

Questions on Lattice theory

1. Consider the POSET(S,R) where $S = \{1,2,4,6,8,10,24,30,60,120\}$ and R be the relation defined as aRb iff a divides b for all $a, b \in S$.
 - (i) Then the greatest lower bound of 10,24 is—
 - (ii) The length of the longest chain in S is ----.
 - (iii) The least upper bound of 8,24 is----
 - (iv) One example for an antichain having maximum number of elements in S is ----
 - (v) Maximal element of S is ----
 - (vi) Minimal element of S is ---
 - (vii) Upper bounds of 8 and 10 are ----
 - (viii) Lower bounds of 8 and 10 are----
 - (ix) $10 \wedge 24 =$ ---
 - (x) $8 \vee 24 =$ --
 - (xi) Is S a Lattice?

2. Consider the POSET(S,R) where $S = \{1,2,6,12,18,24,36,48,72,96,144\}$ and R be the relation defined as aRb iff a divides b for all $a, b \in S$.
 - (xii) Then the greatest lower bound of 24,18 is—
 - (xiii) The longest chain in S has ---- elements.
 - (xiv) The least upper bound of 2,18 is----
 - (xv) One example for an antichain having maximum number of elements in S is ----
 - (xvi) Maximal element of S is ----
 - (xvii) Minimal element of S is ---
 - (xviii) Upper bounds of 12 and 36 are ----
 - (xix) Lower bounds of 12 and 36 are----
 - (xx) $18 \wedge 24 =$ ---
 - (xxi) $2 \vee 18 =$ ---
 - (xxii) Is S a Lattice?

3. Draw a Hasse diagram for the POSET(S,R) where $S = \{1,3,4,8,9,12,24,27,36,72,108\}$ and R be the relation defined as aRb iff a divides b for all $a, b \in S$. Find maximal and minimal elements. Also find the length of the longest chain.