MAT 2155: PROBLEM SET 1

1. Number of ways of selecting one left and one right shoe from six pairs of shoes, without obtaining a pair.
2. Number of ways of picking two books, not both in the same language, from a collection of five different English books, six different German books, and seven different French books.
3. Number of positive integers less than 1 million formed using
   (i) 7s, 8s, and 9s only.
   (ii) 0s, 8s, and 9s only.
4. Sum of all 4-digit numbers that can be obtained using the digits
   (i) 1, 2, 3, 4 once each.
   (ii) 0, 1, 2, 3 once each.
   (iii) 1, 2, 3, 4.
   (iv) 0, 1, 2, 3.
5. Number of 3-letter sequences using the letters a, b, c, d, e, f
   (i) with repetition allowed.
   (ii) without repetition.
   (iii) without repetition, and containing the letter e.
   (iv) with repetition, and containing e.
6. Five people A, B, C, D, E intend to speak at a meeting. In how many ways can they do so
   (i) without B speaking before A?
   (ii) if A must speak immediately before B?
7. Number of ways of selecting three integers from $3n$ consecutive integers so that their sum is a multiple of 3.
8. Number of times the digit 5 is written when listing all numbers from 1 to $100,000$.
9. Number of times 25 is written when listing all numbers from 1 to $100,000$.
10. Number of four-digit numbers divisible be 4 that can be formed using the digits
    (i) 1, 2, 3, 4, 5 with possible repetition.
    (ii) 1, 2, 3, 4, 5 without repetition.
    (iii) 0, 1, 2, 3, 4 with possible repetition.

    (iv) 0, 1, 2, 3, 4 without repetition.

11. Number of ways of wearing five rings on four finger (not including the thumb) of your right hand.

12. Six distinct symbols are transmitted through a communication channel. A total of twelve blanks are to be inserted between the symbols with at least two blanks between every pair of symbols. In how many ways can we arrange the symbols and blanks?

13. Number of ways of placing two black queens on an 8×8 chessboard so that they are not attacking each other.

14. How many points of intersection are formed by n lines drawn in a plane if no two are parallel and no three concurrent? Into how many regions is the plane divided?

15. If a convex decagon is such that no three of its diagonals meet at the same point inside it, then how many segments are the diagonals divided into by their intersections?