MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL

III SEMESTER B. TECH (CSE/ICT/CC) END SEM EXAMINATION

SUBJECT: ENGINEERING MATHEMATICS III (MAT-2105)

Date of Examination: **27-11-2018** Time of Examination: 9 to 12 Max. Marks : 50

Instructions to Candidates: *Answer ALL the questions. Missing data if any may be suitably assumed.*

Q.1A: Determine the number of integers between 1 and 250 (including both 1 and 250) that are divisible by any one of the integers 2, 3, 5 and 7.

Q.1B: Show with generating functions that every positive integer can be written as a unique sum of distinct power of 2.

Q.1C: (i) Find the number of words of length 11 on the alphabet $\{E, N, G, I, R\}$ such that E occurs three times, N occurs three times, G occurs two times, I occurs two times and R occurs one time.

(ii) Show that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$.

(3+3+4=10)

Q.2A: Show that the set G of all 2×2 non singular matrices forms a group under matrix multiplication. Is the group G an Abelian group? Justify your answer.

Q.2B: Is every subgroup of cyclic group cyclic? Justify your answer.

Q.2C: (i) Find the number of ways in which 25 distinct objects can be placed in 3 distinct boxes such that no box is empty.

(ii) Find the number of bit strings of length ten that have exactly three 1's.

(3+3+4=10)

Q.3A: Show that for any two elements a and b in a Boolean algebra,

(i) $\overline{(a \vee b)} = \overline{a} \wedge \overline{b}$

(ii) $\overline{(a \wedge b)} = \overline{a} \vee \overline{b}$

Q.3B: Let a and b be two elements in a lattice $(A, \leq)$. By the notation $a < b$, we mean $a \leq b$ and $a \neq b$. Show that $a \wedge b < a$ and $a \wedge b < b$ if and only if a and b are incomparable.

Q.3C: Let A be a finite Boolean algebra, $b \in A$ and $b \neq 0$. Let $a_1, a_2, \ldots, a_k$ be all the atoms of A such that $a_i \leq b$ for all i where $1 \leq i \leq k$. Then show that

$$b = a_1 \vee a_2 \vee \cdots \vee a_k$$

is the unique way to represent b as a join of atoms.

(3+3+4=10)

Q.4A: Let G be a connected graph with at least 3 vertices. Prove that G is bipartite if and only if all its cycles are even.

Q.4B: Let $R(m, n)$ denote the least positive integer such that every graph with $R(m, n)$ vertices either contains a complete graph on m vertices or a totally disconnected graph on n vertices. Find $R(3, 4)$. Show that

$$R(m, n) \leq R(m-1, n) + R(m, n-1).$$

Q.4C: Consider a graph G with vertex set given by $V(G) = \{s, t, x, y, z\}$. Let $W(u, v)$ denote the directed weight from vertex u to vertex v. Suppose G is such that

$$W(s,t) = 10; W(s,y) = 5; W(t,y) = 2; W(y,t) = 3; W(t,x) = 1;$$

$$W(y,z) = 2; W(y,x) = 9; W(x,z) = 4; W(z,x) = 6; W(z,s) = 7.$$

. Implement the Dijkstra's algorithm on G taking the starting vertex as s.

(3+3+4=10)

Q.5A: Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \rightarrow R, P \rightarrow M$, and $\neg M$.

Q.5B: Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

Q.5C: (i) Let G be a group and $a \in G$ with $a \neq e$. Define $N(a) = \{x \in G \,|\, ax = xa\}$. Show that $N(a)$ is a subgroup of G.

(ii) Let

$$H = \bigcap_{a \in G} N(a).$$

Show that H is a normal subgroup of G.

(3+3+4=10)