In Semester Examinations CCE / IT / CSE

1) If lattice is distributive, then

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \rightarrow (1)$

Since $c \leq a \vee c$ and $a \vee b \leq a \vee b$

we have $(a \vee b) \wedge c \leq a \vee b (a \vee c)$

$= a \vee (b \wedge c)$ [From (1)]

Conversely,

Suppose $(a \vee b) \wedge c \leq a \vee (b \wedge c) \rightarrow (2)$

To prove $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Consider $(a \vee b) \wedge (a \vee c)$

$\leq a \vee (b \wedge (a \vee c))$ [From (2)]

$\leq a \vee [(a \vee c) \wedge b]$ Commutativity

$\leq a \vee [a \vee (c \wedge b)]$ Given

$\leq (a \vee a) \vee (c \wedge b)$ associativity

$\leq a \vee (b \wedge c) \rightarrow (3)$

also $a \leq a \vee b$, $a \leq a \vee c$

$\therefore a \leq (a \vee b) \wedge (a \vee c)$

also $b \leq a \vee b$, $c \leq a \vee c$

$b \wedge c \leq (a \vee b) \wedge (a \vee c)$

$\therefore a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c) \rightarrow (4)$

From (3) and (4)

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

2) Let a_1, a_2 and a_3 are properity that integer n is divisible by p, q and r respectively

$$N(a_1' a_2' a_3') = N - [N(a_1) + N(a_2) + N(a_3)] + [N(a_1 a_2) + N(a_1 a_3) + N(a_2 a_3)] - N(a_1 a_2 a_3)$$

$$= n - \left(\frac{n}{p} + \frac{n}{q} + \frac{n}{r}\right) + \frac{n}{pq} + \frac{n}{pr} + \frac{n}{qr} - \frac{n}{pqr}$$

$$= n\left(1 - \frac{1}{p}\right)\left(1 - \frac{1}{q}\right)\left(1 - \frac{1}{r}\right)$$

3) Consider a partition of n in which no part is greater than k and consider the ferrers graph representation then the no of dots in each row must be less than or equal to k but if we read the same partition columnwise then the no of parts is less than or equal to k.

Conversely, Consider a partition with atmost k parts then the no of rows in ferrers graph is less than or equal to k and in the conjugate partition, size of any column is less than or equal to k. Hence proved.

$$G.F = (1-x)^{-1}(1-x^2)^{-1} \cdots (1-x^k)^{-1}.$$

4) $$\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)^2 \left(x + \frac{x^3}{3!} + \cdots\right)^2 \left(1 + x + \frac{x^2}{2!} + \cdots\right)$$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2 \left(\frac{e^x - e^{-x}}{2}\right)^2 \cdot e^x$$

$$= \frac{e^x}{4}\left[e^{4x} + e^{-4x} - 2\right]$$

$$= \frac{1}{4}\left[e^{5x} + e^{-3x} - 2e^x\right]$$

$\therefore$ co-eff of $\frac{x^{10}}{10!}$ is

$$= \frac{1}{4}\left[5^{10} + (-3)^{10} - 2 \times 1^{10}\right]$$

5)

x_1	x_2	x_3	$E = ((\bar{x}_1 \wedge x_2) \wedge x_3) \vee (\bar{x}_3 \wedge \bar{x}_2) \vee (\bar{x}_2 \vee x_1)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$DNF = (\bar{x}_1 \wedge \bar{x}_2 \wedge \bar{x}_3) \vee (\bar{x}_1 \wedge \bar{x}_2 \wedge x_3) \vee (\bar{x}_1 \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2 \wedge \bar{x}_3)$$
$$\vee (x_1 \wedge \bar{x}_2 \wedge \bar{x}_3) \vee (x_1 \wedge \bar{x}_2 \wedge x_3)$$
$$\vee (x_1 \wedge x_2 \wedge x_3)$$

$$CNF = x_1 \vee \bar{x}_2 \vee x_3$$

6 i) $K = 73$, $n = 5$

$$K - 1 = C_{n-1}(n-1)! + C_{n-2}(n-2)! + \dots + C_1$$
$$+ 0 \times 1!$$

ie $72 = 3 \times 4! + 0 \times 3! + 0 \times 2!$

ie 3000

		2
3000	5 ~~4~~ 3 $\underline{2}$ 1	5
000	$\underline{5}$ 4 3 1	4
00	4 3 1	3
0	3 1	
	1	1

∴ 73^{rd} permutation in reverse lexicographical order is

1 3 4 5 2

Similarly when $k = 97$

$$96 = 4 \times 4! + 0 \times 3! + 0 \times 2! + 0 \times 1!$$

4000

4000	5	4	3	2	1	1
	5	4	3	2		5
	4	3	2			4
	3	2				3
						2

∴ 23451 is the 97^{th} permutation in reverse lexicographical order.

ii) $$K - 1 = c_1 \times \frac{n!}{2!} + c_2 \times \frac{n!}{3!} + \dots + c_{n-1} \times \frac{n!}{n!}$$

ie $$72 = c_1 \times 60 + c_2 \times 20 + c_3 \times 5 + c_4 \times 1$$

ie $$72 = 1 \times 60 + 0 \times 20 + 2 \times 5 + 2 \times 1$$

$\Rightarrow$ 1 0 2 2

∴
$$\begin{array}{r} 1234 \\ 1022 \\ \hline 0212 \end{array}$$
is the pike's sequence

0	1	2	3	4	5
2	2	1	3	4	5
1	2	1	3	4	5
2	2	4	3	1	5
	2	4	5	1	3

$\therefore$ 73^{rd} permutation in Fike's order is 24513

when $k = 97$,

$96 = \cancel{4 \times 4!} \quad \cancel{I} \; 0$

$96 = 1 \times 60 + 1 \times 20 + 3 \times 5 + 1 \times 1$

1 1 3 1

$\therefore$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 1 & 1 & 3 & 1 \\ \hline 0 & 1 & 0 & 3 \end{array}$$

0	1	2	3	4	5
1	2	1	3	4	5
0	2	3	1	4	5
3	4	3	1	2	5
	4	3	1	5	2

$\therefore$ 97^{th} permutation in Fiker order is 43152