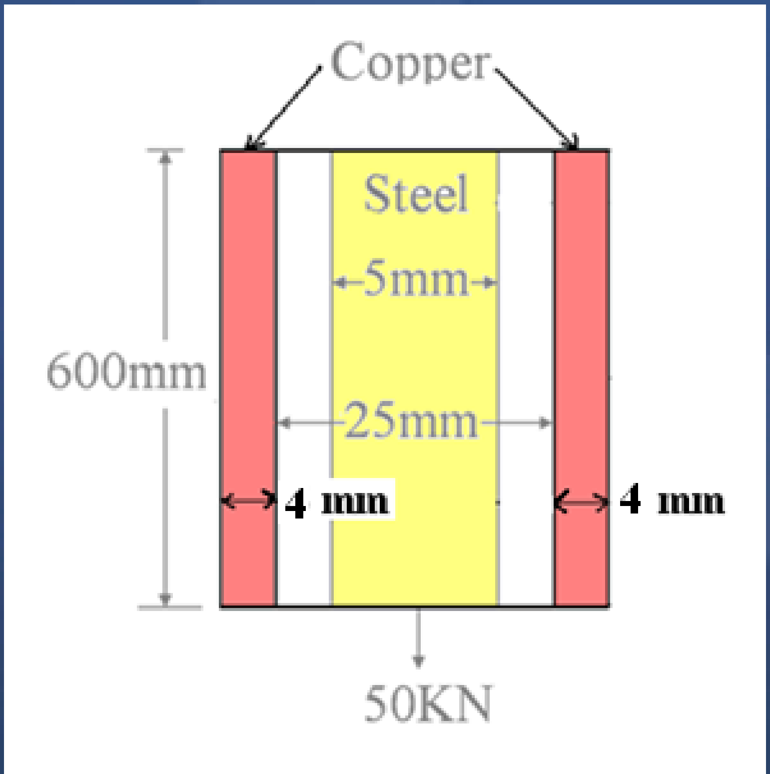
- Structure for which **equilibrium** equations are **sufficient** to obtain the solution are classified as statically **determinate**.

- But for some **combination** of members subjected to axial loads, the solution **cannot** be obtained by merely using equilibrium equations.

- The structural problems with number of **unknowns greater** than the number independent equilibrium **equations** are called **statically indeterminate**.

COMPOUND / COMPOSITE BAR





A compound bar is one which is made of **two** or **more** than two materials rigidly **connected**, so that they sustain **together** an externally applied **load** sharing the load in **correspondence** with their **modulus** of **elasticity** and cross sectional **area.**

Compound Bar – Meaning

Assumptions

Equations to be used

1) **Equilibrium Equation:** based on free body diagram of the structure or part of the structure.

i.e Applied load is equal to sum of the loads carried by each member

$$W = \sigma_1 A_1 + \sigma_2 A_2 \quad \text{——— (1)}$$

2) **Compatibility Equation:** based on geometric relations regarding elastic deformations, produced by the loads.

i.e Change in length in all the materials are same or proportional, depending on the condition

$$(dL)_1 = (dL)_2$$

$$(\sigma_1 / E_1)L_1 = (\sigma_2 / E_2)L_2$$

$$\sigma_1 = \sigma_2 \times (E_1/E_2)(L_2/L_1) \quad \text{—— (2)}$$

E_1/E_2 is called modular ratio

From Equation (1) & (2) σ_1 and σ_2 can be calculated

Steps to be followed

1. Write the given data

2. Apply compatibility condition
   a. Express **σ** of one material in terms of the other

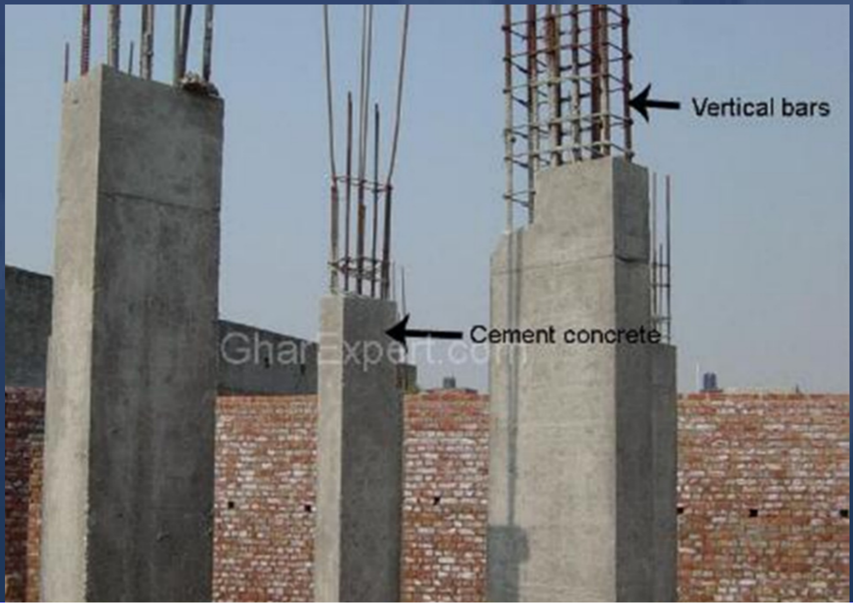
3. Apply equilibrium condition
   a. Substitute **σ** values

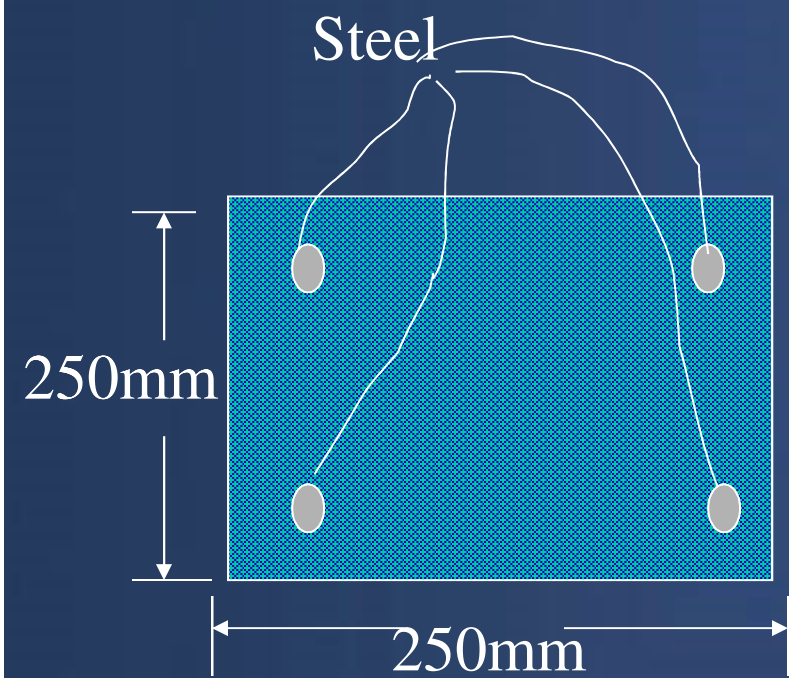
4. Solve the unknowns

N1. A load of 300 kN is supported by a short concrete column 250 mm square. The column is strengthened by 4 steel bars at the corners with total c/s area of 4800 mm^2. If E_s=15E_c,

a) Find the stress in steel and concrete.

b) If the stress in concrete is not to exceed 4 MPa, find the area of steel required so that the column can support a load of 600 kN.





Case(i)

$A_s = 4800 mm^2$

$A_c = (250 \times 250) - 4800$

$= 57{,}700\ mm^2$

$W = \sigma_s A_s + \sigma_c A_c$

$300 \times 10^3 = 15\,\sigma_c \times 4800 + \sigma_c \times 57{,}700$

$\sigma_c = 2.31\ N/mm^2$

$\sigma_s = 15\sigma_c$

$= 15 \times 2.31$

$= 34.69 N/mm^2$

Deformation is same

$(dL)_s = (dL)_c$

$(\sigma_s / E_s) \times L_s = (\sigma_c / E_c) \times L_c$

$\sigma_s / 15E_c = \sigma_c / E_c$

$\boxed{\sigma_s = 15\sigma_c}$ ——— (1)

Case (ii)

$W = \sigma_s A_s + \sigma_c A_c$

$600 \times 10^3 = 15\, \sigma_c \times A_s + \sigma_c A_c$

$600 \times 10^3 = (15 \times 4)\, A_s + 4\,(250 \times 250 - A_s)$

$A_s = 6250\ mm^2$

Summary

LECTURE 23

NUMERICAL PROBLEMS

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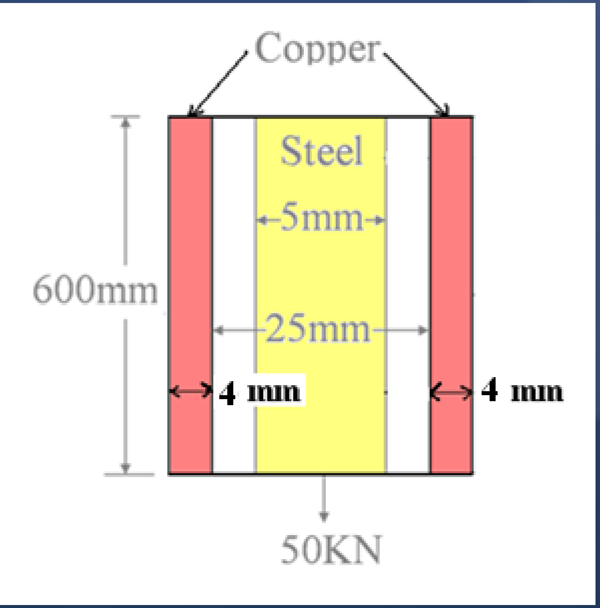
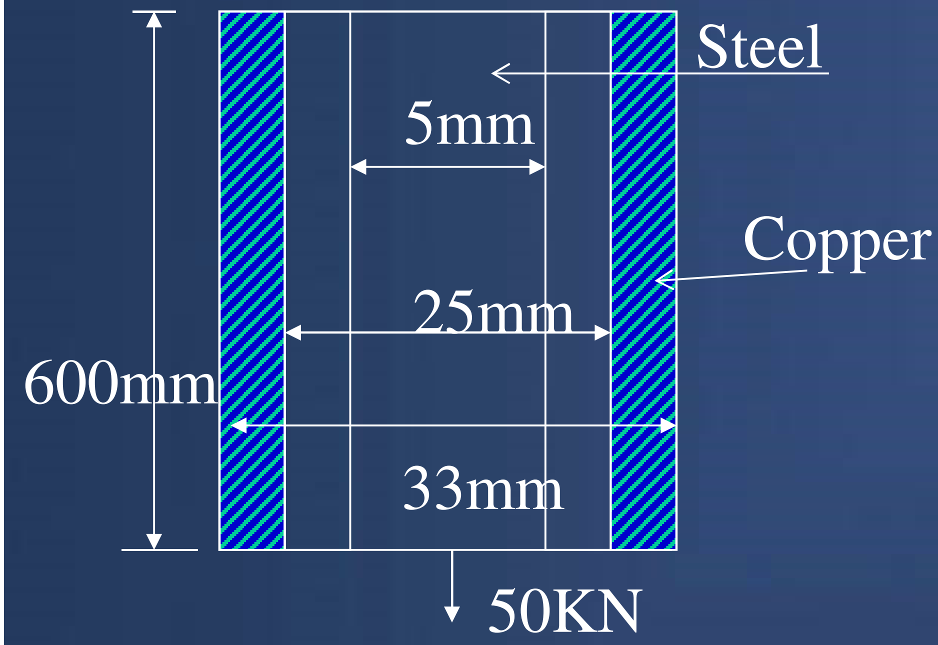
N2. A mild steel rod 5 mm diameter passes centrally through a copper tube of internal diameter 25 mm and thickness 4mm. The composite section is 600 mm long and their ends are rigidly connected. It is then acted upon by an axial tensile load of 50 kN. Find the stresses & deformation in steel and copper. Take E_c = 100 GPa, E_s = 200 GPa





Since deformation are same

$(dL)_s = (dL)_{cu}$

$(\sigma_s / E_s) \times L_s = (\sigma_{cu} / E_{cu}) \times L_{cu}$

$\sigma_s / (200 \times 10^3) = \sigma_{cu} / (100 \times 10^3)$

$\sigma_s = 2\,\sigma_{cu}$

$W = \sigma_s A_s + \sigma_{cu} A_{cu}$

$50 \times 10^3 = 2\sigma_{cu} \times (\pi/4)\,(5)^2 + \sigma_{cu} \times \pi/4\,[(33)^2 - (25)^2]$

$\sigma_{cu} = 123.86\,N/mm^2$

$\sigma_s = 247.72\,N/mm^2$

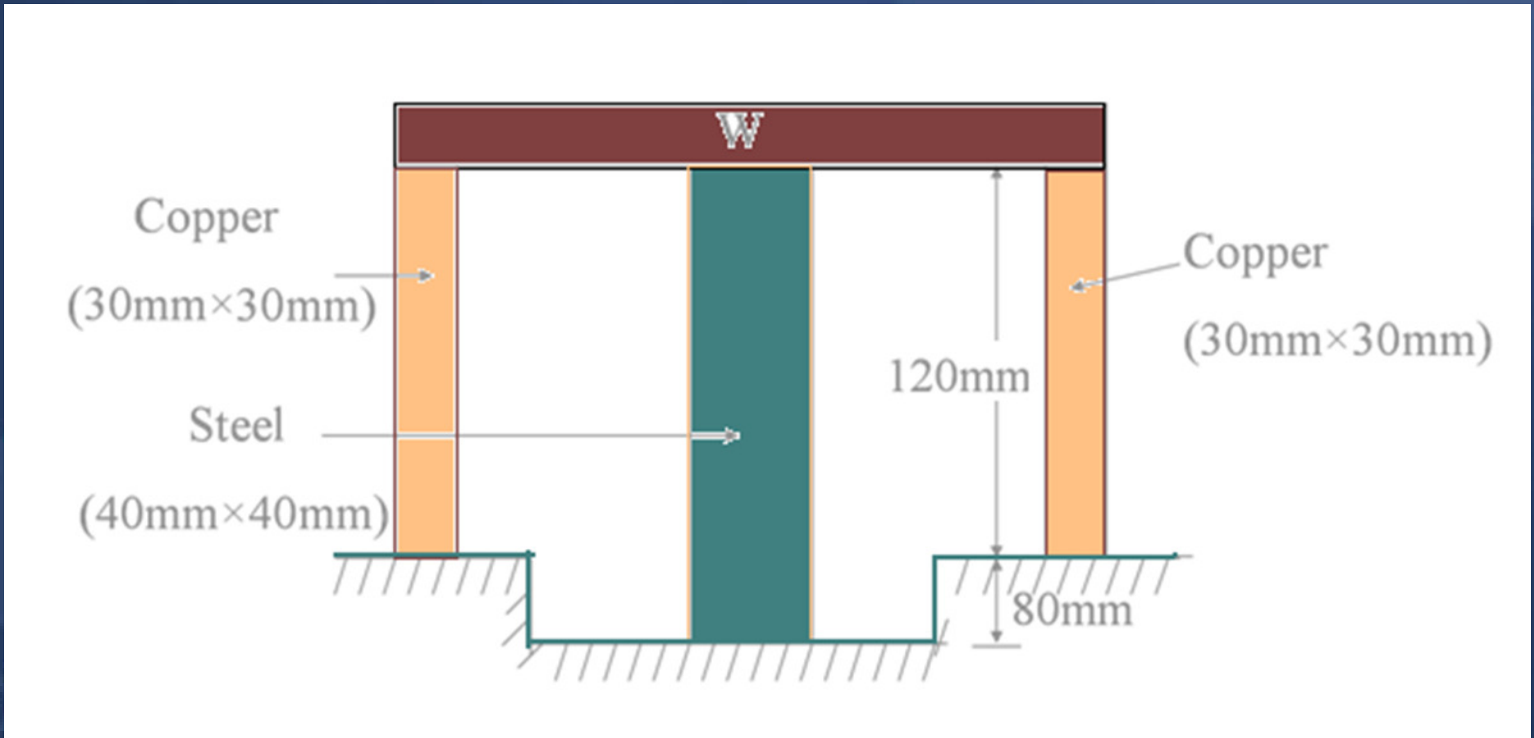
$(dL)_s = (\sigma_s / E_s) \times L_s$

$(dL)_s = [247.72/(200 \times 10^3)] \times 600$

$= 0.74mm$

$= (dL)_{cu}$

N4. Two copper rods and one steel rod together supports a rigid block of unknown weight W as shown in figure. The stress in copper and steel are not to exceed 60 MPa and 120 MPa respectively. Find the safe load that can be supported. Take $E_s = 2E_c$



Solution:

Deformations are same

$(dL)_s = (dL)_{cu}$

$(\sigma_s / E_s) \times L_s = (\sigma_{cu} / E_{cu}) \times L_{cu}$

$(\sigma_s / 2E_{cu}) \times 200 = (\sigma_{cu} / E_{cu}) \times 120$

$$\boxed{\sigma_s = 1.2\,\sigma_{cu}}$$

Let σ_{cu}=60MPa=60N/mm^2, σ_s=1.2x60 = 72N/mm^2 < 120N/mm^2 (safe)

Safe load = W = $\sigma_s \times A_s + 2(\sigma_{cu} \times A_{cu})$

$= 72(40 \times 40) + 2 \times [60 \times (30 \times 30)]$

Safe load = W = 223.2 × 10^3 N = 223.2 kN

TUTORIAL 11

NUMERICAL PROBLEMS

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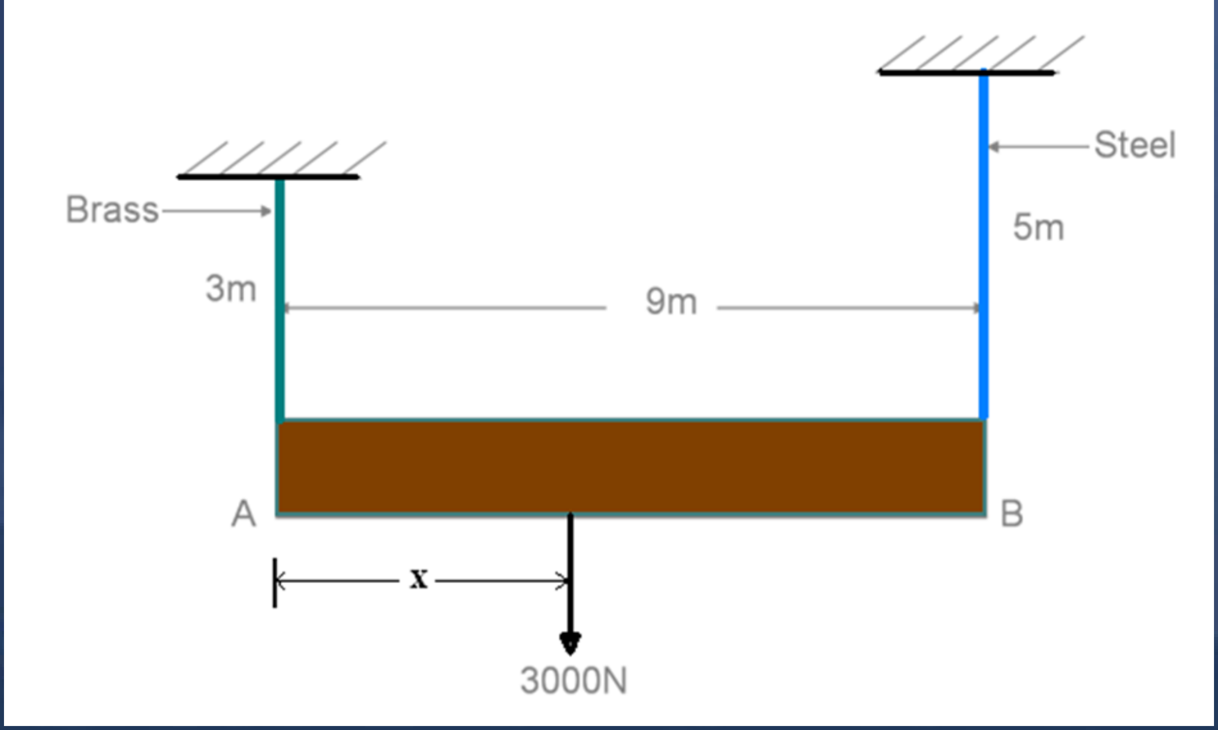
T1. A rigid bar AB 9m long is suspended by two vertical rods at its end A and B and hangs in horizontal position by its own weight. The rod at A is brass, 3m long, 1000 mm^2 c/s area and $E_b = 10^5$ N/mm^2. The rod at B is steel, length 5m, 445 mm^2 c/s area and E_s = 200 GPa. At what distance x from A, if a vertical load P = 3000 N may be applied for the rigid bar to remain horizontal.



Deformations are same

$(dL)_b = (dL)_s$

$(\sigma_b / E_b) \times L_b = (\sigma_s / E_s) \times L_s$

$(\sigma_b / 10^5) \times (3 \times 10^3) = [\sigma_s / (200 \times 10^3)] \times [5 \times 10^3]$

$$\boxed{\sigma_s = 1.2\ \sigma_b}$$

$W = \sigma_s A_s + \sigma_b A_b$

$3000 = (1.2\ \sigma_b \times 445) + (\sigma_b \times 1000)$

$$\boxed{\sigma_b = 1.95\text{N/mm}^2} \quad \boxed{\sigma_s = 2.34\text{N/mm}^2}$$

+ve $\quad \Sigma M_A = 0$

$-(3000)(x) + (2.34 \times 445) \times 9000 = 0$

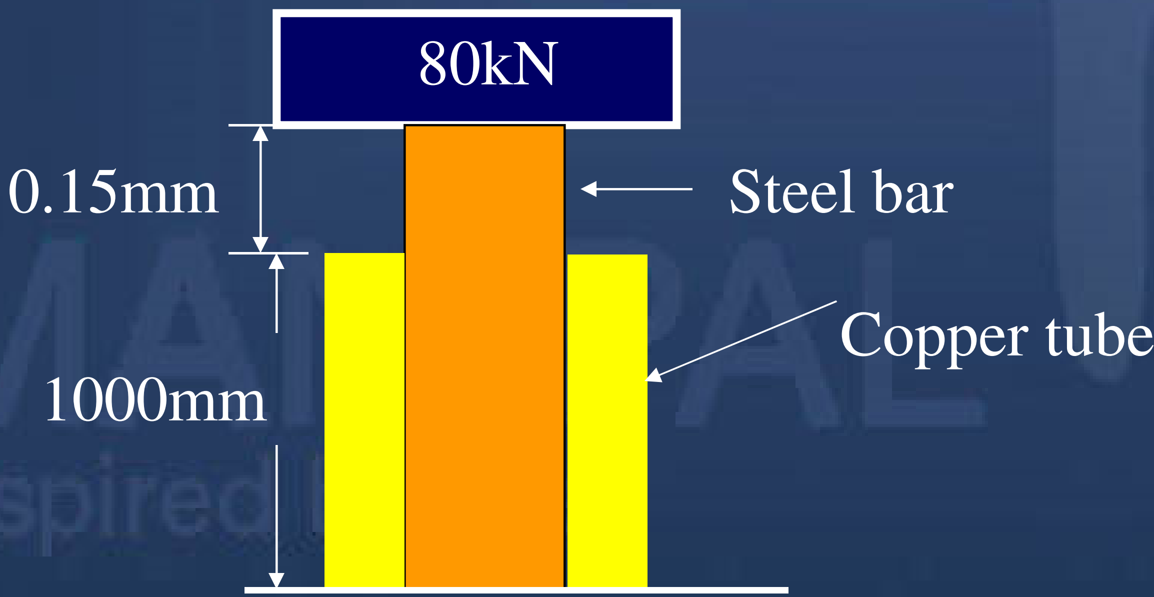
$x = 3123.9$ mm

$= 3.12$m from A

If the load of 3000N is kept at a distance of 3.12m from A, bar AB will remain horizontal.

T2. A mild steel bar of c/s 490 mm^2 is surrounded by a copper tube of c/s 210 mm^2 as shown. When they are placed centrally over a rigid bar, it is found that steel bar is 0.15 mm longer. Over this unit a rigid plate carrying a load of 80 kN is placed. Find the stress in each material.

Take E_s = 200 GPa, E_c = 100 GPa.



Solution:

$(dL)_s = (dL)_{cu} + 0.15$

$(\sigma_s / E_s) \times L_s = (\sigma_{cu} / E_{cu}) \times L_{cu} + 0.15$

$[\sigma_s / (200 \times 10^3)] \times 1000.15 = \{[\sigma_{cu} / (100 \times 10^3)] \times 1000\} + 0.15$

$\sigma_s = 2\,\sigma_{cu} + 30$

$W = \sigma_s \times A_s + \sigma_{cu} \times A_{cu}$

$80 \times 10^3 = [(2\sigma_{cu} + 30) \times 490] + (\sigma_{cu} \times 210)$

$\sigma_{cu} = 54.87 N/mm^2$

$\sigma_s = (2 \times 54.87) + 30$

$= 139.84 N/mm^2$

LECTURE 24

- Temperature stress
- Compound bars subjected to temperature stresses
- Illustrative problems

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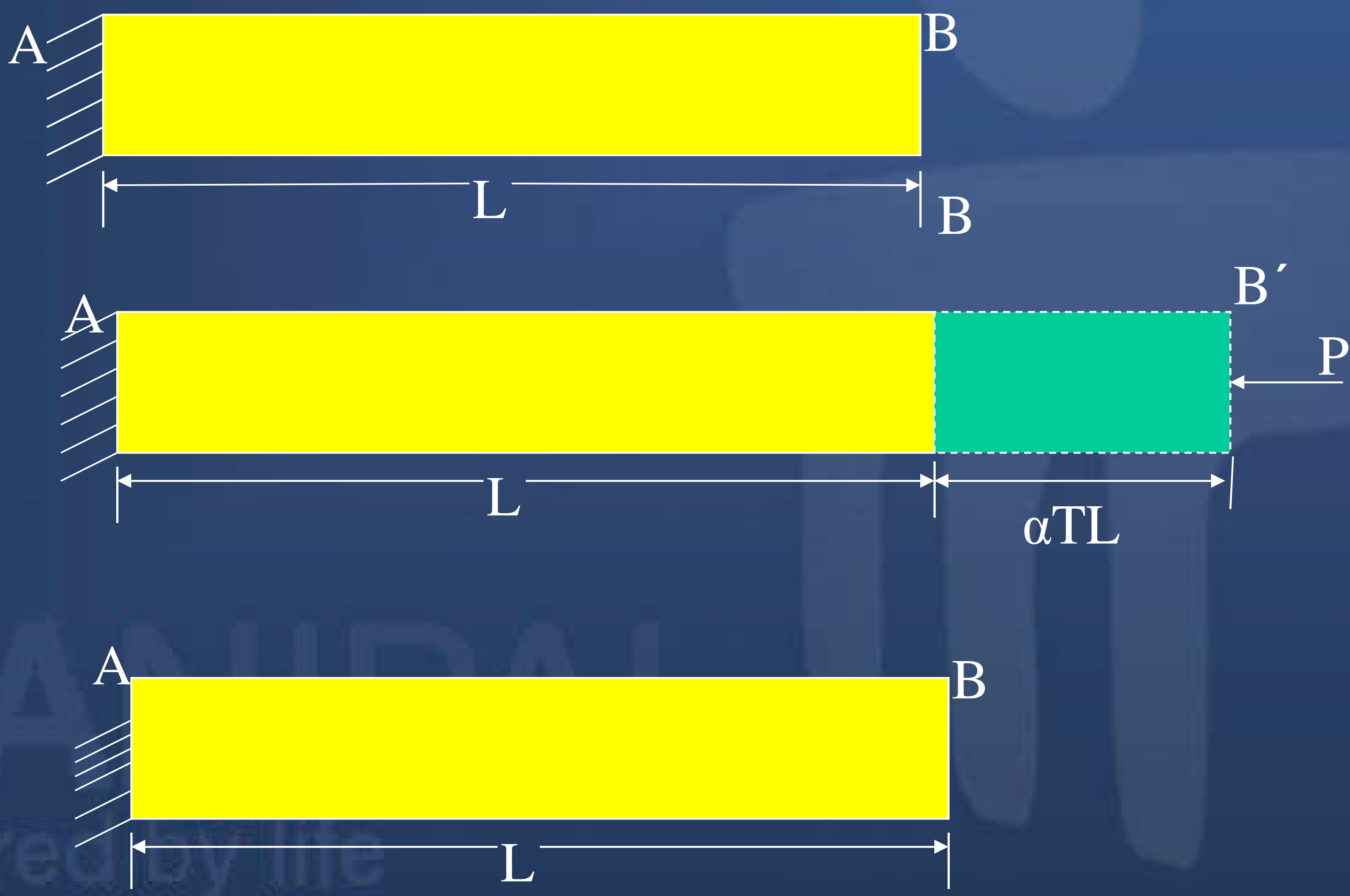
Temperature Stress



From the above figure it is seen that 'B' shifts to B' by an amount 'αTL'. If this expansion is to be prevented a compressive force is required at B'.

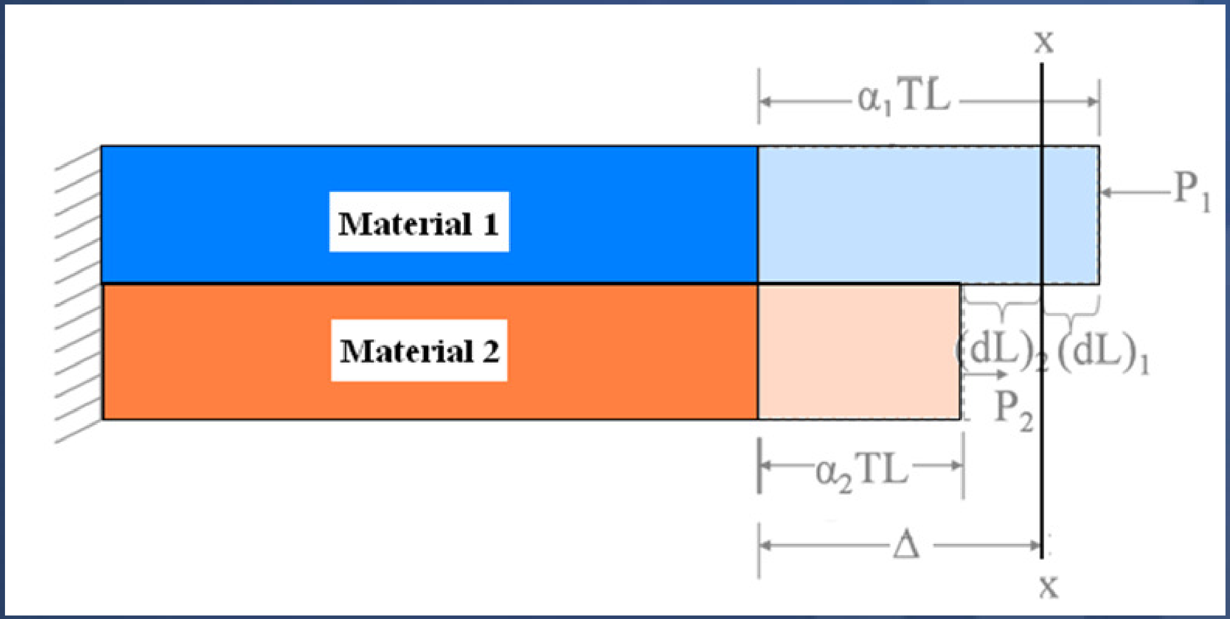
$$\text{Temperature strain} = \alpha TL/(L + \alpha TL) \approx \alpha TL/L = \alpha T$$

$$\text{Temperature stress} = \alpha TE$$

Hence the temperature strain is the ratio of expansion or contraction prevented to its original length.

If a gap δ is provided for expansion then
Temperature strain = $(\alpha TL - \boldsymbol{\delta}) / L$
Temperature stress = $[(\alpha TL - \delta)/L]\, E$

Temperature stress in compound bars:-



When a compound bar is subjected to change in temperature, both the materials will experience stresses of opposite nature.

Compressive force on material (1) = tensile force on material (2)

$\sigma_1 A_1 = \sigma_2 A_2$ (there is no external load)

$$\boxed{\sigma_1 = (\sigma_2 A_2)/A_1} \longleftarrow (1)$$

As the two bars are connected together, the actual position of the bars will be at XX.

Actual expansion in material (1) = actual expansion in material (2)

$$\alpha_1 TL - (dL)_1 = \alpha_2 TL + (dL)_2$$

$$\alpha_1 TL - (\sigma_1 / E_1) L = \alpha_2 TL + (\sigma_2 / E_2) L$$

$$\alpha_1 T - (\sigma_1 / E_1) = \alpha_2 T + \sigma_2 / E_2 \text{ -------------------------(2)}$$

From (1) and (2) magnitude of σ_1 and σ_2 can be found out.

N5. A steel rail 30 m long is at a temperature of 24°C. Estimate the elongation when temperature increases to 44°C.

1) Calculate the thermal stress in the rail under the following two conditions :

   (i) No expansion gap provided

   (ii) A 6 mm gap is provided for expansion

2) If the stress developed is 60 MPa , what is the gap left between the rails?

Take E = 200 GPa, $\alpha = 18 \times 10^{-6}$ /°C

Free expansion $\alpha TL = 18 \times 10^{-6} \times (44\text{-}24) \times 30 \times 10^{3} = 10.8\text{mm}$

i) No expansion gap provided:-

Temperature stress = αTE

$= 18 \times 10^{-6} \times 20 \times 200 \times 10^{3}$

$= 72\text{N/mm}^2$

ii) 6mm gap is provided for expansion

temperature stress $= [(\alpha TL - \delta) / L]\, E$

$= [(10.8 - 6)/(30 \times 10^{3})] \times 200 \times 10^{3}$

$= 32\text{N/mm}^2$

when stress = 60MPa

temperature stress $= [(\alpha TL - \delta) / L]\, E$

$= [(10.8 - \delta) / (30 \times 10^{3})] \times 200 \times 10^{3}$

$\boxed{\delta = 1.8\text{mm}}$

Summary

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TUTORIAL 12

- Numerical problems

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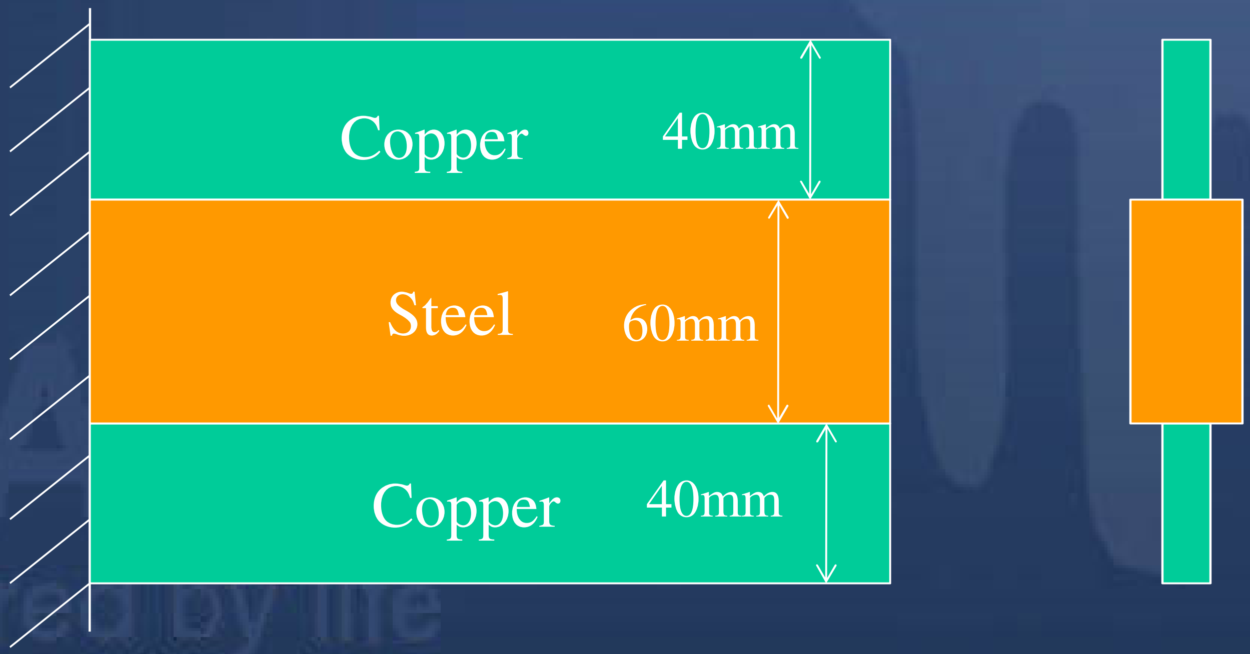
T3. A steel bar is placed between two copper bars. The steel bar and copper bars have c/s 60 mm × 10 mm and 40 mm × 5 mm respectively, and are connected rigidly on each side. If the temperature is raised by 80°C, find stress in each metal and change in length. The length of bar at normal temperature is 1m. E_s = 200 GPa, E_c= 100 GPa, $\alpha_s = 12 \times 10^{-6}/\,°C$, $\alpha_c = 17 \times 10^{-6}/\,°C$



Compressive force on copper bar = tensile force on steel bar

$2\sigma_{cu} \times A_{cu} = \sigma_s \times A_s$

$2\sigma_{cu} (40 \times 5) = \sigma_s (60 \times 10)$

$\boxed{\sigma_{cu} = 1.5\sigma_s}$

Actual expansion in copper = Actual expansion in steel

$\alpha_{cu}TL_{cu} - (dL)_{cu} = \alpha_s TL_s + (dL)_s$

$\alpha_{cu}TL_{cu} - (\sigma_{cu} / E_{cu}) L_{cu} = \alpha_s TL_s + (\sigma_s / E_s) L_s$

Since $L_{cu} = L_s$

$(17 \times 10^{-6} \times 80) - 1.5\sigma_s /(100 \times 10^3) = (12\times 10^{-6} \times 80) + \sigma_s /(200 \times 10^3)$

$\sigma_s = 20N/mm^2(T)$

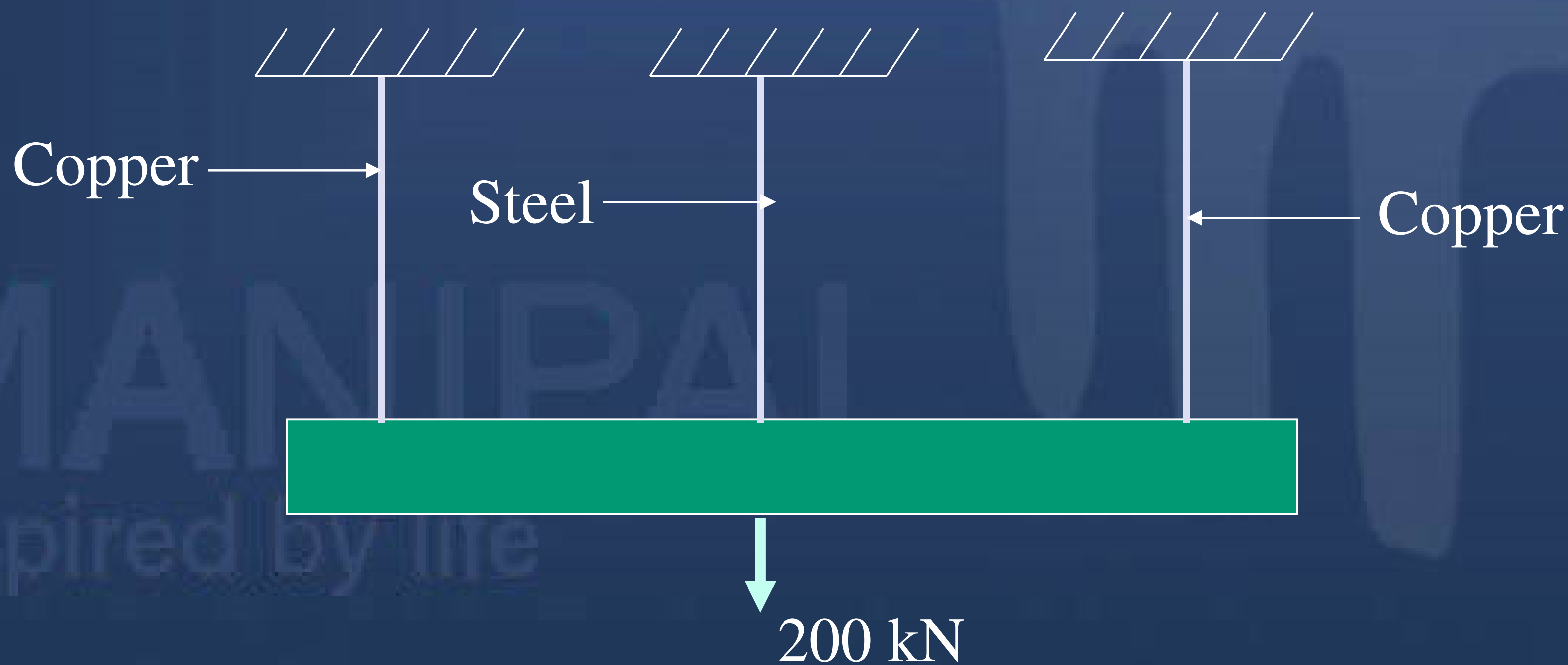
$\sigma_{cu}=1.5 \times 20 = 30N/mm^2 \, (C)$

Δ = Change in length = $\alpha_{cu} \times T \times L_{cu} - (\sigma_{cu} / E_{cu}) L_{cu}$

$= 17\times 10^{-6} \times 80 \times 1000 - [30/(100 \times 10^3)]\times 1000$

$\boxed{\Delta = 1.06mm}$

T4. A horizontal rigid bar weighing 200 kN is hung symmetrically by three vertical rods each of 1 m length and 500 mm^2 c/s symmetrically as shown. Temperature rise is 40º C. Determine the load carried by each rod and by how much the horizontal bar descends. Given E_s = 200 GPa. E_c=100 GPa. α_s =1.2 x 10^{-5}/ ºC. α_c=1.8x 10^{-5}/ ºC. What should be the temperature rise if the entire load of 200 kN is to be carried by steel alone.



$[(dL)_T + (dL)_L]_{cu} = [(dL)_T + (dL)_L]_{st}$

$[\alpha_{cu} T L_{cu} + (P_{cu} L_{cu} / A_{cu} E_{cu})] = [\alpha_s T L_s + (P_s L_s / A_s E_s)]$ -----(1)

For equilibrium condition $\sum F_v = 0$: $P_s + 2P_{cu} = 200 \times 10^3$

$P_s = 200 \times 10^3 - 2P_{cu}$

Substituting in (1)

$[1.8 \times 10^{-5} \times 40 + P_{cu} / (500 \times 100 \times 10^3)]$

$= \{ 1.2 \times 10^{-5} \times 40 + [(200 \times 10^3 - 2P_{cu}) / (500 \times 200 \times 10^3)]\}$

$\boxed{P_{cu} = 44{,}000 N}$

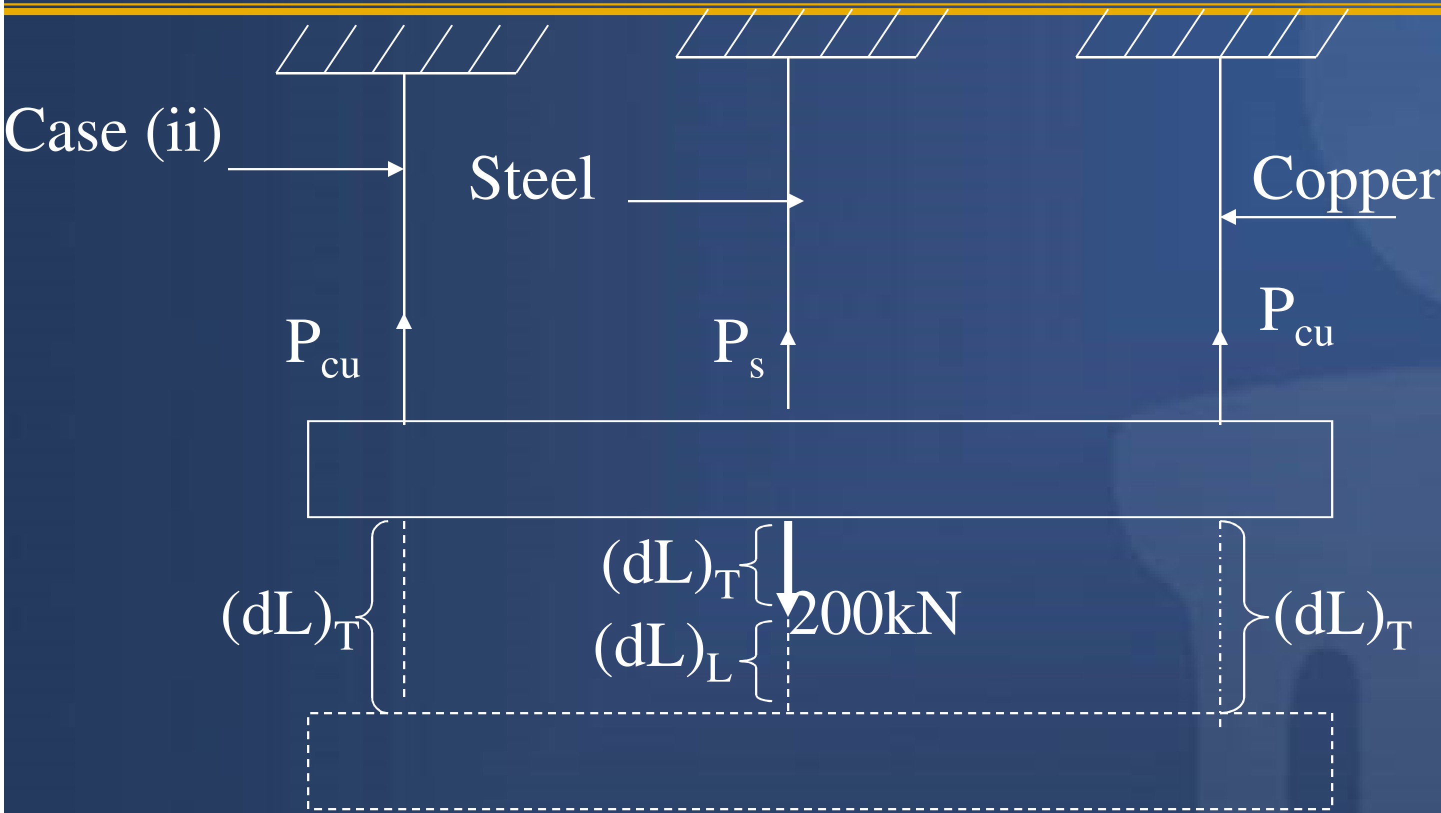
$P_s = 200 \times 10^3 - 2 \times 44{,}000$

$\boxed{P_s = 112 \times 10^3 N}$

Elongation $= \alpha_{cu} T L_{cu} + (P_{cu} \times L_{cu})/(A_{cu} \times E_{cu})$

$= 1.8 \times 10^{-5} \times 40 \times 1000 + [44000 \times 1000/(500 \times 100 \times 10^3)]$

$\boxed{dL = 1.6 mm}$



Note: When entire load is carried by steel, copper will have extension due to temp. only

$$[\,(dL)_T\,]_{cu} = [\,(dL)_L + (dL)_T]_s$$

$$\alpha_{cu} T L_{cu} = P_s L_s / A_s E_s + \alpha_s T L_s$$

$$[1.8 \times 10^{-5} \times T] = [(200 \times 10^3) / (500 \times 200 \times 10^3) + 1.2 \times 10^{-5} \times T]$$

$$T = 333.33^{\circ}C$$

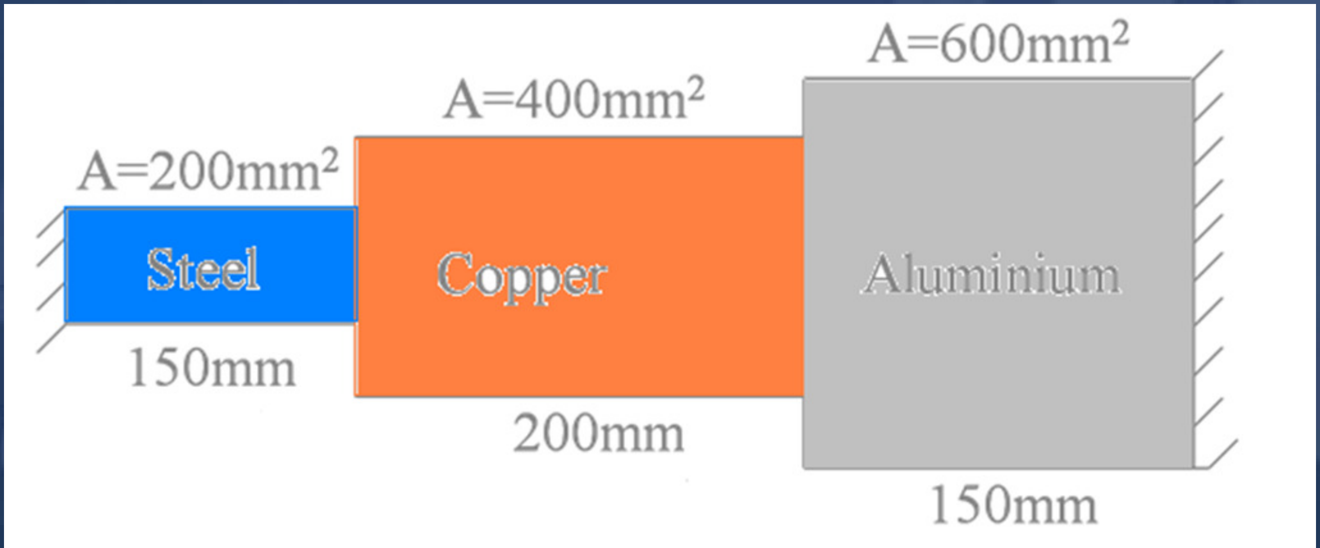
T5. A bar is composed of 3 segments as shown in figure. Find the stress developed in each material when the temperature is raised by 50˚C under two conditions

i)Supports are perfectly rigid

ii) Right hand support yields by 0.2mm

Take E_s = 200 GPa, E_c =100 GPa, E_a = 70 GPa, $\alpha_s = 12 \times 10^{-6}$/ ºC,

$\alpha_c = 18 \times 10^{-6}$/ ºC, $\alpha_a = 24 \times 10^{-6}$/ ºC.



Case(i) Supports are perfectly rigid

$(dL)_s + (dL)_{cu} + (dL)_{al} = \alpha_s TL_s + \alpha_{cu} TL_{cu} + \alpha_{al} TL_{al}$

$= (12\times 10^{-6} \times 50 \times 150) + (18 \times 10^{-6} \times 50 \times 200) + (24 \times 10^{-6} \times 50 \times 150)$

$= 0.45\text{mm}$

$(\sigma_s/E_s)\, L_s + (\sigma_{cu}/E_{cu})\, L_{cu} + (\sigma_{al}/E_{al})\, L_{al} = 0.45\text{mm}$

$[\sigma_s/(200\times 10^3)]\times 150 + [\sigma_{cu}/(100\times 10^3)]\times 200 + [\sigma_{al}/(70\times 10^3)]\times 150 = 0.45$ -(1)

From principle of compound bars

$\sigma_s A_s = \sigma_{cu} A_{cu} = \sigma_{al} A_{al}$

$\sigma_s \times 200 = \sigma_{cu} \times 400 = \sigma_{al} \times 600$

$\boxed{\sigma_s = 2\sigma_{cu}}$ $\boxed{\sigma_{al} = 0.67\sigma_{cu}}$

Substituting in (1)

$[2\sigma_{cu}/(200 \times 10^3)]\times150+[\sigma_{cu}/(100 \times 10^3)]\times200+[0.67\sigma_{cu}/(70\times10^3)]\times150=0.45$

$\sigma_{cu} = 91.27 N/mm^2$

$\sigma_s = 2\sigma_{cu} = 182.54 N/mm^2$, $\sigma_{al} = 0.67\ \sigma_{cu} = 61.15 N/mm^2$

Case (ii) Right hand support yield by 0.2mm

$(\sigma_s/E_s)\ L_s + (\sigma_{cu}/E_{cu})\ L_{cu} + (\sigma_{al}/E_{al})\ L_{al} = 0.45 – 0.2=0.25$

$[2\sigma_{cu}/(200 \times 10^3)]\times150+[\sigma_{cu}/(100 \times 10^3)]\times200+[0.67\sigma_{cu}/(70\times10^3)]\times150=0.25$

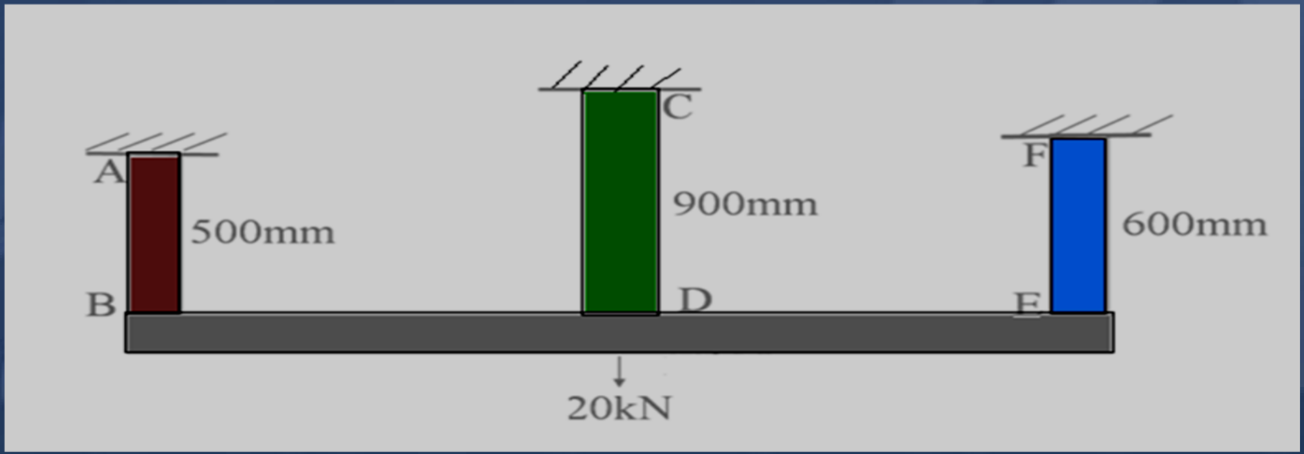
$\sigma_{cu} = 50.61 N/mm^2$

$\sigma_s = 2\sigma_{cu} = 101.22 N/mm^2$, $\sigma_{al} = 0.67\ \sigma_{cu} = 33.91 N/mm^2$

Additional Tutorial Problems

AT1. Three vertical rods AB, CD, EF are hung from rigid supports and connected at their ends by a rigid horizontal bar. The rigid bar carries a vertical load of 20 kN. Details of the bars are as follows:

- Bar AB :- L=500 mm, A=100 mm^2, E=200 GPa
- Bar CD:- L=900 mm, A=300 mm^2, E=100 GPa
- Bar EF:- L=600 mm, A=200 mm^2, E=200 GPa
- If the rigid bar remains horizontal even after loading, determine the stress and elongation in each bar.



Deformations are same

$(dL)_{AB} = (dL)_{CD} = (dL)_{EF}$

$(\sigma_{AB} / E_{AB}) \times L_{AB} = (\sigma_{CD} / E_{CD}) \times L_{CD} = (\sigma_{EF} / E_{EF}) \times L_{EF}$

$[\sigma_{AB}/(200 \times 10^3)] \times 500 = [\sigma_{CD} /(100 \times 10^3)] \times 900 = [\sigma_{EF} /(200 \times 10^3)] \times 600$

$$\sigma_{AB} = 3.6 \times \sigma_{CD},\ \sigma_{EF} = 3 \times \sigma_{CD}$$

$$W = (\sigma_{AB} \times A_{AB}) + (\sigma_{CD} \times A_{CD}) + (\sigma_{EF} \times A_{EF})$$

$$20 \times 10^3 = (3.6 \times \sigma_{CD} \times 100) + (\sigma_{CD} \times 300) + (3\sigma_{CD} \times 200)$$

$$\boldsymbol{\sigma_{CD} = 15.87 N/mm^2}$$

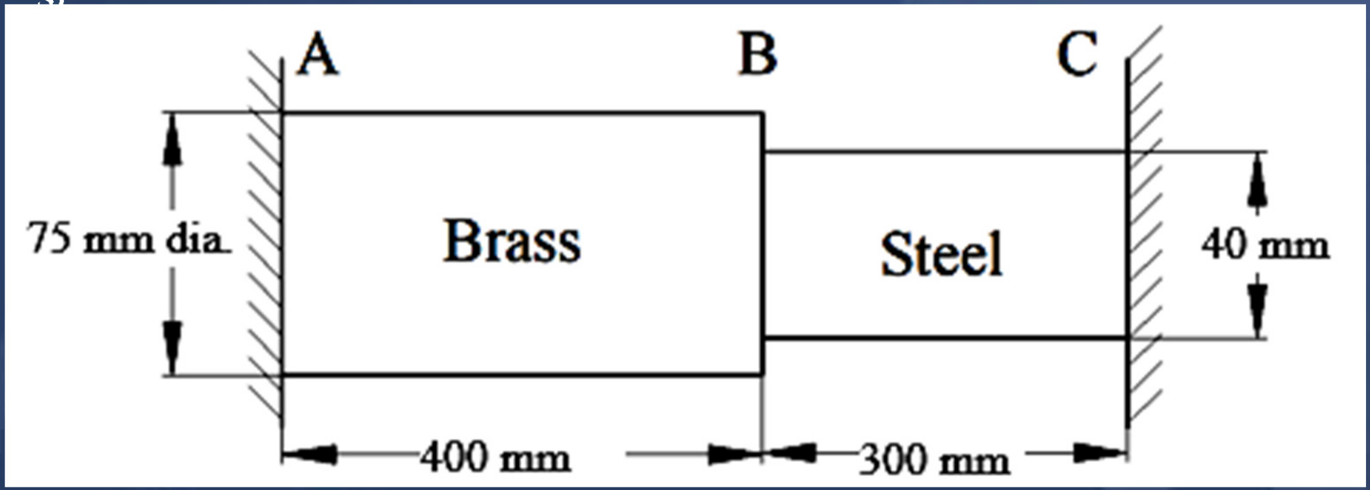
$$\boldsymbol{\sigma_{AB} = 3.6 \times 15.87 = 57.14 N/mm^2}$$

$$\boldsymbol{\sigma_{EF} = 3 \times 15.87 = 47.61 N/mm^2}$$

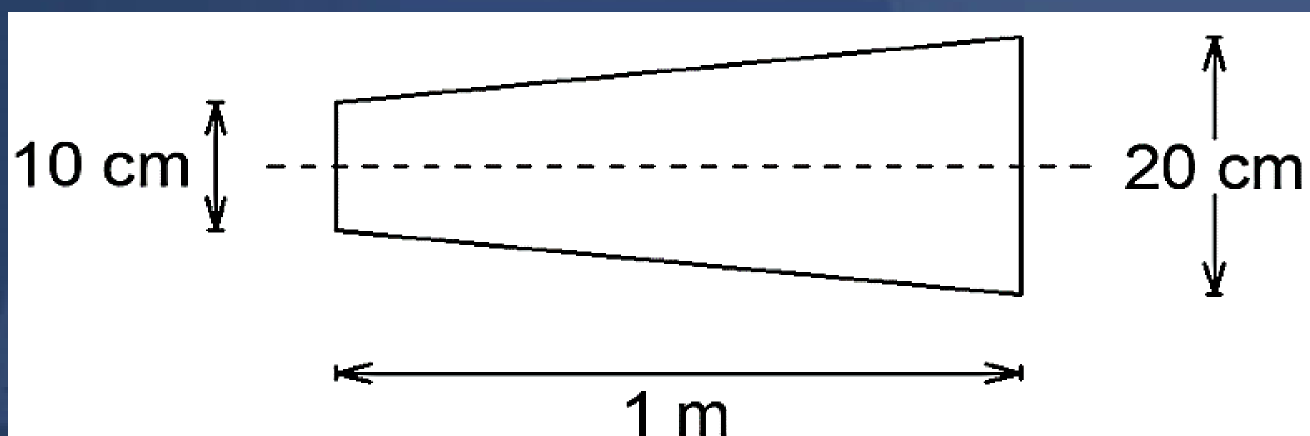
$$dL_{AB} = (\sigma_{AB} / E_{AB}) \times L_{AB} = [57.14/(200 \times 10^3)] \times 500$$

$$\boldsymbol{dL_{AB} = 0.14 = (dL)_{CD} = (dL)_{EF}}$$

AT2. A compound bar of circular cross section consists of a brass portion AB and steel portion BC fixed between two rigid supports as shown in figure. If the temperature is increased by 140° C, find the stress in each segment and change in length of segment AB. Consider E_{br} = 85 GPa; $\alpha_{br} = 20 \times 10^{-6}$ /°C and E_{st} = 210 GPa, $\alpha_{st} = 11 \times 10^{-6}$ /°C.



AT3. A circular cross-section tapered bar is rigidly fixed between two supports at its ends. If the temperature is raised by 30 °C, calculate the max stress in the bar, if, (i) the supports are perfectly rigid; (ii) each support yields by 0.08 mm. Diameter varies from 10 cm to 20 cm; length is 1 m; E = 200 GN/m^2; α = 12$\times$10^{-6}/ °C.



AT4. Two vertical rods are rigidly fixed as shown in the figure. A cross bar fixed to the rods at the lower end carries a load of 5 kN such that the cross bar remains horizontal even after loading. Determine i) stress in each rod ii) position (X) of the load on the cross bar. Take $E_s = 2 \times 10^5$ N/mm^2 and $E_u = 1 \times 10^5$ N/mm^2.