Basic Electrical Technology

2. Magnetic Circuits & Electromagnetism

LECTURE 16 – 18 DEC 2021

Electromagnetic Induction and Coupled Circuits

Faraday's Laws of Electromagnetic Induction

First Law:

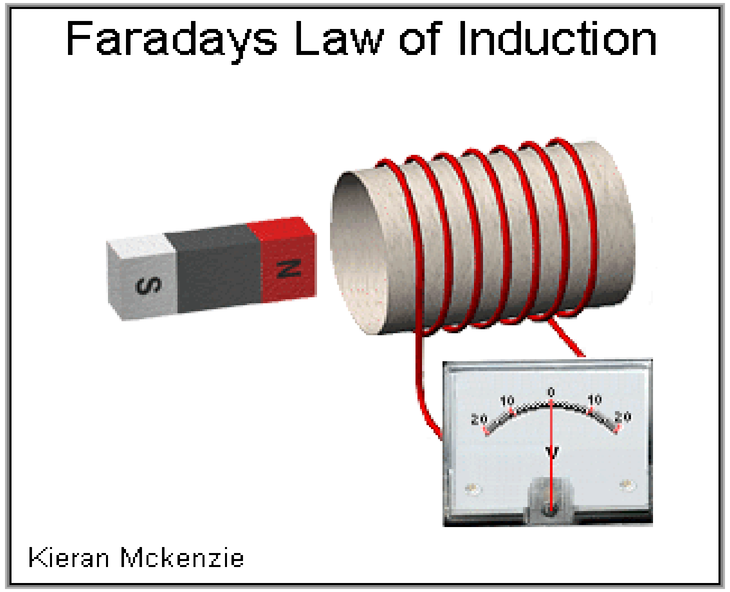
Whenever the magnetic field linking with a conductor changes, an EMF will be induced in that conductor

Second Law:

The magnitude of the induced EMF is proportional to the rate of change of the magnetic flux linking the conductor

$$e = N\frac{d\phi}{dt}$$

Where N = number of turns in the coil



Lenz's Law

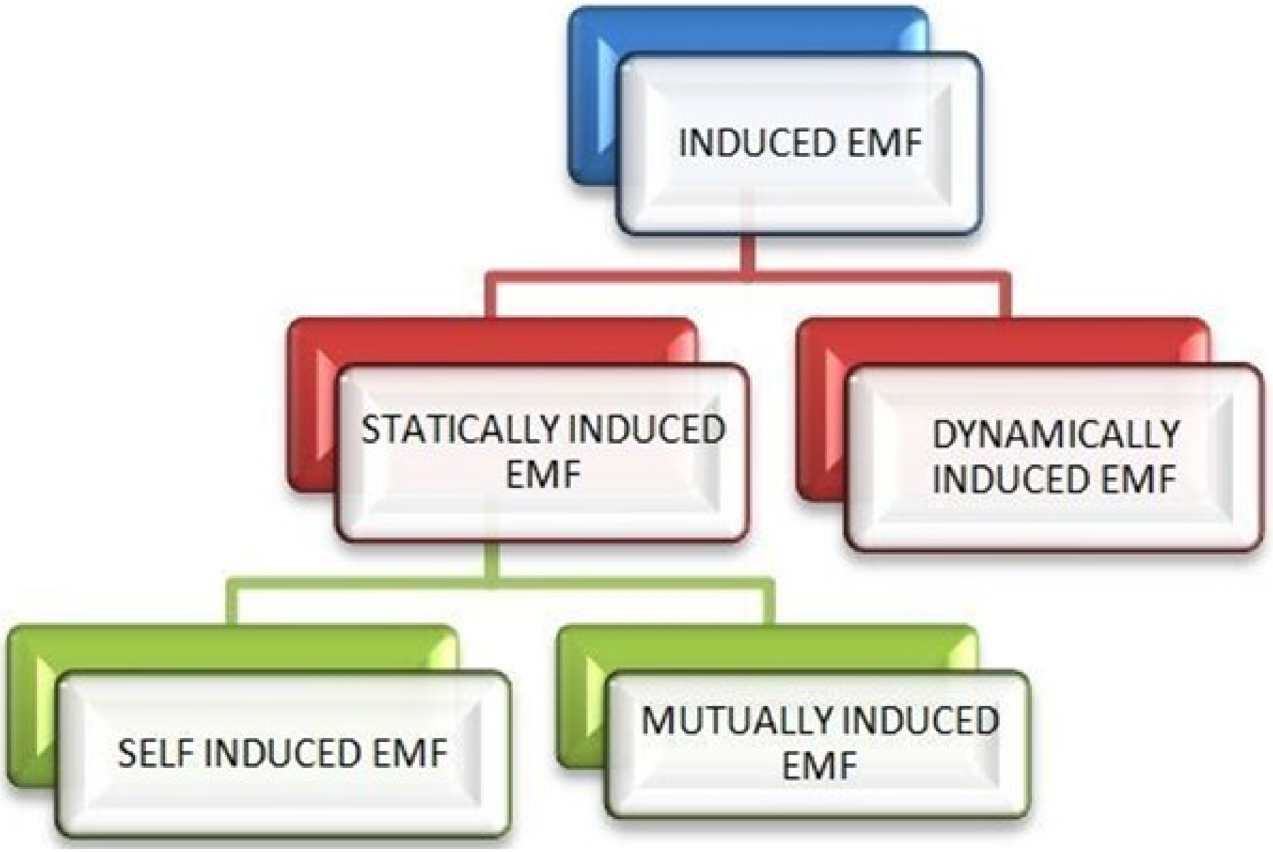
The electro-magnetically induced emf always acts in such a direction to set up a current opposing the motion or change of flux responsible for inducing the emf.

$$e = -N\frac{d\phi}{dt}$$

Fleming's Right Hand Rule

If the first, second and the thumb of the right hand are held at right angles to each other,
first finger indicates the direction of the magnetic flux
and
thumb finger indicates the direction of motion of the conductor relative to the magnetic field,
then
the second finger represents the direction of induced EMF.

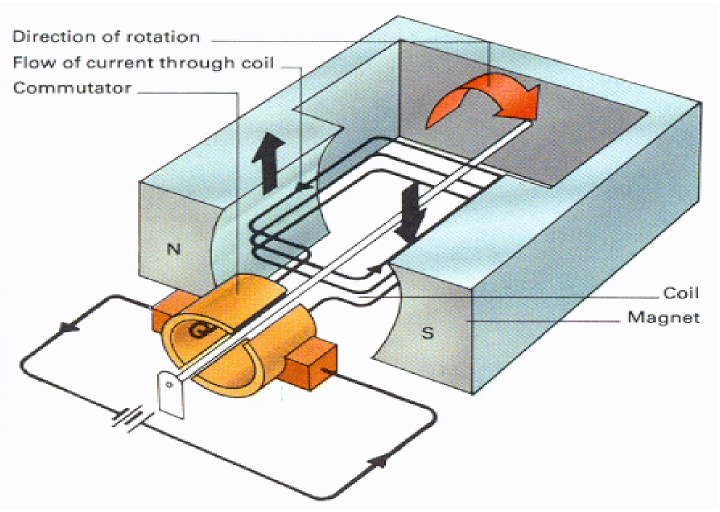
Types of induced EMF

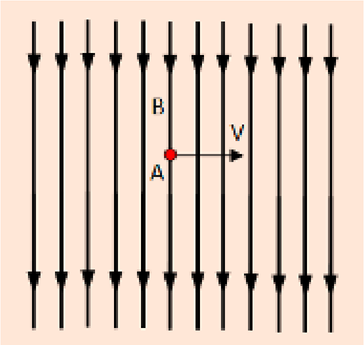


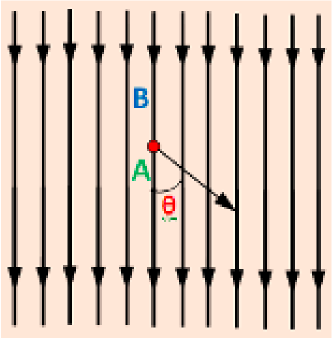
Types of induced EMF

Dynamically induced EMF:

- The voltage induced in the conductor due to relative motion of conductor and magnetic field
- $e = B\,l\,v\,\text{Sin}\theta$
- Either conductor or magnetic field is moving
- The magnetic field system is kept stationary, and the conductor is moving, or the magnetic field system is moving, and the conductor is stationary.
- Principle of **Electric generator**



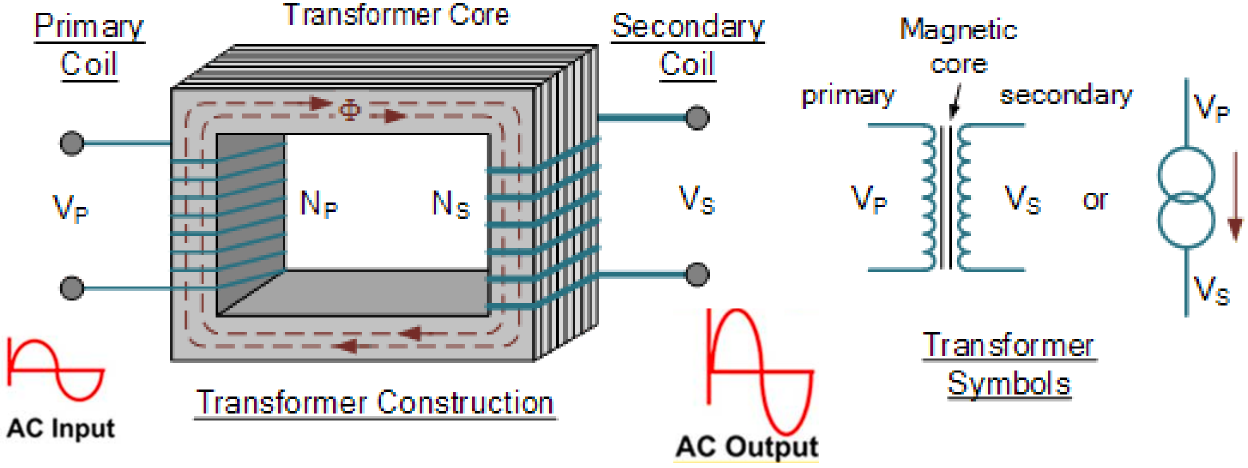




Types of induced EMF

Statically induced EMF:

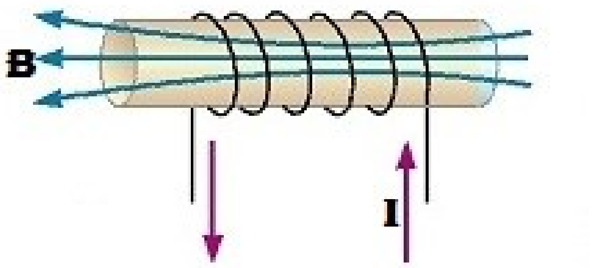
- The voltage induced in the conductor due to change in the magnetic field
- Conductor is stationary
- Magnetic Field is changing in a stationary magnetic system
- The change of flux produced by the field system linking with the coil is obtained by changing the electric current in the field system.
- Example: Transformer



Types of Statically induced EMF

Self Induced Emf:

The induced emf in a coil proportional to the rate of the change of the magnetic flux passing through it due to its own current.



$$\boldsymbol{e = -L\frac{di}{dt}}$$

Self Inductance (L):

The proportionality constant is called the self inductance, L

Unit is Henry (H)

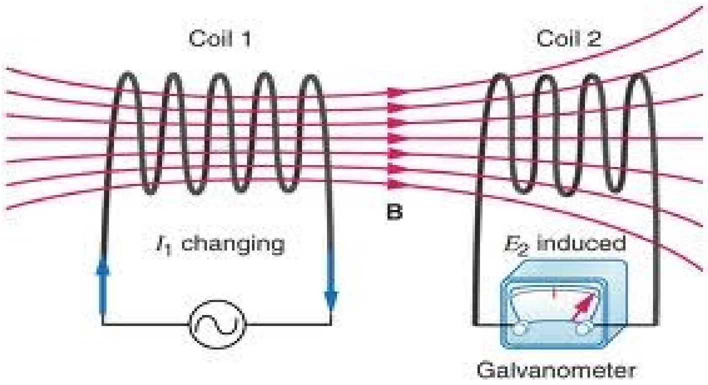
$$e = -N\frac{d\phi}{dt} = -L\frac{di}{dt}$$

$$L = N\frac{d\phi}{di}$$

(i) $\boldsymbol{e = L\frac{di}{dt}}$ (ii) N ϕ = L I (iii) $\mathbf{L} = \frac{\mathbf{N^2}}{\mathbf{Reluctance}} = \frac{\mathbf{N^2 \mu_0\, \mu_r\, A}}{\mathbf{l}}$

Mutually Induced Emf:

The induced emf in a coil due to the change of flux produced by the change of current in the nearby coil



Mutual Inductance M:

This proportionality constant is called the mutual inductance, M

If Coil 1 is excited: Mutually induced emf e_2 in Coil 2,

$$e_2 = N_2 \frac{d\phi_{12}}{dt} = N_2 \frac{d\phi_{12}}{di_1} \times \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}, \text{ Mutual Inductance, } M_{21} = N_2 \frac{d\phi_{12}}{di_1}$$

If coil 2 is excited: Mutually induced emf e_1 in Coil 1,

$$e_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2} \times \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}, \text{ Mutual Inductance, } M_{12} = N_1 \frac{d\phi_{21}}{di_2}$$

Coupling Coefficient (k)

Gives an idea about the degree of magnetic coupling between two coils.

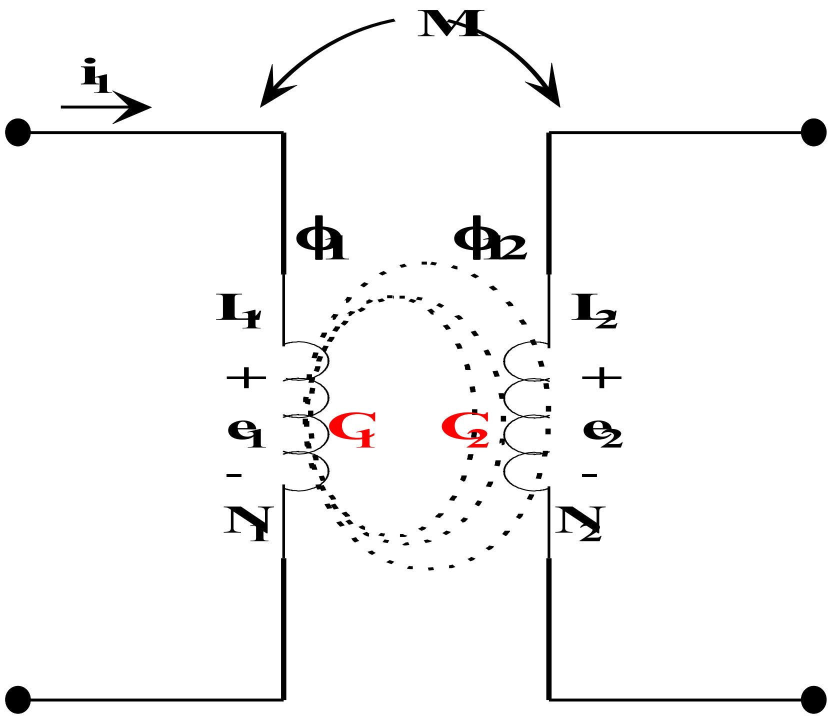
$$\boldsymbol{M_{21} = N_2 \frac{d\phi_{12}}{di_1} = M_{12} = N_1 \frac{d\phi_{21}}{di_2} = M}$$

$$where, \phi_{12} = k\,\phi_1; \;\; \phi_{21} = k\,\phi_2$$

$$M^2 = \left(N_2 k \frac{d\phi_1}{di_1}\right)\left(N_1 k \frac{d\phi_2}{di_2}\right)$$

$$= k^2 L_1 L_2$$

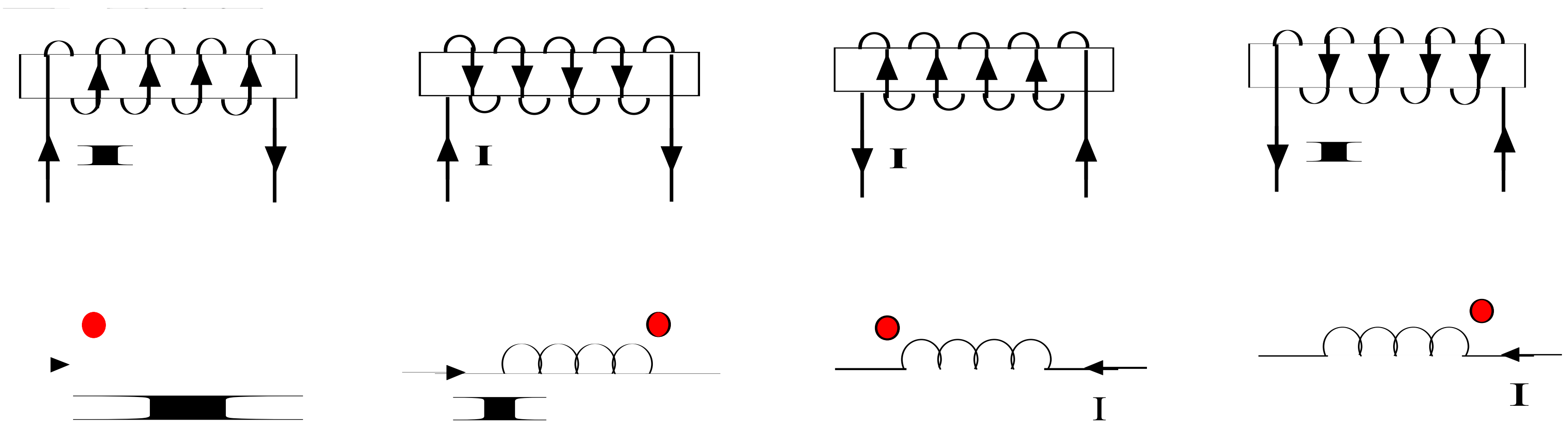
$$\boldsymbol{k = \frac{M}{\sqrt{L_1 L_2}}}$$



Coupled Circuits

- Polarity of mutually induced emf depends on
  - current direction
  - physical construction of the coils
- Obtaining the dotted equivalent: Right Hand Grip Rule

  Place the dot at the terminal directed by the thumb



Dot Rule for coupled coils

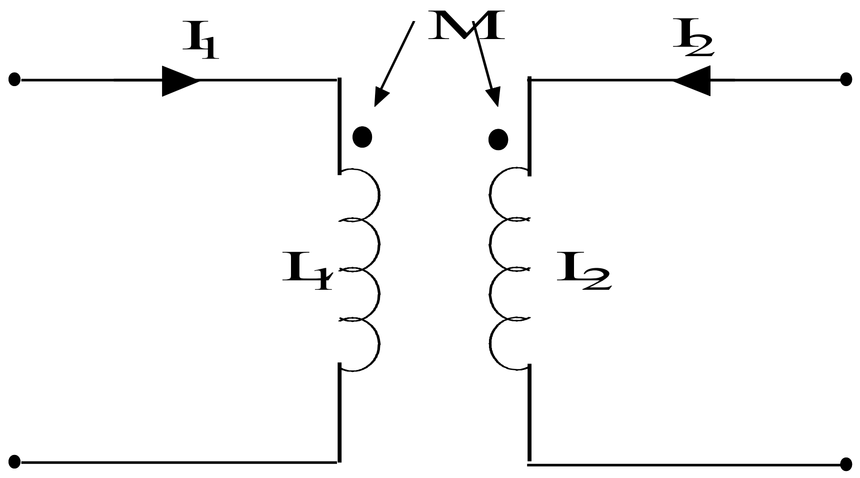
- Dot Rule helps in determining the sign of mutually induced emf without going into the details of physical construction

- **Dot Rule**

  - ✓ If currents enter (or leave) the dotted terminals in both the coils, the sign of mutually induced emf is same as that of sign of self induced emf. **(Additive coupling)**

  - ✓ If the current enters the dotted terminal in one coil and leaves the dotted terminal in the other coil, the sign of mutually induced emf is opposite to that of sign of self induced emf. **(Subtractive coupling)**

Additive Coupling: (Fluxes are aiding)

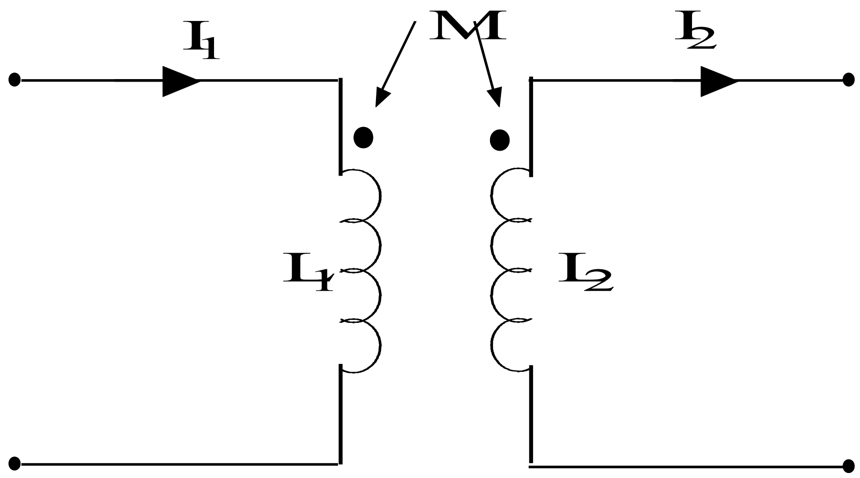


Self induced emf in L_1 = $L_1 \dfrac{di_1}{dt}$

Mutually induced emf in L_1 = $M \dfrac{di_2}{dt}$

Total induced emf in L_1 = $L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt}$

Subtractive Coupling: (Fluxes are opposing)

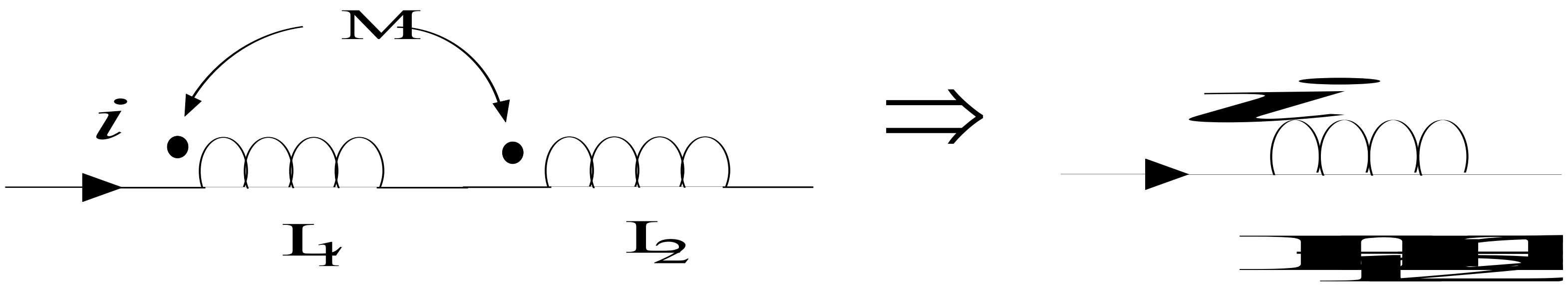


Self induced emf in L_1 = $L_1 \frac{di_1}{dt}$

Mutually induced emf in L_1 = $M \frac{di_2}{dt}$

Total induced emf in L_1 = $L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$

Coupled coils in Series - Aiding



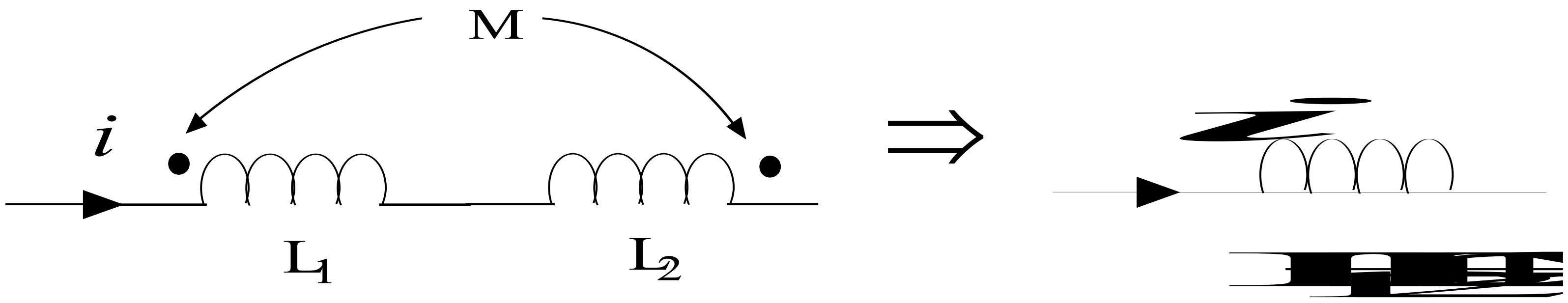
$$e_1 = L_1 \frac{di}{dt} + M \frac{di}{dt}$$

$$e_2 = L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$e = L_{eq} \frac{di}{dt} = e_1 + e_2 = (L_1 + L_2 + 2M) \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + 2M$$

Coupled coils in Series - Opposing



$$e_1 = L_1 \frac{di}{dt} - M \frac{di}{dt}$$

$$e_2 = L_2 \frac{di}{dt} - M \frac{di}{dt}$$

$$e = L_{eq} \frac{di}{dt} = e_1 + e_2 = (L_1 + L_2 - 2M) \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 - 2M$$

Inductances in Series:

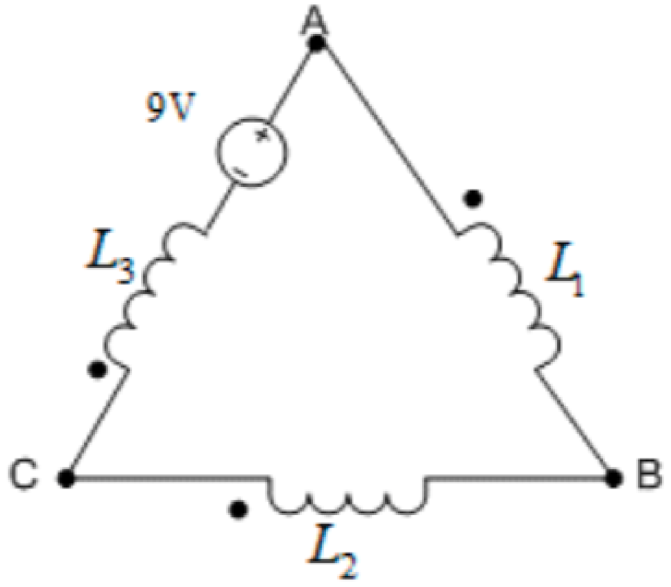
$$L_{eq} = L_1 + L_2 + \; --- \; + L_n$$

Inductances in Parallel:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \; --- \; + \frac{1}{L_3}$$

Illustration 9

Three magnetically coupled inductive coils having the following data are connected as shown in Figure. L_1 = 0.1 H; L_2 = 0.2 H; L_3 = 0.4 H; k_{12} = 0.4; k_{23} = 0.5; k_{31} = 0.5**.** Find the equivalent inductance of the circuit.



Ans: $\mathbf{0.5041}$ $\boldsymbol{H}$

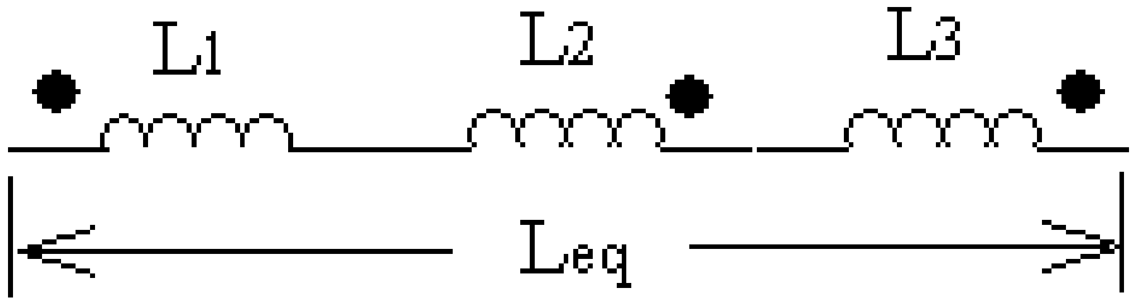
Illustration 10

Three magnetically coupled inductive coils having the following data are connected in series as shown in Figure.

$L_1 = 0.12$ H; $L_2 = 0.14$ H; $L_3 = 0.16$ H

$k_{12} = 0.3$; $k_{23} = 0.6$; $k_{31} = 0.9$

Find the equivalent inductance of the circuit.

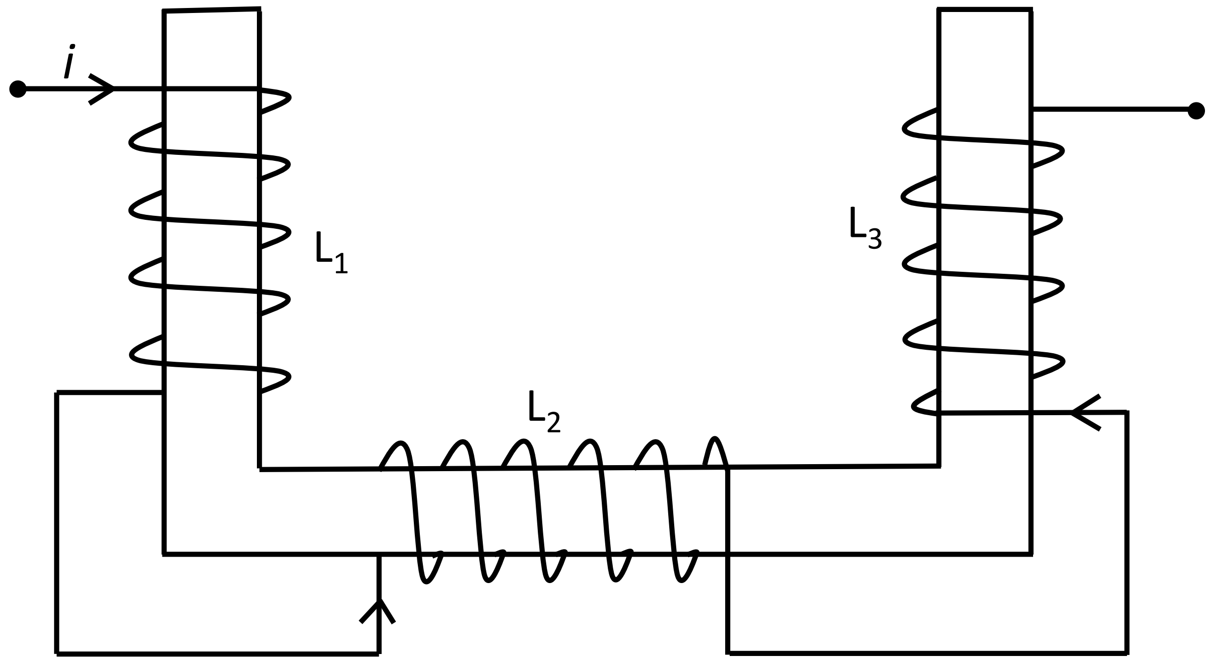


Ans: 0.272 H

Illustration 11

Three magnetically coupled inductive coils having the following data are connected in series as shown in Figure. L_1 = 0.3 H; L_2 = 0.6 H; L_3 = 0.8H and the coefficients of coupling are ,k_{12} = 0.8; k_{23} = 0.75; k_{31} = 0.5

Draw the dotted equivalent circuit of the figure, also find the equivalent inductance of the circuit.



Ans : 0.472 H

Illustration 12

Two similar coils have a coupling coefficient of 0.4. When they are connected in series aiding, the equivalent inductance is 560mH. Calculate:

(i) Self-inductance of both the coils

(ii) Total inductance when the coils are connected in series opposition

(iii) Total energy stored due to a current of 3A when the coils are connected in series opposition.

Ans: 0.2 H, 0.24 H, 1.08 J

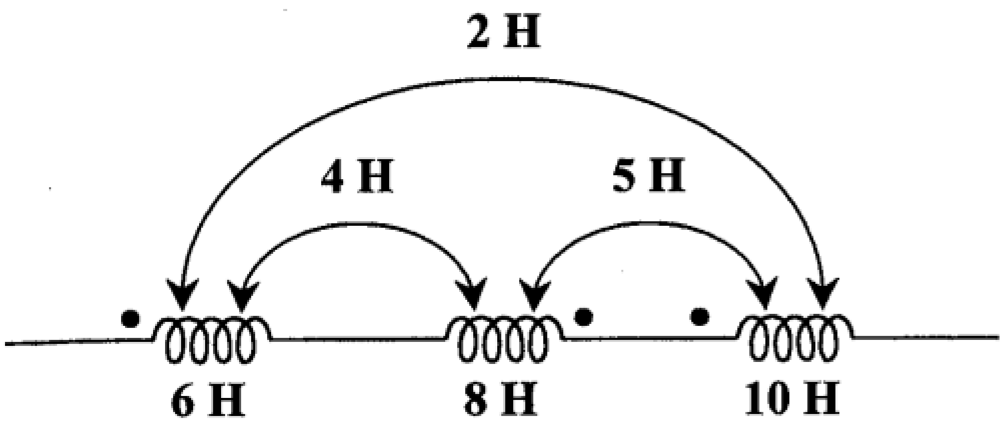
Illustration 13

Two coils of self-inductances 15 H and 20 H are connected in parallel. If the mutual inductance between the coils is 8H, find the total inductance of the circuit when (i) the mutual fluxes aid each other, and (ii) the mutual fluxes opposes each other.

Ans: (i) 12.63 H
(ii) 4.42 H

Illustration 14 (Homework)

For the three coupled coils, calculate the total inductance.



Ans: 10 H