Predicate calculus

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Example

Consider two propositions, Ram is a Bachelor, Shyam is a Bachelor. Both Ram and Shyam have the same property of having bachelor. The part "is a bachelor" is called a predicate.

The predicate is denoted by capital letters and names of individuals or objects by small letters.

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Let "B" denote the predicate "is bachelor", then the sentence "x is a bachelor" can be written as $B(x)$, where x is a predicate variable $B(x)$ is also called a propositional function, which becomes a statement when values are submitted in place of x.

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Example

x is taller than y; T(x,y)- the two place predicate

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Example

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$M(x) : x$ is a human being; $\quad H(x) : x$ is a mortal

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$N(x)$: x is an integer. $P(x)$: x is either positive or negative

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$N(x)$: x is an integer. $P(x)$: x is either positive or negative
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The phrase "there exists" ($\exists$) is called the **EXISTENTIAL QUANTIFIER**

Example

There exists x such that $x^2 = 5$. This can be written as $(\exists x)P(x)$ where $P(x) : x^2 = 5$.

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or There exists at least one x such that x is a man and x is clever.

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$M(x) : x$ is a man. $C(x) : x$ is clever

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or There exists at least one x such that x is a man and x is clever.
$M(x)$: x is a man. $C(x)$: x is clever
$(\exists x)(M(x) \wedge C(x))$

NOTE 1

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- there is an x
- for some x
- there is atleast one x

- $\neg(\forall x)P(x) \Leftrightarrow (\exists x)\neg P(x)$

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Question 4

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& \Leftrightarrow (\exists x)\neg(P(x) \to Q(x)) \\
& \Leftrightarrow (\exists x)\neg(\neg P(x) \vee Q(x)) \\
& \Leftrightarrow (\exists x)(P(x) \wedge \neg Q(x))
\end{aligned}$$

That is there exists a real number x such that $x > 3$ and $x^2 \leq 9$

Exercise Q5 : Negate the following statement " Every city in Canada is clean"

Rules of Inference

The method of derivation involving predicate formulas uses the rules of inference given for the statement calculus and also certain additional rules which are required to deal with the formulas involving quantifiers (In addition to Rule P and T)

(1) Rule US(Universal Specification)
From $(\forall x)A(x)$, we can conclude $A(y)$.

$$(\forall x)A(x) \implies A(y)$$

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From $(\forall x)A(x)$, we can conclude $A(y)$.

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(2) Rule ES (Existential Specification)
From $(\exists x)A(x)$ one can conclude $A(y)$ provided that y is not free in any given premise and also not free in any prior step of the derivation.

$$(\exists x)A(x) \implies A(y)$$

(3) Rule EG(Extential Generalization)
From $A(x)$ one can conclude $(\exists y)A(y)$.

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From $A(x)$ one can conclude $(\exists y)A(y)$.

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(4) Rule UG (Universal Generalization)
From $A(x)$ one can conclude $(\forall y)A(y)$ provided that x is not free in any of the given premises and provided that if x is free in a prior step which resulted from use of ES, then no variables introduced by that use of ES appear free in $A(x)$

Question 5

$$\text{Show that } (\forall x)[H(x) \to M(x)] \wedge H(s) \implies M(s)$$

[Note that this problem is a symbolic representation or translation of a well known arguement known as "Socrates arguement" which is given by
"All men are mortal
Socrates is a man
Therefore Socrates is a mortal"]

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Solution : Denote $H(x)$: x is a man
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$$(1)\ (\forall x)[H(x) \rightarrow M(x)] \qquad \textit{Rule P}$$

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(1) & (\forall x)[H(x) \to M(x)] & \textit{Rule P} \\
(2) & H(s) \to M(s) & \textit{Rule US} \\
(3) & H(s) & \textit{Rule P}
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(1)	$(\forall x)[H(x) \to M(x)]$	*Rule P*
(2)	$H(s) \to M(s)$	*Rule US*
(3)	$H(s)$	*Rule P*
(4)	$M(s)$	*Rule* $T(P \wedge (P \to Q) \Rightarrow Q)$

Question 6

Show that $(\exists x)M(x)$ *follows logically from the premises*

$$(\forall x)[H(x) \to M(x)] \text{ and } (\exists x)H(x)$$

Solution

(1) $(\exists x)H(x)$ *Rule P*

Question 6

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Solution

(1)	$(\exists x)H(x)$	*Rule P*
(2)	$H(y)$	*Rule ES*

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Solution

(1)	$(\exists x)H(x)$	*Rule P*
(2)	$H(y)$	*Rule ES*
(3)	$(\forall x)(H(x) \rightarrow M(x))$	*Rule P*

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Solution

(1)	$(\exists x)H(x)$	*Rule P*
(2)	$H(y)$	*Rule ES*
(3)	$(\forall x)(H(x) \rightarrow M(x))$	*Rule P*
(4)	$(H(y) \rightarrow M(y))$	*Rule US by* (3)
(5)	$M(y)$	*Rule T* $(i.e., P \wedge (P \rightarrow Q) \Rightarrow Q)$

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Solution

(1)	$(\exists x)H(x)$	*Rule P*
(2)	$H(y)$	*Rule ES*
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(4)	$(H(y) \to M(y))$	*Rule US by* (3)
(5)	$M(y)$	*Rule T* $(i.e., P \wedge (P \to Q) \Rightarrow Q)$ *by* (2) *and* (4)

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(4)	$(H(y) \rightarrow M(y))$	*Rule US by* (3)
(5)	$M(y)$	*Rule T* $(i.e., P \wedge (P \rightarrow Q) \Rightarrow Q)$ *by* (2) *and* (4)
(6)	$(\exists x)M(x)$	*Rule EG*

Show that

$$(\forall x)(P(x) \rightarrow Q(x)) \wedge (\forall x)(Q(x) \rightarrow R(x)) \implies (\forall x)(P(x) \rightarrow R(x))$$

Solution

$$(1) \qquad (\forall x)(P(x) \rightarrow Q(x)) \qquad \textit{Rule P}$$

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(2)	$P(y) \to Q(y)$	*Rule US*

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(4)	$Q(y) \to R(y)$	*Rule US by* (3)

Show that

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Solution

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(2)	$P(y) \to Q(y)$	*Rule US*
(3)	$(\forall x)(Q(x) \to R(x))$	*Rule P*
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(5)	$P(y) \to R(y)$	*Rule T*

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(4)	$Q(y) \to R(y)$	*Rule US by* (3)
(5)	$P(y) \to R(y)$	*Rule T* *by* (2) *and* (4)

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(4)	$Q(y) \to R(y)$	*Rule US by* (3)
(5)	$P(y) \to R(y)$	*Rule T*
		by (2) *and* (4)
(6)	$(\forall x)(P(x) \to R(x))$	*Rule UG*

Show that from

$$(a)\ (\exists x)(F(x) \land S(x)) \to (y)(M(y) \to W(y))$$

$$(b)\ (\exists y)(M(y) \land \neg W(y))$$

the conclusion $(x)(F(x) \to \neg S(x))$ follows.

Solution

(1)	$(\exists y)(M(y) \wedge \neg W(y)$	*Rule P*

Solution

$$
\begin{array}{llr}
(1) & (\exists y)(M(y) \wedge \neg W(y) & \textit{Rule P} \\
(2) & M(z) \wedge \neg W(z) & \textit{Rule ES}
\end{array}
$$

Solution

(1)	$(\exists y)(M(y) \wedge \neg W(y)$	*Rule P*
(2)	$M(z) \wedge \neg W(z)$	*Rule ES*
(3)	$\neg(M(z) \rightarrow W(z))$	*Rule T*

Solution

(1)	$(\exists y)(M(y) \wedge \neg W(y)$	*Rule P*
(2)	$M(z) \wedge \neg W(z)$	*Rule ES*
(3)	$\neg(M(z) \rightarrow W(z))$	*Rule T*
(4)	$(\exists y)\neg(M(y) \rightarrow W(y))$	*Rule EG*

Solution

(1)	$(\exists y)(M(y) \wedge \neg W(y)$	*Rule P*
(2)	$M(z) \wedge \neg W(z)$	*Rule ES*
(3)	$\neg(M(z) \rightarrow W(z))$	*Rule T*
(4)	$(\exists y)\neg(M(y) \rightarrow W(y))$	*Rule EG*
(5)	$\neg(y)(M(y) \rightarrow W(y))$	*Rule T*

Solution

$$
\begin{array}{lrl}
(1) & (\exists y)(M(y) \wedge \neg W(y) & Rule\ P \\
(2) & M(z) \wedge \neg W(z) & Rule\ ES \\
(3) & \neg(M(z) \rightarrow W(z)) & Rule\ T \\
(4) & (\exists y)\neg(M(y) \rightarrow W(y)) & Rule\ EG \\
(5) & \neg(y)(M(y) \rightarrow W(y)) & Rule\ T \\
(6) & (\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y)) & Rule\ P
\end{array}
$$

Solution

(1)	$(\exists y)(M(y) \wedge \neg W(y)$	*Rule P*
(2)	$M(z) \wedge \neg W(z)$	*Rule ES*
(3)	$\neg(M(z) \rightarrow W(z))$	*Rule T*
(4)	$(\exists y)\neg(M(y) \rightarrow W(y))$	*Rule EG*
(5)	$\neg(y)(M(y) \rightarrow W(y))$	*Rule T*
(6)	$(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$	*Rule P*
(7)	$\neg(\exists x)(F(x) \wedge S(x))$	*Rule T by* (5)*and*(6)

Solution

(1)	$(\exists y)(M(y) \wedge \neg W(y)$	*Rule P*
(2)	$M(z) \wedge \neg W(z)$	*Rule ES*
(3)	$\neg(M(z) \rightarrow W(z))$	*Rule T*
(4)	$(\exists y)\neg(M(y) \rightarrow W(y))$	*Rule EG*
(5)	$\neg(y)(M(y) \rightarrow W(y))$	*Rule T*
(6)	$(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$	*Rule P*
(7)	$\neg(\exists x)(F(x) \wedge S(x))$	*Rule T by* (5)*and*(6)
(8)	$(x)\neg(F(x) \wedge S(x))$	*Rule T*

Solution

$$
\begin{array}{lrl}
(1) & (\exists y)(M(y) \wedge \neg W(y)) & \textit{Rule P} \\
(2) & M(z) \wedge \neg W(z) & \textit{Rule ES} \\
(3) & \neg(M(z) \rightarrow W(z)) & \textit{Rule T} \\
(4) & (\exists y)\neg(M(y) \rightarrow W(y)) & \textit{Rule EG} \\
(5) & \neg(y)(M(y) \rightarrow W(y)) & \textit{Rule T} \\
(6) & (\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y)) & \textit{Rule P} \\
(7) & \neg(\exists x)(F(x) \wedge S(x)) & \textit{Rule T by } (5) \textit{and} (6) \\
(8) & (x)\neg(F(x) \wedge S(x)) & \textit{Rule T} \\
(9) & \neg(F(x) \wedge S(x)) & \textit{Rule US}
\end{array}
$$

Solution

(1)	$(\exists y)(M(y) \wedge \neg W(y))$	*Rule P*
(2)	$M(z) \wedge \neg W(z)$	*Rule ES*
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(6)	$(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$	*Rule P*
(7)	$\neg(\exists x)(F(x) \wedge S(x))$	*Rule T by* $(5) and (6)$
(8)	$(x)\neg(F(x) \wedge S(x))$	*Rule T*
(9)	$\neg(F(x) \wedge S(x))$	*Rule US*
(10)	$F(x) \rightarrow \neg S(x)$	*Rule T*

Solution

(1)	$(\exists y)(M(y) \wedge \neg W(y)$	*Rule P*
(2)	$M(z) \wedge \neg W(z)$	*Rule ES*
(3)	$\neg(M(z) \rightarrow W(z))$	*Rule T*
(4)	$(\exists y)\neg(M(y) \rightarrow W(y))$	*Rule EG*
(5)	$\neg(y)(M(y) \rightarrow W(y))$	*Rule T*
(6)	$(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$	*Rule P*
(7)	$\neg(\exists x)(F(x) \wedge S(x))$	*Rule T by* (5)*and*(6)
(8)	$(x)\neg(F(x) \wedge S(x))$	*Rule T*
(9)	$\neg(F(x) \wedge S(x))$	*Rule US*
(10)	$F(x) \rightarrow \neg S(x)$	*Rule T*
(11)	$(x)(F(x) \rightarrow \neg S(x))$	*Rule UG*

Show that $(x)(P(x) \vee Q(x)) \implies (x)P(x) \vee (\exists x)Q(x)$