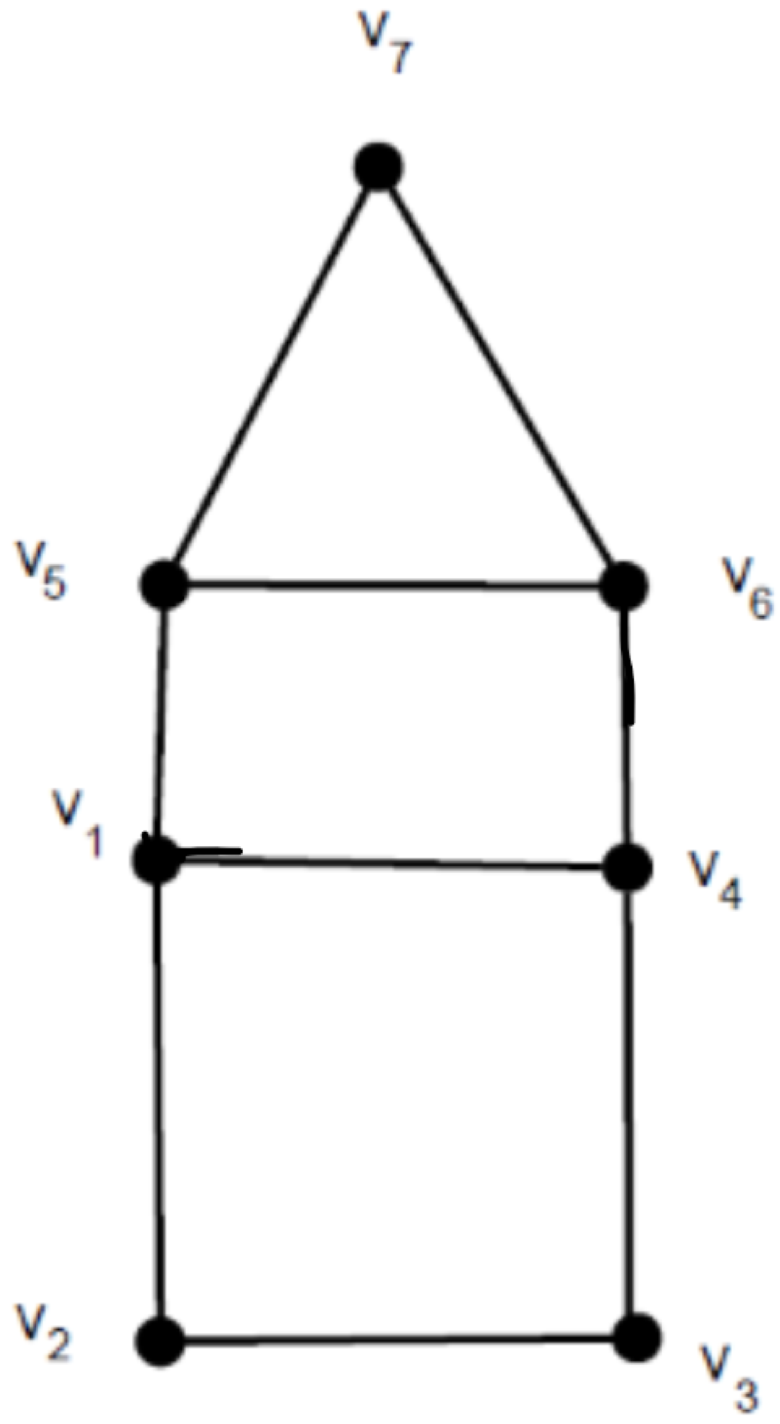
Walk, trail, Path, Cycle:

closed trail : $V_5 V_1 V_4 V_6 V_7 V_5$ (Length: 5)

closed trail : $V_5 V_6 V_4 V_1 V_5$ (cycle)



Walk : $V_6 V_4 V_1 V_4 V_3 V_2$ (Length : 5)

Trail : $V_6 V_4 V_1 V_5 V_6 V_7$ (Length : 5)

Path : $V_4 V_3 V_2 V_1 V_5$ (Length : 4)

closed path : $V_4 V_3 V_2 V_1 V_5 V_6 V_4$ (Length : 6)

Cycle : $V_6 V_5 V_1 V_4 V_6$

$V_6 V_4 V_3 V_4 V_1$ is a walk

$V_2 V_1 V_4 V_6 V_5$ is a path.

$V_5 V_7 V_6 V_5 V_1$ is a trail.

$V_5 V_7 V_6 V_4 V_6 V_5$ is a closed walk.

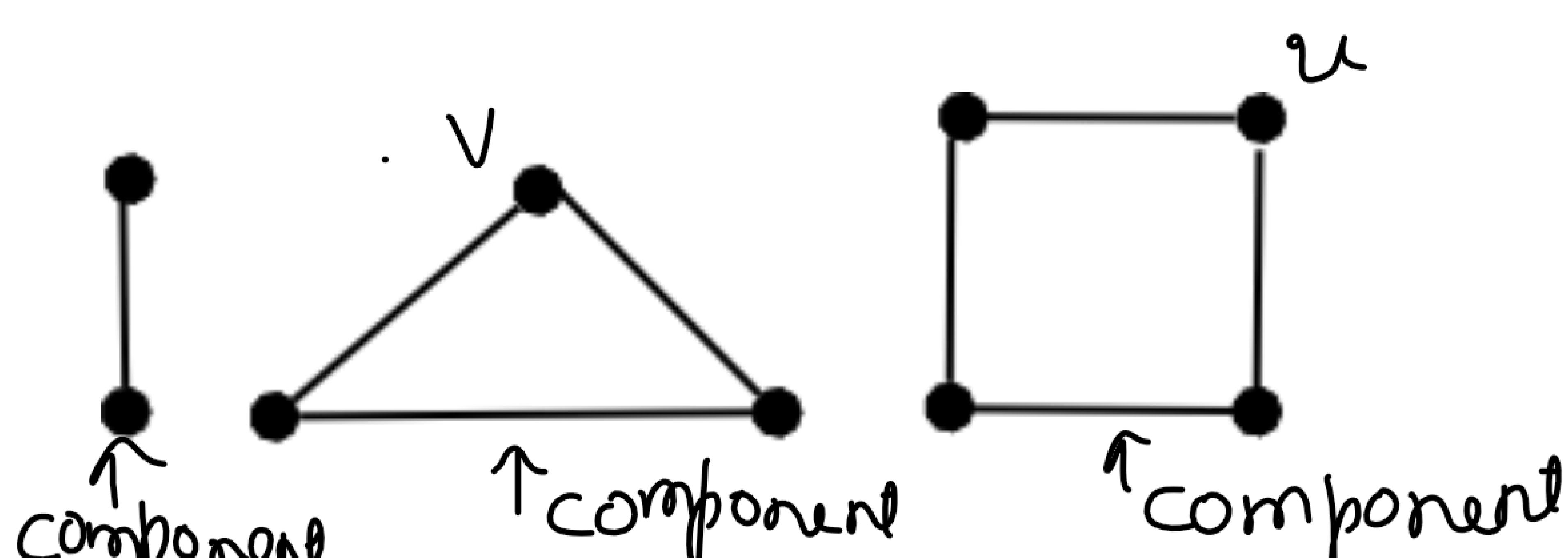
Path $\Rightarrow$ trail

trail $\Rightarrow$ walk

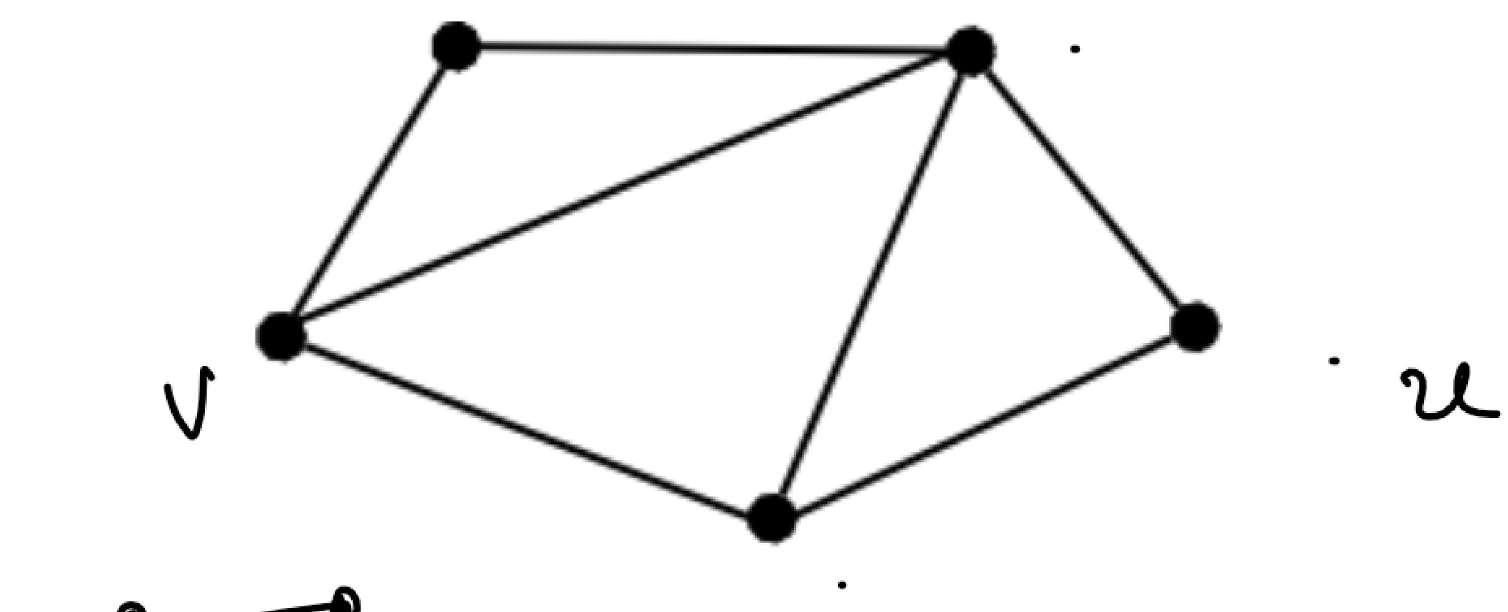
Two vertices u and v of G are said to be connected if there is a (u, v)-path in G.

A graph is said to be connected if there is a path between every pair of vertices in the graph, otherwise the graph is disconnected.

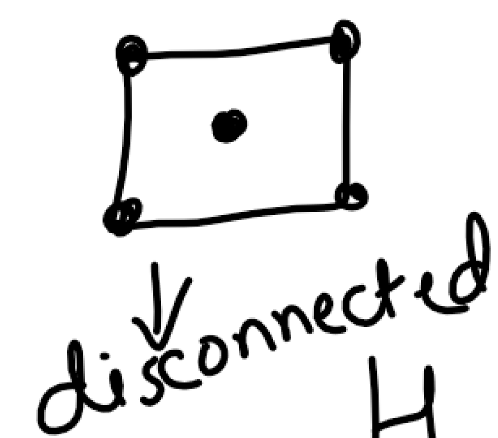
A maximal connected subgraph of G is called component of G. Thus a disconnected graph has at least 2 components.



G



H

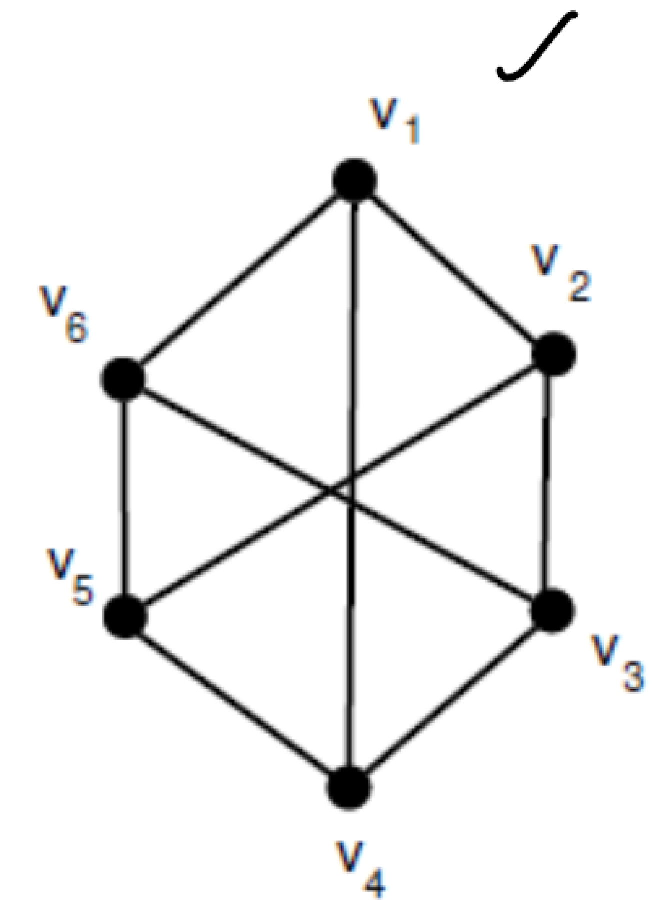


G is Not connected

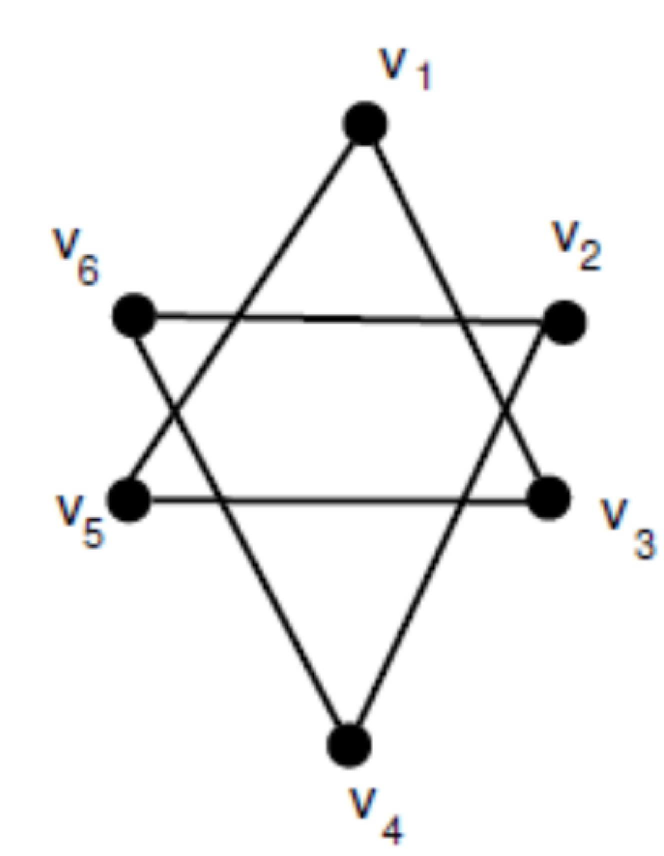
H is connected.

Complement of a graph G :

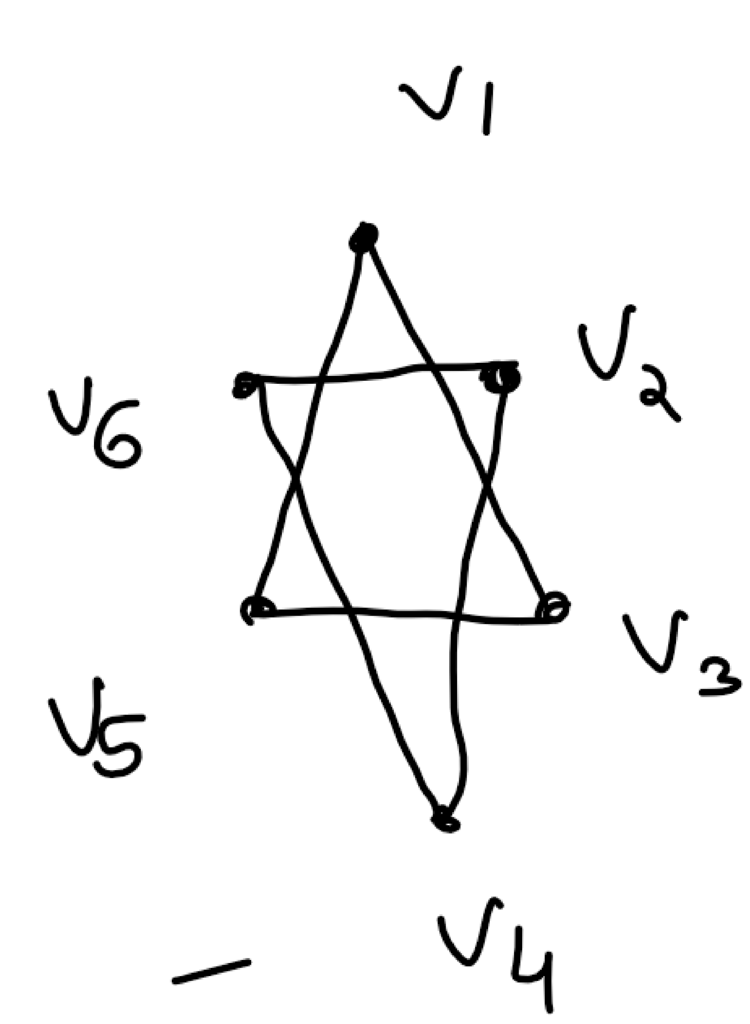
Let G be a graph with vertex set V(G) and edge set E(G). The Complement of G denoted by $\bar{G}$ is a graph with $V(\bar{G}) = V(G)$, and 2 vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G.



G



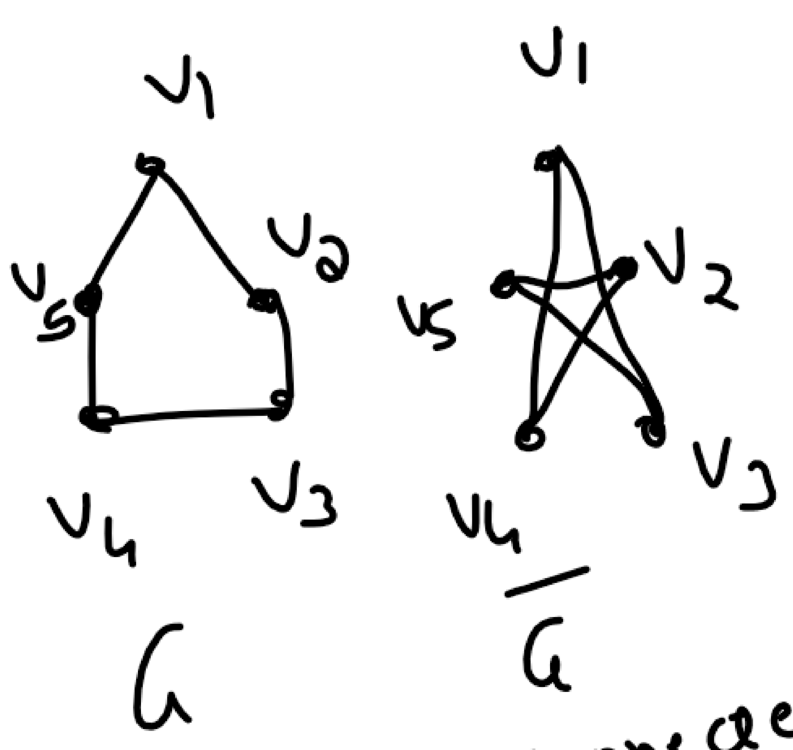
$\bar{G}$



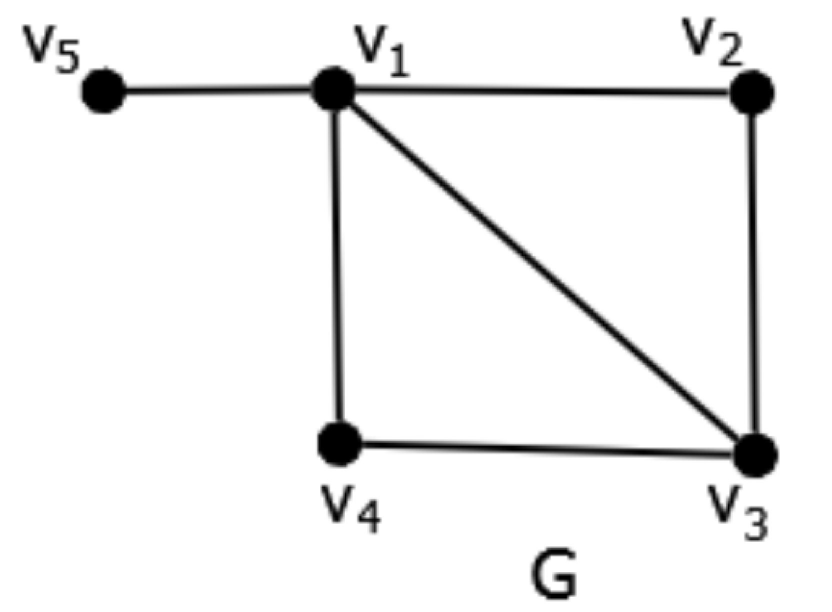
$\bar{G}$

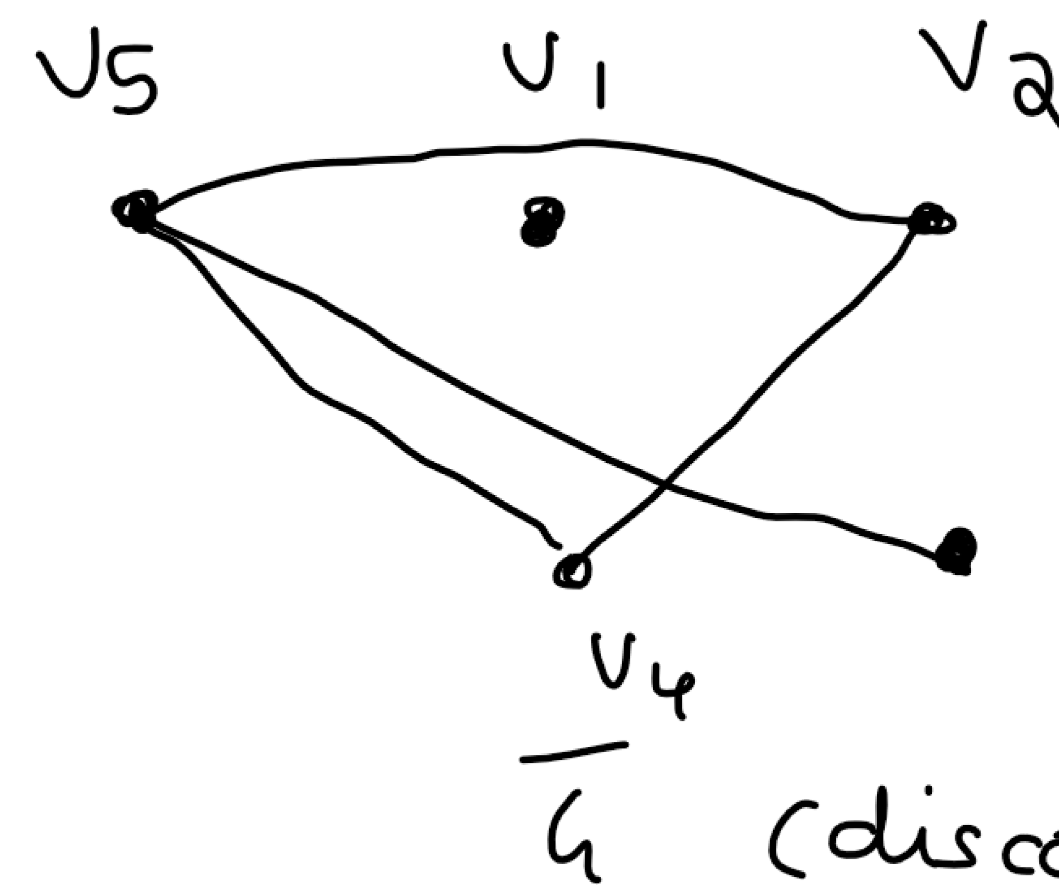
G is connected but $\bar{G}$ is disconnected with 2 components.

Find the complement of the graph G.



(connected.)

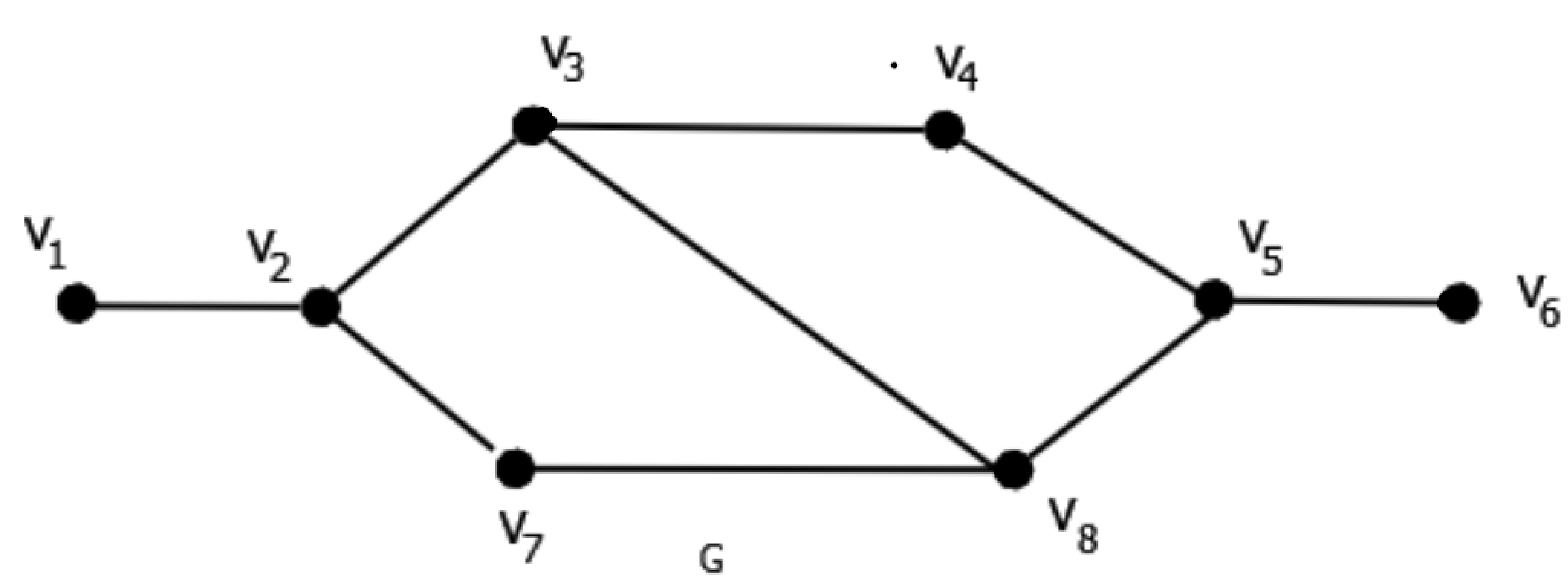


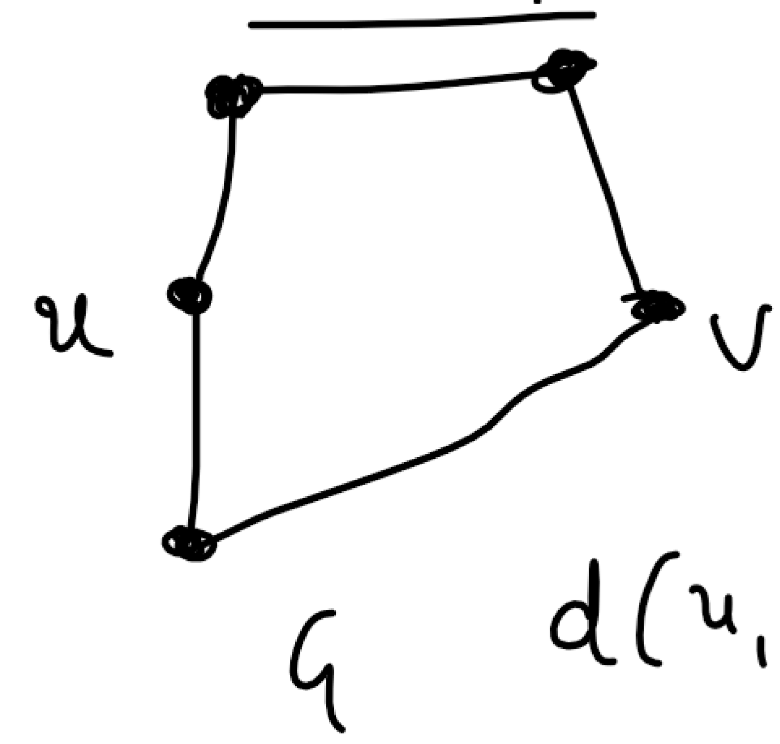


(disconnected)

Let G be a connected graph and let u, v be two vertices in G. A shortest path between u and v in G is a (u, v)– path with minimum number of edges in it.

The distance between u and v in G is denoted by d(u, v) is the length of a shortest path between them.





$d(u,v) = 2$

$d(v_1, v_2) = 1$

$d(v_1, v_8) = 3$

$d(v_4, v_7) = 3$

$d(v_3, v_6) = 3$

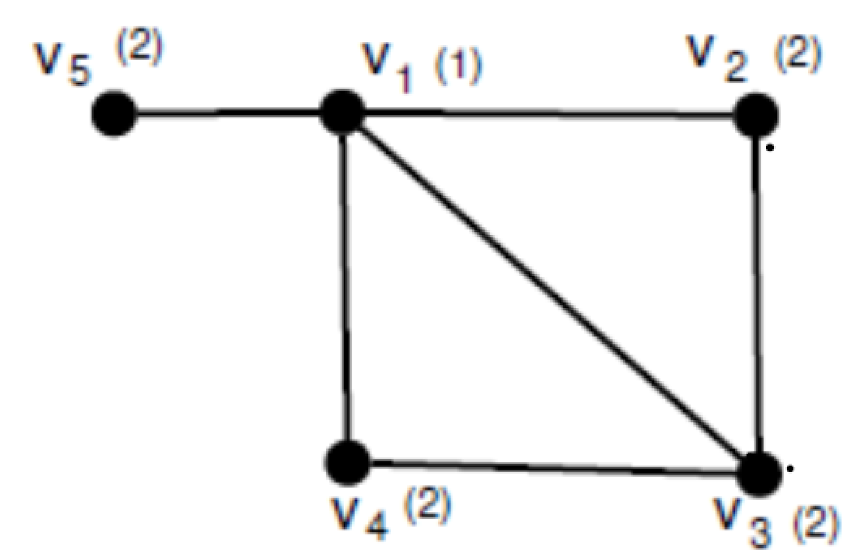
$d(v_7, v_6) = 3$

$d(v_7, v_5) = 2$

Eccentricity of a vertex v, in a connected graph G, denoted by e(v) is defined as follows.

$$e(v) = \max_{u \in V(G)} d(u, v)$$

The minimum and maximum of the eccentricities of vertices of G are radius and diameter of the graph G.
A vertex v in G with minimum eccentricity is called a central vertex and set of all central vertices in G is called the center of G.



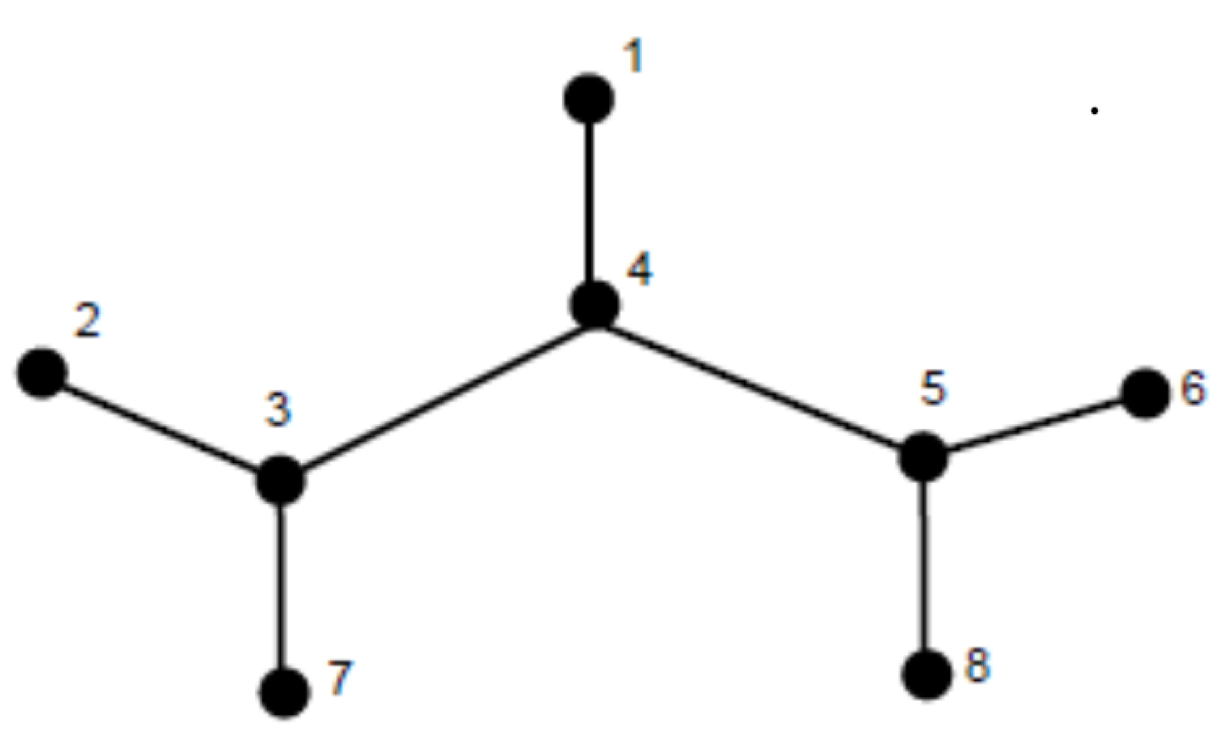
G

$e(v_1) = 1$
$e(v_2) = 2$
$e(v_3) = 2$
$e(v_4) = 2$
$e(v_5) = 2$

$diam(G) = 2$
$radius(G) = 1$
centre : v_1

Diameter of G is denoted by diam(G) is $\max_{u,v \in V(G)} d(u, v)$

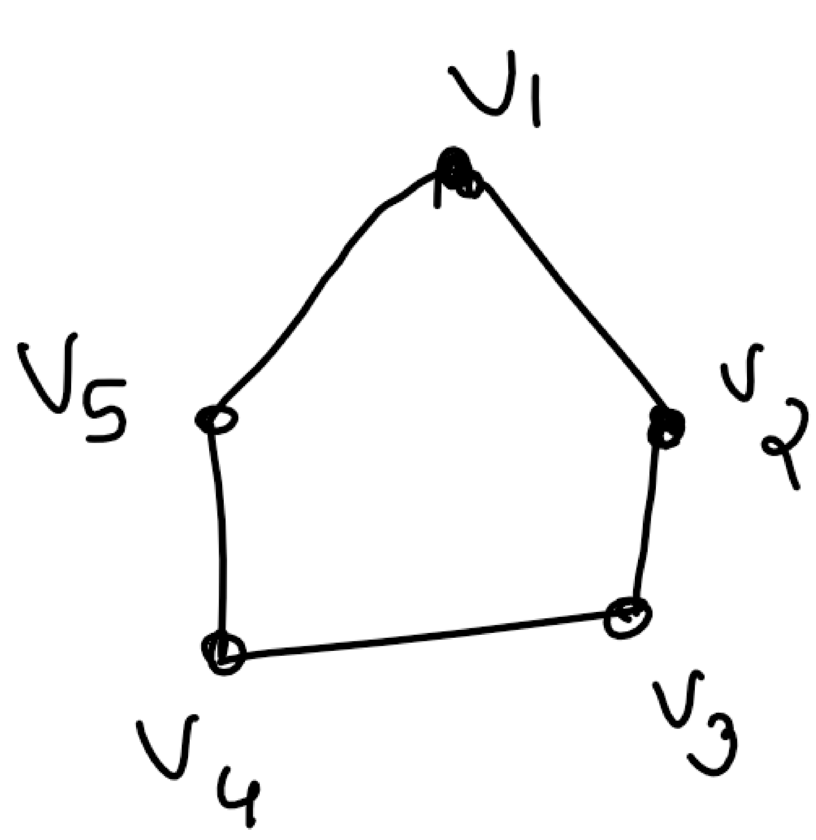
Find radius, diameter and centre of the graph.



$e(1) = 3$
$e(2) = 4$
$e(3) = 3$
$e(4) = 2$
$e(5) = 3$
$e(6) = 4$
$e(7) = 4$
$e(8) = 4$

radius = 2 diameter = 4

centre of G : vertex 4



radius = 2
diameter = 2
Centre of G : V_1, V_2, V_3, V_4, V_5.

Theorem 1: For any graph G with 6 vertices, G or $\bar{G}$ contains a triangle.

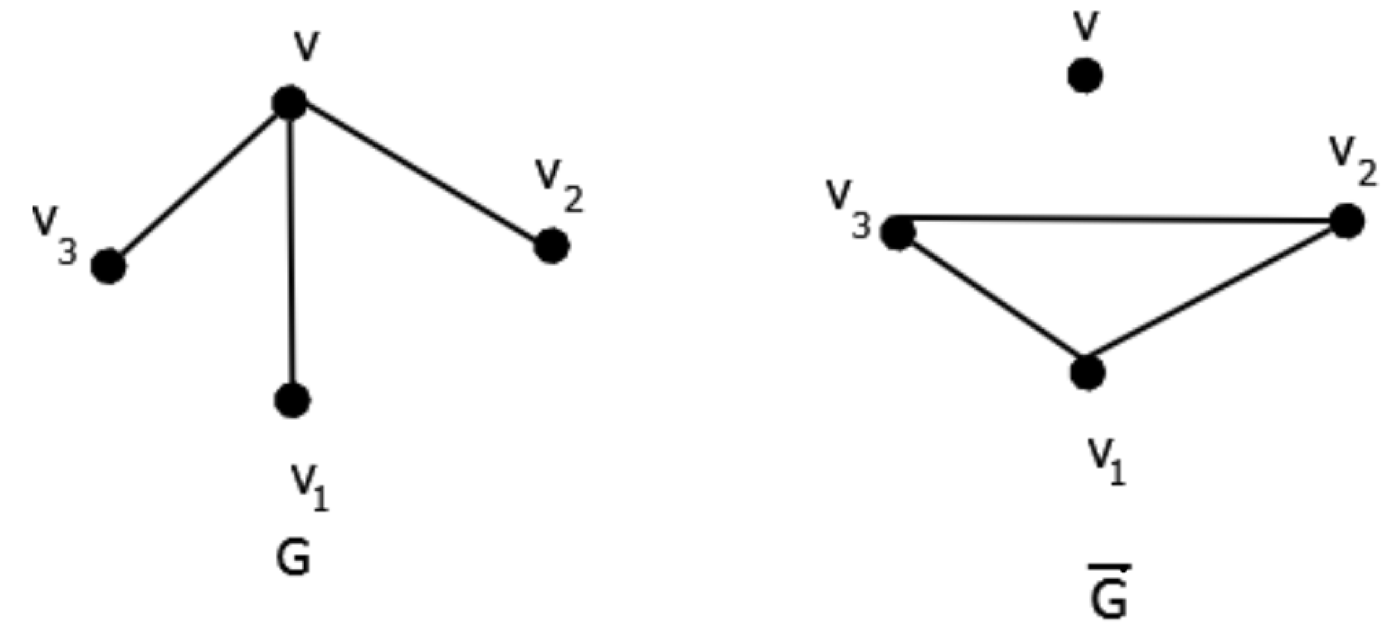
Proof: Let G be a graph with 6 vertices $v, v_1, v_2, v_3, v_4, v_5$.

Since v is adjacent to other 5 vertices in G or $\bar{G}$.

We assume v is adjacent to 3 vertices say v_1, v_2, v_3 in G.

If any of these 2 vertices are adj in G say v_1 and v_2, then $v v_1 v_2$ form a triangle in G.

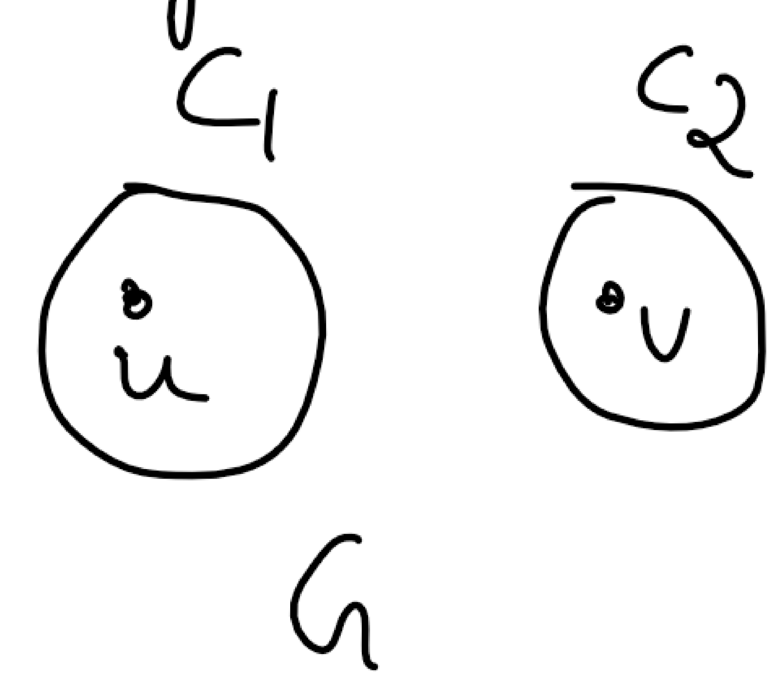
If none of them are adj in G, then in $\bar{G}$ $v_1 v_2 v_3$ form a triangle.



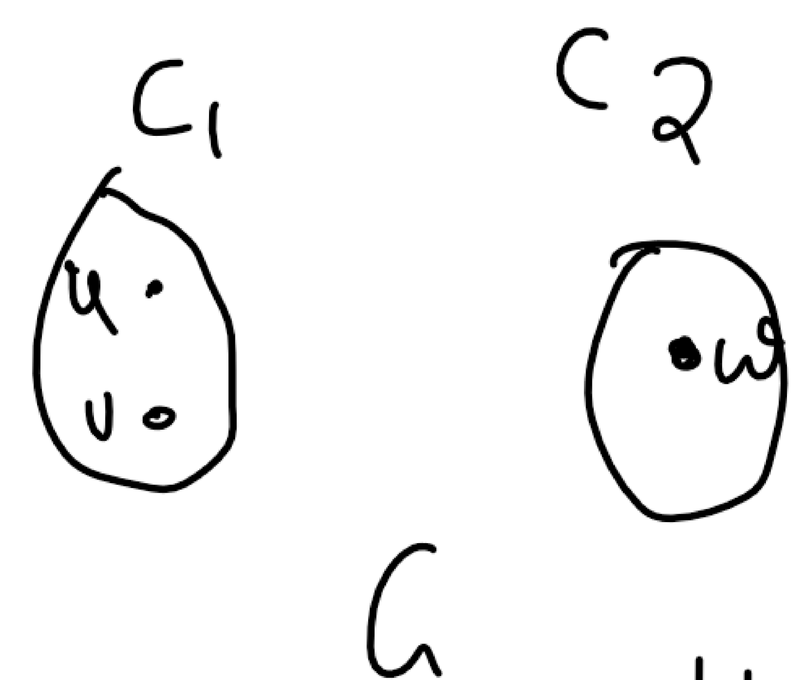
Theorem 2: For any graph G, show that either G or $\overline{G}$ is connected.

Proof: If G is connected, there is nothing to prove. Suppose G is disconnected, then G has 2 components C_1 and C_2. Let u, v be 2 vertices in G.

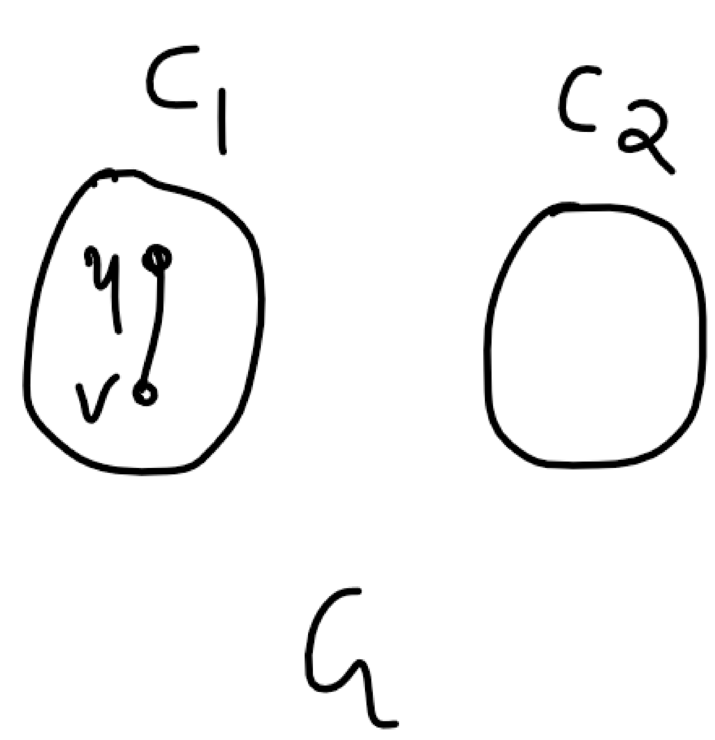
(i) If u and v are in 2 different components, then in $\overline{G}$, u and v are adjacent. Hence $\overline{G}$ is connected.

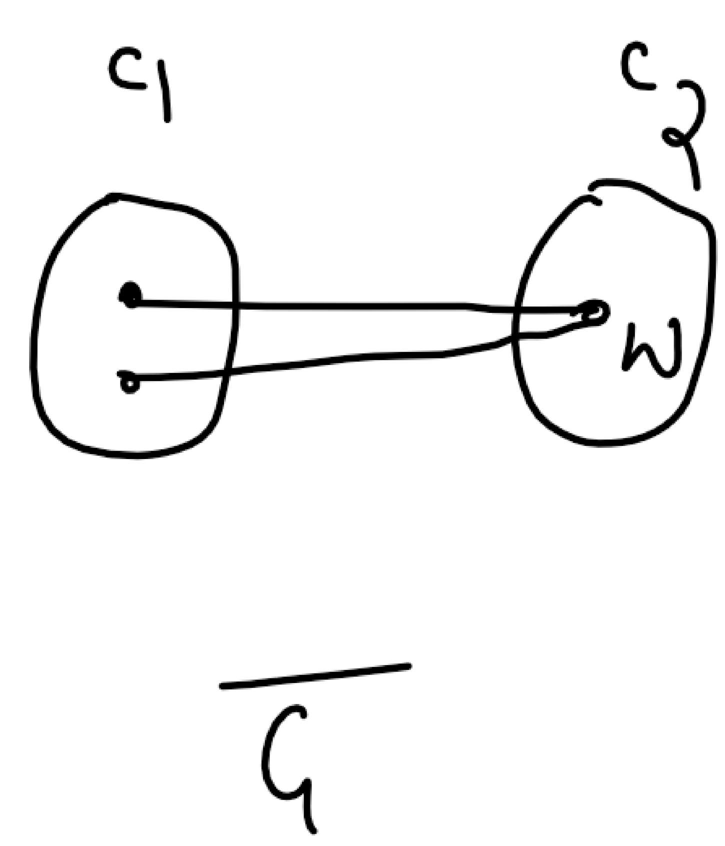


(ii) If u and v are in same component, and they are not adjacent, then in $\overline{G}$ u and v are adjacent in $\overline{G}$. Hence $\overline{G}$ is connected.



(iii) If u and v are in same component, and they are adjacent in G. Then there is a vertex w in another component such that u and v are adj to w in $\overline{G}$. $\Rightarrow$ There is a uv path. $\Rightarrow$ $\overline{G}$ is connected.

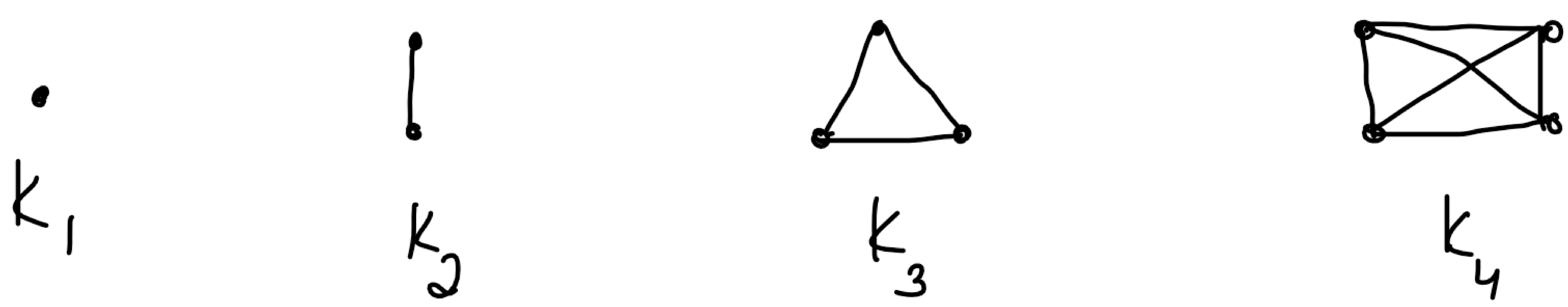


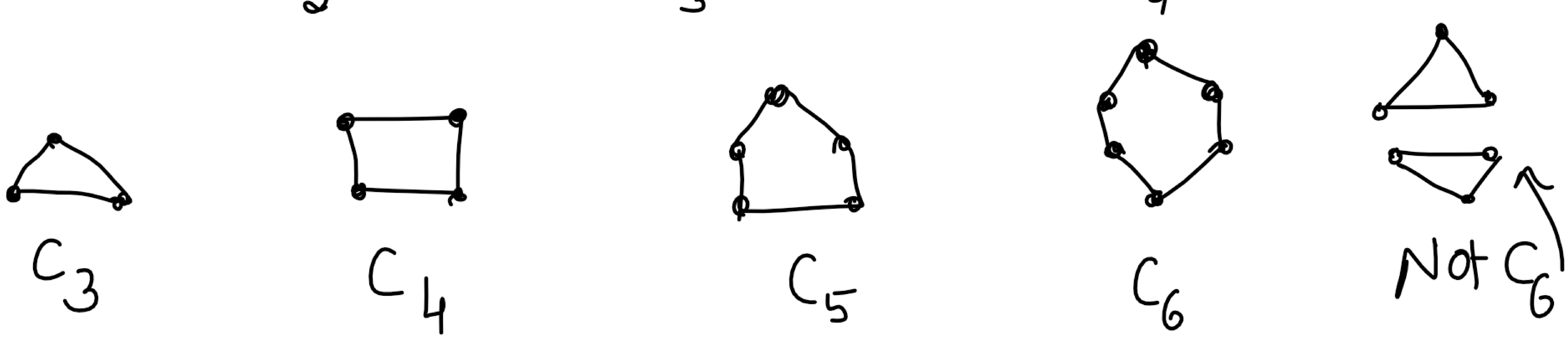


A graph G in which every vertex is of same degree is called a regular graph.
When G is regular, $\delta(G) = \Delta(G)$, and the common value is called regularity of G.
A regular graph with degree 3 is called a cubic graph.
A cubic graph has always even number of vertices.

A graph on n vertices, in which every two vertices are adjacent, is called a complete graph and is denoted by Kn.

A connected regular graph with regularity two is called a cycle.
A cycle on n vertices is denoted by Cn.





Complement of a complete graph on n vertices is called the totally disconnected graph.
A graph G is said to be self centered if every vertex of G has the same eccentricity. In such a graph, radius is equal to the diameter.

The cycle graph Cn is a self-centered graph and is the complete graph Kn.

Question 1: Draw a regular graph on regularity 4 and number of vertices 6.
Question 2: Draw a complete graph on 6 vertices.
Question 3: Draw a cycle graph on 8 vertices.
Question 4: Draw the complement of cycle graph C_8.
Question 5: The complete graph Kp has ________edges.

Question 6: The cycle graph Cn has ______edges.

Question 7: The complete graph Kp has diameter= __________

Question 8: Draw a regular graph on 6 vertices with regularity 1.