



MANIPAL INSTITUTE OF TECHNOLOGY

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Basic Electrical Technology

[ELE 105 I]

Three Phase AC Circuits

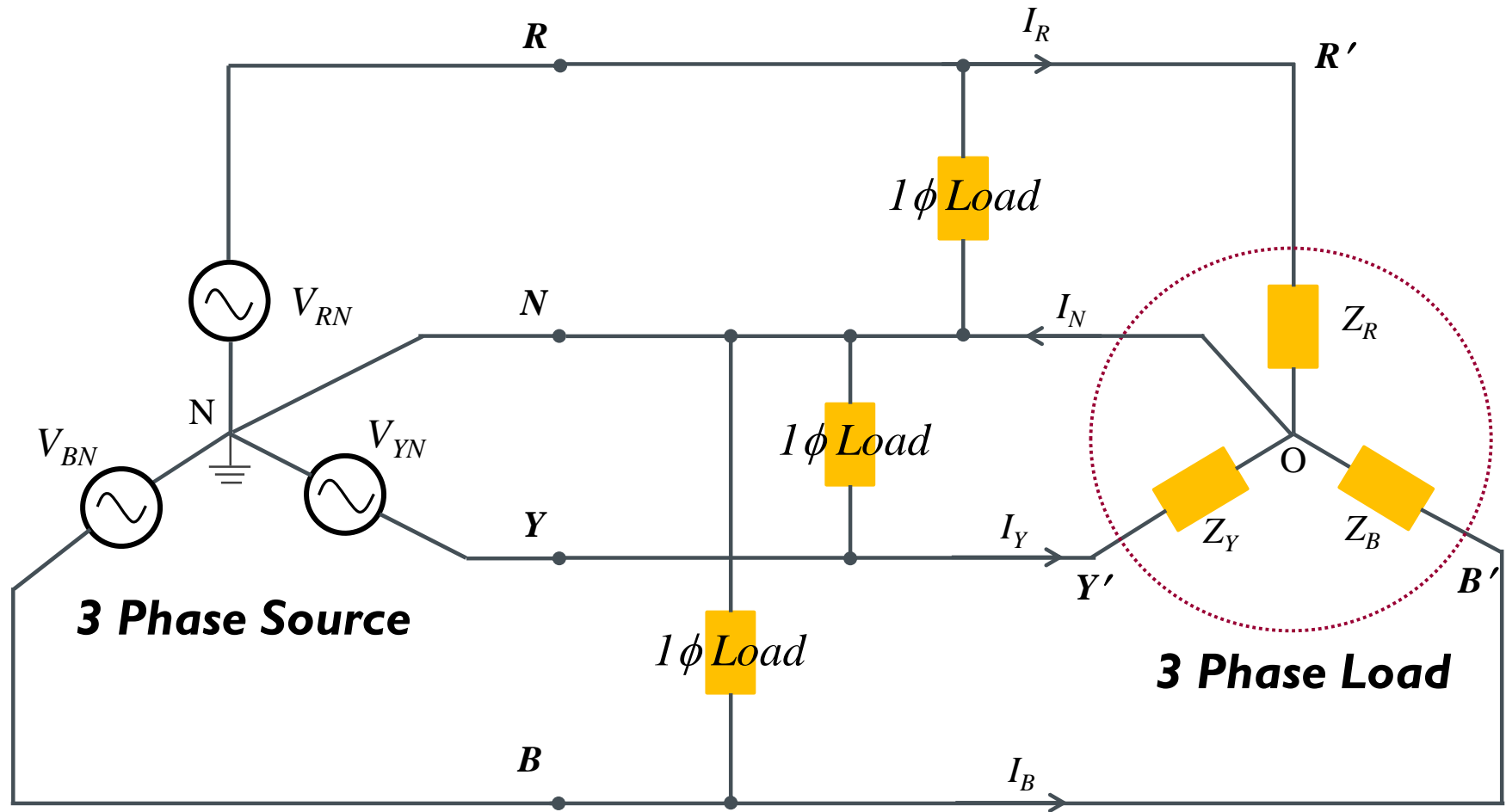
L22 ,L23– Star & Delta Connected Balanced Loads & Unbalanced loads

Topics Covered

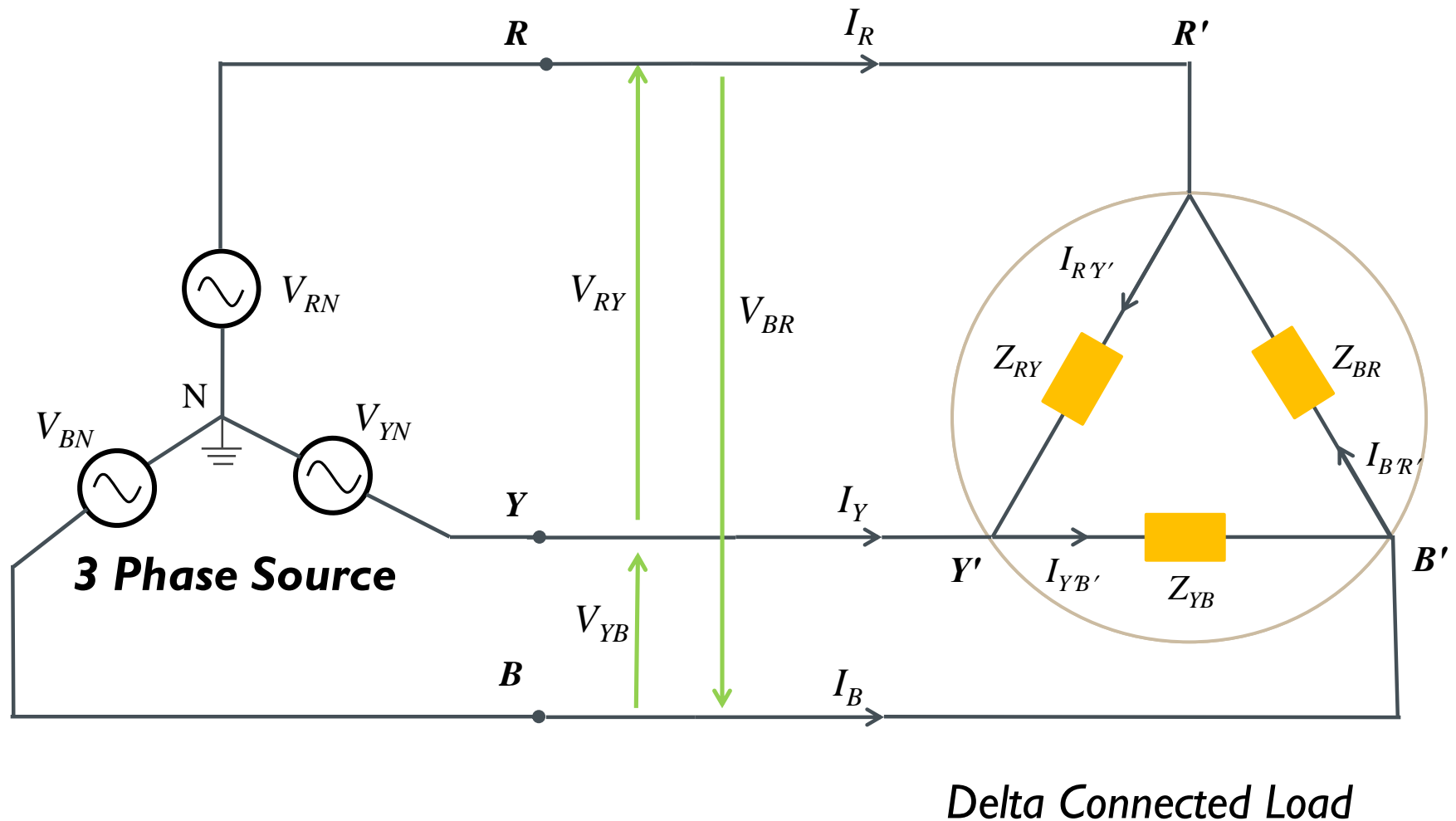


- *Three Phase loads*
- *Analysis of balanced/unbalanced star/delta connected loads with 3 phase excitation.*
- *Phase and line voltage/current relations.*
- *Neutral shift and circulating currents with unbalanced loads.*

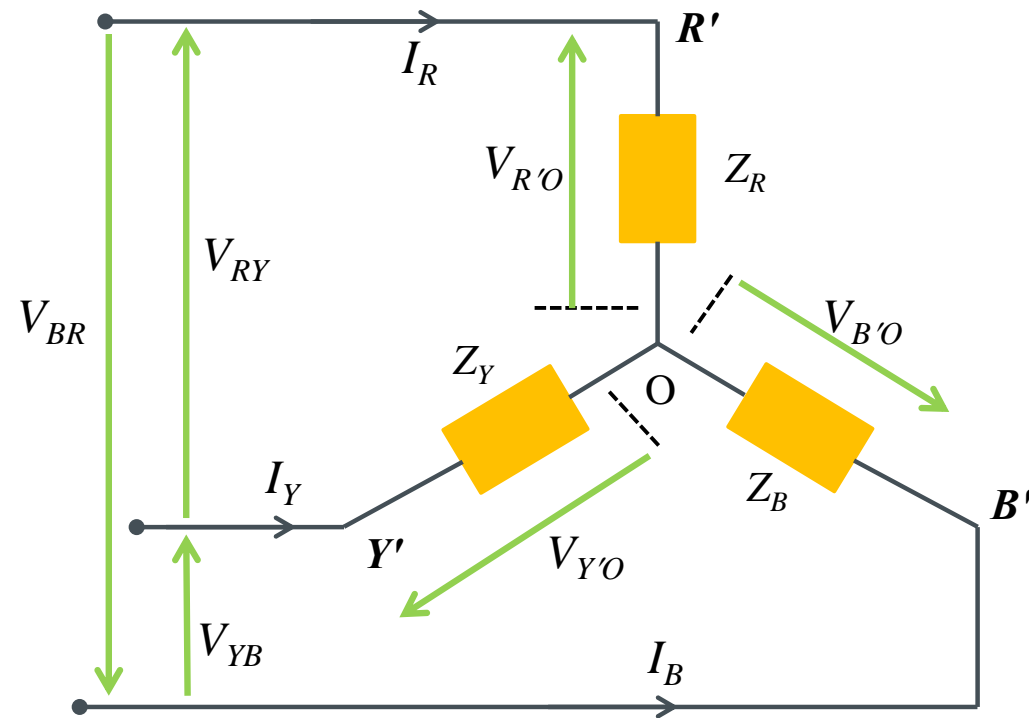
3 Phase 4 Wire Supply



Type of 3 Phase Loads



Star Connected Load

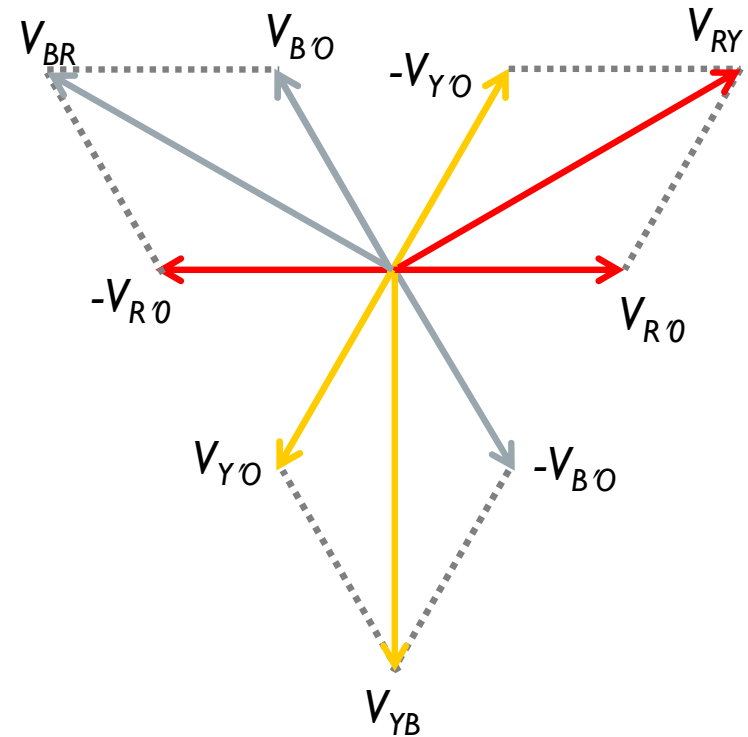


Phase Voltages: $V_{R'O}, V_{Y'O}, V_{B'O}$

Line Voltages: $\hat{V}_{RY} = \hat{V}_{R'O} - \hat{V}_{Y'O}$

$$\hat{V}_{YB} = \hat{V}_{Y'O} - \hat{V}_{B'O}$$

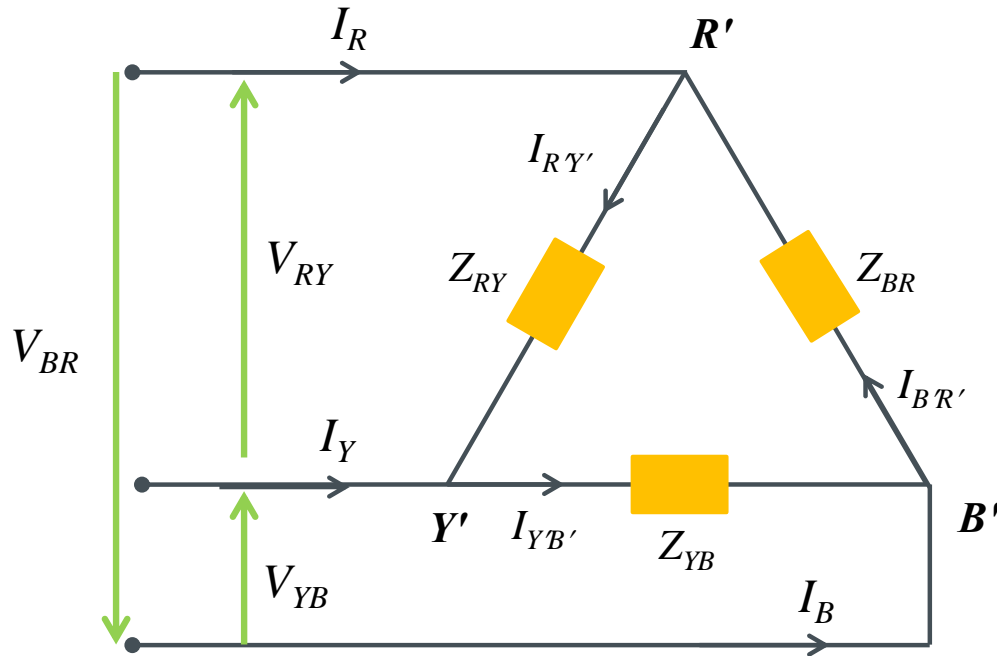
$$\hat{V}_{BR} = \hat{V}_{B'O} - \hat{V}_{R'O}$$



Line Currents = Phase Currents

For Balanced load,
Line Voltage = $\sqrt{3}$ X Phase Voltage

Delta Connected Load

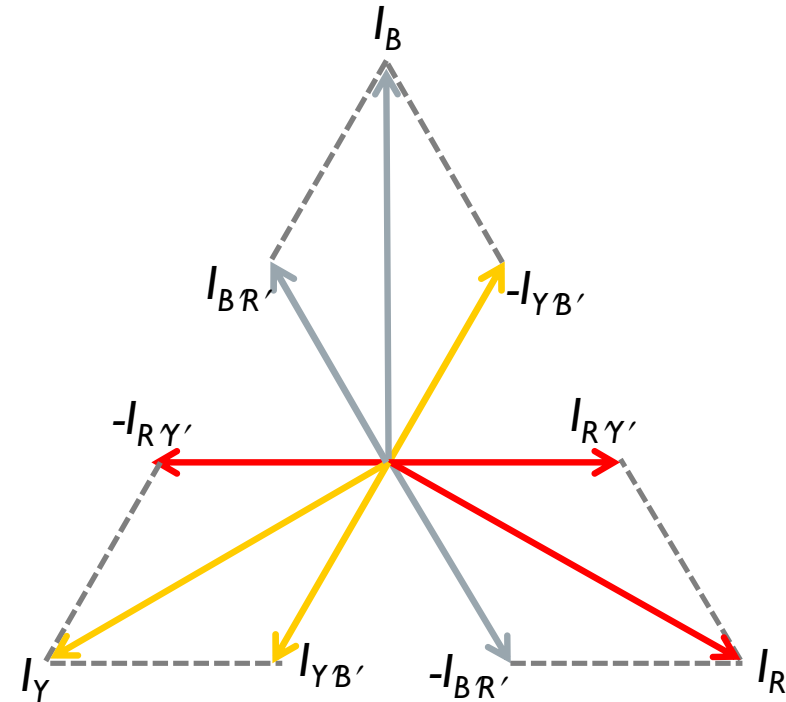


Phase Currents: $I_{RY'}, I_{YB'}, I_{BR'}$

Line Currents: $\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$

$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'}$

$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'}$

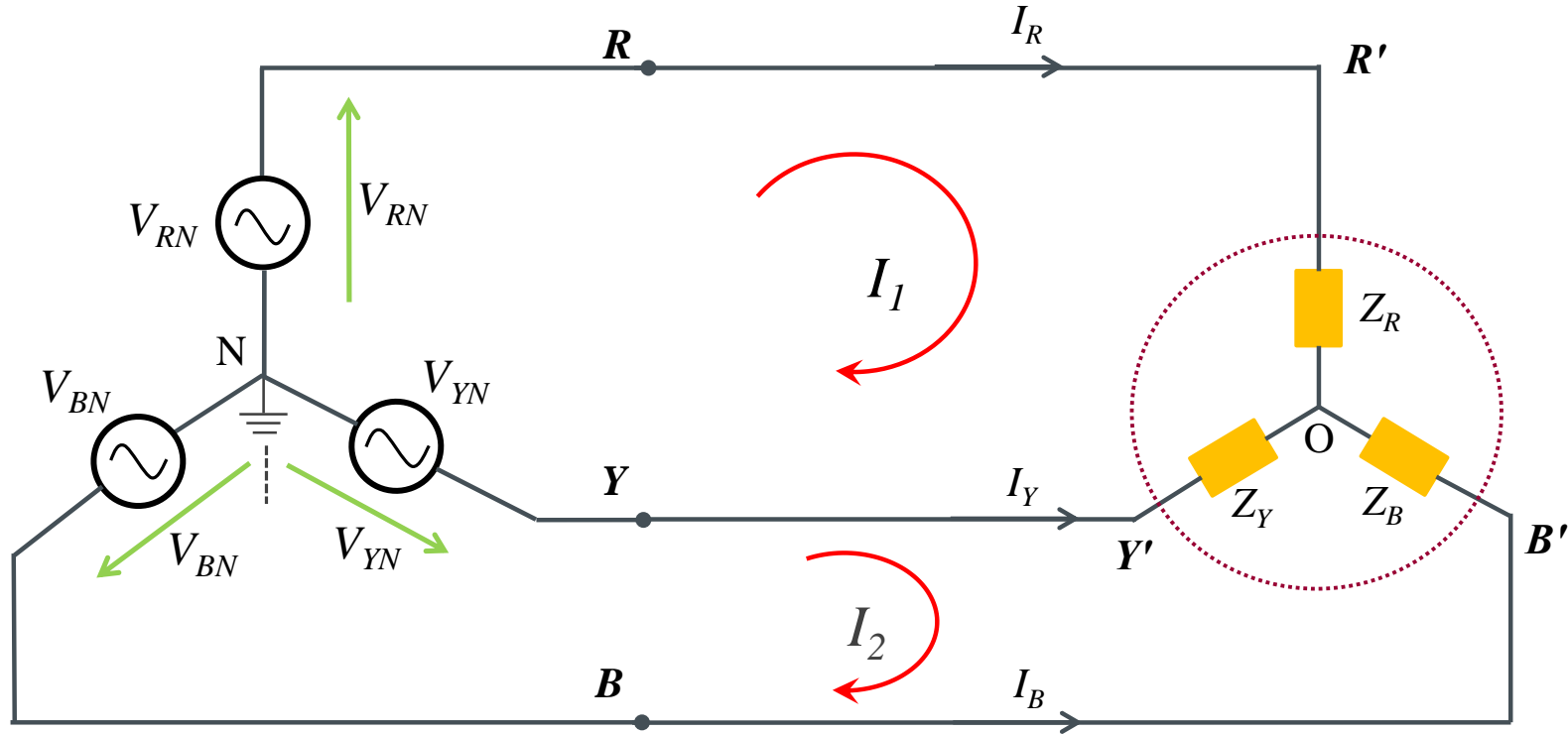


Phase Voltages = Line Voltages

For Balanced load,
Line Current = $\sqrt{3}$ X Phase Current

3 ϕ System with Y Connected Load

Consider the 3 phase star load connected to a 3 wire balanced source. Two mesh currents \hat{I}_1 and \hat{I}_2 are assumed to flow as shown in Fig.



Writing mesh equations,

$$\begin{bmatrix} \bar{Z}_R + \bar{Z}_Y & -\bar{Z}_Y \\ -\bar{Z}_Y & \bar{Z}_Y + \bar{Z}_B \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} \hat{V}_{RN} - \hat{V}_{YN} \\ \hat{V}_{YN} - \hat{V}_{BN} \end{bmatrix}$$

Loop Current Analysis...

Using Cramer's Rule,

$$\hat{I}_1 = \frac{\begin{vmatrix} \hat{V}_{RN} - \hat{V}_{YN} & -\bar{Z}_Y \\ \hat{V}_{YN} - \hat{V}_{BN} & \bar{Z}_Y + \bar{Z}_B \end{vmatrix}}{\begin{vmatrix} \bar{Z}_R + \bar{Z}_Y & -\bar{Z}_Y \\ -\bar{Z}_Y & \bar{Z}_Y + \bar{Z}_B \end{vmatrix}} \quad \hat{I}_2 = \frac{\begin{vmatrix} \bar{Z}_R + \bar{Z}_Y & \hat{V}_{RN} - \hat{V}_{YN} \\ -\bar{Z}_Y & \hat{V}_{YN} - \hat{V}_{BN} \end{vmatrix}}{\begin{vmatrix} \bar{Z}_R + \bar{Z}_Y & -\bar{Z}_Y \\ -\bar{Z}_Y & \bar{Z}_Y + \bar{Z}_B \end{vmatrix}}$$

The line currents are determined using the following equations:

$$\hat{I}_R = \hat{I}_1$$

$$\hat{I}_Y = \hat{I}_2 - \hat{I}_1$$

$$\hat{I}_B = -\hat{I}_2$$

Balanced Star Connected Load

If $Z_R = Z_Y = Z_B = Z \angle \theta$

Then, $|V_{R'O}| = |V_{Y'O}| = |V_{B'O}| = V_{Ph}$

Phase Voltages:

$$\hat{V}_{R'O} = V_{Ph} \angle 0^\circ$$

$$\hat{V}_{Y'O} = V_{Ph} \angle -120^\circ$$

$$\hat{V}_{B'O} = V_{Ph} \angle +120^\circ$$

Line Voltages:

$$\hat{V}_{RY} = \hat{V}_{R'O} - \hat{V}_{Y'O}$$

$$= V_{Ph} \angle 0^\circ - V_{Ph} \angle -120^\circ = \sqrt{3} \times V_{Ph} \angle 30^\circ$$

$$\hat{V}_{YB} = \hat{V}_{Y'O} - \hat{V}_{B'O} = \sqrt{3} \times V_{Ph} \angle -90^\circ$$

$$\hat{V}_{BR} = \hat{V}_{B'O} - \hat{V}_{R'O} = \sqrt{3} \times V_{Ph} \angle 150^\circ$$

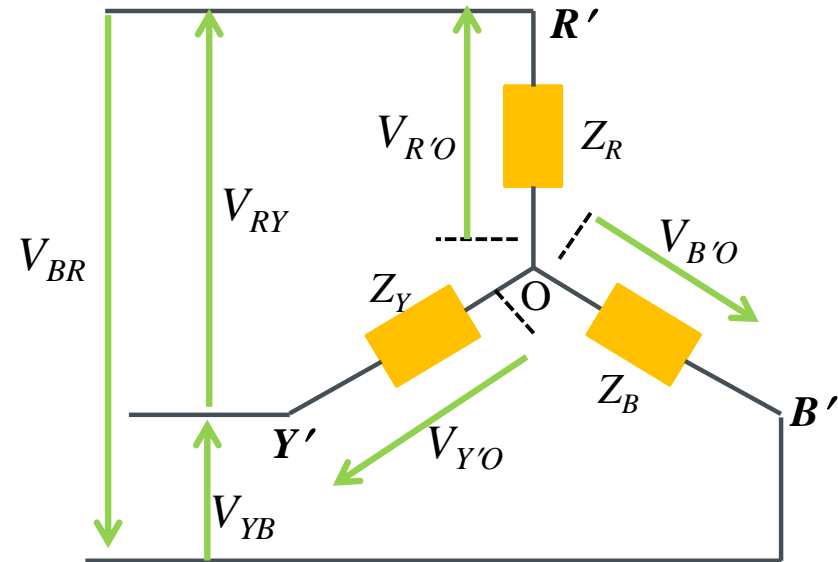


Illustration- I

Three loads $Z_A = 10 \angle 0^\circ \Omega$; $Z_B = 15 \angle -30^\circ \Omega$ and $Z_C = 20 \angle 45^\circ \Omega$ are connected in star across a balanced, 3 phase, 400 V, RYB supply. Determine (a) line currents (b) Phase Voltages (c) Neutral shift voltage, V_{ON} .

Solution:

The three phase load is supplied with a balanced supply of 400V, hence the line voltages appearing across the load are:

$$\hat{V}_{RY} = 400 \angle 0^\circ \text{ (Reference Voltage)}$$

$$\hat{V}_{YB} = 400 \angle -120^\circ$$

$$\hat{V}_{BR} = 400 \angle +120^\circ$$

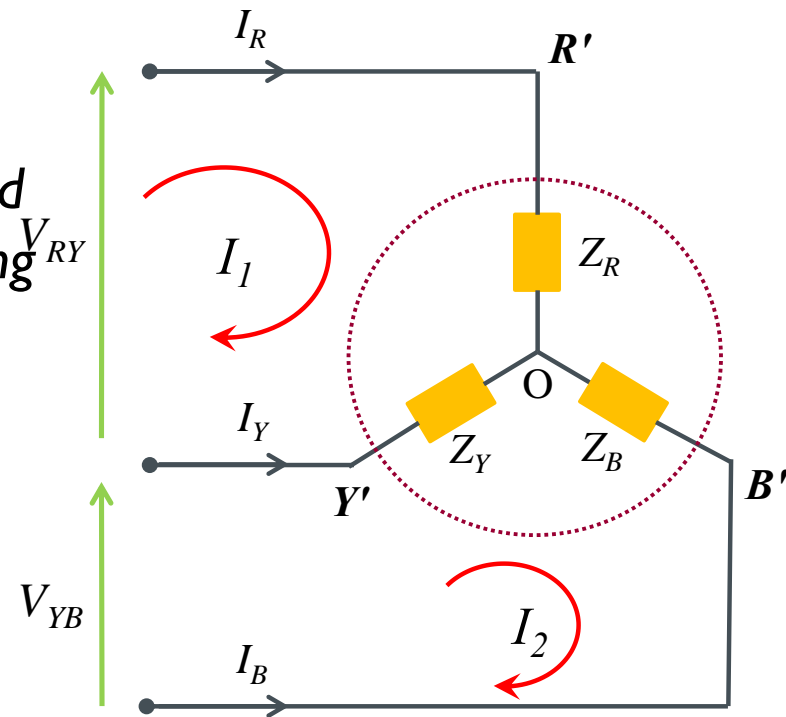


Illustration-I ...

Writing Mesh Equation in Matrix form,

$$\begin{bmatrix} 10\angle 0 + 15\angle -30 & -15\angle -30 \\ -15\angle -30 & 15\angle -30 + 20\angle 45 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 400\angle 0^\circ \\ 400\angle -120^\circ \end{bmatrix}$$

Using Cramer's rule,

$$\hat{I}_1 = 9.783\angle -17.87^\circ \text{ A}$$

$$\hat{I}_2 = 16.69\angle -116.63^\circ \text{ A}$$

(i) The line currents are

$$\hat{I}_R = \hat{I}_1 = 9.783\angle -17.87^\circ \text{ A}$$

$$\hat{I}_Y = \hat{I}_2 - \hat{I}_1 = 20.59\angle -144.63^\circ \text{ A}$$

$$\hat{I}_B = -\hat{I}_2 = 16.69\angle 63.37^\circ \text{ A}$$

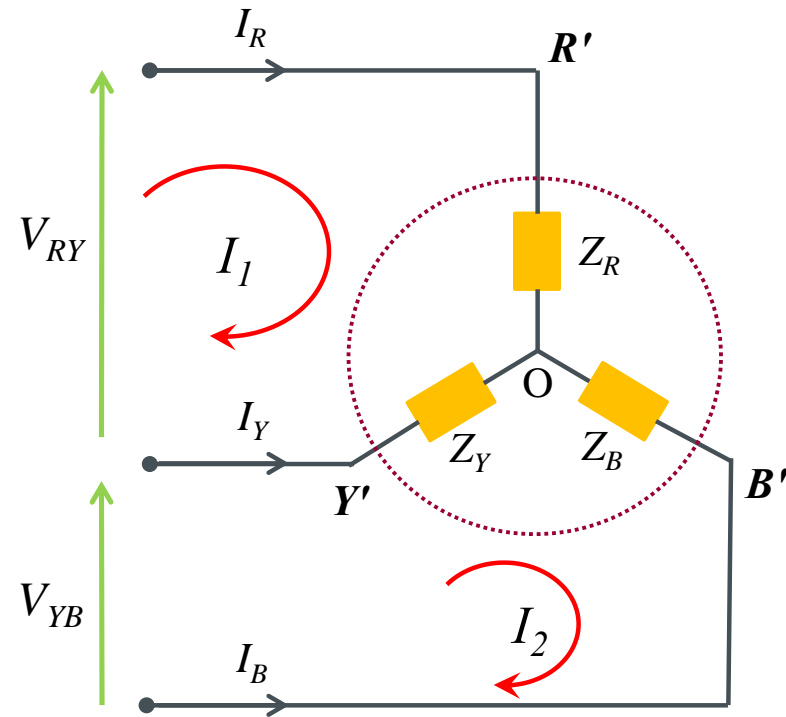


Illustration-I ...

(ii) Phase Voltages are determined using the following equations.

$$\hat{V}_{R'O} = \hat{I}_R \times \bar{Z}_A = 97.83 \angle -7.87^\circ \text{ V}$$

$$\hat{V}_{Y'O} = \hat{I}_Y \times \bar{Z}_B = 308.85 \angle -174.63^\circ \text{ V}$$

$$\hat{V}_{B'O} = \hat{I}_B \times \bar{Z}_C = 338 \angle 108.37^\circ \text{ V}$$

(c) Neutral Shift Voltage (V_{on})

$$\hat{V}_{RY} = 400 \angle 0^\circ \text{ (Reference Voltage)}$$

$$\hat{V}_{RN} = \frac{400}{\sqrt{3}} \angle -30^\circ$$

$$\hat{V}_{YN} = \frac{400}{\sqrt{3}} \angle (-30 - 120)$$

$$\hat{V}_{BN} = \frac{400}{\sqrt{3}} \angle (-30^\circ - 240)$$

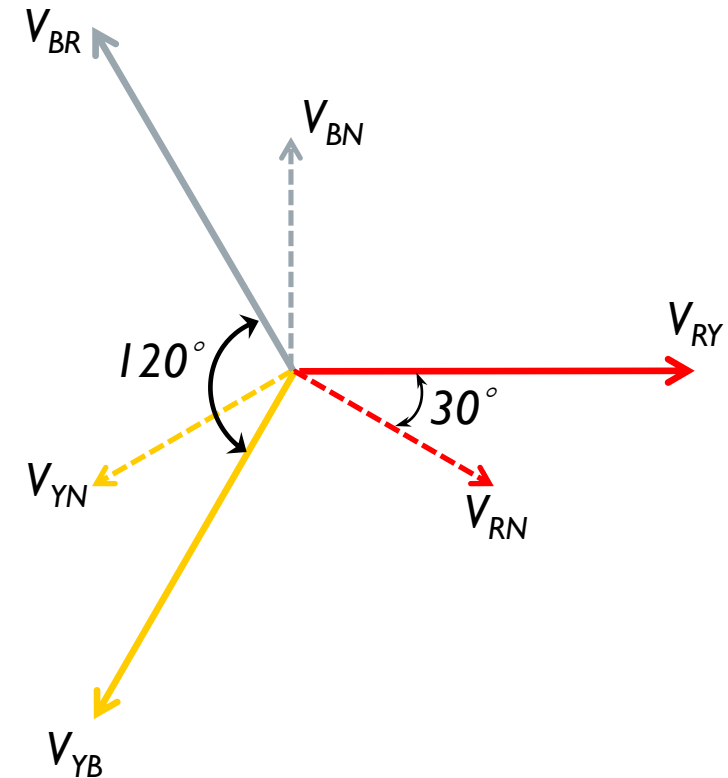
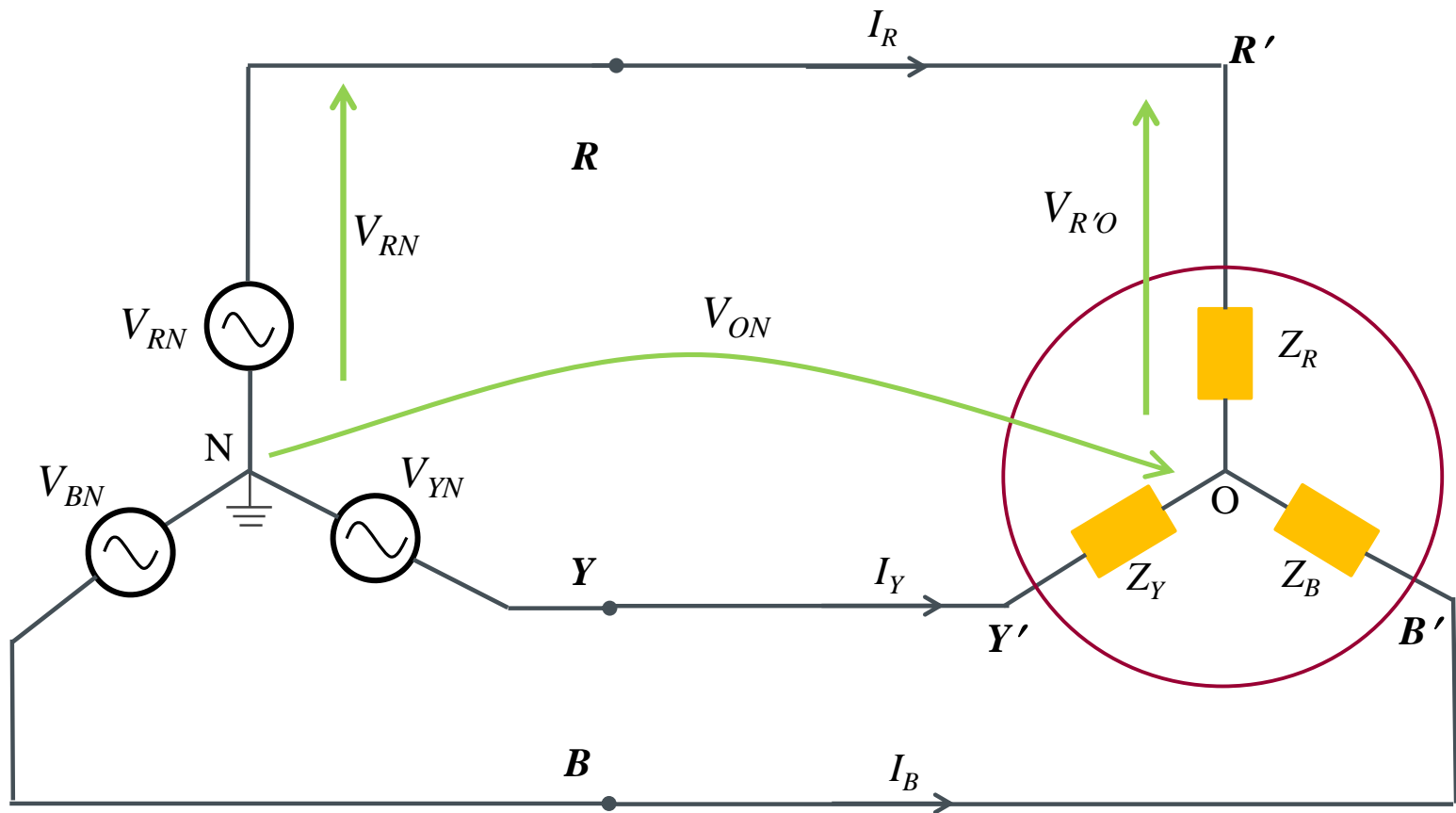


Illustration-I ...

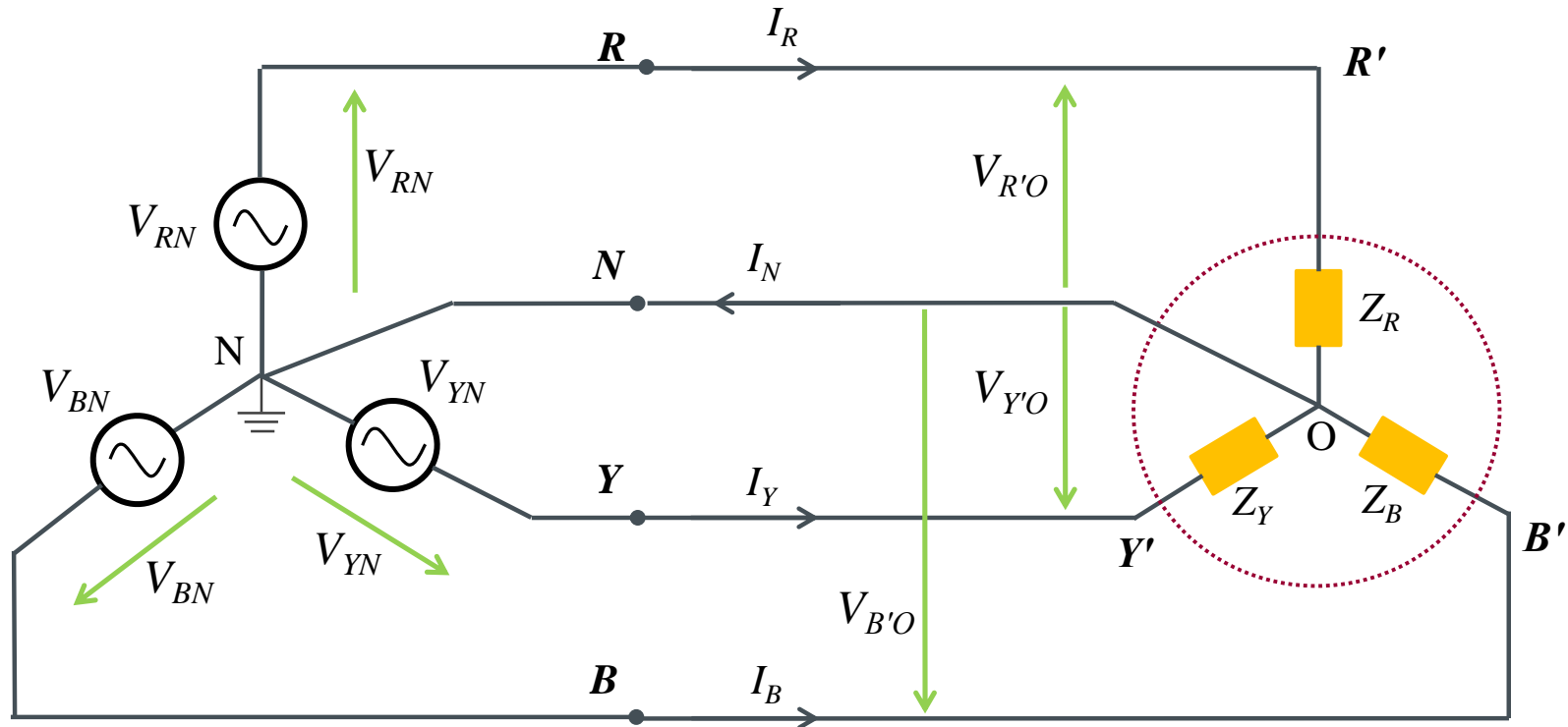


Applying KVL, $\hat{V}_{RN} - \hat{V}_{R'O} - \hat{V}_{ON} = 0$

$$\hat{V}_{ON} = \hat{V}_{RN} - \hat{V}_{R'O} = 145.07 \angle -44.7^\circ \text{ V}$$

3 ϕ , 4 Wire System with Y Load

Consider the 3 phase star load connected to a 4 wire balanced source.



Phase Voltages of Load,

$$\hat{V}_{R'O} = \hat{V}_{RN}$$

$$\hat{V}_{Y'O} = \hat{V}_{YN}$$

$$\hat{V}_{B'O} = \hat{V}_{BN}$$

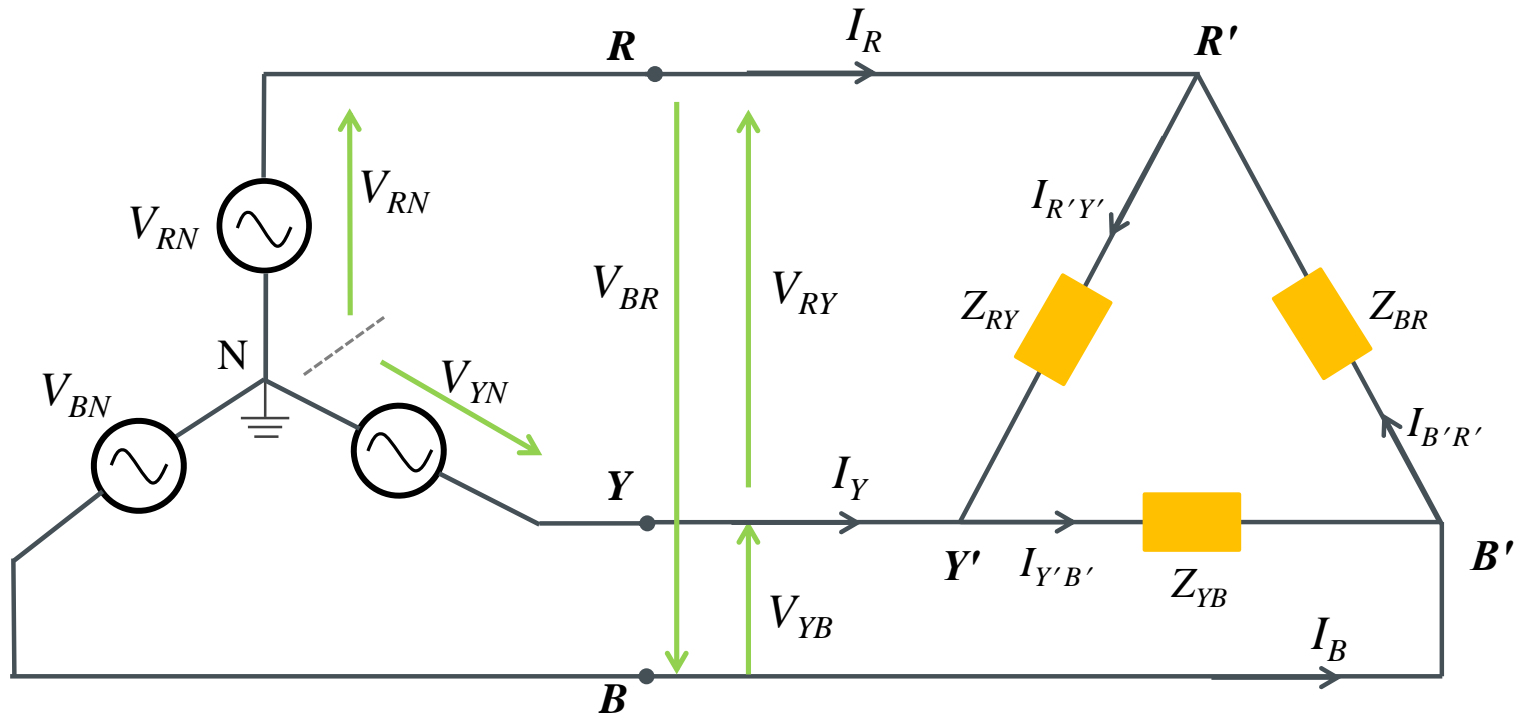
Neutral Current:

$$\hat{I}_N = \hat{I}_R + \hat{I}_Y + \hat{I}_B$$

$$\hat{I}_N = 0; \text{ (If } Z_R = Z_Y = Z_B = Z \angle \theta^\circ \text{)}$$

3 ϕ , 3 Wire System with Δ Load

Consider the 3 phase Delta load connected to a 3 wire balanced source.



Phase Currents,

$$\hat{I}_{R'Y'} = \frac{\hat{V}_{RY}}{\bar{Z}_{RY}} \quad \hat{I}_{Y'B'} = \frac{\hat{V}_{YB}}{\bar{Z}_{YB}} \quad \hat{I}_{B'R'} = \frac{\hat{V}_{B'R'}}{\bar{Z}_{BR}}$$

Line Currents,

$$\begin{aligned} \hat{I}_R &= \hat{I}_{R'Y'} - \hat{I}_{B'R'} \\ \hat{I}_Y &= \hat{I}_{Y'B'} - \hat{I}_{R'Y'} \\ \hat{I}_B &= \hat{I}_{B'R'} - \hat{I}_{Y'B'} \end{aligned}$$

Balanced Δ Load

If $Z_R = Z_Y = Z_B = Z \angle \theta$

Then, $|I_{R'Y'}| = |I_{Y'B'}| = |I_{B'R'}| = I_{Ph}$

Phase Currents:

$$\hat{I}_{R'Y'} = I_{Ph} \angle 0^\circ$$

$$\hat{I}_{Y'B'} = I_{Ph} \angle -120^\circ$$

$$\hat{I}_{B'R'} = I_{Ph} \angle +120^\circ$$

Line Currents:

$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$

$$= I_{Ph} \angle 0^\circ - I_{Ph} \angle +120^\circ = \sqrt{3} \times I \angle -30^\circ$$

$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'} = \sqrt{3} \times I_{Ph} \angle -150^\circ$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'} = \sqrt{3} \times I_{Ph} \angle 90^\circ$$

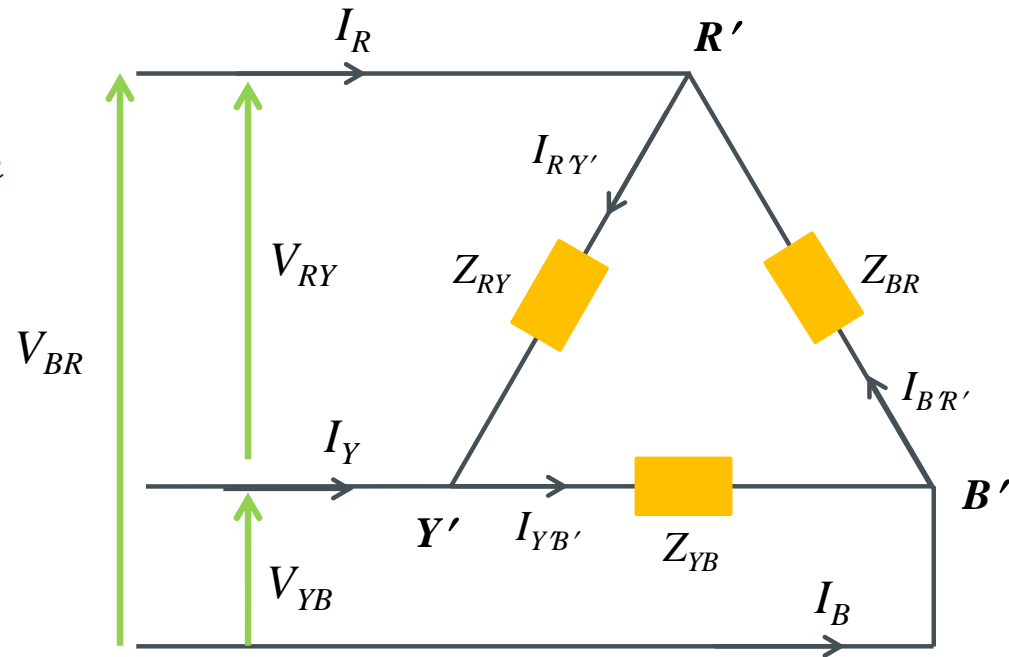


Illustration-2

Three loads, $Z_R = 50 + j40 \, \Omega$, $Z_Y = 100 \, \Omega$ and $Z_B = 80 - j60 \, \Omega$ are connected in Delta across a balanced 3 phase, 415V, 50 Hz supply. Determine

- Phase Currents
 - Line Currents and hence draw the complete phasor diagram.
- Assume a phase sequence of RYB.

Solution:

The three phase load is supplied with a balanced supply of 415V, hence the line voltages appearing across the load are:

$$\hat{V}_{RY} = 415 \angle 0^\circ \text{ (Reference Voltage)}$$

$$\hat{V}_{YB} = 415 \angle -120^\circ$$

$$\hat{V}_{BR} = 415 \angle +120^\circ$$

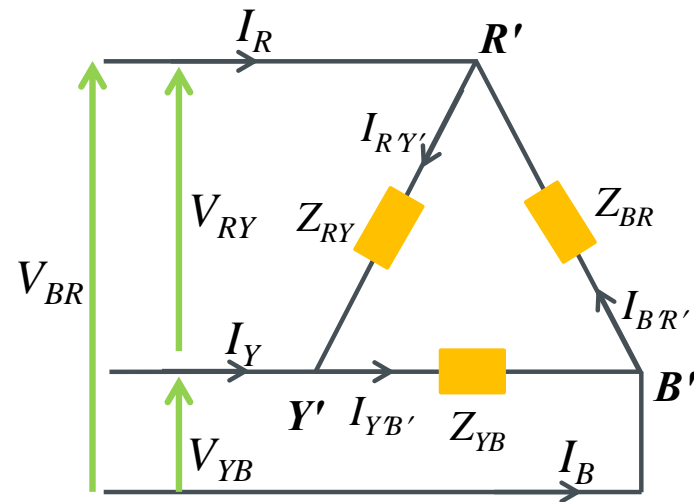


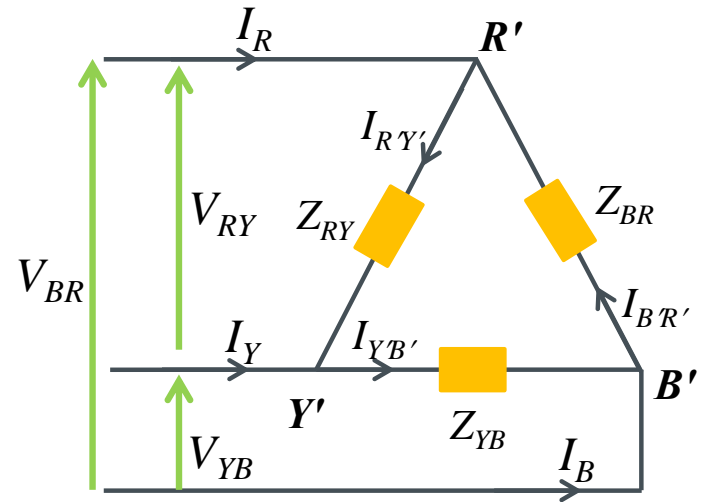
Illustration-2...

(i) Calculating the phase currents,

$$\hat{I}_{R'Y'} = \frac{\hat{V}_{RY}}{\bar{Z}_{RY}} = 6.48 \angle -38.66^\circ \text{ A}$$

$$\hat{I}_{Y'B'} = \frac{\hat{V}_{YB}}{\bar{Z}_{YB}} = 4.15 \angle -120^\circ \text{ A}$$

$$\hat{I}_{B'R'} = \frac{\hat{V}_{BR}}{\bar{Z}_{BR}} = 4.15 \angle 156.87^\circ \text{ A}$$



(ii) Calculating the Line currents,

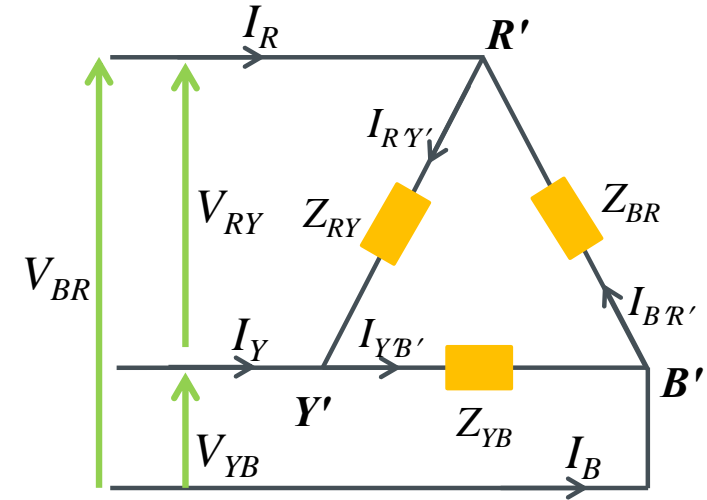
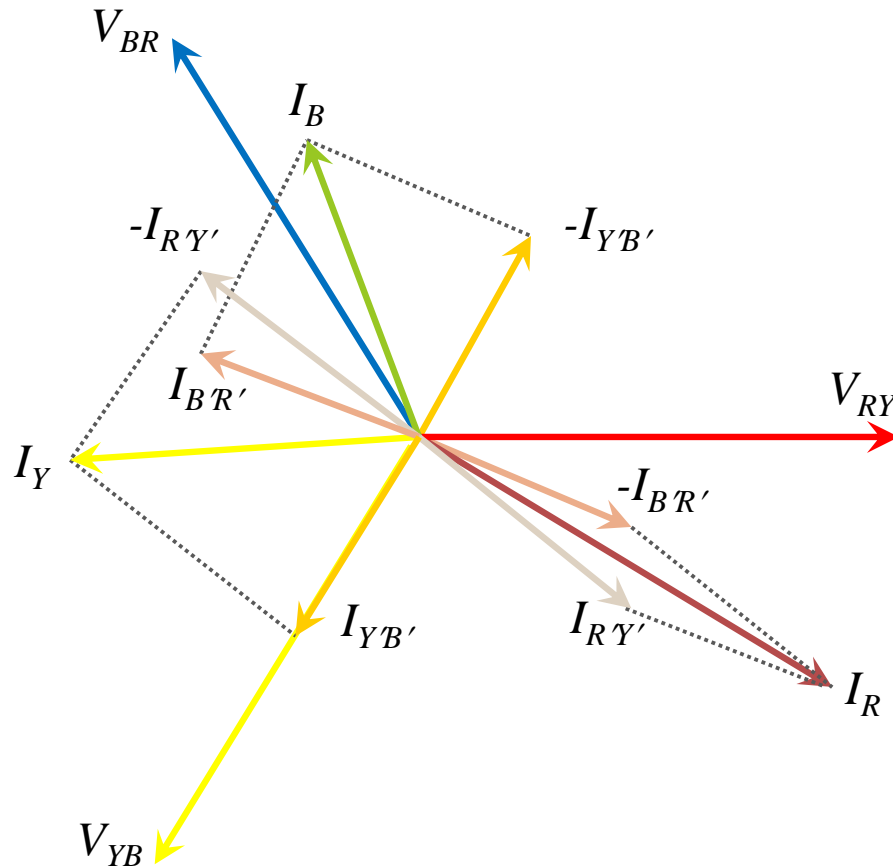
$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'} = 10.537 \angle -32.61^\circ \text{ A}$$

$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'} = 7.149 \angle 176.35^\circ \text{ A}$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'} = 5.506 \angle 108.44^\circ \text{ A}$$

Illustration-2...

(ii) Phasor Diagram,



$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$

$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'}$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'}$$

Analysis of balanced/unbalanced three phase star/delta connected load with 3 phase balanced excitation is performed.

- For Balanced Star connected load, the line voltage = $\sqrt{3}$ x phase voltage.*
- For Balanced Delta connected load, the line current = $\sqrt{3}$ x phase current.*