Binary adders and subtractors

- Half adder, full adder, parallel adder
- Half subtractor , full subtractor, parallel subtractor
- Subtraction using complements, parallel adder/subtractor
- Carry Look ahead adder, Decimal adder

Binary Addition

3bit addition

Carry to Next location

4-bit binary operation

MSB LSB

- 1 1 1 0 — intermediate Carry
- 1st → 1 0 1 0 + — Operand 1 — Binary addition
- 2nd → 0 1 1 1 — Operand2
- 1 0 0 0 1 (sum) — Sum

Final carry

0 1
0 1
1
10

Bit-wise addition

→ two, one-bit addition

→ addition of three one-bits

a_0 →, b_0 → [two one-bit addition] → Sum-bit S, → Carry-bit C_{out}

Half Adder.

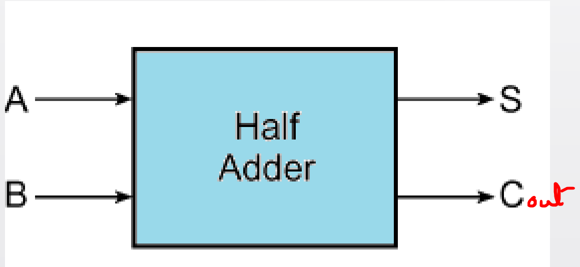
a_0 →, b_0 →, C_{in} → [three one-bit addition] → Sumbit S, → Carry-bit C_{out}

Full-Adder

Half adder(HA)

- Adds 2, 1-bit numbers A and B , generated two outputs sum(S) and carry (C).

I/P



4 combinations

A	B	S	C_{out}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Expression for sum and carry :

SOP ... POS

$S(A,B) = S = \sum_m 1, 2$ or $\prod_M 0, 3$

$C_{out}(A,B) = C_{out} = \sum_m 3$ or $\prod_M 0, 1, 2$

$S = \bar{A}B + A\bar{B} = A \oplus B$

$C_{out} = A.B$

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	0	1
A 1	1	0

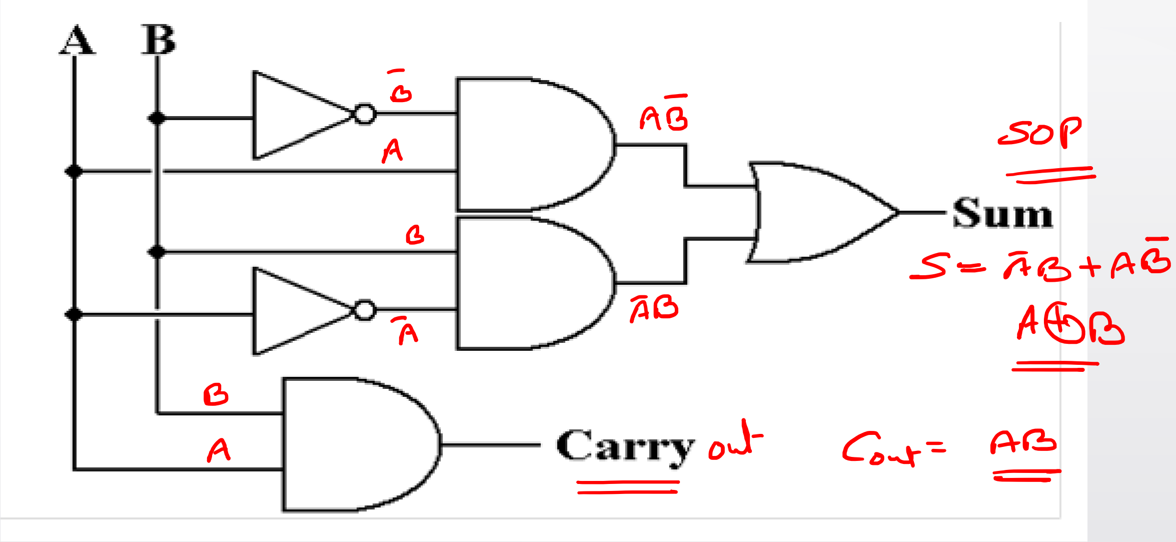
$S = \bar{A}B + A\bar{B}$

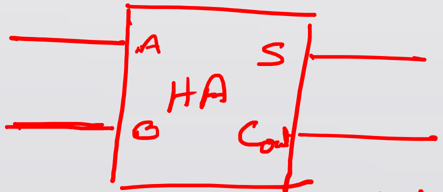
A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	0	0
A 1	0	1

$C_{out} = A.B$

HA circuit

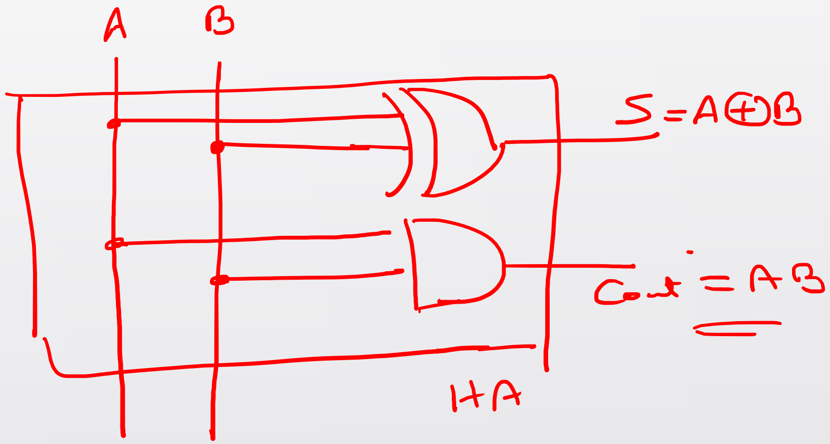
Using basic logic gates

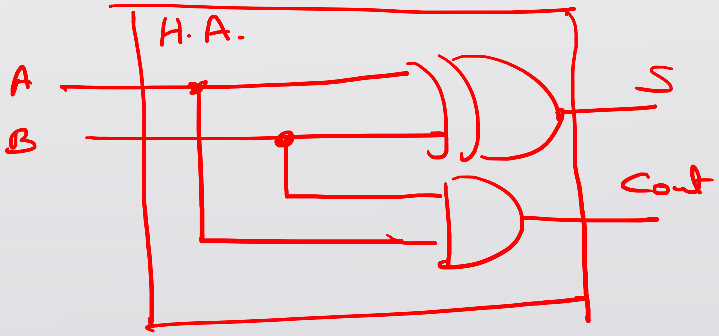




Symbolic Representation

Using XOR and AND gate



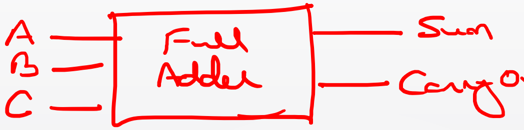


Full adder

Truth Table

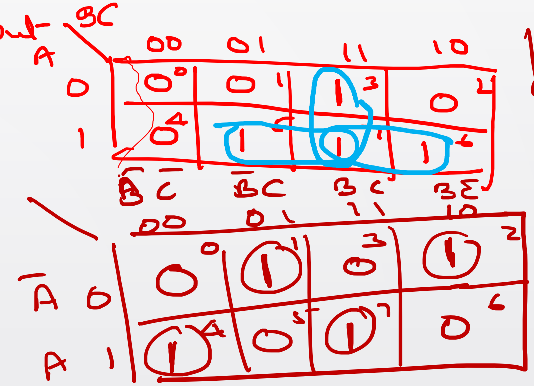
$A, B, C \rightarrow 3$, one-bit inputs. each $\rightarrow$ 0 or 1

A B C (MSB … LSB)	Carry	Sum
0 0 0	0	0
0 0 1	0	1
0 1 0	0	1
0 1 1	1	0
1 0 0	0	1
1 0 1	1	0
1 1 0	1	0
1 1 1	1	1



$$\text{Carry} = \sum_m 3, 5, 6, 7$$

$$\text{Sum} = \sum_m 1, 2, 4, 7$$



$$\text{Carry} = BC + AC + AB$$

$$(m_3, m_7)\quad (m_5, m_7)\quad (m_6, m_7)$$

$$\text{Sum} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

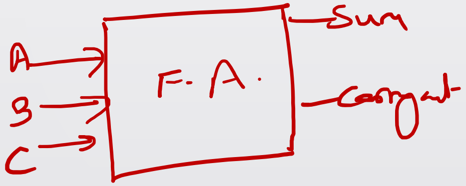
$$= \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC)$$

$$= \bar{A}(\underbrace{B \oplus C}_{x}) + A(\underbrace{\overline{B \oplus C}}_{\bar{x}})$$

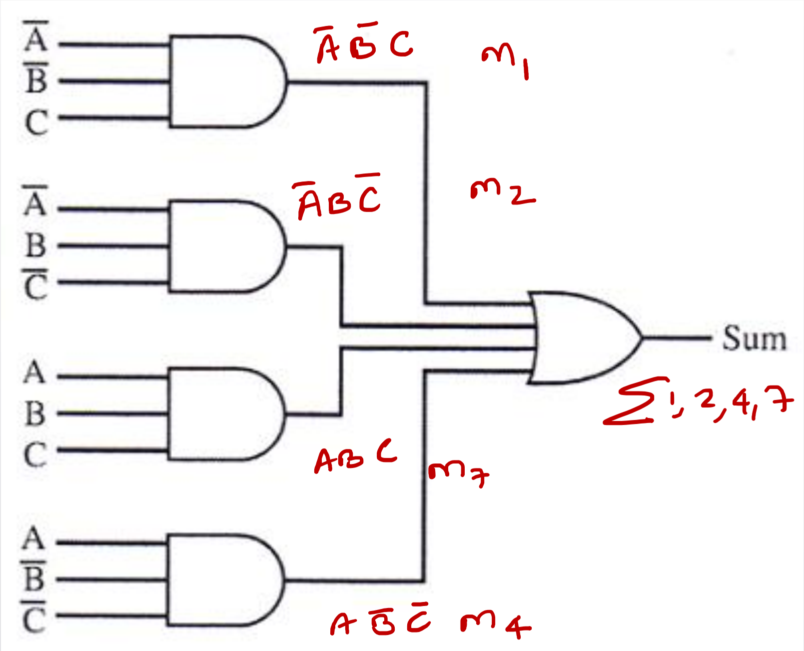
$$= \bar{A}x + A\bar{x}$$

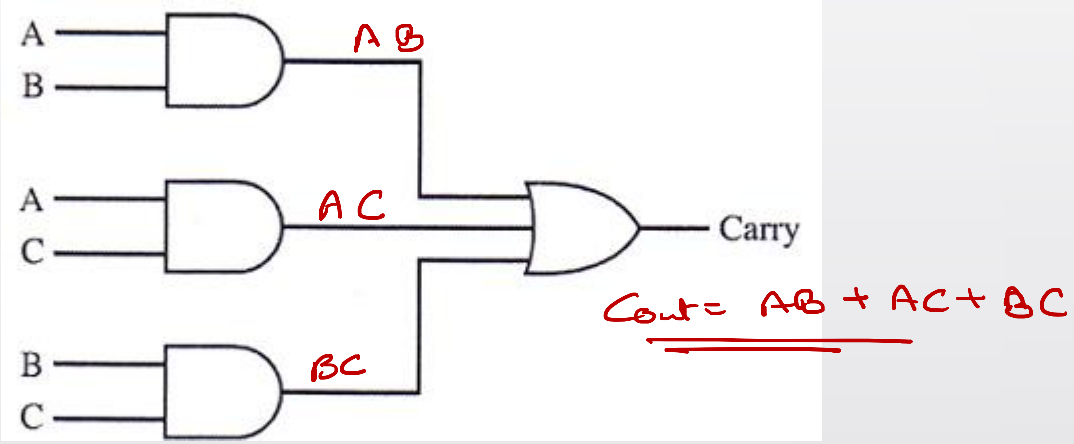
$$= A \oplus x$$

$$\boxed{\text{Sum} = A \oplus B \oplus C}$$



FA circuit using basic logic gates

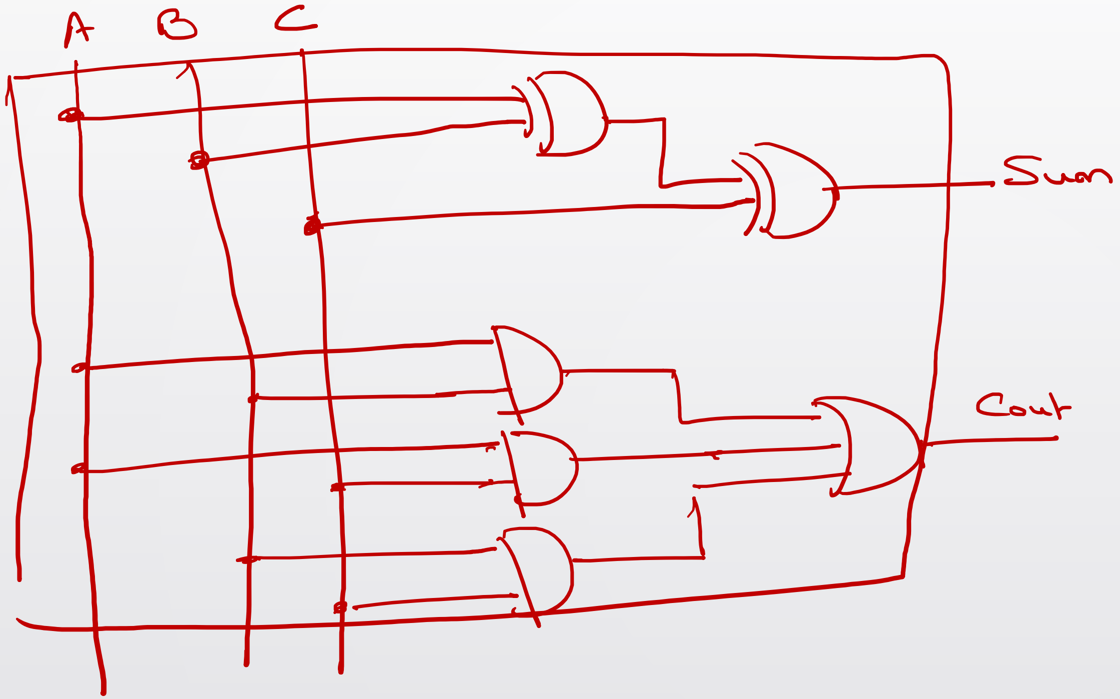


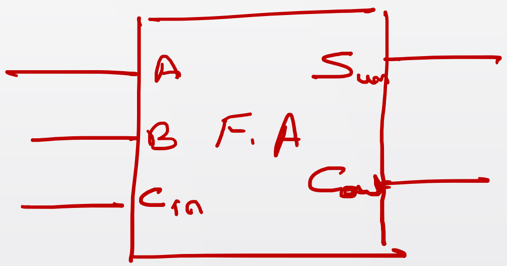


$Sum = \Sigma 1,2,4,7 = A \oplus B \oplus C$

$C_{out} = AB + AC + BC$

Full adder circuit using XOR operations





FA using 2 HA s and one external gate

OR Gate

Expression for F.A

$Sum = A \oplus B \oplus C$

$Carry\ out = AB + BC + AC$

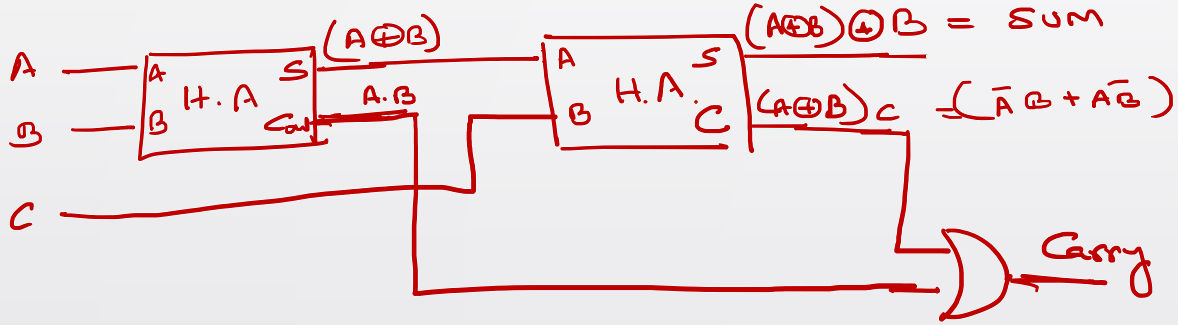
from H.A.

$S = A \oplus B$

$C_{out} = A \cdot B$

$S \oplus C$

$Sum = (A \oplus B) \oplus C$



$= (\bar{A}B + A\bar{B})C = \bar{A}BC + A\bar{B}C$

$m_3 \quad m_5$

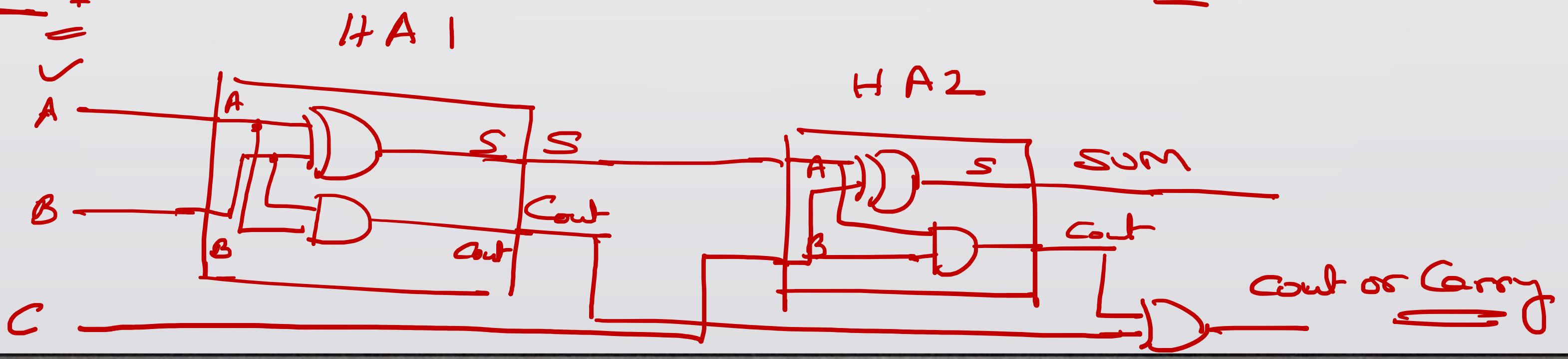
$(m_3 + m_5) + (m_6 + m_7) = C_{out} = Carry$

↓ OR

$AB = AB(\bar{C} + C)$

$= AB\bar{C} + ABC$

$m_6 \quad m_7$



4-bit Parallel adder using FA blocks

- Consider addition of 2, 4-bit numbers: (A_3 A_2 A_1 A_0) and (B_3 B_2 B_1 B_0) → Results a 5-bit Number

1-bit carry + 4-bit sum

A → 4-bit Number

B → 4-bit Number

C_3 C_2 C_1 C_0 0

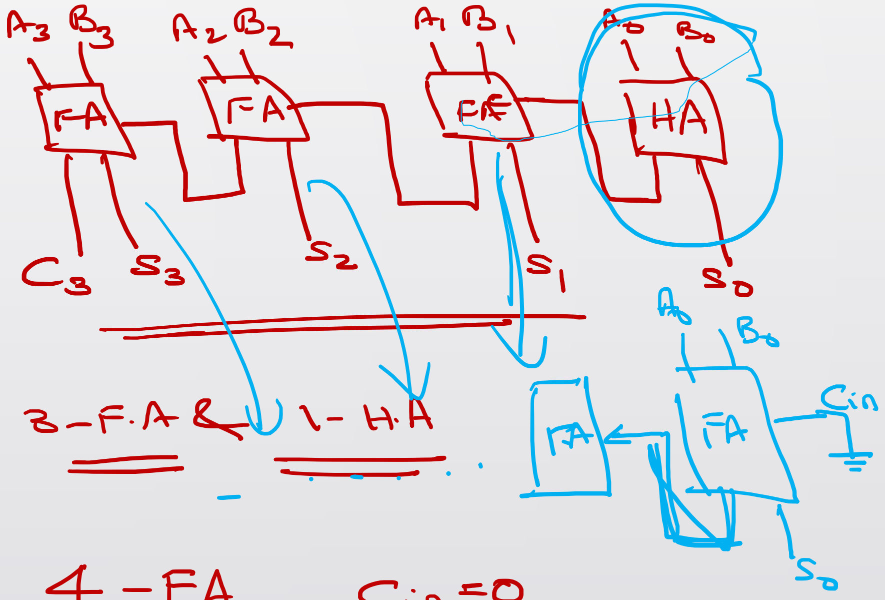
MSB A_3 A_2 A_1 A_0 LSB

B_3 B_2 B_1 B_0

C_3 S_3 S_2 S_1 S_0

Carry 1-bit

4-bit Sum



FA_3 FA_2 FA_1 HA → 3-F.A & 1-H.A

Cin = 0

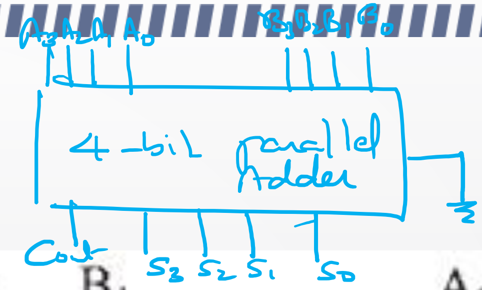
FA_3 FA_2 FA_1 FA_0 → 4-FA

Cin = 0

4-bit parallel adder

Added in parallel

Also called as Carry Propagation Adder (CPA)

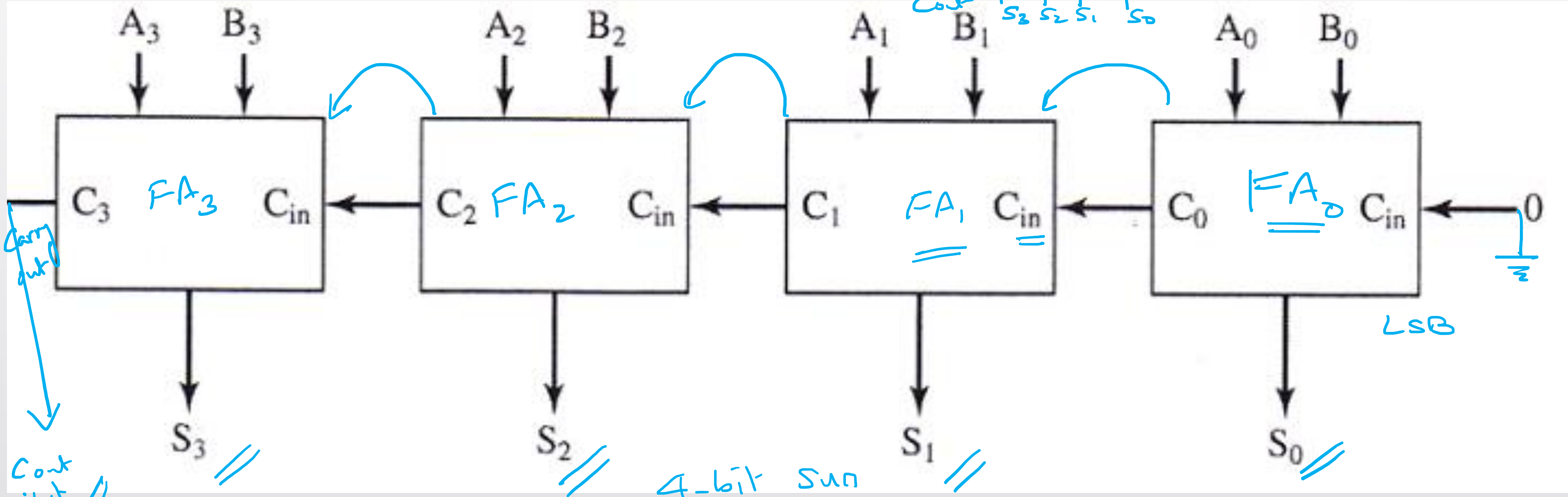


C_0

$A_3\ A_2\ A_1\ A_0$

$B_3\ B_2\ B_1\ B_0$

$S_1\ S_0$



Cout 1 bit

4-bit sum

5 bit output

ripple carry adders

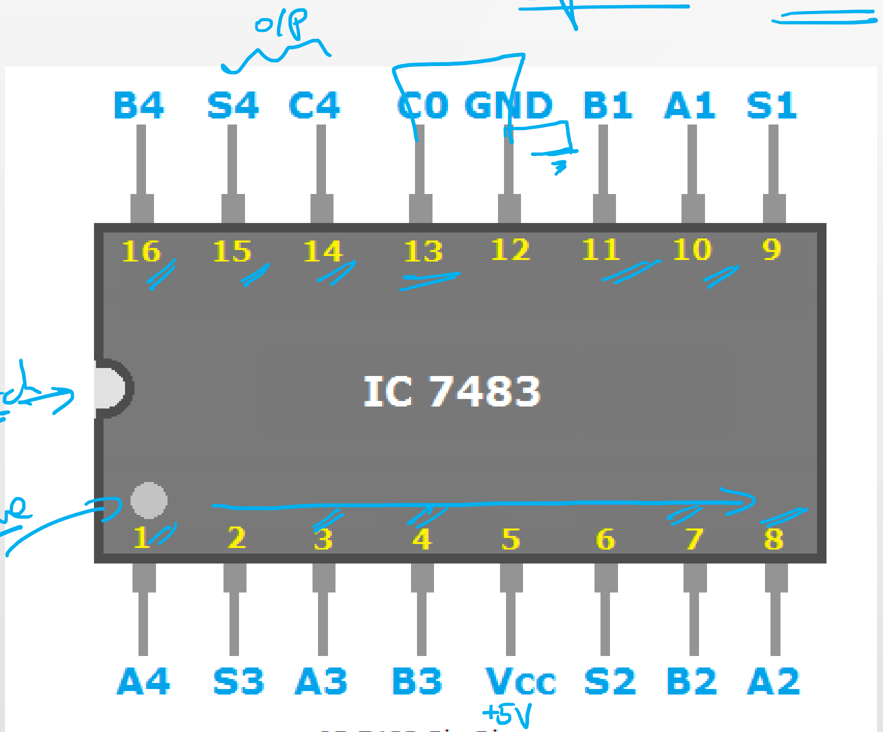
carry ripples

7483 IC : 4-BIT PARALLEL ADDER

Pin diagram of 7483

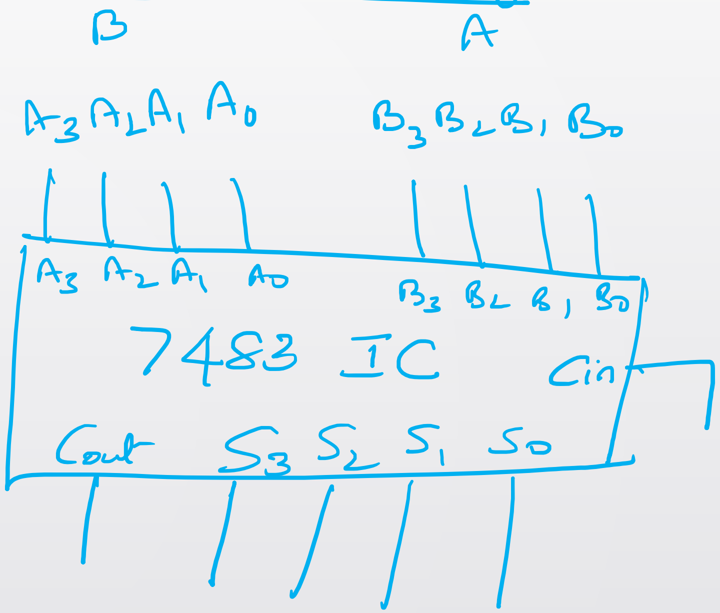
4-bit Binary adders

16 pin IC DIP



IC 7483 Pin Diagram

Schematic-diagram



C_0

A_0 + B_0 → Binary Addition

S_0

NOT an OR logic

Half subtractor

- Write the truth table and circuit for half subtractor

$A - B \rightarrow$ A & B are 1-bit Numbers

Arithmatic minus

otherwise

Truth-Table

A	B	Diff	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

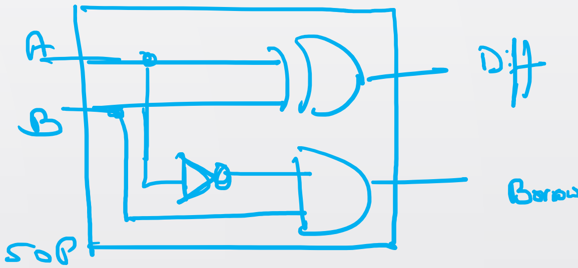
$$\text{Diff} = \sum_m 1, 2 \quad \text{SOP}$$

$$= \prod_M 0, 3 \quad \text{POS}$$

$$\text{Diff} = \bar{A}B + A\bar{B}$$

$$\boxed{\text{Diff} = A \oplus B}$$

H.S.



$$\text{Borrow} = \sum_m 1 \quad \text{SOP}$$

$$= \prod_M 0, 2, 3 \quad \text{POS}$$

$$\boxed{\text{Borrow} = \bar{A}B}$$

Full subtractor

DIFFERENCE (D) = X-Y-Z, Borrow (B)

X	Y	Z	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Expressions for D and B:

$D = \sum_m 1, 2, 4, 7 \qquad \prod_M (0, 3, 5, 6)$

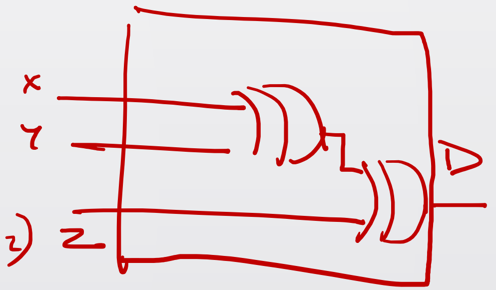
x \ yz	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$D = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

$= \bar{x}(\bar{y}z + y\bar{z}) + x(\bar{y}\bar{z} + yz)$

$= \bar{x}(y \oplus z) + x(\overline{y \oplus z})$

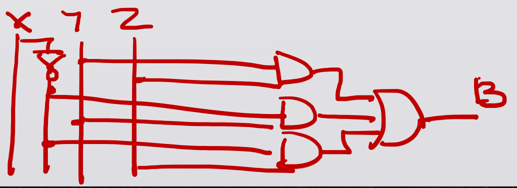
$\boxed{D = x \oplus y \oplus z}$



x \ yz	00	01	11	10
0	0	1	1	1
1	0	0	1	0

$B = \sum_m 1, 2, 3, 7 = \prod_M (0, 4, 5, 6)$

$\boxed{B = \bar{x}z + \bar{x}y + yz}$



FS circuit

- Draw the circuit for FS using
- (i) basic logic gates only
- (ii) XOR and basic logic gates

I/P

x y z

O/P

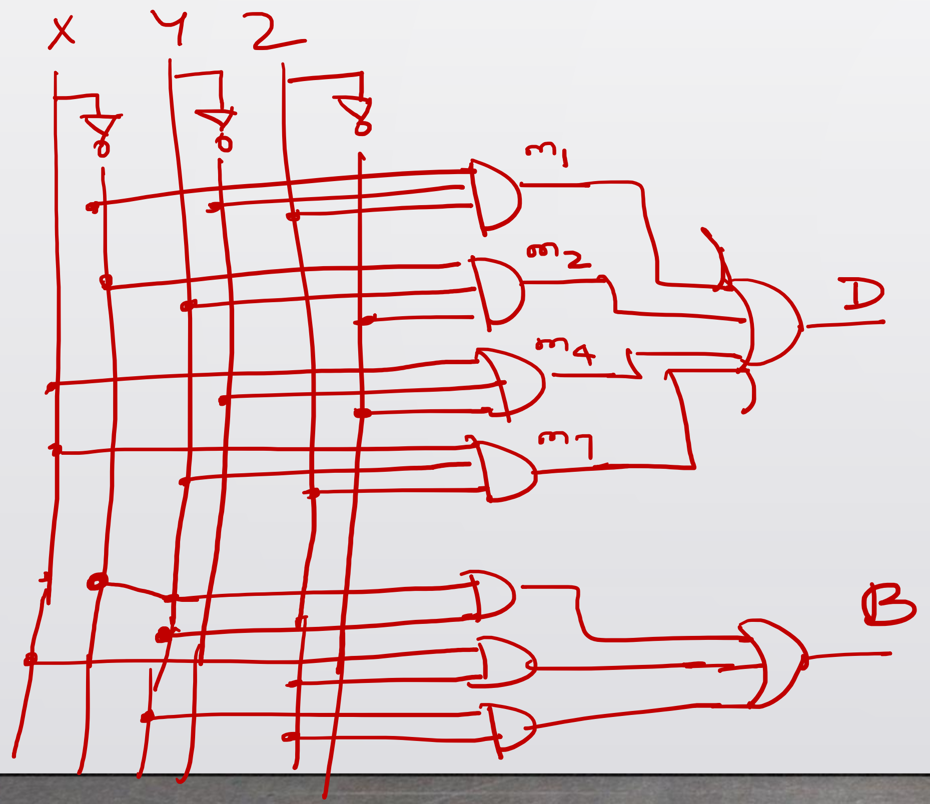
D B

$B = \bar{x}y + \bar{x}z + yz = \sum 1, 2, 3, 7$

$D = x \oplus y \oplus z$

$= \sum 1, 2, 4, 7$

i)



Full subtractor using 2 HS s and one external gate

H.S

$$\boxed{Diff = x \oplus y}$$

$$Borrow = \bar{x}y$$

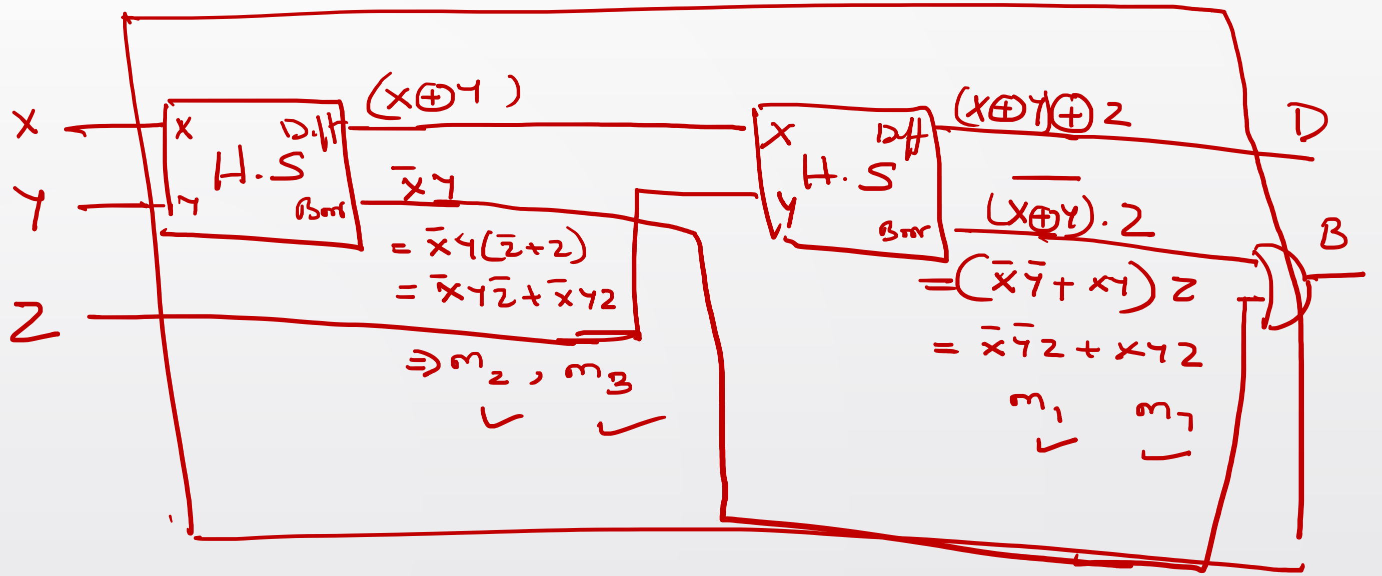
F.S

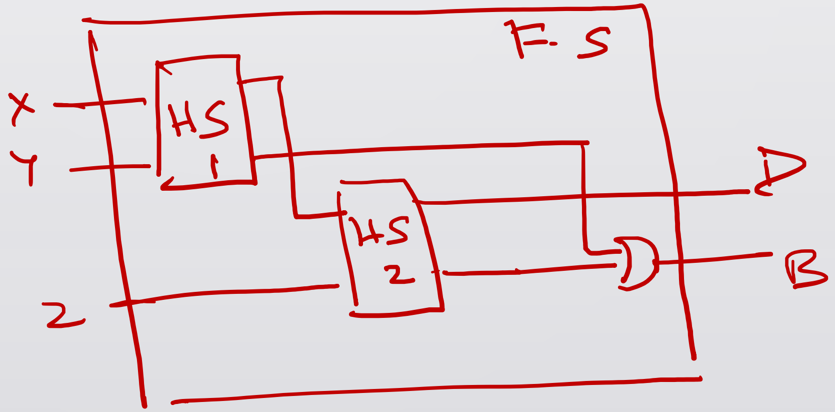
$$\boxed{D = x \oplus y \oplus z}$$

$$B = \bar{x}y + \bar{x}z + yz$$

$$D = \sum_m 1, 2, 4, 7$$

$$B = \sum_m 1, 2, 3, 7$$

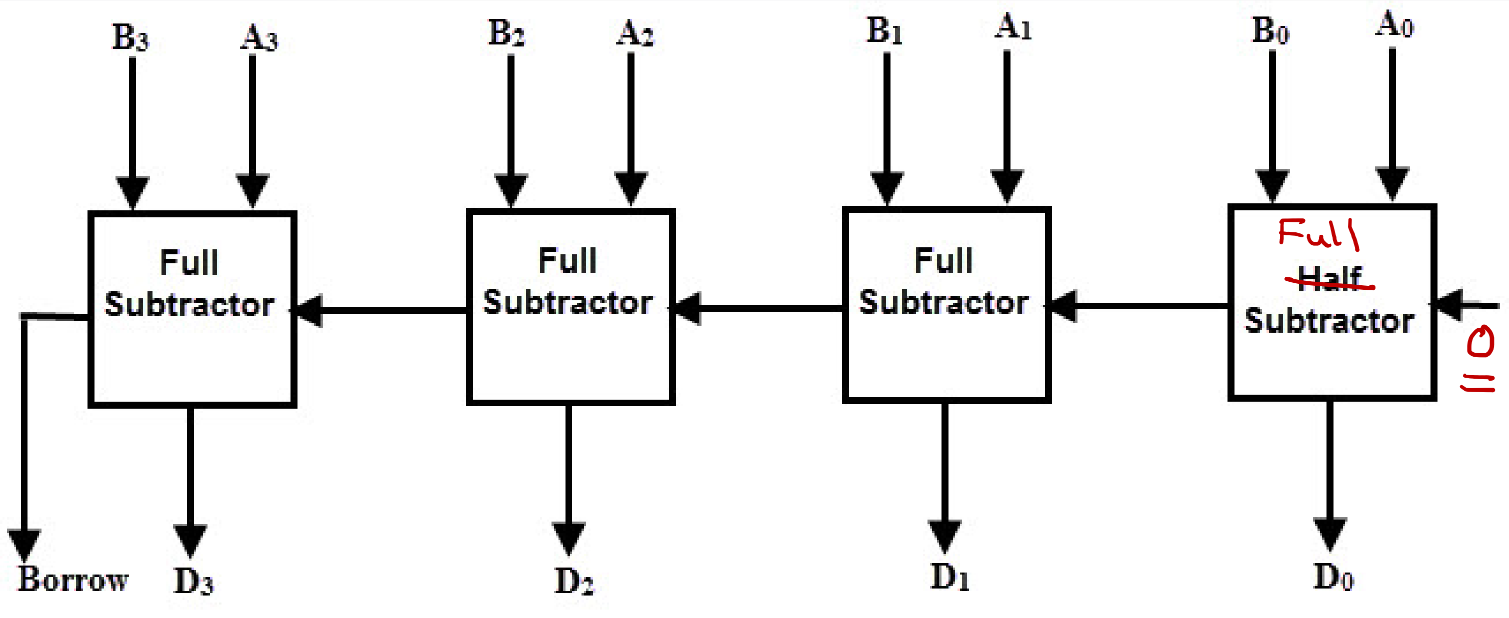




4-bit parallel subtractor using FS blocks

Consider subtraction of 2, 4-bit numbers: (A3 A2 A1 A0) and (B3 B2 B1 B0)

$$\begin{array}{r} A \\ -\ B \\ \hline \end{array} \qquad \begin{array}{cccc} A_3 & A_2 & A_1 & A_0 \\ B_3 & B_2 & B_1 & B_0 \\ \hline \end{array} \quad \text{LSB}$$



1 0 0 0 0 0
1 1 1 1 1

Subtraction using complements

- Using 2's complement method
- Using 1's complement method

2's complement method for realizing subtraction

$$\begin{array}{r} 8 \\ -2 \\ \hline 6 \end{array} \rightarrow \begin{array}{r} 1000 \\ -0010 \\ \hline \end{array}$$

as it is: 1000

2's → 1's comp + 1

1's: 1101 } 2's comp

+ 1

$\boxed{1}\,0110$

+ve & drop carry

Ans +6

1111

0010

1's 1101

0111

$2 = 0010$

$-8 = 1000$

→ 0010

→ 1's 0111 } 2's comp

+1   1

$\boxed{0}$ 1010

-ve

Ans is in 2's complement

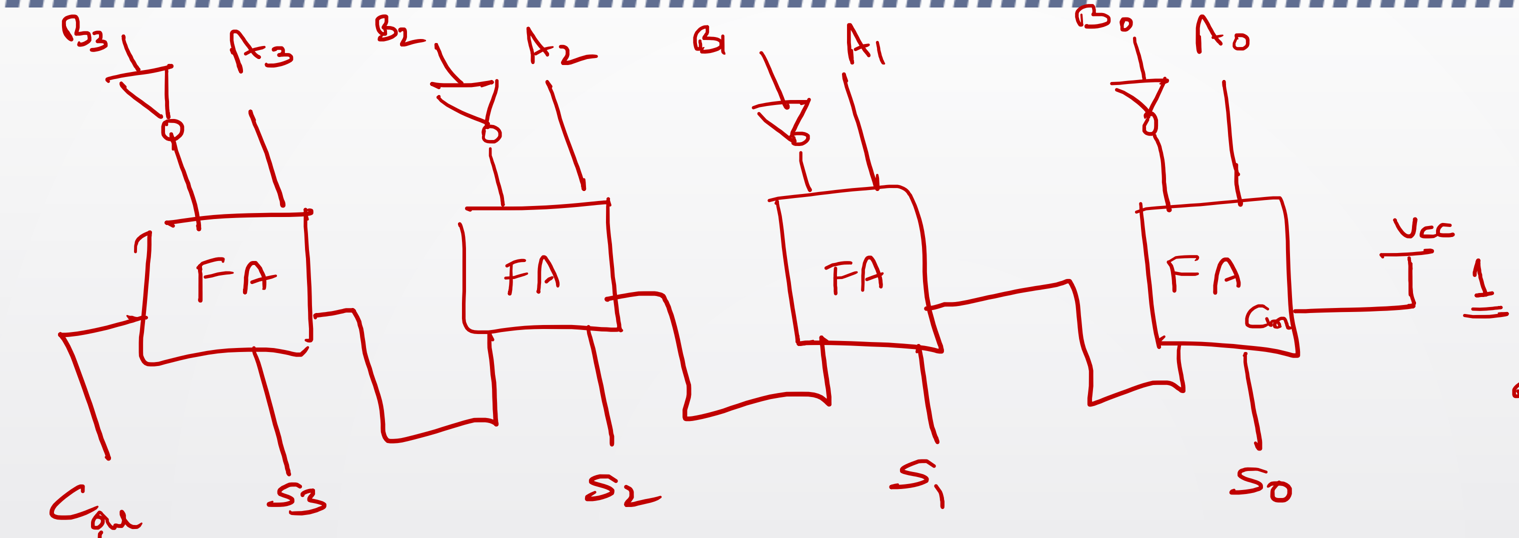
-6

0101

1

0110   6

Subtraction using complements



$A_3\ A_2\ A_1\ A_0$

$\bar{B}_3\ \bar{B}_2\ \bar{B}_1\ \bar{B}_0$

$1 \leftarrow C_{in}$ } 2's cmp of B

+1 → +ve Ans is in its form

drop carry

0 → −ve Ans is in its 2's complement form

drop carry