II SEM - Engg. Mathematics II

MAT -1251 (II sessional)

Time: 1 Hr. **Date: 23.03.2019** **Time: 12.00PM-1.00PM** **Max. Marks: 15**

Answer **ALL** the questions

Note: Questions 1 to 10 are of 0.5 mark and 11 to 15 are of 2 marks each

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1. The value of $L\{\frac{1-e^t}{t}\}$ is ______

   $\boldsymbol{a)\ \log(\frac{s-1}{s})}$ $b)\ \log(\frac{s}{s-1})$ $c)\ \log(\frac{s-1}{s+1})$ $d)\ \log(\frac{s+1}{s-1})$

2. The area between the curves $y = x$ and $y = x^2$ is ________

   $a)\ {}^1/_2$ b) ${}^3/_4$ $c)\ {}^1/_4$ $\boldsymbol{d)\ {}^1/_6}$

3. The value of $\int_0^\infty \frac{x^{12}}{(1+x)^{15}}dx=$ _______

   $a)\ \boldsymbol{\beta(13,2)}$ $b)\ \beta(12,15)$ $c)\ \beta(3,12)$ $d)\ \beta(11,2)$

4. For spherical polar coordinates the value of $J\left(\frac{x,y,z}{r,\theta,\varphi}\right)$ is ____

   $a)\ r\sin\theta$ $b)\ r\cos\theta$ $c)\ \boldsymbol{r^2\sin\theta}$ $d)\ r^2\cos\theta$

5. The value of $L(2^{3t}) =$____________

   $a)\ \frac{1}{s-3}$ $b)\ \boldsymbol{\frac{1}{s-\ln 8}}$ $c)\ \frac{1}{s-\ln 6}$ $d)\ \frac{1}{s-\ln 9}$

6. The limits of θ when finding the area of the region inside $r = 1 + \cos\theta$ and outside the circle $r = 1$ is _______.

   $a)\ -\pi\ to\ \pi$ $b)\ 0\ to\ \pi$ $c)\ 0\ to\ 2\pi$ $\boldsymbol{d)\ -{}^{\pi}/_2\ to\ {}^{\pi}/_2}$

7. If $u = s\cos t$ and $v = s\sin t$ then $\frac{\partial(s,t)}{\partial(u,v)} =$ _______.

   $a)\ s$ $\boldsymbol{b)\ \frac{1}{s}}$ $c)\ t$ $d)\ \frac{1}{t}$

8. Write the given integral in polar form: $\iint_R \sqrt{x^2+y^2}\ dx\,dy$, where R is the region bounded by $x^2+y^2=a^2$ and $x^2+y^2=b^2$ where $a<b$.

   $a)\ \int_0^\pi \int_a^b r^2 dr\,d\theta$ $\boldsymbol{b)\ \int_0^{2\pi} \int_a^b r^2 dr\,d\theta}$

   $c)\ \int_0^\pi \int_a^b r\,dr\,d\theta$ $d)\ \int_0^{2\pi} \int_a^b r\,dr\,d\theta$

9. The value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is ______.

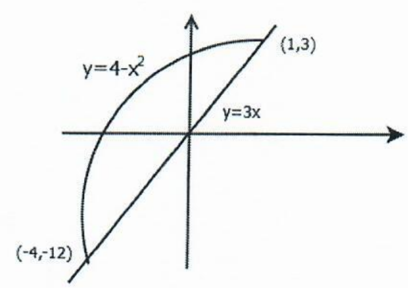
a) $\frac{2\pi}{\sqrt{3}}$ b) 2π c) $\frac{\sqrt{3}\,\pi}{2}$ d) $\frac{\pi}{\sqrt{2}}$

10. The limit of y in the projection of the tetrahedron $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ on XOY plane while finding the volume is ______

a) 1 to $b(1-\frac{x}{a})$ b) 0 to $a\left(1-\frac{x}{a}\right)$

c) 0 to 1 d) **0 to** $\boldsymbol{b\left(1-\frac{x}{a}\right)}$

11. Calculate the volume of a solid whose base is in a xy-plane and is bounded by the parabola $y = 4-x^2$ and the straight line $y = 3x$, while the top of the solid is in the plane $z = x + 4$.



(Fig 0.5M)

$V=\int_{-4}^{1}\int_{3x}^{4-x^2}(x+4)dydx$ (0.5M)

$=\frac{625}{12}$ (1M)

12. Evaluate $\int_{-1}^{1}(1+x)^6(1-x)^7dx$.

Put $x=2t-1, dx=2dt$. (0.5M)

$\int_0^1(2t)^6(2-2t)^7 2dt = 2^{14}\int_0^1(t)^6(1-t)^7dt$ (1M)

$=2^{14}\beta(7,8)=\frac{2^{14}6!7!}{14!}=0.6819.$ (0.5M)

13. Find the Laplace transform of $t^2e^{-3t}sin2t$.

$L(sin2t)=\frac{2}{s^2+4}$, (0.5M)

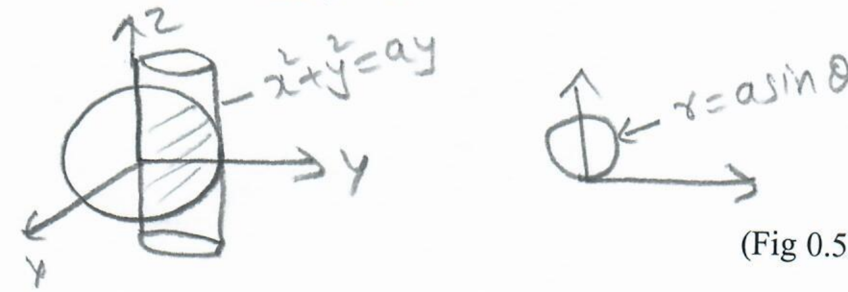
$L(t^2sin2t)=\frac{12s^2-16}{(s^2+4)^3}$ (1M)

$L(t^2e^{-3t}sin2t)=\frac{12(s+3)^2-16}{((s+3)^2+4)^3}$ (0.5M)

14. Find the volume of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ inside the cylinder $x^2 + y^2 = ay$.

Converting to cylindrical polar,

$x = r\cos\theta, y = r\sin\theta, J = r.$



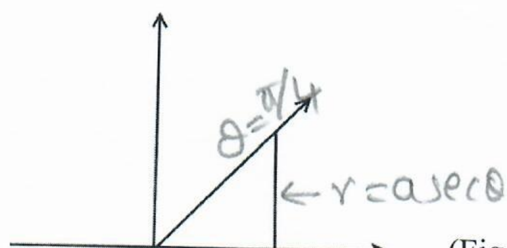
(Fig 0.5M)

$$V = 2\int_0^{\pi}\int_0^{a\sin\theta}\int_0^{\sqrt{a^2-r^2}} r\,dz\,dr\,d\theta \qquad (0.5M)$$

$$= \frac{2a^3(3\pi-4)}{9} \qquad (1M)$$

15. Evaluate $\int_0^a\int_y^a \frac{x}{x^2+y^2}\,dx\,dy$ by changing to polar coordinates.

Put $x = r\cos\theta, y = r\sin\theta, J = r.$



(Fig 0.5M)

$$\int_0^a\int_y^a \frac{x}{x^2+y^2}\,dx\,dy = \int_0^{\frac{\pi}{4}}\int_0^{a\sec\theta} \cos\theta\,dr\,d\theta \qquad (1M)$$

$$= \frac{a\pi}{4} \qquad (0.5M)$$