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Engineering Mathematics I & II

1. Algebra

A quadratic equation is an equation of the form
$ax^2 + bx + c = 0$, $a \neq 0$, a, b, c are real numbers.

1.1 Roots of the quadratic equation are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1.2 $D = b^2 - 4ac$ is called discriminant.

If $D = 0$ then the roots are real and equal
$D > 0$ then the roots are real and distinct
$D < 0$ then the roots are imaginary

1.3 $y = ax^2 + bx + c$ represents a parabola.

If $a > 0$ then the parabola is concave upwards.
If $a < 0$ then the parabola is concave downwards.

2. Complex Numbers

2.1 Representation of Complex Numbers

Complex operator "i"	-	$i = \sqrt{-1}$
Rectangular form	-	$z = x \pm i\,y$
Trigonometric form	-	$z = r\,(\cos\theta \pm i\,\sin\theta)$
Exponential form	-	$z = r\,e^{\pm i\theta}$
Polar form	-	$z = r\angle \pm \theta$
		(r is the modulus and θ is the amplitude of z)

2.2 Rectangular-Polar Conversion

Rectangular to polar	-	$r = \sqrt{x^2 + y^2}$	$\theta = \tan^{-1}\frac{y}{x}$
Polar to Rectangular	-	$x = r\cos\theta$	$y = r\sin\theta$

2.3 Arithmetic Operation

Let $\boldsymbol{z_1 = x_1 + iy_1 = r_1\angle\theta_1}$ & $\boldsymbol{z_2 = x_2 + iy_2 = r_2\angle\theta_2}$

Addition	$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$
Subtraction	$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$
Multiplication	$z_1\,z_2 = r_1\,r_2\angle(\theta_1 + \theta_2)$
Division	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$

3. Logarithm

Let a and x be real numbers such that $a \neq 1$, $a > 0$. Let $y = a^x$. Then x is called the logarithm of y to the base a and we write $x = \log_a y \therefore x = \log_a y \Leftrightarrow a^x = y$

Since $a^x > 0$, for every real x, logarithm of negative real numbers and logarithm of zero are not defined.

4. Progressions

4.1 Arithmetic Progression

First term: a — Common difference: d

Form of sequence - $a, a + d, a + 2d, \dots , a + (n -1)d, \dots$

n^{th} term - $t_n = a + (n – 1) d$

Sum of the first n terms - $S_n = \frac{n}{2}\left[2a + (n - 1)\, d\right] = \frac{n}{2}\left[a + \ell\right]$

where l is the last term.

4.2 Geometric Progression

First term: a — Common ratio: r

Form of sequence - $a, ar, ar^2, \dots, ar^n, \dots$

n^{th} term - $t_n = a\, r^{(n – 1)}$

The sum of the first n terms - $S_n = \frac{a\,(1 - r^n)}{1 - r}$, if $r \neq 1$

- na if $r = 1$

Sum to infinity of the geometric series - $\frac{a}{1 - r}$ (If $| r | < 1$)

4.3 Harmonic Progression

"Sequence in which the reciprocals of the terms are in arithmetic progression"

4.4 Given n numbers $a_1, a_2, a_3, \dots, a_n$, we define

Arithmetic Mean, AM = $\frac{a_1+a_2+\cdots+a_n}{n}$

Geometric Mean, GM = $(a_1 a_2 \dots a_n)^{1/n}$

Harmonic Mean, HM = $\left(\frac{a_1^{-1} + a_2^{-1} + \dots + a_n^{-1}}{n}\right)^{-1}$

5. Matrices

5.1 A system of mn numbers arranged in a rectangular array of m rows & n columns is called a matrix of order m x n. If a_{ij} denotes the element in the i^{th} row and j^{th} column, then the matrix is denoted by $A = (a_{ij})$.

5.2 Transpose

Let A be an $m \times n$ matrix. The matrix of order $n \times m$ obtained by interchanging the rows and columns of A is called the **transpose of A** and is denoted by A'.

5.3 Determinant

Let $A=\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$, then the determinant of A is defined as $|A| = a_1b_2 - b_1 a_2$.

If $A=\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, then $|A| = a_1 (b_2 c_3 - c_2 b_3) - a_2 (b_1 c_3 - c_1 b_3) + a_3 (b_1 c_2 - c_1 b_2)$.

5.4 Types of Matrices

Matrix A is said to be a Square Matrix if it has same number of rows and columns.
A column Matrix has only one column and a row matrix has only one row.
Let A be a Square Matrix of order n given by $A = (a_{ij})$

A is a	Diagonal matrix	if $a_{ij} = 0 \ \forall$ i and j, $i \neq j$
	Scalar Matrix	if it is diagonal and a_{ii} = a constant for every i.
	Identity Matrix	if it is scalar and $a_{ii}=1 \ \forall$ i
	Idempotent	if $A^2=A$
	Symmetric	if $A'=A$
	Skew Symmetric	if $A'=-A$
	Lower Triangular	if $a_{ij}=0 \ \forall \ i<j$
	Upper Triangular	if $a_{ij}=0 \ \forall \ i>j$
	Orthogonal	if $AA'= I$

A square Matrix A of order n is said to be nonsingular or invertible if there exists a square matrix B of order n such that AB=BA=I where I is the Identity Matrix.

A is invertible if and only if $|A| \neq 0$

$A + A'$ and AA' are symmetric

$A - A'$ is skew symmetric.

Every square matrix can be uniquely expressed as sum of two square matrices of which one is symmetric and the other is skew symmetric. i.e. $A = (A + A')/2 + (A-A')/2$

5.5 Elementary Row (Column) Operations

The following three operations on a matrix are called elementary row (column) operations.

Interchange: Interchange of any two rows (columns)

$R_i \leftrightarrow R_j$ denotes interchanging of i th and j th rows.

Scaling: A row (column) can be multiplied by a non-zero scalar

$R_i \to kR_i$ denotes multiplying the i th row by the scalar k .

Row Addition: Adding a multiple of one row of a matrix to any another row (column)

$R_i \to R_i + kR_j$ denotes adding k times the j th row, to the i th row.

Two matrices are said to be equivalent if one can be obtained from the other by a sequence of elementary row (column) operations.

Consider a matrix A of order $m \times n$.
A matrix obtained from A by deleting some rows and some columns of A is called a sub matrix of A. Determinant of a square sub matrix of A of order r is called a minor of A of order r.

A is said to be of rank r
1. If A has at least one non-zero minor of order r and
2. All minors of higher order are zero.

5.6 Echelon Form of a Matrix

Given an (m x n) matrix, it can always be reduced to a so-called Echelon form by a series of row transformations, which has the following structure.

- First k rows, $0 \leq k \leq m$, are non-zero. Remaining m – k rows are zero rows.
- First non-zero element in each non zero row is equal to unity.
- If C_i denotes the column in which 1 of the i^{th} row exists then $C_1 < C_2 < --- < C_k$.

Rank of matrix = number of non-zero rows in the Echelon form of the matrix.

Let A be a (mxn) matrix, x be a (nx1) matrix and B be a (mx1) matrix. Then a system of m linear equations in n unknowns is given by the matrix equation Ax = B

The mx(n+1) matrix obtained by augmenting the column B to the matrix A is called the Augmenting Matrix and is denoted by (A:B)

Consider the linear system Ax = B,

The system is:

Homogeneous	- If B = 0
Consistent	- If rank (A) = rank (A:B) (solution exists)

The system has:

Unique solution	- If rank (A) = rank (A : B) = number of unknowns
Infinite solutions	- If rank (A) = rank (A : B) < number of unknowns
No solution	- If rank (A) $\neq$ rank (A : B)

5.7 Eigen Values

Let A be a square matrix and I be the identity matrix of the same order then,

1. $A - \lambda I$ - Characteristic matrix of A.
2. $|A - \lambda I|$ - Characteristic polynomial of A.
3. $|A - \lambda I| = 0$ - Characteristic equation of A.
4. The roots of $|A - \lambda I| = 0$ - Characteristic roots or eigen values or latent roots of A. If A is of order n, then it has n eigen values.

6. Differential Calculus

6.1 Graph of a function y = f (x) is the set given by g = {(x, y)| y = f(x)}.

A function f is said to be differentiable at a point a, if $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ exists.

The value of the limit is then called derivative of f at a and is denoted by f ′(a).

Geometrically f ′(a) represents the slope of the tangent to the curve y = f(x) at x = a.

6.2 Properties

Linearity Property $$\frac{d\big(af(x)+bg(x)\big)}{dx} = a\frac{d\big(f(x)\big)}{dx} + b\frac{d\big(g(x)\big)}{dx}$$

(a and b are constants)

Product Rule $$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient Rule $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

If y = f (z) and z = g (x), then

Chain Rule $$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$ (for Composite Functions)

6.3 Derivatives of some standard functions

y	$\frac{dy}{dx}$	**y**	$\frac{dy}{dx}$	**y**	$\frac{dy}{dx}$
x^n	nx^{n-1}	sin x	cos x	$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$, |x |< 1
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	cos x	- sin x	$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$, |x |< 1
a^x	$a^x \log a$ $(a > 0, a \neq 1)$	tan x	$\sec^2 x$	$\tan^{-1} x$	$\frac{1}{1+x^2}$
e^x	e^x	sec x	sec x tan x	$\cot^{-1} x$	$\frac{-1}{1+x^2}$
logx	1/x	cosec x	- cosec x cot x	$\sec^{-1} x$	$\frac{1}{\lvert x\rvert \sqrt{x^2-1}}$, |x|> 1

6.4 Mean Value Theorems

6.4.1 Rolle's Theorem

If f(x) is continuous in closed interval [a, b], differential in open interval (a, b) and if f(a)=f(b) then, there exists atleast one value c of x in (a, b) such that f ′(c) = 0.

6.4.2 Lagrange Mean value Theorem

If f(x) is continuous in a closed interval [a, b] and differentiable in the open interval (a, b), then there exists at least one value c of x in (a, b) such that f ′(c) = $\frac{f(b)-f(a)}{b-a}$

6.4.3 Cauchy's Mean Value Theorem

Suppose that two functions f(x) and g(x) are such that

1. f(x) and g(x) are continuous in a closed interval [a, b]
2. f(x) and g(x) are differential in the open interval (a, b)
3. $g'(x) \neq 0$, for all x $\in$(a, b)

   then there exists at least one value c $\in$ (a, b) such that

   $$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$$

6.5 Indeterminate forms

Let $\lim_{x \to a}$ g (x) = 0. Then $\lim_{x \to a} \frac{f(x)}{g(x)}$ has a finite limit only when $\lim_{x \to a}$ f (x) = 0.

Suppose that $\lim_{x \to a}$ f (x) = 0, $\lim_{x \to a}$ g (x) = 0.

Then $\lim_{x \to a} \frac{f(x)}{g(x)}$ is said to be an indeterminate form of the type 0/0.

6.6 L` Hospital's Rule

Suppose that $\lim_{x \to a}$ f (x) = 0, $\lim_{x \to a}$ g (x) = 0. In this case,

$$\lim_{x \to a} \frac{f'(x)}{g'(x)} = l \quad \Rightarrow \quad \lim_{x \to a} \frac{f(x)}{g(x)} = l.$$

If $\lim_{x \to a}$ f ′ (x) = 0, $\lim_{x \to a}$ g′ (x) = 0, then $\lim_{x \to a} \frac{f''(x)}{g''(x)} = l \Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = l$ & so on.

The other indeterminate forms are $\frac{\infty}{\infty}$, $0 \times \infty$, $\infty - \infty$, 0^0, ∞^0 and 1^∞.
Each such form can be reduced to 0/0 form.

6.7 Taylor's series for a function of one variable

If $f(x)$ has derivatives of all orders in an interval containing a, then f(x) can be expressed as series in powers of (x- a) as

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots$$

6.8 Maclaurin's series

It is a particular case of Taylor's series with a = 0 and is given by

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$$

Some standard series

1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$
2. $sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$
3. $tanx = x + \frac{x^3}{3!} + \frac{2}{15}x^5 + \cdots$
4. $cosx = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

6.9 Maxima and Minima of a function of single variable

A necessary condition for $f(c)$ to be an extreme value of $f(x)$ is that $f'(c) = 0$.

1. If $f'(c) = 0$ and $f''(c) > 0$, then $f(c)$ is a minimum value of $f(x)$.
2. If $f'(c) = 0$ and $f''(c) < 0$, then $f(c)$ is a maximum value of $f(x)$.

7. Partial Differentiation

7.1 Let z = f (x, y) be a function of two variables x and y.

Then $\lim_{h \to 0} \frac{f(a+h, b) - f(a, b)}{h}$,

if it exists, is called the first order partial derivative of f w.r.t x at (a, b) and is denoted by

$$\left(\frac{\partial z}{\partial x}\right)_{(a,\, b)} \text{ or } f_x(a, b)$$

Similarly, $\lim_{k \to 0} \frac{f(a, b+k) - f(a, b)}{k}$,

if it exists, is called the first order partial derivative of f w.r.t y at (a, b) and is denoted by

$$\left(\frac{\partial z}{\partial y}\right)_{(a,b)} \text{ or } f_y(a,b).$$

If the partial derivatives of f exist at each point in the domain of definition of f, then they are given by

$$\frac{\partial z}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x, y)}{\Delta x} = f_x,$$

$$\frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x, y)}{\Delta y} = f_y$$

If the partial order derivatives are continuous then the mixed partial derivatives are equal i.e., the order in which we differentiate f is immaterial. For instance

$f_{xy} = f_{yx}$; $f_{xxy} = f_{xyx} = f_{yxx}$, $f_{xyy} = f_{yyx} = f_{yxy}$.

7.2 Homogeneous Function

A function z = f(x, y) is said to be a homogeneous function of degree n in x and y

if we can write z as $z = x^n\phi(y/x)$ or $z = y^n\phi(x/y)$.

Euler theorem for a homogeneous function with degree n

If u is a homogeneous function of degree n, then

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$, (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$

7.3 Composite Functions - Chain Rule

Let z = f(x, y). Then the total differential dz is defined as

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy.$$

Let z = f(x, y) has continuous partial derivatives and x = ϕ(t) and y = ψ(t) have continuous derivatives, then

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} . \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} \cdot$$

If z = f(u, v) has continuous partial derivatives & u = ϕ(x, y) and v = ψ(x, y) has continuous partial derivatives, then

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x} \qquad \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

7.4 Errors and Approximations

Let $f(x, y)$ be a continuous function of x and y. Then
$\delta f = \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y.$

If f is a function of several variables x, y, z, t ..., then
$\delta f = \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y + \frac{\partial f}{\partial z} \delta z + \frac{\partial f}{\partial t} \delta t + \ldots$

7.5 Taylor's series for function of two variables

If f(x,y) is defined and has partial derivatives of all orders at a point (a, b) in the domain of f, and if (a + h, b + k) is a point in the domain of f, then

$$f(a+h, b+k) = f(a,b) + \left(h\frac{\partial f}{\partial x} + k\frac{\partial f}{\partial y}\right) + \frac{1}{2!}\left(h^2\frac{\partial^2 f}{\partial x^2} + 2hk\frac{\partial^2 f}{\partial x \partial y} + k^2\frac{\partial^2 f}{\partial y^2}\right) + \cdots$$

where the partial derivatives are evaluated at (a, b).

7.6 Maclaurin's series for a function of two variables

$$f(x, y) = f(0,0) + \left(x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}\right) + \frac{1}{2!}\left(x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2}\right) + \cdots$$

7.7 Maxima and Minima of a function of two variables

Let $f(x, y)$ be a function of two variables x, y such that it is continuous and finite for all values of x and y in the neighbourhood of a point (a, b). Then the value of $f(a, b)$ is called maximum or minimum value of $f(x, y)$ according as $f(a + h, b + k), < or > f(a, b)$ for all finite and sufficiently small values of h and k.

7.8 Necessary and sufficient conditions for the existence of Maximum and Minimum value of a function f (x,y)

The necessary condition for an extremum is $\frac{\partial}{\partial x} f(a, b) = 0, \quad \frac{\partial}{\partial y} f(a, b) = 0$

Sufficient Conditions: Let A = f_{xx} (a, b), B = f_{xy} (a, b), C = f_{yy} (a, b).

If $AC - B^2 > 0$ and $A > 0$	- then f (a, b) is minimum
If $AC - B^2 > 0$ and $A < 0$	- then f (a, b) is maximum
If $AC - B^2 < 0$	- there is neither maximum nor minimum at (a, b)
If $AC - B^2 = 0$	- then the analysis depends on third degree terms in the Taylor series expansion.

8. Integral Calculus

8.1 If $\frac{d}{dx}(f(x)) = g(x)$, then f (x) is called the indefinite integral of g (x)

and is denoted by $\int g(x)dx = f(x) + C$.

C is called constant of integration.

Given a curve y=f(x) then the definite integral ∫f(x) dx from x=a to x=b represents the area bounded by the curve y, the straight lines x=a, x=b and x axis.

8.2 Integration by parts

$$\int u\,v\,dx = u\int(v\,dx) - \int\left[\frac{du}{dx}\int(v\,dx)\right]dx$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$$

8.3 Integrals of some standard functions

f (x)	$\int f(x)\,dx$	f (x)	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)$	$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\log\left(x+\sqrt{x^2+a^2}\right)$	$\frac{1}{\sqrt{x^2-a^2}}$	$\log\left(x+\sqrt{x^2-a^2}\right)$
$\frac{1}{x\sqrt{x^2-a^2}}$	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)$	$\sqrt{a^2-x^2}$	$\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$
$\sqrt{a^2+x^2}$	$\frac{x\sqrt{a^2+x^2}}{2}+\frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)$	$\sqrt{x^2-a^2}$	$\frac{x\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right)$
$\int\frac{1}{x(x^n+1)}dx$	$\frac{1}{n}\log\left\lvert\frac{x^n}{x^n+1}\right\rvert$		

8.4 Multiple Integrals

8.4.1 Area of the region in the Cartesian form $= \iint_R dx\ dy$

8.4.2 Area of the region in the Polar form $= \iint_R r\, dr\, d\theta$

8.4.3 Volume bounded above by the surface z = f(x,y) and below by the region R which is projection of the surface onto XOY plane is given by $= \iint_R$ f(x,y) dx dy

8.4.4

Jacobian J = $\dfrac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{vmatrix}$

If $J = \dfrac{\partial(u,v)}{\partial(x,y)}$ *and* $J^1 = \dfrac{\partial(x,y)}{\partial(u,v)}$ then $JJ^1 = 1$

If $J_1 = \dfrac{\partial(u,v)}{\partial(x,y)}$ *and* $J_2 = \dfrac{\partial(x,y)}{\partial(z,w)}$ then $J_1 J_2 = \dfrac{\partial(u,v)}{\partial(z,w)}$

8.4.5 The change of Cartesian coordinate to cylindrical polar coordinates

$$\iiint_{R_{xyz}} f(x,y,z)dx\ dy\ dz = \iiint_{Rl_{r\emptyset z}} f(rcos\emptyset, rsin\emptyset, z)r\ drd\emptyset dz$$

8.4.6 The change of Cartesian coordinate to Spherical polar coordinates

$$\iiint_{R_{xyz}} f(x,y,z)dx\ dy\ dz = \iiint_{Rl_{r\theta\emptyset}} f(r\ sin\theta cos\emptyset\ , rsin\theta sin\emptyset, rcos\theta)r^2 sin\theta\ drd\theta d\emptyset$$

9. Beta, Gamma Functions

1. $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$
2. $\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}dx$
3. $\Gamma(n) = (n-1)\Gamma(n-1)$, $\Gamma(n) = (n-1)!$, if n is a positive integer
4. Legendre's duplication formula $\Gamma(n)\Gamma(n+1/2) = \dfrac{\sqrt{\pi}}{2^{2n-1}}\Gamma(2n)$
5. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$
6. $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

10. Interpolation

Suppose a function y = f (x) is given. For a set of values of *x* in the domain, we can tabulate the corresponding values of *y*. The central problem of interpolation is the converse of this:

Given a set of tabular values

$$(x_0, y_0),\ (x_1, y_1),\ \ldots,\ (x_n, y_n)$$

satisfying the relation *y = f(x)* where the explicit nature of *f(x)* is not known, it is required to find a simpler function, say $\phi(x)$, which approximates *f(x)*, such that *f(x)* and $\phi(x)$ agree at the set of tabulated points. Such a process of approximation of an unknown function by a known function is called interpolation. If $\phi(x)$ is a polynomial, then the process is called polynomial interpolation and $\phi(x)$ is called the interpolating polynomial.

10.1 Weierstrass Theorem

If *f(x)* is continuous in $x_0 \le x \le x_n$, then given any ε > 0,

there exists a polynomial *P(x)* such that $\left| f(x) - P(x) \right| < \varepsilon$ for all x in $(x_0,\ x_n)$.

This approximation theorem justifies the polynomial approximation.

10.2 Interpolation formulae

Let $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ be a set of tabulated points satisfying y = f(x), where explicit nature of *y* is not known and values of *x* are equally spaced,

10.2.1 Newton-Gregory Forward Difference Interpolation Formula

$$y_n(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \ldots + \frac{p(p-1)\ldots(p-n+1)}{n!}\Delta^n y_0$$

where $x = x_0 + ph$.

Δ is the forward difference operator & the r^th order forward differences are obtained using the relation $\Delta^r y_k = \Delta^{r-1} y_{k+1} - \Delta^{r-1} y_k$, *r* = 1, 2,…

10.2.2 Newton-Gregory Backward Difference Interpolation Formula

$$y_n(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \ldots + \frac{p(p+1)\ldots(p+n-1)}{n!}\nabla^n y_n$$

where $x = x_n + ph$

∇ is the backward difference operator & the r^th order backward differences are obtained using the relation $\nabla^r y_k = \nabla^{r-1} y_k - \nabla^{r-1} y_{k-1}$, *r* = 1, 2, …

10.2.3 Interpolation for unevenly spaced values of x

Lagrange's formula

$$y_n(x) = \frac{(x - x\,x_1)x - x_2)\ldots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)\ldots(x_0 - x_n)} y_0 + \frac{(x - x_0)(x - x_2)\ldots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)\ldots(x_1 - x_n)} y_1 + \ldots$$
$$+ \frac{(x - x_0)(x - x_1)\ldots(x - x_{n-1})}{(x_n - x_0)(x_n - x_1)\ldots(x_n - x_{n-1})} y_n.$$

Newton's divided difference formula

$$y = y_0 + (x - x_0)[x_0,\ x_1] + (x - x_0)(x - x_1)[x_0, x_1,\ x_2]$$
$$+(x - x_0)(x - x_1)(x - x_2)\ [x, x_0, x_1,\ x_2] + \ldots$$
$$+(x - x_0)(x - x_1)(x - x_2)\ldots(x - x_{n-1})\,[x_0, x_1, \ldots, x_n]$$
$$+(x - x_0)(x - x_1)(x - x_2)\ldots(x - x_n)\,[x, x_0, x_1, \ldots, x_n]$$

where $[x_0,\ x_1] = \frac{y_1 - y_0}{x_1 - x_0}$

$[x_1,\ x_2] = \frac{y_2 - y_1}{x_2 - x_1}$, etc., are the first order divided differences,

$[x_0, x_1, x_2] = \frac{[x_1,\ x_2] - [x_0,\ x_1]}{x_2 - x_0}$,

$[x_1, x_2, x_3] = \frac{[x_2,\ x_3] - [x_1,\ x_2]}{x_3 - x_1}$ are the second order divided differences & so on.

11. Numerical Differentiation

11.1 Derivatives using Newton's forward interpolation formula

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2p-1}{2!}\Delta^2 y_0 + \frac{3p^2 - 6p + 2}{3!}\Delta^3 y_0 + \frac{4p^3 - 18p^2 + 22p - 6}{4!}\Delta^4 y_0 + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{6p-6}{3!}\Delta^3 y_0 + \frac{12p^2 - 36p + 22}{4!}\Delta^4 y_0 + \cdots\right]$$

where $p = \frac{x - x_0}{h}$.

At $x = x_0, p = 0$

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 \ldots\right]$$

11.2 Derivatives using Newton's backward interpolation formula

$$\frac{dy}{dx}=\frac{1}{h}\left[\nabla y_n+\frac{2p+1}{2!}\nabla^2 y_n+\frac{3p^2+6p+2}{3!}\nabla^3 y_n+\frac{4p^3+18p^2+22p+6}{4!}\nabla^4 y_n+\cdots\right]$$

$$\frac{d^2y}{dx^2}=\frac{1}{h^2}\left[\nabla^2 y_n+\frac{6p+6}{3!}\nabla^3 y_n+\frac{12p^2+36p+22}{4!}\nabla^4 y_n+\cdots\right]$$

where $p=\frac{x-x_n}{h}$.

At $x=x_n, p=0$

$$\left(\frac{dy}{dx}\right)_{x=x_n}=\frac{1}{h}\left[\nabla y_n+\frac{1}{2}\nabla^2 y_n+\frac{1}{3}\nabla^3 y_n+\frac{1}{4}\nabla^4 y_n+\frac{1}{5}\nabla^5 y_n+\cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n}=\frac{1}{h^2}\left[\nabla^2 y_n+\nabla^3 y_n+\frac{11}{12}\nabla^4 y_n+\frac{5}{6}\nabla^5 y_n \ldots\right]$$

12. Numerical Integration

12.1 Newton-Cotes quadrature formula

$$\int_{x_0}^{x_0+nh} f(x)dx=nh\left[y_0+\frac{n}{2}\Delta y_0+\frac{n(2n-3)}{12}\Delta^2 y_0+\frac{n(n-2)^2}{24}\Delta^3 y_0+\cdots\ldots\right]$$

For n = 1, 2, 3,… we get the following integration formulae.

12.2 Trapezoidal rule

$$\int_{x_0}^{x_0+nh} f(x)dx=\frac{h}{2}[(y_0+y_n)+2(y_1+y_2+y_3+\cdots\ldots+y_{n-1})]$$

12.3 Simson's one-third rule

$$\int_{x_0}^{x_0+nh} f(x)dx=\frac{h}{3}[(y_0+y_n)+4(y_1+y_3+\cdots\ldots+y_{n-1}) \\ +2(y_2+y_4+\cdots\ldots+y_{n-2})]$$

12.4 Simson's three-eigth rule

$$\int_{x_0}^{x_0+nh} f(x)dx=\frac{3h}{8}\left[\begin{matrix}(y_0+y_n)+3(y_1+y_2+y_4+y_5\ldots+y_{n-1})\\ +2(y_3+y_6+\cdots+y_{n-3})\end{matrix}\right]$$

Simpson's 1/3rd rule is applicable when the number subintervals is even and Simpson's 3/8th rule applicable when the number of subintervals is a multiple of 3.

12.5 Techniques for finding roots of Algebraic and Transcendental Equations

If f (x) is continuous on some interval [a, b] and f (a) f (b) < 0,
then from intermediate value theorem,
the equation f (x) = 0 has at least one real root in the interval (a, b).

12.5.1 Bisection Method

Let $f(a)$ be negative and $f(b)$ be positive. Then the root lies between a and b and its approximate value be given by $x_0 = \frac{a+b}{2}$.

12.5.2 Regula Falsi method / Method of false position

Let $f(a)$ be negative and $f(b)$ be positive. Then the approximate value of the root is given by $x_1 = \dfrac{a\,f(b) - b\,f(a)}{b-a}$

12.5.3 Newton- Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\ ,\ n = 0,1,2,3.....$$

Non-linear simultaneous equations by Newton-Raphson (or Newton's) method:

$$f_0 + h\frac{\partial f}{\partial x_0} + k\frac{\partial f}{\partial y_0} = 0, \qquad g_0 + h\frac{\partial g}{\partial x_0} + k\frac{\partial g}{\partial y_0} = 0$$

13. Numerical Methods for Differential Equations

Consider the equation $dy/dx = f(x, y)$, $y(x_0) = y_0$.

13.1 Taylor Series Method

$$y = y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + ...$$

13.2 Euler's Method

$y_{n+1} = y_n + hf(x_n, y_n)$, $x_n = x_0 + nh$, n = 1, 2, 3, …, h > 0, $y(x_n) = y_n$.

13.3 Modified Euler's method

$$\boldsymbol{y_1^{(0)}} = y_0 + hf(x_0, y_0)$$

$$y_1^{(n+1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(n)})\right],\ n = 0,1,2,...$$

13.4 Runge-Kutta Method of Second Order

$y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$, where $k_1 = hf(x_0, y_0)$ and $k_2 = hf(x_0 + h, y_0 + k_1)$.

13.5 Runge-Kutta Method of Fourth Order

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$, where

$k_1 = hf(x_0, y_0)$, $\qquad k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$,

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$, $\qquad k_4 = hf(x_0 + h, y_0 + k_3)$.

14. Two Dimensional Geometry

14.1 The general form of the equation of a straight line is ax + by + c = 0

14.2 The normal form of the straight line is $x\cos\alpha + y\sin\alpha = p$, where p is the length of the perpendicular from O (0,0) to the line, α is the inclination of the perpendicular

14.3 The equation of the line with slope **m** and y-intercept **c** is **y = mx + c**.

14.4 The distance of the point (x_1, y_1) from the line ax + by + c = 0 is $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

14.5 The equation of the line passing through (x_1,y_1) and having slope m is **$y - y_1 = m(x - x_1)$**, which is called the slope-point form

14.6 The equation of the line having a and b as the x-intercept and y-intercept is $\frac{x}{a} + \frac{y}{b} = 1$ and is called the equation of the line intercept form

14.7 The equation of the line passing through two points (x_1, y_1) and (x_2, y_2) is $\frac{y-y1}{y2-y1} = \frac{x-x1}{x2-x1}$

14.8 Consider two points P (x_1, y_1) and Q (x_2, y_2) then

14.8.1 The distance formula: PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

14.8.2 The point R (x, y) dividing PQ in the ratio $\frac{k1}{k2}$ is :

$$x = \frac{k_1x_2+k_2x_1}{k_1+k_2}, \quad y = \frac{k_1y_2+k_2y_1}{k_1+k_2}$$

14.8.3 If R is the mid point, $k_1 = k_2$ and $\mathrm{x} = \left(\frac{\mathrm{x}_1+\mathrm{x}_2}{2}\right)$ and $\mathrm{y} = \left(\frac{\mathrm{y}_1+\mathrm{y}_2}{2}\right)$

(mid point formula)

14.8.4 The Slope of PQ is: $m = \frac{y_2-y_1}{x_2-x_1}$

14.9 Consider two lines l_1 and l_2 having the slopes m_1 and m_2 respectively.

14.9.1 The angle θ from l_1 to l_2 is $\tan\theta = \frac{\mathrm{m}_2-\mathrm{m}_1}{1+\mathrm{m}_1\mathrm{m}_2}$

14.9.2 If two lines l_1 and l_2 are parallel, then $m_1 = m_2$.

14.9.3 If two lines l_1 and l_2 are perpendicular, then $m_1 \times m_2 = -1$.

15. Three Dimensional Geometry

15.1 If α, β, γ are the angles which a given line OP makes with the positive directions of co-ordinate axes, then $l = \cos\alpha, \mathrm{m} = \cos\beta, \mathrm{n} = \cos\gamma$ are called the direction cosines (d.c.'s) of the given line.

15.2 Three numbers a, b, c which are propotional to the directional cosines l, m, n respectively of a line are called direction ratios (d.r's)of a line

15.2.1 $l^2 + m^2 + n^2 = 1$.

15.2.2 The co-ordinates of P are (lr, mr, nr) (OP = r)

15.2.3 $l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}$, $m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}$, $n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$

15.3 If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two given points then the direction ratios of PQ are, $x_2 - x_1, y_2 - y_1, z_2 - z_1$.

15.4 If l_1, m_1, n_1 and l_2, m_2, n_2 are the d.c.'s of two lines, then the angle θ between the two lines is given by $\theta = \cos^{-1}(l_1 l_2 + m_1 m_2 + n_1 n_2)$

15.5 If a_1, b_1, c_1 and a_2, b_2, c_2 are the d.r.'s of two lines, then the angle θ between the lines is given by $\cos\theta = \frac{(a_1a_2+b_1b_2+c_1c_2)}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

15.6 Equations of the line through the point $A\,(x_1,\ y_1,\ z_1)$ and having direction cosines $l,\ m,\ n$ are $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$. (Symmetrical form)

15.7 Any point on the symmetrical form of a line is $(x_1 + kl, y_1 + km, z_1 + kn)$

15.8 The equations of the line joining the points (x_1, y_1, z_1) *and* (x_2, y_2, z_2) are $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

15.9 The lines are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ and

parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

15.10 The projection of the join of two points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ on a line whose d.c.'s are l, m, n is $|l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$.

15.11 General Form: The two linear equations in x, y and z together represent a straight line $ax + by + cz + d = 0 \quad and\ a'x + b'y + c'z + d' = 0$.

15.12 Sphere

- A sphere is the locus of a point which moves in space such that its distance from a fixed point remains a constant.
- The fixed point is called the centre and the constant distance is the radius of the sphere.
- Equation of a sphere with centre at (x_0, y_0, z_0) and radius r is
  $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$

15.13 Right Circular Cone

- It is a surface generated by a straight line which passes through a fixed point and makes a constant angle with a fixed line.
- The constant angle θ is called the semi vertical angle
- The fixed point is called a vertex
- The fixed line is called the axis.

15.14 Right Circular Cylinder

- It is a surface generated by a straight line which is parallel to a fixed straight line and is at a constant distance from it.
- The constant distance is called the radius of the cylinder.
- The fixed line is called the axis of the cylinder.

16. Mensuration

Figure	Area / Volume	Perimeter
2D Figures		
Equilateral triangle with side a	$\sqrt{3}\ a^2/4$	3a
Triangle with sides a, b, c	(base ×height)/2	a + b + c
Rectangle with sides l and b	l × b	2 (l + b)
Parallelogram	base × height	2 (l + b)
Trapezoid	(sum of parallel sides)×h/2	
Circle with radius r	πr^2	$2\pi r$
3D Figures		
Sphere with radius r	$\frac{4}{3}\pi r^3$	$4\ \pi r^2$
Cube with side a	a^3	$6a^2$
Right circular cylinder	$\pi r^2\ h$	$2\ \pi r\ (r + h)$
Cuboid	l ×b×h	2(l×b + b×h + h×l)
Right circular cone	$\frac{1}{3} * \pi r^2 h$	$S = \pi\ r\ l + \pi\ r^2$
Spherical shell with R and r as the outer and inner radii	$\frac{4}{3}\pi(R^3 - r^3)$	

17. Infinite Sequence

A sequence is a function u from the set of natural numbers N into the set of all real numbers R. u(n) is denoted by u_n and is called n^{th} term of the sequence and the sequence is denoted by $\{u_n\}$.

The sequence is said to be

- Convergent - if $\lim_{n\to\infty} u_n$ is finite
- Divergent - if $\lim_{n\to\infty} u_n$ is infinite
- Oscillatory - if $\lim_{n\to\infty} u_n$ is not unique

Infinite Series

If $\{u_n\}$ is an infinite sequence, then $\sum_{n=1}^{\infty} u_n = u_1 + u_2 + \cdots . + u_n + \cdots$. is called an infinite series.

$S_n = \sum_{k=1}^{n} u_k = u_1 + u_2 + \ldots\ldots + u_n$, is called n^{th} partial sum of the series.

The series is said to be

- Convergent - if $\lim_{n\to\infty} S_n$ is finite and unique
- Divergent - if $\lim_{n\to\infty} S_n$ is infinite
- Oscillatory - if $\lim_{n\to\infty} S_n$ is not unique.

Nature of a series can be obtained by using one of the following tests:

Geometric series test

The series $\sum_{r=0}^{\infty} r^n$

- Converges - if $|r| < 1$
- Diverges - if $r \geq 1$
- Oscillates - finitely, if $r = -1$ & infinitely if $r < -1$.

Comparison Test

Let $\sum u_n$ and $\sum v_n$ be two positive term series

- If $\sum v_n$ is convergent and $u_n \leq v_n$, for all n, then $\sum u_n$ is also convergent.
- If $\sum v_n$ is divergent and $u_n \geq v_n$, for all n, then $\sum u_n$ is also divergent.

Quotient test

Suppose $\sum u_n$ and $\sum v_n$ be two positive term series. If $\lim_{n\to\infty} \frac{u_n}{v_n} = k(\neq 0$ a constant), then $\sum u_n$ and $\sum v_n$ converge or diverge together.

Integral Test

A positive term series $f(1) + \cdots + f(n) +..$ where $f(n)$ decreases as n increases, converges or diverges according as the integral $\int_1^{\infty} f(x)dx$ is finite or infinite.

p-series (harmonic series) test

A positive term series $\sum u_n = \sum \frac{1}{n^p}$ is

- Convergent - if $p > 1$
- Divergent - if $p \leq 1$.

D'Alemberts's Ratio test

A positive term series $\sum u_n$

- Converges if $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} < 1$
- Diverges if $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} > 1$
- Test fails if $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = 1$

Raabe's Test

A positive term series $\sum u_n$

- Converges if $\lim_{n\to\infty} n(\frac{u_n}{u_{n+1}} - 1) > 1$
- Diverges if $\lim_{n\to\infty} n(\frac{u_n}{u_{n+1}} - 1) < 1$
- Fails if the limit = 1

Cauchy's Root Test

In a positive series $\sum u_n$, if $\lim_{n\to\infty} (u_n)^{1/n}$ =λ, the series

- Converges for $\lambda < 1$
- Diverges for $\lambda > 1$
- Test fails for $\lambda = 1$

Leibnitz's rule

An alternating series $u_1 - u_2 + u_3 - u_4 + \cdots$

- Converges - If each term is numerically less than its preceding term - and $\lim_{n\to\infty} u_n = 0$
- Oscillatory - If $\lim_{n\to\infty} u_n \neq$

Absolutely convergent

If the series of arbitrary terms $u_1 + u_2 + u_3 + \cdots + u_n + \cdots$ be such that the series $|u_1|+|u_2|+|u_3|+.....+|u_n|+.....$ is convergent, then the series $\sum u_n$ is said to be absolutely convergent.

Conditionally convergent

If $\sum|u_n|$ is divergent but $\sum u_n$ is convergent, then $\sum u_n$ is said to be conditionally convergent.

Note: An absolutely convergent series is necessarily convergent but not conversely.

18. Laplace Transform

$$L\{f(t)\} = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

Properties of Laplace transform

- $L\{af(t)+bg(t)-ch(t)\} = aL\{f(t)\} + bL\{g(t)\} - cL\{h(t)\}$
  (**Linearity Property)**
- If L{f (t)} = F(s) then $L\{e^{at} f(t)\} = F(s-a)$ **(First Shifting property)**
- If $L\{f(t)\} = F(s)$ then $L\left\{\int_0^t f(u)du\right\} = \frac{1}{s}F(s)$
- If $L\{f(t)\} = F(s)$

  then $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s),\ n = 1,2,3,...$
- If $L\{f(t)\} = F(s)$ then $L\left\{\frac{1}{t} f(t)\right\} = \int_s^\infty F(\beta)d\beta$
- If F(t) has a Laplace Transform and if $F(t+w) = F(t)$,

  then, $L\{F(t)\} = \dfrac{\int_0^w e^{-s\beta} F(\beta) d\beta}{1 - e^{-sw}}$

Laplace transforms of standard function :

1	L {1} = $\frac{1}{s}$, s > 0.
2	$L\{t^n\} = \begin{cases} \frac{n!}{s^{n+1}} \text{ when } n = 0,1,2,\ldots \\ \text{otherwise } \frac{\Gamma(n+1)}{s^{n+1}} \end{cases}$
3	$L\{e^{at}\} = \frac{1}{s-a}, \quad s > a$
4	$L\{\sin at\} = \frac{a}{s^2+a^2}, \quad s>0$
5	$L\{\cos at\} = \frac{s}{s^2+a^2}, \quad s>0$
6	$L\{\sinh a\,t\} = \frac{a}{s^2-a^2}, \quad s>\lvert a\rvert$
7	$L\{\cosh at\} = \frac{s}{s^2-a^2}, \quad s>\lvert a\rvert$

Inverse Laplace transforms :

Let L{f(t)} = F(s).
Then f(t) is defined as the inverse Laplace transform of F(s) and is denoted by L^{-1} {F(s)}.
Thus L^{-1} F(s) = f(t)

Shifting Property :

If L^{-1} {F(s)}=f(t) then L^{-1} [F(s-a)]= $e^{at}L^{-1}$ {F(s)}

Inverse transform of derivative :

$L^{-1}(F^{(n)}(s)) = (-1)^n t^n L^{-1}(F(s))$

$L^{-1}[e^{-as} F(s)] = f(t-a)\, H(t-a)$

$L^{-1}[F(s)G(s)] = \int_0^t f(t-u)g(u)du = f(t) * g(t)$ (Convolution Theorem)

Inverse Laplace Transform of some standard functions:

F(s)	$f(t) = L^{-1}F(s)$
$\frac{1}{s}, s > 0$	1
$\frac{1}{s-a}, s > a$	e^{at}
$\frac{s}{s^2+a^2}, s > 0$	Cos at
$\frac{1}{s^2+a^2}, s > 0$	$\frac{\text{Sin at}}{\text{a}}$
$\frac{1}{s^2-a^2}, s > \|a\|$	$\frac{\text{Sin h at}}{\text{a}}$
$\frac{s}{s^2-a^2}, s > \|a\|$	Cos h at
$\frac{1}{s^{n+1}}, s > 0$ *n = 0, 1, 2, 3, . . .*	$\frac{t^n}{n!}$
$\frac{1}{s^{n+1}}, s > 0$	$\frac{t^n}{\Gamma(n+1)}$

1. Interference of Light Waves

Young's double slit expt.:
Condition for constructive and destructive interference

$$d \sin \theta_{\text{bright}} = m\lambda \; ; \quad (m = 0, \pm 1, \pm 2, \ldots)$$

$$d \sin \theta_{\text{dark}} = \left(m + \tfrac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

Relation between phase difference and path difference

$$\varphi = \frac{2\pi}{\lambda}\delta$$

Linear positions of bright and dark fringes

$$y_{\text{bright}} = L\frac{m\lambda}{d} \quad \text{(small angle approximation)}$$

$$y_{dark} = L\frac{\left(m + \frac{1}{2}\right)\lambda}{d} \quad (m = 0, \pm 1, \pm 2, \ldots)$$

Average light intensity at a point on the screen

$$I = I_{max}\, cos^2\left(\frac{\varphi}{2}\right)$$

$$I = I_{max}\, cos^2\left(\frac{\pi d \sin \theta}{\lambda}\right) \qquad I = I_{max}\, cos^2\left(\frac{\pi d}{\lambda L} y\right)$$

Condition for interference in thin films in air
(reflective system)
Constructive interference:

$$2nt = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots)$$

Destructive interference:

$$2nt = m\lambda \quad (m = 0, 1, 2, \ldots)$$

Radius of m[th] order Newton's ring

$$r_{dark} \approx \sqrt{mR\lambda} \quad (m = 0, 1, 2, \ldots)$$

$$r_{bright} \approx \sqrt{\frac{\left(m + \frac{1}{2}\right)R\lambda}{n_{film}}} \quad (m = 0, 1, 2, \ldots)$$

d : distance between the two slits
λ : wavelength of light used
θ : angular position on the screen
m : order number
φ : phase difference
δ : path difference
y : linear position on the screen
L : distance between the slit and the screen
I_{max}: maximum intensity on the screen
n : refractive index
t : thickness of the film
r : radius of curvature of lens

2. Diffraction Patterns and Polarization

Single slit diffraction: condition for minima $sin\ \ \theta_{\text{dark}} = m\frac{\lambda}{a} \qquad m = \pm 1,\ \pm 2,\ \pm 3,\ \ldots$	a : width of single slit I_{max} : maximum intensity [Central maxima] d : distance between the two slits D : diameter of the aperture
Intensity due to single slit diffraction $I = I_{max}\left[\frac{sin\ \ (\pi a\ \ sin\ \ \theta/\lambda)}{(\pi a\ \ sin\ \ \theta)/\lambda}\right]^2$	
Intensity of two slit diffraction pattern [combined effect] $I = I_{max}\, cos^2\left(\frac{\pi d\ \ sin\ \ \theta}{\lambda}\right)\left[\frac{sin\ \ (\pi a\ \ sin\ \ \theta/\lambda)}{(\pi a\ \ sin\ \ \theta)/\lambda}\right]^2$	
Rayleigh's criterion: limiting angle of resolution $\theta_{min} = 1.22\ \frac{\lambda}{D}$ [for circular aperture] $\theta_{min} = \frac{\lambda}{a}$ [for rectangular aperture]	
Grating equation for maxima $d\, sin\ \ \theta_{\text{bright}} = m\lambda\ ;\ \ m = 0,\ \pm 1,\ \pm 2,\ \pm 3,\ \ldots$	
X-ray diffraction: Bragg's law $2d\ \ sin\ \ \theta = m\lambda \qquad m = 1,\ 2,\ 3,\ \ldots$	d : Inter-planar spacing in the crystal
Malus's law $I = I_{max}\, cos^2\,\theta$	I_{max} : intensity of the polarized beam incident on the analyzer θ : angle made by the analyzer transmission axis with the polarizer axis
Brewster's law $\tan\theta_p = \frac{n_2}{n_1}$	θ_p : polarizing or Brewster's angle n_1, n_2: refractive indices of first and second medium

3. Quantum Physics

Wien's Displacement Law $\lambda_m T = 2.898 \times 10^{-3}$ m.K	λ_m : wavelength corresponding to peak intensity. T : equilibrium temperature of the blackbody.
Stefan's Law $P = \sigma A\, e\, T^4$	P : power radiated from the surface area A of the object. T : equilibrium surface temperature. Σ : Stefan-Boltzmann constant. e : emissivity of the surface
Planck's law $I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$	$I(\lambda, T)$: intensity or power per unit area emitted in the wavelength interval $d\lambda$ from a blackbody at the equilibrium temperature T h : Planck's constant. k_B : Boltzmann's constant c : speed of light in vacuum

Einstein's photoelectric equation $K_{max} = hf - \phi$	f : frequency of incident photon. K_{max} : kinetic energy of the most energetic photoelectron. ϕ : work function of the photocathode material.
Relativistic momentum of a particle $p = \gamma m v \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$	p : momentum of the particle m : mass of the particle v : speed of the particle c : speed of light in vacuum
Relativistic kinetic energy of a particle $K = (\gamma - 1)\, m c^2$	
Total energy (relativistic) of the particle $E = \gamma m c^2 \qquad E^2 = p^2 c^2 + m^2 c^4$	
Compton shift equation $\lambda' - \lambda_o = \frac{h}{mc}(1 - \cos\theta)$	λ_o : wavelength of the incident photon. λ' : wavelength of the scattered photon, θ :angle of scattering
de Broglie wavelength, λ $\lambda = \frac{h}{p} = \frac{h}{mv} \qquad p = m v = \sqrt{2 m q \Delta V}$	h : Planck's constant p : momentum of the quantum particle. m : mass of the particle v : speed of the particle q : charge of the particle ΔV : accelerating voltage
Relation between group speed and phase speed $v_g = v_P - \lambda\left(\frac{dv_P}{d\lambda}\right)$	v_g : group speed v_P : phase speed
Heisenberg uncertainty relations. $(\Delta x)(\Delta p_x) \geq h/4\pi$ $(\Delta E)(\Delta t) \geq h/4\pi$	Δx : uncertainty in the measurement of position x of the particle. Δp_x : uncertainty in the measurement of momentum p_x of the particle. ΔE : uncertainty in the measurement of energy E Δt : time interval in the measurement of E.

4. Quantum Mechanics

One dimensional time independent Schrödinger equation $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi$	$\hbar$: Reduced Planck's constant m : mass of the particle ψ : *wave function* $U(x)$: potential energy function E : total energy of the system h : Planck's constant L : length of the "box". n : integers T : tunneling probability
Expectation value of x $\langle x \rangle \equiv \int_{-\infty}^{+\infty} \psi^* x \psi \, dx$	
Particle in a "box" E_n - quantized energy values of the particle. $E_n = \left(\frac{h^2}{8 m L^2}\right) n^2$	
Transmission coefficient $T \approx e^{-2CL} \qquad C = \frac{\sqrt{2m(U-E)}}{\hbar}$	

5. Atomic Physics

Time independent 3-dimensional Schrödinger equation $-\frac{\hbar^2}{2\,m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+U\,\psi\;=\;E\,\psi$	$\hbar$ = h/2π k_e= 9 x 10^9 Nm^{2}C^{-2} m : mass of the particle ψ : wave function a_o: Bohr radius = 0.0529 nm n : principal quantum number r : radial distance
Expression for energy as per quantum model $E_n=-\frac{k_e e^2}{2a_0}\left(\frac{Z^2}{n^2}\right)=-\frac{(13.6\text{ eV})Z^2}{n^2}$	
Wave function for H-atom in the 1s-state(ground state) $\psi_{1s}(r)\;=\;\frac{1}{\sqrt{\pi\,a_o^3}}exp\left(-\frac{r}{a_o}\right)$	
Wave function for H-atom in the 2s-state $\psi_{2S}(r)\;=\;\frac{1}{\sqrt{32\pi a_o^3}}\left(2-\frac{r}{a_o}\right)exp\left(-\frac{r}{a_o}\right)$	
λ_{MIN} of X-ray $\lambda_{MIN}\;=\;\frac{hc}{e\Delta V}$	ΔV : tube voltage f : frequency of K_α *X-ray* Z : atomic number C : constant k_B :Boltzmann constant T : Temperature
Moseley's formula $\sqrt{f}\;=\;C\;(Z\;-\;1)$	
Ratio of population of atoms in two energy states E_1 and E_2 $\frac{n(E_2)}{n(E_1)}\;=\;exp\left(-\frac{E_2-E_1}{k_B\;T}\right)$	

6. Molecules and Solids

Total potential energy of the crystal $U_{\text{total}}=-\alpha k_e\frac{e^2}{r}+\frac{B}{r^m}$	α : Madelung constant r : separation distance between ions m : small integer E_F : Fermi energy k_B : Boltzmann constant
Probability of a particular energy state E being occupied by an electron: Fermi-Dirac distribution function $f(E)\;=\;\frac{1}{exp\;\left(\frac{E\;-\;E_F}{k_BT}\right)\;+\;1}$	
Density-of-states function $g(E)\;dE\;=\;\frac{8\sqrt{2}\;\pi\;m^{\frac{3}{2}}}{h^3}\;E^{\frac{1}{2}}\;dE$	m : mass of the particle h : Planck's constant
Fermi energy at 0K $E_F(0)\;=\;\frac{h^2}{2\;m}\left(\frac{3\;n_e}{8\;\pi}\right)^{\frac{2}{3}}$	n_e: electron density m : mass of the electron

Lennard–Jones potential equation $U(r) = -\frac{A}{r^n} + \frac{B}{r^m}$	
Molecule's angular momentum $L = \sqrt{J(J+1)}\,\hbar \qquad J = 0,\ 1,\ 2,\ \ldots$	
Energies of the absorbed photons in rotational transitions $E_{\text{photon}} = \frac{\hbar^2}{I}J = \frac{h^2}{4\pi^2 I}J \qquad J = 1,\ 2,\ 3,\ \ldots$	*r* : inter nuclear separation distance between the two atoms *A* : is associated with the attractive force *B* : with the repulsive force *J* : rotational quantum number I : Moment of inertia of the molecule *k* : effective spring constant μ : reduced mass *f* : frequency of spectra n_0 : number of molecules in the *J*=0 state k_B : Boltzmann constant *n, m* : small integers
Vibrational energies $E_{\text{vib}} = \left(v + \frac{1}{2}\right)\frac{h}{2\pi}\sqrt{\frac{k}{\mu}} \qquad v = 0,\ 1,\ 2,\ \ldots$	
Combined [rotational + vibrational] spectra $E_{\text{photon}} = \Delta E = hf + \frac{\hbar^2}{I}(J+1)$ $J = 0,\ 1,\ 2,\ ..$	
Number of molecules in an excited rotational state $n = n_0 e^{\frac{-\hbar^2 J(J+1)}{2I k_B T}}$ Intensity of spectral lines $Intensity \propto (2J+1)e^{\frac{-\hbar^2 J(J+1)}{2I k_B T}}$	

Engineering Chemistry

1. Thermodynamics

$dq = du + dw$	$\Delta G = \Delta H - T\Delta S$	$\Delta G = \Delta G° + RT \ln K$	q : heat absorbed by the system w : work done on the system G : Gibbs free energy H : enthalpy S : entropy F : Faraday P : pressure V : volume K : equilibrium constant T : temperature E : electromotive force U : internal energy C : heat capacity R : gas constant A : work function
$\frac{dq}{T} = dS$	$\Delta C_P = \frac{\Delta H}{T}$	$W_{max} = -\Delta G$	
$C_P - C_V = R$	$\Delta C_V = \frac{\Delta S}{T}$	$\Delta G = -nFE$	
$PV = nRT$	$\Delta E = q + w$	$\Delta G = \Delta H + T\left[\frac{d(\Delta G)}{dT}\right]_P$	
$A = U - TS$	$\Delta H = \Delta E + P\Delta V$	$\Delta H = nF\left[T\left(\frac{\Delta E}{dT}\right)_P - E\right]$	
$G = H - TS$	$\Delta S = nR \ln \frac{V_2}{V_1}$	$\Delta S = nF\left(\frac{\Delta E}{dT}\right)_P$	
$dG = VdP - SdT$	$\Delta S = C_P \ln \frac{T_2}{T_1}$	$\ln \frac{P_2}{P_1} = \frac{\Delta H_V}{R}\left[\frac{(T_2 - T_1)}{T_1 T_2}\right]$	

2. Expression for molecular weight of polymer

$$\overline{M}_n = \frac{\sum_{i=1}^{N} N_i M_i}{\sum_{i=1}^{N} N_i} = \frac{\sum_{i=1}^{N} w_i}{\sum_{i=1}^{N} \frac{w_i}{M_i}}$$

$$\overline{M}_w = \frac{\sum_{i=1}^{N} N_i M_i^2}{\sum_{i=1}^{N} N_i M_i} = \frac{\sum_{i=1}^{N} w_i M_i}{\sum_{i=1}^{N} w_i}$$

$\overline{M}_n$ = Number average molecular weight
$\overline{M}_w$ = Weight average molecular weight
N = Total number of molecules
N_i = Number of molecules with molecular weight M_i
W_i = weight fraction of all molecules with molecular weight M_i

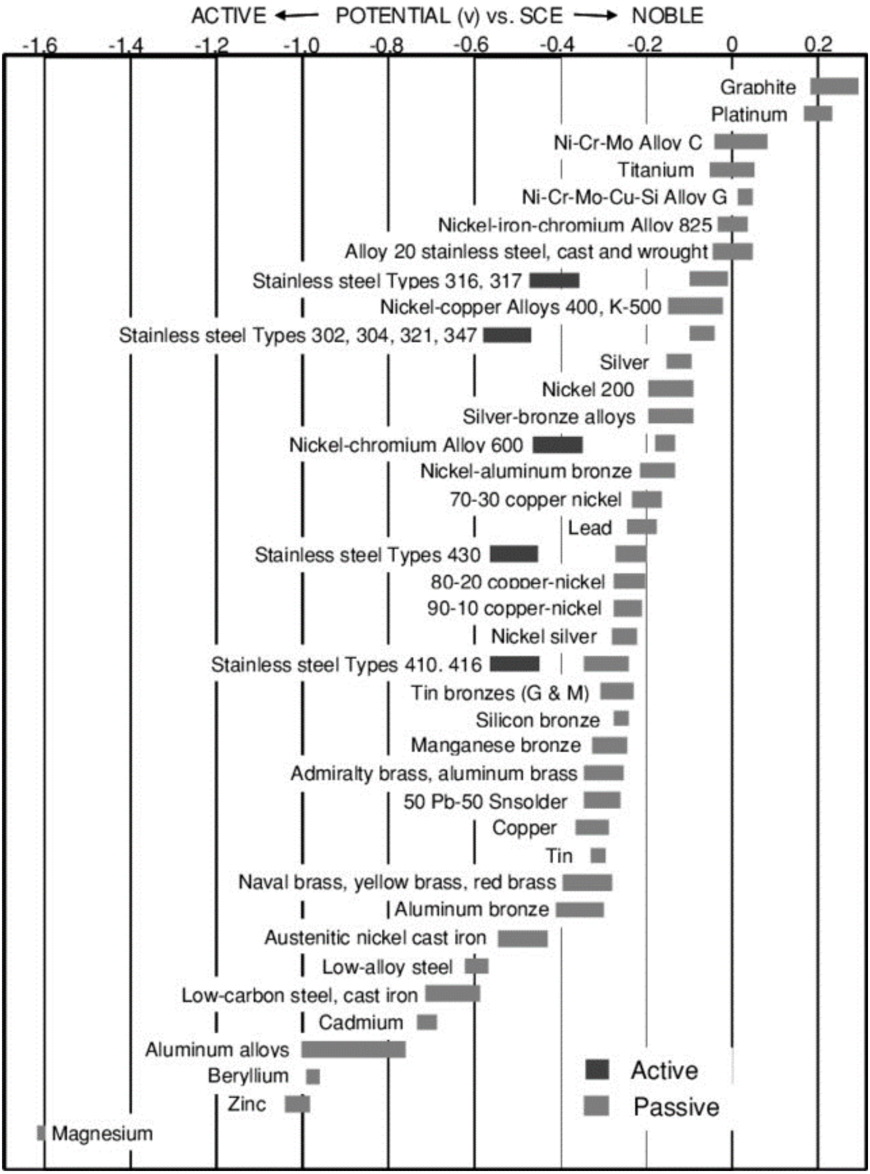
3. Calculation of calorific value of chemical fuels

NCV = GCV – (0.09 × H × 587)

Where,

NCV is net calorific value
GCV is gross calorific value
H is percentage of hydrogen in fuel
587 cal/g is the latent heat of steam

4. Galvanic Series



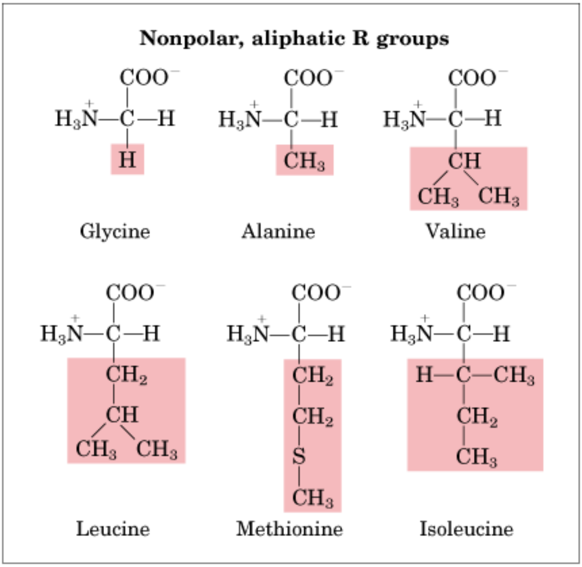
5. Electrochemical Series

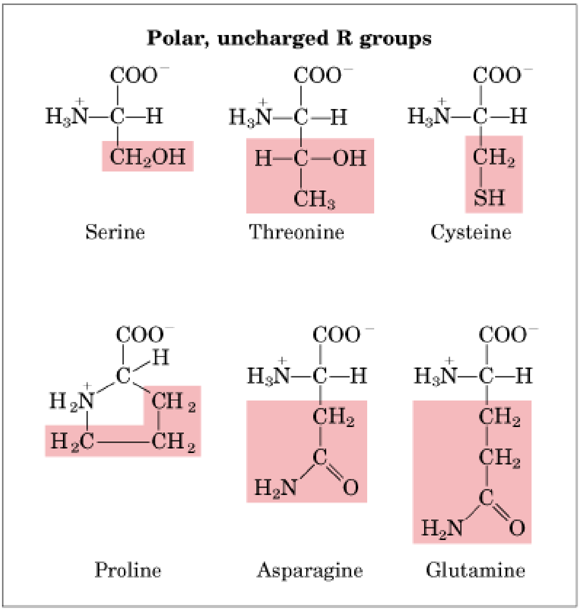
(a) Tendency for oxidation to occur (b) Power as reducing agent INCREASES

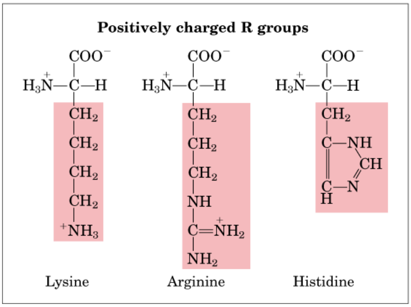
Elements	Electrode reaction	$E_{red.}$ (Volts)
	Oxidised form + ne^- ------------> Reduced form	
Li	$Li^+_{(aq)} + e^-$ ------------> $Li_{(s)}$	-3.05
K	$K^+_{(aq)} + e^-$ ------------> $K_{(s)}$	-2.93
Ba	$Ba^{2+}_{(aq)} + 2e^-$ ------------> $Ba_{(s)}$	-2.90
Ca	$Ca^{2+}_{(aq)} + 2e^-$ ------------> $Ca_{(s)}$	-2.87
Na	$Na^+_{(aq)} + e^-$ ------------> $Na_{(s)}$	-2.71
Mg	$Mg^{2+}_{(aq)} + 2e^-$ ------------> $Mg_{(s)}$	-2.37
Al	$Al^{3+}_{(aq)} + 3e^-$ ------------> $Al_{(s)}$	-1.66
Zn	$Zn^{2+}_{(aq)} + 2e^-$ ------------> $Zn_{(s)}$	-0.76
Cr	$Cr^{3+}_{(aq)} + 3e^-$ ------------> $Cr_{(s)}$	-0.74
Fe	$Fe^{2+}_{(aq)} + 2e^-$ ------------> $Fe_{(s)}$	-0.44
	$H_2O_{(l)} + e^-$ ------------> $½H_{2(g)} + OH^-_{(aq)}$	-0.41
Cd	$Cd^{2+}_{(aq)} + 2e^-$ ------------> $Cd_{(s)}$	-0.40
Pb	$PbSO_{4(s)} + 2e^-$ ------------> $Pb_{(s)} + SO_4^{2-}{}_{(aq)}$	-0.31
Co	$Co^{2+}_{(aq)} + 2e^-$ ------------> $Co_{(s)}$	-0.28
Ni	$Ni^{2+}_{(aq)} + 2e^-$ ------------> $Ni_{(s)}$	-0.25
Sn	$Sn^{2+}_{(aq)} + 2e^-$ ------------> $Sn_{(s)}$	-0.14
Pb	$Pb^{2+}_{(aq)} + 2e^-$ ------------> $Pb_{(s)}$	-0.13
H_2	**$2H^+ + 2e^-$ ------------> $H_{2(g)}$ (Standard Electrode)**	**0.00**
Cu	$Cu^{2+}_{(aq)} + 2e^-$ ------------> $Cu_{(s)}$	0.34
I_2	$I_{2(s)} + 2e^-$ ------------> $2I^-_{(aq)}$	0.54
Fe	$Fe^{3+}_{(aq)} + e^-$ ------------> $Fe^{2+}_{(aq)}$	0.77
Hg	$Hg_2^{2+}{}_{(aq)} + 2e^-$ ------------> $2Hg_{(l)}$	0.79
Ag	$Ag^+_{(aq)} + e^-$ ------------> $Ag_{(s)}$	0.80
Hg	$Hg^{2+}_{(aq)} + 2e^-$ ------------> $Hg_{(l)}$	0.85
N_2	$NO_3^- + 4H^+ + 3e^-$ ------------> $NO_{(g)} + 2H_2O$	0.97
Br_2	$Br_{2(aq)} + 2e^-$ ------------> $2Br^-_{(aq)}$	1.08
O_2	$O_{2(g)} + 2H_3O^+_{(aq)} + 2e^-$ ------------> $3H_2O$	1.23
C_r	$Cr_2O_7^{2-} + 14H^+ + e^-$ ------------> $2Cr^{3-} + 7H_2O$	1.33
Cl_2	$Cl_{2(g)} + 2e^-$ ------------> $2Cl^-_{(aq)}$	1.36
Au	$Au^{3+}_{(aq)} + 3e^-$ ------------> $Au_{(s)}$	1.42
M_n	$MnO_4^-{}_{(aq)} + 8H_3O^+_{(aq)} + 5e^-$ ------------> $Mn^{2+}_{(aq)} + 12H_2O$	1.51
F_2	$F_{2(g)} + 2e^-$ ------------> $2F^-_{(aq)}$	2.87

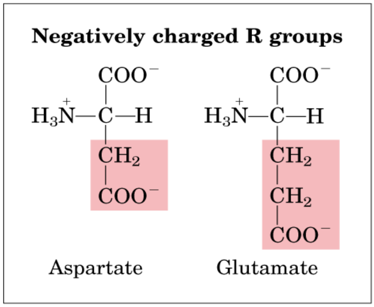
(a) Tendency for oxidation to occur (b) Power as reducing agent INCREASES

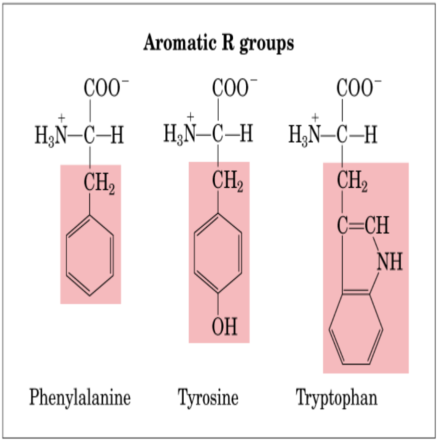
AMINO ACIDS


Nonpolar, aliphatic R groups
Glycine
Alanine
Valine
Leucine
Methionine
Isoleucine


Polar, uncharged R groups
Serine
Threonine
Cysteine
Proline
Asparagine
Glutamine

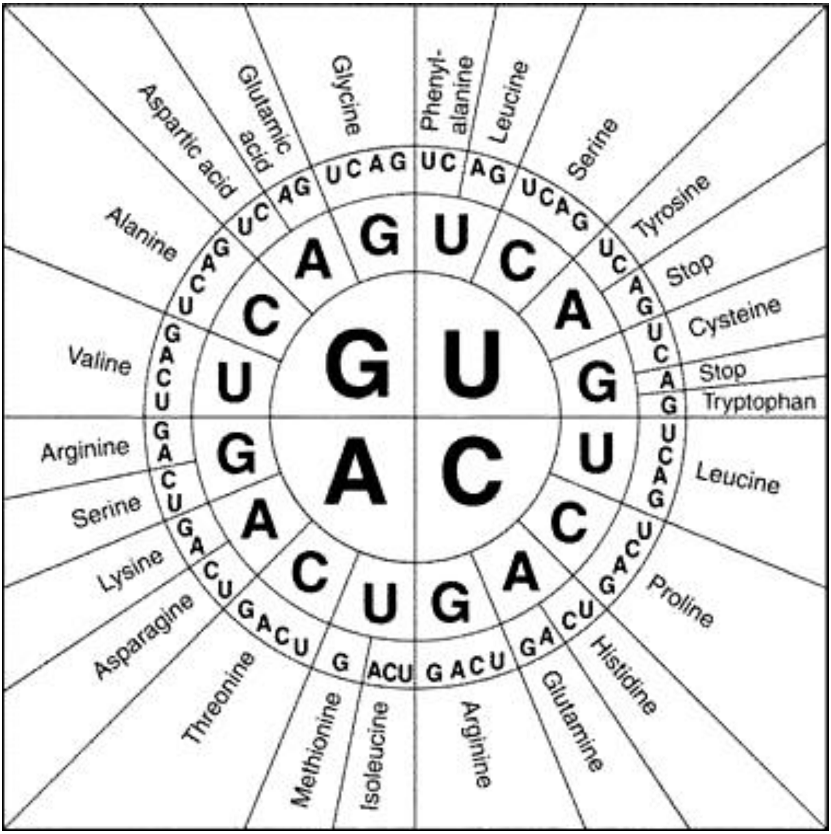






Amino Acid	Abbreviations
Alanine	Ala; A
Arginine	Arg; R
Asparagine	Asn; N
Aspartic acid	Asp; D
Cysteine	Cys; C
Glutamic acid	Glu; E
Glutamine	Gln; Q
Glycine	Gly; G
Histidine	His; H
Isoleucine	Ile; I
Leucine	Leu; L
Lysine	Lys; K
Methionine	Met; M
Phenylalanine	Phe; F
Proline	Pro; P
Serine	Ser; S
Threonine	Thr; T
Tyrosine	Tyr; Y
Tryptophan	Trp; W
Valine	Val; V

Genetic code



Mechanics of Solids

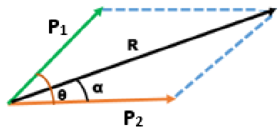
1. Resultant & Equilibrium of Coplanar Concurrent Force System

1.1. Law of Parallelogram of Forces

P_1 & P_2- concurrent forces acting on a particle
R- Resultant of P_1 & P_2
Θ- Angle between P_1 & P_2
α - angle between resultant and P_2

$$R^2 = P_1+P_2+2\,P_1P_2\cos\theta$$

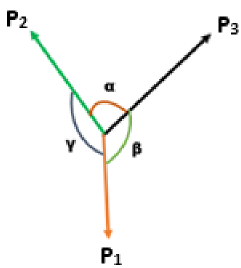
$$\alpha = \tan^{-1}\left[\frac{P_2\sin\theta}{P_1+P_2\cos\theta}\right]$$



1.2. Lami's Theorem

P_1, P_2 and P_3 -Three concurrent forces in equilibrium
α, β and γ - The angles between forces

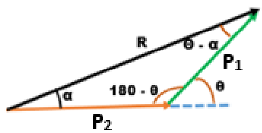
$$\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$$



1.3. Law of Triangle of Forces

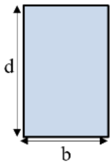
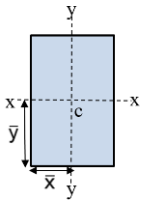
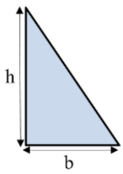
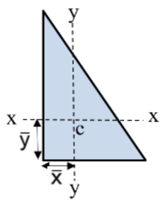
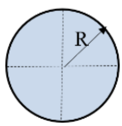
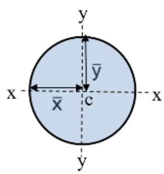
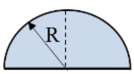
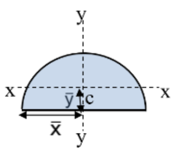
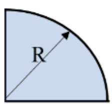
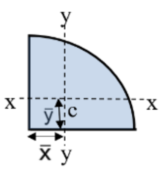
$$R^2 = P_1+P_2+2\,P_1P_2\cos\theta$$

$$\alpha = \tan^{-1}\left[\frac{P_2\sin\theta}{P_1+P_2\cos\theta}\right]$$



2. Centroid and Moment of Inertia

2.1. Formulae for Standard Geometrical Figures

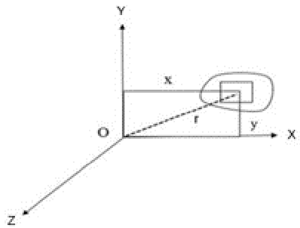
Shape	Location of centroidal axis	$\bar{x}$	$\bar{y}$	$I_{\overline{xx}}$	$I_{\overline{yy}}$
		$\frac{b}{2}$	$\frac{d}{2}$	$\frac{bd^3}{12}$	$\frac{db^3}{12}$
		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh^3}{36}$	$\frac{hb^3}{36}$
		R	R	$\frac{\pi R^4}{4}$	$\frac{\pi R^4}{4}$
		R	$\frac{4R}{3\pi}$	$0.11R^4$	$\frac{\pi R^4}{8}$
		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$0.055R^4$	$0.055R^4$

2.2. Perpendicular Axis Theorem

OX and OY represents reference axis

X - Perpendicular distance of elemental area from OY axis
Y - Perpendicular distance of elemental area from OX axis
r - Perpendicular distance of elemental area from axis perpendicular to OZ
I_{ZZ} - Polar moment of inertia
I_{XX} - Moment of inertia w.r.t OX axis
I_{YY} - Moment of inertia w.r.t OY axis

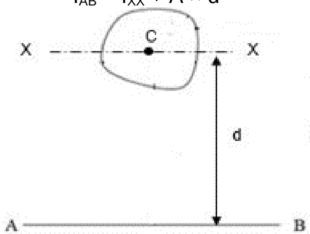
$$I_{zz} = I_{xx} + I_{yy}$$



2.3. Parallel Axis Theorem

X-X - Centroidal axis
AB - axis parallel to centroidal axis
A - Total area
d - Perpendicular distance between centroidal axis and axis parallel to centroidal axis

$$I_{AB} = I_{XX} + A \times d^2$$

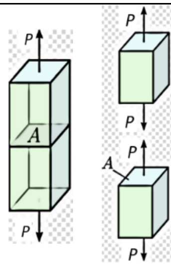


3. Simple Stress and Strain

3.1. Axial / Linear Stress

General Expression: $\sigma = \frac{P}{A}$

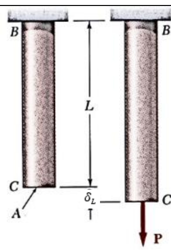
σ - Normal stress
P – Axial force
A – Cross- section area



3.2. Axial / Linear strain

General Expression: $\varepsilon_L = \frac{\delta_L}{L}$

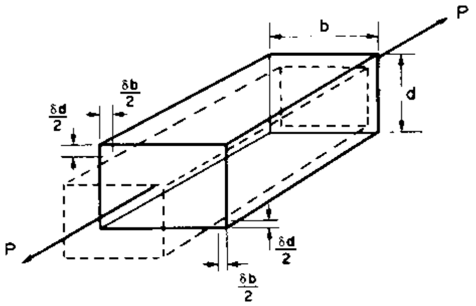
ε - Linear strain
δ_L - Change in length
L - Length of the member



3.3. Lateral Strain

$$\varepsilon_b = \frac{\delta_b}{b} \quad \text{and} \quad \varepsilon_d = \frac{\delta_d}{d}$$

ε_b - lateral strain in breadth of the member
ε_d - lateral strain in depth of the member
δ_b - change in breadth of the member
δ_d - change in depth of the member



3.4. Poisson's Ratio

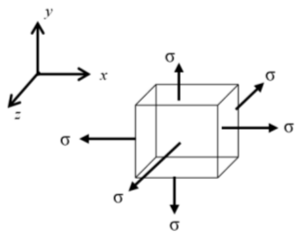
$$\mu = \frac{Lateral\ Strain}{Linear\ Strain}$$

Where, μ - Poisson's ratio

3.5. Volumetric Strain

$$\varepsilon_v = \frac{\delta_v}{V}$$

ε_v - volumetric strain
δ_v - change in volume
V - Original volume

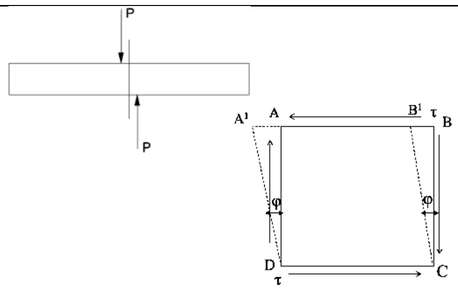


3.6. Shear Stress and Shear Strain

$$\tau = \frac{P}{A}$$

$$\varphi = \frac{BB'}{AB}$$

τ	-	Shear stress
P	-	Shear resistance
A	-	Area resisting shear
φ	-	Shear strain



Cross Sectional Area

3.7. Modulus of Elasticity or Young's Modulus

$$E = \frac{\sigma}{\varepsilon}$$

E	- Modulus of elasticity
σ	- Normal stress
ε	- Linear strain

3.8. Bulk Modulus

$$K = \frac{\sigma}{\left(\frac{\delta_V}{V}\right)}$$

K	- Bulk modulus
σ	- Normal stress in all three directions
$\frac{\delta_V}{V}$	- Volumetric strain

3.9. Modulus of Rigidity

$$G = \frac{\tau}{\varphi}$$

G	- Modulus of rigidity
φ	- Shear strain
τ	- Shear stress

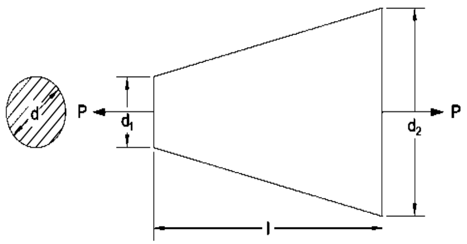
3.10. Relation between Elastic Constants

$E = 3K(1-2\mu)$ $\quad\quad$ $E = 2G(1+\mu)$

3.11. Deformation of Circular Tapered Bar

$$\delta = \frac{4Pl}{\pi E d_1 d_2}$$

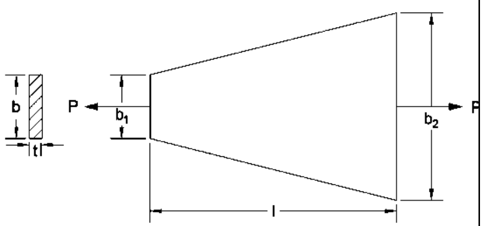
P	-	Axial force
E	-	Modulus of elasticity
l	-	Length of the member
d_1	-	Smaller diameter
d_2	-	Larger diameter
δ	-	Change in length of tapered bar



3.12. Deformation of Rectangular Tapered Bar

$$\delta = \frac{2.302\ P\ l}{t\ E\ (b_1 - b_2)}(\log b_2 - \log b_1)$$

P	-	Axial force
E	-	Modulus of elasticity
l	-	Length of the member
b_1	-	Smaller depth
b_2	-	Larger depth
t	-	Thickness of the member
δ	-	Change in length of tapered bar

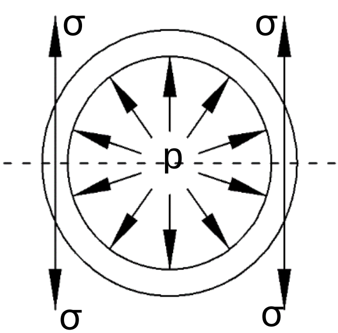


4. Thin Cylinders

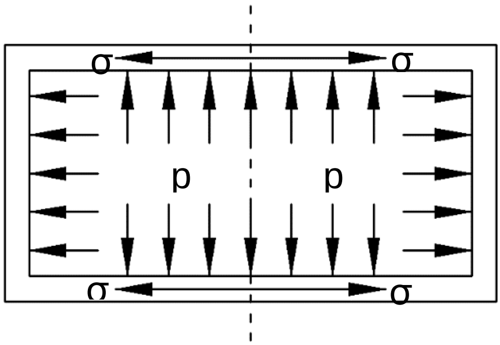
Symbols and Notations

σ_C	-	Circumferential / hoop stress
σ_L	-	Longitudinal stress
p	-	Internal pressure in the cylinder
d	-	Internal diameter of the cylinder
t	-	Thickness of the wall of the cylinder
E	-	Modulus of elasticity
ϵ_C	-	Circumferential strain
ϵ_L	-	Longitudinal strain
ϵ_V	-	Volumetric strain

4.1. Longitudinal and Circumferential Stresses and Strains



Cross Section of thin cylinder



Longitudinal Section of thin cylinder

$$\sigma_C = \frac{pd}{2t} \qquad \sigma_L = \frac{pd}{4t}$$

$$\epsilon_C = \frac{pd}{4tE}(2-\mu) \qquad \epsilon_L = \frac{pd}{4tE}(1-2\mu)$$

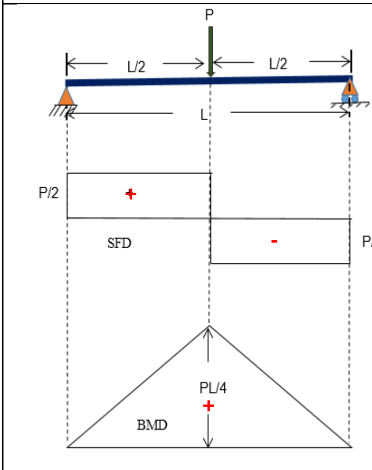
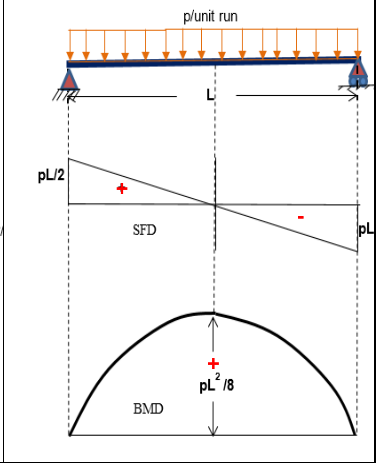
$$\tau_{max} = \frac{pd}{8t} \qquad \epsilon_V = \frac{pd}{4tE}(5-4\mu)$$

4.2. Joint Efficiency (η)

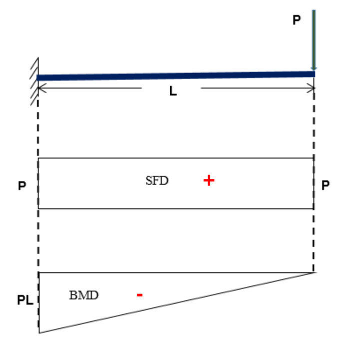
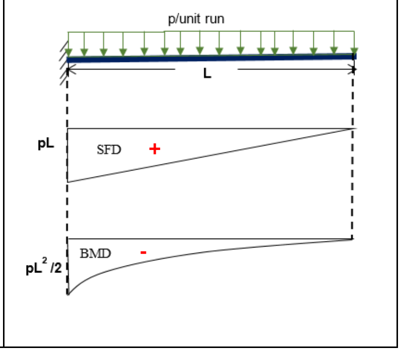
$$\eta = \frac{\text{strength of joint}}{\text{strength of plate}} \times 100$$

5. Shear Force and Bending Moment

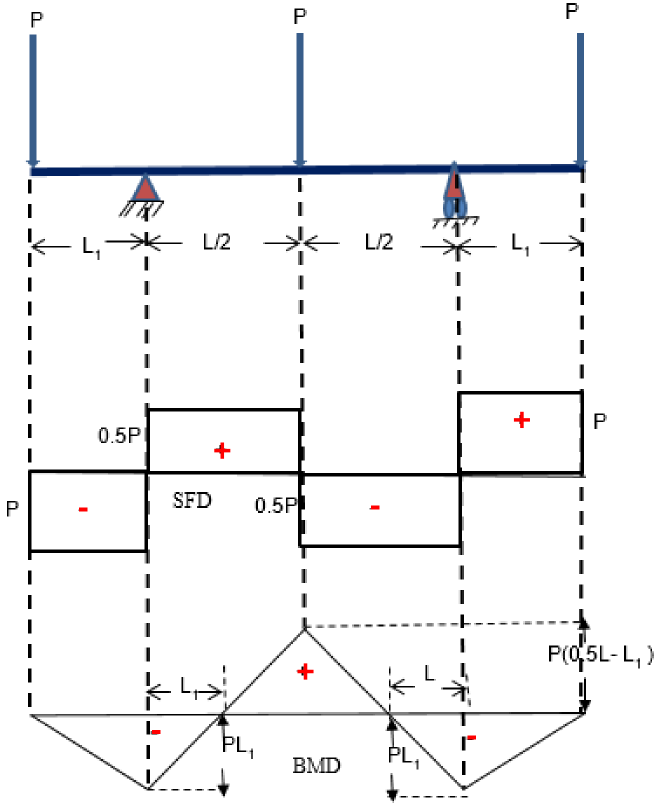
5.1. Simply Supported Beam

5.1.1 Point Load at Mid Span	5.1.2 Uniformly Distributed Load throughout the Span
	

5.2. Cantilever Beam

5.2.1 Point Load at Free End	5.2.2 Uniformly Distributed Load throughout the Span
	

5.3. Overhanging Beam



Basic Mechanical Engineering

1. Properties of Steam

1.1. Enthalpy of Dry Saturated Steam (kJ/kg)

$$h_g = h_f + h_{fg}$$

h_g = Enthalpy of dry steam in kJ/kg
h_f = Sensible heat in kJ/kg
h_{fg} = Enthalpy of evaporation or latent heat in kJ/kg

1.2. Enthalpy of Wet Steam (kJ/kg)

$$h = h_f + xh_{fg}$$

x = Dryness fraction of the steam 0<x<1

1.3. Enthalpy of Superheated Steam (kJ/kg)

$$h_{sup} = h_g + C_{sup}\left(T_{sup} - T_{sat}\right)$$

h_{sup} = Enthalpy of superheated steam in kJ/kg
C_{sup} = Specific heat of superheated steam =2.25 kJ/kg K
T_{sup} = Superheated temperature in °C
T_{sat} = Saturation temperature in °C

1.4. Degree of Superheat (DOS) (°C)

$$DOS = \left(T_{sup} - T_{sat}\right)$$

1.5. Amount of Superheat (AOS) (kJ/kg)

$$AOS = C_{sup}\left(T_{sup} - T_{sat}\right)$$

1.6. Enthalpy of Feed Water (kJ)

$$h_w = mC_p(T - 0)$$

m = Mass of water in kg
C_p = Specific heat of water = 4.187 kJ/kg K
T = Temperature of feed water in °C

1.7. Dryness Fraction

$$x = \frac{m_g}{m_f + m_g}$$

m_g = Mass of dry steam present in the sample quantity of wet steam
m_f = Mass of suspended water molecules in the sample quantity of wet steam

1.8. Boiler Efficiency

$$\eta = \frac{Q(h_s - h_w)}{m_{fu} \times GCV} \times 100$$

Q = Quantity of steam generated per unit time in kg/hr
h_s = Enthalpy of steam generated in kJ/kg
h_w = Enthalpy of feed water in kJ/kg
m_{fu} = Quantity of fuel consumed per unit time in kg/hr
GCV = Gross calorific value of the fuel in kJ/kg

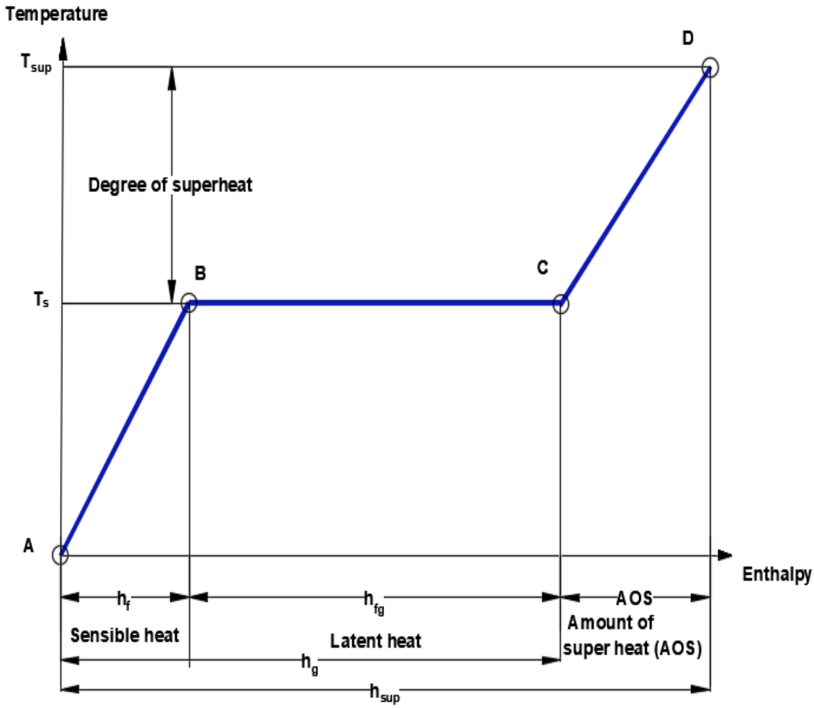


Figure1.1 Temperature – Enthalpy diagram

Staturated Water and Steam (Pressure) Tables

Absolute pressure in bar	Temperature in °C	Specific volume in m³/kg		Specific enthalpy in kJ/kg			Specific entropy in kJ/kg K			Absolute pressure in bar
(p)	(t)	water (v_f)	Steam (v_g)	Water (h_f)	Evaporation (h_{fg})	Steam (h_g)	Water (s_f)	Evaporation (s_{fg})	Steam (s_g)	(p)
0.006 1	0.000	0.001 000	206.31	0.0	2 501.6	2 501.6	0.000	9.158	9.158	**0.006 1**
0.010	6.983	0.001 000	129.21	29.3	2 485.1	2 514.4	0.106	8.871	8.977	**0.010**
0.015	13.04	0.001 001	87.982	54.7	2 470.8	2 525.5	0.196	8.634	8.830	**0.015**
0.020	17.51	0.001 001	67.006	73.5	2 460.1	2 533.6	0.261	8.464	8.725	**0.020**
0.025	21.10	0.001 002	54.256	88.4	2 451.8	2 540.2	0.312	8.333	8.645	**0.025**
0.030	24.10	0.001 003	45.667	101.0	2 444.6	2 545.6	0.354	8.224	8.578	**0.030**
0.035	26.69	0.001 003	39.479	111.8	2 438.6	2 550.4	0.391	8.132	8.523	**0.035**
0.040	28.98	0.001 004	34.802	121.4	2 433.1	2 554.5	0.423	8.053	8.476	**0.040**
0.045	31.03	0.001 005	31.141	130.0	2 428.2	2 558.2	0.451	7.983	8.434	**0.045**
0.050	32.90	0.001 005	28.194	137.8	2 423.8	2 561.6	0.476	7.920	8.396	**0.050**
0.060	36.18	0.001 006	23.741	151.5	2 416.0	2 567.5	0.521	7.810	8.331	**0.06**
0.070	39.03	0.001 007	20.531	163.4	2 409.2	2 572.6	0.559	7.718	8.277	**0.07**
0.080	41.53	0.001 008	18.105	173.9	2 403.2	2 577.1	0.593	7.637	8.230	**0.08**
0.090	43.79	0.001 009	16.204	183.3	2 397.8	2 581.1	0.622	7.566	8.188	**0.09**
0.100	45.83	0.001 010	14.675	191.8	2 392.9	2 584.7	0.649	7.502	8.151	**0.10**
0.11	47.71	0.001 011	13.416	199.7	2 388.4	2 588.1	0.674	7.444	8.118	**0.11**
0.12	49.45	0.001 012	12.362	206.9	2 384.3	2 591.2	0.696	7.391	8.087	**0.12**
0.13	51.06	0.001 013	11.466	213.7	2 380.3	2 594.0	0.717	7.342	8.059	**0.13**
0.14	52.57	0.001 013	10.694	220.0	2 376.7	2 596.7	0.737	7.296	8.033	**0.14**
0.15	54.00	0.001 014	10.023	226.0	2 373.2	2 599.2	0.755	7.254	8.009	**0.15**

Saturated Water and Steam (Pressure) Tables

(p)	(t)	(v_f)	(v_g)	(h_f)	(h_{fg})	(h_g)	(s_f)	(s_{fg})	(s_g)	(p)
0.16	55.34	0.001 015	9.433 1	231.6	2 370.0	2 601.6	0.772	7.215	7.987	**0.16**
0.17	56.62	0.001 015	8.911 1	236.9	2 366.9	2 603.8	0.788	7.178	7.966	**0.17**
0.18	57.83	0.001 016	8.445 2	242.0	2 363.9	2 605.9	0.804	7.142	7.946	**0.18**
0.19	58.98	0.001 017	8.027 2	246.8	2 361.1	2 607.9	0.818	7.109	7.927	**0.19**
0.20	60.09	0.001 017	7.649 8	251.5	2 358.4	2 609.9	0.832	7.077	7.909	**0.20**
0.21	61.15	0.001 018	7.307 3	255.9	2 355.8	2 611.7	0.845	7.047	7.892	**0.21**
0.22	62.16	0.001 018	6.995 1	260.1	2 353.4	2 613.5	0.858	7.018	7.876	**0.22**
0.23	63.14	0.001 019	6.709 3	264.2	2 351.0	2 615.2	0.870	6.991	7.861	**0.23**
0.24	64.08	0.001 019	6.446 7	268.2	2 348.6	2 616.8	0.882	6.964	7.846	**0.24**
0.25	64.99	0.001 020	6.204 5	272.0	2 346.3	2 618.3	0.893	6.939	7.832	**0.25**
0.26	65.87	0.001 020	5.980 3	275.7	2 344.2	2 619.9	0.904	6.915	7.819	**0.26**
0.27	66.72	0.001 021	5.772 4	279.2	2 342.1	2 621.3	0.915	6.891	7.806	**0.27**
0.28	67.55	0.001 021	5.577 8	282.7	2 340.0	2 622.7	0.925	6.868	7.793	**0.28**
0.29	68.35	0.001 022	5.398 2	286.0	2 338.1	2 624.1	0.935	6.847	7.781	**0.29**
0.30	69.12	0.001 022	5.229 3	289.3	2 336.1	2 625.4	0.944	6.825	7.769	**0.30**
0.32	70.62	0.001 023	4.922 0	295.6	2 332.4	2 628.0	0.962	6.785	7.747	**0.32**
0.34	72.03	0.001 024	4.650 4	301.5	2 328.9	2 630.4	0.980	6.747	7.727	**0.34**
0.36	73.37	0.001 025	4.407 6	307.1	2 325.5	2 632.6	0.996	6.711	7.707	**0.36**
0.38	74.66	0.001 026	4.190 0	312.5	2 322.3	2 634.8	1.011	6.677	7.688	**0.38**
0.40	75.89	0.001 027	3.993 4	317.7	2 319.2	2 636.9	1.026	6.645	7.671	**0.40**
0.42	77.06	0.001 027	3.814 8	322.6	2 316.3	2 638.9	1.040	6.614	7.654	**0.42**
0.44	78.19	0.001 028	3.652 2	327.3	2 313.4	2 640.7	1.054	6.584	7.638	**0.44**
0.46	79.28	0.001 029	3.503 2	331.9	2 310.7	2 642.6	1.067	6.556	7.623	**0.46**
0.48	80.33	0.001 029	3.366 3	336.3	2 308.0	2 644.3	1.079	6.530	7.609	**0.48**
0.50	81.35	0.001 030	3.240 1	340.6	2 305.4	2 646.0	1.091	6.504	7.595	**0.50**
0.52	82.33	0.001 031	3.123 3	344.7	2 302.9	2 647.6	1.103	6.478	7.581	**0.52**
0.54	83.28	0.001 031	3.014 8	348.7	2 300.5	2 649.2	1.114	6.455	7.569	**0.54**
0.56	84.19	0.001 032	2.913 9	352.5	2 298.2	2 650.7	1.125	6.431	7.556	**0.56**
0.58	85.09	0.001 033	2.819 7	356.3	2 295.8	2 652.1	1.135	6.409	7.544	**0.58**
0.60	85.95	0.001 033	2.731 7	359.9	2 293.7	2 653.6	1.145	6.388	7.533	**0.60**

Saturated Water and Steam (Pressure) Tables

(p)	(t)	(v_f)	(v_g)	(h_f)	(h_{fg})	(h_g)	(s_f)	(s_{fg})	(s_g)	(p)
0.62	86.80	0.001 034	2.649 1	363.5	2 291.4	2 654.9	1.155	6.367	7.522	**0.62**
0.64	87.62	0.001 034	2.571 5	366.9	2 289.4	2 656.3	1.165	6.346	7.511	**0.64**
0.66	88.42	0.001 035	2.498 5	370.3	2 287.3	2 657.6	1.174	6.326	7.500	**0.66**
0.68	89.20	0.001 036	2.429 7	373.6	2 285.2	2 658.8	1.183	6.307	7.490	**0.68**
0.70	89.96	0.001 036	2.364 7	376.8	2 283.3	2 660.1	1.192	6.288	7.480	**0.70**
0.72	90.70	0.001 037	2.303 1	379.9	2 281.4	2 661.3	1.201	6.270	7.471	**0.72**
0.74	91.43	0.001 037	2.244 8	382.9	2 279.5	2 662.4	1.209	6.253	7.462	**0.74**
0.76	92.14	0.001 038	2.189 5	385.9	2 277.7	2 663.6	1.217	6.235	7.452	**0.76**
0.78	92.83	0.001 038	2.136 9	388.9	2 275.8	2 664.7	1.225	6.219	7.444	**0.78**
0.80	93.51	0.001 039	2.086 9	391.7	2 274.1	2 665.8	1.233	6.202	7.435	**0.80**
0.85	95.15	0.001 040	1.972 1	398.6	2 269.8	2 668.4	1.252	6.163	7.415	**0.85**
0.90	96.71	0.001 041	1.869 1	405.2	2 265.7	2 670.9	1.270	6.125	7.395	**0.90**
0.95	98.20	0.001 042	1.777 1	411.5	2 261.7	2 673.2	1.287	6.091	7.378	**0.95**
1.00	99.63	0.001 043	1.693 8	417.5	2 257.9	2 675.4	1.303	6.057	7.360	**1.00**
1.013 25	100.00	0.001 044	1.673 0	419.1	2 256.9	2 676.0	1.307	6.048	7.355	**1.013 25**
1.05	101.0	0.001 045	1.618 1	423.3	2 254.3	2 677.6	1.318	6.025	7.343	**1.05**
1.10	102.3	0.001 046	1.549 2	428.8	2 250.8	2 679.6	1.333	5.995	7.328	**1.10**
1.15	103.6	0.001 047	1.486 1	434.2	2 247.4	2 681.6	1.347	5.966	7.313	**1.15**
1.20	104.8	0.001 048	1.428 1	439.3	2 244.1	2 683.4	1.361	5.937	7.298	**1.20**
1.25	106.0	0.001 049	1.374 6	444.4	2 240.8	2 685.2	1.374	5.911	7.285	**1.25**
1.30	107.1	0.001 050	1.325 0	449.2	2 237.8	2 687.0	1.387	5.885	7.272	**1.30**
1.35	108.2	0.001 050	1.279 1	453.4	2 234.8	2 688.7	1.399	5.860	7.259	**1.35**
1.40	109.3	0.001 051	1.236 3	458.4	2 231.9	2 690.3	1.411	5.836	7.247	**1.40**
1.45	110.4	0.001 052	1.196 3	462.8	2 229.0	2 691.8	1.423	5.812	7.235	**1.45**
1.50	111.4	0.001 053	1.159 0	467.1	2 226.3	2 693.4	1.433	5.790	7.223	**1.50**
1.60	113.3	0.001 055	1.091 1	475.4	2 220.8	2 696.2	1.455	5.747	7.202	**1.60**
1.70	115.2	0.001 056	1.030 9	483.2	2 215.8	2 699.0	1.475	5.706	7.181	**1.70**
1.80	116.9	0.001 058	0.977 18	490.7	2 210.8	2 701.5	1.494	5.668	7.162	**1.80**
1.90	118.6	0.001 059	0.928 95	497.9	2 206.1	2 704.0	1.513	5.631	7.144	**1.90**
2.00	120.2	0.001 061	0.885 40	504.7	2 201.6	2 706.3	1.530	5.597	7.127	**2.00**

Saturated Water and Steam (Pressure) Tables

(p)	(t)	(v_f)	(v_g)	(h_f)	(h_{fg})	(h_g)	(s_f)	(s_{fg})	(s_g)	(t)
2.1	121.8	0.001 062	0.845 86	511.3	2 197.2	2 708.5	1.547	5.564	7.111	**2.1**
2.2	123.3	0.001 064	0.809 80	517.6	2 193.0	2 710.6	1.563	5.532	7.095	**2.2**
2.3	124.7	0.001 065	0.776 77	523.7	2 188.9	2 712.6	1.578	5.502	7.080	**2.3**
2.4	126.1	0.001 066	0.746 41	529.6	2 184.9	2 714.5	1.593	5.473	7.066	**2.4**
2.5	127.4	0.001 068	0.718 40	535.3	2 181.1	2 716.4	1.607	5.445	7.052	**2.5**
2.6	128.7	0.001 069	0.692 47	540.9	2 177.3	2 718.2	1.621	5.418	7.039	**2.6**
2.7	130.0	0.001 070	0.668 40	546.2	2 173.7	2 719.9	1.634	5.392	7.026	**2.7**
2.8	131.2	0.001 071	0.646 00	551.4	2 170.1	2 721.5	1.647	5.367	7.014	**2.8**
2.9	132.4	0.001 072	0.625 09	556.5	2 166.6	2 723.1	1.660	5.342	7.002	**2.9**
3.0	133.5	0.001 074	0.605 53	561.5	2 163.2	2 724.7	1.672	5.319	6.991	**3.0**
3.1	134.7	0.001 075	0.587 18	566.2	2 159.9	2 726.1	1.683	5.297	6.980	**3.1**
3.2	135.8	0.001 076	0.569 95	570.9	2 156.7	2 727.6	1.695	5.274	6.969	**3.2**
3.3	136.8	0.001 077	0.553 73	575.5	2 153.5	2 729.0	1.706	5.253	6.959	**3.3**
3.4	137.9	0.001 078	0.538 43	579.9	2 150.4	2 730.3	1.717	5.232	6.949	**3.4**
3.5	138.9	0.001 079	0.523 97	584.3	2 147.3	2 731.6	1.727	5.212	6.939	**3.5**
3.6	139.9	0.001 080	0.510 29	588.5	2 144.4	2 732.9	1.738	5.192	6.930	**3.6**
3.7	140.8	0.001 081	0.497 33	592.7	2 141.4	2 734.1	1.748	5.173	6.921	**3.7**
3.8	141.8	0.001 082	0.485 02	596.7	2 138.6	2 735.3	1.758	5.154	6.912	**3.8**
3.9	142.7	0.001 083	0.473 33	600.8	2 135.7	2 736.5	1.767	5.136	6.903	**3.9**
4.0	143.6	0.001 084	0.462 20	604.7	2 132.9	2 737.6	1.776	5.118	6.894	**4.0**
4.1	144.5	0.001 085	0.451 59	608.5	2 130.2	2 738.7	1.786	5.100	6.886	**4.1**
4.2	145.4	0.001 086	0.441 47	612.3	2 127.5	2 739.8	1.795	5.083	6.878	**4.2**
4.3	146.3	0.001 087	0.431 81	616.0	2 124.9	2 740.9	1.803	5.067	6.870	**4.3**
4.4	147.1	0.001 088	0.422 57	619.6	2 122.3	2 741.9	1.812	5.050	6.862	**4.4**
4.5	147.9	0.001 089	0.413 73	623.2	2 119.7	2 742.9	1.820	5.035	6.855	**4.5**
4.6	148.7	0.001 090	0.405 26	626.7	2 117.2	2 743.9	1.829	5.018	6.847	**4.6**
4.7	149.5	0.001 090	0.397 14	630.1	2 114.7	2 744.8	1.837	5.003	6.840	**4.7**
4.8	150.3	0.001 091	0.389 34	633.5	2 112.2	2 745.7	1.845	4.988	6.833	**4.8**
4.9	151.1	0.001 092	0.381 86	636.8	2 109.8	2 746.6	1.853	4.973	6.826	**4.9**
5.0	151.8	0.001 093	0.374 66	640.1	2 107.4	2 747.5	1.860	4.959	6.819	**5.0**

Saturated Water and Steam (Pressure) Tables

(p)	(t)	(v_f)	(v_g)	(h_f)	(h_{fg})	(h_g)	(s_f)	(s_{fg})	(s_g)	(p)
5.2	153.3	0.001 095	0.361 06	646.5	2 102.7	2 749.2	1.875	4.931	6.806	**5.2**
5.4	154.8	0.001 096	0.348 44	652.8	2 098.1	2 750.9	1.890	4.903	6.793	**5.4**
5.6	156.2	0.001 098	0.336 69	658.8	2 093.7	2 752.5	1.904	4.877	6.781	**5.6**
5.8	157.5	0.001 099	0.325 72	664.7	2 089.3	2 754.0	1.918	4.851	6.769	**5.8**
6.0	158.8	0.001 101	0.315 46	670.4	2 085.1	2 755.5	1.931	4.827	6.758	**6.0**
6.2	160.1	0.001 102	0.305 84	676.1	2 080.8	2 756.9	1.944	4.803	6.747	**6.2**
6.4	161.4	0.001 104	0.296 80	681.5	2 076.7	2 758.2	1.956	4.780	6.736	**6.4**
6.6	162.6	0.001 105	0.288 29	686.8	2 072.7	2 759.5	1.968	4.757	6.725	**6.6**
6.8	163.8	0.001 107	0.280 26	692.0	2 068.8	2 760.8	1.980	4.735	6.715	**6.8**
7.0	165.0	0.001 108	0.272 68	697.1	2 064.9	2 762.0	1 992	4.713	6.705	**7.0**
7.2	166.1	0.001 110	0.265 50	702.0	2 061.2	2 763.2	2 003	4.693	6.696	**7.2**
7.4	167.2	0.001 111	0.258 70	706.9	2 057.4	2 764.3	2 014	4.672	6.686	**7.4**
7.6	168.3	0.001 112	0.252 24	711.7	2 053.7	2 765.4	2 025	4.652	6.677	**7.6**
7.8	169.4	0.001 114	0.246 10	716.3	2 050.1	2 766.4	2 035	4.633	6.668	**7.8**
8.0	170.4	0.001 115	0.240 26	720.9	2 046.5	2 767.4	2.046	4.614	6.660	**8.0**
8.2	171.4	0.001 116	0.234 69	725.4	2 043.0	2 768.4	2.056	4.595	6.651	**8.2**
8.4	172.4	0.001 118	0.229 38	729.9	2 039.6	2 769.4	2.066	4.577	6.643	**8.4**
8.6	173.4	0.001 119	0.224 31	734.2	2 036.2	2 770.4	2.075	4.560	6.635	**8.6**
8.8	174.4	0.001 120	0.219 46	738.5	2 032.8	2 771.3	2.085	4.542	6.627	**8.8**
9.0	175.4	0.001 121	0.214 82	742.6	2 029.5	2 772.1	2.094	4.525	6.619	**9.0**
9.2	176.3	0.001 123	0.210 37	746.8	2 026.2	2 773.0	2.103	4.509	6.612	**9.2**
9.4	177.2	0.001 124	0.206 10	750.8	2 023.0	2 773.8	2.112	4.492	6.604	**9.4**
9.6	178.1	0.001 125	0.202 01	754.8	2 019.8	2 774.6	2.121	4.476	6.597	**9.6**
9.8	179.0	0.001 126	0.198 08	758.7	2 016.7	2 775.4	2.130	4.460	6.590	**9.8**
10.0	179.9	0.001 127	0.194 30	762.6	2 013.6	2 776.2	2.138	4.445	6.583	**10.0**
10.5	182.0	0.001 130	0.185 48	772.0	2 006.0	2 778.0	2.159	4.407	6.566	**10.5**
11.0	184.1	0.001 133	0.177 39	781.1	1 998.6	2 779.7	2.179	4.371	6.550	**11.0**
11.5	186.0	0.001 136	0.170 02	789.9	1 991.4	2 781.3	2.198	4.336	6.534	**11.5**
12.0	188.0	0.001 139	0.163 21	798.4	1 984.3	2 782.7	2.216	4.303	6.519	**12.0**
12.5	189.8	0.001 141	0.156 96	806.7	1 977.5	2 784.2	2.234	4.271	6.505	**12.5**

Saturated Water and Steam (Pressure) Tables

(p)	(t)	(v_f)	(v_g)	(h_f)	(h_{fg})	(h_g)	(s_f)	(s_{fg})	(s_g)	(p)
13.0	191.6	0.001 144	0.151 14	814.7	1 970.7	2 785.4	2.251	4.240	6.491	**13.0**
13.5	193.3	0.001 146	0.145 76	822.5	1 964.2	2 786.7	2.267	4.211	6.478	**13.5**
14.0	195.0	0.001 149	0.140 73	830.1	1 957.7	2 787.8	2.284	4.181	6.465	**14.0**
14.5	196.7	0.001 151	0.136 06	837.5	1 951.4	2 788.9	2.299	4.154	6.453	**14.5**
15.0	198.3	0.001 154	0.131 67	844.6	1 945.3	2 789.9	2.314	4.127	6.441	**15.0**
15.5	199.8	0.001 156	0.127 56	851.6	1 939.2	2 790.8	2.329	4.100	6.429	**15.5**
16.0	201.4	0.001 159	0.123 70	858.5	1 933.2	2 791.7	2.344	4.074	6.418	**16.0**
16.5	202.9	0.001 161	0.120 06	865.3	1 927.3	2 792.6	2.358	4.049	6.407	**16.5**
17.0	204.3	0.001 163	0.116 64	871.8	1 921.6	2 793.4	2.371	4.025	6.396	**17.0**
17.5	205.7	0.001 166	0.113 40	878.2	1 915.9	2 794.1	2.384	4.001	6.385	**17.5**
18.0	207.1	0.001 168	0.110 33	884.5	1 910.3	2 794.8	2.398	3.977	6.375	**18.0**
18.5	208.5	0.001 170	0.107 42	890.7	1 904.8	2 795.5	2.410	3.955	6.365	**18.5**
19.0	209.8	0.001 172	0.104 67	896.8	1 899.3	2 796.1	2.423	3.933	6.356	**19.0**
19.5	211.1	0.001 174	0.102 04	902.7	1 894.0	2 796.7	2.435	3.911	6.346	**19.5**
20.0	212.4	0.001 177	0.099 55	908.5	1 888.7	2 797.2	2.447	3.890	6.337	**20.0**
21.0	214.8	0.001 181	0.094 902	919.9	1 878.3	2 798.2	2.470	3.849	6.319	**21.0**
22.0	217.2	0.001 185	0.090 663	930.9	1 868.1	2 799.1	2.492	3.809	6.301	**22.0**
23.0	219.6	0.001 189	0.086 780	941.6	1 858.2	2 799.8	2.514	3.771	6.285	**23.0**
24.0	221.8	0.001 193	0.083 209	951.9	1 848.5	2 800.4	2.534	3.735	6.269	**24.0**
25.0	223.9	0.001 197	0.079 915	961.9	1 839.1	2 801.0	2.554	3.699	6.253	**25.0**
26.0	226.0	0.001 201	0.076 865	971.7	1 829.7	2 801.4	2.574	3.665	6.239	**26.0**
27.0	228.1	0.001 205	0.074 033	981.2	1 820.5	2 801.7	2.592	3.632	6.224	**27.0**
28.0	230.0	0.001 209	0.071 396	990.5	1 811.5	2 802.0	2.611	3.600	6.211	**28.0**
29.0	232.0	0.001 213	0.068 935	999.5	1 802.7	2 802.2	2.628	3.569	6.197	**29.0**
30.0	233.8	0.001 216	0.066 632	1 008.3	1 794.0	2 802.3	2.646	3.538	6.184	**30.0**
31.0	235.7	0.001 220	0.064 473	1 017.1	1 785.4	2 802.3	2.662	3.509	6.171	**31.0**
32.0	237.4	0.001 224	0.062 443	1 025.4	1 776.9	2 802.3	2.679	3.480	6.159	**32.0**
33.0	239.2	0.001 227	0.060 533	1 033.7	1 768.6	2 802.3	2.694	3.452	6.146	**33.0**
34.0	240.9	0.001 231	0.058 731	1 041.8	1 760.3	2 802.1	2 710	3.424	6.134	**34.0**
35.0	242.5	0.001 235	0.057 028	1 049.7	1 752.3	2 802.0	2.725	3.398	6.123	**35.0**

Saturated Water and Steam (Pressure) Tables

(p)	(t)	(v_f)	(v_g)	(h_f)	(h_{fg})	(h_g)	(s_f)	(s_{fg})	(s_g)	(p)
36.0	244.2	0.001 238	0.055 417	1 057.5	1 744.2	2 801.7	2.740	3.371	6.111	**36.0**
37.0	245.8	0.001 242	0.053 889	1 065.2	1 736.2	2 801.4	2.755	3.345	6.100	**37.0**
38.0	247.3	0.001 245	0.052 439	1 072.7	1 728.4	2 801.1	2.769	3.321	6.090	**38.0**
39.0	248.8	0.001 249	0.051 061	1 080.1	1 720.7	2 800.8	2.783	3.296	6.079	**39.0**
40.0	250.3	0.001 252	0.049 749	1 087.4	1 712.9	2 800.3	2.797	3.272	6.069	**40.0**
42.0	253.2	0.001 259	0.047 306	1 101.6	1 697.8	2 799.4	2.823	3.225	6.048	**42.0**
44.0	256.1	0.001 266	0.045 078	1 115.4	1 682.9	2 798.3	2.849	3.180	6.029	**44.0**
46.0	258.8	0.001 273	0.043 036	1 128.8	1 668.2	2 797.0	2.874	3.136	6.010	**46.0**
48.0	261.4	0.001 279	0.041 158	1 141.8	1 653.9	2 795.7	2.897	3.094	5.991	**48.0**
50.0	263.9	0.001 286	0.039 425	1 154.5	1 639.7	2 794.2	2.921	3.053	5.974	**50.0**
52.0	266.4	0.001 293	0.037 820	1 166.9	1 625.7	2 792.6	2.943	3.013	5.956	**52.0**
54.0	268.8	0.001 299	0.036 330	1 179.0	1 611.8	2 790.8	2.965	2.974	5.939	**54.0**
56.0	271.1	0.001 306	0.034 942	1 190.8	1 598.2	2 789.0	2.986	2.937	5.923	**56.0**
58.0	273.4	0.001 312	0.033 646	1 202.4	1 584.6	2 787.0	3.007	2.899	5.906	**58.0**
60.0	275.6	0.001 319	0.032 433	1 213.7	1 571.3	2 785.0	3.027	2.863	5.890	**60.0**
62.0	277.7	0.001 325	0.031 295	1 224.9	1 558.0	2 782.9	3.047	2.828	5.875	**62.0**
64.0	279.8	0.001 332	0.030 225	1 235.8	1 544.8	2 780.6	3.066	2.794	5.860	**64.0**
66.0	281.9	0.001 338	0.029 218	1 246.5	1 531.8	2 778.3	3.085	2.760	5.845	**66.0**
68.0	283.9	0.001 345	0.028 267	1 257.1	1 518.8	2 775.9	3.104	2.727	5.831	**68.0**
70.0	285.8	0.001 351	0.027 368	1 267.4	1 506.0	2 773.4	3.122	2.694	5.816	**70.0**
72.0	287.7	0.001 358	0.026 517	1 277.7	1 493.2	2 770.9	3.140	2.662	5.802	**72.0**
74.0	289.6	0.001 365	0.025 711	1 287.8	1 480.4	2 768.2	3.157	2.631	5.788	**74.0**
76.0	291.4	0.001 371	0.024 944	1 297.7	1 467.8	2 765.5	3.174	2.600	5.774	**76.0**
78.0	293.2	0.001 378	0.024 215	1 307.5	1 455.4	2 762.7	3.191	2.569	5.760	**78.0**
80.0	295.0	0.001 384	0.023 521	1 317.2	1 442.7	2 759.9	3.208	2.539	5.747	**80.0**
82.0	296.7	0.001 391	0.022 860	1 326.7	1 430.3	2 757.0	3.224	2.510	5.734	**82.0**
84.0	298.4	0.001 398	0.022 228	1 336.2	1 417.8	2 754.0	3.240	2.481	5.721	**84.0**
86.0	300.1	0.001 404	0.021 624	1 345.4	1 405.5	2 750.9	3.256	2.452	5.708	**86.0**
88.0	301.7	0.001 411	0.021 046	1 354.7	1 393.1	2 747.8	3.271	2.424	5.695	**88.0**
90.0	303.3	0.001 418	0.020 493	1 363.8	1 380.8	2 744.6	3.287	2.395	5.682	**90.0**

Saturated Water and Steam (Pressure) Tables

(p)	(t)	(v_f)	(v_g)	(h_f)	(h_{fg})	(h_g)	(s_f)	(s_{fg})	(s_g)	(p)
92	304.9	0.001 425	0.019 962	1 372.8	1 368.5	2 741.3	3.302	2.367	5.669	**92**
94	306.5	0.001 432	0.019 453	1 381.7	1 356.3	2 738.0	3.317	2.340	5.657	**94**
96	308.0	0.001 439	0.018 964	1 390.6	1 344.1	2 734.7	3.332	2.313	5.644	**96**
98	309.5	0.001 446	0.018 493	1 399.4	1 331.9	2 731.2	3.346	2.286	5.632	**98**
100	311.0	0.001 453	0.018 041	1 408.0	1 319.7	2 727.7	3.361	2.259	5.620	**100**
105	314.6	0.001 470	0.016 981	1 429.5	1 289.2	2 718.7	3.396	2.194	5.590	**105**
110	318.0	0.001 489	0.016 007	1 450.5	1 258.8	2 709.3	3.430	2.129	5.560	**110**
115	321.4	0.001 508	0.015 114	1 471.3	1 228.2	2 699.5	3.464	2.066	5.530	**115**
120	324.6	0.001 527	0.014 285	1 491.7	1 197.5	2 698.2	3.497	2.003	5.500	**120**
125	327.8	0.001 547	0.013 518	1 511.9	1 166.5	2 678.4	3.530	1.941	5.471	**125**
130	330.8	0.001 567	0.012 800	1 531.9	1 135.1	2 667.0	3.561	1.880	5.441	**130**
135	333.8	0.001 588	0.012 130	1 551.8	1 103.3	2 655.1	3.593	1.818	5.411	**135**
140	336.6	0.001 611	0.011 498	1 571.5	1 070.9	2 642.4	3.624	1.756	5.380	**140**
145	339.4	0.001 634	0.010 905	1 591.3	1 037.9	2 629.2	3.655	1.694	5.349	**145**
150	342.1	0.001 658	0.010 343	1 610.9	1 004.2	2 615.1	3.686	1.632	5.318	**150**
155	344.8	0.001 683	0.009 813	1 630.7	969.7	2 600.4	3.716	1.570	5.286	**155**
160	347.3	0.001 710	0.009 310	1 650.4	934.5	2 584.9	3.747	1.506	5.253	**160**
165	349.7	0.001 739	0.008 833	1 670.4	898.5	2 568.9	3.778	1.442	5.220	**165**
170	352.3	0.001 770	0.008 372	1 691.6	860.0	2 551.6	3.811	1.375	5.186	**170**
175	354.6	0.001 803	0.007 927	1 713.3	820.0	2 533.3	3.844	1.306	5.150	**175**
180	357.0	0.001 840	0.007 497	1 734.8	779.1	2 513.9	3.877	1.236	5.113	**180**
185	359.2	0.001 881	0.007 082	1 756.5	736.5	2 493.0	3.910	1.164	5.074	**185**
190	361.4	0.001 926	0.006 676	1 778.7	691.8	2 470.5	3.943	1.090	5.033	**190**
195	363.6	0.001 978	0.006 276	1 801.9	643.9	2 445.8	3.978	1.011	4.989	**195**
200	365.7	0.002 037	0.005 875	1 826.6	591.6	2 418.2	4.015	0.926	4.941	**200**
205	367.8	0.002 110	0.005 462	1 854.2	532.0	2 386.2	4.056	0.830	4.886	**205**
210	369.8	0.002 202	0.005 023	1 886.3	461.2	2 347.5	4.105	0.717	4.822	**210**
215	371.8	0.002 342	0.004 509	1 929.4	365.2	2 294.6	4.170	0.566	4.736	**215**
220	373.7	0.002 668	0.003 735	2 010.3	186.3	2 196.6	4.293	0.288	4.581	**220**
221.2	374.15	0.003 170	0.003 170	2 107.4	000.0	2 107.4	4.443	0.000	4.443	**221.2**

2. Power Transmission

2.1. Belt Drive

2.1.1. Velocity Ratio

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

N_1 = Speed of the driving pulley in rpm
N_2 = Speed of the driven pulley in rpm
d_1 = Diameter of the driving pulley in mm
d_2 = Diameter of the driven pulley in mm

2.1.2. Velocity Ratio (Considering slip)

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left[\frac{100 - S}{100}\right]$$

S = percentage slip

2.1.3. Initial Tension in the Belt

$$T_0 = \frac{T_1 + T_2}{2}$$

T_0 = Initial tension in the belt in N
T_1 = Tension on the tight side of belt in N
T_2 = Tension on the slack side of belt in N

2.1.4. Length and Angle of Lap of a Belt Drive

2.1.4.1. Open Belt Drive

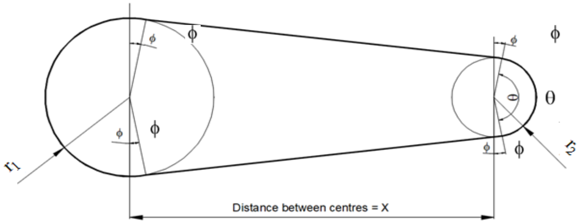


Figure 2.1 Schematic of an open belt drive

Length

$$L = \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{X} + 2X$$

Angle of Lap

$$\theta = \pi - 2\emptyset$$

$$\emptyset = \sin^{-1}\left[\frac{r_1 - r_2}{X}\right]$$

L = Total length of the belt in mm
r_1 = Radius of driving pulley in mm
r_2 = Radius of driven pulley in mm
X = Centre distance between pulleys in mm
θ = Angle of lap in radians
ϕ is measured in radians

2.1.4.2. Crossed Belt Drive

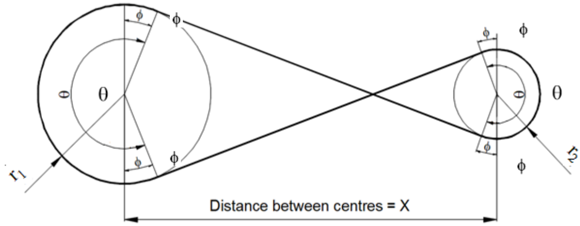


Figure 2.2 Schematic of a crossed belt drive

Length

$$L = \pi(r_1 + r_2) + \frac{(r_1 + r_2)^2}{X} + 2X$$

Angle of Lap

$$\theta = \pi + 2\emptyset$$

$$\emptyset = \sin^{-1}\left[\frac{r_1 + r_2}{X}\right]$$

L = Total length of the belt in mm
r_1 = Radius of driving pulley in mm
r_2 = Radius of driven pulley in mm
X = Centre distance between pulleys in mm
θ = Angle of lap in radians
ϕ is measured in radians

2.1.5. Ratio of Tensions in a Belt Drive

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

μ is coefficient of friction

2.1.6. Linear Velocity of the Belt (m/min)

$$v = \frac{\pi d_1 N_1}{1000} = \frac{\pi d_2 N_2}{1000}$$

2.1.7. Power Transmitted by a Belt Drive (kW)

$$P = \frac{(T_1 - T_2)\, v}{60000}$$

T_1 = Tension on the tight side of belt in N
T_2 = Tension on the slack side of belt in N
v = Linear velocity of the belt in m/min

2.1.8. Belt width, when permissible tension is known

$$Belt\ Width = \frac{Maximum\ tension\ in\ the\ belt}{Permisible\ belt\ tension\ per\ unit\ length\ of\ belt\ width}$$

2.2. Gear Drive

2.2.1 Circular Pitch $P_C = \dfrac{\pi D}{T}$

2.2.2 Diametral Pitch $P_d = \dfrac{T}{D}$

D = Pitch circle diameter in mm
T = Number of teeth

2.2.3 Module $m = \dfrac{D}{T} \quad or \quad m = \dfrac{1}{P_d}$

2.2.4 Simple Gear Train

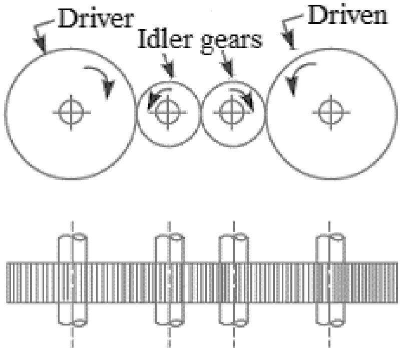


Figure 2.3 Schematic of a simple gear train

2.2.4.1 Velocity Ratio

$$VR = \frac{Speed\ of\ the\ driven\ gear}{Speed\ of\ the\ driver\ gear} = \frac{Number\ of\ teeth\ on\ the\ driver\ gear}{Number\ of\ teeth\ on\ the\ driven\ gear}$$

2.2.5 Compound Gear Train

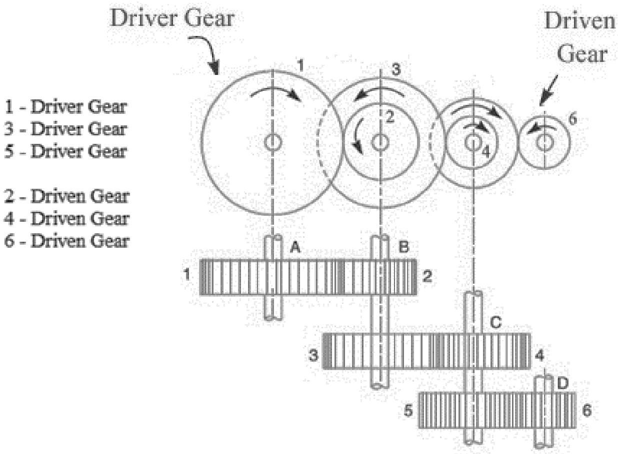


Figure 2.4 Schematic of a compound gear drive

2.2.5.1 Velocity Ratio

$$VR = \frac{Speed\ of\ the\ driven\ gear}{Speed\ of\ the\ driver\ gear} = \frac{Product\ of\ number\ of\ teeth\ on\ the\ driver\ gears}{Product\ of\ number\ of\ teeth\ on\ the\ driven\ gears}$$

3. Internal Combustion Engine

3.1. Swept Volume (m^3)

$$V_s = \frac{\pi d^2 L}{4}$$

d = Bore diameter in m
L = Stroke length in m

3.2. Compression Ratio

$$CR = \frac{(V_s + V_C)}{V_C}$$

V_c = Clearance volume in m^3

3.3. Piston Speed (S) (m/min)

$$S = 2LN$$

L = Stroke length in m
N = Rotational speed of crankshaft in rpm

3.4. Indicated Power (IP) (kW)

$$IP = \frac{iP_m LAn}{60000}$$

i = Number of cylinders
P_m = Mean effective pressure in N/m^2
L = Length of stroke in m
A = Area of cross section of cylinder in m^2
N = Rotational speed of the crank shaft in rpm
n = Number of cycles per minute

n = N for 2 stroke I.C. Engines and n = N / 2 for 4 stroke I.C.Engines

3.5. Brake Power (BP) (kW)

$$BP = \frac{2\pi NT}{60000}$$

N = Rotational speed of the crank shaft in rpm
T = Torque applied due to net load on the brake drum in Nm

3.6. Torque on the Brake Drum (T) (Nm)

$$T = W_{net} \times R$$

W_{net} = Net load acting on the brake drum in N
R = Effective radius in m

$$W_{net} = Load\ acting\ on\ the\ brake\ drum - Spring\ balance\ reading$$

$$R = \frac{(Brake\ drum\ diameter + Rope\ diameter)}{2}$$

3.7. Frictional Power (FP) (kW)

$$FP = Indicated\ Power\ (IP) - Brake\ Power(BP)$$

3.8. Mechanical Efficiency (η_{mech})

$$\eta_{mech} = \frac{Brake\ Power\ (BP)}{Indicated\ Power\ (IP)} \times 100$$

3.9. Brake Thermal Efficiency (η_{bth})

$$\eta_{bth} = \frac{BP \times 3600}{CV \times m_{fu}}$$

3.10. Indicated Thermal Efficiency (η_{ith})

$$\eta_{ith} = \frac{IP \times 3600}{CV \times m_{fu}}$$

m_{fu} = Mass of fuel consumed in kg / hr
CV = Calorific value of the fuel in kJ / kg
BP = Brake power in kW
IP = Indicated power in kW

3.11. Brake Specific Fuel Consumption (SFC) (kg/kWh)

$$SFC = \frac{m_{fu}}{BP}$$

4. Machine Tools

4.1. Taper Turning by Swivelling the Compound Rest

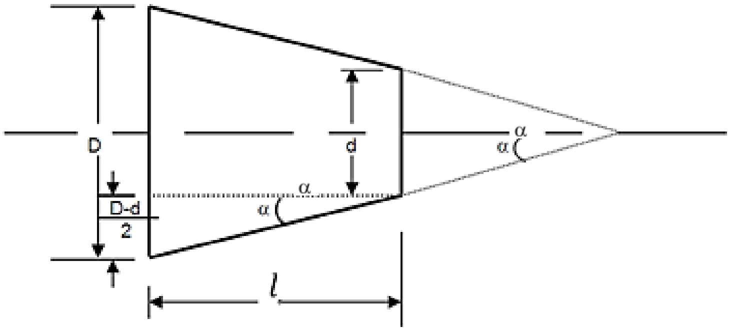


Figure 4.1 Schematic of calculating the swivel angle for taper turning

$$\tan\alpha = \frac{(D-d)}{2l}$$

α = Half taper angle or (simply taper angle) in degrees
D = Larger diameter of taper in mm
d = Smaller diameter of taper in mm
l = Length of the taper in mm

4.2. Taper Turning by Tailstock Set-Over Method

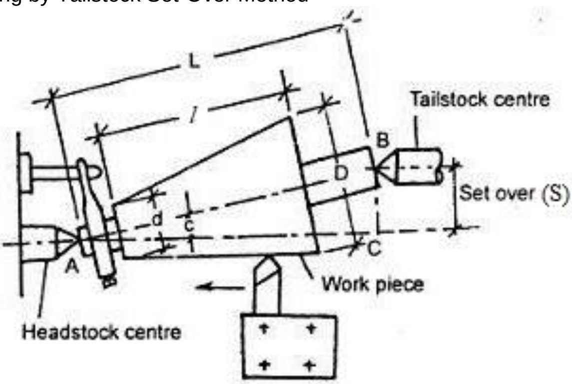


Figure 4.2 Schematic of taper turning by tailstock set-over method

$$S = \frac{L\,(D-d)}{2l}$$

S = Set-over distance in mm
D = Larger diameter of taper in mm
d = Smaller diameter of taper in mm
L = Length of the job in mm
l = Length of the taper in mm

4.3. Machining Time Calculation

4.3.1. Cutting Speed

$$V = \frac{\pi D N}{1000}$$

V = Cutting speed (surface) in m/min
D = Diameter of the workpiece in mm
N = Rotational speed of the workpiece in rpm

4.3.2. Time for a single pass

$$t = \frac{L + L_O}{fN}$$

t = Time required for single pass in min
L = Length of the job in mm
L_O = Over travel of the tool beyond the length of the job in mm
f = Feed rate in mm/rev

4.3.3. Number of roughing passes

$$P_r = \frac{A - A_f}{d_r}$$

P_r = Number of passes for roughing operation
A = Total machining allowance in mm
A_f = Finish machining allowance in mm
d_r = Depth of cut for roughing in mm

4.3.4. Number of finishing passes

$$P_f = \frac{A_f}{d_f}$$

P_f = Number of passes for finishing operation
A_f = Finish machining allowance in mm
d_f = Depth of cut for finishing in mm

Table 4.1 Suggested cutting process parameters for turning

Work Material	Hardness (BHN)	High-Speed Steel Tool		Carbide Tool	
		Speed m/min	Feed mm/rev	Speed m/min	Feed mm/rev
Grey cast iron	150 – 180	30	0.25	140	0.30
Grey cast iron	220 – 260	20	0.25	90	0.30
Malleable iron	160 – 220	33	0.25	50	0.25
Malleable iron	240 – 270	-	-	45	0.30
Cast steel	140 – 180	40	0.25	150	0.30
Cast steel	190 – 240	26	0.25	125	0.30
C20 steel	110 – 160	40	0.30	150	0.38
C40 steel	120 – 185	30	0.30	145	0.38
C80 steel	170 – 200	26	0.30	130	0.30
Alloy steel	150 – 240	30	0.25	110	0.38
Alloy steel	240 – 310	20	0.25	100	0.30
Alloy steel	315 – 370	15	0.25	85	0.25
Alloy steel	380 – 440	10	0.20	75	0.25
Alloy steel	450 – 500	8	0.20	55	0.25
Tool steel	150 – 200	18	0.25	70	0.25
Hot work die steel	160 – 220	25	0.25	120	0.25
Hot work die steel	340 – 375	15	0.25	75	0.25
Hot work die steel	515 – 560	5	0.20	23	0.20
Stainless steel	160 – 220	30	0.20	120	0.25
Stainless steel	300 – 350	14	0.20	70	0.25
Stainless steel	375 – 440	10	0.20	30	0.25
Aluminium alloys	70 – 105	210	0.30	400	0.38
Copper alloys	120 – 160	200	0.25	300	0.25
Copper alloys	165 – 180	85	0.25	230	0.25

1. D.C. Circuit Analysis

Symbols and Notations

Symbol	Name	Unit
ρ	Resistivity	Ωm
l	Length of the material	m
R_0	Resistance at 0°C	Ω
α_0	Temp coefficient of resistivity at 0°C	$°C^{-1}$
A	Cross-sectional area of the material	m^2
L	Self-inductance	H
A	Cross-sectional area	m^2
N	Number of turns in the coil	
φ	Flux produced	Wb
l	Length of the magnetic circuits	m
i	Current following through inductor	A
μ_0	Permeability of free space = $4\pi \times 10^{-7}$	H/m
C	Capacitance	F
v_c	Capacitor voltage	V
q	Charge	C
d	Distance between the parallel plates	m
A	Cross-sectional area of the parallel plates	m^2
ε_0	Permittivity of free space= 8.85×10^{-12}	F/m
ε_r	Relative permittivity of the dielectric medium	
I_0	Initial current	A
τ	Time constant	
V_0	Initial voltage	V

1.1. Passive Circuit Elements

1.1.1. Resistors

Resistance of a material	$R = \rho \frac{l}{A}$
Resistance varies with temperature	$R_t = R_0(1 + \alpha_0 t)$
Resistors in series	$R_{eq} = R_1 + R_2 + \cdots \ldots + R_n$
Resistors in parallel	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \ldots . + \frac{1}{R_n}$
Power dissipated in a resistor	I^2R

1.1.2. Inductors

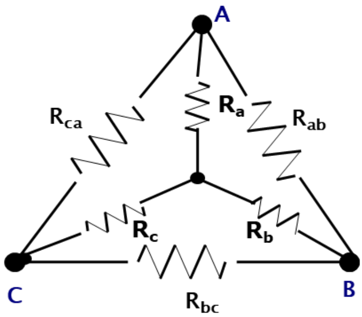
Inductance of a coil with non-magnetic core	$L = \frac{\mu_0 A N^2}{l}$
Inductors in series	$L_{eq} = L_1 + L_2 + \cdots \ldots + L_n$
Inductors in parallel	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \ldots . + \frac{1}{L_n}$
Emf induced in an inductor	$e_L = L\left(\frac{di}{dt}\right) = N\left(\frac{d\varphi}{dt}\right)$
Energy stored in an inductor	$\frac{1}{2} LI^2$

1.1.3. Capacitors

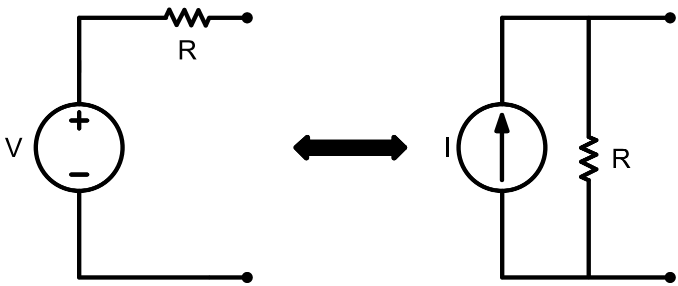
Capacitance of parallel plate capacitor	$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$
Capacitors in series	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \ldots + \frac{1}{C_n}$
Capacitors in parallel	$C_{eq} = C_1 + C_2 + \ldots .. + C_n$
Current in a capacitor	$i_c = \frac{dq}{dt} = C\left(\frac{dv_c}{dt}\right)$
Energy stored in a capacitor	$\frac{1}{2} CV^2$

1.2. DELTA-STAR / STAR-DELTA TRANSFORMATION

Delta to Star	Star to Delta
$R_a = \dfrac{R_{ab}R_{ca}}{\sum R_{ab}}$	$R_{ab} = \dfrac{\sum R_a R_b}{R_c}$
$R_b = \dfrac{R_{bc}R_{ab}}{\sum R_{ab}}$	$R_{bc} = \dfrac{\sum R_a R_b}{R_a}$
$R_c = \dfrac{R_{ca}R_{bc}}{\sum R_{ab}}$	$R_{ca} = \dfrac{\sum R_a R_b}{R_c}$



1.3. SOURCE TRANSFORMTION



$\boldsymbol{V = IR}$ $\qquad\qquad$ $\boldsymbol{I = V/R}$

1.4. NETWORK EQUATIONS

KVL equation (matrix form)

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} R_{11} & \cdots & R_{1N} \\ \vdots & \ddots & \vdots \\ R_{N1} & \cdots & R_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

$$[\boldsymbol{V}] = [\boldsymbol{R}][\boldsymbol{I}]$$

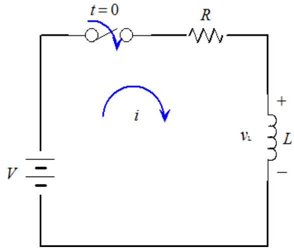
KCL equation (matrix form)

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} G_{11} & \cdots & G_{1N} \\ \vdots & \ddots & \vdots \\ G_{N1} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

$$[\boldsymbol{I}] = [\boldsymbol{G}][\boldsymbol{V}]$$

1.5. DC TRANSIENTS

1.5.1. RL Transient Circuit



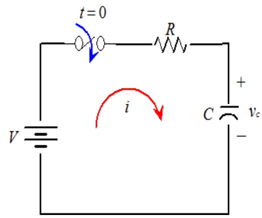
Circuit equation $V = R\,i + L\dfrac{di}{dt}$

Current growth $i = \dfrac{V}{R}\left(1 - e^{-(t/\tau)}\right)$

Current decay $i = I_0 e^{-(t/\tau)}$

Time constant $\tau = L/R$

1.5.2. RC Transient Circuit



Circuit equation $C\left(\dfrac{dv_c}{dt}\right) + \dfrac{v_c - V}{R} = 0$

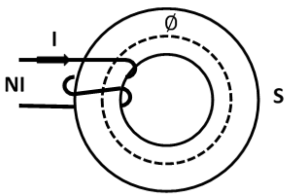
Capacitor voltage during charging $v_c = V\left(1 - e^{-(t/\tau)}\right)$

Capacitor voltage during discharging $v_c = V_0 e^{-(t/\tau)}$

Time constant $\tau = RC$

2. Magnetic Circuits

Symbol	Name	Unit
φ	Magnetic flux	Wb
H	Magnetic field strength	Am
S	Reluctance	A/Wb
μ_r	Relative permeability	
N	No. of turns of the coil	
I	Current flowing through the coil	
l	Length of the magnetic path	



Magneto-motive force $F = N \times I = \varphi \times S$

Magnetic field strength $H = \dfrac{N \times I}{l}$

Reluctance of the magnetic path $S = \dfrac{l}{\mu_0 \mu_r A}$

Permeance of the magnetic path $\mathcal{P} = \dfrac{1}{S}$

3. Electromagnetism

Faraday's law $$e = N\frac{d\varphi}{dt}$$

Induced EMF $$e = L\frac{di}{dt}$$

Self-inductance $$L = N\frac{d\varphi}{di}$$

Mutual-inductance $$M = k\sqrt{L_1 L_2}$$

4. Coupled Circuits

Equivalent inductance (series addition) $$L_{eq} = L_1 + L_2 + 2M$$

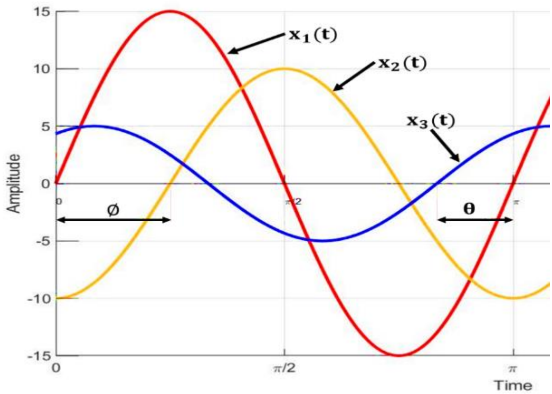
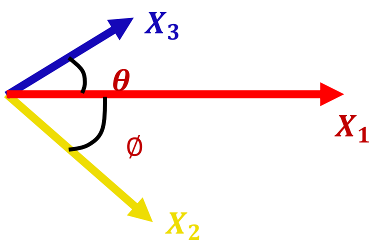
Equivalent inductance (series opposition) $$L_{eq} = L_1 + L_2 - 2M$$

5. A.C. Circuits

Symbol	Name	Unit
T	Time period of the signal $\boldsymbol{f}(\boldsymbol{t})$	s
F_m	Maximum amplitude of the sinusoidal signal $\boldsymbol{f}(\boldsymbol{t})$	
G	Conductance	S
B	Susceptance	S
P	Active power	W
Q	Reactive power	VAR
S	Apparent power	VA
$\cos\emptyset$	Power factor	
ω_1	Lower Half-power Frequency	rad/s
ω_2	Upper Half-power Frequency	rad/s
ω_0	Resonant Frequency	rad/s
f	Supply frequency	Hz
φ_m	Magnetic flux	Wb
N_1	Number of turns on primary	
N_2	Number of turns on secondary	

5.1. Single Phase

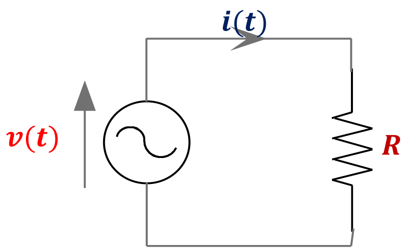
5.1.1. Representations of Sinusoidal Signal

Graphical Representation	
Mathematical Representation	$x_1(\mathrm{t}) = \mathrm{X}_{1\mathrm{m}}\sin(\omega \mathrm{t})$ $x_2(\mathrm{t}) = \mathrm{X}_{2\mathrm{m}}\sin(\omega \mathrm{t} - \emptyset)$ $x_3(\mathrm{t}) = \mathrm{X}_{3\mathrm{m}}\sin(\omega \mathrm{t} + \theta)$
Phasor Representation	 Where **X_1, X_2** and **X_3** are the RMS values

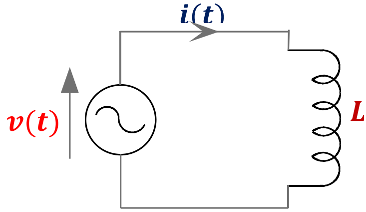
5.1.2. Average and RMS Values of a Periodic Signal

For a periodic signal $\boldsymbol{f(t)}$	$\mathrm{F_{avg}} = \frac{2}{\mathrm{T}}\int_0^{\mathrm{T}/2} \mathrm{f(t)dt}$ $\mathrm{F_{rms}} = \sqrt{\frac{1}{T}\int_0^{\mathrm{T}} \mathrm{f^2(t)dt}}$
For a sinusoidal signal $\boldsymbol{f(t)}$	$\mathrm{F_{avg}} = \frac{2F_m}{\pi}$ $\mathrm{F_{rms}} = \frac{F_m}{\sqrt{2}}$
Form factor for a sinusoidal signal	$\mathrm{FF} = \frac{\text{RMS Value}}{\text{Average Value}} = 1.11$
Peak factor for a sinusoidal signal	$\mathrm{PF} = \frac{\text{Maximum Value}}{\text{RMS Value}} = \sqrt{2}$

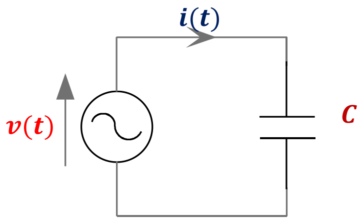
5.1.3. Purely Resistive Circuit

Circuit Diagram	
Mathematical Representation	$v(t) = V_m \sin(\omega t)$ $i(t) = I_m \sin(\omega t)$
Circuit Impedance	$\mathrm{R} = \frac{\overline{\mathrm{V}}}{\overline{\mathrm{I}}} = \frac{\mathrm{V}\angle 0°}{\mathrm{I}\angle 0°}$
Average Power	$\mathrm{P_{avg}} = \frac{\mathrm{V_m I_m}}{2} = \mathrm{VI} = \frac{\mathrm{V^2}}{\mathrm{R}} = \mathrm{I^2R}$
Phasor Representation	

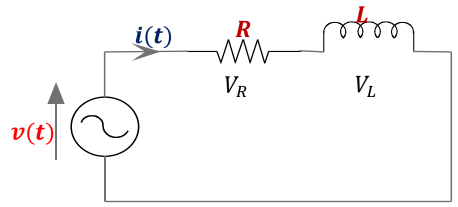
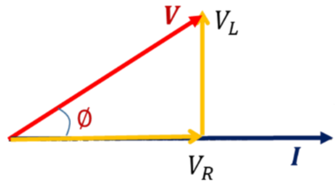
5.1.4. Purely Inductive Circuit

Circuit Diagram	
Mathematical Representation	$v(t) = V_m \sin(\omega t)$ $\quad i(t) = I_m \sin(\omega t - 90°)$
Circuit Impedance	$\frac{\overline{V}}{\overline{I}} = \frac{V\angle 0°}{I\angle -90°} = jX_L$
Average Power	$P_{avg} = 0$
Phasor Representation	

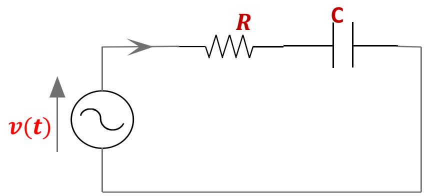
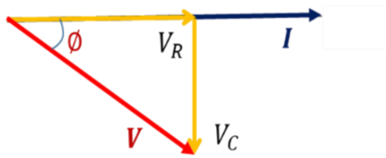
5.1.5. Purely Capacitive Circuit

Circuit Diagram	
Mathematical Representation	$v(t) = V_m \sin(\omega t)$ $\quad i(t) = I_m \sin(\omega t + 90°)$
Circuit Impedance	$\frac{\overline{V}}{\overline{I}} = \frac{V\angle 0°}{I\angle 90°} = -jX_C$
Average Power	$P_{avg} = 0$
Phasor Representation	

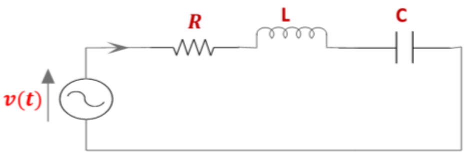
5.1.6. RL Circuit

Circuit Diagram	
Mathematical Representation	$(t) = V_m \sin(\omega t) \qquad i(t) = I_m \sin(\omega t - \emptyset)$
Circuit Impedance	$\frac{\bar{V}}{\bar{I}} = \frac{\bar{\mathrm{I}}(\mathrm{R} + j\mathrm{X_L})}{\bar{\mathrm{I}}} = \mathrm{R} + j\mathrm{X_L} = \mathrm{Z}\angle\emptyset$
Average Power	$\mathrm{P_{avg}} = \mathrm{VI}\cos\emptyset$
Phasor Representation	

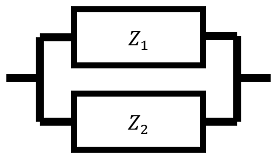
5.1.7. RC Circuit

Circuit Diagram	
Mathematical Representation	$v(t) = V_m \sin(\omega t) \qquad i(t) = I_m \sin(\omega t + \emptyset)$
Circuit Impedance	$\frac{\bar{V}}{\bar{I}} = \frac{\bar{\mathrm{I}}(\mathrm{R} - j\mathrm{X_C})}{\bar{\mathrm{I}}} = \mathrm{R} - j\mathrm{X_C} = \mathrm{Z}\angle - \emptyset$
Average Power	$\mathrm{P_{avg}} = \mathrm{VI}\cos\emptyset$
Phasor Representation	

5.1.8. RLC Circuit

Circuit Diagram	
Circuit Impedance	$\mathrm{Z = R + j(X_L \sim X_c)}$ $if\ X_L = X_C \Longrightarrow Resistive\ circuit$ $if\ X_L > X_C \Longrightarrow RL\ series\ circuit$ $if\ X_L < X_C \Longrightarrow RC\ series\ circuit$

5.1.9. Impedance and Admittance

Impedance	$\mathrm{Z = R \pm jX}$	
Total Impedance	$\mathrm{Z}_{eq} = \mathrm{Z_1 + Z_2}$	
Admittance	$\mathrm{Y = 1/Z = G \mp jB}$	
Total Admittance	$\mathrm{Y}_{eq} = \frac{1}{\mathrm{Z_1}} + \frac{1}{\mathrm{Z_2}} = \mathrm{Y_1 + Y_2}$	

5.1.10. Network Equations

KVL Equation (Matrix Form)

$$\begin{bmatrix} \mathbf{V_1} \\ \vdots \\ \mathbf{V_N} \end{bmatrix} = \begin{bmatrix} \mathbf{Z_{11}} & \cdots & \mathbf{Z_{1N}} \\ \vdots & \ddots & \vdots \\ \mathbf{Z_{N1}} & \cdots & \mathbf{Z_{NN}} \end{bmatrix} \begin{bmatrix} \mathbf{I_1} \\ \vdots \\ \mathbf{I_N} \end{bmatrix}$$

$$[\boldsymbol{V}] = [\boldsymbol{Z}][\boldsymbol{I}]$$

KCL Equation (Matrix Form)

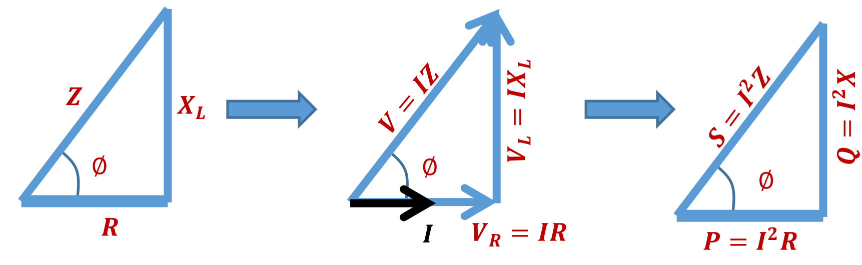
$$\begin{bmatrix} \mathbf{I_1} \\ \vdots \\ \mathbf{I_N} \end{bmatrix} = \begin{bmatrix} \mathbf{Y_{11}} & \cdots & \mathbf{Y_{1N}} \\ \vdots & \ddots & \vdots \\ \mathbf{Y_{N1}} & \cdots & \mathbf{Y_{NN}} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1 \\ \vdots \\ \boldsymbol{V}_N \end{bmatrix}$$

$$[\boldsymbol{I}] = [\boldsymbol{Y}][\boldsymbol{V}]$$

5.1.11. Power in AC Circuits

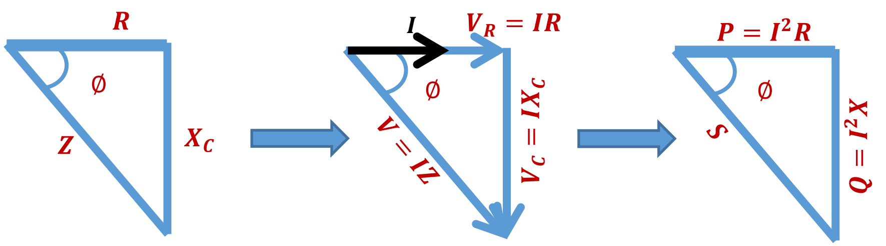
Instantaneous Power in an AC System	$p(t) = v(t) \times i(t) = V_m I_m \sin(\omega t) \sin(\omega t \pm \emptyset)$
Active Power	$P = VI \cos\emptyset = I^2 Z \cos\emptyset = I^2 R$
Reactive Power	$Q = VI \sin\emptyset = I^2 Z \sin\emptyset = I^2 X$
Complex Power	$S = P \pm jQ$

RL Load



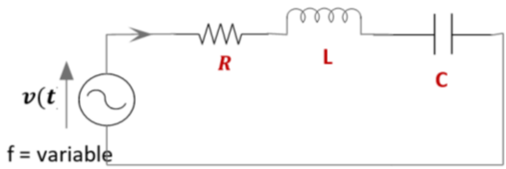
Impedance Diagram Phasor Diagram Power Diagram

RC Load



Impedance Diagram Phasor Diagram Power Diagram

5.1.12. Series Resonance

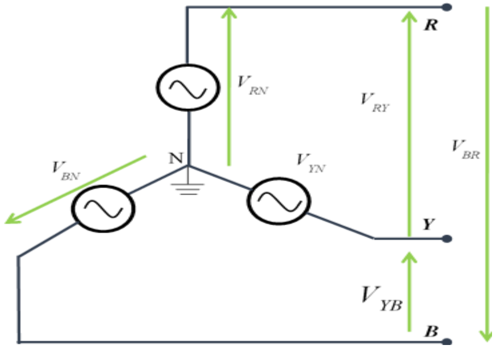
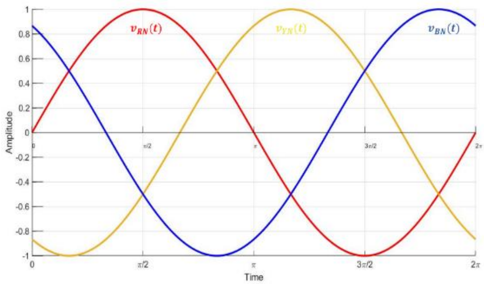
Circuit Diagram	
At Resonance	$X_L = X_C$ $\Rightarrow Z = R$
Resonant Frequency	$\omega_0 = \frac{1}{\sqrt{LC}}$
Bandwidth (BW)	$\mathrm{BW} = \omega_2 - \omega_1 = R/L$
Quality factor for Series RLC Circuit	$Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}} = \frac{1}{\omega_0 CR} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

5.1.13. Parallel Resonance

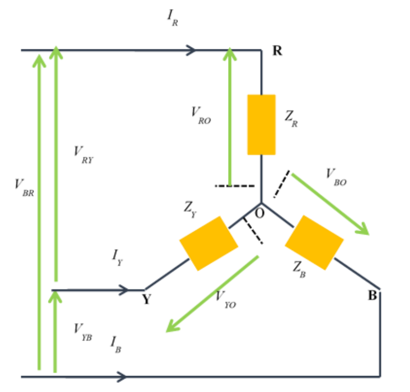
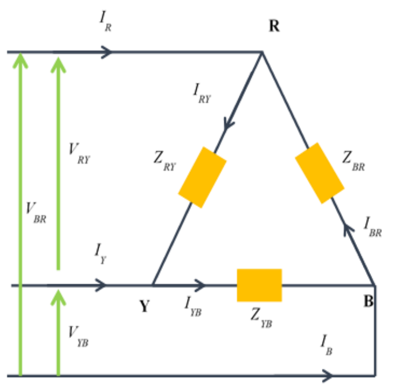
Equivalent Admittance	$Y_{eq} = Y_1 + Y_2 + \cdots + Y_n = G \pm jB$
At resonance	$\mathrm{Img}(Y_{eq}) = B = 0$

5.2. Three Phase

5.2.1. Three Phase Source Voltage Representation

Circuit Diagram	
Instantaneous Phase Voltages	$v_{RN} = V_m \, Sin(\omega t)$ $v_{YN} = V_m \, Sin(\omega t - 120°)$ $\quad v_{BN} = V_m \, Sin(\omega t - 240°)$
Polar Representation Of Phase Voltages	$\overline{V}_{RN} = V_{ph}\angle 0^{\circ}$ $\overline{V}_{YN} = V_{ph}\angle{-120}^{\circ}$ $\quad$ where, $V_{ph} = \frac{V_m}{\sqrt{2}}$ $\overline{V}_{BN} = V_{ph}\angle - 240^{\circ}$
Graphical Representation	
Polar Representation of Line Voltages	$\overline{V}_{RY} = \overline{V}_{RN} - \overline{V}_{YN} = \sqrt{3} \times V_{ph}\angle 30^{\circ}$ $\overline{V}_{YB} = \overline{V}_{YN} - \overline{V}_{BN} = \sqrt{3} \times V_{ph}\angle{-90}^{\circ}$ $\overline{V}_{BR} = \overline{V}_{BN} - \overline{V}_{RN} = \sqrt{3} \times V_{ph}\angle{-210}^{\circ}$

5.2.2. Three Phase Loads

<table>
<tr><th>Star Connected Loads</th><th>Delta Connected Loads</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td><u>For Balanced Loads</u>

$V_L = \sqrt{3} \times V_{Ph}$
$I_L = I_{Ph}$
$P = \sqrt{3} \times V_L \times I_L \times \cos \emptyset$</td><td><u>For Balanced Loads</u>

$V_L = V_{Ph}$
$I_L = \sqrt{3} \times I_{Ph}$
$P = \sqrt{3} \times V_L \times I_L \times \cos \emptyset$</td></tr>
<tr><td><u>For Unbalanced Loads</u>

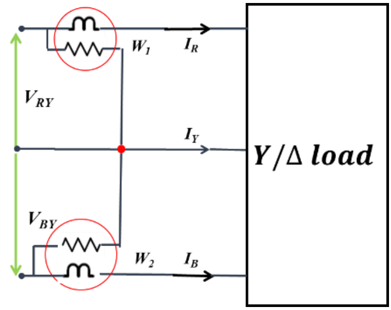
$I_L = I_{Ph}$
$V_{RY} = V_{RO} - V_{YO}$
$V_{YB} = V_{YO} - V_{BO}$
$V_{BR} = V_{BO} - V_{RO}$

$P = V_{RO}I_R Cos\angle(V_{RO}\&I_R) +$
$V_{YO}I_Y Cos\angle(V_{YO}\&I_Y) + V_{BO}I_B Cos\angle(V_{BO}\&I_B)$</td><td><u>For Unbalanced Loads</u>

$V_L = V_{Ph}$
$I_R = I_{RY} - I_{BR}$
$I_Y = I_{YB} - I_{RY}$
$I_B = I_{BR} - I_{YB}$

$P = V_{RY}I_{RY}Cos\angle(V_{RY}\&I_{RY}) +$
$V_{YB}I_{YB}Cos\angle(V_{YB}\&I_{YB}) +$
$V_{BR}I_{BR}Cos\angle(V_{BR}\&_{BR})$</td></tr>
</table>

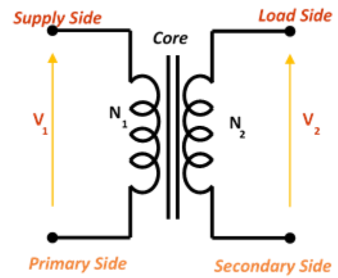
5.2.3. Power Measurement For 3-Phase Balanced Loads Using 2-Wattmeter Method

Circuit Diagram	
Wattmeter Readings	$W_1 = V_{RY}\, I_R\, Cos\angle(V_{RY} \& I_R)$ $W_1 = V_L I_L Cos(30° + \emptyset)$ $W_2 = V_{BY} I_B Cos\angle(V_{BY} \& I_B)$ $W_2 = V_L I_L Cos(30° - \emptyset)$
Total Power Consumed	$P = W_1 + W_2 = \sqrt{3} \times V_L\, I_L\, Cos\emptyset$
Power Factor of Balanced Star-Connected Loads	$Cos\emptyset = Cos\left\{tan^{-1}\left[\sqrt{3} \times \frac{W_2 - W_1}{W_2 + W_1}\right]\right\}$

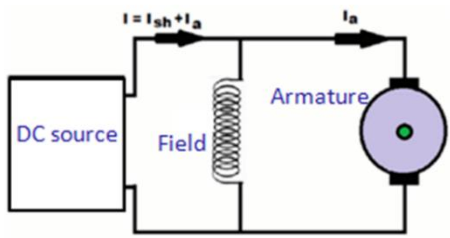
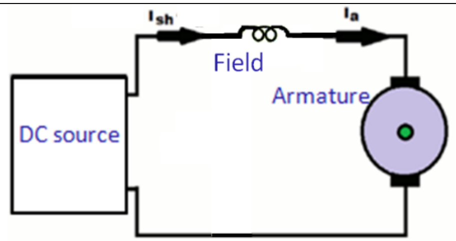
6. Electrical Power Systems

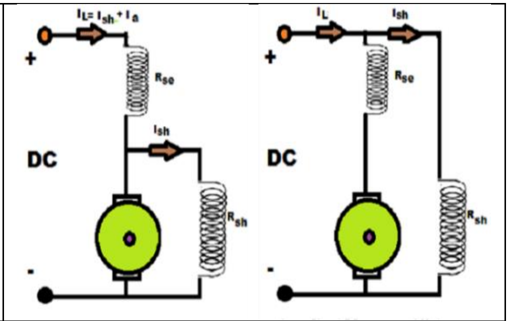
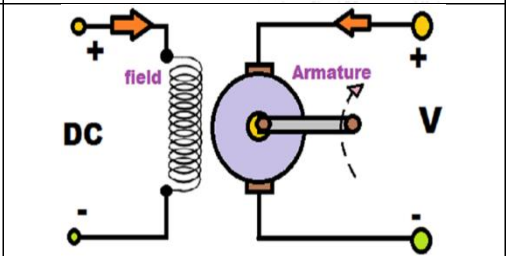
Symbol	Name	Unit
P	Number of poles	
φ	Flux per pole	Wb
I_a	Armature Current	A
N	Armature speed	RPM
R_{se}	Series Resistance of the field winding	Ω
R_a	Armature resistance	Ω
a	Number of parallel paths in armature $a = \begin{cases} 2, \text{for wave winding} \\ P, \text{for lap winding} \end{cases}$	
Z	Total number of armature conductors	
N_s	Synchronous speed	RPM
f	Supply frequency	Hz

6.1. Transformers

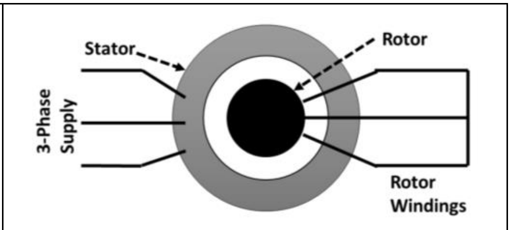
Circuit diagram	
Primary induced EMF	$E_1 = 4.44\, N_1 f \varphi_m$
Secondary induced EMF	$E_2 = 4.44\, N_2 f \varphi_m$
Turns Ratio	$\frac{V_1}{V_2} \cong \frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$

6.2. D C Motors

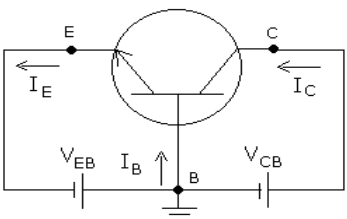
Circuit Diagram of DC Shunt Motor	
Voltage Equation for DC Shunt Motor	$V = E_b + I_a R_a$
Circuit Diagram of a DC Series Motor	
Voltage Equation for DC Series Motor	$V = E_b + I_a(R_a + R_{se})$

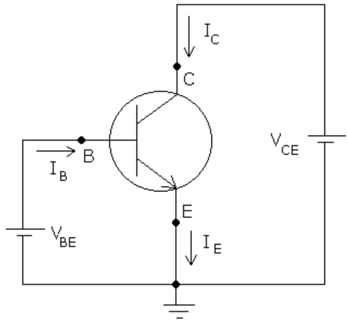
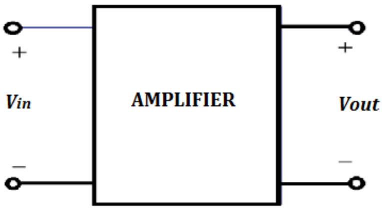
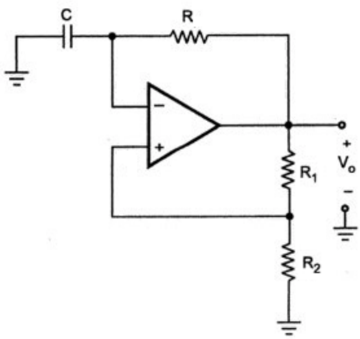
Circuit Diagram of a DC Compound Motor	
Circuit Diagram of a DC Separately Excited Motor	
Back EMF Induced in Armature	$\mathrm{E_b} = \frac{\mathrm{PZ}}{60a}\varphi\mathrm{N}$
Torque Developed in DC Motor	$\mathrm{T} = \frac{\mathrm{PZ}}{2\pi a}\varphi\, \mathrm{I}_a$

6.3. Three Phase Induction Motors

Block Diagram of Three Phase Induction Motor	
Speed of rotating magnetic field	$\mathrm{N_s} = \frac{120\mathrm{f}}{\mathrm{P}}$
Percentage of operating slip	$s = \frac{\mathrm{N_s} - \mathrm{N}}{\mathrm{N_s}} \times 100$

Basic Electronics

Diode Current $$I_D = I_s\left\{e^{\frac{kV_D}{T_K}} - 1\right\}$$	I_s : Reverse saturation current V_D : Voltage applied across the diode K = 11,600/η; n=1 for Ge and 2 for Si T_K : Operating temperature in kelvin
Ripple Factor of Half-wave rectifier with capacitor filter $$\frac{1}{2\sqrt{3}fCR_L}$$	f : frequency of input AC signal C : Capacitance R_L : Load resistance
Ripple Factor of full-wave rectifier with capacitor filter $$\frac{1}{4\sqrt{3}fCR_L}$$	
DC output voltage of Half-wave rectifier $$\frac{2fCR_L}{1+2fCR_L}$$	
DC output voltage of full-wave rectifier $$\frac{4fCR_L}{1+4fCR_L}$$	
Emitter Current of Bipolar junction transistor $I_E=I_C+I_B$	I_C : Collector current I_B : Base current
Collector Current of Bipolar junction transistor in CB Configuration  $I_C=\alpha I_E+I_{CBO}$	α : Common-base current gain I_{CBO} : Common base leakage current

Collector Current of Bipolar junction transistor in CE Configuration  $I_C = \beta I_B + I_{CEO}$	β : Common-emitter current gain I_{CEO} : Common emitter leakage current
Voltage Gain of Amplifier in decibels  $(A_V)_{dB} = 20log_{10}\left(\frac{V_{Out}}{V_{In}}\right)$	V_{Out} : Output voltage V_{In} : Input voltage
Common Mode Rejection Ratio (CMRR) $CMRR = 20log_{10}\left(\frac{A_d}{A_{cm}}\right)$	A_d : Differential voltage gain A_{cm} : Common mode voltage gain
Time period of output of Operational amplifier based square wave generator  $T = 2RCln\left[\frac{1+\beta}{1-\beta}\right]$	β : Feedback factor = $\frac{R_2}{R_1+R_2}$ R : Resistance. C : Capacitance

Output voltage of Differential Amplifier $V_O = A_d V_d + A_{cm} V_{cm}$	A_d : Differential voltage gain A_{cm} : Common mode voltage gain. V_d : Differential voltage. V_{cm} : Common mode voltage.
Commutative Law Property 1: $(x + y) = (y + x)$ Property 2: $x. y = y. x$	
Associative Law Property 1: $(x + y) + z = x + (y + z)$ Property 2: $(x. y)z = x(y. z)$	*x,y,z*… are Boolean Variables
Distributive Law Property 1: $x(y + z) = x. y + x. z$ Property 2: $x + x. y = x + y$	
DeMorgan's Theorem Theorem 1: $\overline{x + y} = \bar{x}. \bar{y}$ Theorem 2: $\overline{x. y} = \bar{x} + \bar{y}$	
Amplitude Modulated signal $S(t) = A_C[1 + k_a m(t)]Cos(2\pi f_c t)$	
Modulation Index of AM Signal $\mu = k_a m(t)$	A_c : Amplitude of carrier signal. k_a : Amplitude sensitivity. $m(t)$: Modulating signal. f_c : frequency of carrier signal. k_f : Frequency sensitivity. Δf : Frequency deviation. f_m : frequency of modulating signal
Frequency Modulated signal $S(t) = A_c Cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right]$	
Transmission bandwidth of FM Signal $B_T = 2(\Delta f + f_m)$	

Basic Structure of a C program:

```
#include<stdio.h> // pre-processor directive
int main( )
 {
 statement(s); // program body
return 0; // return statement
 }
```

Preprocessor Directives:

Standard preprocessor directives	
# (null directive)	#if
#define	#ifdef
#elif	#ifndef
#else	#include
#endif	#line
#error	#undef

Keywords in C:

asm	**enum**	**signed**	**const**	**if**	**typedef**
auto	**extern**	**sizeof**	**continue**	**int**	**union**
break	**float**	**static**	**default**	**long**	**unsigned**
case	**for**	**struct**	**do**	**register**	**void**
char	**goto**	**switch**	**double**	**return**	**volatile**
else	**short**	**while**			

Variable Specifiers (Format Specifiers) in C:

Data Type		**Format specifier**
Integer	short signed	%d or %I
	short unsigned	%u
	long singed	%ld
	long unsigned	%lu
	unsigned hexadecimal	%x
	unsigned octal	%o
Real	float	%f
	double	%lf
Character	Singed character	%c
	Unsigned character	%c
String		%s

Print in C:

Syntax:

printf("Text");
printf("Text variableSpecifier Text", Variable); *//with a variable*

User Input in C:

Syntax:

scanf("variableSpecifier", variable);

C comments:

```
// Single line comment
/* Block
   comment
*/
```

Including Header Files in C:

Syntax:

//for standard system header files, like stdio.h will search standard system directories
#include <file>

//for header files built by the user, within the source code's directory
#include "file"

Local Variables in C:

Syntax:

dataType variableName;

Constants in C:

Syntax:

#define VARIABLENAME value

const dataType VARIABLENAME = value;

Arrays in C:

Syntax:

dataType arrayName [arraySize]; *//Declaring array*

dataType arrayName[arraySize] = { arrayElement1, arrayElement2, … ,arrayElementN };
//Initializing an array all in one line, can omit the arraySize with this method

arrayName[elementPosition] = arrayElement; *//To set a value at a specific position in the array*

variableType Variable = arrayName[elementPosition]; *//To access a specific element and store*

Pointers in C:

Syntax:

dataType *pointerName = &Variable; *//Pointer*
dataType &referenceName = Variable; *//Reference*

Structures in C:

Syntax:

struct structureName
{
//member declarations
};
//to access a structure member
struct structureName structureInstanceName; *//creating a structure instance*
givenName.member; *//standard members*
givenName → member; *//pointer members*

Type Definition in C:

Syntax:

typedef oldTypeName newTypeName;

Enumeration Constants:

Syntax:

enum *tag* {*enumeration-list*};

Conditional Operator?:

Syntax:

expr1 ? expr2 : expr3

Simple Assignment Operator:

Syntax:

expression1 = expression2

Compound Assignment Operators:

Syntax:

expression1 op= expression2
where op can be one of binary operators +, -, *, /, %, &, |, ^, <<, or >>

Compound assignment has the same effect as
expression1 = expression1 ***op*** *expression2*

Size of Operator:

Syntax:

int ***sizeof*** *(Datatype or Variable) ;*

Comma Operator:

Syntax:

***expression_1*,*expression_2, ... expression_n*;**

Escape Sequences:

Sequence	Value	Char	What it does
\a	0 x 07	BEL	Audible bell
\b	0 x 08	BS	Backspace
\f	0 x 0C	FF	Form feed
\n	0 x 0A	LF	Newline (Linefeed)
\r	0 x 0D	CR	Carriage Return
\t	0 x 09	HT	Tab (horizontal)
\v	0 x 0B	VT	Vertical Tab
\ \	0 x 5C	\	Backslash
\ ’	0 x 27	‘	Single quote (Apostrophe)
\ “	0 x 22	“	Double quote
\ ?	0 x 3F	?	Question mark
\ O		any	O = string of up to 3 octal digits
\ xH		any	H – string of hex digits
\ XH		any	H – string of hex digits
\0		NULL	Null character. Marks end f the string

Operators Precedence and Associativity:

Precedence	Operands	Operators	Associativity
15	2	( ) [ ] . < >	left-to-right
14	1	! ~ ++ -- + - * & (*type*) sizeof	right-to-left
13	2	* / %	left-to-right
12	2	+ -	left-to-right
11	2	<< >>	left-to-right
10	2	< <= > >=	left-to-right
9	2	= = !=	left-to-right
8	2	&	left-to-right
7	2	^	left-to-right
6	2	|	left-to-right
5	2	&&	left-to-right
4	2	||	left-to-right
3	3	? :	left-to-right
2	2	= *= /= %= += -= &= ^= |= <<= >>=	right-to-left
1	2	,	left-to-right

Fundamental Datatypes:

Type	Size*	Range
(unsigned) char	8-bit	0 … 255
signed char	8-bit	- 128 … 127
(signed) short (int)	8-bit	- 128 … 127
unsigned short (int)	8-bit	0 … 255
(signed) int	16-bit	-32768 … 32767
unsigned (int)	16-bit	0 … 65535
(signed) long (int)	32-bit	-2147483648 … 2147483647
unsigned long (int)	32-bit	0 … 4294967295
float	32-bit	±1.17549435082E-38 … ±6.80564774407E38
double	32-bit	±1.17549435082E-38 … ±6.80564774407E38
long double	32-bit	±1.17549435082E-38 … ±6.80564774407E38

** For 16 bit Architecture*

Functions:

Syntax:

```
returnType functionName(parameterTypes); //function prototype
//main code

returnType functionName (functionParameters) { //function implementation
//statements that execute when called
return value;
}
```

***if* Statement:**

Syntax:

```
if(booleanExpression) {
  //Executes when booleanExpression holds true
}
else if(booleanExpression2) {
   //Executes when booleanExpression2 holds true
}
else {
        //Executes when neither boolean Expression nor boolean Expression2 are true
       }
```

***Switch* Statement:**

Syntax:

```
switch(variable) {
    case valueOne:
                        //statements
                        break;
    case valueTwo:
                        //statements
                        break;
    default: //optional
                        //statements
}
```

While Statement:

Syntax:

```
while(booleanExpression) {
    //block of code to be executed
}
```

Do Statement:

Syntax:

```
do {
 //block of code to be executed
} while(booleanExpression);
```

***For* Statement:**

Syntax:

```
for(initialization; booleanExpression; update) {
   //statements
} //end for loop
```

General format of Algorithm:

Name of the algorithm [mandatory]
[gives a meaningful name to the algorithm based on the problem]

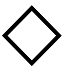
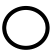
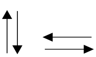
Start
[Beginning of the algorithm]

Step Number [mandatory]
[indicate each individual simple task]

Explanatory comment [optional]
[gives an explanation for each step, if needed]

Termination [mandatory]
[tells the end of algorithm]

Flowchart Symbols:

Symbol	Name	Function
	**Process**	Indicates any type of internal operation inside the Processor or Memory
	**Input/output**	Used for any Input / Output (I/O) operation. Indicates that the computer is to obtain data or output results
	**Decision**	Used to ask a question than can be answered in a binary format (Yes / No, True/ False)
	**Connector**	Allows the flowchart to be drawn without intersecting lines or without a reverse flow.
	**Predefined Process**	Used to invoke a subroutine or an Interrupt program
	**Terminal**	Indicates the starting or ending of the program, process, or interrupt program
	**Flow Lines**	Shows direction of flow

ASCII TABLE

Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char
0	0	0		32	20	40	[space]	64	40	100	@	96	60	140	`
1	1	1		33	21	41	!	65	41	101	A	97	61	141	a
2	2	2		34	22	42	“	66	42	102	B	98	62	142	b
3	3	3		35	23	43	#	67	43	103	C	99	63	143	c
4	4	4		36	24	44	$	68	44	104	D	100	64	144	d
5	5	5		37	25	45	%	69	45	105	E	101	65	145	e
6	6	6		38	26	46	&	70	46	106	F	102	66	146	f
7	7	7		39	27	47	‘	71	47	107	G	103	67	147	g
8	8	10		40	28	50	(	72	48	110	H	104	68	150	h
9	9	11		41	29	51	)	73	49	111	I	105	69	151	i
10	A	12		42	2A	52	*	74	4A	112	J	106	6A	152	j
11	B	13		43	2B	53	+	75	4B	113	K	107	6B	153	k
12	C	14		44	2C	54	,	76	4C	114	L	108	6C	154	l
13	D	15		45	2D	55	-	77	4D	115	M	109	6D	155	m
14	E	16		46	2E	56	.	78	4E	116	N	110	6E	156	n
15	F	17		47	2F	57	/	79	4F	117	O	111	6F	157	o
16	10	20		48	30	60	0	80	50	120	P	112	70	160	p
17	11	21		49	31	61	1	81	51	121	Q	113	71	161	q
18	12	22		50	32	62	2	82	52	122	R	114	72	162	r
19	13	23		51	33	63	3	83	53	123	S	115	73	163	s
20	14	24		52	34	64	4	84	54	124	T	116	74	164	t
21	15	25		53	35	65	5	85	55	125	U	117	75	165	u
22	16	26		54	36	66	6	86	56	126	V	118	76	166	v
23	17	27		55	37	67	7	87	57	127	W	119	77	167	w
24	18	30		56	38	70	8	88	58	130	X	120	78	170	x
25	19	31		57	39	71	9	89	59	131	Y	121	79	171	y
26	1A	32		58	3A	72	:	90	5A	132	Z	122	7A	172	z
27	1B	33		59	3B	73	;	91	5B	133	[	123	7B	173	{
28	1C	34		60	3C	74	<	92	5C	134	\	124	7C	174	|
29	1D	35		61	3D	75	=	93	5D	135	]	125	7D	175	}
30	1E	36		62	3E	76	>	94	5E	136	^	126	7E	176	~
31	1F	37		63	3F	77	?	95	5F	137	_	127	7F	177	