MECHANICS OF DEFORMABLE BODIES

LECTURE 14

Contents:

Introduction

Mechanical properties of materials

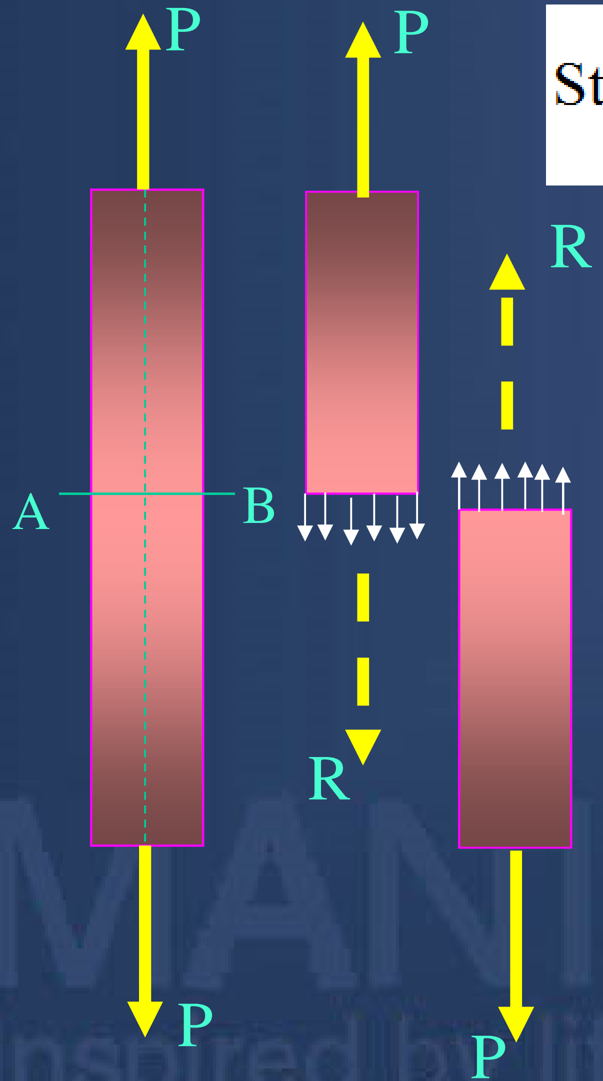
Normal stress and strain

Hooke's law

Modulus of elasticity

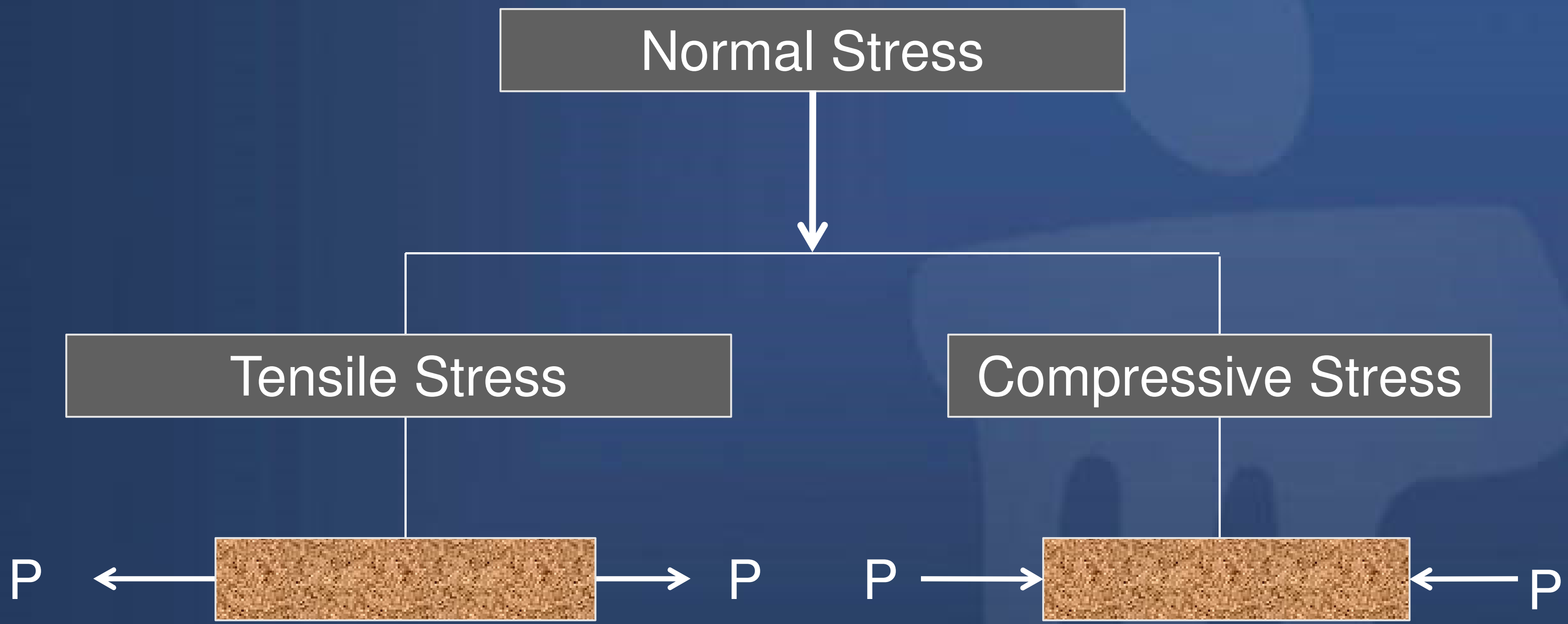
HOME

Normal stress



$$\text{Stress} = \frac{\text{Internal resisting force}}{\text{Resisting cross sectional area}} = \frac{R}{A}$$

The body resists the deformation by developing stresses.



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Units:

SI unit for stress is Pascal (Pa)

Pa= N/m^2

	N/m^2	N/mm^2
1kPa	10^3	10^{-3}
1MPa	10^6	1
1GPa	10^9	10^3

Kilopascal, 1kPa= 1000 N/m^2

Megapascal , 1MPa = $1x\ 10^6\ N/m^2$

$= 1x\ 10^6\ N/(10^6\ mm^2) = 1N/mm^2$

1MPa = 1 N/mm^2

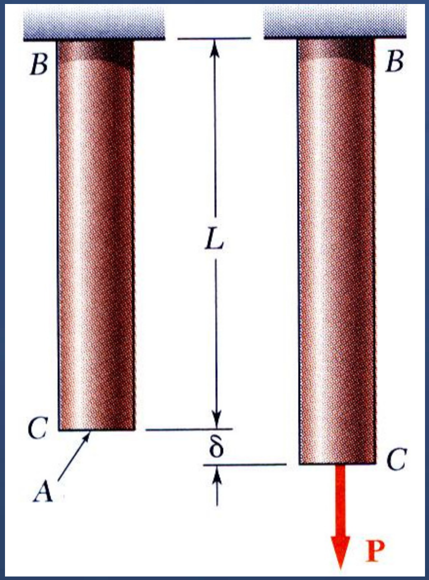
Gigapascal, 1GPa = $1x\ 10^9\ N/m^2$

$= 1x\ 10^3$ MPa

$=1x\ 10^3\ N/mm^2$

STRAIN

$$\varepsilon = \frac{\delta L}{L} = \frac{\text{Change in the length}}{\text{Original length}}$$



$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{Linear strain}$$

Hooks law and Modulus of elasticity

Hooks law:

$$\frac{Stress(\sigma)}{Strain(\varepsilon)} = \text{constant}$$

Modulus of elasticity:

$$\frac{Stress(\sigma)}{Strain(\varepsilon)} = \frac{\text{PL}}{\text{Adl}}$$

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The following table shows modulus of elasticity of important materials:

Material	**Modulus of elasticity**
Steel	210 GPa
Aluminium	73Gpa
Brass	96 – 110 GPa
Cast Iron	83 – 170 GPa
Concrete	17 – 31 GPa
Rubber	0.0007 – 0.004 GPa
Tungsten	340 – 380 GPa

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Tension test on ductile and brittle material

Factor of safety

Allowable stress

HOME

Tension test on ductile and brittle material

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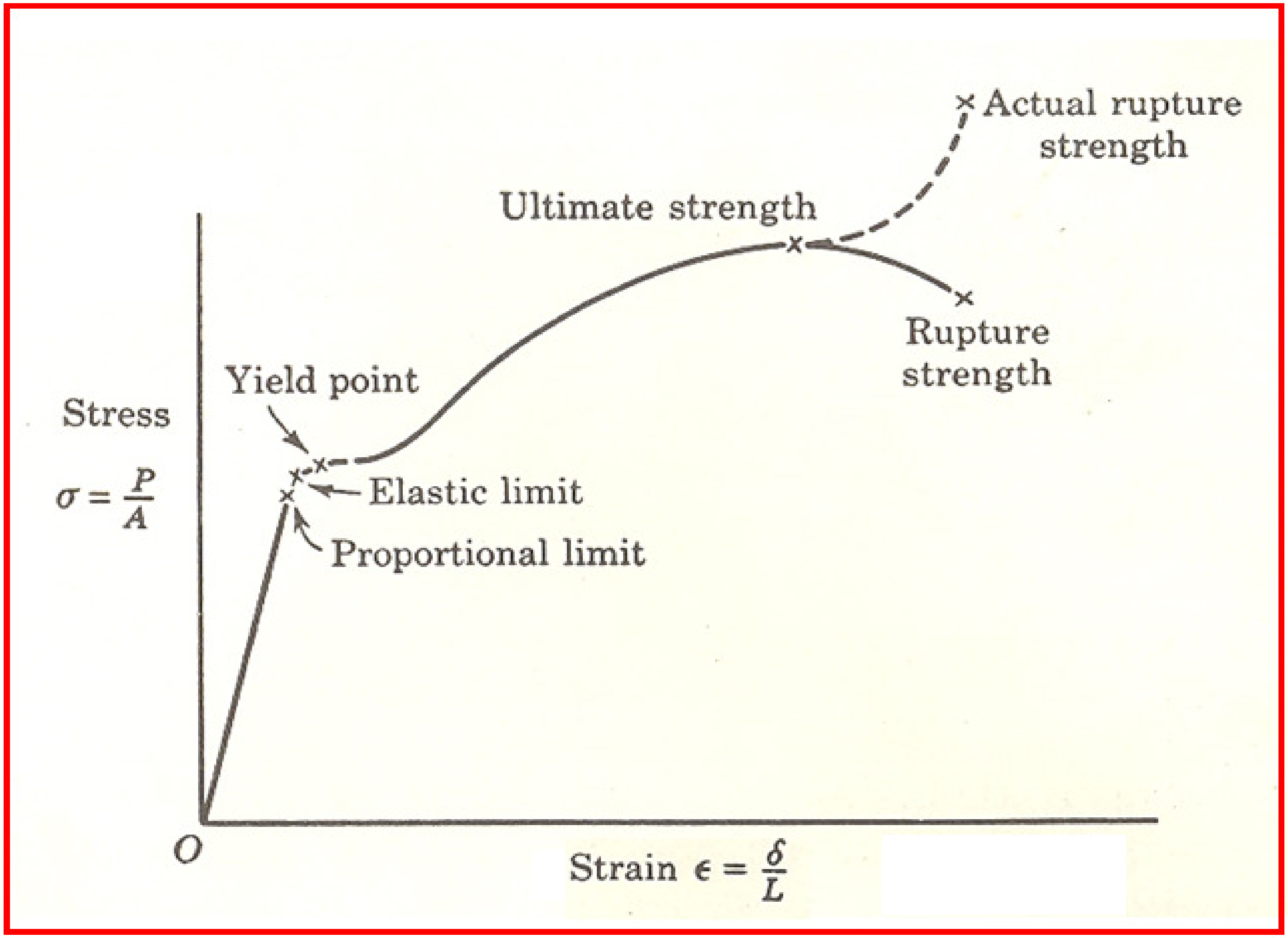
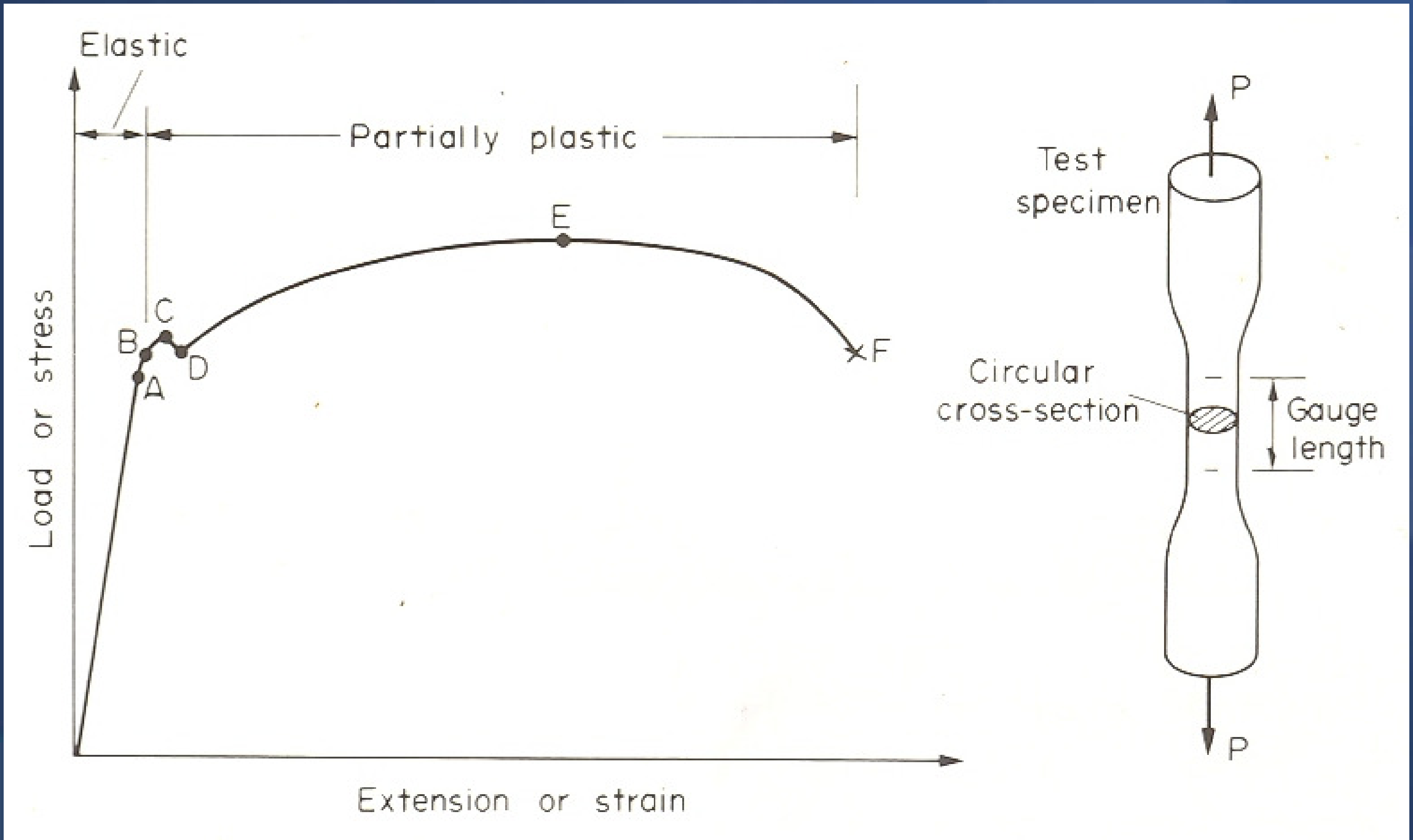
STRESS-STRAIN DIAGRAM



Typical tensile test curve for mild steel

STRESS-STRAIN DIAGRAM



Typical tensile test curve for mild steel showing upper yield point and lower yield point and also the elastic range and plastic range

Limit of Proportionality: $$\sigma_P = \frac{\text{Load at proportionality limit}}{\text{Original crosssectional area}} = \frac{P_P}{A}$$

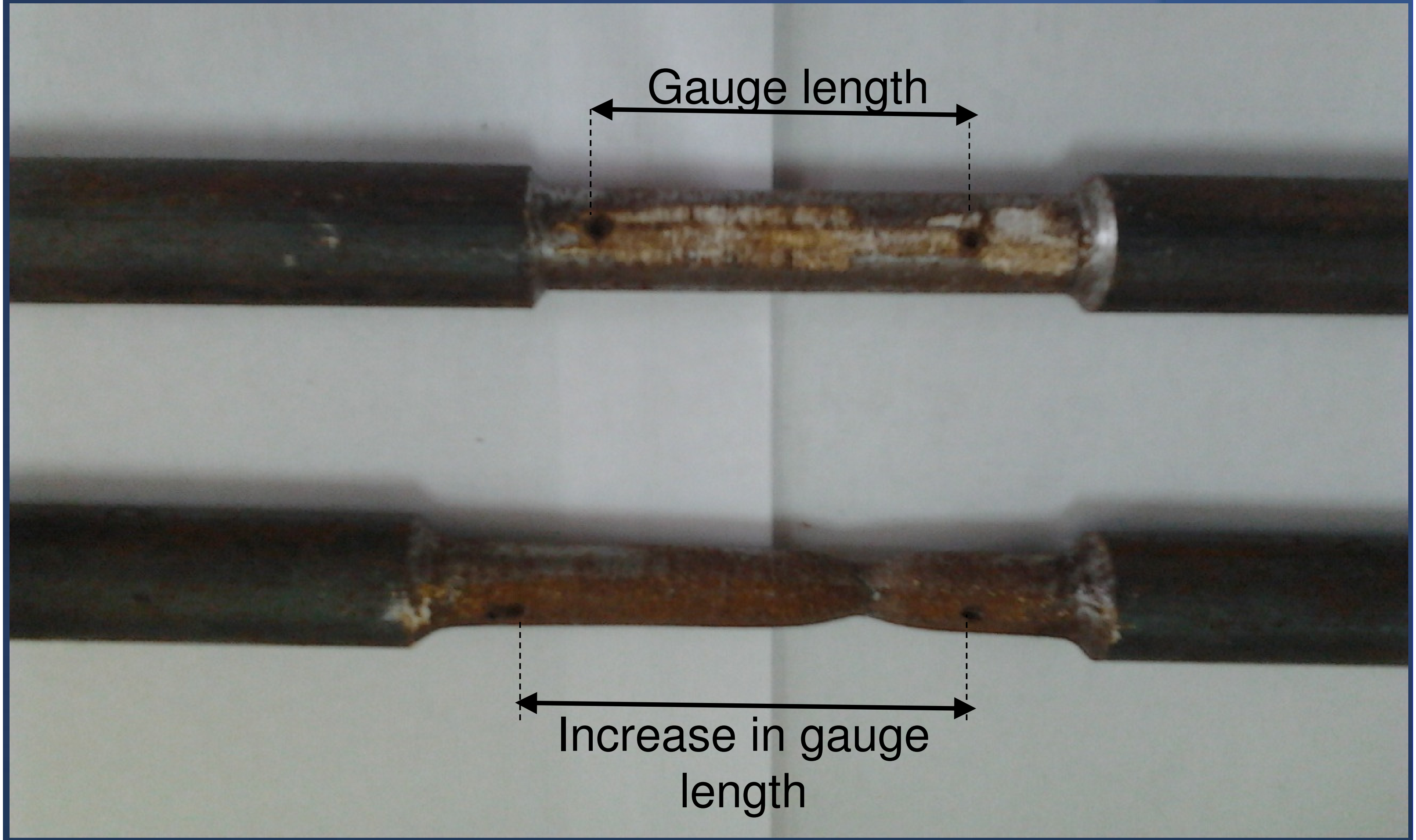
Elastic limit: $$\sigma_E = \frac{\text{Load at elatic limit}}{\text{Original cross sectional area}} = \frac{P_E}{A}$$

Yield point: $$\sigma_Y = \frac{\text{Load at yield point}}{\text{Original cross sectional area}} = \frac{P_Y}{A}$$

Ultimate strength: $$\sigma_U = \frac{\text{Maximum load taken by the material}}{\text{Original cross sectional area}} = \frac{P_U}{A}$$

Rupture strength
(Nominal Breaking stress): $$\sigma_B = \frac{\text{Load at failure}}{\text{Original cross sectional area}} = \frac{P_B}{A}$$

True breaking stress: $$\sigma_B = \frac{\text{Load at failure}}{\text{Actual cross sectional area}} = \frac{P_B}{A}$$


Gauge length
Increase in gauge length

Ductile Materials

Percentage elongation

Percentage reduction in area

Measures of ductility

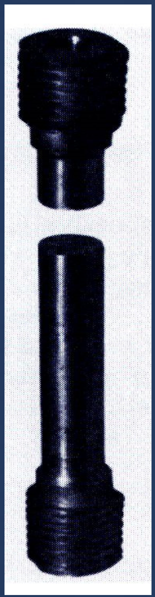
Cup and cone fracture for a Ductile Material ►

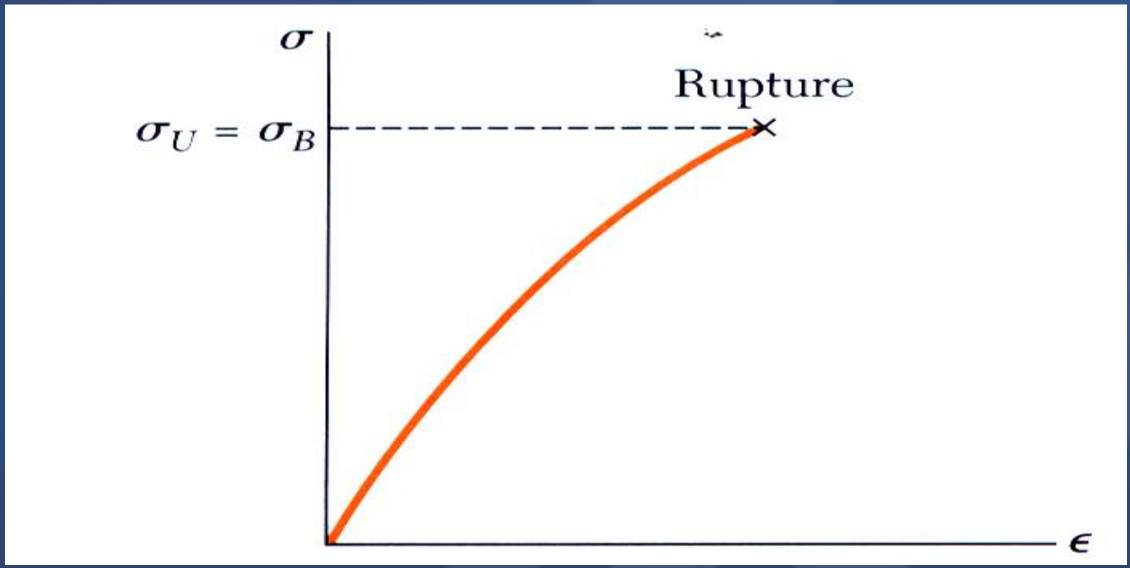
$$\text{Percentage elongation} = \frac{\text{Increase in the gauge length (upto fracture)}}{\text{Original gauge length}} \times 100$$

$$\text{Percentage reduction in area} = \frac{\text{Reduction in cross sectional area of neck portion (at fracture)}}{\text{Original cross sectional area}} \times 100$$

Example: Low carbon steel, mild steel, gold, silver, aluminum

Brittle Materials :





Stress-strain diagram for a typical brittle material

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Working stress & Factor of safety

<u>Ductile Material:</u>

Working stress = Yield Stress / Factor of Safety

<u>Brittle Material:</u>

Working stress = Ultimate Stress / Factor of Safety

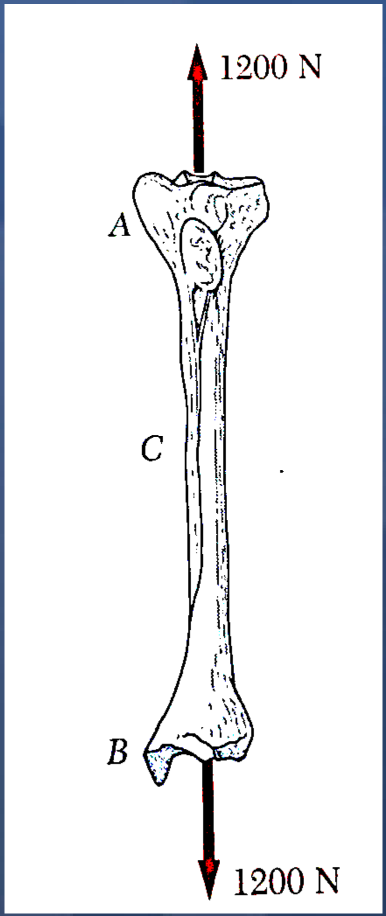
Factor of Safety = Maximum stress / Allowable working stress

LECTURE 16

Contents:

Numerical problems

N1. A strain gauge located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200 N forces as shown. Assuming the cross section of the bone at C to be annular and Knowing that its outer diameter is 25mm, determine the inner diameter of the bones cross section at C.



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$$\sigma = P/A$$

$$A = P/\sigma$$

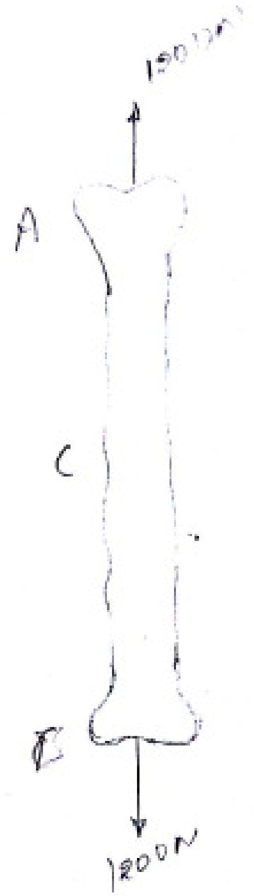
$$A = \frac{\pi}{4}\left(d_1^2 - d_2^2\right)$$

$$d_2^2 = d_1^2 - \frac{4A}{\pi}$$

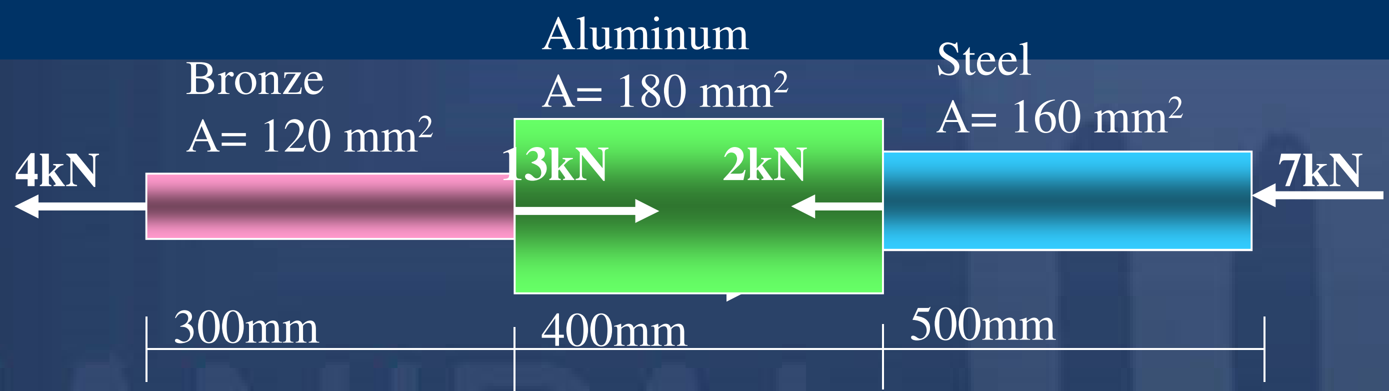
$$= 25^2 - \frac{4 \times 1200}{\pi \times 380}$$

$$d_2^2 = 222.92 \text{ mm}^2$$

$$d_2 = 14.93 \text{ mm.}$$

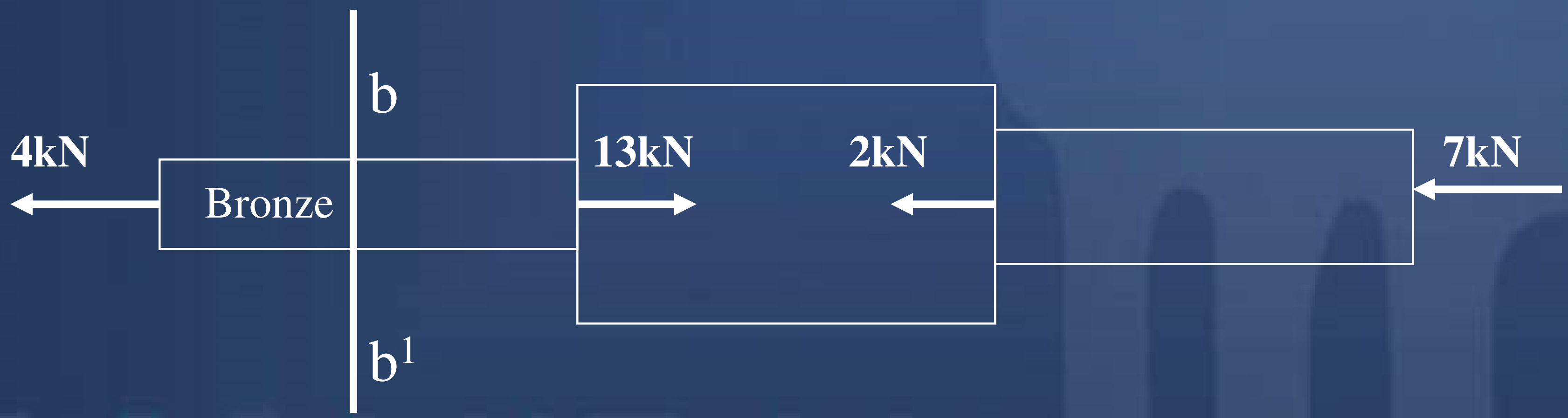


N2 A composite bar consists of an aluminum section rigidly fastened between a bronze section and a steel section as shown in figure. Axial loads are applied at the positions indicated. Determine the stress in each section. Also determine the change in each section and the change in total length.

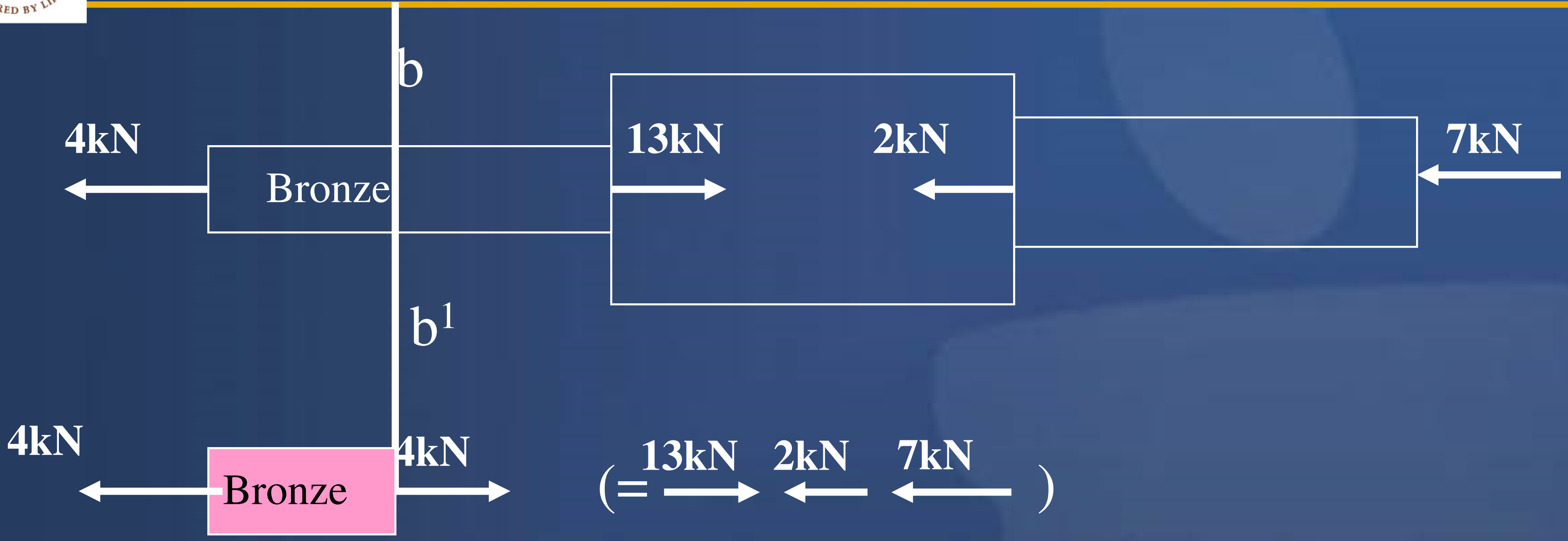


To calculate the stresses, first determine the forces in each section.

To find the Force in bronze section,
consider a section bb^1 as shown in the figure



For equilibrium condition algebraic sum of forces on LHS of the section must be equal to that of RHS



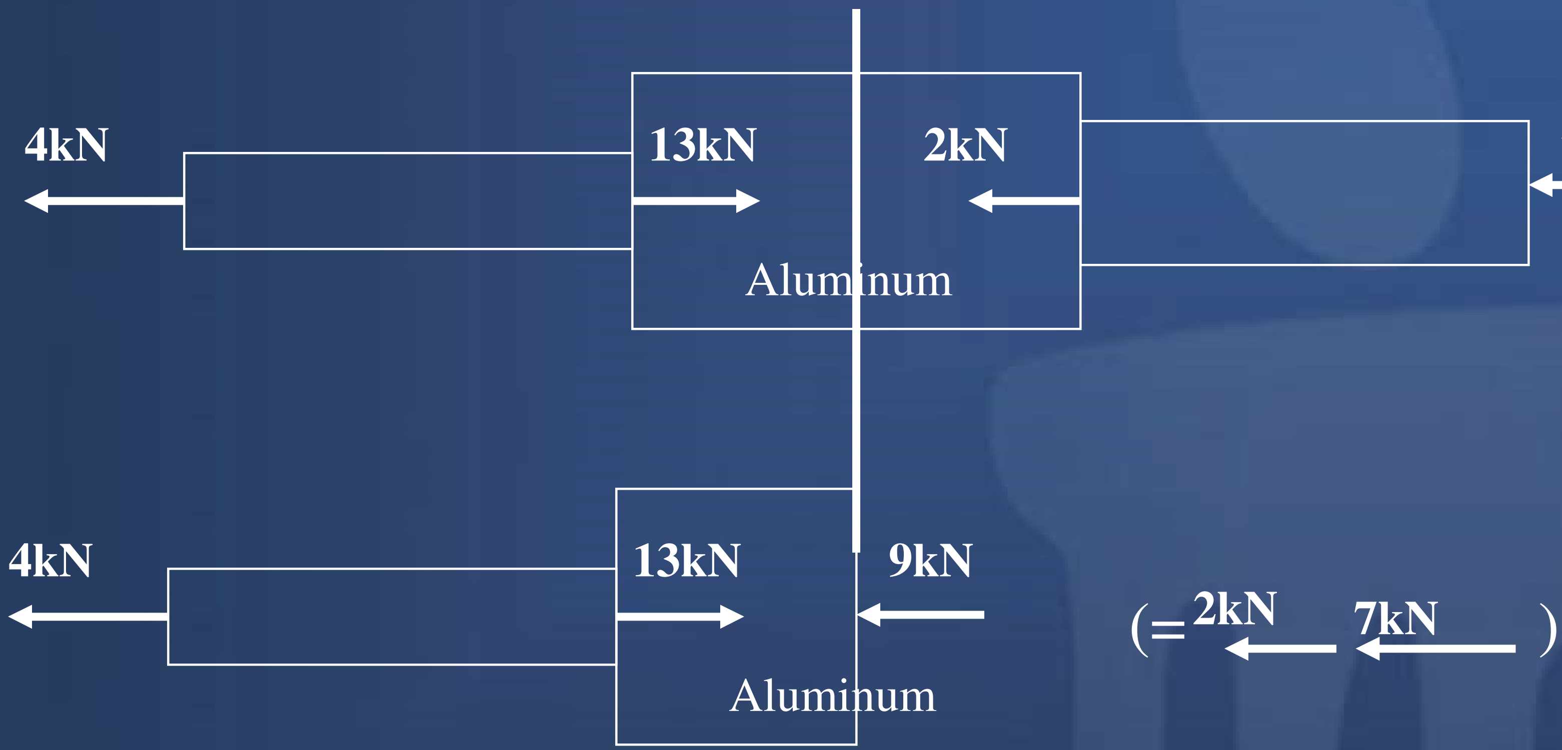
Force acting on Bronze section is 4kN, tensile

$$\text{Stress in Bronze section} = \frac{\text{Force in Bronze section}}{\text{Resisting cross sectional area of the Bronze section}}$$

$$= \frac{4kN}{120mm^2} = \frac{4\times 1000N}{120mm^2} = 33.33N/mm^2 = 33.33\text{MPa}$$

(Tensile stress)

Force in Aluminum section



Force acting on Aluminum section is 9kN, (Compressive)

Force in steel section

Force acting on Steel section is 7kN, (Compressive)

Stress in Aluminum section $= \dfrac{\text{Force in } Al \text{ section}}{\text{Resisting cross sectional area of the } Al \text{ section}}$

$$= \frac{9kN}{180mm^2} = \frac{9\times1000N}{180mm^2} = 50N/mm^2 = 50\text{MPa}$$

Compressive stress

Stress in Steel section $= \dfrac{\text{Force in Steel section}}{\text{Resisting cross sectional area of the Steel section}}$

$$= \frac{7kN}{160mm^2} = \frac{7\times1000N}{160mm^2} = 43.75N/mm^2 = 43.75\text{MPa}$$

(Compressive stress)

we know that,

P_{br} = +4kN (Tension)

P_{al} = -9kN (Compression)

P_{st} = -7kN (Compression)

Deformation due to compressive force is shortening in length, and is considered as -ve

$$E = \frac{\text{stress } (\sigma)}{\text{strain } (\varepsilon)} = \boxed{= \frac{PL}{A\,\delta L}}$$

Change in length = $\delta L = \dfrac{PL}{AE}$

Change in length of bronze = $\delta L_{br} = \dfrac{4000N \times 300mm}{120mm^2 \times 100 \times 10^3 (N/mm^2)}$

= 0.1mm

Change in length of aluminum section = $\delta L_{al} = \dfrac{-9000N \times 400mm}{180mm^2 \times 70 \times 10^3 (N/mm^2)}$ = -0.286mm

Change in length of steel section = $\delta L_{st} = \dfrac{-7000N \times 500mm}{160mm^2 \times 200 \times 10^3 (N/mm^2)}$ = -0.109mm

Change in total length = $\delta L_{br} + \delta L_{al} + \delta L_{st} =$ +0.1 – 0.286 - 0.109

= -0.295mm

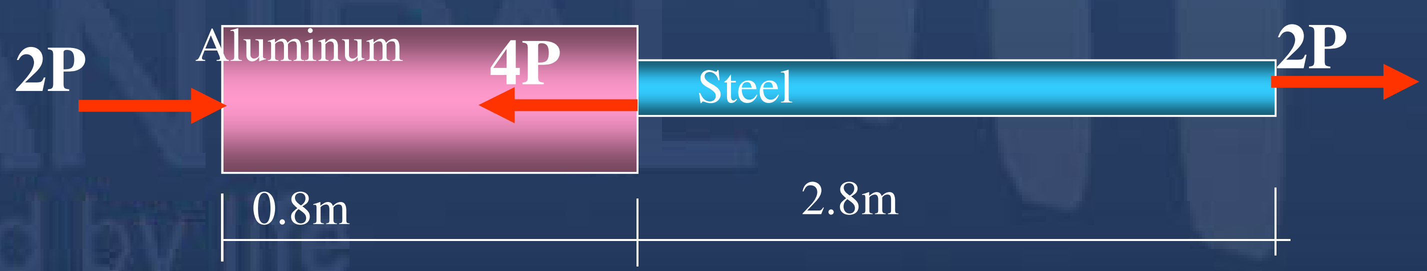
N3. An aluminum rod is fastened to a steel rod as shown. Axial loads are applied at the positions shown. The area of cross section of aluminum and steel rods are $600 mm^2$ and $300 mm^2$ respectively. Find maximum value of P that will satisfy the following conditions.

a) $\sigma_{st} \leq 140$ MPa

b) $\sigma_{al} \leq 80$ MPa

c) Total elongation ≤ 1mm,

Take $E_{al} = 70$GPa, $E_{st} = 200$GPa

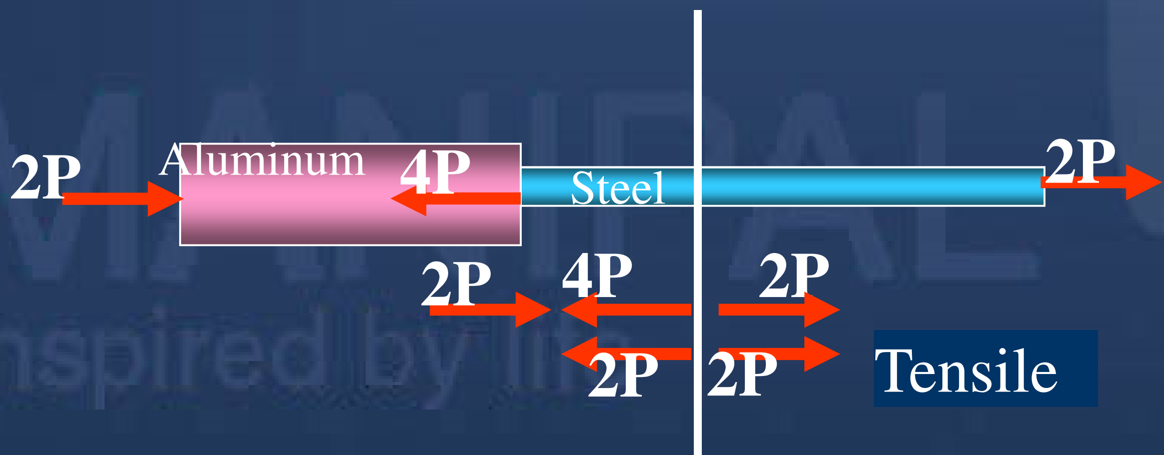


To find P, based on the condition, $\sigma_{st} \leq$ 140 MPa

Stress in steel must be less than or equal to 140MPa. Hence, σ_{st} = = 140MPa

$$= \frac{P_{st}}{A_{st}} = \frac{2P}{A_{st}} = 140N/mm^2$$

$$P = \frac{140 \times A_{st}}{2} = 21000N = 21kN$$



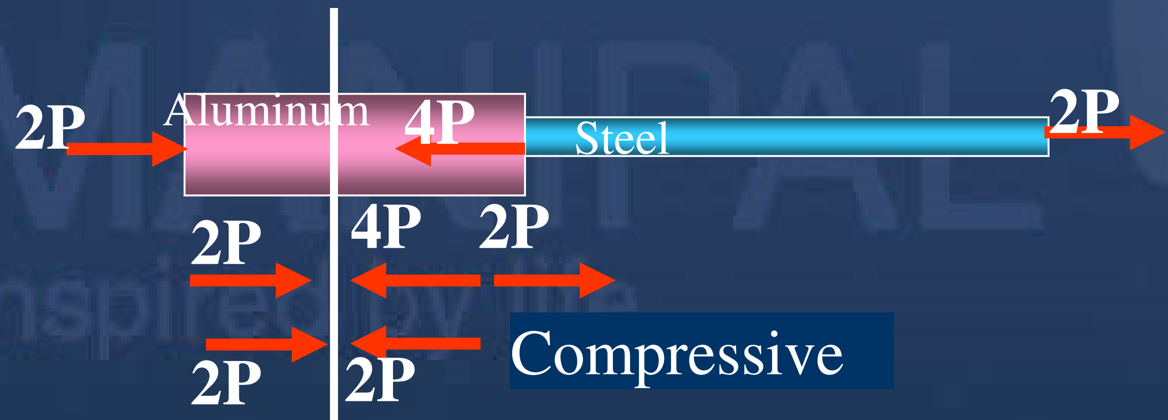
To find P, based on the condition, $\sigma_{al} \leq$ 80 MPa

Stress in aluminum must be less than or equal to 80MPa.

Hence, σ_{al} =

$$= 80\text{MPa} = \frac{P_{al}}{A_{al}} = \frac{2P}{A_{al}} = 80N/mm^2$$

$$P = \frac{80 \times A_{al}}{2} = 24000N = 24kN$$



To find P, based on the condition, total elongation ≤ 1mm

Total elongation = elongation in aluminum + elongation in steel.

$$1\text{mm} = \left(\frac{PL}{AE}\right)_{al} + \left(\frac{PL}{AE}\right)_{st}$$

$$1\text{mm} = \left(\frac{-2PL_{al}}{A_{al}E_{al}}\right) + \left(\frac{+2PL_{st}}{A_{st}E_{st}}\right)$$

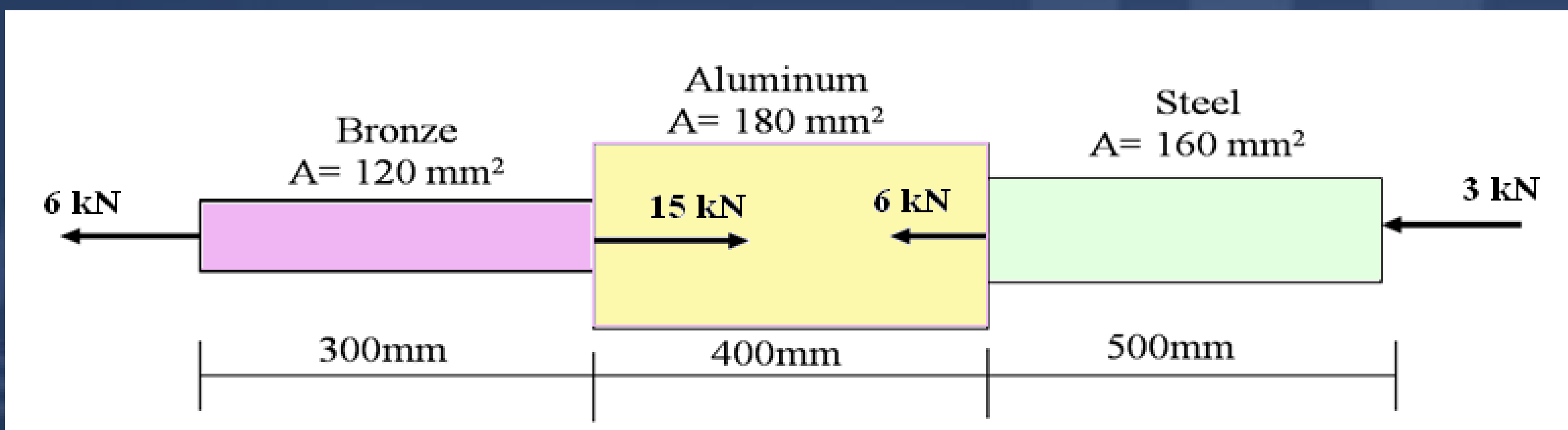
$$1\text{mm} = \left(\frac{-2P\times 800}{600\times 70\times 10^3}\right) + \left(\frac{+2P\times 2800}{300\times 200\times 10^3}\right)$$

$$\text{P} = 18.1\text{kN}$$

Ans: P = 18.1kN (minimum of the three values)

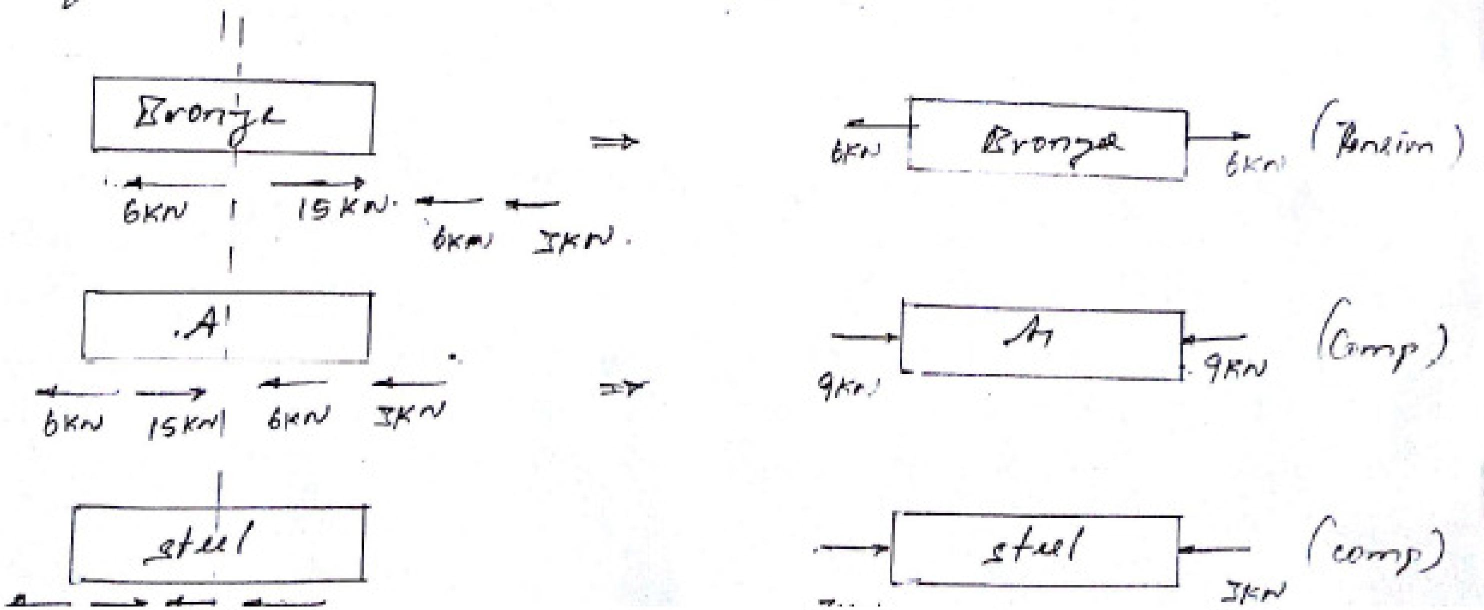
N4. A composite bar consists of an aluminum section rigidly fastened between a bronze section and a steel section as shown in figure. Axial loads are applied at the positions indicated. Determine the stress in each section. Also determine the change in each section and the change in total length.

Given $E_b = 100$ GPa, $E_a = 70$ GPa, $E_s = 200$ GPa



Sol:

To calculate the stresses, first determine the forces in each section.



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stress in bronze section :-

$$= \frac{\text{Force in bronze section}}{\text{c/s area (resisting)}}$$

$$= \frac{6 \times 10^3}{120} = 50\ N/mm^2\ (T)$$

stress in Alluminium section :

$$= \frac{\text{Force in alluminium section}}{\text{c/s area}}$$

$$= \frac{9 \times 10^3}{180} = 50\ N/mm^2\ (C)$$

stress in steel s/n

$$= \frac{\text{Force in steel s/n}}{\text{c/s area}}$$

$$= \frac{3 \times 10^3}{160} = 18.75\ N/mm^2\ (C)$$

∴ change in length $\delta = PL/AE$

change in length of bronze, $\delta_{br} = \dfrac{6000 \times 300}{120 \times 100 \times 10^3} = 0.15\,mm$ (+)

change in length of Alluminium $= \dfrac{-9000 \times 400}{180 \times 70 \times 10^3} = -0.286\,mm$

change in length of steel $= \dfrac{-3000 \times 500}{160 \times 200 \times 10^3} = -0.469\,mm$

Total change in length

$= 0.15 - 0.286 - 0.469$

$= -0.605\,mm$

$= 0.605\,mm$ (shortening)

Inspired by life

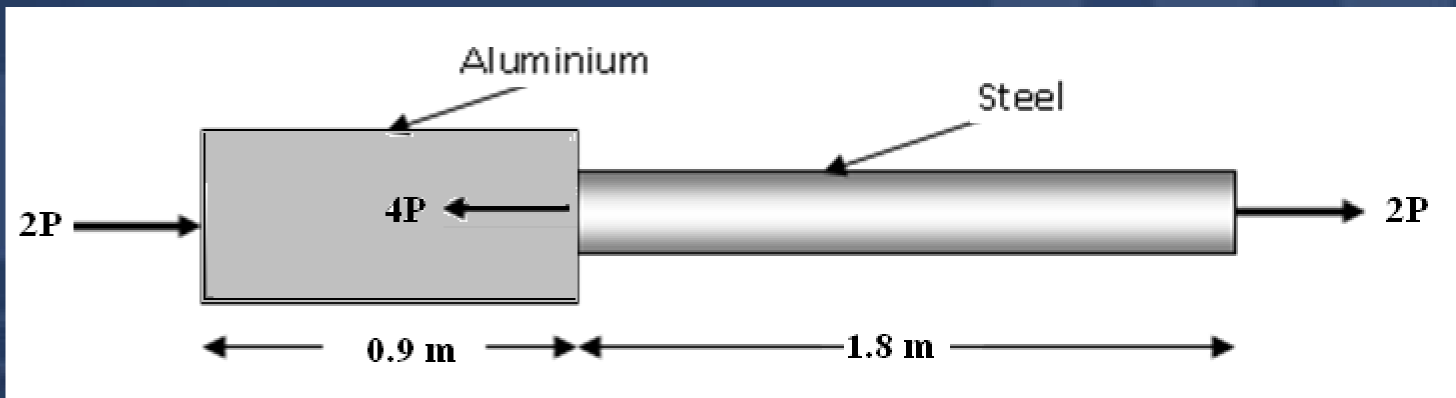
N5. An aluminum rod is fastened to a steel rod as shown. Axial loads are applied at the positions shown. The area of cross section of aluminum and steel rods are 400 mm^2 and 200mm^2 respectively. Find maximum value of P that will satisfy the following conditions.

a) $\sigma_s \leq$ 140 MPa

b) $\sigma_a \leq$ 80 MPa

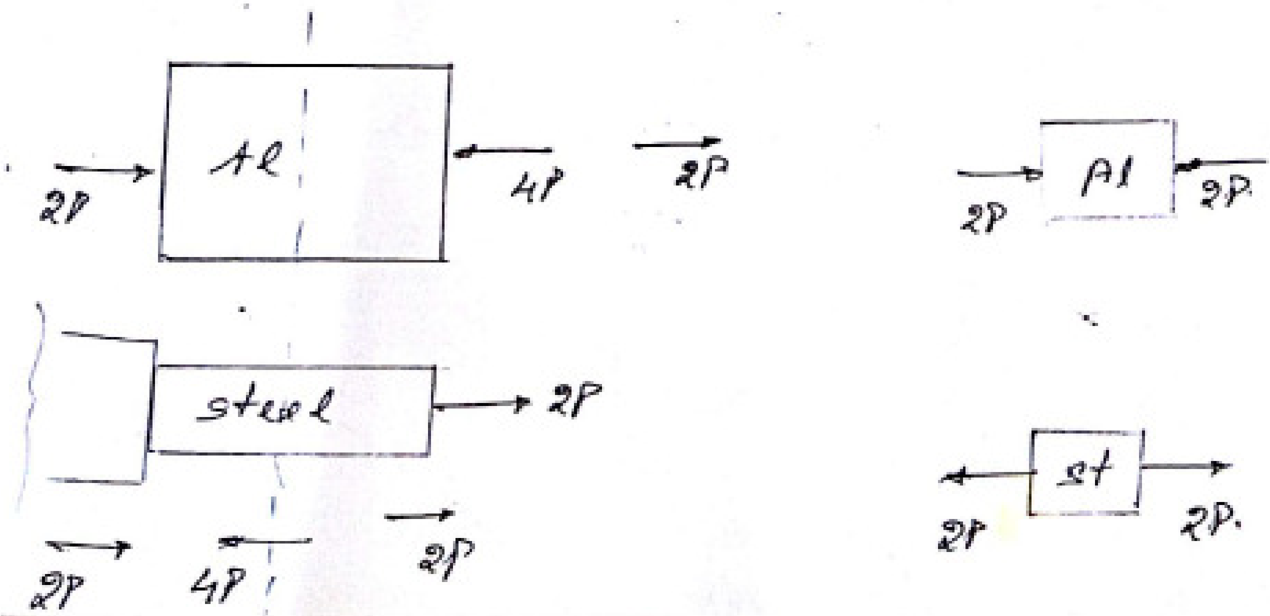
c) Total elongation $\leq$ 0.5 mm,

Take E_a = 70 GPa and E_s = 210 GPa



(a) To find 'P' based on the condition $\sigma_{st} \leq 140\,MPa$

$$\sigma_{st} = 140 = \frac{P_{st}}{A_{st}} = \frac{2P_{st}}{200} \quad \therefore \; P = 14000\,N = 14\,kN$$



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(b) To find 'P' based on the condition $\sigma_{al} \leq 80$

$$\sigma_{al} = \frac{2P_{al}}{A_{al}} = \frac{2P_{al}}{400} = 80$$

$$P_{al} = 16000\ N = 16\ KN$$

(c) To find 'P' based on the condition elongation ≤ 0.5 mm

$$0.5 = \left(\frac{PL}{AE}\right)_{AL} + \left(\frac{PL}{AE}\right)_{st}$$

$$0.5 = \frac{-2P \times 900}{400 \times 70 \times 10^3} + \frac{2P \times 1800}{200 \times 210 \times 10^3}$$

$$0.5 = -6.43 \times 10^{-5} P + 857 \times 10^{-5} P$$

$$0.5 = 2.14 \times 10^{-5} P$$

$$P = 23364.5\ N$$

$$P = 23.4\ N$$

$\therefore$ $P = 14$ KN (min of all the three)

N6. A member ABCD is subjected to point loads P1, P2,P3 and P4 as shown in figure below.

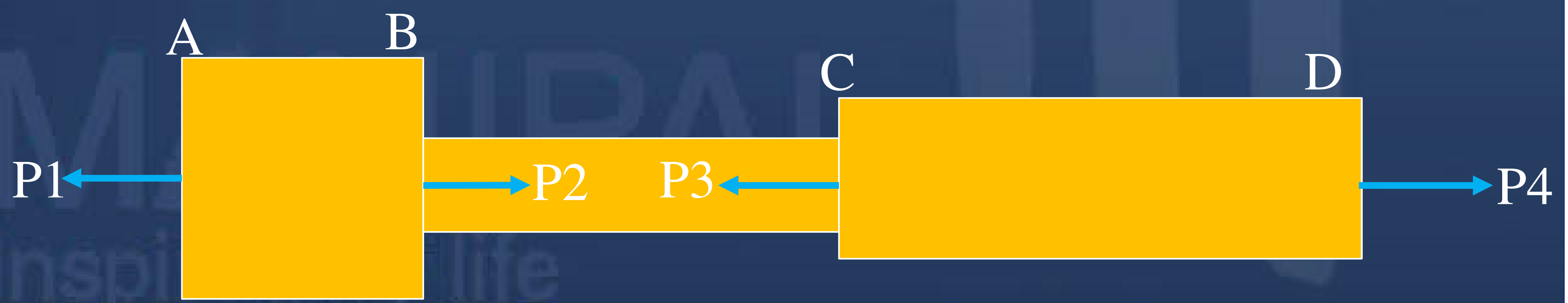
Calculate the force P3 necessary for equilibrium if P1 = 120 kN, P2 = 220 kN and P4 = 160 kN.

Determine the net change in the length of the member. Take $E = 200\ GN/m^2$.

Given: area and length of AB: $1600\ mm^2$, 0.75 m ;

area and length of BC: $625\ mm^2$, 1.0 m;

area and length of CD: $900 mm^2$, 1.2 m.



solu!

Value of P_3 necessary for equilibrium!

Equate the forces acting towards right to those acting towards left

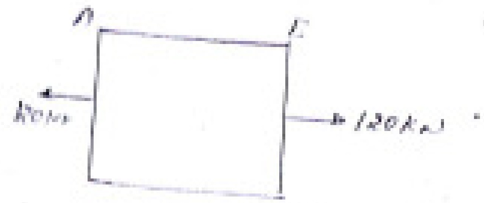
$$P_1 + P_3 = P_2 + P_4$$

$$120 + P_3 = 220 + 160$$

$$P_3 = 260 \text{ kN}$$

Applying method of sections.

Part AB!



Tensile (T)

Part BC!

Compression (C)

Part CD

Tensile (T)

Net change in length of the member = Increase in length of AB + decrease in length of BC + Increase in length of CD

$$\delta L_{net} = \delta L_{AB} + \delta L_{BC} + \delta L_{CD}$$

$$= \frac{P_1 L_1}{A_1 E} - \frac{P_2 L_2}{A_2 E} + \frac{P_3 L_3}{A_3 E}$$

$$= \frac{120 \times 10^3 \times 450}{1600 \times 200 \times 10^3} - \frac{100 \times 10^3 \times 1000}{625 \times 200 \times 10^3} + \frac{160 \times 10^3 \times 1200}{900 \times 200 \times 10^3}$$

$$= +0.28 - 0.8 + 1.07 = \underline{-0.55\,mm}$$

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LECTURE 17

Contents:

Expression for deformation of a tapered bar

Expression for deformation of a tapered flat

Application problems

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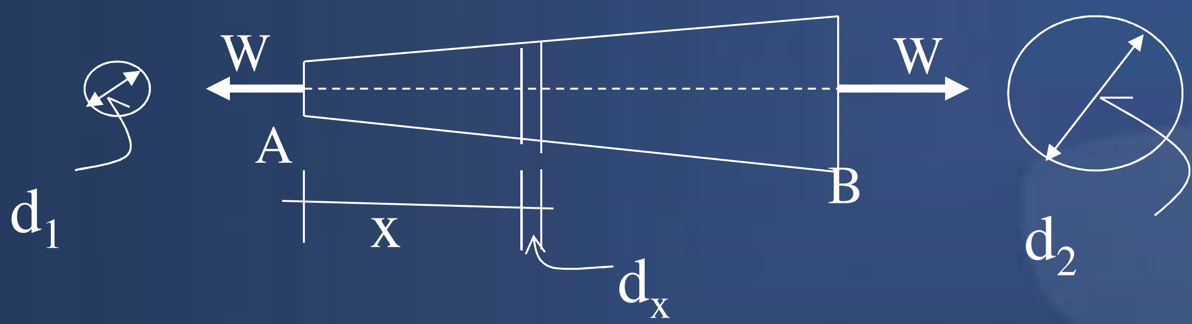
Derive an expression for the total extension of the tapered bar of circular cross section shown in the figure, when subjected to an axial tensile load , W

Consider an element of length, δx at a distance x from A



Diameter at x, $= d_1 + \frac{(d_2 - d_1)}{L} \times x$

$= d_1 + k \times x$

c/s area at x, $= \pi d_1^2 / 4 = \frac{\pi}{4}(d_1 + kx)^2$

Change in length over a length dx is $= \left(\frac{PL}{AE}\right)_{dx} = \left(\frac{Wdx}{\frac{\pi}{4}(d_1 + kx)^2 \times E}\right)$

Change in length over a length L is $= \int_0^L \left(\frac{Wdx}{\frac{\pi}{4}(d_1 + kx)^2 \times E}\right)$

Consider an element of length, δx at a distance x from A

Change in length over a length L is

$$= \int_0^L \left(\frac{W dx}{\frac{\pi}{4}(d_1 + kx)^2 \times E} \right)$$

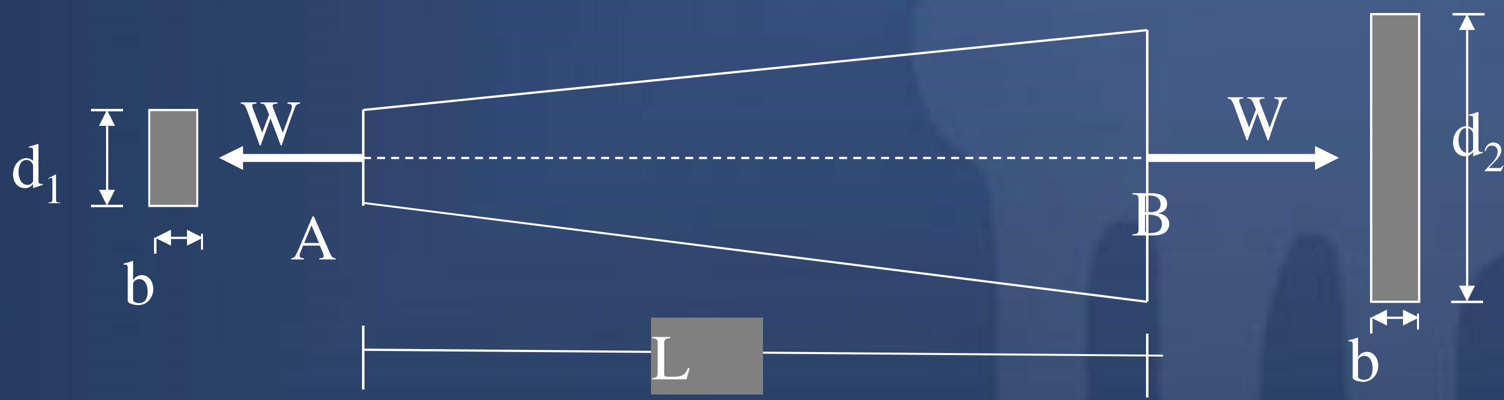
$$= \int_0^L \left(\frac{W \frac{dt}{k}}{\frac{\pi}{4}(t)^2 \times E} \right)$$

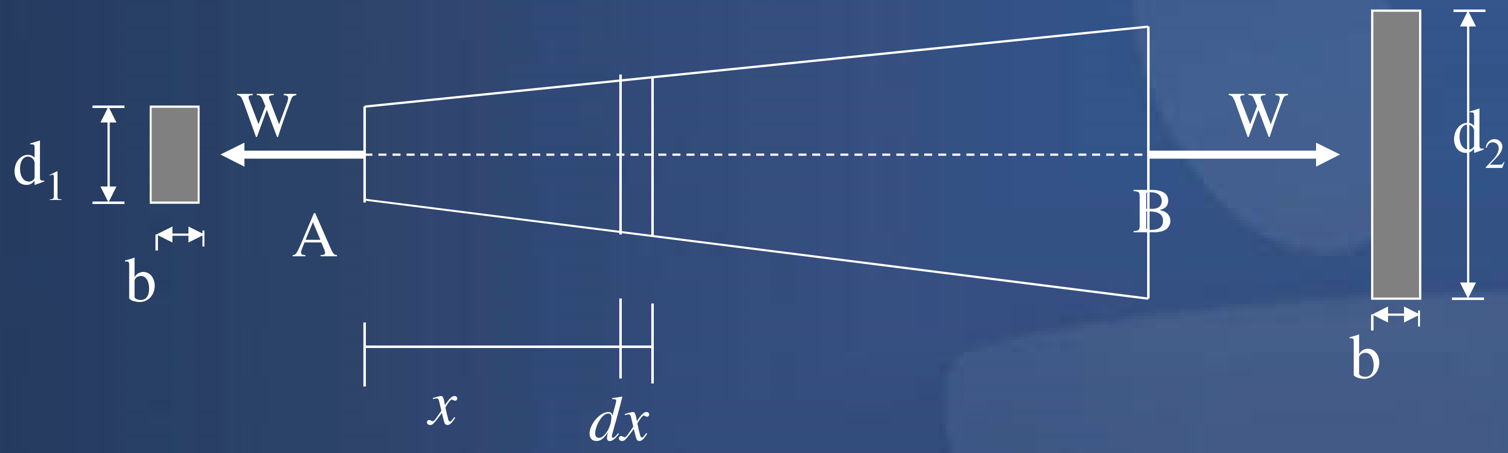
Put $d_1 + kx = t$,
Then $k\,dx = dt$

$$= \frac{4W}{\pi E k}\left[\frac{t^{-2+1}}{-1}\right]_0^L = \frac{4W}{\pi E k}\left[\frac{-1}{t}\right]_0^L = \frac{-4W}{\pi E k}\left[\frac{1}{(d_1 + kx)}\right]_0^L$$

$$= \frac{4WL}{\pi E d_1 d_2} = \frac{WL}{\frac{\pi d_1 d_2}{4} \times E}$$

Derive an expression for the total extension of the tapered bar AB of rectangular cross section and uniform thickness, as shown in the figure, when subjected to an axial tensile load ,W.





Consider an element of length, δx at a distance x from A

depth at x, $= d_1 + \frac{(d_2 - d_1)}{L} \times x$

$= d_1 + k \times x$

c/s area at x, $= (d_1 + kx)b$

Change in length over a length dx is $= \left(\frac{PL}{AE}\right)_{dx} = \left(\frac{Wdx}{(d_1 + kx)b \times E}\right)$

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Change in length over a length L is

$$= \int_0^L \left(\frac{W dx}{(d_1 + kx) b \times E} \right)$$

$$= \frac{P}{b \times E \times k} \left(\log_e d_2 - \log_e d_1 \right)$$

$$= \frac{2.302 \times P \times L}{b \times E \times (d_2 - d_1)} \left(\log d_2 - \log d_1 \right)$$

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N7. Find the modulus of elasticity of the material of a tapering bar from the following data: The bar has 20 mm diameter at one end, 40 mm diameter at the other, length 1.0 m and axial load of 10 kN. The elongation observed was 0.1 mm.

Sol

$$\Delta L = \frac{4PL}{\pi d_1 d_2 E}$$

$$0.1 = \frac{4 \times 10 \times 10^3 \times 1 \times 10^3}{\pi \times 20 \times 40 \times E}$$

$$E = 159{,}155 \text{ N/mm}^2 \simeq 159 \text{ GPa}$$

N8. A tapered bar of rectangular cross section is 20 mm wide at one end and 40 mm wide at the other, 8 mm thick and 800 mm long. The elongation of 0.08 mm was observed under load P. find the load P, if the modulus of elasticity of the material of the bar is 100 GPa.

Sol:

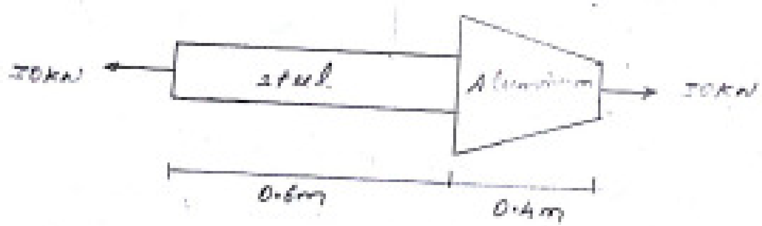
$$\Delta L = \frac{PL}{Et(b_2 - b_1)} \log_e\left(b_2/b_1\right)$$

$$0.08 = \frac{P \times 800}{100 \times 10^3 \times 8 (40-20)} \log_e\left(40/20\right)$$

$$P = 2308 \text{ N}$$

$$P = 2.3 \text{ kN}$$

N9. A uniform steel rod of diameter 20 mm is connected to an aluminium rod of diameter 60 mm at one end. The aluminium rod tapers to a diameter of 20 mm at the other end. The steel rod is 0.6 m long and is connected rigidly to 60 mm diameter end of the aluminium rod which is 0.4 m long. If E = 200 GPa for steel and 70 GPa for aluminium, find the total extension under an axial load of 30 kN.



Sol

Total extension of the rod = Extension in steel + Extension in aluminium

$$= \frac{PL}{AE} + \frac{4PL}{\pi E D_1 D_2}$$

$$= \frac{30\times10^3 \times 0.6\times10^3}{(\pi/4 \times 20^2) \times 200\times10^3} + \frac{4\times30\times10^3\times0.4\times10^3}{\pi \times 70\times10^3\times60\times20}$$

$$\delta l_{total} = 0.27 + 0.182$$

$$= 0.452 \text{ mm}$$

LECTURE 18

Contents:

Shear stress

Shear strain

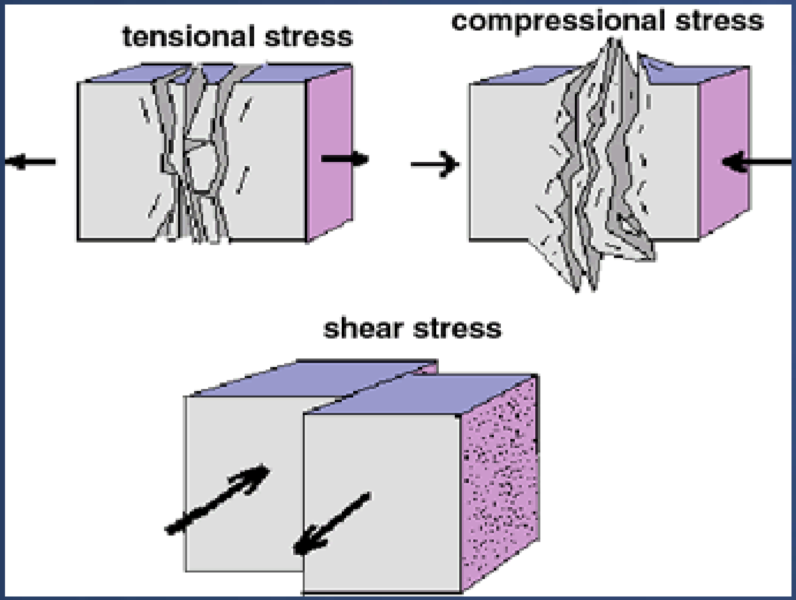
Modulus of rigidity

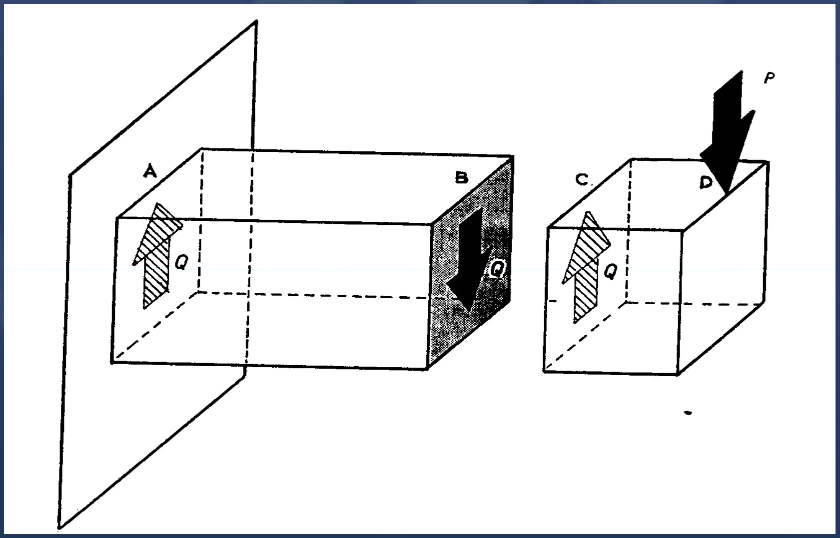
State of simple shear & Complementary shear

Direct stress due to pure shear

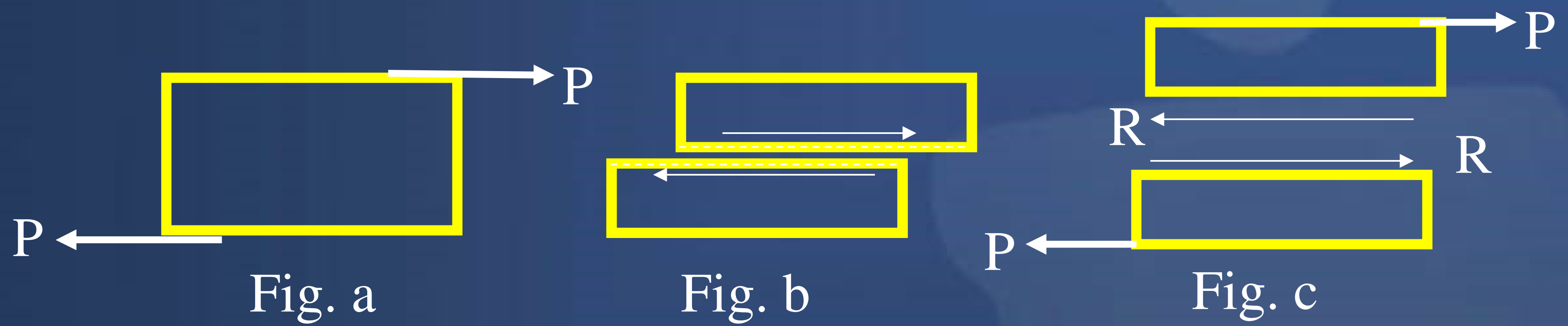
MANIPAL
Inspired by life

SHEAR STRESS





SHEAR STRESS



$$\text{Shear stress}(\tau) = \frac{\text{Shear resistance}}{\text{Area resisting shear}} = \frac{R}{A} = \frac{P}{A}$$

This shear stress will always be tangential to the area on which it acts

SHEAR STRAIN

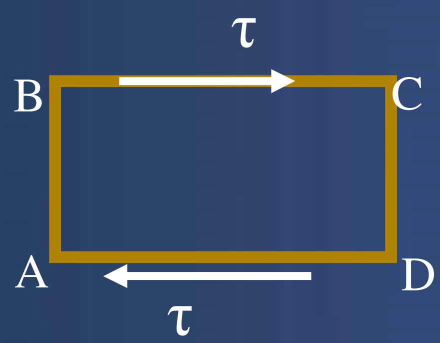


Fig. d

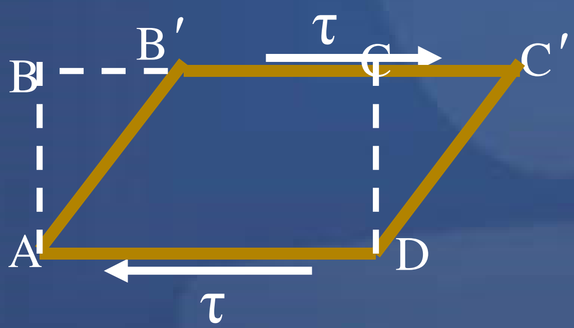
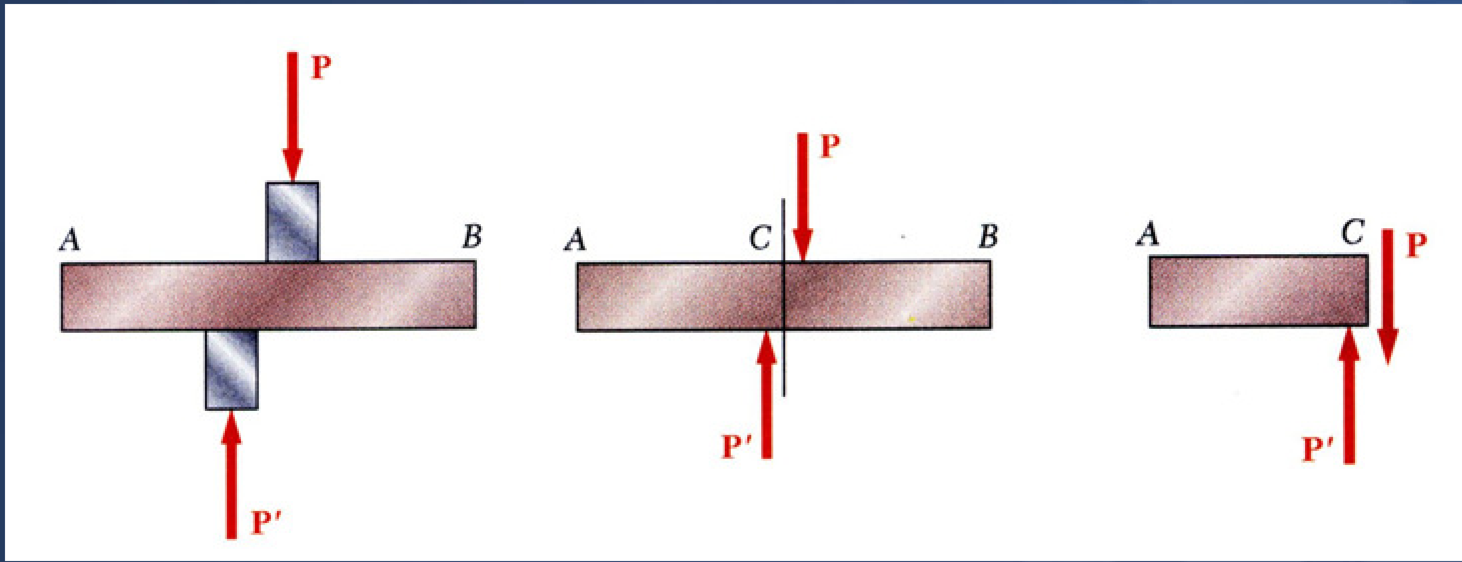


Fig. e

$$\text{shear strain} = \frac{BB'}{AB} = \tan\phi \approx \phi$$

Shear modulus:

$$\frac{\text{Shear stress } (\tau)}{\text{Shear strain } (\phi)} = \text{constant} = \text{G} = \text{Shear Modulus or Modulus of Rigidity}$$



$$\tau_{ave} = \frac{P}{A}$$

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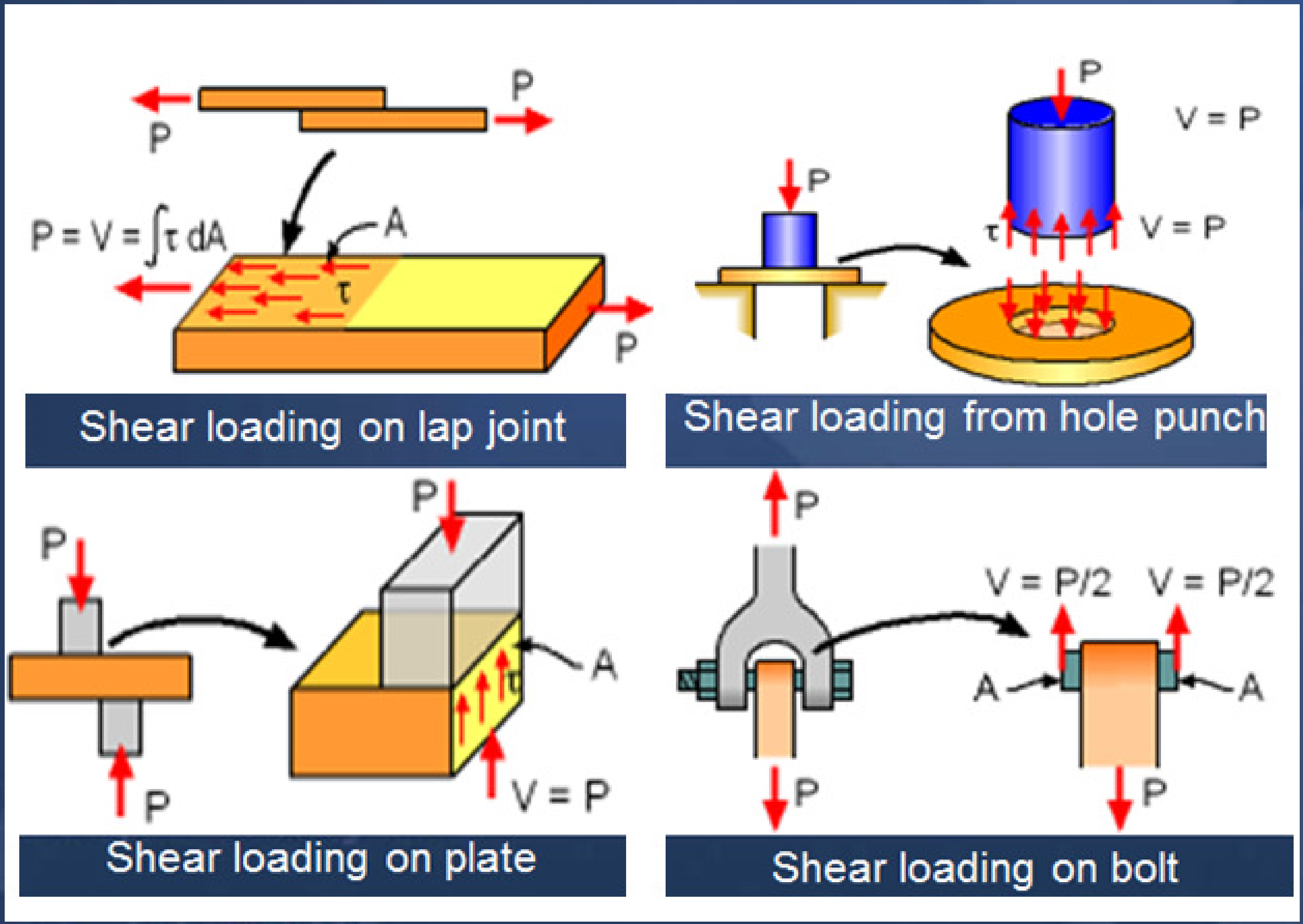
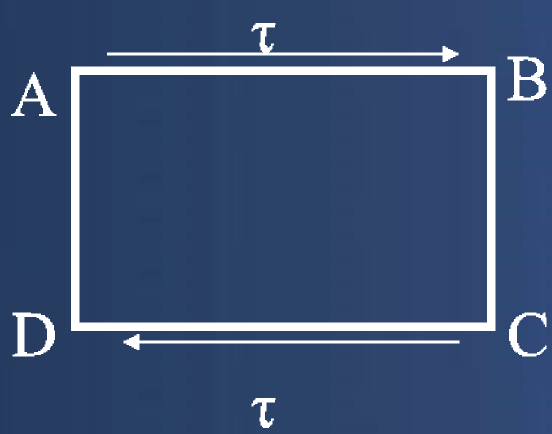
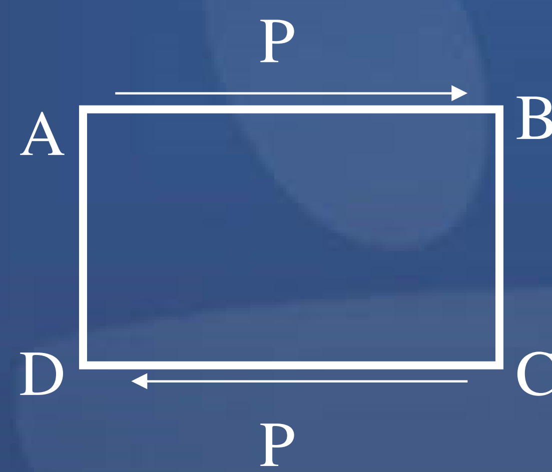
Examples of Shear



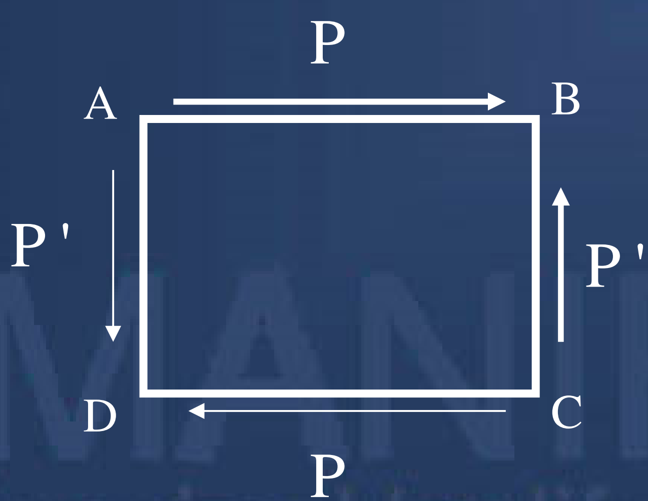
State of simple shear



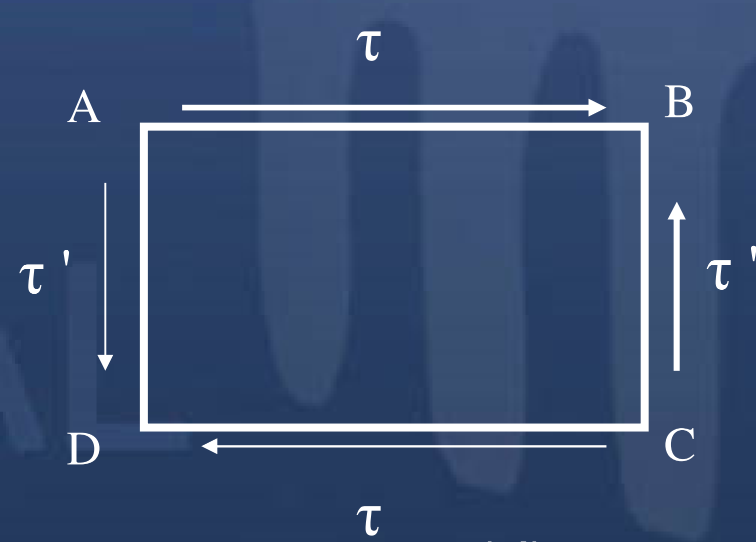
(a)



(b)



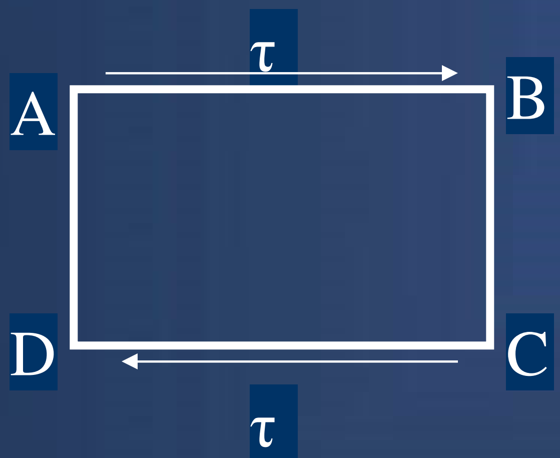
(c)



(d)

State of simple shear

Consider an element ABCD in a strained material subjected to shear stress, τ as shown in the figure



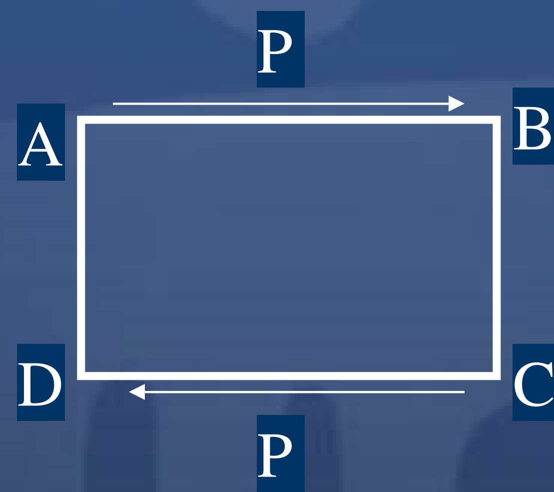
Force on the face AB = P = $\tau \times AB \times t$

Where, t is the thickness of the element.

Force on the face DC is also equal to P

State of simple shear

Now consider the equilibrium of the element.
(i.e., $\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma M = 0$.)



For the force diagram shown,
$\Sigma Fx = 0$, & $\Sigma Fy = 0$,
But $\Sigma M \neq 0$

The element is subjected
to a clockwise moment

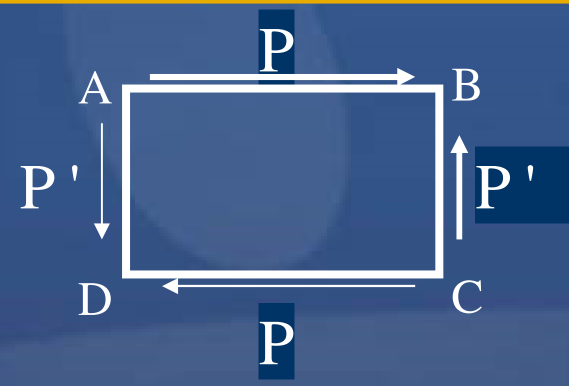
force

$$P \times AD = (\tau \times AB \times t) \times AD$$

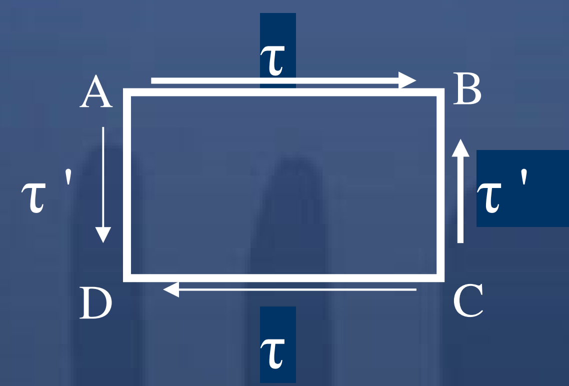
But, as the element is actually in equilibrium, there must be another pair of forces say P' acting on faces AD and BC, such that they produce a anticlockwise moment equal to (P × AD)

State of simple shear

$$P' \times AB = P \times AD$$
$$= (\tau \times AB \times t) \times AD \quad \text{-----} \quad (1)$$



If τ^1 is the intensity of the shear stress on the faces AD and BC, then P ' can be written as,
$P' = \tau' \times AD \times t$



Equn.(1) can be written as

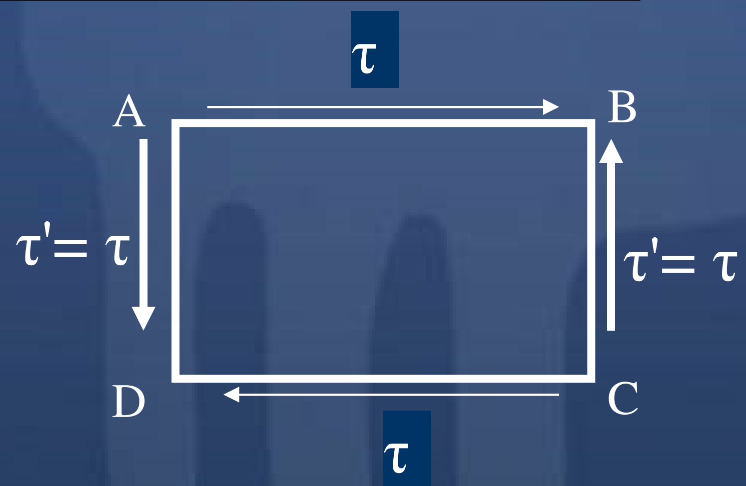
$$(\tau' \times AD \times t) \times AB = (\tau \times AB \times t) \times AD \quad \text{-----} \quad (1)$$

$$\tau' = \tau$$

State of simple shear

Thus in a strained material a shear stress is always accompanied by a balancing shear of same intensity at right angles to itself. This balancing shear is called "complementary shear".

The shear and the complementary shear together constitute a state of simple shear



Direct stress due to pure shear

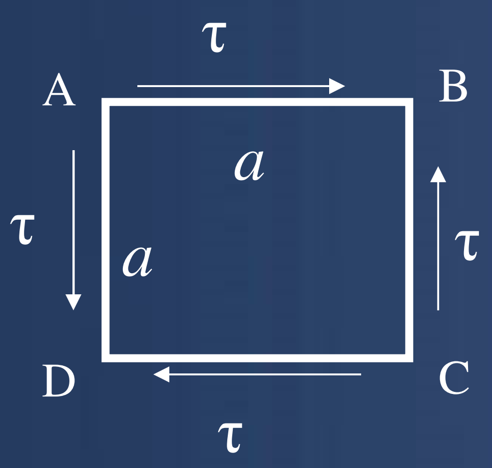


Fig.(a).

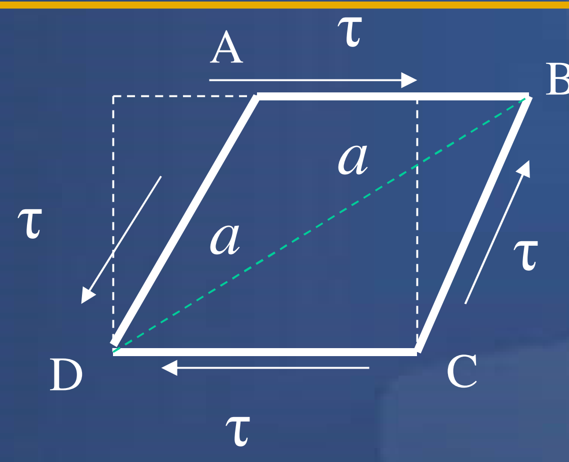


Fig.(b).

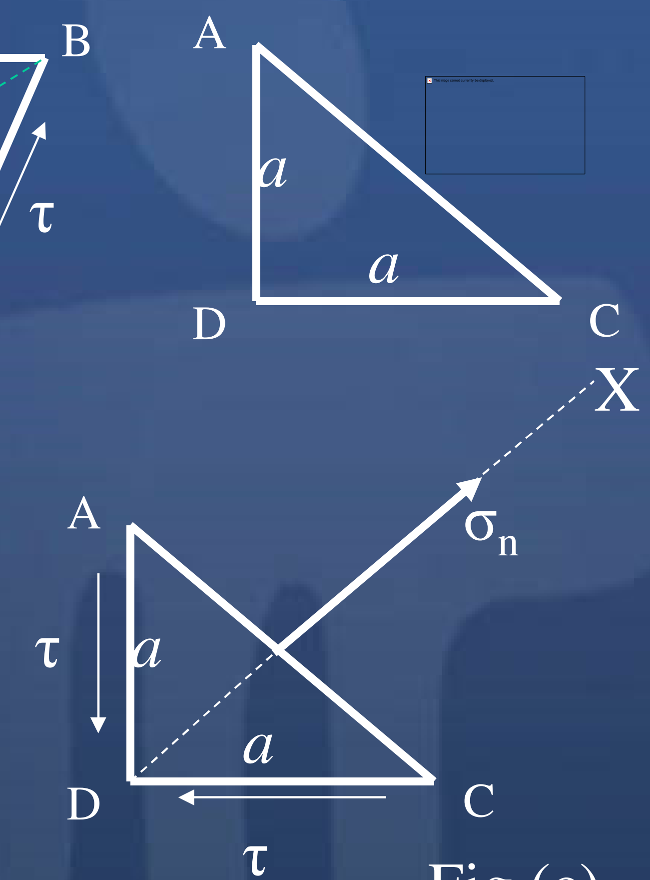


Fig.(c).

$$\Sigma F_x = 0$$

$$= \sigma_n \times \left(\sqrt{2} \times a \times 1\right) - 2 \times (\tau \times a \times \cos 45)$$

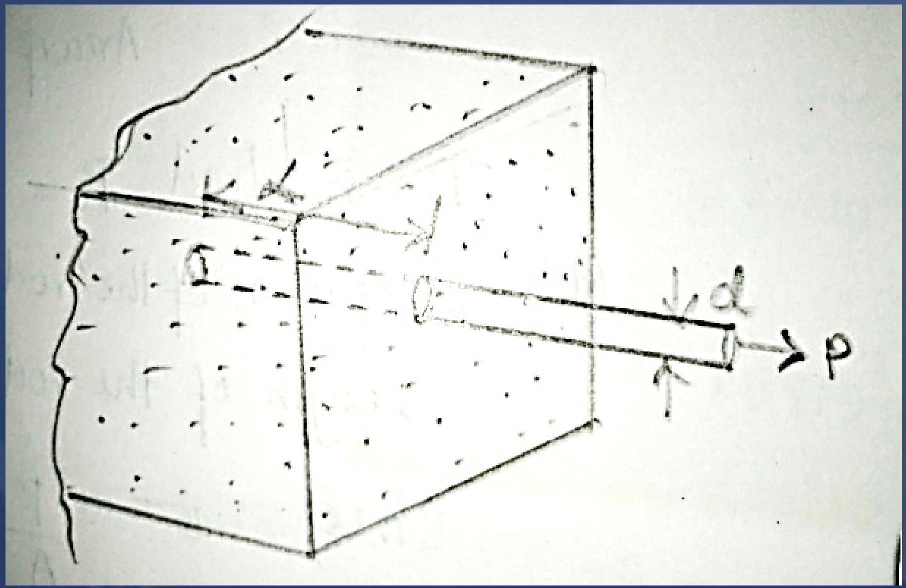
For equilibrium, $\sigma_n = \tau$

Direct stress due to pure shear

Therefore the intensity of normal tensile stress developed on plane BD is numerically equal to the intensity of shear stress.

Similarly it can be proved that the intensity of compressive stress developed on plane AC is numerically equal to the intensity of shear stress.

N10. To check the bond strength between reinforcing bars and concrete, a tensile force of P=30 kN is applied to the end of the bar of diameter d=12 mm and length L= 100 mm. Calculate the average shear stress developed between steel and concrete.

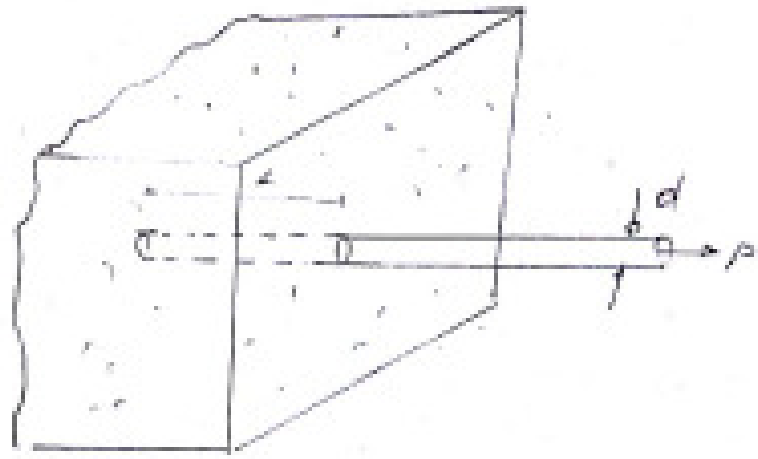


$$\text{shear stress} = \frac{\text{shear force}}{\text{Area of resisting shear}}$$

$$\tau = \frac{30 \times 10^3}{\pi \times d \times \ell}$$

$$= \frac{30 \times 10^3}{\pi \times 12 \times 100}$$

$$\underline{\tau = 7.96 \text{ MPa}}$$



MANIPAL
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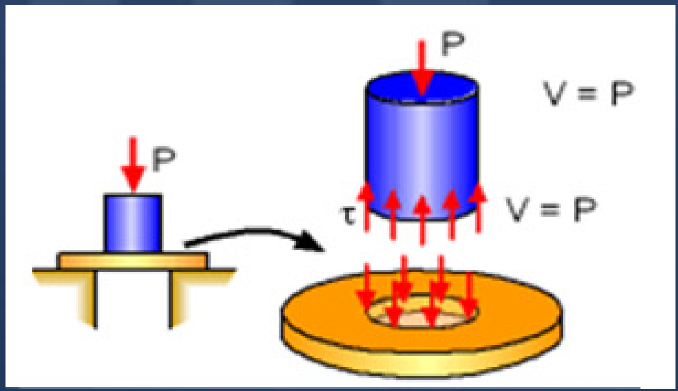
N11. A hole is to be punched out of a plate having an ultimate shear stress of 300 MPa. If the compressive stress in the punch is limited to 400 MPa,

determine:

(a) Maximum thickness of the plate for which a 100 mm dia hole can be punched.

(b) If the plate is 10mm thick, smallest diameter hole that can be punched.



Ans:
t=33.33 mm
d=30 mm

Sol^n (a) Shear stress = $\dfrac{\text{Shear force}}{\text{Area resisting shear}}$

$$300 = \frac{F_s}{2\pi r \times t}$$

Comp. stress = $\dfrac{\text{load}}{\text{c/s area}}$

$$400 = \frac{F}{\pi d^2/4} \Rightarrow 400 = \frac{F}{\frac{\pi}{4} \times 100^2}$$

$$\therefore F = 3141.6 \times 10^3 \text{ N}$$

substituting the value of F in eqn (1)

$$300 = \frac{3141.6 \times 10^3}{2\pi(50) \times t}$$

$$t = 33.33 \text{ mm}$$

(b) $t = 10\,mm$

$$300 = \frac{\text{force}}{\text{area resisting shear}} \quad ; \quad 400 = \frac{\text{Force}}{\pi r^2}$$

$$F = 400\pi r^2$$

Area resisting shear = $2\pi r t = 2\pi r \times 10$

$$\therefore 300 = \frac{400 \times \pi r^2}{2\pi r \times 10}$$

$$r = 15\,mm \; ; \; d = 30\,mm$$

LECTURE 19

Contents:

Poisson's ratio
Volumetric strain
Bulk modulus
Relationship between volumetric strain and linear strain

HOME

POISSON'S RATIO

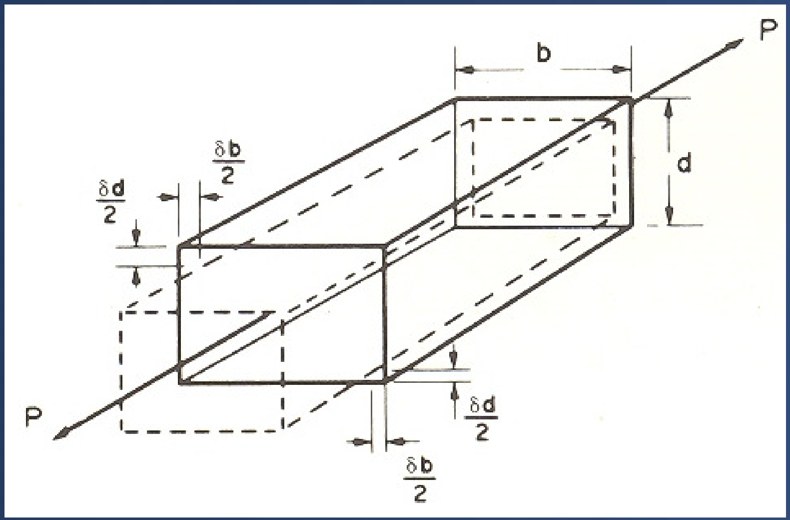


Fig.(a)

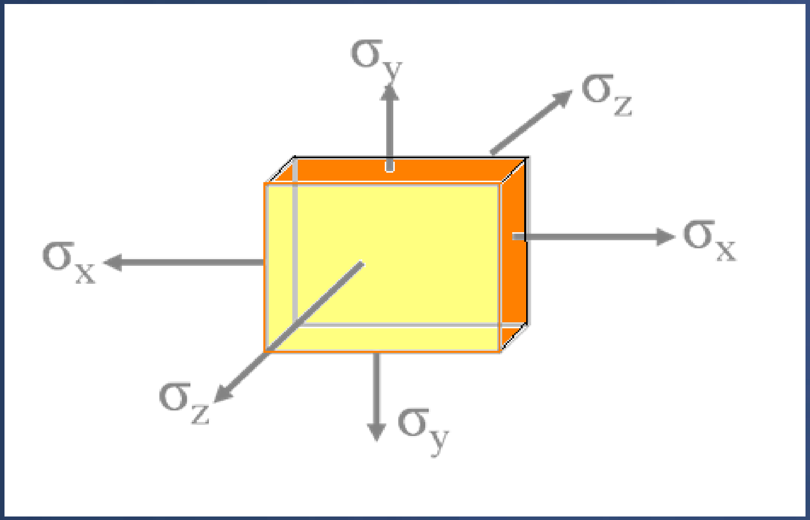
$$\varepsilon_{lat} = -\frac{\delta b}{b} = -\frac{\delta d}{d}$$

$$\epsilon_l = \frac{\delta l}{l}$$

$$\text{Poisson's ratio} = \frac{Lateral\ Strain}{Longitudinal\ Strain}$$

$$= \frac{\left(-\frac{\delta b}{b}\right)}{\frac{\delta l}{l}} \quad \text{or} \quad \frac{\left(-\frac{\delta d}{d}\right)}{\frac{\delta l}{l}}$$

General case:



Strain in X-direction = $$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

Strain in Y-direction = $$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

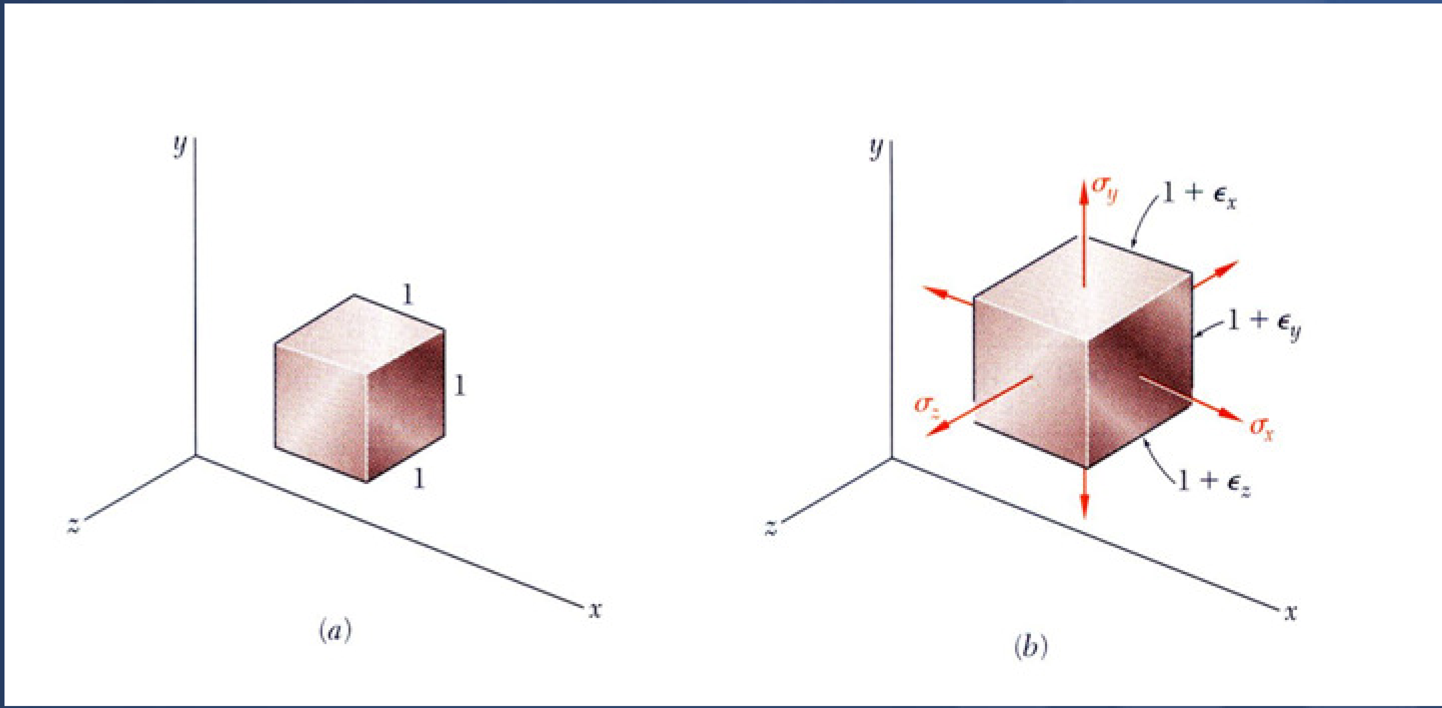
Strain in Z-direction = $$\varepsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$$

Bulk Modulus

Bulk modulus, K $= \dfrac{\sigma}{\left(\dfrac{dV}{V}\right)}$

A body subjected to three mutually perpendicular equal direct stresses then the ratio of stress to volumetric strain is called Bulk Modulus.

Relationship between volumetric strain and linear strain



$$\frac{dV}{1} = \left[\left(1+\varepsilon_x\right)\left(1+\varepsilon_y\right)\left(1+\varepsilon_z\right)\right]-1 = \left[1+\varepsilon_x+\varepsilon_y+\varepsilon_z\right]-1$$

$$= \varepsilon_x + \varepsilon_y + \varepsilon_z$$

= change in volume per unit volume

Relationship between volumetric strain and linear strain

Volumetric Strain

$$\frac{dV}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \left(\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}\right) + \left(\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E}\right) + \left(\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}\right)$$

$$= \frac{1-2\mu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

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For element subjected to uniform hydrostatic pressure,

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

$$\frac{dV}{V} = \frac{1-2\mu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\frac{dV}{V} = \frac{1-2\mu}{E}(3\sigma)$$

$$K = \frac{\sigma}{\left(\frac{dV}{V}\right)}$$

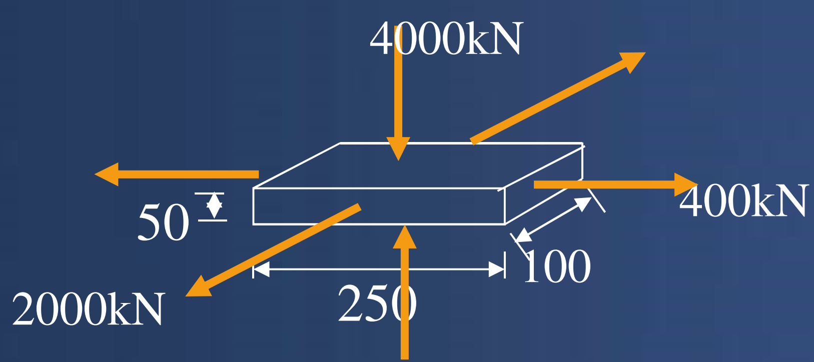
$$E = 3K(1-2\mu)$$

N12. A bar of metal 100x50 mm in cross section is 250 mm long. It carries a tensile load of 400 kN in the direction of its length, a compressive load of 4000 kN on its 100 mm x 250 mm faces and a tensile load of 2000 kN on its 50 mm x 250 mm faces. If $E=2x10^5$ N/mm^2 and poisson's ratio is 0.25, find the change in volume of the bar.

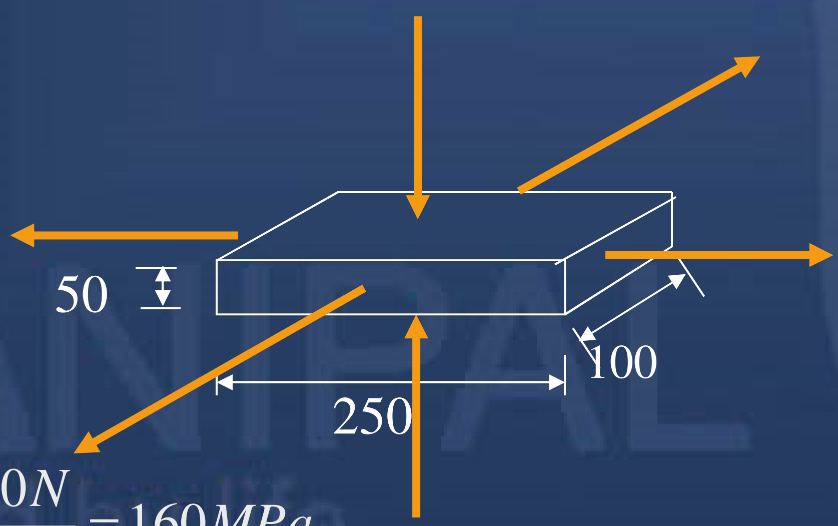
What change must be made in the 4000 kN load in order that there shall be no change in volume of the bar.

MANIPAL
Inspired by life

Stresses in different directions



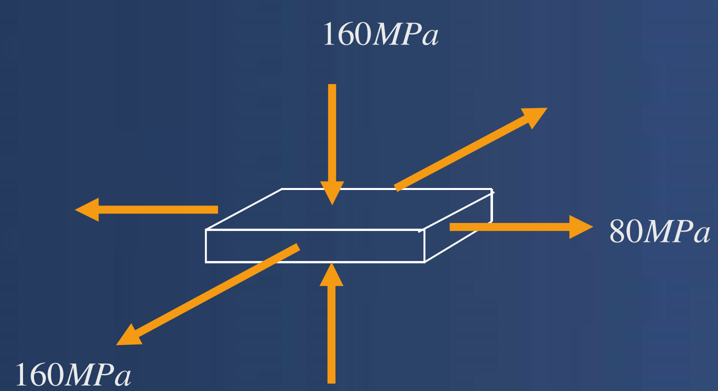
$$\sigma_y = \frac{4000 \times 1000 N}{250 \times 100 mm^2} = 160 MPa$$



$$\sigma_x = \frac{400 \times 1000 N}{100 \times 50 mm^2} = 80 MPa$$

$$\sigma_z = \frac{2000 \times 1000 N}{250 \times 50 mm^2} = 160 MPa$$

Stresses in different direction

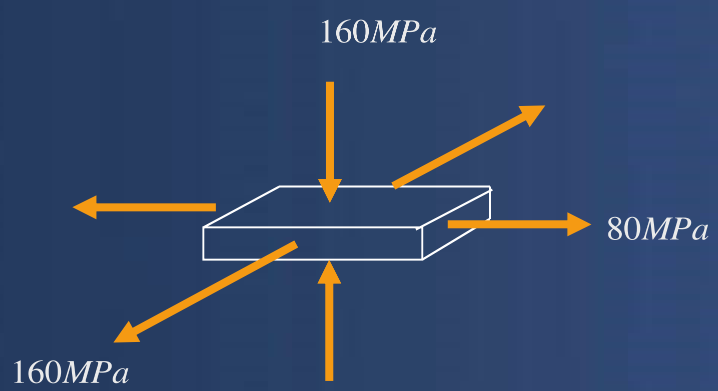


$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\boxed{\varepsilon_x = \frac{+80}{E} - \mu\frac{-160}{E} - \mu\frac{+160}{E} = 4\times10^{-4}}$$

$$\frac{\delta l_x}{l_x} = \frac{\delta l_x}{250} = 4\times10^{-4}$$

$$\delta l_x = 0.1mm$$

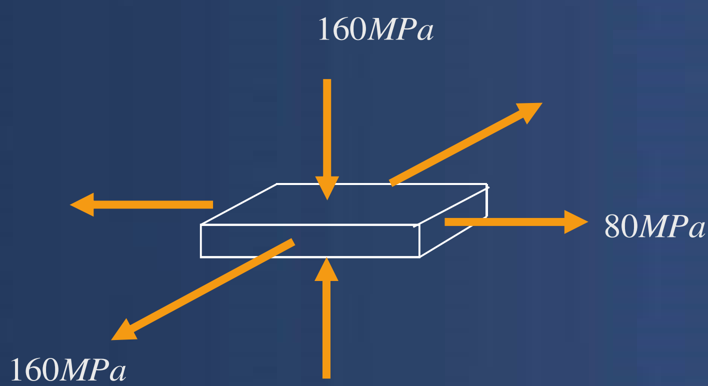


$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{-160}{E} - \mu\frac{+80}{E} - \mu\frac{+160}{E} = -\left(1.1\times 10^{-3}\right)$$

$$\frac{\delta l_y}{l_y} = \frac{\delta l_y}{50} = -\left(1.1\times 10^{-3}\right)$$

$$\delta l_y = -0.005mm$$



$$\varepsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$$

$$\boxed{\varepsilon_z = \frac{+160}{E} - \mu\frac{-160}{E} - \mu\frac{+80}{E} = +\left(9\times10^{-4}\right)}$$

$$\frac{\delta l_z}{l_z} = \frac{\delta l_z}{250} = +\left(9\times10^{-4}\right)$$

$$\delta l_z = +0.09mm$$

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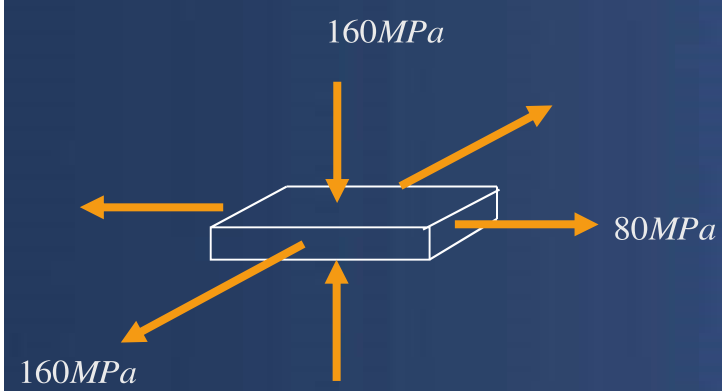


To find change in volume

$$\frac{dV}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\frac{dV}{V} = (4 - 11 + 9) \times 10^{-4} = 2 \times 10^{-4}$$

$$dV = (2 \times 10^{-4}) \times V = (2 \times 10^{-4}) \times 250 \times 100 \times 50$$

$$dV = +250 mm^3$$

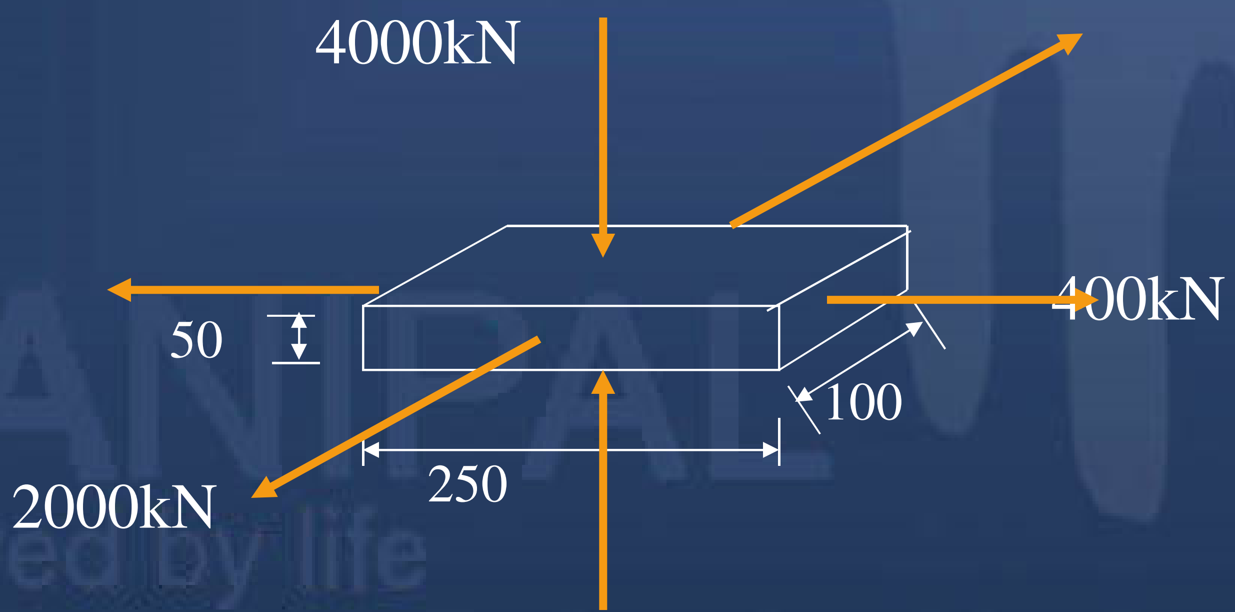
Alternatively,

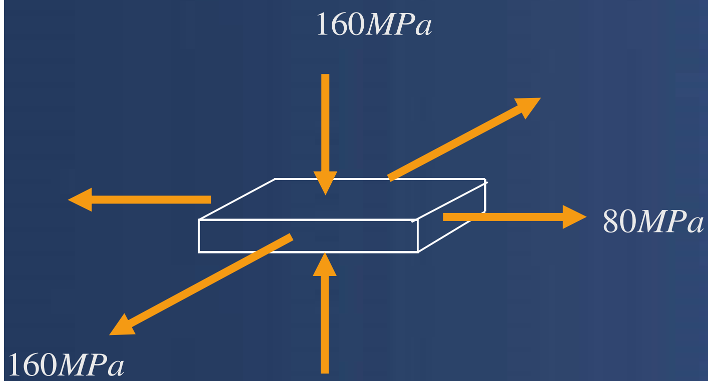
$$\frac{dV}{V} = \frac{1 - 2\mu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$\frac{dV}{V} = \frac{1 - 2\mu}{E}(+80 - 160 + 160)$$

$$= \frac{1 - 2\mu}{E}(80) = 2 \times 10^{-4}$$

The change in value that should be made in 4000kN load, in order that there should be no change in the volume of the bar.





We know that

$$\frac{dV}{V} = \frac{1-2\upsilon}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

In order that change in volume to be zero

$$0 = \frac{1-2\upsilon}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\left(\sigma_x + \sigma_y + \sigma_z\right) = 0$$

$$\left(+80 + \sigma_y + 160\right) = 0$$

$$\sigma_y = -240MPa$$

$$-240 = \frac{P_y}{250 \times 100}$$

$$P_y = -6000kN$$

The change in value should be an addition of 2000kN compressive force in Y-direction

N13. A bar of steel 40 mm x 40 mm cross section and 150 mm long is subjected to a tensile load of 200 kN along its longitudinal axis and tensile load of 600 kN and 400 kN along lateral axis.

Find,

(a) Change in each dimension and change in volume

(b) What longitudinal force alone can produce same longitudinal strain as in case (a).

Given E= 200 GPa μ=0.3

Sol :-

stress in x, y, z direction.

$$\sigma_x = \frac{P_x}{A_x} = \frac{200\times10^3}{40\times40} = 125\ N/mm^2$$

$$\sigma_y = \frac{P_y}{A_y} = \frac{600\times10^3}{150\times40} = 100\ N/mm^2$$

$$\sigma_z = \frac{P_z}{A_z} = \frac{410\times10^3}{150\times40} = 66.67\ N/mm^2$$

change in dimension along x, y & z direction

$$e_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$= \frac{125}{2\times10^5} - \frac{0.3\times100}{2\times10^5} - \frac{0.3\times66.67}{2\times10^5} = 3.75\times10^{-4}$$

$$e_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E}$$

$$= \frac{100}{2\times10^5} - 0.3\times\frac{125}{2\times10^5} - 0.3\times\frac{66.67}{2\times10^5} = 2.125\times10^{-4}$$

$$e_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

$$= \frac{66.67}{2\times10^5} - 0.3\times\frac{125}{2\times10^5} - 0.3\times\frac{100}{2\times10^5}$$

$$= -4.167\times10^{-6}$$

$\delta l_x = 0.056\ mm$

$\delta l_y = 8.5\times10^{-3}\ mm$

$\delta l_z = -1.667\times10^{-4}\ mm$

change in volume, $dv = (e_x + e_y + e_z)v$

$$= (3.75\times10^{-4} + 2.125\times10^{-4} - 4.167\times10^{-6})\times(150\times40\times40)$$

$$= 140\ mm^3$$

MANIPAL
Inspired by life

LECTURE 20

Contents:

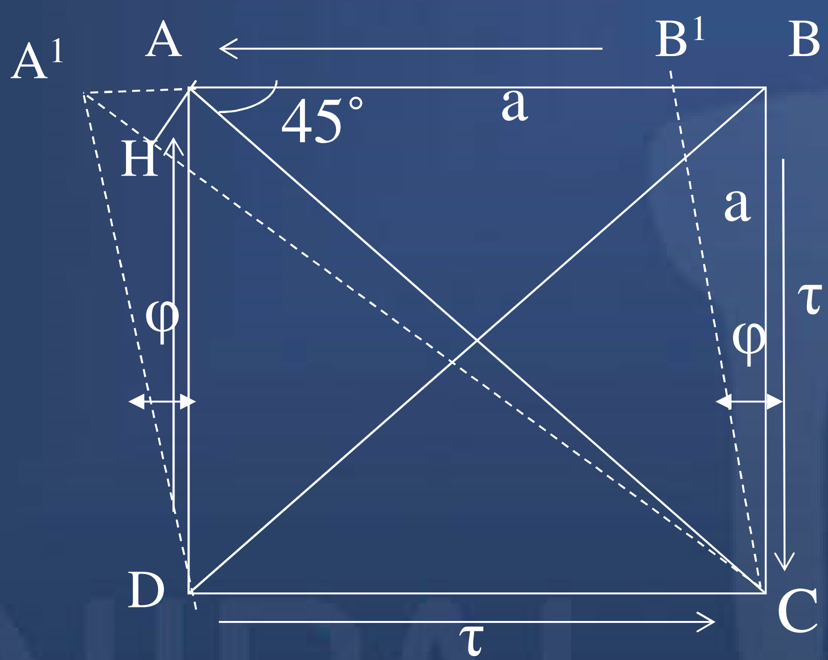
Relationship between modulus of elasticity and modulus of rigidity

Relationship between E, G and K

Application problems

HOME

Relationship between young's modulus of elasticity (E) and modulus of rigidity (G) :-



MANIPAL
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Consider a square element ABCD of side ‘a’ subjected to pure shear‘τ’. DA'B'C is the deformed shape due to shear τ.

Drop a perpendicular AH to diagonal A'C.

Strain in the diagonal AC = $\tau/E - \mu(-\tau/E)$ [$\sigma_n = \tau$]

$= \tau/E\,[1 + \mu]$ -----------(1)

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Strain along the diagonal $AC=(A'C–AC)/AC=(A'C–CH)/AC=A'H/AC$

In Δle $AA'H$

$\cos 45^\circ = A'H/AA'$

$A'H= AA' \times 1/\sqrt{2}$

$AC = \sqrt{2} \times AD$ $\quad$ $(AC = \sqrt{AD^2 +AD^2})$

Strain along the diagonal $AC = AA'/ (\sqrt{2} \times \sqrt{2} \times AD)=\varphi/2$ ----(2)

Modulus of rigidity $= G = \tau /\varphi$

$$\varphi = \tau /G$$

Substituting in (2)

Strain along the diagonal $AC = \tau /2G$ -----------(3)

Equating (1) & (3)

$\tau /2G = \tau /E[1+\mu]$

$$\boxed{E=2G(1+ \mu)}$$

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Relationship between E, G, and K:-

We have

$E = 2G(1+\mu)$ -----------(1)

$E = 3K(1-2\mu)$ -----------(2)

Equating (1) & (2)

$2G(1+\mu) = 3K(1-2\mu)$

$2G + 2G\mu = 3K - 6K\mu$

$\mu = (3K - 2G)/(2G + 6K)$

Substituting in (1)

$E = 2G[1 + (3K - 2G)/(2G + 6K)]$

$E = 18GK/(2G + 6K)$

$$\boxed{E = 9GK/(G+3K)}$$

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N14. A circular rod of 100 mm dia and 500 mm length is subjected to a tensile force of 2000 kN. Determine the modulus of rigidity, bulk modulus and the change in volume, if the poisson's ratio=0.3 and $E=2\times10^5$ N/mm^2.

Ans:

$G=0.77\times10^5$ N/mm^2

$K=1.67\times10^5$ N/mm^2

$dv=1994.9$ mm^3

Sol

$A = \pi/4 \times 100^2 = 7853.98\ mm^2$

$L = 500mm$

$P = 2000 \times 10^3 N$

$\mu = 0.3$ & $E = 2 \times 10^5\ N/mm^2$

(i) $E = 2G(1+\mu)$ $\therefore G = \frac{E}{2(1+\mu)}$

$$G = \frac{2 \times 10^5}{2(1+0.3)} = 0.77 \times 10^5\ N/mm^2$$

(ii) $$K = \frac{E}{3(1-2\mu)} = \frac{2 \times 10^5}{3(1-2 \times 0.3)} = 1.67 \times 10^5\ N/mm^2$$

(iii) To find the change in volume using the relation for volumetric strain we have

$$\frac{dv}{V} = \epsilon_x + \epsilon_y + \epsilon_z$$

Longitudinal strain is given by

$$\epsilon_x = \sigma/E = \frac{P}{AE} = \frac{2000 \times 10^3}{2 \times 10^5 \times 7853.98} = 1.27 \times 10^{-3}$$

lateral strain is given by.

$$\epsilon_z = \epsilon_y = -\mu \epsilon_x = -0.3 \times 1.27 \times 10^{-3} = -3.82 \times 10^{-4}$$

$$e = \frac{dv}{V} = \epsilon_x - \mu \epsilon_x - \mu \epsilon_x$$

$$e = 1.27 \times 10^{-3}(1 - 2 \times 0.3)$$

$$= 5.08 \times 10^{-4}$$

change in volume is given by $\therefore dv = V.e$

$$dv = (7853.98 \times 500) \times 5.08 \times 10^{-4}$$

$$= 1994.9\ mm^3$$

N15. The modulus of rigidity of a material is 0.8×10^5 N/mm^2 . When a 6 mm x 6 mm bar of this material is subjected to an axial pull of 3600 N, it was found that the lateral dimension of bar is changed to 5.9991 mm x 5.9991 mm.

Find μ and E.

Sol/ Lateral strain $= \frac{\delta b}{b} = \frac{6-5.9991}{6} = 1.5\times10^{-4}$

Axial stress, $\sigma = P/A = \frac{3600}{6\times6} = 100\ N/mm^2$

Lateral strain $= \mu\,(\sigma/E)$

$$\mu = \frac{1.5\times10^{-4}}{(110/E)} = \frac{1.5\times10^{-4}}{110}E$$

or $E = \frac{110\,\mu}{1.5\times10^{-4}}$ ← ①

also $G = \frac{E}{2(1+\mu)} \Rightarrow 0.8\times10^{5} = \frac{E}{2(1+\mu)}$

$\therefore E = 160\times10^{3}(1+\mu)$ ← ②

Equating ① & ②

$$\frac{100\,\mu}{1.5\times10^{-4}} = 160\times10^{3}(1+\mu)$$

$\therefore \mu = 0.316$

Substituting $\mu = 0.316$ in ①

$$E = \frac{100\times0.316}{1.5\times10^{-4}} = 2.105\times10^{5}\ N/mm^2$$

TUTORIAL 8

Contents:

Tutorial problems

HOME

T1. Find the Young's modulus of a brass rod of diameter 25 mm and of length 250 mm which is subjected to a tensile load of 75 kN when the extension of the rod is equal to 0.3 mm.

Sol^n Given, diameter of rod = 25 mm

Area of rod $= \frac{\pi}{4} \times 25^2 = 490.87\ mm^2$

tensile load, $P = 75\ kN = 75000\ N$

(dL) extension of rod = 0.3 mm

length of rod = 250 mm

Stress $(\sigma) = \frac{P}{A} = \frac{75,000}{490.87} = 152.79\ N/mm^2$

Strain $(\epsilon) = \frac{dL}{L} = \frac{0.3}{250} = 1.2 \times 10^{-3}$

Young's modulus $(E) = \frac{\sigma}{\epsilon} = \frac{152.79}{1.2 \times 10^{-3}} = 127325\ N/m$

T2. The ultimate stress, for a hollow steel column which carries an axial load of 2.0 MN is 480 N/mm². If the external diameter of the column is 200 mm, determine the internal diameter. Take factor of safety as 3.

Sol^n. ultimate

Axial load = 2 MN = 2×10^6 N

External diameter (D) = 200 mm

Factor of safety = 3

let 'd' be internal diameter

∴ Area of c/s of the column;

$$A_c = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(200^2 - d^2)$$

$$Fos = \frac{\text{ultimate stress}}{\text{working stress / allowable stress}}$$

$$3 = \frac{480}{\text{working stress}}$$

$$\sigma_{working} = 160 \text{ N/mm}^2$$

TUTORIAL 9

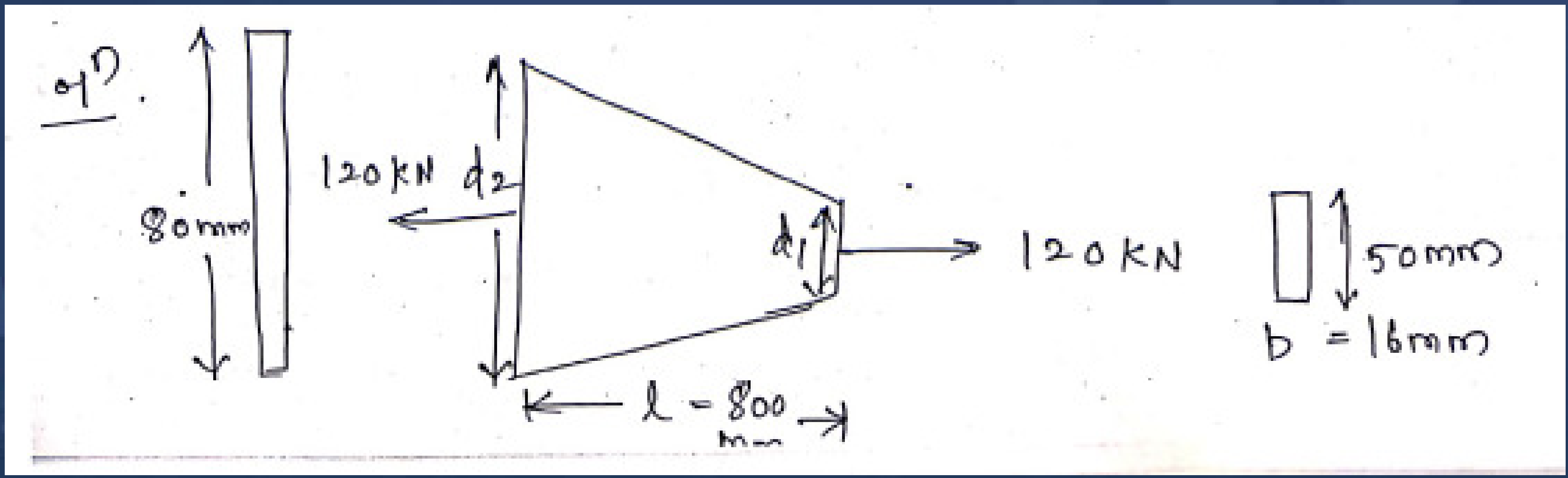
Contents:

Tutorial problems

HOME

T3. A steel flat of thickness 16 mm tapers uniformly from 80 mm at one end to 50 mm at the other end in a length of 800 mm, If the flat is subjected to a load of 120 kN, find the extension of the flat. Also calculate the percentage error if average area is used for calculating its extension. Take $E=2x10^5$ Mpa.

Solution:



$$\delta = \frac{2.302\, PL}{b \times E(d_2 - d_1)} (\log d_2 - \log d_1)$$

$$= \frac{2.302 \times 120 \times 10^3 \times 800}{16 \times 2\times10^5 \times (80-50)} (\log 80 - \log 50)$$

$$= 0.469\,mm$$

Area of c/s at Larger end $A_2 = 80 \times 16 = 1280\,mm^2$

Area of c/s at smaller end $A_1 = 50 \times 16 = 800\,mm^2$

Average c/s area $= A = \frac{A_1 + A_2}{2} = \frac{1280+800}{2}$

$$= 1040\,mm^2$$

Total extension of bar, if average area of c/s of uniform width of bar is used

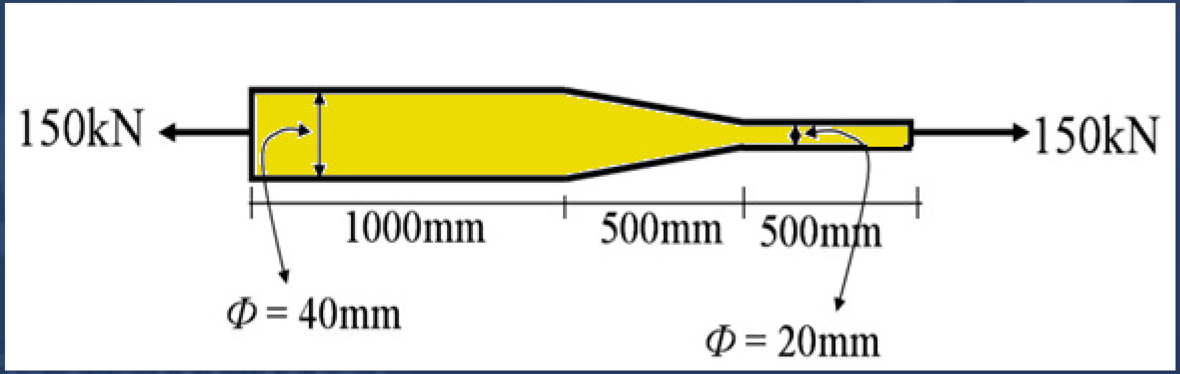
$$\delta a = \frac{PL}{AE} = \frac{120 \times 10^3 \times 800}{1040 \times 2\times10^5} = 0.462\,m$$

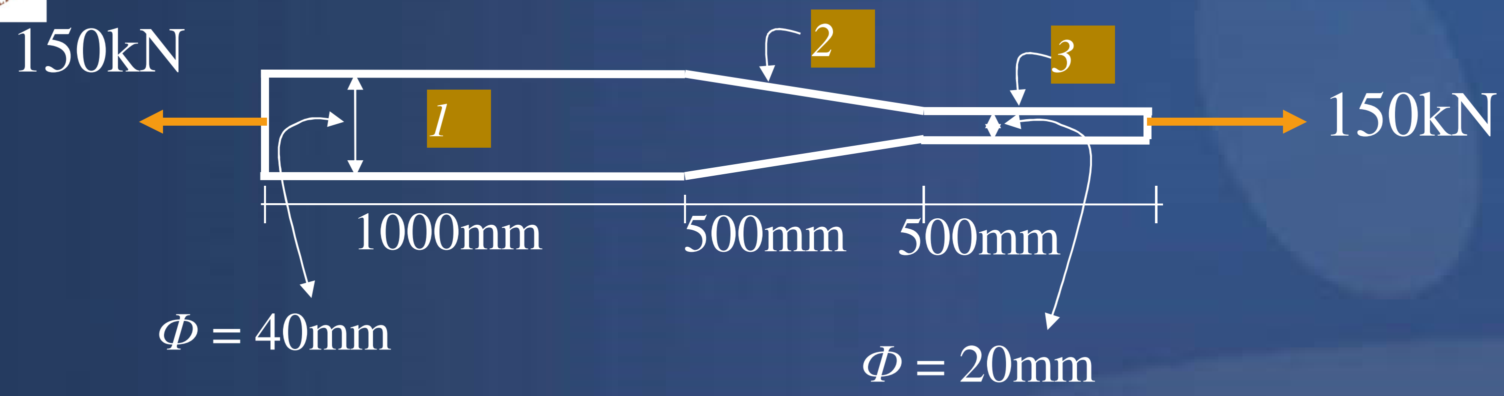
Percentage error in extension

$$= \frac{\delta - \delta a}{\delta} = \frac{0.469 - 0.462}{0.469}$$

$$= 1.5\%$$

T4. A two meter long steel bar is having uniform diameter of 40 mm for a length of 1 m, in the next 0.5 m its diameter gradually reduces to 20 mm and for remaining 0.5 m length diameter remains 20 mm uniform as shown in the figure. If a load of 150 kN is applied at the ends, find the stresses in each section of the bar and total extension of the bar. Take E = 200 GPa.





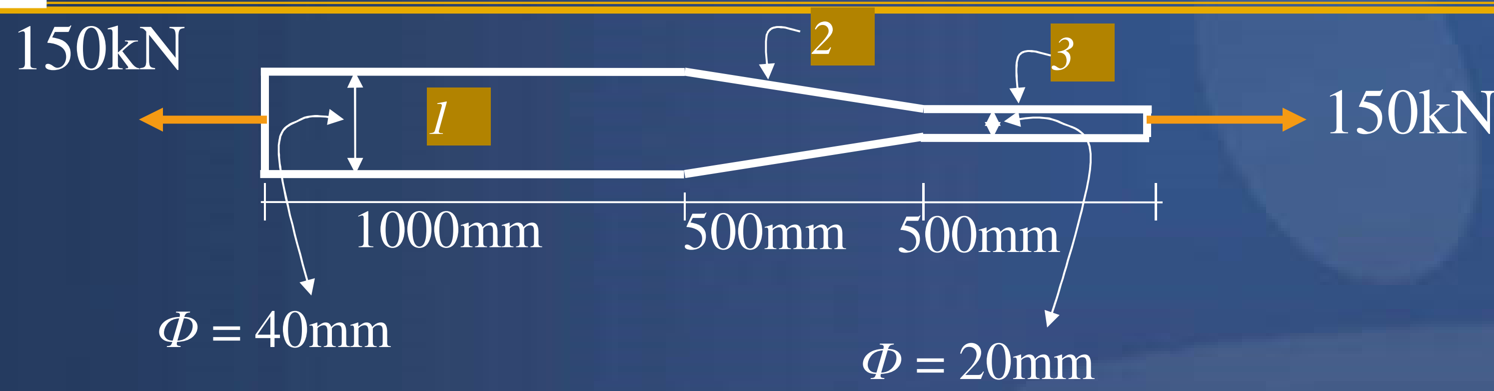
If we take a section any where along the length of the bar, it is subjected to a load of 150kN.

$$\sigma_1 = \frac{150kN}{\pi 40^2 / 4} = 119.37MPa$$

$$\sigma_2 = \frac{150kN}{\pi d^2 / 4} \Rightarrow \quad \sigma_{2,\max.} = \frac{150kN}{\pi 40^2 / 4} = 119.37MPa$$

$$\sigma_{2,\min.} = \frac{150kN}{\pi 20^2 / 4} = 477.46MPa$$

$$\sigma_3 = \frac{150kN}{\pi 20^2 / 4} = 477.46MPa$$



If we take a section any where along the length of the bar, it is subjected to a load of 150kN.

$$\delta l_1 = \frac{150kN \times 1000}{\left(\pi 40^2 / 4\right) \times E} = 0.597mm$$

$$\delta l_2 = \frac{4PL}{\pi E d_1 d_2} = \frac{4 \times 150kN \times 500}{\pi \times E \times 40 \times 20} = 0.597mm$$

$$\delta l_3 = \frac{150kN \times 500}{\left(\pi 20^2 / 4\right) \times E} = 1.194mm$$

$$\text{total, } \delta l = 2.388mm$$

TUTORIAL 10

Contents:

Tutorial problems

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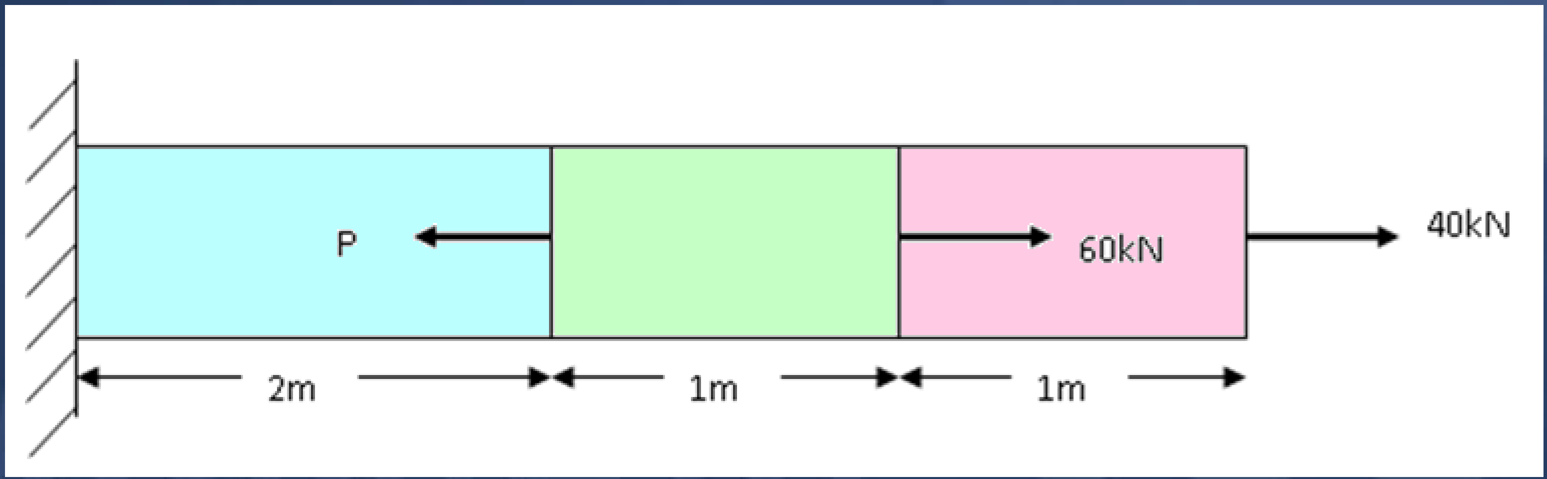
T5. Determine the magnitude of the load P necessary to produce zero net change in the length of the bar shown in the figure below. Take A=400 mm^2.

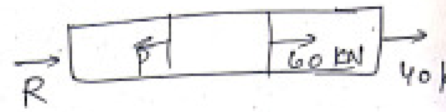


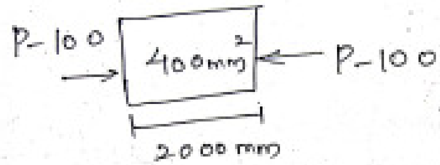
Q.3 Sol^n let the reaction 'R' at the support acts towards right.

for the equillibrium of the bar

$$R + 60 + 40 = P$$

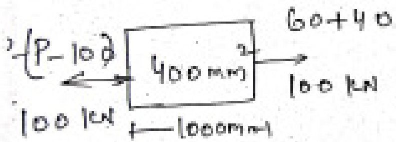
$$R = P - 100$$



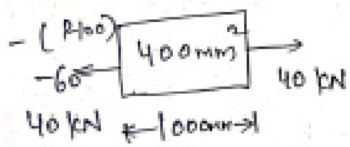


$$\delta_{AB} = \frac{-PL}{AE} = \frac{(P-100)\times 2000}{400 \times E}$$

$$= \frac{5P - 500}{E}$$



$$\delta_{BC} = \frac{100 \times 1000}{400 \times E} = \frac{250}{E}$$



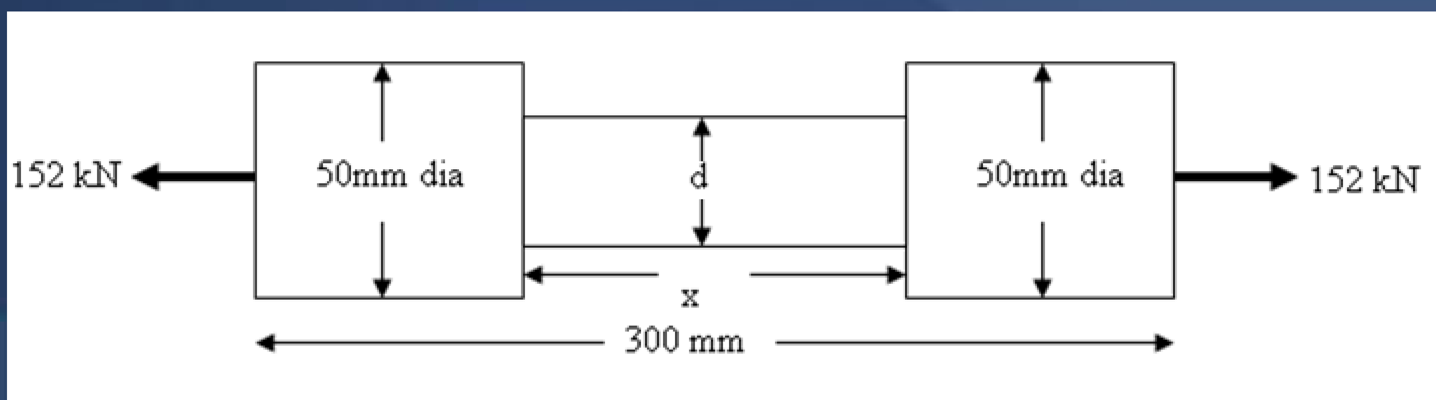
$$\delta_{CD} = \frac{40 \times 1000}{400 \times E} = \frac{100}{E}$$

Total extension $\delta = \delta_{AB} + \delta_{BC} + \delta_{CD}$

$$0 = \frac{5P - 500}{E} + \frac{250}{E} + \frac{100}{E}$$

$$\therefore P = 170 \text{ kN}$$

T6. For the bar shown below, determine diameter of the central portion and its length, if the total extension of the bar is 0.16 mm. Take E=200 GPa. Stress at central portion is limited to 140 N/mm^2



HOME

Sol) Extension of end portion + extension of middle portion

$$\frac{\sigma}{E}(300-x) + \frac{\sigma}{E}(x) = 0.16$$

$$x = 140.28 \text{ mm}$$

C/s area of the middle portion, stress = $\frac{\text{Force}}{\text{C/s area}}$

$$\text{C/s area} = \frac{F}{\sigma} = \frac{15200}{140}$$

$$A = 1085.71 \text{ mm}^2 = \frac{\pi d^2}{4}$$

$$d = 37.18 \text{ mm}$$

let the length of the middle portion be 'x' mm

stress in the end portion, $\sigma = \frac{P}{A} = \frac{152000}{\frac{\pi}{4} \times 50^2}$

$$= 77.41 \text{ N/mm}^2$$

$a = 0.16$ mm

T7. A tension test is carried out subjected on a mild steel tube of external diameter 18 mm and internal diameter 12 mm. An an axial load of 2 kN produces an extension of 3.36×10^{-3} mm on a length of 50 mm and a lateral contraction of 3.62×10^{-4} mm of outer diameter.

Determine E, μ,G and K.

Ans:
$E=2.11 \times 10^{5}$ N/mm^2
$\mu=0.3$
$G=81.15 \times 10^{3}$ N/mm^2
$K=175.42 \times 10^{3}$ N/mm^2

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ADDITIONAL TUTORIAL PROBLEMS

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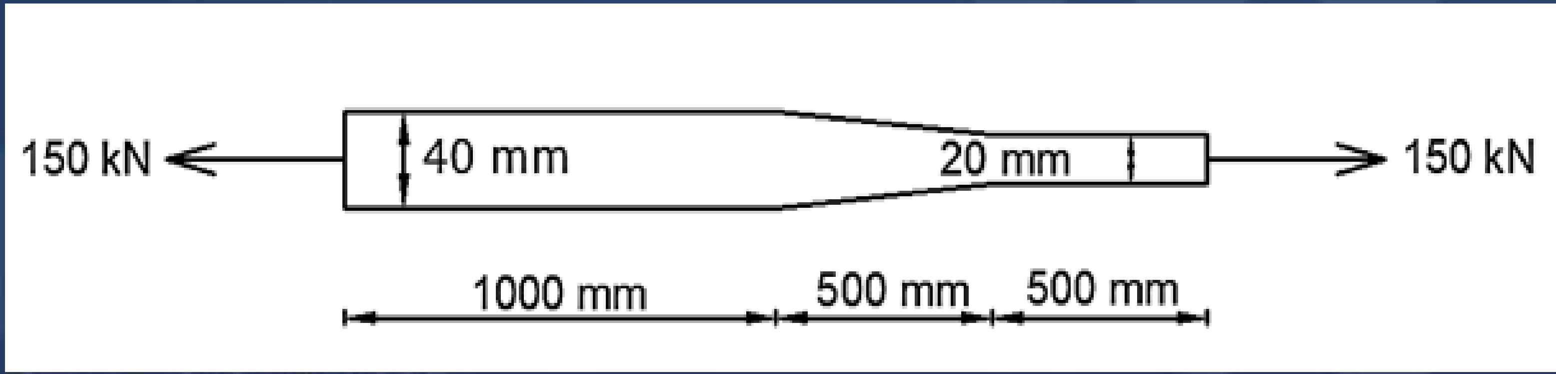
HOME

AT1. A two meter long steel bar is having uniform diameter of 40 mm for a length of 1 m, in the next 0.5 m its diameter gradually reduces to 20 mm and for remaining 0.5 m length diameter remains 20 mm uniform as shown in the figure. If a load of 150 kN is applied at the ends, find the stress in each section and total extension of the bar. Take E = 200 Gpa



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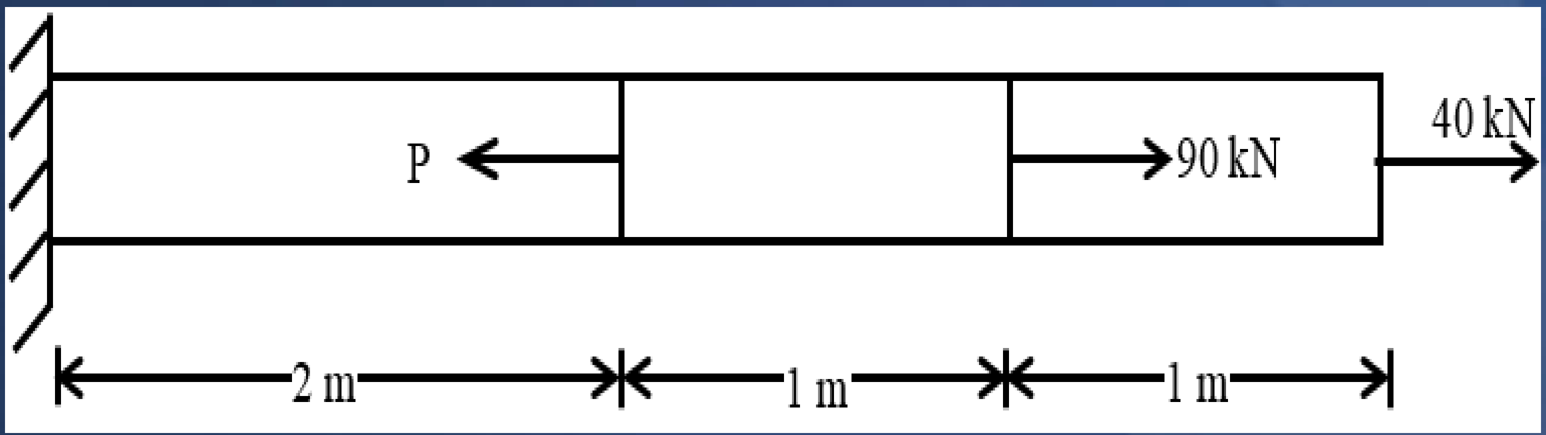
AT2. Determine the magnitude of the load P necessary to produce zero net change in the length of the bar shown in the figure below. Take A=400 mm^2.



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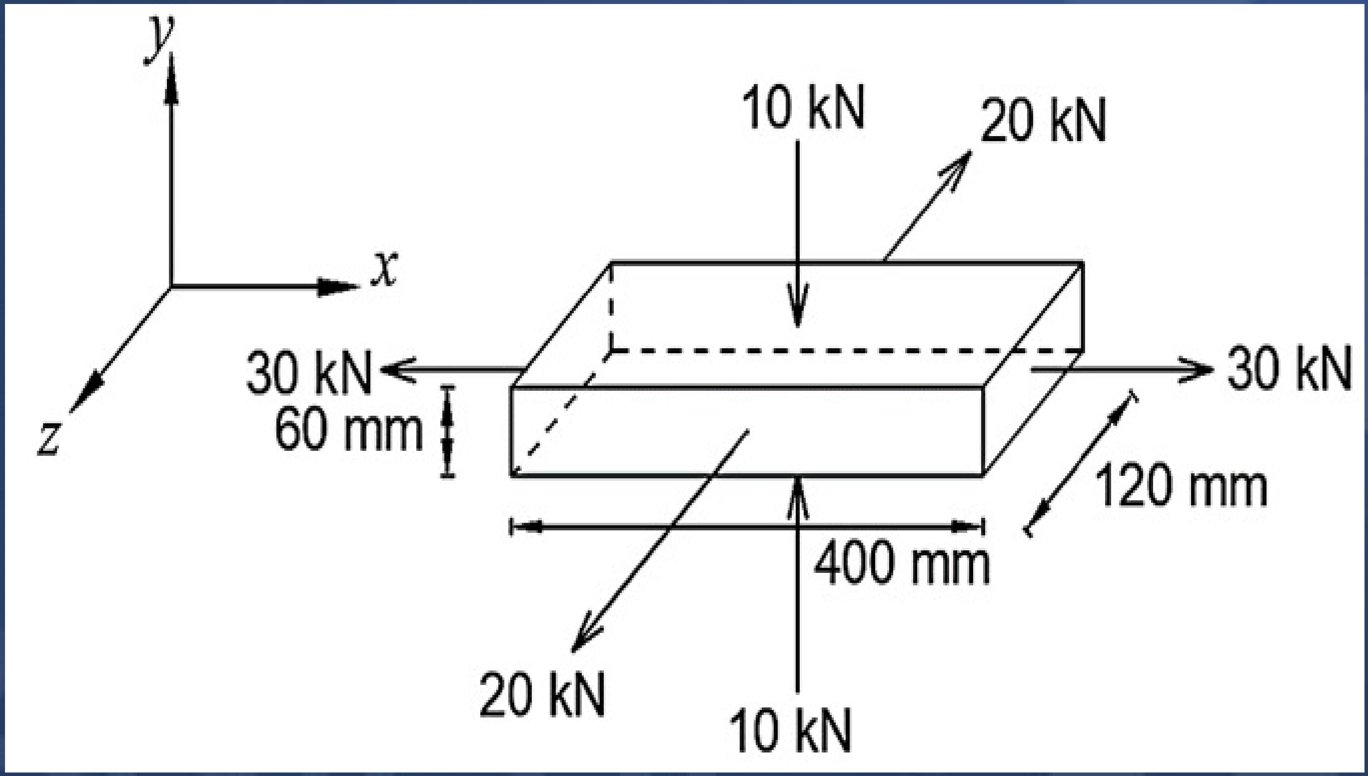
AT3. A steel bar of 400 mm x 120 mm x 60 mm is subjected to forces as shown in the figure. Find the change in dimension. Taking E = 200 GPa and μ = 0.25.



Inspired by life

AT4. A circular concrete pillar consists of six steel rods of total area 2280 mm^2. Determine the area of concrete required when it has to carry a load of 1000 kN. Take allowable stresses for steel & concrete as 140 MPa & 8 MPa respectively. Take $E_s = 15\ E_c$.

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