An edge with identical ends is called a loop and two edges with same end vertices are called parallel edges.

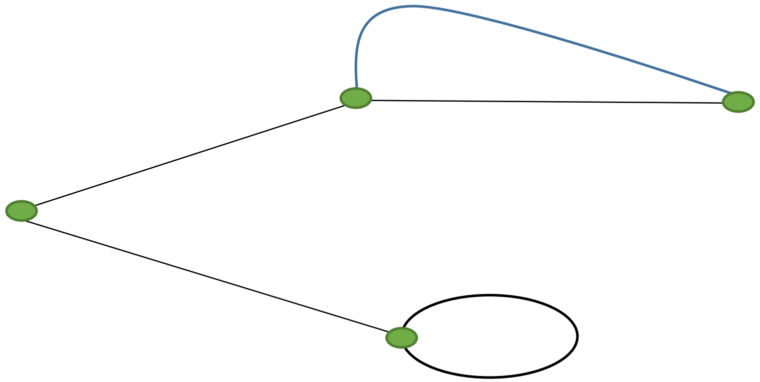
A graph is finite if both its vertex set and edge set are finite.

A graph is simple if it has no loops or parallel edges

In a **multigraph**, no loops are allowed but more than one line can join two points; these are called multiple lines.

If both loops and multiple lines are permitted, we have a **pseudograph**.

Ex: A graph *G* with loops and multiple edges



Distance between two vertices: The distance *d*(*u, v*) between two vertices *u* and *v* in G is the length of the shortest path joining them, if any; otherwise d(*u, v*) = ∞.

In a connected graph G,

$d(u, v) \geq 0$ with $d(u, v) = 0$ if and only if $u = v$.

$$d(u, v) = d(v, u)$$

$$d(u, v) + d(v, w) \geq d(u, w)$$

A shortest $u - v$ path is called a **geodesic.**

Eccentricities: The eccentricity $e(v)$ of a vertex v in a connected graph G is maximum of $d(u, v)$ for all u in G.

The radius $r(G)$ is the minimum eccentricity of the vertices of G.

The maximum eccentricity is the **diameter.** A vertex v is a central vertex if $e(v) = r(G)$, and the center of G is the set of all central vertices.

The ***girth*** of a graph G, denoted $g(G)$, is the length of a shortest cycle in G; the ***circumference*** $c(G)$ the length of any longest cycle.

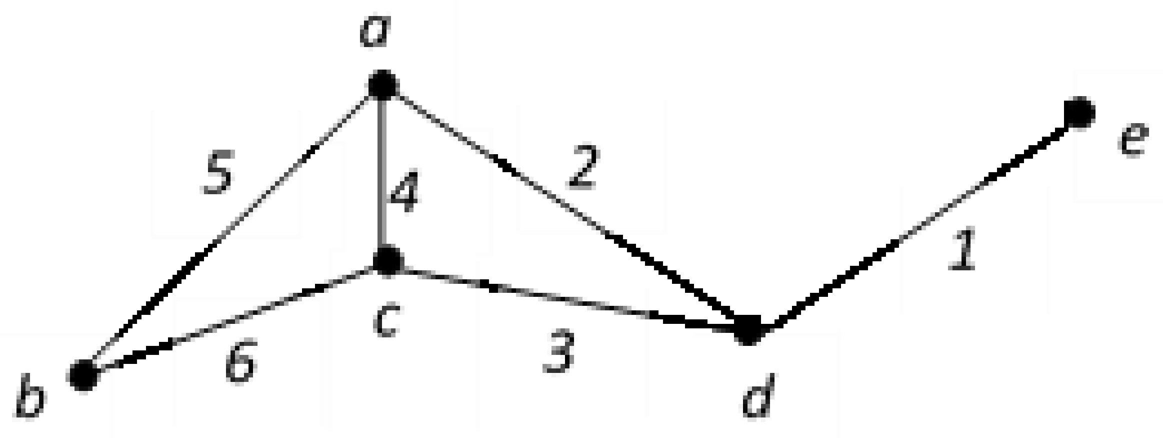


Figure 6. Graph G

$d(b,d) = 2,$

diameter of G = 3, radius of G =2

girth *g*(G) = 3, *circumference c*(G) = 4

Degree: The degree of a vertex v in a graph G, denoted $\deg(v)$, is the number of edges incident with v.

A vertex in a graph G is said to be **isolated** when its degree is ‘0’.

A vertex in a graph G is said to be an **end vertex** or **pendent vertex** if its degree is 1.

The minimum degree among the vertices of G is denoted by δG, the maximum degree among the vertices of G is denoted by ΔG.

Example: In a graph G shown in fig.6, $\delta G = 1$ and $\Delta G = 3$.

Regular graph: A graph in which all vertices are of equal degree is called a *regular graph*.

A regular graph of degree 3 is called **cubic** graph.

A cubic graph has always even number of vertices

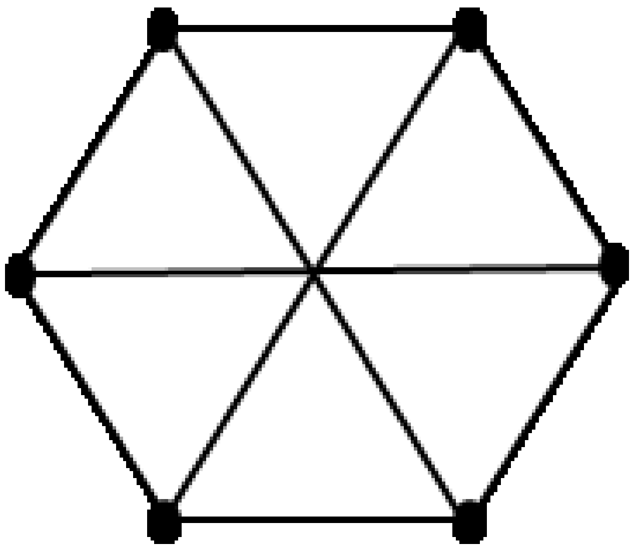


Figure 7. Regular graph

Hand shaking lemma: The sum of the degree of all vertices in a graph G is an even number, and this number is equal to twice the number of edges in the graph.

Proof: Let us consider a graph G with q edges and *n* vertices v_1, v_2, v_3, …, v_n. Since each edge contributes two degrees, the sum of the degrees of all vertices in G is twice the number of edges in G. i.e., $\sum_{i=1}^{n} \deg v_i$ = 2*q*.

Theorem : In any graph, the number of vertices of odd degree is even.

Proof: Let Se= Sum of all degree of all even degree vertices.

Let So= Sum of all degree of all odd degree vertices.

By Hand shaking lemma, $So + Se = 2q$.

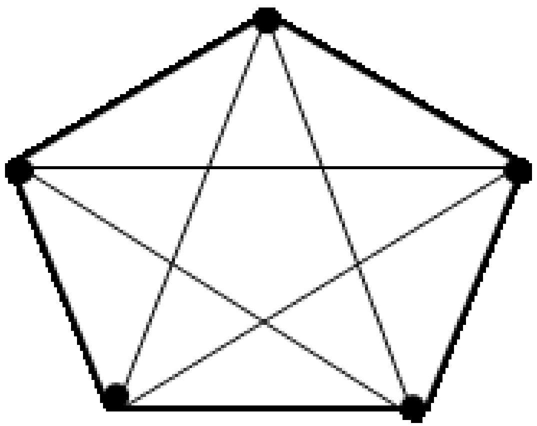
i.e, $So = 2q - Se = even$.

Each term in the sum So is odd.

Therefore, So can be even, only if even number of terms in So. Hence, the theorem.

Complete graph: A simple graph in which there exists an edge between every pair of vertices is called a ***complete graph***.

A complete graph with p vertices is denoted by K_p. The graph K_p has $\binom{p}{2} = \frac{p(p-1)}{2}$ edges and K_p is a regular graph of degree $p - 1$.



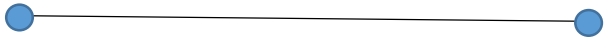


Figure 8. Complete graph K_5, K_2

Definition: A graph is said to be **perfect** if no two vertices are of same degree.

Question: Show that no graph is perfect.

Ans: Let G be a (p, q) graph.

For any vertex v in G, $0 \leq \deg(v) \leq p - 1$.

If we have a vertex with degree 0, then we cannot have a vertex with degree $p - 1$.

Similarly, if we have a vertex with degree $p - 1$, then we cannot have a vertex with degree 0. Hence degree of a vertex has $p - 1$ choices.

The $p - 1$ integers are to be associated as degrees to p vertices.

From the Pigeonhole principle, there are at least two vertices which are of same degree. Hence no graph is perfect.