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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER B.E DEGREE MAKEUP EXAMINATION – January, 2009

SUB: ENGG.MATHEMATICS I (MAT – 101)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

Note : a) Answer any FIVE full questions.
b) All question carry equal marks.

1A. Find the n^{th} derivative of

(i) $\dfrac{x^2 - x - 1}{2x^3 + 5x^2 + 4x + 1}$ (ii) $y = \sinh 2x \cos^2 x$

1B. Trace the following curve with explanation $3ay^2 = x(x - a)^2$.

1C. A plane meets the coordinate axes at A, B, C such that the centroid of the triangle ABC is the point (a, b, c). Show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$.

(4 + 3+ 3)

2A. If $y = (x^2 - 1)^n$, prove that
$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)\, y_n = 0$

2B. Obtain the reduction formula for $\int \sin^m x \cos^n x dx$ and hence evaluate
$\int_0^{\pi/2} \sin^m x \cos^n x dx$ for all positive integers m and n.

2C. Show that the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$ are coplanar. Find their point of intersection and the plane in which they lie.

(4 + 3+ 3)

3A. Find the nature of the series

(i) $1 + \frac{3}{7} + \frac{3.6}{7.10} + \frac{3.6.9}{7.10.13} + \ldots$ (ii) $1 + \frac{x}{2} + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \ldots.$

3B. Sketch and find the area bounded by the curve $x = a(\theta + \sin\theta)$, y a $(1 - \cos\theta)$ and it base.

3C. Find the evolute of $2xy = a^2$.

(4 + 3+ 3)

4A. Evaluate :

(i) $\lim_{x \to 0} \frac{x \cos x - \log(1+x)}{x^2}$ (ii) $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{1/x}$

4B. Find the angle between the curves
$r^m = a^m \cos m\theta, \quad r^m = a^m \sin m\theta, \quad a > 0$

4C. The radius of a normal selection of a right circular cylinder is 2 units, the axis lies along the straight line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$. Find its equations.

(4 + 3+ 3)

5A. Find the first three nonzero terms in the Maclaurin's series of $\log\left(x + \sqrt{1+x^2}\right)$.

5B. Prove that if, ρ be the radius of curvature at any point P on the parabola $y^2 = 4ax$ and S be its focus, then prove that ρ^2 varies as $(SP)^3$.

5C. Find the volume bounded by revolving the curve $x^{2/3} + y^{2/3} = a^{2/3}$ about x – axis.

(4 + 3+ 3)

6A. (i) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y} \right)$ then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$$

(ii) If H = f (y – z, z – x, x – y) , then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$

6B. State and prove Lagrange's mean value theorem.

6C. If the sides of a plane triangle ABC vary in such a way that is circum – radius remains a constant, then prove that

$$\frac{\delta a}{\cos A} + \frac{\delta b}{\cos B} + \frac{\delta c}{\cos C} = 0$$

(4 + 3+ 3)

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