No of compositions of 'n' : 2^n

No of compositions of 'n' into 'm' parts : ${}^{n-1}C_{m-1}$

} ordered

Generating functions

No of compositions of 'n' : $x^m(1-x)^{-m}$

No of compositions on 'n' into 'm' parts : $\frac{x}{1-2x}$

PARTITIONS

No of partitions of 'n' : $(1-x)^{-1}(1-x^2)^{-1}(1-x^3)^{-1}\dots\dots$

No of partitions of 'n' into 'K' parts : $x^k(1-x)^{-1}(1-x^2)^{-1}\dots(1-x^k)^{-1}$

Ferrers graph

No of partitions of 'n' with no part $>$ K = No of partitions on 'n' with atmost 'K' parts

No of partitions of n = No of partitions of '2n' into 'n' parts.

* Generating function for partitions of 'n' into 'm' parts

Let $f_m(x)$ be the required g·f

$$f_m(x) = \begin{bmatrix}\text{Enumerator for partitions} \\ \text{with atmost } k \text{ parts}\end{bmatrix} - \begin{bmatrix}\text{Enumerator for partitions} \\ \text{with atmost } (k-1) \text{ parts}\end{bmatrix}$$

$\left\{ \begin{pmatrix}\text{set of all partitns with} \\ \text{atmost } k \text{ parts}\end{pmatrix} - \begin{pmatrix}\text{set of all partitns} \\ \text{with atmost } (k-1) \\ \text{parts}\end{pmatrix} = \begin{pmatrix}\text{set of all partitions} \\ \text{with exactly } k \text{ parts}\end{pmatrix}\right.$

$$= \begin{bmatrix}\text{Enumerator for partitions} \\ \text{with no part} > k\end{bmatrix} - \begin{bmatrix}\text{Enumerator for partitions} \\ \text{with no part} > k-1\end{bmatrix}$$

(∵ No of partitions of 'n' with atmost 'k' parts = the no of part^n's of 'n' in which no part > k)

$$= \left[(1-x)^{-1}(1-x^2)^{-1}(1-x^3)^{-1}\cdots(1-x^k)^{-1}\right] - \left[(1-x)^{-1}(1-x^2)^{-1}\cdots(1-x^{k-1})^{-1}\right]$$

$$= (1-x)^{-1}(1-x^2)^{-1}\cdots(1-x^{k-1})^{-1}\left[(1-x^k)^{-1} - 1\right]$$

$$= (1-x)^{-1}(1-x^2)^{-1}\cdots(1-x^{k-1})^{-1}\left(\frac{1}{1-x^k} - 1\right]$$

$$= (1-x)^{-1}\cdots(1-x^{k-1})^{-1}\left(\frac{\cancel{1}-\cancel{1}+x^k}{(1-x^k)}\right)$$

$$\boxed{f_m(x) = x^k (1-x)^{-1}(1-x^2)^{-1}\cdots(1-x^{k-1})^{-1}(1-x^k)^{-1}}$$

To get the no of partitions of 100 into 6 parts, we have take the coeff of x^{100} from the fn :

$$x^6(1-x)^{-1}(1-x^2)^{-1}\cdots\cdots(1-x^6)^{-1}$$

(3) PoT the no of partitions of 'n' with exactly 'k' parts is equal to the no of partitions of $(n-k)$ with no part greater than k.

Soln

Partitions of 'n' which have exactly 'k' parts = Partitions of $(n-K)$ in which no part greater than 'K'

Ex:- $n=11$ $K=3$

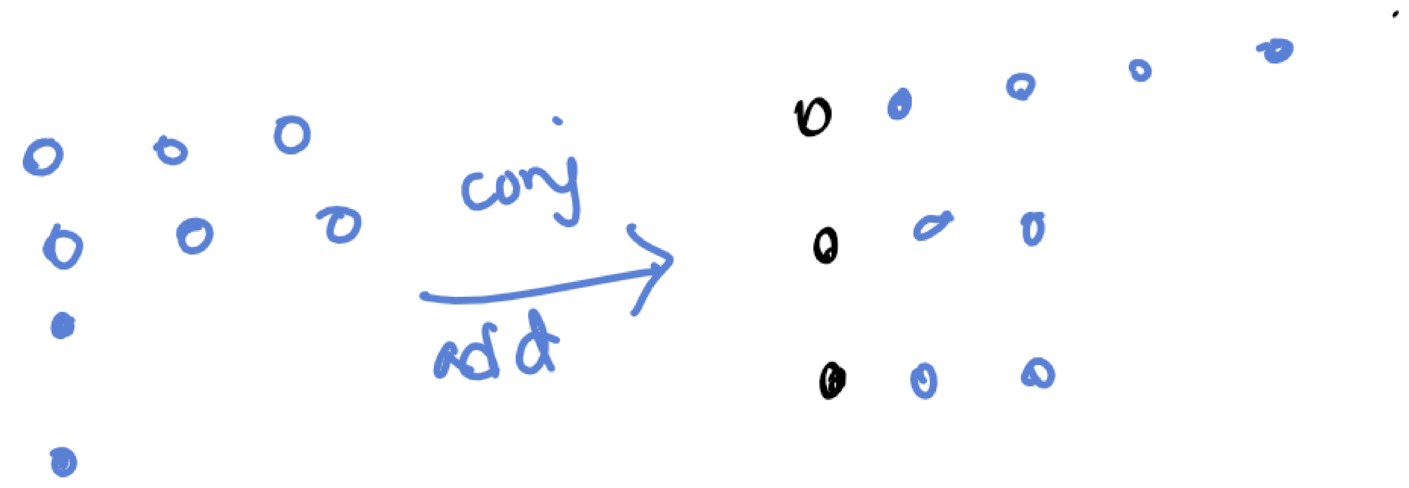
L: partitions of 11 into 3 parts

R: partitions 8 in which no part is > 3

1,2,3

L		R	
542	821	11 11 11 11	3221
551	911	2 11 11 11	332
443	533	2 2 11 11	3311
641		2 2 2 11	31111
632		2 2 2 2	3311
731			
722			

$443 \xrightarrow[\text{Dele row}]{\text{conjugatn}} 332$



Proof:-

Consider a partition of 'n' with exactly 'k' parts and consider its Ferrers graph representation. It has exactly 'k rows. Also, the first column has exactly 'k' dots (since I've to get partition for (n-k), I've to delete k dots)

consider the conjugate of the given partition, In the conjugate the first row has exactly k dots. On deleting this first row which has k dots, we get a partition of (n-k) in which no part > k.

For a given partition of 'n' with 'k' parts; there exists a partition for (n-k) with no partition > k.

conversely, consider a partition of (n-k) with no part > k. consider its Ferrers graph rep^n. The no of dots in any row $\leq k$. If we consider the conjugate, the no of cols will be $\leq k$. If we add a column containing k dots on the leftside, we get a partition of n which has exactly 'k' parts

$\therefore$ For every partition of (n-k) with no part > k, there exists a unique partition of n with k parts

$\therefore$ There is one-to-one correspondance

Hence the proof

ORDERING

① Lexicographical Order : (Dict Ordering)

WPKR

1234

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
1423	2413	3412	4312
1432	2431	3421	4321

② Reverse Lexicographical Order :-

1234	1243	1342	2341
2134	2143	3142	3241
1324	1423	1432	2431
3124	4123	4132	4231
2314	2413	3412	3421
3214	4213	4312	4321

15th :- 1432

21st : 2431

1 2 3 4   3214

15th lexico of WPKR ⟹ KPWR

1432

15th reverse lexico of ○ □ △ | ⟹ ○ | △ □

1 2 3 4

③ Fike's ordering :

for a permutation $(1\ 2\ 3\ 4 \cdots n)$ in Fike's ordering :- suppose to get k^{th} permutation

① Get a sequence $(d_2\ d_3\ d_4 \cdots d_n)$:-

To get the seq :- write $(k-1)$ as follows

$$(k-1) = c_1\left[\frac{n!}{2}\right] + c_2\left[\frac{n!}{3!}\right] + c_3\left[\frac{n!}{4!}\right] + \cdots + c_{n-1}\left[\frac{n!}{n!}\right]$$

② From the sequence, get k^{th} permutation:

i) subtract the seq from the $1\ 2\ 3 \ldots (n-1)$

ii) In every step, interchange the elt in k^{th} position with the elt in $(d_k+1)^{th}$ position for $k = 2, 3, \ldots n$

* What is 50th permutatⁿ of 12345 in Fike's order

Soln

$n=5$ (total no of digits)

$K=50$

i) Get the seq $(d_2\, d_3\, d_4\, d_5)$

$$49 = \_\left(\frac{5!}{2!}\right) + \_\left(\frac{5!}{3!}\right) + \_\left(\frac{5!}{4!}\right) + \_\left(\frac{5!}{5!}\right)$$

$$49 = \underset{d_2}{0}(60) + \underset{d_3}{2}(20) + \underset{d_4}{1}(5) + \underset{d_5}{4}(1)$$

$$49 = 0(60) + 2(20) + 1(5) + 4(1)$$

$\therefore\ (C_1\, C_2\, C_3\, C_4) \equiv (0\,2\,1\,4)$

ii)

$$\begin{array}{r} 1\ 2\ 3\ 4 \\ -\ 0\ 2\ 1\ 4 \\ \hline 1\ 0\ 2\ 0 \\ \hline \end{array}$$

(subtract)

$$\begin{array}{r} 1\ 2\ \ldots\ (n-1) \\ -\ C_1\, C_2\ \ldots\ C_n \\ \hline \\ \hline \end{array}$$

$\therefore\ (d_2\, d_3\, d_4\, d_5) \equiv (1\,0\,2\,0)$

12345

iii)

$d_2 + 1 = 2 \xrightarrow[2\ \&\ 2]{\text{interchang}} 12345$

$d_3 + 1 = 1 \xrightarrow{3rd\ \&\ 1st} 32145$

$d_4 + 1 = 3 \xrightarrow{4th\ \&\ 3rd} 32415$

$d_5 + 1 = 1 \xrightarrow{5th\ \&\ 1st} 52413$

* Find 35^{th} permutatn of PQRST in Fike's ordering

Soln

$n=5 \quad K=35$

i) $34 = \frac{5!}{2!} + \frac{5!}{3!} + \frac{5!}{4!} + \frac{5!}{5!}$

$34 = 0(60) + 1(20) + 2(5) + 4(1)$

Now subtract 0124 from 1234

$$\begin{array}{r} 1\ 2\ 3\ 4 \\ -\ 0\ 1\ 2\ 4 \\ \hline 1\ 1\ 1\ 0 \end{array}$$

$\therefore (d_2\, d_3\, d_4\, d_5) \equiv (1\,1\,1\,0)$

PQRST

$d_2+1 = 2 \xrightarrow{2^{nd} \,\&\, 2^{nd}}$ PQRST

$d_3+1 = 2 \xrightarrow{3^{rd} \,\&\, 2^{nd}}$ PRQST

$d_4+1 = 2 \xrightarrow{4^{th} \,\&\, 2^{nd}}$ PSQRT

$d_5+1 = 1 \xrightarrow{5^{th} \,\&\, 1^{st}}$ TSQRP

Probs

① S.T the no of partitions of 'n' in which no part is smaller than 3, is equal to the no of partitions of (n+3) in which the three largest parts are consecutive integers

② S.T the no of partitions of an integer 'n' with exactly 'k' parts is equal to the no of partitions of 'n' whose largest part is 'k'

③ Write 50th, 100th permutations of 01234 (with initial permutation) in

i) Lexicographical
ii) Reverse lexicographical
iii) Fike's ordering

Soln

Lexico → 50th : 20143
100th : 40234

Reverse Lexico → 50th : 10342
100th : 31240

Fike's → 50th : 41302
100th : 42013