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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

FIRST SEMESTER B.E DEGREE END SEMESTER EXAMINATION – NOVEMBER, 2008

SUB: ENGG.MATHEMATICS I (MAT – 101)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All question carry equal marks.

1A. Find the n^{th} derivative of

(i) $\dfrac{4x}{(x-1)^2(x+1)}$ (ii) $e^{2x}\sin^2 x\cos 3x$

1B. Trace the curve with explanation $a^2 y^2 = x^2(a^2 + x^2 + y^2)$ $(a > 0)$.

1C. The plane $x - y - z = 2$ is rotated through 90° about its line of intersection with the plane $x + 2y + z = 2$. Find its equation in the new position.

(4 + 3+ 3)

2A. If $y^{1/m} + y^{-1/m} = 2x$, show that

$\left(x^2-1\right)y_{n+2} + (2n+1)xy_{n+1} + \left(n^2-m^2\right)y_n = 0$.

2B. Evaluate :

(i) $\displaystyle\int_0^1 x^5\sqrt{\frac{1+x^2}{1-x^2}}\,dx$ (ii) $\displaystyle\int_0^{2a} x^5\left(2ax-x^2\right)^{-1/2}dx$

2C. Prove that the line

$\dfrac{x+1}{1} = \dfrac{y+1}{2} = \dfrac{z+1}{3}$

and $x + 2y + 3z - 8 = 0 = 2x + 3y + 4z - 11$ are coplanar and find the co-ordinates of the point of intersection. Find the equation of the plane containing them.

(4 + 3+ 3)

3A. Test the nature of the following series

(i) $1 + \dfrac{2}{5}x + \dfrac{6}{9}x^2 + \dfrac{14}{17}x^3 + ...$

(ii) (ii) $\left(\dfrac{2^2}{1^2}-\dfrac{2}{1}\right)^{-1} + \left(\dfrac{3^3}{2^3}-\dfrac{3}{2}\right)^{-2} + \left(\dfrac{4^4}{3^4}-\dfrac{4}{3}\right)^{-3} + \ldots$

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3B. Sketch and find the length of the curve
$8a^2y^2 = x^2 (a^2 - x^2)$; $(a > 0)$

3C. Find the evolute of
$x = a\left(\cos t + \log\left(\tan t/2\right)\right)$, $y = a \sin t$

(4 + 3+ 3)

4A. Evaluate :

(i) $\lim_{x \to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$ (ii) $\lim_{x \to 0}\left(\frac{1}{x^2} - \cot^2 x\right)$

4B. Find the angle between the curves $r = a\log\theta$, $r = \frac{a}{\log\theta}$.

4C. A plane passes through a fixed point (a, b, c) and cuts the axes at A, B, C. Show that the locus of the center of sphere OABC is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$.

(4 + 3+ 3)

5A. Find the fist three nonzero terms in the Maclurin's series expansion of $f(x) = \log(\sec x)$

5B. Find the radius of curvature at point (– 2a, 2a) on the curve $x^2y = a (x^2 + y^2)$.

5C. Find the volume bounded by revolving the curve $r = a (1 - \cos\theta)$ about the initial line.

(4 + 3+ 3)

6A. (i) Verify Euler's theorem for $z = \sin^{-1}\left(x/y\right) + \tan^{-1}\left(y/x\right)$

(ii)If $v = r^m$, $r^2 = x^2 + y^2 + z^2$, show that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = m(m+1)r^{m-2}$.

6B. Verify Cauchy's mean value theorem for $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{x}$ in [a, b]

6C. At a distance of 50 meters from foot of the tower the elevation of its top is 30^o. If the possible errors in measuring the distance and elevation are 2cm and 0.05 degrees, find the approximate error in calculating the height.

(4 + 3+ 3)
