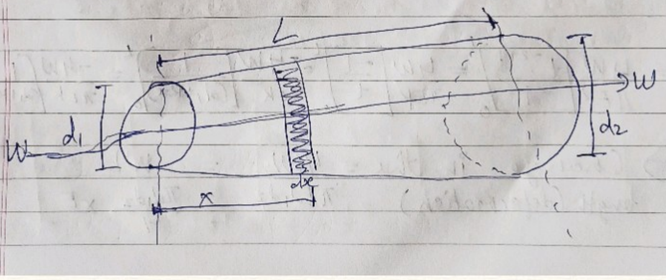
Deformation in Circular tapered bar

Deformation of tapered bar with circular cross-section.

$$\delta = \frac{4WL}{\pi E d_1 d_2}$$

Where, W = Axial force, E = Modulus of elasticity, L = Length of the member, d_1 = smaller diameter, d_2 = Larger diameter, δ = Change in length of tapered bar.



Consider an element of length, dx at a distance x for A.

$$\text{Diameter at } x = d_1 + \frac{(d_2 - d_1)}{L} \times x = d_1 + K \times x$$

$$\text{C/S area at } x = \frac{\pi d_x^2}{4} = \frac{\pi}{4}(d_1 + Kx)^2$$

$$\text{Change in length over a length dx is} = \frac{PE}{E}\left(\frac{PL}{AE}\right)_{dx}$$

$$= \left(\frac{W dx}{\frac{\pi}{4}(d_1 + Kx)^2 \times E}\right)$$

$$\text{Change in Length over a length } L \text{ is} = \int_0^L \left(\frac{W dx}{\frac{\pi}{4}(d_1 + Kx)^2 \times E}\right)$$

$$= \int_0^L \left(\frac{W \frac{dt}{K}}{\frac{\pi}{4}(t)^2 \times E}\right)$$

Put $d_1 + Kx = t$,
Differentiating w.r.t x
Then $K dx = dt$.

$$= \frac{4W}{\pi E K}\left[\frac{t^{-2+1}}{-1}\right]_0^L = \frac{4W}{\pi E K}\left[\frac{-1}{t}\right]_0^L = \frac{-4W}{\pi E K}\left[\frac{1}{(d_1 + Kx)}\right]_0^L = \frac{-4W}{\pi E K}\left[\frac{1}{(d_1 + KL)} - \frac{1}{(d_1)}\right]$$

$$\Rightarrow \text{Change in the length (deformation)} = \frac{4WL}{\pi E d_1 d_2} = \frac{WL}{\frac{\pi d_1 d_2}{4} \times E}$$