

Practice problems: Reducible to exact equation

1. Solve: $x dy - y dx + a (x^2 + y^2) dx = 0$

Hint: dividing by $x^2 + y^2$

Ans: $\tan^{-1} (y/x) + ax = c$

2. Solve : $x dx + y dy = \frac{a^2 (x dy - y dx)}{x^2 + y^2}$

Ans: $\frac{x^2}{2} + \frac{y^2}{2} = a^2 \tan^{-1} \frac{y}{x}$

3. Solve: $(x^2 + y^2 + x) dx - (2x^2 + 2y^2 - y) dy = 0$

Hint: dividing by $x^2 + y^2$

Ans: $x - 2y + \frac{1}{2} \log(x^2 + y^2) = c$

4. Solve: $(x^n y^{n+1} + ay) dx + x^{n+1} y^n + bx) dy = 0$

Hint: dividing by xy

Ans: $\frac{x^n y^n}{n} + a \ln x + b \ln y = c$

5. Solve: $y dx + (x + x^3 y^2) dy = 0$

Hint: dividing by $x^3 y^3$

Ans: $\frac{-1}{2x^2 y^2} - \ln y = c$

6. Solve: $y(8x - 9y)dx + 2x(x - 3y)dy = 0$

Hint: $\frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = \frac{2}{x}$

Ans: $2x^4 y - 3x^3 y^2 = c$

7. Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

Hint $\frac{1}{M} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = -\frac{3}{y}$

Ans $\left(y + \frac{2}{y^2} \right) x + y^3 = C$

8. Solve $(6x^2 + 4y^3 + 12y)dx + 3x(1 + y^2)dy = 0$

Hint: $\frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = \frac{3}{x}$

Ans: $x^6 + x^4 y^3 + 3yx^4 = C$