PERMUTATIONS: (Arrangements order is considered)

No of ways of arranging r objects out of total of n objects:

Case 1: With no repetition: $\longrightarrow {}^{n}P_{r}$

Case 2: With unlimited repetition: $\longrightarrow n^{r}$

No of ways of arrangements of n objects where n_1 objects are of kind 1, n_2 objects are of kind 2, …., n_k objects are of kind k, such that $n_1 + n_2 + .. + n_k = n$ $\longrightarrow \dfrac{n!}{n_1!\, n_2!\, n_3! \cdots n_k!}$

COMBINATIONS: (selection order is immaterial)

No of ways of selecting r objects out of total of n objects:

Case 1: With no repetition: $\longrightarrow {}^{n}C_{r}$

Case 2: With unlimited repetition: $\longrightarrow {}^{n+r-1}C_{r}$

(Assignment of 'r' obj's into 'n' boxes)

CASE 1: When the objects are distinct

Permutatn

Number of distributions of r distinct objects to n different cells

select 'r' boxes out of 'n' boxes with order in mind

Subcase 1: If each box can hold at most one object

(selecting r boxes out of tot of n boxes s.t there is no rep of boxes, (order ✓))

$\boxed{{}^{n}P_{r}}$

Subcase 2: If each box can hold any number of objects

$\boxed{n^{r}}$

CASE 2: When the objects are identical

Combn

Number of distributions of r identical objects to n different cells

For 'r' objects, I've to select r boxes out of total of n boxes, but order doesn't matter.

Subcase 1: If each box can hold at most one object

$\boxed{{}^{n}C_{r}}$

Subcase 2: If each box can hold any number of objects

$\boxed{{}^{n+r-1}C_{r}}$

Note:

Number of non-negative solutions of the equation

$\underbrace{x_1 + x_2 + \cdots + x_n}_{} = r$

n x_i's summing up to r

"No of ways of selecting r x_i's out of total of n x_i's with repetition allowed"

$\boxed{{}^{n+r-1}C_{r}}$

(Distn of r 1's to n boxes with rep allowed)

$x_1 + x_2 + x_3 = 6$

($n = 3$, $r = 6$.

select 6 x_i's out of $x_1\ x_2\ x_3$? with rep allowed

${}^{3+6-1}C_6 = {}^{8}C_6$

$\underline{x_1 + x_2 + x_3 = 6}$

nonneg solns → (1 1 4), (2 2 2)

$x_1\ x_1\ x_2\ x_2\ x_2\ x_2 \equiv (2, 4, 0)$

selecting any 6 x_i's out of $x_1\ x_2\ x_3$ with repetn

Q2: Find the no of permutations of the word INSTITUTION?

(i) How many of them begin with I and end with N?

(ii) How many permutations are with 3 Ts not together?

soln

11 letters — I(3), T(3), N(2) $\longrightarrow \dfrac{11!}{3!\,3!\,2!}$

i) $\boxed{I \mid \ \mid \ \mid \ \mid \cdots \mid N}$ $\longrightarrow \dfrac{9!}{2!\,3!}$

iii) 3 Ts not together = Total − 3Ts together $= \dfrac{11}{3!\,3!\,2!} - \dfrac{9!}{3!\,2!}$

3 Ts together : $\boxed{TTT \mid \quad}$

Q3: In how many ways 3 integers can be selected from $3n$ consecutive integers such that the sum is a multiple of 3?

soln

1, 2, ... 3n

ex:- 3+6+9 = 18 ✓

S_0 (Rem 0) → 3 6 9 ..... 3n

S_1 (Rem 2) → 2 5 8 ...... 3n−1

S_2 (Rem 1) → 1 4 7 ...... 3n−2

~~2 2 2 = 6~~

~~x+y+z~~

~~$S_1\ S_2\ S_2$~~

~~2 3 3 = 8~~

(All 3 elts from S_0) + (All 3 elts from S_1) + (All from S_3) + (one from S_1, one from S_2, One from S_3)

$^nC_3 + {}^nC_3 + {}^nC_3 + {}^nC_1\,{}^nC_1\,{}^nC_1$

$$\text{Ans} = 3\left({}^nC_3\right) + \left({}^nC_1\right)^3$$

Q4: In how many ways one right and one left shoe can be selected from 6 pairs of shoes without obtaining a pair?

soln

6 pairs $\Rightarrow \underbrace{6 \text{ left}}_{A} + \underbrace{6 \text{ right}}$

$${}^6C_1 \cdot {}^5C_1 = \boxed{30}$$

Q5: If 5 men A, B, C, D, E intend to speak at a meeting,

i. in how man orders can they do so without B speaking before A?

ii. How many orders are there in which A speaks immediately before B?

soln

i) B shouldn't spk before A :

A				
	4	3	2	1

3	A	3	2	1

C, D, E

3	2	A	2	1

			A	
3	2	1		1

$$\boxed{4! + 18 + 12 + 6}$$

ii) A speaks imediatly before B

A	B			

Ans = $\underline{\underline{4!}}$

Q6: A new national flag has to be designed with 6 vertical strips in yellow, green, blue and red. In how many ways can this be done s.t no two adjacent strips have same color?

soln

Y, G, B, R / 6 strips

(4)
(3)

(3)

$\boxed{Ans = 4 \times 3^5}$

Q7: In how many ways can 2 squares be selected one by one from 8 × 8 chess board such that they are not in the same row and same columns?

soln

8×8 board

64	35

$\rightarrow$ $^{64}C_1 \times {^{49}C_1} = \underline{64 \times 49}$ $(\because 64-8-7$

Q8: In how many ways can 5 different msgs be delivered by 3 messengers if no messenger is left unemployed. The order in which a messenger delivers his msg is immaterial?

soln

5 msgs $\longrightarrow$ 3 ppl s.t nobody is left unemployed

i) One person 3 msgs & other two del one each

(3 1 1, 1 3 1, 1 1 3)

$\frac{5!}{3!} + \frac{5!}{3!} + \frac{5!}{3!} = 3\left(\frac{5!}{3!}\right)$

$P_1\ P_2\ P_3 \equiv \overset{m_1 m_2 m_3}{(3, 1, 1)}$

$\neq (3, 2, 0) \times$

$(3,1,1) \neq (1,3,1)$

$P_1 \rightarrow 3 (m_1 m_2 m_3)$
$P_2 \rightarrow 1 (m_4)$
$P_3 \rightarrow 1 (m_5)$

$P_1 \rightarrow 1$ msg
$P_2 \rightarrow 3$
$P_3 \rightarrow 1$

ii) 2 ppl deliver 2 msgs and one person del one msg

(2 2 1, 1 2 2, 2 1 2)

$\boxed{\frac{5!}{2!\,2!} \times 3}$

$P_1 \rightarrow m_1, m_2 \longrightarrow (2)$
$P_2 \rightarrow m_3\ m_4 \longrightarrow (2!$
$P_3 \rightarrow \underline{m_5}$

$Ans = 3\left(\frac{5!}{3!}\right) + 3\left(\frac{5!}{2!\,2!}\right)$

If the second was'nt there, then

$3(5!) + 3(5!)$

Q9: Find the sum of all 4 digit numbers that can be obtained using the digits 1,2,3,4 once in each?

sum of all 4 dig nos formed by 1,2,3,4

1234
2134
1342
⋮
4321

Ans = ?

soln

Total no of 4 dig nos $\Rightarrow 4! = 24$

in every col, each no occurs exactly 6 times

unit : $1(6) + 2(6) + 3(6) + 4(6) = 60$

10^{th} place : $[1(6) + 2(6) + 3(6) + 4(6)] \times 10 = 600$

100^{th} place : $[1(6) + 2(6) + 3(6) + 4(6)] 100 = 6000$

1000^{th} place : $[1(6) + 2(6) + 3(6) + 4(6)] \times 1000 = 60000$

$\therefore$ Sum $= (60 + 600 + 6000 + 60000)$

$= \underline{66660}$

Q10: In how many ways can an examiner assign 30M to 8 questions such that no question receives less than 2 marks?

soln

Dist 30M to 8 Qs s.t each Q recieves ≥ 2 marks

$Q_1, Q_2 \ldots Q_8$
$\downarrow \ \downarrow \ \ \downarrow$
$2 \ \ 2 \ \ \ 2$

Give 2M each to all 8 questn first. $(30-16)$ are left

$\therefore$ Rem 14 marks should be distd to 8 questn

s.t each questn recieves any no of marks

$n = 8 \quad r = 14, \quad {}^{n+r-1}C_r = \boxed{{}^{14+8-1}C_{14}}$

Q11: 3 identical dice are rolled. How many outcomes can be recorded?

soln

$n = 6 \quad r = 3$

$\boxed{{}^{6+3-1}C_3}$

selecting 3 nos out of tot of 6 nos (1,2,3,4,5,6) with repetition

Dist of r obj to n boxes s.t
i) every box can get any no of objs
ii) every box can get atmost one obj

or

selecting r objs out of n objs s.t
i) with rep
ii) no rep

if order is considered

6^3

1. How many first thousand +ve integers have distinct digits?

Ans = 738

2. In how many ways can 12 white and 12 black pawns can be placed on black squares of 8x8 chessboard?

$$Ans = {}^{32}C_{12}\,{}^{20}C_{12}$$

3. How many 7 letters palindromes are there?

$$Ans = 26 \times 25 \times 24 \times 23 \longrightarrow \text{wdt rep}$$

$$26^4 \longrightarrow \text{wd ''}$$

4. A shop sells 6 diff flavours of icecream. In how many ways, a customer can choose 4 icecream cones if

i) they are all of diff flavours
ii) they are not necessarily of diff flavours
iii) they contain only 2 or 3 flavours

soln

i) ${}^{6}C_4$ ii) ${}^{6+4-1}C_4$ iii) 105

5. How many integer solns are there to the eqn

$$x_1 + x_2 + x_3 + x_4 = 12 \quad \text{with } x_i \geq 0$$

i) How many of them are s.t $x_i \geq 1$
ii) How many of them with $x_1 \geq 2$, $x_2 \geq 2$, $x_3 \geq 4$ & $x_4 \geq 0$

soln

455

i) ${}^{11}C_8$ ii) ${}^{7}C_4$

6. In how many ways can 2 adjacent squares be selected from 8x8 chess board?

soln

112