KARNAUGH MAP (K – MAP)

LECTURE 5 & 6

K – MAP

- Pictorial form of a truth table.
- Graphical tool to simplify a logical equation by forming groups of cells.

TWO VARIABLE K – MAP

$F(A, B) = 2^2$

A	B	SOP	POS
0	0	$\overline{A}\,\overline{B}$	$A + B$
0	1	$\overline{A}B$	$A + \overline{B}$
1	0	$A\overline{B}$	$\overline{A} + B$
1	1	AB	$\overline{A} + \overline{B}$

A. SOP: -

A \ B	$\overline{B}$ 0	B 1
$\overline{A}$ 0	$\overline{A}.\overline{B}$	$\overline{A}.B$
A 1	$A.\overline{B}$	$A.B$

B. POS: -

A \ B	B 0	$\overline{B}$ 1
A 0	$A+B$	$A+\overline{B}$
$\overline{A}$ 1	$\overline{A}+B$	$\overline{A}+\overline{B}$

EXAMPLE 1:

A	B	F
0	0	1
0	1	1
1	0	0
1	1	0

$2^0, 2^1, 2^2, 2^3, 2^4, 2^5$

$1, 2, 4, 8, 16, 32 \ldots$

SOP

POS

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	1	1
A 1	0	0

$\bar{A}$

$$F = \bar{A}\bar{B} + \bar{A}B$$

$$= \bar{A}[\bar{B} + B] = \bar{A}$$

$$F = \bar{A}$$

A \ B	B 0	B 1
A 0	1	1
$\bar{A}$ 1	0	0

$\bar{A}$

$$(\bar{A} + B)(\bar{A} + \bar{B})$$

$$F = \bar{A}$$

EXAMPLE 2:

A	B	F
0	0	1
0	1	1
1	0	0
1	1	1

SOP

$$F = \bar{A}\bar{B} + \bar{A}B + AB$$

$$= \bar{A}\bar{B} + \bar{A}B + \bar{A}B + AB$$

$$= \bar{A}[\bar{B} + B] + B[\bar{A} + A]$$

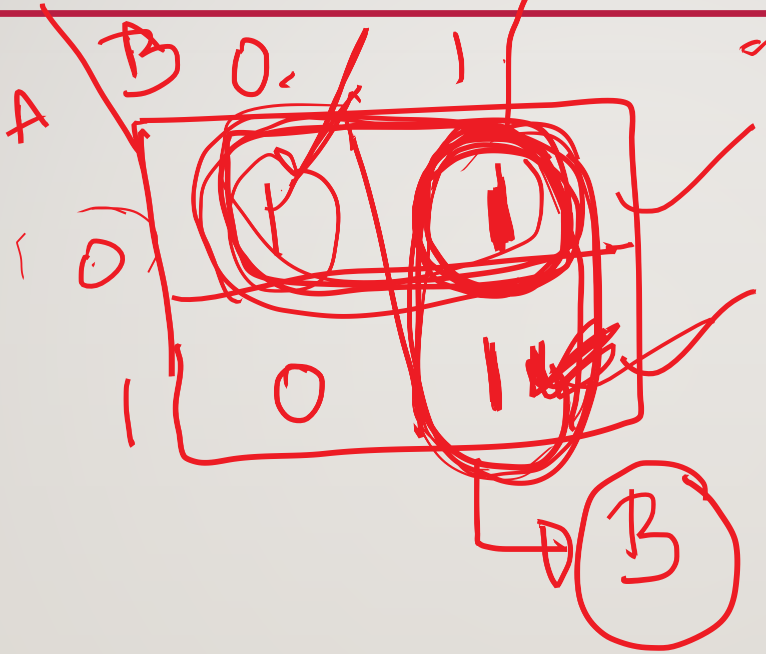
$$= \bar{A} + B$$

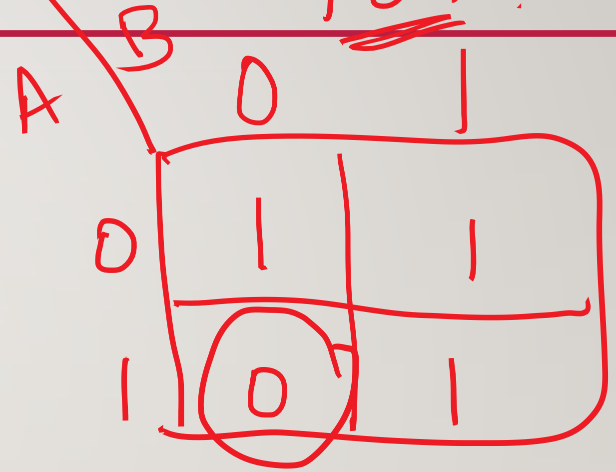
POS

$A + A +$

$+ A +$

$= A$





$$F = \bar{A} + B$$

$$F = \bar{A} + B$$

EXAMPLE 3:

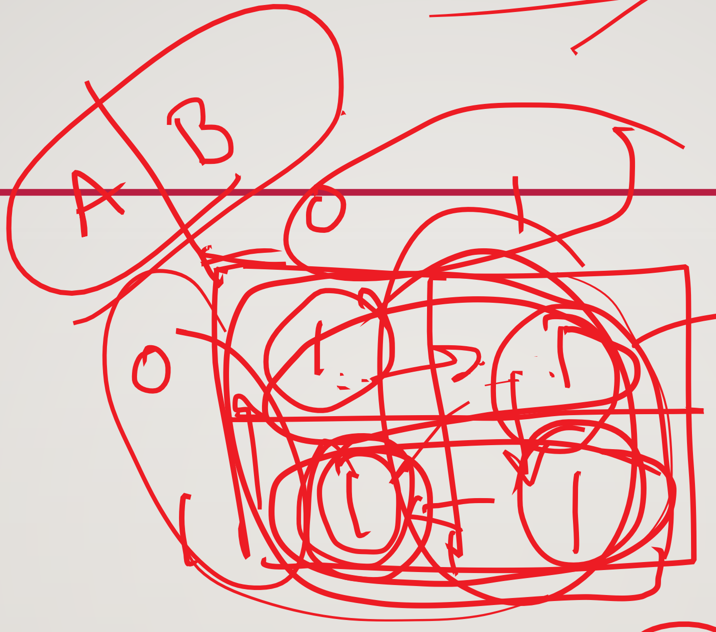
A	B	F
0	0	1
0	1	1
1	0	1
1	1	1

$$F = \bar{A}\bar{B} + A\bar{B} + \bar{A}B + AB$$

$$= \bar{B}[\bar{A} + A] + B[\bar{A} + A]$$

$$= \bar{B} + B$$

$$= 1$$



$$F = 1$$

THREE VARIABLE K – MAP

$2^3 = 8$

00 01 11 10

	A	B	C	SOP	POS
0	0	0	0	$\overline{A}\,\overline{B}\,\overline{C}$	$A+B+C$
1	0	0	I	$\overline{A}\,\overline{B}C$	$A+B+\overline{C}$
2	0	I	0	$\overline{A}B\overline{C}$	$A+\overline{B}+C$
3	0	I	I	$\overline{A}BC$	$A+\overline{B}+\overline{C}$
4	I	0	0	$A\overline{B}\,\overline{C}$	$\overline{A}+B+C$
5	I	0	I	$A\overline{B}C$	$\overline{A}+B+\overline{C}$
6	I	I	0	$AB\overline{C}$	$\overline{A}+\overline{B}+C$
7	I	I	I	ABC	$\overline{A}+\overline{B}+\overline{C}$

BC \ A	0	1
00	0	4
01	1	5
11	3	7
10	2	6

A. SOP: -

A \ BC	$\overline{B}\,\overline{C}$ 0 0	$\overline{B}C$ 0 1	BC 1 1	$B\overline{C}$ 1 0
$\overline{A}$ 0	$\overline{A}\,\overline{B}\,\overline{C}$ 0	$\overline{A}\,\overline{B}C$ 1	$\overline{A}BC$ 3	$\overline{A}B\overline{C}$ 2
A 1	$A\overline{B}\,\overline{C}$ 4	$A\overline{B}C$ 5	ABC 7	$AB\overline{C}$ 6

B. POS: -

A \ B+C	$B+C$ 0 0	$B+\overline{C}$ 0 1	$\overline{B}+\overline{C}$ 1 1	$\overline{B}+C$ 1 0
A 0	$A+B+C$ 0	$A+B+\overline{C}$ 1	$A+\overline{B}+\overline{C}$ 3	$A+\overline{B}+C$ 2
$\overline{A}$ 1	$\overline{A}+B+C$ 4	$\overline{A}+B+\overline{C}$ 5	$\overline{A}+\overline{B}+\overline{C}$ 7	$\overline{A}+\overline{B}+C$ 6

EXAMPLE 1:

Given the Boolean function:

$$F = \bar{A}C + \bar{A}B + A\bar{B}C + BC$$

- Express it in Sum of minterms form.
- Find the minimal sum of products expression.

0 1 3 2

00 01 11 10

3 - Variable

$A\bar{B}C = 101 = 5 = 4+1 = 5$

$\bar{A}\bar{B}C = 001 = 1$

$\bar{A}C[B+\bar{B}] + \bar{A}B[C+\bar{C}] + A\bar{B}C + BC[\bar{A}+A]$

$= \bar{A}BC + \bar{A}\bar{B}C + A\bar{B}C + \bar{A}BC + ABC + \bar{A}B\bar{C} + AB$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0 (0)	1 (1)	1 (3)	1 (2)
A 1	0 (4)	1 (5)	1 (7)	0 (6)

C, $\bar{A}B$

$$F = \Sigma(1, 2, 3, 5, 7)$$

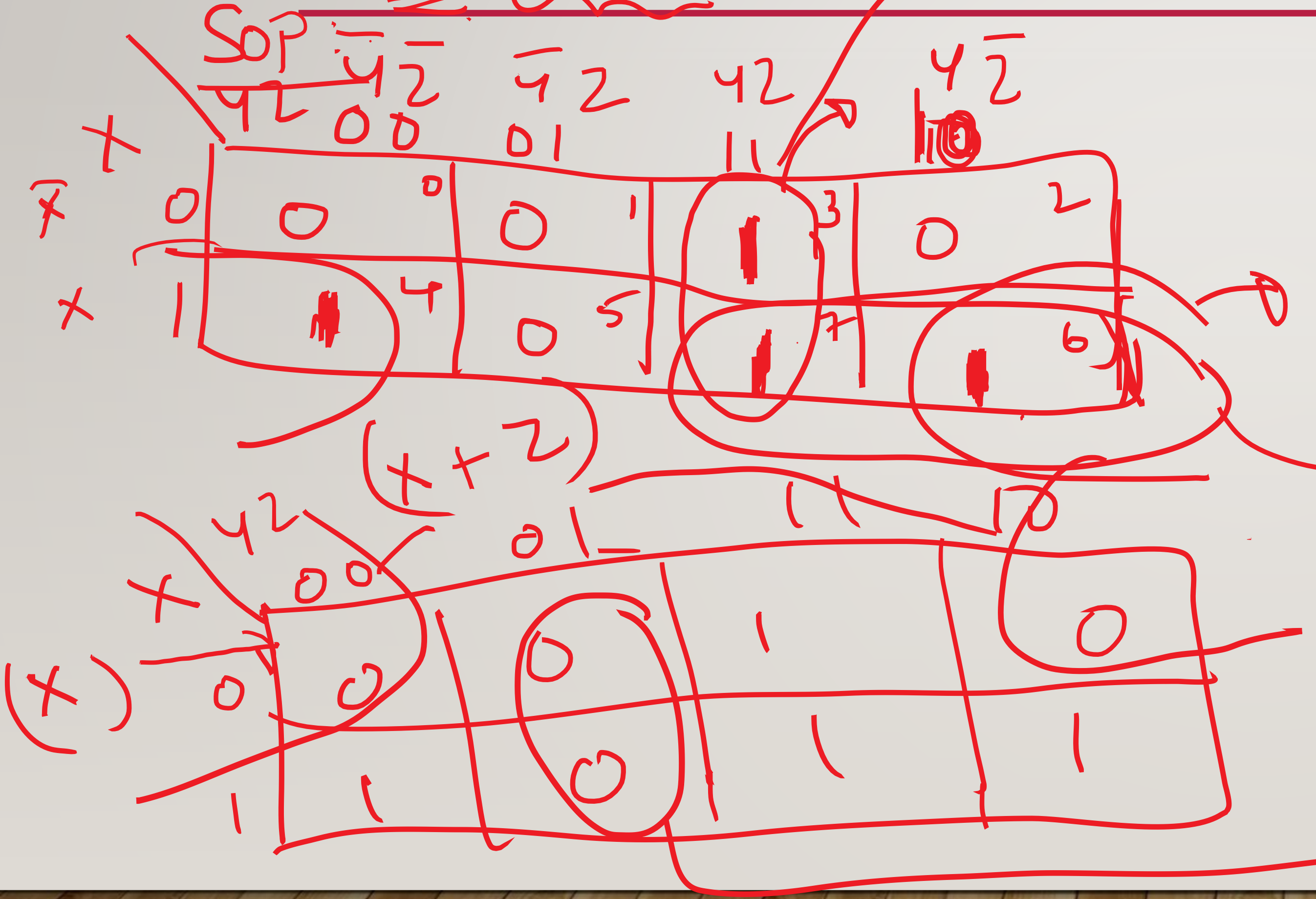
$$F = C + \bar{A}B$$

EXAMPLE 2:

Simplify the Boolean expression into (i) SOP and implement using NAND gates (ii) POS and implement using NOR gates:

$$F(x, y, z) = \sum(3,4,6,7)$$

SOP



$F = yz + x\bar{z}$ → NAND

$F = (y+\bar{z})(x+z)$ → NOR

EXAMPLE 3:

$$F(x,y,z) = \sum(0,2,4,5,6)$$

$\bar{z}$

SOP :-

x \ yz	00	01	11	10
0	1	0	0	1
1	1	1	0	1

$$F = \bar{z} + x\bar{y}$$

$x\bar{y}$

$(x+\bar{z})$

x \ yz	00	01	11	10
0	1	0	0	1
1	1	1	0	1

$$F = (x+\bar{z})(\bar{y}+\bar{z})$$

$(\bar{y}+\bar{z})$

EXAMPLE 4:

$$F(x,y,z) = \prod(0,2,5,7)$$

$\bar{x}z$

NAND

x \ yz	00	01	11	10
0	0	1	1	0
1	1	0	0	1

$\to x\bar{z}$

SOP

$$F = (\bar{x}z + x\bar{z})$$

x \ yz	00	01	11	10
0	0	1	1	0
1	1	0	0	1

POS :

$$F = (x+z)(\bar{x}+\bar{z})$$

NOR