LECTURE 5 & 6

KARNAUGH MAP (K – MAP)

K – MAP

Karnaugh - Map

- Pictorial form of a truth table.
- Graphical tool to simplify a logical equation by forming groups of cells.

One-variable

$F = X$ or $F(x)$

$F = \bar{X}$

Two-variable

$F(x,y)$ or $F(A,B)$

$2^2 = 4$ variations i/p

minterms	(maxterms)
$\bar{x}\bar{y}$	$x+y$
$\bar{x}y$	$x+\bar{y}$
$x\bar{y}$	$\bar{x}+y$
xy	$\bar{x}+\bar{y}$

$SOP = \sum_m (0,1,2) = \bar{x}\bar{y} + \bar{x}y + x\bar{y} =$

$POS = \prod_M 3 = (\bar{x}+\bar{y})$ ← simplified

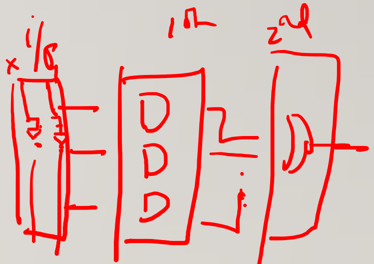
Why minimization?

Cost minimization

Cost ∝ Number of i/ps to the gates + Number of gates

i/p level	1st level	2nd level	
2 Not gates	3 AND	1 OR	= 6
2	6	3	= 11
			17
2	—	1 OR	= 3
2	—	2	= 4
			7

SoP



AND-OR

Cost = 17 unit cost

Cost = 7 unit cost

K – MAP

3 Variable function $f(x,y,z)$

$2^3 = 8$ combinations

	minterms				MAX terms	F
0	$\bar{x}\bar{y}\bar{z}$	000	m_0	M_0	$x+y+z$	0
1	$\bar{x}\bar{y}z$	001	m_1	M_1	$x+y+\bar{z}$	1
2	$\bar{x}y\bar{z}$	010	m_2	M_2	$x+\bar{y}+z$	1
3	$\bar{x}yz$	011	m_3	M_3	$x+\bar{y}+\bar{z}$	1
4	$x\bar{y}\bar{z}$	100	m_4	M_4	$\bar{x}+y+z$	0
5	$x\bar{y}z$	101	m_5	M_5	$\bar{x}+y+\bar{z}$	0
6	$xy\bar{z}$	110	m_6	M_6	$\bar{x}+\bar{y}+z$	1
7	xyz	111	m_7	M_7	$\bar{x}+\bar{y}+\bar{z}$	0

m_i $\qquad$ M_i

$$\boxed{M_i = \overline{m_i}}$$

Any function involving 3 Variables

SOP $\quad f(x,y,z) = \sum(1,2,3,6) = \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + xy\bar{z}$

canonical

POS $\quad f(x,y,z) = \prod(0,4,5,7) = (x+y+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+\bar{z})$

Boolean law's

TWO VARIABLE K – MAP

Two_Variable

$2^2 = 4$ combination ⇒ 4 cells

$\bar{A}\bar{B} + AB$

	A	B	SOP	POS
m_0	0	0	$\bar{A}\bar{B}$	$A+B$
m_1	0	1	$\bar{A}B$	$A+\bar{B}$
m_2	1	0	$A\bar{B}$	$\bar{A}+B$
m_3	1	1	AB	$\bar{A}+\bar{B}$

A. SOP: - $\bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB$

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	$\bar{A}.\bar{B}$ (m_0)	$\bar{A}.B$ (m_1)
A 1	$A.\bar{B}$ (m_2)	$A.B$ (m_3)

$\bar{A}\bar{B} + \bar{A}B = \bar{A}(\bar{B}+B) = \bar{A}$

minterms

B. POS: - $(A+B)(A+\bar{B})$

$A + AB + A\bar{B} + B\bar{B}$

A \ B	B 0	$\bar{B}$ 1
A 0	A+B (M_0)	A+$\bar{B}$ (M_1)
$\bar{A}$ 1	$\bar{A}$+B (M_2)	$\bar{A}$+$\bar{B}$ (M_3)

$A + 0 = A$

$F(AB) = A\bar{B} + \bar{B}A = A\bar{B}$

$A\bar{B} + A\bar{B} = A\bar{B}$

$2^{10} = 1024$

$F(A,B)$

MSB	LSB	
0	0	0
0	1	1
1	0	2
1	1	3

$F = \bar{A}B + AB$

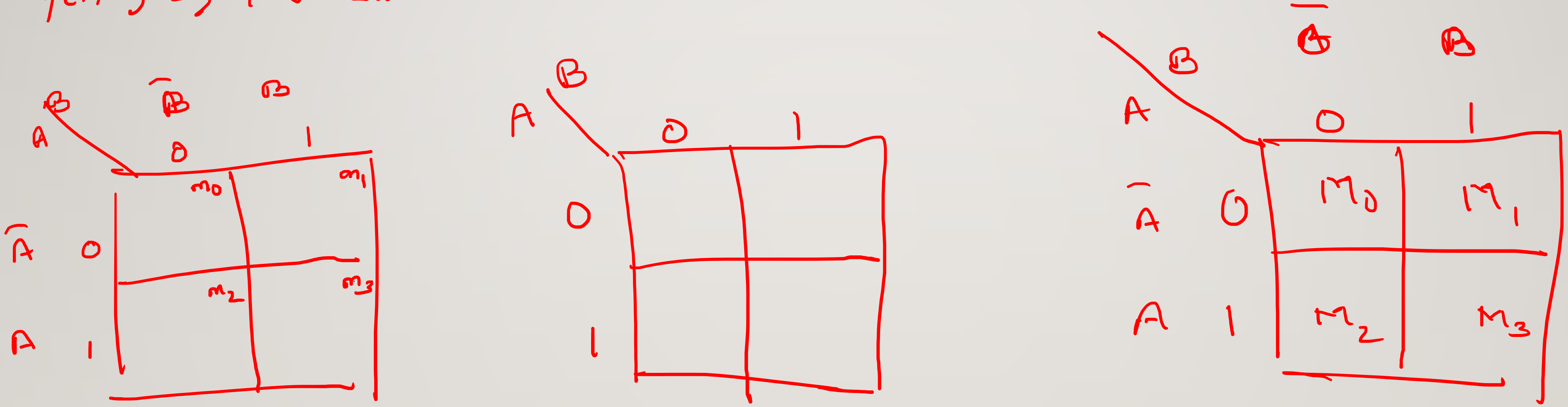
$F(A,B) = \sum 1,3$

$F = B\bar{A} + AB$

$F(A,B) = \sum(1,3)$

TWO VARIABLE K – MAP

$f(x, y) \Rightarrow 4$ variation



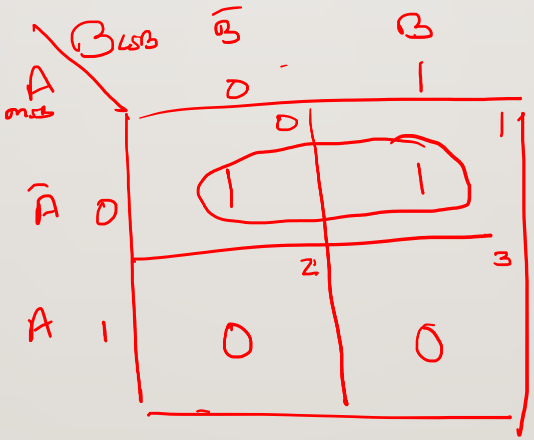
EXAMPLE 1: Solve using K-Map, Draw simplified Circuit

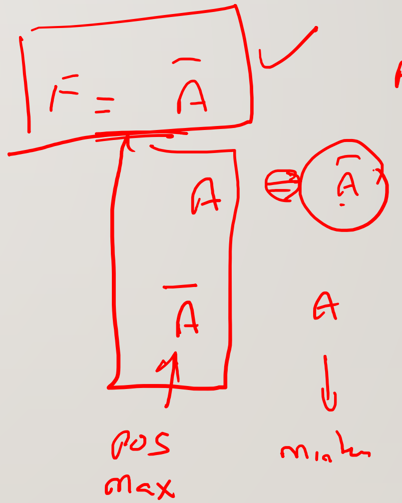
Give SOP, POS & simplified Expression

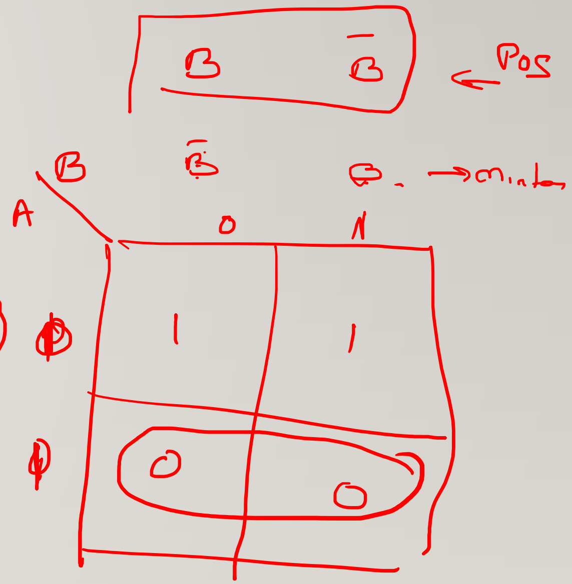
	A	B	F		
0	0	0	1	$\bar A \bar B$	$A+B$
1	0	1	1	$\bar A B$	$A+\bar B$
2	1	0	0	$A \bar B$	$\bar A+B$
3	1	1	0	AB	$\bar A+\bar B$

$F(A,B)$. SOP $= \sum_m(0,1) = \bar A \bar B + \bar A B$

POS $= \prod_M(2,3) = (\bar A + B)(\bar A + \bar B)$







SOP

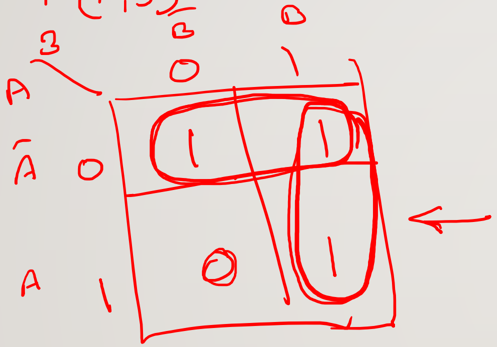
$\boxed{F = \bar A}$ ✓

POS

EXAMPLE 2:

A	B	F
0	0	1
0	1	1
1	0	0
1	1	1

$F(A,B)$



SOP

$F = \bar{A} + AB \Rightarrow \bar{A} + B$

$F = \bar{A} + B$ ✓

Combination Rule

Note: K-map

* To combine the max number of 1's SOP
  0's POS

2^0 2^1 2^2 2^3 2^4 2^5 . . . .

1 2 4 8 16 32

Aim is to eliminate variables

$F = \bar{A}\bar{B} + \bar{A}B + AB$

$= (\bar{A}\bar{B} + \bar{A}B) + (\bar{A}B + AB)$

$= \bar{A}(\bar{B}+B) + (\bar{A}+A)B$

$\boxed{F = \bar{A} + B}$

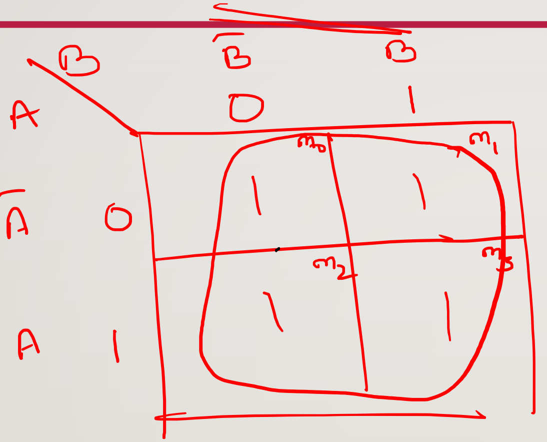
$x + x + x \dots = x$

EXAMPLE 3:

Solve using K-MAP

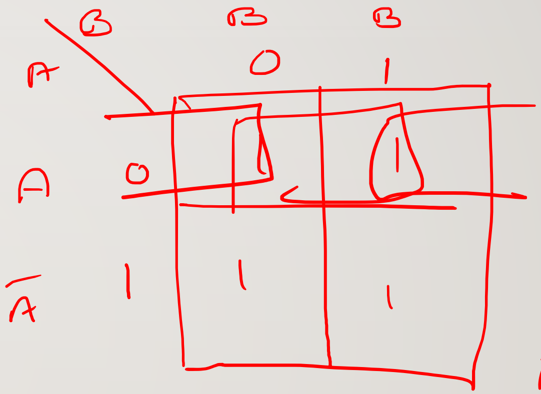
$12 = 1100 = f(ABCD) = AB\bar{C}\bar{D}$

A	B	F	
0	0	1	0
0	1	1	0
1	0	1	0
1	1	1	0



SOP

$F = 1$



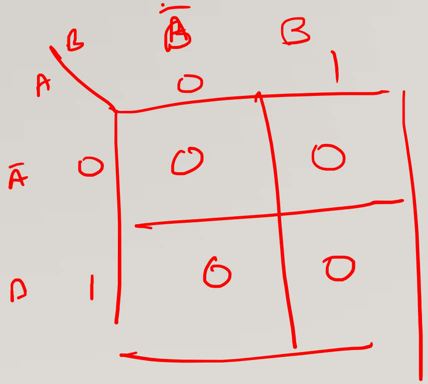
$\bar{F}$

POS $F = 1$

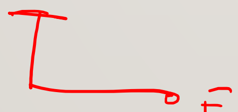
$F = 1$

POS

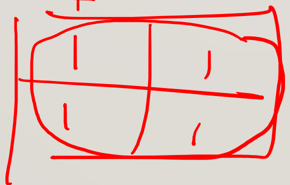
$\boxed{F = 1}$



$\bar{F} = 0$



High = Vcc



$\bar{F} =$

$F = SOP = \sum_m (\ )$

$F = POS = \prod (\ )$

$\bar{F}$

SOP $= \sum_m (\ )$

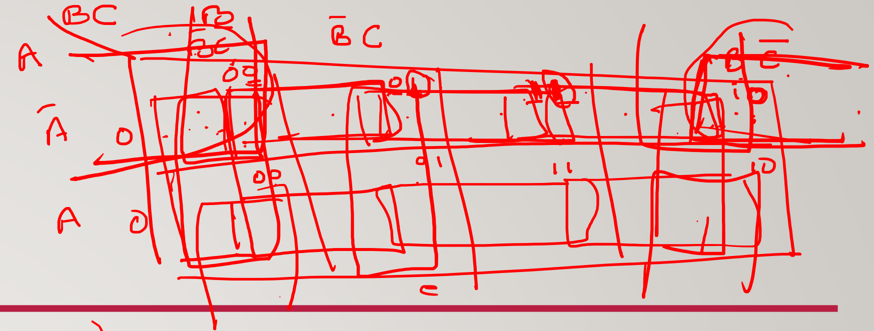
POS $= \prod (\ )$

THREE VARIABLE K – MAP

Variables = 3 Combinations = 8

BC
00
01
10
11

$\bar{A}\bar{B} + AB$ $\bar{A}B + A\bar{B}$



	A	B	C	SOP	POS	
0	0	0	0	m_0 $\bar{A}\bar{B}\bar{C}$	$A+B+C$	M_0
1	0	0	1	m_1 $\bar{A}\bar{B}C$	$A+B+\bar{C}$	M_1
2	0	1	0	m_2 $\bar{A}B\bar{C}$	$A+\bar{B}+C$	M
3	0	1	1	m_3 $\bar{A}BC$	$A+\bar{B}+\bar{C}$	
4	1	0	0	m_4 $A\bar{B}\bar{C}$	$\bar{A}+B+C$	
5	1	0	1	m_5 $A\bar{B}C$	$\bar{A}+B+\bar{C}$	
6	1	1	0	m_6 $AB\bar{C}$	$\bar{A}+\bar{B}+C$	
7	1	1	1	m_7 ABC	$\bar{A}+\bar{B}+\bar{C}$	M_7

$F(A\,B\,C)$ MSB LSB

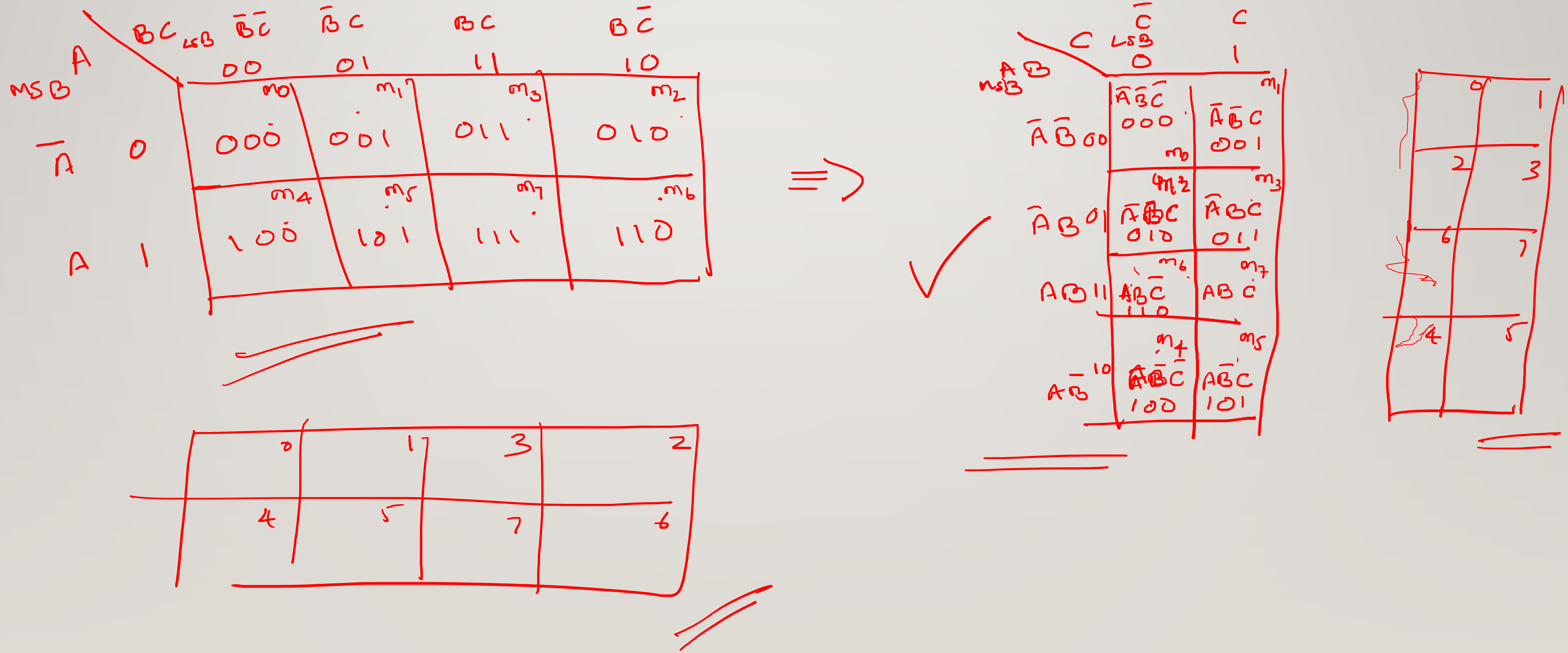
A. SOP: -

MSB A \ BC LSB	$\bar{B}\bar{C}$ 0 0	$\bar{B}C$ 0 1	BC 1 1	$B\bar{C}$ 1 0
$\bar{A}$ 0	$\bar{A}\bar{B}\bar{C}$ 0	$\bar{A}\bar{B}C$ 1	$\bar{A}BC$ 3	$\bar{A}B\bar{C}$ 2
A 1	$A\bar{B}\bar{C}$ 4	$A\bar{B}C$ 5	ABC 7	$AB\bar{C}$ 6

B. POS: -

A \ B+C	$B+C$ 0 0	$B+\bar{C}$ 0 1	$\bar{B}+\bar{C}$ 1 1	$\bar{B}+C$ 1 0
A 0	$A+B+C$ 0	$A+B+\bar{C}$ 1	$A+\bar{B}+\bar{C}$ 3	$A+\bar{B}+C$ 2
$\bar{A}$ 1	$\bar{A}+B+C$ 4	$\bar{A}+B+\bar{C}$ 5	$\bar{A}+\bar{B}+\bar{C}$ 7	$\bar{A}+\bar{B}+C$ 6

3-VARIABLE K-MAPS



EXAMPLE 1:

Given the Boolean function:

$$F = \bar{A}C + \bar{A}B + A\bar{B}C + BC$$

3 Variable $= 2^3 = 8$

- Express it in Sum of minterms form.
- Find the minimal sum of products expression using k-map.

$F = \bar{A}C + \bar{A}B + A\bar{B}C + BC$

$F(A,B,C) = \bar{A}(\bar{B}+B)C + \bar{A}B(\bar{C}+C) + A\bar{B}C + (\bar{A}+A)BC$

$= \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + \bar{A}BC + ABC$

$\quad 001 \quad 011 + \quad 101$

$= m_1 + m_3 + m_2 + m_3 + m_5 + m_3 + m_7$

$F = \sum_m (1, 2, 3, 5, 7)$ SOP

$\bar{F} \Rightarrow$ SOP $= \sum_m (0, 4, 6)$

$F = \prod (0, 4, 6)$ POS

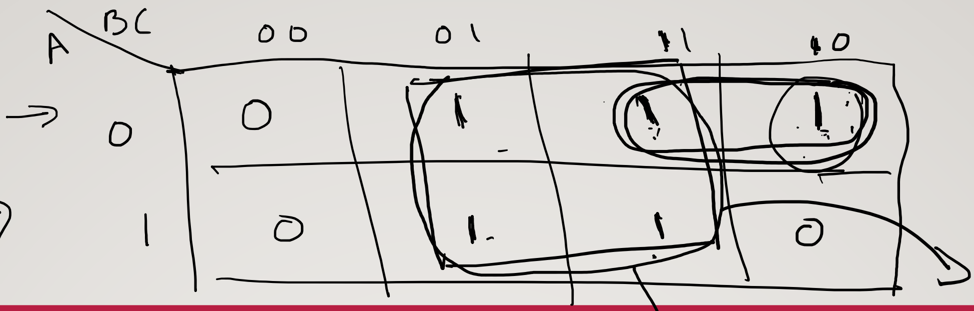
$\bar{F}$ POS $= \prod_M (1, 2, 3, 5, 7)$

A	B	C	F	
0	0	0	0	0
0	0	1	1	1
0	1	0	1	2
0	1	1	1	3
1	0	0	0	4
1	0	1	1	5
1	1	0	0	6
1	1	1	1	7

EXAMPLE 1:

Given the Boolean function:

$$F = \bar{A}C + \bar{A}B + A\bar{B}C + BC$$



$\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

$\Rightarrow \bar{A}C + AC$

$= (\bar{A}+A)C = C$

$C + \bar{A}B\bar{C}$

$\boxed{C + \bar{A}B}$

$\bar{A}BC + \bar{A}B\bar{C}$

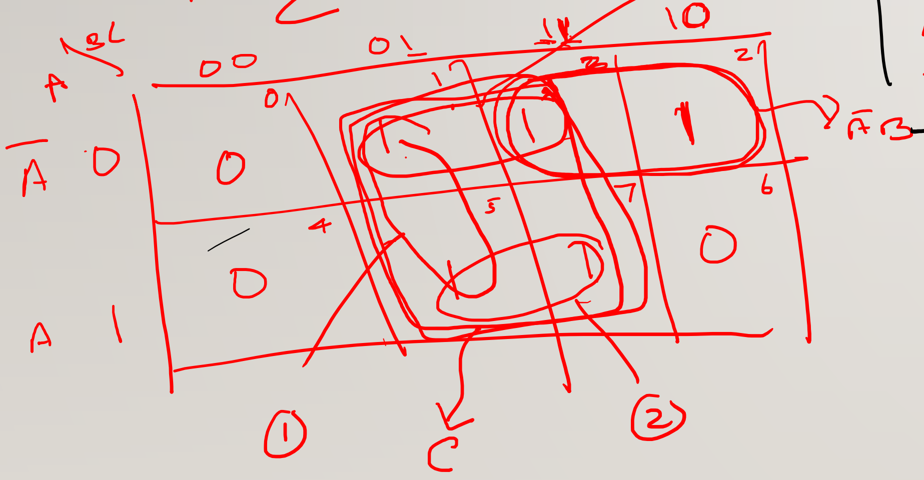
$\bar{A}B(C+\bar{C})$

$= \bar{A}B$

1st term

- Express it in Sum of minterms form.
- Find the minimal sum of products expression using k-map.

$F = \sum 1, 2, 3, 5, 7$



$\boxed{F = C + \bar{A}B} + ① + ② + ③$

Redundant

$\boxed{X + \bar{X}Y = X + Y}$

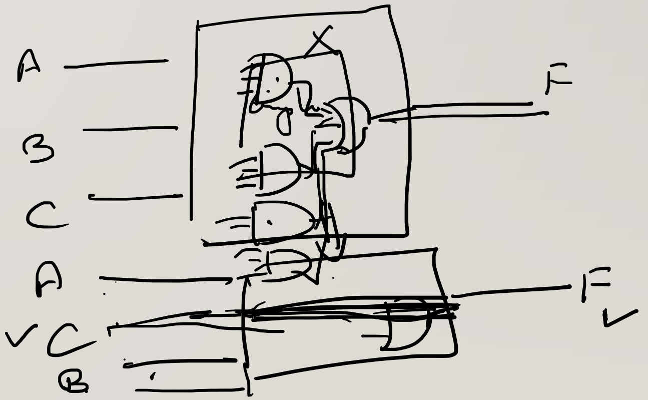
Grouping Rule

① Try combine max No. of cells

② Try to cover uncovered 1's

$F = C + \bar{A}B$

$C + \bar{A}B\bar{C}$

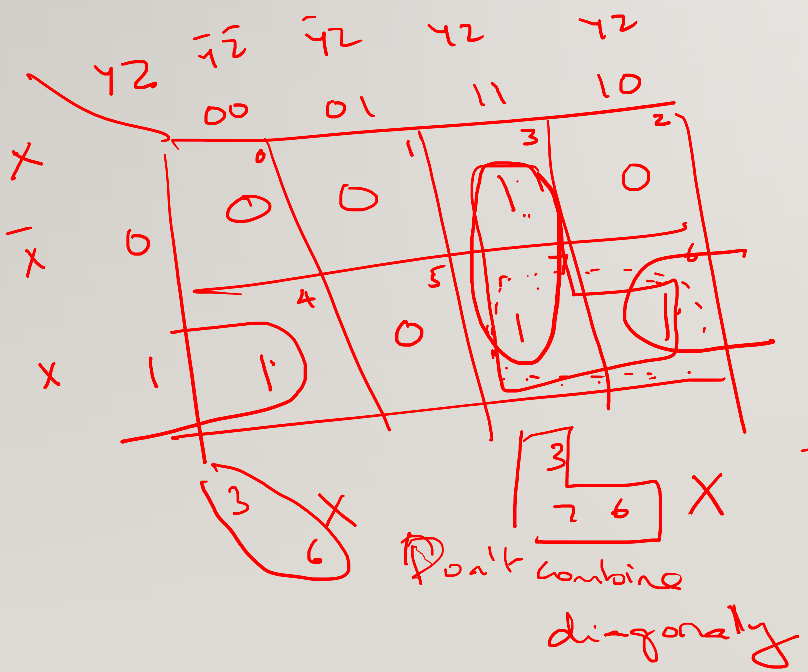


EXAMPLE 2:

Simplify the Boolean expression:

$$F(x, y, z) = \sum(3,4,6,7)$$

SOP $F = \sum(3,4,6,7)$



$F = yz + x\bar{z} + (\quad)$

redundant

$\bar{x}yz + xyz$

$(\bar{x}+x)yz$

m_3 m_6

$\bar{x}yz + xy\bar{z}$

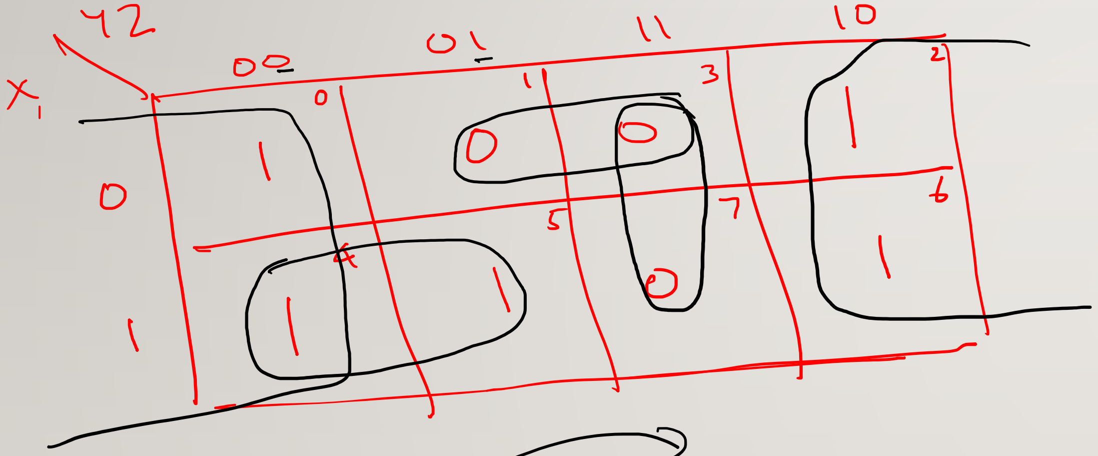
$y.(\bar{x}z + x\bar{z})$

2 3 i/p AND
1 2 i/p OR

2 2 i/p AND
1 2 i/p OR
1 2 i/p AND

EXAMPLE 3:

$$F(x, y, z) = \sum (0,2,4,5,6)$$



M_1 M_3

$(x + y + \bar{z})(x + \bar{y} + \bar{z})$

$y\bar{y}$

$a \cdot (b + c) = a.b + a.c$

$a + bc = (a+b)(a+c)$

$F = \bar{z} + x\bar{y}$ $\Longleftrightarrow$ POS $= (x + \bar{z})(\bar{y} + \bar{z})$

SOP

$(\bar{z} + x)(\bar{z} + \bar{y})$

EXAMPLE 4:

$$F(x,y,z) = \prod(0,2,5,7)$$

x \ yz	00	01	11	10
0	0	1	1	0
1	1	0	0	1

$$F = (x + z)(\bar{x} + \bar{z})$$