Engineering Mathematics – III

Chapter 1 : Permutation and Combination

Permutations and Combinations: With and without repetition, identical objects, examples, Distributions, Problems on permutations and combinations,Principle of Inclusion and Exclusion (statement only), problems, derangement,Partitions and Compositions, Ferrers Graph, Generating Functions

Ordering of permutations – Lexicographical and reverse Lexicographical, Fike's ordering of permutations

Chapter 2 : Boolean Algebra

Partial Ordering Relations and Posets, Chains and anti-Chains, Lattices and Algebraic Systems, Distributive and Complemented Lattices, Boolean Lattices and Boolean Algebra, Boolean Functions and Boolean Expressions

Chapter 3 : Graph Theory

Graphs – Basic definitions, Basic properties and problems, Isomorphism and self-complementary graphs, Connectedness of a graph, Eulerian and Hamiltonian graphs, Center, radius, diameter of a graph, Trees and Properties

Matrices related to graphs, Dijkstra's algorithm for finding the shortest path

Chapter 4 : Group Theory

Semi-groups, Monoids and Groups – Definitions and examples, Elementary properties of groups and problems, Subgroups and related problems, Cosets of a group and related problems, Lagrange's Theorem and related problems, Cyclic groups and proerties

Normal subgroups and properties

Chapter 5 : Proposition and Predicate Calculus

Propositional calculus – Basic definitions, Connectives Well-formed formulas and tautologies, Equivalence formulas and tautological implications, inference theory of propositional calculus, Predicate calculus – Basic denfifitions, quantifications, Inference theory of Predicated calculus

Reference Books:

Permutations & Combinatorics: An Introduction to Computational Combinatorics – Page & Wilson Applied Combinatorics – Alan Tucker

Lattice Theory: Elements of Discrete Mathematics – C.L. Liu

Graph Theory: Graph Theory – Harary
Graph theory with Application to computer science, PHI, 1987, by Narasingh Deo,

Group Theory: Topics in Algebra – Herstein

Propositional and Predicate Calculus: Discrete Mathematical Structures with Applications to Computer Science – Tremblay & Manohar

Chapter 1 : Permutation and Combination

Rule of Sum : If the object 'A' may be chosen in 'm' ways and 'B' in 'n' ways, then either A or B (exactly one) chosen in

$m+n$ ways.

Rule of Product : If the object A may be chosen in 'm' ways and 'B' in 'n' ways, then one of A and one of B chosen in

mn ways.

Distribution : is defined as a separation of a set of objects into a number of classes.

Permutation (Arrangement)

Let $a_1, a_2, \ldots a_n$ be n distinct elements.

An r-permutation of the n elements is an ordered selection of r of the n elements.

1 2 3 4 5

5 2 _ _ _

↑

2 5

Distribution of distinct objects is permutation

Case i : Consider 'n' distinct objects.

The arrangement of 'r' of the n objects without repetition can be done is $\underline{{}^{n}p_{r}}$ ways.

First object can be chosen in n ways.

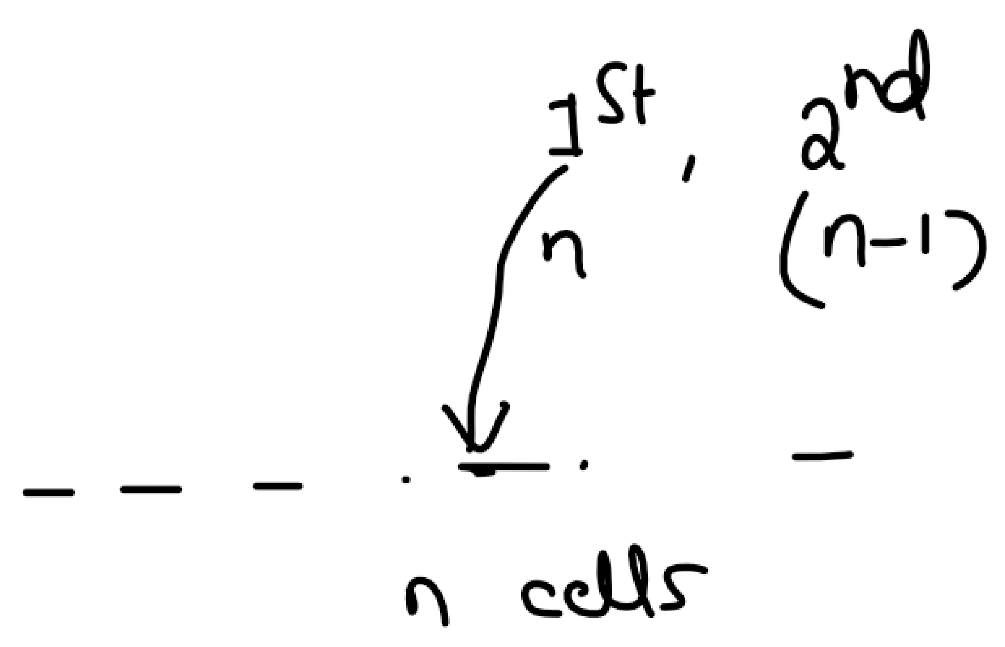
2nd " " " (n-1) "

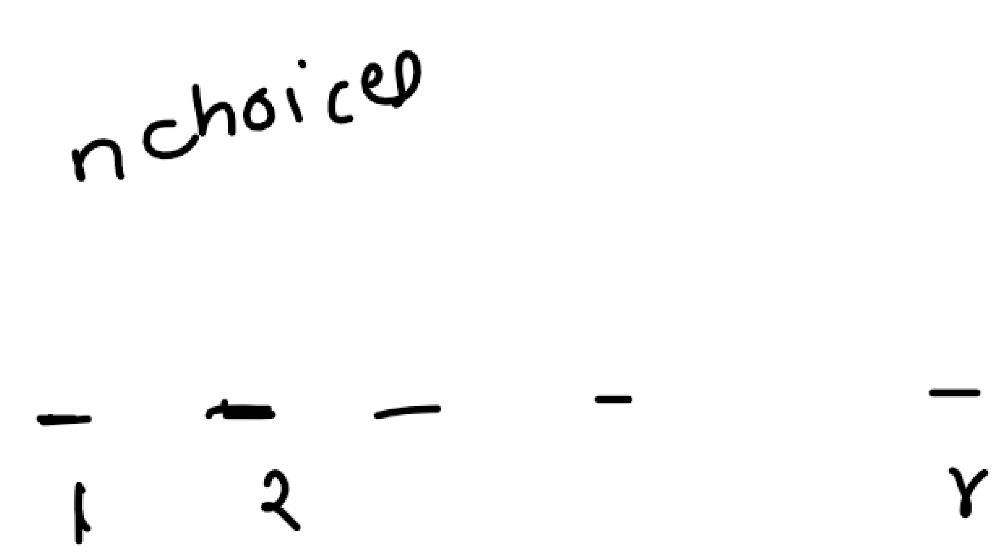
⋮

r^{th} " " " $(n-(r-1))$ "

$${}^{n}p_{r} = n \times (n-1) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!}$$

Distribution of r distinct objects into n distinct cells such that each cell has at most one object is ${}^{n}p_{r}$





Case ii : Consider 'n' distinct objects. The arrangement of r of these n objects with repetition is $\underline{n^{r}}$

1st object in n ways

2nd " " n "

⋮

r^{th} " " n "

Distribution of r- distinct objects in to n distinct cells such that each cell can hold any no of objects is n^r.

(iii) Suppose that we have 'n' distinct objects. Out of which m_1 are of one kind, m_2 are of the 2nd kind, ..., m_k are of k^{th} kind. Then the number of permutation of the object taken at a time is

$$\frac{n!}{m_1!\, m_2! \ldots m_k!}$$

MADAM ABCD

$5 \times \overset{A}{4} \times 3 \times 2 \times \overset{A}{1}$ $\frac{5!}{2!\,2!}$

Combination (Selection)

An r- combination of n objects is an un ordered selection of r objects from n objects.

$$nc_r = \frac{n!}{(n-r)!\,r!}$$

D D D D

$$\frac{n \times (n-1) \times \cdots \times (n-r+1)}{r!}$$

A
B
C

ABC
$=$ BAC $=$ CAB

selecting r objects at a time from n objects without repetition is $nc_r = \frac{n!}{(n-r)!\,r!}$

Distribution of r identical objects in to n distinct cells such that each cell hold atmost one object is nc_r.

Problems

1) Find the number of ways in which 3 exams can be scheduled in a 5 day period such that

(i) no two exams are scheduled on the same day

(ii) there is no restriction on number of exams conducted on a day.

M T W Th F

$\underline{5} \times \underline{4} \times \underline{3}$

$\uparrow$ I^{st}

Soln: (i) 60

(ii) $5 \times 5 \times 5 = 125$ ways

2. Find the number of different letter arrangement that can be formed using SYSTEMS ?

Soln: $\frac{7!}{3!} = 840$

3. A new flag is to be designed with 6 vertical strips in yellow, green, blue and red. In how many ways can this be done so that no two adjacent strips have the same color ?

Soln: 4×3^5

$\underline{4} \times \underline{3} \times \underline{3} \times \underline{3} \times \underline{3} \times \underline{3}$

4. The number of ways in which one right and one left shoe be selected from 6 pair of shoes without obtaining a pair is $\underline{6 \times 5 = 30}$

5. In how many ways can 2 integers be selected from the integers 1, 2, ... , 100 so that their difference is exactly 7?

Soln : (1, 8) (2, 9) ... ... (10, 17) ... (93, 100)

(9 16) –

1, 2, ... 93 $93C_1 \times 1C_1$

$\underline{\underline{93}}$

(ii) If the difference is 7 or less

diff 7 : 93

6 : 94

1 : 99 (1 1)

0 : Not possible

(∵ we are considering no repetition)

93 + 94 + ... + 99 =

6. How many +ve integers less than one million can be formed using 7's, 8's and 9's only? (ii) How many using 0's, 8's and 9's only?

Soln: $< 10,00,000$

(i) 6 digit: $\underline{3} \times \underline{3} \times \cdots \underline{\ }\ \underline{\ }\ \underline{3} \longrightarrow 3^6$

5 digit $\_\ \_\ \_\ \_\ \_ \longrightarrow 3^5$

4 digit $\_\ \_\ \_\ \_ \longrightarrow 3^4$

3 digit $\_\ \_\ \_ \longrightarrow 3^3$

2 digit $\_\ \_ \longrightarrow 3^2$

one digit no: $\_ \longrightarrow 3$

Ans: $\overline{1092}$

(ii) 6 digit: $\underset{7 \text{ or } 8}{2} \times \underset{0\,7\,8}{3} \times 3 \times 3 \times 3 \times 3 \rightarrow 2 \times 3^5$

5 digit: $\rightarrow 2 \times 3^4$

4 digit: $\rightarrow 2 \times 3^3$

3 digit: $\rightarrow 2 \times 3^2$

2 digit: $\rightarrow 2 \times 3$

1 digit: $\rightarrow 2$

$\underline{\underline{728}}$

7. Find the sum of 4 digit numbers that can be formed using the digits 1, 2, 3, 4 once in each.

Soln: $4 \times 3 \times 2 \times 1 : 4!$ → Total 24 numbers.

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3	4
1342	2341	3	4
1423	2413	3	4
1432	2431	3	4

Each digit occupies each place

$\frac{24}{4} = 6$ times

$$6 \{ (1+2+3+4) 1000 + (1+2+3+4) 100 + (1+2+3+4) 10 + (1+2+3+4) \} = 66660$$

Note: Suppose any digit can occur any number of times then sum of all 4 digit numbers is ______

Total: $4^4 = 256$

Each digit occupies each place $\frac{256}{4} = 64$ times.

Ans: $64 \{ (1+2+3+4) 1000 + (1+2+3+4) 100 + (1+2+3+4) 10 + (1+2+3+4) \} =$