LECTURE 5 & 6

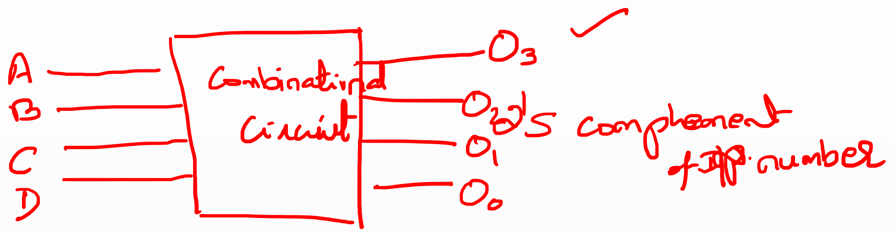
Karnaugh MAP (K – Map)

Contd…. Examples from last class

EXAMPLE 5:

Design a combinational circuit with 4- input lines that represents a decimal digit in BCD and 4- output lines that generates 2's complement of input digit.

8421 code



Binary Coded Decimal No. = 8421

	2^3 (8) A	2^2 (4) B	2^1 (2) C	2^0 (1) D	O_3	O_2	O_1	O_0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	1	1
2	0	0	1	0	1	1	1	0
3	0	0	1	1	1	1	0	1
4	0	1	0	0	1	1	0	0
5	0	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	0
7	0	1	1	1	1	0	0	1
8	1	0	0	0	1	0	0	0
9	1	0	0	1	0	1	1	1

Valid Numbers

1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1

both {1110, 1111}

Invalid → φ, d, D

0011
1's 1100
+ 1
1101

8 → 1000
1's 0111
2's { +1
1000

9 → 1001
1's 0110
+1
0111

pls note: use K-map

O_3

AB \ CD	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	φ	φ	φ	φ
10	1	0	φ	φ

$O_3 = B + \bar{A}D + C + A\bar{D}$

O_2

AB \ CD	00	01	11	10
00	0	1	1	1
01	1	0	0	0
11	φ	φ	φ	φ
10	0	1	φ	φ

$O_2 = \bar{B}D + \bar{B}C + B\bar{C}\bar{D}$

O_1

AB \ CD	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	φ	φ	φ	φ
10	0	1	φ	φ

$O_1 = \bar{C}D + C\bar{D}$

$O_1 = C \oplus D$

O_0

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	φ	φ	φ	φ
10	0	1	φ	φ

$O_0 = D$

EX[redacted]AMPLE 7:

Des[redacted]n a combinational circuit that multiplies by '5' an input decimal digit represented in BCD. The out[redacted]t is also in BCD.

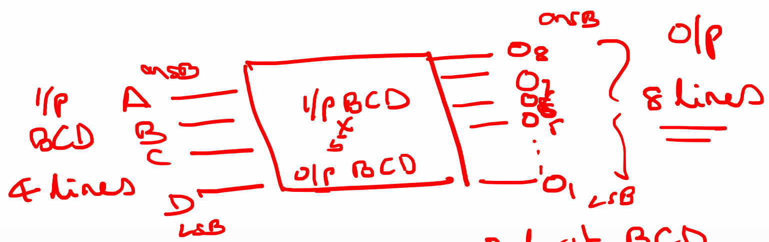
8421 Every digit is represented using BCD

No. o/p variables

How many K-maps ? ⇒ 8 one each for each o/p

what variable K-maps should be used ?

4 variable i/p variable



$O_8 = 0$

$O_4 = 0,\ O_2 = 0$

O_1 & $O_3 = D$

2 digit BCD

ABCD	×5		$O_8O_7O_6O_5$	$O_4O_3O_2O_1$
0000	0 × 5	= 0	0000	0000
0001	1 × 5	= 5	0000	0101
0010	2 × 5	= 10	0001	0000
0011	3 × 5	= 15	0001	0101
0100	4 × 5	= 20	0010	0000
0101	5 × 5	= 25	0010	0101
0110	6 × 5	= 30	0011	0000
0111	7 × 5	= 35	0011	0101
1000	8 × 5	= 40	0100	0000
1001 max	9 × 5	= 45	0100	0101
1010	×5		ϕϕϕϕ	ϕϕϕϕ
1011			‖	‖
1100				
1101				
1110				
1111			ϕϕϕϕ	ϕϕϕϕ

Invalid BCD

CODE CONVERTERS

Lecture 7 & 8

Numbering System with Complements

Complements: Are used for simplifying the subtraction operation and for logical manipulation.

There are two complements for each base:

- (R-1)'s complement (Diminished radix complement)
- R's complement (Radix complement)

	Base		
BINARY	2	2-1	1's compl & 2's
QUAD	4	4-1	3's & 4's
OCTAL	8	8-1	7's & 8's
DECIMAL	10	10-1	9's & 10's
HEXADECIMAL	16	16-1	15's & 16's
⋮	⋮		
BASE	R	(R-1)	

- (R-1)'s complement:

  (R-1)'s complement of a number is $(R^n - 1) - N$

  Where R → base

  N → number who's complement is to be taken

  n → number of digits/bits in the number N

Numbering System with Complements

BINARY: Base = 2 = R   Diminished Radix = R − 1 = 1 = (R − 1)

(R-1)'s

1's Complements: Example $(1\ 0\ 0\ 1)_2 = N$ $= 1\ 0\ 0\ 1_2$ 4 digit base 2 Number

Diminished Radix complement   in binary bits → 0, → 1   4-bit Number

1001 $2^4 - 1$ 1001

1's complement of a number is $(R^n - 1) - N$

Where R → base 2

N → number who's complement is to be taken $1\ 0\ 0\ 1_2$ 1's $\begin{array}{r} 1111 \\ -\ 1001 \\ \hline 0110_2 \end{array}$

n → number of digits/bits in the number N = 4

$(2^4 - 1) - (1\ 0\ 0\ 1)_2$

$(16-1) - 1001_2$ $= 15_{10} - 1\ 0\ 0\ 1_2 = 1\ 1\ 1\ 1_2 - 1\ 0\ 0\ 1 = 0\ 1\ 1\ 0$ = 1's complement

Try for 1010_2 ⇒ 4 bit $(2^4 - 1) = 15 - 1 = 1111 - 1010 = 0101$

Numbering System with Complements

DECIMAL $R=10$, $R-1=10-1=9$

9's Complements: Example N = 1234 $n=4$

9's complement of a number is $(R^n - 1) - N$ $= (10^4 - 1) - 1234$

Where R → base 9

N → number who's complement is to be taken 1234

n → number of digits/bits in the number N = 4

$(10^4 - 1) - (1234)$

$(10000-1)_{10} - 1234_{10} = 9999_{10} - 1234_{10} = 8765_{10}$ = 9's complement

Try for 38607 $n=5$ $(10^5-1) - 38607 = (100000-1) - 38607$

$= 99999 - 38607 = 61392_{10}$

Numbering System with Complements

HEXADECIMAL   R = 16   R - 1 = 15

4 digit

15's Complements: Example N = $123B_{16}$

15s complement of a number is $(R^n - 1) – N$ $(16^4 - 1) - 123B$

Where R → base 16

N → number who's complement is to be taken 123B

n → number of digits/bits in the number N = 4

A+1 B
+1 C
+1 D
+1 E
+1 F

$(16^4 - 1) – (123B)$

$(FFFF-1)_{16} – 123B$ = EDC4 = 15's complement

Try for A3EDC   FFFFF - A3EDC = $5C123_{16}$

Numbering System with Complements

R’s complement (Radix complement)

(R)’s complement of a number is $R^n - N$

Where R → base

N → number who's complement is to be taken

n → number of digits/bits in the number N

- Note in (R-1)’s complement:

R's complement

from

- (R-1)’s complement of a number is $\underbrace{(R^n - 1) - N}_{+1} = [\underbrace{(R^n - 1)}_{(R-1)\text{'s complement}} - N] + 1 = R^n - N$

R's complement

R's complement

Numbering System with Complements

Binary

2's complement (Radix complement)   0 1 1 0

(R)'s complement of a number is $R^n - N$

Where R → base 2

N → number who's complement is to be taken 0 1 1 0

n → number of digits/bits in the number N = 4

$R^n - N = 2^4 - 0110_2 = 16 - 0110_2 = 10000_2 - 0110_2 = 1010$

Using 1's complement

1111 – 0110 +1

1's 2's

1001+ 1 = 1010

Numbering System with Complements

Decimal

10’s complement (Radix complement) 562_{10}

(R)’s complement of a number is $R^n - N$

Where R → base 10

N → number who's complement is to be taken 562

n → number of digits/bits in the number N = 3

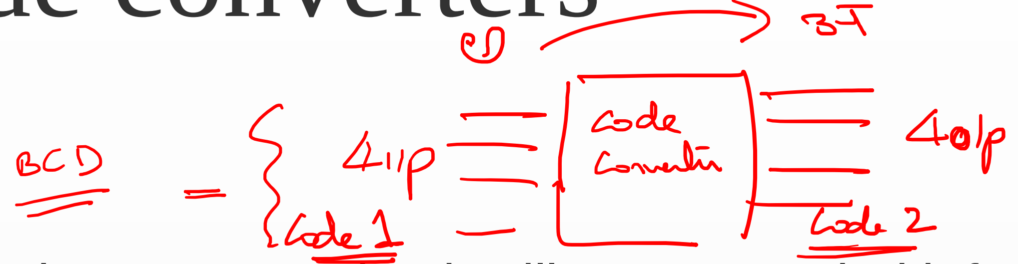
$R^n - N = 10^3 - 562 = 1000 - 562 = 438$

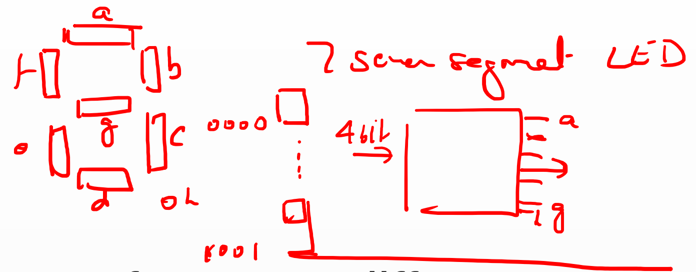
Using 9’s complement

999 – 562 +1

437 + 1 = 438

Code converters





- A code converter circuit will convert coded information in one form to a different coding form.
- Coded representation for 10 decimal symbols is known as binary coded decimal (or BCD) or decimal codes.
- Minimum 4-bits are required to represent decimal symbol.
- Out of 16 , 4-bit combinations, only 10 combinations are used to represent 10 decimal symbols and remaining 6 will not be used (don't cares)

BCD

A
b
C
d
E
1111 F

ABCD
0 0000
9 1001
6 unused

Difference between binary and BCD representation

- $(28)_{10}$ 2 digit BCD Number

Binary representation : $(11100)_2$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

16 8 4 ⇒ 28

8421 BCD representation : $(0010\ 1000)_2$

2 digit

2 8

4 bit BCD 4 bit BCD

Ex:

$(5843)_{10}$ BCD

Each digit must be replaced with BCD equivalent

4 digit BCD

$2^3\ 2^2\ 2^1\ 2^0$ $2^3\ 2^2\ 2^1\ 2^0$

0101 1000 $\boxed{0100}$ $\boxed{0011}$ BCD

5 8 4 3 5843

Introduction to BCD codes

	Weighted Number, 4-digit, 4-bit	Non Weighted Number, Self-complementary	Weighted, Self-complementary	weighted	Non weighted, Non Self complementary
Decimal digit	**8421 (BCD)**	**Excess 3**	**84-2-1**	**2421**	**Gray code**
0	0000	0011	0000	0000	0000
1	0001	0100	0111	0001	0001
2	0010	0101	0110	0010	0011
3	0011	0110	0101	0011	0010
4	0100	0111	0100	0100	0110
5	0101	1000	1011	1011	0111
6	0110	1001	1010	1100	0101
7	0111	1010	1001	1101	0100
8	1000	1011	1000	1110	1100
9	1001	1100	1111	1111	1101
Don't cares	1010,1011, 1100,1101, 1110,1111	0000,0001, 0010,1101, 1110,1111	0001,0010, 0011,1100, 1101,1110	0101,0110, 0111,1000, 1001,1010	1000, 1001, 1010, 1011, 1110, 1111

invalid

Invalid.

5 Symbols

5 Symbols

9−4=5

9−3=6

9−2=7

Note: BCD 8421
① Weighted Number
Every 4-bit representation of any digit
$2^3 2^2 2^1 2^0$

Excess3 = BCD + 3
0 → 0+3 = 3
1 → 1+3 = 4
① weighted Number → Non weighted
② BCD + 3
Self-complementary

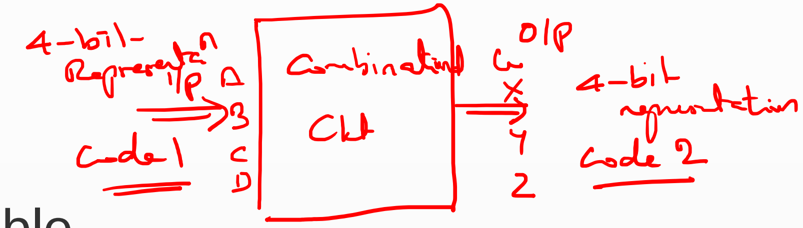
84-2-1
① Weighted Number
② Self-complementary
9 − N =

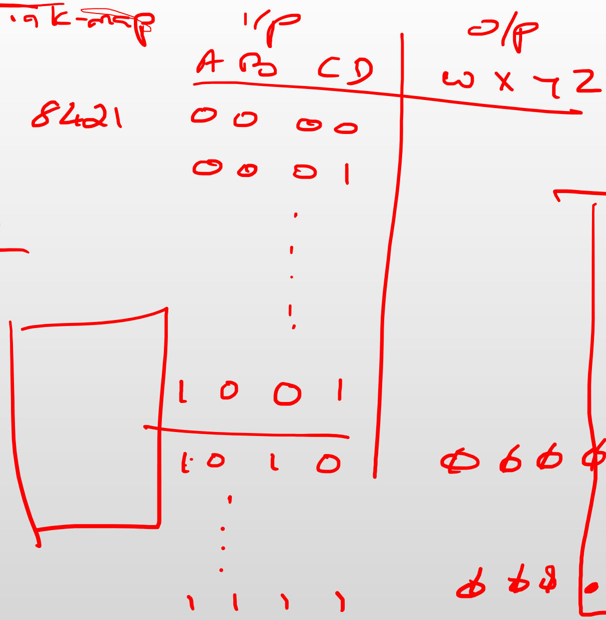
2421
Till digit 4, BCD
2 → 1000 ✗
0010 ✓

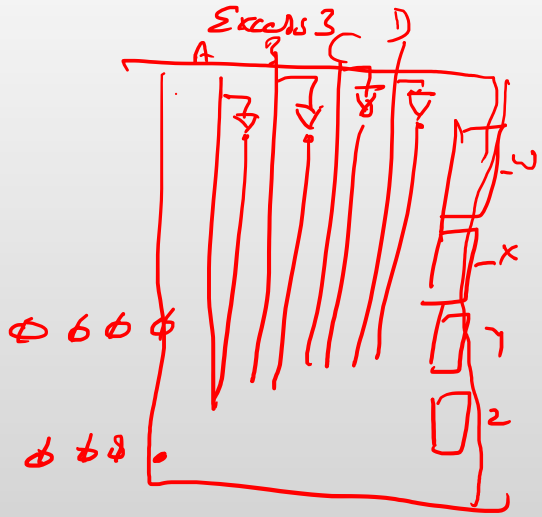
Gray
Adjacent two codes varies with only 1 bit position

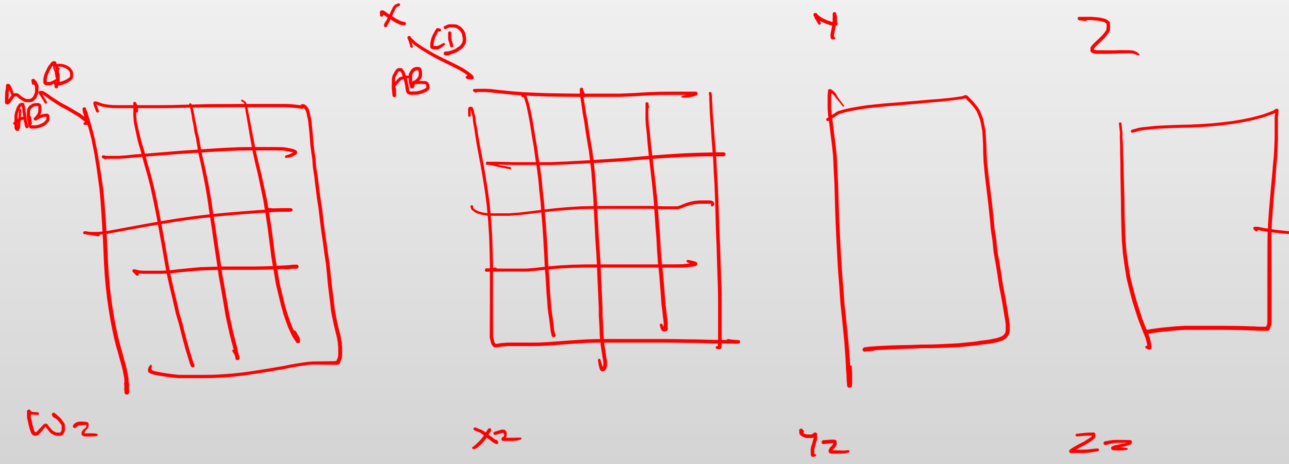
Code converter design steps:

- 1. Write the truth table
- 2. Identify the don't care inputs from input code
- 3. Write the minterms/maxterms for every output variable
- 4. Simplify the expressions for output variables
- 5. Draw the circuit using the specified gates.



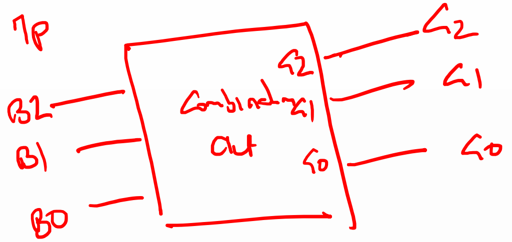






Ex:1

Design a 3 bit binary to gray code converter.

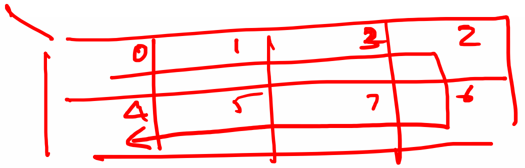


$G_2 = \sum_m 4,5,6,7$

$G_1 = \sum_m 2,3,4,5$

$G_0 = \sum_m 1,2,5,6$

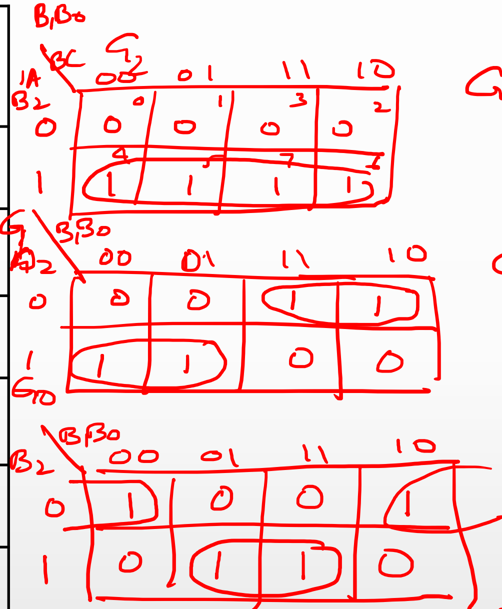
minterms	3-bit Binary B2 B1 B0		Gray G2 G1 G0
0	0 0 0	m_0	0 0 0
1	0 0 1	m_1	0 0 1
2	0 1 0	m_2	0 1 1
3	0 1 1	m_3	0 1 0
4	1 0 0	m_4	1 1 0
5	1 0 1	m_5	1 1 1
6	1 1 0	m_6	1 0 1
7	1 1 1	m_7	1 0 0



Grey Code

0	000
1	001
3	011
2	010
6	110
7	111
5	101
4	100

Solve for $G_2\ G_1\ G_0$



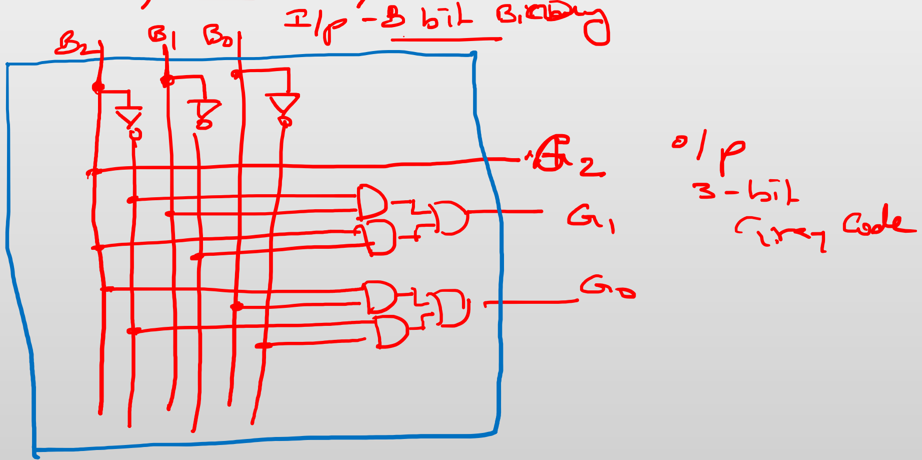
$G_2 = A = B_2$

$G_1 = \bar{B}_2 B_1 + B_2 \bar{B}_1$

$= B_2 \oplus B_1$

$G_0 = B_2 B_0 + \bar{B}_2 \bar{B}_0$

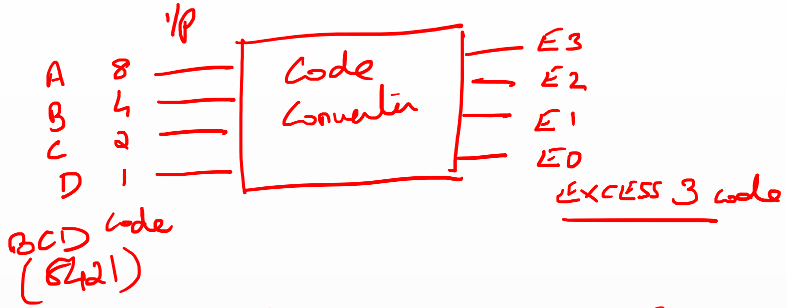
$= B_2 \oplus B_0$



Design a code converter to convert a decimal digit represented in 8421 code to a decimal digit represented in Excess 3 code.

Decimal digit	8 4 2 1 A B C D	Excess 3 code E3 E2 E1 E0
0	0 0 0 0	0 0 1 1
1	0 0 0 1	0 1 0 0
2	0 0 1 0	0 1 0 1
3	0 0 1 1	0 1 1 0
4	0 1 0 0	0 1 1 1
5	0 1 0 1	1 0 0 0
6	0 1 1 0	1 0 0 1
7	0 1 1 1	1 0 1 0
8	1 0 0 0	1 0 1 1
9	1 0 0 1	1 1 0 0
Don't cares	1010,1011, 1100,1101,1110,1111	ϕ ϕ ϕ ϕ -------------

I/p
ϕ 1010
ϕ 1011
ϕ 1100
ϕ 1101
ϕ 1110
ϕ 1111



$E_3 = \sum_m 5, 6, 7, 8, 9 + \sum_\phi 10, 11, 12, 13, 14, 15$ → SOP ← NAND

$E_3 = \prod_M 0, 1, 2, 3, 4 \cdot \prod_d 10, 11, 12, 13, 14, 15$ → POS ← NOR

SOP

$E_2 = \sum_m 1, 2, 3, 4, 9 + \sum_\phi 10, 11, 12, 13, 14, 15$

$E_1 = \sum_m 0, 3, 4, 7, 8 + \sum_\phi 10, 11, 12, 13, 14, 15$

$E_0 = \sum_m 0, 2, 4, 6, 8 + \sum_\phi 10, 11, 12, 13, 14, 15$

Pls note! I/p → 4 Variable → 4-Variable K-MAP

How many K-maps? observe o/p → 4 Variable
we require to solve 4 K-maps
one each for o/p E_3, E_2, E_1, E_0