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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER B.E DEGREE MAKE UP EXAMINATION- DECEMBER 2010

SUB: ENGG. MATHEMATICS I (MAT – 101)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All questions carry equal marks

1A. Find the n^{th} derivatives of the following

(i) $\dfrac{x^2}{2x^2+7x+6}$ (ii) $\cosh x.\cos 3x$

1B. Trace the following curve with explanations $y^2(a-x) = x^3$, $a > 0$

1C. Find the image of the line $\dfrac{x-1}{2} = \dfrac{y-2}{1} = \dfrac{z-3}{4}$ in the plane $2x + y + z = 6$.

(4 + 3+3)

2A. If $y = \sin\left(m\sin^{-1}x\right)$, show that $(1-x^2)y_{n+2} = (2n+1)\,x\,y_{n+1} + (n^2-m^2)\,y_n$

2B. Obtain the reduction formula for $\int \sin^n x\,dx$ and hence evaluate $\int_0^{\pi/2} \cos^n x\,dx$.

2C. A variable plane at a constant distance p from the origin meets the coordinate axes at A, B, C. Through A, B, C planes are drawn parallel to coordinate planes. Show that locus of their point of intersection is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

(4 + 3+ 3)

3A. Find the nature of the series

(i) $\sum_{n=1}^{\infty} \dfrac{n!2^n}{n^n}$ (ii) $\dfrac{x}{1} + \dfrac{1}{2}\dfrac{x^3}{3} + \dfrac{1.3}{2.4}\dfrac{x^5}{5} + \dfrac{1.3.5}{2.4.6}\dfrac{x^7}{7} + \ldots$

3B. Sketch and find perimeter of the curve $r = a(1 - \cos\theta)$, $a > 0$

3C. Find the evolute of $y^2 = 4ax$.

(4 + 3+ 3)

4A. Evaluate :

(i) $\lim_{x \to 0} \left(\frac{a^x + b^x}{2} \right)^{1/x}$

(ii) $\lim_{x \to 0} \frac{\tan x - x}{x^2 \tan x}$

4B. Find the angle between the curves
$r^m = a^m \cos m\theta, \quad r^m = a^m \sin m\theta, \quad a > 0$.

4C. Find the centre and the radius of the circle of intersection by the plane x+4y+z = 4 and the sphere $x^2 + y^2 + z^2 - x - z - 2 = 0$.

(4 + 3+ 3)

5A. Find the first three nonzero terms in the Maclaurin's series expansion $\tan x$.

5B. The tangents at two points P, Q on the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ are at right angles. Show that if ρ_1 and ρ_2 be the radii of curvature at the points, then show that $\rho_1^2 + \rho_2^2 = 16a^2$.

5C. Find the volume of the solid generated by revolution of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ about the x – axis.

(4 + 3+ 3)

6A. (i) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y} \right)$ then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$$

6B. State and prove Lagrange's mean value theorem.

6C. Find the maximum possible error in calculating g if $T = 2\pi\sqrt{\frac{l}{g}}$, given 1% and 0.5% errors in l and T respectively.

(4 + 3+ 3)

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