Basic Electrical Technology 7

1. D.C. Circuit Analysis

Symbols and Notations

Symbol	Name	Unit
ρ	Resistivity	Ωm
l	Length of the material	m
R_0	Resistance at 0°C	Ω
α_0	Temp coefficient of resistivity at 0°C	$°C^{-1}$
A	Cross-sectional area of the material	m^2
L	Self-inductance	H
A	Cross-sectional area	m^2
N	Number of turns in the coil	
φ	Flux produced	Wb
l	Length of the magnetic circuits	m
i	Current following through inductor	A
μ_0	Permeability of free space = $4\pi \times 10^{-7}$	H/m
C	Capacitance	F
v_c	Capacitor voltage	V
q	Charge	C
d	Distance between the parallel plates	m
A	Cross-sectional area of the parallel plates	m^2
ε_0	Permittivity of free space= 8.85×10^{-12}	F/m
ε_r	Relative permittivity of the dielectric medium	
I_0	Initial current	A
τ	Time constant	
V_0	Initial voltage	V

1.1. Passive Circuit Elements

1.1.1. Resistors

Resistance of a material	$R = \rho \frac{l}{A}$
Resistance varies with temperature	$R_t = R_0(1 + \alpha_0 t)$
Resistors in series	$R_{eq} = R_1 + R_2 + \cdots \ldots + R_n$
Resistors in parallel	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \ldots . + \frac{1}{R_n}$
Power dissipated in a resistor	$I^2 R$

1.1.2. Inductors

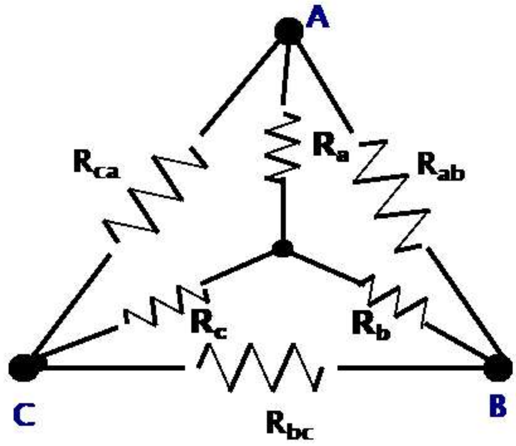
Inductance of a coil with non-magnetic core	$L = \frac{\mu_0 A N^2}{l}$
Inductors in series	$L_{eq} = L_1 + L_2 + \cdots \ldots + L_n$
Inductors in parallel	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \ldots . + \frac{1}{L_n}$
Emf induced in an inductor	$e_L = L\left(\frac{di}{dt}\right) = N\left(\frac{d\varphi}{dt}\right)$
Energy stored in an inductor	$\frac{1}{2} L I^2$

1.1.3. Capacitors

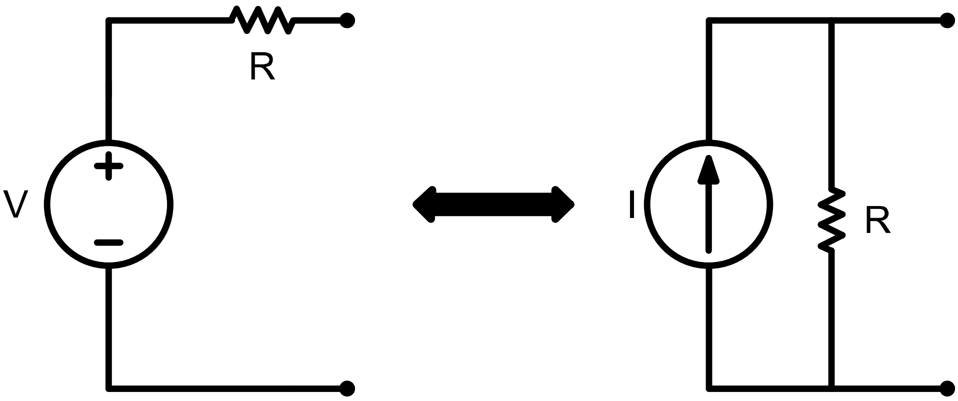
Capacitance of parallel plate capacitor	$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$
Capacitors in series	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \ldots + \frac{1}{C_n}$
Capacitors in parallel	$C_{eq} = C_1 + C_2 + \ldots .. + C_n$
Current in a capacitor	$i_c = \frac{dq}{dt} = C\left(\frac{dv_c}{dt}\right)$
Energy stored in a capacitor	$\frac{1}{2} C V^2$

1.2. DELTA-STAR / STAR-DELTA TRANSFORMATION

Delta to Star	Star to Delta
$R_a = \dfrac{R_{ab}R_{ca}}{\sum R_{ab}}$	$R_{ab} = \dfrac{\sum R_a R_b}{R_c}$
$R_b = \dfrac{R_{bc}R_{ab}}{\sum R_{ab}}$	$R_{bc} = \dfrac{\sum R_a R_b}{R_a}$
$R_c = \dfrac{R_{ca}R_{bc}}{\sum R_{ab}}$	$R_{ca} = \dfrac{\sum R_a R_b}{R_c}$



1.3. SOURCE TRANSFORMTION



$\boldsymbol{V = IR}$

$\boldsymbol{I = V/R}$

1.4. NETWORK EQUATIONS

KVL equation (matrix form)

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} R_{11} & \cdots & R_{1N} \\ \vdots & \ddots & \vdots \\ R_{N1} & \cdots & R_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

$$[\boldsymbol{V}] = [\boldsymbol{R}][\boldsymbol{I}]$$

KCL equation (matrix form)

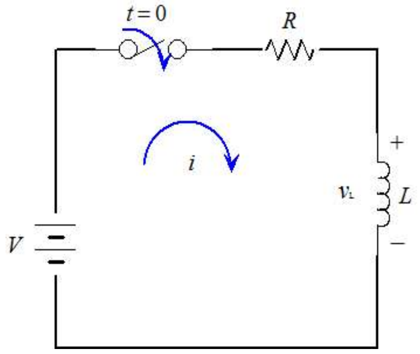
$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} G_{11} & \cdots & G_{1N} \\ \vdots & \ddots & \vdots \\ G_{N1} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

$$[\boldsymbol{I}] = [\boldsymbol{G}][\boldsymbol{V}]$$

1.5. DC TRANSIENTS

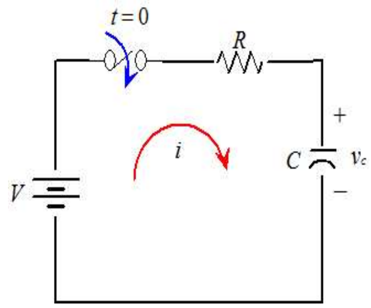
1.5.1. RL Transient Circuit

Circuit equation	$V = R\,i + L\dfrac{di}{dt}$
Current growth	$i = \dfrac{V}{R}\left(1 - e^{-(t/\tau)}\right)$
Current decay	$i = I_0 e^{-(t/\tau)}$
Time constant	$\tau = L/R$



1.5.2. RC Transient Circuit

Circuit equation	$C\left(\dfrac{dv_c}{dt}\right) + \dfrac{v_c - V}{R} = 0$
Capacitor voltage during charging	$v_c = V\left(1 - e^{-(t/\tau)}\right)$
Capacitor voltage during discharging	$v_c = V_0 e^{-(t/\tau)}$
Time constant	$\tau = RC$

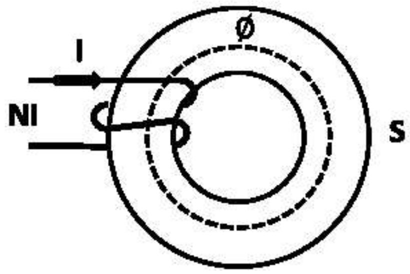


2. Magnetic Circuits Analysis

Symbol	Name	Unit
φ	Magnetic flux	Wb
H	Magnetic field strength	Am
S	Reluctance	A/Wb
μ_r	Relative permeability	
N	No. of turns of the coil	
I	Current flowing through the coil	
l	Length of the magnetic path	

2.1 Magnetic Circuits

Magneto-motive force	$F = N \times I = \varphi \times S$
Magnetic field strength	$H = \dfrac{N \times I}{l}$
Reluctance of the magnetic path	$S = \dfrac{l}{\mu_0 \mu_r A}$
Permeance of the magnetic path	$\mathcal{P} = \dfrac{1}{S}$



2.2 Electromagnetism

Faraday's law $$e = N\frac{d\varphi}{dt}$$

Induced EMF $$e = L\frac{di}{dt}$$

Self-inductance $$L = N\frac{d\varphi}{di}$$

Mutual-inductance $$M = k\sqrt{L_1L_2}$$

2.3 Coupled Circuits

Equivalent inductance (series addition) $$L_{eq} = L_1 + L_2 + 2M$$

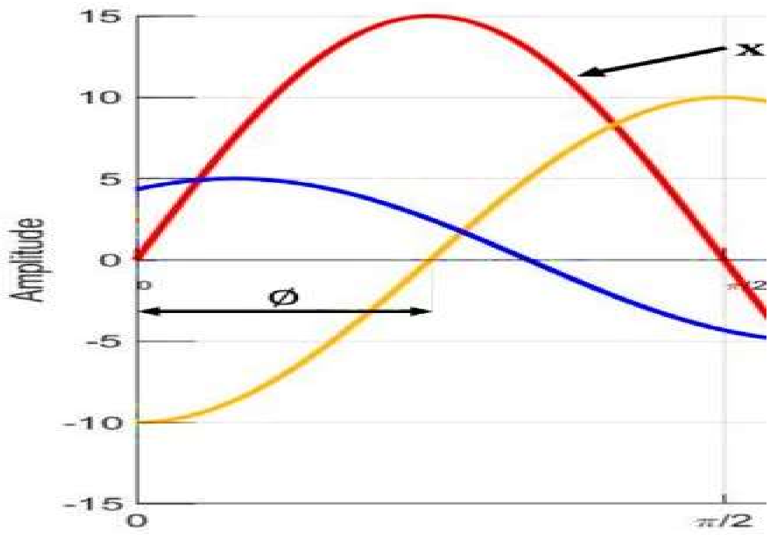
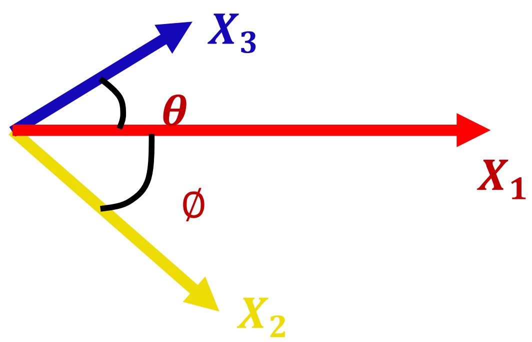
Equivalent inductance (series opposition) $$L_{eq} = L_1 + L_2 - 2M$$

3. Single Phase A.C. Circuit Analysis

Symbol	Name	Unit
T	Time period of the signal $\boldsymbol{f(t)}$	s
F_m	Maximum amplitude of the sinusoidal signal $\boldsymbol{f(t)}$	
G	Conductance	S
B	Susceptance	S
P	Active power	W
Q	Reactive power	VAR
S	Apparent power	VA
$\cos\emptyset$	Power factor	
ω_1	Lower Half-power Frequency	rad/s
ω_2	Upper Half-power Frequency	rad/s
ω_0	Resonant Frequency	rad/s
f	Supply frequency	Hz
φ_m	Magnetic flux	Wb
N_1	Number of turns on primary	
N_2	Number of turns on secondary	

3.1. Single Phase AC

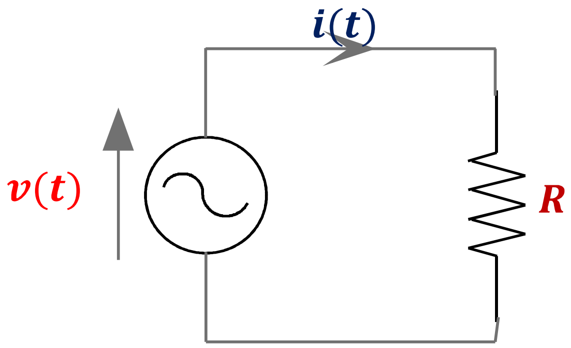
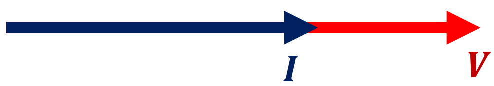
3.1.1. Representations of Sinusoidal Signal

Graphical Representation	
Mathematical Representation	$x_1(\mathrm{t}) = \mathrm{X}_{1\mathrm{m}}\sin(\omega \mathrm{t})$ $x_2(\mathrm{t}) = \mathrm{X}_{2\mathrm{m}}\sin(\omega \mathrm{t} - \emptyset)$ $x_3(\mathrm{t}) = \mathrm{X}_{3\mathrm{m}}\sin(\omega \mathrm{t} + \theta)$
Phasor Representation	 Where $\mathbf{X_1}$, $\mathbf{X_2}$ and $\mathbf{X_3}$ are the RMS values

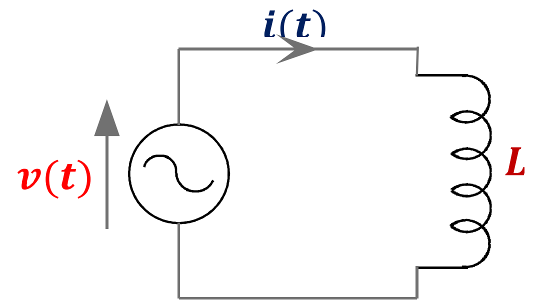
3.1.2. Average and RMS Values of a Periodic Signal

For a periodic signal $\boldsymbol{f(t)}$	$\mathrm{F_{avg}} = \dfrac{2}{\mathrm{T}} \displaystyle\int_{0}^{\mathrm{T}/2} \mathrm{f(t)dt}$ $\mathrm{F_{rms}} = \sqrt{\dfrac{1}{T} \displaystyle\int_{0}^{\mathrm{T}} \mathrm{f^2(t)dt}}$
For a sinusoidal signal $\boldsymbol{f(t)}$	$\mathrm{F_{avg}} = \dfrac{2F_m}{\pi}$ $\mathrm{F_{rms}} = \dfrac{F_m}{\sqrt{2}}$
Form factor for a sinusoidal signal	$\mathrm{FF} = \dfrac{\text{RMS Value}}{\text{Average Value}} = 1.11$
Peak factor for a sinusoidal signal	$\mathrm{PF} = \dfrac{\text{Maximum Value}}{\text{RMS Value}} = \sqrt{2}$

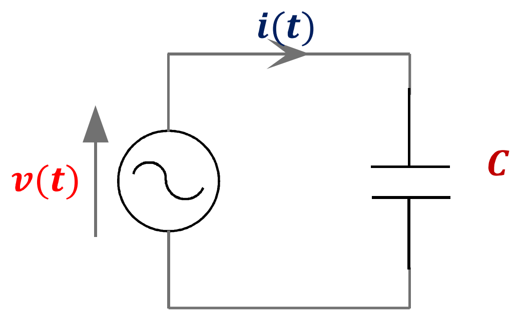
3.1.3. Purely Resistive Circuit

Circuit Diagram	
Mathematical Representation	$v(t) = V_m \sin(\omega t) \qquad i(t) = I_m \sin(\omega t)$
Circuit Impedance	$\mathrm{R} = \dfrac{\overline{\mathrm{V}}}{\overline{\mathrm{I}}} = \dfrac{\mathrm{V}\angle 0°}{\mathrm{I}\angle 0°}$
Average Power	$\mathrm{P_{avg}} = \dfrac{\mathrm{V_m I_m}}{2} = \mathrm{VI} = \dfrac{\mathrm{V^2}}{\mathrm{R}} = \mathrm{I^2R}$
Phasor Representation	

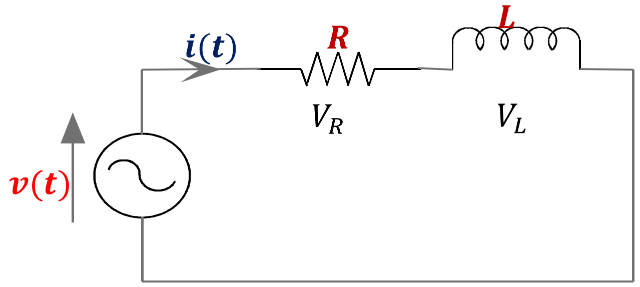
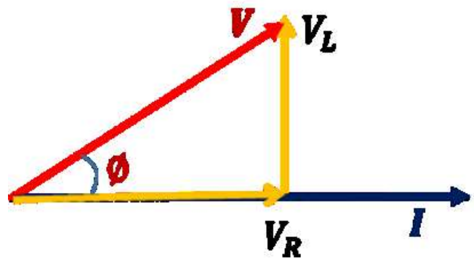
3.1.4. Purely Inductive Circuit

Circuit Diagram	
Mathematical Representation	$v(t) = V_m \sin(\omega t) \qquad i(t) = I_m \sin(\omega t - 90°)$
Circuit Impedance	$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0°}{I\angle -90°} = jX_L$
Average Power	$P_{avg} = 0$
Phasor Representation	

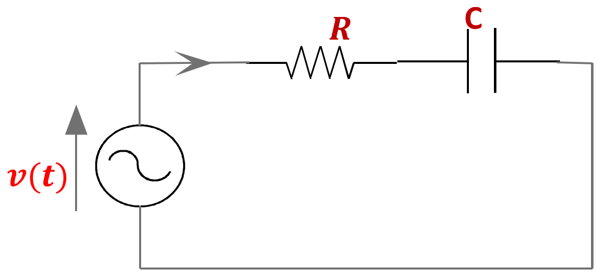
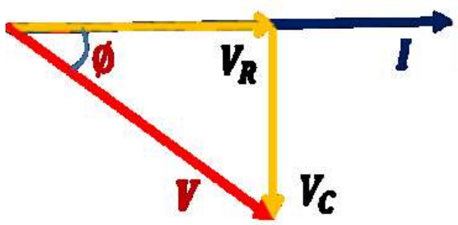
3.1.5. Purely Capacitive Circuit

Circuit Diagram	
Mathematical Representation	$v(t) = V_m \sin(\omega t) \qquad i(t) = I_m \sin(\omega t + 90°)$
Circuit Impedance	$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0°}{I\angle 90°} = -jX_C$
Average Power	$P_{avg} = 0$
Phasor Representation	

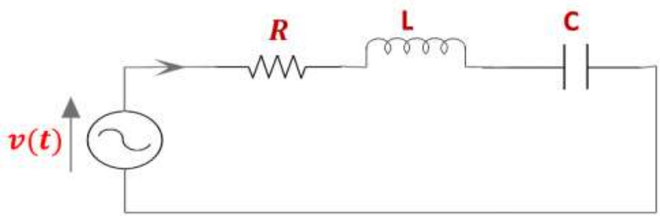
3.1.6. RL Circuit

Circuit Diagram	
Mathematical Representation	$(t) = V_m \sin(\omega t)$ $\quad\quad i(t) = I_m \sin(\omega t - \emptyset)$
Circuit Impedance	$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R + jX_L)}{\bar{I}} = R + jX_L = Z\angle\emptyset$
Average Power	$P_{avg} = VI\cos\emptyset$
Phasor Representation	

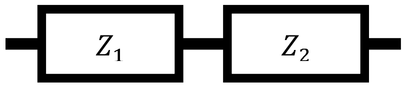
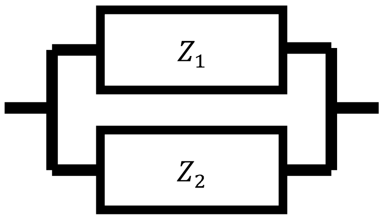
3.1.7. RC Circuit

Circuit Diagram	
Mathematical Representation	$v(t) = V_m \sin(\omega t)$ $\quad\quad i(t) = I_m \sin(\omega t + \emptyset)$
Circuit Impedance	$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R - jX_C)}{\bar{I}} = R - jX_C = Z\angle - \emptyset$
Average Power	$P_{avg} = VI\cos\emptyset$
Phasor Representation	

3.1.8. RLC Circuit

Circuit Diagram	
Circuit Impedance	$Z = R + j(X_L \sim X_c)$ $if\ X_L = X_C \Longrightarrow Resistive\ circuit$ $if\ X_L > X_C \Longrightarrow RL\ series\ circuit$ $if\ X_L < X_C \Longrightarrow RC\ series\ circuit$

3.1.9. Impedance and Admittance

Impedance	$Z = R \pm jX$	
Total Impedance	$Z_{eq} = Z_1 + Z_2$	
Admittance	$Y = 1/Z = G \mp jB$	
Total Admittance	$Y_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2} = Y_1 + Y_2$	

3.1.10. Network Equations

KVL Equation (Matrix Form)

$$\begin{bmatrix} \mathbf{V_1} \\ \vdots \\ \mathbf{V_N} \end{bmatrix} = \begin{bmatrix} \mathbf{Z_{11}} & \cdots & \mathbf{Z_{1N}} \\ \vdots & \ddots & \vdots \\ \mathbf{Z_{N1}} & \cdots & \mathbf{Z_{NN}} \end{bmatrix} \begin{bmatrix} \mathbf{I_1} \\ \vdots \\ \mathbf{I_N} \end{bmatrix}$$

$$[\boldsymbol{V}] = [\boldsymbol{Z}][\boldsymbol{I}]$$

KCL Equation (Matrix Form)

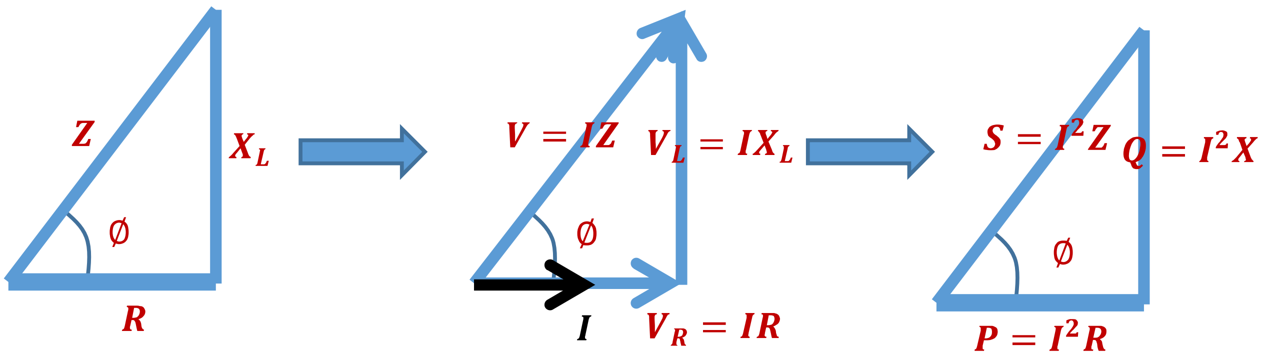
$$\begin{bmatrix} \mathbf{I_1} \\ \vdots \\ \mathbf{I_N} \end{bmatrix} = \begin{bmatrix} \mathbf{Y_{11}} & \cdots & \mathbf{Y_{1N}} \\ \vdots & \ddots & \vdots \\ \mathbf{Y_{N1}} & \cdots & \mathbf{Y_{NN}} \end{bmatrix} \begin{bmatrix} \boldsymbol{V_1} \\ \vdots \\ \boldsymbol{V_N} \end{bmatrix}$$

$$[\boldsymbol{I}] = [\boldsymbol{Y}][\boldsymbol{V}]$$

3.1.11. Power in AC Circuits

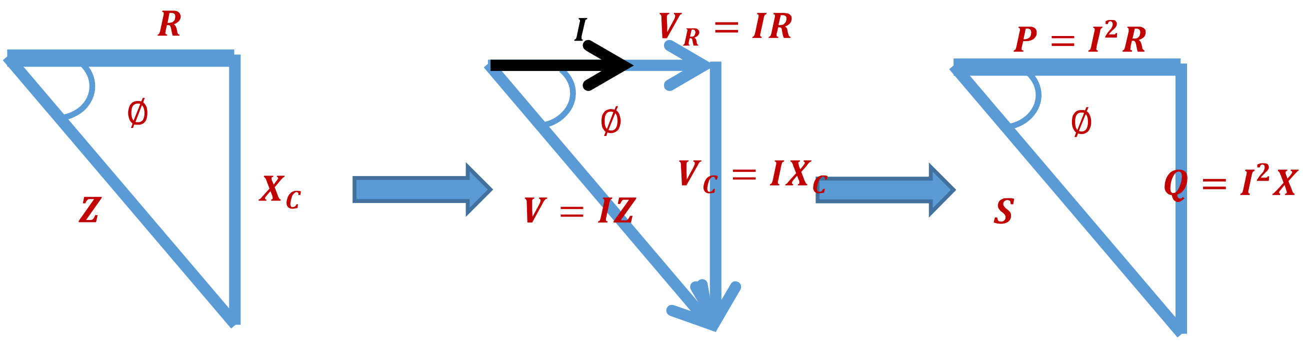
Instantaneous Power in an AC System	$p(t) = v(t) \times i(t) = V_m I_m \sin(\omega t)\sin(\omega t \pm \emptyset)$
Active Power	$P = VI\cos\emptyset = I^2 Z\cos\emptyset = I^2 R$
Reactive Power	$Q = VI\sin\emptyset = I^2 Z\sin\emptyset = I^2 X$
Complex Power	$S = P \pm jQ$

RL Load



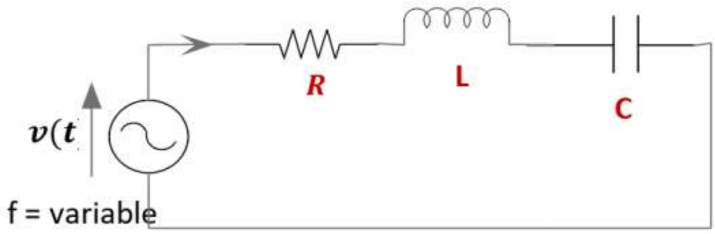
Impedance Diagram — Phasor Diagram — Power Diagram

RC Load



Impedance Diagram — Phasor Diagram — Power Diagram

3.1.12. Series Resonance

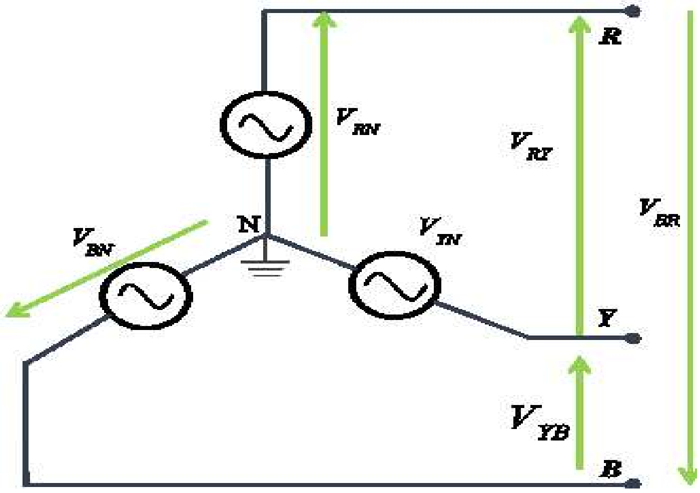
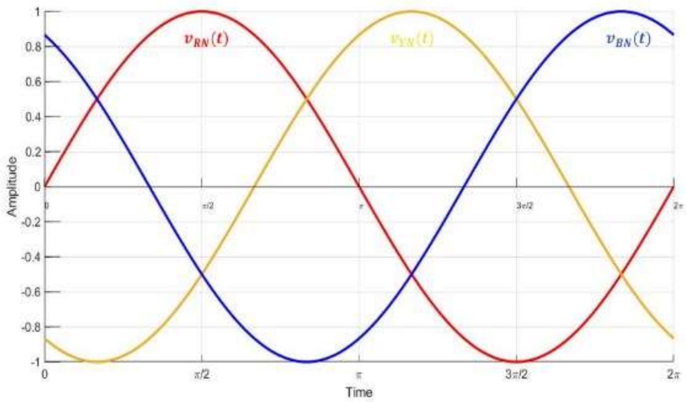
Circuit Diagram	
At Resonance	$X_L = X_C$ $\Rightarrow Z = R$
Resonant Frequency	$\omega_0 = \frac{1}{\sqrt{LC}}$
Bandwidth (BW)	$\mathrm{BW} = \omega_2 - \omega_1 = R/L$
Quality factor for Series RLC Circuit	$Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}} = \frac{1}{\omega_0 CR} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

3.1.13. Parallel Resonance

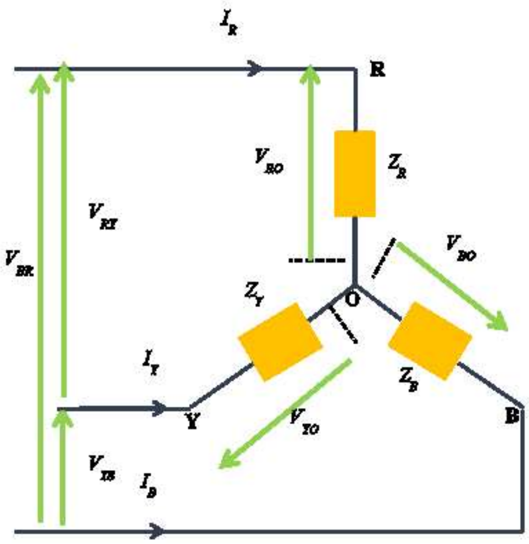
Equivalent Admittance	$Y_{eq} = Y_1 + Y_2 + \cdots + Y_n = G \pm jB$
At resonance	$\mathrm{Img}(Y_{eq}) = B = 0$

4. Three Phase AC Circuit Analysis

4.1 Three Phase Source Voltage Representation

Circuit Diagram	
Instantaneous Phase Voltages	$v_{RN} = V_m \, \mathrm{Sin}(\omega t)$ $v_{YN} = V_m \, \mathrm{Sin}(\omega t - 120°)$ $v_{BN} = V_m \, \mathrm{Sin}(\omega t - 240°)$
Polar Representation Of Phase Voltages	$\overline{V}_{RN} = V_{ph}\angle 0^{\circ}$ $\overline{V}_{YN} = V_{ph}\angle{-120}^{\circ}$ where, $V_{ph} = \frac{V_m}{\sqrt{2}}$ $\overline{V}_{BN} = V_{ph}\angle - 240^{\circ}$
Graphical Representation	
Polar Representation of Line Voltages	$\overline{V}_{RY} = \overline{V}_{RN} - \overline{V}_{YN} = \sqrt{3} \times V_{ph}\angle 30^{\circ}$ $\overline{V}_{YB} = \overline{V}_{YN} - \overline{V}_{BN} = \sqrt{3} \times V_{ph}\angle{-90}^{\circ}$ $\overline{V}_{BR} = \overline{V}_{BN} - \overline{V}_{RN} = \sqrt{3} \times V_{ph}\angle{-210}^{\circ}$

4.2 Three Phase Loads

<table>
<tr><th>Star Connected Loads</th><th>Delta Connected Loads</th></tr>
<tr><td>


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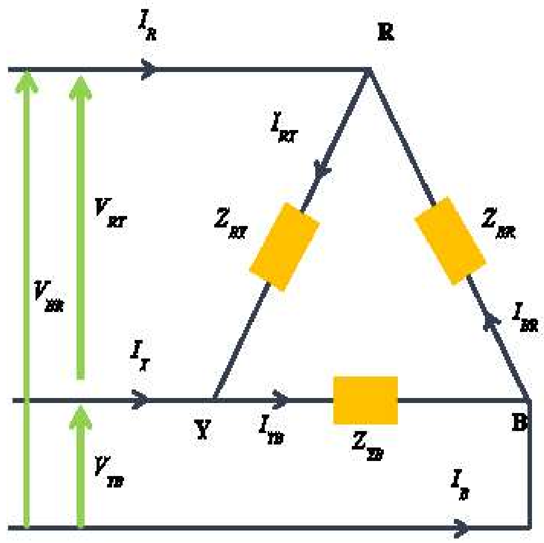


</td></tr>
<tr><td>
<u>For Balanced Loads</u>

$V_L = \sqrt{3} \times V_{Ph}$
$I_L = I_{Ph}$
$P = \sqrt{3} \times V_L \times I_L \times \cos \emptyset$
</td><td>
<u>For Balanced Loads</u>

$V_L = V_{Ph}$
$I_L = \sqrt{3} \times I_{Ph}$
$P = \sqrt{3} \times V_L \times I_L \times \cos \emptyset$
</td></tr>
<tr><td>
<u>For Unbalanced Loads</u>

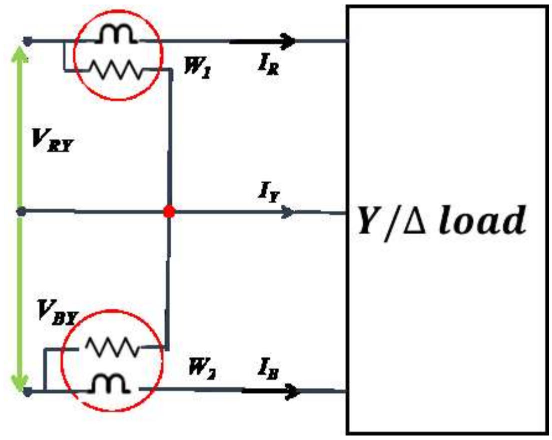
$I_L = I_{Ph}$
$V_{RY} = V_{RO} - V_{YO}$
$V_{YB} = V_{YO} - V_{BO}$
$V_{BR} = V_{BO} - V_{RO}$

$P = V_{RO}I_R Cos\angle(V_{RO}\&I_R) +$
$V_{YO}I_Y Cos\angle(V_{YO}\&I_Y) + V_{BO}I_B Cos\angle(V_{BO}\&I_B)$
</td><td>
<u>For Unbalanced Loads</u>

$V_L = V_{Ph}$
$I_R = I_{RY} - I_{BR}$
$I_Y = I_{YB} - I_{RY}$
$I_B = I_{BR} - I_{YB}$

$P = V_{RY}I_{RY}Cos\angle(V_{RY}\&I_{RY}) +$
$V_{YB}I_{YB}Cos\angle(V_{YB}\&I_{YB}) +$
$V_{BR}I_{BR}Cos\angle(V_{BR}\&{}_{BR})$
</td></tr>
</table>

4.3 Power Measurement For 3-Phase Balanced Loads Using 2-Wattmeter Method

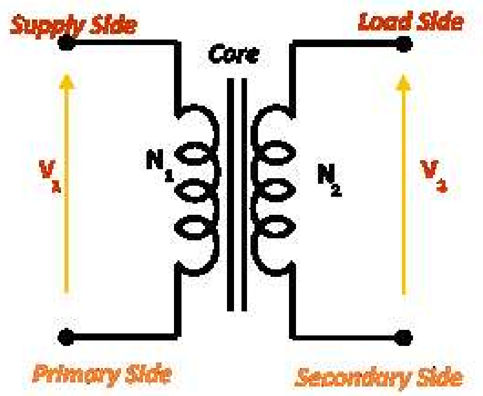
<table>
<tr><td>Circuit Diagram</td><td>
</td></tr>
<tr><td>Wattmeter Readings</td><td>$W_1 = V_{RY} I_R \, Cos\angle(V_{RY} \& I_R)$
$W_1 = V_L I_L Cos(30° + \emptyset)$

$W_2 = V_{BY} I_B Cos\angle(V_{BY} \& I_B)$
$W_2 = V_L I_L Cos(30° - \emptyset)$</td></tr>
<tr><td>Total Power Consumed</td><td>$P = W_1 + W_2 = \sqrt{3} \times V_L \, I_L \, Cos\emptyset$</td></tr>
<tr><td>Power Factor of Balanced Star-Connected Loads</td><td>$Cos\emptyset = Cos\left\{tan^{-1}\left[\sqrt{3} \times \frac{W_2 - W_1}{W_2 + W_1}\right]\right\}$</td></tr>
</table>

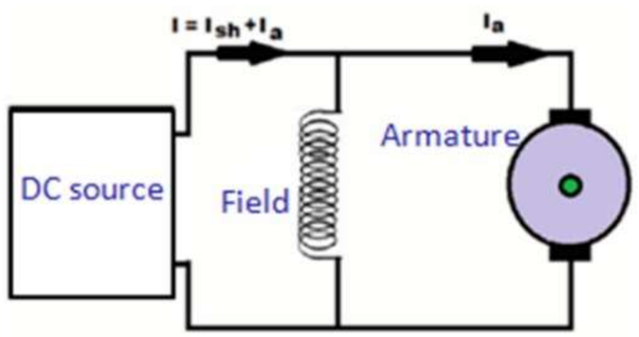
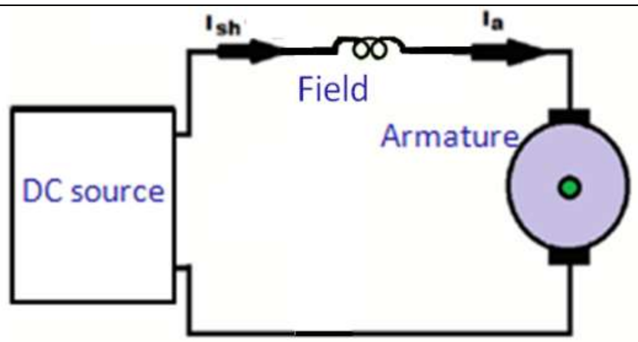
5. Electric Machines

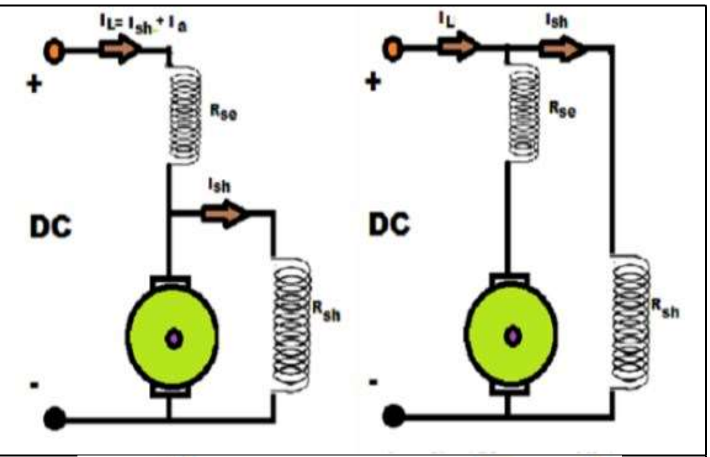
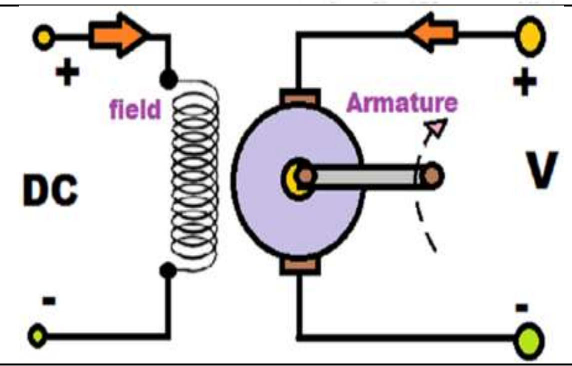
Symbol	Name	Unit
P	Number of poles	
φ	Flux per pole	Wb
I_a	Armature Current	A
N	Armature speed	RPM
R_{se}	Series Resistance of the field winding	Ω
R_a	Armature resistance	Ω
a	Number of parallel paths in armature $a = \begin{cases} 2, \text{for wave winding} \\ P, \text{for lap winding} \end{cases}$	
Z	Total number of armature conductors	
N_s	Synchronous speed	RPM
f	Supply frequency	Hz

5.1 Transformers

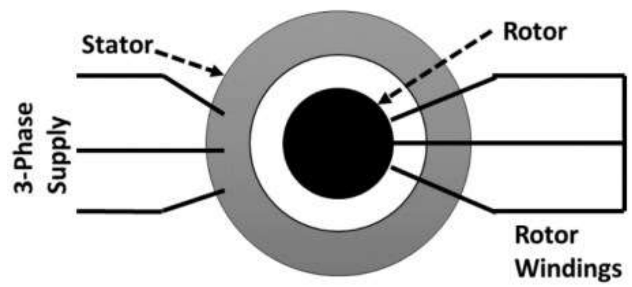
Circuit diagram	
Primary induced EMF	$E_1 = 4.44\ N_1 f \varphi_m$
Secondary induced EMF	$E_2 = 4.44\ N_2 f \varphi_m$
Turns Ratio	$\frac{V_1}{V_2} \cong \frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$

5.2 D C Motors

Circuit Diagram of DC Shunt Motor	
Voltage Equation for DC Shunt Motor	$V = E_b + I_a R_a$
Circuit Diagram of a DC Series Motor	
Voltage Equation for DC Series Motor	$V = E_b + I_a(R_a + R_{se})$

<table>
<tr><td>Circuit Diagram
of a DC Compound Motor</td><td>
</td></tr>
<tr><td>Circuit Diagram of a
DC Separately Excited Motor</td><td>
</td></tr>
<tr><td>Back EMF
Induced in Armature</td><td>$E_b = \frac{PZ}{60a} \varphi N$</td></tr>
<tr><td>Torque Developed
in DC Motor</td><td>$T = \frac{PZ}{2\pi a} \varphi\, I_a$</td></tr>
</table>

5.3 Three Phase Induction Motors

<table>
<tr><td>Block Diagram of
Three Phase Induction Motor</td><td>
</td></tr>
<tr><td>Speed of rotating magnetic field</td><td>$N_s = \frac{120f}{P}$</td></tr>
<tr><td>Percentage of operating slip</td><td>$s = \frac{N_s - N}{N_s} \times 100$</td></tr>
</table>

Basic Electrical Technology

CLASS 1 – 19 OCTOBER 2021

- INTRODUCTION TO THE COURSE
- CIRCUIT ELEMENTS

Course Outline

Basic Electrical Technology

DC Circuit Analysis

- Circuit elements
  - → Sources
  - → Resistor
  - → Inductor
  - → Capacitor
- Mesh current analysis
- Node voltage analysis
- Superposition Theorem
- Thevenin's Theorem
- Max. Power Transfer Theorem

Magnetic Circuits Analysis

- Magnetism
- Laws of magnetism
- Series and parallel magnetic circuits
- Electromagnetic induction
- Magnetic coupling
- Induced EMF
- Mesh analysis

Single Phase AC Circuit Analysis

- Generation
- Representation
- AC through R, L and C
- Series and parallel circuits
- Power & power factor
- Resonance

Three Phase AC Circuit Analysis

- Generation
- Representation
- Types of load connection
  - → Star
  - → Delta
- Analysis of balanced and unbalanced loads
- Measurement of Power

Power System Components

- Generation – Transmission - Distribution
- Utilization of Electric power
- Electrical machines
  - → Overview
  - → Types
  - → Working principle
  - → Application
- Energy meters

[L T P C] = [2 1 0 3]

Course Outcome

CO1	Analyze DC Circuit
CO2	Analyze Magnetic Circuit
CO3	Analyze Single Phase AC Circuit
CO4	Analyze Three Phase AC Circuit
CO5	Describe Electrical Power System Components

BET Online Class Structure

Chemistry Cycle

Group 1 (Class on Wed & Sat)		Group 2 (Class on Tue & Fri)		
Team 6	Team 7	Team 8	Team 9	Team 10

Group 2 Faculty Team:


Dr. Muralidhar Killi


Ms. Suprabha Padiyar

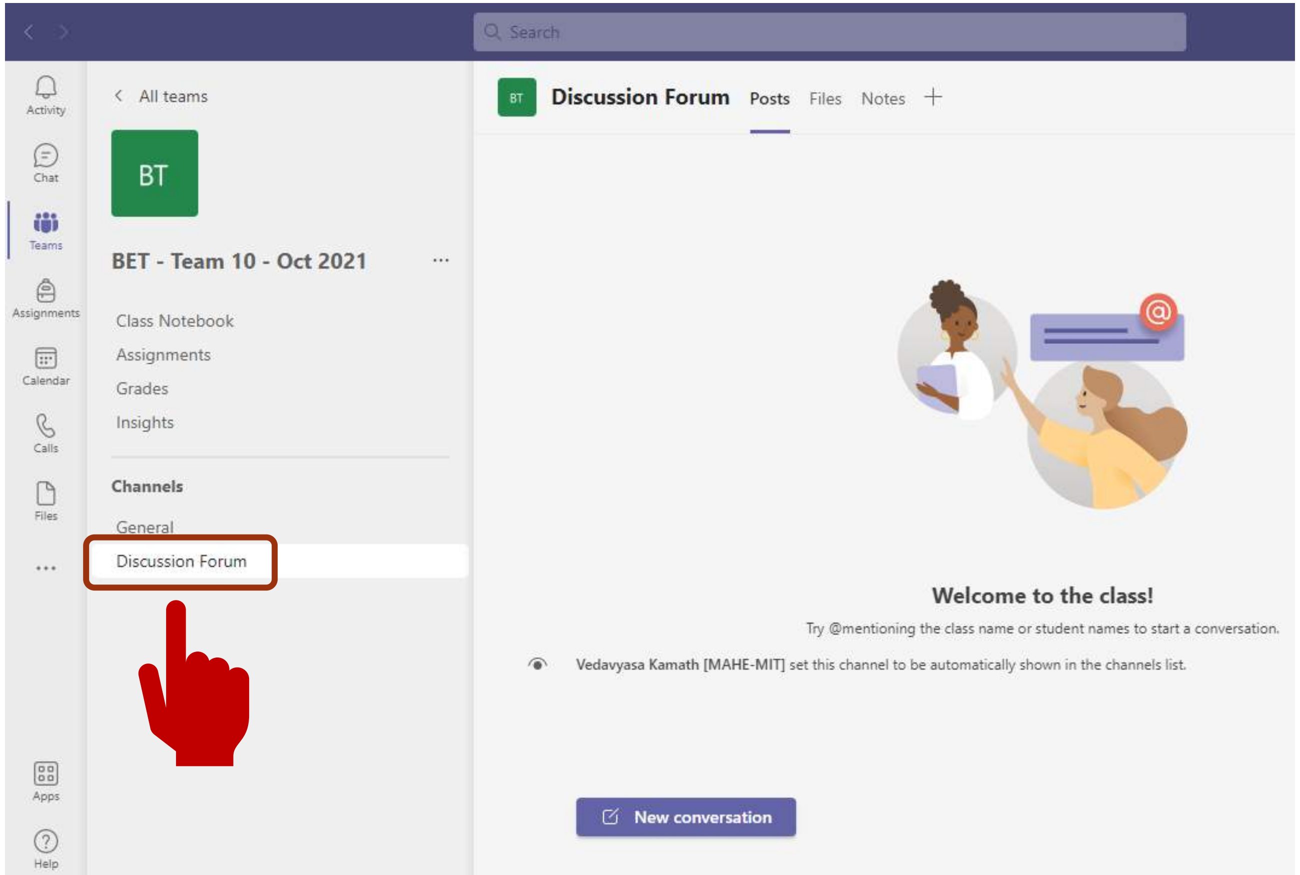

Ms. Namratha Pai


Mr. Siddaraju N.


Mr. Adarsh S.


Mr. Vedavyasa Kamath

Student Query Resolution



- Know the background of the question
- If possible, mention your approach

Assessment

Quiz	Continuous Assessment	In-Semester	End-Semester
• 10% • 5th calendar week • 30 minutes	• 20% • 2 marks each quiz • Immediately after every class, 5th calendar week onwards • 10 to 15 minutes	• 20% • 90 minutes • 4 questions – 10 marks each	• 50% • 180 minutes • 5 questions – 10 marks each

Note:

Course plan will be shared in due course of time

Scientific Calculator

- Should be **non – programmable**
- Should be **non - graphical**
- Suggestions:
  - Casio fx-991ES plus (2nd edition)
  - Casio fx-991MS (2nd edition)
  - Casio fx-991ES plus
  - Casio fx-991MS

- Android/Apple OS based apps available
- Windows app (trial for 3 months) available from Casio

Casio fx-991ES plus (2nd edition)

Quiz 1 of 5

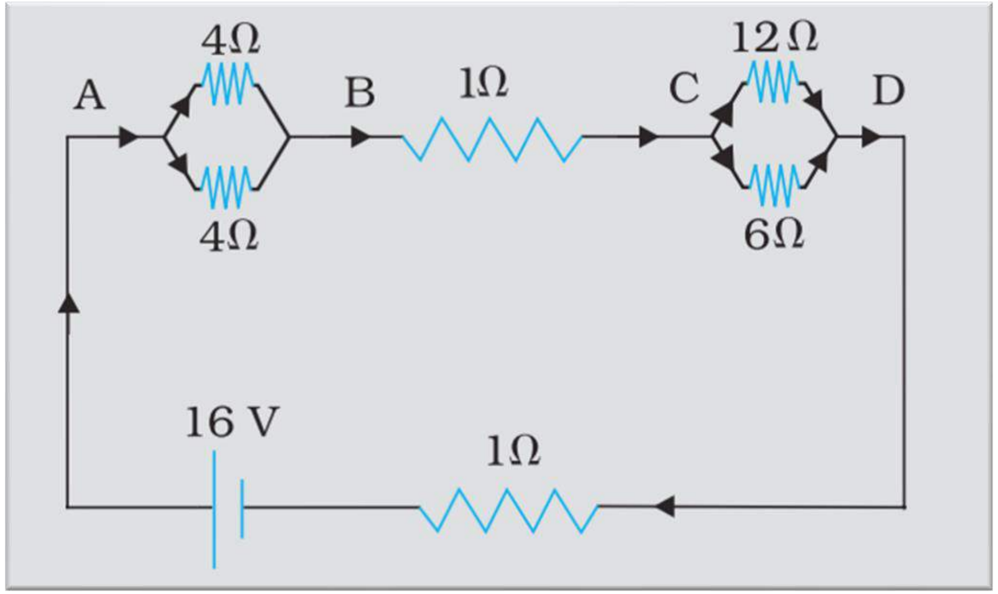
The electrical installations at our home are connected in _______

A) Series

B) Parallel

Quiz 2 of 5

A network of resistors is connected to a 16 V battery with internal resistance of 1Ω, as shown below. The voltage drop V_{CD} is ____



a) 4 V

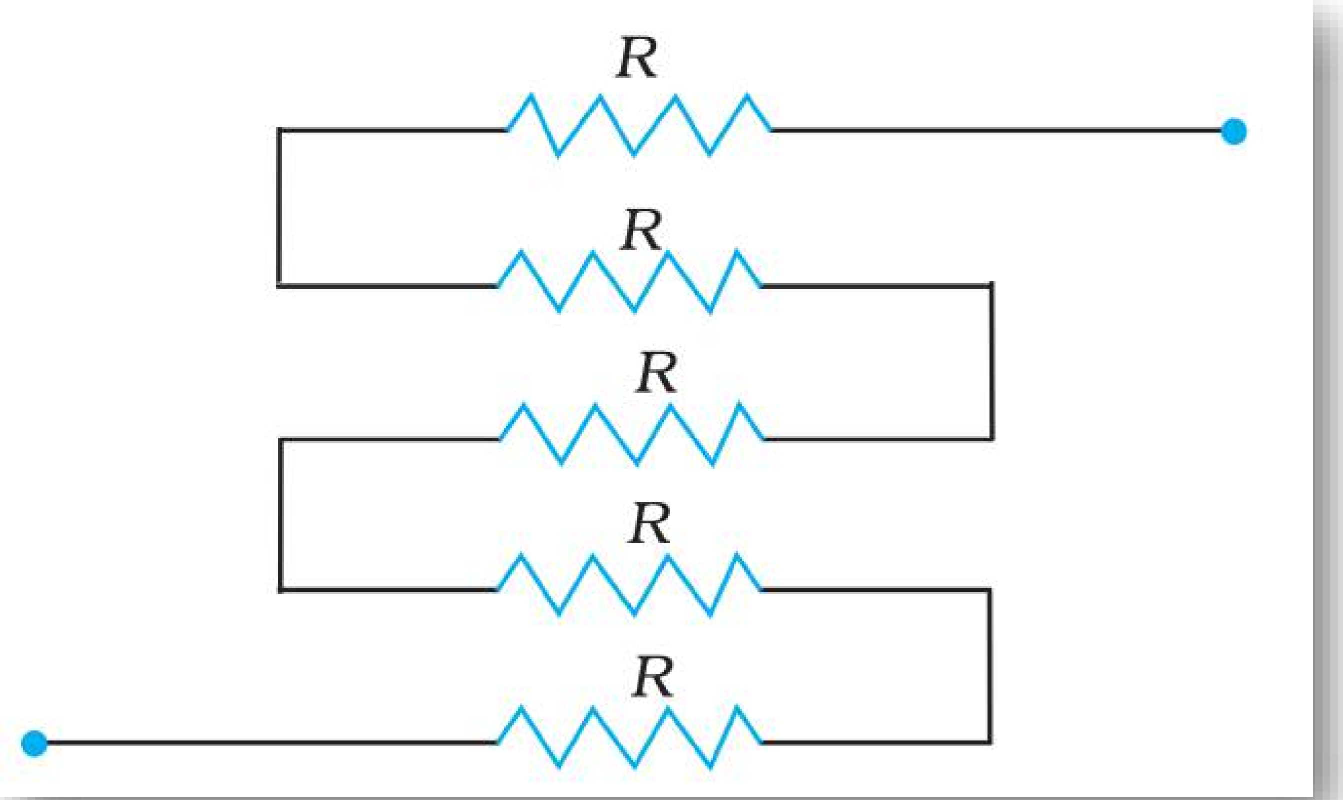
b) 2 V

c) 8 V

d) 16 V

Quiz 3 of 5

In the circuit shown, the equivalent resistance of the network shown is ________



A) R/5 Ω

B) 5R Ω

C) 6R/5 Ω

D) 2R Ω

Quiz 4 of 5

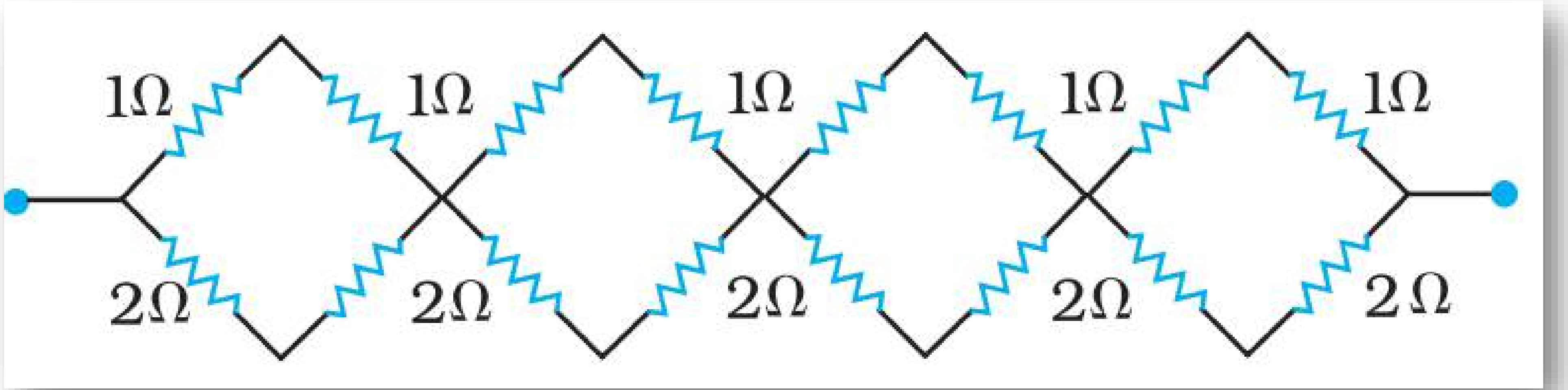
The equivalent resistance of the network shown is _______

A) 16/3 Ω

B) 10/3 Ω

C) 15/3 Ω

D) 15 Ω



Quiz 5 of 5

Two electric bulbs have filaments of same thickness. When connected to the same source, one of them consumes 60W and other one consumes 100W. Then

a) 60W lamp filament has shorter length
b) 100W lamp filament has longer length
c) 60W lamp filament has longer length
d) Both have equal length

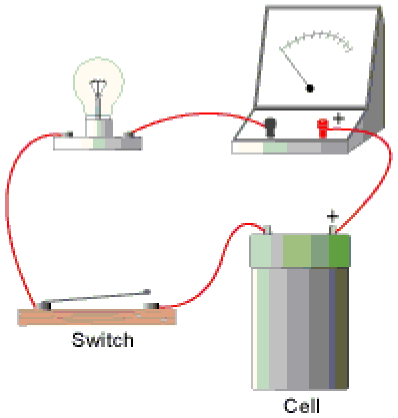
Basic Electrical Technology

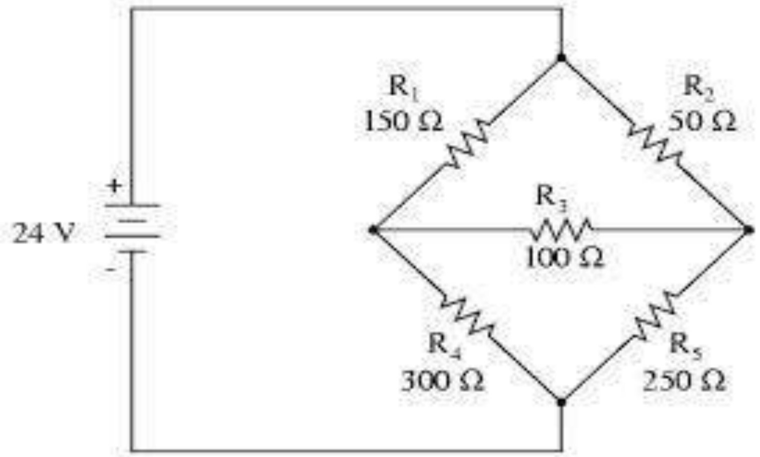
DC CIRCUIT ANALYSIS

What is an Electric Circuit?

Definition:

“An interconnection of simple electrical devices with at least one closed path in which current may flow”

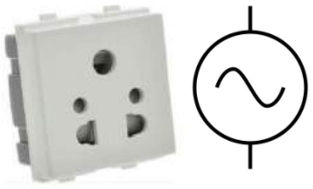
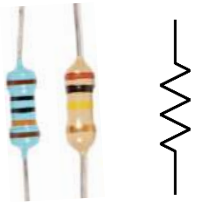
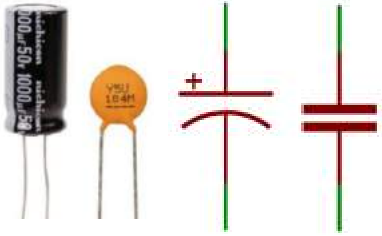


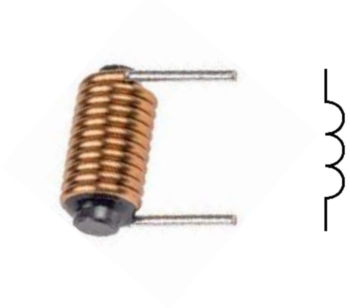


Circuit Elements

Active & Passive	• Active Elements: *Voltage & Current Sources* • Passive Elements: *Resistor, Inductor, Capacitor*
Linear & Non-linear Elements	• Linear: *Resistor, Inductor, Capacitor* • Nonlinear: *Diode, LDR (Light Dependent Resistor), Thermistor, transistor*
Unilateral & Bilateral Elements	• Unilateral (Current Flow in one direction): *Diode, Transistor* • Bilateral: *Resistor, Inductor, Capacitor**
Lumped & Distributed	• Lumped elements are simplified version of distributed elements

Our study is limited to **lumped linear bilateral** circuit elements



Basic Electrical Technology

CLASS 2 – 22 OCTOBER 2021

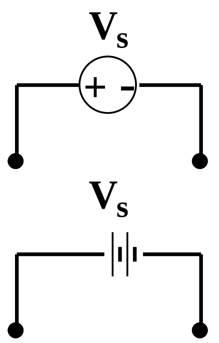
- ACTIVE & PASSIVE ELEMENTS

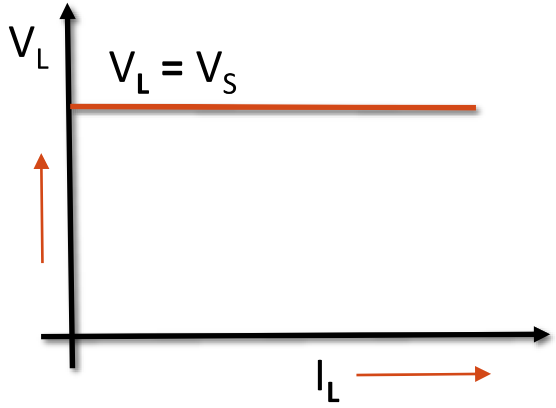
Active Elements - Sources

Voltage Source:

- **Ideal:**
  - Maintains constant voltage irrespective of connected load
  - Internal resistance $\boldsymbol{R_s = 0}$

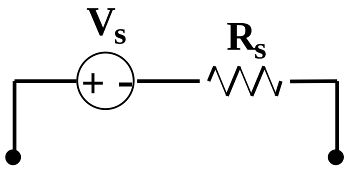
Ideal Voltage Source (DC)

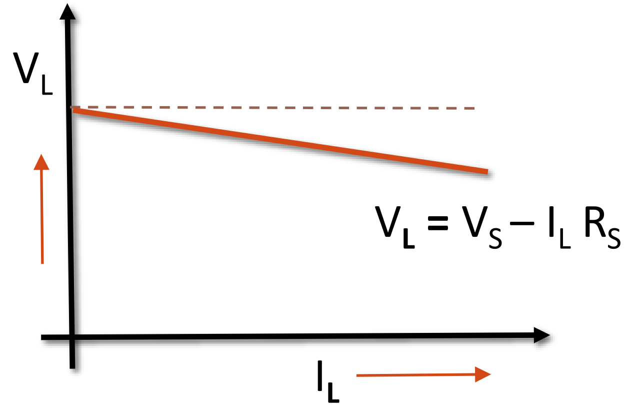




- **Practical:**
  - Terminal voltage changes based on the connected load
  - Internal resistance $\boldsymbol{R_s \neq 0}$

Practical Voltage Source



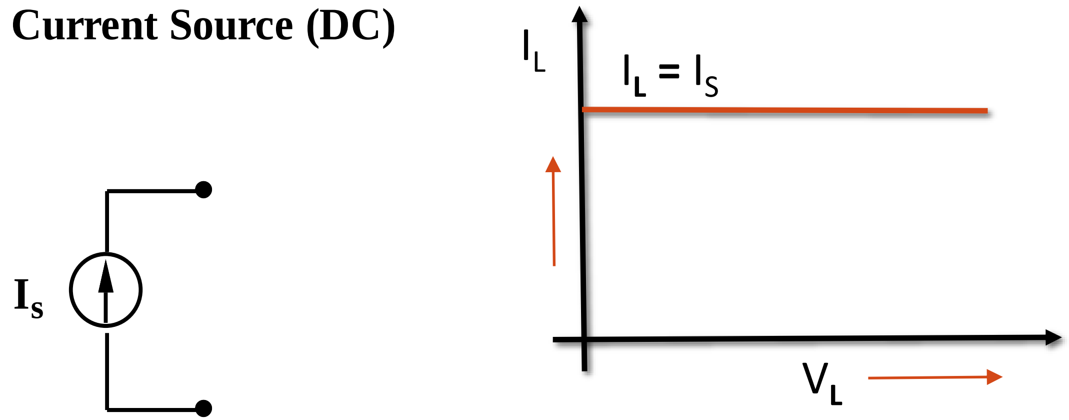


Active Elements - Sources

Current Source:

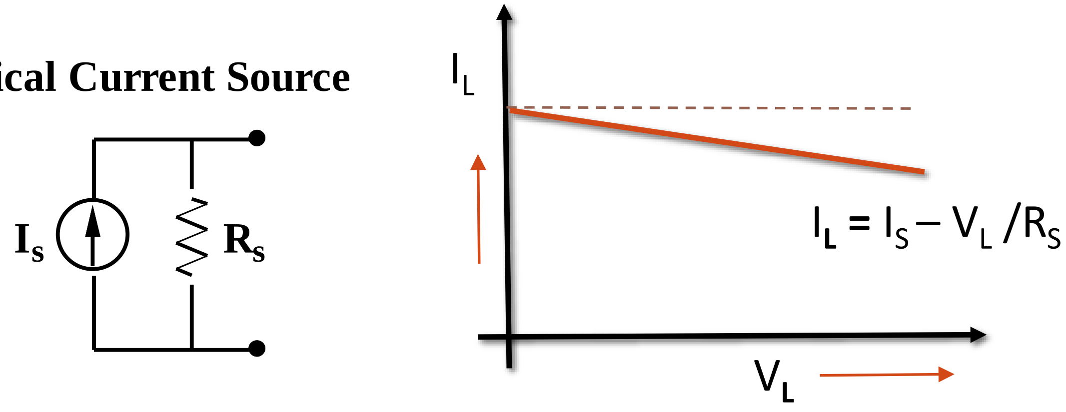
- **Ideal:**
  - Maintains constant current irrespective of the load connected
  - Internal resistance $\boldsymbol{R_s} = \infty$

Ideal Current Source (DC)



- **Practical:**
  - Output current changes based on the connected load
  - Internal resistance $\boldsymbol{R_s} < \infty$

Practical Current Source



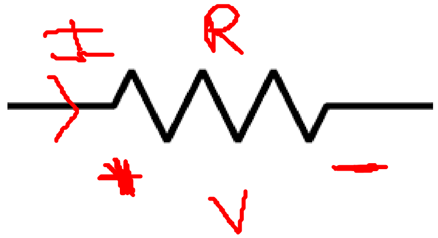
Resistor

Energy Consuming Element

Resistor

- **Passive electric device** that **dissipates energy**

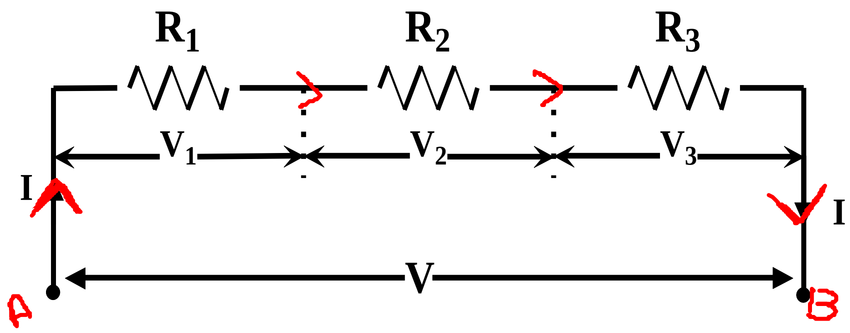
- **Resistance:** Property which opposes flow of current
  - Symbol: **R**
  - Unit: Ohms (Ω)
  - Power Consumed $= \boldsymbol{I^2 R}$ W



- **Conductance**
  - Reciprocal of resistance
  - Symbol: G
  - Unit – Siemens (S)

Resistors

Series connection of Resistors



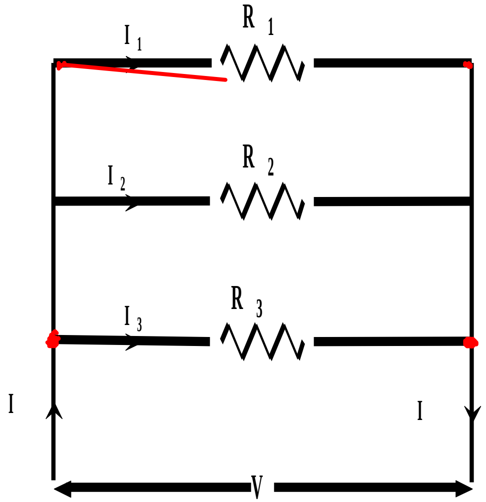
$V_1 = IR_1$ $V_2 = IR_2$ $V_3 = IR_3$

$V = V_1 + V_2 + V_3$

$IR_{eq} = IR_1 + IR_2 + IR_3$

- Current (I) in the all the resistors remains same
- $V = V_1 + V_2 + V_3$
- $R_{eq} = R_1 + R_2 + R_3$

Parallel connection of Resistors



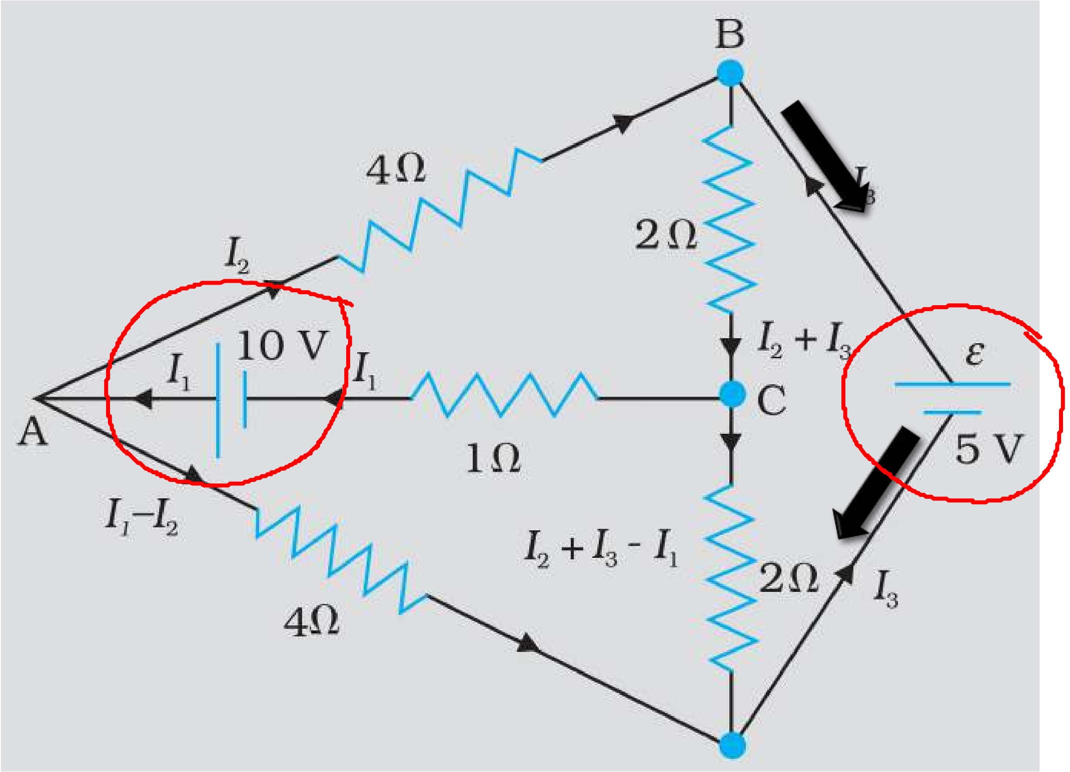
$I = I_1 + I_2 + I_3$

$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + V/R_3$

- Voltage (V) is same
- $I = I_1 + I_2 + I_3$
- $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Delivering and absorbing power by a source

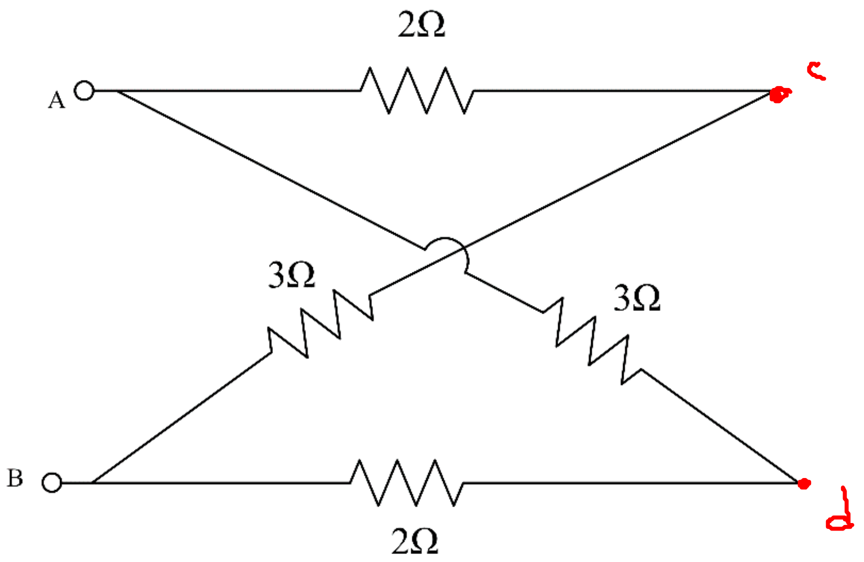
- A battery is **discharging** (delivering) if,
  - Current flows from the **+ve terminal to –ve terminal**
- A battery is **charging** (absorbing) if,
  - Current flows from the **-ve terminal to +ve terminal**
- When current flows through a resistor,
  - Power is dissipated

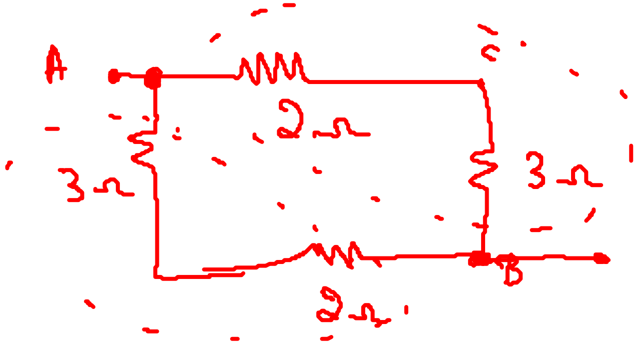


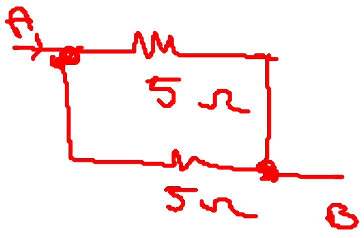
10 V battery is discharging
5V battery is charging

Illustration 1

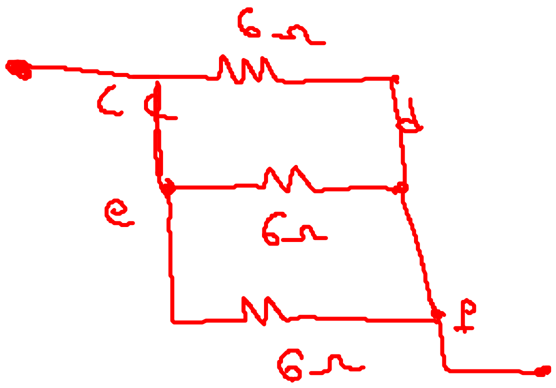
Find the equivalent resistance of the networks given below.







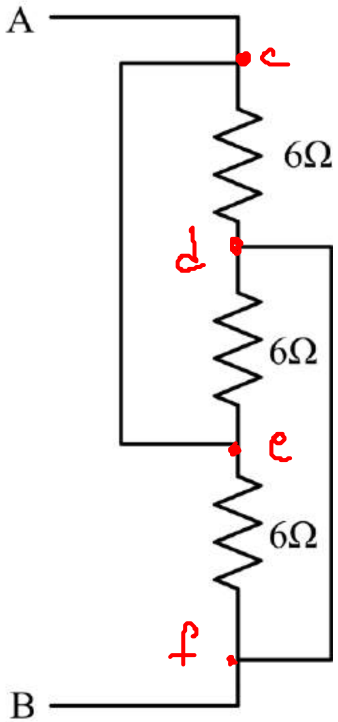
$R_{AB} = 5 \| 5 = 2.5\,\Omega$



$R_{AB} = \dfrac{1}{\frac{1}{6} + \frac{1}{6} + \frac{1}{6}}$

$R_{AB} = \dfrac{1}{0.5\,\Omega}$

$= \underline{\underline{2\,\Omega}}$



Quiz 1

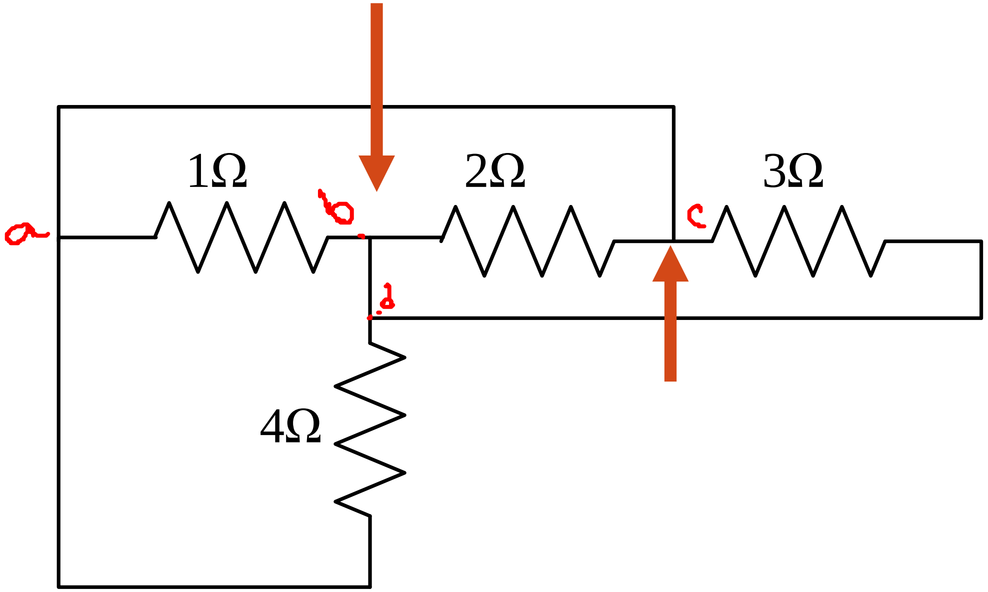
Resistors in the following circuit are in,

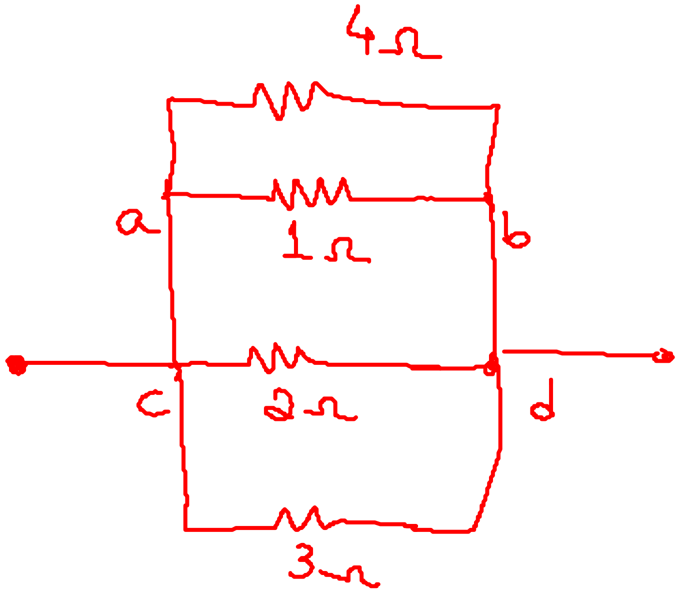
a) Series

b) Parallel

c) Combination of series and parallel

d) None of the above





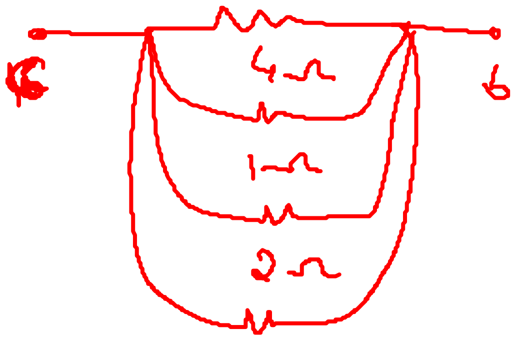


Illustration 2

Determine the equivalent resistance between the points A and B for the given resistive network with 1 Ω resistors

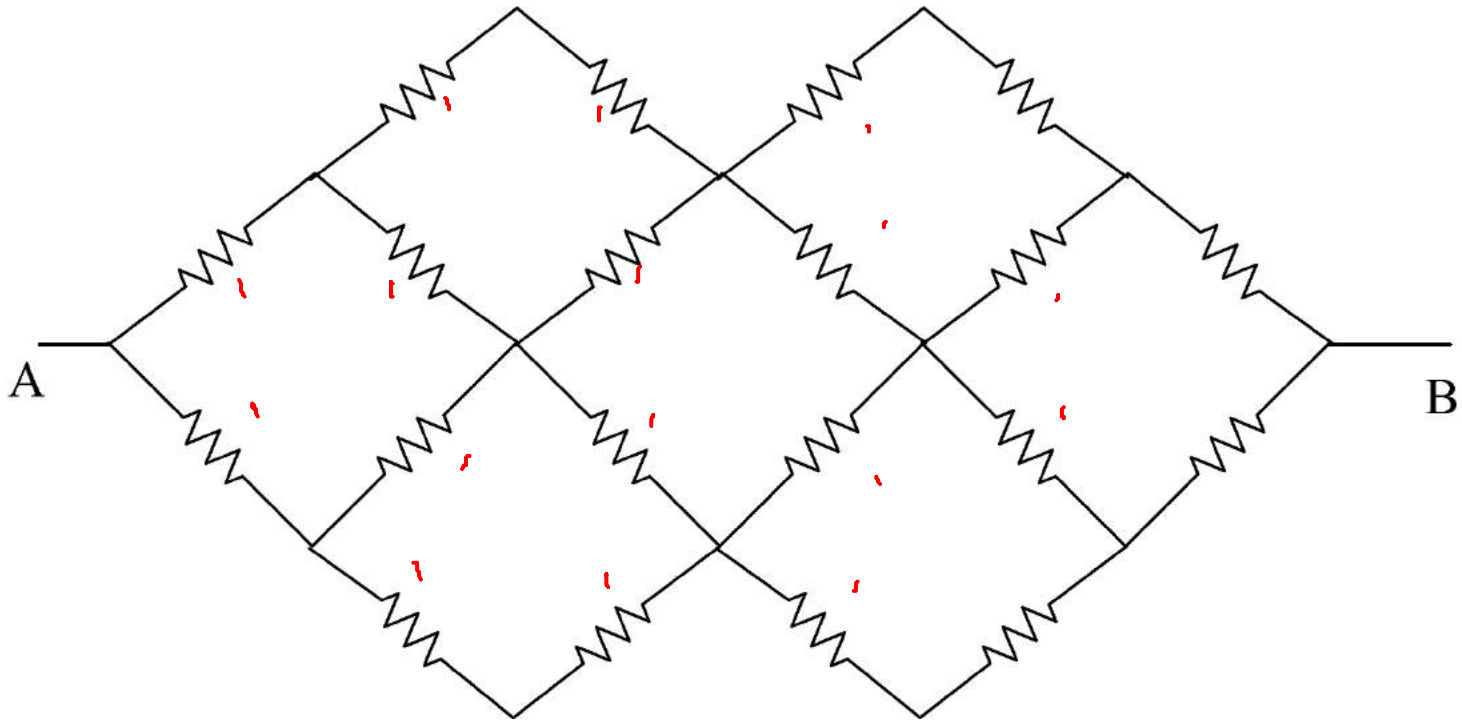
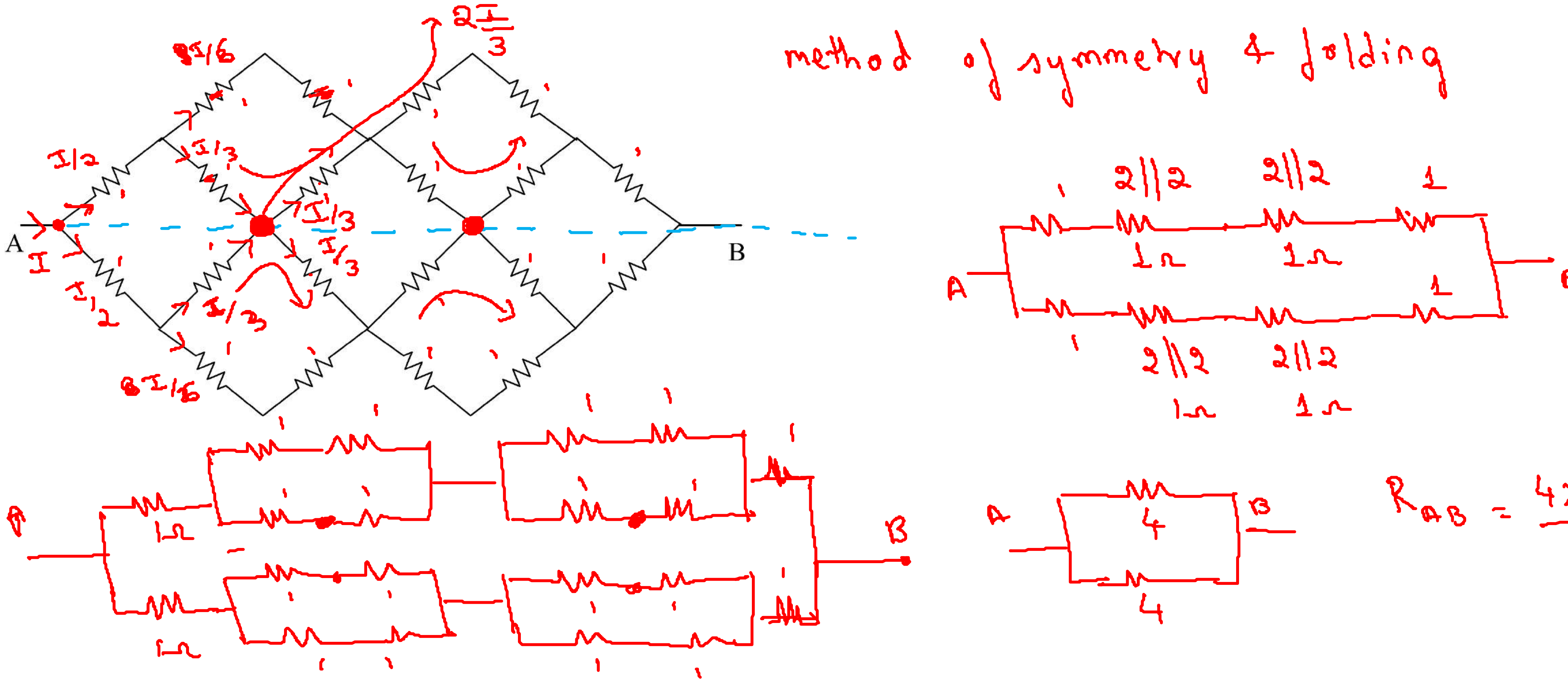


Illustration 2

method of symmetry & folding



$$R_{AB} = \frac{4\times 4}{8} = 2\,\Omega$$

Inductor

Energy Storing Element

Inductor

- **Passive** electric device that **stores energy in its magnetic field** when current flows through it

- A coil of wire wound on a core
  - Eg.: Air core Inductor, iron core inductor

- **Inductance**: property which opposes rate of change of current
  - Symbol: **L**
  - Unit: Henry (H)

$V_L = L\frac{di}{dt}$

- The voltage across inductor is proportional to the rate of change of current through it

$$v_L = L\frac{di}{dt}$$

Inductive Circuit

For a coil uniformly wound on **a non-magnetic core** of uniform cross section, self-inductance is given by

$$L = \frac{\mu_0 A N^2}{l}$$

where,

$l =$ length of the magnetic circuit in meters

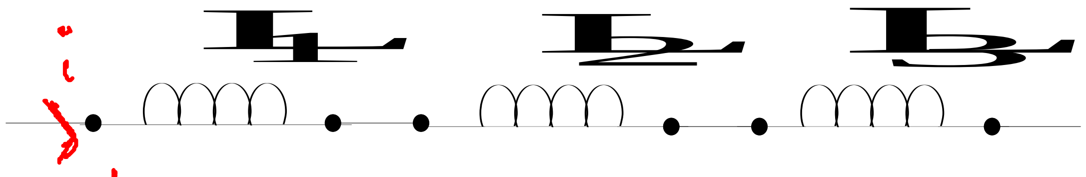
$A =$ cross sectional area in square meters

$\mu_o =$ Permeability of air $= 4\pi \times 10^{-7}$

$N = No.of\ turns\ in\ the\ coil$

Equivalent Inductance

Inductors in series



$$L_{eq} = L_1 + L_2 + \dots\dots + L_n$$

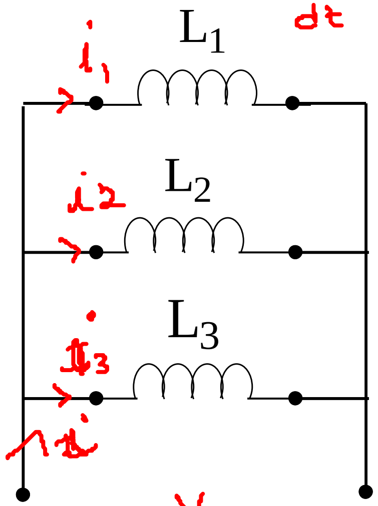
+ V_1 − + V_2 − + V_3 −

$V = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt}$

$V_1 = L_1\frac{di}{dt}$ $V_2 = L_2\frac{di}{dt}$ $V_3 = L_3\frac{di}{dt}$

$L_{eq}\frac{di}{dt} =$

Inductors in Parallel



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots\dots + \frac{1}{L_n}$$

$i = i_1 + i_2 + i_3$

$V = L_{eq}\frac{di}{dt} = L_{eq}\frac{d}{dt}(i_1 + i_2 + i_3)$

$= L_{eq}\frac{di_1}{dt} + L_{eq}\frac{di_2}{dt} + L_{eq}\frac{di_3}{dt}$

$V = L_{eq}\left(\frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}\right)$

$\frac{1}{L_{eq}} = 1/L_1 + 1/L_2 + 1/L_3$

Energy Stored in an Inductor

- Instantaneous power,

$$p = v_L . i = L\, i \frac{di}{dt}$$

- Energy absorbed in 'dt' time is

$$dw = L\, i\, di$$

- Energy absorbed by the magnetic field when current increases from $\mathbf{0}$ to $\boldsymbol{I}$ amperes, is

$$W = \int_0^I L\, i\, di = \frac{1}{2} L\, I^2$$

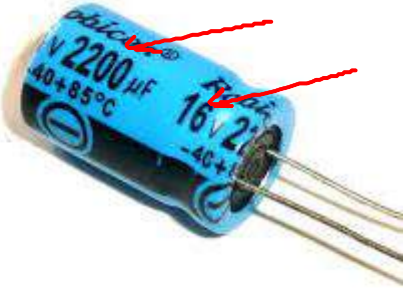
Capacitor

Energy Storing Element

Capacitors

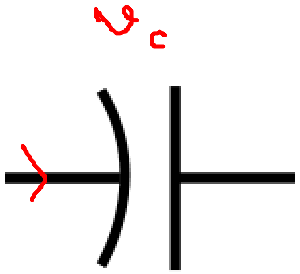
- **Passive electric device** that **stores energy in the electric field** between a pair of closely spaced conductors

- **Capacitance**: Property which opposes the rate of change of voltage
  - Symbol: **C**
  - Unit: Farad (F)



- The capacitive current is proportional to the rate of change of voltage across it

$$i_c = C\frac{dv_c}{dt}$$



- Charge stored in a capacitor whose plates are maintained at constant voltage:

$$Q = CV$$

Terminologies

- Electric field strength,

$$E = \frac{V}{d}\ volts/m$$

- Electric flux density,

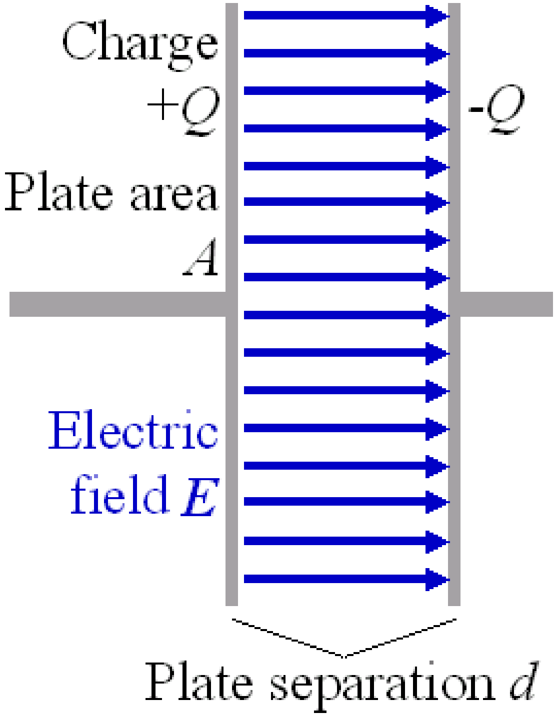
$$D = \frac{Q}{A}\ C/m^2$$

- Permittivity of free space,

$$\varepsilon_0 = 8.854 \times 10^{-12}\ F/m$$

- Relative permittivity, ε_r
- Capacitance of parallel plate capacitor

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

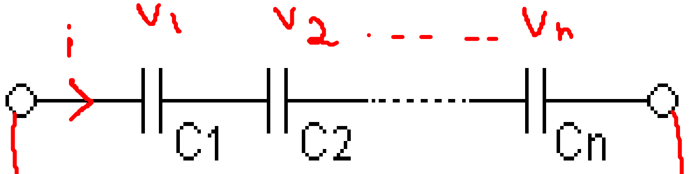


Material	Relative permittivity
Vacuum	1.0
Air	1.0006
Paper (dry)	2–2.5
Polythene	2–2.5
Insulating oil	3–4
Bakelite	4.5–5.5
Glass	5–10
Rubber	2–3.5
Mica	3–7
Porcelain	6–7
Distilled water	80
Barium titanate	6000+

Equivalent Capacitance

Capacitors in Series

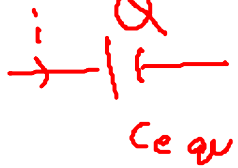
$$\frac{1}{C_{eq}} = \frac{1}{C1} + \frac{1}{C2} + \;\dots\dots + \frac{1}{Cn}$$



$Q = C_1V_1 = C_2V_2 \cdots = C_nV_n$

$V = V_1 + V_2 + V_3$

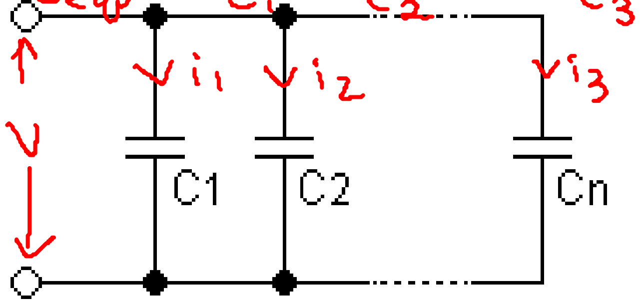
$\frac{Q}{C_{equ}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots \frac{Q}{C_3}$



$Q = C_{equ} V$

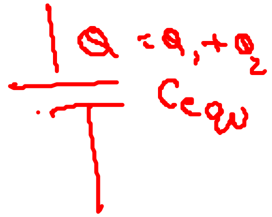
Capacitors in Parallel

$$C_{eq} = C_1 + C_2 + \;\dots\dots + C_n$$



$Q_1 = C_1 V$

$Q_2 = C_2 V$



$Q_1 + Q_2 + \cdots = C_{equ} V$

$C_1V + C_2V + \cdots$

Air dielectric strength 5.75 MV/m 0.2 mm

Mica 200 MV/m 0.01 mm

$\boxed{\text{Hughes}}$

Energy stored in a Capacitor

- Instantaneous power

$$p = v_c \times i = C\, v_c \frac{dv_c}{dt}$$

- Energy supplied during ‘dt’ time is:

$$dw = C\, v_c\, dv_c$$

- Energy stored in the electric field when potential rises from 0 to V volts is,

$$W = \int_0^V C\, v_c dv_c = \frac{1}{2} C V^2 \text{ Joules}$$

Quiz 2

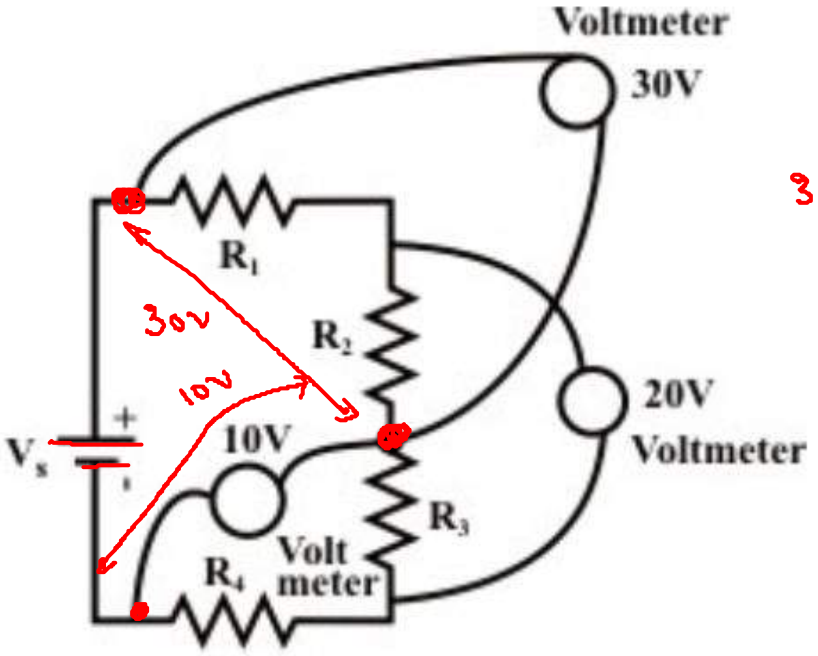
An inductor and a resistor opposes _____ & _____ respectively

a) flow of current, rate of change of current
b) rate of change of current, flow of current
c) rate of change of current, rate of change of current
d) flow of current, flow of current

Quiz 3

The source voltage is ___

a) 10 V
b) 20 V
c) 30 V
d) 40 V



$30 + 10 = \underline{40V}$

Illustration 3

Two incandescent bulbs have the following ratings:

Bulb-1: 120 V, 60 W; **Bulb-2:** 240 V, 480 W

a) Both of them are connected in series with a voltage source.
   i. Which bulb will glow brighter and why?
   ii. What is the maximum voltage that can be applied so that none of the bulbs fuse?

b) Now both of them are connected in parallel with a voltage source.
   i. Which bulb will glow brighter and why?
   ii. What is the maximum voltage that can be applied so that none of the bulbs fuse?

Assume that the incandescent bulbs are purely resistive.

Illustration 3

Two incandescent bulbs have the following ratings:

Bulb-1: 120 V, 60 W; **Bulb-2:** 240 V, 480 W

a) Both of them are connected in series with a voltage source.

i. Which bulb will glow brighter and why? 60W bulb

ii. What is the maximum voltage that can be applied so that non of the bulbs fuse?

$$V_{max} = (0.5 \times 240) + (0.5 \times 120) = \underline{\underline{180\,V}}$$

Bulb 1

$$I_{r1} = \frac{P_{r1}}{V_{r1}} = \frac{60}{120} = \underline{\underline{0.5\,A}}$$

$$R_1 = \frac{P_{r1}}{I_{r1}} \text{ or } \frac{V_{r1}^2}{P_{r1}} = \frac{120^2}{60} = \underline{\underline{240\,\Omega}}$$

Bulb2

$$I_{r2} = \frac{P_{r2}}{V_{r2}} = \frac{480}{240} = \underline{\underline{2\,A}}$$

$$R_2 = \frac{V_{r2}^2}{P_{r2}} = \frac{240^2}{480} = \underline{\underline{120\,\Omega}}$$

$$P_{B1} = 0.5^2 \times 240 = \boxed{60\,W}$$

$$P_{B2} = 0.5^2 \times 120 = 30\,W$$

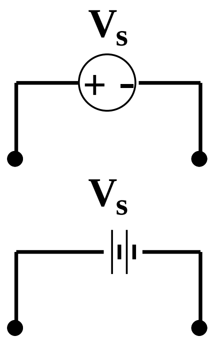
Basic Electrical Technology

CLASS 3 – 26 OCTOBER 2021

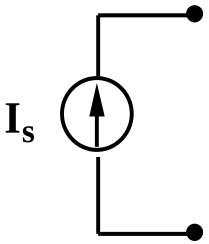
- NETWORK REDUCTION

RECAP

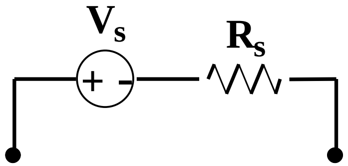
Ideal Voltage Source (DC)



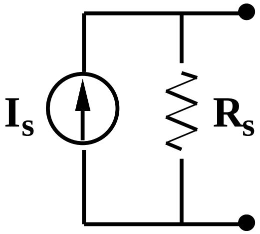
Ideal Current Source (DC)



Practical Voltage Source



Practical Current Source



Illustration

Two incandescent bulbs have the following ratings:

Bulb-1: 120 V, 60 W; **Bulb-2:** 240 V, 480 W

a) Both of them are connected in series with a voltage source.
   i. Which bulb will glow brighter and why?
   ii. What is the maximum voltage that can be applied so that non of the bulbs fuse?

b) Now both of them are connected in parallel with a voltage source.
   i. Which bulb will glow brighter and why?
   ii. What is the maximum voltage that can be applied so that non of the bulbs fuse?

Assume that the incandescent bulbs are purely resistive.

Quiz 3

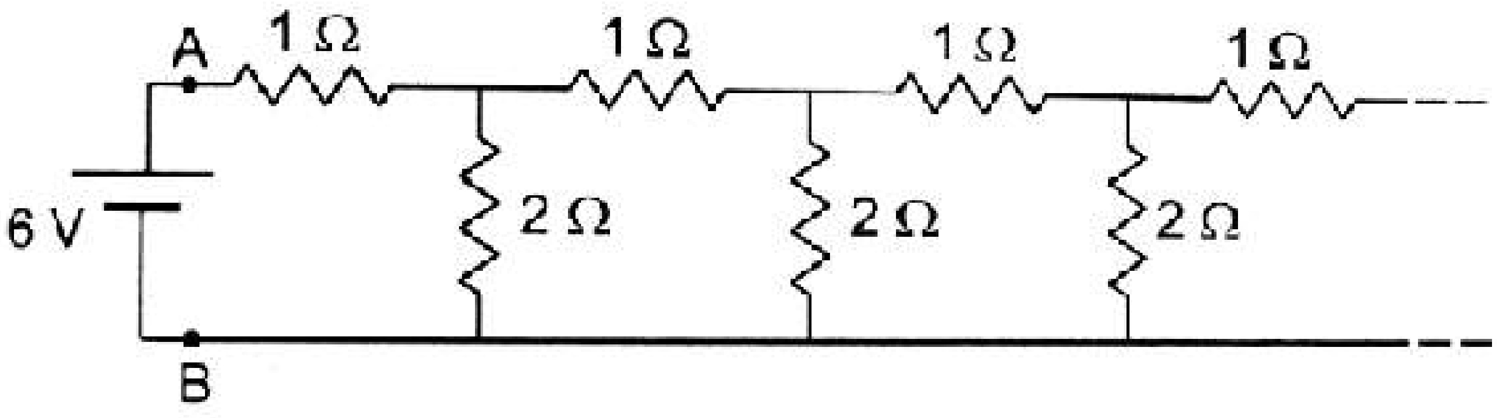
Two incandescent bulbs of 40 W and 60 W ratings are connected in series across the mains. Then which of the following statement(s) is(are) correct?

a) The bulbs together will consume 100 W
b) The bulbs together will consume 50 W
c) The 60 W bulb glows brighter
d) The 40 W bulb glows brighter

Assume the voltage rating of both the bulbs to be same

Illustration

What is the equivalent resistance across the terminals A & B in the network shown?

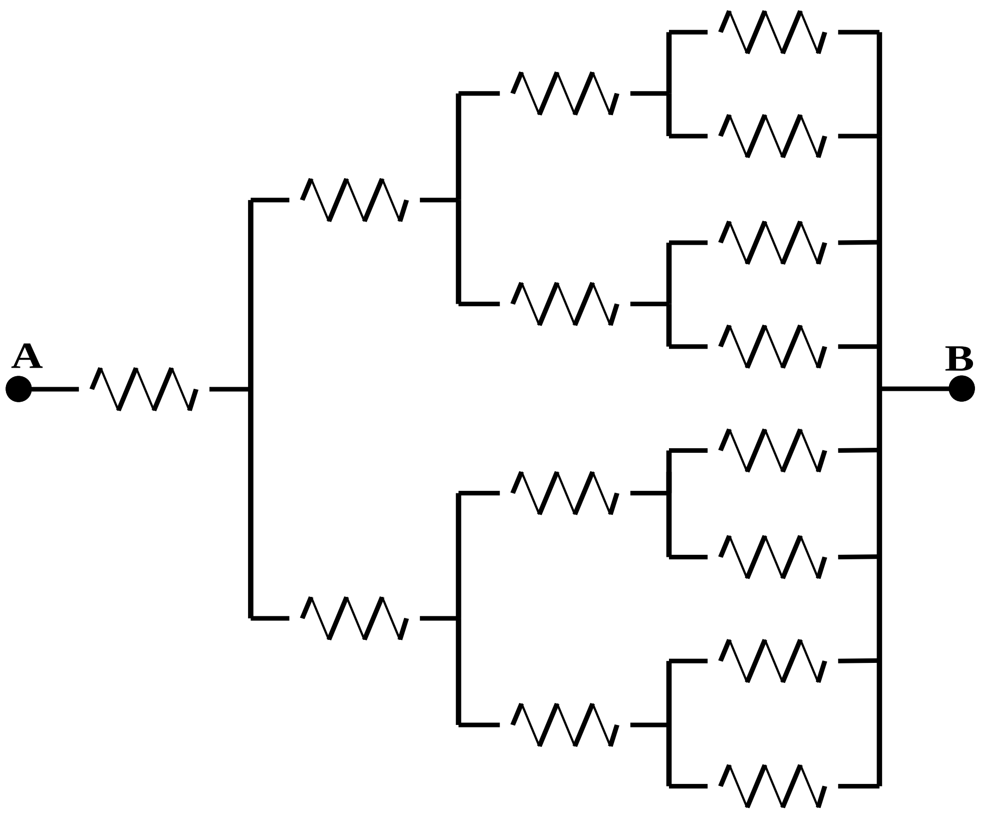


Illustration

15 resistors are connected as shown in the diagram. Each of the resistors has resistance 1 Ω.

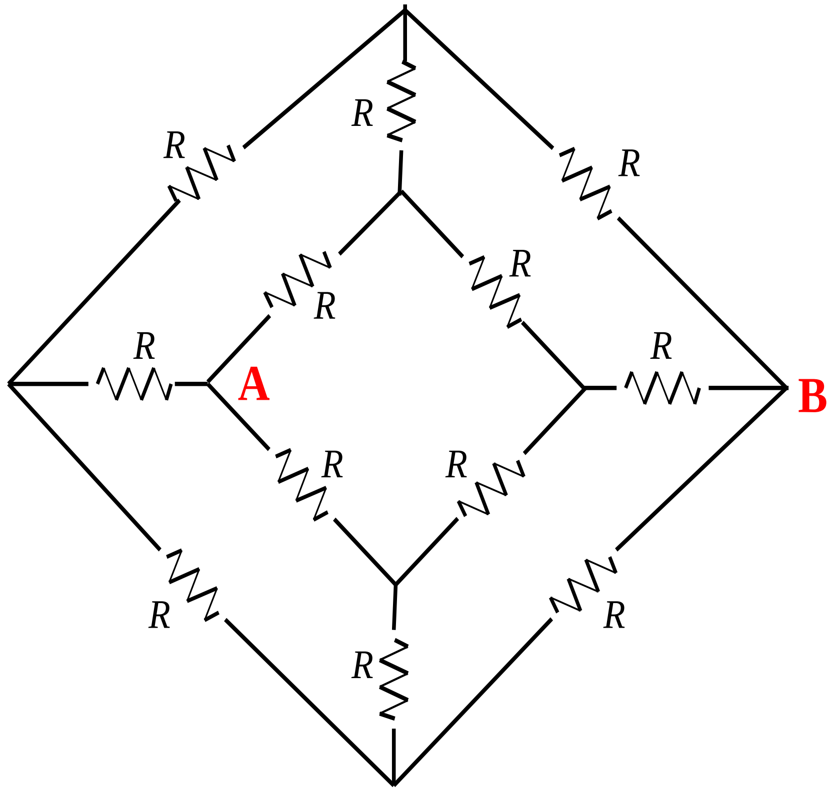
a) Find the equivalent resistance of the network between A & B.

b) What will be the equivalent resistance of this network if the resistors arranged in the sequence extends to infinity?



Homework

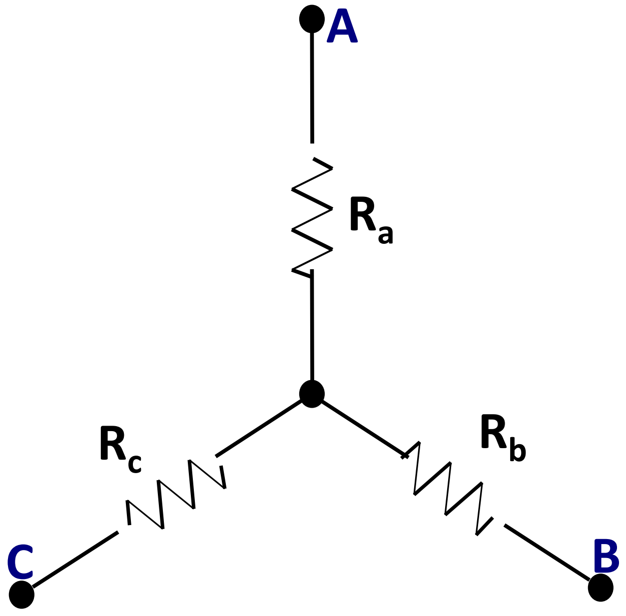
Reduce the network to its equivalent resistance between terminals A and B



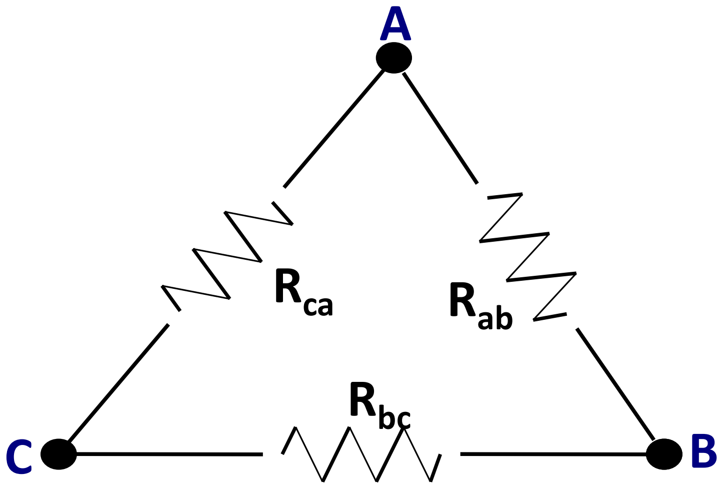
Star – delta transformation

NETWORK REDUCTION TECHNIQUE

Star & Delta Connections



Star Connection

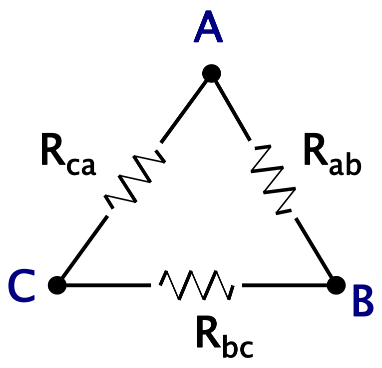


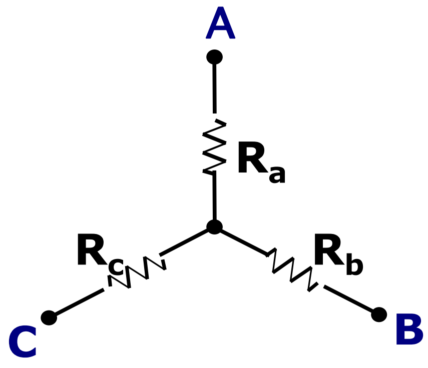
Delta Connection

Link for the formula derivation:
https://nptel.ac.in/content/storage2/courses/108105053/pdf/L-06(GDR)(ET)%20((EE)NPTEL).pdf

Star-Delta Transformation

Delta to Star Transformation





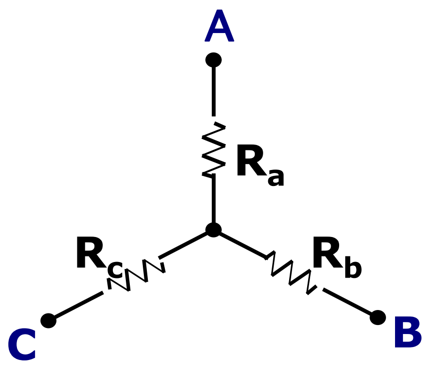
$$R_a = \frac{R_{ab}\, R_{ca}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ab}\, R_{ca}}{\sum R_\Delta}$$

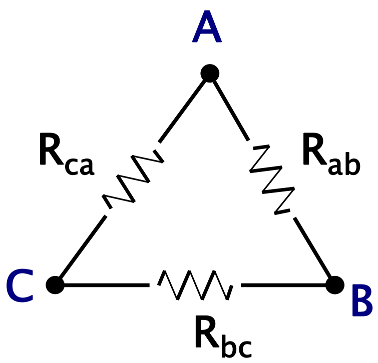
$$R_b = \frac{R_{bc}\, R_{ab}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{bc}\, R_{ab}}{\sum R_\Delta}$$

$$R_c = \frac{R_{ca}\, R_{bc}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ca}\, R_{bc}}{\sum R_\Delta}$$

Star-Delta Transformation

Star to Delta Transformation





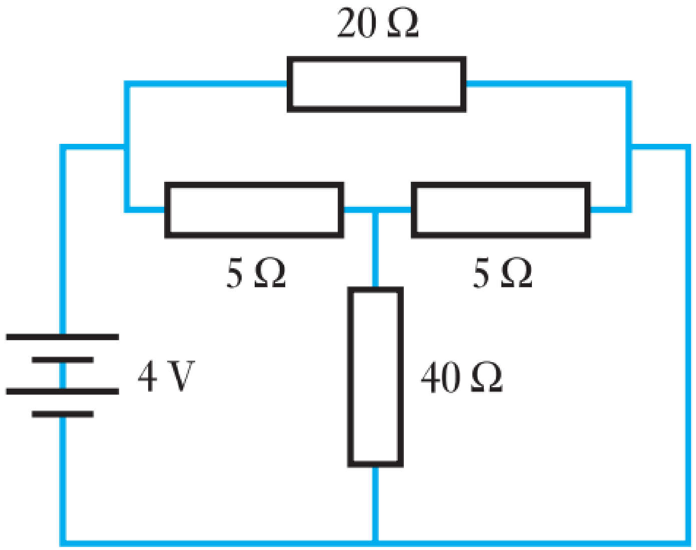
$$R_{ab} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_c} = R_a + R_b + \frac{R_aR_b}{R_c}$$

$$R_{bc} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_a} = R_b + R_c + \frac{R_bR_c}{R_a}$$

$$R_{ca} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_b} = R_c + R_a + \frac{R_aR_c}{R_b}$$

Illustration 1

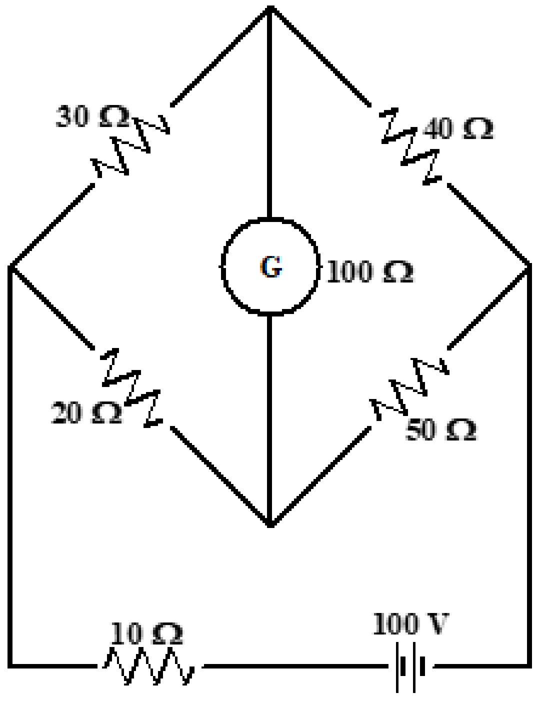
Find the current through 40 ohms resistor in the circuit shown



Current through 40 ohms =0.047 A

Illustration 2

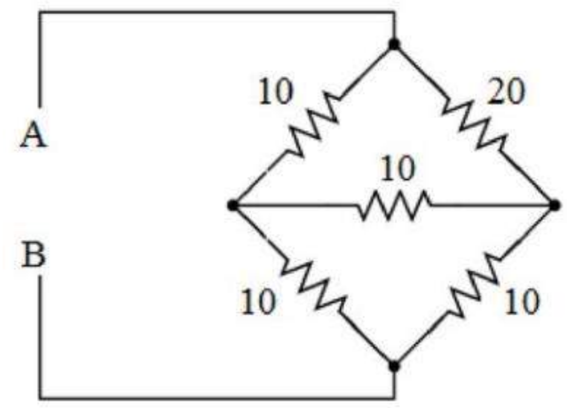
For the circuit shown, determine the total power supplied by the source using star-delta transformation



Ans: $P_{supplied}$ = 223.1608 W

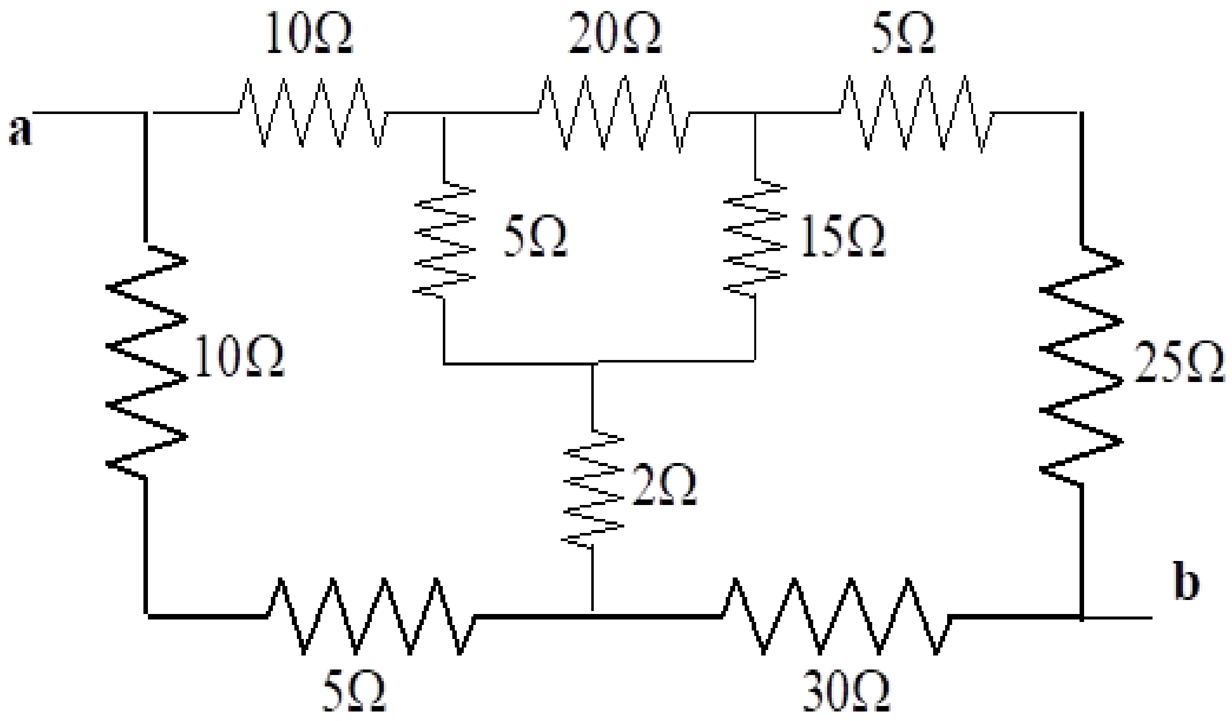
Quiz 1

Find R_{AB}



Homework

Determine the resistance between terminals a & b of the network shown in figure, using Star-Delta transformation.



Ans: 23.518 Ω

Source Transformation

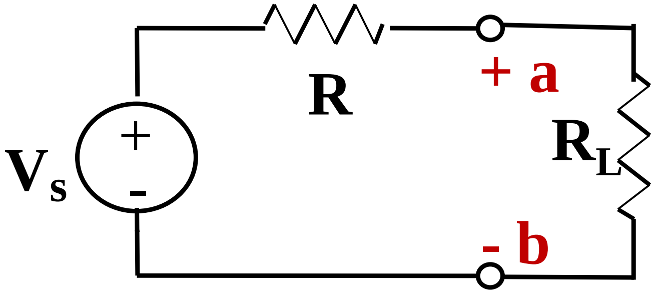
NETWORK REDUCTION TECHNIQUE

Source Transformation

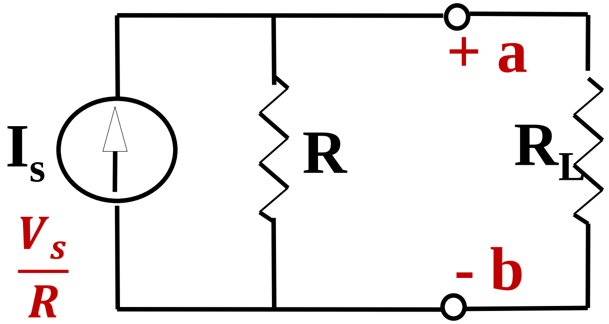
Practical Voltage Source ⟷ **Practical Current Source**

Source Transformation

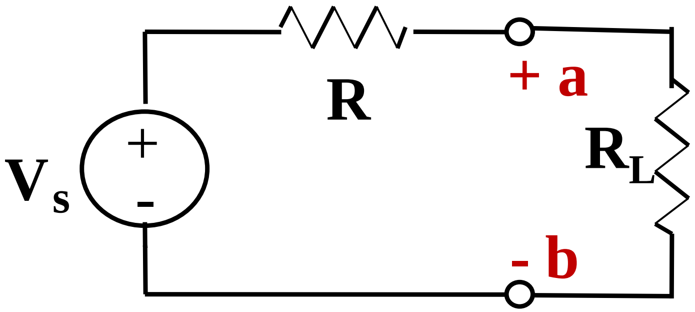
Practical Voltage source



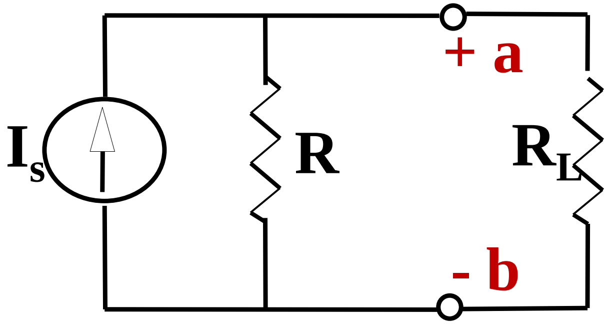
Practical Current source



$$I_s = \frac{V_s}{R}$$



$$V_s = R \times I_s$$



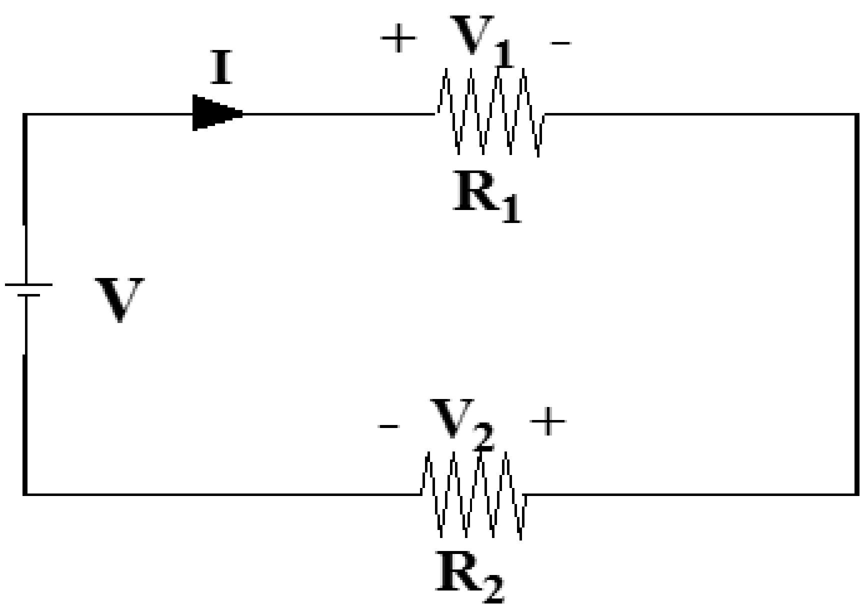
Voltage Division (in Series Circuit)

$$V = V_1 + V_2$$

$$V = V_1 + V_1 \frac{R_2}{R_1}$$

$$V_1 = V \frac{R_1}{R_1 + R_2}$$

$$V_2 = V \frac{R_2}{R_1 + R_2}$$



Quiz 2

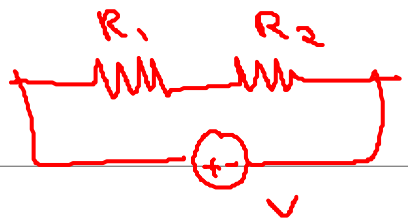
What is the voltage across 10 ohms

Basic Electrical Technology

CLASS 4 – 29 OCTOBER 2021

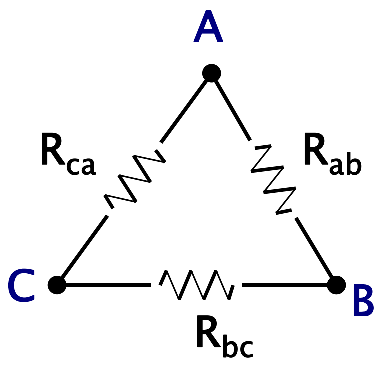
- NETWORK REDUCTION

Recap



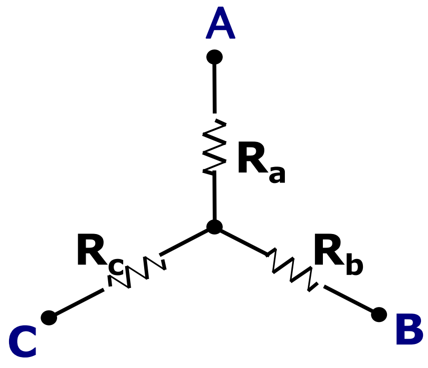
$V_{R_1} = V\frac{R_1}{R_1+R_2}$

- **Star delta transformation**

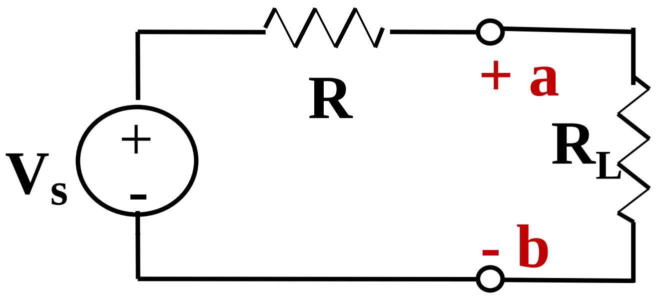


$$R_a = \frac{R_{ab}\,R_{ca}}{R_{ab}+R_{bc}+R_{ca}} = \frac{R_{ab}\,R_{ca}}{\sum R_\Delta}$$

$$R_{ab} = \frac{R_aR_b+R_bR_c+R_cR_a}{R_c} = R_a+R_b+\frac{R_aR_b}{R_c}$$

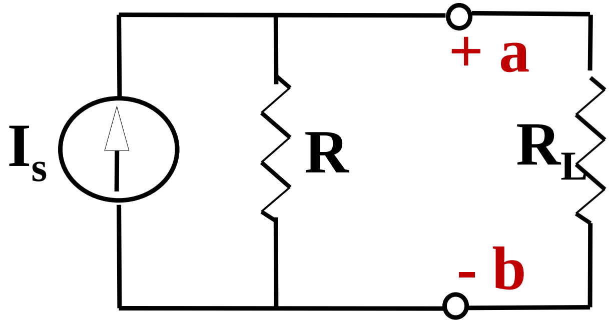


- **Source transformation**



$$I_s = \frac{V_s}{R}$$

$$V_s = R \times I_s$$



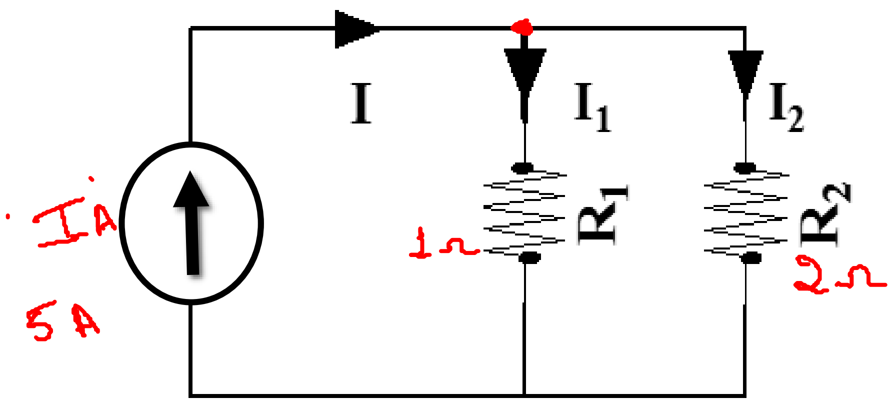
Current Division (in Parallel Circuit)

$$I = I_1 + I_2$$

$$I = I_1 + I_1 \frac{R_1}{R_2}$$

$$I_1 = I \frac{R_2}{R_1 + R_2}$$

$$\boxed{I_2 = I \frac{R_1}{R_1 + R_2}}$$



$$V_{R_1} = I_1 R_1 = V_{R_2} = I_2 R_2$$

$$I_{1\Omega} = 5 \times \frac{2}{1+2}$$

$$= 10/3 \text{ A}$$

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_{R_1}}{R_2} = \frac{I_1 R_1}{R_2}$$

$$I_{2\Omega} = \frac{5 \times 1}{3}$$

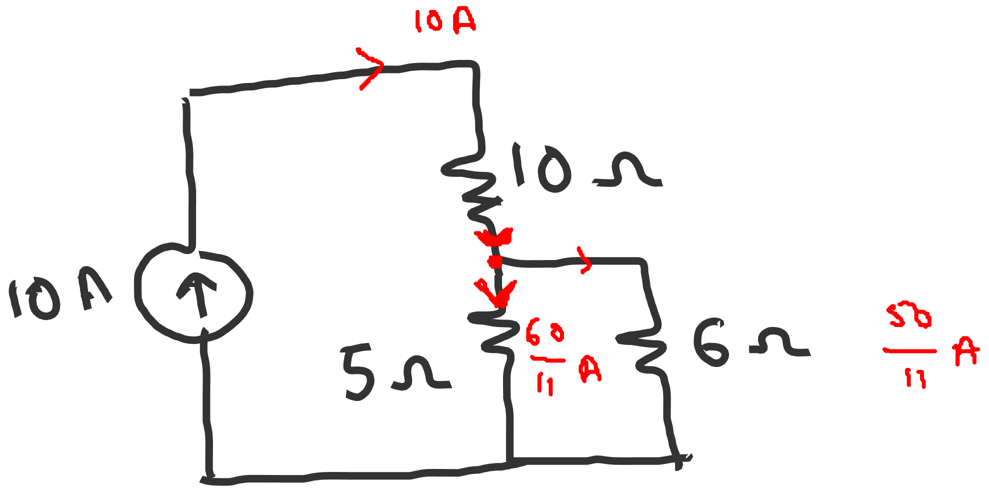
$$= 5/3 \text{ A}$$

Illustration

What is the current through 5 ohm

$$I_{5\Omega} = \frac{10 \times 6}{5+6} = \frac{60}{11}\,A$$

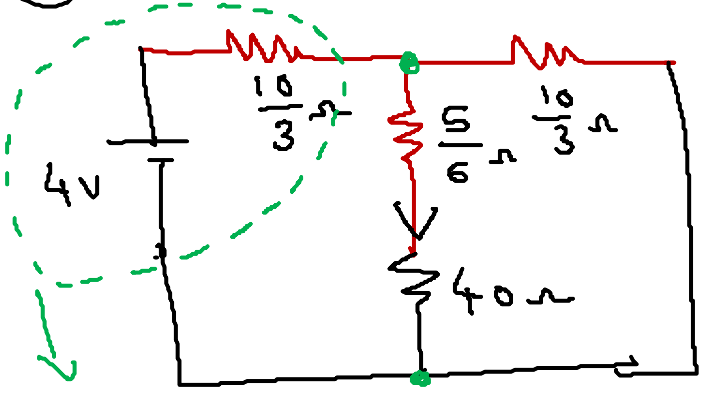
$$I_{6\Omega} = \frac{10 \times 5}{5+6} = \frac{50}{11}\,A$$



Illustration

Find the current through 40 ohms resistor.

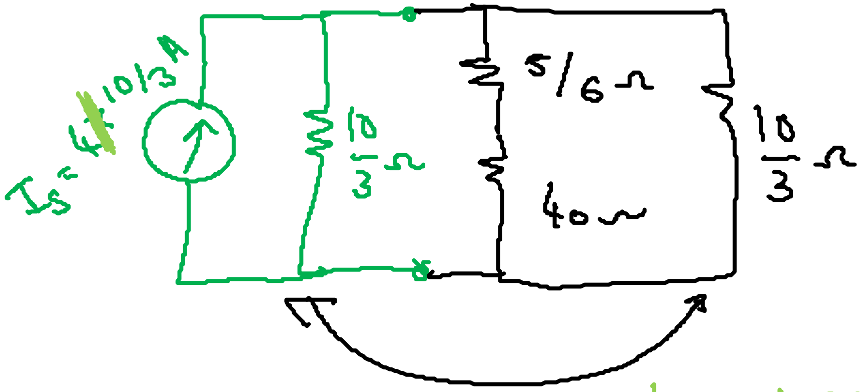
① Δ to Y



practical voltage source

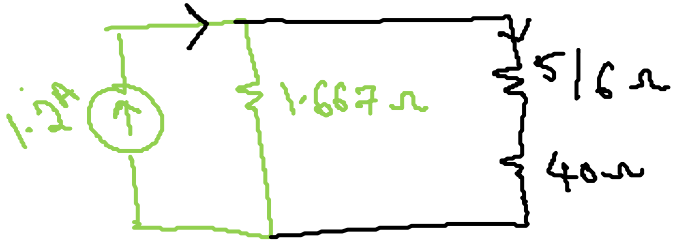
② source transformation

Practical voltage source to current source



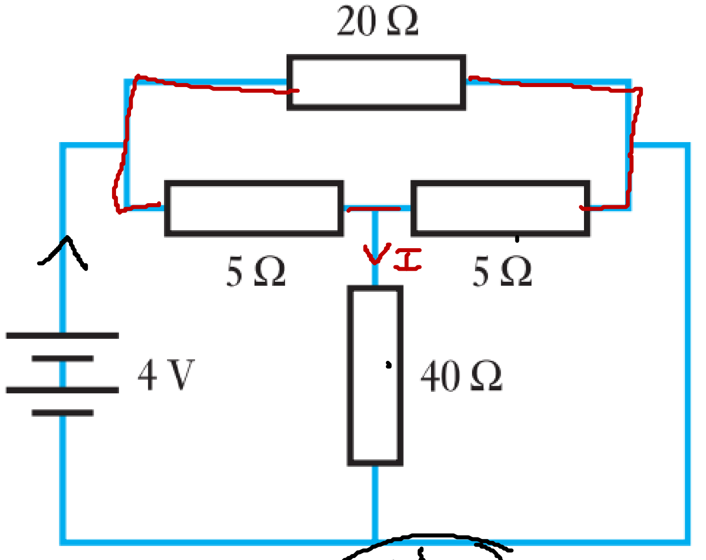
$10/3 \,||\, 10/3 = 1.667\Omega$

③ Series & parallel



④ Current division formula

$$I_{40\Omega} = \frac{1.2 \times 1.667}{1.667 + 5/6 + 40} = 0.047\,A$$

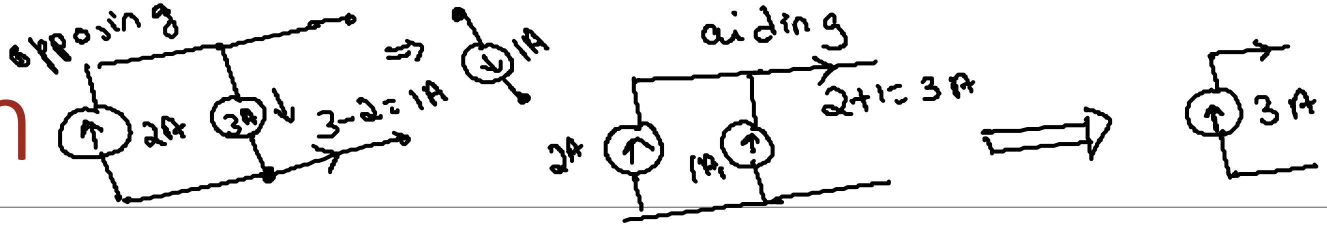


HW

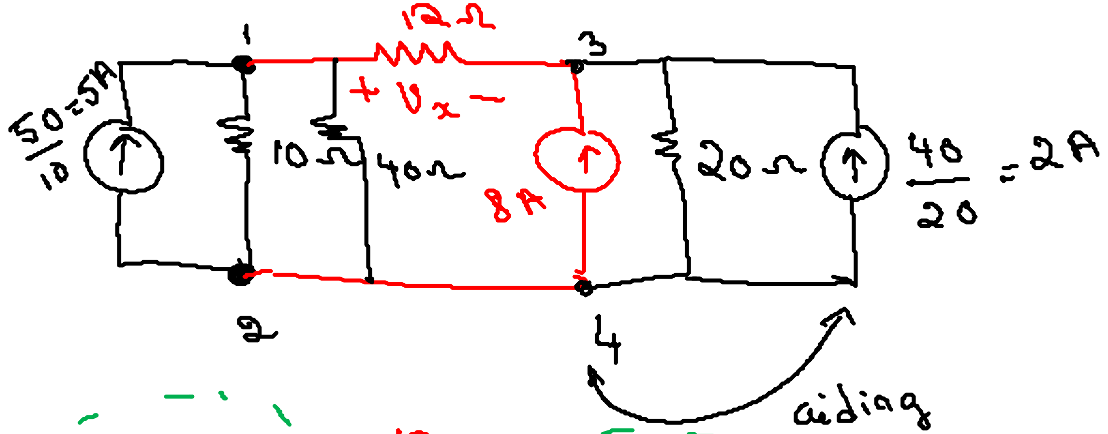
Find current supplied by 4V battery using only series parallel reduction of resistance.

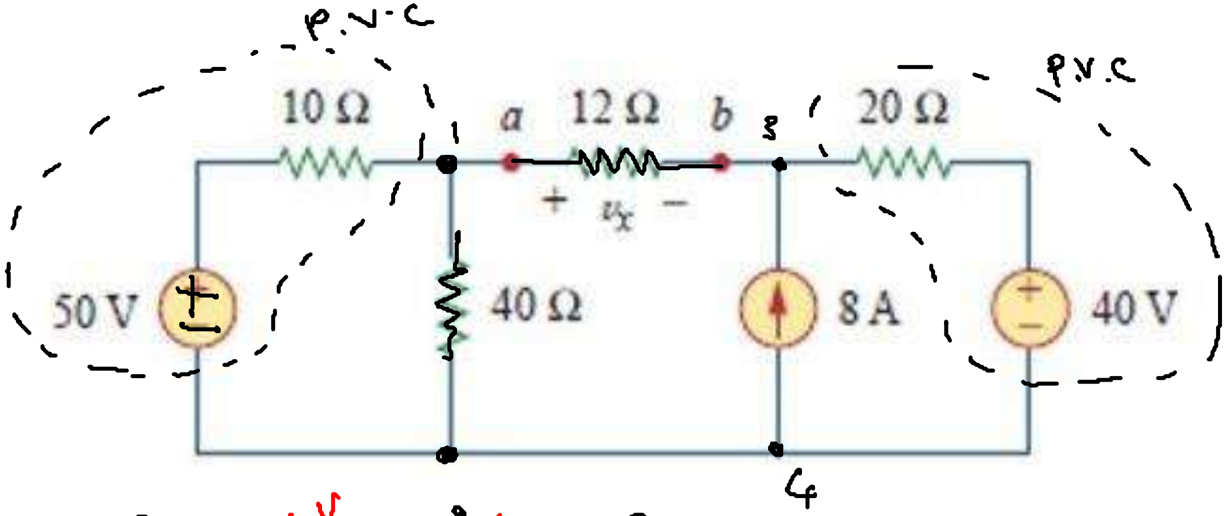
Current through 40 ohms =0.047 A

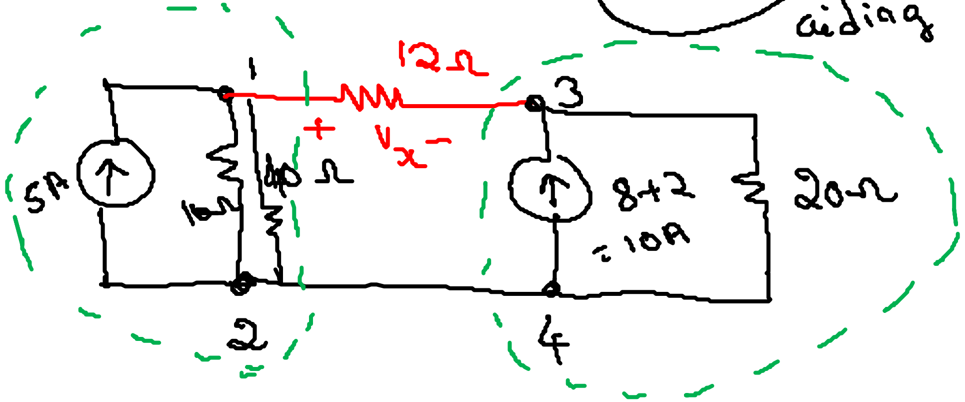
Illustration

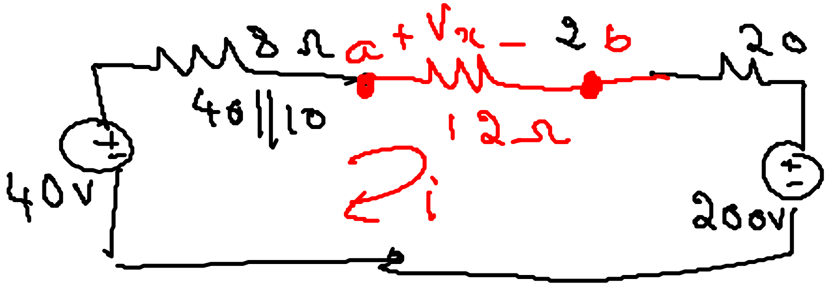


Find the voltage across 12 Ω resistor (i.e., Vx) by source transformation method.









'b' is at higher potential wrt 'a'.

KVL : $40 - i(8+12+20) - 200 = 0$

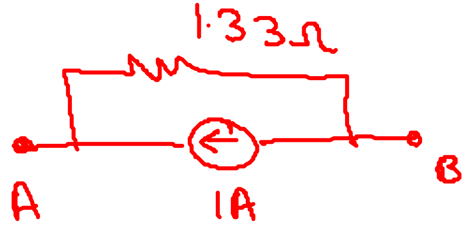
$i = -4A$, $V_{12\Omega} = 4 \times 12 = -48V$

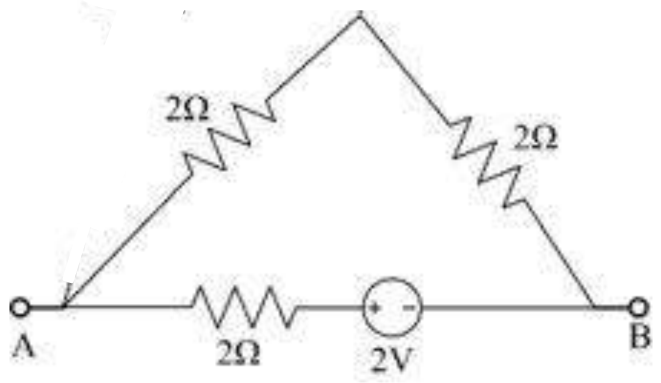
Ans. - 48 V

Quiz

Reduce the network to a single current source and a resistor across A and B

1.33Ω and 1A

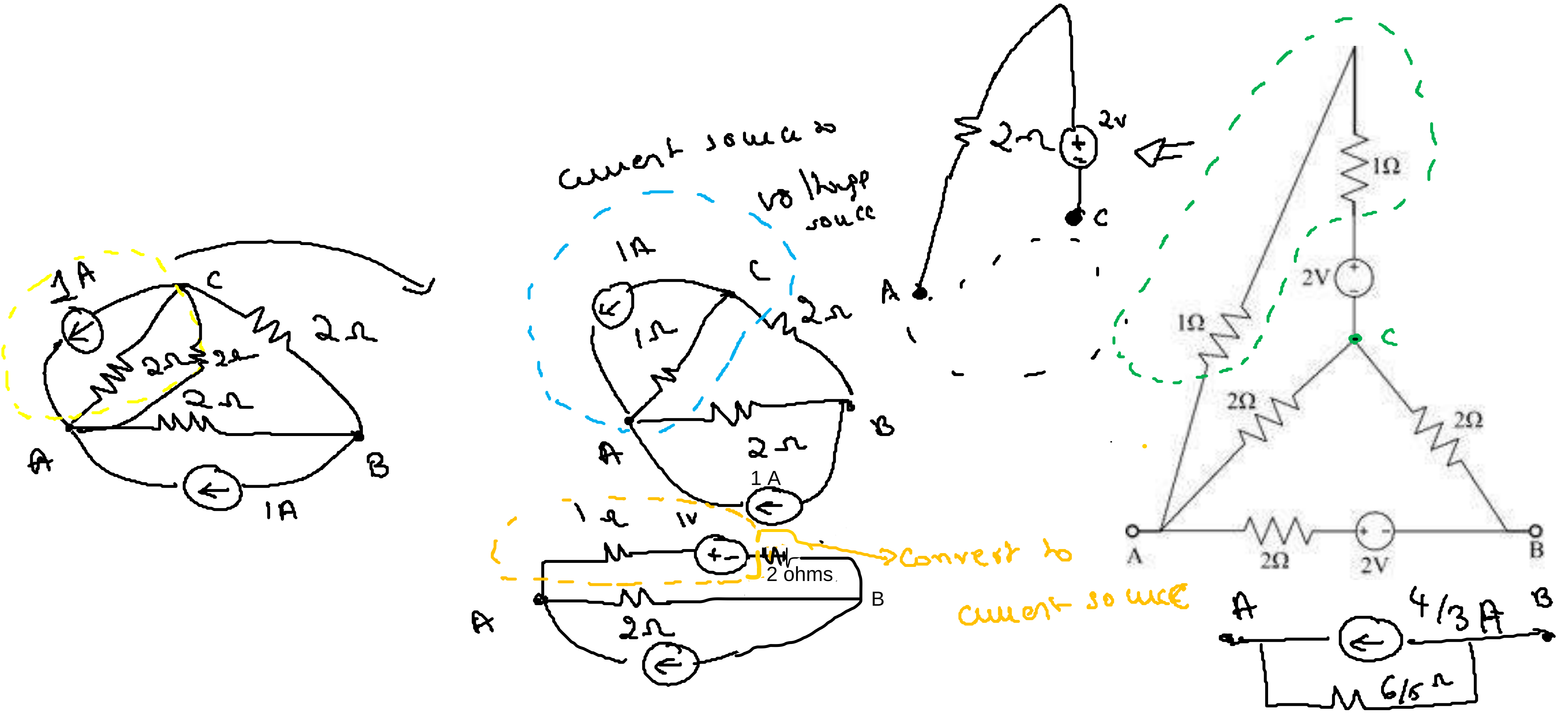




Illustration

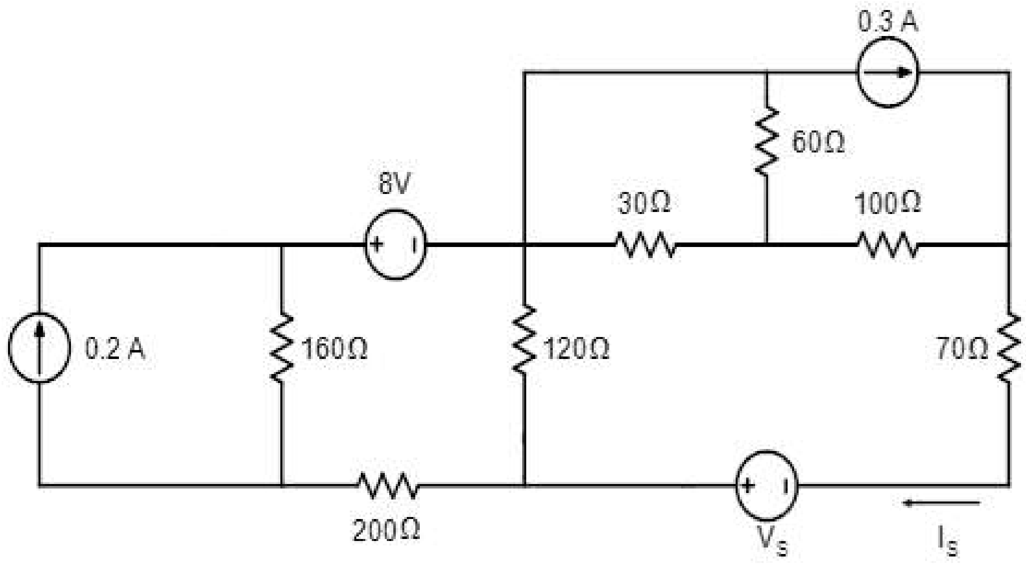
HW: Reduce the ckt to one voltage source & one resistance

Reduce the following circuit to a current source in parallel with a resistor across the terminals A & B.



Homework

In the circuit shown, compute the value of V_S to deliver a current of I_S = 0.25 A using source transformation.

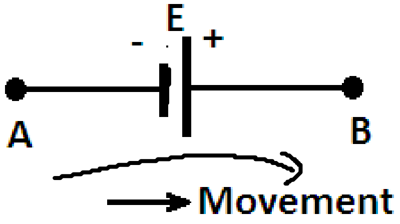


Ans: $V_s = 28\ V$

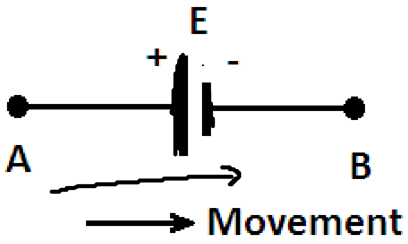
Mesh Current Analysis

NETWORK REDUCTION TECHNIQUE

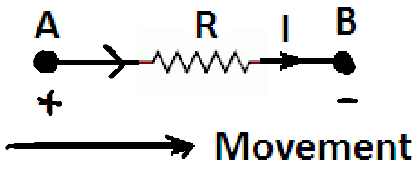
Sign Conventions for Kirchoff's Voltage Law (KVL)



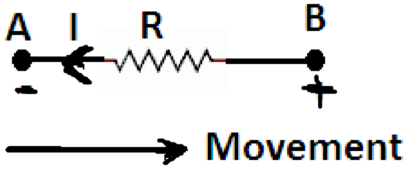
Rise in potential, because we are going negative terminal of the battery to postive terminal.
Therefore, EMF = + E



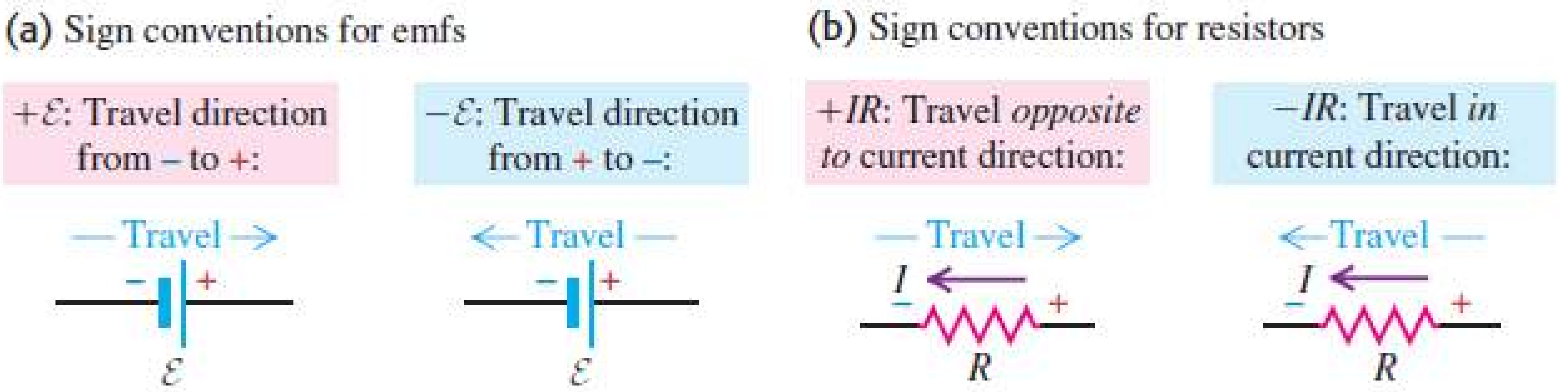
Fall in potential, because we are going postive terminal of the battery to negative terminal.
Therefore, EMF = - E



Fall in potential, because we are going in the direction of current.
Therefore, voltage drop = - IR



Rise in potential, because we are going in opposite direction of current.
Therefore, voltage drop = + IR



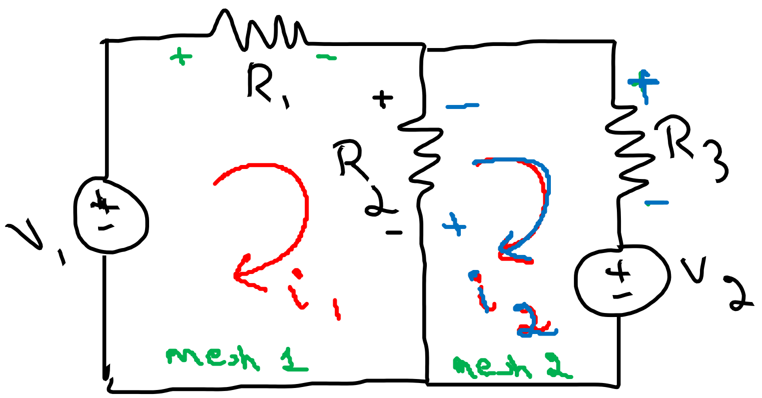
Introduction

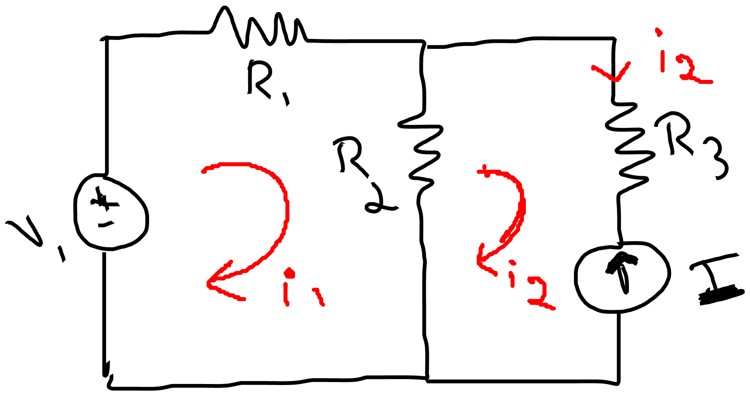
Mesh

- A closed path for the flow of current

Kirchhoff's Voltage Law (KVL)

- The algebraic sum of voltages in a mesh is zero





mesh 1: $V_1 - i_1R_1 - (i_1 - i_2)R_2 = 0$

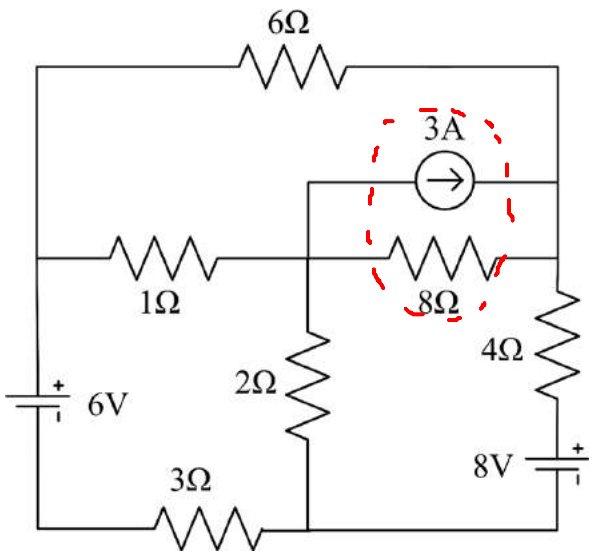
mesh 2: $-V_2 - (i_2 - i_1)R_2 - i_2R_3 = 0$

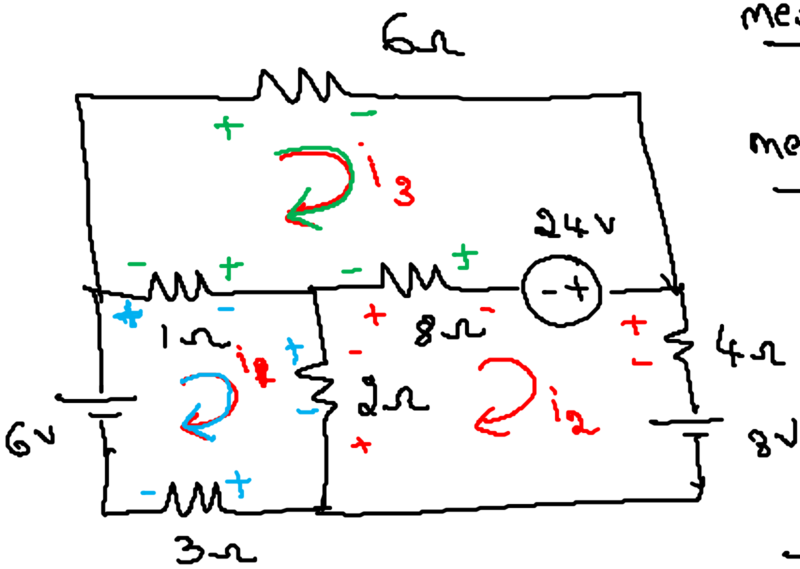
mesh 1: $V_1 - i_1R_1 - (i_1 - i_2)R_2 = 0$

mesh 2: $I = -i_2$ or $i_2 = -I$

Mesh Current Analysis Method

- Transform all the current sources present in the circuit to voltage sources
- Mark different currents in all the independent meshes of the given network
- Write KVL equations for these independent meshes
- Solve for the currents





Determine the power drawn by 2Ω resistor using mesh current analysis

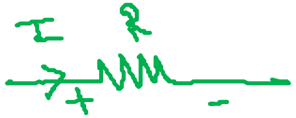
$= 3.775W$

$i_1 = 2.301A$

$i_2 = 0.927A$

$i_3 = -0.951A$

$P_{2\Omega} =$



$(i_1 - i_2)^2 2$

$= (2.301 - 0.927)^2 \times 2$

mesh 1: $6 - 1(i_1 - i_3) - 2(i_1 - i_2) - 3i_1 = 0$

$3i_1 - 2i_2 - i_3 = 6$ — ①

a b c d
3 −2 −1 6

mesh 2:

$-8 - 2(i_2 - i_1) - 8(i_2 - i_3) + 24 - 4(i_2) = 0$

$-2i_1 + 14i_2 - 8i_3 = 16$ — ②

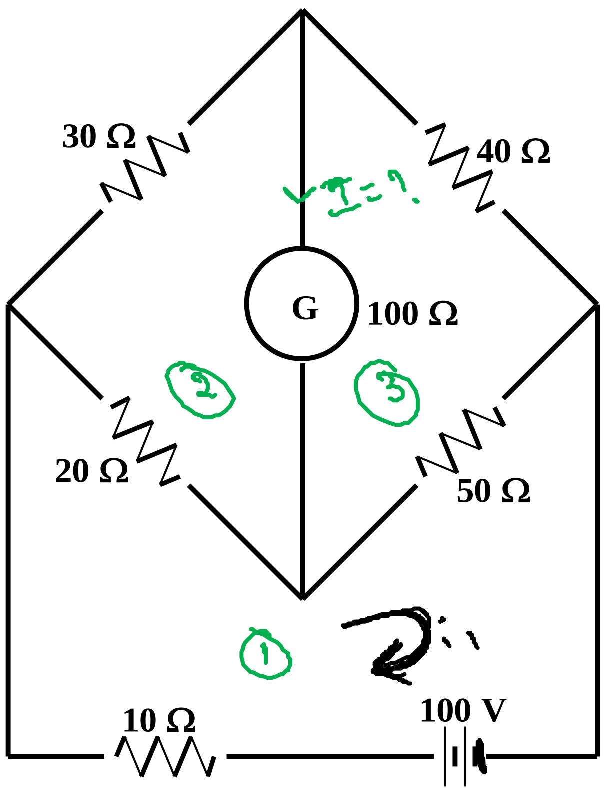
mesh 3:

$-24 - (i_3 - i_2)8 - (i_3 - i_1)1 - i_3 6 = 0$

$-i_1 - 8i_2 + 15i_3 = -24$ — ③

Illustration 1

Determine the current through the galvanometer "**G**". Also, write network equations using inspection method



$$\begin{array}{c|ccc|}
 & a & b & c \\
(2,1) & 80 & -20 & -50 \\
 & -20 & 150 & -100 \\
 & -50 & -100 & 190
\end{array}
\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

R I V d

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) \\ (2,1) & (2,2) & (2,3) \\ (3,1) & (3,2) & (3,3) \end{bmatrix}$$

mesh 1 :

$100 - 10\, i_1 - 20(i_1 - i_2) - 50(i_1 - i_3) = 0$

$100 - 10\, i_1 - 20\, i_2 - 50\, i_3 = 100$

$\Rightarrow 80\, i_1 - 20\, i_2 - 50\, i_3 = 100$

$i_g = i_{100\Omega} = (i_3 - i_2) = 1.1459 - 1.0615 = 0.0844\text{ A}$

I_G = 0.0844 A

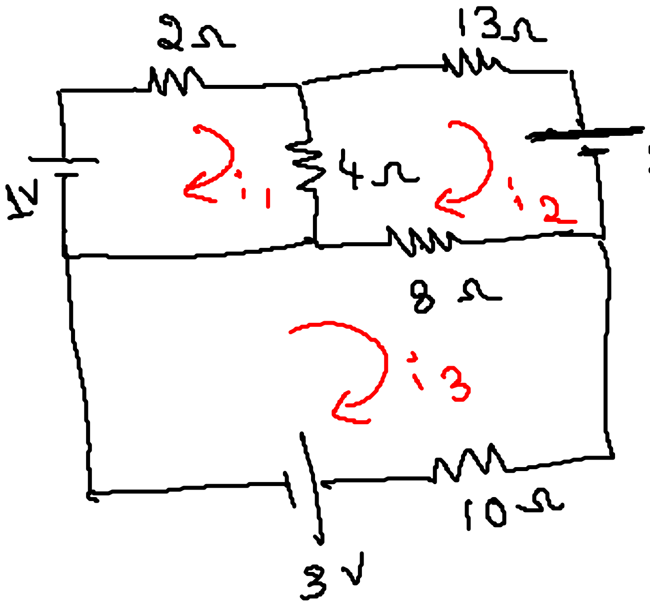
Illustration 2

$x = i$

Realize the network defined by mesh current equation

$$\begin{bmatrix} 6 & -4 & 0 \\ -4 & 25 & -8 \\ 0 & -8 & 18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$$

$$\begin{array}{ccc} R & i & V \end{array}$$

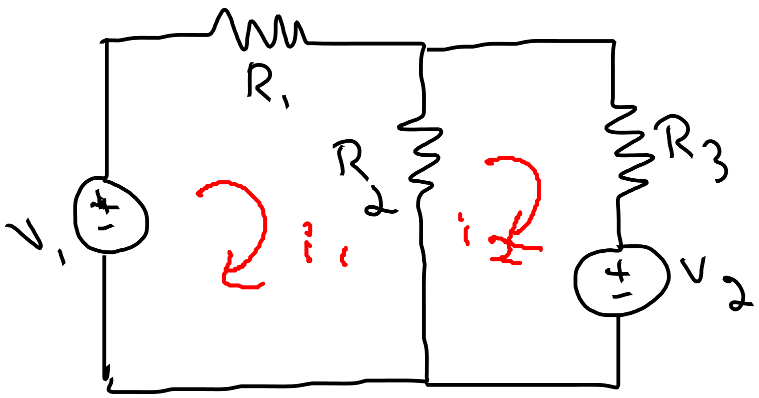


* assume all mesh currents to be in same direction

* write eqn. in the direction of current.

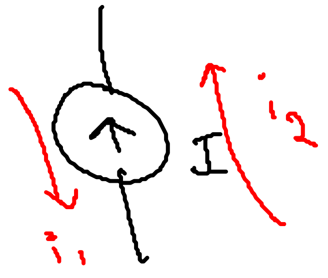
Supermesh

$I = 2A \quad R_I = ? \quad V_I = ?$

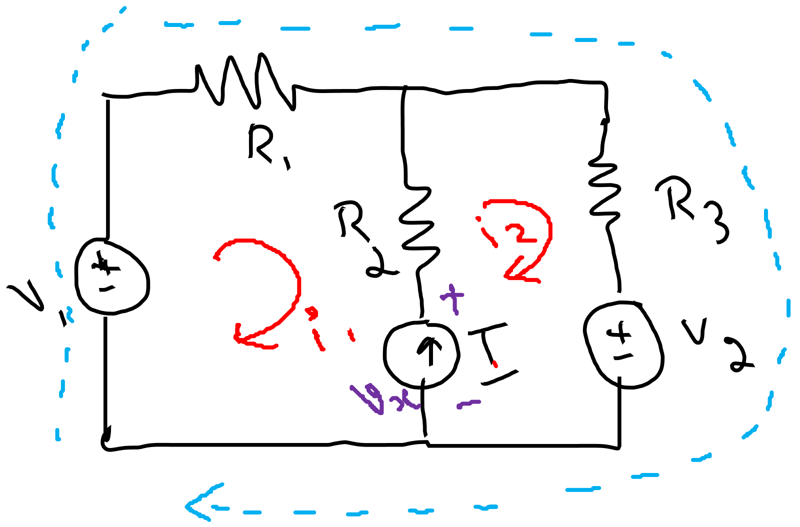


$V_1 - i_1 R_1 - (i_1 - i_2) R_2 = 0$

$-V_2 - (i_2 - i_1) R_2 - i_2 R_3 = 0$



$i_2 - i_1 = I$



$V_1 - i_1 R_1 - (i_1 - i_2) R_2 - \cancel{v_x} = 0 \quad -(1)$

$-V_2 + \cancel{v_x} - (i_2 - i_1) R_2 - i_2 R_3 = 0 \quad -(2)$

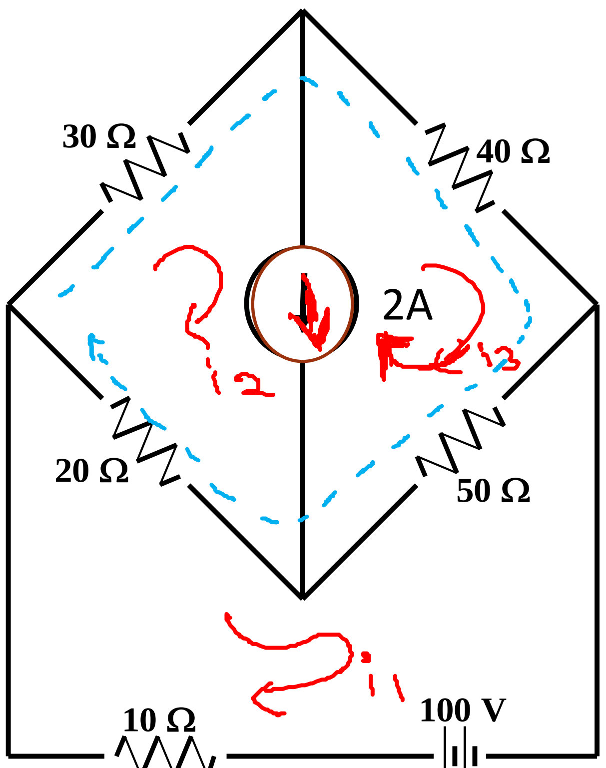
$(1) + (2)$ OR

$V_1 - i_1 R_1 - V_2 - i_2 R_3 = 0$ Supermesh

$i_2 - i_1 = I$

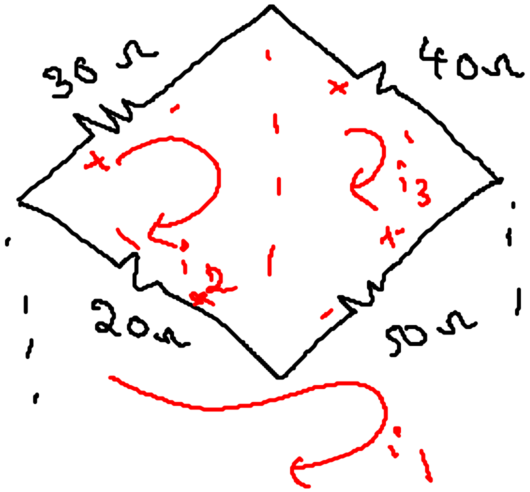
Illustration 3

Write the mesh equations for the circuit shown below.



mesh 1: $100 - 10i_1 - 20(i_1 - i_2) - 50(i_1 - i_3) = 0$ — (1)

Mesh 2 and mesh 3 form supermesh.



$-i_2(30) - i_3(40) - (i_3 - i_1)50$

$- (i_2 - i_1)20 = 0$ — (2)

$i_2 - i_3 = 2A$ — (3)

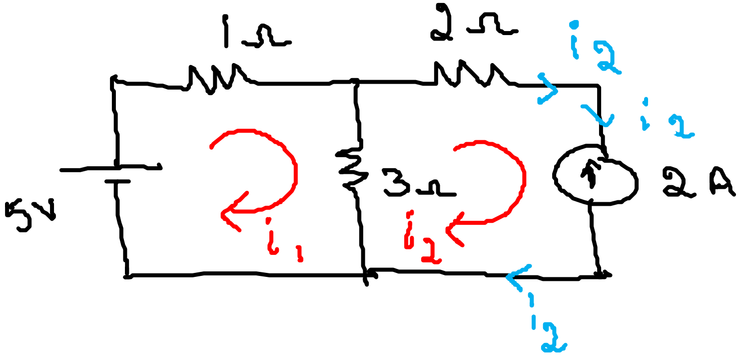
$i_3 - i_2 = -2A$

Basic Electrical Technology

CLASS 5 – 2 NOVEMBER 2021

- MESH CURRENT ANALYSIS AND NODE VOLTAGE ANALYSIS

Recap



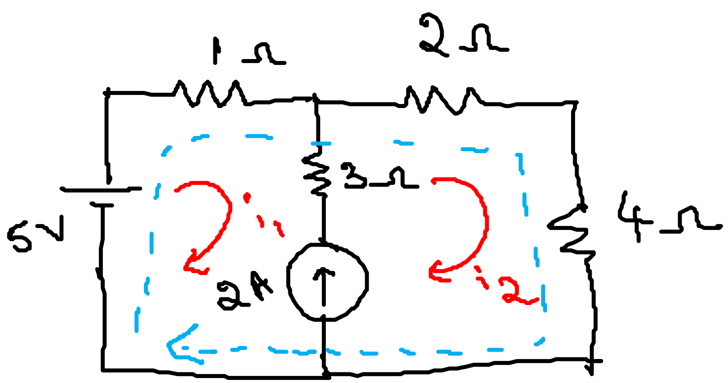
KVL : find mesh current

mesh 1 : $5 - i_1 1 - (i_1 - i_2)3 = 0$ — (1)

mesh 2 : $\cancel{-3(i_2 - i_1) - 2 i_2}$ $\frac{?}{2A}$ volt. across

mesh 2: $i_2 = -2A$ — (2)

Current source is common to two mesh – Supermesh



Supermesh

Eqn of supermesh:

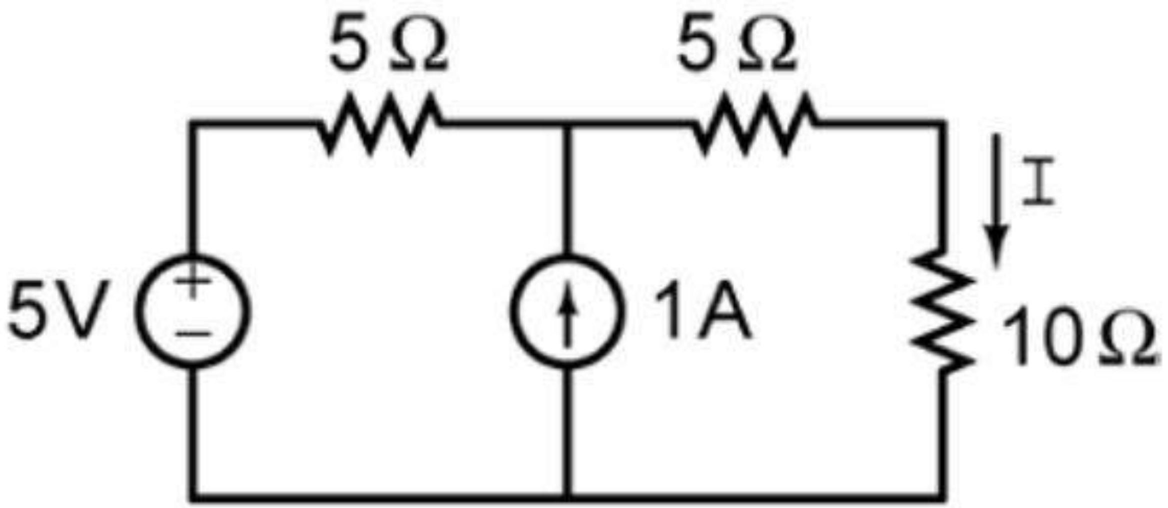
$5 - i_1 1 - i_2 2 - 4 i_2 = 0$ — (1)

$i_2 - i_1 = 2A$ — (2)

Quiz

The value of current I in the circuit is

a) 1 A
b) 2 A
c) 0.5 A
d) 0.25 A



Quiz

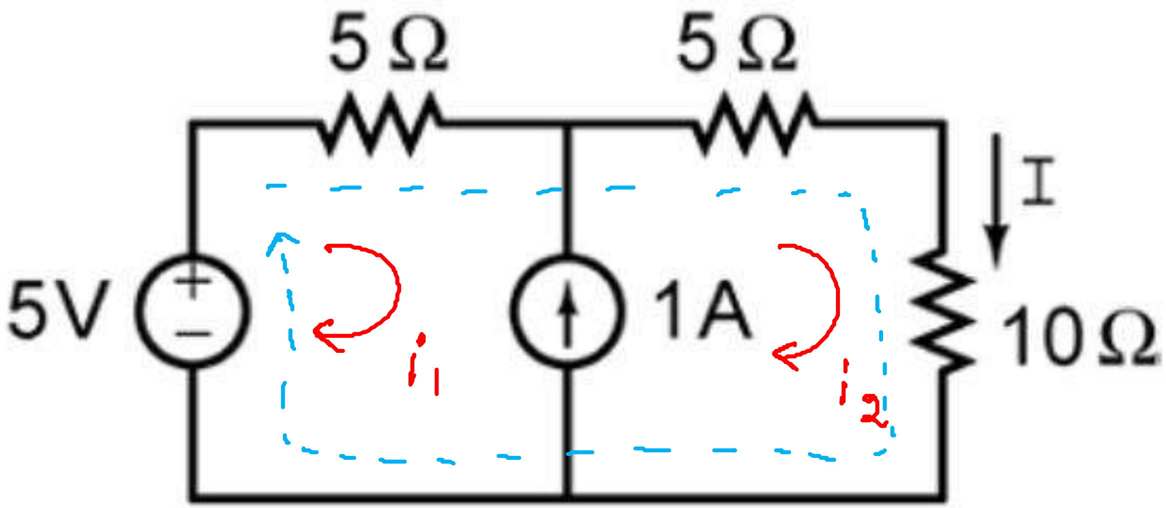
The value of current I in the circuit is

a) 1 A

b) 2 A

c) 0.5 A ✓

d) 0.25 A



$i_2 = I$

$5 - 5i_1 - 5i_2 - 10i_2 = 0$

$5 - 5i_1 - 15I = 0 \quad -(1)$

$i_2 - i_1 = 1A \Rightarrow I - i_1 = 1A \quad -(2)$

$5 - 5(I - 1) - 15I = 0$

$I = \underline{\underline{0.5A}}$

Illustration 4

Find the power supplied by 2A current source using mesh current analysis.

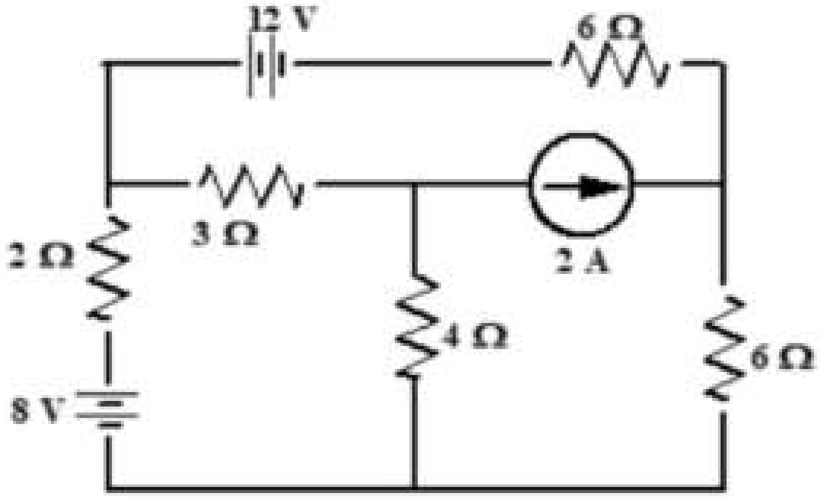
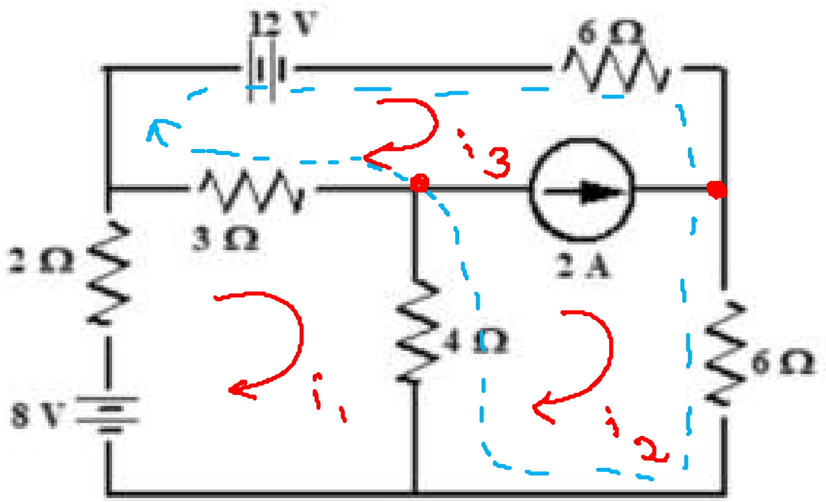


Illustration 4

Find the power supplied by 2A current source using mesh current analysis.



mesh 2 and mesh 3 together form supermesh

Supermesh equ:

$-12 - 6i_3 - 6i_2 - 4(i_2 - i_1) - 3(i_3 - i_1) = 0 \quad (2)$

$i_2 - i_3 = 2A \quad (3)$

$i_1 = 0.655A \qquad i_2 = 0.557A \qquad i_3 = -1.44A$

$P = V^2/R = I^2R = VI = V(2)$

R for 2A is unknown.

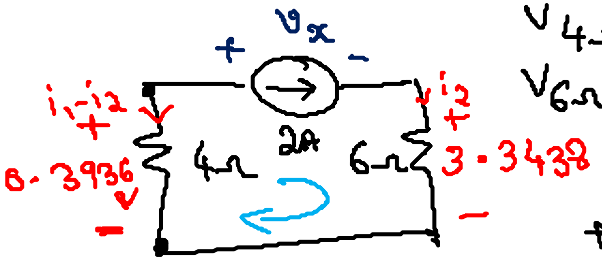
mesh 1:

$8 - 2i_1 - 3(i_1 - i_3) - 4(i_1 - i_2) = 0$

$9i_1 - 4i_2 - 3i_3 = 8 \quad (1)$

$7i_1 - 10i_2 - 9i_3 = 12 \quad (2)$

$0i_1 + i_2 - i_3 = 2 \quad (3)$



$V_{4\Omega} = (i_1 - i_2)4 = 0.3936V$

$V_{6\Omega} = i_2 6 = 3.3438V$

Apply KVL

$+0.3936 - v_x - 3.3438 = 0$

$v_x = 2.950V \qquad P_{2A} = v_x I = 5.9W$

Quiz

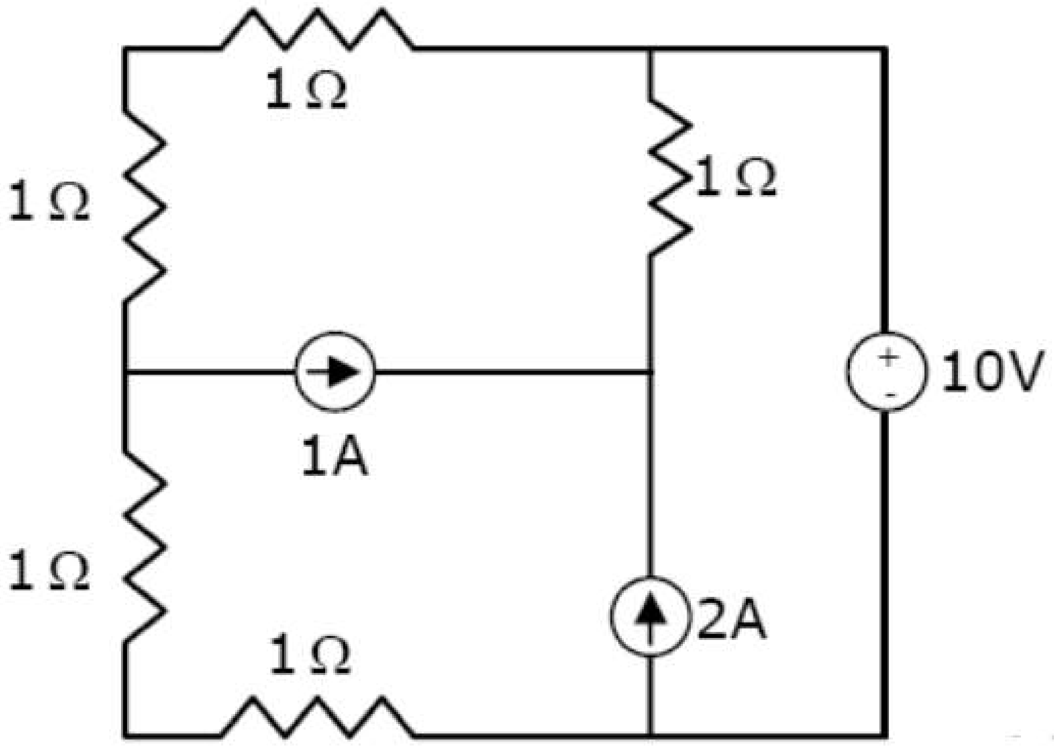
The current supplied by the voltage source is

a) 2 A

b) 3 A

c) 1 A

d) 0 A



Quiz

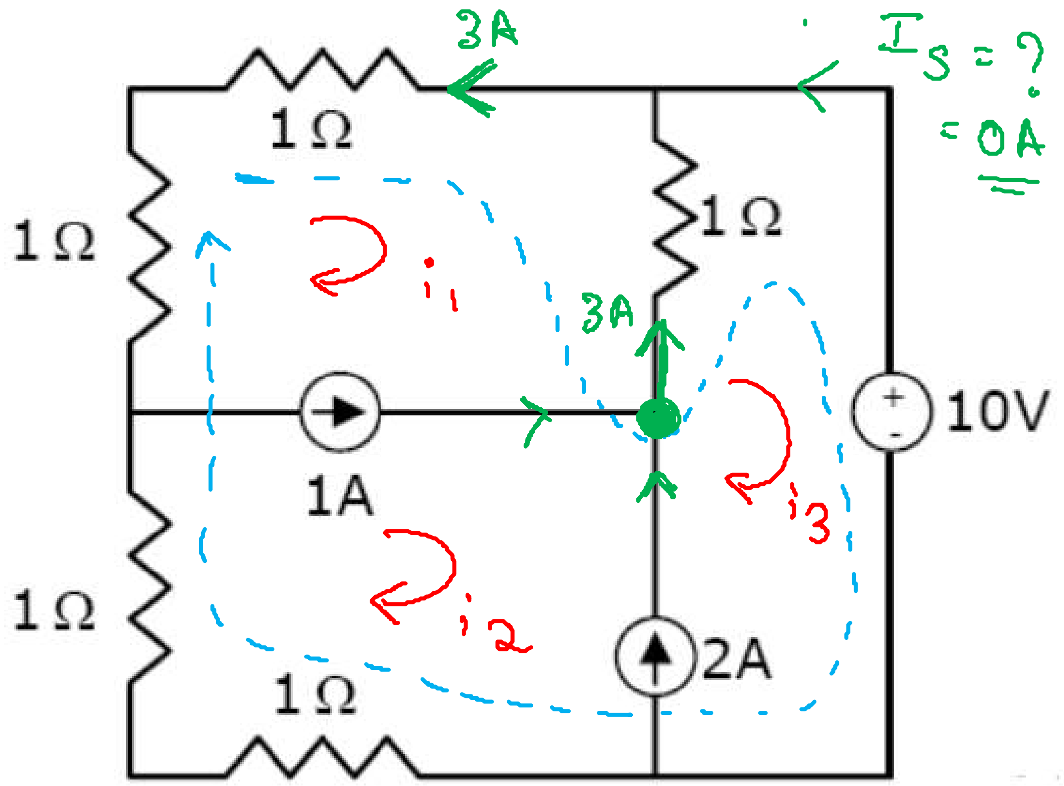
The current supplied by the voltage source is

a) 2 A

b) 3 A

c) 1 A

d) 0 A ✓



mesh 1, 2, 3 form supermesh

Supermesh eqn.

$$-1i_1 - 10 - 1i_2 - 1i_2 - 1i_1 = 0$$

$$-2i_1 - 2i_2 - 10 = 0 \quad -(1)$$

$$i_2 - i_1 = 1A$$

$$i_3 - i_2 = 2A$$

$$-2i_1 - 2(i_1+1) - 10 = 0$$

$$-4i_1 - 12 = 0 \qquad i_1 = -3A$$

$$I_s + 3 = 3 \qquad I_s = 0A$$

Node Voltage Analysis

DC CIRCUIT ANALYSIS

Introduction

Kirchhoff's Current Law (KCL)

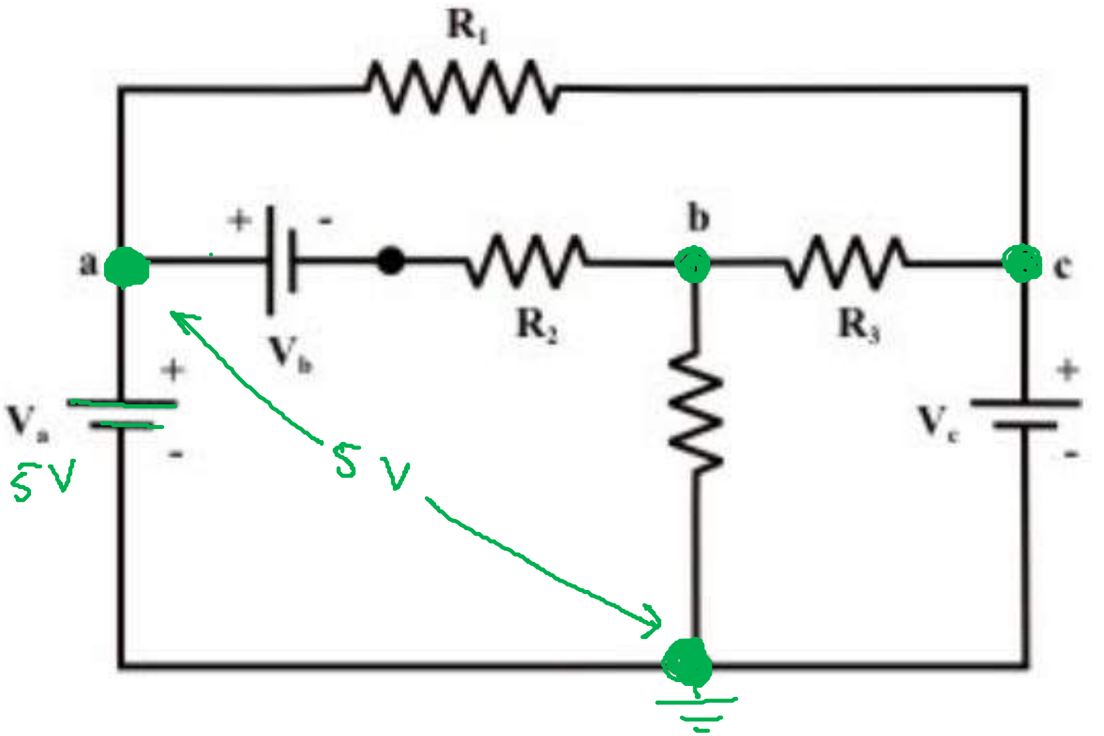
- At any node (junction) in a circuit, the algebraic sum of currents entering and leaving the node at any instant of time must be equal to zero

Node

- A point in an electric circuit where 3 or more elements are connected

Branch

- A conducting path between two nodes in a circuit containing circuit elements



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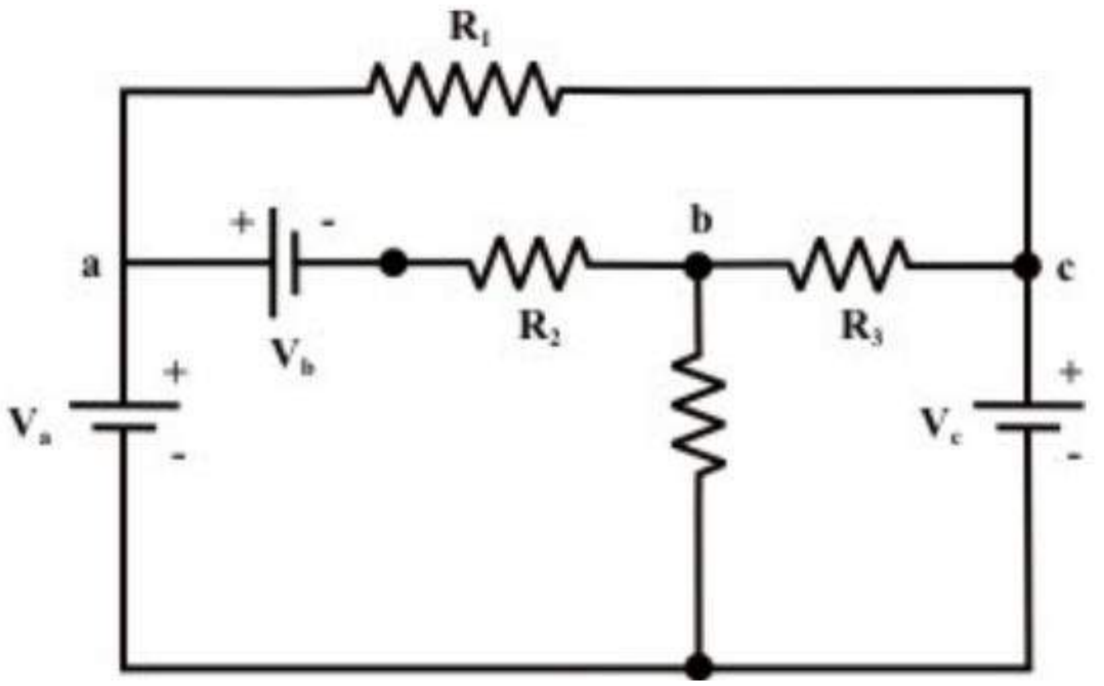


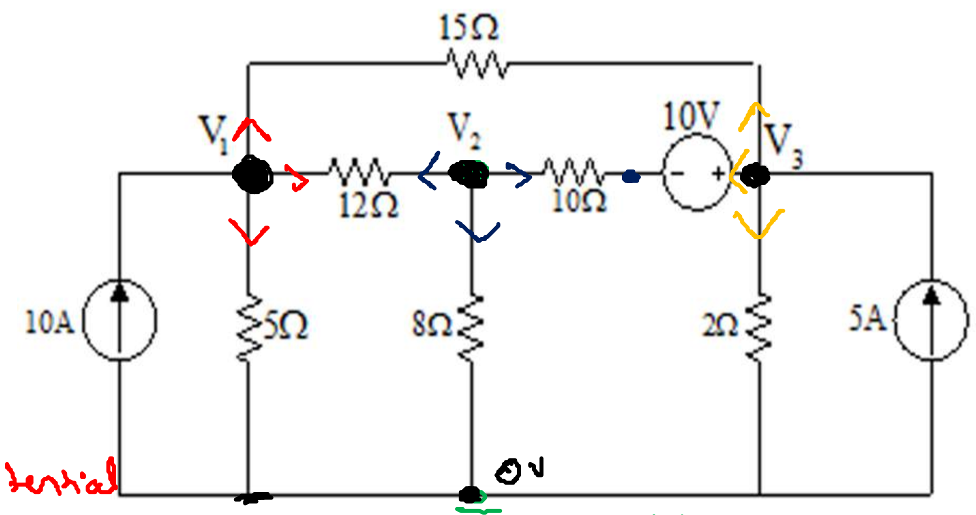
Node Voltage Analysis Method

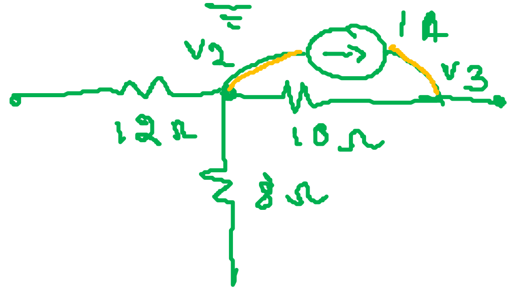
- Convert all the practical voltage sources to current sources
- Identify nodes in the circuit
- One of the nodes is taken as reference node
- Assign a voltage to each of the remaining nodes

  assume node under consideration to be at higher potential

  $I = \frac{V}{R}$

- Write KCL equations for all the nodes (excluding the reference node)
- Solve for voltages





for node 1 : $\frac{V_1-0}{5} + \frac{V_1-V_2}{12} + \frac{V_1-V_3}{15} = 10$

for node 2: $\frac{V_2-V_1}{12} + \frac{V_2-0}{8} + \frac{V_2-V_3}{10} + 1 = 0$

node 3: $\frac{V_3-0}{2} + \frac{V_3-V_2}{10} + \frac{V_3-V_1}{15} = 6\,A$

Node Voltage Analysis Method

- Convert all the practical voltage sources to current sources
- Identify nodes in the circuit
- One of the nodes is taken as reference node
- Assign a voltage to each of the remaining nodes
- Write KCL equations for all the nodes (excluding the reference node)
- Solve for voltages

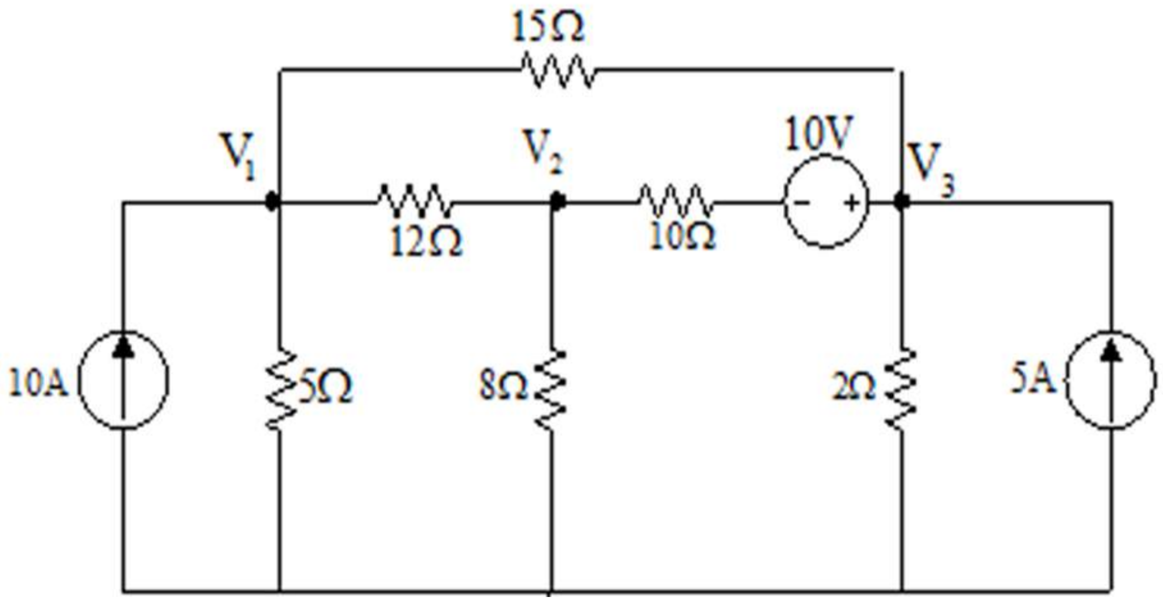


Illustration 1

Determine the current through the galvanometer “**G**”. Also, write network equations using inspection method

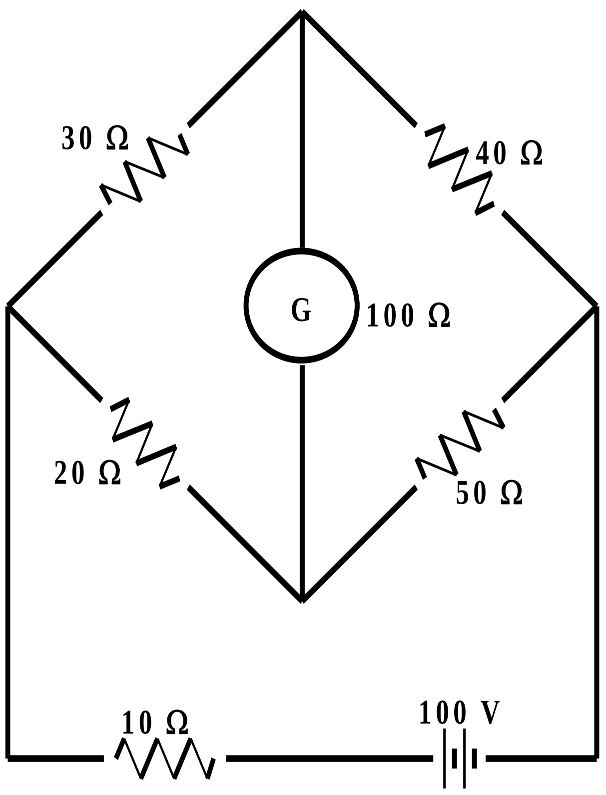
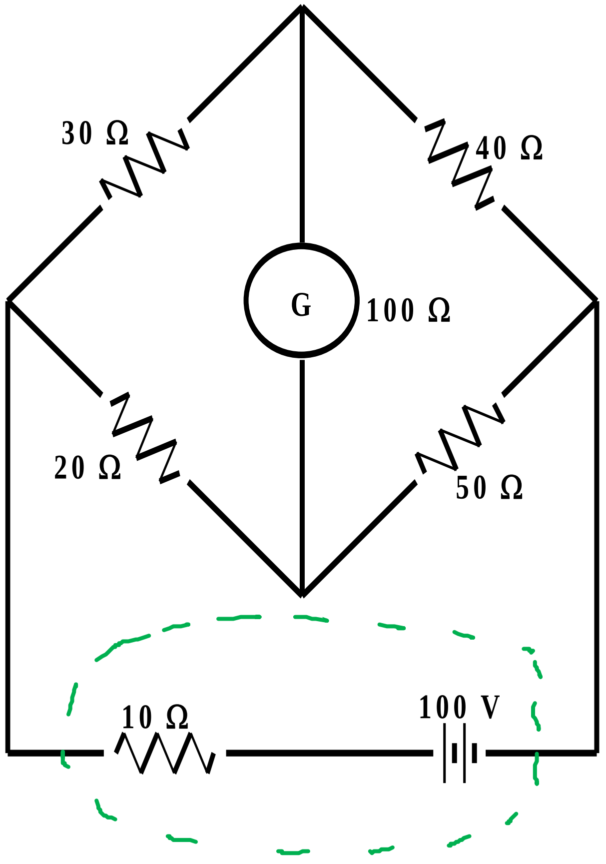
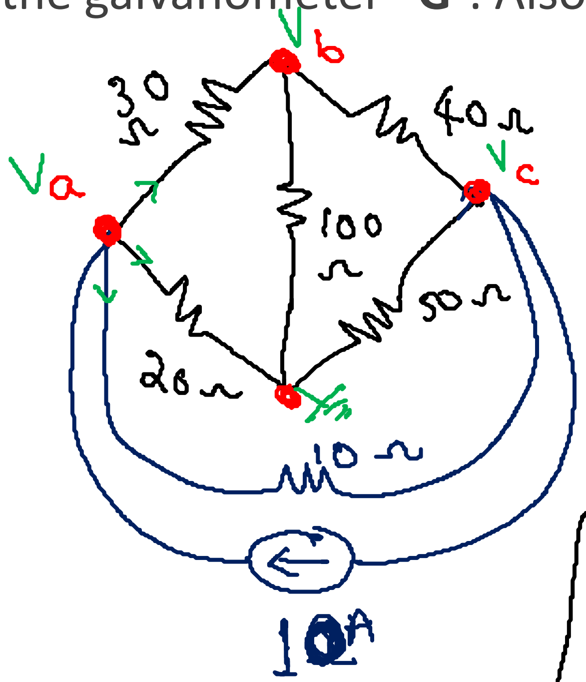


Illustration 1

Determine the current through the galvanometer "**G**". Also, write network equations using inspection method





node a:

$$\frac{V_a - V_b}{30} + \frac{V_a - 0}{20} + \frac{V_a - V_c}{10} = 10$$

$$V_a\left[\frac{1}{30} + \frac{1}{20} + \frac{1}{10}\right] + V_b\left[-\frac{1}{30}\right] + V_c\left[-\frac{1}{10}\right] = 10$$

$$Y_R \cdot V_{node} = I_{source}$$

$$\begin{bmatrix} \left(\frac{1}{30}+\frac{1}{20}+\frac{1}{10}\right) & -\frac{1}{30} & -\frac{1}{10} \\ -\frac{1}{30} & \left(\frac{1}{30}+\frac{1}{100}+\frac{1}{40}\right) & -\frac{1}{40} \\ -\frac{1}{10} & -\frac{1}{40} & \left(\frac{1}{10}+\frac{1}{50}+\frac{1}{40}\right) \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -10 \end{bmatrix}$$

$V_a = 23.36\,V$ $\quad V_b = -8.599\,V$ $\quad V_c = -54.33\,V$

$$I_{100\Omega} = \frac{V_b - 0}{100} = \underline{\underline{-0.08\,A}}$$

Illustration 2

Realize the network defined by node voltage equation

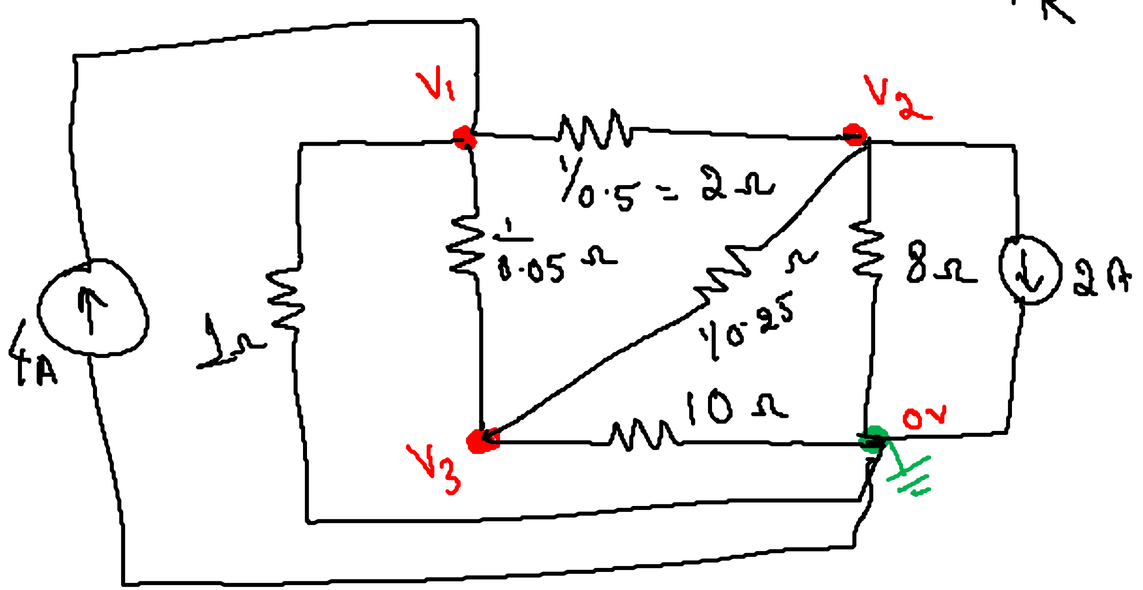
$$\begin{bmatrix} 1.55 & -0.5 & -0.05 \\ -0.5 & 0.875 & -0.25 \\ -0.05 & -0.25 & 0.4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$$

Illustration 2

Realize the network defined by node voltage equation

$$\begin{bmatrix} 1.55 & -0.5 & -0.05 \\ -0.5 & 0.875 & -0.25 \\ -0.05 & -0.25 & 0.4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$$

$Y_R \cdot V_{node} \quad I_{source}$



node 2

2 and 3   2 and 1

$0.875 - (0.25 + 0.5) = 0.125\Omega$

$1/0.125 = 8\Omega$ between node 2 and ground

Quiz

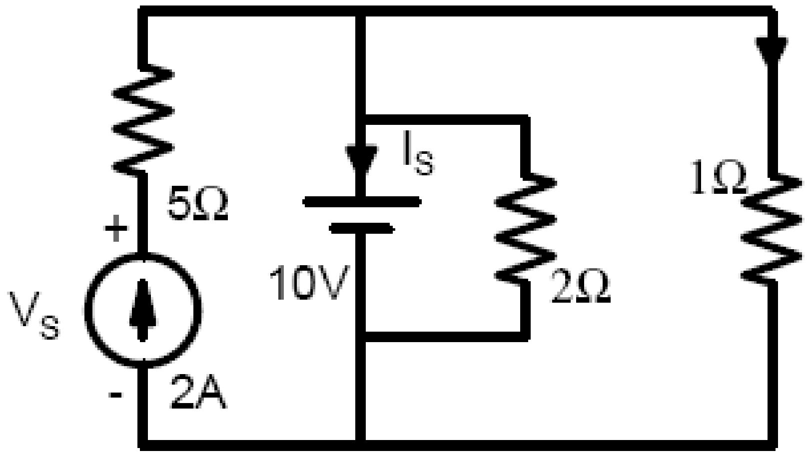
The current I_s in amperes in voltage source and voltage V_s across the current source is

a) -13 A, 20 V

b) -13 A, 10 V

c) 13 A, -10 V

d) 13 A, -20 V



Quiz

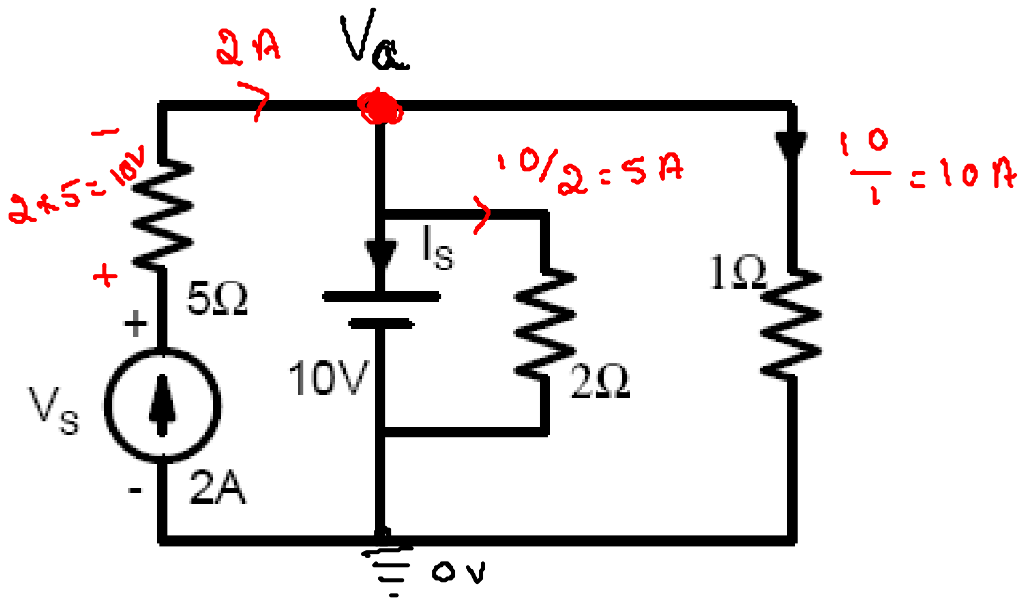
The current I_s in amperes in voltage source and voltage V_s across the current source is

a) -13 A, 20 V ✓

b) -13 A, 10 V

c) 13 A, -10 V

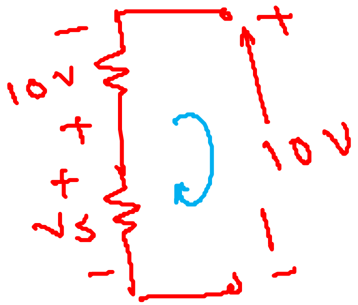
d) 13 A, -20 V



$V_a = 10V$

KCL At node a: $2 = I_s + 5 + 10$

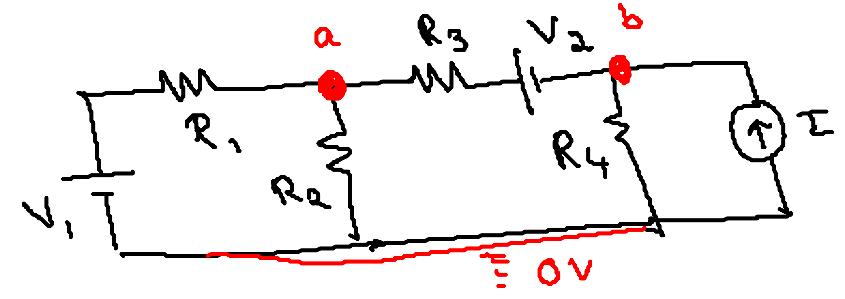
$I_s = -13\,A$



KVL: $-10 + V_s - 10 = 0$

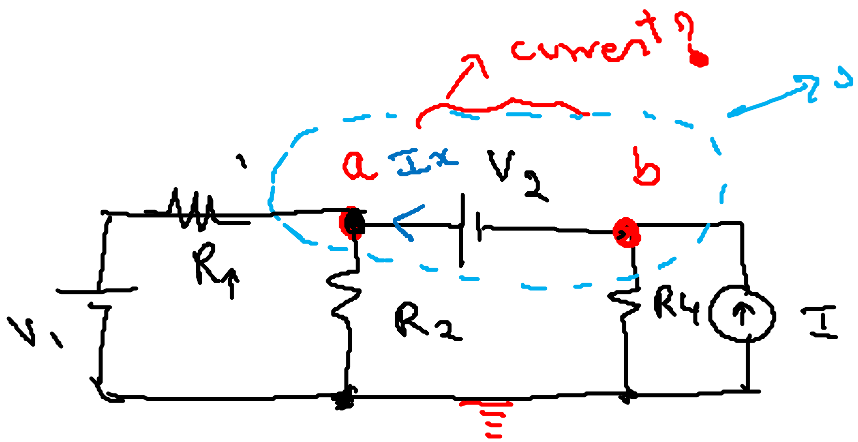
$V_s = 20V$

Supernode



node a: $$\frac{V_a}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = \overbrace{\frac{V_2}{R_3} + \frac{V_1}{R_1}}^{\text{current enters node a}} \quad -①$$

node b: $$\frac{V_b}{R_4} + \frac{V_b - V_a}{R_3} + \underbrace{\frac{V_2}{R_3}}_{\text{current}} = I \quad -②$$



→ supernode → Ideal voltage source between two nodes of which none of them are reference nodes.

node a: $$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} = I_x \quad -①$$

node b: $$\frac{V_b}{R_4} + I_x = I \quad ②$$

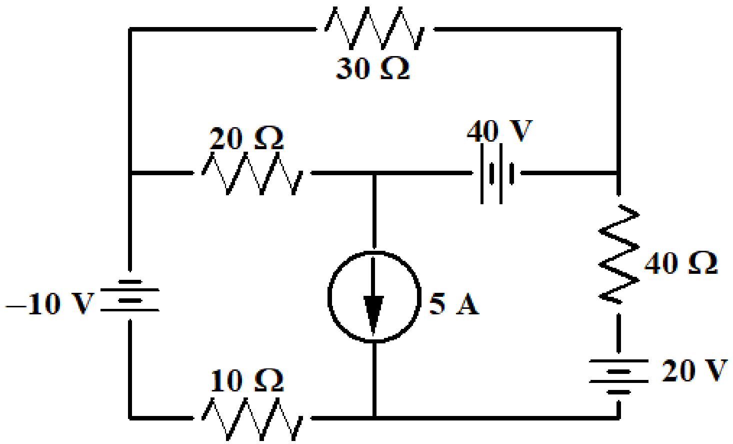
OR

① + ② $$\frac{V_a}{R_1} + \frac{V_a}{R_2} + \frac{V_b}{R_4} = I + \frac{V_1}{R} \quad -①$$

and $$V_a - V_b = V_2 \quad -②$$

Supernode

Find the current through 40 V battery. Is the battery charging or discharging?



Ans: 4.19 A, Discharging

Supernode

node b equation
($V_b = 40V$)
HW: Consider node c as reference & solve using normal node eqn.

Find the current through 40 V battery. Is the battery charging or discharging?

node a

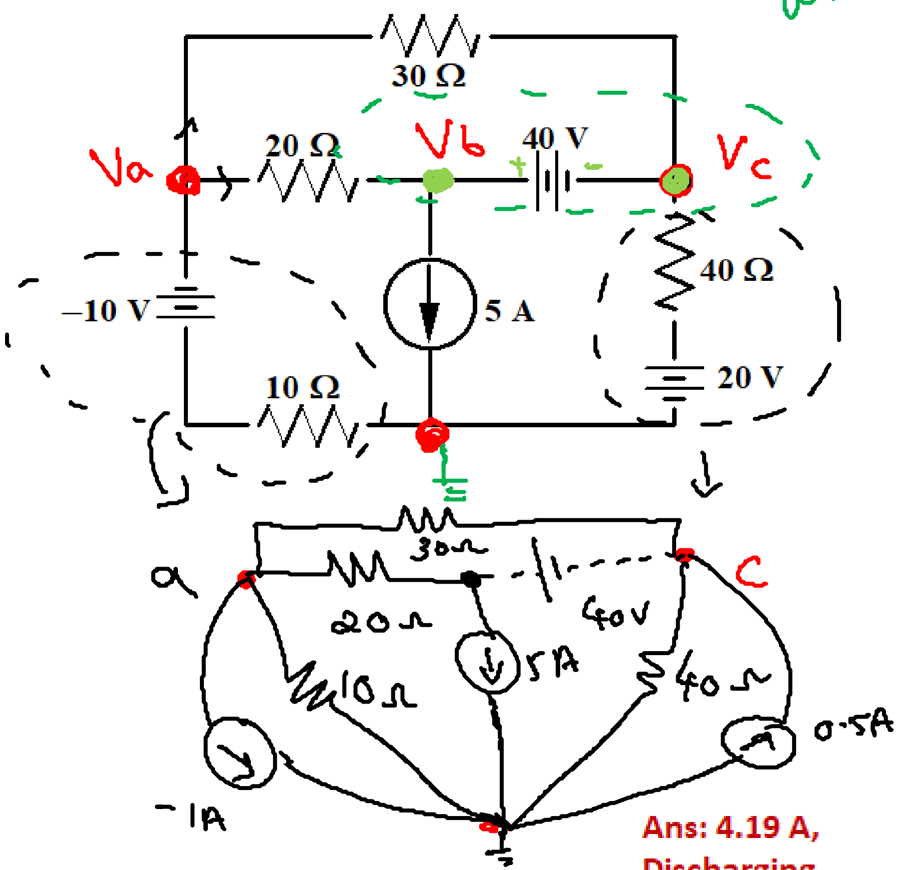
$$\frac{V_a - V_c}{30} + \frac{V_a - V_b}{20} + \frac{V_a - 0}{10} - 1 = 0 \quad -(1)$$

node b and node c form supernode

$$\frac{V_b - V_a}{20} + 5 \;\Big|\; + \frac{V_c - V_a}{30} + \frac{V_c}{40} - 0.5 = 0 \quad (2)$$

node b | node c

$$V_b - V_c = 40\text{ V} \quad -(3)$$



Ans: 4.19 A, Discharging

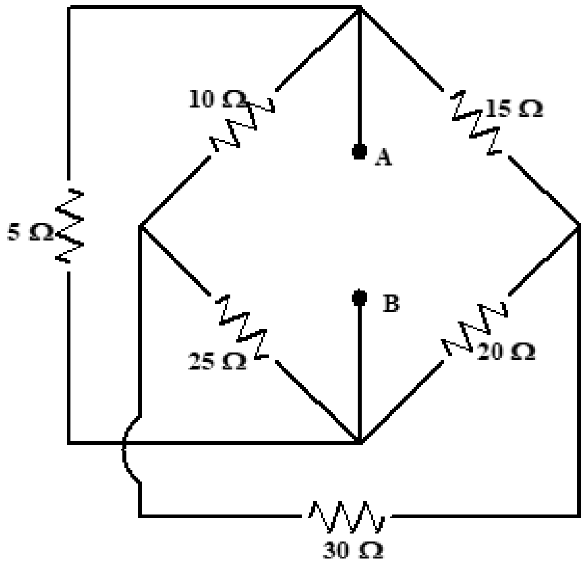
Basic Electrical Technology

CLASS 6 – 5 NOVEMBER 2021

- TUTORIAL 1

Star – Delta Transformation

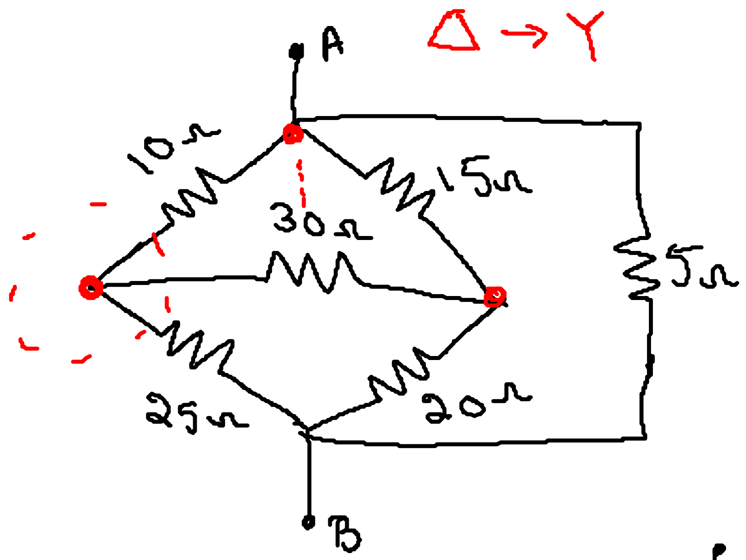
Determine the resistance between A and B in the network shown below

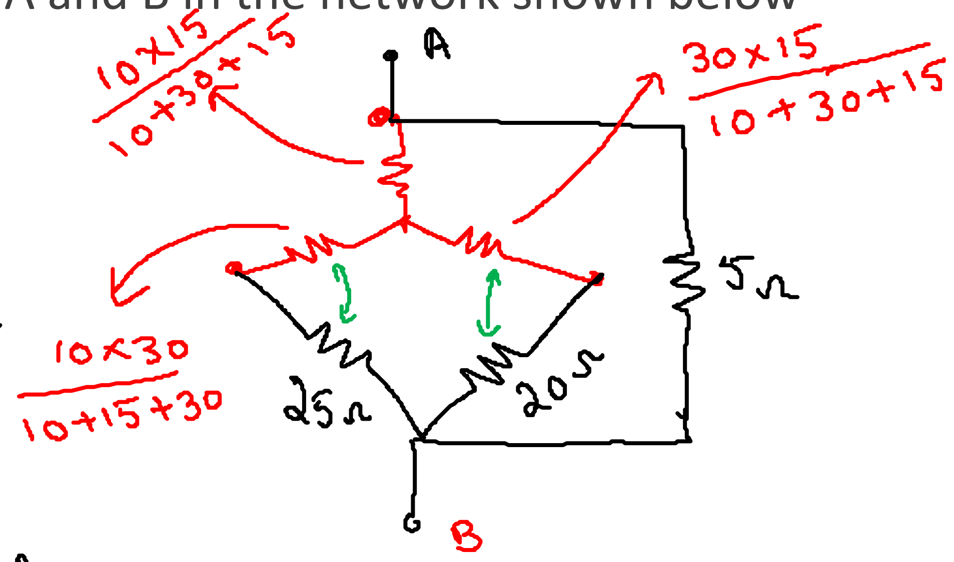


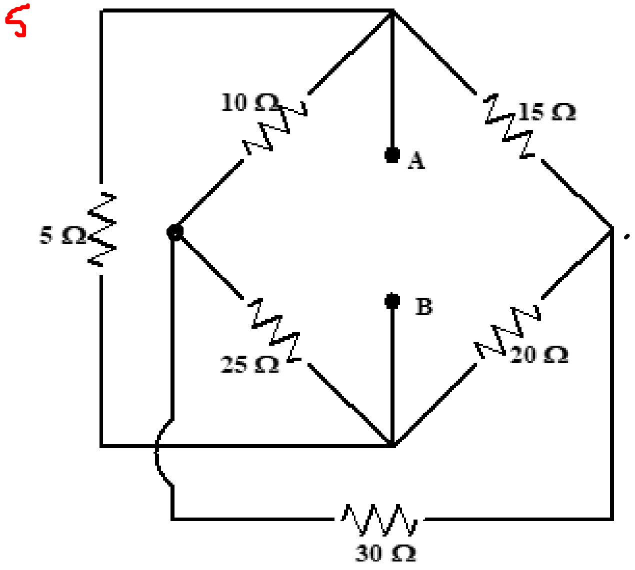
$R_{AB} = 3.89\ \Omega$

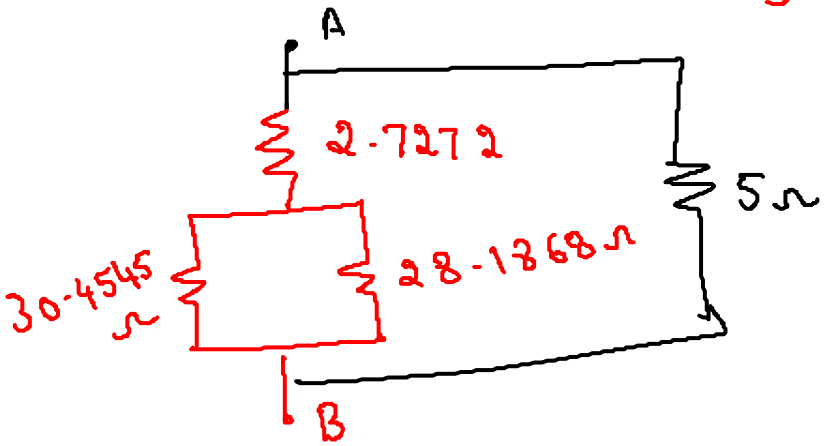
Star – Delta Transformation

Determine the resistance between A and B in the network shown below







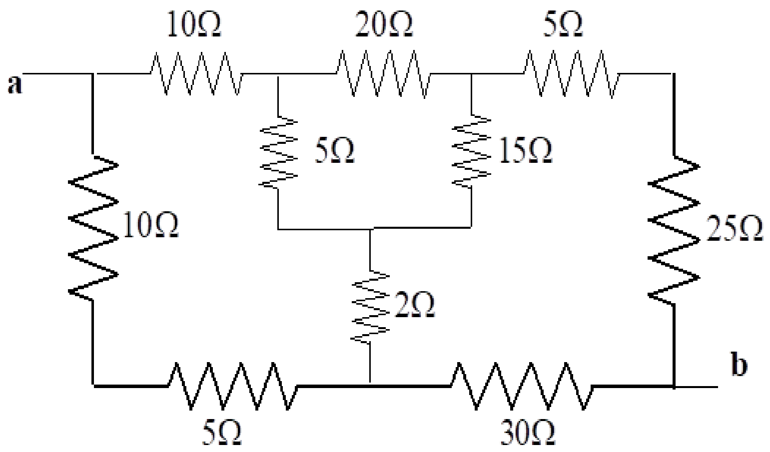


$R_{equ} = 3.882\,\Omega$

$R_{AB} = 3.89\ \Omega$

Homework

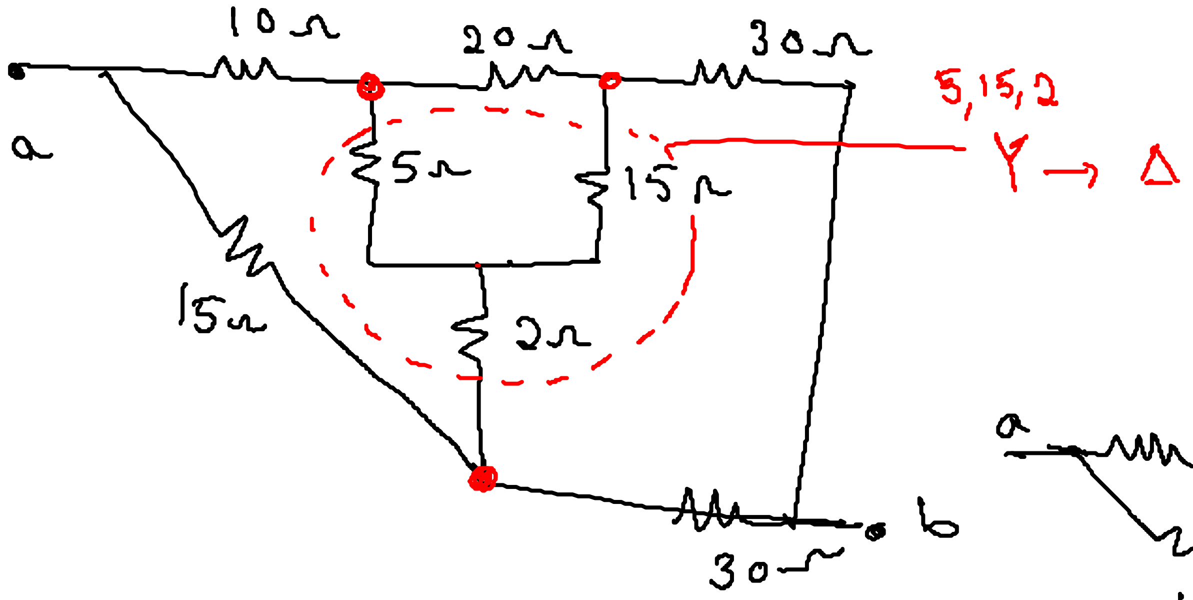
Determine the resistance between terminals a & b of the network shown in figure, using Star-Delta transformation.

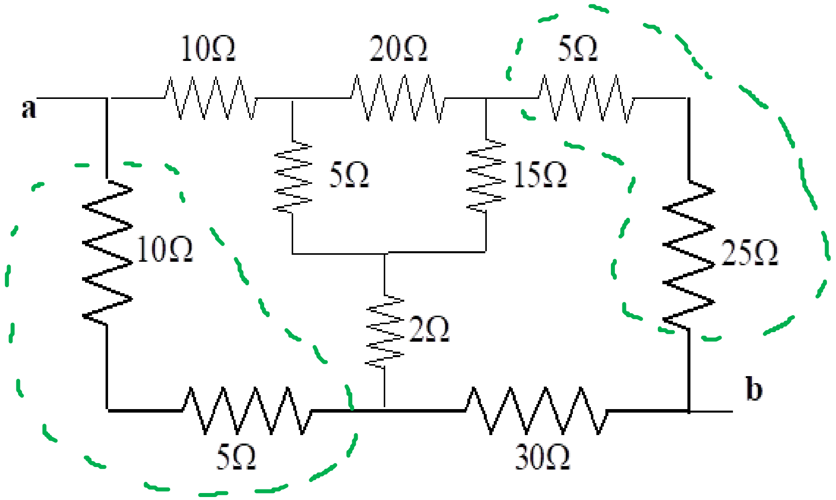


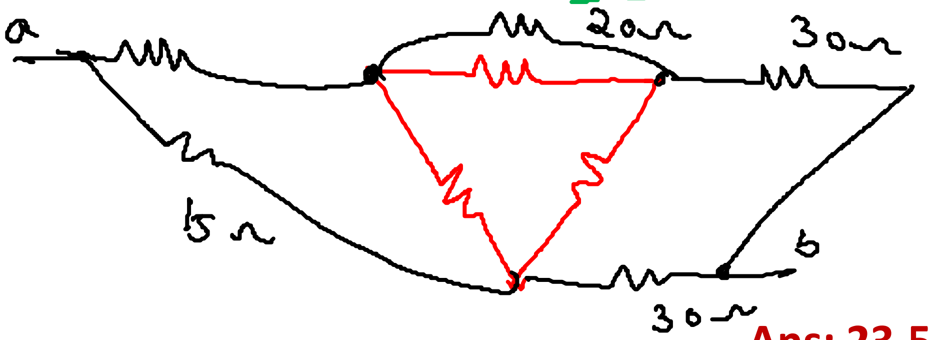
Ans: 23.518 Ω

Homework

Determine the resistance between terminals a & b of the network shown in figure, using Star-Delta transformation.





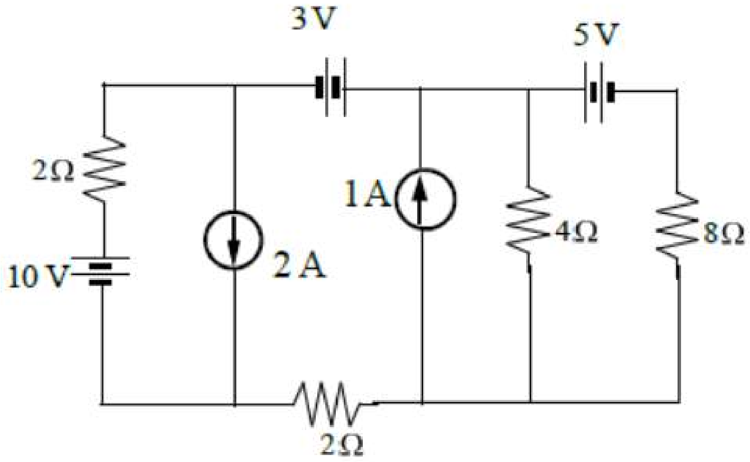


One more transformation required.

Ans: 23.518 Ω

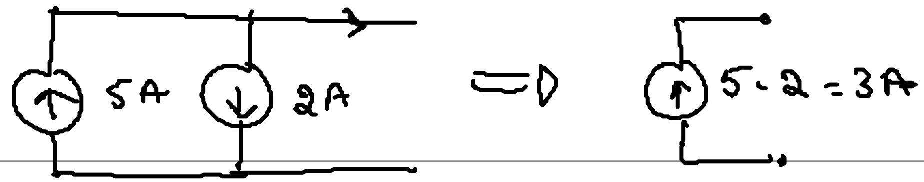
Source Transformation

Find the current through 8Ω resistor by source transformation method, in the circuit shown below

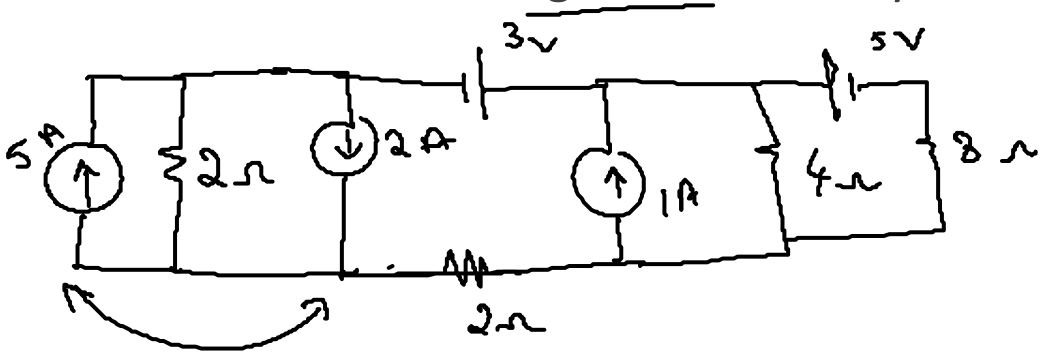


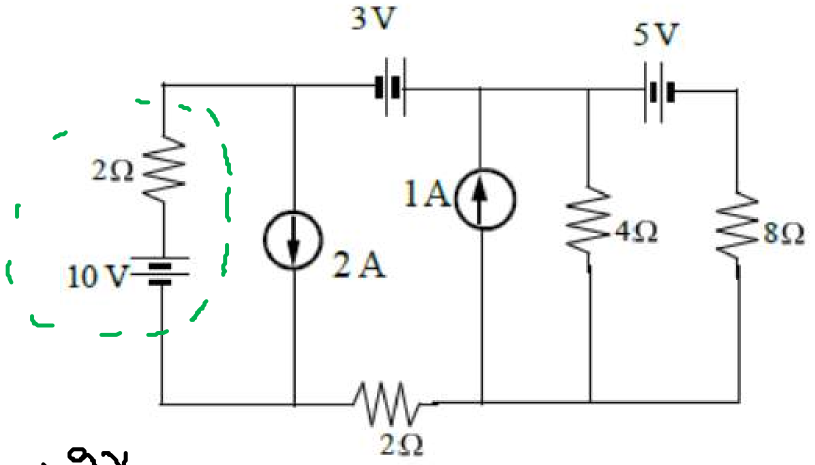
Ans: 150 mA

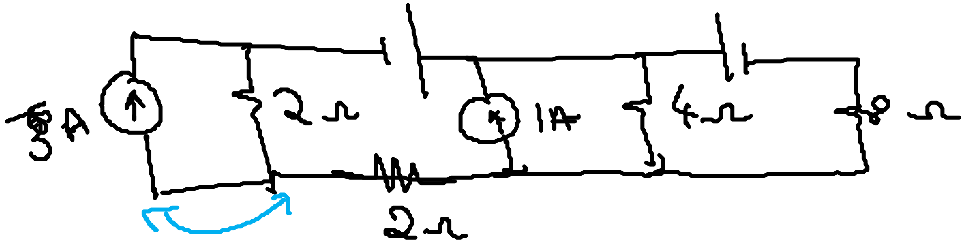
Source Transformation

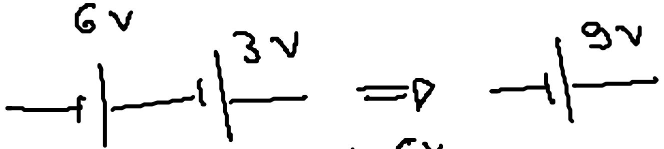


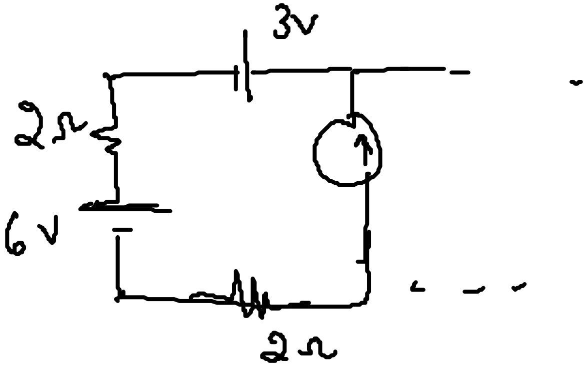
Find the current through 8Ω resistor by source transformation method, in the circuit shown below

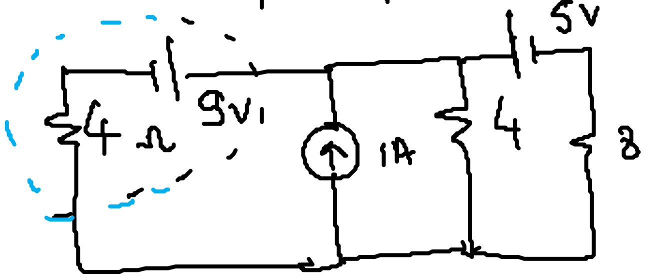


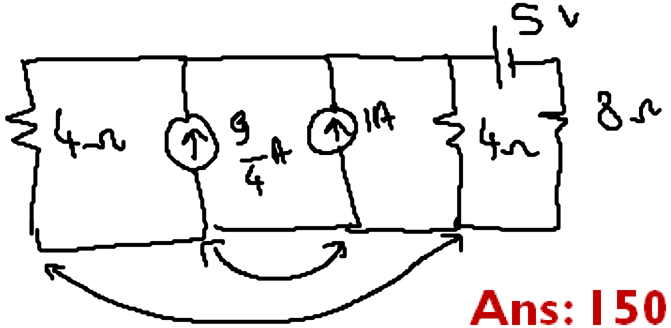








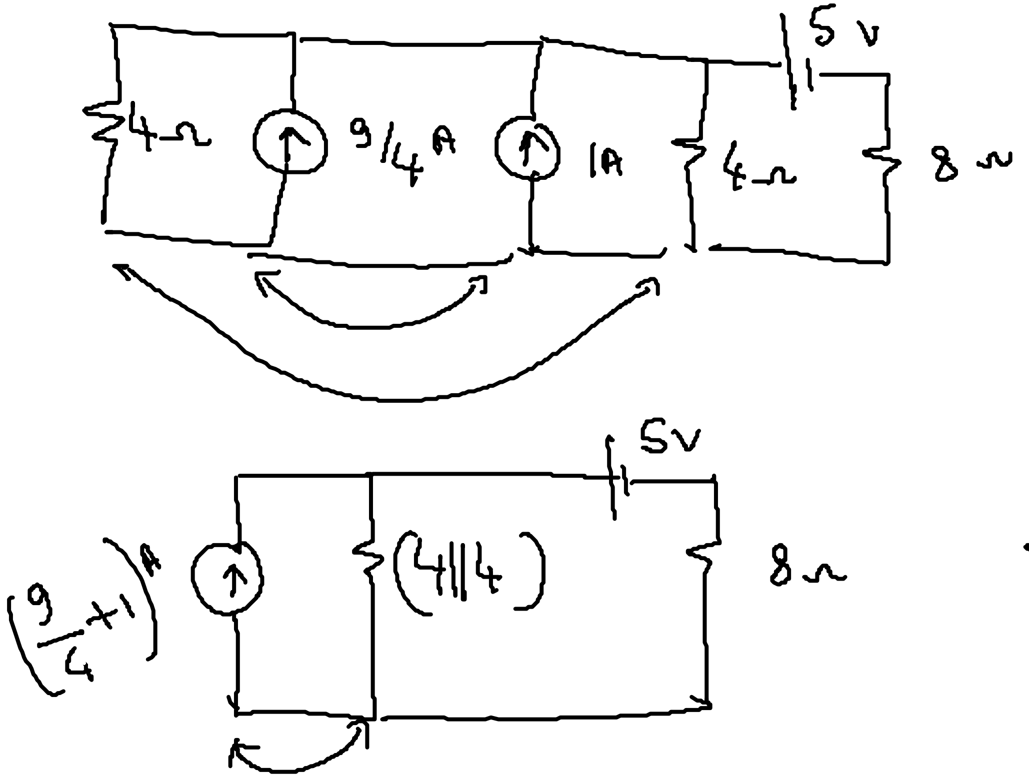




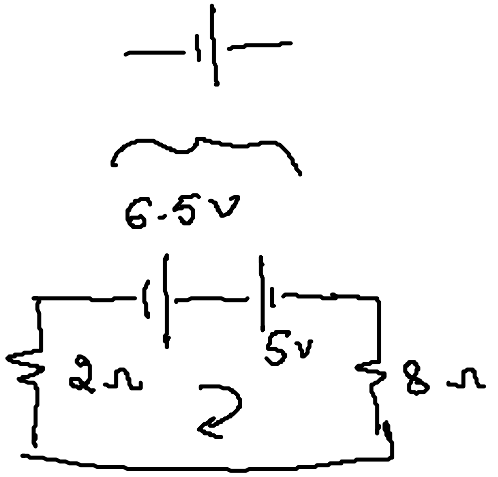
Ans: 150 mA

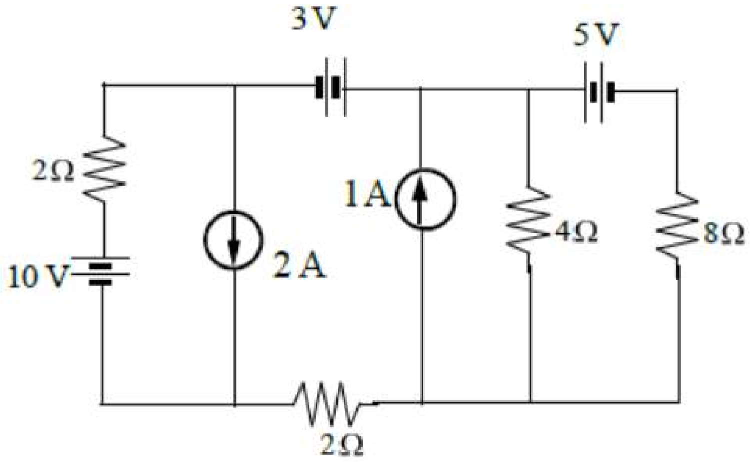
Source Transformation

Find the current through 8Ω resistor by source transformation method, in the circuit shown below



$6.5 - 5 = 1.5V$



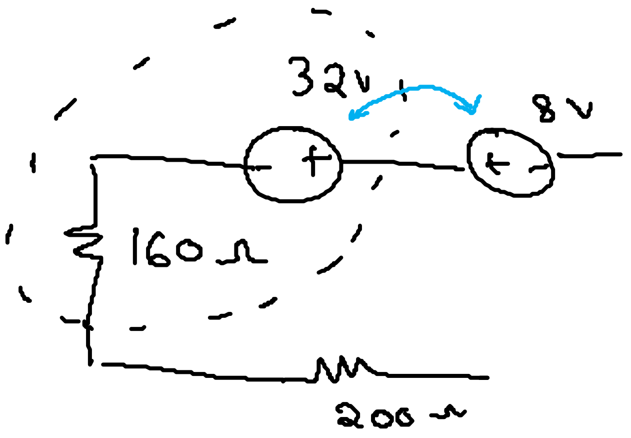


$$I_{8\Omega} = \frac{1.5}{10} = 150mA$$

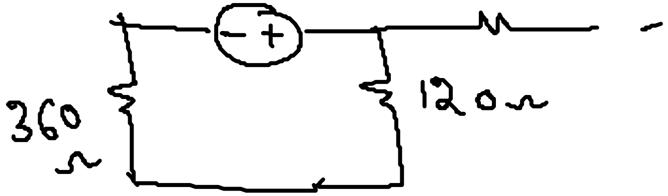
Ans: 150 mA

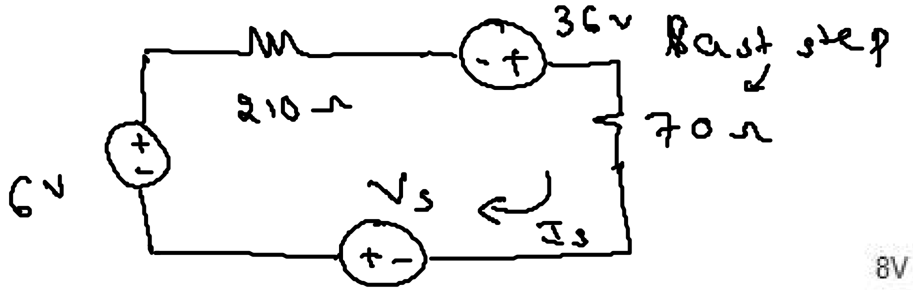
Homework

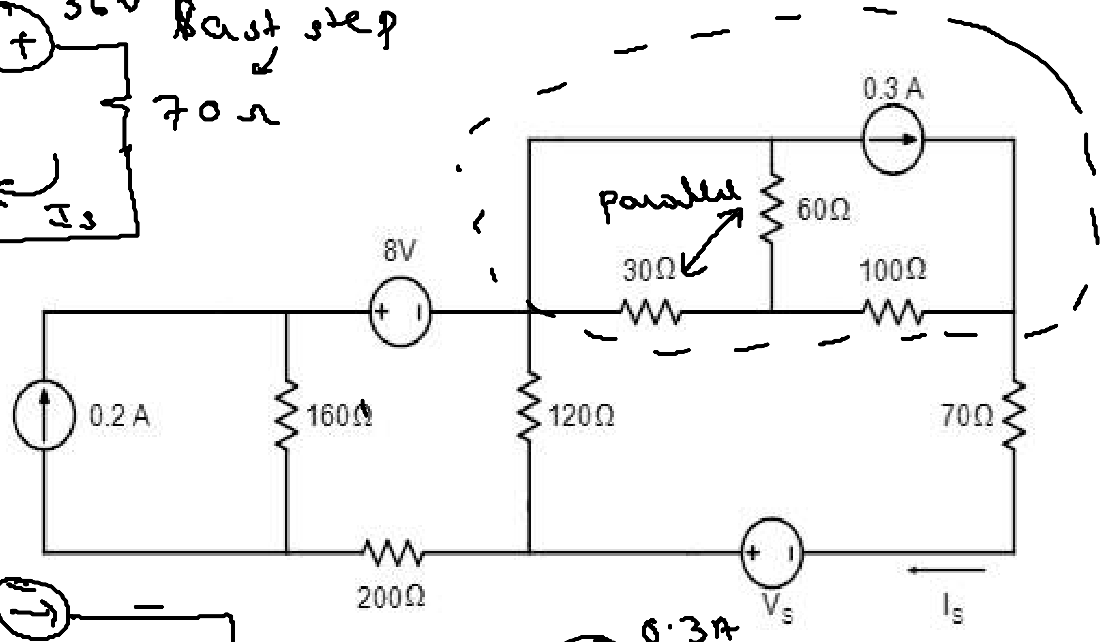
In the circuit shown, compute the value of V_S to deliver a current of I_S = 0.25 A using source transformation.

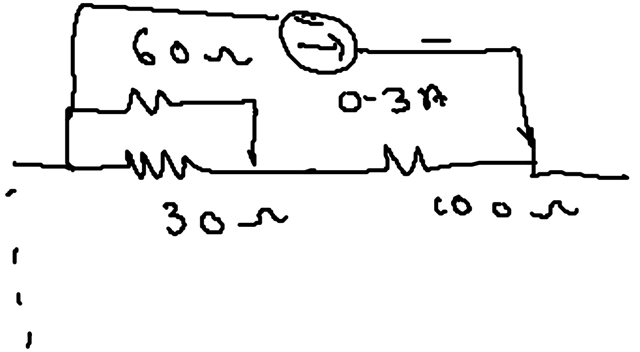


32 - 8 = 24V

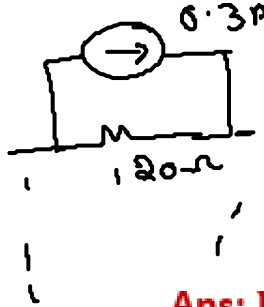








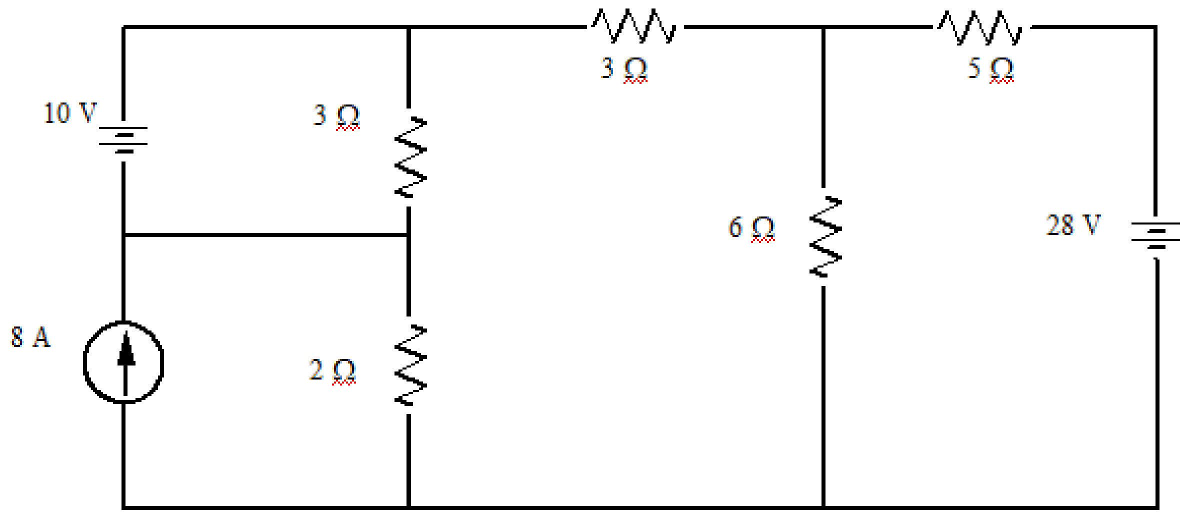
=>



Ans: $V_s = 28\ V$

Mesh Current Analysis

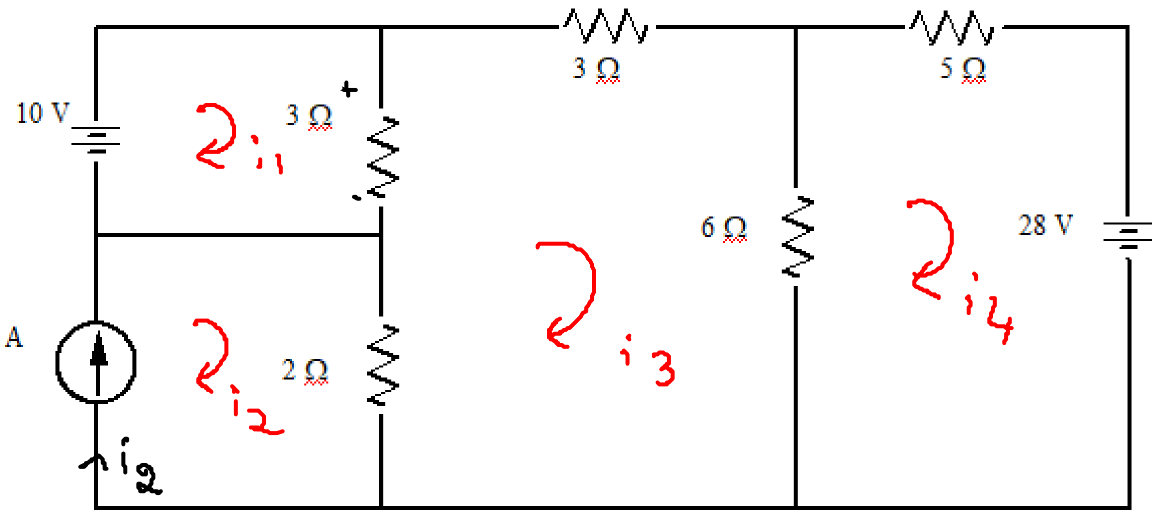
Find the voltage across the current source using mesh current analysis.



Mesh Current Analysis $\rightarrow i_1, i_2, i_3, i_4$

Find the voltage across the current source using mesh current analysis.

$V_{8A} = I_{cs} R_{cs}$?



mesh 1:

$$10 - 3(i_1 - i_3) = 0$$

$$3i_1 - 3i_3 = 10 \quad -(1)$$

mesh 2:

$$i_2 = 8A \quad -(2)$$

mesh 3:

$$-3(i_3 - i_1) - 3i_3 - 6(i_3 - i_4) - 2(i_3 - i_2) = 0$$

$$3i_1 - 14i_3 + 6i_4 = -16 \quad -(3)$$

mesh 4:

$$-28 - 6(i_4 - i_3) - 5i_4 = 0$$

$$6i_3 - 11i_4 = 28 \quad (4)$$

$$V_{8A} = V_{2\Omega} = (i_2 - i_3)2 \text{ or } (i_3 - i_2)2$$

$$= 13.224V$$

i_1, i_3, i_4

a b c d

$$3i_1 - 3i_3 + 0i_4 = 10 \quad -(1)$$

$$3i_1 - 14i_3 + 6i_4 = -16 \quad -(2)$$

$$0i_1 + 6i_3 - 11i_4 = 28 \quad -(3)$$

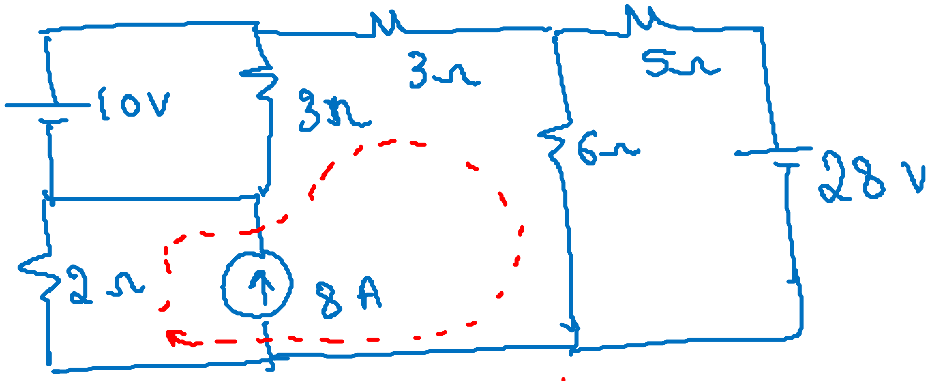
$i_1 = 4.721A$

$i_3 = 1.388A$

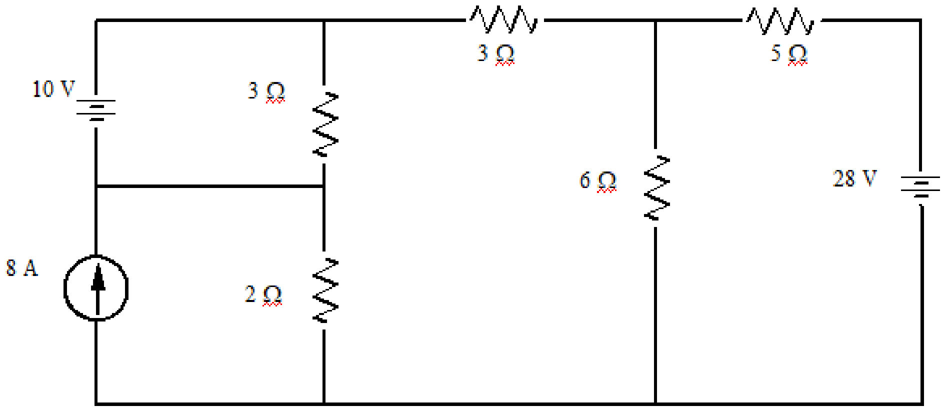
$i_4 = 1.788A$

Mesh Current Analysis

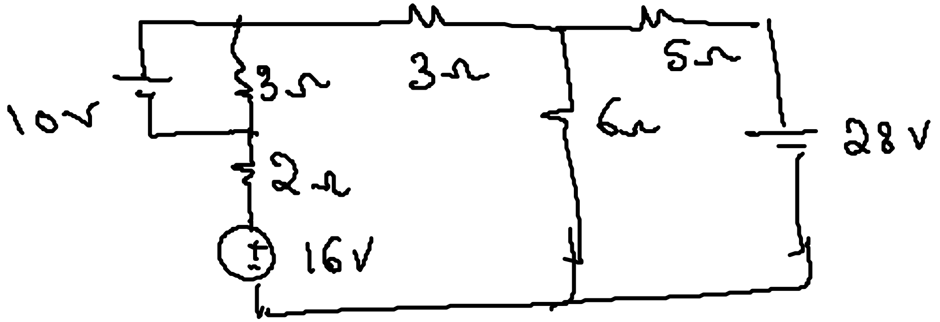
Find the voltage across the current source using mesh current analysis.



Supermesh.

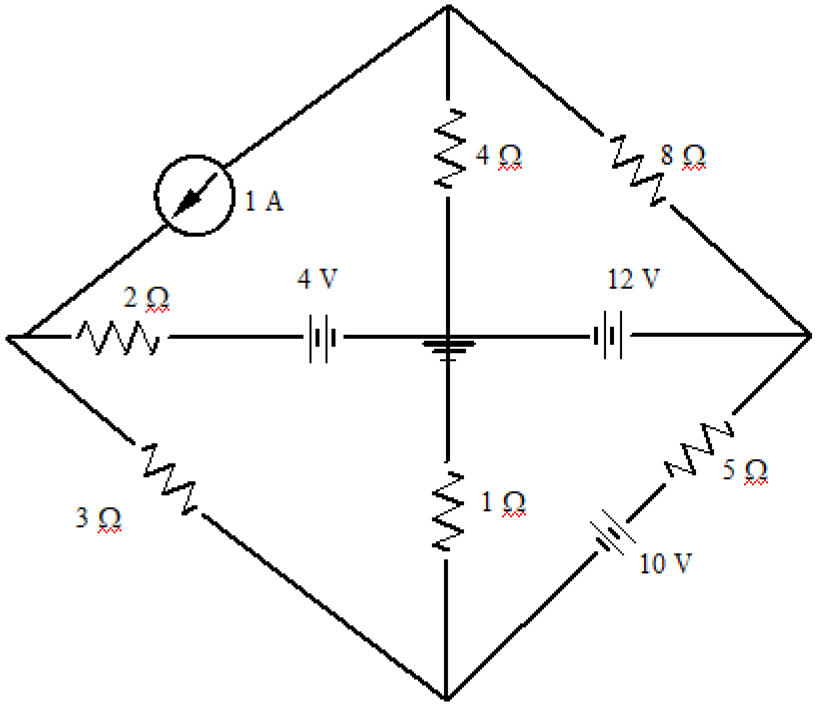


Q) Find current through 28 V battery ; convert practical current source to voltage source as shown here



Node Voltage Analysis

Find the voltage of all nodes using node voltage analysis



Node Voltage Analysis

V_a, V_b, V_c, V_d w.r.t reference.

Find the voltage of all nodes using node voltage analysis

node a: $\frac{V_a - V_d}{3} + \frac{V_a - 0}{2} = 1 + 2 = 3$

$0.833V_a - 0.3333V_d = 3$ — (1)

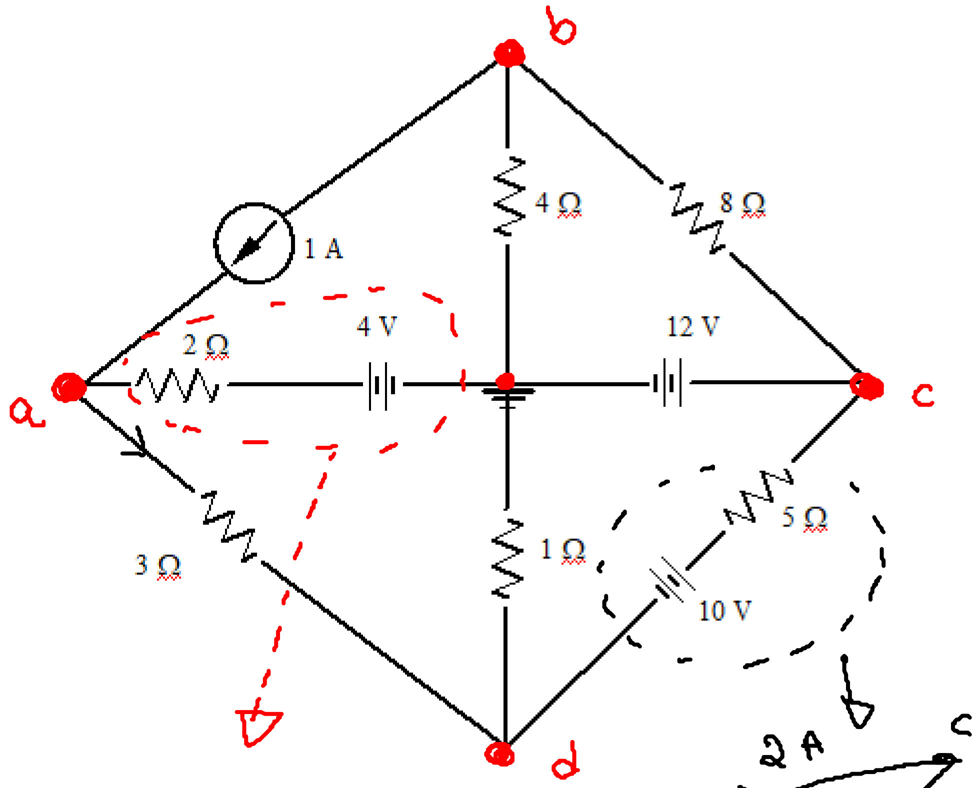
node b: $\frac{V_b - 0}{4} + \frac{V_b - V_c}{8} + 1 = 0$

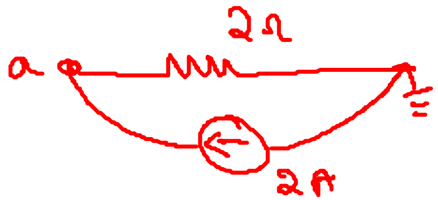
$V_b = 1.33V$

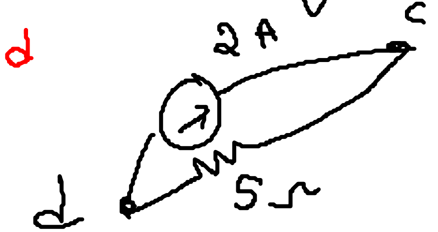
node c: $V_c = 12V$

node d: $\frac{V_d - V_a}{3} + \frac{V_d - 0}{1} + \frac{V_d - V_c}{5} + 2 = 0$

$-V_a(1/3) + V_d(1.5333) = 0.4$ — (2)







$V_a = 4.0581V$ $V_d = 1.1424V$

Node Voltage Analysis

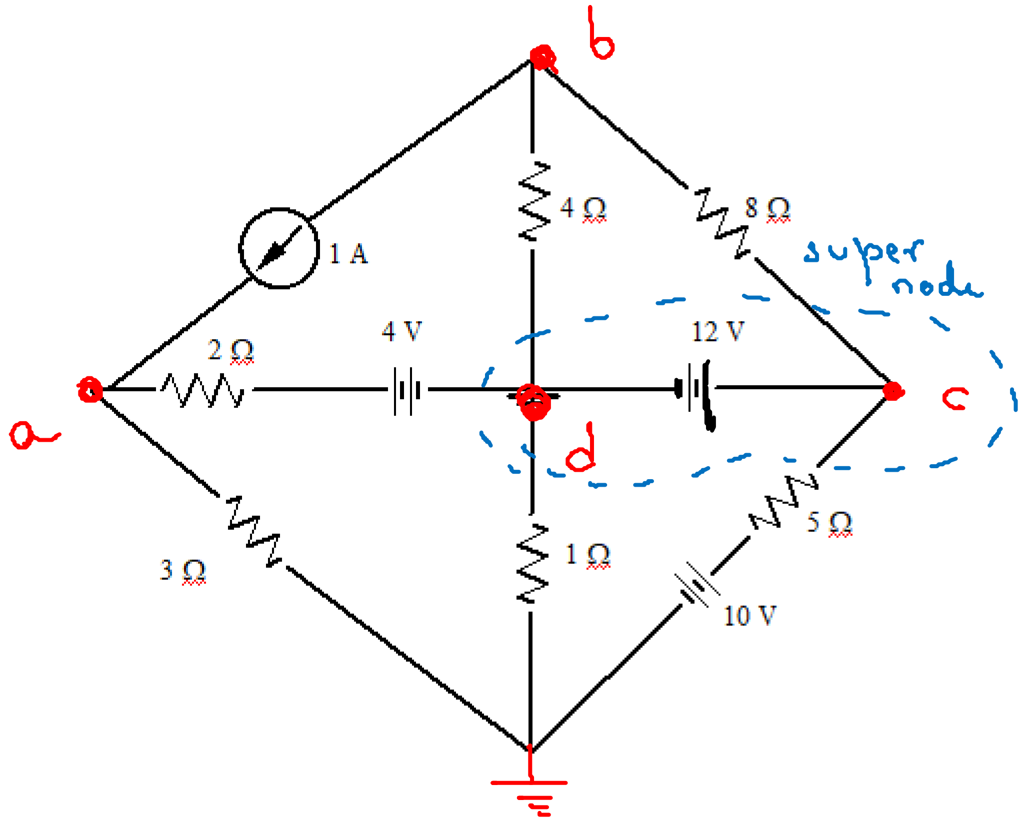
Find the voltage of all nodes using node voltage analysis

Supernode d & c together.

$$\frac{V_d - 0}{1} + \frac{V_d - V_a}{2} + \frac{V_d - V_b}{4} + 2 + \frac{V_c - V_b}{8} + \frac{V_c - 0}{5} = 2$$

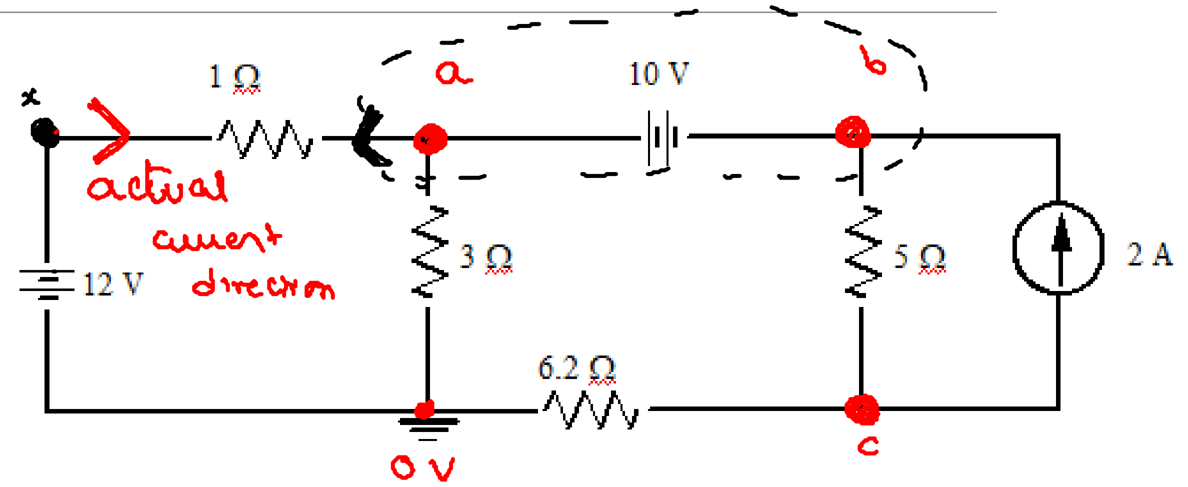
Constraint eqn: $V_c - V_d = 12V$

rest node eqn (a, b) remains same as previous.



Node Voltage Analysis

Find the node voltages and also current through 1 ohm.



Super node a&b

$V_x = 12V$

$$\frac{V_a - 0}{3} + \frac{V_a - V_x}{1} + \frac{V_b - V_c}{5} = 2$$

vs to cs

$$\frac{V_a}{3} + \frac{V_a}{1} - 12 + \frac{V_b - V_c}{5} = 2$$

$$V_a(1.333) + V_b(0.2) - V_c(0.2) = 14 \quad \text{—(1)}$$

$$V_a - V_b = 10 \quad \text{—(2)}$$

node c

$$\frac{V_c - V_b}{5} + \frac{V_c - 0}{6.2} + 2 = 0$$

$$V_c(0.361) - V_b(0.2) = -2 \quad \text{—(3)}$$

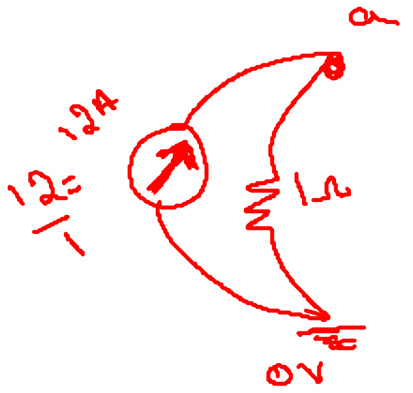
$V_a = 9.69\,V$ $V_b = -0.307\,V$

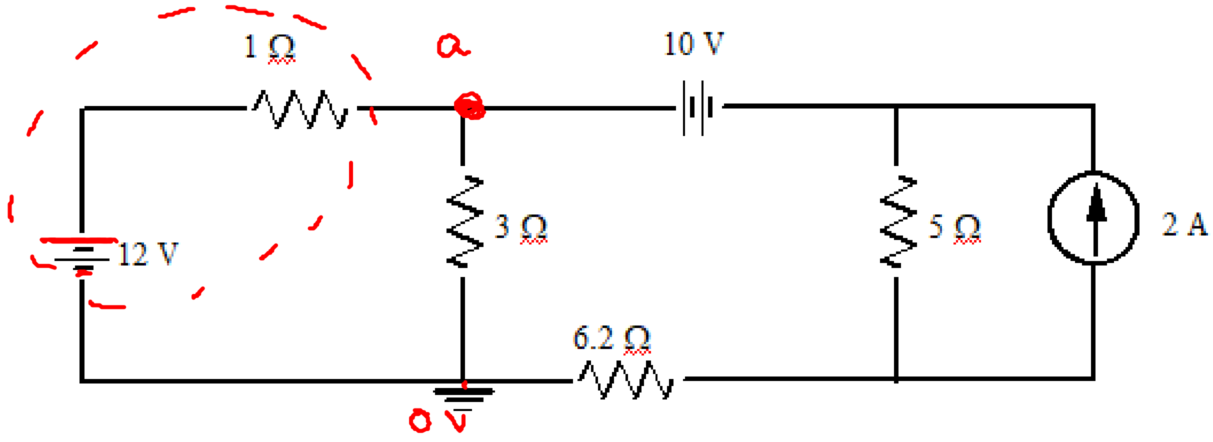
$V_c = -5.71V$

$$I_1 = \frac{V_a - V_x}{1} = \frac{9.69 - 12}{1} = -2.4A$$

Node Voltage Analysis

Find the node voltages and also current through 1 ohm.

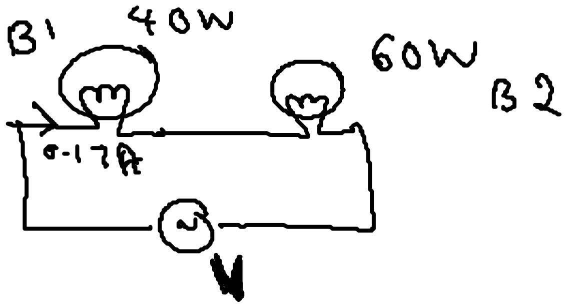




Illustration

Two incandescent bulbs of 40 W and 60 W ratings are connected in series across the voltage V. Then which of the following statement(s) is(are) correct?

a) The bulbs together will consume 100 W
b) The bulbs together will consume 50 W
c) The 60 W bulb glows brighter
d) The 40 W bulb glows brighter ✓



$P_B = I_B^2 R_B$

$\propto$

$V_B^2/R_B \propto V_B \propto I_B$

Assume the voltage rating of both the bulbs to be same 120 V, 100 V

Bulb 1 : 40W = P_{r1}

Voltage rating : 230V = V_{r1}

$I_{B1} = \frac{P_{r1}}{V_{r1}} = 0.17\,A$

$R_{B1} = \frac{V_{r1}}{I_{B1}} = 1352.94\,\Omega$

Bulb 2 : 60W = P_{r2}

230V = V_{r2}

$I_{B2} = 0.26\,A$

$R_{B2} = \frac{V_{r2}}{I_{B2}} = 884.615\,\Omega$

$P_{B1} = I_{B1}^2 R_{B1} = 40W$

$P_{B2} = (0.17)^2 (884.615) = 25.56\,W$

Basic Electrical Technology

Class 7 – 9 November 2021

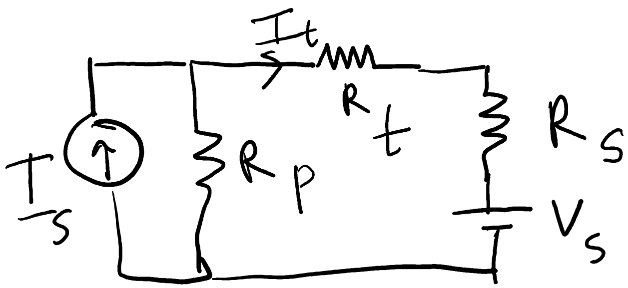
- Network Theorems 1

Terminologies Used

- Linear element: V-I characteristics is linear. E.g: R, L, C
- Non-linear element: V-I characteristics is non-linear. E.g: Diode
- Bi-lateral element: Property does not change with direction of operation. E.g: R, L, C
- Unilateral element: Property changes with the direction of operation. E.g: Diode
- Linear Circuit: Circuit with linear elements only
- Bi-lateral circuit: Circuit with bi-lateral elements only.
- Response: The output of the network. E.g: current, voltage

Superposition Theorem

- **Definition** : In **any linear, bi-lateral** network, **total response** is the **sum** of partial responses.



$$\Rightarrow I_t = \left(I_s \times \frac{R_p}{R_s + R_t + R_p} \right) + \left(-V_s \times \frac{1}{R_s + R_t + R_p} \right)$$

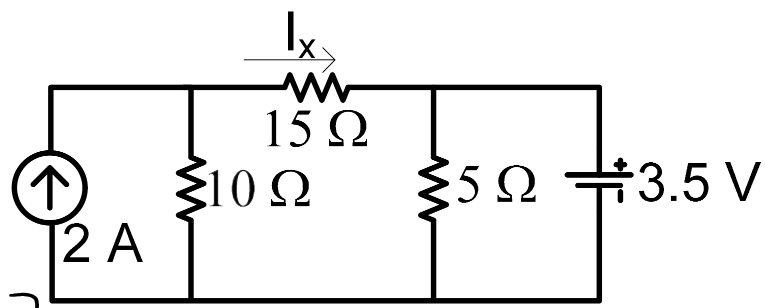
- In any linear, bilateral network, the total response may be determined by adding the responses due to individual sources, considering one source at a time and replacing the other sources by their internal resistances.

Procedure to apply Superposition theorem to solve a DC Circuit

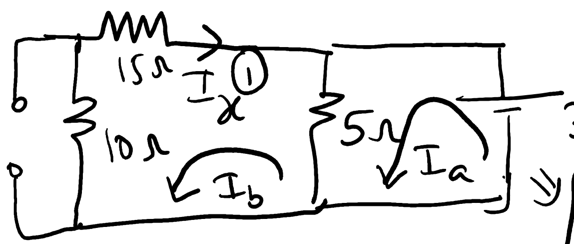
1. Draw the circuit with passive elements only.
2. Place one of the sources in its position.
3. Replace the other sources by their internal resistances.
   a. Ideal **voltage** source by short circuit,
   b. Ideal **current** source by open circuit.
4. Find the response using one of the methods, i.e., network reduction, mesh current, node voltage methods.
5. Repeat the procedure for all the sources.
6. Add the responses due to individual sources.

Illustration 1

Find the current I_x using Superposition theorem



1.

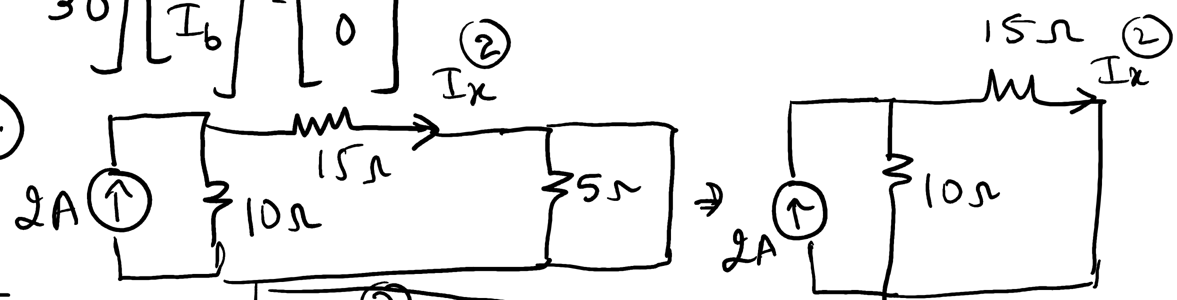


$$\Rightarrow \begin{bmatrix} 5 & -5 \\ -5 & 30 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} 3.5 \\ 0 \end{bmatrix}$$

$$I_a = 840\,\text{mA} \quad I_b = 140\,\text{mA}$$

$$\boxed{I_x^{(1)} = -I_b = -140\,\text{mA}}$$

2.



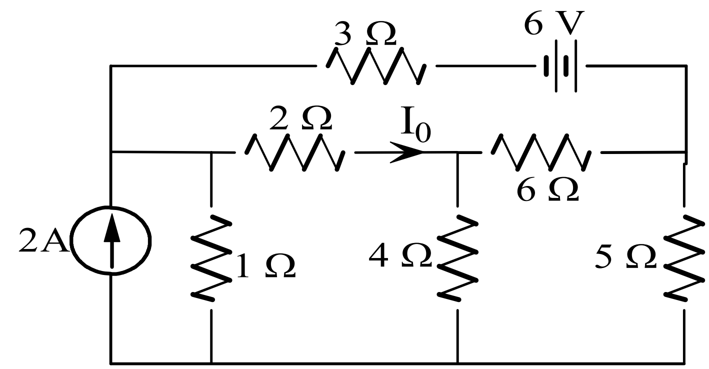
$$\boxed{I_x^{(2)} = 2 \times \frac{10}{10+15} = 800\,\text{mA}}$$

$$I_x = I_x^{(1)} + I_x^{(2)} = 660\,\text{mA}$$

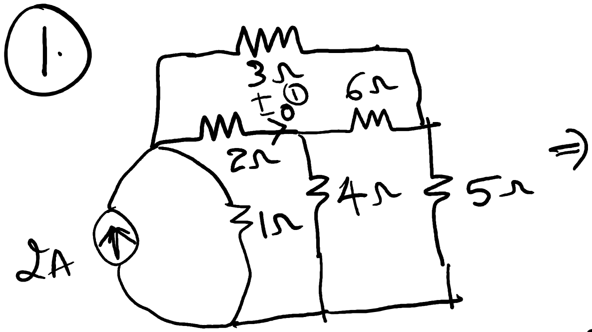
Ans: I_x = 660 mA

Illustration 2

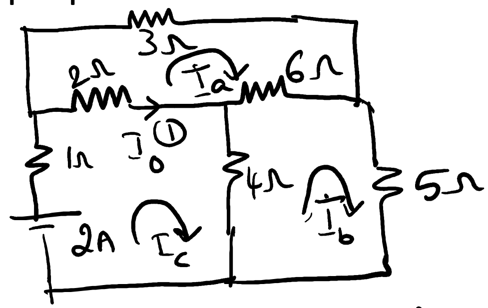
Find the current I_0 using Superposition theorem



1.



⇒



$$\begin{bmatrix}11 & -6 & -2\\ -6 & 15 & -4\\ -2 & -4 & 7\end{bmatrix}\begin{bmatrix}I_a\\ I_b\\ I_c\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 2\end{bmatrix}$$

$I_a = 189.142\,mA$

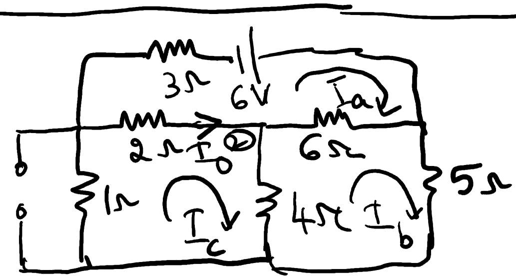
$I_b = 196.147\,mA$

$I_c = 451.839\,mA$

$I_0^{(1)} = I_c - I_a = 262.697\,mA$

$I_0^{(2)} = I_c - I_a = -367.775\,mA$

2.



$$\begin{bmatrix}11 & -6 & -2\\ -6 & 15 & -4\\ -2 & -4 & 7\end{bmatrix}\begin{bmatrix}I_a\\ I_b\\ I_c\end{bmatrix}=\begin{bmatrix}6\\ 0\\ 0\end{bmatrix}$$

$I_a = 935.201\,mA$

$I_b = 525.394\,mA$

$I_c = 567.426\,mA$

$I_0 = I_0^{(1)} + I_0^{(2)}$

$= -105.078\,mA$

Ans: I_0 = -105.078mA

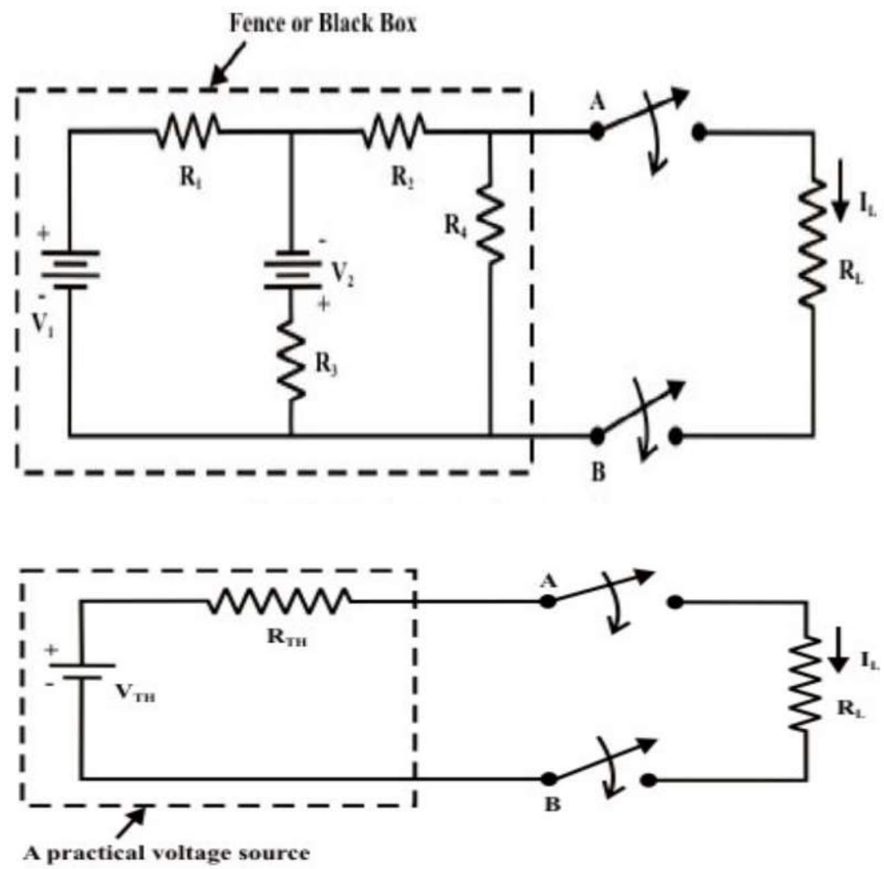
Limitations of superposition Theorem

- **Doesn't work for power calculation**
  - Involves product of voltage and current,
  - the square of current or
  - the square of the voltage,
  - which are **non-linear** operations

  - **cannot be applied for nonlinear circuit**

Why Thevenin's Theorem

- In many applications, a network may contain a variable component or element while other elements in the circuit are kept constant.
- If the solution for current or voltage or power in any component of network is desired, in such cases the whole circuit need to be analyzed each time with the change in component value.
- In order to avoid such repeated computation, it is desirable to introduce a method that will not have to be repeated for each value of variable component.
- For the circuit shown,
  - Find
    - Mesh current method needs 3 equations to be solved
    - Node voltage method requires 2 equations to be solved

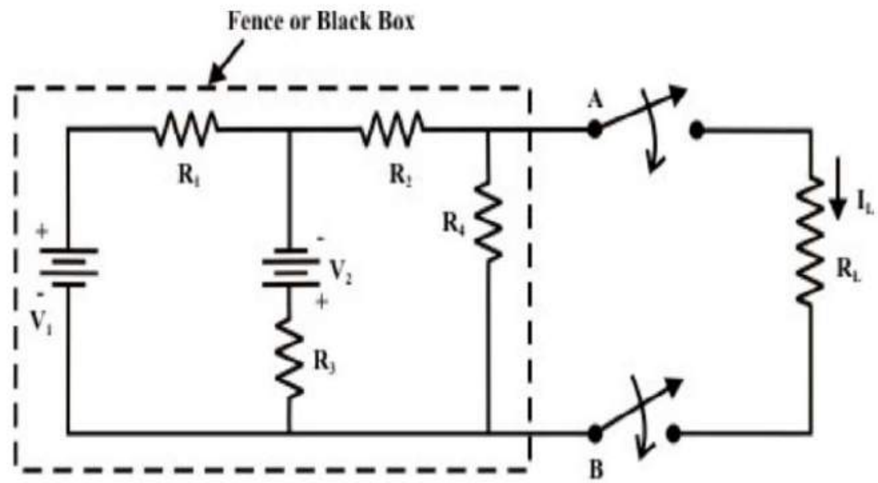


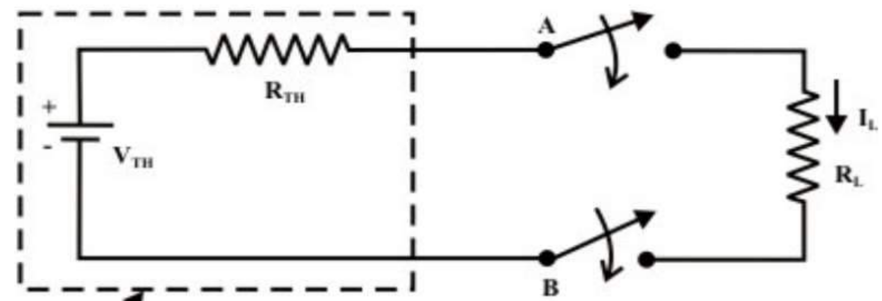
Definition of Thevenin's Theorem

- Any **linear, bilateral** network may be replaced by a single voltage source (called Thevenin's equivalent voltage, $\mathbf{V_{Th}}$) in series with one resistance (called Thevenin's equivalent resistance, $\mathbf{R_{Th}}$) across the load terminals.

- Thevenin's equivalent voltage, $\mathbf{V_{Th}}$, is the open circuit voltage at the load terminals.

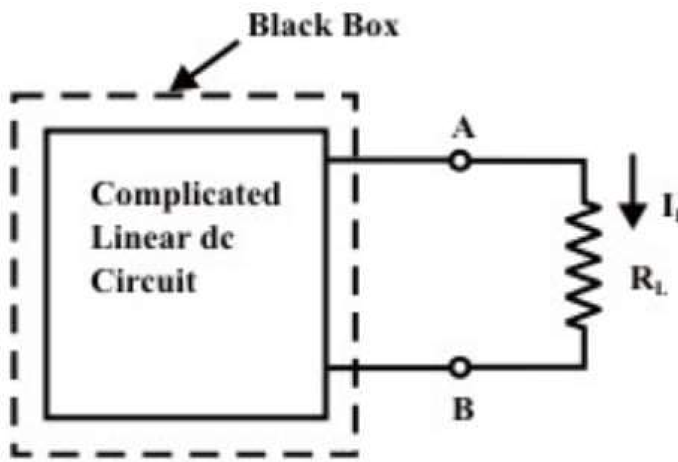
- Thevenin's equivalent resistance, $\mathbf{R_{Th}}$, is the equivalent resistance at the load terminals, after replacing the sources by their internal resistances.





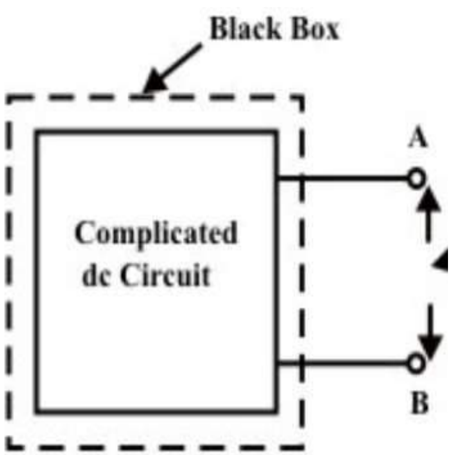
Procedure of Thevenin's Theorem

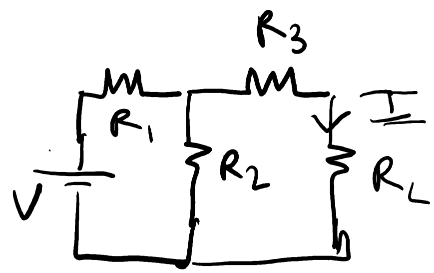
Suppose: Find I_L through R_L.



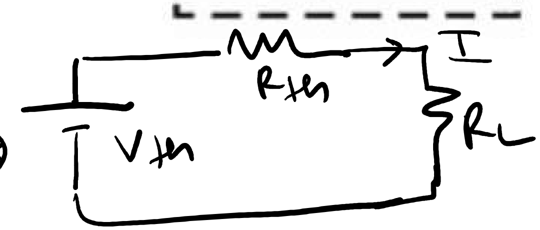
➢ **Step-1: Disconnect R_L**

- Remove the load
- Keep the terminals open circuited as shown in 2nd figure below.





⇒

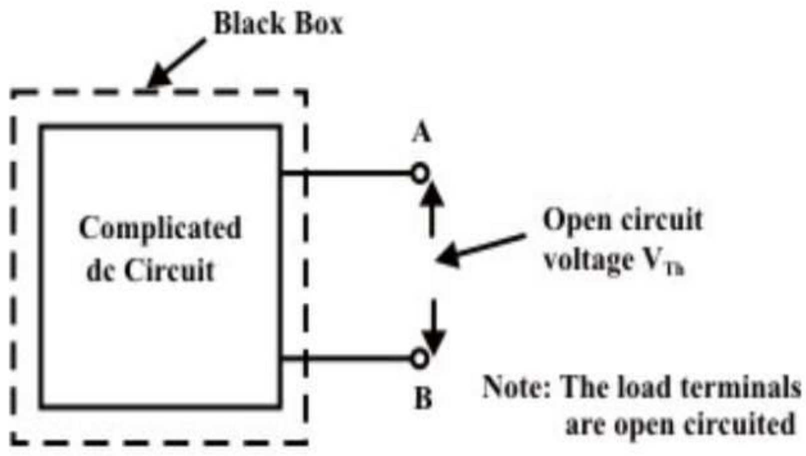


$$\boxed{V_{th} = \frac{V R_2}{R_1+R_2}}$$

$$\boxed{R_{th} = \frac{R_1 R_2}{R_1+R_2} + R_3}$$

$$\boxed{I = \frac{V}{R_L + R_{th}}}$$

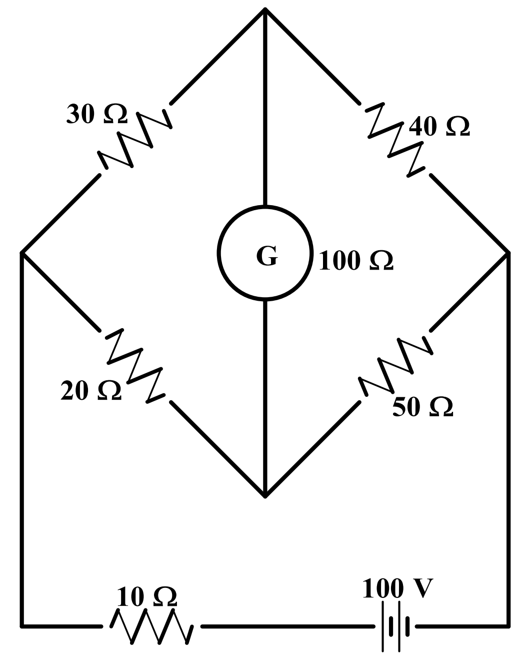
- **Step-2: Find V_{Th}**
  - Apply mesh current / node voltage method
  - Find the voltage across the open circuited terminals.



- **Step-3: To find R_{Th}**
  - Keep the load terminals open.
  - Replace all the sources by their internal resistances.
    - **Voltage sources** should be **short-circuited (just remove them and replace with plain wire)**
    - **Current sources** should be **open-circuited (just remove them)**
  - Find the equivalent resistance with respect to open circuited load terminals.

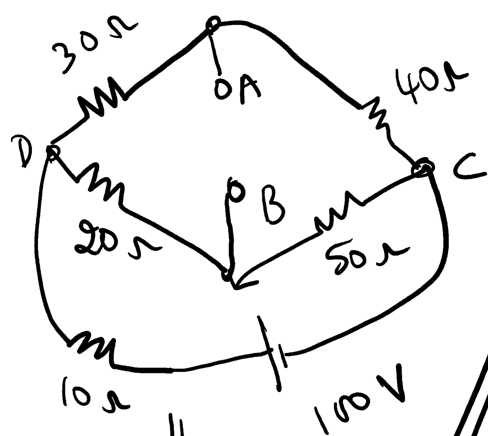
Illustration 3

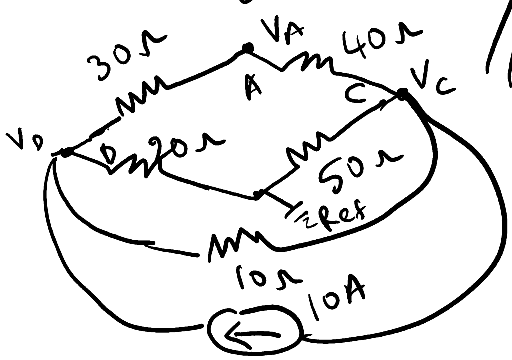
Determine the current through the galvanometer using Thevenin's Theorem



1.

Find V_{TH}





node analysis

@ A $\quad \dfrac{V_A - V_D}{30} + \dfrac{V_A - V_C}{40} = 0$

@ C $\quad \dfrac{V_C - V_A}{40} + \dfrac{V_C - V_D}{10} + \dfrac{V_C}{50} = -10$

@ D $\quad \dfrac{V_D - V_C}{10} + \dfrac{V_D - V_A}{30} + \dfrac{V_D}{20} = 10$

$$\Downarrow$$

$$\begin{bmatrix} \frac{7}{120} & -\frac{1}{40} & -\frac{1}{30} \\ -\frac{1}{40} & \frac{29}{200} & -\frac{1}{10} \\ -\frac{1}{30} & -\frac{1}{10} & \frac{11}{60} \end{bmatrix} \begin{bmatrix} V_A \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \\ 10 \end{bmatrix}$$

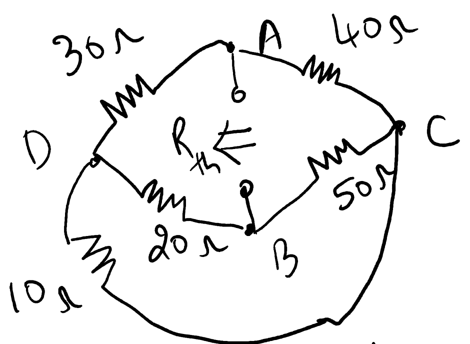
(cont ...)

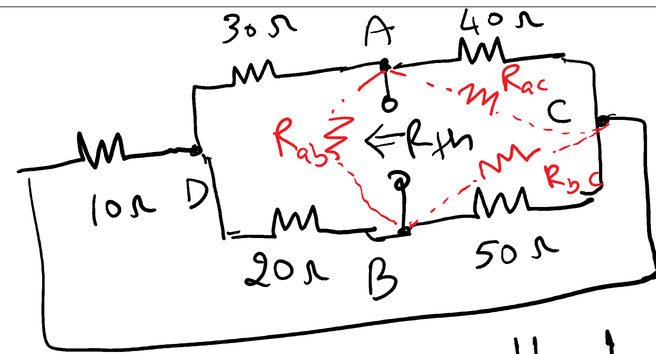
$V_A = -11.11 \quad V_C = 22.22$

$V_D = -55.56 \quad \boxed{V_{Th} = V_A = -11.11\ V}$

2.

Find R_{th}





$Y - \Delta$ transformation

$$R_{ab} = 20+30+\frac{20\times30}{10}$$

$$R_{ab} = 110\ \Omega$$

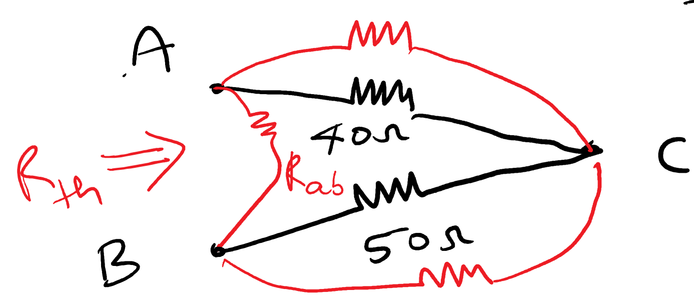
$$R_{bc} = 20+10+\frac{20\times10}{30}$$

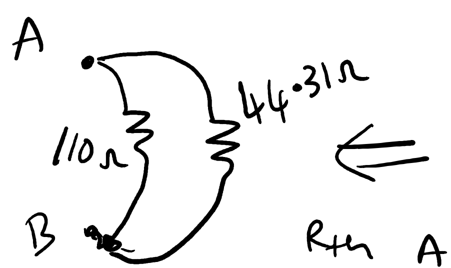
$$R_{bc} = 36.67\ \Omega$$

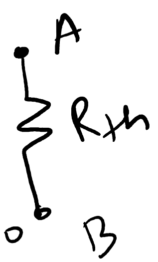
$$R_{ac} = 10+30+\frac{10\times30}{20}$$

$$R_{ac} = 55\ \Omega$$

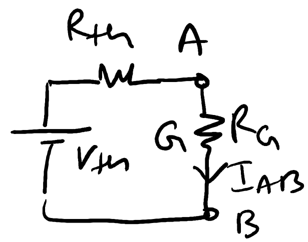
Short circuit the source







$$\boxed{R_{th} = 31.59\ \Omega}$$



Current $I_{AB} = \frac{V_{th}}{R_{th}+R_G} = -84.429\ mA$

Answer : Current is I_{BA} = 84.249mA

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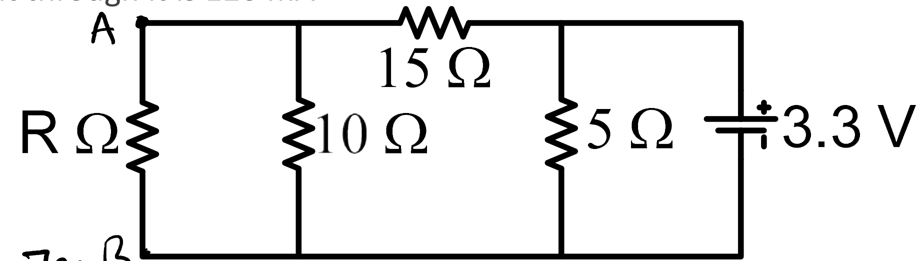
Basic Electrical Technology

Class 8 – 12 November 2021

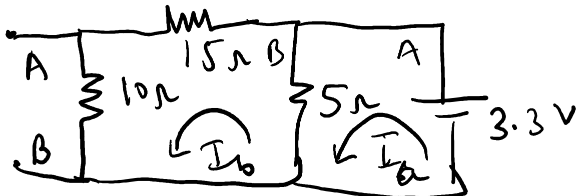
- Network Theorems 2

Illustration 4

Using Thevenin's theorem, find the value of **R** such that the current through it is 120 mA



1. Find V_{Th}



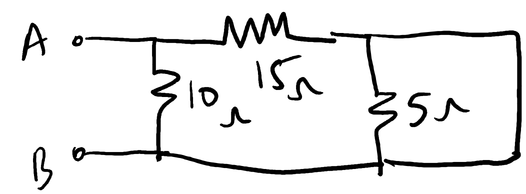
$$\begin{bmatrix} 5 & -5 \\ -5 & 30 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 3.3 \\ 0 \end{bmatrix}$$

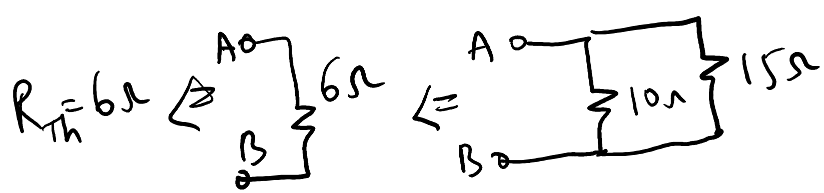
$I_a = 0.792\,A$

$I_b = 0.132\,A$

$$\boxed{V_{Th} = V_{AB} = 10\,I_b = 1.32\,V}$$

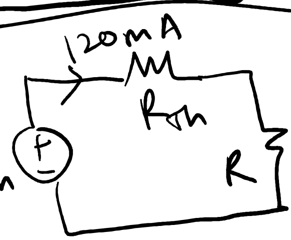
2. Find R_{th}





$$120\,mA = \frac{1.32}{R+6}$$

$$\boxed{R = 5\Omega}$$

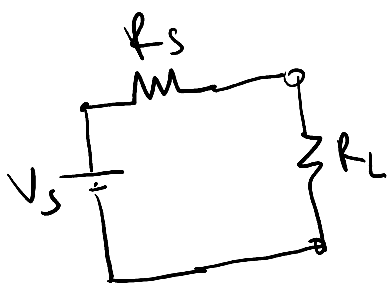


Ans: 5 Ω

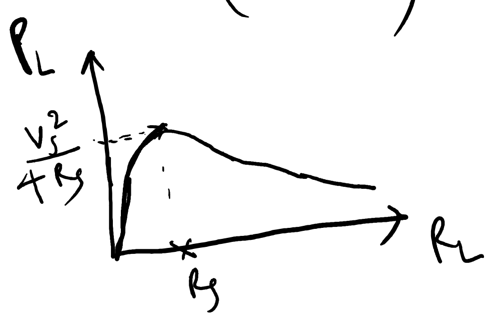
Maximum Power Transfer Theorem

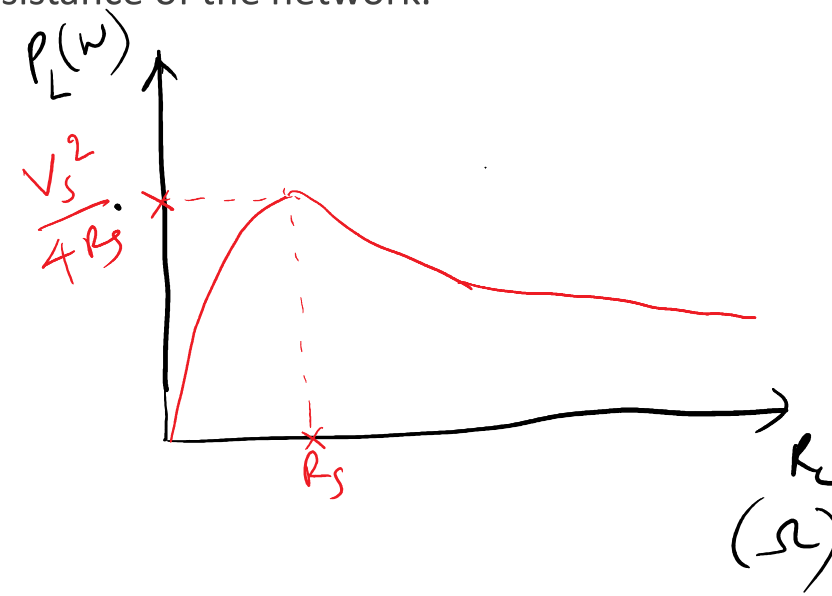
Definition:

In **any linear, bi-lateral network**, maximum power will be transferred to the load from the network when the load resistance is equal to the internal resistance of the network.



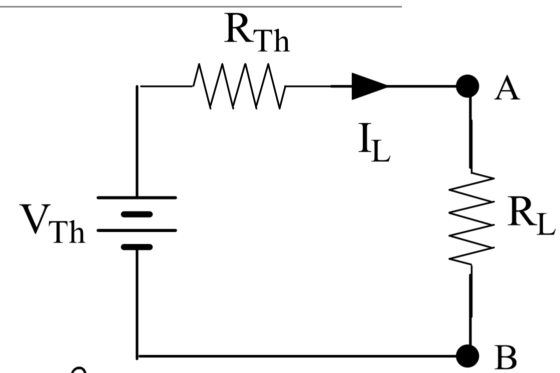
$$P_L = \frac{V_S^2 R_L}{(R_S + R_L)^2} \Rightarrow$$





Proof

Consider the Thevenin's equivalent circuit of a network



$$I_L = \frac{V_{Th}}{R_{Th}+R_L}$$

$$P_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

$For P_L\ to\ be\ maximum, \frac{dP_L}{dR_L} = 0$

$$\frac{dP_L}{dR_L} = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 - 2\frac{V_{Th}^2}{(R_{Th}+R_L)^3}R_L = 0$$

$$\Downarrow$$

$$\frac{2R_L}{R_{Th}+R_L} = 1 \Rightarrow \boxed{R_L = R_{Th}}$$

$which\ yields, \boldsymbol{R_L = R_{Th}}$

@ $R_L = R_{Th}$ hence power is maximum

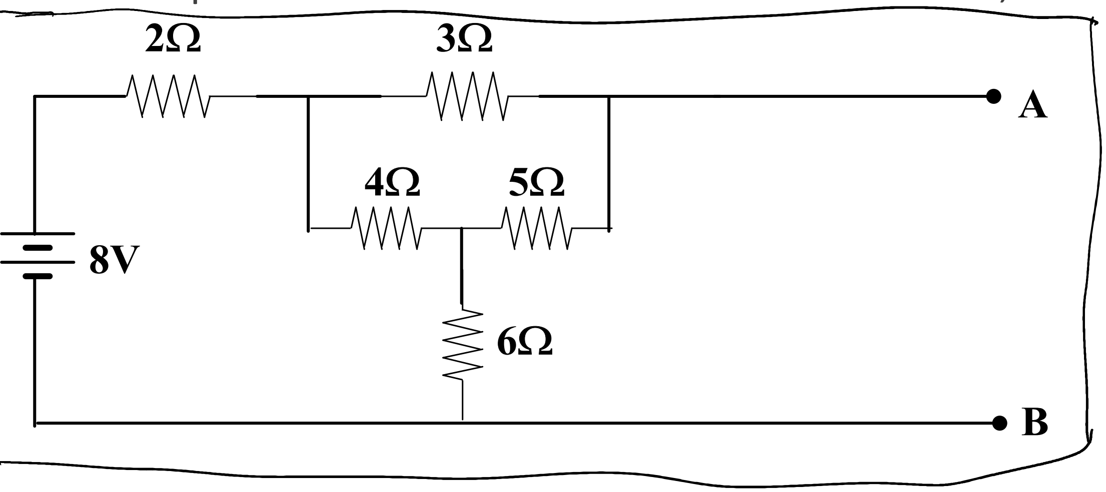
$Maximum\ Power,$

$$P_{L-max} = \left(\frac{V_{Th}}{R_{Th}+R_{Th}}\right)^2 R_{Th} = \frac{V_{Th}^2}{4R_{Th}}$$

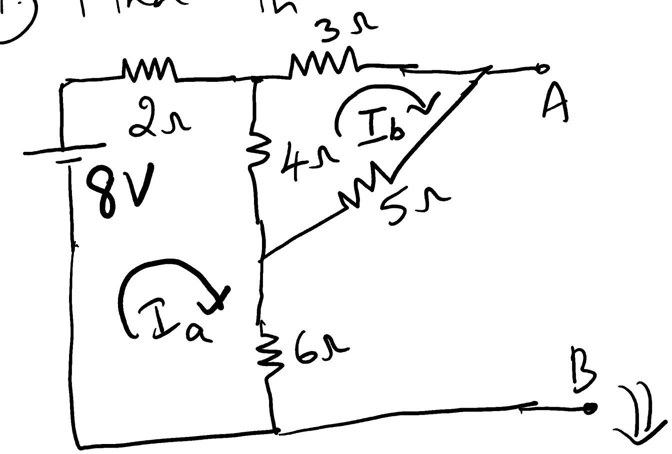
$$\frac{d^2P_L}{dP_L}\Big|_{R_{Th}} = \left\{ 4\frac{V_{Th}^2}{(R_{Th}+R_L)^3} + \frac{6V_{Th}^2R_L}{(R_{Th}+R_L)^4}\right\}\Big|_{R_{Th}^2} - \frac{V_{Th}^2}{8R_{Th}^3} < 0$$

Illustration 5

Determine the value of resistor to be connected across the terminals A & B such that maximum power is transferred to the that resistor. Also, find the value of maximum power.



1. Find V_{Th}



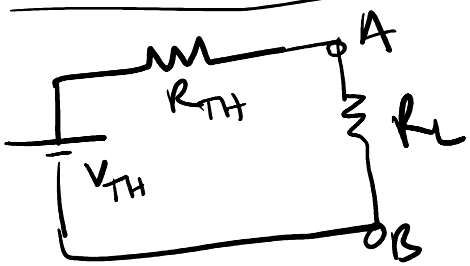
$$\begin{bmatrix} 12 & -4 \\ -4 & 12 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$$

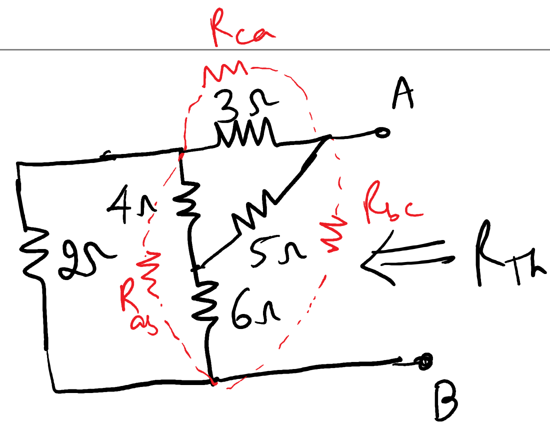
$I_a = 0.75\,A$

$I_b = 0.25\,A$

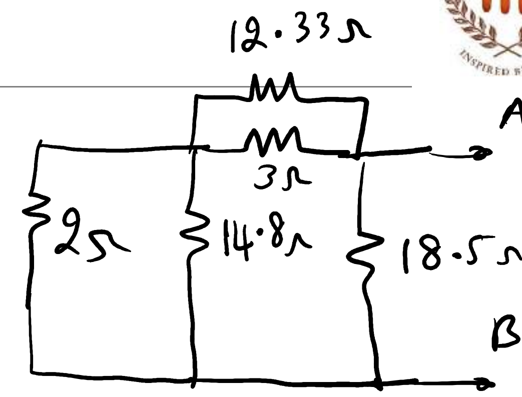
$$\boxed{V_{Th} = 5I_b + 6I_a = 5.75\,V}$$

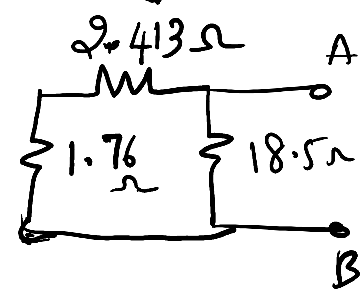
2. Find R_{th}

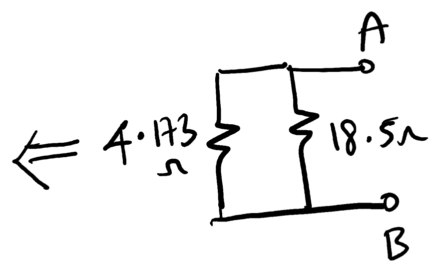


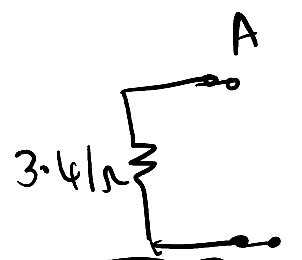


Y to Δ Convert









$\boxed{R_{TH} = 3.41\Omega}$

For maximum powers

$$\boxed{R_L = R_{TH} = 3.41\Omega}$$

maximum power P_{Lmax}

$$\boxed{P_{Lmax} = \frac{V_{Th}^2}{4R_{th}} = \frac{5.75^2}{4\times 3.41} = 2.43}$$

Ans: 3.41 Ω, 2.43 W

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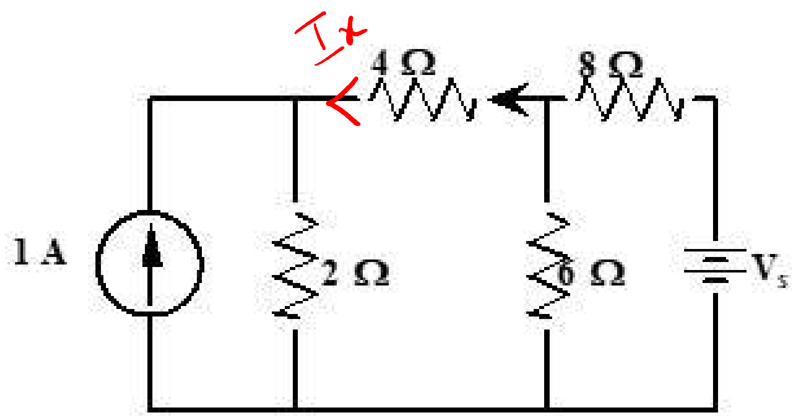
CLASS 9 – 16 NOVEMBER 2021

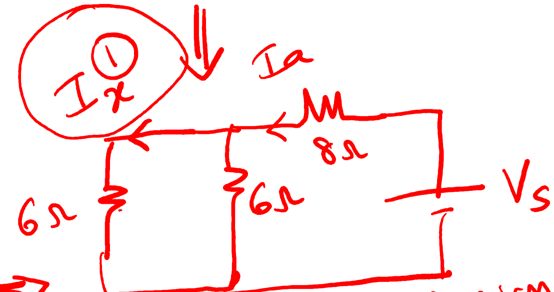
- Tutorial 2 : Network Theorems

Question 1

Using Superposition theorem, find the value of Vs if the current in 4 Ω is 0.515 A and the direction is as shown in the diagram below.

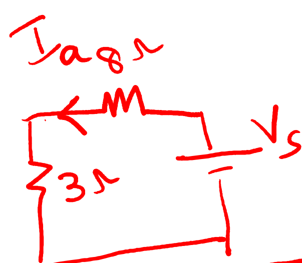


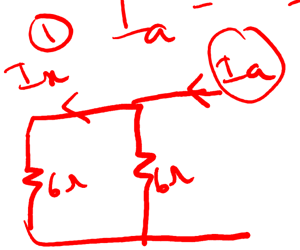
Step 1 Partial response due to voltage source mag : V_s V $\Rightarrow I_x^{(1)}$



current division rule

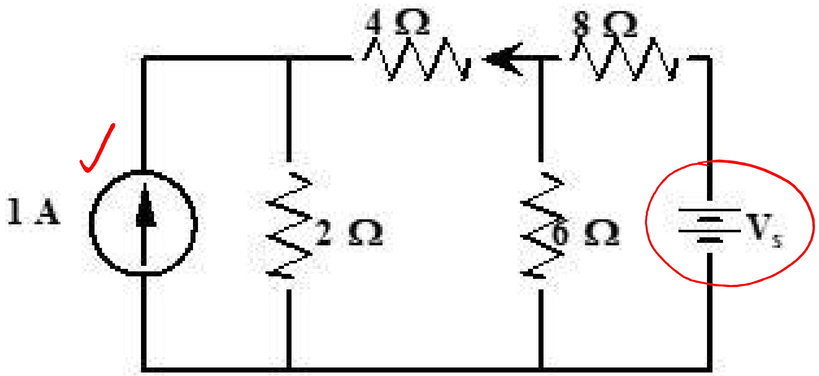
$$I_x^{(1)} = I_a \times \frac{6}{6+6}$$



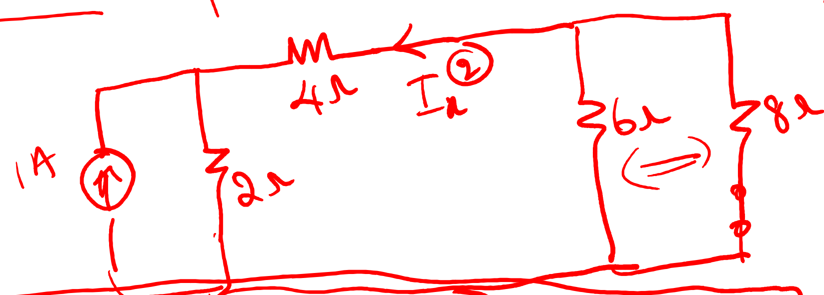


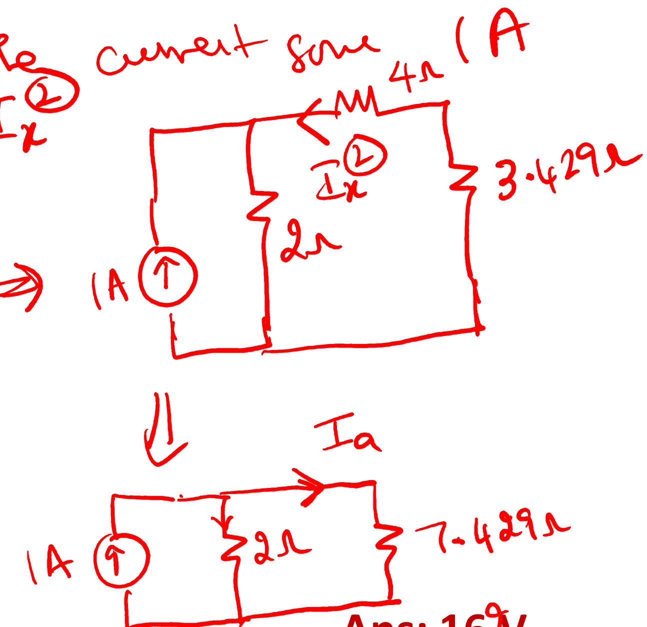
$$I_a = \frac{V_s}{11}$$

$$\boxed{I_x^{(1)} = \frac{V_s}{22}}$$



Step II — partial response due to the current source $I_x^{(2)}$





$0.212\,A = I_a = \frac{1 \times 2}{9.429\Omega}$

$I_x^{(2)} = -I_a = -0.212\,A$

Step III

$I_x = I_x^{(1)} + I_x^{(2)}$

Solve for Vs

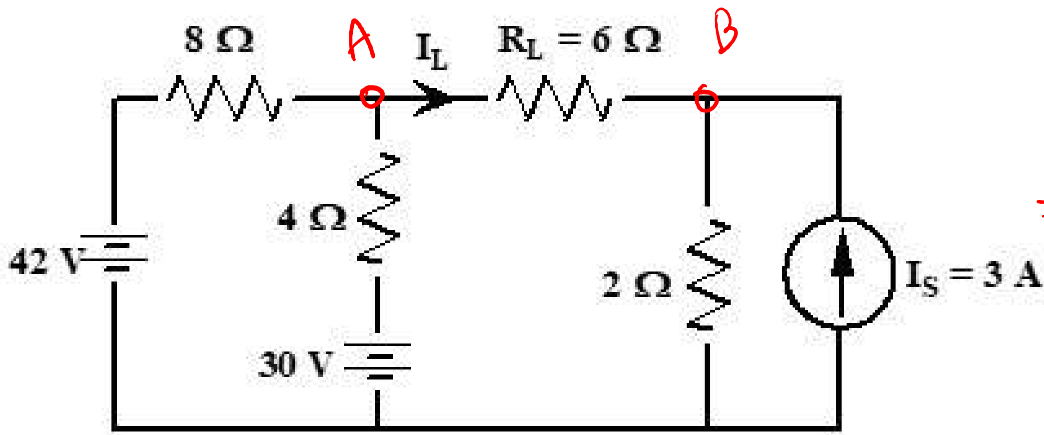
$0.515 = \frac{V_s}{22} - 0.212$

$$\boxed{V_s = 15.994\ V}$$

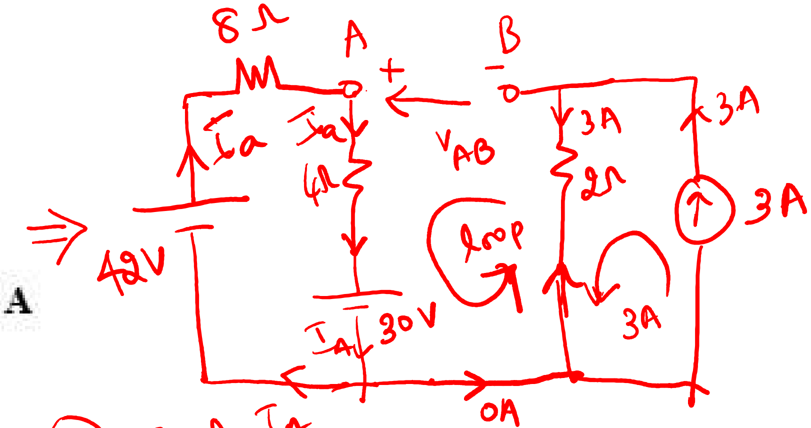
Ans: 16V

Question 2

For the circuit shown find the current I_L through 6 Ω resistor using Thevenin's theorem



⇒

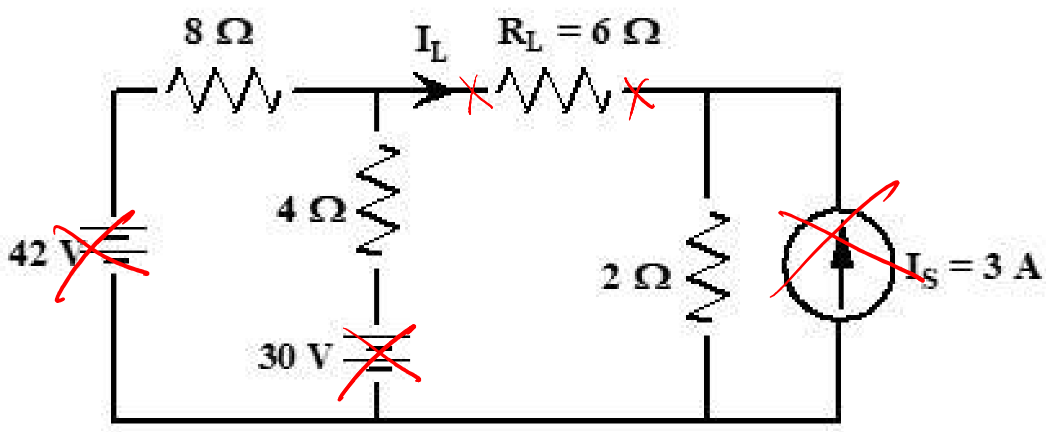


① Find I_A $V_{Th} = V_{AB}$

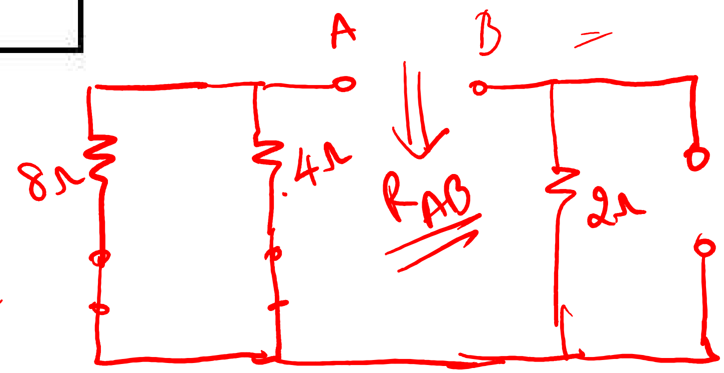
$$I_a = \frac{42 - 30V}{8+4} = \frac{12}{12}\,A = 1A$$

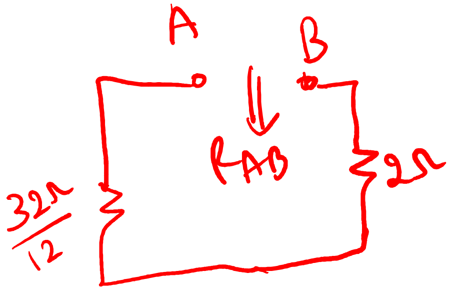
Applying KVL for loop : $V_{AB} = 4I_a + 30 - (3 \times 2) = 4 + 30 - 6$

$V_{AB} = 28V = V_{Th}$



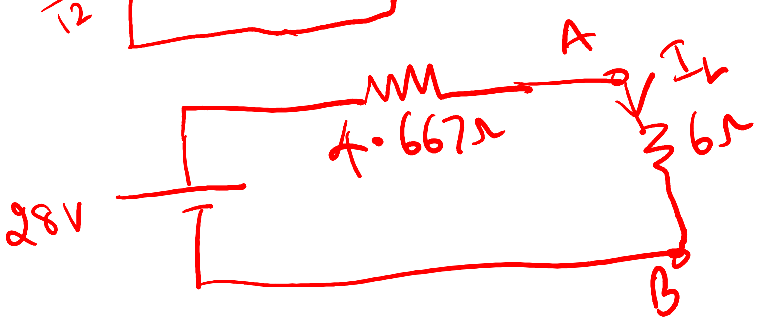
2. To find R_{Th}





$\Rightarrow$ $R_{AB} = 4.667\,\Omega = R_{Th}$

3.

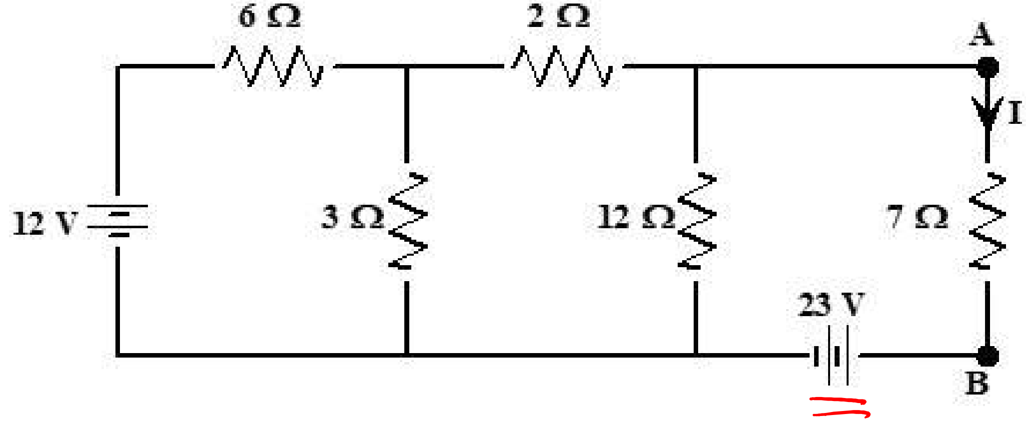


$I_L = 2.625\text{ A}$

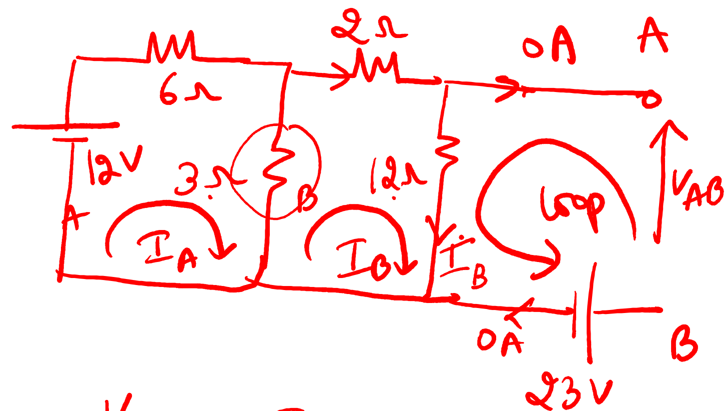
Ans: I_L = 2.625 A

Question 3

In the figure shown below replace the network to the left of terminals A & B by its Thevenin equivalent circuit. Hence, determine I



Step I Find V_{TH}



$$V_{AB} = I_B \times 12 - 23$$

$$\begin{array}{c} \\ A: \\ B: \end{array} \begin{bmatrix} 9 & -3 \\ -3 & 17 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$$

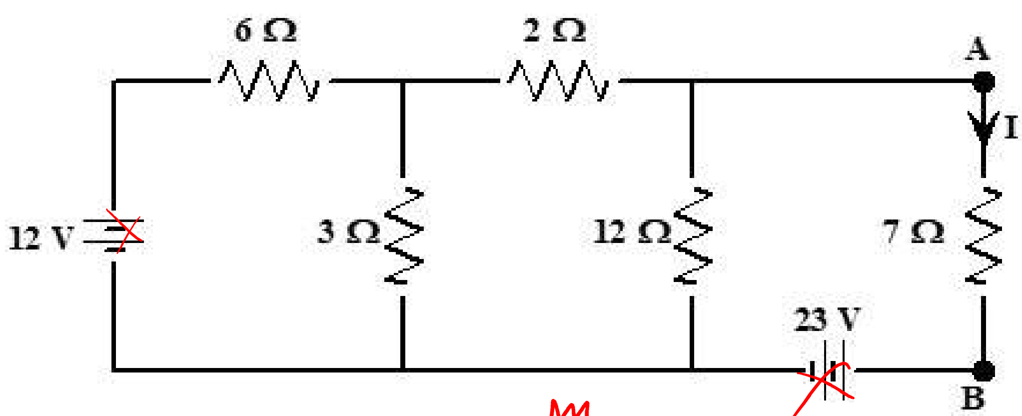
Solve for I_A & I_B

$$I_A = 1.417\,A$$

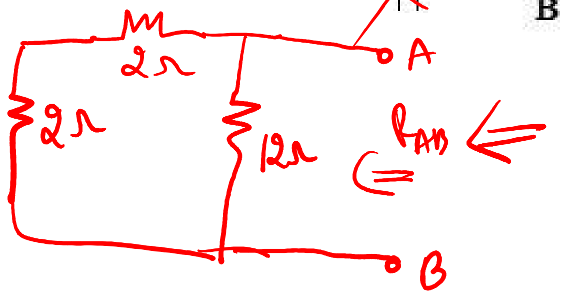
$$I_B = 0.25\,A$$

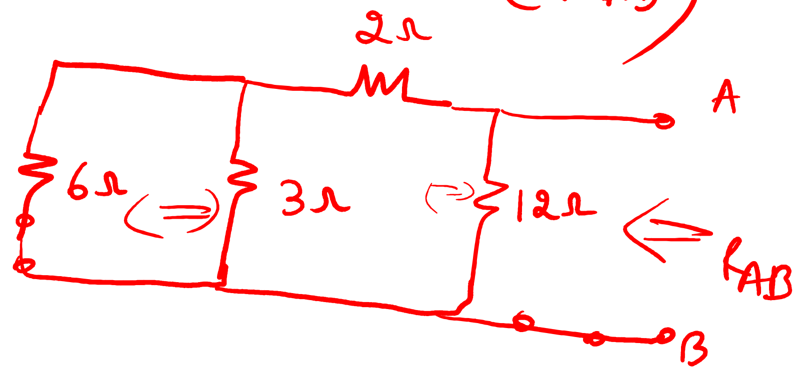
$$V_{AB} = 0.25 \times 12 - 23$$

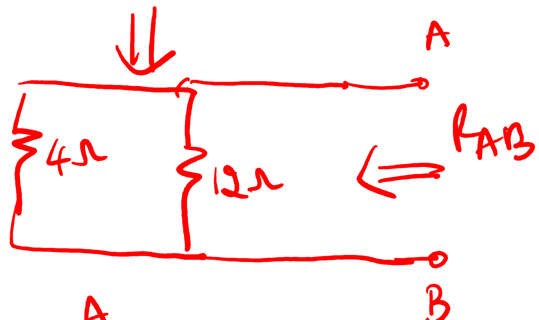
$$V_{AB} = V_{TH} = -20\,V$$



Step II: Find R_{Th} (R_{AB})

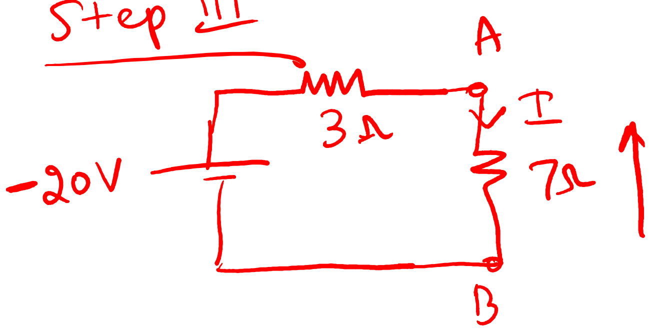






$\Rightarrow R_{TH} = R_{AB} = 3\,\Omega$

Step III



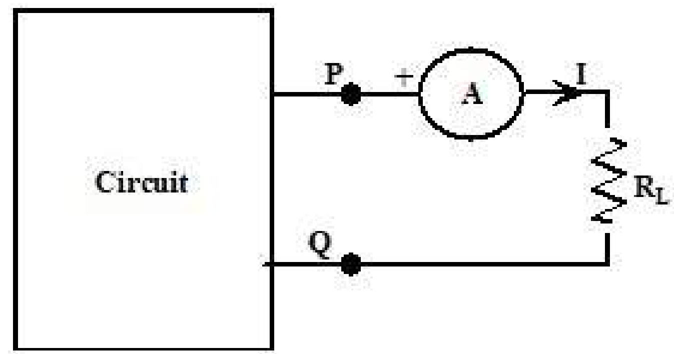
$$I = \frac{-20}{10} = -2\,A$$

Answer:
R_{th} = 3 Ω,
V_{th} = 20 V
I = -2 A

Question 4

The box shown in the adjacent figure consists of independent dc sources and resistances. Measurements are taken by connecting an ammeter in series with the resistor R_L and the results are shown in the table below. Find the value of R_L for which the current is 0.6 A

R_L	I
10 Ω	2.0 A
20 Ω	1.5 A
?	0.6 A

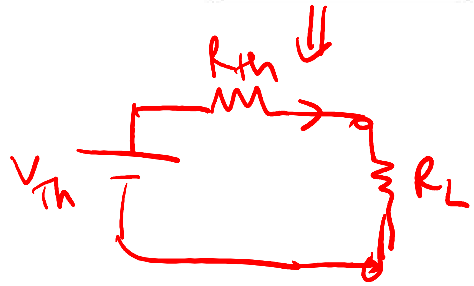


from 1.

$$2 = \frac{V_{Th}}{R_{Th}+10} \quad \text{—— (1)}$$

from 2.

$$1.5 = \frac{V_{Th}}{R_{Th}+20} \quad \text{—— (2)} \Rightarrow$$



solving (1) & (2)

$$V_{Th} - 2R_{Th} = 20 \quad \text{— (1)}$$
$$V_{Th} - 1.5R_{Th} = 30 \quad \text{— (2)}$$
$$\Rightarrow V_{Th} = 60V, \quad R_{Th} = 20\Omega$$

3. $0.6 = \frac{60}{20+R_L}$

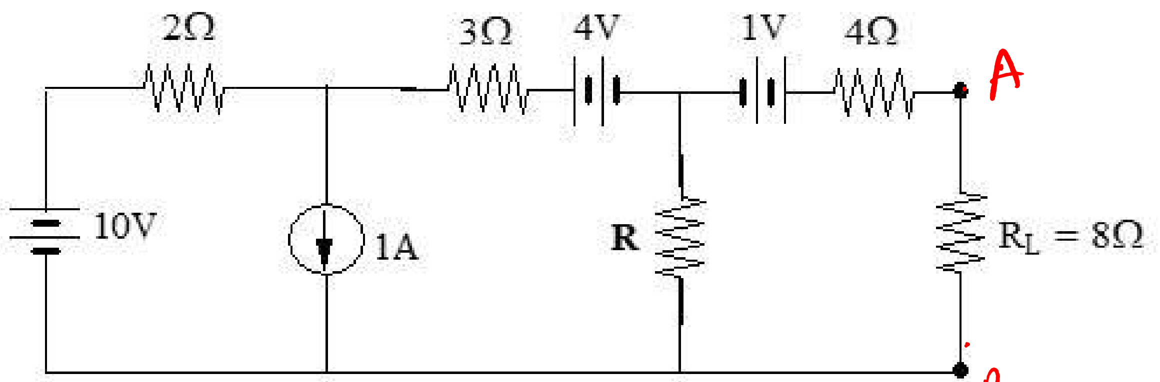
$$\Downarrow$$

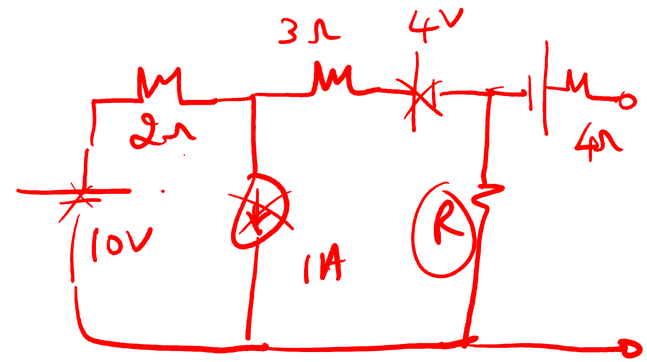
$$\boxed{R_L = 80\Omega}$$

Ans: $R_L = 80\ \Omega$

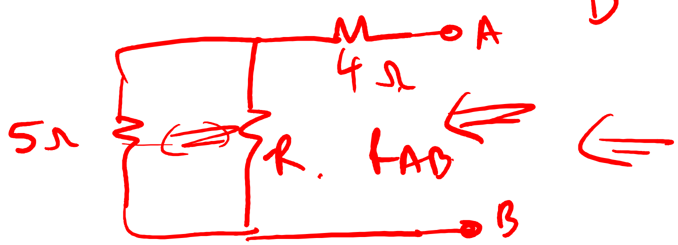
Question 5

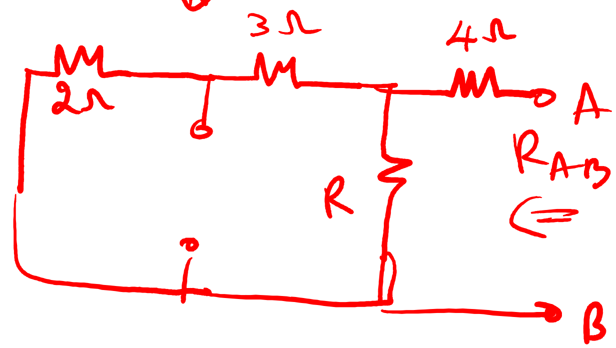
Find the value of **R** such that maximum power is transferred to **8 Ω** resistor.





⇓ Find $R_{Th} = 8\,\Omega$



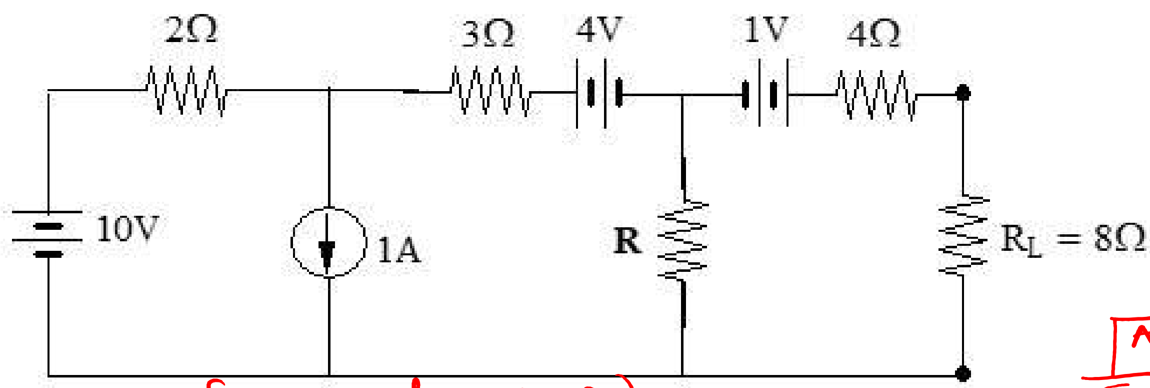


$$R_{AB} = \frac{5R}{5+R} + 4 = 8\,\Omega = R_{Th}$$

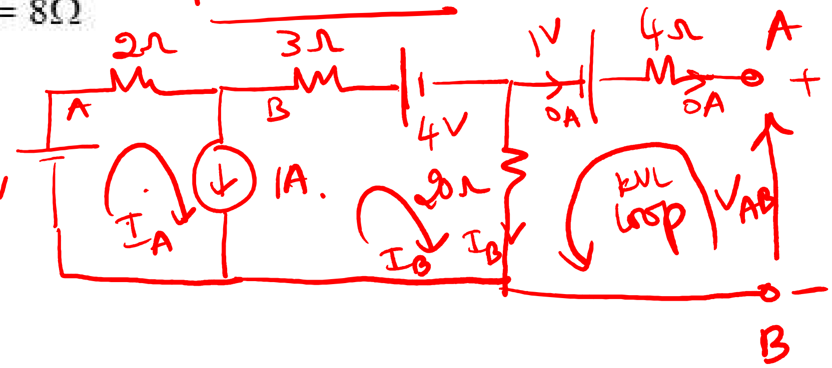
⇓ solve for R

$$\boxed{R = 20\,\Omega}$$

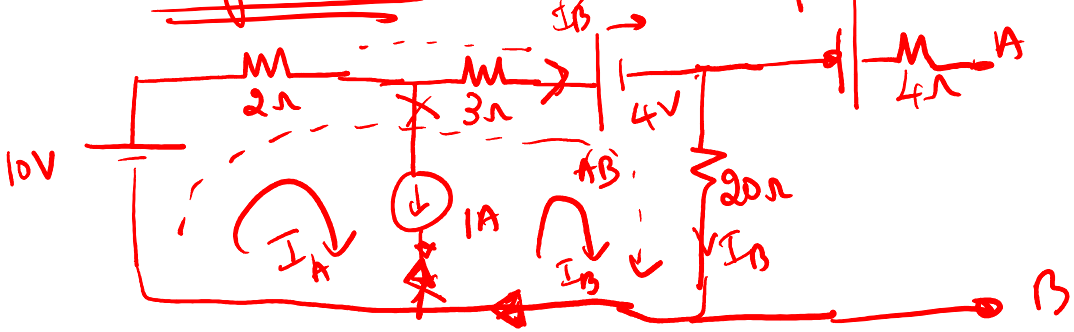
$R_{Th} = R_{AB}$



Find V_{Th}



Supermesh (A B)



$V_{AB} = 1 + 20I_B$

KVL for supermesh : $10 - 2I_A - 3I_B - 4 - 20I_B = 0$ —— (1)

for 1A current source KCL : $I_A - I_B = 1$ —— (2)

⇓ solving

$I_A = 1.16\,A$ $\quad$ $I_B = 0.16\,A$

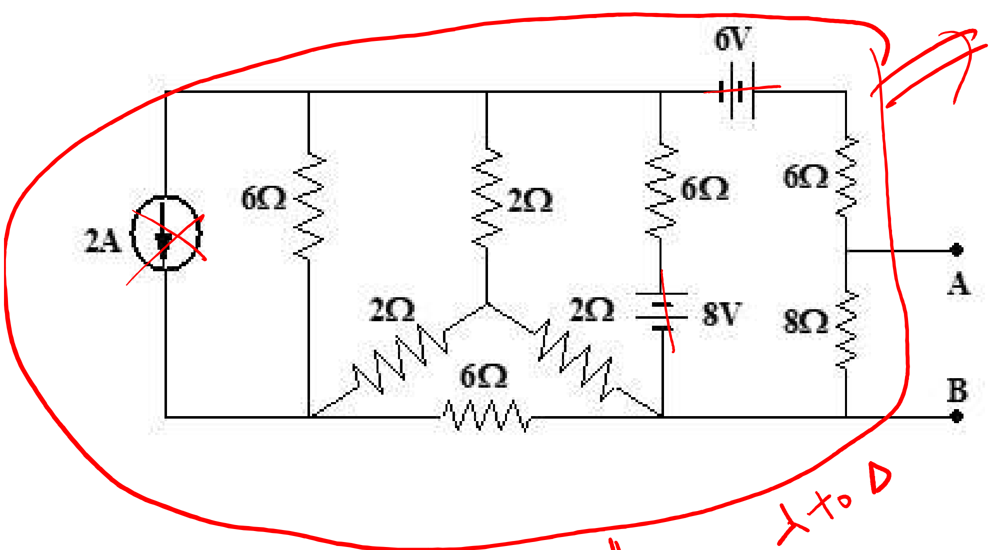
$$V_{TH} = 1 + (20 \times 0.16) = 4.2\,V$$

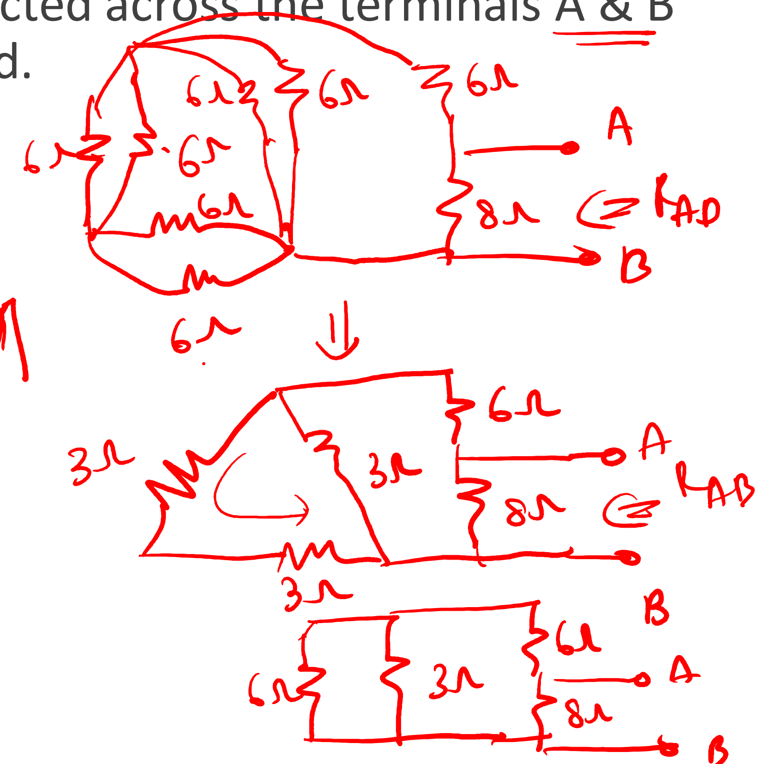
$$P_{max}\ (\text{load}) = \frac{V_{TH}^2}{4R_{TH}} = 0.551\,W$$

Ans: 20 Ω

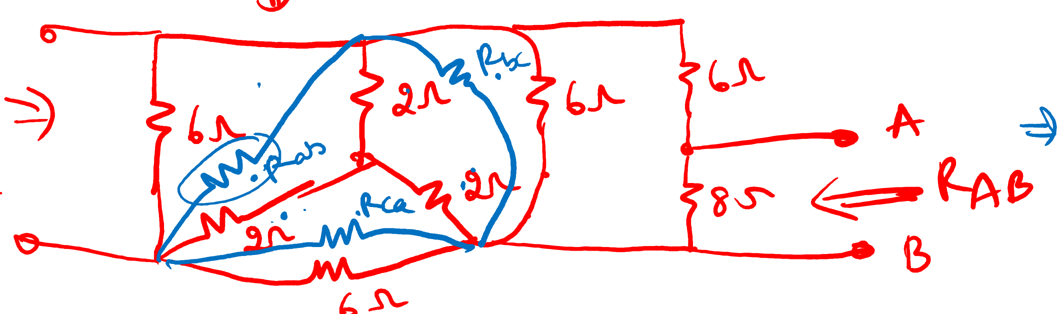
Question 06

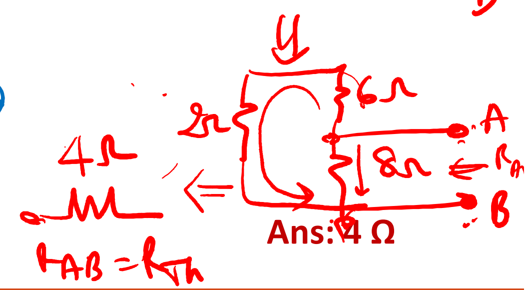
Determine the value of load resistance to be connected across the terminals A & B such that maximum power is transferred to the load.





$R_{ab} = 2+2+\frac{2\times 2}{2}$
$= R_{bc} = R_{ca}$





Ans: 4 Ω

Basic Electrical Technology

Class 10 – 19 November 2021

- R L Transients

Growth of Current in an Inductive Circuit

Applying KVL,

$$V - R\,i - L\frac{di}{dt} = 0$$

Initial Conditions,

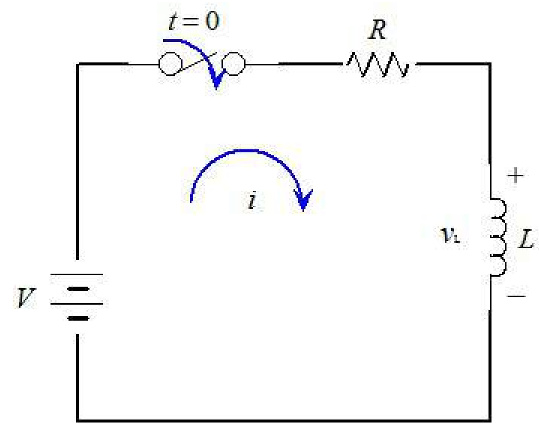
$$At\ t = 0\ sec, i = 0\ A$$

Final current & voltage equation,

$$i = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$v_L = L\frac{di}{dt}$

$$v_L = V\,e^{-\left(\frac{R}{L}\right)t}$$



Apply KVL in the loop

$V = iR + V_L$

$\Rightarrow V = iR + L\frac{di}{dt}$

$\Rightarrow \frac{di}{dt} = \left(-\frac{R}{L}\right)i + \left(\frac{1}{L}\right)V$

$\frac{dx}{dt} = Ax + Bu$

$x(t) = e^{At}x(0) + e^{At}\int_0^t e^{-A\tau}Bu\,d\tau$

$A = -\frac{R}{L}$ $x = i$ $u = V$ $B = \frac{1}{L}$

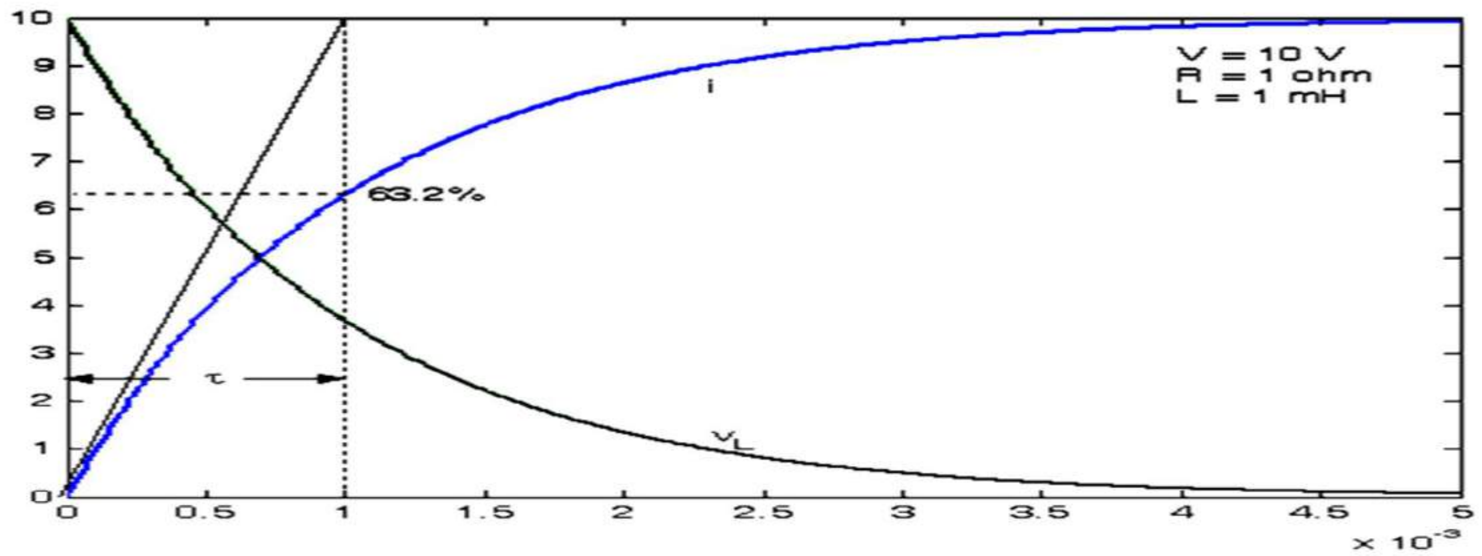
$I(0) = 0$

$e^{At}\int_0^t e^{-A\tau}Bu\,d\tau = \frac{V}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$

Growth of current in an inductive circuit

Time Constant (τ): Time taken by the current through the inductor to reach its final steady state value, had the initial rate of rise been maintained constant

$$\tau = \frac{L}{R}$$

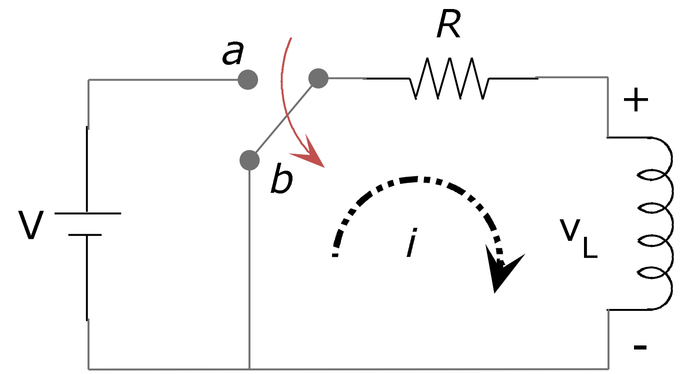


Decay of current in an Inductive Circuit

- Initial current is through inductor is

$$I_0 = V/R$$

- At $t = 0$, switch is moved from position ***a*** to ***b***



Applying KVL,

$$L\frac{di}{dt} + R\,i = 0$$

$$0 = iR + L\frac{di}{dt} \Leftarrow t > t_0$$

$$\frac{di}{dt} = \left(-\frac{R}{L}\right) i$$

Using initial conditions and then solving

$$i = I_0 e^{\left(\frac{-Rt}{L}\right)}$$

$$i = I(t_0)\, e^{-\frac{R}{L}(t-t_0)}$$

$$v_L = -V\, e^{-\left(\frac{Rt}{L}\right)}$$

Decay of current in an Inductive Circuit

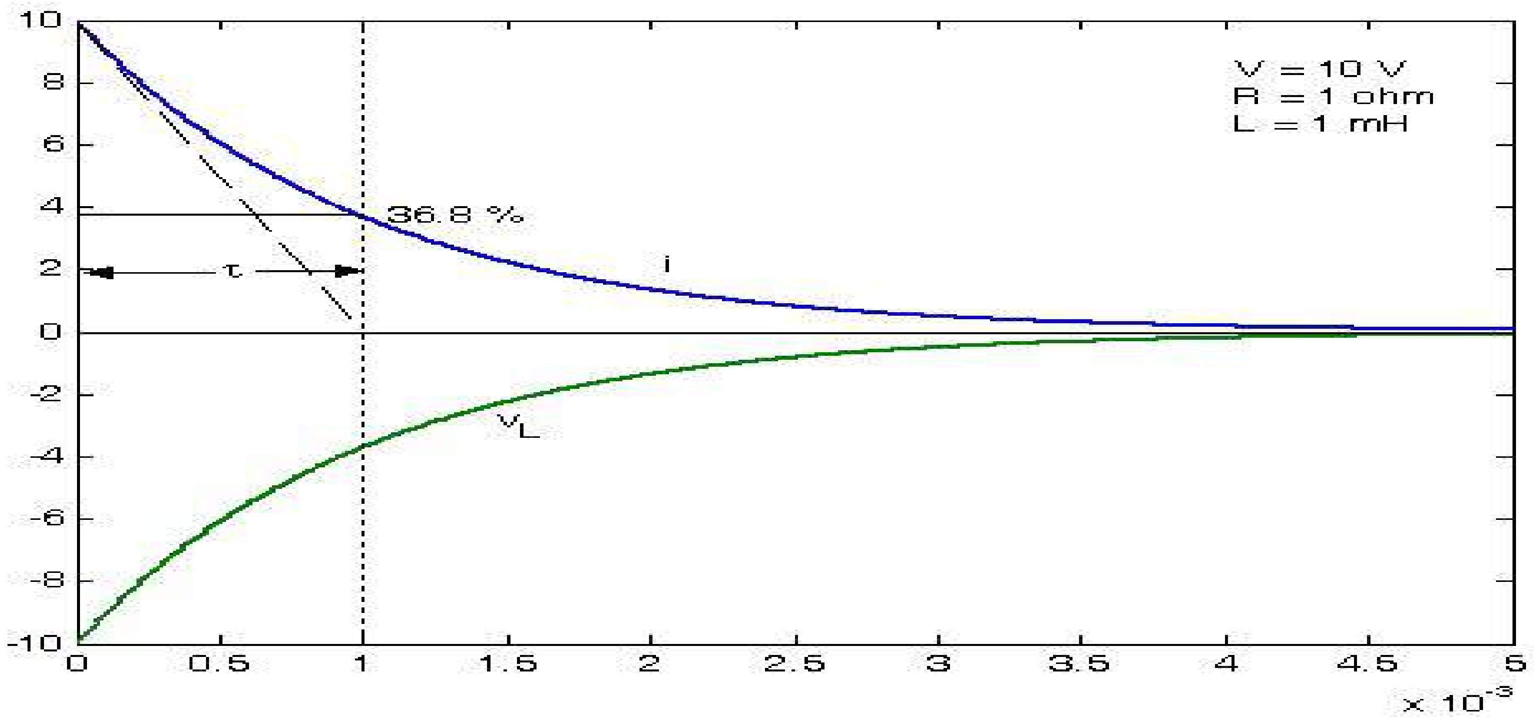


Illustration 1

A coil of inductance 0.04 H and resistance 10 is connected to a 120 V, d.c. supply. Determine

$V = 120\,V \quad R = 10\,\Omega \quad L = 40\,mH$

(a) the final value of current.

(b) the time constant of the circuit.

(c) the value of current after a time equal to the time constant from the instant the supply voltage is connected.

(d) the expected time for the current to rise to within 1% of its final value.

(a) $I = \frac{V}{R} = 12\,A$ (b) $\tau = \frac{L}{R} = \frac{40\,mH}{10} = 4\,ms$

(c) $i(\tau) = \frac{V}{R}\left(1 - e^{-1}\right) = 7.58\,A$

(d) $i(t) = (0.99) \times \frac{V}{R} = 11.88\,A \quad t = ?$

$11.88 = 12\left(1 - e^{-\frac{t}{4ms}}\right) \Rightarrow t = -4ms \times \ln\left(1 - \frac{11.88}{12}\right) = 18.421\,ms$

Ans: (a) 12A ; (b) 4ms (c) 7.58A (d) 18.421ms

Illustration 2

An R-L series circuit is designed for a steady current of 250mA. A current of 120 mA flows in the circuit at an instant 0.1 sec after connecting the supply voltage. Calculate i) time constant of the circuit ii) the time from closing the circuit at which the circuit current has reached 200 mA.

$$i(t) = 250\,\text{mA} \times \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$120\,\text{mA} = 250\,\text{mA} \times \left(1 - e^{-\frac{0.1}{\tau}}\right)$$

$$\Rightarrow \tau = \frac{-0.1}{\ln\left(1 - \frac{120\,\text{mA}}{250\,\text{mA}}\right)}$$

$$\tau = 0.1529\,\text{s}$$

$$200\,\text{mA} = 250\,\text{mA}\left(1 - e^{-\frac{t}{0.1529}}\right)$$

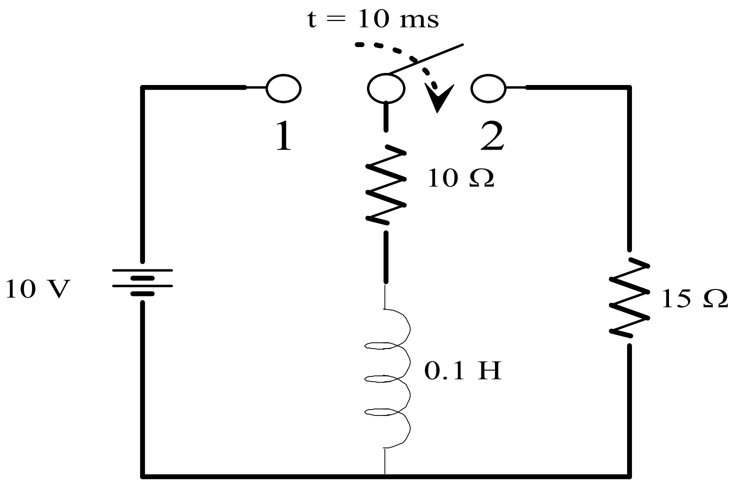
$$t = -0.1529 \ln\left(1 - \frac{200\,\text{mA}}{250\,\text{mA}}\right)$$

$$t = 0.2461\,\text{s}$$

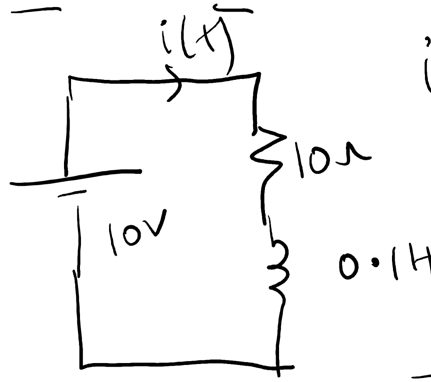
Ans: i)Time constant = 0.1529 s ii) t= 0.2461 s

Illustration 3

In the network shown in figure, the switch is closed to position 1 at t = 0 and is moved to position 2 at 10 ms. Determine iL(t) & sketch it.



$0 \leq t \leq 10\,ms$



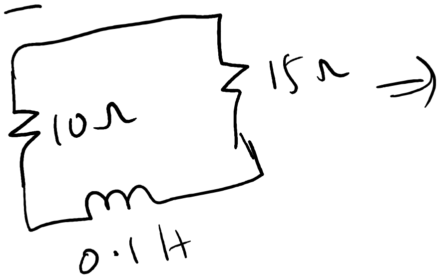
$$i(t) = 1 - e^{-\frac{t}{0.01}}$$

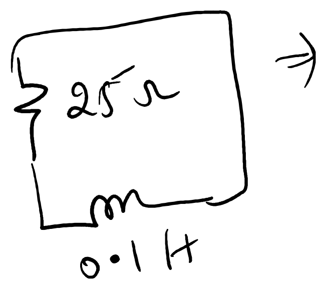
$$i(10ms) = 0.6321\ A$$

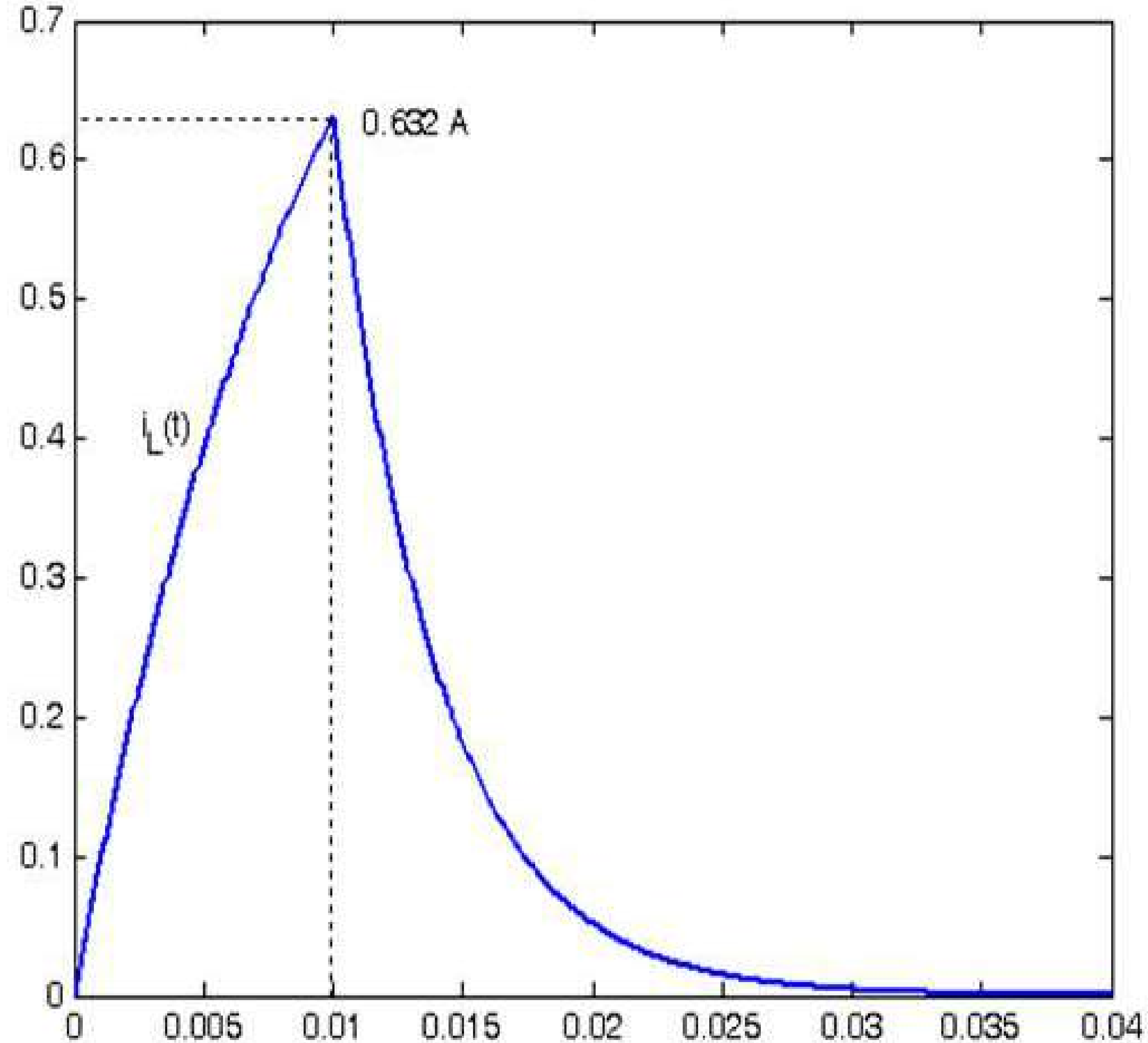
$$i(t - 10ms) = I(10ms)\, e^{-\frac{(t-10ms)}{4ms}}$$

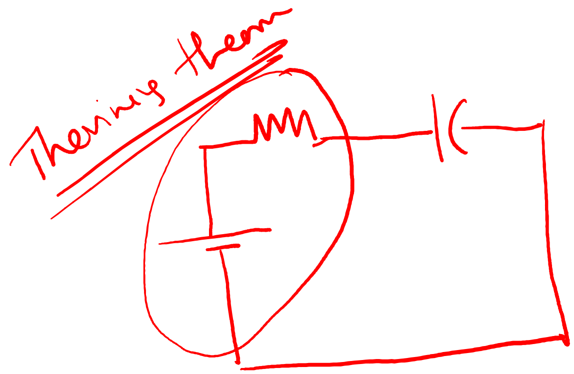
$$= 0.6321\, e^{-\frac{(t-10ms)}{4ms}}$$

$t \geq 10\,ms$






0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.632 A
$i_L(t)$
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04



Basic Electrical Technology

CLASS 11 – 23 NOVEMBER 2021

Charging of a Capacitor through a Resistor

$V_c(0) = 0$

Applying KVL,

$$V - Ri - v_c = 0$$

where, $i = C\dfrac{dv_c}{dt}$

Initial Conditions,

$$At\ t = 0\ sec, V_c = 0\ V$$

Final current & voltage equation,

$$v_c = V\left(1 - e^{-\frac{1}{RC}t}\right)$$

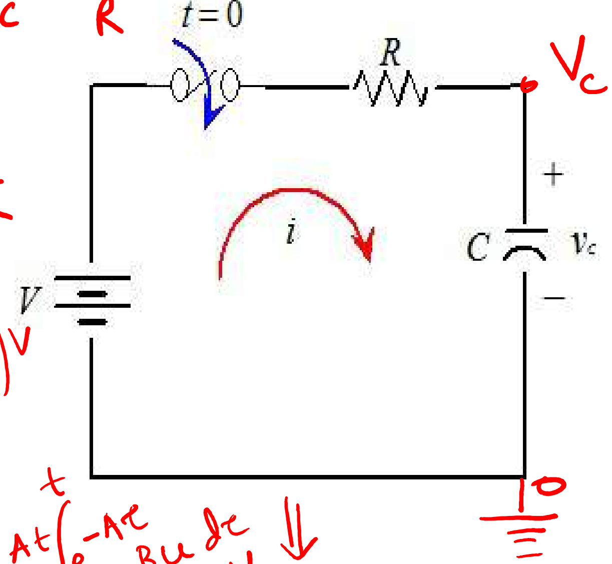
$i_c = C\dfrac{dv_c}{dt}$

$$i_c = \left(\frac{V}{R}\right) e^{-\left(\frac{1}{RC}\right)t}$$

$i_c(0) = \dfrac{V}{R}$

Applying KCL (Node analysis)

$\dfrac{V_c}{R} + i_c = \dfrac{V}{R}$ $\quad i_c = C\dfrac{dv_c}{dt}$



$C\dfrac{dv_c}{dt} + \dfrac{V_c}{R} = \dfrac{V}{R}$

$\dfrac{dv_c}{dt} = \left(-\dfrac{1}{RC}\right)v_c + \left(\dfrac{1}{RC}\right)V$

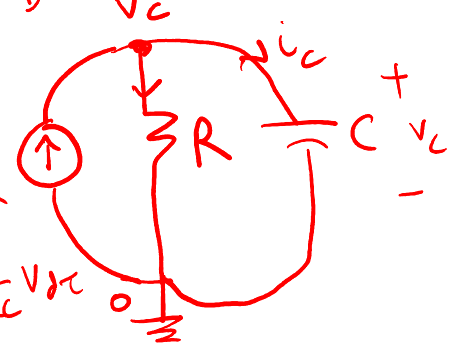
$\dfrac{dx}{dt} = Ax + Bu$

$x(t) = e^{At}x(0) + e^{At}\int_0^t e^{-A\tau} Bu\, d\tau$

$A = -\dfrac{1}{RC}$ $\quad B = \dfrac{1}{RC}$

$x = v_c$ $\quad u = V$

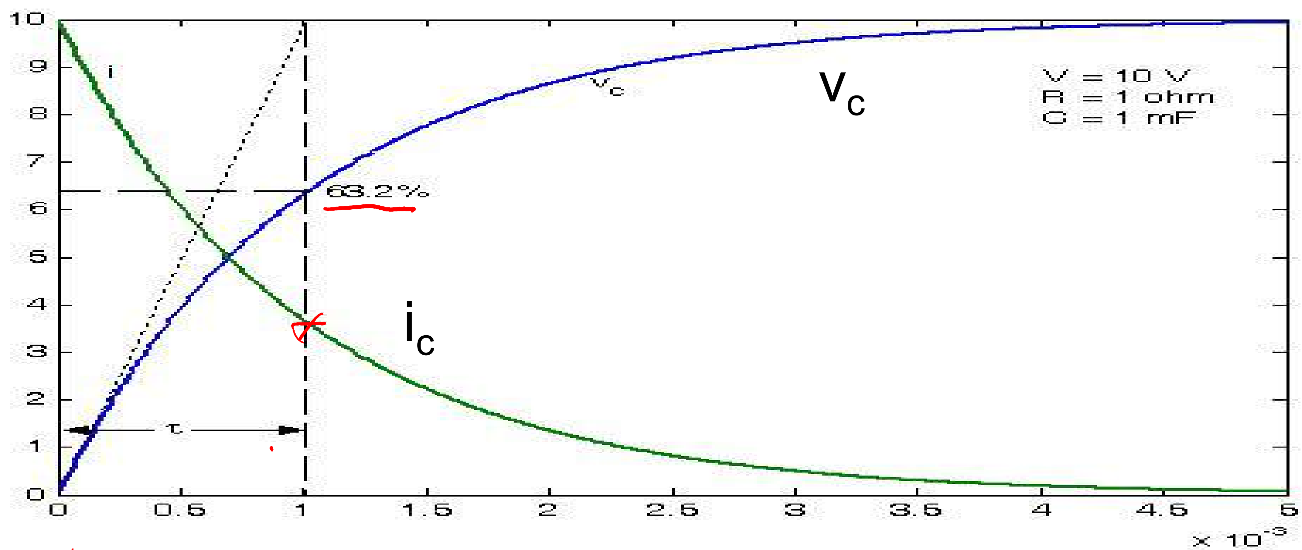
$v_c(t) = e^{-\frac{t}{RC}} v_c(0) + e^{-\frac{t}{RC}} \int_0^t e^{\frac{\tau}{RC}} \dfrac{1}{RC} V\, d\tau$



Time Constant (τ): Time taken by the voltage of the capacitor to reach its final steady state value, had the initial rate of rise been maintained constant

$V(1 - e^{-t/\tau})$

$$\boldsymbol{\tau = RC}$$

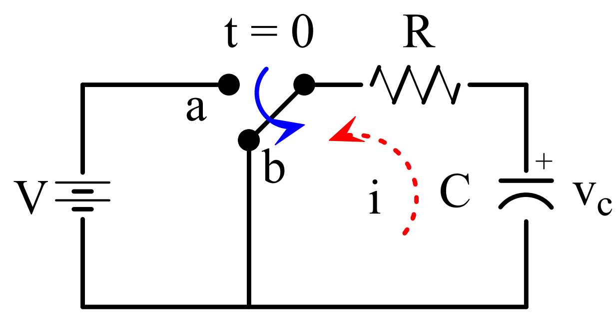


Discharging of a Capacitor through a Resistor

- Capacitor is initially charged to a voltage $\boldsymbol{V}$

$i_c(t) = C\frac{dv_c}{dt} = -\frac{V_c(0)}{R} e^{-\frac{t}{RC}}$

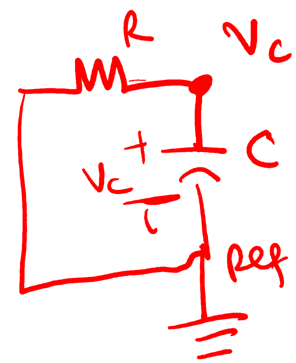
- At $\boldsymbol{t = 0}$, switch is moved from position ***a*** to ***b***



Applying KVL,

$$v_c - Ri = 0$$

Where, $\boldsymbol{i = -C\frac{dv_c}{dt}}$



Using initial conditions and then solving

$V_c(0) = V$

$I = \frac{V_c(0)}{R} = \frac{V}{R}$

$v_c(t) = V_c(0) e^{-\frac{t}{RC}}$

$$v_c = V\, e^{-\left(\frac{1}{RC}\right)t}$$

$$i_c = -I\, e^{-\left(\frac{1}{RC}\right)t}$$

$\frac{V_c}{R} + i_c = 0$

$\frac{dv_c}{dt} = -\frac{V_c}{RC}$

Discharging of a Capacitor through a Resistor

$\tau = RC$

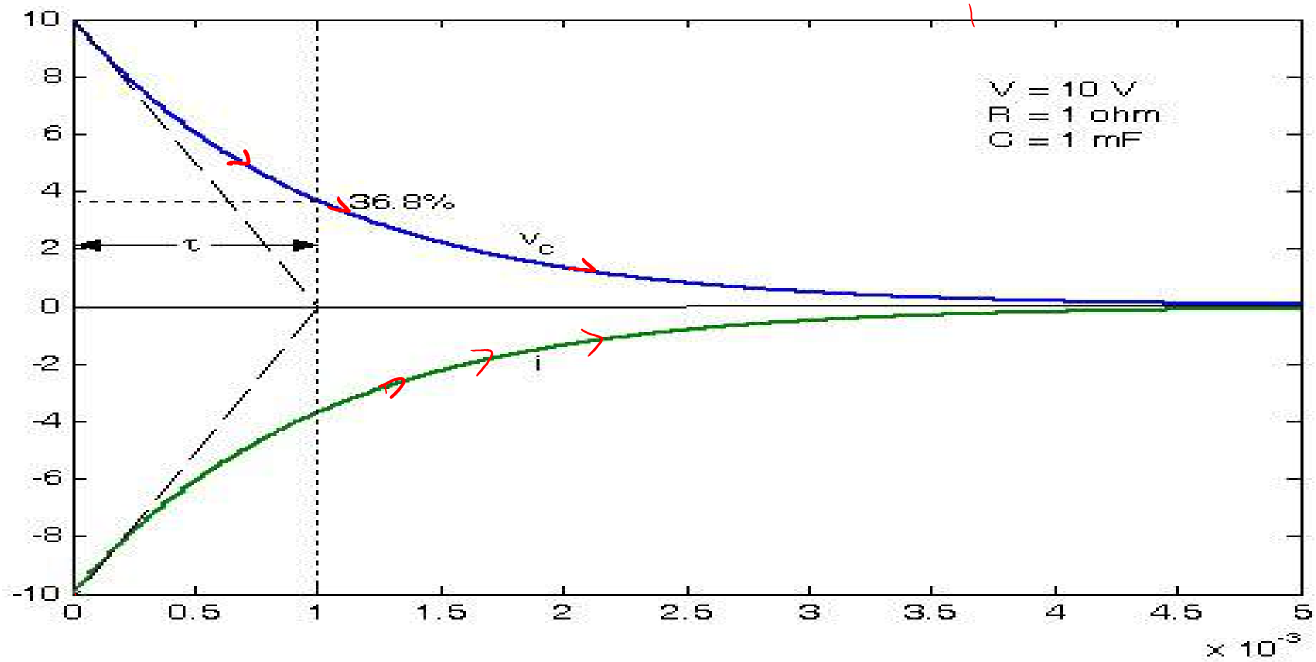


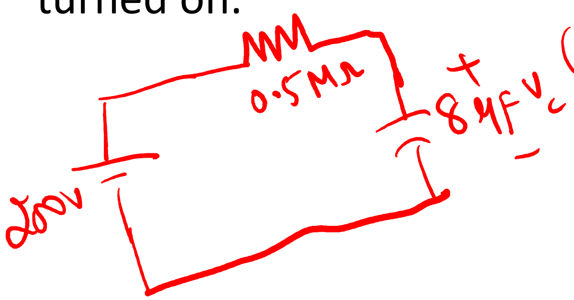
Illustration 1

An 8 μF capacitor is connected in series with a 0.5 MΩ resistor, across a 200 V dc supply through a switch. At t=0 sec, the switch is turned on. Calculate

i. Time constant of the circuit
ii. Initial charging current.
iii. Time taken for the potential difference across the capacitor to grow to 160 V.
iv. Current & potential difference across the capacitor 4.0 seconds after the switch is turned on.

(iv) $v_c(4s) = 200(1 - e^{-1}) = 126.424\ V$

$i_c(4s) = 400\mu A \times e^{-1} = 147.152\ \mu A$



(i) $\tau = RC = 0.5M\Omega \times 8\mu F = 4s$

(ii) $i_c(t) = \frac{V}{R} e^{-\frac{t}{RC}} = \frac{200}{0.5M\Omega} e^{-\frac{t}{4s}} \Big|_{t=0}$

$i_c(0) = \frac{200}{0.5M\Omega} = 400\mu A$

(iii) $v_c(t) = V\left(1 - e^{-\frac{t}{RC}}\right) = 200\left(1 - e^{-\frac{t}{4}}\right) \Rightarrow 160 = 200\left(1 - e^{-\frac{t}{4}}\right)$ $t = ?$

$t = -4\ln\left(1 - \frac{160}{200}\right) = 6.438s$

Ans: (i) 4 seconds, (ii) 400 μA , (iii) 6.44 seconds (iv) 126.42 V & 147.15 μA

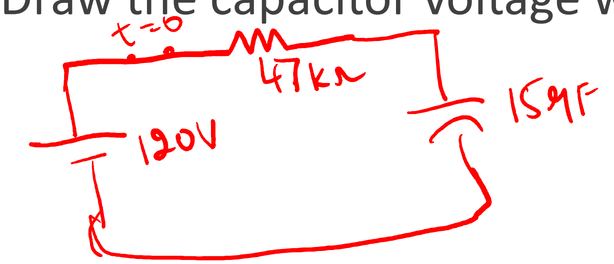
Illustration 2

$V_c(0) = 0$

A 15 µF uncharged capacitor is connected in series with a 47 kΩ resistor across a 120 V, d.c. supply.

a) Determine the capacitor voltage at a time equal to one time constant after being connected to the supply and also two seconds after being connected to the supply.

b)Find the time for the capacitor voltage to reach one half of its steady state value. Draw the capacitor voltage waveform.



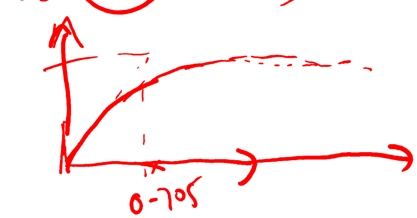
$V_c(t) = V\left(1 - e^{-\frac{t}{RC}}\right)$ RC = 0.705 s

(a) $V_c(\tau) = 120\left(1 - e^{-1}\right) = 75.854\ V$

$V_c(2s) = 120\left(1 - e^{-\frac{2}{0.705}}\right) = 112.967\ V$

(b) $60 = 120\left(1 - e^{-\frac{t}{0.705}}\right)$ $t = ?$

$t = -0.705 \ln\left(1 - \frac{60}{120}\right)$
$= 0.489\ s$

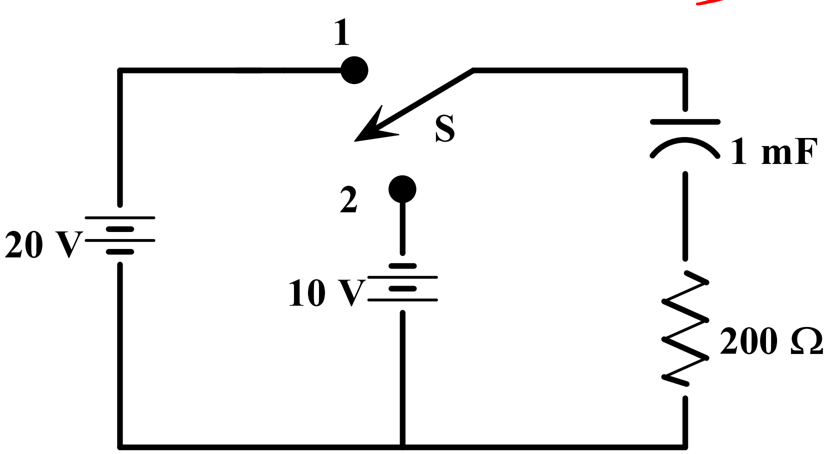


Ans: a) 75.84 V, 112.97 V b) 0.49 s.

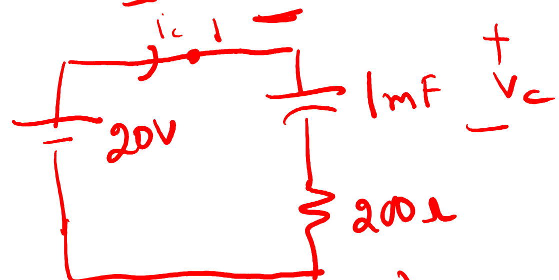
Illustration 3

In the network shown below, the switch is closed to position 1 at t = 0 & is moved to position 2 at t = 0.4 sec. Determine the voltage across the capacitor $v_c(t)$ & sketch it for 0 ≤ t ≤1 sec

Also find the value of ‘t’ for which $v_c(t) = 0$



$0 \le t(s) < 0.4$



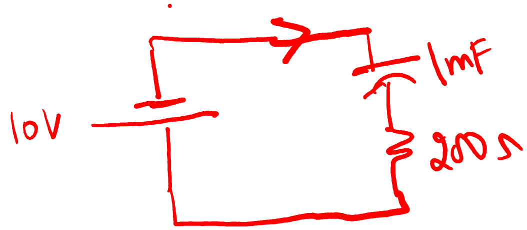
$V_c(t) = V\left(1 - e^{-\frac{t}{RC}}\right)$

$RC = 0.2$ $\quad V_c(t) = 20\left(1 - e^{-\frac{t}{0.2}}\right)$

$V_c(0.4s) = 20\left(1 - e^{-\frac{0.4}{0.2}}\right)$

$= 17.293V$

$0.4 \le t(s) \le 1$



$V_c(t) = e^{-\frac{t}{RC}} V_c(0) + V\left(1 - e^{-\frac{t}{RC}}\right)$

$V = -10V$

$V_c(0) = 17.293V$

$\boldsymbol{Ans: At\ t = 0.6\ sec, v_c = 0\ V}$

$$v_c(t) = 17.293\, e^{-\frac{(t-0.4)}{0.2}} - 10\left(1 - e^{-\frac{(t-0.4)}{2}}\right)$$

$$\Downarrow$$

$$v_c(t) = -10 + 27.293\, e^{-\left(\frac{t-0.4}{0.2}\right)}$$

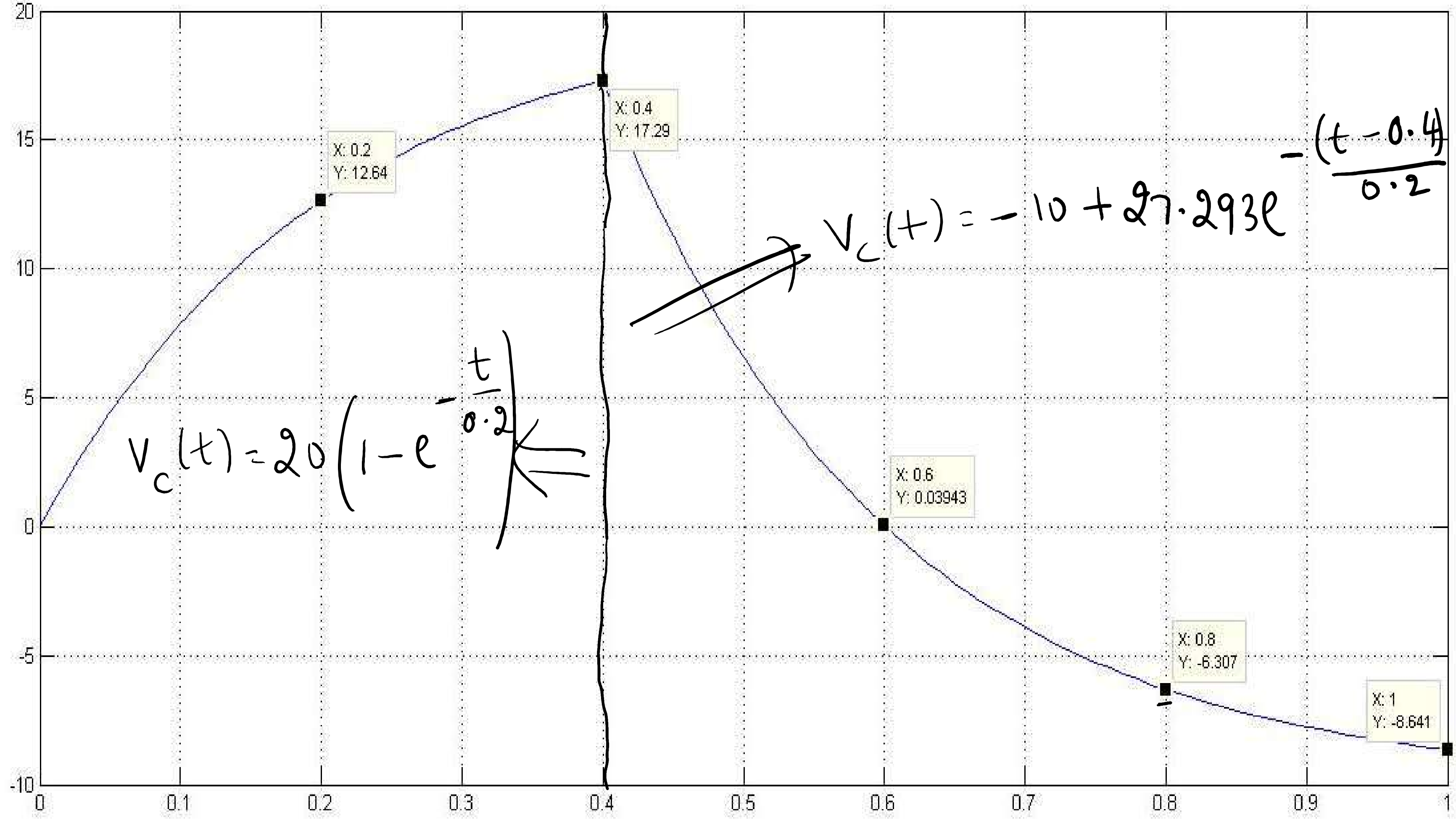
$$-10 + 27.293\, e^{-\left(\frac{t-0.4}{0.2}\right)} = 0$$

$$\frac{t-0.4}{0.2} = -\ln \frac{10}{27.293} \Rightarrow \underline{t = 0.6008\ s}$$

$v_c = 20(1 - e^{-t/0.2})$
$At\ t = 0.4\ sec, v_c = 17.29\ V$

$After\ 0.4\ second, the\ switch\ is\ in\ postion\ 2$
$v_c = -10 + 27.29e^{-(t-0.4)/0.2}$
$At\ t = 1\ sec, v_c = -8.64\ V$

Solution


X: 0.2
Y: 12.64
X: 0.4
Y: 17.29
X: 0.6
Y: 0.03943
X: 0.8
Y: -6.307
X: 1
Y: -8.641
$V_c(t) = 20\left(1 - e^{-\frac{t}{0.2}}\right)$
$V_c(t) = -10 + 27.293e^{-\frac{(t-0.4)}{0.2}}$

Basic Electrical Technology

Class 11 – 23 November 2021

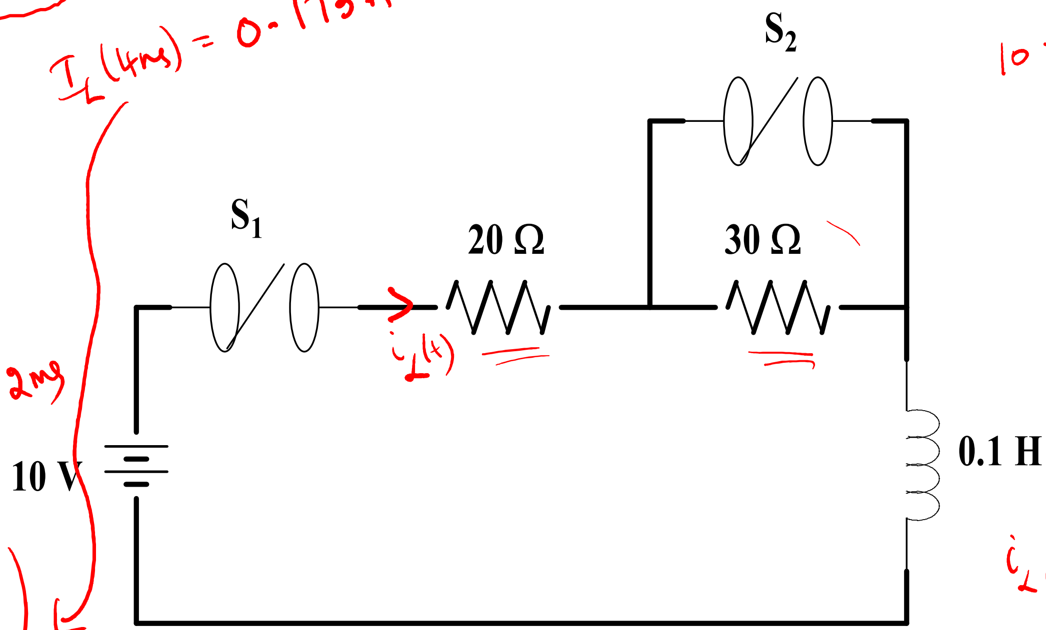
- Tutorial 3 : DC Transients

Question 01

$t - 4\text{ms}$ $\quad i_L(t) =$ $\quad t=0$

$x(t) = a^{t}$ $\quad = a^{(t - t_0)}$

In the circuit shown below, both the switches, S_1 & S_2 , are open initially. At t = 0 sec, S_1 is closed (& S_2 remains open). At t = 4 ms S_2 is closed. Sketch the inductor current $\boldsymbol{i(t)}$ for $0 \le t \le 25$ ms.



$0 \le t \le 4\text{ms}$

$i_L(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$

$\tau = \frac{0.1}{50} = 2\text{ms}$

$i_L(t) = 0.2\left(1 - e^{-\frac{t}{2\text{ms}}}\right)$

$I_L(4\text{ms}) = 0.173\text{ A}$

$4\text{ms} \le t \le 25\text{ms}$

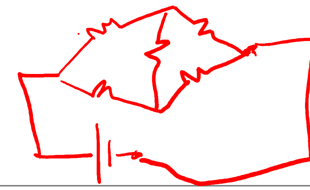
$\frac{V}{R} = \frac{10}{20} = 0.5$

$\tau = \frac{0.1}{20} = 5\text{ms}$

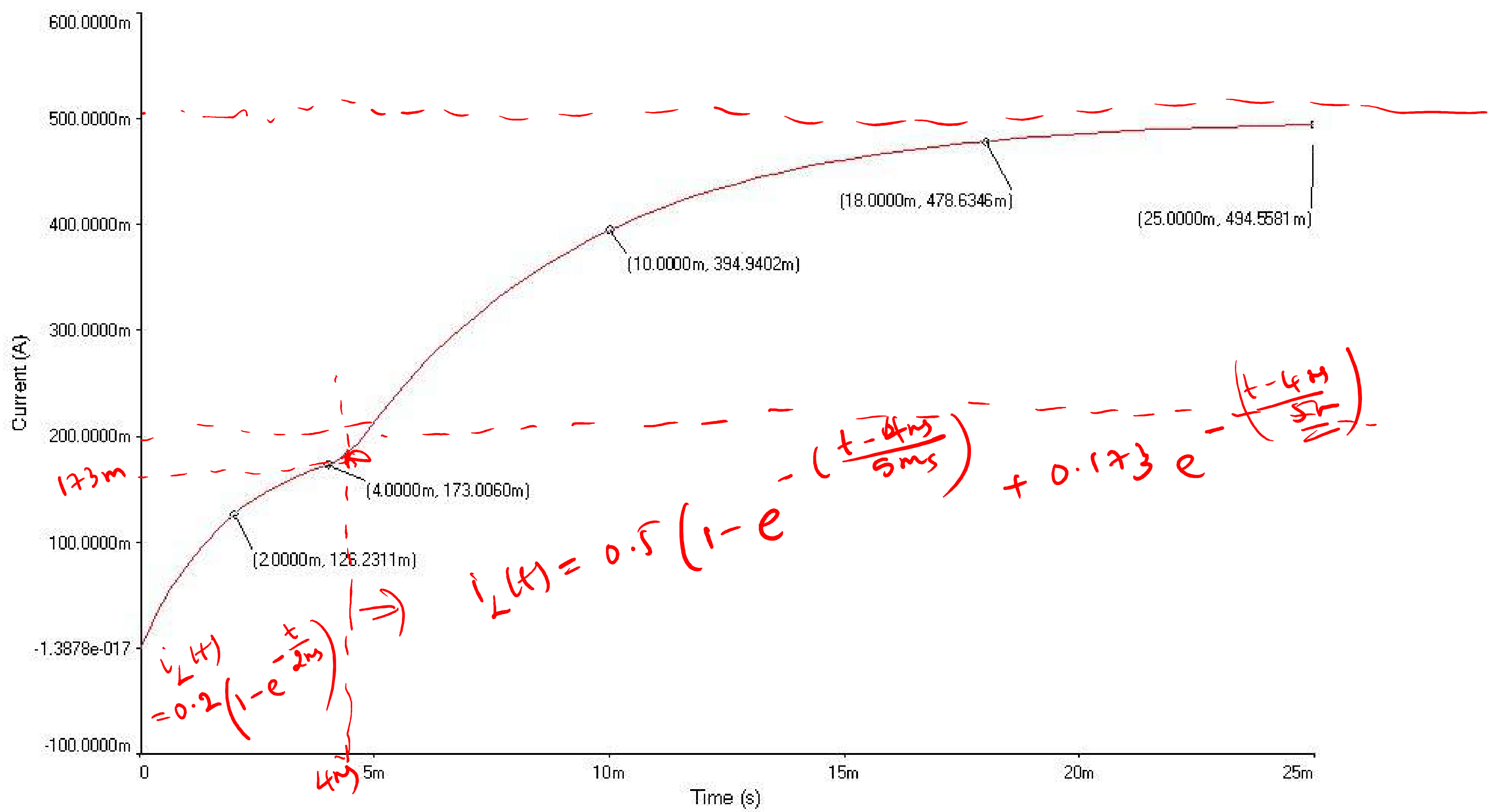
$i_L(t) = 0.5\left(1 - e^{-\frac{(t-4\text{ms})}{5\text{ms}}}\right) + I_L(4\text{ms})\, e^{-\frac{(t-4\text{ms})}{5\text{ms}}}$

$i_L(t) = 0.5\left(1 - e^{-\frac{(t-4\text{ms})}{5\text{ms}}}\right) + 0.173 e^{-\frac{(t-4\text{ms})}{5\text{ms}}}$

Solution



① ⇒ Thevn Th



$i_L(t) = 0.2\left(1-e^{-\frac{t}{2ms}}\right) \Rightarrow$

$i_L(t) = 0.5\left(1-e^{-\left(\frac{t-4ms}{5ms}\right)}\right) + 0.173\, e^{-\left(\frac{t-4m}{5m}\right)}$

Question 02

For the circuit shown in figure below, the switch **S** has been closed for a long time and then opens at $t = 0$.

Find,

i. $v_{ab}(0^-)$ just before the switch is opened

ii. $i_x(0^-)$

iii. $i_L(0^-)$

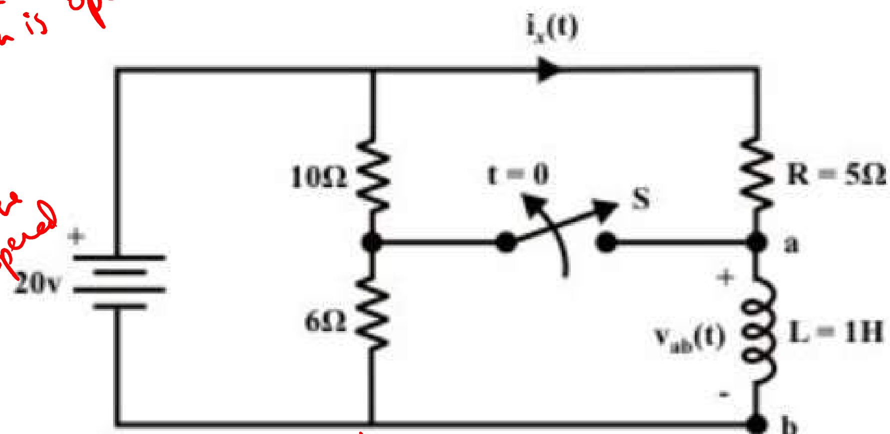
iv. $i_x(0^+)$ ⇒ just after the switch is opened

v. $v_{ab}(0^+)$

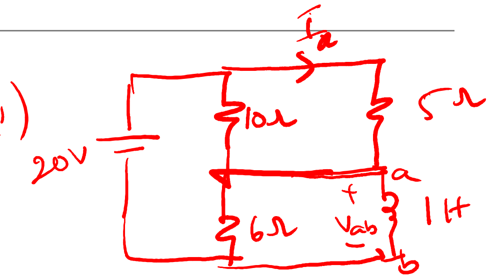
vi. $i_x(t = \infty)$

vii. $v_{ab}(t = \infty)$

viii. $i_x(t)$, for $t > 0$

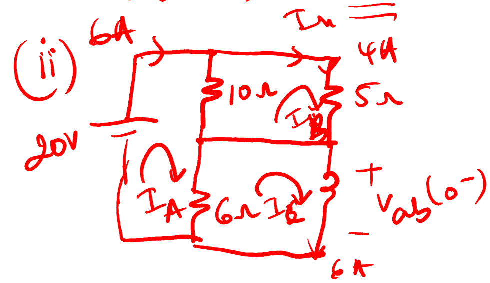


(i)



$V_{ab}(0^-) = 0V$

(ii)



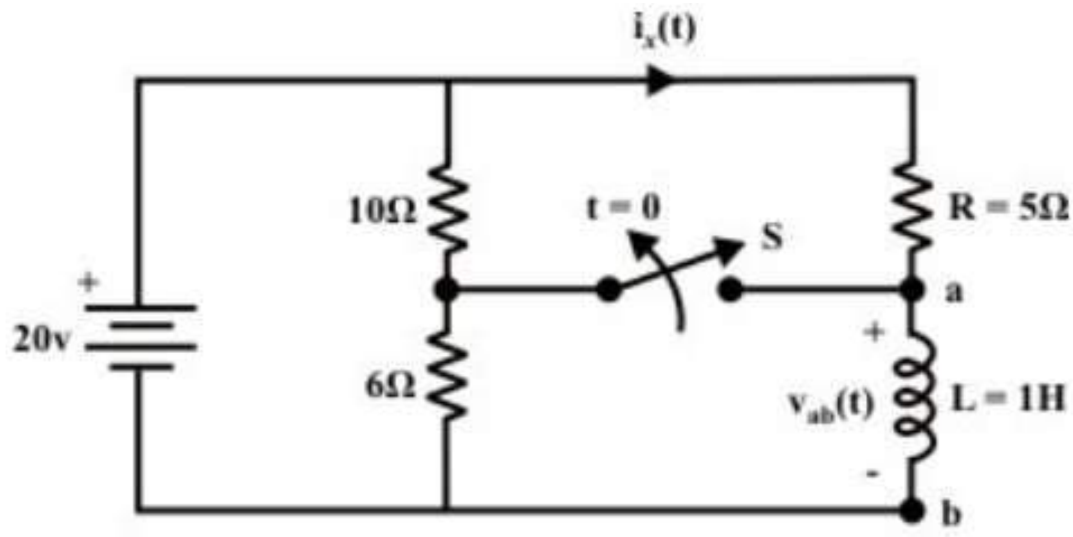
(ii) $I_x(0^-) = I_B = 4A$

(iii) $I_L(0^-) = I_C = 6A$

$$\begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 16 & -10 & -6 \\ -10 & 15 & 0 \\ -6 & 0 & 6 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

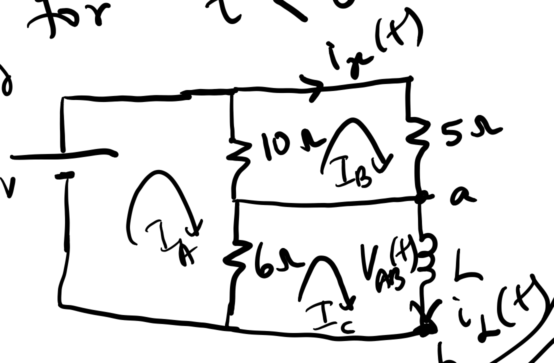
$I_A = 6A$ $I_B = 4A$ $I_C = 6A$

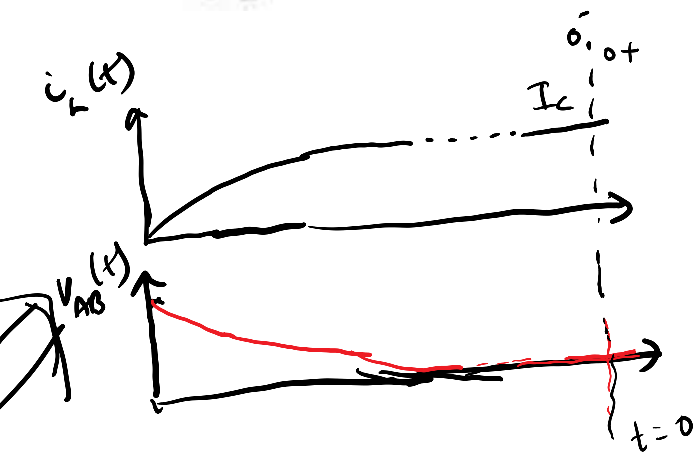
For the circuit shown in figure below, the switch S has been closed for a long time and then opens at $\mathbf{t = 0}$.



for $t < 0$

For a long time the circuit is in this state hence





$$\begin{bmatrix} 16 & -10 & -6 \\ -10 & 15 & 0 \\ -6 & 0 & 6 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

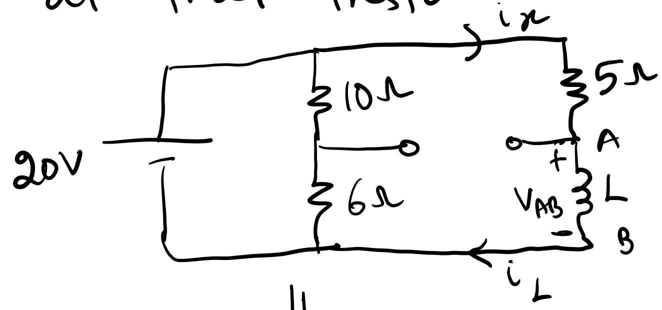
$I_A = 6A \quad I_B = 4A \quad I_C = 6A$

$\therefore V_{AB}(0^-) = 0\,V, \quad I_x(0^-) = I_B = 4A$

$I_L(0^-) = I_C = 6A$

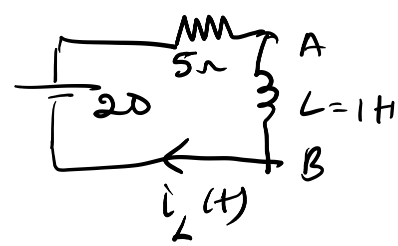
In this question the instant at which the switch is opened is $t=0^+$

at that instant —



$\Downarrow$

$V_{Th} = 20V$ $\quad$ $R_{Th} = 5Ω$



$\underline{I_L(0^-) = 6A}$

$$i_x(t) = i_L(t) = 4\left(1 - e^{-\frac{t}{0.2}}\right) + 6e^{-\frac{t}{0.2}} \quad A$$

$$V_{AB}(t) = 20e^{-\frac{t}{0.2}} - 30e^{-\frac{t}{0.2}} \quad V$$

$$\because V_{AB}(t) = L\frac{d\,i_L(t)}{dt}$$

$$i_L(0+) = 6A \qquad V_{AB}(0+) = -10V$$

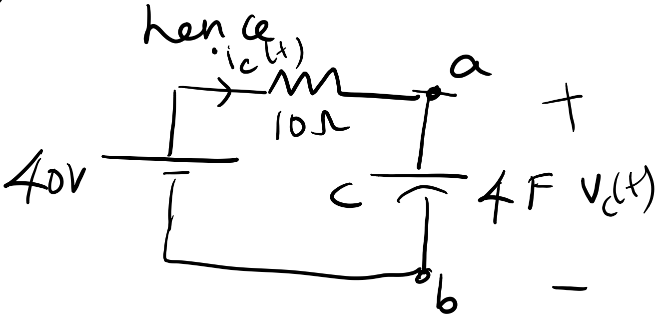
$$i_x(0+) = i_L(0^+) = 6A$$

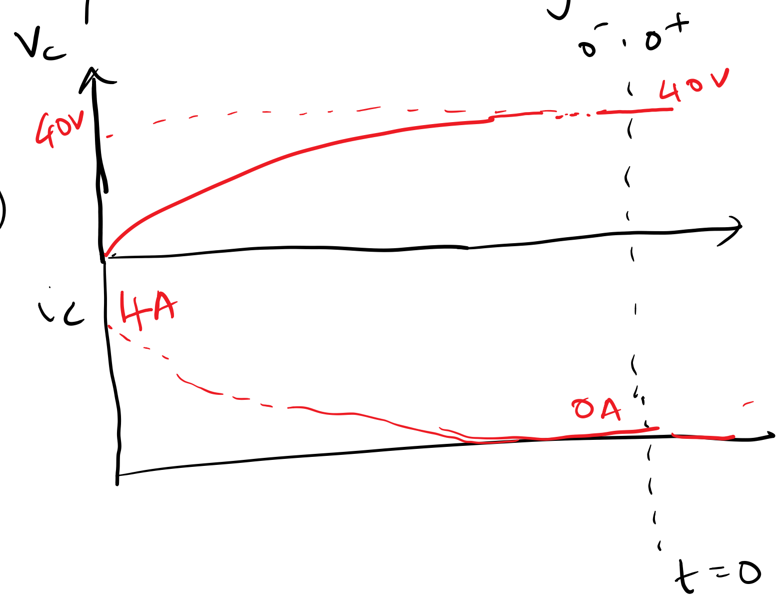
$$i_x(t \Rightarrow \infty) = 4A = i_L(t \Rightarrow \infty)$$

$$V_{AB}(t \Rightarrow \infty) = 0V$$

$$i_x(t) = \left\{4 + 2e^{-\frac{t}{0.2}}\right\} A$$

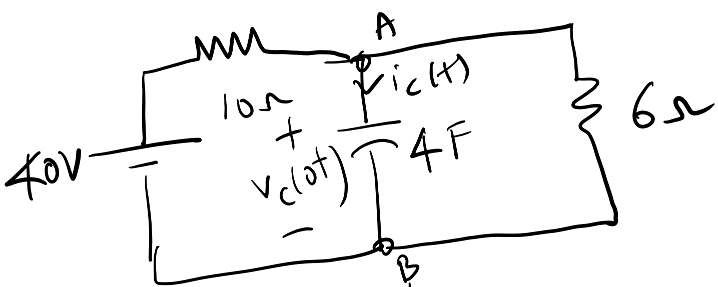
3. The switch is open for a long time hence



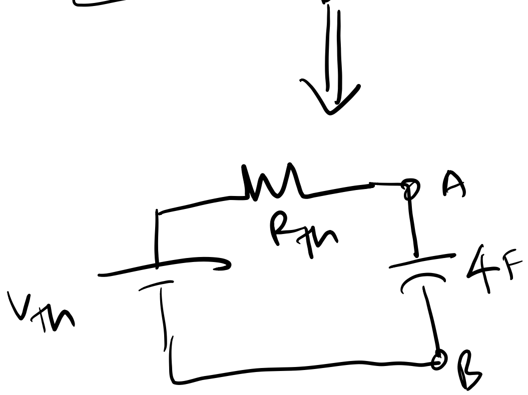


$$V_c(0^-) = 40V$$
$$i_c(0^-) = 0A$$

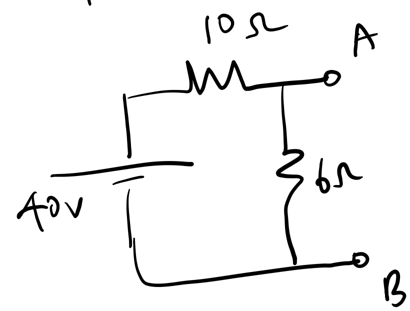
At t=0 the switch is closed hence at the instant $t=0^+$



$$V_c(0^+) = V_c(0^-) = 40V$$

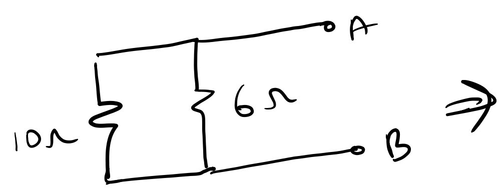


Find V_{th}



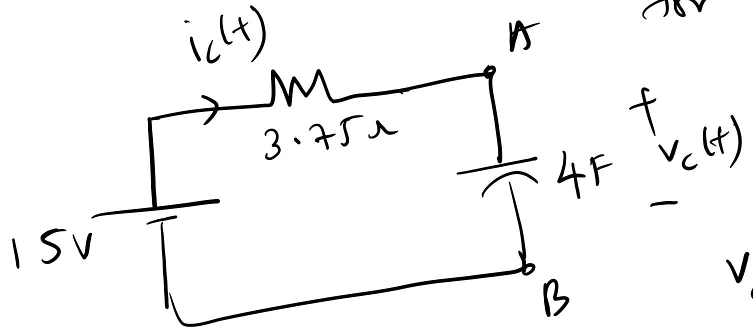
$$V_{Th} = V_{AB} = 40 \times \frac{6}{10+6} = 15\,V$$

Find R_{Th}



$\Rightarrow$ $R_{Th} = R_{AB} = 3.75\Omega$

for $t \geq 0^+$



$$V_c(0^+) = V_c(0^-) = 40V$$

$$V_c(t) = 15\left(1 - e^{-\frac{t}{15}}\right) + 40e^{-\frac{t}{15}}$$

$$\Rightarrow V_c(t) = \left\{15 + 25e^{-\frac{t}{15}}\right\} \text{ V}$$

$$i_c(t) = C\frac{dv_c(t)}{dt} = -4 \times \frac{25}{15} e^{-\frac{t}{15}} = -6.667e^{-\frac{t}{15}} \text{ A}$$

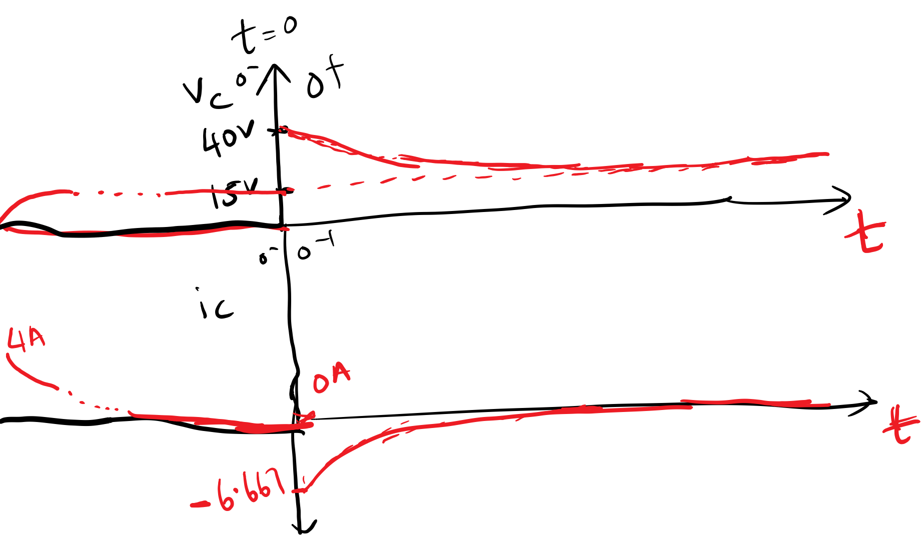
$$\boxed{i_c(0^+) = -6.667 \text{ A}}$$

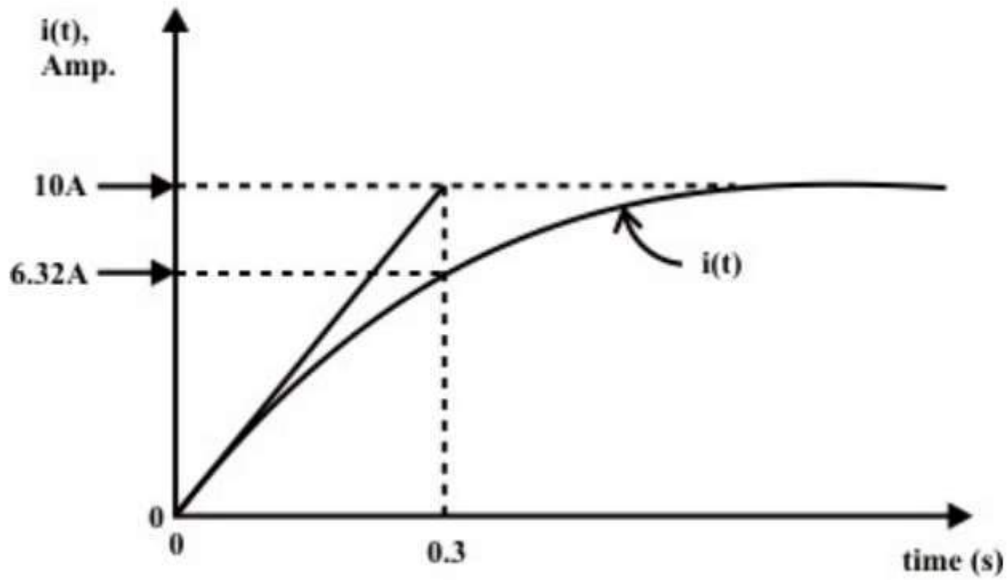
$$\frac{dv_c}{dt} = -1.667e^{-\frac{t}{15}}$$

$$\therefore \left.\frac{dv_c}{dt}\right|_{t=0^+} = -1.667 \text{ V/s}$$

$$V_c(t \Rightarrow \infty) = 15 \text{ V}$$

$$\tau = R_{Th}C = 15 \text{ s}$$





Find L and R

Figure above shows the plot of current $i(t)$ through a series R-L circuit when a constant voltage of magnitude 50 V is applied to it. Calculate the values of resistance R and inductance L .

$$i_L(t) = 10\left(1 - e^{-\frac{t}{\tau}}\right) \quad \tau = 0.3$$

$$10 = \frac{V}{R} = \frac{50}{R} \Rightarrow \underline{\underline{R = 5\,\Omega}}$$

$$\tau = \frac{L}{R} = 0.3 \Rightarrow \underline{\underline{L = 1.5\,H}}$$

Basic Electrical Technology

2. Magnetic Circuits & Electromagnetism

Introduction to Magnetism
Series Magnetic Circuits

Magnetism

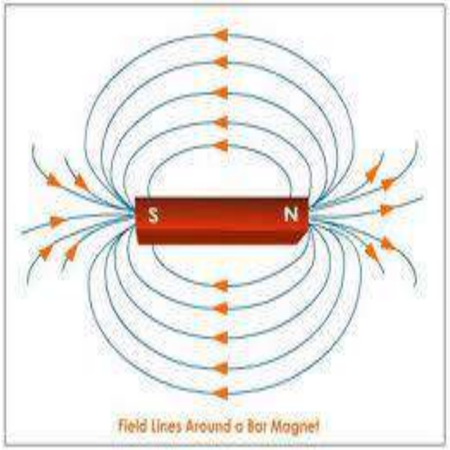
- A physical phenomena by which materials exert attractive or repulsive force on other materials

- **Magnetic Materials**
  - Properties:
    - Points in the direction of magnetic north and south pole when suspended freely and attracts iron fillings
  - Classification:
    - Natural Magnets: Lodestone
    - Temporary magnets (exhibits these properties when subjected to external force)

Definitions

- **Magnetic Line of Force**
  - Closed path radiating from north pole, passes through the surrounding, terminates at south pole and is from south to north pole within the body of the magnet

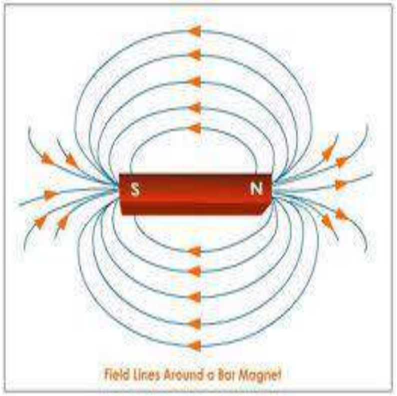
- **Magnetic Field**
  - The space around which magnetic lines of force act
  - Strong near the magnet and weakens at points away from the magnet



- **Magnetic Flux (ϕ)**
  - Analogous to Electric Current
  - Number of magnetic lines of force created in a magnetic circuit.
  - Unit : Weber (Wb)

Definitions

- **Magnetic Flux Density (B)**
  - Analogous to Current Density
  - No. of magnetic lines of force created in a magnetic circuit per unit area normal to the direction of flux lines
  - $\boldsymbol{B} = \boldsymbol{\Phi}/\boldsymbol{A}$
  - Unit : Wb/m^2 (Tesla)



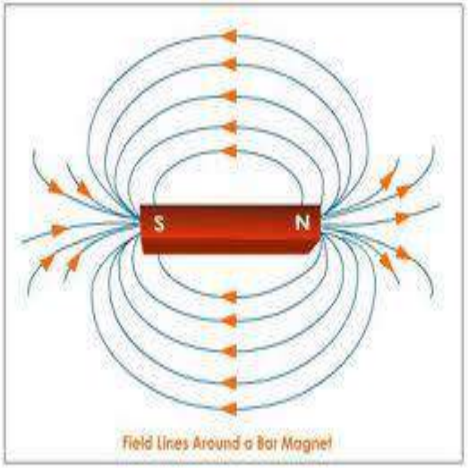
- **Magneto Motive Force (F)**
  - Analogous to EMF
  - Force which drives the magnetic lines of force through a magnetic circuit
  - $\boldsymbol{F} = \boldsymbol{\Phi} \times \boldsymbol{S} = \boldsymbol{N} \times \boldsymbol{I}$

    Where, $\boldsymbol{\Phi}$ = Magnetic flux, $\boldsymbol{S}$ = Reluctance of the magnetic path

    $\boldsymbol{N}$ = No. of turns of the coil, $\boldsymbol{I}$ = Current flowing through the coil
  - Unit: AT (Ampere-Turns)

Definitions

- **Magnetic Field Strength (H)**
  - Analogous to Electric Field Strength
  - The magneto motive force per meter length of the magnetic circuit
  - $\boldsymbol{H = (N \times I)/l}$
  - Unit: AT/m

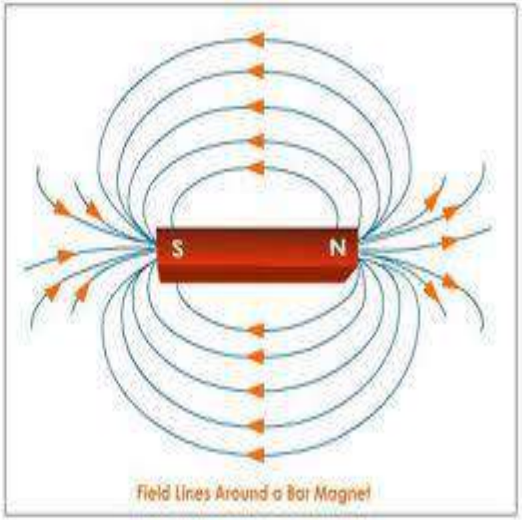


- **Permeability (μ)**
  - Analogous to Conductivity
  - A property of a magnetic material which indicates the ability of magnetic circuit to carry magnetic flux.
  - $\boldsymbol{\mu = B \;/\; H}$
  - $\mu_0 = 4\pi \times 10^{-7} \Longrightarrow$ Permeability of free space or air or non magnetic material
  - Unit: H/m

- **Relative Permeability (μ_r)**
  - Permeability of the material with reference to air / vacuum
  - $\boldsymbol{\mu_r = \mu/\mu_0}$

Definitions

- **Reluctance (S)**
  - Analogous to Resistance
  - Opposition of a magnetic circuit to the setting up of magnetic flux in it
  - Unit: AT/Wb



- **Derivation of an expression for reluctance**

$$H = (N \times I)/l$$

$$\mu = B\ /\ H$$

$$B\ = \Phi /\ A$$

$$F\ = N\ \times\ I = H\ \times l = (B/\mu) \times l = ((\Phi /\ A\ )/\mu) \times l = \frac{\Phi}{\mu A} \times l$$

$$F = \frac{\Phi}{\mu_0 \mu_r A} \times l = \Phi \times S$$

$$S = \frac{l}{\mu_0 \mu_r A}$$

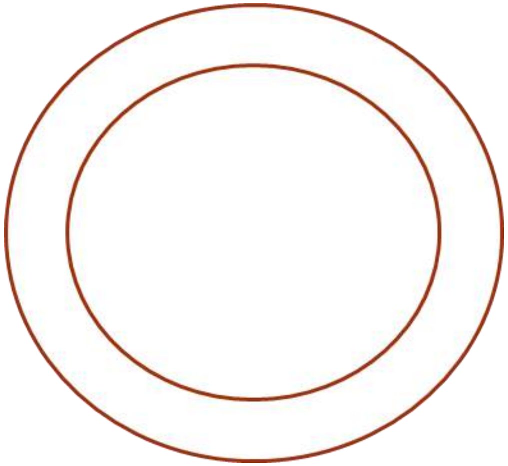
Illustration 01

A ring made of ferromagnetic material has 500 mm^2 as cross-sectional area and 400 mm as mean circumference. A coil of 600 turns is wound uniformly around it. Calculate:

a) The reluctance of the ring

b) The current required to produce a flux density of 1.6 T in the ring

Take μ_r of the ferromagnetic material as 800 for flux density of 1.6 T



Ans:

a) 795774.72 AT/Wb

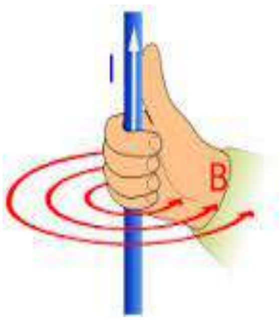
b) 1.06 A

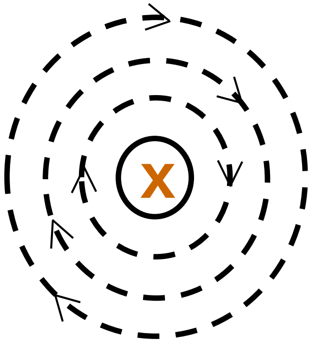
Magnetic Field (in a Current-Carrying Conductor)

- An electric current flowing in a conductor creates a magnetic field around it

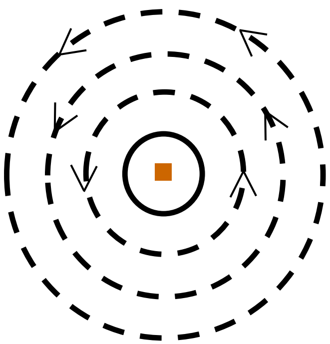
- **Direction of magnetic field**
  - **By Maxwell's Right Hand Grip Rule:**

    Assume that the current carrying conductor is held in right hand so that the fingers wrap around the conductor and the thumb is stretched along the direction of current. Wrapped fingers will show the direction of circular magnetic field lines





Current inwards

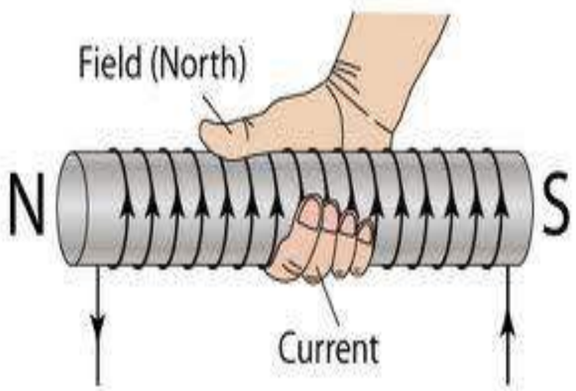


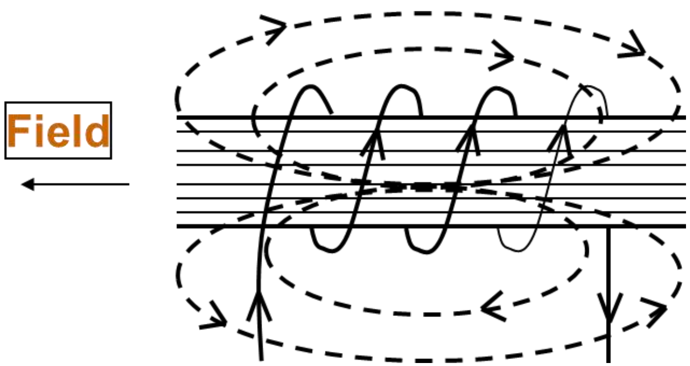
Current outwards

Magnetic Field (in a Solenoid)

- **Direction of magnetic field**
  - **By Right Hand Grip Rule:**

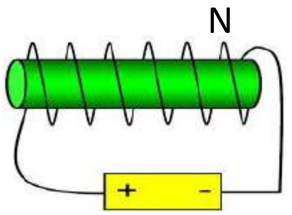
    If the coil is gripped with the right hand, with the fingers pointing in the direction of the current, then the thumb, outstretched parallel to the axis of the solenoid, points in the direction of the magnetic field inside the solenoid

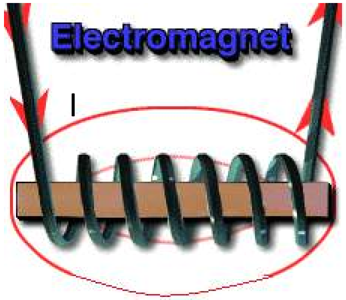




Electromagnets

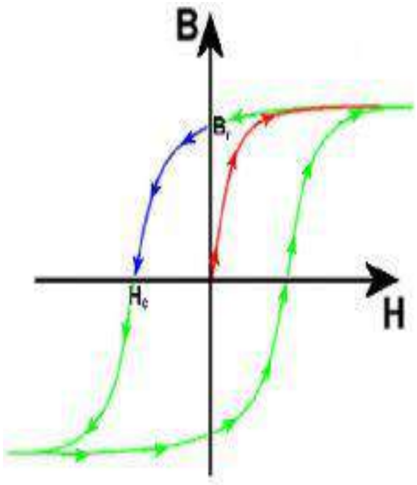
- **Principle:** An electric current flowing in a conductor creates a magnetic field around it
- Strength of the field is proportional to the amount of current in the coil
- The field disappears when the current is turned off
- A simple electromagnet consists of a coil of insulated wire wrapped around an iron core
- Widely used as components of motors, generators, relays etc.





Losses in Magnetic Circuit

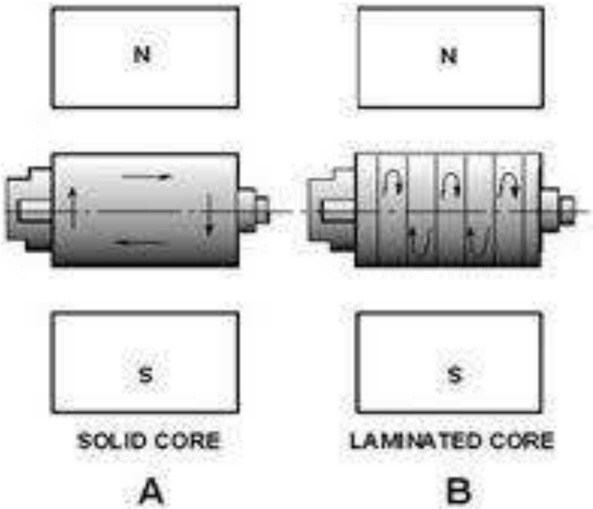
- **Hysteresis Loss**
  - Lagging of magnetization or flux density behind the magnetizing force is called **Magnetic Hysteresis**
  - The energy dissipated as heat in the process of magnetization and demagnetization which is proportional to the area of hysteresis loop is the **Hysteresis Loss**

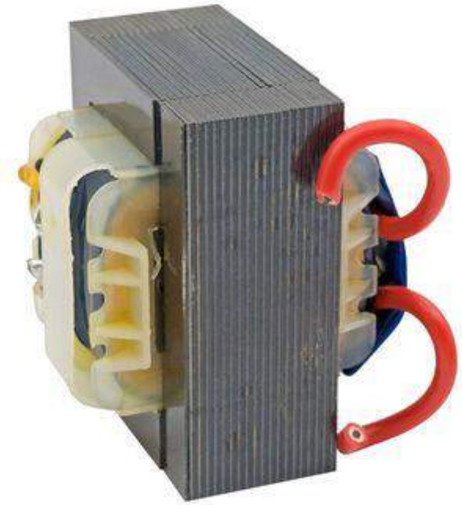


Hysteresis loop

Losses in Magnetic Circuit

- **Eddy Current Loss**
  - The varying flux in the magnetic core induces emf and hence eddy current within the material
  - Flow in closed loops in planes perpendicular to the magnetic field
  - Results in loss of power and heating of the material
  - Cores of electric machines are laminated to reduce eddy current loss





Comparison of Electric and Magnetic Circuits

Analogy:

Electric Circuits	Magnetic Circuits
Current	Flux
Current Density	Flux Density
EMF	MMF
Conductivity	Permeability
Resistance	Reluctance

Differences:

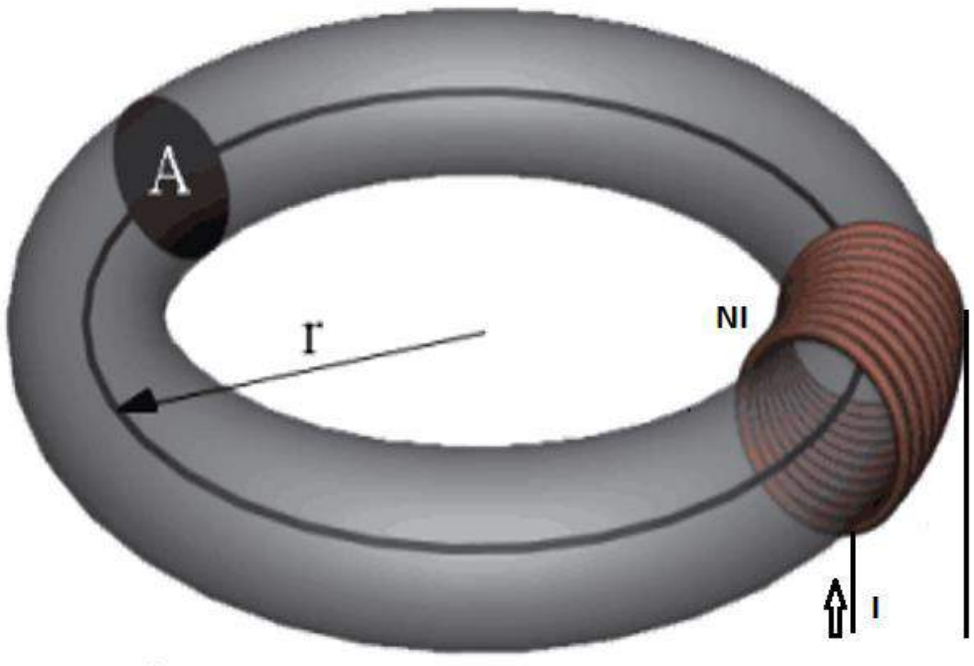
Electric Circuits	Magnetic Circuits
Current actually flows	Flux does not flow
Current can not flow in air / vacuum	Flux can be created in air / vacuum

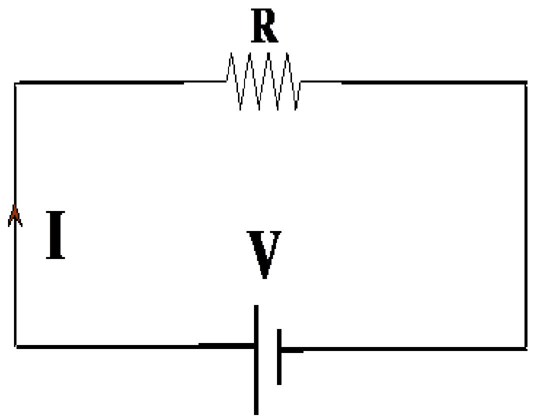
Basic Electrical Technology

Series Magnetic Circuits

Magnetic Circuits

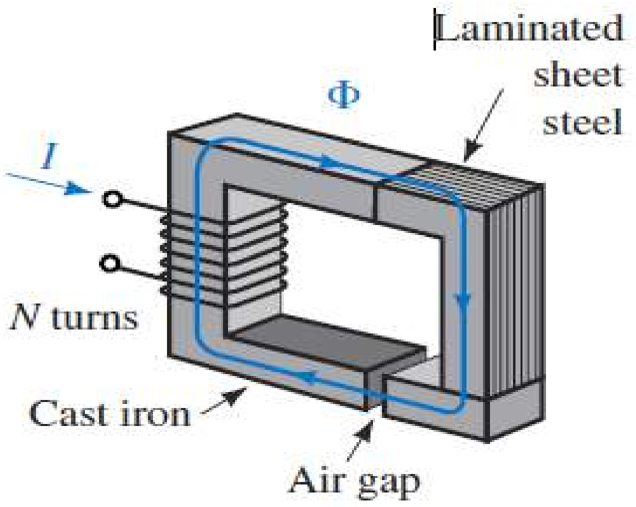
The complete closed path followed by any group of magnetic lines of flux





Series Magnetic Circuit

- Flux ϕ is the same in all sections if leakage flux is neglected.
- Flux density and reluctance in each section may vary, depending on its effective cross-sectional area and material.
- Equivalent reluctance is the sum of reluctance of different parts/elements.
- The resultant MMF is the sum of MMFs in each individual parts/elements



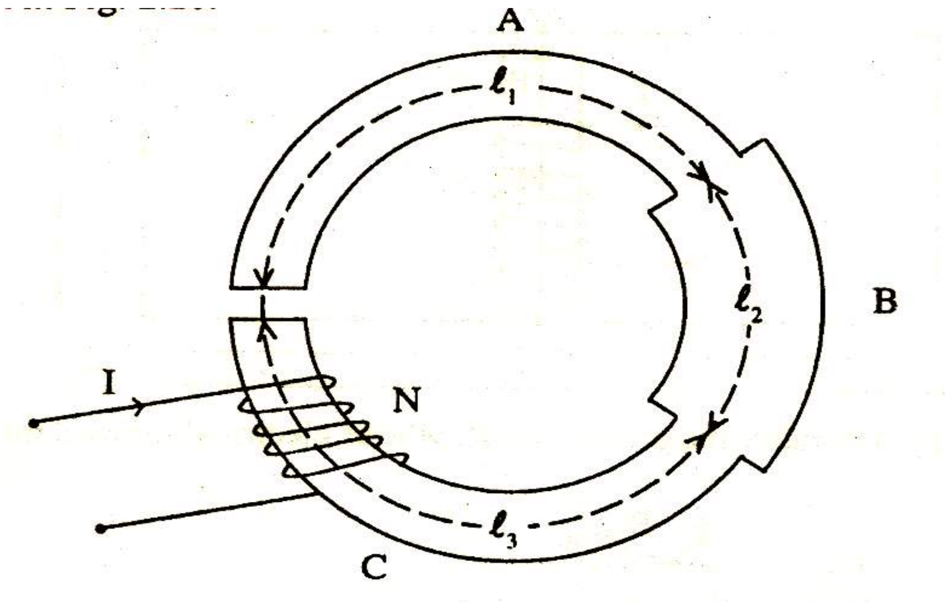
Rectangular shaped series magnetic circuit with air gap.

Series Magnetic Circuit

$$S_1 = \frac{l_1}{\mu_0\,\mu_{r1}\,A_1},\ S_2 = \frac{l_2}{\mu_0\,\mu_{r2}\,A_2}$$

$$S_3 = \frac{l_3}{\mu_0\,\mu_{r3}\,A_1}, S_g = \frac{l_g}{\mu_0\,A_1}$$

$$S_T = S_1 + S_2 + S_3 + S_g$$



$$Total\ mmf\ = \emptyset(S_1 + S_2 + S_3 + S_g)$$

$$= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

$$= \left(\frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_3 l_3}{\mu_0 \mu_{r3}} + \frac{B_g l_g}{\mu_0}\right)$$

Useful & Leakage Flux

- **Magnetic leakage:**
  - The passage of magnetic flux outside the path along which it can do useful work.

- Total flux of coil = Useful flux + Leakage flux

- **Leakage Coefficient**:

$$\lambda = \frac{\text{Total Flux of the Coil}}{\text{Useful Flux}}$$

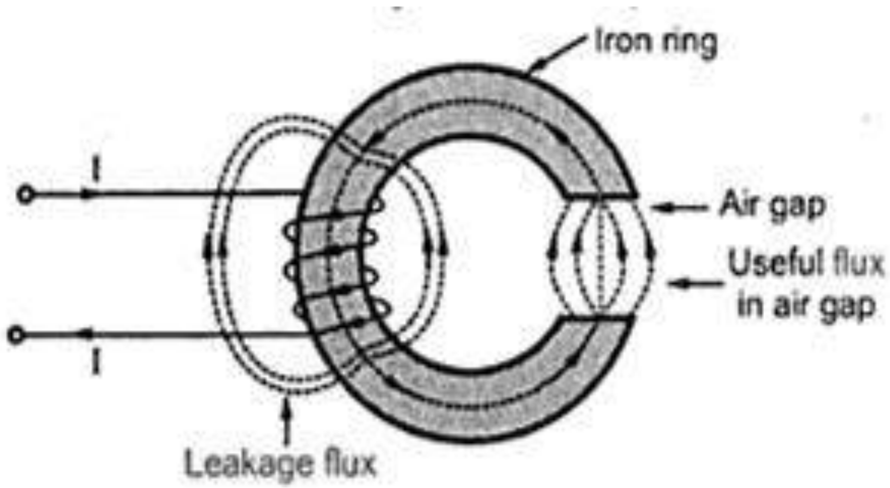


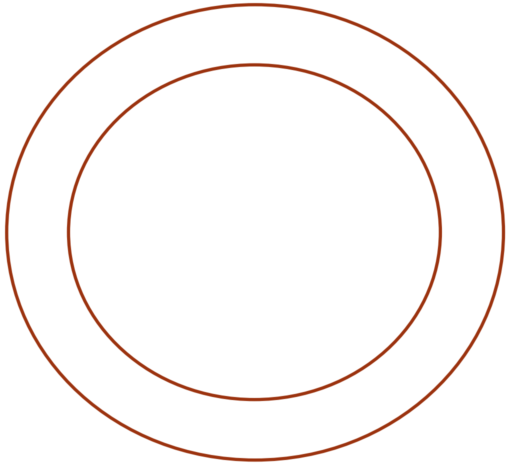
Illustration 1

An iron ring has a circular cross- sectional area of 5 cm^2 and a mean circumference of 100 cm. The ring is uniformly wound with a coil of 1000 turns. Relative permeability of iron is 800.

a) Find the current required to produce a flux of 1 mWb in the ring.

b) If a saw cut of 2 mm wide is made in the ring, find the flux produced, if the current is same as that found in **part a**.

c) Find the current required to produce the same flux as in **part a** for the cut made in the ring in **part b**.



Ans:

a) 1.99 A

b) 0.385 mWb

c) 5.17 A

Illustration 2

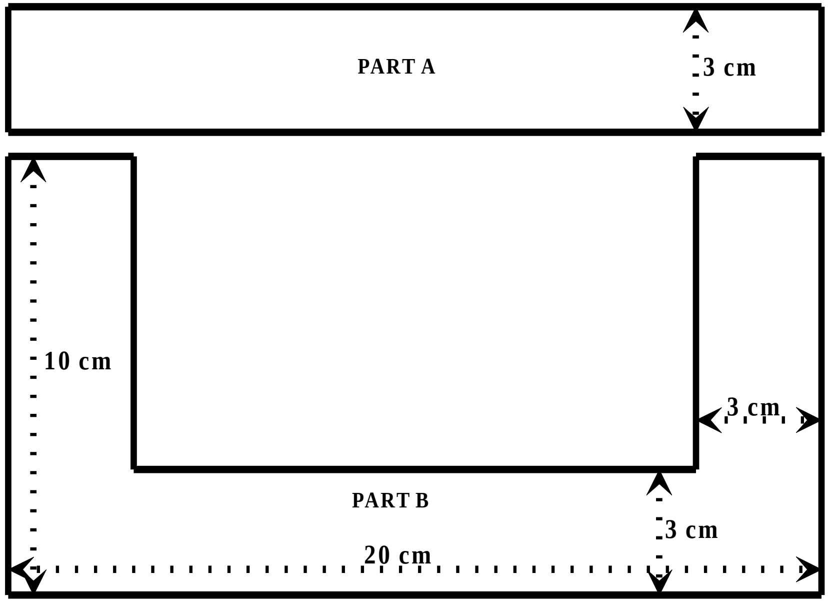
The magnetic circuit shown in the figure is made of iron having a square cross-section of 3 cm side. It has two parts A and B, with relative permeabilities of 1000 and 1200 respectively, separated by two air gaps, each 2 mm wide. The part B is wound with a total of 1000 turns of wire on the two side limbs carrying a current of 2.5 A. Calculate

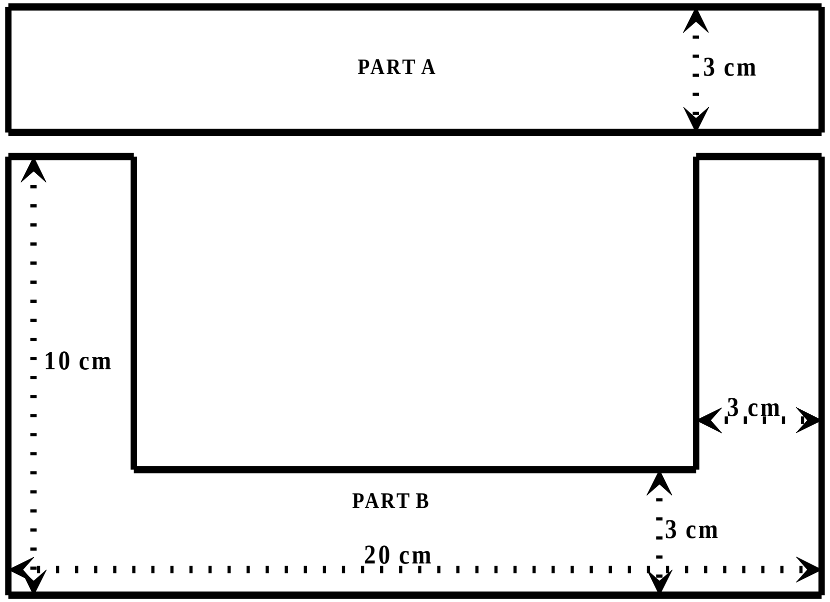
a) The reluctances of Part-A, Part-B & air gaps,
b) the total reluctance
c) the mmf
d) the flux and the flux density.

Hint:

Length of Part A = 20cm

Length of Part B = (10-1.5)+(20-1.5-1.5)+(10-1.5) = 34 cm




PART A
3 cm
10 cm
3 cm
PART B
3 cm
20 cm

Ans:

S_A = 176838.83AT/Wb,

S_B = 250521.67AT/Wb

S_g = 3536776.51AT/Wb

S_T = 3964137AT/Wb

mmf = 2500 AT

Φ = 0.63 mWb, B = 0.7 T

Illustration 3

A ring of cross sectional area 12 cm^2 has 3 parts made of following materials:

Part	Material	Length	Relative Permeability
A	Iron	25 cm	800
B	Steel	18 cm	1100
C	Air	2 mm	---

A coil of 660 turns carrying a current of 2.1 A is wound uniformly on the ring. Determine the flux density in the air gap. Assume no leakage and fringing effect.

Ans: 0.703 Wb/m^2

Basic Electrical Technology

Parallel Magnetic Circuits

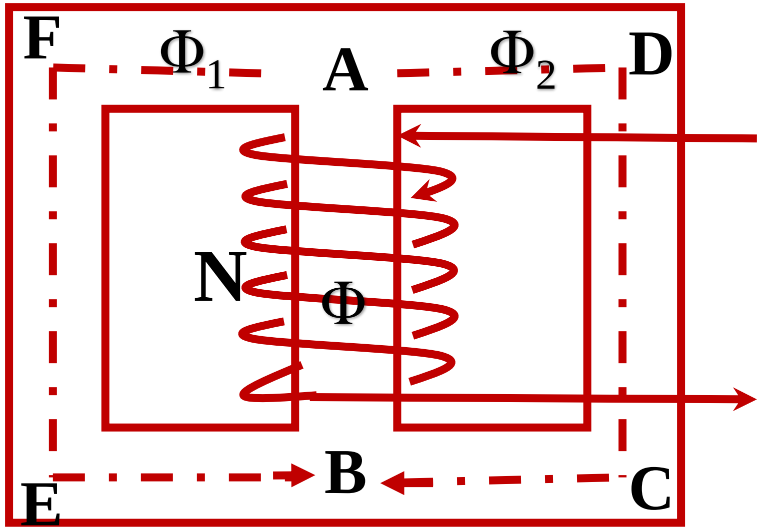
Parallel Magnetic Circuit

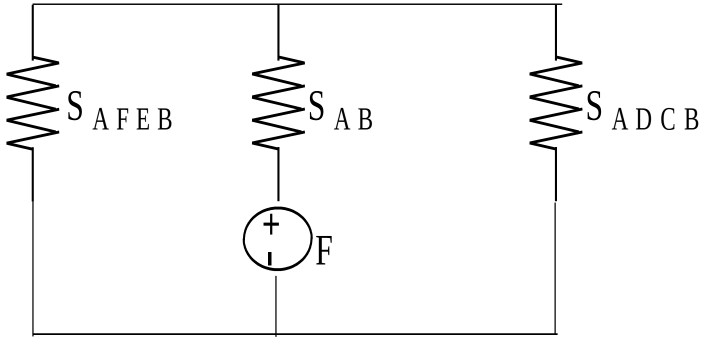
- More than one path for flux
- $\boldsymbol{\Phi = \Phi_1 + \Phi_2}$

$$S_{AB} = \frac{l_{AB}}{\mu_0\,\mu_{rAB}\,A_{AB}}$$

$$S_{ADCB} = \frac{l_{ADCB}}{\mu_0\,\mu_{rADCB}\,A_{ADCB}}$$

$$S_{AFEB} = \frac{l_{AFEB}}{\mu_0\,\mu_{rAFEB}\,A_{AFEB}}$$





Analogous Electrical Circuit

Parallel Magnetic Circuit

- $(\text{Mmf})_{Total} = (\text{Mmf})_{AB} + (\text{Mmf})_{ADCB}$

  OR

  $(\text{Mmf})_{Total} = (\text{Mmf})_{AB} + (\text{Mmf})_{AFEB}$

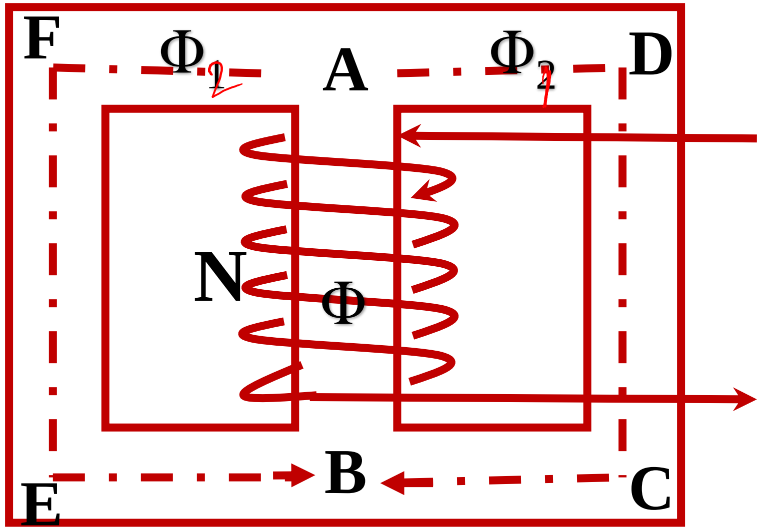
$F = NI = \phi \cdot S$

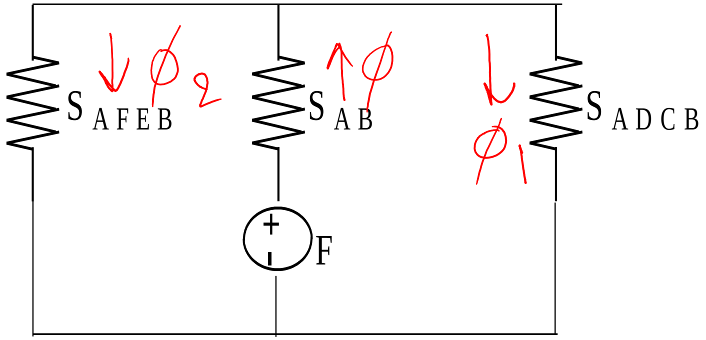
- $(\text{Mmf})_{Total} = \Phi\, S_{AB} + \Phi_1\, S_{ADCB}$

  OR

  $(\text{Mmf})_{Total} = \Phi\, S_{AB} + \Phi_2\, S_{AFEB}$

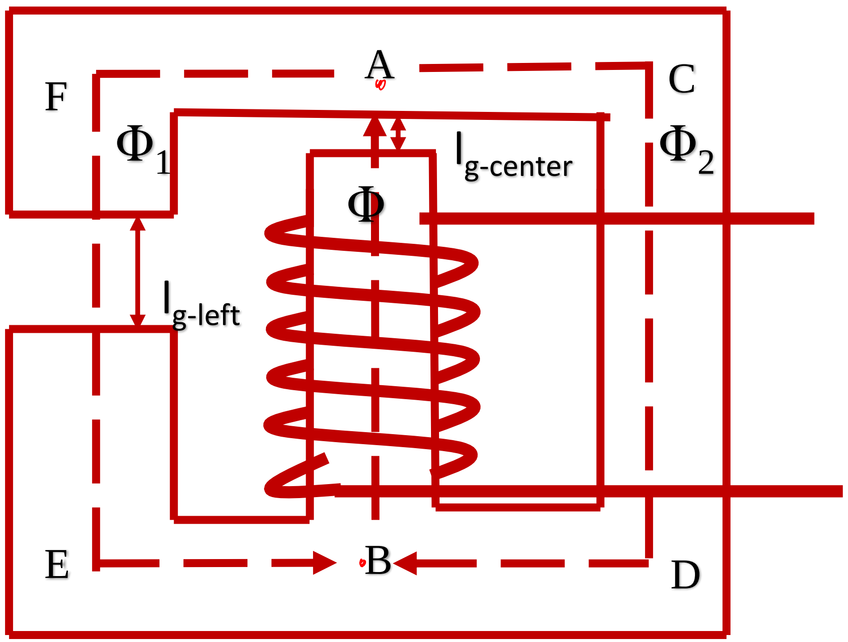
$F =$





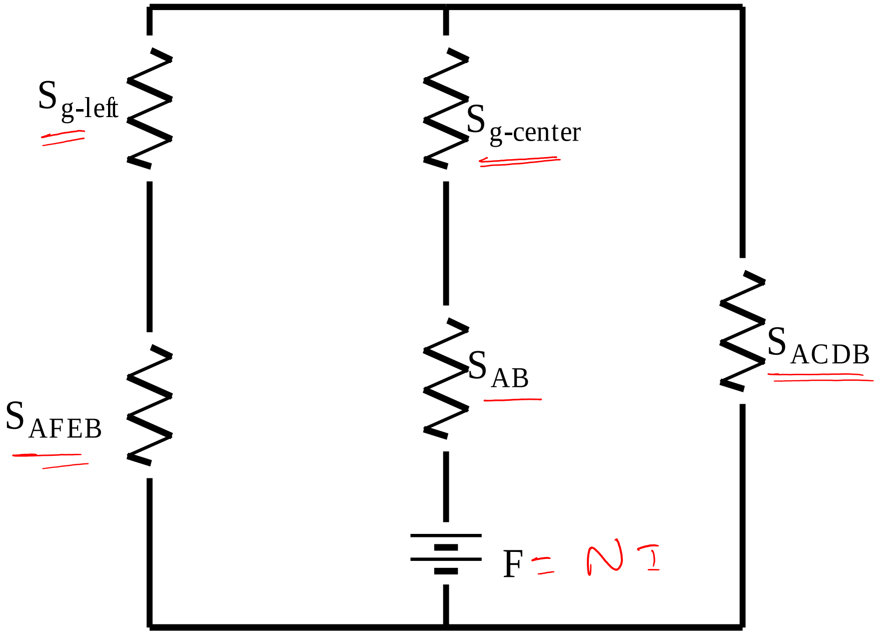
Analogous Electrical Circuit

Parallel Magnetic Circuit with Air Gap



$S = \dfrac{l}{\mu_0 \mu_r \cdot A}$

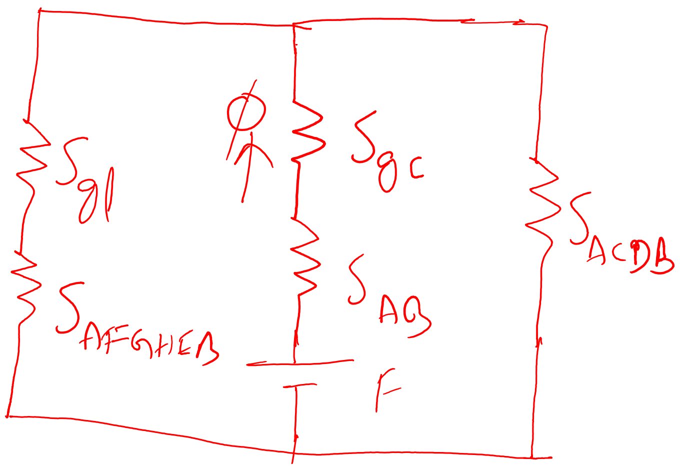
$l = (10 - 0.1)$ cm.

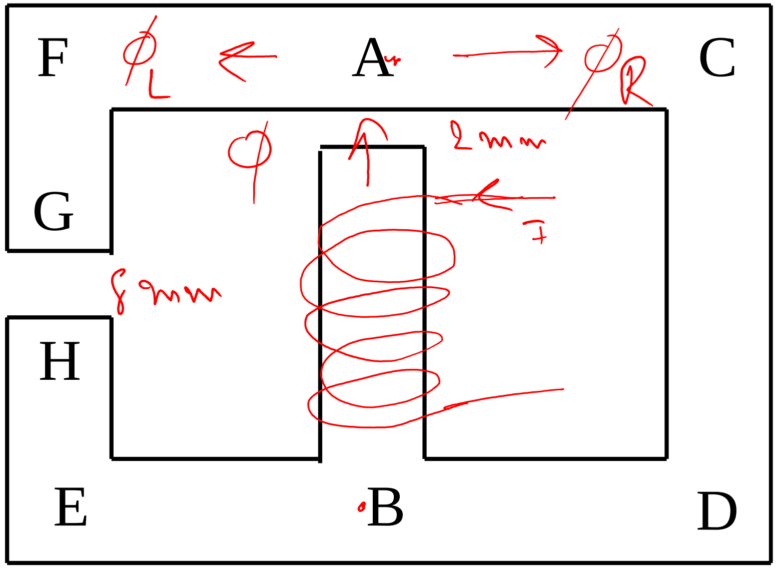


$$S_{AFEB} = \frac{(l_{AFEB} - l_{gleft})}{\mu_0\, \mu_{rAFEB}\, A_{AFEB}}; \qquad S_{AB} = \frac{(l_{AB} - l_{gcenter})}{\mu_0\, \mu_{rAB}\, A_{AB}}$$

Illustration 1

The magnetic circuit shown in Fig. is made of a material having relative permeability of 2000. The central limb is wound with 1000 turns and has an airgap of length of 2mm. The side limb airgap is 8 mm. Calculate the current required to set up a flux of 2.6 mWb in the central limb. Mean lengths of various sections are as follows: AB = 24 cm, ACDB = AFGHEB = 60 cm. Cross sectional area of the structure is 10 cm^2.





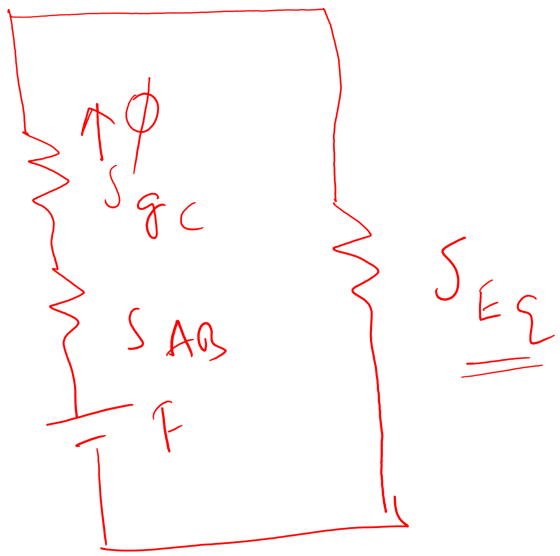
$I = ?$

$\mu_r = 2000$

$N = 1000$

$\phi = 2.6$ mWb

$A = 10$ cm^2



$$\boxed{F = \phi\left(S_{AB} + S_{gc} + S_{E\Sigma}\right)}$$

$$N \cdot I = \phi\left(S_{AB} + S_{gc} + S_{E\Sigma}\right)$$

$$I = \_\_\_$$

$$S_{AB} = \frac{l_{A \cdot B}}{\mu_0 \mu_r \cdot A} = \frac{(24 - 0.2) \times 10^{-2}}{\mu_0 \mu_r \cdot A} = 94697.1914 \ AT/wb.$$

$$S_{gc} = \frac{l_{gc}}{\mu_0 \cdot A} = \frac{0.2 \times 10^{-2}}{\mu_0 \cdot A} = 1591549.431 \ AT/wb.$$

$$S_{gl} = \frac{l_{gl}}{\mu_0 \cdot A} = \frac{0.8 \times 10^{-2}}{\mu_0 \cdot A} = 6366197.724A$$

$$S_{AFGHEB} = \frac{(60-0.8)\times 10^{-2}}{\mu_0 \mu_r \cdot A} = 235549.3158 \text{ AT/Wb.}$$

$$S_{ACDB} = \frac{60\times 10^{-2}}{\mu_0 \mu_r \cdot A} = 238732.4146 \text{ AT/Wb.}$$

$$S_{EQ} = (S_{gl} + S_{AFGHEB}) \parallel S_{ACDB} = 230400.6644 \text{ AT/Wb.}$$

$$F = \phi\,(S_{AB} + S_{gc} + S_{EQ})$$

$$N \cdot I = 2.6\times 10^{-3}\,(1916647.266)$$

$$\underline{I = 4.983 A}$$

Ans: 4.98 A

Illustration 2

A coil carrying a current of 2.8 A is wound on the left limb of the cast steel symmetrical frame of uniform square cross section 16 cm^2 as shown in Fig. Calculate the number of turns in the coil to produce a flux of 1.8 mWb in the air gap of 0.2 cm length. The relative permeability of cast steel is 1200.

$I = 2.8\,A$

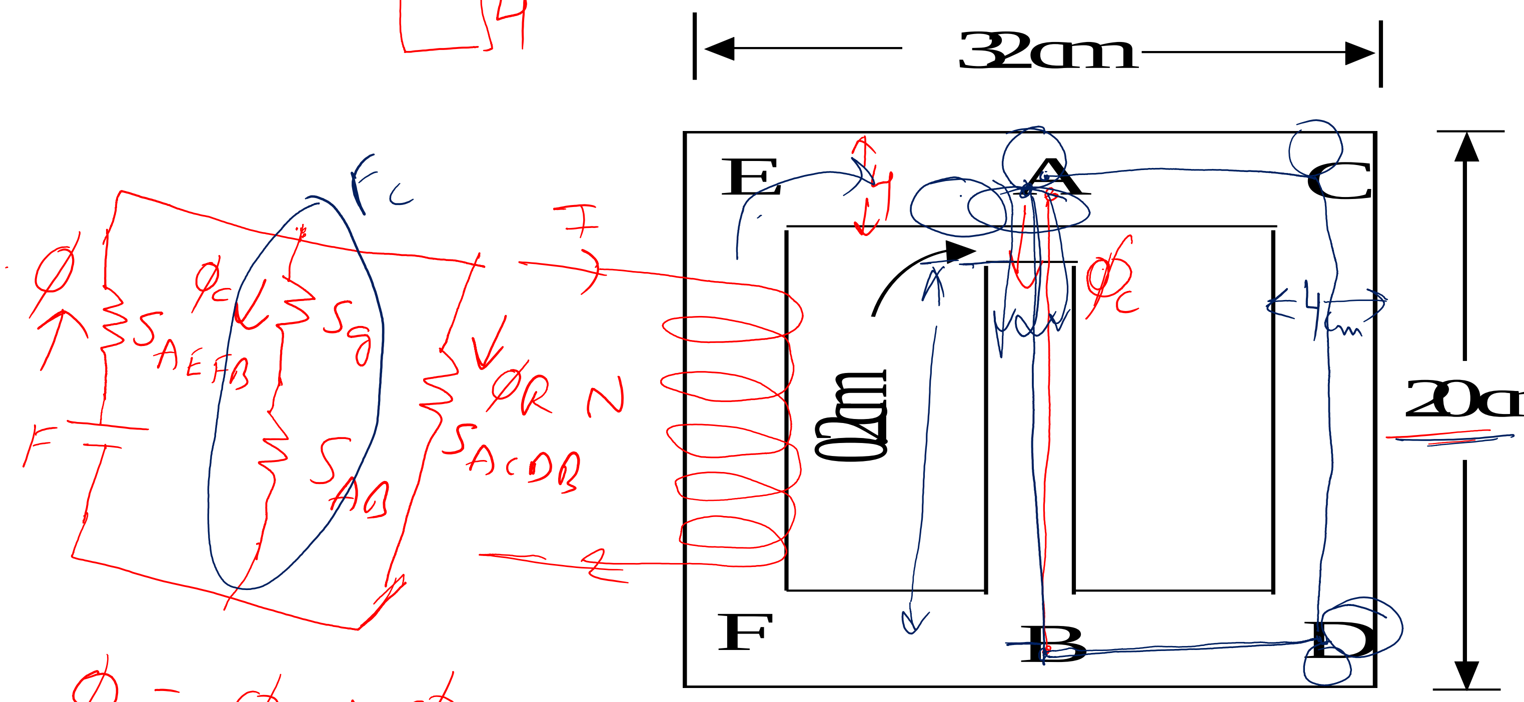
$N = ?$

$A = 16\,cm^2$

$l_g = 0.2\,cm$

$\mu_r = 1200$

$\phi_c = 1.8\,mWb$



$20-2-2$

$=16-0.2$

$AB = 15.8\,cm$

$\phi = \phi_c + \phi_R$

$l_{AEFB} = 16-2+20-2-2+16-2 = 44\,cm$

$l_{AEFB} = 44\,cm.$

$$S_{AEFB} = \frac{l_{AEFB}}{\mu_0 \mu_r \cdot A} = 182365.039\ AT/wb.$$

$$S_{ACDB} = S_{AEFB}$$

$$S_g = \frac{0.2 \times 10^{-2}}{\mu_0 \cdot A} = 994718.3943\ AT/wb.$$

$$S_{AB} = \frac{(16 - 0.2) \times 10^{-2}}{\mu_0 \cdot \mu_r \cdot A} = 65485.627\ AT/wb$$

$$\left(F_c = \phi_R \cdot S_{AEDB}\right)$$

$$F_c = \phi_c (S_g + S_{AB}) = 1908.3672\ AT/wb$$

$$\phi_R = \frac{F_c}{S_{ACDB}} = 10.464\ mwb$$

$$\phi = \phi_c + \phi_R = 12.264 \text{ mWb}$$

$$F = \phi(S_{AEFB}) + \phi_c(S_g + S_{AB})$$

$$N.I = \phi(S_{AEFB}) + F_c$$

$$N.(2.8) = 2236.624 + 1908.3672$$

$$\boxed{N \simeq 1480}$$

Ans: 1480

Basic Electrical Technology

[ELE 1051]

Electromagnetic induction

Faraday's Laws of Electromagnetic Induction

First Law:

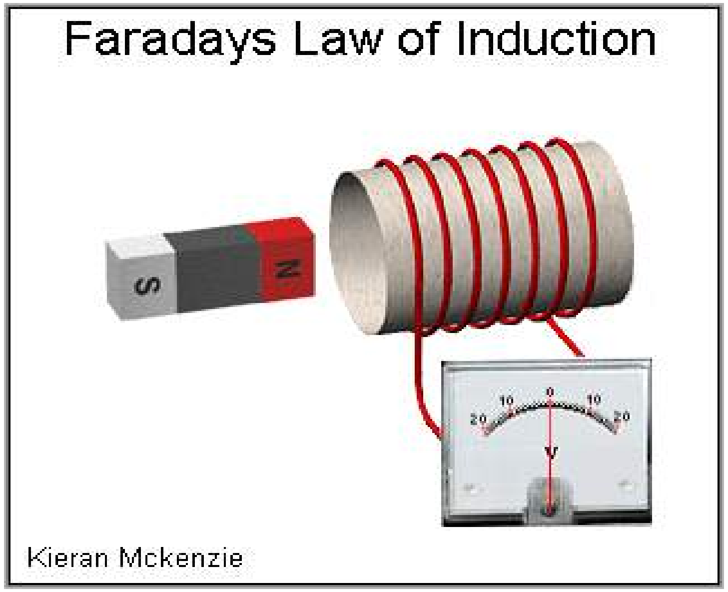
Whenever the magnetic field linking with a conductor changes, an EMF will be induced in that conductor

Second Law:

The magnitude of the induced EMF is proportional to the rate of change of the magnetic flux linking the conductor

$$e = N\frac{d\phi}{dt}$$

Where N = number of turns in the coil



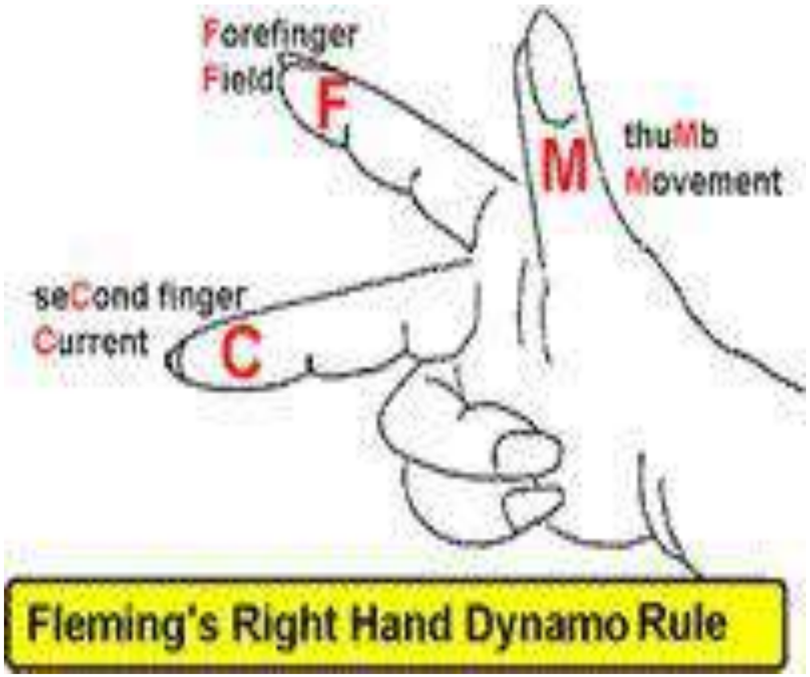
Lenz's Law

The electro-magnetically induced emf always acts in such a direction to set up a current opposing the motion or change of flux responsible for inducing the emf.

$$e = -N\frac{d\phi}{dt}$$

Fleming’s Right Hand Rule

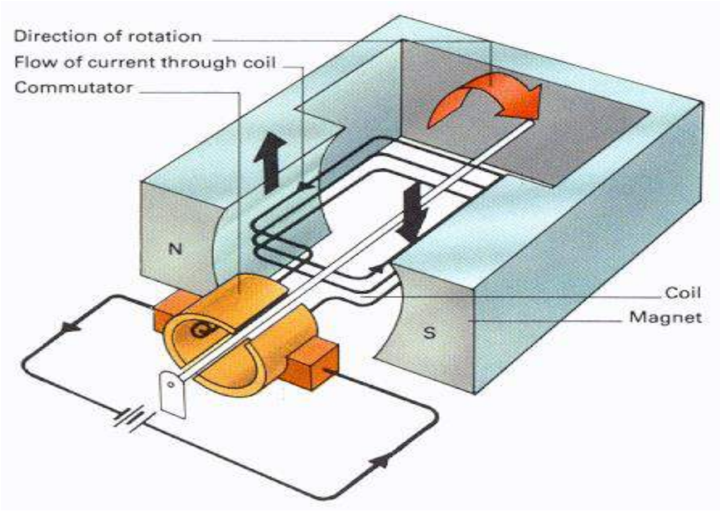
If the first, second and the thumb of the right hand are held at right angles to each other,
first finger indicates the direction of the magnetic flux
and
thumb finger indicates the direction of motion of the conductor relative to the magnetic field,
then
the second finger represents the direction of induced EMF.



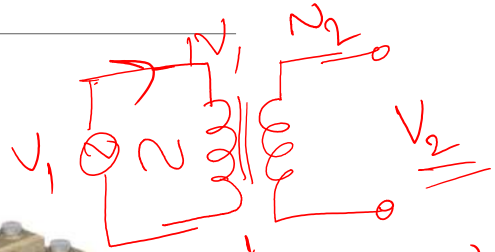
Types of induced EMF

Dynamically induced EMF:

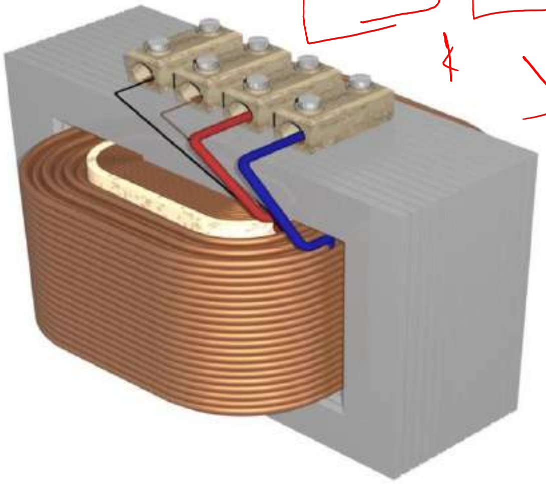
- The voltage induced in the conductor due to relative motion of conductor and magnetic field
- $e = B\, l\, v\, Sin\theta$
- Either conductor or magnetic field is moving
- Principle of **Electric generator**



Types of induced EMF



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$



Statically Induced EMF:

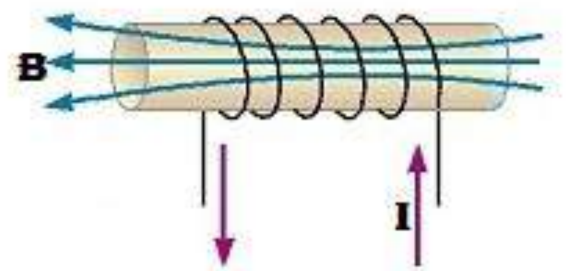
- The voltage induced in the conductor due to change in the magnetic field
- Conductor is stationary
- Magnetic Field is changing in a stationary magnetic system
- Eg: Transformer

Types of Statically induced EMF

Self Induced Emf:

The induced emf in a coil proportional to the rate of the change of the magnetic flux passing through it due to its own current.

$$\boxed{e = -L\frac{di}{dt}}$$



Self Inductance L:

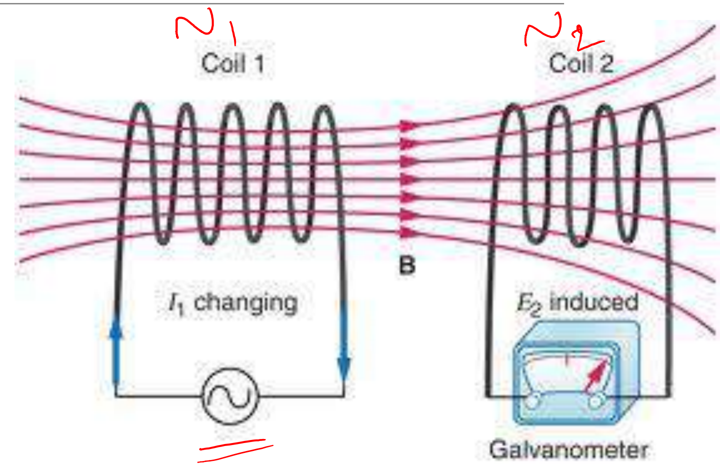
The proportionality constant is called the self inductance, L. Unit is Henry

$$e = -N\frac{d\phi}{dt} = -L\frac{di}{dt}$$

$$L = N\frac{d\phi}{di}$$

Mutually Induced Emf:

The induced emf in a coil due to the change of flux produced by the change of current in the nearby coil



Mutual Inductance M:

This proportionality constant is called the mutual inductance, M

If Coil 1 is excited:

Mutually induced emf e_2 in Coil 2, $\quad e_2 = N_2 \frac{d\phi_{12}}{dt} = M \frac{di_1}{dt}$

Mutual Inductance, $\quad M = N_2 \frac{d\phi_{12}}{di_1}$

If coil 2 is excited: $M = N_1 \frac{d\phi_{21}}{di_2}$

Coupling Coefficient (k)

Gives an idea about the degree of magnetic coupling between two coils.

$$M = N_2 \frac{d\phi_{12}}{di_1} = N_1 \frac{d\phi_{21}}{di_2}$$

$$where, \phi_{12} = k\,\phi_1; \;\; \phi_{21} = k\,\phi_2$$

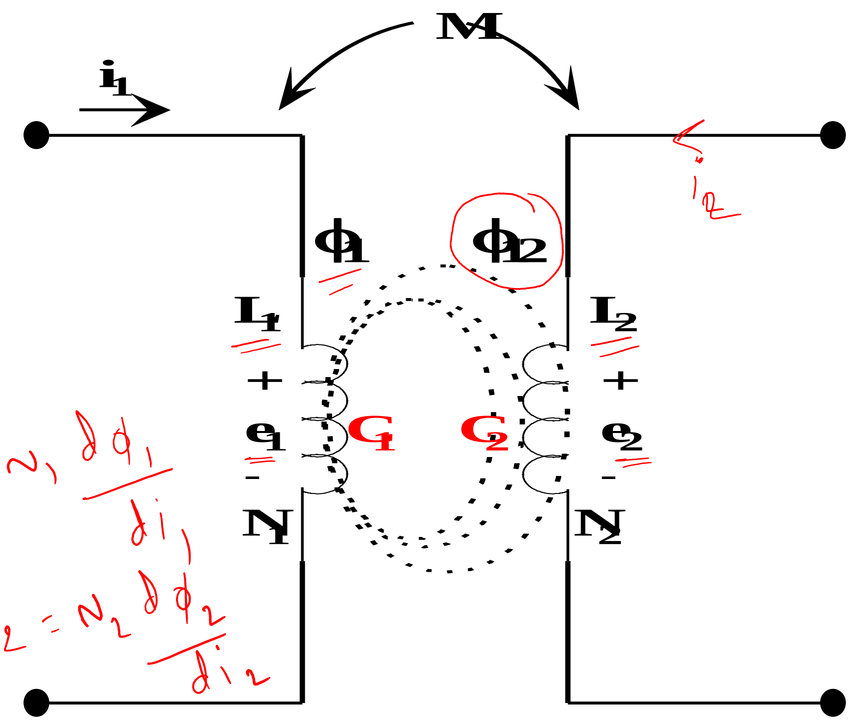
$$M^2 = \left(N_2 k \frac{d\phi_1}{di_1}\right)\left(N_1 k \frac{d\phi_2}{di_2}\right)$$

$; \; L_1 = N_1 \frac{d\phi_1}{di_1}$

$L_2 = N_2 \frac{d\phi_2}{di_2}$

$$M^2 = k^2 L_1 L_2$$

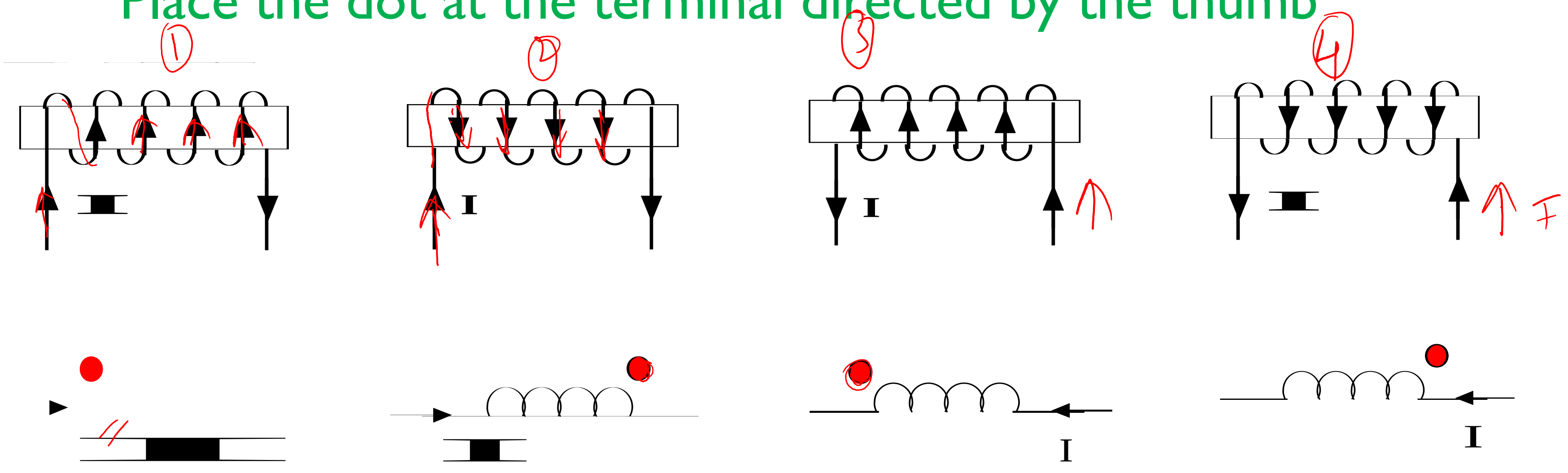
$$\boldsymbol{k = \frac{M}{\sqrt{L_1 L_2}}}$$



Coupled Circuits

- Polarity of mutually induced emf depends on
  - current direction
  - physical construction of the coils
- Obtaining the dotted equivalent: Right Hand Grip Rule

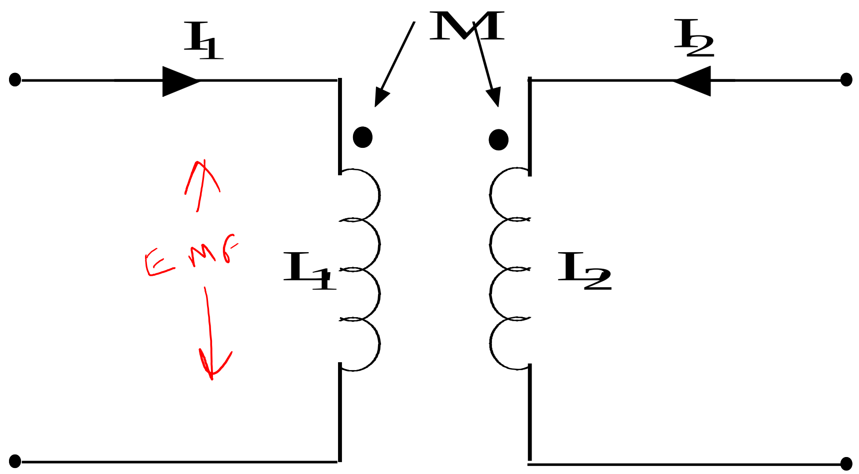
Place the dot at the terminal directed by the thumb



Dot Rule for coupled coils

- Dot Rule helps in determining the sign of mutually induced emf without going into the details of physical construction
- **Dot Rule:**
  - ✓ If currents enter (or leave) the dotted terminals in both the coils, the sign of mutually induced emf is same as that of sign of self induced emf. (Additive coupling)
  - ✓ If the current enters the dotted terminal in one coil and leaves the dotted terminal in the other coil, the sign of mutually induced emf is opposite to that of sign of self induced emf. (Subtractive coupling)

Additive Coupling: (Fluxes are aiding)

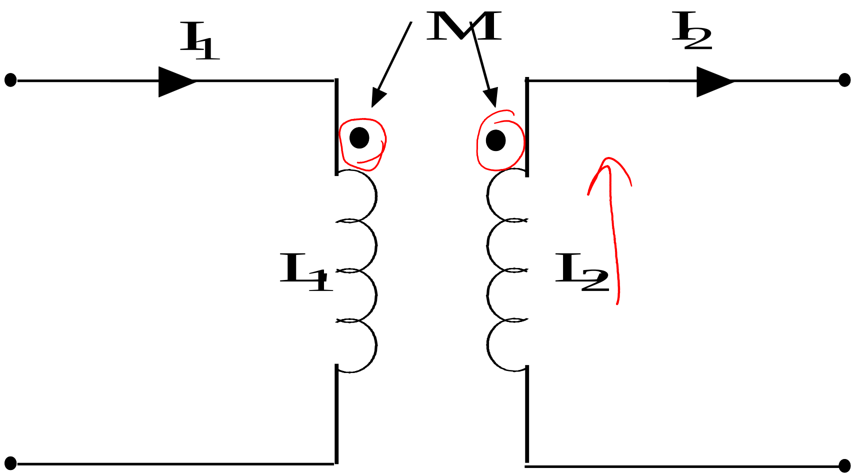


Self induced emf in L_1 = $$-L_1 \frac{di_1}{dt}$$

Mutually induced emf in L_1 = $$-M \frac{di_2}{dt}$$

Total induced emf in L_1 = $$-\left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \right)$$

Subtractive Coupling: (Fluxes are opposing)

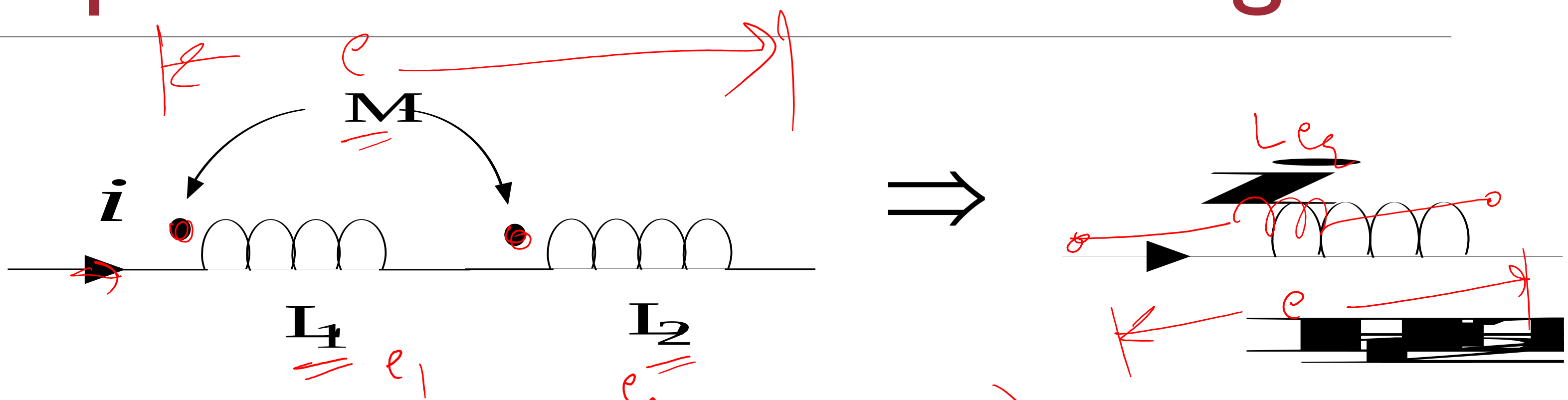


Self induced emf in L₁ = $-L_1 \frac{di_1}{dt}$

Mutually induced emf in L₁ = $+M \frac{di_2}{dt}$

Total induced emf in L₁ = $\left(-L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}\right)$

Coupled coils in Series - Aiding



$$e_1 = L_1 \frac{di}{dt} + M \frac{di}{dt}$$

$$\left(-L_1 \frac{di}{dt} - M \frac{di}{dt} \right)$$

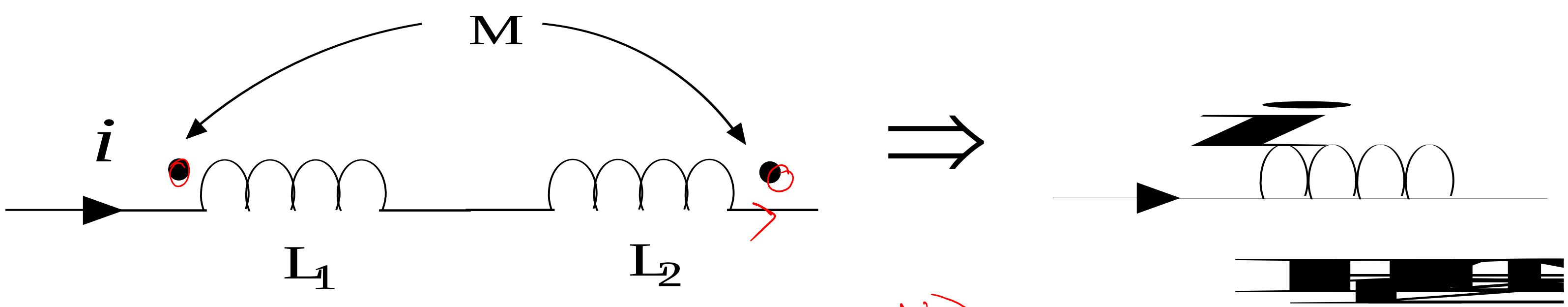
$$= -(L_1 + M) \frac{di}{dt}$$

$$e_2 = L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$e = L_{eq} \frac{di}{dt} = e_1 + e_2 = (L_1 + L_2 + 2M) \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + 2M$$

Coupled coils in Series - Opposing



$$e_1 = L_1\frac{di}{dt} - M\frac{di}{dt}$$ $\Rightarrow -\left(L_1\frac{di}{dt} + M\frac{di}{dt}\right)$

$$e_2 = L_2\frac{di}{dt} - M\frac{di}{dt}$$

$$e = L_{eq}\frac{di}{dt} = e_1 + e_2 = (L_1 + L_2 - 2M)\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 - 2M$$

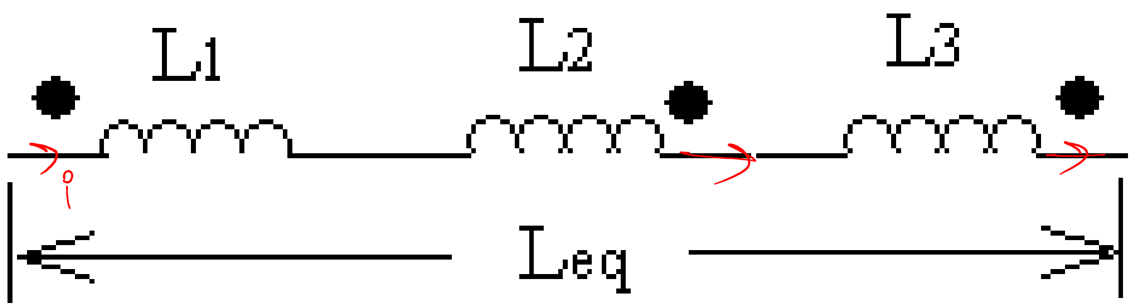
Example 1

Three magnetically coupled inductive coils having the following data are connected in series as shown in Figure.

$L_1 = 0.12$ H; $L_2 = 0.14$ H; $L_3 = 0.16$ H

$k_{12} = 0.3$; $k_{23} = 0.6$; $k_{31} = 0.9$

Find the equivalent inductance of the circuit.



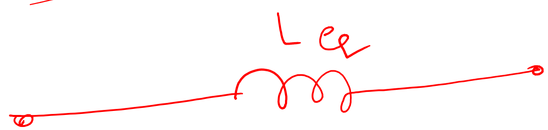
$M_{12} = k_{12}\sqrt{L_1 L_2} = 0.0388$

$M_{23} = k_{23}\sqrt{L_2 L_3} = 0.0897$

$M_{31} = k_{31}\sqrt{L_3 L_1} = 0.124$

$$L_{eq} = L_1 + L_2 + L_3 - 2M_{12} + 2M_{23} - 2M_{31}$$

$$= \underline{\underline{0.2738\,H}}$$

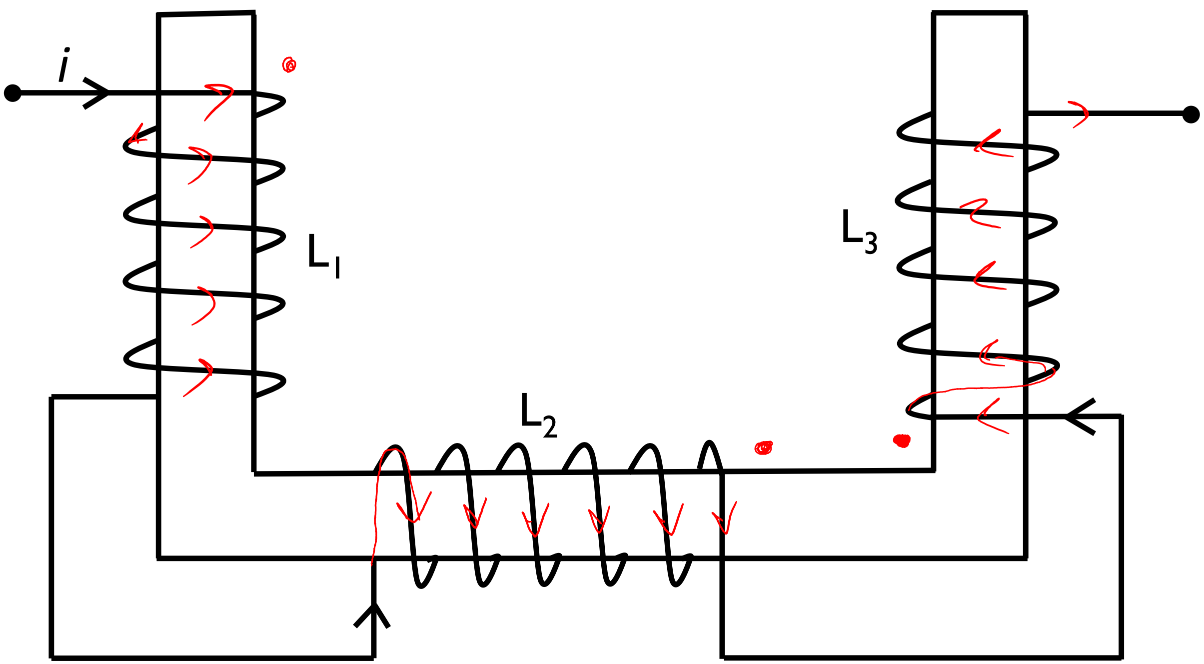


Ans: 0.272 H

Example 2

Three magnetically coupled inductive coils having the following data are connected in series as shown in Figure. $L_1 = 0.3$ H; $L_2 = 0.6$ H; $L_3 = 0.8$H and the coefficients of coupling are ,$k_{12} = 0.8$; $k_{23} = 0.75$; $k_{31} = 0.5$
Draw the dotted equivalent circuit of the figure, also find the equivalent inductance of the circuit.

Leq



$M_{12} = 0.339$ H

$M_{23} = 0.519$ H

$M_{31} = 0.2449$ H

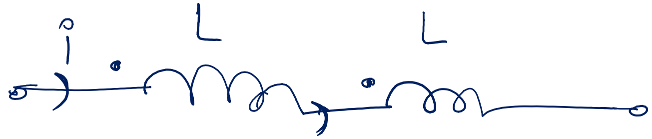
$$L_{eq} = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{31}$$

$$L_{eq} = 0.4738\,H$$

Ans : 0.472 H

Example 3

Two similar coils have a coupling coefficient of 0.4. When they are connected in series aiding, the equivalent inductance is 560mH. Calculate: i) self-inductance of both the coils. ii) Total inductance when the coils are connected in series opposition. iii) total energy stored due to a current of 3A when the coils are connected in series opposition.

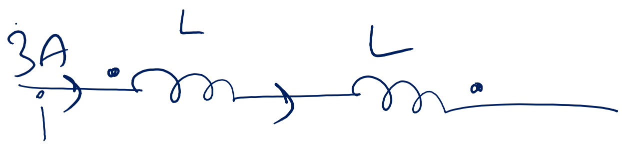


$M = K\cdot\sqrt{L\cdot L} = K\cdot L = 0.4L$

$L_{eq} = L + L + 2M = 560\,mH$

$L_{eq} = 2.8L = 560\,mH$

1 $L = 200\,mH$



2 $L_{eq} = L + L - 2M = 1.2L = 240\,mH$

3 $E = \frac{1}{2} L_{eq} I^2 = 1.08\,J$

Ans: 0.2 H, 0.24 H, 1.08 J

Basic Electrical Technology

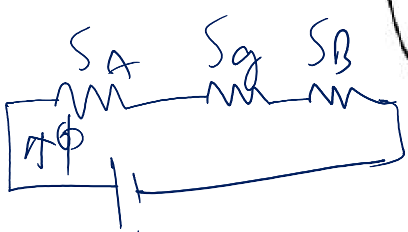
Tutorial 03 & 04
Magnetic Circuits & Electromagnetism

Question 01

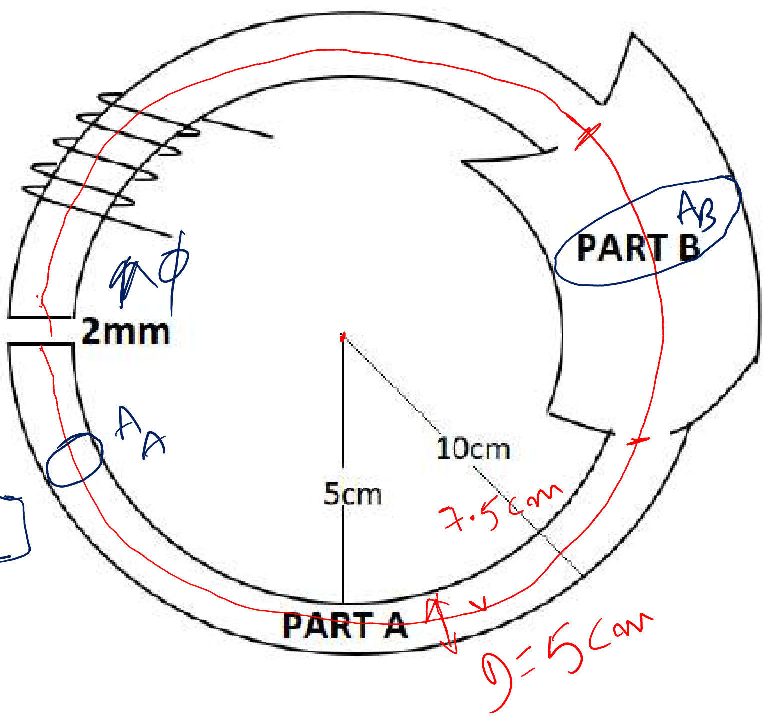
In the figure below, Part A is a toroid core with inner radius of 5cm & outer radius of 10 cm. A portion of Part A is cut & Part B is sandwiched. Mean length of Part B is 9.5 CM & area of cross section of 80 cm2. The airgap shown has a length of 2mm. Find the current required in the coil to set a flux of 2 mWb in the airgap. Total number of turns in the coil is 500. Relative permeability of Part A & Part B 1000 & 1500 respectively.

$F = \phi(S_A + S_g + S_B)$

$I = \_\_\_\_ A$



$F = NI$



$l_A = 2\pi(7.5) - 9.5 - 0.2$

$= 37.424\text{ cm}$

$l_B = 9.5\text{ cm}$

$N = 500$

$A_B = 80\text{ cm}^2$

$A_A = \frac{\pi \cdot D^2}{4}$

$= 19.635\text{ cm}^2$

$$S_A = \frac{l_A}{\mu_0 \mu_{r1} \cdot A_A} = 151673.404 \text{ AT/Wb}$$

$$S_B = \frac{l_B}{\mu_0 \mu_{r2} \cdot A_B} = 6299.883 \text{ AT/Wb}$$

$$S_g = \frac{l_g}{\mu_0 \cdot A_A} = 810567.5737 \text{ AT/Wb}$$

$$F = \phi(S_A + S_g + S_B)$$

$$NI = 2\times10^{-3}(968540.8643)$$

$$\underline{\underline{I = 3.874 \text{ A}}}$$

Question 02

A series magnetic circuit comprises of three sections **(i)** length of 80 mm with cross-sectional area 60 mm2, **(ii)** length of 70 mm with cross-sectional area 84 mm2 and **(iii)** and airgap of length 0.5 mm with cross-sectional area of 60 mm2. Sections **(i)** and **(ii)** are of a material having magnetic characteristics given by the following table. Determine the current necessary in a coil of 4000 turns wound on section (ii) to produce a flux density of 0.7 Tesla in the air-gap. Neglect magnetic leakage.

H (AT/m)	**100**	**210**	**290**	**420**	**800**	**1500**
B (Tesla)	0.2	0.4	0.5	0.7	1.0	1.2

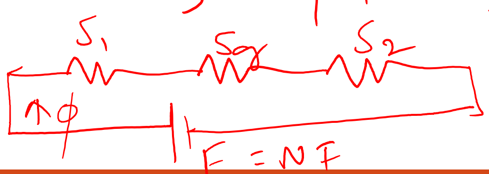
$\mu_r = $ ——

$S =$

$B = 0.7\,T = \phi/A \Rightarrow \phi = B \cdot A = 0.7 \times 60 \times 10^{-6} = 42\,\mu Wb$

$B = \mu H \Rightarrow \mu = \mu_0 \cdot \mu_r = B/H = \dfrac{0.7}{420}$

$\mu = 1.66666 \times 10^{-3}$



$$S_1 = \frac{l_1}{\mu_0 \mu_r \cdot A_1} = \frac{80\times10^{-3}}{1.66666\times10^{-3} \times 60\times10^{-6}} = 800003.2 \, AT/wb$$

$$S_2 = \frac{l_2}{\mu_0 \mu_r \cdot A_2} = \frac{70\times10^{-3}}{1.66666\times10^{-3} \times 84\times10^{-6}} = 500002 \, AT/wb$$

$$S_g = \frac{l_g}{\mu_0 A_1} = \frac{0.5\times10^{-3}}{4\pi\times10^{-7}\times60\times10^{-6}} = 6631455.962 \, AT/wb$$

$$F = \phi(S_1 + S_2 + S_g) \qquad (\because N = 4000 \text{ Turns})$$

$$NI = 42\times10^{-6}(79{,}31461.162)$$

$$\underline{\underline{I = 83.280 \, mA}}$$

Numerical 3

A 710 turns coil is wound on the central limb of the cast steel symmetrical frame of uniform cross section 16 cm^2 is as shown. Calculate the current required to produce a flux of 1.8 mWb in an air gap of 0.2 cm length. Given l_{AFEB} = l_{ACDB} = 25 cm, l_{AB} = 12.5 cm. The magnetization details is as follows. Neglect the fringing and leakage effects

.

H	300	500	600	700	900	1092
B	0.1	0.45	0.562	0.775	1	1.125

$\mu = \mu_0 \cdot \mu_r$

$\mu_r = ?$

$\mu = 1.0302 \times 10^{-3}$

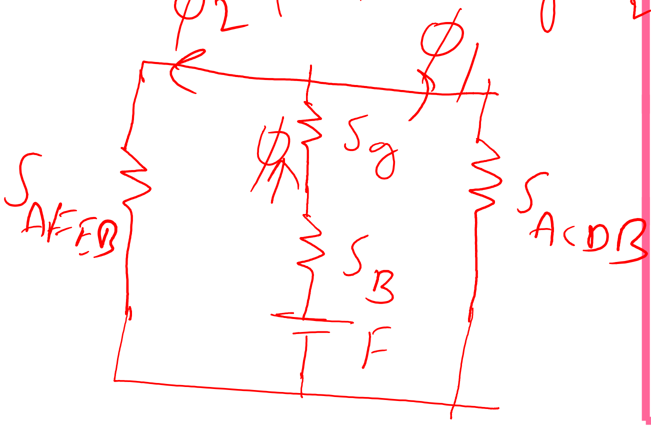
$\phi_1 = \phi_2 = \frac{\phi}{2}$

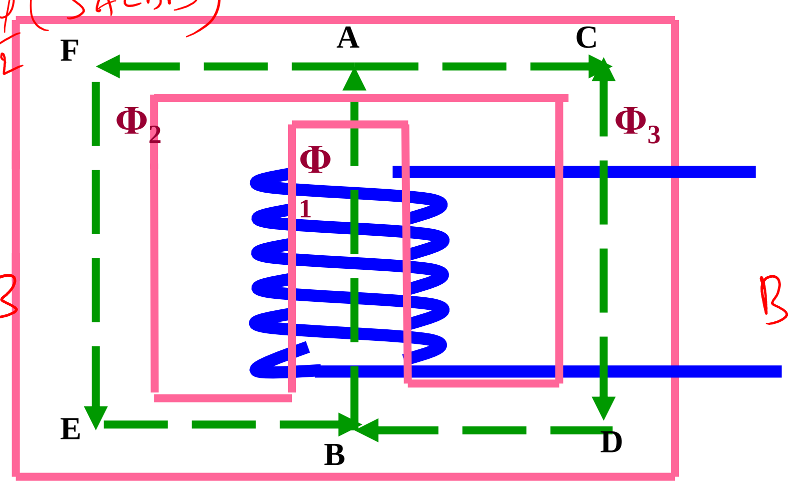
$\phi_2 \quad F = \phi(S_B + S_g) + \frac{\phi}{2}(S_{ACDB})$

$\phi = 1.8$ mWb.

$B = \phi / A$

$B = 1.125$ Wb/m^2





$B = \mu H$

$\mu = B/H$

Ans: I = 2.92 A

$$S_{AFEB} = S_{ACDB} = \frac{l}{\mu_0 \mu_r \cdot A} = \frac{25 \times 10^{-2}}{1.0302 \times 10^{-3} \times 16 \times 10^{-4}}$$

$$= 151.669 \times 10^{3} \ AT/Wb$$

$$S_g = \frac{l_g}{\mu_0 A} = \frac{0.2 \times 10^{-2}}{4\pi \times 10^{-7} \times 16 \times 10^{-4}} = 994.718 \times 10^{3} \ AT/Wb$$

$$S_{AB} = \frac{l_{AB}}{\mu_0 \mu_r \cdot A} = \frac{(12.5 - 0.2) \times 10^{-2}}{1.0302 \times 10^{-3} \times 16 \times 10^{-4}} = 74.621 \times 10^{3} \ AT/Wb$$

$$S_{Eq} = \frac{S_{ACDB}}{2}$$

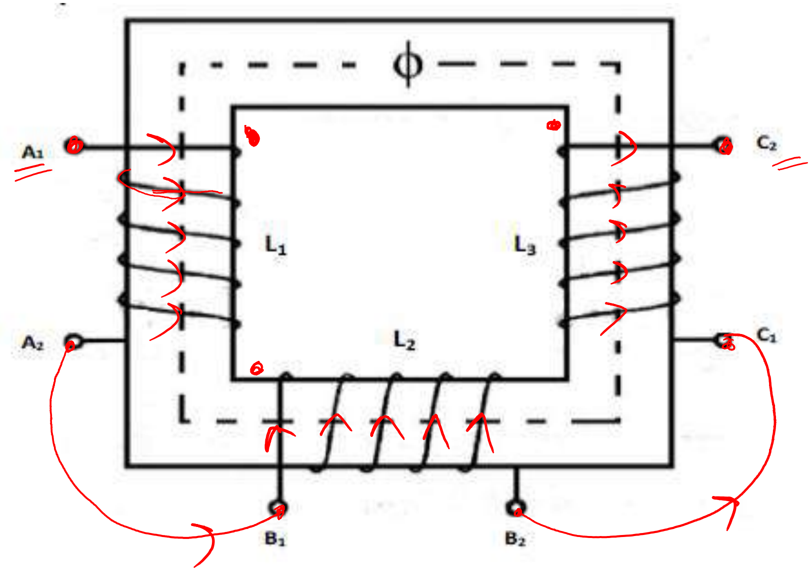
$$F = \phi\left(S_{AB} + S_g + S_{Eq}\right)$$

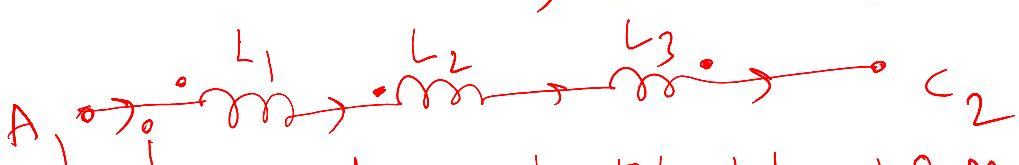
$$NI = \phi\left(1145173.5\right) \qquad (\because N = 710)$$

$$\boxed{I = 2.903 \ A}$$

Question 04

Three coupled coils L1 = 0.4 H, L2 = 0.5 H and L3 = 0.8 H wounded on the same core as shown in the **Fig.** are connected in series by joining the terminals A2 to B1 and B2 to C1 and the coefficient of coupling k12 = k13 = k23 = 0.8. Sketch the dotted equivalent circuit of the coils connected in series and find the equivalent inductance measured across terminals A1 and C2.





$L_{eq} = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{31}$

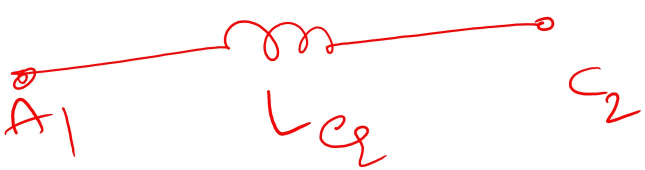
$$M_{12} = k_{12}\sqrt{L_1 \cdot L_2} = 0.3577\,H$$

$$M_{23} = k_{23}\sqrt{L_2 \cdot L_3} = 0.50596\,H$$

$$M_{31} = k_{31}\sqrt{L_3 L_1} = 0.45254\,H$$

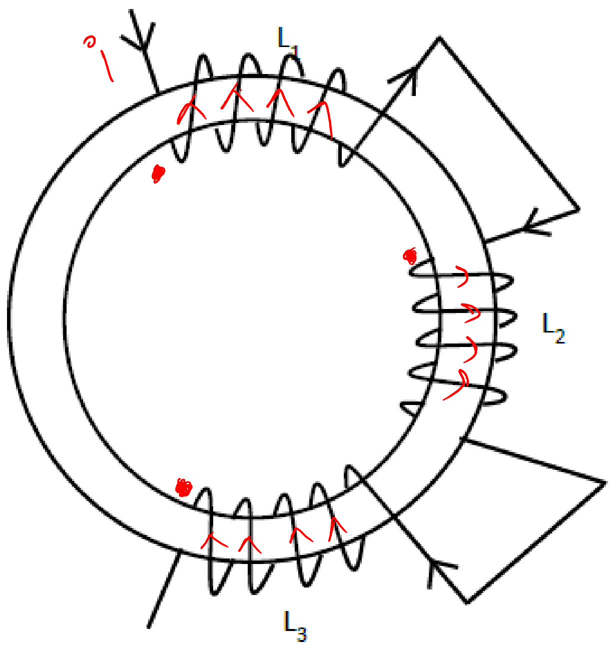
$$L_{eq} = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{31}$$

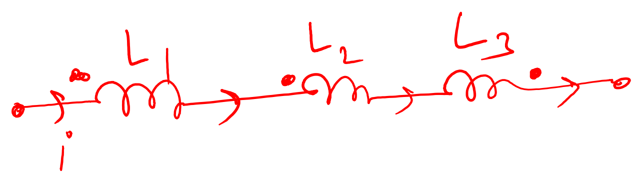
$$L_{eq} = 0.4984\,H$$



Question 05

Three magnetically coupled inductive coils shown in figure having the following data. L1 = 0.4 H; L2 = 0.8 H; L3 = 0.2H and the coefficients of coupling are ,k12 = 0.6; k23 = 0.55; k31 = 0.9 Draw the dotted equivalent circuit of the figure, also find the equivalent inductance of the circuit.





$$L_{eq} = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{31}$$

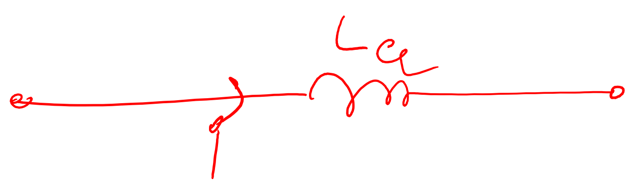
$$M_{12} = k_{12}\sqrt{L_1 L_2} = 0.3394\,H$$

$$M_{23} = k_{23}\sqrt{L_2 L_3} = 0.22\,H$$

$$M_{31} = k_{31}\sqrt{L_3 L_1} = 0.2545\,H$$

$$L_{eq} = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{31}$$

$$L_{eq} = 1.1298\,H$$



Basic Electrical Technology

[ELE 1051]

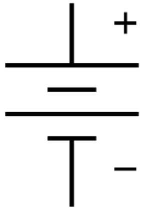
CHAPTER 3 - SINGLE PHASE AC CIRCUITS

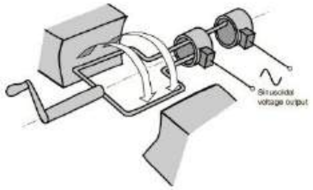
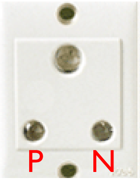
(3.1)

Topics covered…

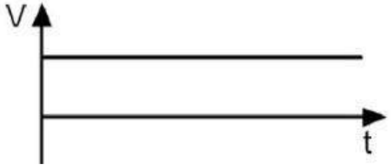
- Comparison of DC & AC
- How is AC generated?
- Terminologies of AC
- Average value of an alternating waveform
- RMS value of an alternating waveform

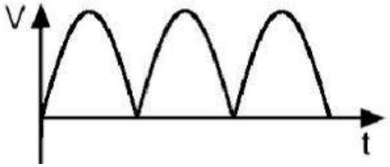
DC vs. AC

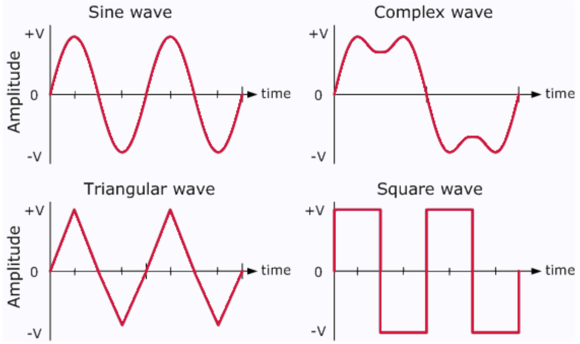


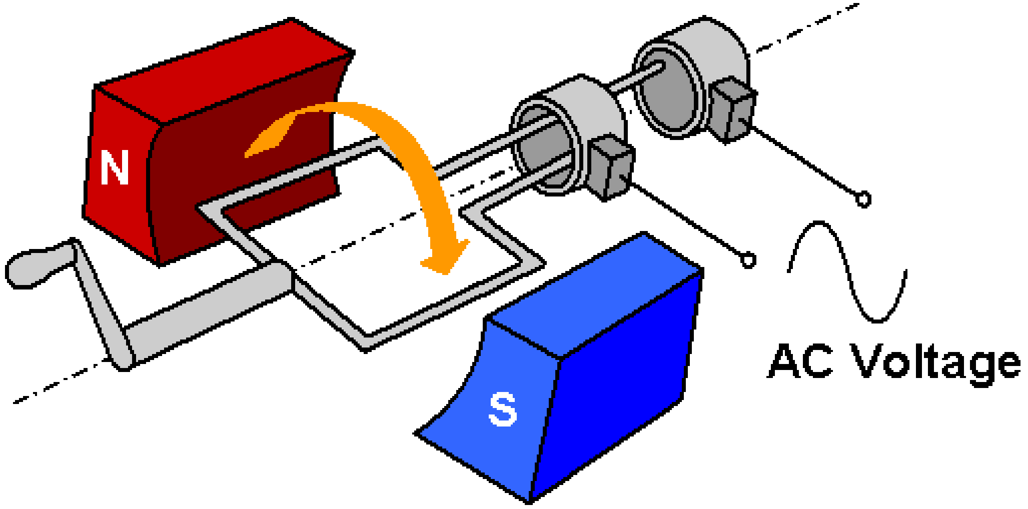
	DC	AC
Obtained from	Battery / cell / derived from AC	AC Generator
Polarity	Positive and Negative	Oscillatory
Frequency	Zero	50Hz or 60Hz
Types	Constant or pulsating	**Sinusoidal**, Trapezoidal, Triangular, Square



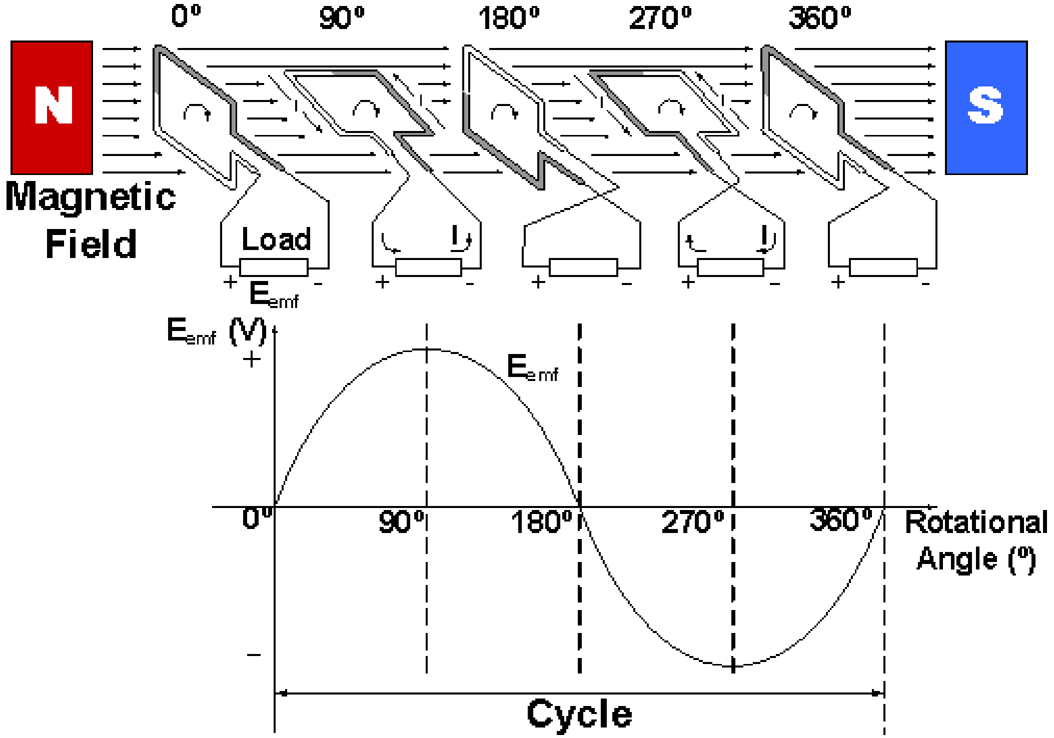




Generation of Alternating EMF



Generator working principle



EMF Equation

EMF induced per conductor is

$$e = B\,l\,v\sin\theta$$

EMF Induced in one turn of a coil is

$$e = 2\,B\,l\,v\sin\theta$$

If, b = width of the coil,

$$v = \pi\,b\,n$$ *'n' is the speed in revolutions per sec.*

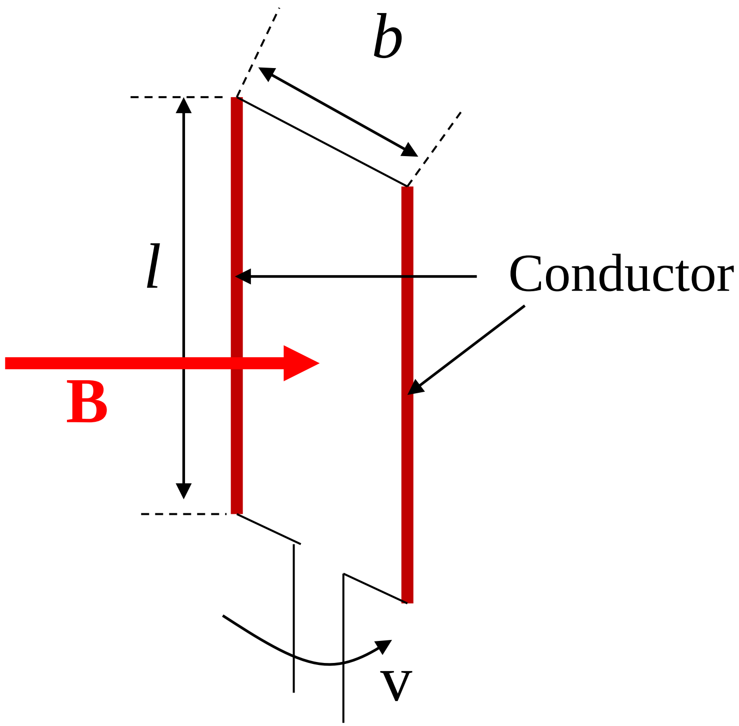
$$e = 2\,B\,l\,b\,\pi\,n\sin\theta$$

$$= 2\,B\,A\,\pi\,n\sin\theta$$

If there are N turns in the coil, the emf induced is,

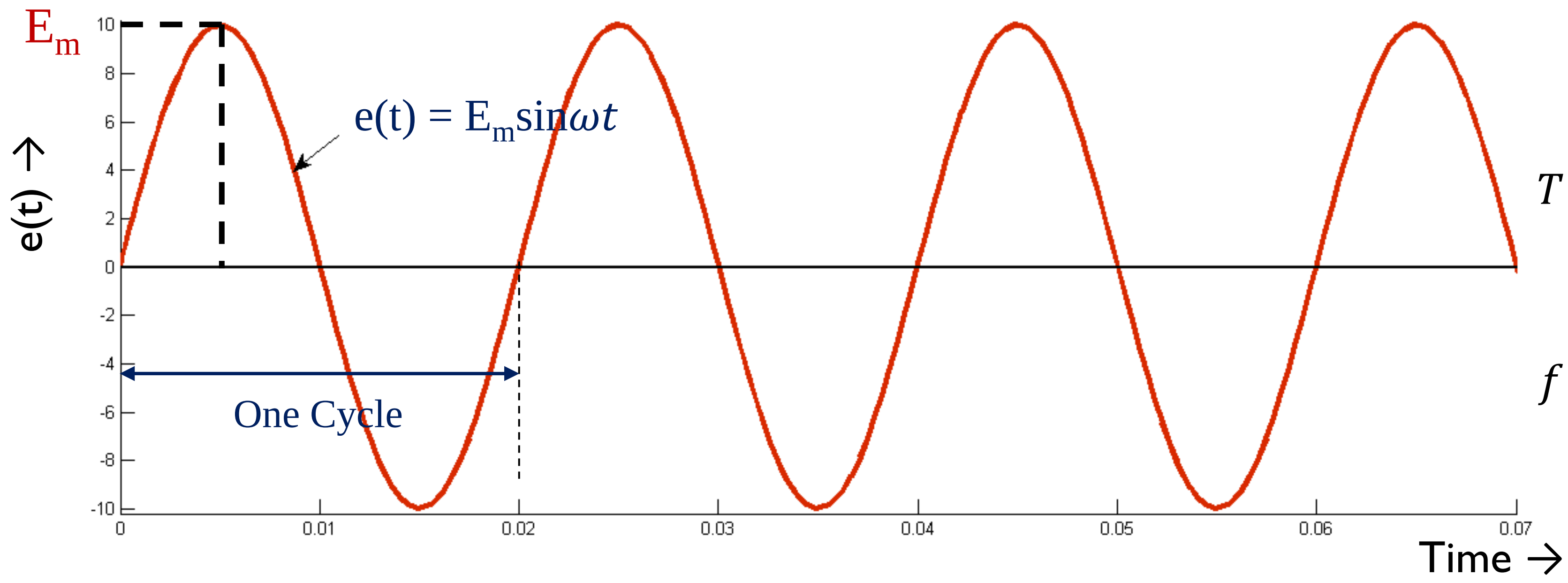
$$e = 2\,\pi\,n\,B\,A\,N\sin\theta$$

$$\boxed{e = E_m \sin\theta}$$



Turn of a coil

Terminologies in AC waveform



$T = 0.02s$

$f = 50Hz$

Cycle: Each repetition of the alternating quantity, recurring at equal intervals

Period (T): Duration of one cycle

Instantaneous Value (e(t)): The magnitude of a waveform at any instant in time

Peak Amplitude: Maximum value or peak value of alternating quantity

Frequency (f): Number of cycles in one second (Hz) $f = \dfrac{1}{T}$

Average value of Sinusoidal Alternating Current

Definition: *"It is that steady current which transfers the same amount of charge to any circuit during the given interval of time, as is transferred by the alternating current to the same circuit during the same time"*

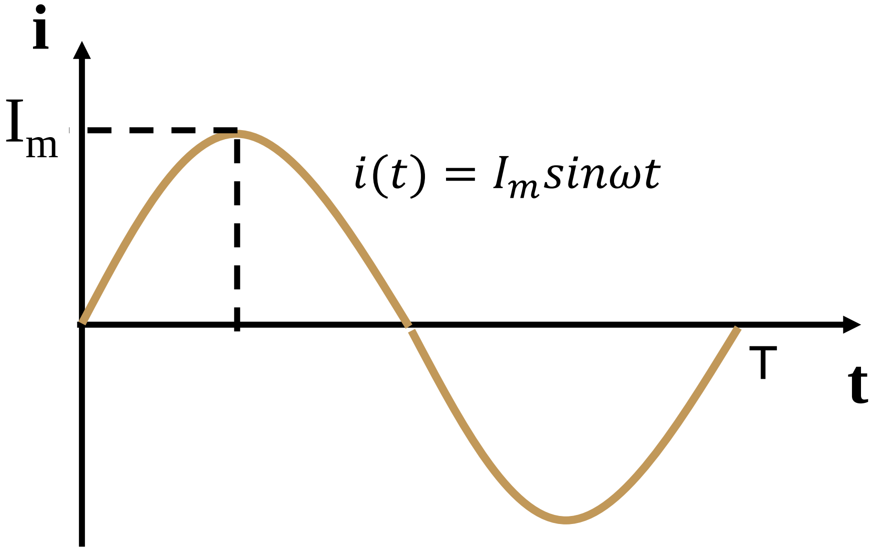
For a periodic function f(t) with period T,

$$\boldsymbol{F_{avg} = \frac{1}{T}\int_0^T f(t)dt}$$

For sinusoidal signal,

$$I_{avg} = \frac{1}{T/2}\int_0^{T/2} I_m \sin\omega t dt$$

$$\boxed{\boldsymbol{I_{avg} = \frac{2I_m}{\pi}}}$$



RMS value of Sinusoidal Alternating Current

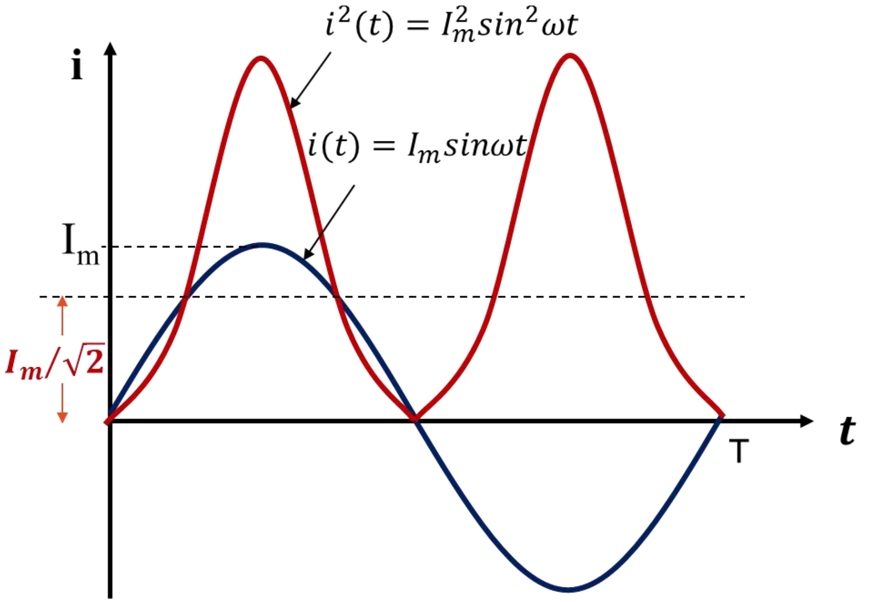
Definition: *"It is that value of direct current which when flowing through a circuit produces the same amount of heat for a given interval of time as that of the alternating current flowing through the same circuit during the same time"*

For a periodic function f(t) with period T,

$$\boldsymbol{F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 sin^2\omega t \; dt}$$

$$\boxed{\boldsymbol{I_{RMS} = \frac{I_m}{\sqrt{2}}}}$$

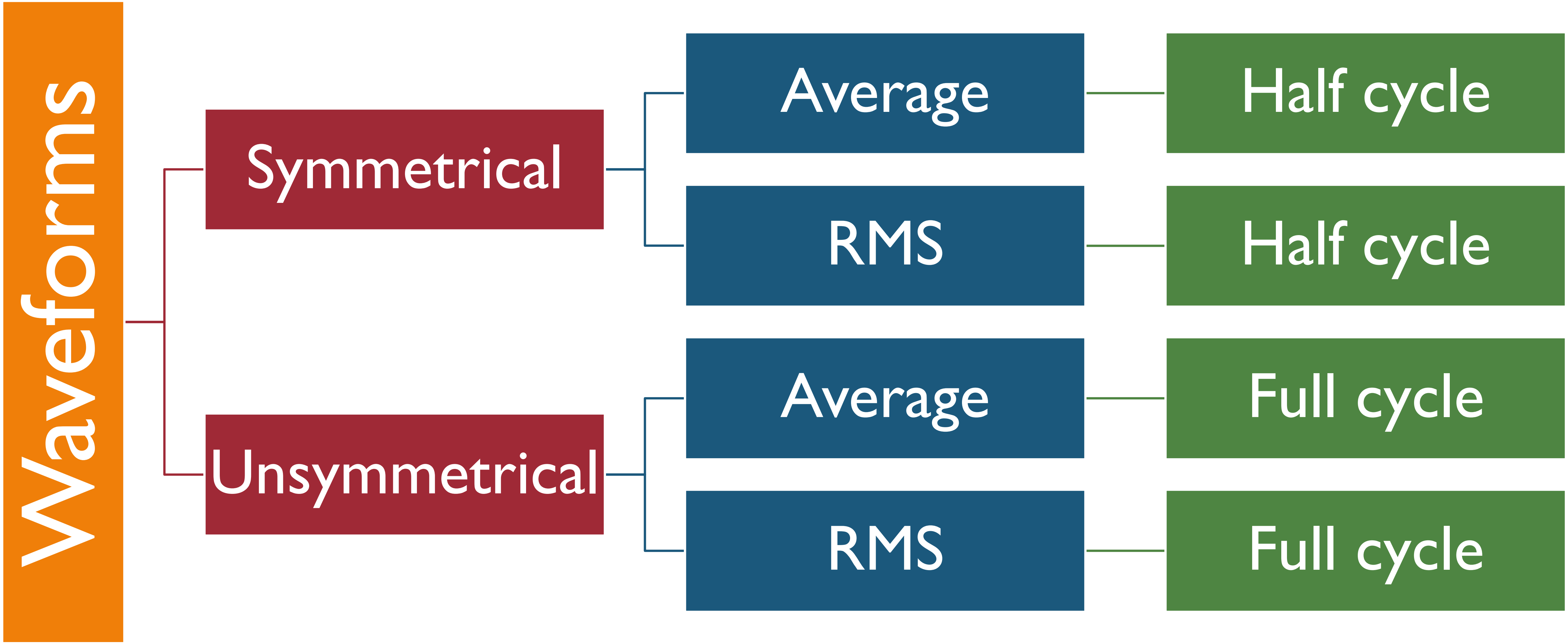


Form Factor & Peak Factor

$$\boldsymbol{Form\ Factor = \frac{RMS\ Value}{Average\ Value} = 1.11\ for\ sinusoidal}$$

$$\boldsymbol{Peak\ Factor = \frac{Maximum\ Value}{RMS\ Value} = \sqrt{2}\ for\ sinusoidal}$$

Full cycle and half cycle - considerations



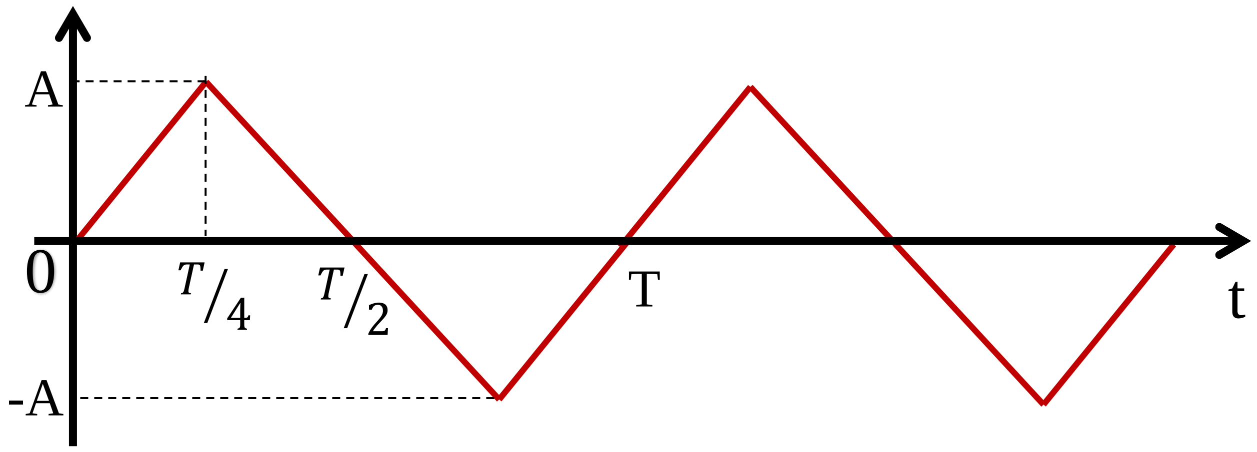
Exercise 1

If an alternating voltage has the equation **$v(t) = 141.4 \sin 377t$**, calculate

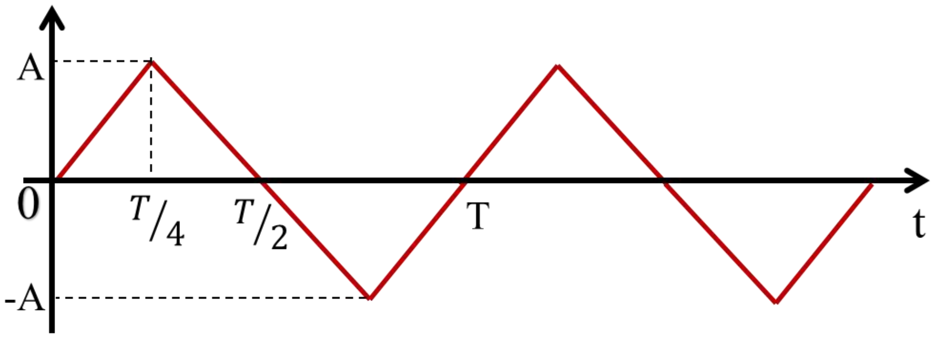
a. Maximum voltage value
b. RMS value of the voltage
c. Frequency
d. The instantaneous voltage when $t = 3ms$

Exercise 2

Find the Average value and RMS value of the given non-sinusoidal waveform



Solution:



Average Value

$$I_{avg} = \frac{1}{T/4}\int_0^{T/4} f(t).dt$$

$$I_{avg} = \frac{4}{T}\int_0^{T/4} \frac{4At}{T}.dt$$

$$I_{avg} = \frac{4}{T} \times \frac{4A}{T} \times \left[\frac{t^2}{2}\right]_0^{\frac{T}{4}}$$

$$I_{avg} = \frac{8A}{T^2} \times \left[\frac{T^2}{16}\right]$$

$$\boldsymbol{I_{avg} = \frac{A}{2}}$$

RMS Value

$$I_{rms}^2 = \frac{1}{T/4}\int_0^{T/4} f^2(t).dt$$

$$I_{rms}^2 = \frac{4}{T}\int_0^{T/4} \frac{16A^2t^2}{T^2}.dt$$

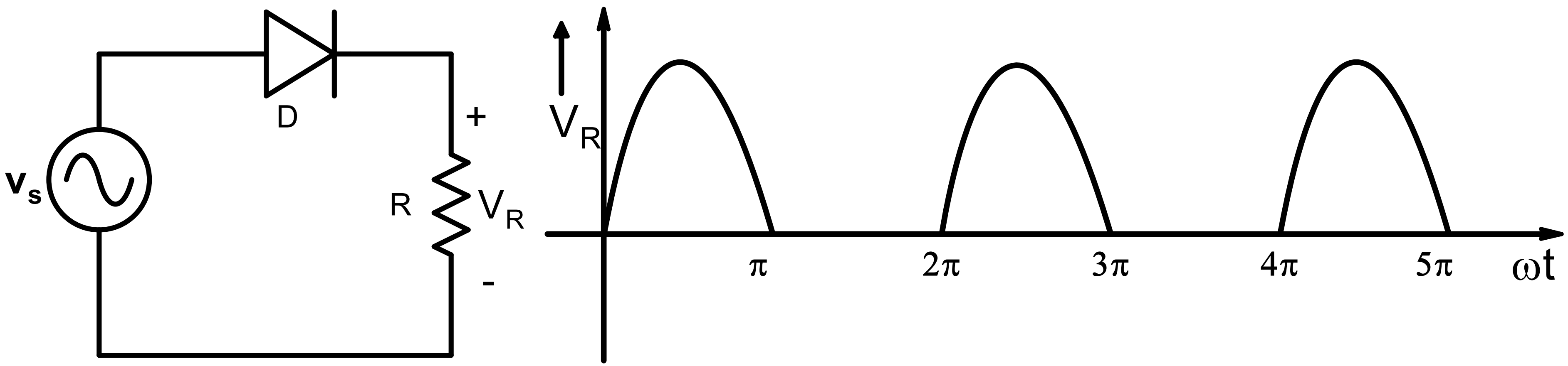
$$I_{rms}^2 = \frac{4}{T} \times \frac{16A^2}{T^2} \times \left[\frac{t^3}{3}\right]_0^{\frac{T}{4}}$$

$$I_{rms}^2 = \frac{4}{T} \times \frac{16A^2}{T^2} \times \frac{1}{3} \times \left[\frac{T^3}{4^3}\right]$$

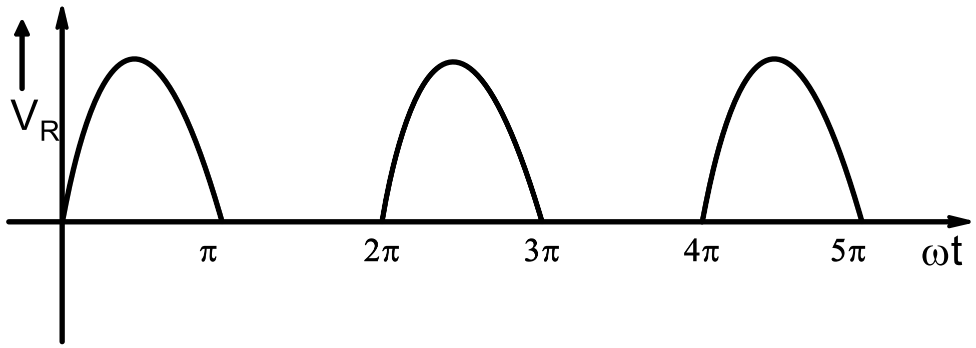
$$\boldsymbol{I_{rms} = \frac{A}{\sqrt{3}}}$$

Exercise 3

For the circuit shown below, sketch the voltage across the resistance, & then find the Average value and RMS value of the same.



Solution:



Average Value

$$V_{avg} = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\omega t\,.\,d\omega t + \int_{\pi}^{2\pi} 0.\,d\omega t\right]$$

$$V_{avg} = \frac{V_m}{2\pi}(-\cos\omega t)_0^{\pi}$$

$$V_{avg} = \frac{-V_m}{2\pi}(-1-1)$$

$$\boldsymbol{V_{avg} = \frac{V_m}{\pi}}$$

RMS Value

$$V_{rms}^2 = \frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 sin^2\omega t.\,d\omega t + \int_{\pi}^{2\pi} 0.\,d\omega t\right]$$

$$V_{rms}^2 = \frac{V_m^2}{2\pi}\left[\int_0^{\pi} \frac{1-\cos 2\omega t}{2}.\,d\omega t\right]$$

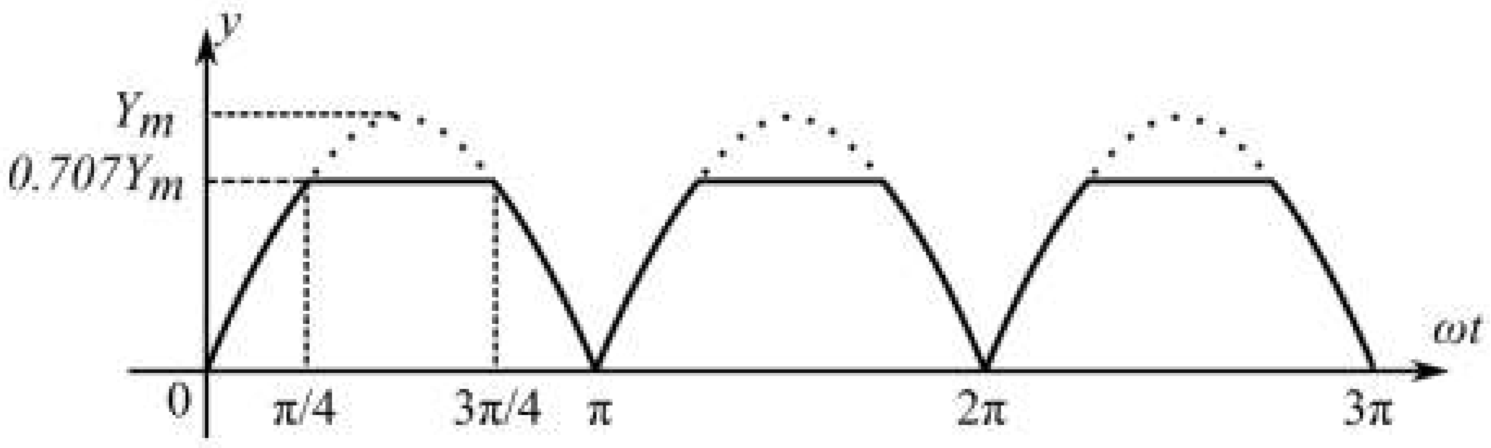
$$V_{rms}^2 = \frac{V_m^2}{4\pi}\left[\omega t|_0^{\pi} - \sin 2\omega t|_0^{\pi}\right]$$

$$V_{rms}^2 = \frac{V_m^2}{4\pi}[\pi]$$

$$\boldsymbol{V_{rms} = \frac{V_m}{2}}$$

Homework

Find the average and RMS value of the waveform



Basic Electrical Technology

[ELE 1051]

CHAPTER 3 - SINGLE PHASE AC CIRCUITS

(3.2)

Topics covered…

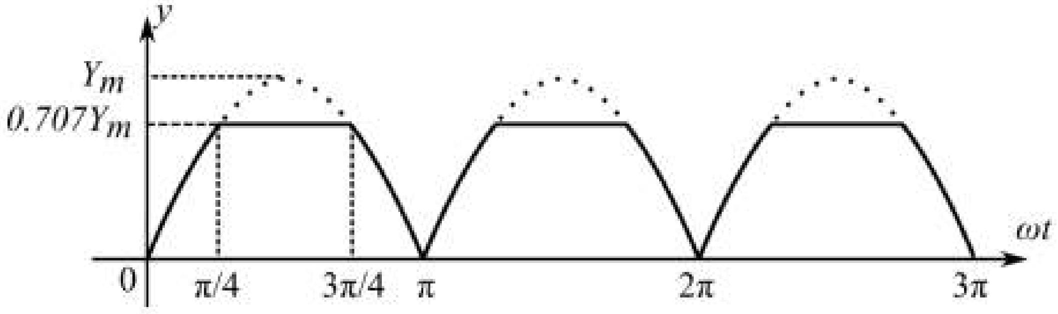
Last class

- Comparison of DC & AC
- How is AC generated?
- Terminologies of AC
- Average value of an alternating waveform
- RMS value of an alternating waveform

Today

- Complex numbers
- Representing AC
- R, L, C circuit response with AC supply
- Power associated with a pure R, L, C

Homework Solution:



Average Value

$$V_{avg} = \frac{1}{\pi}\left[\int_{0}^{\pi/4} Y_m \sin\omega t \,.\, d\omega t \;+\; \int_{\pi/4}^{3\pi/4} 0.707Y_m.\, d\omega t + \int_{3\pi/4}^{\pi} Y_m \sin\omega t \,.\, d\omega t\right]$$

$$V_{avg} = \frac{1}{\pi}[0.2928Y_m + 1.1105Y_m + 0.2928Y_m]$$

$$\boldsymbol{V_{avg} = 0.5398Y_m}$$

RMS Value

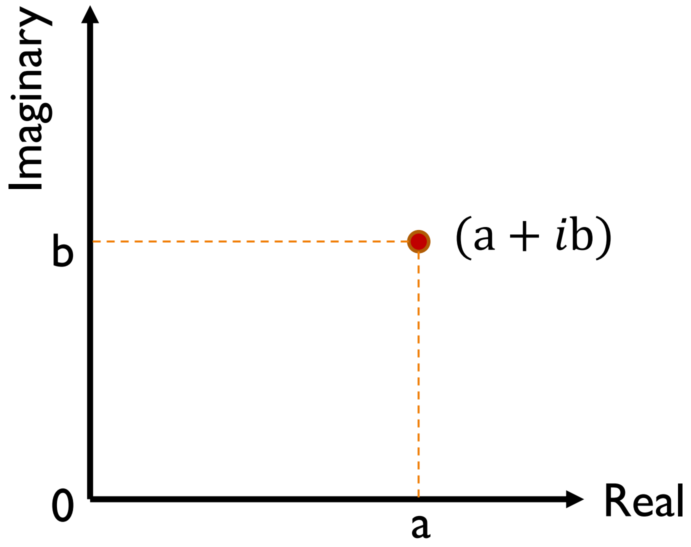
$$V_{rms}^2 = \frac{Y_m^2}{\pi}\left[\int_{0}^{\pi/4} sin^2\omega t.\, d\omega t \;+\; \int_{\pi/4}^{3\pi/4} 0.707^2\, d\omega t + \int_{3\pi/4}^{\pi} sin^2\omega t.\, d\omega t\right]$$

$$\boldsymbol{V_{rms} = 0.5837Y_m}$$

Complex Number

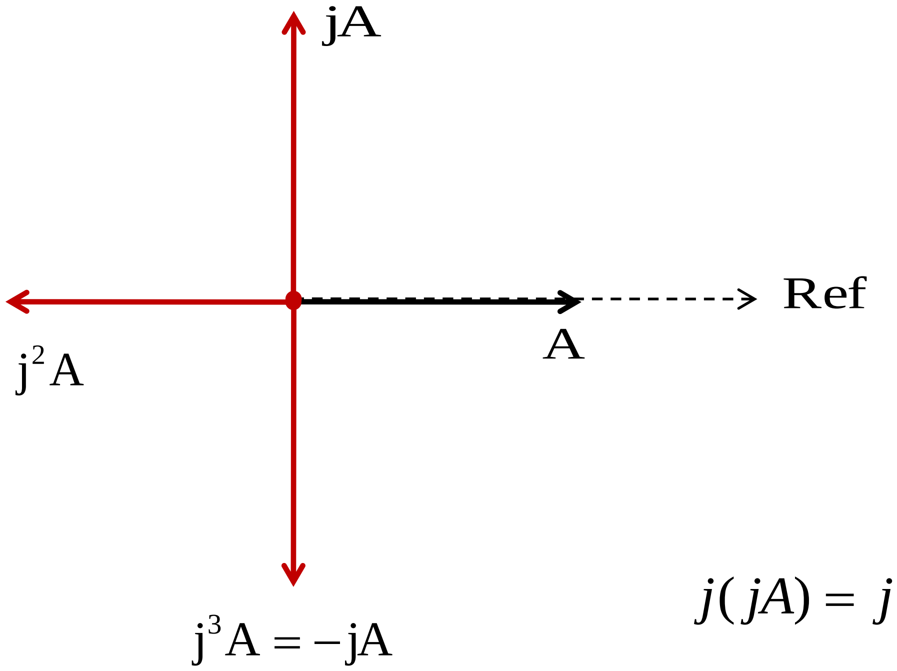
- A **complex number** is of the form **a + *i* b**

- Represented on complex plane as:



The operator 'j'

$$j = 1\angle 90°$$



$$j(jA) = j^2 A = -A$$

$$\boxed{\text{Therefore,} \quad j^2 = -1; \quad j = \sqrt{-1}}$$

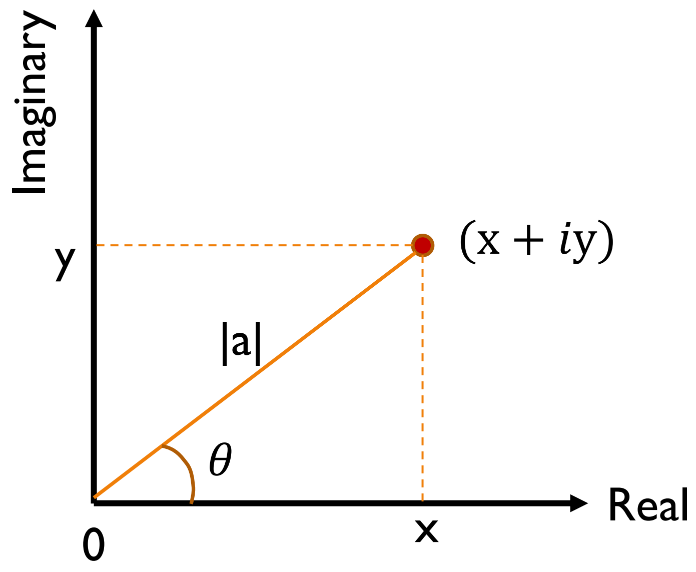
The operator 'j' rotates the given vector by 90 degrees in anti-clockwise direction

Rectangular ↔ Polar conversion

- **Rectangular to polar:**

$$|a| = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$



- **Polar to Rectangular:**

$$x = |a|\cos\theta$$

$$y = |a|\sin\theta$$

Representation of a complex number

- **Rectangular form:** $\mathbf{a = x \pm \mathit{jy}}$
- **Polar form:** $\mathbf{a = |\mathit{a}|\angle \pm \theta}$
- **Exponential form:** $\mathbf{a = |\mathit{a}|e^{\pm \mathit{j}\theta}}$
- **Trigonometric form:** $\mathbf{a = |a|(\cos\theta \pm \mathit{j}\sin\theta)}$

Rectangular ↔ Polar conversion

- Convert the following into polar form

1) $3 + j\,4$ $= 5\ \angle 53.13°$

2) $8 + j\,6$ $= 10\ \angle 36.87°$

3) $8 - j\,6$ $= 10\ \angle -36.87°$

- Convert the following into rectangular form

1) $5\ \angle 30°$ $= 4.33 + j\,2.5$

2) $3\ \angle -60°$ $= 1.5 - j\,2.59$

3) $-(10\ \angle 45°)$ $= -\ 7.07 - j\,7.07$

Representing AC

- Consider three sinusoidal signals $x(t), y(t)$ & $z(t)$ with same frequency

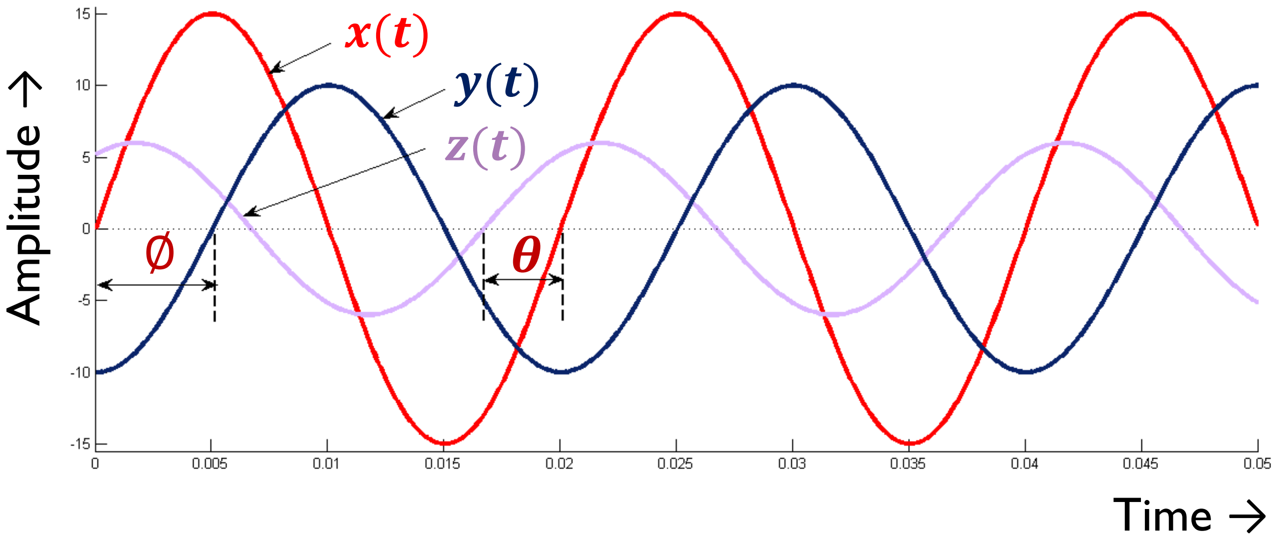
Mathematical Representation

$$x(t) = X_m sin(\omega t)$$

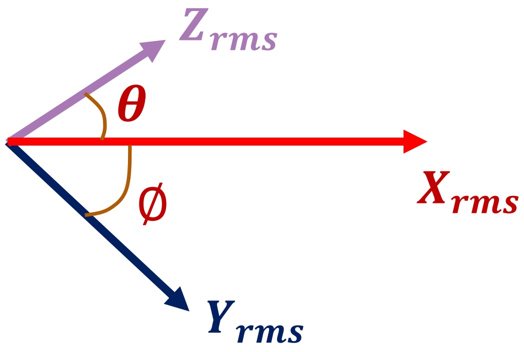
$$y(t) = Y_m \sin(\omega t - \emptyset)$$

$$z(t) = Z_m \sin(\omega t + \theta)$$

Graphical Representation

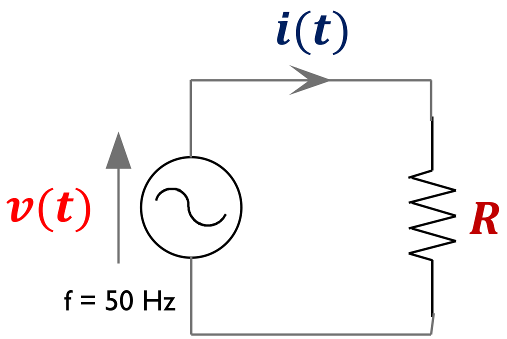


Phasor Representation



- Representing the relationship between sinusoidal signals with same frequency in graphical or mathematical form is tedious
- Phasor representation is often used

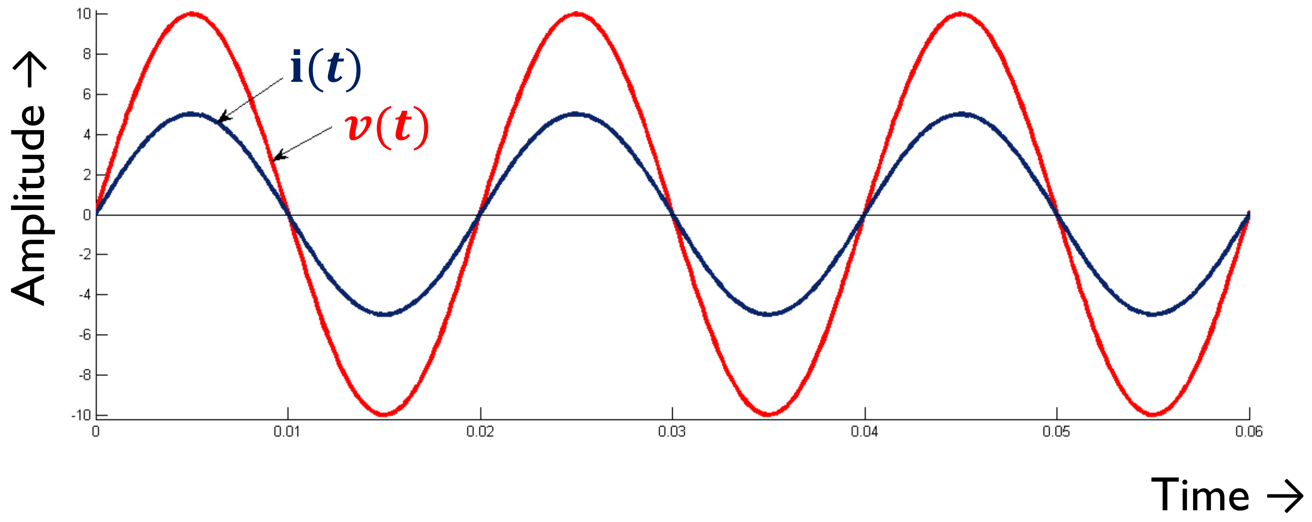
R circuit response with AC supply



$$i(t) = \frac{v(t)}{R}$$

'Current through the resistor is in phase with the voltage across it'

Graphical Representation

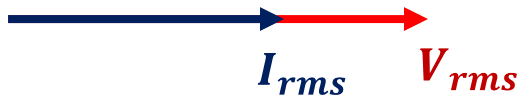


Mathematical Representation

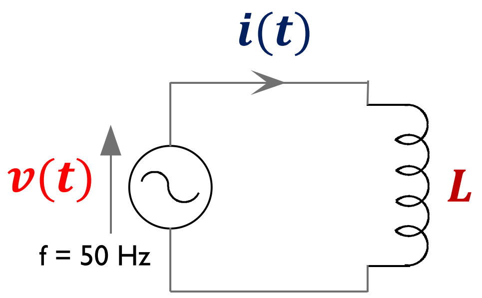
$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t)$$

Phasor Representation



L circuit response with AC supply



$$i(t) = \frac{1}{L}\int v(t)dt$$

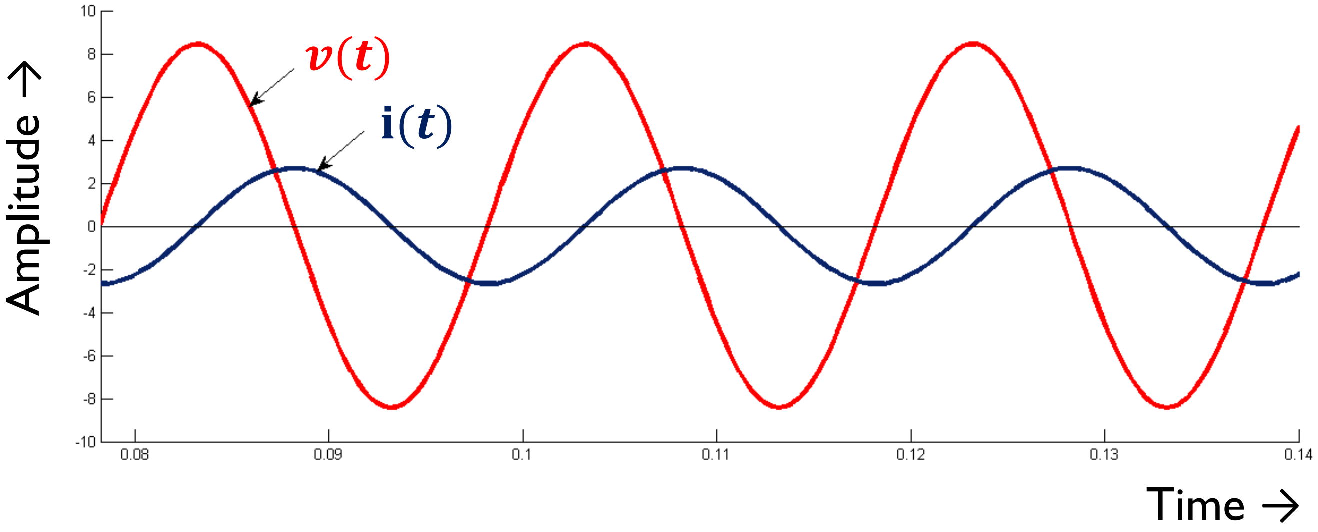
'Current through the inductor lags the voltage across it by 90°'

$\bar{V} = V\angle 0°$ $\quad$ $\bar{I} = I\angle -90°$

$$\frac{\bar{\boldsymbol{V}}}{\bar{\boldsymbol{I}}} = \frac{\boldsymbol{V\angle 0°}}{\boldsymbol{I\angle -90°}} = \boldsymbol{jX_L} \quad where\ \frac{V}{I} = X_L$$

$\boldsymbol{X_L}$ *is called* ***Inductive Reactance***

Graphical Representation



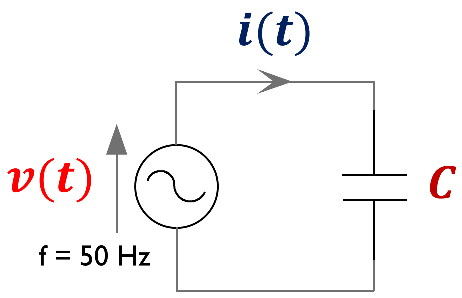
Mathematical Representation

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t - 90°)$$

Phasor Representation

C circuit response with AC supply



$$i(t) = C\frac{dv(t)}{dt}$$

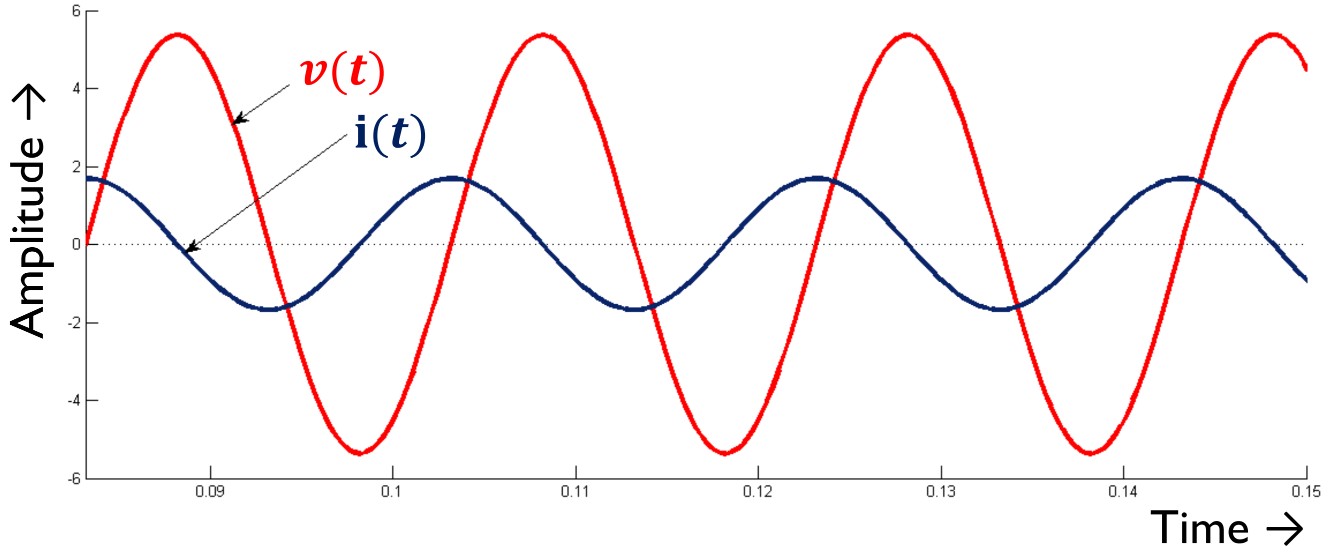
'Current through the capacitor leads the voltage across it by 90°'

$\bar{V} = V\angle 0°$ $\qquad$ $\bar{I} = I\angle 90°$

$$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0°}{I\angle 90°} = -jX_C \quad where\ \frac{V}{I} = X_C$$

X_C *is called* ***Capacitive Reactance***

<u>*Graphical Representation*</u>



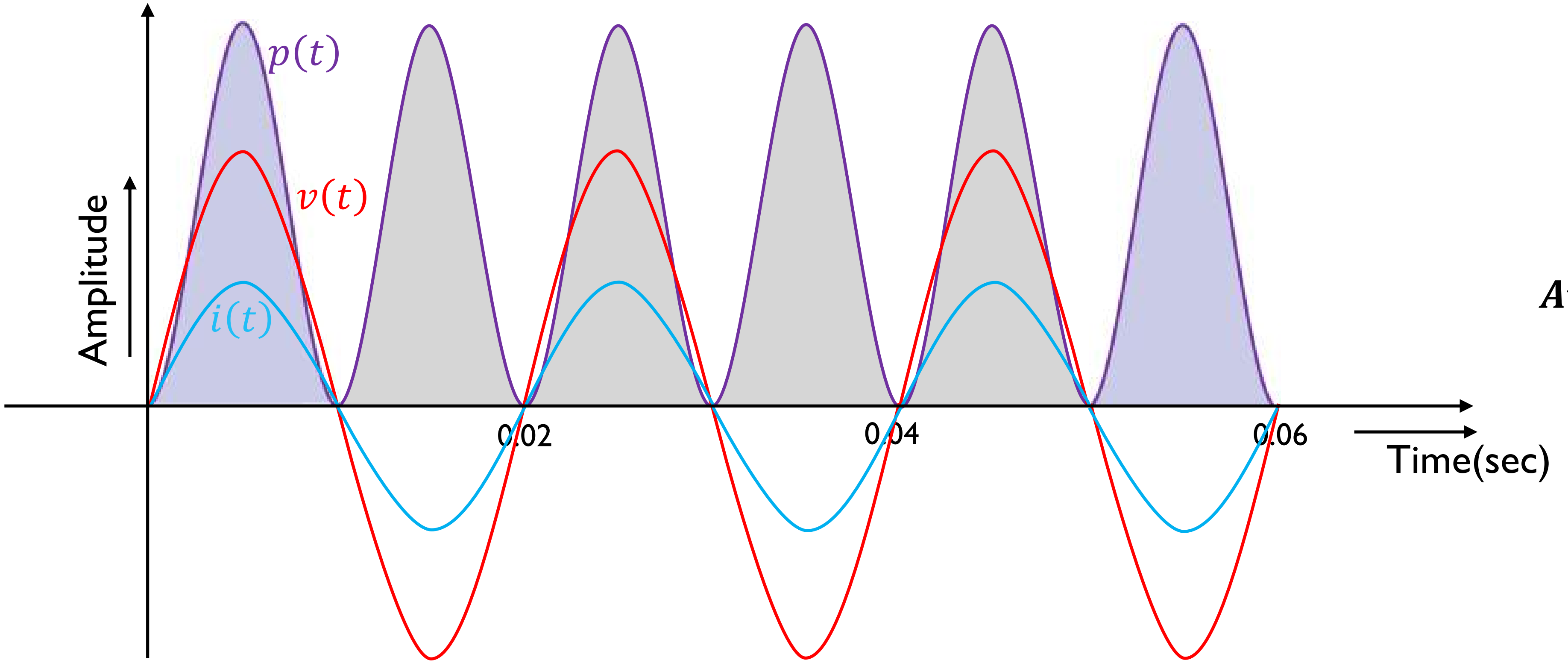
<u>*Mathematical Representation*</u>

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t + 90°)$$

<u>*Phasor Representation*</u>

Power Associated - Pure Resistive Circuit



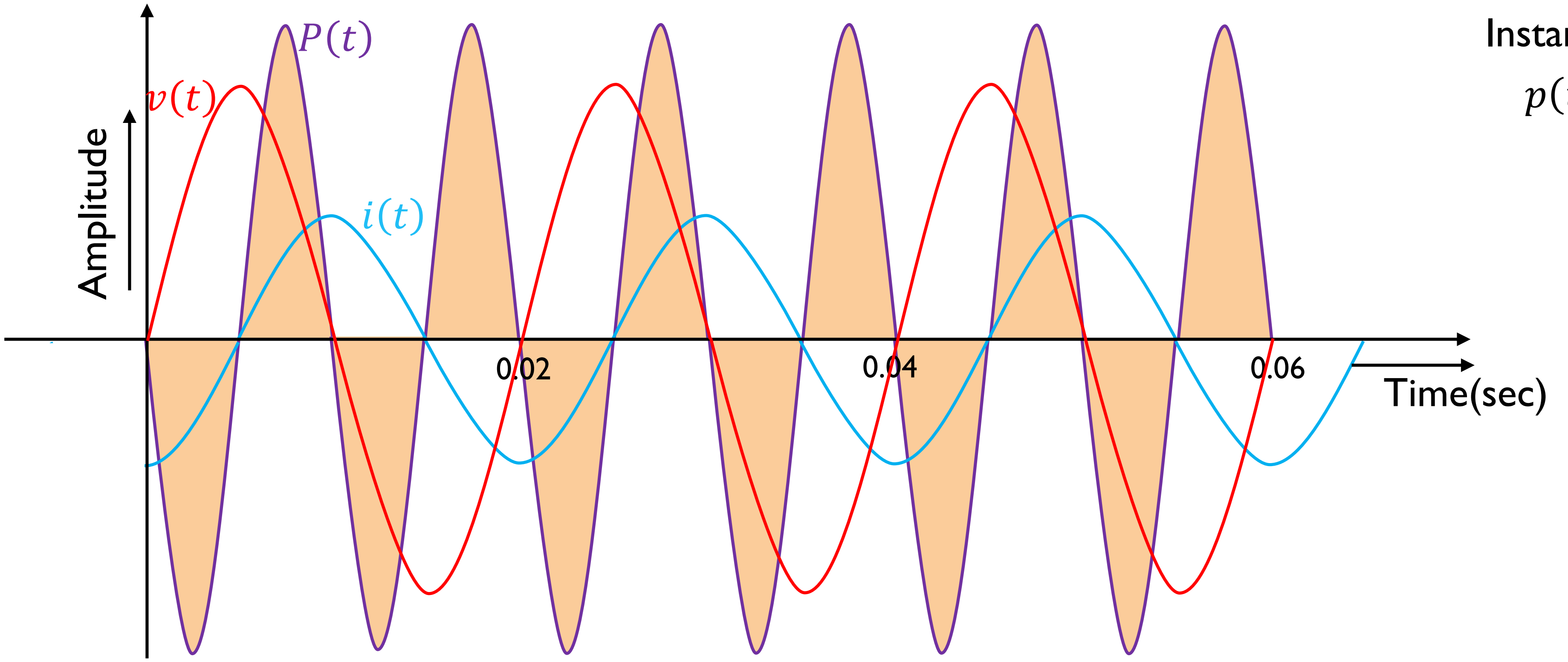
Instantaneous power,

$$p(t) = v(t).\,i(t) = V_m I_m sin^2 \omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = \frac{V_m I_m}{2} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R}}$$

Power Associated – Pure Inductive Circuit



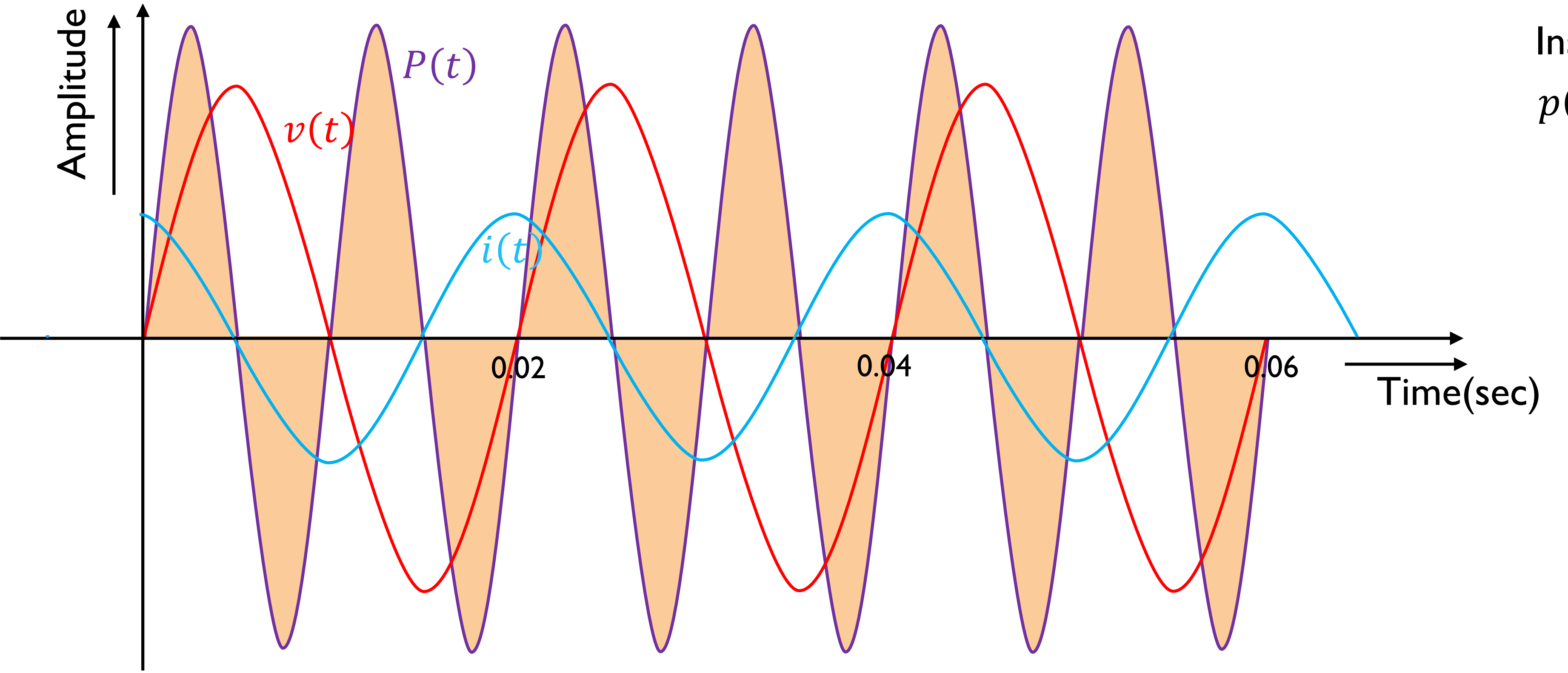
Instantaneous power,

$$p(t) = v(t).\, i(t)$$
$$= V_m I_m \sin \omega t \, . \sin(\omega t - 90\,°)$$
$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = 0}}$$

Power Associated – Pure capacitive Circuit



Instantaneous power,

$$p(t) = v(t).\,i(t)$$
$$= V_m I_m \sin \omega t \,.\, \sin(\omega t + 90\,°)$$
$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{P_{avg} = 0}$$

Basic Electrical Technology

[ELE 1051]

CHAPTER 3 - SINGLE PHASE AC CIRCUITS

(3.3)

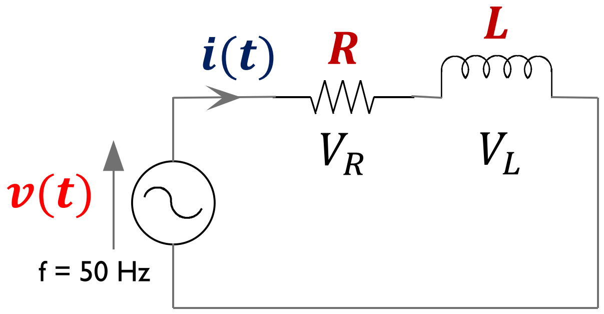
Summary of previous classes

- Comparison of DC & AC
- How is AC generated?
- Terminologies of AC
- Average value of an alternating waveform
- RMS value of an alternating waveform
- Complex numbers
- Representing AC
- R, L, C circuit response with AC supply
- Power associated with a pure R, L, C

Topics covered today

- RL, RC, RLC circuit response with AC supply
- Power associated with a series RL, RC circuits
- Example problems
- Loads in parallel

RL circuit analysis



Let $\bar{I}$ *be along the reference*

$$\overline{V_R} = \bar{I}R$$

$$\overline{V_L} = j\bar{I}X_L$$

$$\bar{V} = \overline{V_R} + \overline{V_L} = |V|\angle\emptyset$$

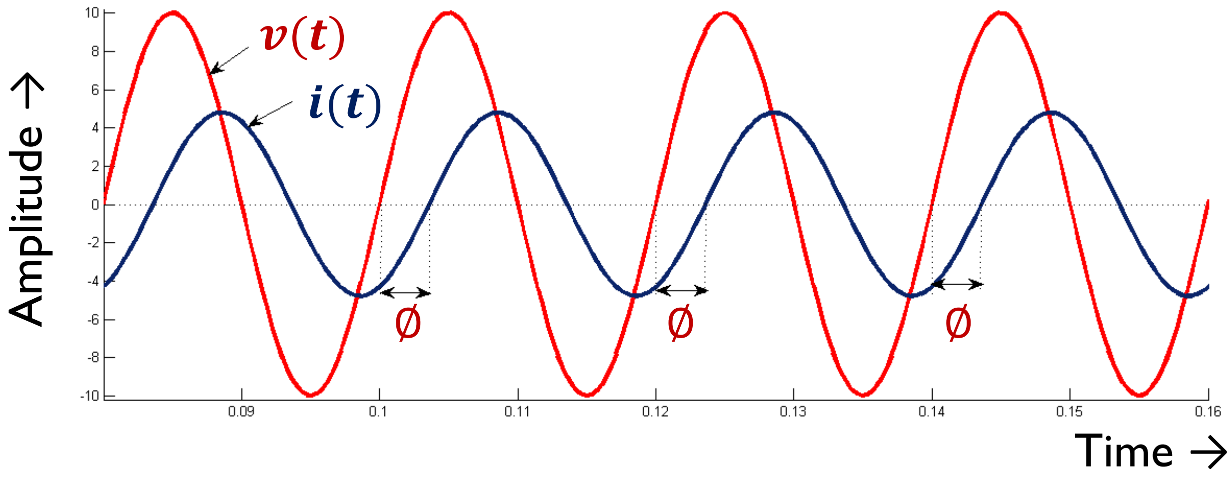
Mathematical Representation

$$i(t) = I_m \sin(\omega t)$$

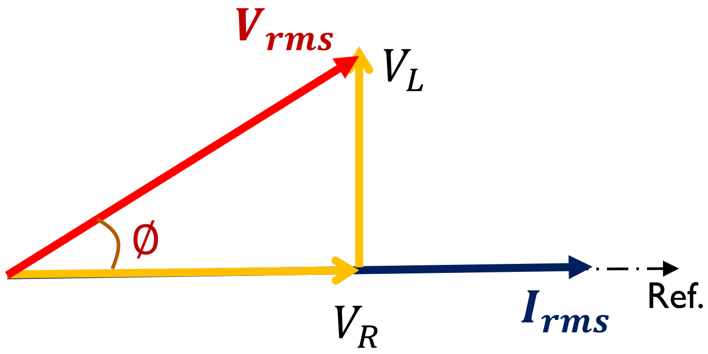
$$v(t) = V_m \sin(\omega t + \emptyset)$$

$\emptyset$ – *Phase Angle*

Graphical Representation



Phasor Representation



Impedance

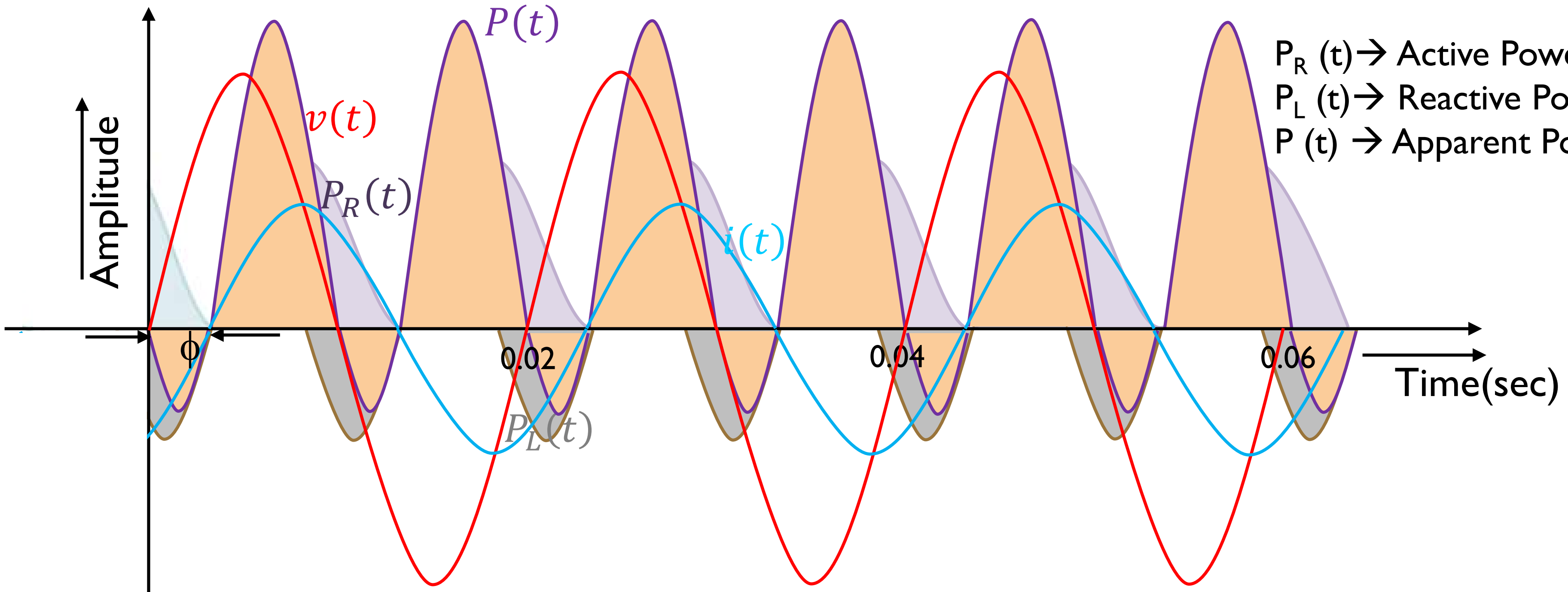
$$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R + jX_L)}{\bar{I}} = R + jX_L = |Z|\angle\emptyset$$

Z – *Impedance of the circuit*

$$\therefore \boldsymbol{R = |Z|\cos\emptyset} \qquad \boldsymbol{X_L = |Z|\sin\emptyset}$$

$$|Z| = \sqrt{R^2 + X_L^2} \qquad \emptyset = \tan^{-1}\frac{X_L}{R}$$

Power associated - RL circuit



P_R (t)→ Active Power
P_L (t)→ Reactive Power
P (t) → Apparent Power

Instantaneous power,

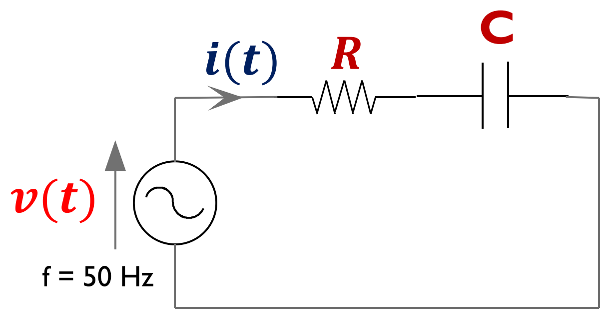
$$p(t) = v(t).i(t)$$
$$= V_m I_m \sin\omega t \,.\sin(\omega t + \emptyset)$$
$$= V_{rms} I_{rms}[\cos\emptyset - \cos(2\omega t + \emptyset)]$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2}\cos\emptyset}$$

$$\boxed{\mathbf{P_{avg} = V_{rms} I_{rms} \cos\emptyset}}$$

$\mathbf{\cos\emptyset}$ *is called the* ***Power Factor***

RC circuit analysis



Let $\bar{I}$ *be along the reference*

$$\overline{V_R} = \bar{I}R$$

$$\overline{V_C} = -j\bar{I}X_C$$

$$\bar{V} = \overline{V_R} + \overline{V_C} = |V|\angle - \emptyset$$

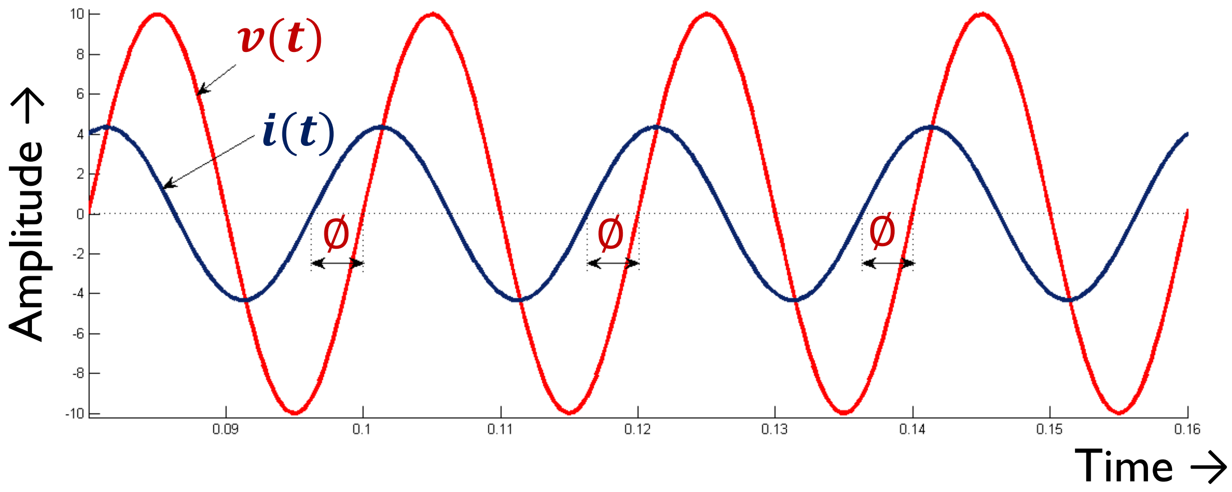
Mathematical Representation

$$i(t) = I_m \sin(\omega t)$$

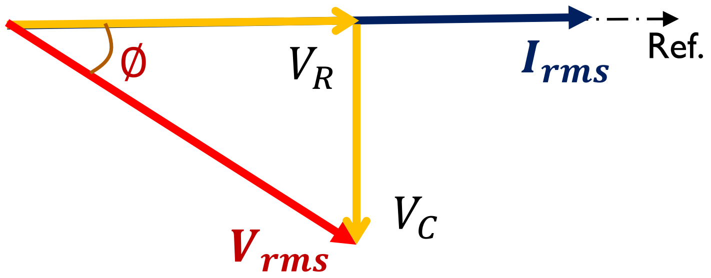
$$v(t) = V_m \sin(\omega t - \emptyset)$$

$\emptyset$ *– Phase Angle*

Graphical Representation



Phasor Representation



Impedance

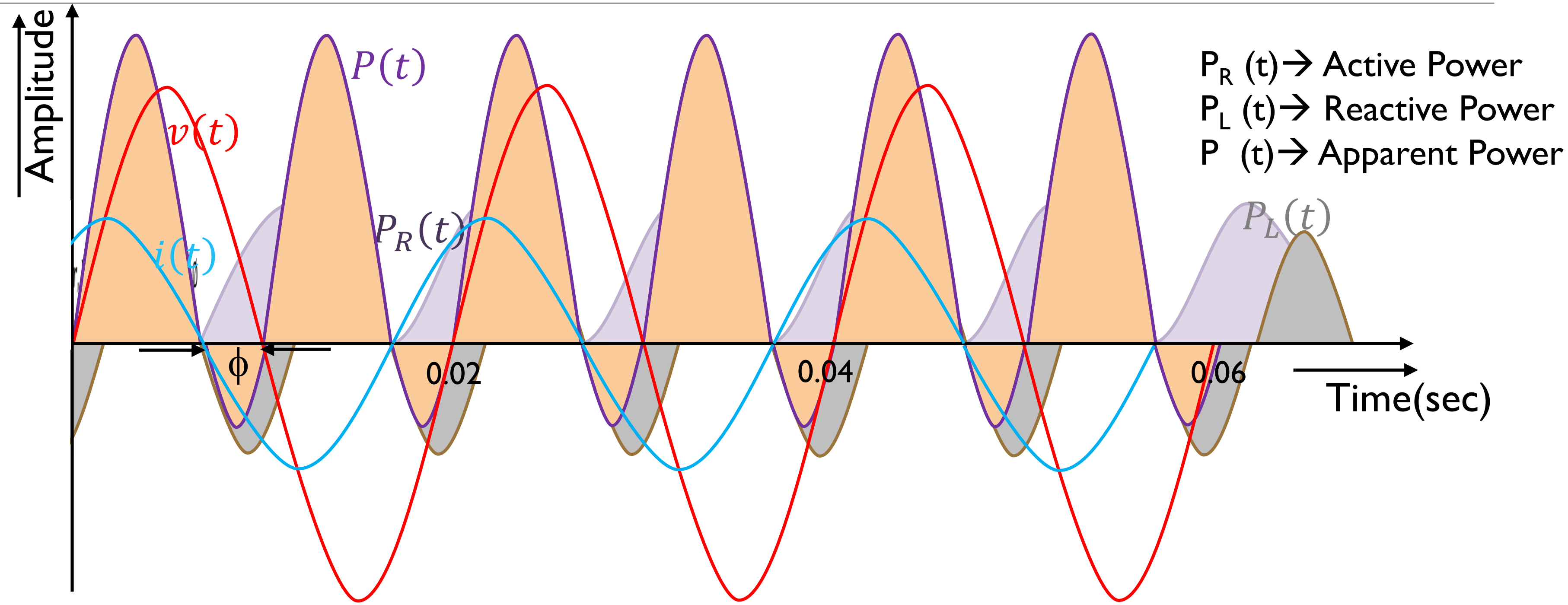
$$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R - jX_L)}{\bar{I}} = R - jX_L = |Z|\angle - \emptyset$$

Z *– Impedance of the circuit*

$$\therefore R = |Z|\cos\emptyset \qquad X_C = |Z|\sin\emptyset$$

$$|Z| = \sqrt{R^2 + X_C^2} \qquad \emptyset = \tan^{-1}\frac{X_C}{R}$$

Power associated - RC circuit



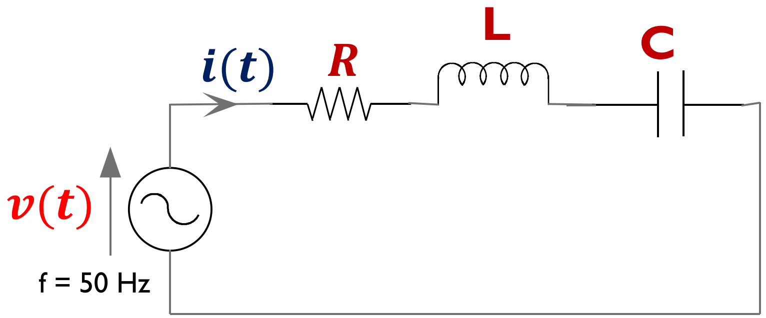
Instantaneous power,

$$p(t) = v(t).\, i(t)$$
$$= V_m I_m \sin \omega t\, .\sin(\omega t - \emptyset)$$
$$= V_{rms} I_{rms}[\cos \emptyset - \cos(2\omega t - \emptyset)]$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2}\cos \emptyset}$$

$$\boxed{\mathbf{P_{avg} = V_{rms} I_{rms} \cos \emptyset}}$$

RLC circuit



$Let\ i(t)\ be\ the\ reference$

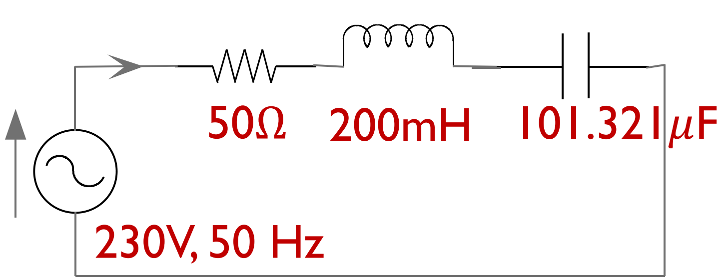
$$\boldsymbol{Impedance, Z = R + j(X_L \sim X_c)}$$

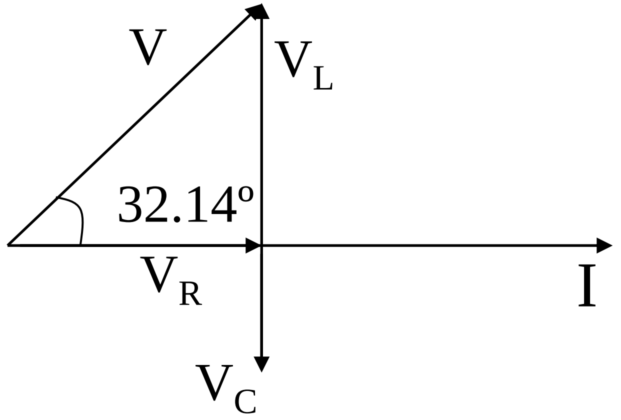
$if\ X_L = X_C$	$\Longrightarrow$	$Resistive\ circuit$ $(Resonance\ condition)$
$if\ X_L > X_C$	$\Longrightarrow$	$RL\ series\ circuit$
$if\ X_L < X_C$	$\Longrightarrow$	$RC\ series\ circuit$

Illustration 1

A resistance of 50Ω is connected in series with an inductance of 200mH and capacitance of 101.321μF across a 230V, 50 Hz, single phase AC supply. Obtain,

a) *Impedance of the circuit*
b) *Current drawn*
c) *Power factor*
d) *Power consumed*
e) *Phasor diagram*





$$X_L = 2 \times \pi \times 50 \times 0.2 = 62.8315\Omega$$

$$X_c = \frac{1}{2 \times \pi \times 50 \times 101.321\mu} = 31.4159\Omega$$

$$PF = \cos(32.14) = \mathbf{0.846\ lag}$$

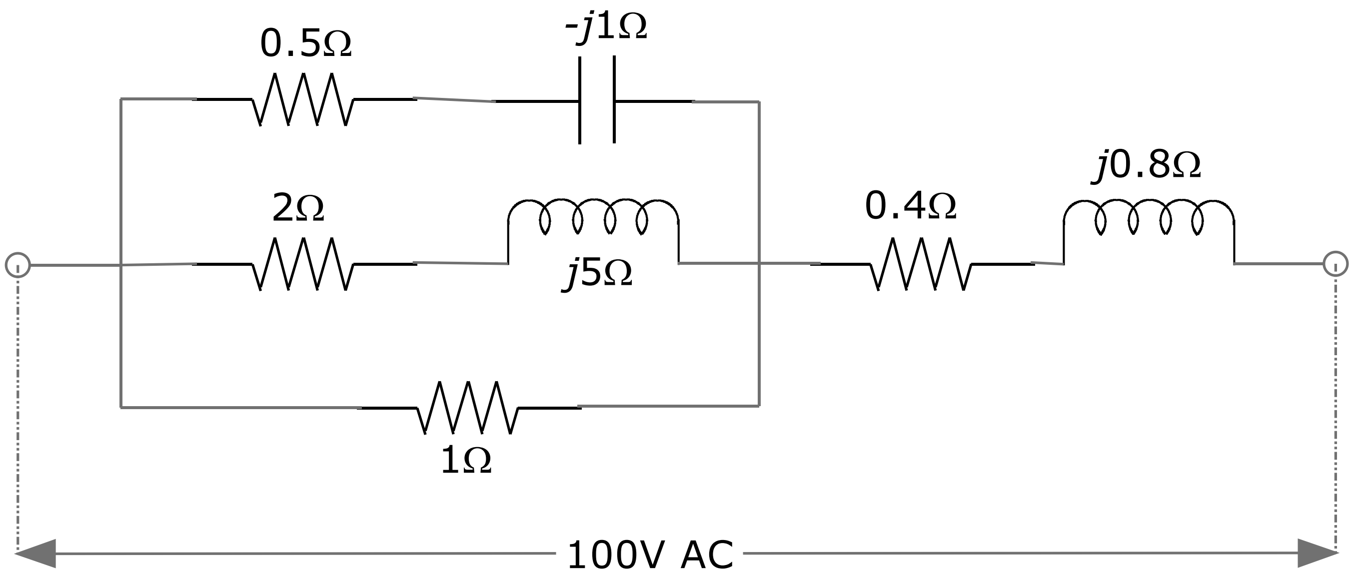
$$Z = R + jX_L - jX_c = \mathbf{50 + j31.4156\Omega = 59.050\angle 32.14°\ \Omega}$$

$$I = \frac{230\angle 0}{59.05\angle 32.14} = \mathbf{3.898\angle -32.14°A}$$

$$P = |V_{rms}||I_{rms}|cos\emptyset$$
$$= 230 \times 3.898 \times 0.846 = \mathbf{759.15W}$$

Illustration 2

Determine the impedance of the circuit shown and the power consumed in each branch



$Z_1 = 0.5 - j1\Omega$ $\quad Z_3 = 1\Omega$

$Z_2 = 2 + j5\Omega$ $\quad Z_4 = 0.4 + j0.8\Omega$

$$\bar{I} = \frac{100\angle 0}{1.12\angle 29.5} = \mathbf{89.285\angle - 29.5A} = \bar{I_4}$$

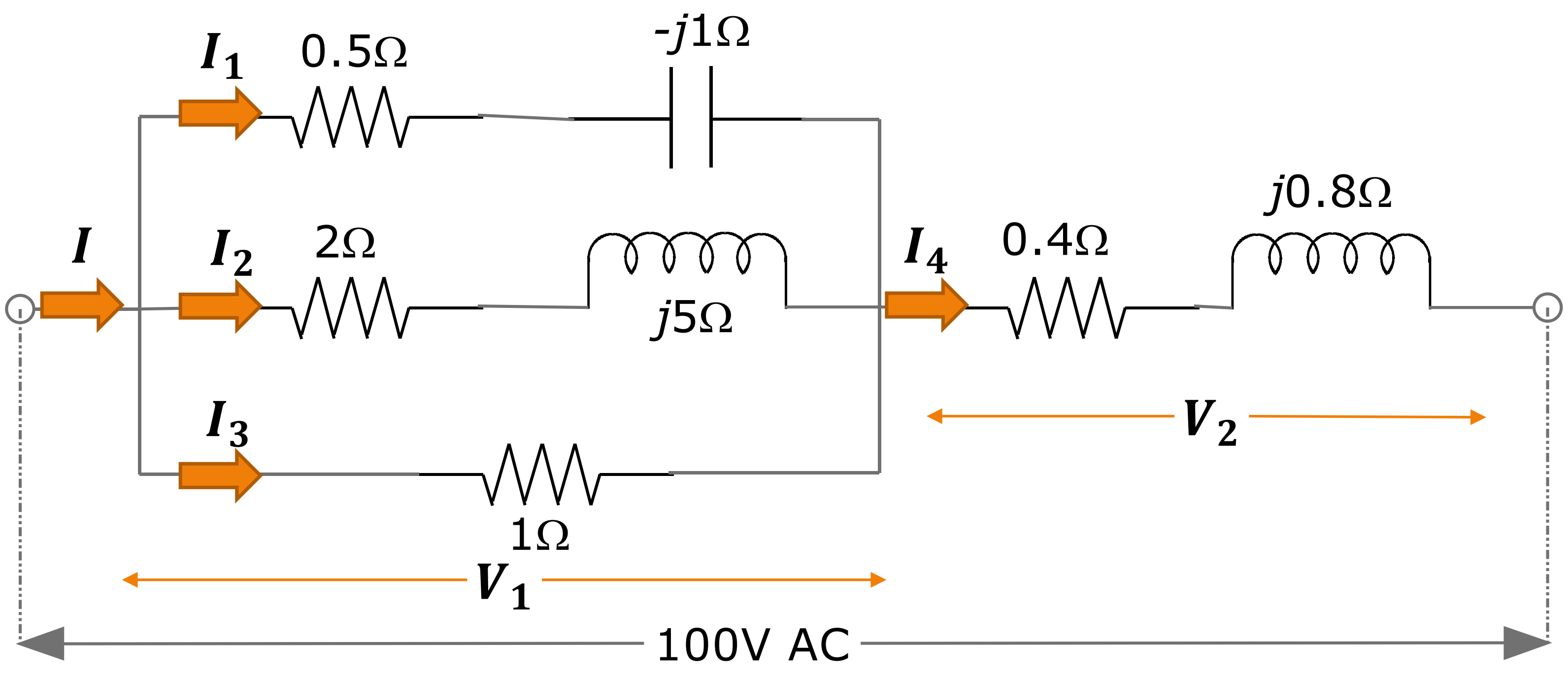
$$Z_{eq} = (Z_1||Z_2||Z_3) + Z_4 = \mathbf{1.12\angle 29.5°\Omega}$$

$$\overline{\boldsymbol{V_2}} = \overline{\boldsymbol{I_4}} \times \boldsymbol{Z_4} = \mathbf{79.85\angle 33.934\ V}$$

$$\bar{V} = \bar{V_1} + \bar{V_2} \qquad \overline{\boldsymbol{V_1}} = \mathbf{55.91\angle - 52.868\ V}$$

$$\overline{\boldsymbol{I_1}} = \frac{\overline{\boldsymbol{V_1}}}{\boldsymbol{Z_1}} = \mathbf{50.00\angle 10.565\ A} \qquad \overline{\boldsymbol{I_2}} = \mathbf{10.38\angle - 121.068\ A} \qquad \overline{\boldsymbol{I_3}} = \mathbf{55.91\angle - 52.868\ A}$$

$$\boldsymbol{P_1} = |\boldsymbol{I_1}|^2 \times \boldsymbol{R_1} = \mathbf{1.25\ kW} \qquad P_2 = 0.215\ kW \qquad P_3 = 3.125\ kW \qquad P_4 = 3.188\ kW$$



Basic Electrical Technology

[ELE 1051]

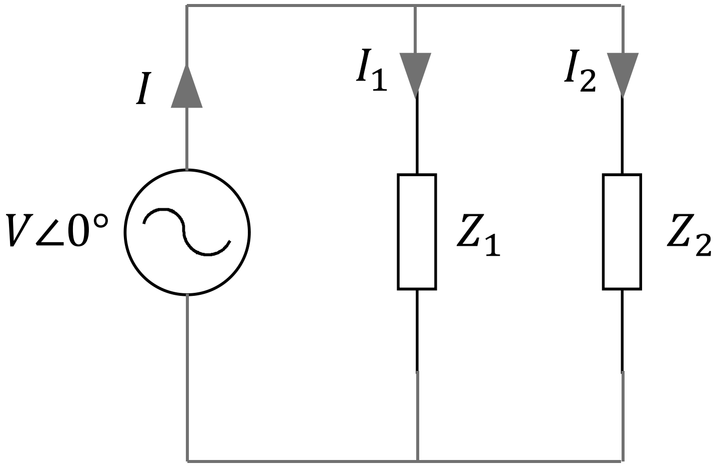
CHAPTER 3 - SINGLE PHASE AC CIRCUITS

(3.4)

Topics covered today

- Loads in parallel
- AC circuit equations and solving
- Tutorial 1

Impedance in parallel



Let $\quad Z_1 = R_1 + jX_1 \qquad Z_2 = R_2 + jX_2$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad I_1 = I * \frac{Z_2}{Z_1 + Z_2} \qquad I_2 = I * \frac{Z_1}{Z_1 + Z_2}$$

$Y_{eq} = Y_1 + Y_2 \quad$ Y: *Admittance*

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_1} = \frac{1}{(R_1 + jX_1)} * \frac{(R_1 - jX_1)}{(R_1 - jX_1)} = \frac{R_1}{(R_1^2 + X_1^2)} - j\frac{X_1}{(R_1^2 + X_1^2)} = G_1 - jB_1$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 + jX_2} = \frac{1}{(R_2 + jX_2)} * \frac{(R_2 - jX_2)}{(R_2 - jX_2)} = \frac{R_2}{(R_2^2 + X_2^2)} - j\frac{X_2}{(R_2^2 + X_2^2)} = G_2 - jB_2$$

G: *Conductance* $\qquad B$: *Susceptance*

$$\mathbf{Y_{eq} = (G_1 + G_2) - j(B_1 + B_2) = G_{eq} - jB_{eq}}$$

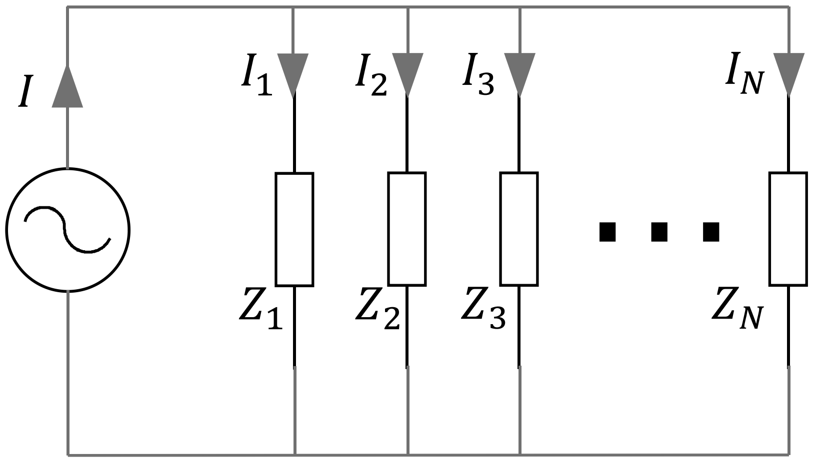
Impedance in parallel

For 'N' impedances connected in parallel,

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}$$

$$Y_{eq} = Y_1 + Y_2 + Y_3 + \cdots + Y_N$$

$$\mathbf{Y_{eq} = G_{eq} \pm \boldsymbol{j}B_{eq}}$$



$$I_1 = VY_1; I_2 = VY_2; I_3 = VY_3; \ldots \ldots I_N = VY_N$$

$$I = I_1 + I_2 + I_3 + \cdots + I_N = VY_{eq}$$

Network equations for AC circuits

KVL Equation

(Matrix form)

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

$$[V] = [Z][I]$$

KCL Equation

(Matrix form)

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

$$[I] = [Y][V]$$

All the other theorems are applicable to the AC circuits

Crammers rule

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

Solution for the linear simultaneous equations above is as follows

Step 1: finding the determinant

$$\Delta = \begin{vmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{vmatrix}$$

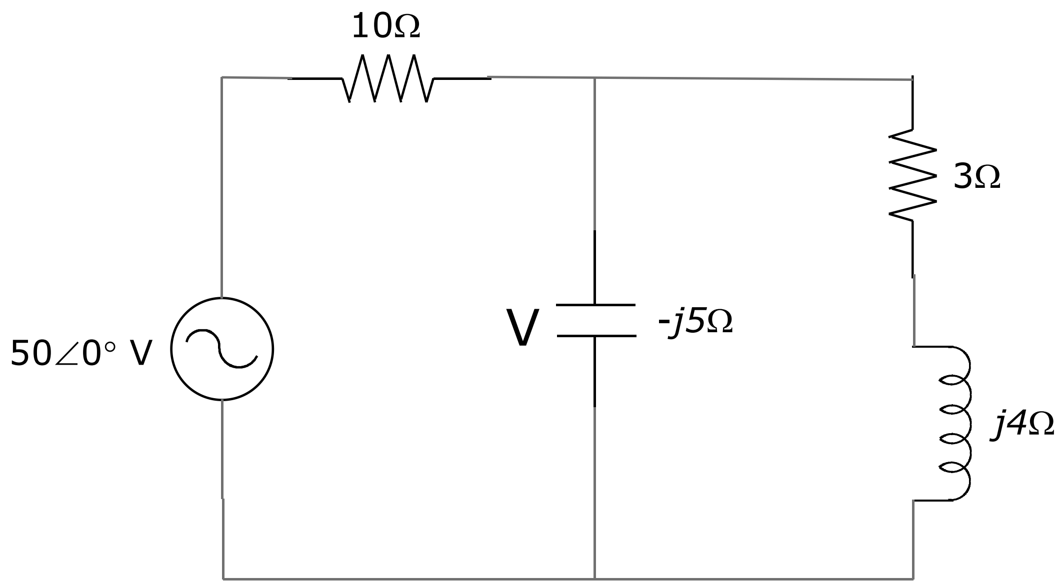
Step 2: finding the determinant after substituting first column with RHS column matrix

$$\Delta_1 = \begin{vmatrix} \boldsymbol{V_1} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ \boldsymbol{V_N} & \cdots & Z_{NN} \end{vmatrix}$$

Step 3 :Solution for I_1 $\quad I_1 = \dfrac{\Delta_1}{\Delta}$

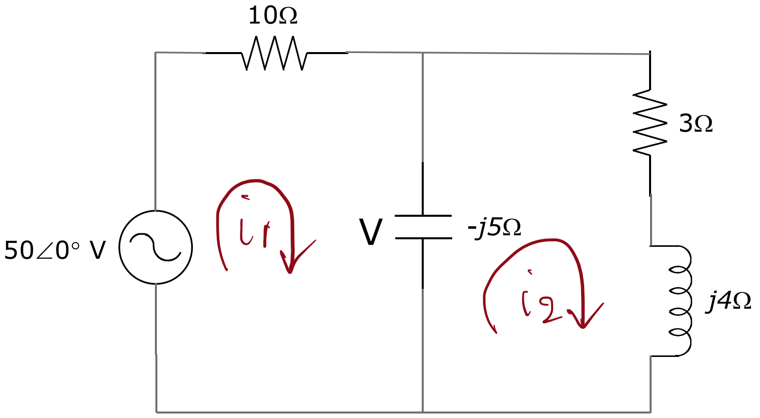
Illustration 1

Assigning two mesh currents, find the voltage V across the capacitor in the following circuit



Ans:

$$V = 22.36\angle -10.30° V$$



$$\begin{bmatrix} 10-j5 & j5 \\ j5 & 3-j \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0 \\ 0 \end{bmatrix}$$

$$\Delta = 50 - j25$$

$$\Delta_{i_1} = 150 - 50j$$

$$i_1 = \frac{\Delta_{i_1}}{\Delta} = 2.8 + 0.4j \text{ A}$$
$$(2.8284 \angle 8.1301 \text{ A})$$

$$\Delta_{i_2} = -250j$$

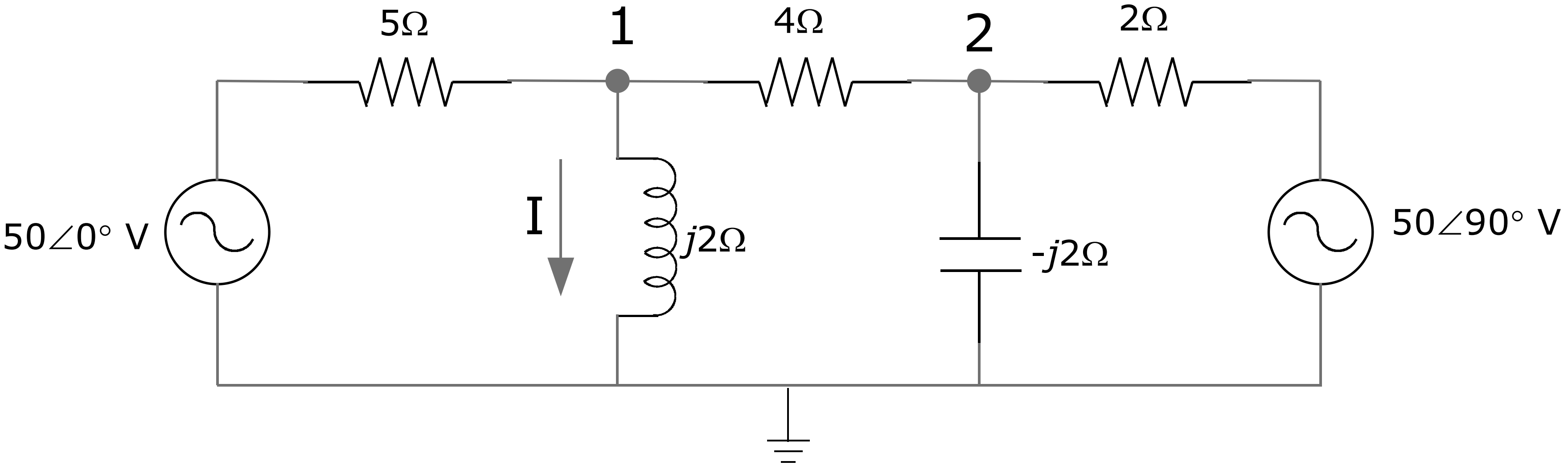
$$i_2 = \frac{\Delta_{i_2}}{\Delta} = 2 - 4j \text{ A}$$
$$(4.4721 \angle -63.43 \text{ A}$$

$$V = (i_2 - i_1)(-j5)$$

$$V = 22.36 \angle 169.69 \text{ V}$$

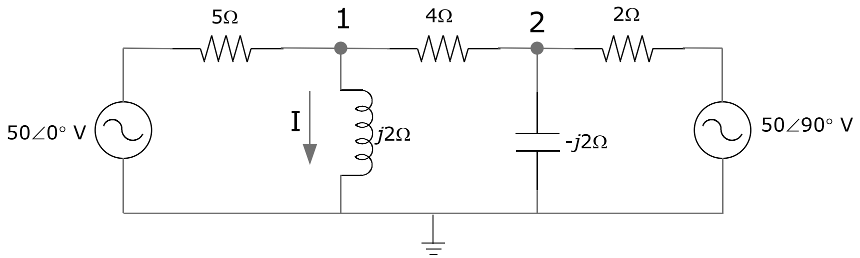
Illustration 2

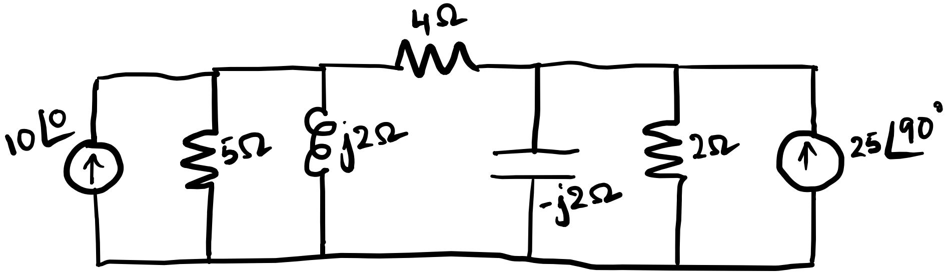
Use node voltage method to obtain the current I in the network



Ans:

$I = 12.38\angle -17.75° \ A$





$$\begin{bmatrix} \left(\frac{1}{5}+\frac{1}{2j}+\frac{1}{4}\right) & -\frac{1}{4} \\ -\frac{1}{4} & \left(\frac{1}{4}+\frac{1}{2}-\frac{1}{2j}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ 25\angle 90 \end{bmatrix}$$

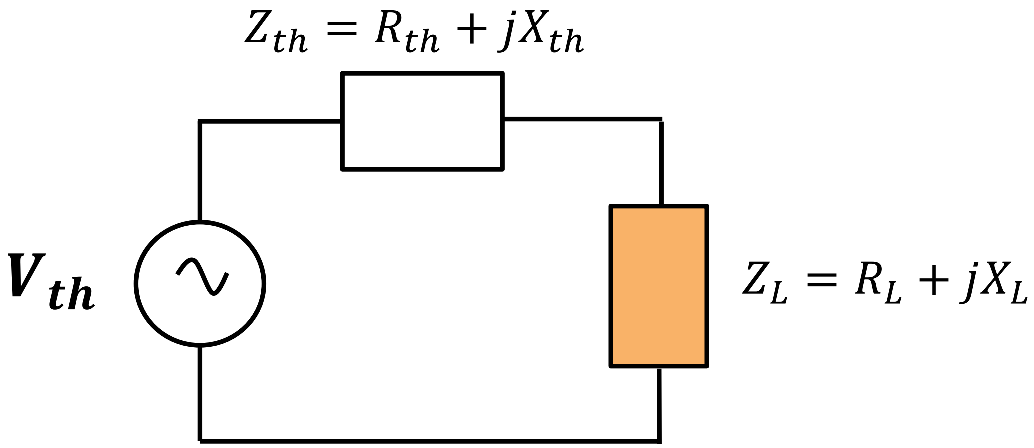
$$\Delta = 0.525 - 0.15j$$

$$\Delta_{V_1} = 7.5 + 11.25j$$

$$V_1 = \frac{\Delta_{V_1}}{\Delta} = 24.763\angle 72.25\,V$$

$$I = \frac{V_1}{2j} = 12.38\angle -17.75\,A$$

Maximum power transfer theorem



	Type of load	Condition of maximum power transfer
Case 1	Load is purely resistive	$R_L = \sqrt{R_{th}^2 + X_{th}^2}$
Case 2	Both R_L & X_L are variable	$Z_L = Z_{TH}^*$
Case 3	X_L is fixed & R_L is variable	$R_L = \sqrt{R_{th}^2 + (X_{th} + X_L)^2}$

Basic Electrical Technology

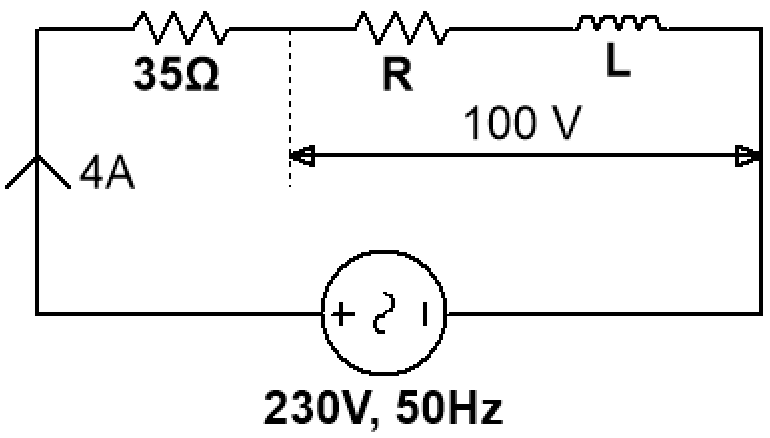
[ELE 1051]

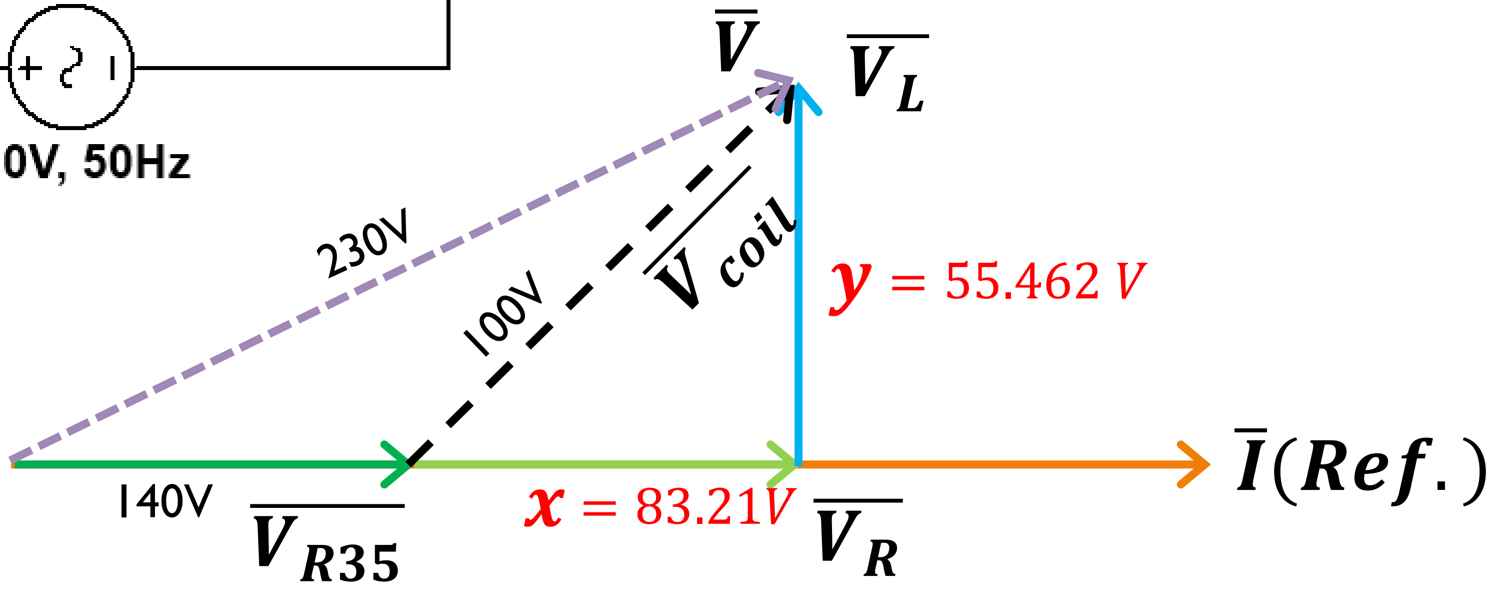
SINGLE PHASE AC CIRCUITS

Tutorial 1

Exercise 1

A resistance of 35Ω is connected in series with an inductive coil having an internal resistance 'R' and inductance 'L'. When connected across 230V, 50Hz single phase supply, voltage across the coil is 100V and the current drawn is 4 A. Find the unknowns 'R' and 'L'.





$$(140 + x)^2 + y^2 = 230^2$$
$$x^2 + y^2 = 100^2$$

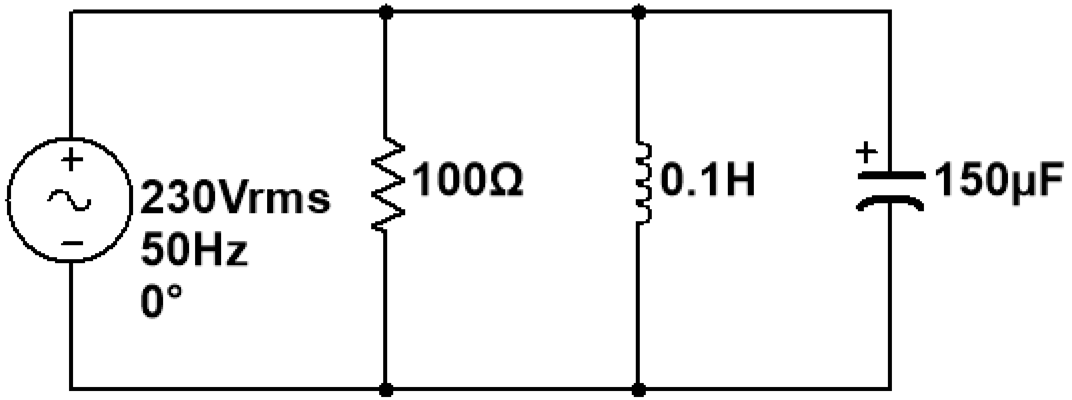
$$\boldsymbol{R} = \frac{V_R}{I} = \frac{x}{I} = \frac{83.21}{4} = \boldsymbol{20.80\Omega}$$

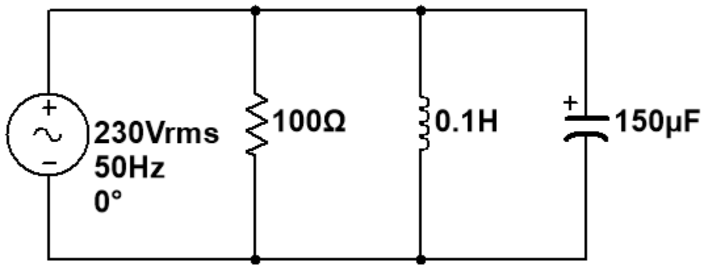
$$X_L = \frac{V_L}{I} = \frac{y}{I} = \frac{55.462}{4} = 13.8655\Omega$$

$$\therefore \boldsymbol{L = 0.044H}$$

Exercise 2

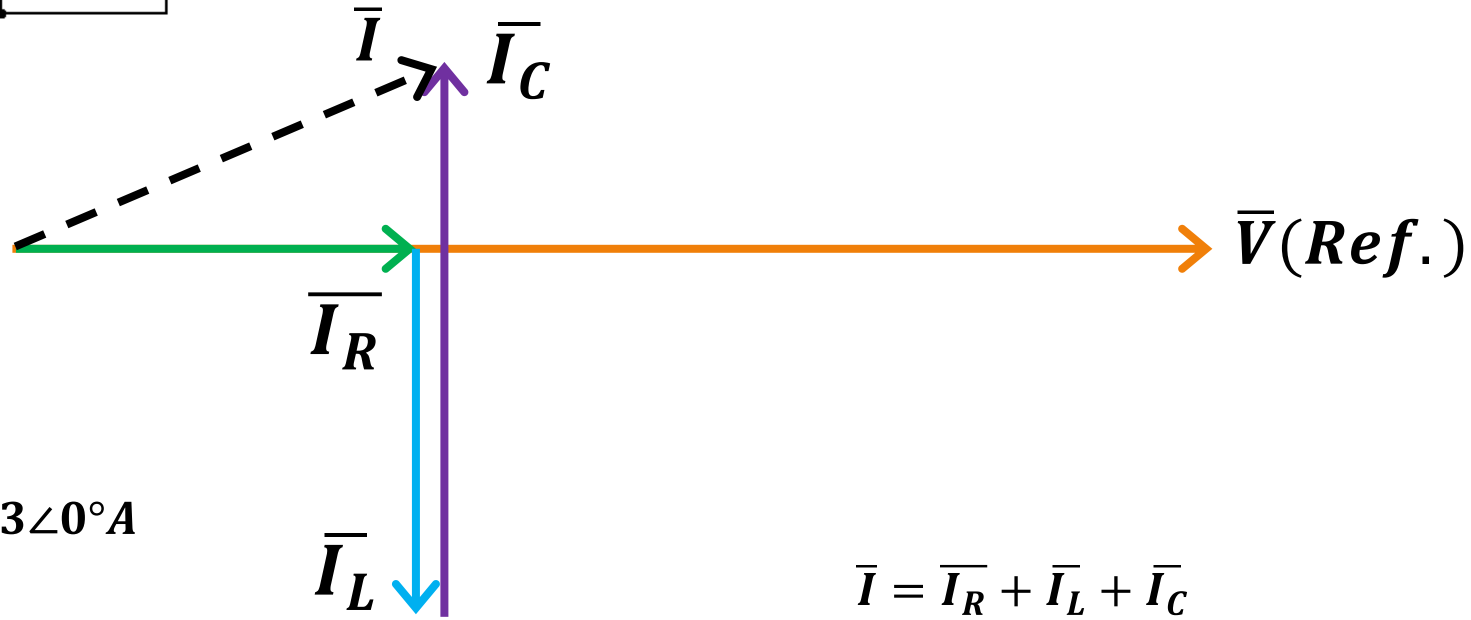
Three elements, a resistance of 100Ω, an inductance of 0.1H and a capacitance of 150μF are connected in parallel to a 230V, 50Hz supply. Calculate the current in each element and the supply current. Draw the phasor diagram.





$X_L = 31.4159\Omega$

$X_C = 21.2206\Omega$



$$\bar{I}_R = \frac{230\angle 0°}{100} = \mathbf{2.3\angle 0° A}$$

$$\bar{I}_L = \frac{\bar{V}}{jX_L} = \frac{230\angle 0°}{j31.4159} = \mathbf{7.3211\angle -90° A}$$

$$\bar{I}_C = \frac{\bar{V}}{-jX_C} = \frac{230\angle 0°}{-j21.2206} = \mathbf{10.83\angle 90° A}$$

$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$$\bar{I} = \mathbf{4.2\angle 56.819° A}$$

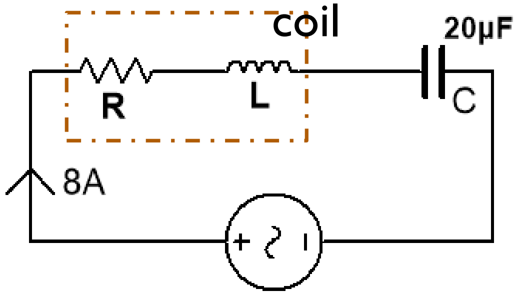
Exercise 3

A coil is in series with a 20μF capacitor across a 230 V 50 Hz supply. The current taken by the circuit is 8A and power consumed is 200W.

Calculate the inductance of the coil if the power factor of the circuit is lagging.

Calculate the inductance of the coil if the power factor of the circuit is leading.

Draw the phasor diagram.

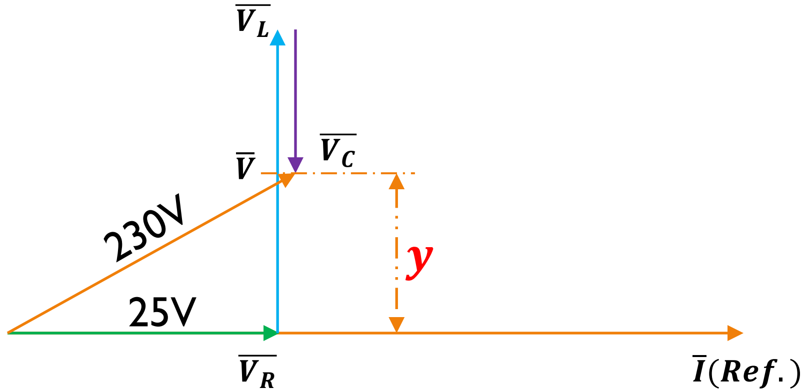


$$R = \frac{P_R}{I^2} = \frac{200}{8^2} = \mathbf{3.125\ \Omega}$$

$$\boldsymbol{V_R = 25\ V}$$

$$V_C = IX_C = \frac{8}{2\pi \times 50 \times 20\mu} = 1273.2395\ V$$

Case 1
(p.f. is lagging)



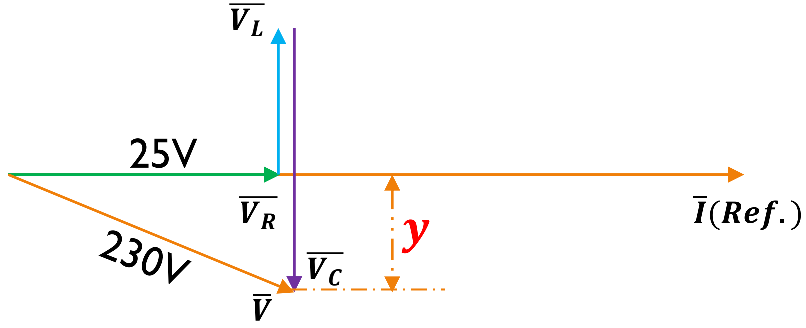
$$25^2 + y^2 = 230^2$$

$$y = 228.6372$$

$$\therefore V_L = V_C + y = 1501.8695\ V$$

$$X_L = \frac{V_L}{I} = 187.7336\Omega \qquad \boldsymbol{L = 0.5975H}$$

Case 2
(p.f. is leading)



$$25^2 + y^2 = 230^2$$

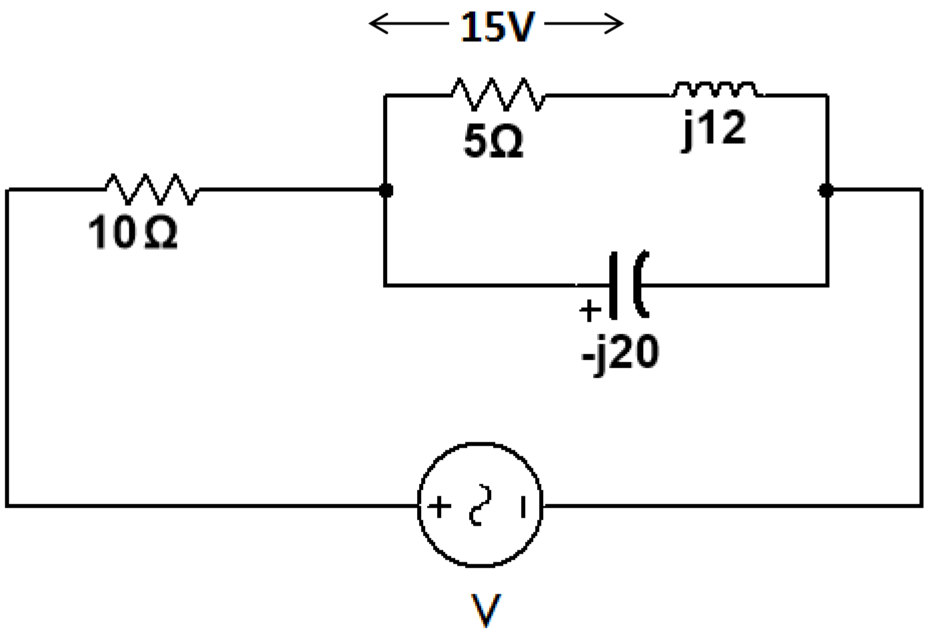
$$y = 228.6372$$

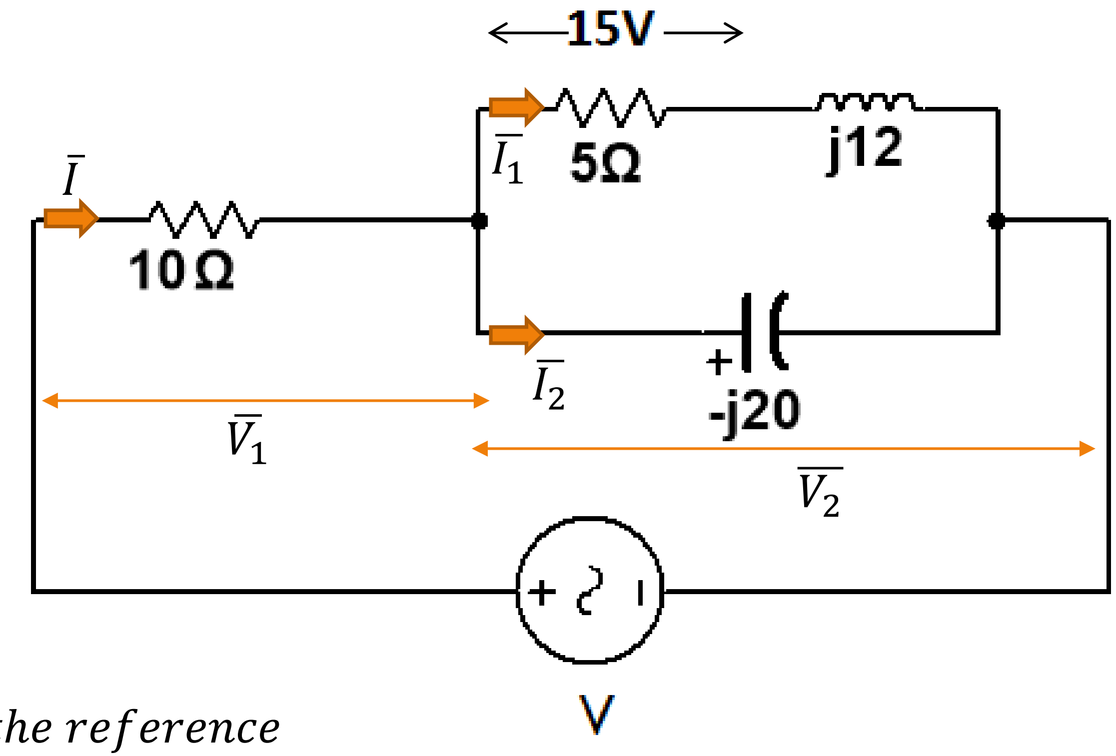
$$\therefore V_L = V_C - y = 1044.6023\ V$$

$$X_L = \frac{V_L}{I} = 130.575\Omega \qquad \boldsymbol{L = 0.4156H}$$

Exercise 4

Find the supply voltage, total current and the value of the power consumed in each arm of the series parallel circuit shown. The voltage across the 5Ω resistor is 15V.





Assume $\overline{V_2}$ *as the reference*

$$|I_1| = \frac{15}{5} = 3A \qquad \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{12}{5} = 67.38°$$

$$\therefore \bar{I_1} = 3\angle -67.38°\ A \qquad \bar{V_1} = \bar{I} \times 10 = 14.15\angle -35.3746°\ A$$

$$\overline{V_2} = \bar{I_1} \times (5 + j12) = 39\angle 0°\ V \qquad \bar{V} = \bar{V_1} + \bar{V_2} = 51.1972\angle -9.207°\ V$$

$$\therefore \bar{I_2} = \frac{\overline{V_2}}{-j20} = 1.95\angle 90°\ A \qquad P_{5\Omega} = I_1^2 \times 5 = 45W$$

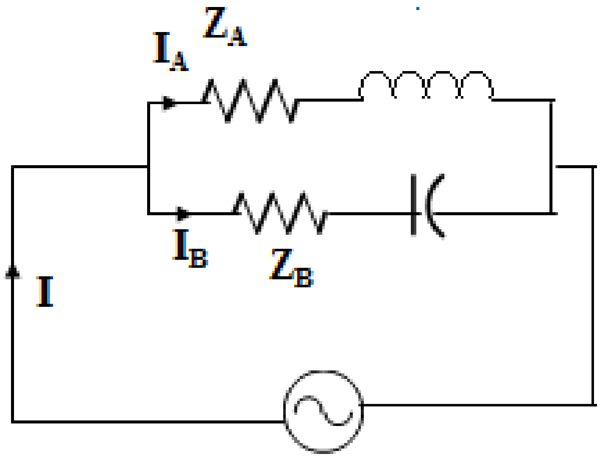
$$\bar{I} = \bar{I_1} + \bar{I_2} = 1.415\angle -35.3746°\ A \qquad P_{10\Omega} = I^2 \times 10 = 20.022\ W$$

Exercise 5

Two impedances Z_A and Z_B are connected in parallel across a 115V, 50Hz supply. The total current taken by the combination is 10A at unity p.f. Z_B has resistance of 10Ω and 200μF capacitor connected in series. Z_B consists of a resistor and inductor is series. Find

(a) The current in each branch

(b) The resistance and inductance of Z_A



Assume supply voltage as the reference $\Longrightarrow$ $\bar{V} = 115\angle 0°\ V$

Given, supply current, $\bar{I} = 10\angle 0°\ A$

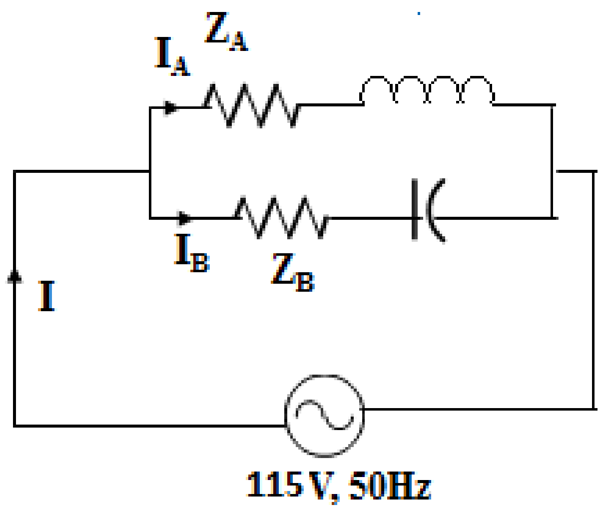
$X_C = 15.9154\ \Omega$ $\qquad Z_B = 10 - j15.9154\ \Omega$

$$\overline{\boldsymbol{I_B}} = \frac{\overline{\boldsymbol{V}}}{\boldsymbol{Z_B}} = \boldsymbol{6.1182\angle 57.8579°\ A}$$

$\bar{I} = \overline{I_A} + \overline{I_B}$

$$\overline{\boldsymbol{I_A}} = \boldsymbol{8.5048\angle -37.5261°\ A}$$

$$Z_A = \frac{\bar{V}}{\overline{I_A}} = \underbrace{10.7237}_{\boldsymbol{R_A}} + \underbrace{j8.2364}_{\boldsymbol{jX_A}}\ \Omega$$



$$\boldsymbol{R_A = 10.72\ \Omega}$$

$$\boldsymbol{L_A = 0.0262\ \Omega}$$

Basic Electrical Technology

[ELE 1051]

CHAPTER 3 - SINGLE PHASE AC CIRCUITS

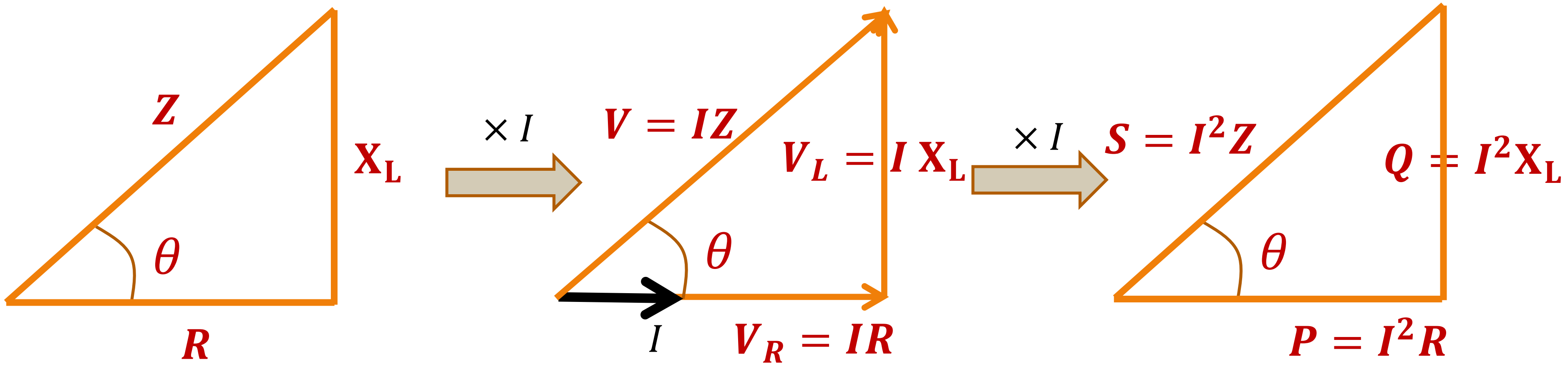
(3.5)

Topics covered

- Impedance, phasor & power triangles
- Concept of power factor and its significance
- Need for power factor improvement
- Tutorial 2a

Power associated in RL load

For RL load:



Impedance diagram | Phasor diagram | Power diagram

$$S = P + jQ$$

Where,
S = Apparent Power (VA)
P = Active Power (W)
Q = Reactive Power (var)

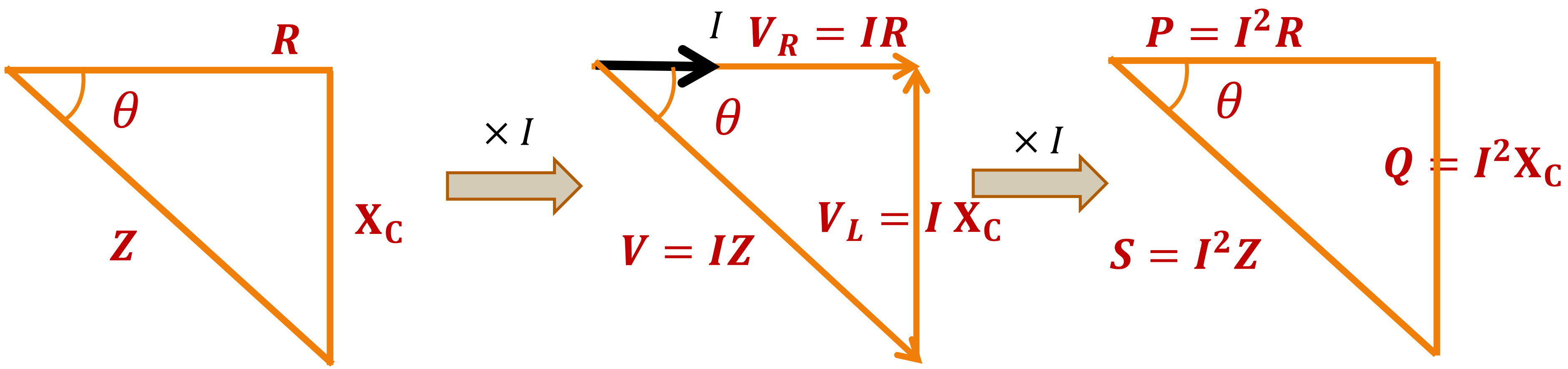
$S = |V||I|$

$P = VI\cos\emptyset$

$Q = VI\sin\emptyset$

Power associated in RC load

For RC load:



Impedance diagram — Phasor diagram — Power diagram

$$S = P - jQ$$

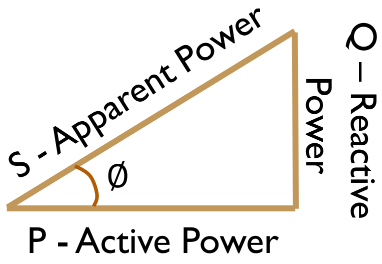
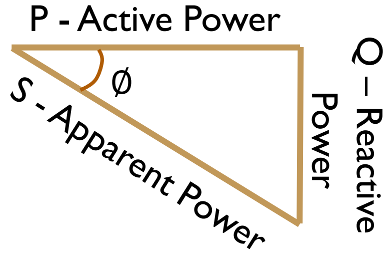
Where,
S = Apparent Power (VA)
P = Active Power (W)
Q = Reactive Power (var)

$$S = |V||I|$$
$$P = VI\cos\emptyset$$
$$Q = VI\sin\emptyset$$

Power in AC circuits

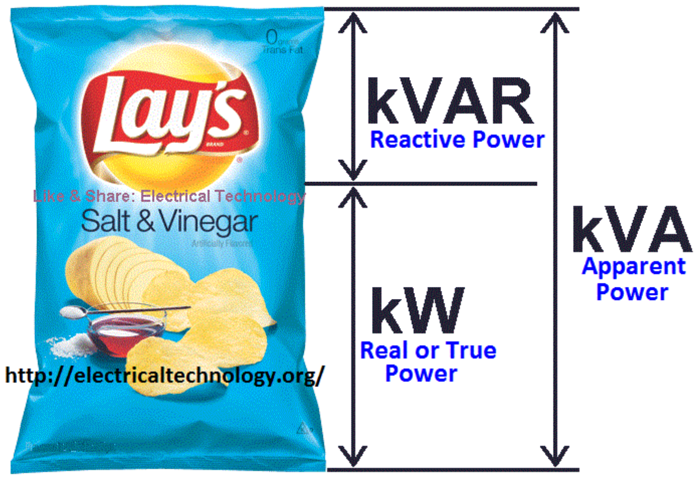
Power in AC circuit can be written as,

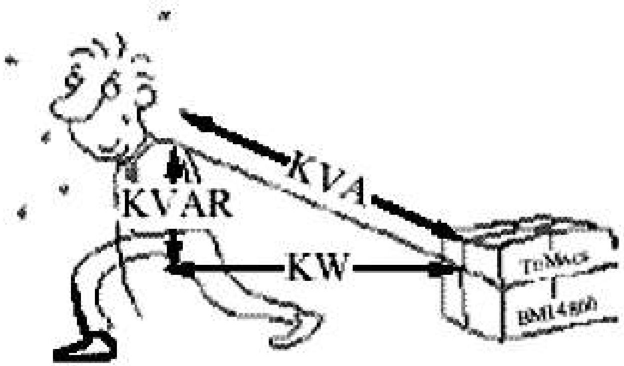
$$S = (\bar{V})(\bar{I}^*)$$

For RL Load	$Z = \|Z\|\angle\emptyset$ $if\ \bar{V} = \|V\|\angle 0°$ $\bar{I} = \|I\|\angle -\emptyset$ $I^* = \|I\|\angle\emptyset$	$S = VI(\cos\emptyset + j\sin\emptyset)$ $\boldsymbol{S = P + jQ}$ $P = V_{rms}I_{rms}\cos\emptyset$ $Q = V_{rms}I_{rms}\sin\emptyset$	
For RC Load	$Z = \|Z\|\angle -\emptyset$ $if\ \bar{V} = \|V\|\angle 0°$ $\bar{I} = \|I\|\angle\emptyset$ $I^* = \|I\|\angle -\emptyset$	$S = VI(\cos\emptyset - j\sin\emptyset)$ $\boldsymbol{S = P - jQ}$ $P = V_{rms}I_{rms}\cos\emptyset$ $Q = V_{rms}I_{rms}\sin\emptyset$	

$\underline{Units:}$ $Apparent\ Power(S)$ $\boldsymbol{VA}$ $Active\ Power(P)$ $\boldsymbol{W}$ $Reactive\ Power(Q)$ $\boldsymbol{var}$

Complex Power Analogy





Power Factor

$$\boldsymbol{Power\ Factor = \frac{Active\ Power\ P\ in\ watts}{Apparent\ Power\ S\ in\ voltamperes}}$$

$$\cos\theta = \frac{P}{S} = \frac{P}{VI}$$

- For an impedance Z,

$$\cos\theta = \frac{IR}{V} = \frac{IR}{IZ} = \frac{resistance}{impedance}$$

- Power factor is *lagging* when the *current lags the supply voltage*
- Power factor is *leading* when the *current leads the supply voltage*
- For a resistive load, power factor is Unity

Disadvantages of Low Power Factor

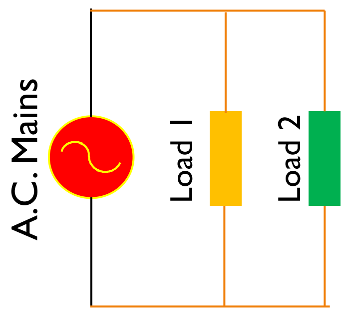
- Under utilisation of power system network
- Increased transmission losses
- Hence bulk consumers are advised to maintain the power factor close to unity by power utilities

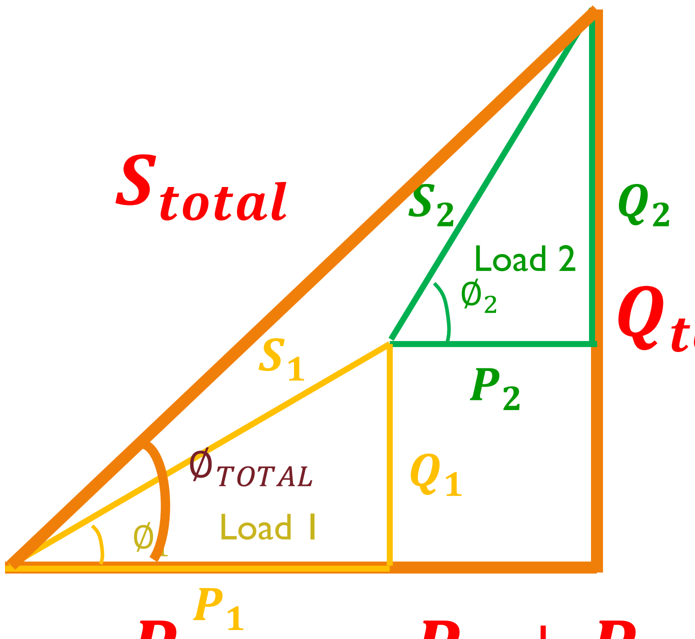
Remedial Measures

- Reactive power demand of Inductive loads can be compensated with capacitive loads
- It is possible to localise reactive power requirement by connecting parallel capacitors across the load

Power Triangle

- Practically, loads are in connected parallel
- Majority of the loads are inductive in nature





$$Q_{total} = Q_1 + Q_2$$

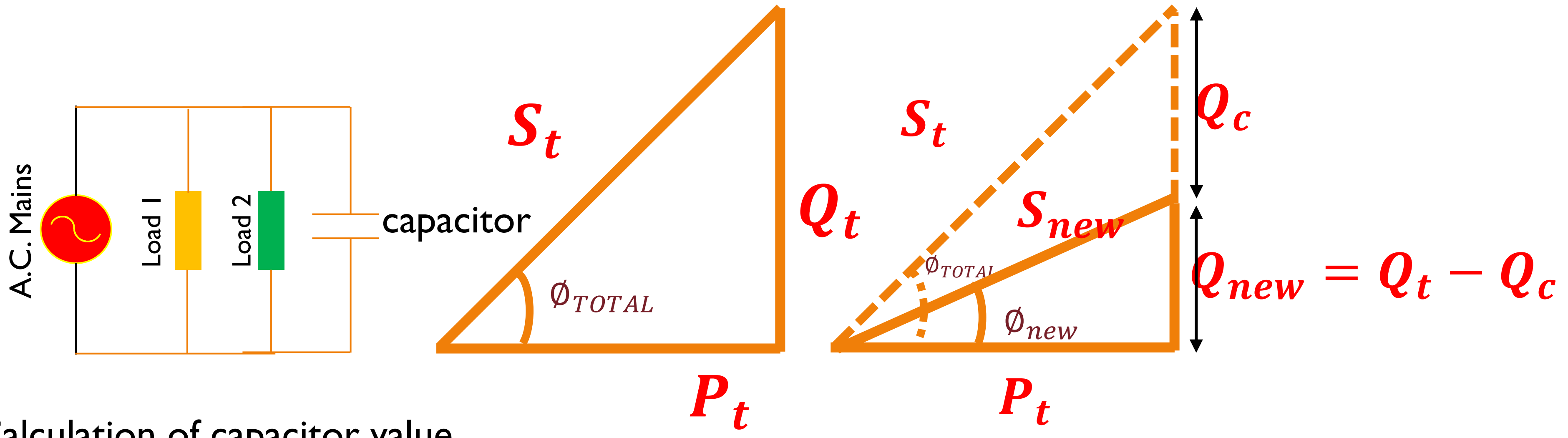
$$S_{total} = P_{total} + jQ_{total}$$

$$P_{total} = P_1 + P_2$$

http://www.kptcl.com/save.htm

Power Factor Improvement

- Connect capacitor parallel to the load
- Energy stored by the capacitor provides the required reactive power by the load



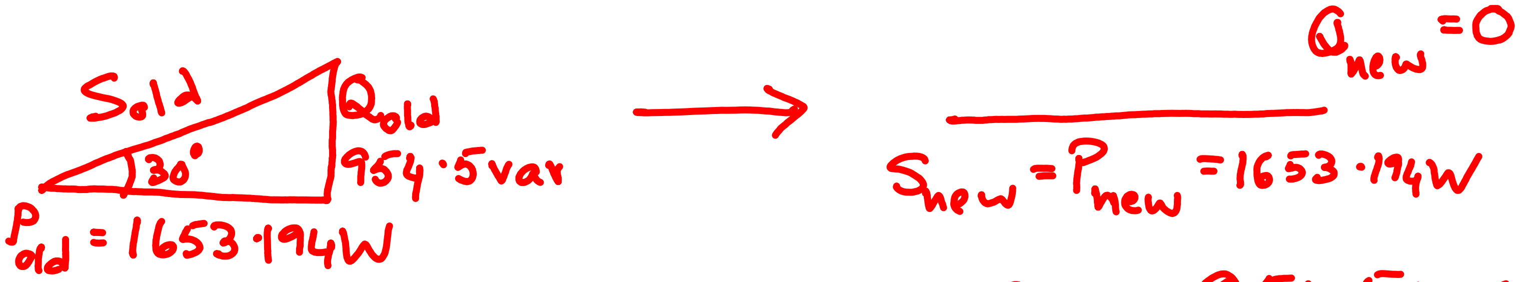
Calculation of capacitor value

- Calculate Q_c needed to improve power factor to $cos\emptyset_{new}$
- Calculate $X_c = \frac{V^2}{Q_C}$ & $C = \frac{1}{2\pi f X_c}$

Illustration 1

A single-phase motor takes 8.3 A at a power factor of 0.866 lagging when connected to a 230 V, 50 Hz supply. Capacitance bank is now connected in parallel with the motor to raise the power factor to unity. Determine the capacitance value

Solution:



$P_{old} = 230 \times 8.3 \times 0.866$

$Q_{old} = 230 \times 8.3 \times \sin 30$

$Q_C = Q_{old} - Q_{new} = 954.5\ var$

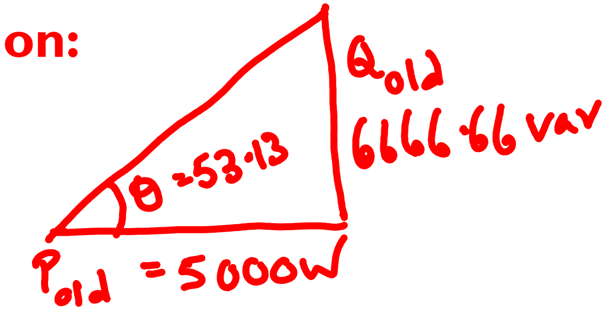
$Q_C = \frac{V^2}{X_C} \Rightarrow X_C = \frac{230^2}{954.5} = 55.4216\ \Omega$

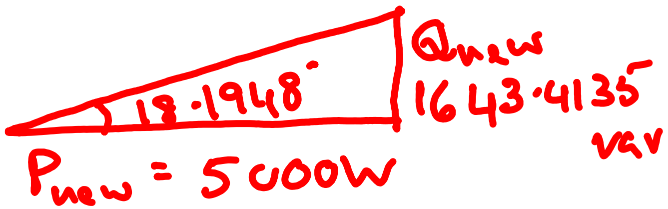
$\Rightarrow C = \frac{1}{2\pi f X_C} = 57.43 \mu F$

Illustration 2

A single-phase load of 5 kW operates at a power factor of 0.6 lagging. It is proposed to improve this power factor to 0.95 lagging by connecting a capacitor across the load. Calculate the kvar rating of the capacitor

Solution:





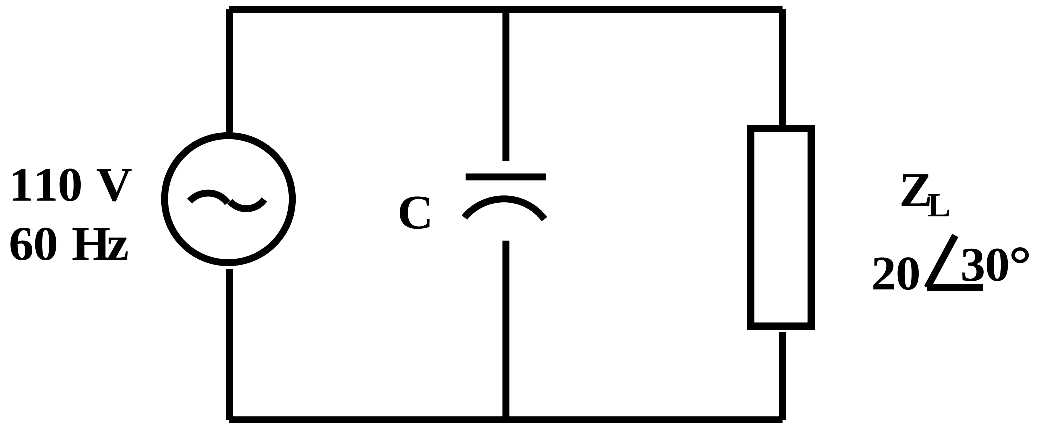
$$\tan 53.13 = \frac{Q_{old}}{5000}$$

$$Q_c = (6.666 - 1.643)k$$

$$\underline{Q_c = 5.023 \text{kvar}}$$

Exercise 1

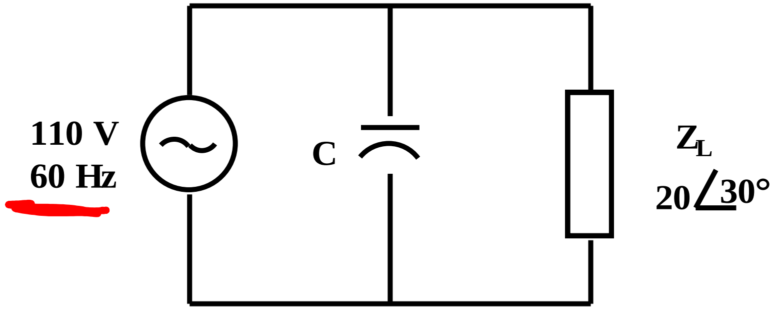
In the parallel circuit shown, Find the value of Capacitance C, necessary to correct the power factor to 0.95 lagging



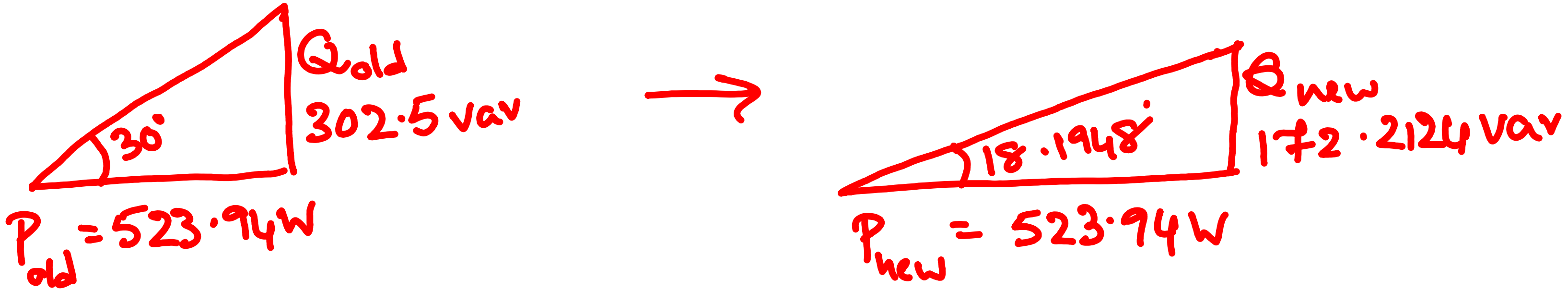
Solution:

$$I = \frac{V}{Z} = \frac{110\angle 0}{20\angle 30} = 5.5\angle -30 \text{ A}$$

$$P_{old} = 110 \times 5.5 \times \cos 30 = 523.94 \text{ W}$$



Find C to improve p.f. to 0.95 lag



$$Q_c = Q_{old} - Q_{new}$$

$$Q_c = 130.2875 \text{ var}$$

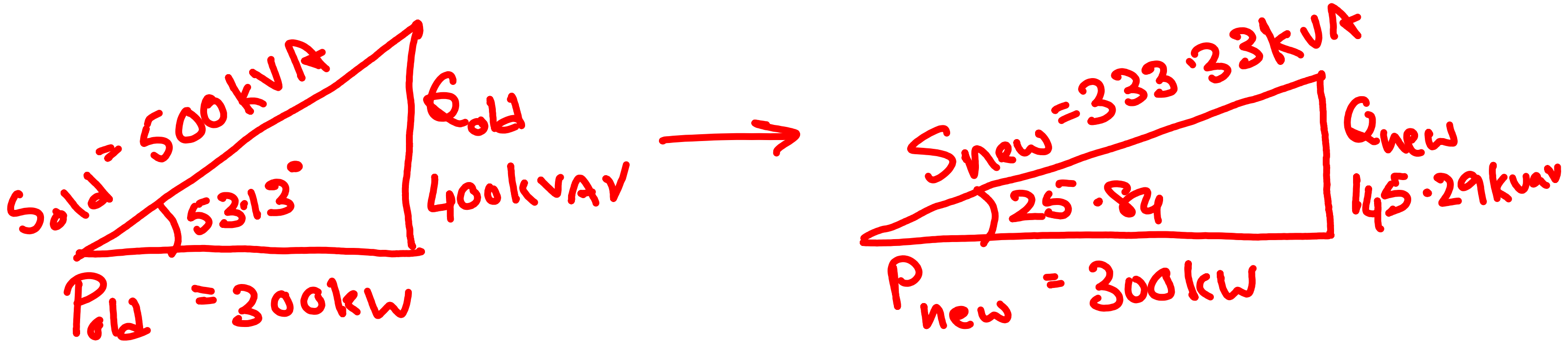
$$X_c = \frac{110^2}{130.2875} = 92.8715\,\Omega$$

$$C = \frac{1}{2\pi \times 60 \times 92.87} = 28.56\,\mu F$$

Exercise 2

A 500 kVA transformer is at full load with power factor 0.6 lagging. What should be the kVAR rating of the shunt capacitor needed to improve its operating power factor to 0.9 lagging? What will be the percentage loading of the transformer after power factor correction?

Solution:



$$Q_c = Q_{old} - Q_{new} = 254.7033\,\text{kvar}.$$

$$\%\ \text{loading} = \frac{333.33}{500} \times 100 = \underline{66.}$$

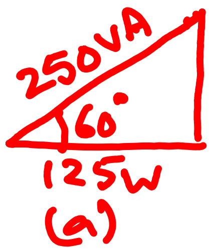
Exercise 3

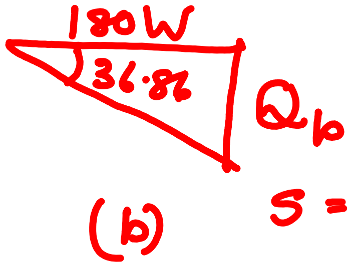
Obtain the complete power triangle for three parallel-connected loads:

(a) 250VA, 0.5 p.f lagging

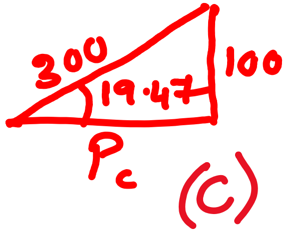
(b) 180W ,0.8 p.f leading

(c) 300VA, 100 var (inductive)





$S = P - jQ$



Solution:

$P_a = 0.5 \times 250 = 125W$

$Q_a = \tan 60 \times 125 = 216.50\ var$

$P_b = 180W$

$Q_b = -135\ var$

$P_c = \sqrt{300^2 - 100^2}$
$= 282.84W$

$Q_c = 100\ var$

$P_{total} = P_a + P_b + P_c = 587.84W$

$Q_{total} = Q_a - Q_b + Q_c = 181.50\ var$

$S_{total} = P_{total} + jQ_{total} = 615.22\angle 17.15\ VA$

Homework 1

An inductive circuit supplied with 250V, 50Hz has an active power of 11.9 KW and apparent power of 17 KVA

a) Find the power factor of the circuit

b) Draw the power triangle

c) Find the value of the capacitance required to improve the p.f. to unity, 0.9 lagging ,0.9 leading

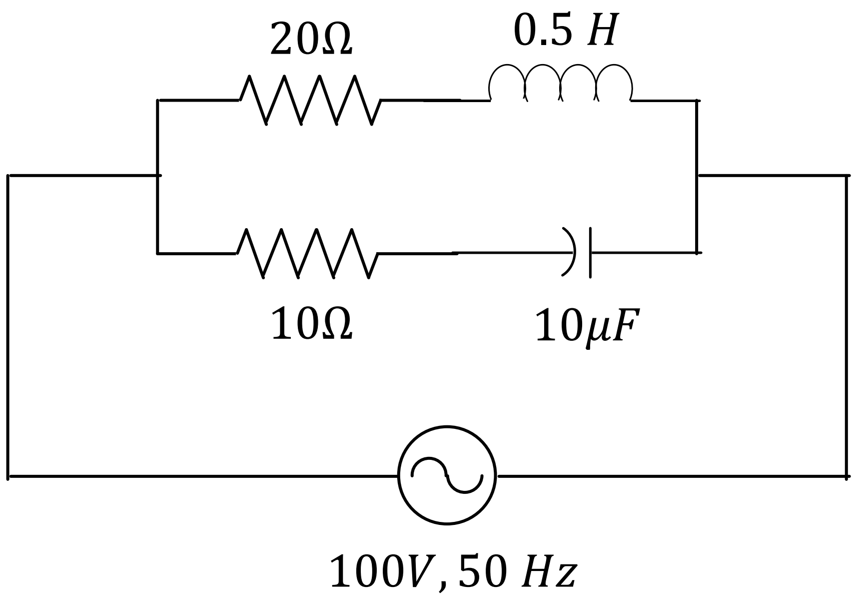
Ans:

a) p.f = 0.7 lag

c) C= 618.3 μF,
324.9 μF,
911.6μF

Homework 2

Find the power factor of the circuit shown below. Also, find the value of the capacitor to be connected in series with the circuit to increase the power factor to unity.



Ans: 0.276 lag, 9.95μF

Basic Electrical Technology

[ELE 1051]

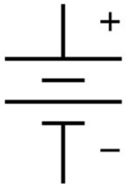
CHAPTER 3 - SINGLE PHASE AC CIRCUITS

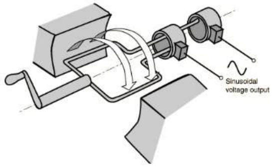
(3.1)

Topics covered…

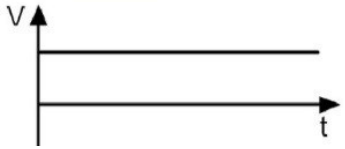
- Comparison of DC & AC
- How is AC generated?
- Terminologies of AC
- Average value of an alternating waveform
- RMS value of an alternating waveform

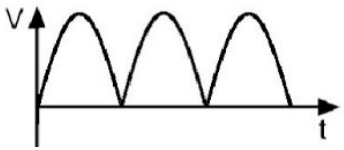
DC vs. AC

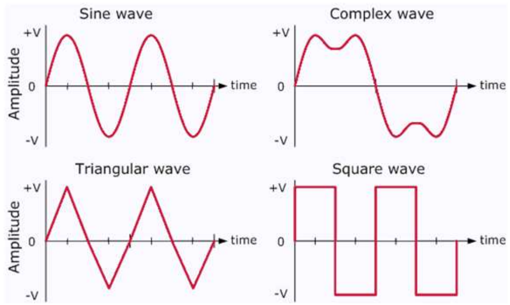




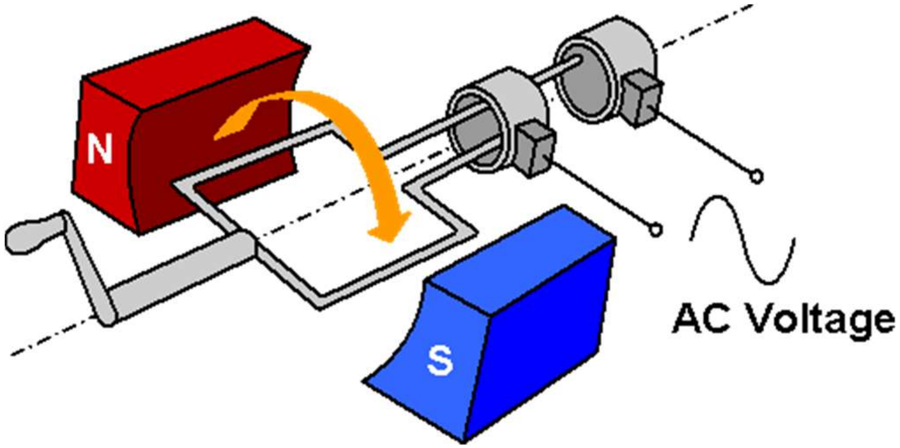
	DC	AC
Obtained from	Battery / cell / derived from AC	AC Generator
Polarity	Positive and Negative	Oscillatory
Frequency	Zero	50Hz or 60Hz
Types	Constant or pulsating	**Sinusoidal**, Trapezoidal, Triangular, Square



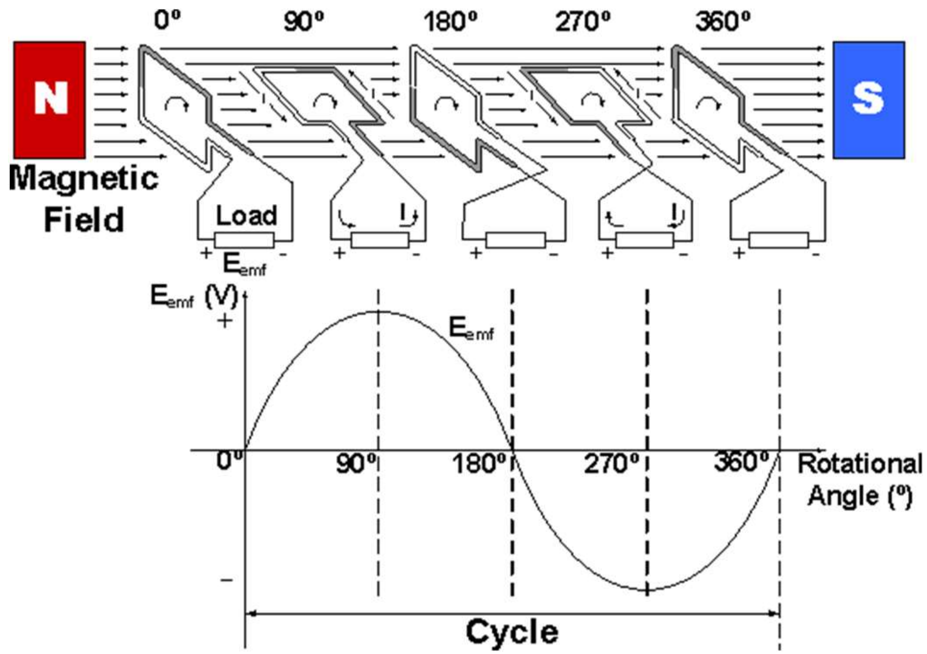




Generation of Alternating EMF



Generator working principle



EMF Equation

EMF induced per conductor is

$$e = B\, l\, v \sin\theta$$

EMF Induced in one turn of a coil is

$$e = 2\, B\, l\, v \sin\theta$$

If, b = width of the coil,

$$v = \pi\, b\, n$$ *'n' is the speed in revolutions per sec.*

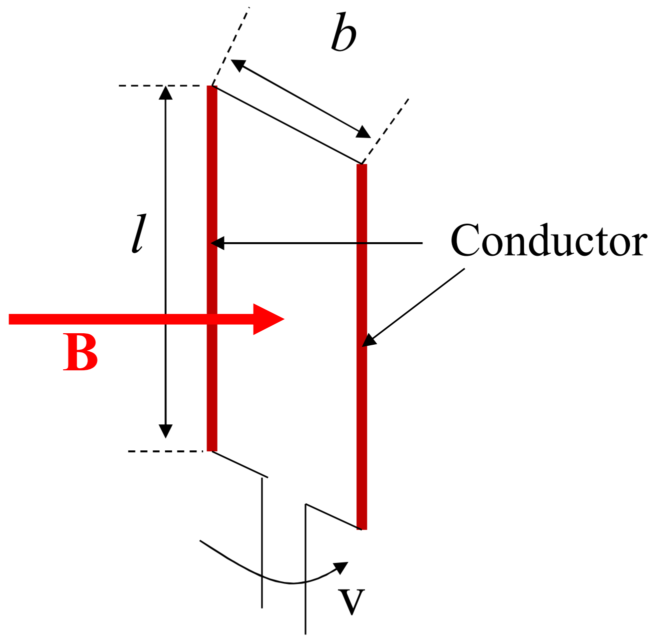
$$e = 2\, B\, l\, b\, \pi\, n \sin\theta$$

$$= 2\, B\, A\, \pi\, n \sin\theta$$

If there are N turns in the coil, the emf induced is,

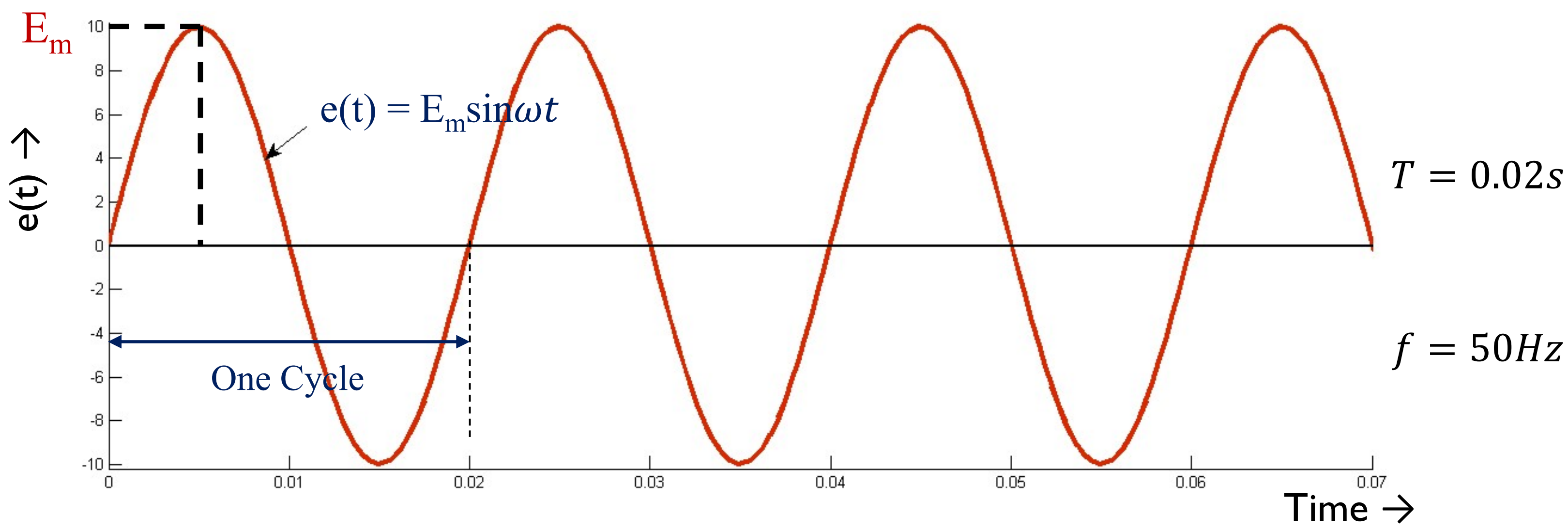
$$e = 2\, \pi\, n\, B\, A\, N \sin\theta$$

$$\boxed{e = E_m \sin\theta}$$



Turn of a coil

Terminologies in AC waveform



Cycle: Each repetition of the alternating quantity, recurring at equal intervals

Period (T): Duration of one cycle

Instantaneous Value (e(t)): The magnitude of a waveform at any instant in time

Peak Amplitude: Maximum value or peak value of alternating quantity

Frequency (f): Number of cycles in one second (Hz) $f = \frac{1}{T}$

Average value of Sinusoidal Alternating Current

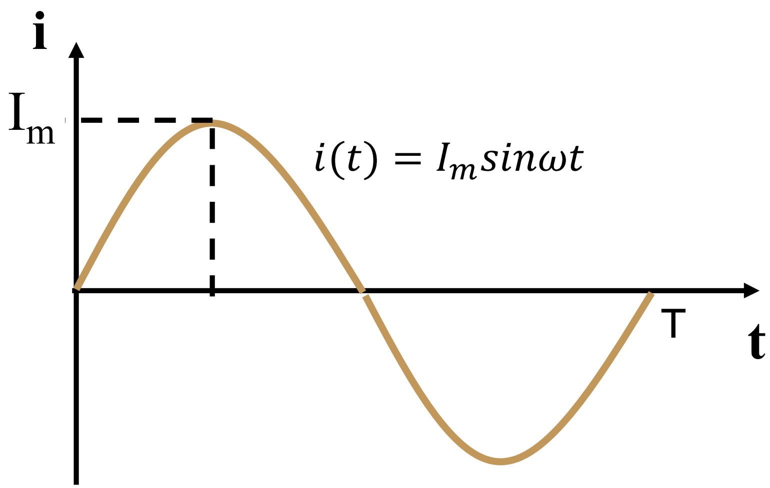
Definition: *"It is that steady current which transfers the same amount of charge to any circuit during the given interval of time, as is transferred by the alternating current to the same circuit during the same time"*

For a periodic function f(t) with period T, $$\boldsymbol{F_{avg} = \frac{1}{T}\int_0^T f(t)dt}$$

For sinusoidal signal,

$$I_{avg} = \frac{1}{T/2}\int_0^{T/2} I_m \sin\omega t dt$$

$$\boxed{\boldsymbol{I_{avg} = \frac{2I_m}{\pi}}}$$



RMS value of Sinusoidal Alternating Current

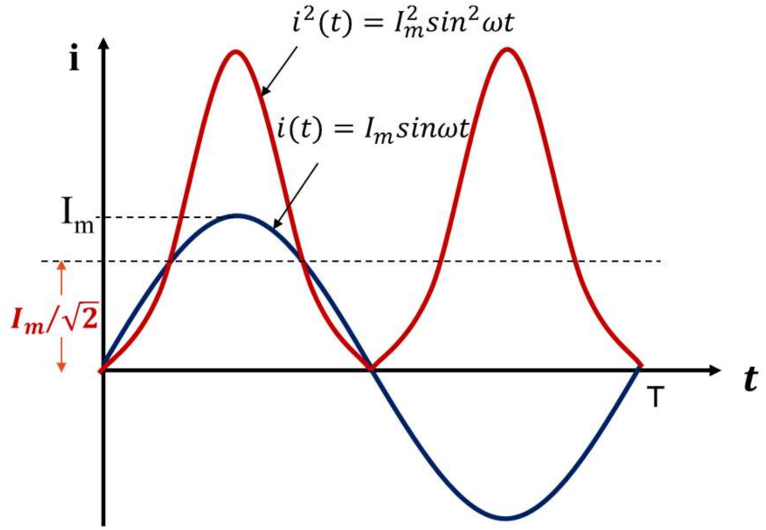
Definition: *"It is that value of direct current which when flowing through a circuit produces the same amount of heat for a given interval of time as that of the alternating current flowing through the same circuit during the same time"*

For a periodic function f(t) with period T,

$$F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 sin^2\omega t \; dt}$$

$$\boxed{I_{RMS} = \frac{I_m}{\sqrt{2}}}$$

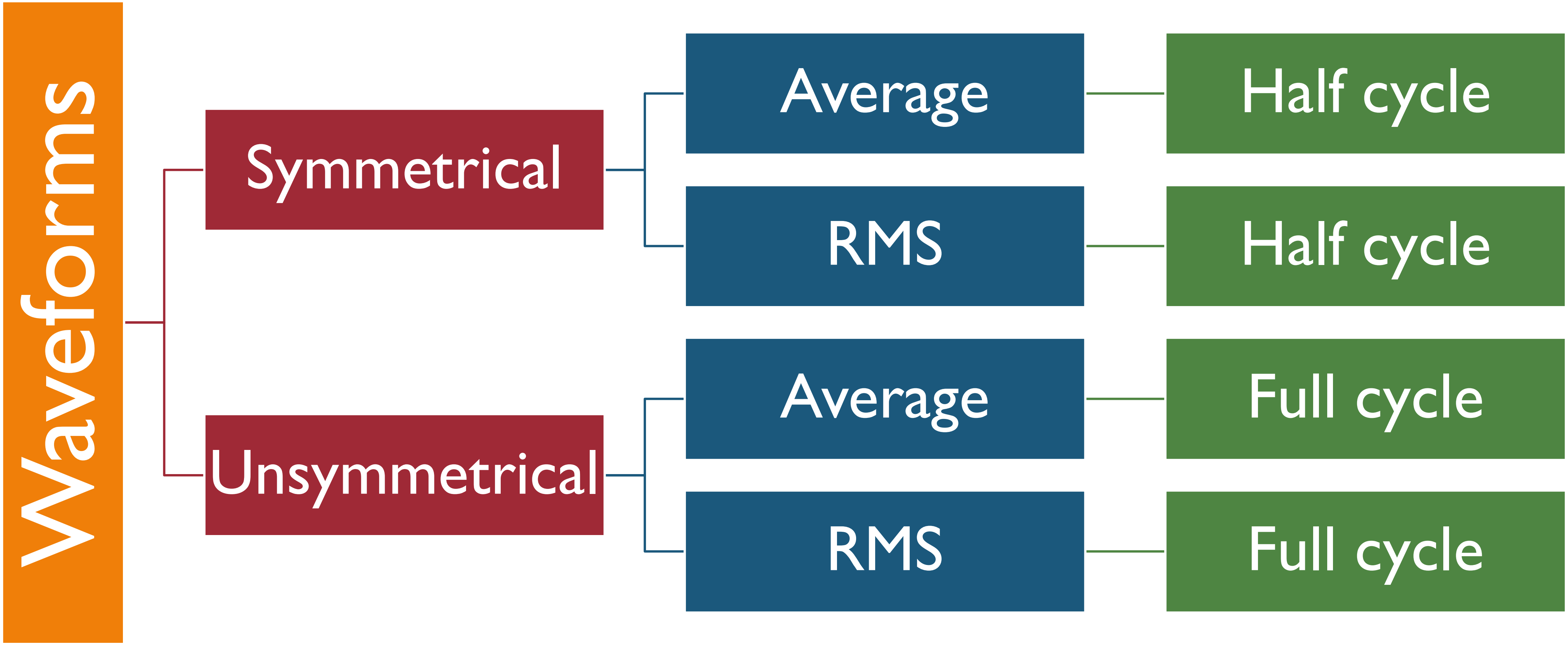


Form Factor & Peak Factor

$$Form\ Factor = \frac{RMS\ Value}{Average\ Value} = 1.11\ for\ sinusoidal$$

$$Peak\ Factor = \frac{Maximum\ Value}{RMS\ Value} = \sqrt{2}\ for\ sinusoidal$$

Full cycle and half cycle - considerations



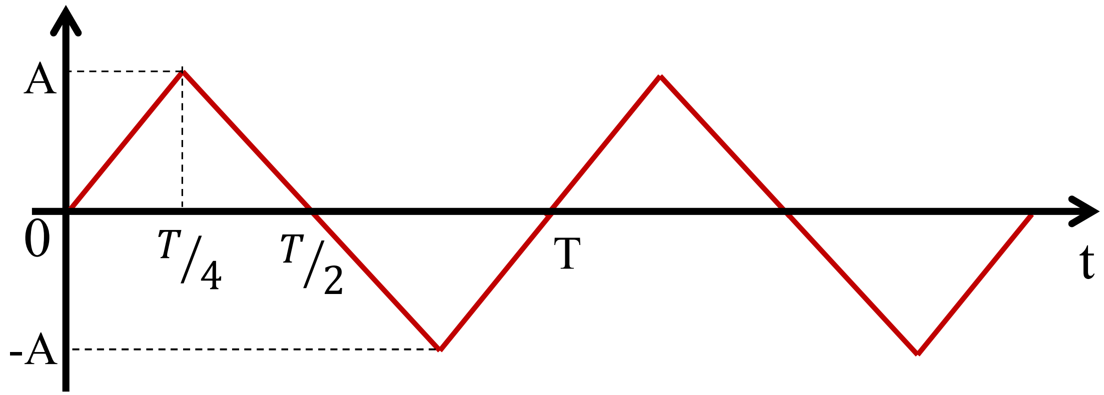
Exercise 1

If an alternating voltage has the equation
$v(t) = 141.4 \sin 377t$, calculate

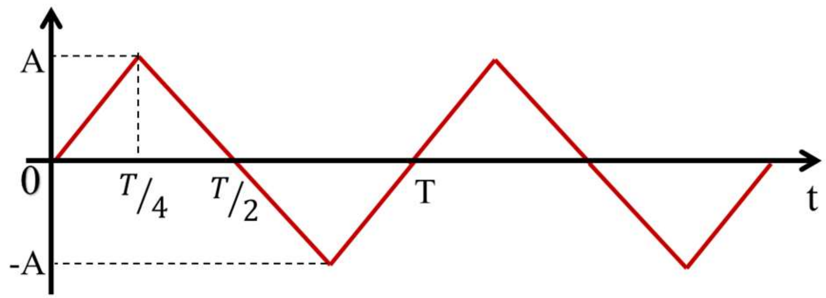
a. Maximum voltage value = 141.4V
b. RMS value of the voltage = 100V
c. Frequency = 60 Hz
d. The instantaneous voltage when t = 3ms is 127.9V (calculator should be in 'rad' mode)

Exercise 2

Find the Average value and RMS value of the given non-sinusoidal waveform



Solution:



Average Value

$$I_{avg} = \frac{1}{T/4}\int_0^{T/4} f(t).dt$$

$$I_{avg} = \frac{4}{T}\int_0^{T/4} \frac{4At}{T}.dt$$

$$I_{avg} = \frac{4}{T}\times\frac{4A}{T}\times\left[\frac{t^2}{2}\right]_0^{\frac{T}{4}}$$

$$I_{avg} = \frac{8A}{T^2}\times\left[\frac{T^2}{16}\right]$$

$$\boldsymbol{I_{avg} = \frac{A}{2}}$$

RMS Value

$$I_{rms}^2 = \frac{1}{T/4}\int_0^{T/4} f^2(t).dt$$

$$I_{rms}^2 = \frac{4}{T}\int_0^{T/4} \frac{16A^2t^2}{T^2}.dt$$

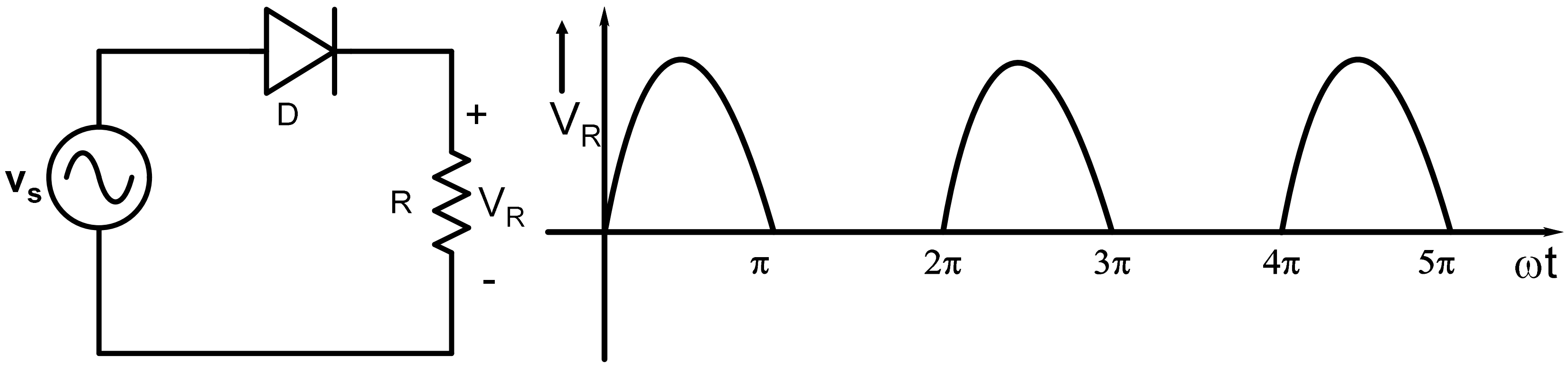
$$I_{rms}^2 = \frac{4}{T}\times\frac{16A^2}{T^2}\times\left[\frac{t^3}{3}\right]_0^{\frac{T}{4}}$$

$$I_{rms}^2 = \frac{4}{T}\times\frac{16A^2}{T^2}\times\frac{1}{3}\times\left[\frac{T^3}{4^3}\right]$$

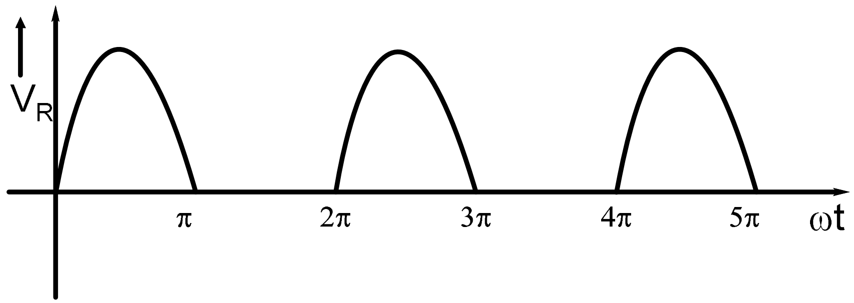
$$\boldsymbol{I_{rms} = \frac{A}{\sqrt{3}}}$$

Exercise 3

For the circuit shown below, sketch the voltage across the resistance, & then find the Average value and RMS value of the same.



Solution:



Average Value

$$V_{avg} = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\omega t\,.\,d\omega t \;+\; \int_{\pi}^{2\pi} 0.\,d\omega t\right]$$

$$V_{avg} = \frac{V_m}{2\pi}(-\cos\omega t)_0^{\pi}$$

$$V_{avg} = \frac{-V_m}{2\pi}(-1-1)$$

$$\boldsymbol{V_{avg} = \frac{V_m}{\pi}}$$

RMS Value

$$V_{rms}^2 = \frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 sin^2\omega t.\,d\omega t \;+\; \int_{\pi}^{2\pi} 0.\,d\omega t\right]$$

$$V_{rms}^2 = \frac{V_m^2}{2\pi}\left[\int_0^{\pi} \frac{1-\cos 2\omega t}{2}.\,d\omega t\right]$$

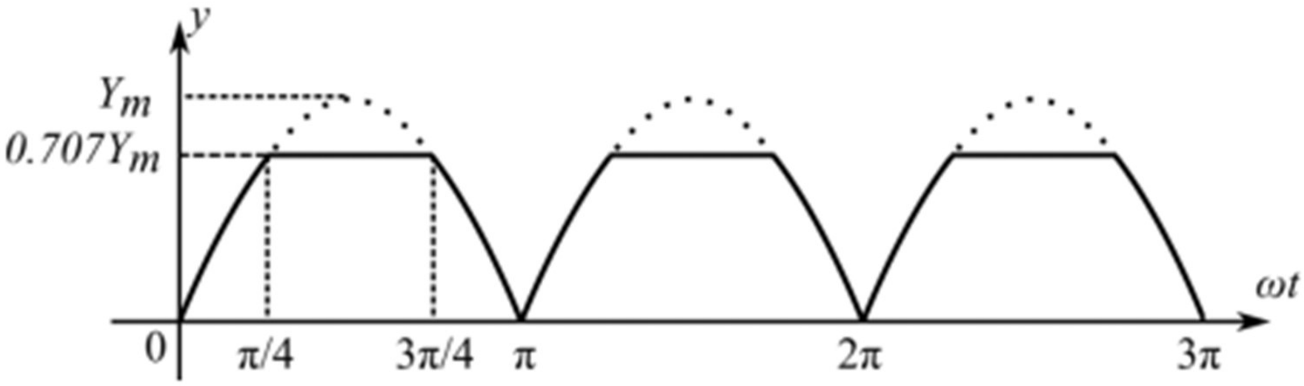
$$V_{rms}^2 = \frac{V_m^2}{4\pi}\left[\omega t|_0^{\pi} - \sin 2\omega t|_0^{\pi}\right]$$

$$V_{rms}^2 = \frac{V_m^2}{4\pi}[\pi]$$

$$\boldsymbol{V_{rms} = \frac{V_m}{2}}$$

Homework

Find the average and RMS value of the waveform



Basic Electrical Technology

[ELE 1051]

CHAPTER 3 - SINGLE PHASE AC CIRCUITS

(3.2)

Topics covered…

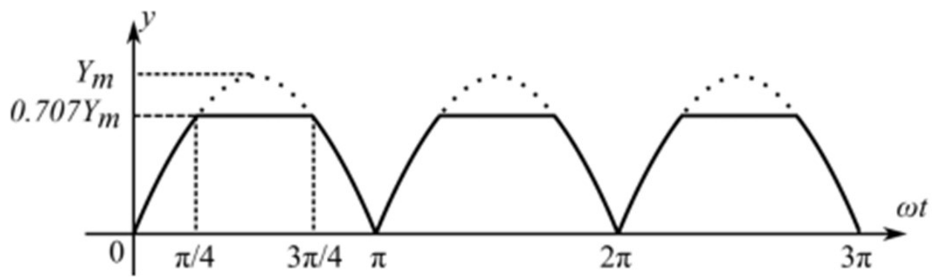
Last class

- Comparison of DC & AC
- How is AC generated?
- Terminologies of AC
- Average value of an alternating waveform
- RMS value of an alternating waveform

Today

- Complex numbers
- Representing AC
- R, L, C circuit response with AC supply
- Power associated with a pure R, L, C

Homework Solution:



Average Value

$$V_{avg} = \frac{1}{\pi}\left[\int_{0}^{\pi/4} Y_m \sin\omega t\,.\,d\omega t\; + \int_{\pi/4}^{3\pi/4} 0.707Y_m.\,d\omega t + \int_{3\pi/4}^{\pi} Y_m \sin\omega t\,.\,d\omega t\right]$$

$$V_{avg} = \frac{1}{\pi}[0.2928Y_m + 1.1105Y_m + 0.2928Y_m]$$

$$\boldsymbol{V_{avg} = 0.5398Y_m}$$

RMS Value

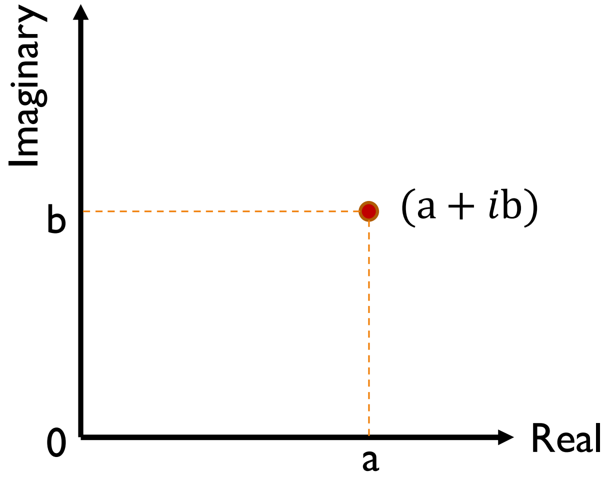
$$V_{rms}^2 = \frac{Y_m^2}{\pi}\left[\int_{0}^{\pi/4} sin^2\omega t.\,d\omega t\; + \int_{\pi/4}^{3\pi/4} 0.707^2\; d\omega t + \int_{3\pi/4}^{\pi} sin^2\omega t.\,d\omega t\right]$$

$$\boldsymbol{V_{rms} = 0.5837Y_m}$$

Complex Number

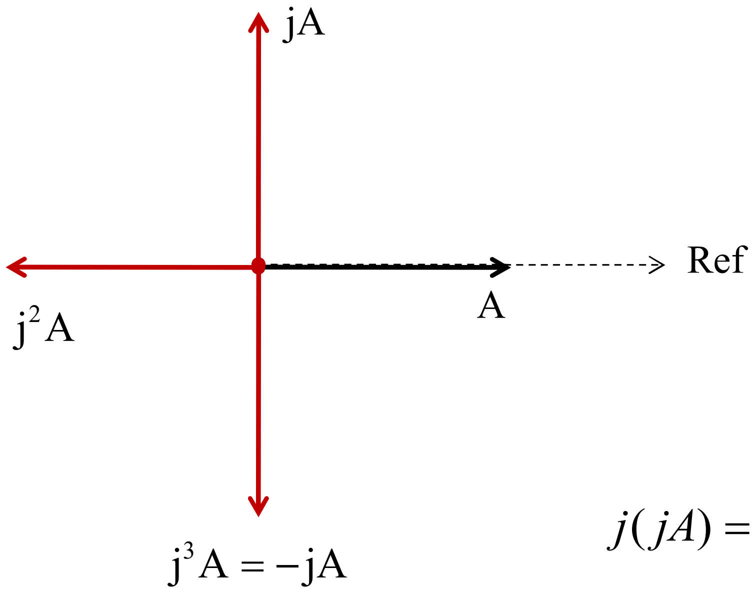
- A **complex number** is of the form **a + *i* b**

- Represented on complex plane as:



The operator 'j'

$j = 1\angle 90°$



$$j(jA) = j^2 A = -A$$

$$\boxed{\text{Therefore,} \quad j^2 = -1; \quad j = \sqrt{-1}}$$

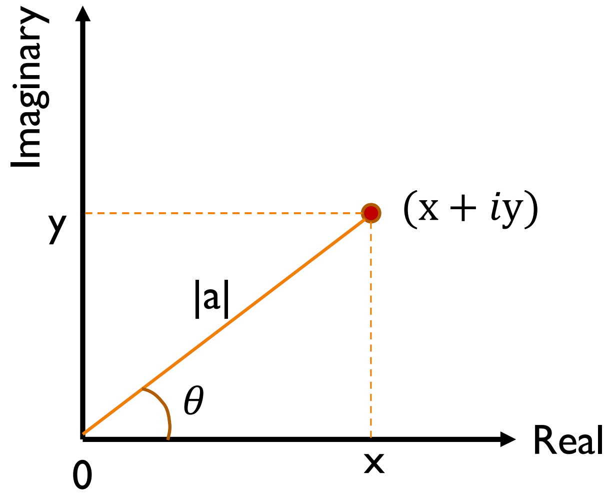
The operator 'j' rotates the given vector by 90 degrees in anti-clockwise direction

Rectangular ↔ Polar conversion

- **Rectangular to polar:**

$$|a| = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$



- **Polar to Rectangular:**

$$x = |a| \cos\theta$$

$$\mathrm{y} = |a| \sin\theta$$

Representation of a complex number

- **Rectangular form**: $\mathbf{a = x \pm \mathit{j}y}$
- **Polar form**: $\mathbf{a = |\mathit{a}| \angle \pm \theta}$
- **Exponential form**: $\mathbf{a = |\mathit{a}| e^{\pm \mathit{j}\theta}}$
- **Trigonometric form:** $\mathbf{a = |a|(cos\theta \pm \mathit{j}sin\theta)}$

Rectangular ↔ Polar conversion

- Convert the following into polar form
  1) $3 + j\,4 \quad = 5 \angle 53.13°$
  2) $8 + j\,6 \quad = 10 \angle 36.87°$
  3) $8 - j\,6 \quad = 10 \angle -36.87°$

- Convert the following into rectangular form
  1) $5 \angle 30° \quad = 4.33 + j\,2.5$
  2) $3 \angle -60° \quad = 1.5 - j\,2.59$
  3) $-(10 \angle 45°) \quad = -\,7.07 - j\,7.07$

Representing AC

- Consider three sinusoidal signals $x(t), y(t)$ & $z(t)$ with same frequency

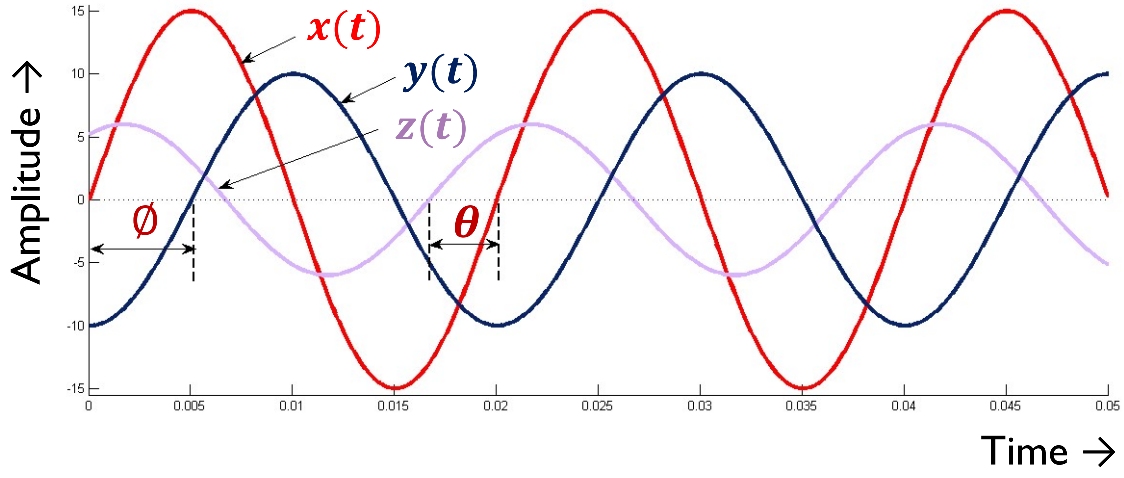
Mathematical Representation

$$x(t) = X_m sin(\omega t)$$

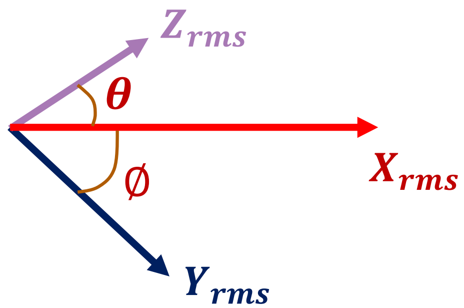
$$y(t) = Y_m \sin(\omega t - \emptyset)$$

$$z(t) = Z_m \sin(\omega t + \theta)$$

Graphical Representation

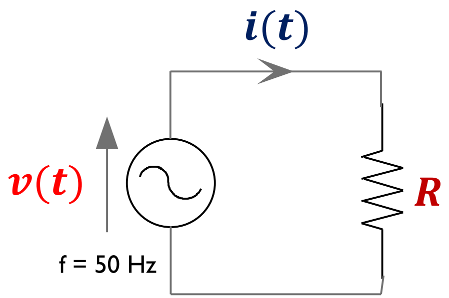


Phasor Representation



- Representing the relationship between sinusoidal signals with same frequency in graphical or mathematical form is tedious
- Phasor representation is often used

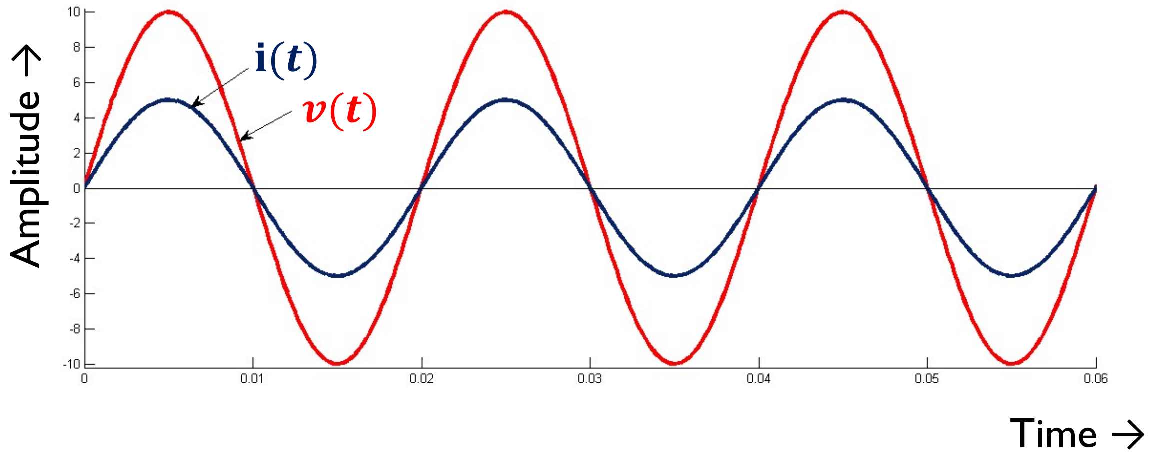
R circuit response with AC supply



$$i(t) = \frac{v(t)}{R}$$

'Current through the resistor is in phase with the voltage across it'

<u>*Graphical Representation*</u>

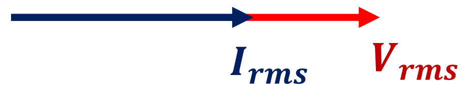


<u>*Mathematical Representation*</u>

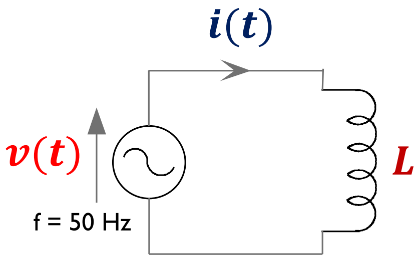
$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t)$$

<u>*Phasor Representation*</u>



L circuit response with AC supply



$$i(t) = \frac{1}{L}\int v(t)dt$$

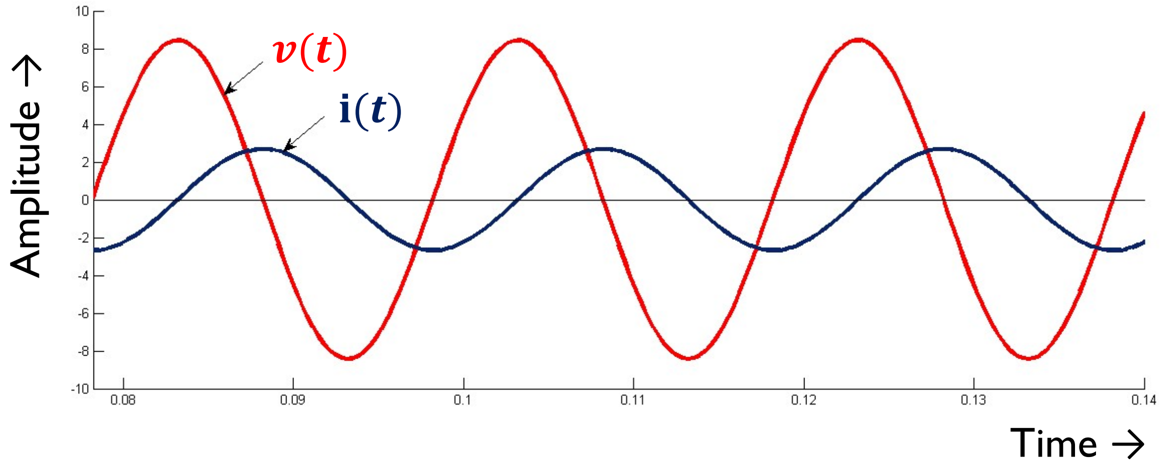
'Current through the inductor lags the voltage across it by 90°'

$$\bar{V} = V\angle 0° \qquad \bar{I} = I\angle -90°$$

$$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0°}{I\angle -90°} = jX_L \quad where\ \frac{V}{I} = X_L$$

X_L *is called* ***Inductive Reactance***

<u>*Graphical Representation*</u>



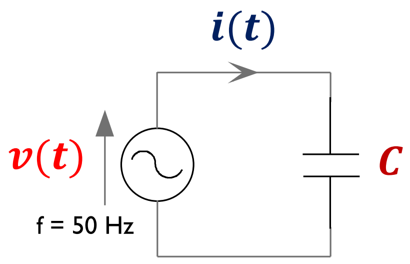
<u>*Mathematical Representation*</u>

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t - 90°)$$

<u>*Phasor Representation*</u>

C circuit response with AC supply



$$i(t) = C\frac{dv(t)}{dt}$$

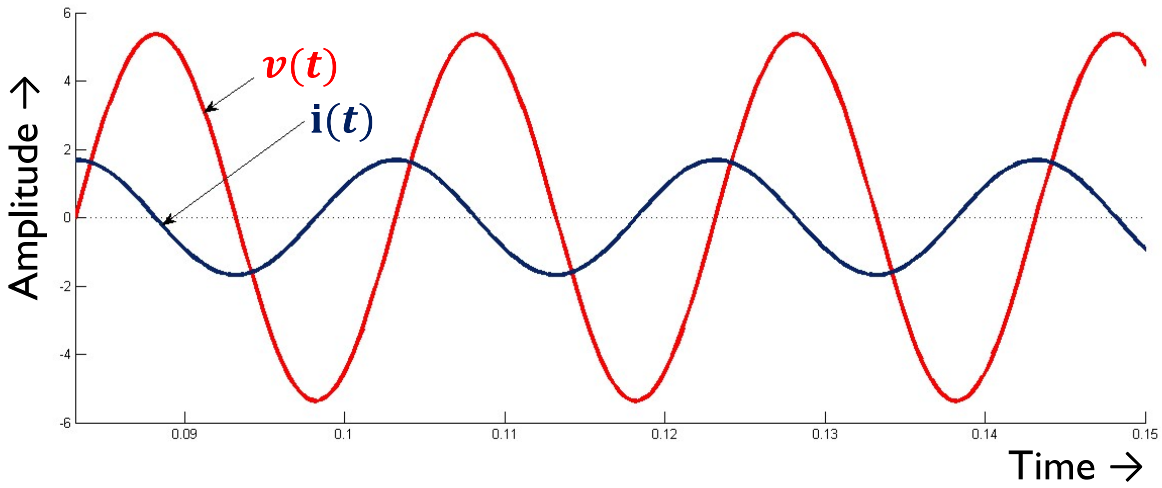
'Current through the capacitor leads the voltage across it by 90°'

$\bar{V} = V\angle 0°$ $\bar{I} = I\angle 90°$

$$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0°}{I\angle 90°} = -jX_C \quad where\ \frac{V}{I} = X_C$$

X_C is called ***Capacitive Reactance***

Graphical Representation



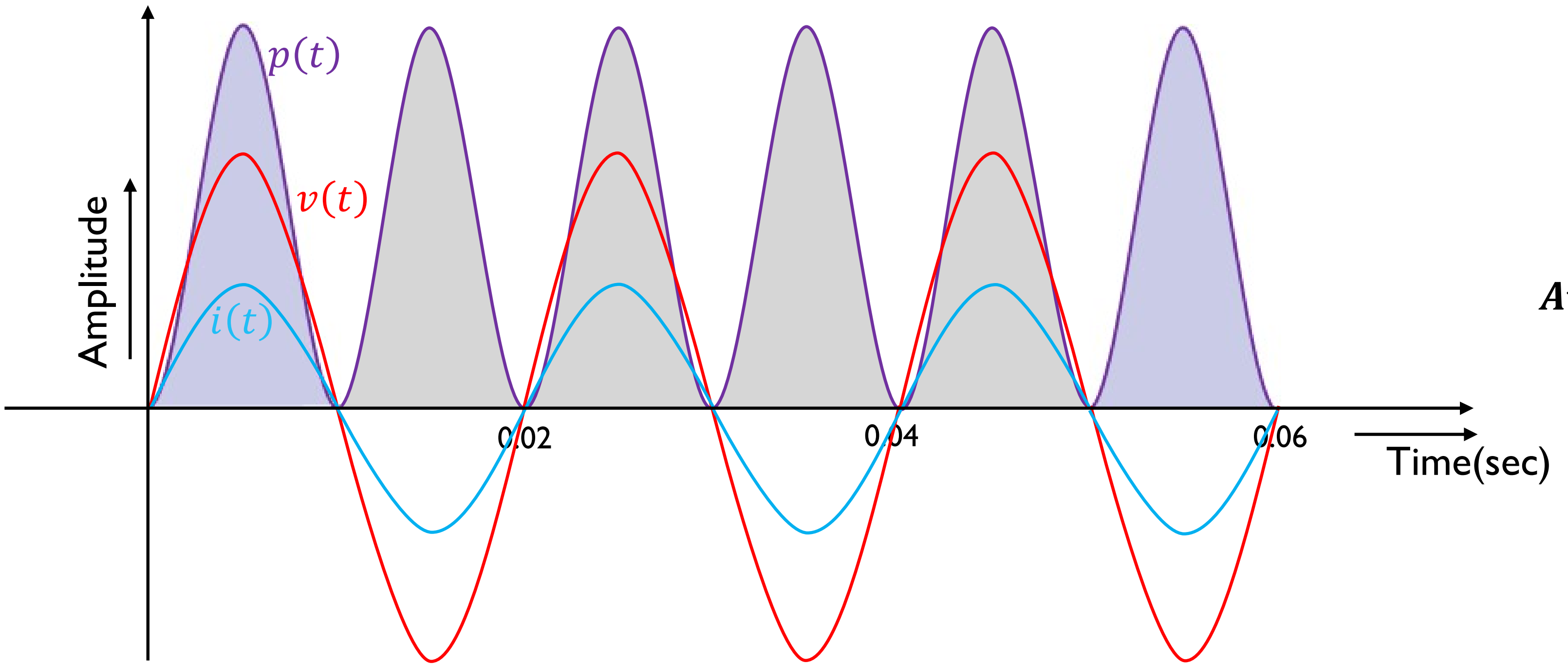
Mathematical Representation

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t + 90°)$$

Phasor Representation

Power Associated - Pure Resistive Circuit



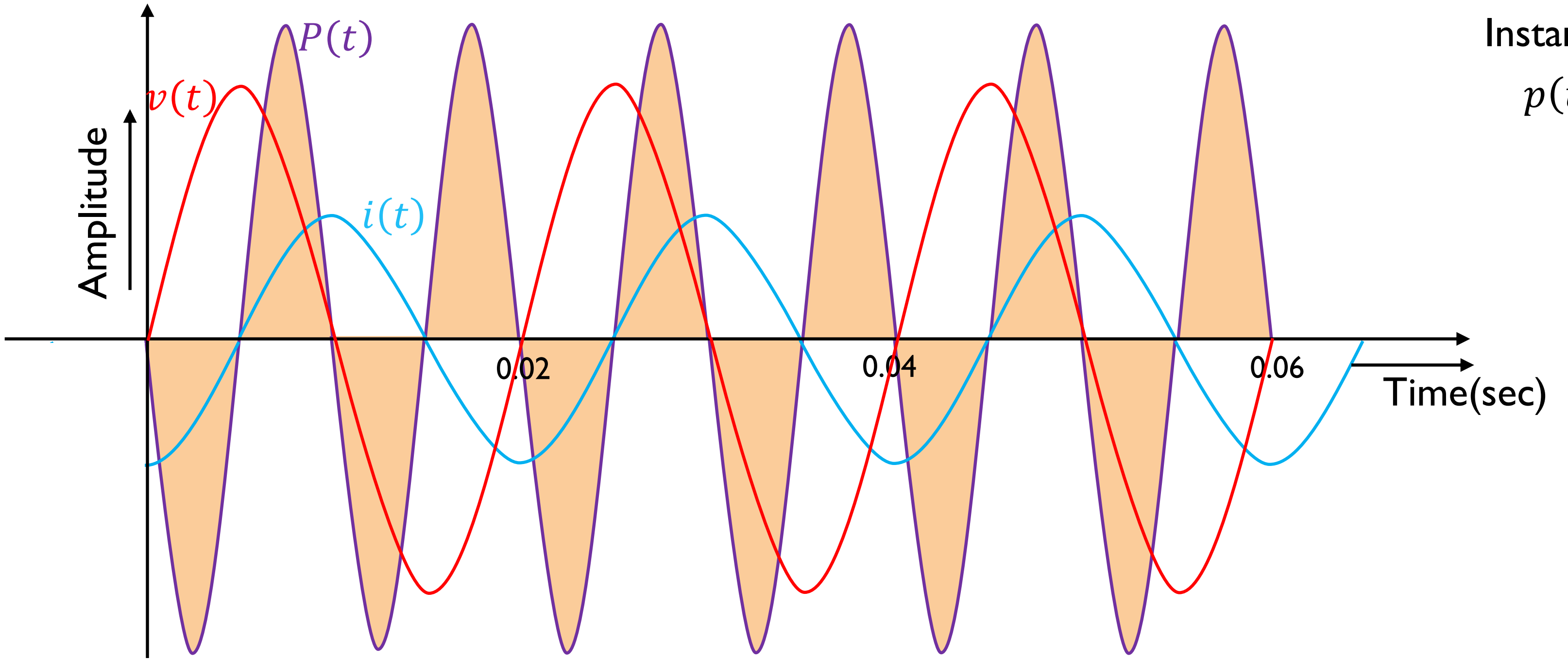
Instantaneous power,

$p(t) = v(t).i(t) = V_m I_m sin^2 \omega t$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = \frac{V_m I_m}{2} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R}}$$

Power Associated – Pure Inductive Circuit



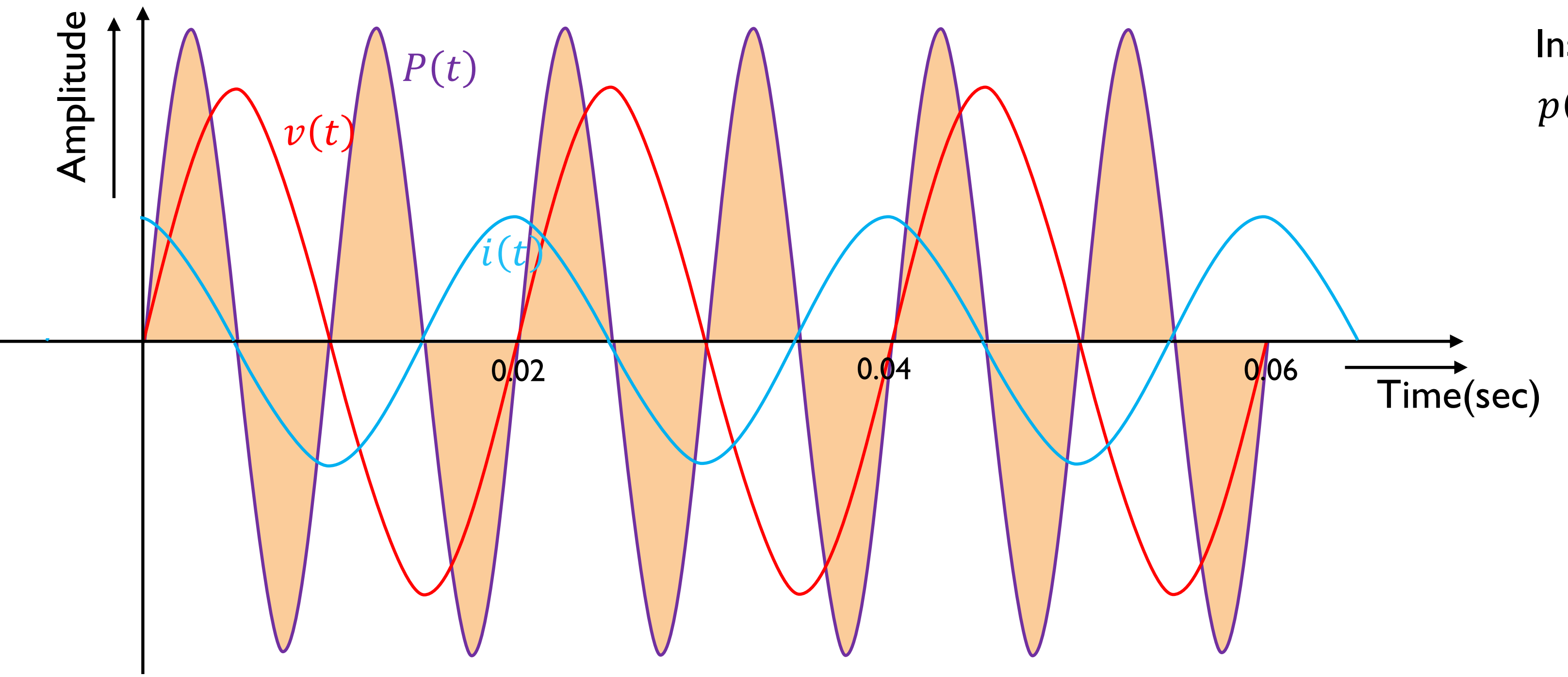
Instantaneous power,

$$p(t) = v(t).\,i(t)$$
$$= V_m I_m \sin\omega t\,.\sin(\omega t - 90\,°)$$
$$= -\frac{V_m I_m}{2}\sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = 0}}$$

Power Associated – Pure capacitive Circuit



Instantaneous power,

$$p(t) = v(t).\,i(t)$$

$$= V_m I_m \sin \omega t\,.\sin(\omega t + 90\,°)$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt}$$

$$\boxed{\boldsymbol{P_{avg} = 0}}$$

Basic Electrical Technology

[ELE 1051]

CHAPTER 3 - SINGLE PHASE AC CIRCUITS

(3.3)

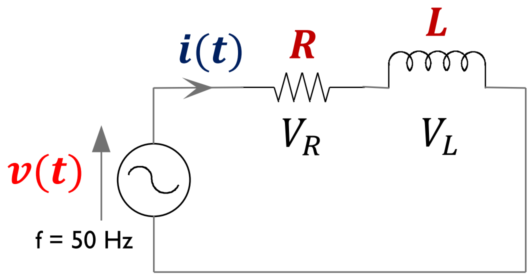
Summary of previous classes

- Comparison of DC & AC
- How is AC generated?
- Terminologies of AC
- Average value of an alternating waveform
- RMS value of an alternating waveform
- Complex numbers
- Representing AC
- R, L, C circuit response with AC supply
- Power associated with a pure R, L, C

Topics covered today

- RL, RC, RLC circuit response with AC supply
- Power associated with a series RL, RC circuits
- Loads in parallel

RL circuit analysis



Let $\bar{I}$ *be along the reference*

$$\overline{V_R} = \bar{I}R$$

$$\overline{V_L} = j\bar{I}X_L$$

$$\boldsymbol{\bar{V} = \overline{V_R} + \overline{V_L} = |V|\angle\emptyset}$$

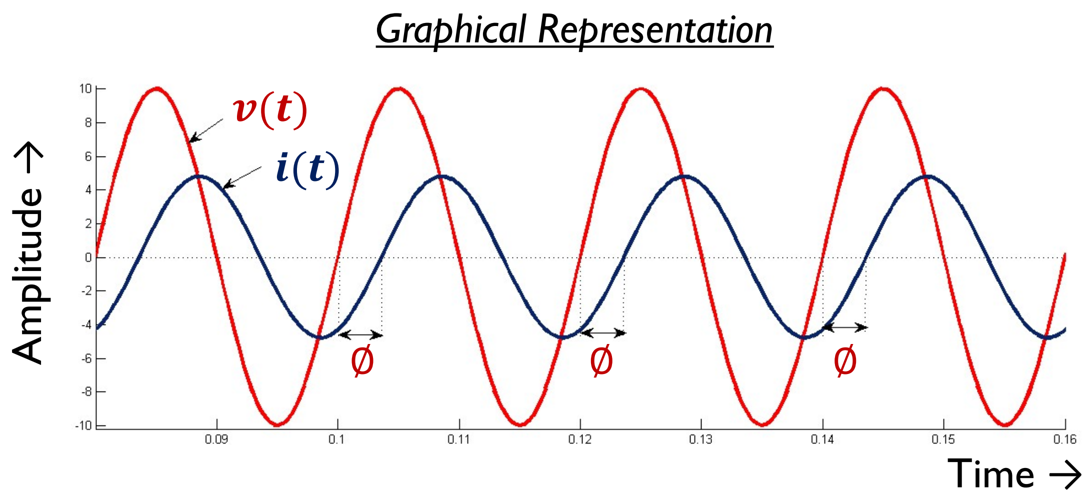
Mathematical Representation

$$i(t) = I_m \sin(\omega t)$$

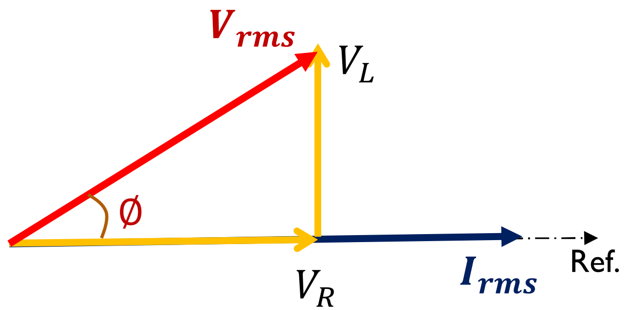
$$v(t) = V_m \sin(\omega t + \emptyset)$$

$\emptyset$ $-$ *Phase Angle*

Graphical Representation



Phasor Representation



Impedance

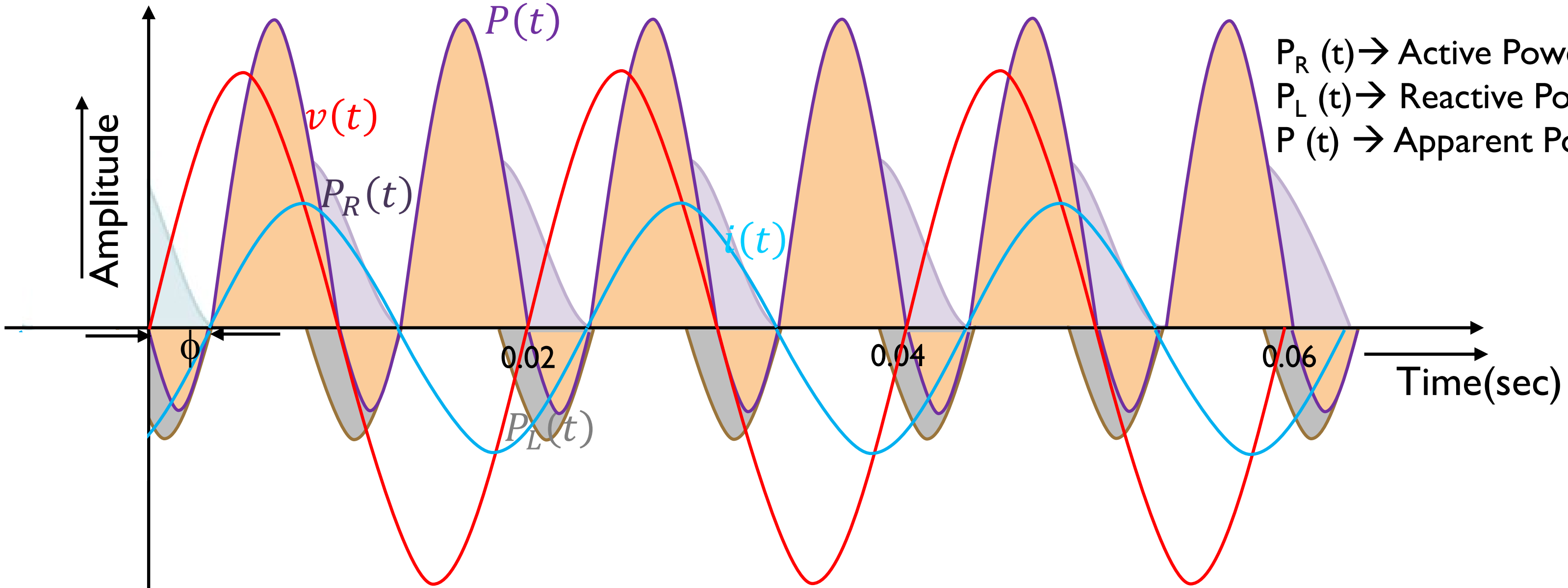
$$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R + jX_L)}{\bar{I}} = R + jX_L = |Z|\angle\emptyset$$

Z $-$ *Impedance of the circuit*

$$\therefore \boldsymbol{R = |Z|\cos\emptyset} \qquad \boldsymbol{X_L = |Z|\sin\emptyset}$$

$$|\boldsymbol{Z}| = \sqrt{\boldsymbol{R^2 + X_L^2}} \qquad \emptyset = \tan^{-1}\frac{\boldsymbol{X_L}}{\boldsymbol{R}}$$

Power associated - RL circuit



P_R (t)→ Active Power
P_L (t)→ Reactive Power
P (t) → Apparent Power

Instantaneous power,

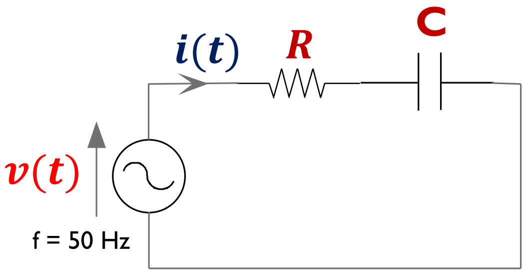
$$p(t) = v(t).i(t)$$
$$= V_m I_m \sin \omega t . \sin(\omega t + \emptyset)$$
$$= V_{rms} I_{rms} [\cos \emptyset - \cos(2\omega t + \emptyset)]$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2}\cos \emptyset}$$

$$\boxed{\mathbf{P_{avg} = V_{rms} I_{rms} \cos \emptyset}}$$

$\cos \emptyset$ *is called the* ***Power Factor***

RC circuit analysis



Let $\bar{I}$ *be along the reference*

$$\overline{V_R} = \bar{I}R$$

$$\overline{V_C} = -j\bar{I}X_C$$

$$\bar{V} = \overline{V_R} + \overline{V_C} = |V|\angle -\emptyset$$

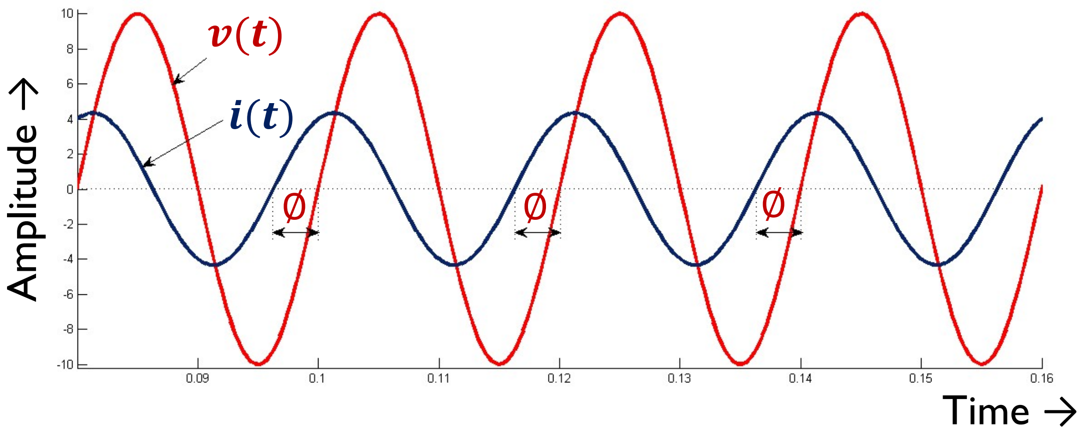
Mathematical Representation

$$i(t) = I_m \sin(\omega t)$$

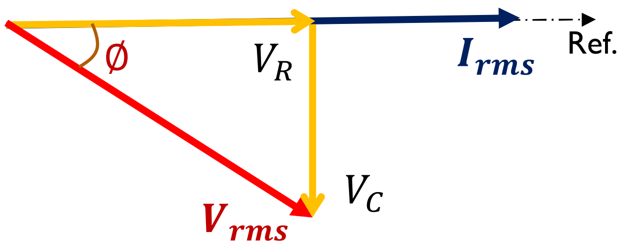
$$v(t) = V_m \sin(\omega t - \emptyset)$$

$\emptyset$ − *Phase Angle*

Graphical Representation



Phasor Representation



Impedance

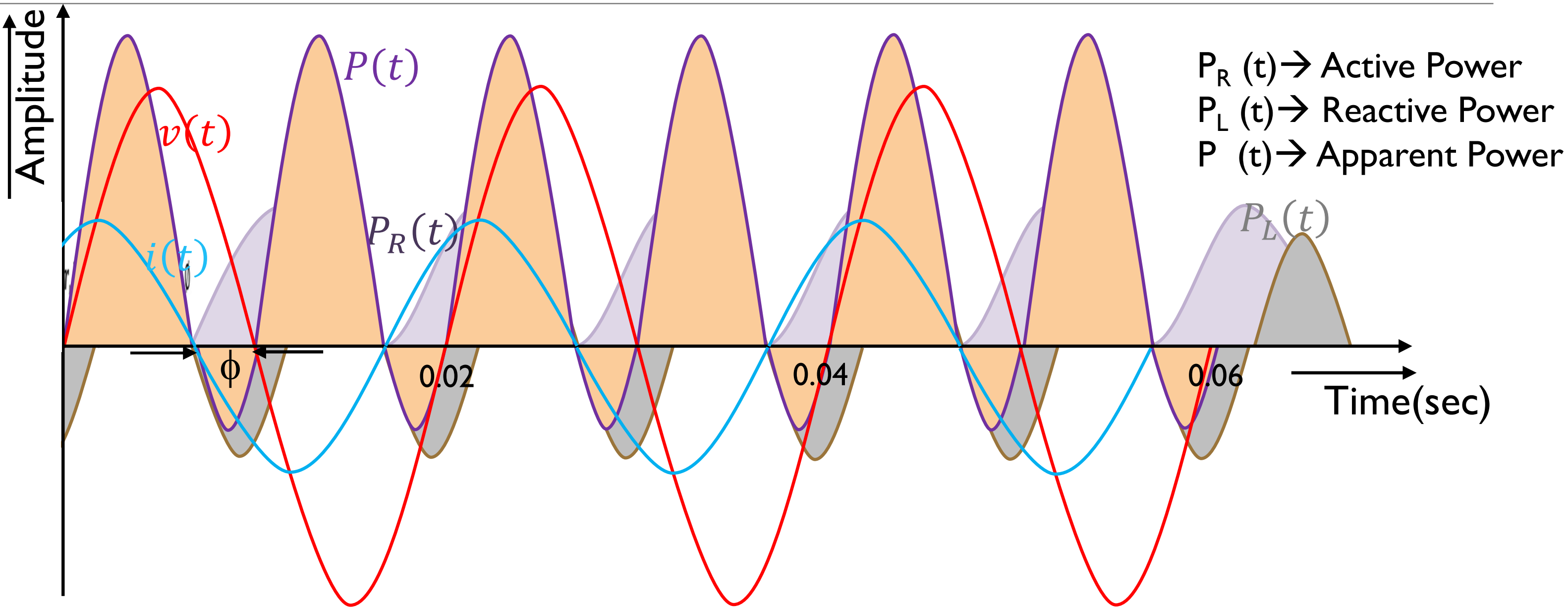
$$\frac{\bar{V}}{\bar{I}} = \frac{\bar{I}(R - jX_L)}{\bar{I}} = R - jX_L = |Z|\angle -\emptyset$$

Z − *Impedance of the circuit*

$\therefore\ \boldsymbol{R = |Z|\cos\emptyset}$ $\quad \boldsymbol{X_C = |Z|\sin\emptyset}$

$|\boldsymbol{Z}| = \sqrt{\boldsymbol{R^2 + X_C^2}}$ $\quad \emptyset = \tan^{-1}\frac{\boldsymbol{X_C}}{\boldsymbol{R}}$

Power associated - RC circuit



Instantaneous power,

$$p(t) = v(t).i(t)$$

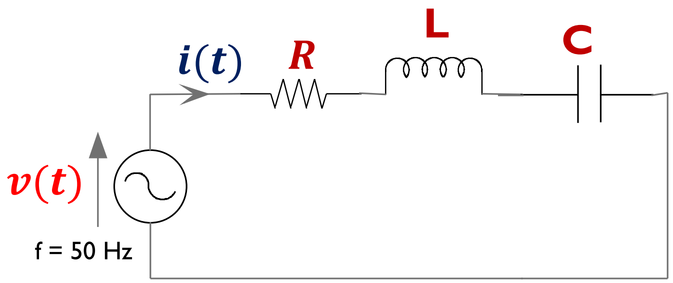
$$= V_m I_m \sin\omega t\,.\sin(\omega t - \emptyset)$$

$$= V_{rms} I_{rms}[\cos\emptyset - \cos(2\omega t - \emptyset)]$$

$$\boldsymbol{Average\ Power, P = \frac{1}{T}\int_0^T p(t)dt = \frac{V_m I_m}{2}\cos\emptyset}$$

$$\boxed{\mathbf{P_{avg} = V_{rms} I_{rms} \cos\emptyset}}$$

RLC circuit



$Let\ i(t)\ be\ the\ reference$

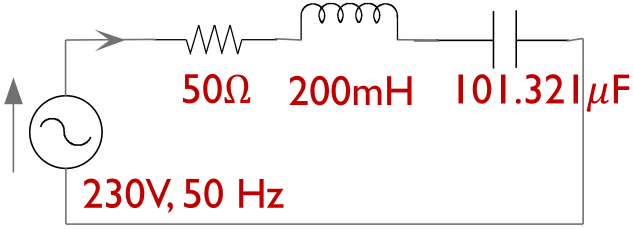
$\boldsymbol{Impedance, Z = R + j(X_L \sim X_c)}$

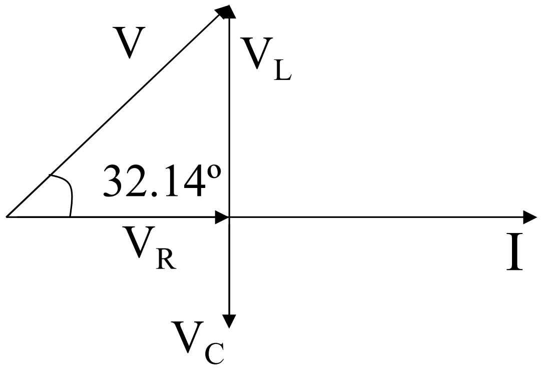
$if\ X_L = X_C$	$\Longrightarrow$	$Resistive\ circuit$ $(Resonance\ condition)$
$if\ X_L > X_C$	$\Longrightarrow$	$RL\ series\ circuit$
$if\ X_L < X_C$	$\Longrightarrow$	$RC\ series\ circuit$

Illustration 1

A resistance of 50Ω is connected in series with an inductance of 200mH and capacitance of 101.321μF across a 230V, 50 Hz, single phase AC supply. Obtain,

a) *Impedance of the circuit*
b) *Current drawn*
c) *Power factor*
d) *Power consumed*
e) *Phasor diagram*





$$X_L = 2 \times \pi \times 50 \times 0.2 = 62.8315\Omega$$

$$X_c = \frac{1}{2 \times \pi \times 50 \times 101.321\mu} = 31.4159\Omega \qquad PF = \cos(32.14) = \mathbf{0.846\ lag}$$

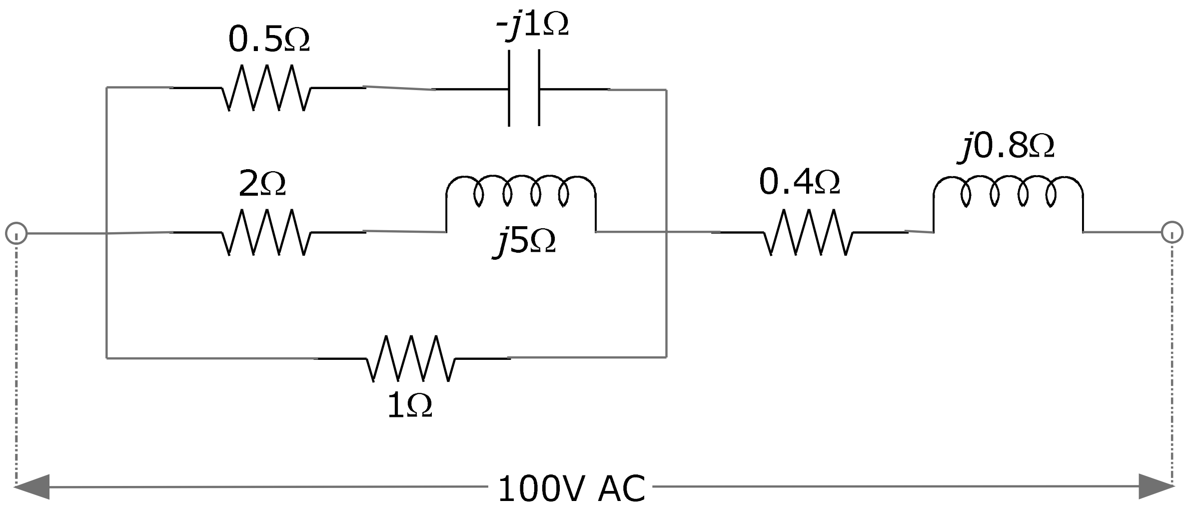
$$Z = R + jX_L - jX_c = \mathbf{50 + j31.4156\Omega = 59.050\angle 32.14^\circ\ \Omega}$$

$$I = \frac{230\angle 0}{59.05\angle 32.14} = \mathbf{3.898\angle - 32.14^\circ A}$$

$$P = |V_{rms}||I_{rms}|cos\emptyset$$
$$= 230 \times 3.898 \times 0.846 = \mathbf{759.15W}$$

Illustration 2

Determine the impedance of the circuit shown and the power consumed in each branch



$Z_1 = 0.5 - j1\Omega$ $\quad Z_3 = 1\Omega$

$Z_2 = 2 + j5\Omega$ $\quad Z_4 = 0.4 + j0.8\Omega$

$$\bar{I} = \frac{100\angle 0}{1.12\angle 29.5} = \mathbf{89.285\angle - 29.5A} = \bar{I_4}$$

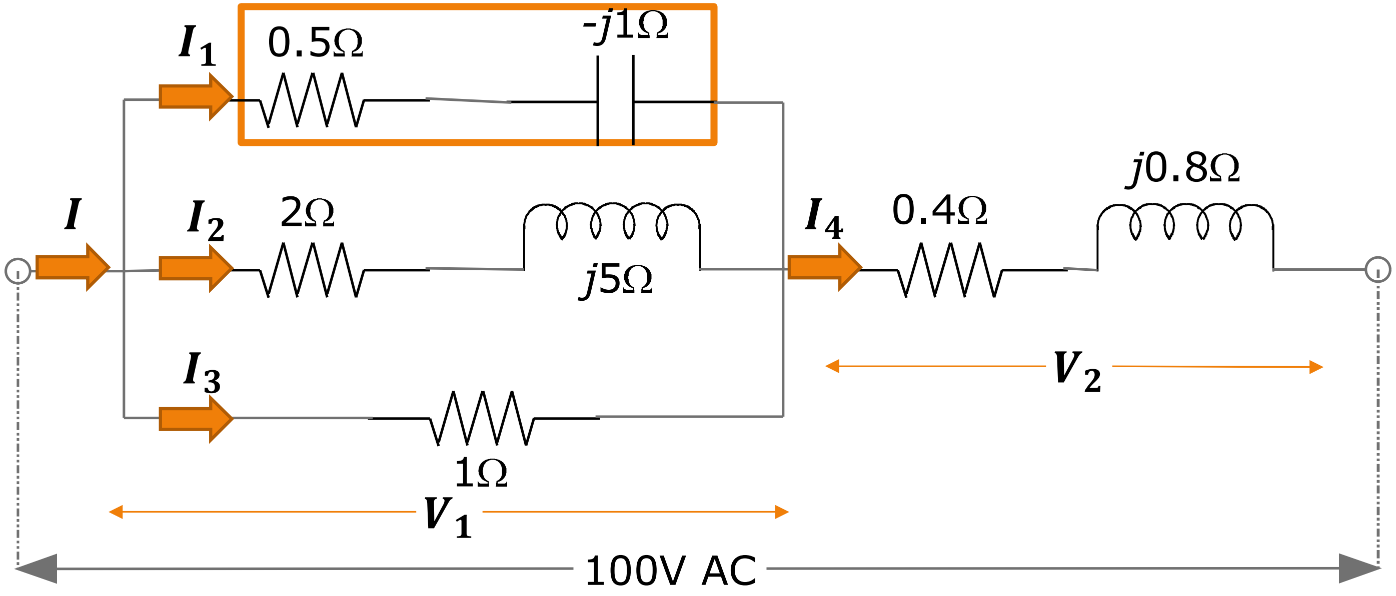
$$Z_{eq} = (Z_1||Z_2||Z_3) + Z_4 = \mathbf{1.12\angle 29.5°\Omega}$$

$$\overline{\boldsymbol{V_2}} = \overline{\boldsymbol{I_4}} \times \boldsymbol{Z_4} = \mathbf{79.85\angle 33.934\ V}$$

$$\bar{V} = \overline{V_1} + \overline{V_2} \qquad \overline{\boldsymbol{V_1}} = \mathbf{55.91\angle - 52.868\ V}$$

$$\overline{\boldsymbol{I_1}} = \frac{\overline{\boldsymbol{V_1}}}{\boldsymbol{Z_1}} = \mathbf{50.00\angle 10.565\ A} \qquad \overline{\boldsymbol{I_2}} = \mathbf{10.38\angle - 121.068\ A} \qquad \overline{\boldsymbol{I_3}} = \mathbf{55.91\angle - 52.868\ A}$$

$$\boldsymbol{P_1} = |\boldsymbol{I_1}|^2 \times \boldsymbol{R_1} = \mathbf{1.25\ kW} \qquad P_2 = 0.215\ kW \qquad P_3 = 3.125\ kW \qquad P_4 = 3.188\ kW$$



Basic Electrical Technology

[ELE 1051]

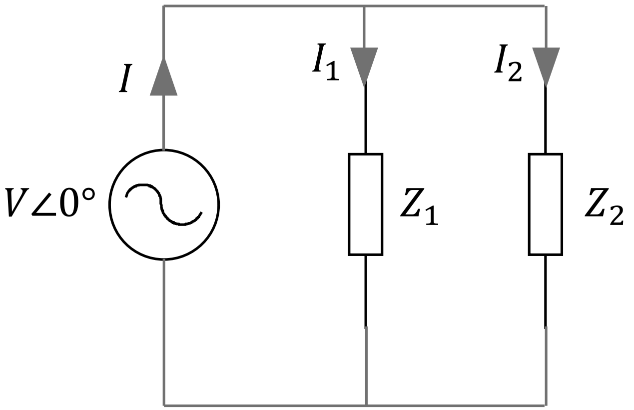
CHAPTER 3 - SINGLE PHASE AC CIRCUITS

(3.4)

Topics covered today

- Loads in parallel
- AC circuit equations and solving
- Tutorial 1

Impedance in parallel



Let $Z_1 = R_1 + jX_1$ $\qquad Z_2 = R_2 + jX_2$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad I_1 = I * \frac{Z_2}{Z_1 + Z_2} \qquad I_2 = I * \frac{Z_1}{Z_1 + Z_2}$$

$$Y_{eq} = Y_1 + Y_2 \qquad Y: \ Admittance$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_1} = \frac{1}{(R_1 + jX_1)} * \frac{(R_1 - jX_1)}{(R_1 - jX_1)} = \frac{R_1}{(R_1^2 + X_1^2)} - j\frac{X_1}{(R_1^2 + X_1^2)} = G_1 - jB_1$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 + jX_2} = \frac{1}{(R_2 + jX_2)} * \frac{(R_2 - jX_2)}{(R_2 - jX_2)} = \frac{R_2}{(R_2^2 + X_2^2)} - j\frac{X_2}{(R_2^2 + X_2^2)} = G_2 - jB_2$$

$$G: \ Conductance \qquad B: \ Susceptance$$

$$\mathbf{Y_{eq} = (G_1 + G_2) - \mathit{j}(B_1 + B_2) = G_{eq} - \mathit{j}B_{eq}}$$

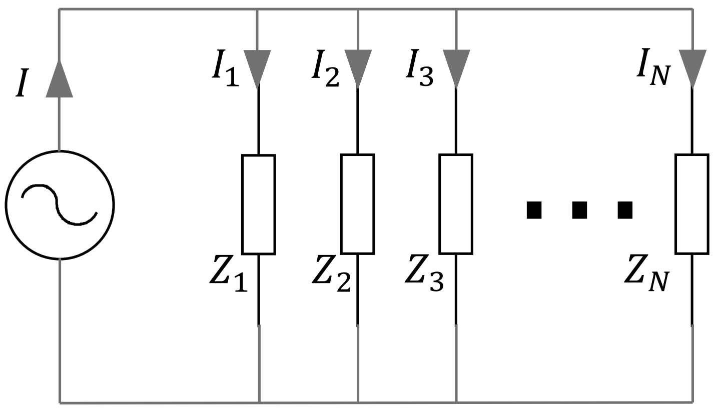
Impedance in parallel

For 'N' impedances connected in parallel,

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}$$

$$Y_{eq} = Y_1 + Y_2 + Y_3 + \cdots + Y_N$$

$$\mathbf{Y_{eq} = G_{eq} \pm \mathit{j}B_{eq}}$$



$$I_1 = VY_1; I_2 = VY_2; I_3 = VY_3; \ldots \ldots I_N = VY_N$$

$$I = I_1 + I_2 + I_3 + \cdots + I_N = VY_{eq}$$

Network equations for AC circuits

KVL Equation

(Matrix form)

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

$$[V] = [Z][I]$$

KCL Equation

(Matrix form)

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

$$[I] = [Y][V]$$

All the other theorems are applicable to the AC circuits

Crammers rule

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

Solution for the linear simultaneous equations above is as follows

Step 1: finding the determinant

$$\Delta = \begin{vmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{vmatrix}$$

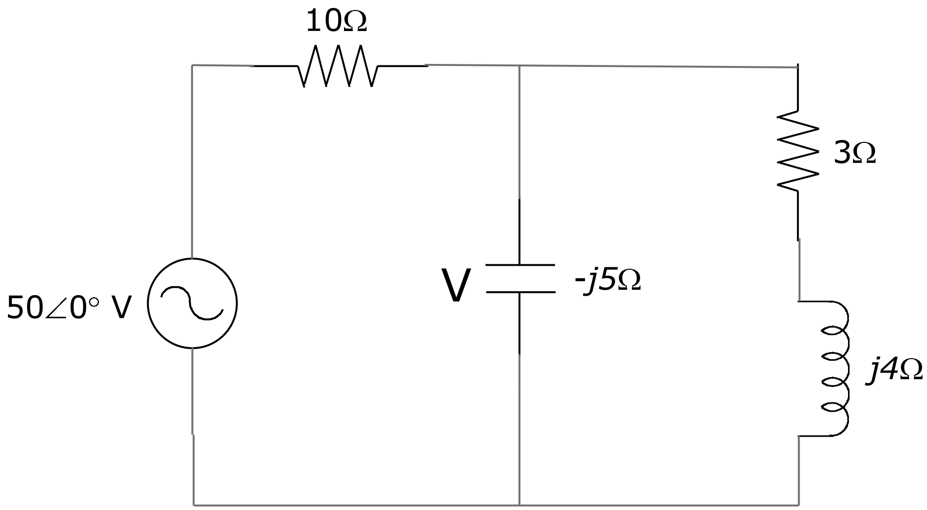
Step 2: finding the determinant after substituting first column with RHS column matrix

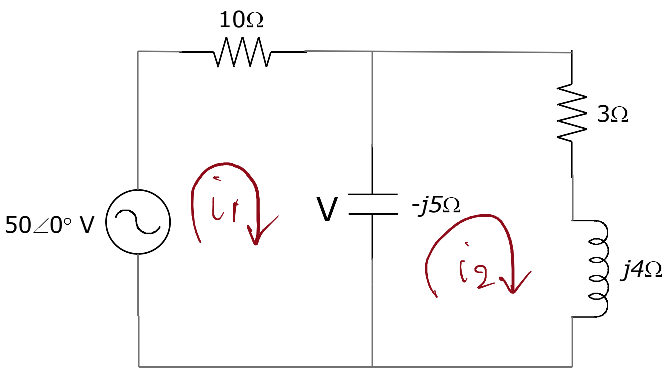
$$\Delta_1 = \begin{vmatrix} \boldsymbol{V_1} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ \boldsymbol{V_N} & \cdots & Z_{NN} \end{vmatrix}$$

Step 3 :Solution for I_1 $\quad I_1 = \frac{\Delta_1}{\Delta}$

Illustration 1

Assigning two mesh currents, find the voltage V across the capacitor in the following circuit





$$\begin{bmatrix} 10-j5 & j5 \\ j5 & 3-j \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50\angle 0 \\ 0 \end{bmatrix}$$

$$\Delta = 50 - j25$$

$$\Delta_{i_2} = -250j$$

$$\Delta_{i_1} = 150 - 50j$$

$$i_2 = \frac{\Delta_{i_2}}{\Delta} = 2 - 4j\ A \quad (4.4721\angle -63.43\ A)$$

$$i_1 = \frac{\Delta_{i_1}}{\Delta} = 2.8 + 0.4j\ A \quad (2.8284\angle 8.1301\ A)$$

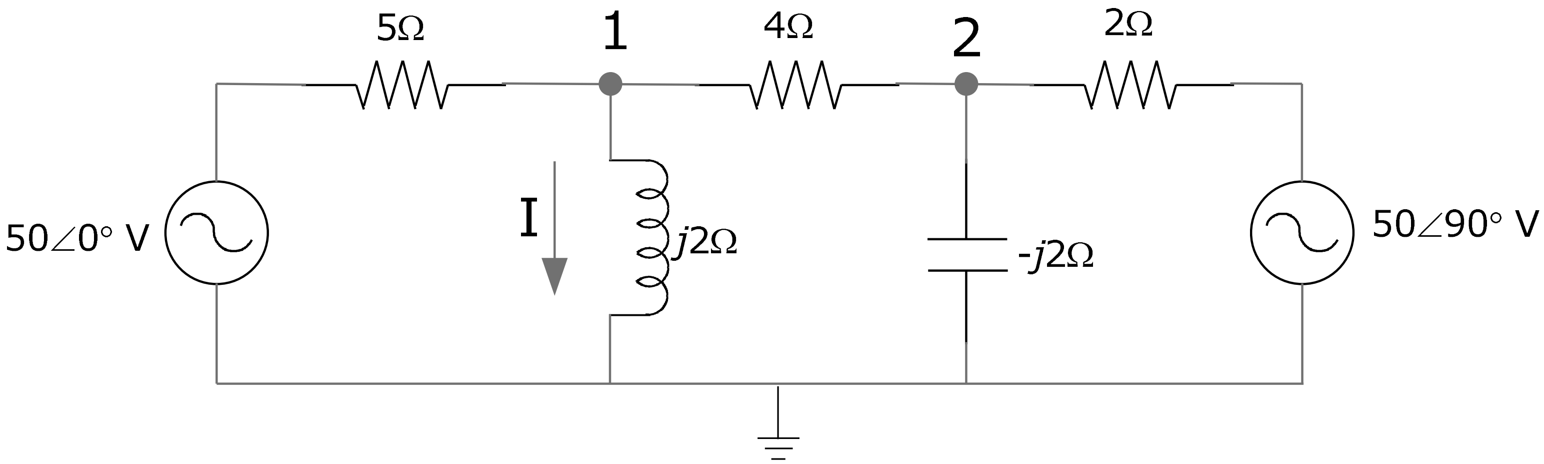
$$V = (i_2 - i_1)(-j5)$$

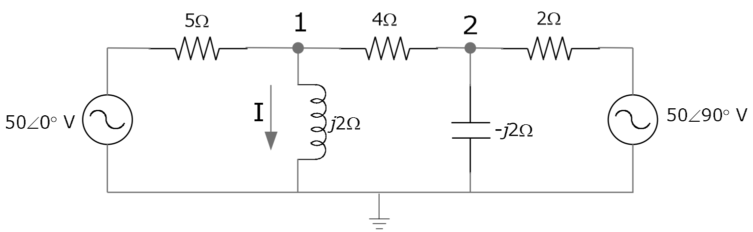
$$\underline{\underline{V = 22.36\angle 169.69\ V}}$$

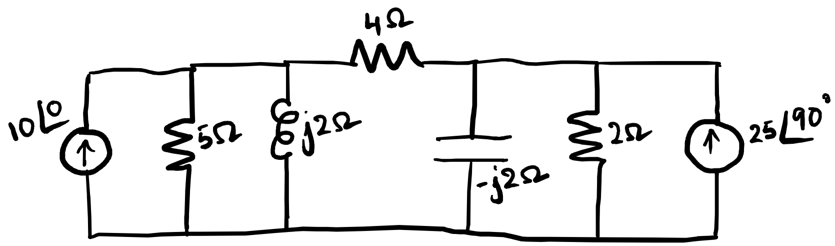
Ans:
$V = 22.36\angle 169.69°\ V$

Illustration 2

Use node voltage method to obtain the current I in the network







$$\begin{bmatrix} \left(\frac{1}{5}+\frac{1}{2j}+\frac{1}{4}\right) & -\frac{1}{4} \\ -\frac{1}{4} & \left(\frac{1}{4}+\frac{1}{2}-\frac{1}{2j}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ 25\angle 90 \end{bmatrix}$$

$$\Delta = 0.525 - 0.15j$$

$$\Delta_{V_1} = 7.5 + 11.25j$$

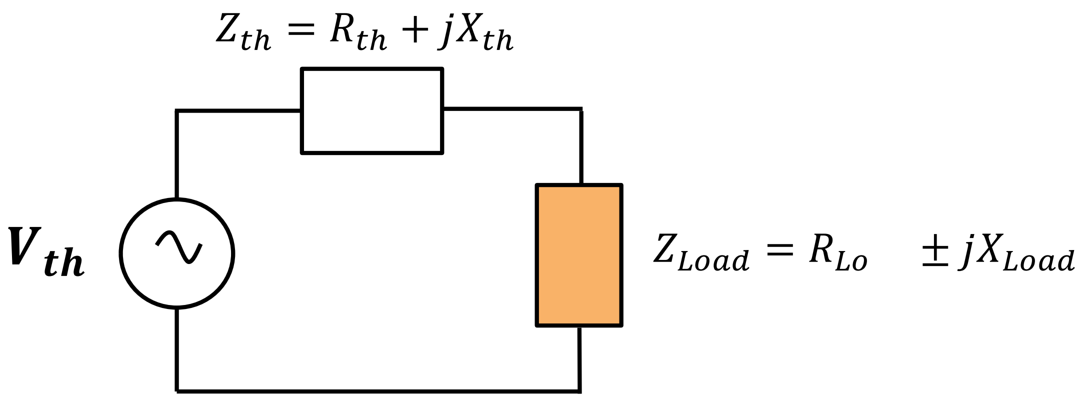
$$V_1 = \frac{\Delta_{V_1}}{\Delta} = 24.763\angle 72.25\,V$$

$$I = \frac{V_1}{2j} = \underline{\underline{12.38\angle -17.75\,A}}$$

Ans:

$I = 12.38\angle -17.75°\ A$

Maximum power transfer theorem

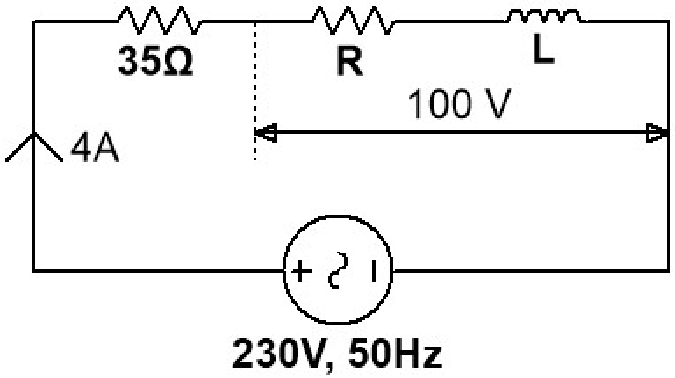


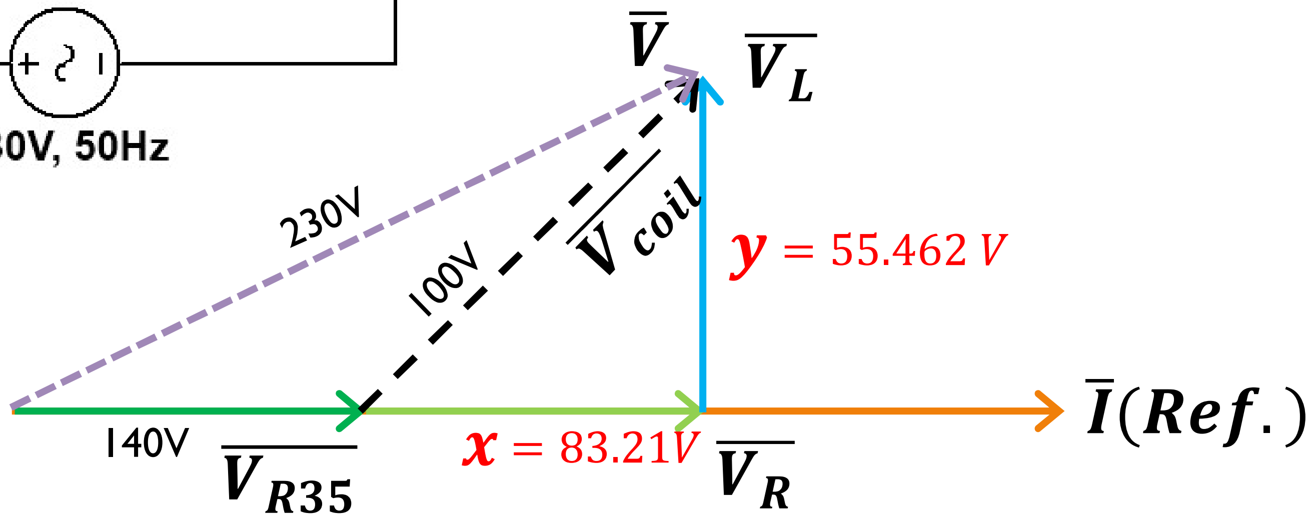
	Type of load	Condition of maximum power transfer
Case 1	Load is purely resistive	$R_L = \sqrt{R_{th}^2 + X_{th}^2}$
Case 2	Both R_L & X_L are variable	$Z_L = Z_{TH}^*$
Case 3	X_L is fixed & R_L is variable	$R_L = \sqrt{R_{th}^2 + (X_{th} + X_L)^2}$

Tutorial Questions

Exercise 1

A resistance of 35Ω is connected in series with an inductive coil having an internal resistance 'R' and inductance 'L'. When connected across 230V, 50Hz single phase supply, voltage across the coil is 100V and the current drawn is 4 A. Find the unknowns 'R' and 'L'.





$$(140 + x)^2 + y^2 = 230^2$$
$$x^2 + y^2 = 100^2$$

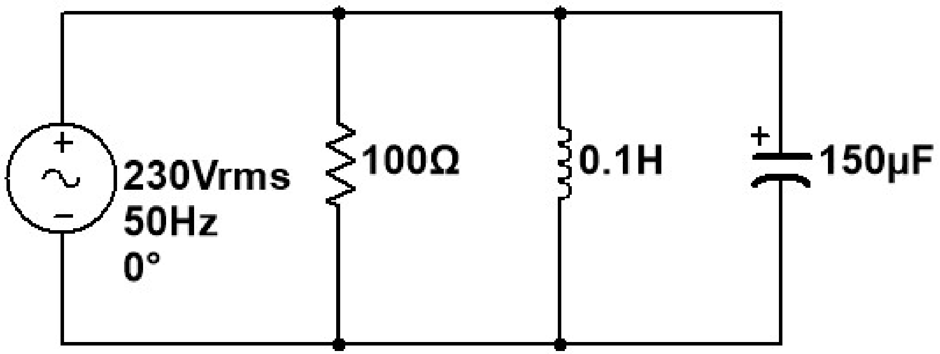
$$\boldsymbol{R} = \frac{V_R}{I} = \frac{x}{I} = \frac{83.21}{4} = \mathbf{20.80\Omega}$$

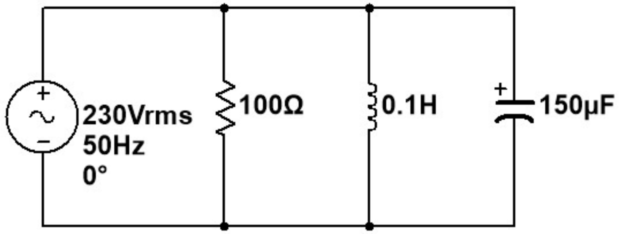
$$X_L = \frac{V_L}{I} = \frac{y}{I} = \frac{55.462}{4} = 13.8655\Omega$$

$$\therefore \boldsymbol{L = 0.044H}$$

Exercise 2

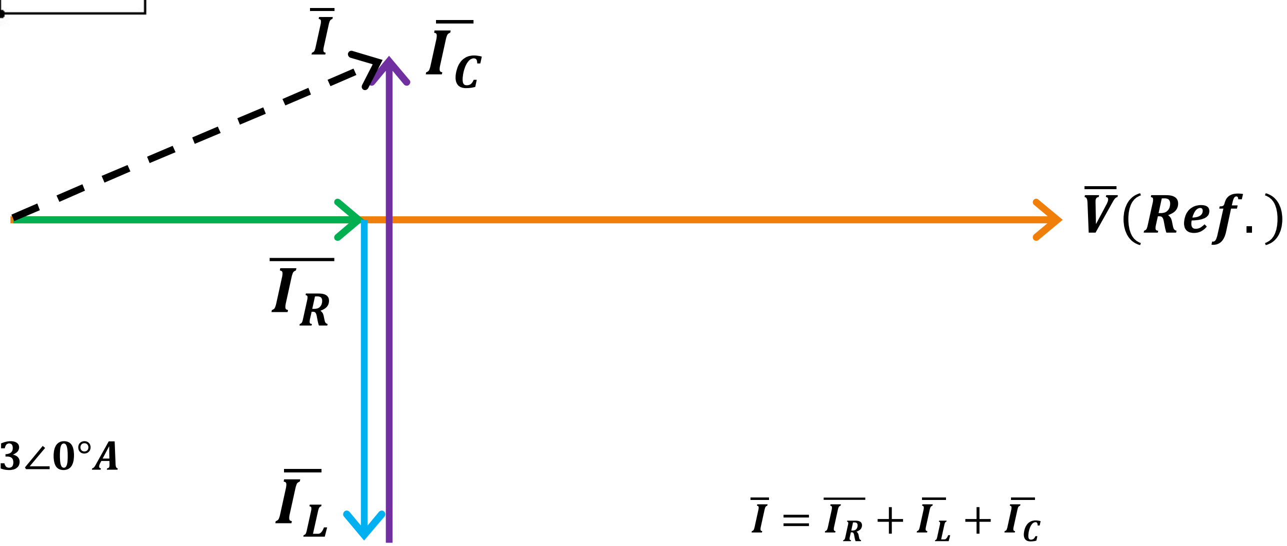
Three elements, a resistance of 100Ω, an inductance of 0.1H and a capacitance of 150μF are connected in parallel to a 230V, 50Hz supply. Calculate the current in each element and the supply current. Draw the phasor diagram.





$X_L = 31.4159\Omega$

$X_C = 21.2206\Omega$



$$\overline{I_R} = \frac{230\angle 0°}{100} = \mathbf{2.3\angle 0°A}$$

$$\overline{I_L} = \frac{\overline{V}}{jX_L} = \frac{230\angle 0°}{j31.4159} = \mathbf{7.3211\angle -90°A}$$

$$\overline{I_C} = \frac{\overline{V}}{-jX_C} = \frac{230\angle 0°}{-j21.2206} = \mathbf{10.83\angle 90°A}$$

$$\overline{I} = \overline{I_R} + \overline{I_L} + \overline{I_C}$$

$$\overline{I} = \mathbf{4.2\angle 56.819°A}$$

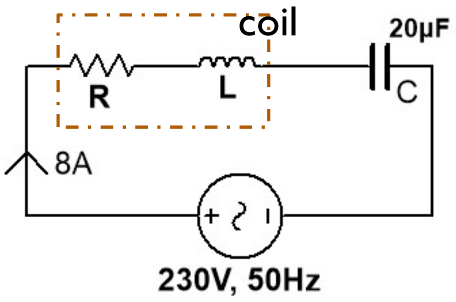
Exercise 3

A coil is in series with a 20μF capacitor across a 230V 50 Hz supply. The current taken by the circuit is 8A and power consumed is 200W.

Calculate the inductance of the coil if the power factor of the circuit is lagging.

Calculate the inductance of the coil if the power factor of the circuit is leading.

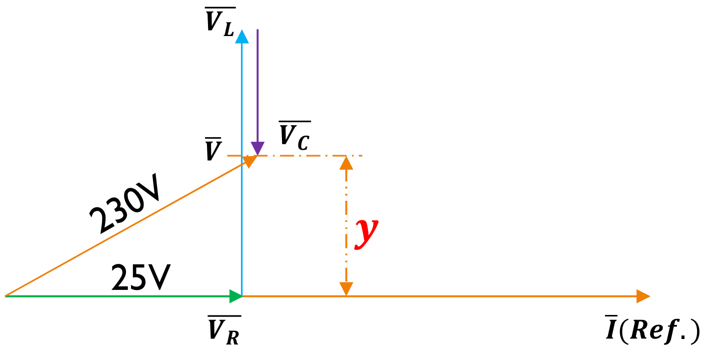
Draw the phasor diagram.



$$R = \frac{P_R}{I^2} = \frac{200}{8^2} = \mathbf{3.125\ \Omega} \qquad \boldsymbol{V_R = 25\ V}$$

$$V_C = IX_C = \frac{8}{2\pi \times 50 \times 20\mu} = 1273.2395\ V$$

Case 1
(p.f. is lagging)



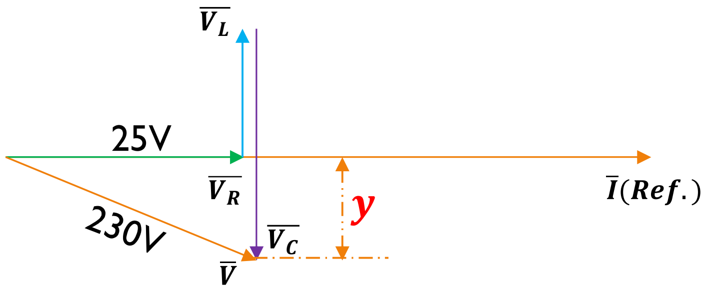
$$25^2 + y^2 = 230^2$$

$$y = 228.6372$$

$$\therefore V_L = V_C + y = 1501.8695\ V$$

$$X_L = \frac{V_L}{I} = 187.7336\Omega \qquad \boldsymbol{L = 0.5975H}$$

Case 2
(p.f. is leading)



$$25^2 + y^2 = 230^2$$

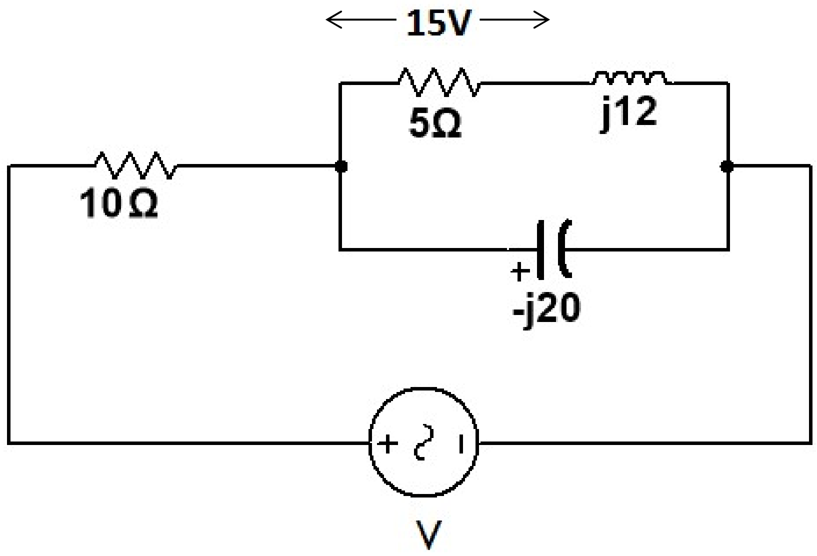
$$y = 228.6372$$

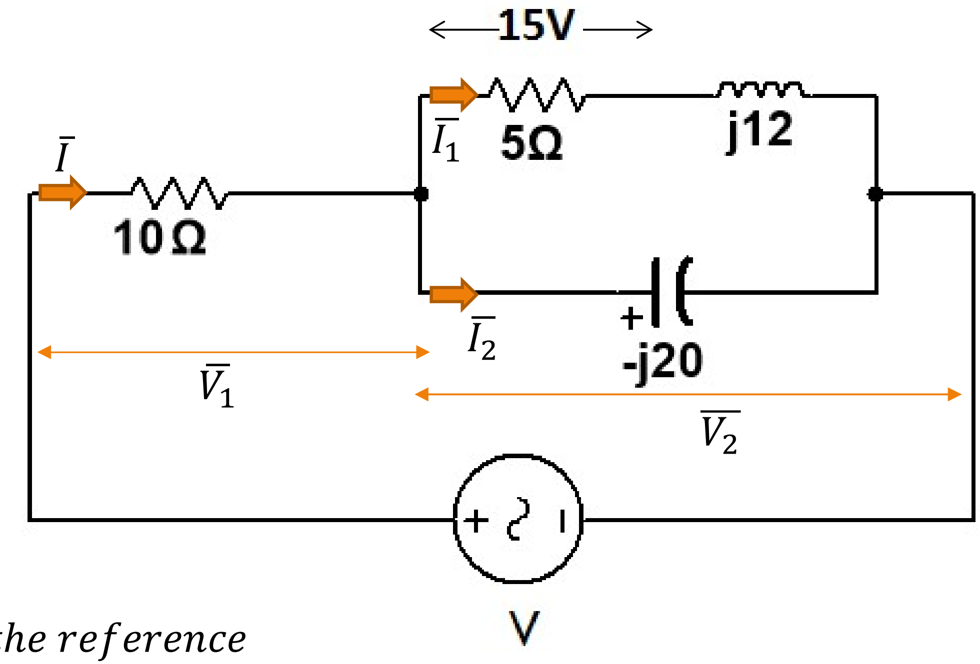
$$\therefore V_L = V_C - y = 1044.6023\ V$$

$$X_L = \frac{V_L}{I} = 130.575\Omega \qquad \boldsymbol{L = 0.4156H}$$

Exercise 4

Find the supply voltage, total current and the value of the power consumed in each arm of the series parallel circuit shown. The voltage across the 5Ω resistor is 15V.





$Assume\ \overline{V_2}\ as\ the\ reference$

$|I_1| = \frac{15}{5} = 3A$ $\quad \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{12}{5} = 67.38°$

$\therefore \boldsymbol{\bar{I}_1 = 3\angle - 67.38°\ A}$

$\overline{V_2} = \bar{I_1} \times (5 + j12) = 39\angle 0°\ V$

$\therefore \boldsymbol{\bar{I}_2 = \frac{\overline{V_2}}{-j20} = 1.95\angle 90°\ A}$

$\boldsymbol{\bar{I} = \bar{I}_1 + \bar{I}_2 = 1.415\angle - 35.3746°\ A}$

$\bar{V_1} = \bar{I} \times 10 = 14.15\angle - 35.3746°\ A$

$\boldsymbol{\bar{V} = \overline{V_1} + \overline{V_2} = 51.1972\angle - 9.207°\ V}$

$\boldsymbol{P_{5\Omega} = I_1^2 \times 5 = 45W}$

$\boldsymbol{P_{10\Omega} = I^2 \times 10 = 20.022\ W}$

Basic Electrical Technology

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CHAPTER 3 - SINGLE PHASE AC CIRCUITS

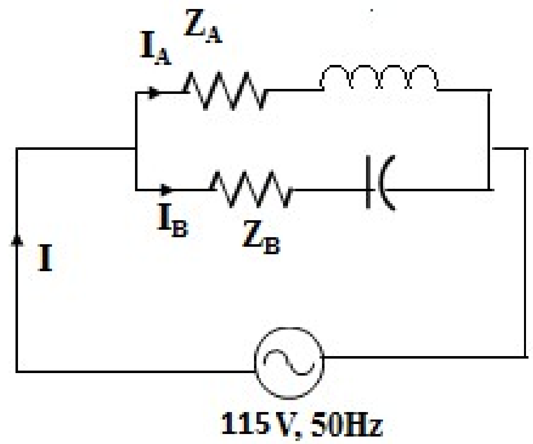
(3.5)

Exercise 5

Two impedances Z_A and Z_B are connected in parallel across a 115V, 50Hz supply. The total current taken by the combination is 10A at unity p.f. Z_B has resistance of 10Ω and 200μF capacitor connected in series. Z_A consists of a resistor and inductor is series. Find

(a) The current in each branch

(b) The resistance and inductance of Z_A



Assume supply voltage as the reference $\Longrightarrow$ $\bar{V} = 115\angle 0° \, V$

Given, supply current, $\bar{I} = 10\angle 0° \, A$

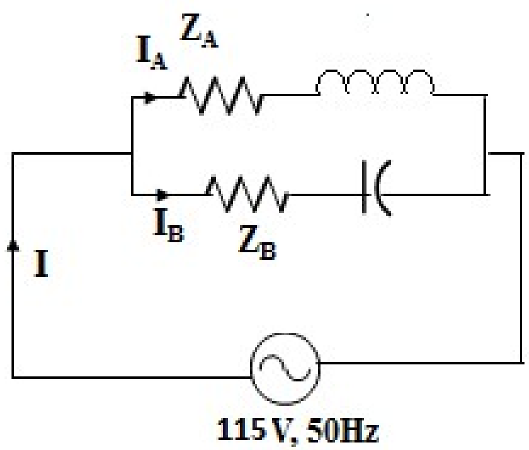
$X_C = 15.9154 \, \Omega$ $\qquad$ $Z_B = 10 - j15.9154 \, \Omega$

$$\overline{\boldsymbol{I_B}} = \frac{\bar{\boldsymbol{V}}}{\boldsymbol{Z_B}} = \mathbf{6.1182\angle 57.8579°} \, \boldsymbol{A}$$

$\bar{I} = \bar{I_A} + \bar{I_B}$

$\overline{\boldsymbol{I_A}} = \mathbf{8.5048\angle - 37.5261°} \, \boldsymbol{A}$

$$Z_A = \frac{\bar{V}}{\bar{I_A}} = \underbrace{10.7237}_{\boldsymbol{R_A}} + \underbrace{j8.2364}_{\boldsymbol{jX_A}} \, \Omega$$



$$\boldsymbol{R_A} = \mathbf{10.72} \, \boldsymbol{\Omega}$$

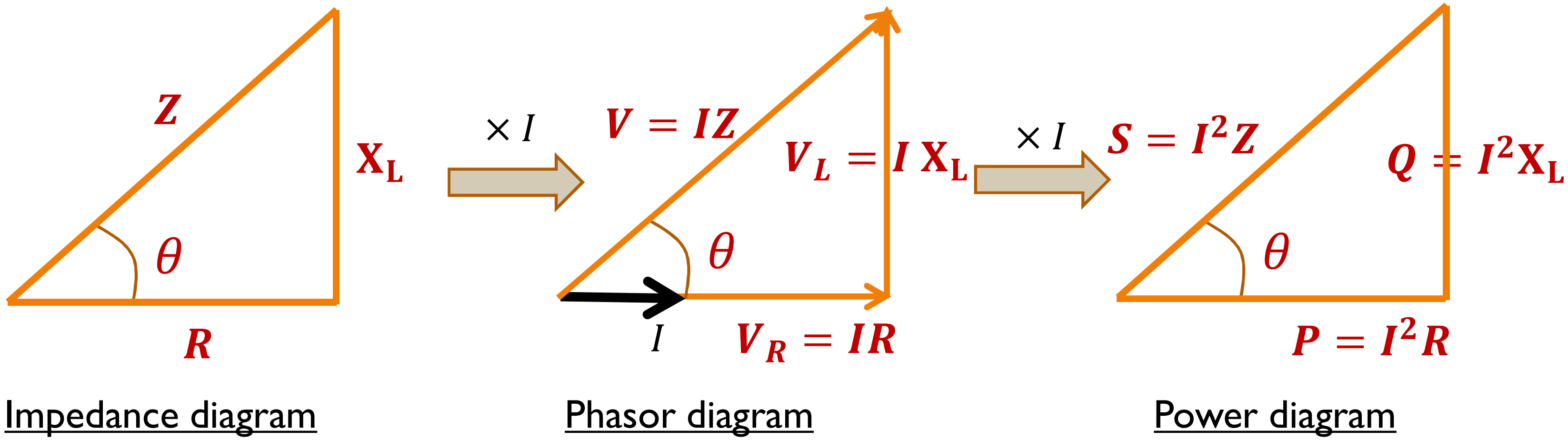
$$\boldsymbol{L_A} = \mathbf{0.0262} \, \boldsymbol{H}$$

Topics covered

- Impedance, phasor & power triangles
- Concept of power factor and its significance
- Need for power factor improvement
- Tutorial 2a

Power associated in RL load

For RL load:



$$S = P + jQ$$

Where,
S = Apparent Power (VA)
P = Active Power (W)
Q = Reactive Power (var)

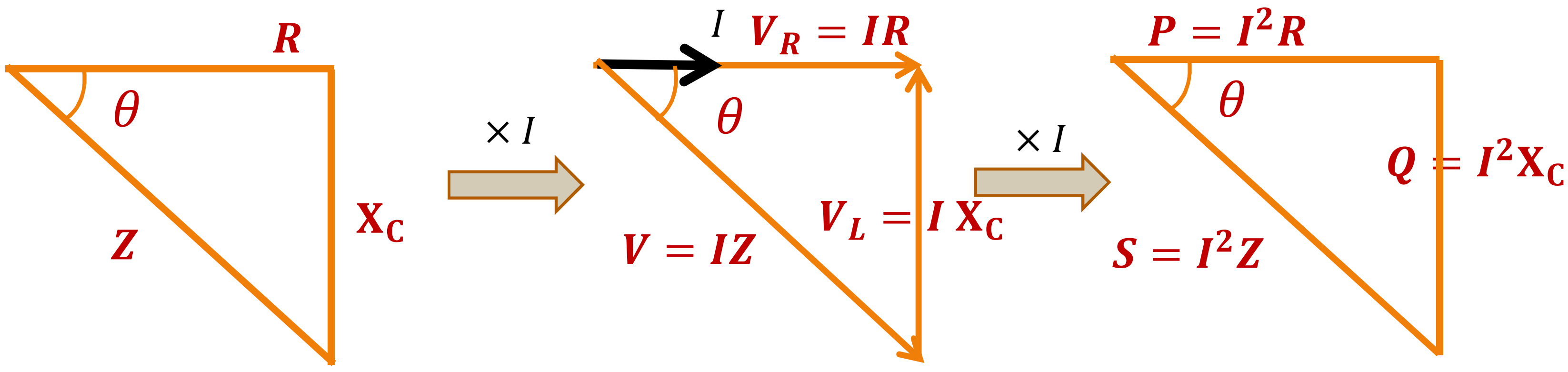
$$S = |V||I|$$

$$P = VI\cos\emptyset$$

$$Q = VI\sin\emptyset$$

Power associated in RC load

For RC load:



Impedance diagram | Phasor diagram | Power diagram

$$S = P - jQ$$

Where,
S = Apparent Power (VA)
P = Active Power (W)
Q = Reactive Power (var)

$$S = |V||I|$$
$$P = VI\cos\emptyset$$
$$Q = VI\sin\emptyset$$

Power in AC circuits

Power in AC circuit can be written as,

$$\boldsymbol{S = (\bar{V})(\bar{I}^*)} = \boxed{S = V.I^*}$$

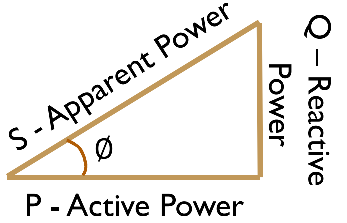
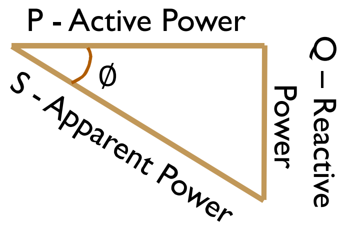
$S = V.I\angle +\phi$

$Z = R + jX_L$ ↳ $|Z|\angle\phi$

$V = V\angle 0$

$I = I\angle -\phi$

$I^* = I\angle +\phi$

For **RL Load**	$Z = \|Z\|\angle\emptyset$ $if\ \bar{V} = \|V\|\angle 0°$ $\bar{I} = \|I\|\angle -\emptyset$ $I^* = \|I\|\angle\emptyset$	$S = VI(\cos\emptyset + j\sin\emptyset)$ $\boldsymbol{S = P + jQ}$ $P = V_{rms}I_{rms}\cos\emptyset$ $Q = V_{rms}I_{rms}\sin\emptyset$	
For RC Load	$Z = \|Z\|\angle -\emptyset$ $if\ \bar{V} = \|V\|\angle 0°$ $\bar{I} = \|I\|\angle\emptyset$ $I^* = \|I\|\angle -\emptyset$	$S = VI(\cos\emptyset - j\sin\emptyset)$ $\boldsymbol{S = P - jQ}$ $P = V_{rms}I_{rms}\cos\emptyset$ $Q = V_{rms}I_{rms}\sin\emptyset$	

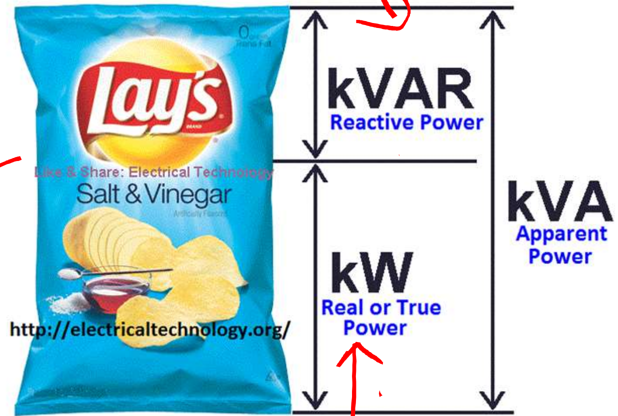
Units: — *Apparent Power(S)*: ***VA*** — *Active Power(P)*: ***W*** — *Reactive Power(Q)*: ***var***

Complex Power Analogy

Air

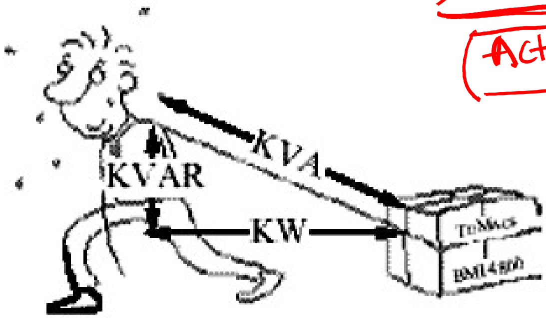
20%

80%



wattfall

usefull pw



S

Reactive pow

Active power

RL load

Active

S

Reach

Power Factor

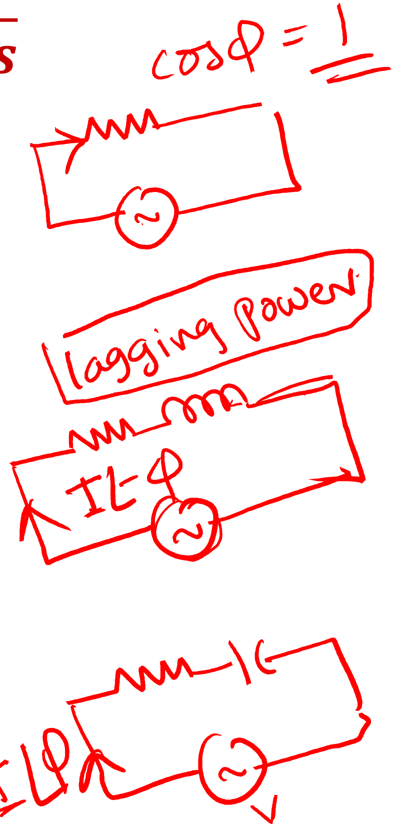
$$\textit{Power Factor} = \frac{\textit{Active Power P in watts}}{\textit{Apparent Power S in voltamperes}}$$

$$\cos\theta = \frac{P}{S} = \frac{P}{VI}$$

- For an impedance Z,

$$\cos\theta = \frac{IR}{V} = \frac{IR}{IZ} = \frac{resistance}{impedance}$$

- Power factor is *lagging* when the *current lags the supply voltage*
- Power factor is *leading* when the *current leads the supply voltage*
- For a resistive load, power factor is Unity

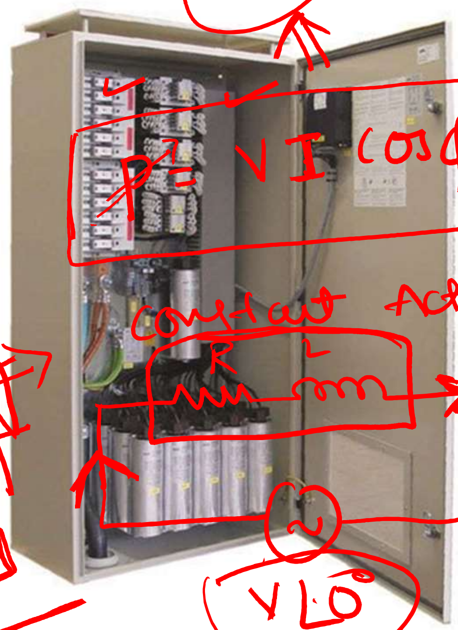


Disadvantages of Low Power Factor

- Under utilisation of power system network
- Increased transmission losses
- Hence bulk consumers are advised to maintain the power factor close to unity by power utilities

0 to 1

low



$P = VI\cos\phi$

Constant Active power tran.

I^2R

$V\angle 0^\circ$

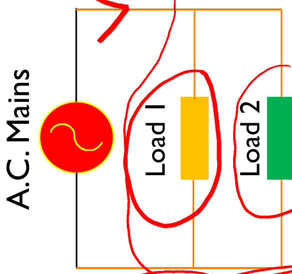
Remedial Measures

- Reactive power demand of Inductive loads can be compensated with capacitive loads
- It is possible to localise reactive power requirement by connecting parallel capacitors across the load

$I \propto \frac{1}{\cos\phi}$

Power Triangle

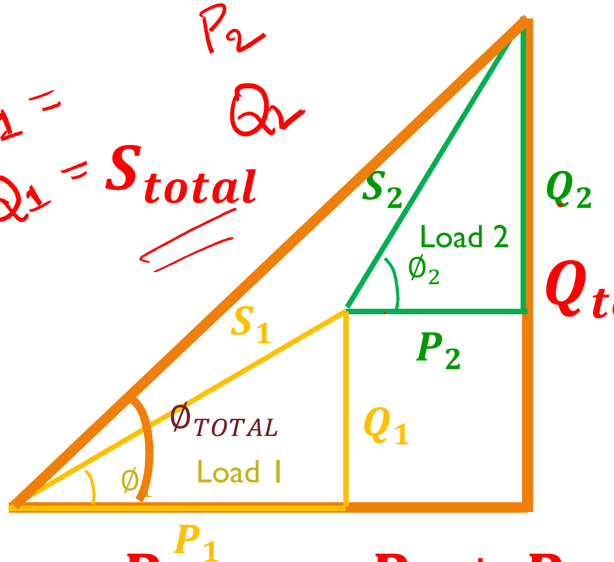
- Practically, loads are in connected parallel
- Majority of the loads are inductive in nature



$P_1 =$ P_2

$Q_1 =$ Q_2

S_{total}



$$Q_{total} = Q_1 + Q_2$$

socket

$$S_{total} = P_{total} + jQ_{total}$$

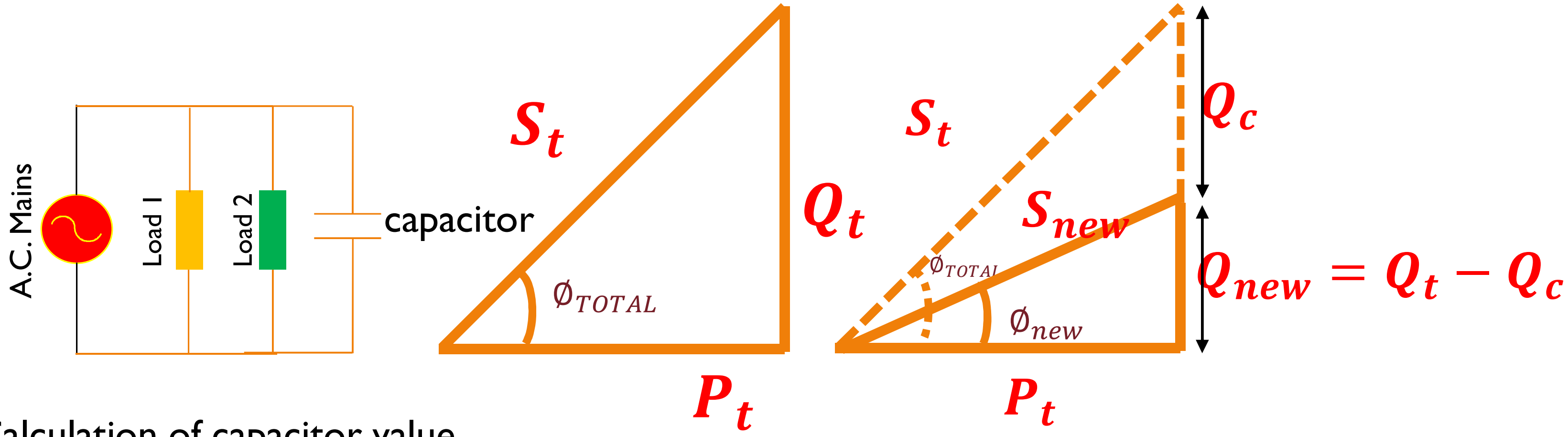
$$P_{total} = P_1 + P_2$$

$$S_{tot} = (P_1+P_2) + j(Q_1+Q_2)$$

http://www.kptcl.com/save.htm

Power Factor Improvement

- Connect capacitor parallel to the load
- Energy stored by the capacitor provides the required reactive power by the load



<u>Calculation of capacitor value</u>

- Calculate Q_c needed to improve power factor to $cos\emptyset_{new}$
- Calculate $X_c = \frac{V^2}{Q_C}$ & $C = \frac{1}{2\pi f X_c}$

Illustration 1

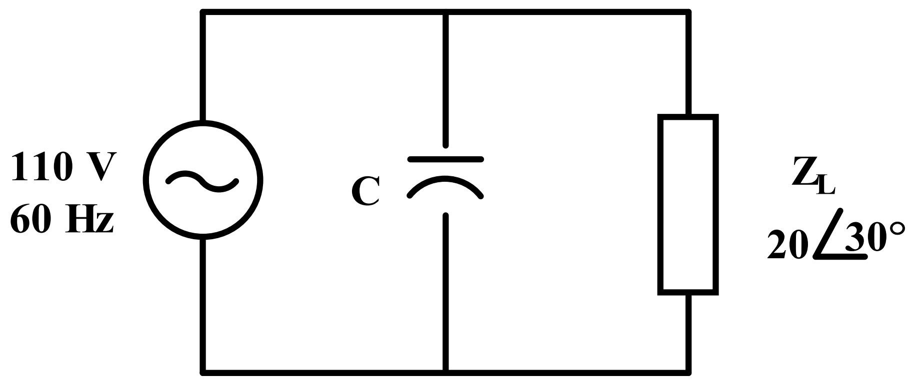
A single-phase motor takes 8.3 A at a power factor of 0.866 lagging when connected to a 230 V, 50 Hz supply. Capacitance bank is now connected in parallel with the motor to raise the power factor to unity. Determine the capacitance value

Illustration 2

A single-phase load of 5 kW operates at a power factor of 0.6 lagging. It is proposed to improve this power factor to 0.95 lagging by connecting a capacitor across the load. Calculate the kvar rating of the capacitor

Exercise 1

In the parallel circuit shown, Find the value of Capacitance C, necessary to correct the power factor to 0.95 lagging



Basic Electrical Technology

[ELE 1051]

CHAPTER 3 - SINGLE PHASE AC CIRCUITS

(3.6)

Exercise 2

A 500 kVA transformer is at full load with power factor 0.6 lagging. What should be the kVAR rating of the shunt capacitor needed to improve its operating power factor to 0.9 lagging? What will be the percentage loading of the transformer after power factor correction?

Exercise 3

Obtain the complete power triangle for three parallel-connected loads:

(a) 250VA, 0.5 p.f lagging

(b) 180W ,0.8 p.f leading

(c) 300VA,100 var (inductive)

Homework 1

An inductive circuit supplied with 250V, 50Hz has an active power of 11.9 KW and apparent power of 17 KVA

a) Find the power factor of the circuit

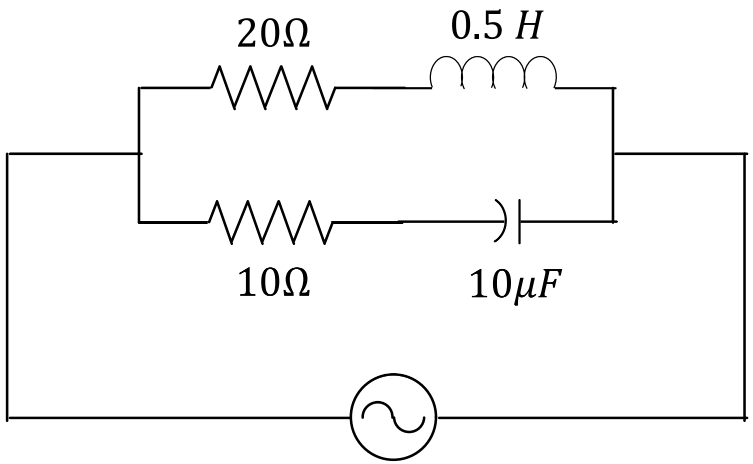
b) Draw the power triangle

c) Find the value of the capacitance required to improve the p.f. to unity, 0.9 lagging ,0.9 leading

Ans:
a) p.f = 0.7 lag
c) C= 618.3 μF,
324.9 μF,
911.6μF

Homework 2

Find the power factor of the circuit shown below. Also, find the value of the capacitor to be connected in series with the circuit to increase the power factor to unity.



$100V, 50\ Hz$

Ans: 0.276 lag, 9.95μF

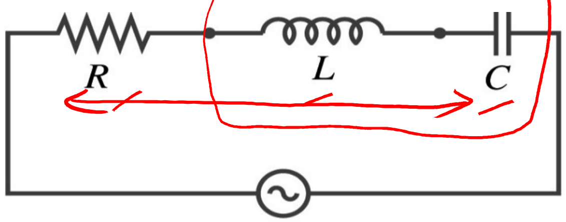
Topics covered

- Resonance in series RLC circuit
- Half power frequency & bandwidth
- Resonance in parallel circuits

Series Resonance

V & I are in phase.

Resistive



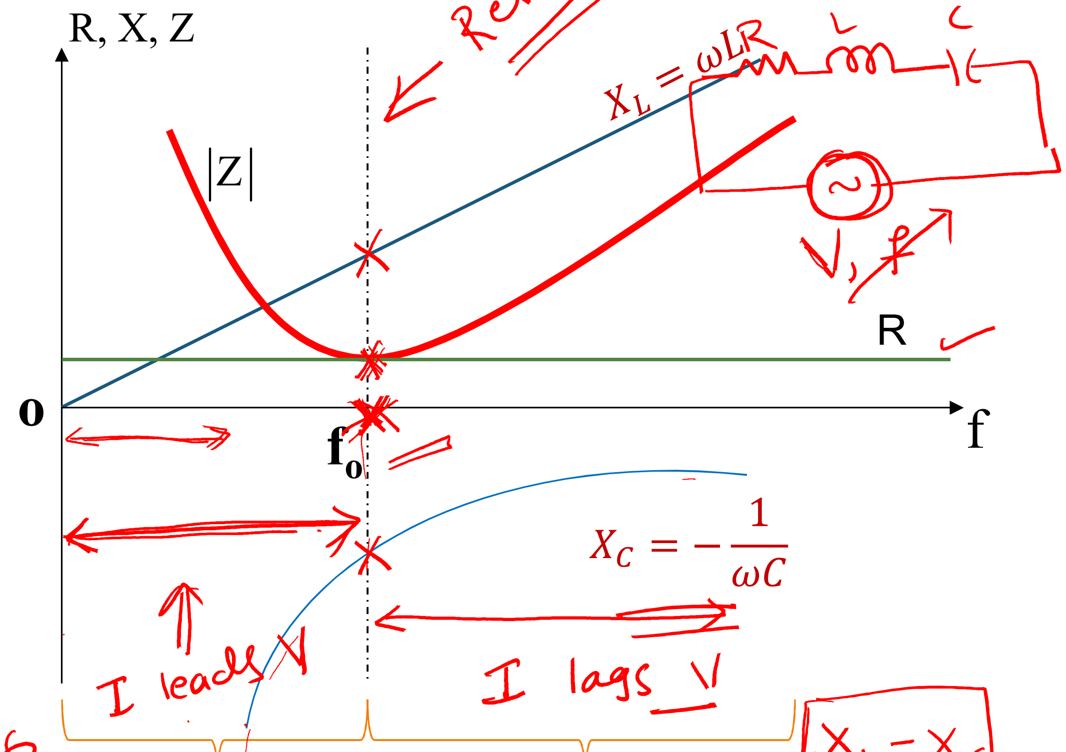
v(t), variable frequency

$z = R + j(X_L - X_C)$

$= R + j\left(2\pi f L - \frac{1}{2\pi f C}\right)$

$$Z = R + j(X_L \sim X_C)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



Z is capacitive

RC

Z is inductive

RL

$Z = R + j(X_L - X_C)$

$\boxed{Z = R}$ minimum

'f_0 is called the resonant frequency'

Series Resonance

- **When series RLC circuit is at resonance,**
  - Current is in phase with voltage
  - Circuit power factor is unity
  - $X_L = X_C$
  - $Z = R$

- **Resonant frequency for a series RLC circuit is obtained as follows:**

$Z = R + j(X_L - X_C)$

$$Imaginary\ part\ of\ Z_{eq} = 0$$

$X_L = X_C$

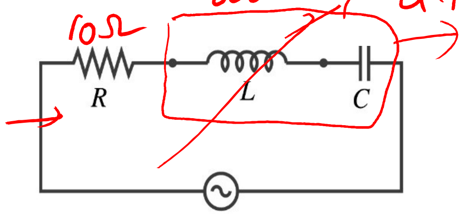
$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$2\pi f_0 L = \frac{1}{2\pi f_0 C}$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}\ hertz}$$

$\uparrow f_0$



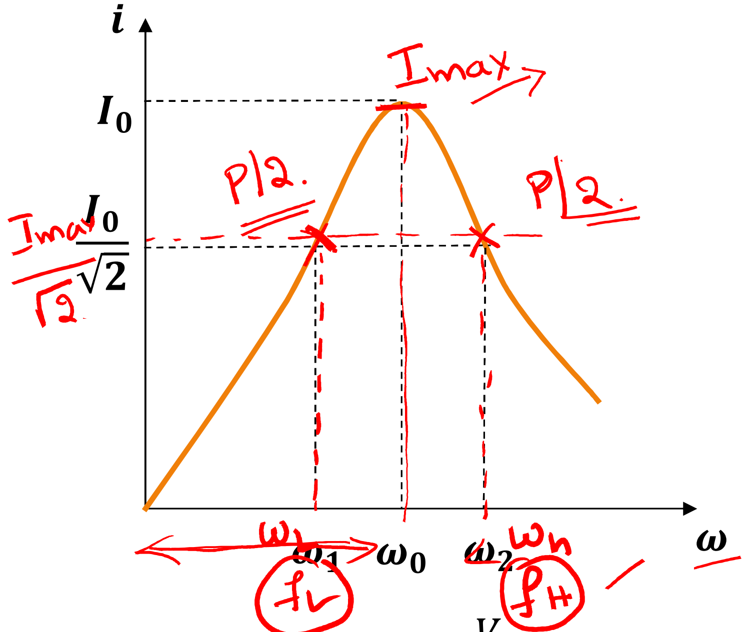
v(t), variable frequency

$Z = R + j(X_L \sim X_C)$

f_0 = resonant frequency

Current vs. Frequency in RLC Series Circuit

Variation of current with frequency



$$I_0 = I_{max} = \frac{V_{rms}}{R}$$

- **Half Power Frequency**

$\frac{P}{2} = \frac{I_{max}^2 R}{2}$

'Frequency at which the power is half of the power at resonant frequency'

$$\boxed{Power = \frac{1}{2} I_0^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R \qquad At\ \omega_1\ and\ \omega_2, I = \frac{I_0}{\sqrt{2}}}$$

$\frac{P}{2} = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R$

$\omega_1 = Lower\ half\ power\ frequency$

$\omega_2 = Upper\ half\ power\ frequency$

Bandwidth $= \omega_2 - \omega_1$

f_L & f_H

$\omega_2 - \omega_1$

$f_H - f_L$

$f_2 - f_1$

at f_0 $P = (I_{max})^2 \cdot R.$ Half power frequency

In practice the curve of $|I|$ against ω is not symmetrical about the resonant frequency

$\boxed{Z = R}$ $\quad P = I_{max}^2 \cdot R \quad f_H - f_L = \text{Bandwidth}$

Half Power Frequency

$$Impedance\ at\ \omega_1 and\ \omega_2, |Z| = V_0 \Big/ \frac{I_0}{\sqrt{2}} = \sqrt{2}R$$

at f_0 $Z = R$

$\frac{V}{I_{max}} = R$

Below Resonant frequency ω_0, $|X_C| > |X_L|$

At ω_1,

$$\sqrt{R^2 + (X_C - X_L)^2} = \sqrt{2}R$$

at f_L

$Z = |R + j(X_C - X_L)| = \sqrt{2}R$

$$X_C - X_L = R$$

$$\frac{1}{\omega_1 C} - \omega_1 L = R$$

$$\boxed{\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

$f_L = \frac{1}{2\pi}\left(\frac{-R}{2L} + \right.$

Band

$f_H - f_L$

$\omega_2 - \omega_L$

Above Resonant frequency ω_0, $|X_L| > |X_C|$

At ω_2,

$$\sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2}R$$

at f_H

$Z = |R +$

$$X_L - X_C = R$$

$f < f_0$

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\boxed{\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}}$$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

ω_0

$2\pi f_0 = \frac{1}{\sqrt{LC}}$

Resonant freq.

$$\omega_2\omega_1 = \frac{1}{LC} = \omega_0^2$$

Band

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$f_H = \frac{1}{2\pi}\left(\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + 1/LC}\right)$

Quality Factor for series circuit

- At resonance, V_C and V_L can be very much greater than applied voltage

$$|V_C| = |I|X_C = \frac{V.X_C}{\sqrt{R^2 + (X_L - X_C)^2}}$$

At resonance, $X_L = X_C$

$$V_C = \frac{V}{R}X_C$$

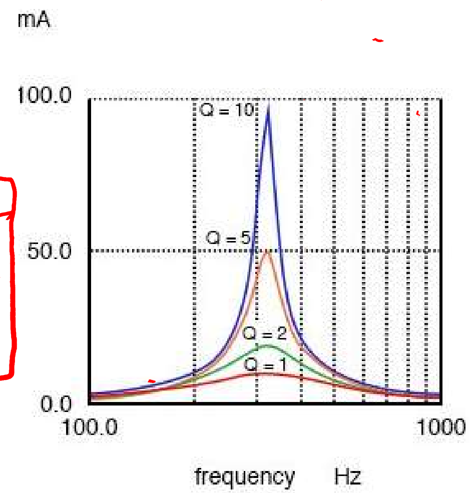
$$V_C = \frac{V}{\omega_0 CR} = \boldsymbol{Q}V$$

Q is termed the Q factor or voltage magnification

$$\boxed{\boldsymbol{Q = \frac{1}{\omega_0 CR} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}}}$$

$$\boldsymbol{Q = \frac{Resonant\ frequency}{Bandwidth}} = \frac{f_0}{f_H - f_L} = \frac{\omega_0}{\omega_H - f_L} = \frac{\frac{1}{2\pi\sqrt{LC}}}{f_H - f_L}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$



- High value of Q can lead to component damage
- Careful design necessary
- Larger the value of Q, more symmetrical the curve appears about the resonant frequency

$$\boxed{Q = \frac{1}{2\pi f_0 RC} = \frac{2\pi f_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$V_C = I . X_C = \frac{V}{R} . \frac{1}{2\pi f_0 C} = \frac{V}{2\pi f_0 RC}$$

$$V_C = QV$$

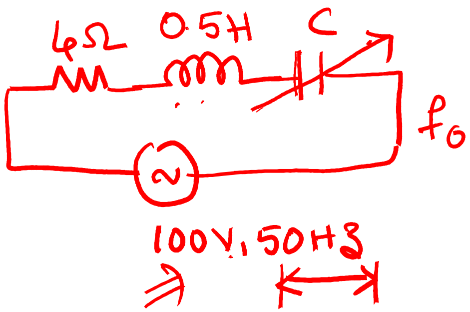
Illustration 1

A circuit having a resistance of 4Ω and inductance of 0.5H and a variable capacitance in series, is connected across a 100V, 50Hz supply. Calculate:

a) The capacitance to give resonance ✓
b) The voltages across the inductor and the capacitor ✓
c) The Q factor of the circuit ✓

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$V_L = Q.V$



$X_L = X_C$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

$X_L = X_C$

$V_C = Q.V$

$2\pi f L = \frac{1}{2\pi f C}$ (0.5H, 50)

$C = 20.2\mu F$

$Q = \frac{3926.75}{100} = 39.26$

V_L & V_C

$V_L = Q.V$

$= I.X_L = \frac{V}{R}.X_L = \frac{100}{4} \times 2\pi \times 50 \times 0.5 = 3926.75V$

$V_C = I.X_C = \frac{100}{4} \times X_L = 3926.75V$ ($X_C = X_L$)

Illustration 2

The bandwidth of a series resonant circuit is 500 Hz. If the resonant frequency is 6000 Hz, what is the value of Q? If R =10 Ω, what is the value of the inductive reactance at resonance? Calculate the inductance and capacitance of the circuit

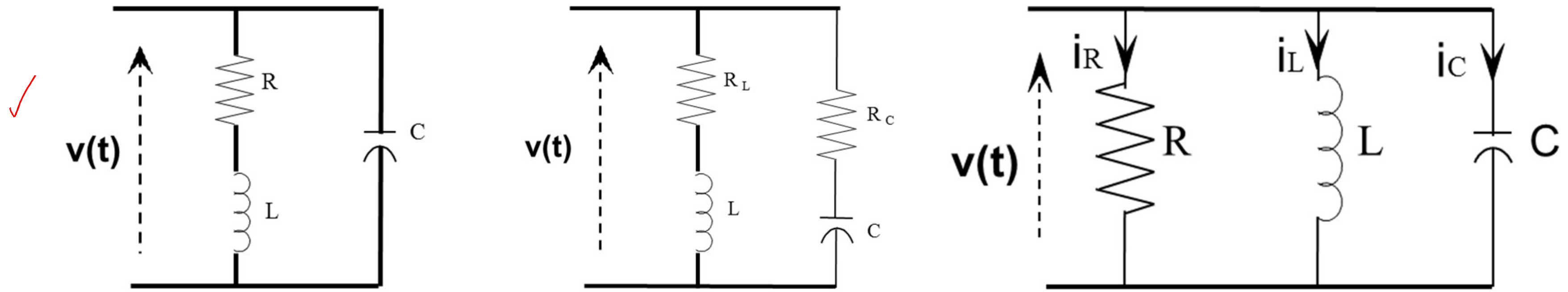
Basic Electrical Technology

[ELE 1051]

CHAPTER 3 - SINGLE PHASE AC CIRCUITS

(3.6)

Resonance in parallel circuits



<u>Steps to obtain the expression of resonant frequency in parallel circuits</u>

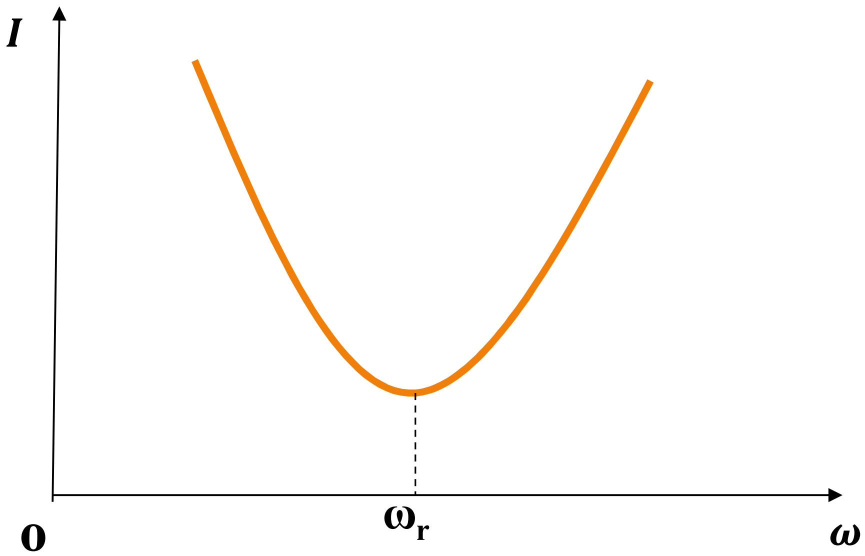
- Obtain the net admittance of the circuit ; $\boldsymbol{Y_{eq} = y_1 + y_2 + \cdots}$

$$\mathbf{Y_{eq} = G_{eq} \pm \mathit{j}B_{eq}}$$

- Equate the imaginary part (susceptance) to zero; $B_{eq} = 0$ and obtain the expression of ω_r

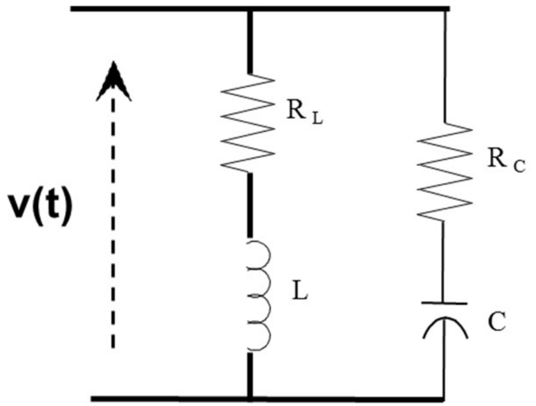
The expression for resonant frequency depends on circuit configuration

Current vs. Frequency in parallel Circuits



- At resonance
  - Impedance is maximum
  - Resultant current minimum

Parallel resonance circuits



$$y_{eq} = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_c}$$

$$y_{eq} = \frac{R_C - jX_c + R_L + jX_L}{(R_L R_C + X_L X_c) - j(R_L X_C - X_L R_C)}$$

Rationalizing ;

$$y_{eq} = \frac{\left((R_L R_C + X_L X_c) + j(R_L X_C - X_L R_C)\right)(R_C - jX_c + R_L + jX_L)}{(R_L R_C + X_L X_c)^2 + (R_L X_C - X_L R_C)^2}$$

Separating the real & imaginary terms ;

$$y_{eq} = \frac{1}{(R_L R_C + X_L X_c)^2 + (R_L X_C - X_L R_C)^2}(R_L^2 R_C + R_L R_C^2 - R_L X_C^2 - X_L^2 R_C) + \boldsymbol{j}(R_L^2 X_C + X_C X_L^2 - X_L R_C^2 - X_C^2 X_L)$$

Parallel resonance circuits

Equating the imaginary part to zero;

$B_{eq} = 0$;

$1/\omega_0 C$ $\quad$ $\omega_0 L$

$R_L^2 X_C + X_C X_L^2 - X_L R_C^2 - X_C^2 X_L = 0$

Solving for ω_0;

$$\boldsymbol{\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 C - L}{R_C^2 C - L}}}$$

If $R_L = R_c$:

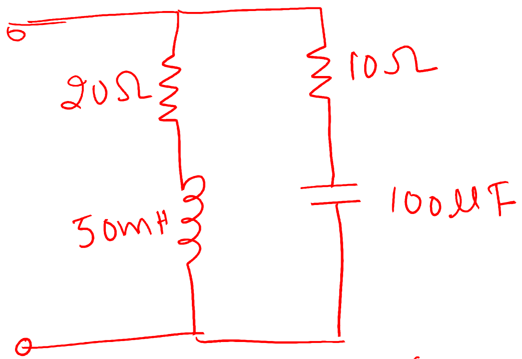
$\omega_0 = \frac{1}{\sqrt{LC}}$

$$\boldsymbol{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

Exercise 1

A parallel circuit with an RL series branch (R = 20 Ω and L = 50 mH) and an RC series branch (R = 10 Ω and C = 100 μF) are connected to a variable frequency voltage source. Find at what frequency the circuit will resonate?



$Y = G + jB \rightarrow 0$

Ans:

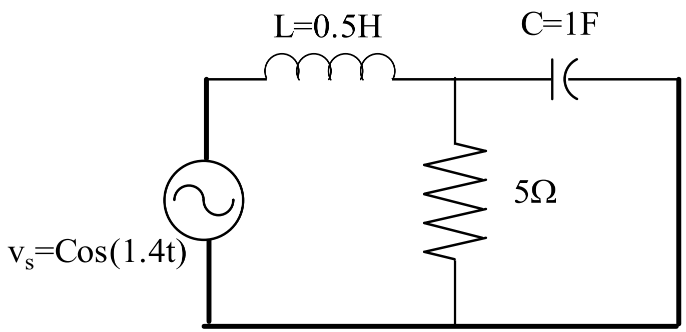
$$\omega_c = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2C-L}{R_C^2C-L}} = 223.6067\, rad/sec = 35.58\, Hz$$

$\omega_0 = 223.60$ rad/sec

$= 35.58$ Hz.

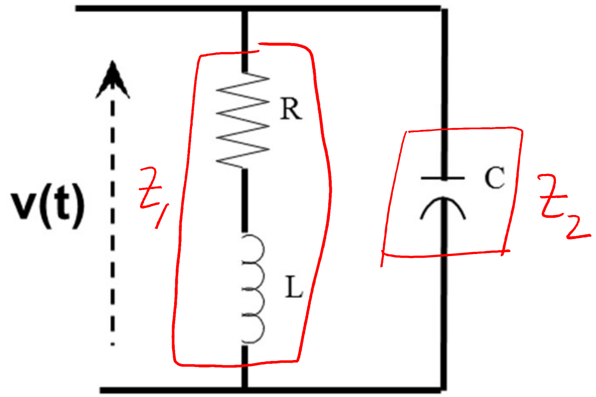
Exercise 2

Show that circuit given in figure will be at resonance at supply frequency



Exercise 3

Obtain the expression for resonant frequency for the given parallel circuit



$Z_1 = R + jX_L$

$Z_2 = -jX_C$

$$Y = \frac{1}{R + jX_L} - \frac{1}{jX_C}$$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{(R - jX_L)X_C + j(R^2 + X_L^2)}{(R^2 + X_L^2)X_C} \Rightarrow$$

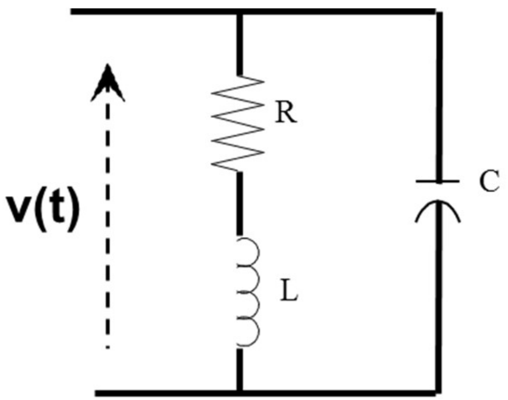
$$j\left(X_L^2 + R^2 - X_C X_L\right) = 0$$

$$\Downarrow$$

$$X_C X_L = R^2 + X_L^2$$

$$\frac{\cancel{\omega_0} L}{\cancel{\omega_0} C} = R^2 + (\omega_0 L)^2 \rightarrow$$

$$\hookrightarrow \omega_0 = \underline{\qquad}$$


v(t)
R
L
C

Homework 1

A series circuit comprises a 10 ohm resistance, a 5 μF capacitor and a variable inductance L. The supply voltage is $20\angle 0^\circ$ volts at a frequency of 318.3 Hz. The inductance is adjusted until the voltage drop across the 10ohm resistance is a maximum. Determine for this condition,

a) Value of inductance L
b) Voltage across each component and
c) Q-factor

Ans:

a) 50mH
b) $V_L = 200\angle 90\ V$ $\qquad V_C = 200\angle -90\ V$
c) 10

Homework 2

A coil of resistance R and inductance L is connected in series with a capacitor C across a variable frequency source. The voltage is maintained constant at 100V and the frequency is varied until a maximum current of 4A flows through the circuit at 10KHz. Under these conditions, the Q factor of the circuit is 10. Calculate,

a) Voltage across the capacitor
b) The values of R, L and C

Ans:
a) $1000\angle -90$ V
b) $R = 25\Omega, L = 3.97mH, C = 63.66nF$

Homework 3

An R–L–C series circuit has a resonant frequency of 1.2 kHz and a Q-factor at resonance of 30. If the impedance of the circuit at resonance is 50ohm determine the values of,

(a) Inductance

(b) Capacitance

(c) Bandwidth

Ans:
198mH, 88nF, 40

Homework 4

$R = 10\Omega, L = 0.02H$ and a variable capacitor are connected in parallel across a 100V, 50Hz AC supply. Calculate the value of the capacitance for which the current taken from the supply is in phase with the supply voltage

Ans:
506.61μF

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Basic Electrical Technology

[ELE 1051]

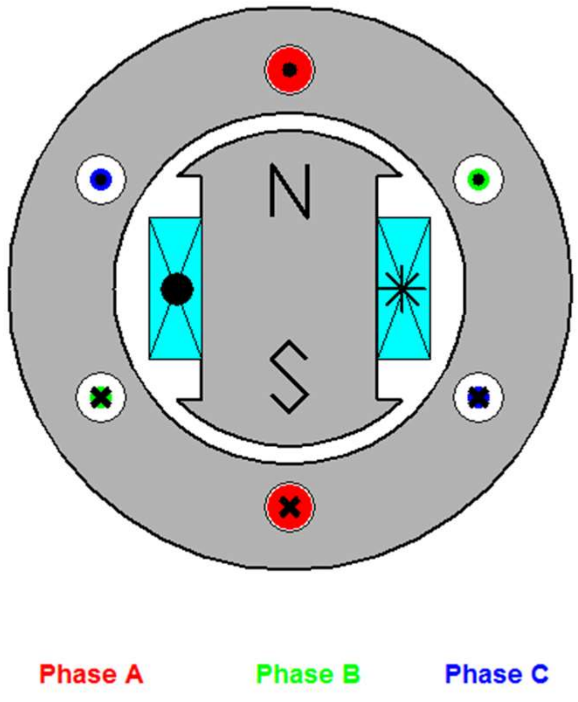
THREE PHASE AC CIRCUITS

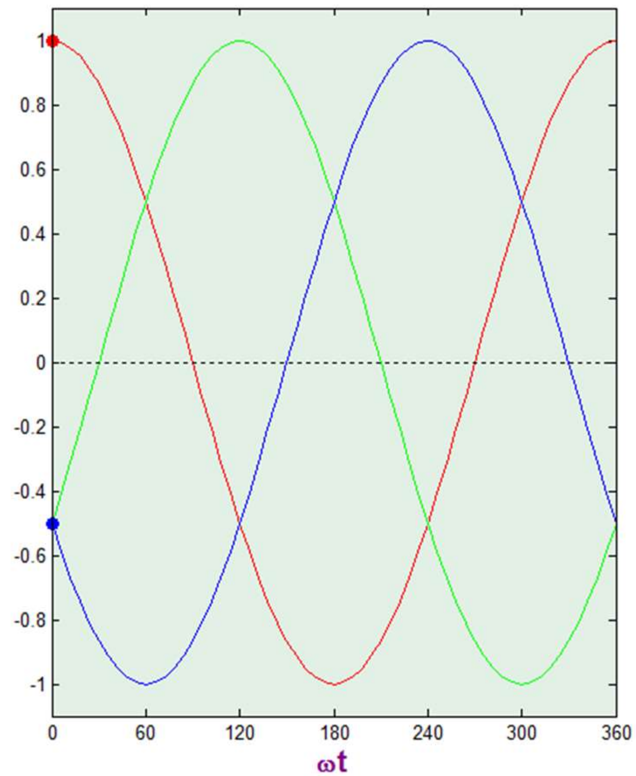
Topics Covered

- Generation of Three Phase Supply
- Representation of Three Phase Excitation
- Relationship between Phase and Line Voltages
- 3 Phase Supply & Loads

Generation of 3-Phase Power

Generation of 3-Phase Power





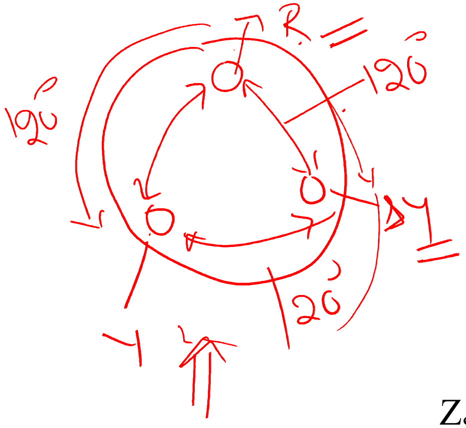
3 Phase Excitation (Phase Voltages)

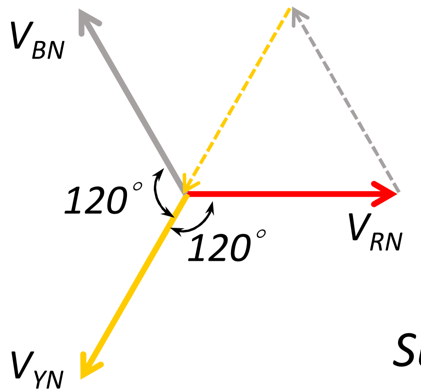
Phase Voltages,

$$\hat{V}_{RN} = V_m\, Sin(\omega t)$$

$$\hat{V}_{YN} = V_m\, Sin(\omega t - 120°)$$

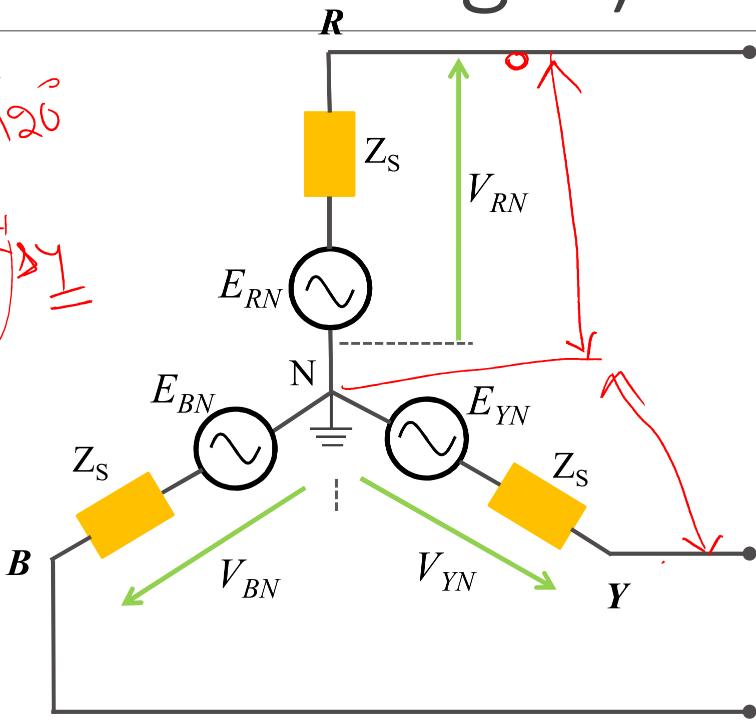
$$\hat{V}_{BN} = V_m\, Sin(\omega t - 240°)$$





Balanced 3-φ supply

= 0V



Three Phase Source

$V_{Bn} = V_m\, Sin(\omega t - 240°)$ ✓

$= V_m\, Sin(\omega t + 120°)$ ✓

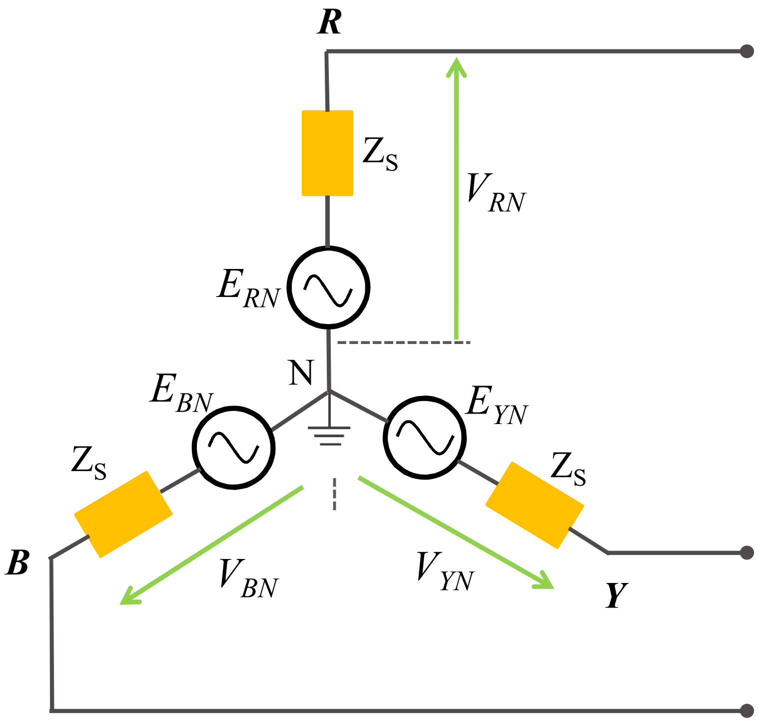
Summing up the phase voltages,

$V_{RN} = V_m\, Sin(\omega t)$

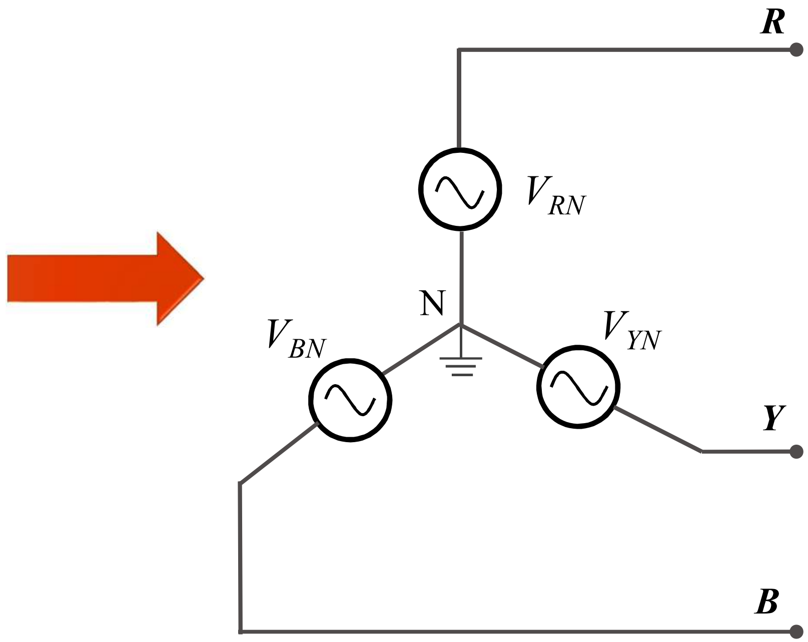
$V_{YN} = V_m\,(\omega t - 120°)$

$$\hat{V}_{RN} + \hat{V}_{YN} + \hat{V}_{BN} = 0$$

3 Phase Excitation (Phase Voltages)

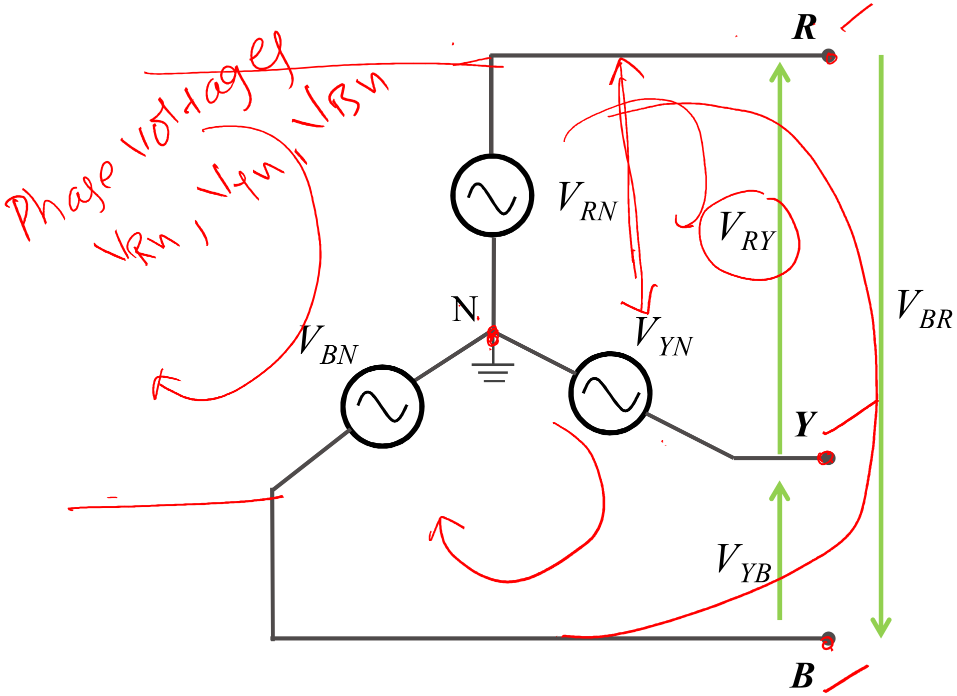


Three Phase Source



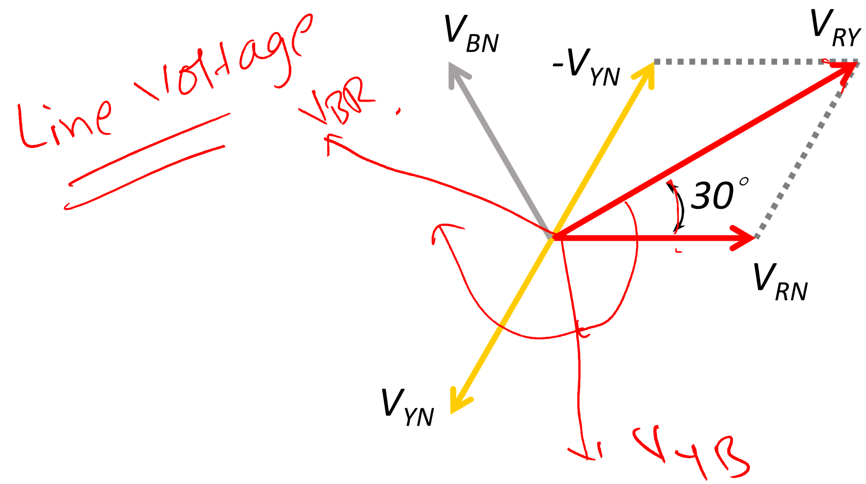
Three Phase Source

3 Phase Excitation (Line Voltages)



Three Phase Source

Lines.



Line Voltages,

$$\hat{V}_{RY} = \hat{V}_{RN} - \hat{V}_{YN}$$

$$= V_m\, Sin(\omega t) - V_m\, Sin(\omega t - 120°)$$

$$= \sqrt{3} \times V_m\, Sin(\omega t + 30)$$

3 Phase Excitation (Line Voltages)…

Similarly,

$$\hat{V}_{YB} = \hat{V}_{YN} - \hat{V}_{BN}$$

$$= V_m\,Sin(\omega t - 120) - V_m\,Sin(\omega t - 240°)$$

$$= \cancel{\sqrt{3} \times V_m\,Sin(\omega t - 120)}$$

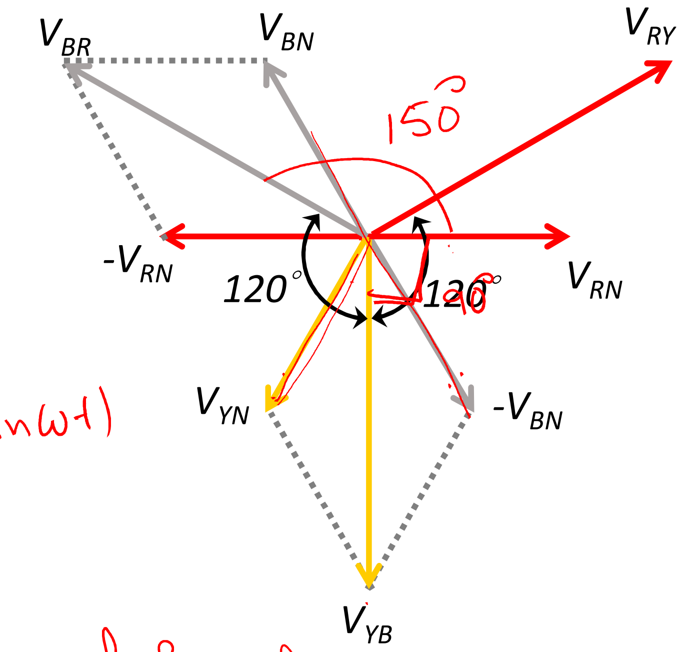
$$= \boxed{\sqrt{3}\,V_m\,Sin(\omega t - 90°)}$$

$$\hat{V}_{BR} = \hat{V}_{BN} - \hat{V}_{RN}$$

$V_m\,Sin(\omega t - 240) - V_m\,Sin\,\omega t$

$$= \cancel{\sqrt{3} \times V_m\,Sin(\omega t + 120)}$$

$$= \sqrt{3}\,V_m\,Sin(\omega t + 150°)$$

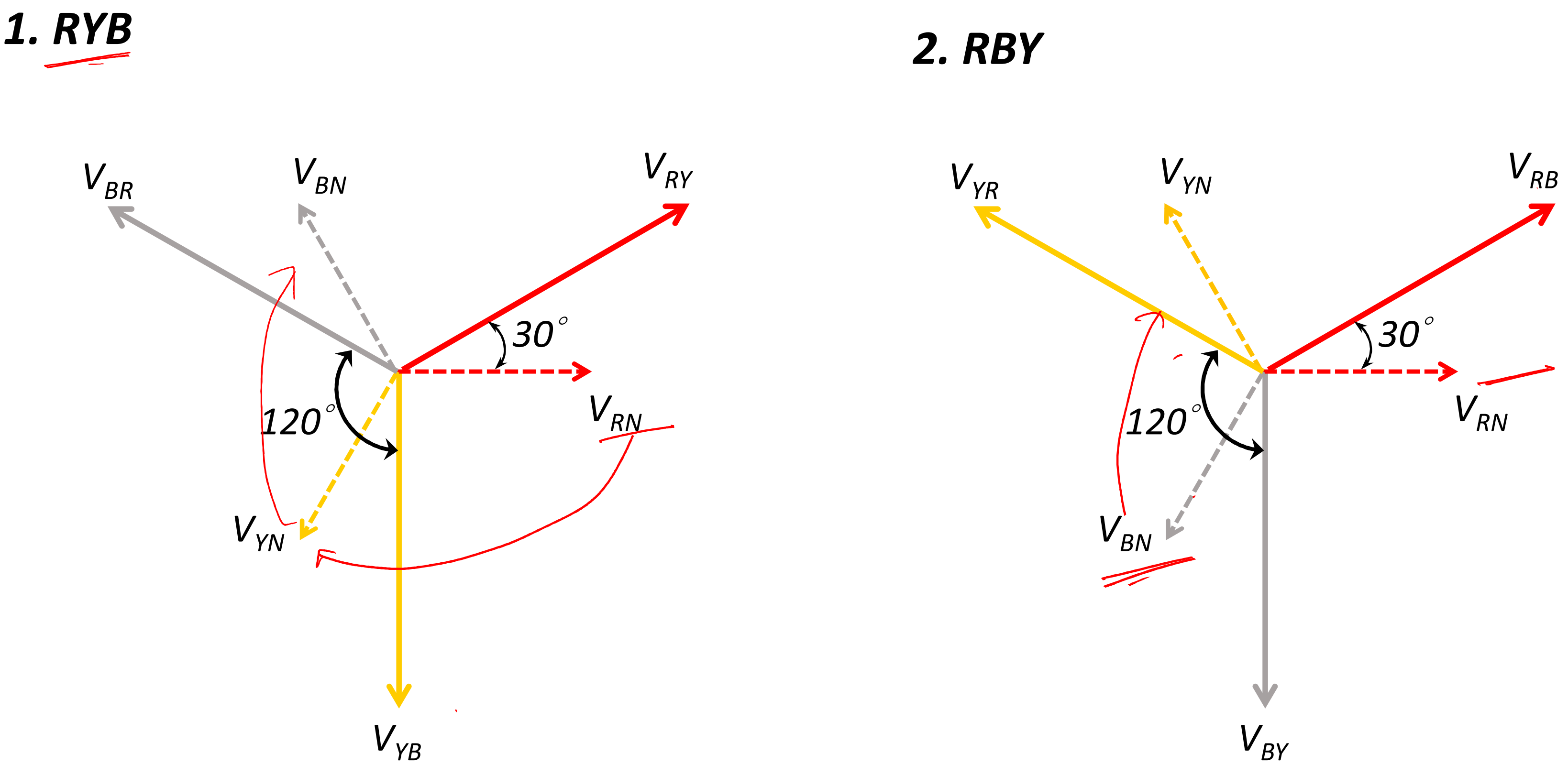


Summing up the Line voltages, $= 0$ Balanced supply.

$$\hat{V}_{RY} + \hat{V}_{YB} + \hat{V}_{BR} = 0$$

In a Three Phase balanced Supply, the summation of Phase voltages and summation of Line Voltages is zero.

Phase Sequence



Phase Sequence is the order in which the three phases attain their peak or maximum values

Basic Electrical Technology

[ELE 1051]

Three Phase AC Circuits

L25– Star & Delta Connected Balanced Loads & Unbalanced loads

Illustration-1

Three loads $Z_A = 10\angle 0^0\ \Omega$; $Z_B = 15\angle{-30^0}\ \Omega$ and $Z_C = 20\angle 45^0$ are connected in star across a balanced, 3 phase, 400 V, RYB supply. Determine (a) line currents (b) Phase Voltages (c) Neutral shift voltage, $V_{ON.}$

Solution:

The three phase load is supplied with a balanced supply of 400V, hence the line voltages appearing across the load are:

$$\hat{V}_{RY} = 400\angle 0° \text{ (Reference Voltage)}$$

$$\hat{V}_{YB} = 400\angle -120°$$

$$\hat{V}_{BR} = 400\angle +120°$$

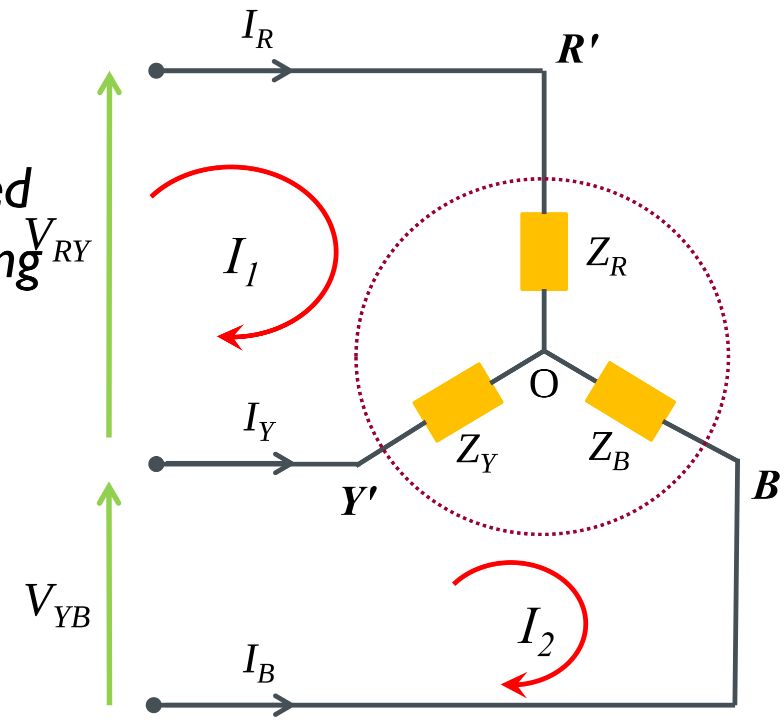


Illustration-I …

Writing Mesh Equation in Matrix form,

$$\begin{bmatrix} 10\angle 0 + 15\angle -30 & -15\angle -30 \\ -15\angle -30 & 15\angle -30\ + 20\angle 45 \end{bmatrix}\begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 400\angle\ 0° \\ 400\angle\ -120° \end{bmatrix}$$

Using Cramer's rule,

$$\hat{I}_1 = \mathbf{9.783\angle -17.87}\ A$$

$$\hat{I}_2 = \mathbf{16.69\angle -116.63}\ A$$

(i) The line currents are

$$\hat{I}_R = \hat{I}_1 = \mathbf{9.783\angle -17.87}\ A$$

$$\hat{I}_Y = \hat{I}_2 - \hat{I}_1 = \mathbf{20.59\angle -144.63}\ A$$

$$\hat{I}_B = -\hat{I}_2 = \mathbf{16.69\angle 63.37}\ A$$

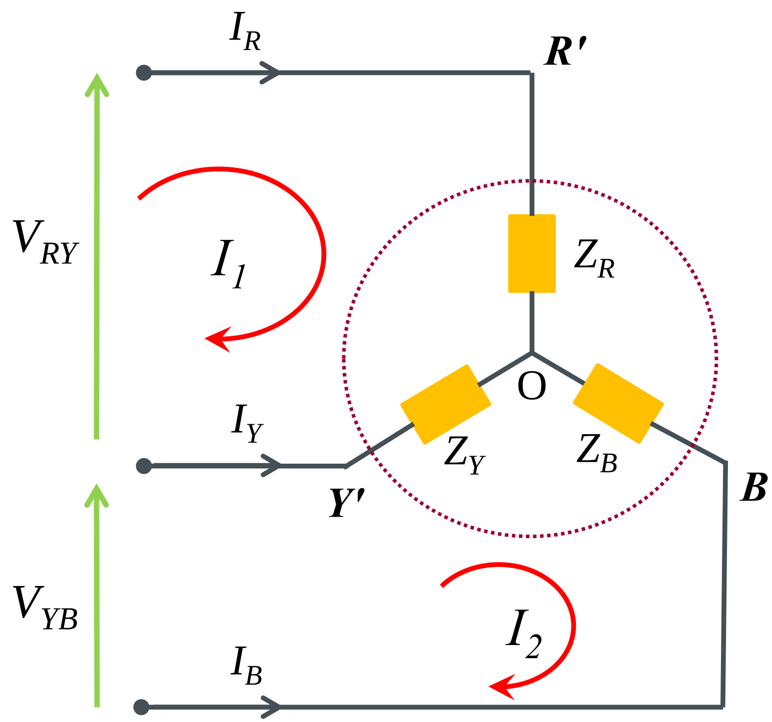


Illustration-I …

(ii) Phase Voltages are determined using the following equations.

$$\hat{V}_{R'O} = \hat{I}_R \times \bar{Z}_A = \mathbf{97.83\angle-7.87\,V}$$

$$\hat{V}_{Y'O} = \hat{I}_Y \times \bar{Z}_B = \mathbf{308.85\angle-174.63\,V}$$

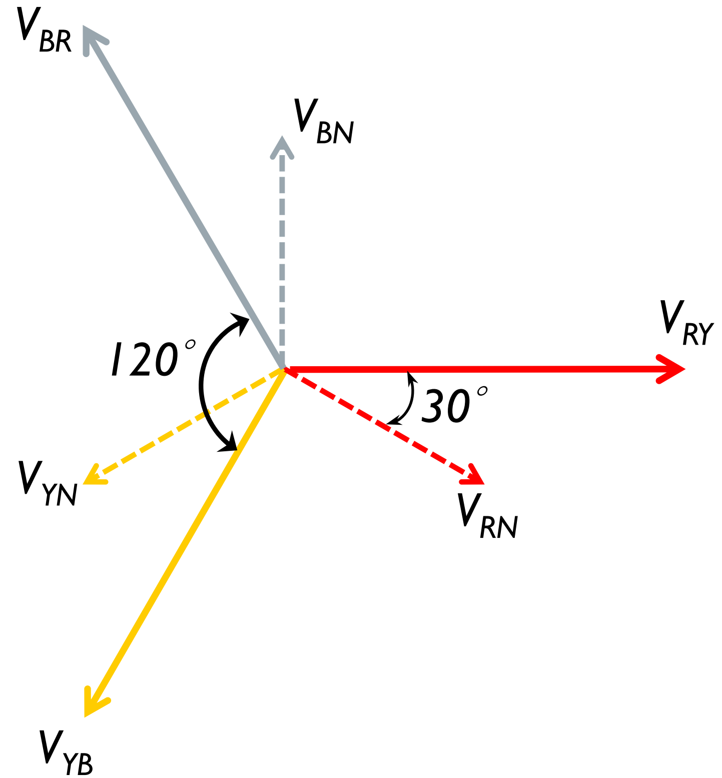
$$\hat{V}_{B'O} = \hat{I}_B \times \bar{Z}_C = \mathbf{338\angle108.37\,V}$$

(c)Neutral Shift Voltage (Von)

$$\hat{V}_{RY} = 400\angle 0° \text{ (Reference Voltage)}$$

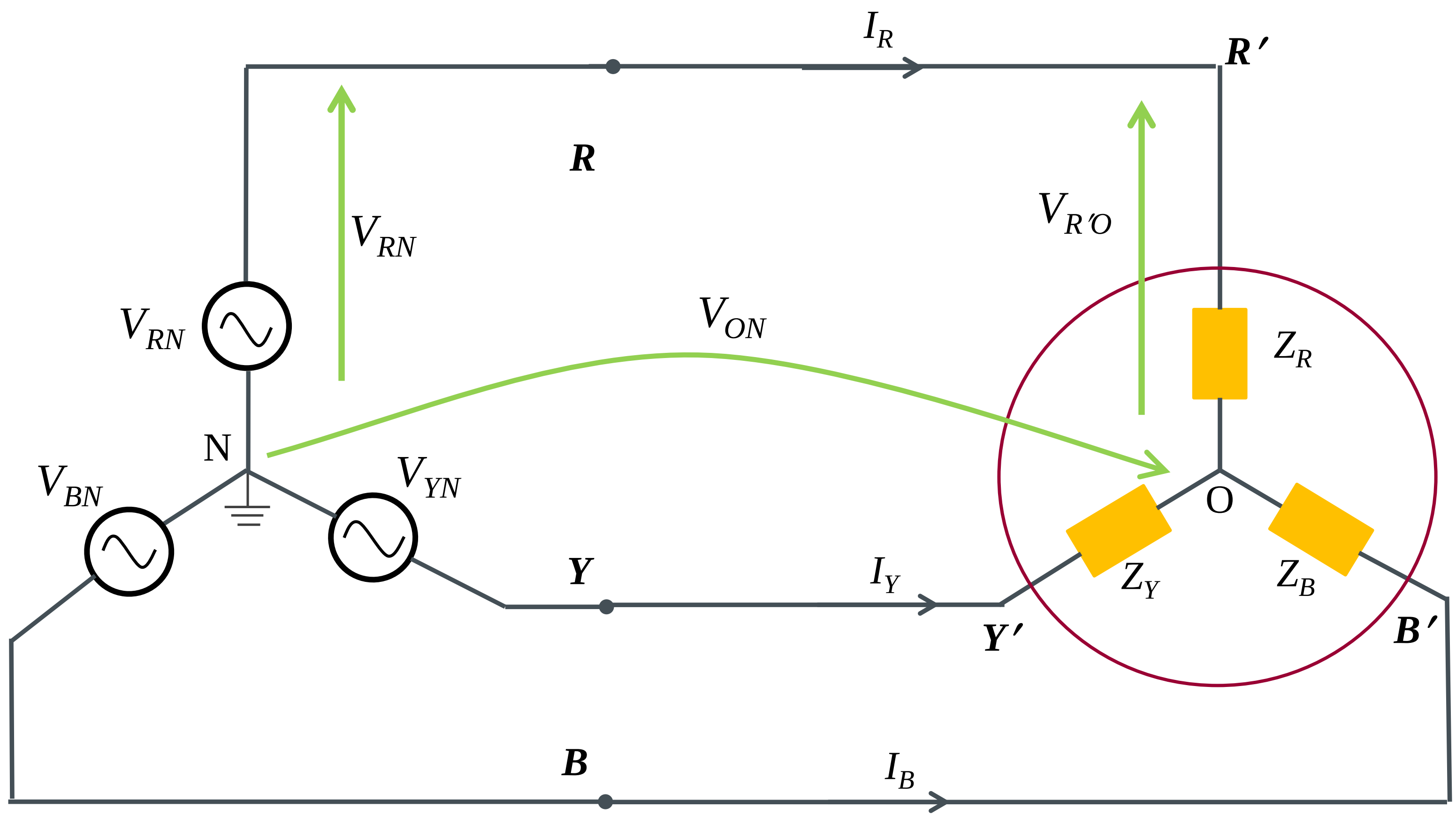
$$\hat{V}_{RN} = \frac{400}{\sqrt{3}}\angle-30°$$

$$\hat{V}_{YN} = \frac{400}{\sqrt{3}}\angle(-30-120)$$



$$\hat{V}_{BN} = \frac{400}{\sqrt{3}}\angle(-30°-240)$$

Illustration-I …

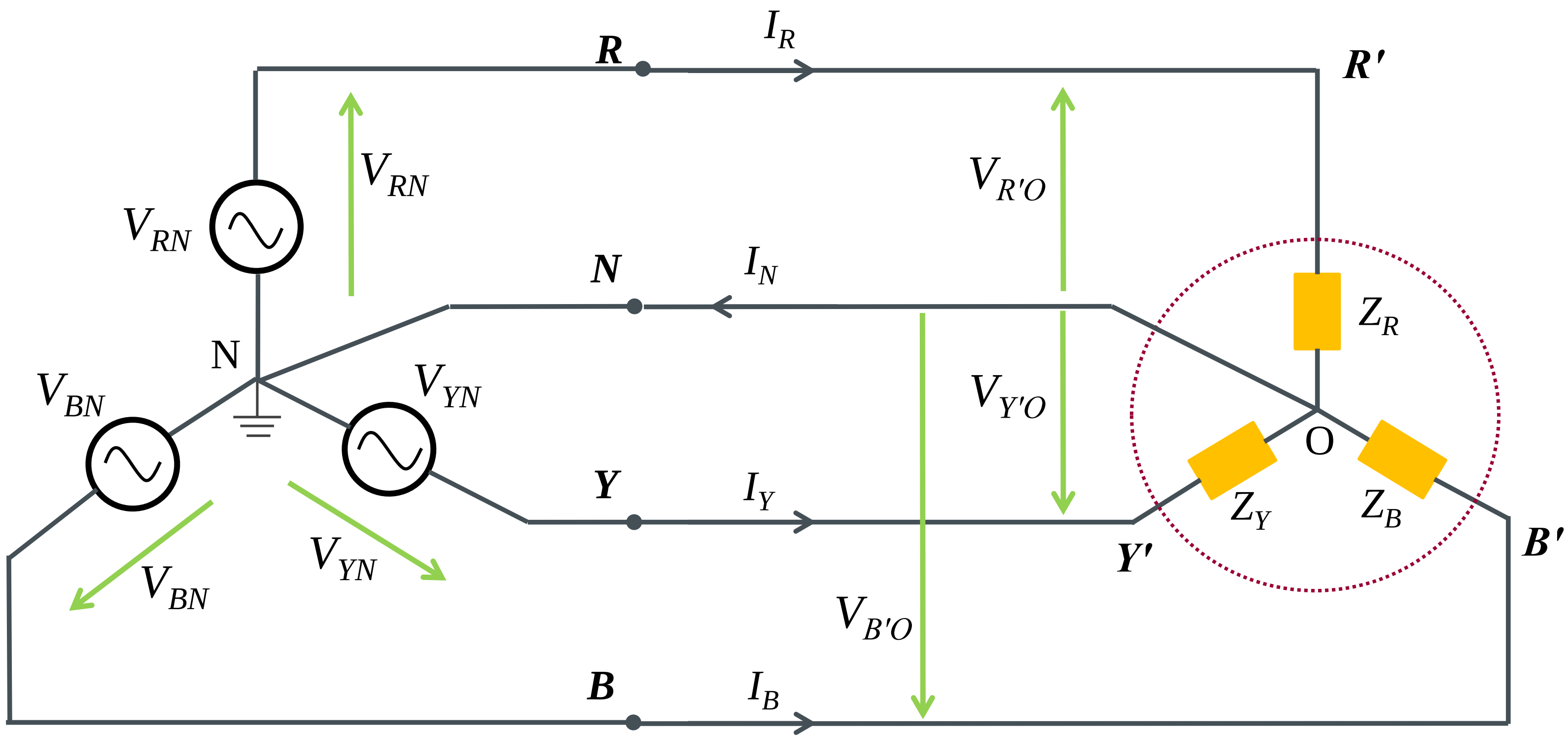


Applying KVL, $\hat{V}_{RN} - \hat{V}_{R'O} - \hat{V}_{ON} = 0$

$$\hat{V}_{ON} = \hat{V}_{RN} - \hat{V}_{R'O} = 145.07\angle -44.7^\circ\ V$$

3φ , 4 Wire System with Y Load

Consider the 3 phase star load connected to a 4 wire balanced source.



Phase Voltages of Load,

$$\hat{V}_{R'O} = \hat{V}_{RN}$$

$$\hat{V}_{Y'O} = \hat{V}_{YN}$$

$$\hat{V}_{B'O} = \hat{V}_{BN}$$

Neutral Current:

$$\hat{I}_N = \hat{I}_R + \hat{I}_Y + \hat{I}_B$$

$\hat{I}_N = 0$; (If $Z_R = Z_Y = Z_B = Z\angle\theta°$)

Illustration 1

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Illustration 2

If the impedances are $Z_R = 10+j20\Omega$, $Z_Y = 15-j30\Omega$ and $Z_B = 50\Omega$ in Illustration 01, find the current neutral current of the circuit.

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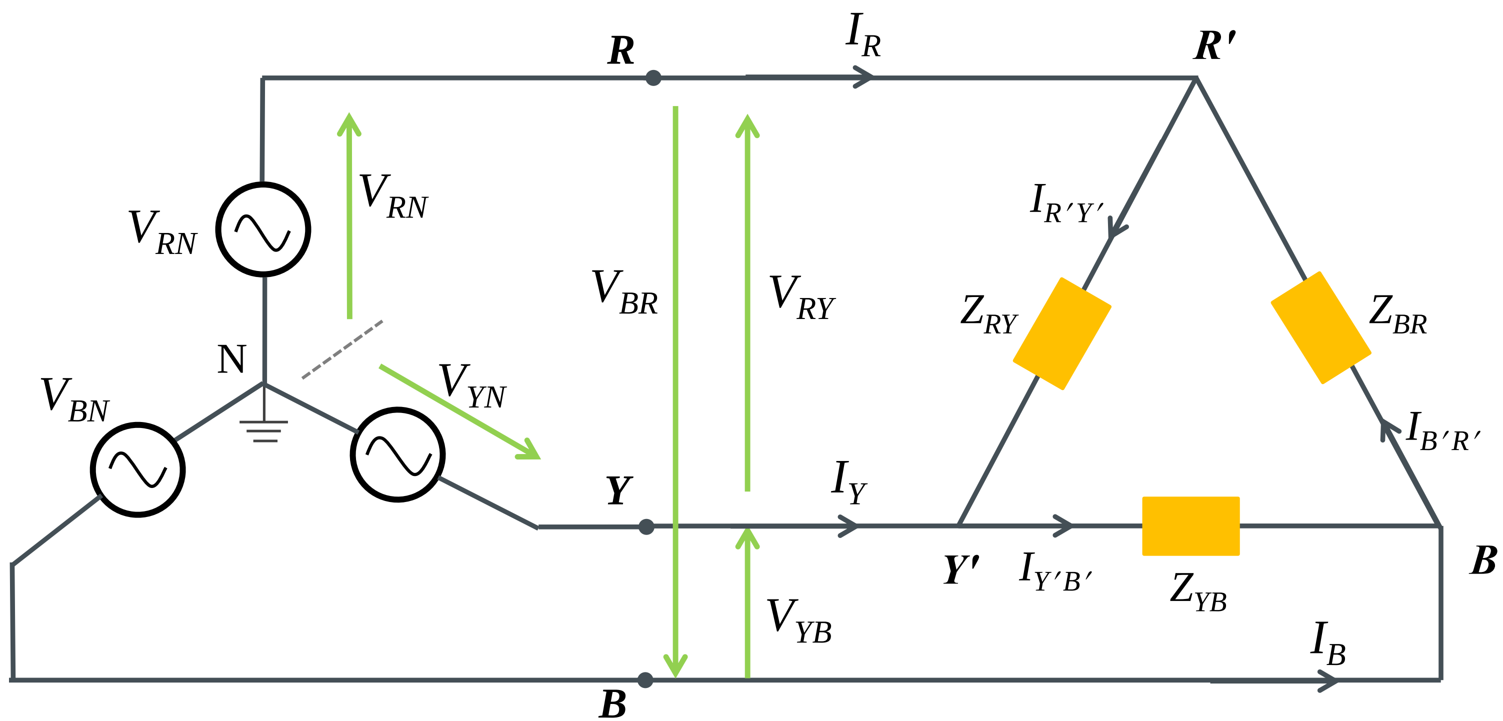
3φ , 3 Wire System with Δ Load

Consider the 3 phase Delta load connected to a 3 wire balanced source.



Phase Currents,

$$\hat{I}_{R'Y'} = \frac{\hat{V}_{RY}}{\bar{Z}_{RY}} \qquad \hat{I}_{Y'B'} = \frac{\hat{V}_{YB}}{\bar{Z}_{YB}} \qquad \hat{I}_{B'R'} = \frac{\hat{V}_{B'R'}}{\bar{Z}_{BR}}$$

Line Currents,

$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$

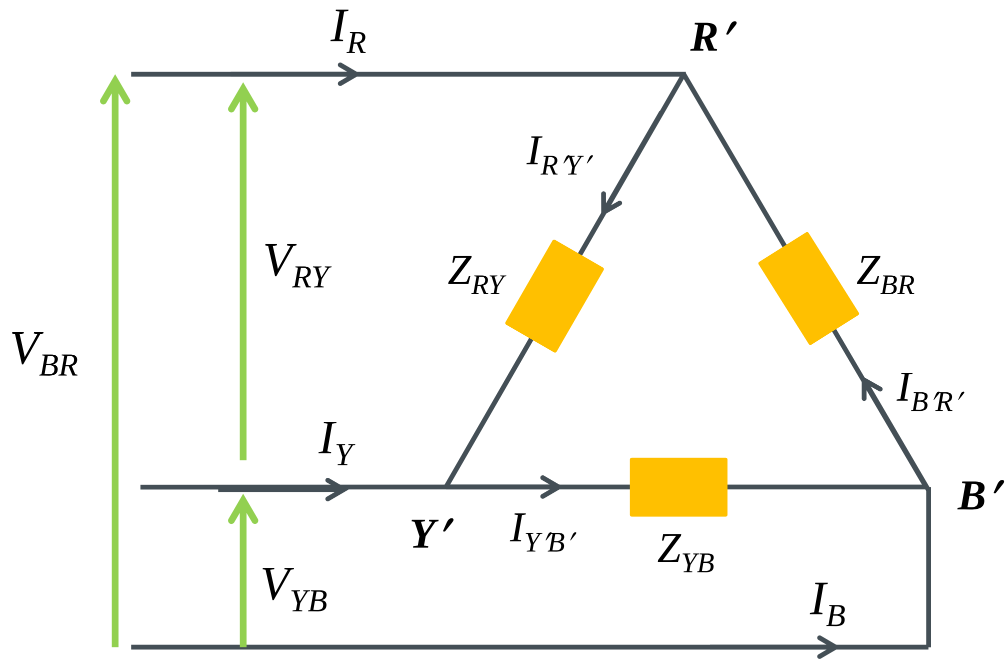
$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'}$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'}$$

Balanced Δ Load

If $Z_R = Z_Y = Z_B = Z\angle\theta$

Then, $|I_{R'Y'}| = |I_{Y'B'}| = |I_{B'R'}| = I_{Ph}$



Phase Currents:

$$\hat{I}_{R'Y'} = I_{Ph}\angle 0°$$

$$\hat{I}_{Y'B'} = I_{Ph}\angle -120°$$

$$\hat{I}_{B'R'} = I_{Ph}\angle +120°$$

Line Currents:

$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$

$$= I_{Ph}\angle 0° - I_{Ph}\angle +120° = \sqrt{3}\times I\angle -30°$$

$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'} = \sqrt{3}\times I_{Ph}\angle -150°$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'} = \sqrt{3}\times I_{Ph}\angle 90°$$

Illustration-3

Three loads, Z_R=50+j40 Ω, Z_Y=100 Ω and Z_B=80-j60 Ω are connected in Delta across a balanced 3 phase, 415V, 50 Hz supply. Determine

(i) Phase Currents

(ii) Line Currents and hence draw the complete phasor diagram.

Assume a phase sequence of RYB.

Solution:

The three phase load is supplied with a balanced supply of 415V, hence the line voltages appearing across the load are:

$$\hat{V}_{RY} = 415\angle 0° \text{ (Reference Voltage)}$$

$$\hat{V}_{YB} = 415\angle -120°$$

$$\hat{V}_{BR} = 415\angle +120°$$

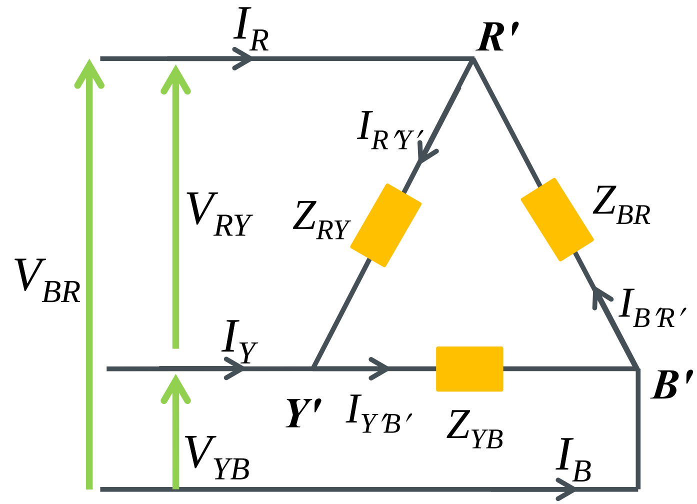


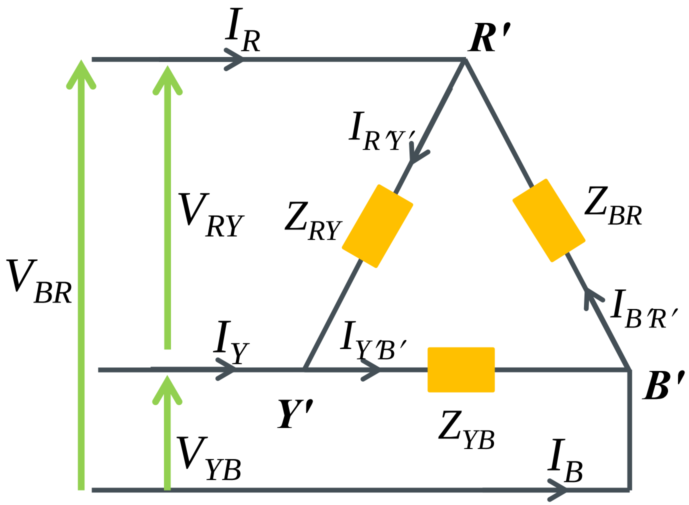
Illustration-3…

(i) Calculating the phase currents,

$$\hat{I}_{R'Y'} = \frac{\hat{V}_{RY}}{\bar{Z}_{RY}} = \mathbf{6.48\angle -38.66°}\ A$$

$$\hat{I}_{Y'B'} = \frac{\hat{V}_{YB}}{\bar{Z}_{YB}} = \mathbf{4.15\angle -120°}\ A$$

$$\hat{I}_{B'R'} = \frac{\hat{V}_{BR}}{\bar{Z}_{BR}} = \mathbf{4.15\angle 156.87°}\ A$$



(iI) Calculating the Line currents,

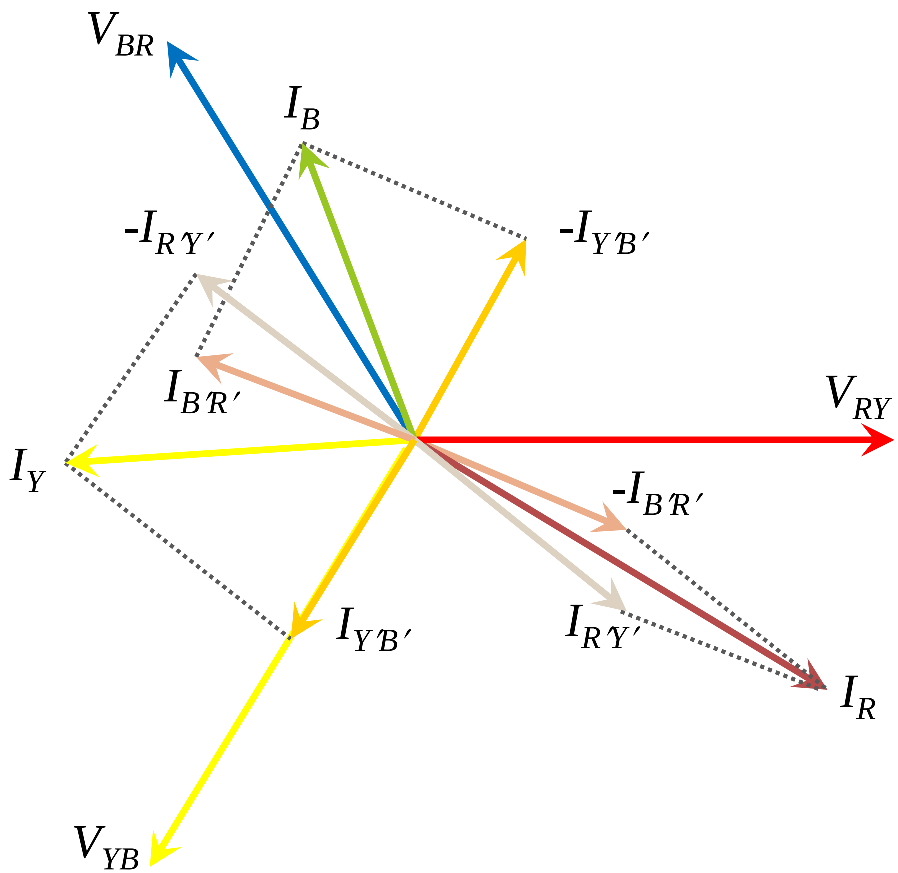
$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'} = \mathbf{10.537\angle -32.61°}\ A$$

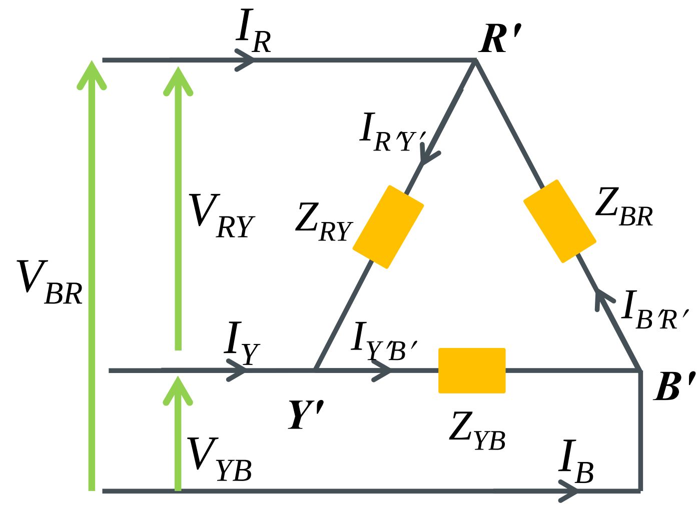
$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'} = \mathbf{7.149\angle 176.35°}\ A$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'} = \mathbf{5.506\angle 108.44°}\ A$$

Illustration-3…

(ii) Phasor Diagram,





$$\hat{I}_R = \hat{I}_{R'Y'} - \hat{I}_{B'R'}$$

$$\hat{I}_Y = \hat{I}_{Y'B'} - \hat{I}_{R'Y'}$$

$$\hat{I}_B = \hat{I}_{B'R'} - \hat{I}_{Y'B'}$$

Illustration 4

A balanced star connected load of 8+j6 Ω per phase is connected to a 3 phase, 415V supply. Find the line currents, power factor, power, reactive volt amperes and total volt amperes.

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Illustration 5

A star connected load is supplied from a symmetrical three phase, 440V system. The branch impedances of the load are ,$Z_R = 5\angle 30^\circ\ \Omega$, $Z_Y = 10\angle 45^\circ\ \Omega$, $Z_B = 10\angle 60^\circ\ \Omega$. Find the active power supplied by the source.

Monday, February 7, 2022

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Summary

Analysis of balanced/unbalanced three phase star/delta connected load with 3 phase balanced excitation is performed.

- *For Balanced Star connected load, the line voltage = √3 x phase voltage.*
- *For Balanced Delta connected load, the line current = √3 x phase current.*

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Basic Electrical Technology

[ELE 1051]

Three Phase AC Circuits

L26– Power Measurement in Three Phase System and Power System Components

Balanced, unbalanced Star & Delta connected loads

→ $P = \sqrt{3} \cdot V_L I_L \cos\phi$
$= 3 \cdot V_{ph} I_{ph} \cos\phi$

load is balance.

if load is unbalanced

$V_{ph1} I_{ph1} \cos\phi_1 + V_{ph2} I_{ph2} \cos\phi_2 + V_{ph3} I_{ph3} \cos\phi_3$

$Q = \sqrt{3} V_L I_L \sin\phi$
$= 3 V_{ph} I_{ph} \sin\phi$

$S = \sqrt{3} V_L I_L^*$ //

1) wattmeter.
↳
1) 3 - wattmeters ✓
2) 2 - wattmeter
3) 1 - wattmeter.

Wednesday, February 9, 2022 DEPT. OF ELECTRICAL & ELECTRONICS ENGG., MIT - MANIPAL 2

Measurement of 3 Ph. Active Power

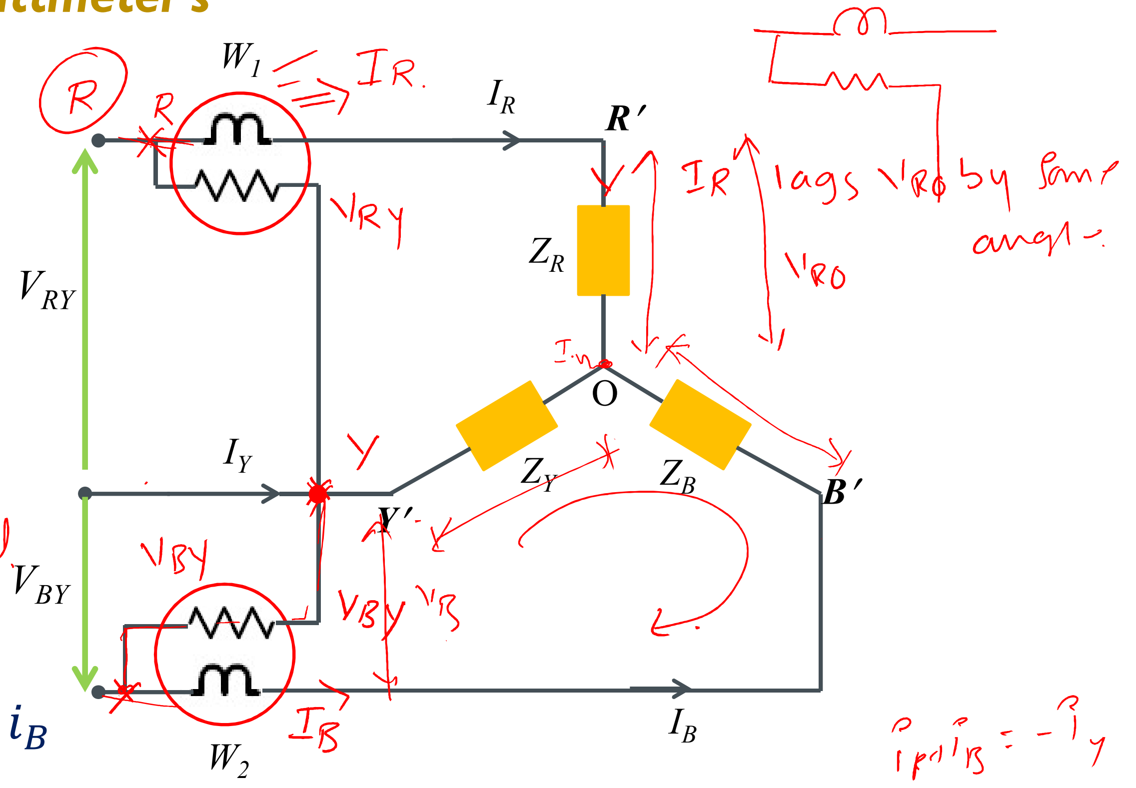
I. Star Connected Load using 2 Wattmeter's

Wattmeter Reading, → Source
line voltage

load voltage → Phase value

$$W_1 = v_{RY}\ i_R = (v_{R'O} - v_{Y'O})\ i_R$$

$$W_2 = v_{BY}\ i_B = (v_{B'O} - v_{Y'O})\ i_B$$



Total Active Power, → Total Power consumed by 3-φ load

$$= W_1 + W_2$$

$$= (v_{R'O} - v_{Y'O})\ i_R + (v_{B'O} - v_{Y'O})\ i_B$$

$\Rightarrow -i_Y$

$$= v_{R'O}\ i_R - v_{Y'O}(i_R + i_B) + v_{B'O}\ i_B$$

$$= v_{R'O}\ i_R + v_{Y'O}\ i_Y + v_{B'O}\ i_B \quad Since,\ i_R + i_Y + i_B = 0$$

$i_R + i_Y + i_B = 0 \Rightarrow i_R + i_B = -i_Y$

Measurement of 3 Ph. Active Power

2. Balanced Load (Star Connected) using 2 Wattmeter's

phase voltage as Ref RYB.

load is RL load

Wattmeter Reading,

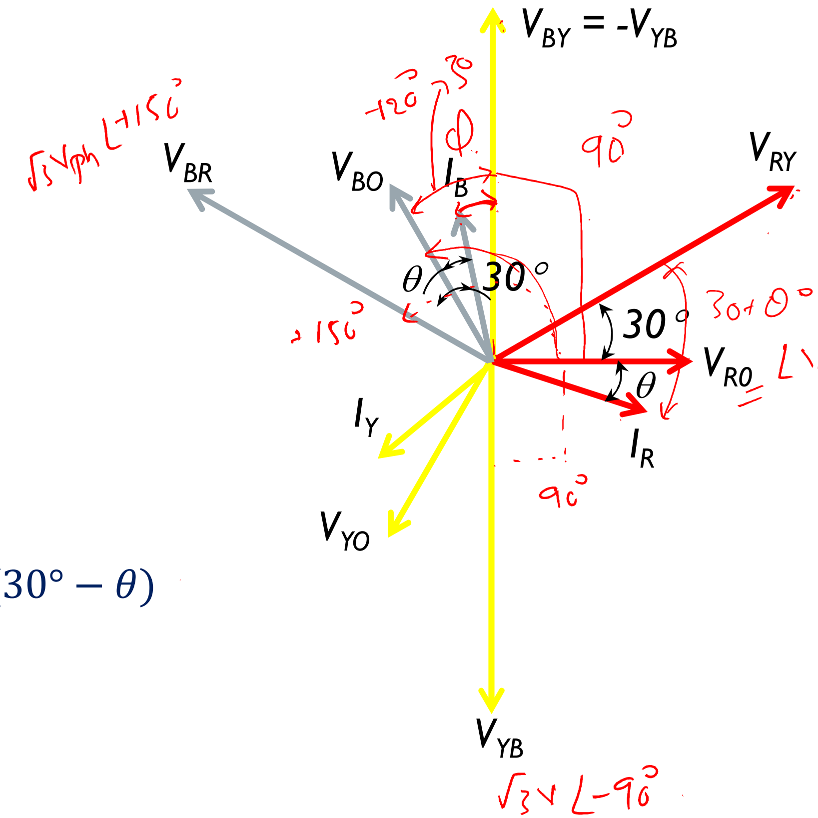
$W_1 + W_2$ = total active power drawn by 3-φ load.

$$W_1 = V_{RY}\, I_R\, Cos\angle(\, V_{RY}\;\&\; I_R)$$
$$= V_L\, I_L\, Cos(30° + \theta)$$

$$W_2 = V_{BY}\, I_B\, Cos\angle(\, V_{BY}\;\&\; I_B)$$
$$= V_L\, I_L\, Cos(30° - \theta)$$

Total active power consumed,

$$P = W_1 + W_2$$
$$= V_L\, I_L\, Cos(30° + \theta) + V_L\, I_L\, Cos(30° - \theta)$$
$$P \;= \boxed{\sqrt{3} \times V_L\, I_L\, Cos\theta}$$



$V_{YB} = V_{YO} - V_{BO}$

$V_{RY} = V_{RO} - V_{YO}$
$= V\angle 0° - V\angle -120°$
$= \sqrt{3}V\angle +30°$

Meas. of 3 Ph. Active Power…

Summation of two wattmeters,

$$W_1 + W_2 = \sqrt{3} \times V_L \times I_L \times Cos\ \theta$$

θ

$V_L I_L \cos(30-\theta) - V_L I_L \cos(30+\theta)$

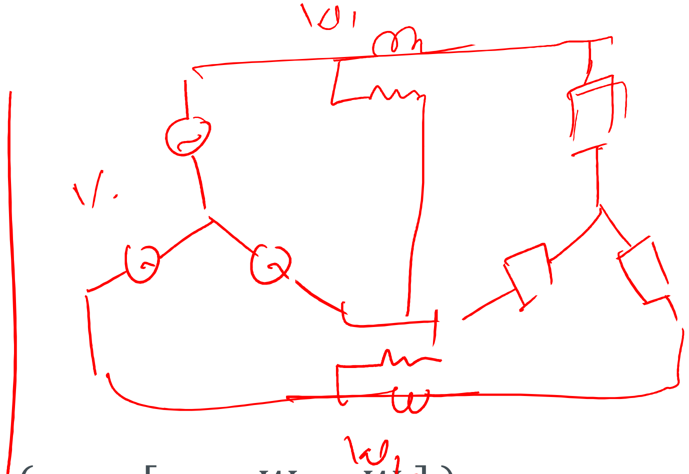
Difference in the reading of two wattmeters,

$$W_2 - W_1 = \boxed{V_L \times I_L \times Sin\ \theta}$$

Hence,

$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{Sin\ \theta}{\sqrt{3} \times Cos\ \theta}$$

$$\theta = \tan^{-1}\left[\sqrt{3} \times \frac{W_2 - W_1}{W_2 + W_1}\right]$$



Power factor of the Balanced Load = $Cos\theta = Cos\left\{\tan^{-1}\left[\sqrt{3} \times \frac{W_2 - W_1}{W_2 + W_1}\right]\right\}$

θ $\cos\theta$

Measurement of 3 Ph. Active Power

3. Balanced Load (Star Connected) using 1 Wattmeter

Wattmeter Reading,

→ power factor

$$W_1 = V_{R'O}\, I_R\, Cos\angle(V_{R'O}\ \&\ I_R)$$

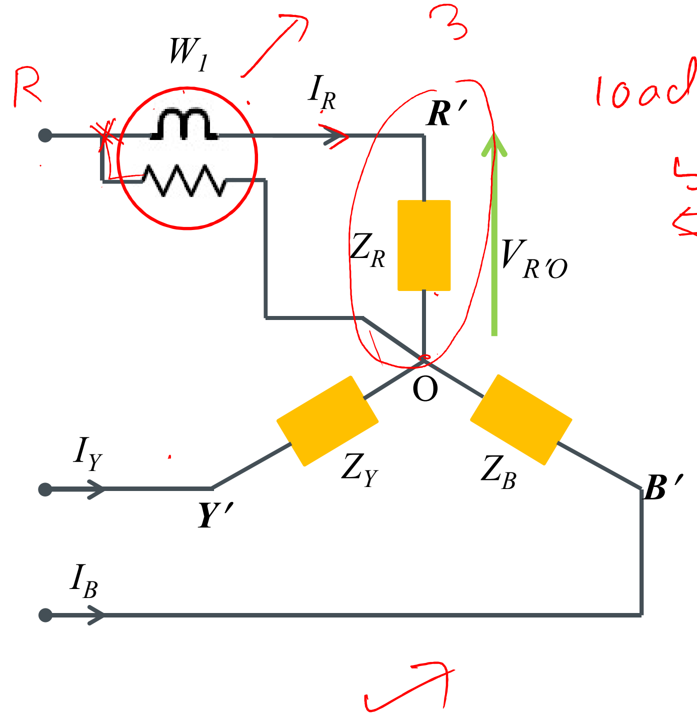
$$= V_{Ph}\, I_{Ph}\, Cos\theta$$

Total active power consumed,

$$= 3 \times W_1$$

$$= 3 \times V_{Ph}\, I_{Ph}\, Cos\theta$$

$$= \sqrt{3} \times V_L I_L Cos\theta$$



load should be balanced

Measurement of 3 Ph. Active Power

4. Balanced Load (Delta Connected) using 1 Wattmeter

Wattmeter Reading,

$$W_1 = V_{RY}\, I_{R'Y'}\, Cos\angle(V_{RY}\ \&\ I_{R'Y'})$$

$$= V_L\, I_{Ph}\, Cos\theta$$

$= V_{ph} I_{ph} \cos\theta$

Total active power consumed,

$$= 3 \times W_1$$

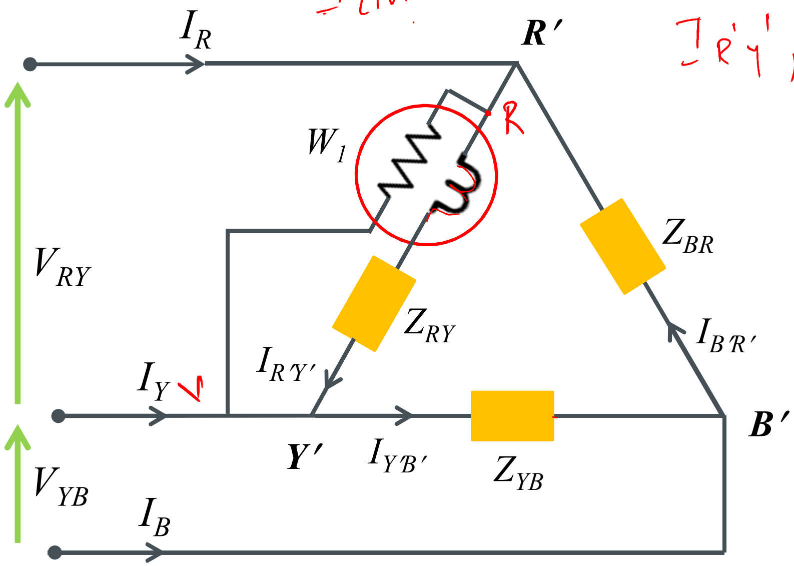
$$= 3 \times V_L\, I_{Ph}\, Cos\theta$$

$$= \sqrt{3} \times V_L\, I_L\, Cos\theta$$

$V_{line} = V_{phase}$

$I_{line} = \sqrt{3} I_{ph}$

$I_{R'Y'}$, V_{RY} $\cos$



Exercise-3

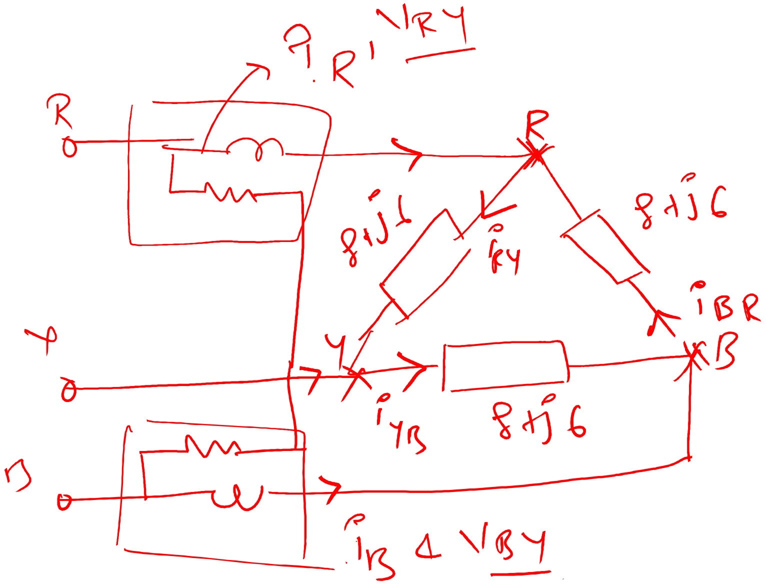
Three identical impedances of (8+j6) Ω are connected in delta across a symmetrical 3 phase, 3 wire 400 V system. Calculate the power factor using wattmeter readings.

$$\theta = \tan^{-1}\left[\sqrt{3}\left[\frac{W_2 - W_1}{W_2 + W_1}\right]\right]$$

$$i_{RY} = \frac{V_{RY}}{Z_R} = \frac{400\angle 0}{10\angle 36.86^\circ} = 40\angle -36.86^\circ$$

$$i_{YB} = \frac{V_{YB}}{Z_Y} = 40\angle -156.86^\circ$$

$$i_{BR} = \frac{V_{BR}}{Z_B} = 40\angle -276.86^\circ$$



R Y B

3-φ

$V_{Line} = 400V$

Line volt as Reference

$V_{RY} = 400\angle 0^\circ$

$V_{YB} = 400\angle -120^\circ$

$V_{BR} = 400\angle -240^\circ$

$$i_R = i_{RY} - i_{BR} = 69.283\angle -66.86$$

$$i_B = i_{BR} - i_{YB} = 69.28\angle 53.145^\circ$$

$W_1 + W_2$

$\cos\phi$

$$W_1 = V_{RY} \cdot \hat{I}_R \cos \angle V_{RY} \,\&\, \hat{I}_R$$

$$= 400 \times 69.28 \times \cos \angle 66.86^\circ$$

$$= 10.89\,kW.$$

$$W_2 = V_{BY} \cdot \hat{I}_B \cos \angle V_{BY} \,\&\, \hat{I}_B$$

$$= 400 \times 69.28 \cos \angle -120^\circ - 53.14^\circ$$

$$= -27.513\,kW$$

$$= 27.513\,kW.$$

$$\theta = \tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_2 + W_1}\right)\right] =$$

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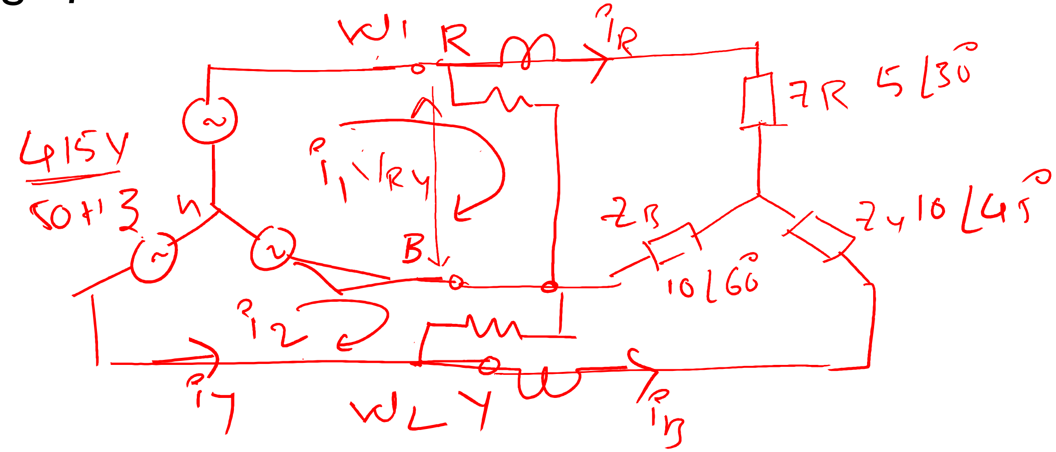
Exercise-4

Three loads $Z_R = 5\angle 30^\circ\ \Omega, Z_Y = 10\angle 45^\circ\ \Omega, Z_B = 10\angle 60^\circ\ \Omega$ are connected in Star to R,Y and B Phase respectively. The current coils of the two wattmeters are connected in R & Y lines. If the supply voltage is 415V, 50 Hz, determine the reading of the two wattmeters. Assume the phase sequence is RBY.



$W_1 = V_{RY} \times i_R \times \cos \angle V_{RY} \,\&\, i_R$

$W_2 = V_{BY} \times i_Y \times \cos \angle V_{BY} \times i_Y$

$$\begin{bmatrix} Z_R + Z_B & -Z_B \\ -Z_B & Z_R + Z_Y \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \overbrace{V_{Rn} - V_{Bn}}^{V_{RB}} \\ \underbrace{V_{By} - V_{Yn}}_{V_{BY}} \end{bmatrix}$$

$V_L = 415\,V$

Line voltages as Ref

$V_{RB} = 415\angle 0^\circ$

$V_{BY} = 415\angle -120^\circ$, $V_{YR} = 415\angle -240^\circ$

$i_R = i_1$

$i_Y = -i_2$

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Summary

Measurement of Active Power for a three phase Star/Delta connected balanced/unbalanced load can be performed by using two wattmeters.

2- watt/m.

For a balanced Load, the Load Power factor can be measured by using one or two wattmeter method.

Power-factor
only
Balance.

1 →

Measurement of power for a balanced Star/Delta load can be performed using one wattmeter.

Tutorial

A balanced 3 phase star connected load of 150kW takes a leading current of 100A with a line voltage of 1100V, 50Hz. Find the circuit constants of the load per phase.

$P = \sqrt{3} V_L I_L \cos\phi = 150\ kW$

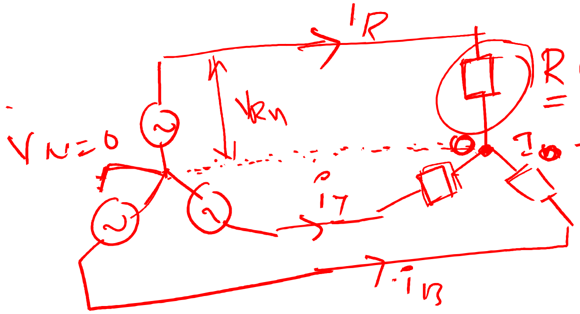
$I_{Line} = 100\ A$

$V_{Line} = 1100\ V$

$\cos\phi = \dfrac{150\ kW}{\sqrt{3} \times 1100 \times 100} = 0.787$

$i_R = \dfrac{V_{Rn}}{Z_R}$

$Z_R = \dfrac{V_{Rn}}{i_R} = \dfrac{635.08}{100} = 6.35\ \Omega$



R C

$P = 150\ kW$

$I_n = I_R + I_Y + I_B$

$= I\angle 0 + I\angle -120^\circ + I\angle -240^\circ = 0$

$V_n = 0\ V$

$V_{Line} = 1100\ V$

$V_{phase} = V_{line}/\sqrt{3} = 635.08\ V$

Phase voltages as Ref.

$V_{Rn} = 635.08\angle 0^\circ$

$V_{Yn} = 635.08\angle -120^\circ$

$V_{Bn} = 635.08\angle -240^\circ$

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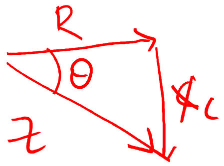
RC - load



$= 6.35\ \Omega$

$R = Z\cos\theta = 6.35 \times 0.787$

$X_C = Z\sin\theta = 6.35 \times \sin(38.06^\circ)$

$=$

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Power of a 3 phase motor load, connected to 400 V, 50 Hz supply, is to be measured by 2 watt-meters. The readings of the watt-meters are 2500 W and 1200 W respectively. Determine the power factor, active power, reactive power and apparent power of the load. Also, find the load impedance per phase if the stator windings are star connected.

$V_{Line} = 400V,$

$W_1 = 2500W.$

$W_2 = 1200W$

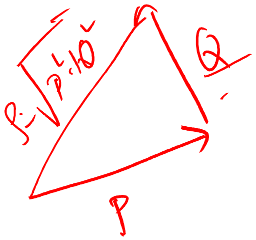
$\cos\phi = ?$

$P = \_\_\_$

$Q = \_\_\_$

$S = \_\_\_$

$Z = \_\_\_$



$$\tan\phi = \sqrt{3}\left[\frac{W_2 - W_1}{W_2 + W_1}\right]$$

$$\tan\phi = \sqrt{3}\left[\frac{2500 - 1200}{2500 + 1200}\right]$$

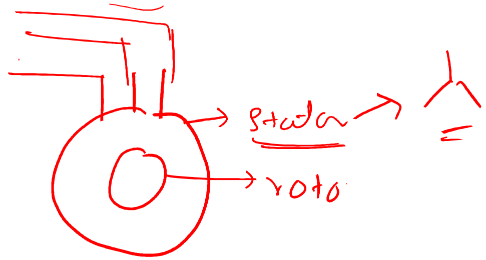
$\phi = 31.32°$

$P.f = \cos\phi = 0.854$

$P = \sqrt{3}\, V_L I_L \cos\phi = W_1 + W_2$

$= 3700W$ //

$Q = \sqrt{3}\, V_L I_L \sin\phi =$



$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi$

$\sqrt{3}(W_2 - W_1) = \sqrt{3}\, V_L I_L \sin\phi$

$= Q.$

$Q = \sqrt{3}\,(W_2 - W_1)$

$= \sqrt{3}(2500 - 1200)$ VAR

$V_{line} = 400V$

$\sqrt{3} \cdot V_L I_L \cos\phi = 3760\,W.$

$\sqrt{3} \times 400 \times I_L \times 0.854 = 3700$

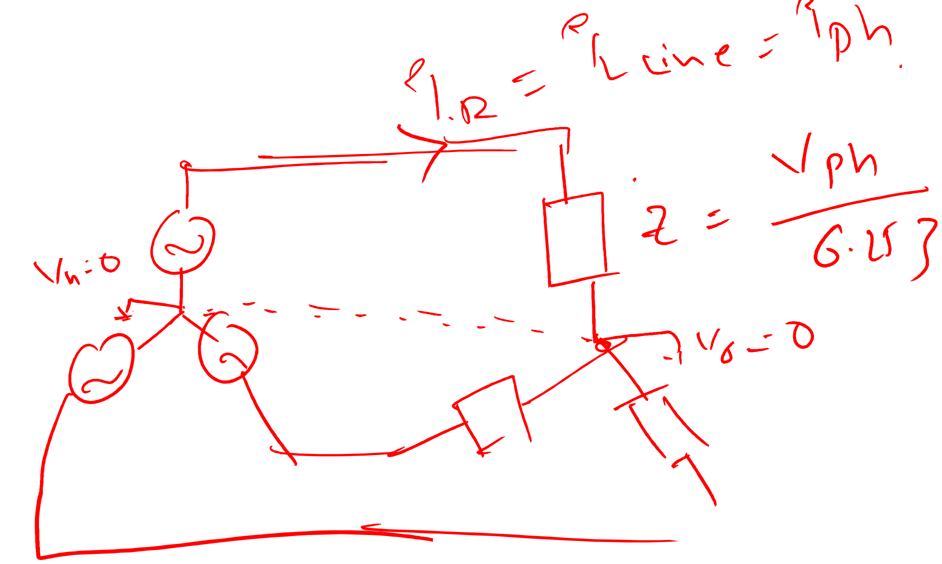
$I_L = 6.253\,A$

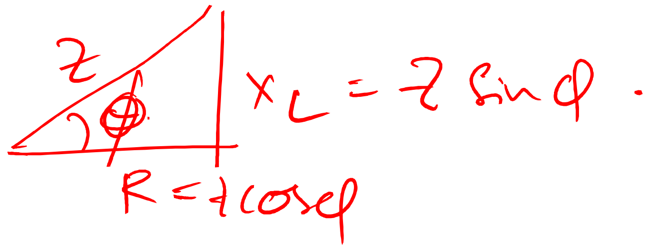
$Z = \dfrac{400/\sqrt{3}}{6.253} = 36.92\,\Omega$

36.92×0.854

$R = Z\cos\phi$

$X = Z\sin\phi$





A symmetrical 3 phase 400V system supplies a balanced delta connected load. The current in each branch is 20 A and the phase angle is 40 degree lagging. Find the line current, total power. Draw the phasor diagram showing the voltages and currents in the lines and phases for all the 3 phases.

Phase currents are given

Phase currents as Ref.

$I_{RY} = 20\angle{-40^\circ} = 20\angle{-40^\circ}$

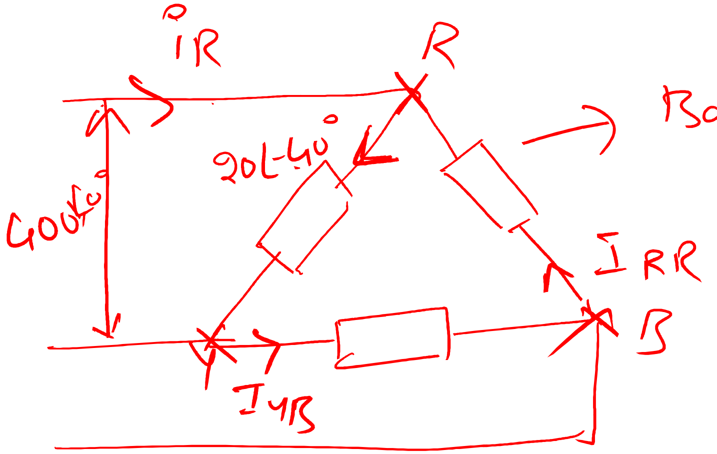
$I_{YB} = 20\angle{-120-40^\circ} = 20\angle{-160^\circ}$

$I_{BR} = 20\angle{-240^\circ - 40^\circ} = 20\angle{-280^\circ}$

$i_R = I_{RY} - I_{BR} = 34.64\angle{-70^\circ}$

$i_Y =$

$i_B =$



$P = \sqrt{3}\, V_L I_L \cos\phi$

$= 3 \times V_{ph} \times I_{ph} \cos\phi$

$= 3 \times 400 \times 20 \times \cos\angle{-40^\circ}$

$=$

Basic Electrical Technology

[ELE 1051]

CHAPTER 5 – ELECTRICAL POWER SYSTEM COMPONENTS & MACHINES

Topics covered…

- Electrical power system
  - An overview
  - Types of generation
  - Loads
  - Digital energy meter
- Electrical machines
  - Transformer
  - DC motor
  - Induction motor
  - Synchronous motor

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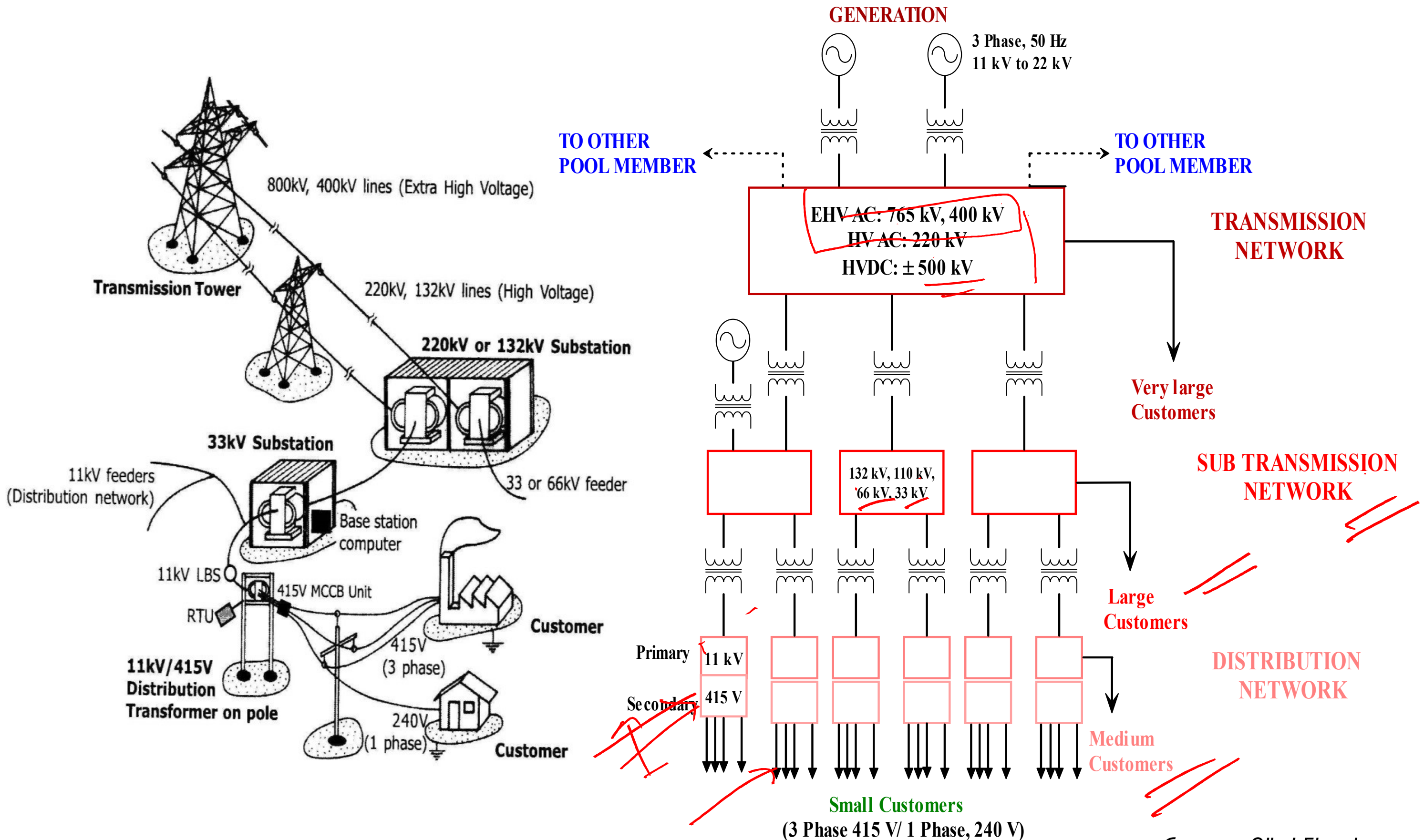
Power system structure



Courtesy: Olle I Elgerd

Power system components

- Generation subsystem
- Transmission subsystem
- Sub-transmission subsystem
- Distribution subsystem
- Protection and Control subsystem

Transmission networks - EHV AC or HVDC

- Operates @765 kV/400 kV/220 kV AC or 500 kV DC.

AC Sub-Transmission networks

- Operates @ 132 kV/110kV/66 kV/ 33 kV

AC Distribution Network

- Primary side: 11 kV
- Secondary side: 415 V, 4 Wire

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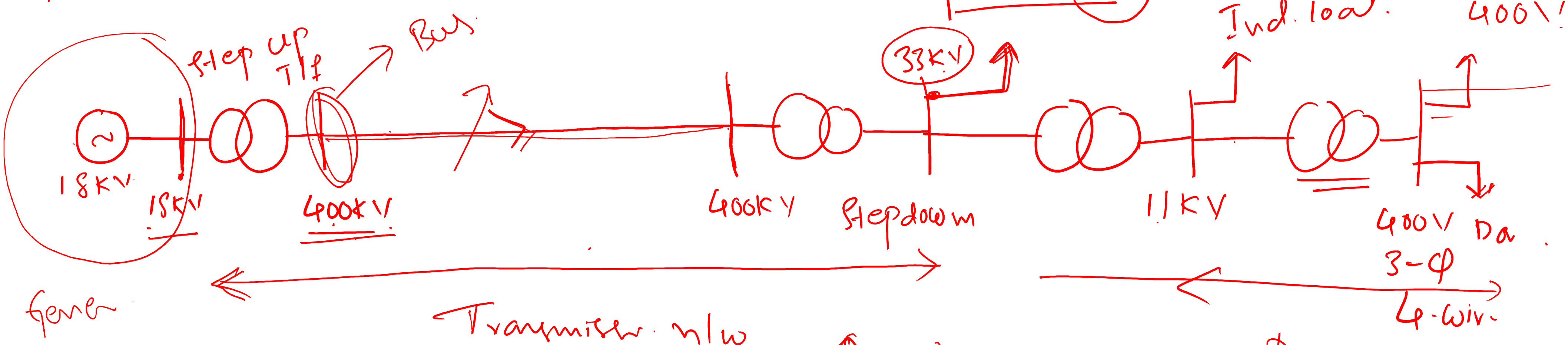
Power System Struct

$\boxed{\text{3-}\phi \text{ power}}$ 18KV to 22KV, 200MW



220KV
400KV
765KV
800KV HVDC

$\sqrt{3}\, V_L I_L \cos\phi$

$I^2 \times R_{trans}$ loss.

2A 2Ω load

$\boxed{P = 4 \times 2 = 8W}$

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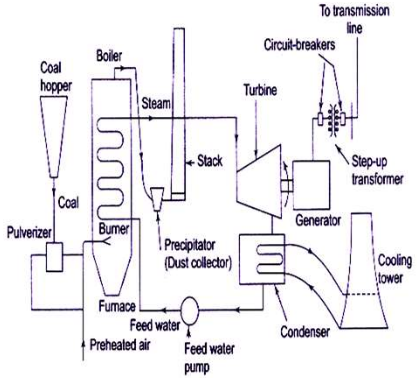
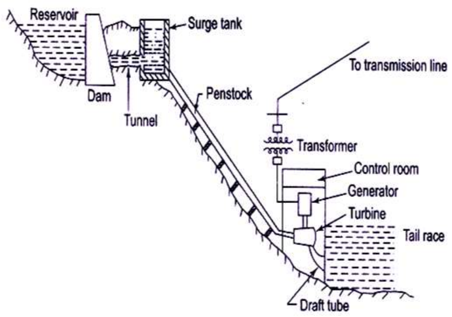
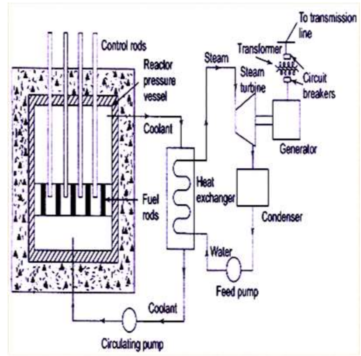
Generation

Primary sources of energy

- Fossil fuel
  - Coal, oil, natural gas
- Renewable energy
  - Water, solar, wind, tidal, geo-thermal etc.
- Nuclear energy





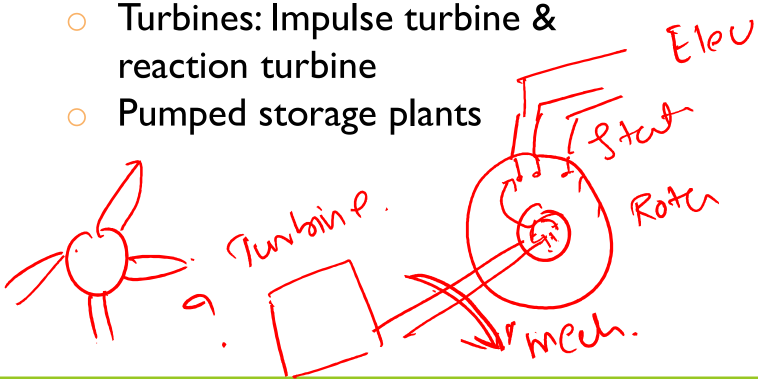


Thermal power stations

- Coal fired
  - Turbo alternators driven by steam turbine
- Oil fired
  - Crude oil or residual oil
- Gas fired
  - Combined cycle- First stage: Gas turbine, Second stage: Steam turbine
- Diesel fired
  - Internal combustion engines as prime mover
  - Standby power plants

Hydroelectric power station

- Salient pole alternators driven by turbines
- Turbines: Impulse turbine & reaction turbine
- Pumped storage plants



Nuclear power plant

- Fissile material
- Moderator
  - D2O, Graphite
  - Control rods
  - Boron OR Cadmium
- Fast breeder reactors
  - Liquid metal (alloy of Na & K) is coolant

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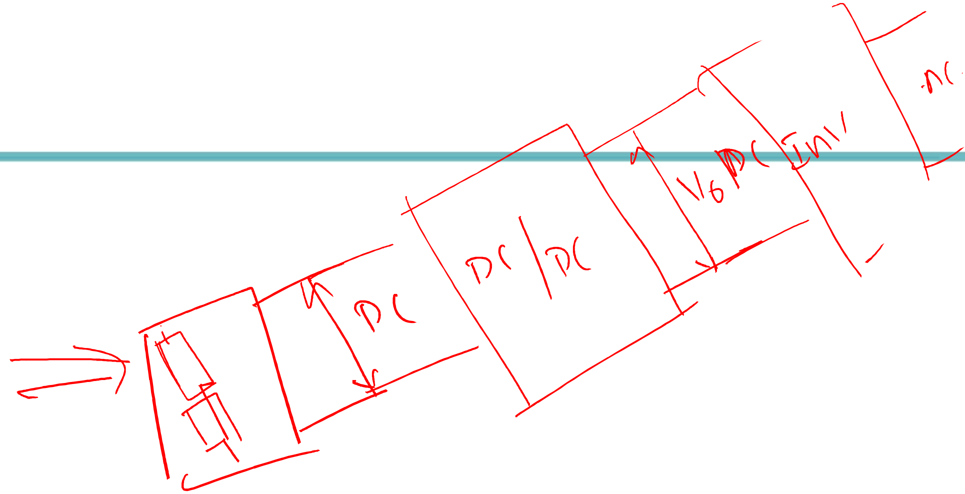
Generation



Non-conventional power stations

- Wind power stations
- Solar power stations
- Micro-hydel power stations
- Bio-mass power stations
- Geothermal power stations

Non-C.


Wind Farm in Karnataka


20 MW hydro plant, HP


Bio-mass Plant, Chattisgarh


Solar Park, Charanka Village, Gujarat

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Substation

Substation components

- Lightning arrester
- Carrier line communication equipment (Wave trap)
- Instrument transformers (CT, PT)
- Circuit breakers
- Isolators
- Bus bars
- Power transformers
- Control room

L-G fault

R

Y

B

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Protection & Control Subsystem

- Fail free power is *hypothetical*
- Faults: Open circuit & short circuit
- Faults detection: *Relays*
- Fault Isolation: *Circuit breakers*
- Modern Trend: Supervisory Control And Data Acquisition (SCADA) systems

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Types of loads

Industrial Loads
- 3 Phase
- Complex Tariff Structure

Domestic Loads/Commercial Loads
- Single Phase
- Tariff based on energy consumed- kWH

Reduce Electricity bill by minimizing the use of heating / environmental conditioning gadgets

Domestic loads	Typical power rating
Incandescent lamps	5 W to 100 W
Fluorescent lamps	20 W to 40 W
CFL	5 W to 25 W
LED Lamps	1W to 100 W
Air Conditioner (1.5 T)	1800 W
Electric Iron	750 W
Heaters/ Geysers	2000 W
Ceiling Fan	70 W
Washing Machine (with heater)	2500 W
Refrigerator	160 W
Desktop PC	200 W
Office Laptop	65 W

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Exercise

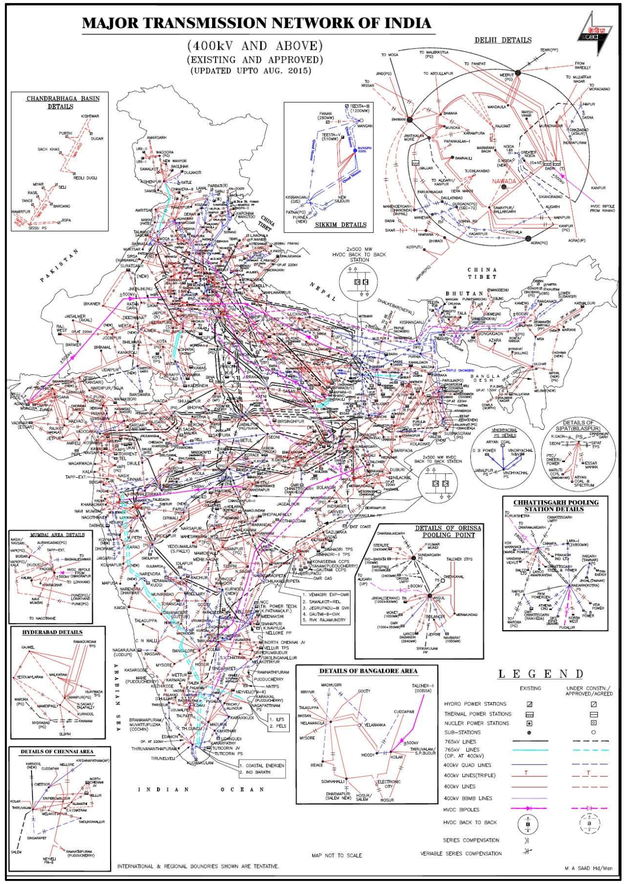
Power transmission maps of India

- https://cea.nic.in/old/powermaps.html

Indian Power Sector – A Glance

Sector	MW	%
State		
Central		
Private		
Total		

Fuel	MW	%
Total Thermal		
Coal		
Lignite		
Gas		
Oil		
Hydro (Renewable)		
Nuclear		
RES		
Total		



- https://www.niti.gov.in/edm/
- https://cea.nic.in/dashboard/?lang=en

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$P = \sqrt{3} V_L I_L \cos\phi$

$\boxed{E = P \times t}$

Energy Meters

Working principle

- Energy is the total power delivered or consumed over a time interval,

Energy = Power x Time

- Electrical energy developed as work or dissipated as heat over an interval of time 't' may be expressed as:

$$Energy = \int_0^t v\,i\;dt$$

P×t

v – Applied voltage in (volts)
i – current (A)
t – time (hr)

- *Unit of Energy:* ***kWh or Units***

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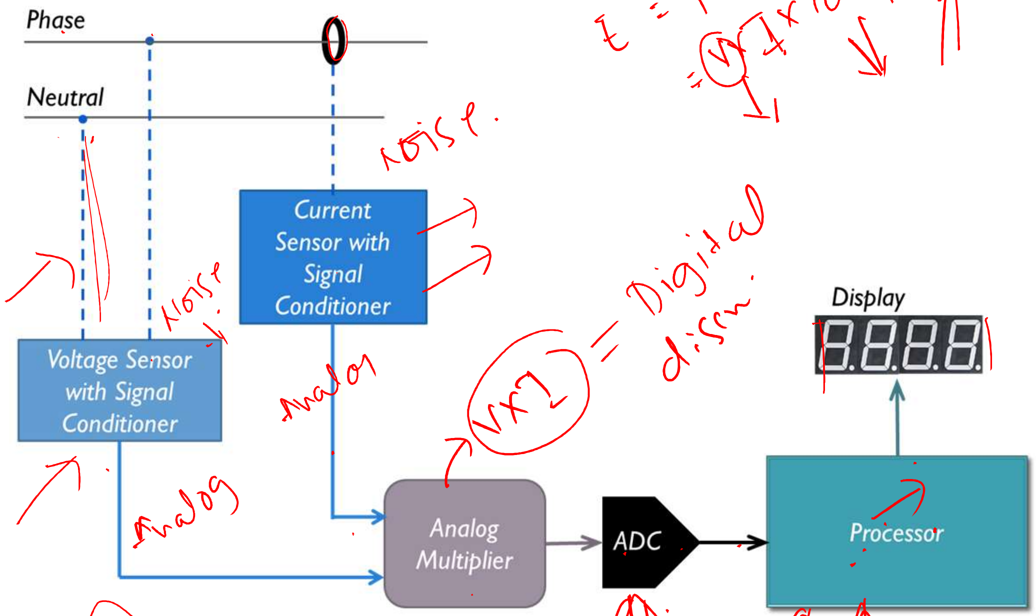
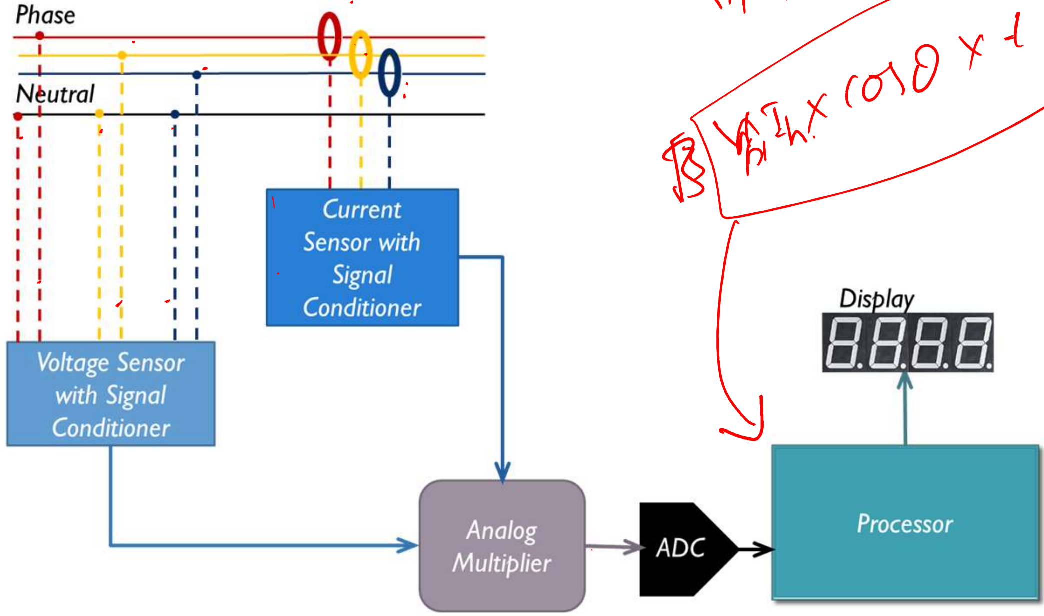
Digital energy meter



Single phase digital energy meter schematic



Three phase digital energy meter schematic

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Electricity tariff

1 kW × 10 hr = 10 kWh × 30 = 3000 kWh = 3000 unit

Installation	Power supply	Tariff
Industries	11kV and above	Demand Charges (per kVA) Power Factor Surcharge (per unit) Energy Charges (per kWh)
Hotels, Restaurant Cinemas Petrol Bunks Banks Commercial complexes	400V Three Phase 230V Single Phase	Sanctioned Load (per kW) Power Factor Surcharge (per unit) Energy Charges (per kWh)
Residential	400V Three Phase 230V Single Phase	Sanctioned Load (per kW) Energy Charges (per kWh) Rebate for Solar Installations

Fixed cha

= 200RS +

1 – 500 → 2.5RS

500 – 1000 - 5/un

>1000 - 7.

0.9

0.8

= 200 + (500) 2.5 + (1000) 5 + 1500 × 7

=

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Basic Electrical Technology

[ELE 1051]

CHAPTER 5 – ELECTRICAL MACHINES

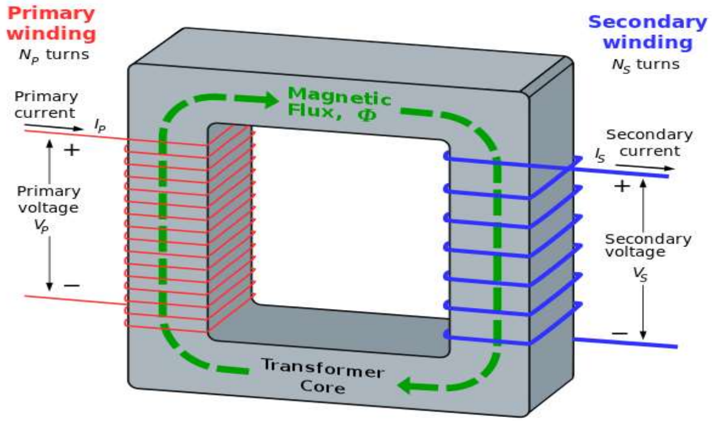
Topics covered…

- Electrical machines
  - Transformer
  - DC motor
  - Induction motor
  - Synchronous motor

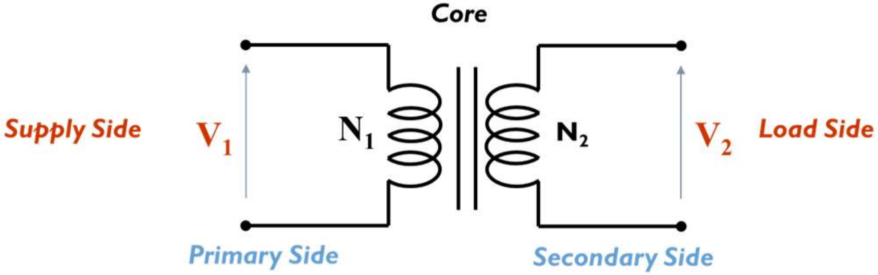
Transformers

Introduction

- Static device with AC excitation
- Transfers energy between two or more magnetically coupled circuits without change in frequency
- Principle of operation: Electromagnetic Induction
- Electric circuits are linked by a common ferromagnetic core
- Ferromagnetic core ensures maximum magnetic flux linkage



Representation:

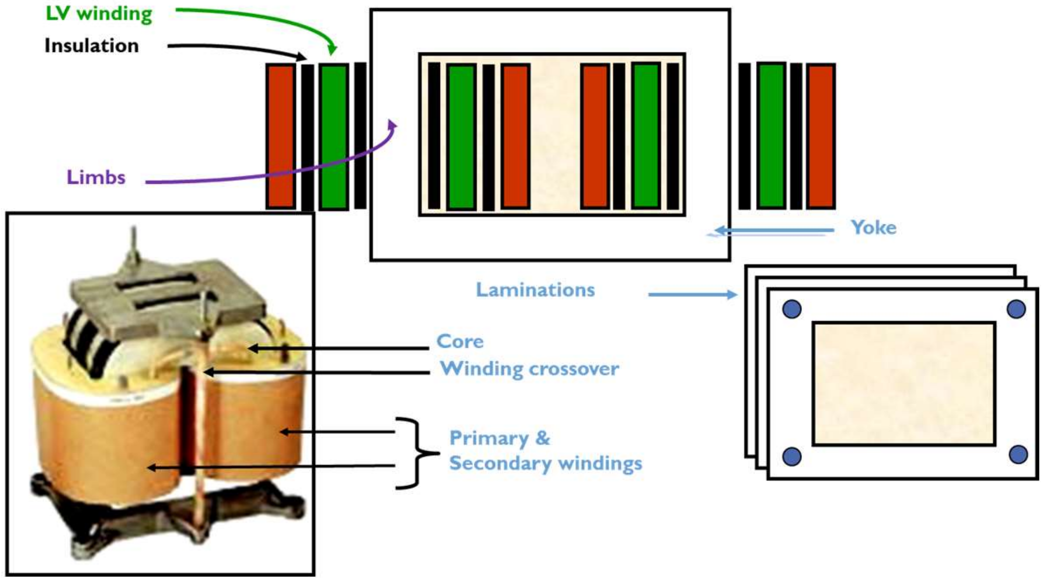


N_1 = Number of turns on primary
N_2 = Number of turns on secondary

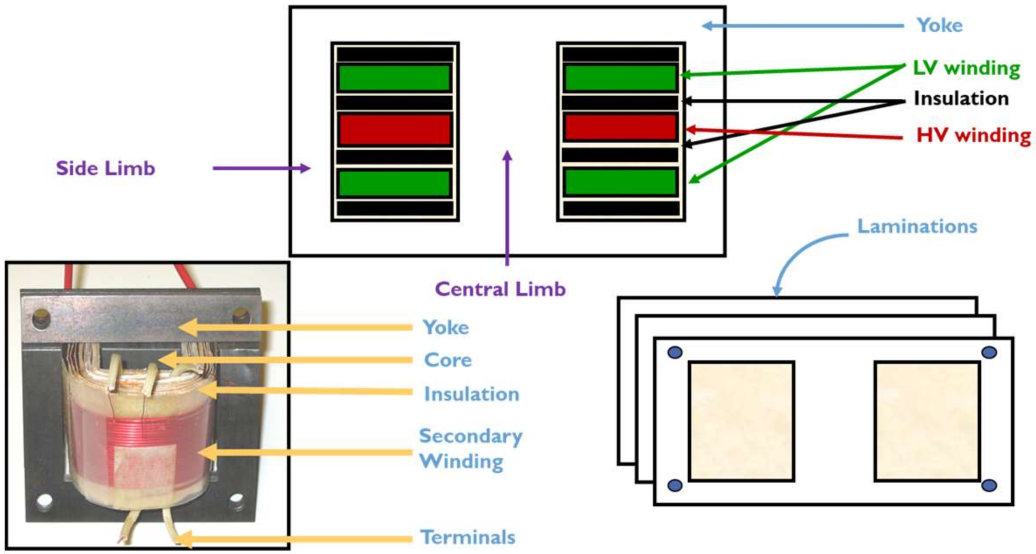
Types

Based on Construction	***Based on Function***	***Based on Windings***
Core Type	Step Up	Single Winding
Shell Type	Step Down	2 or 3 Windings

Core & shell type

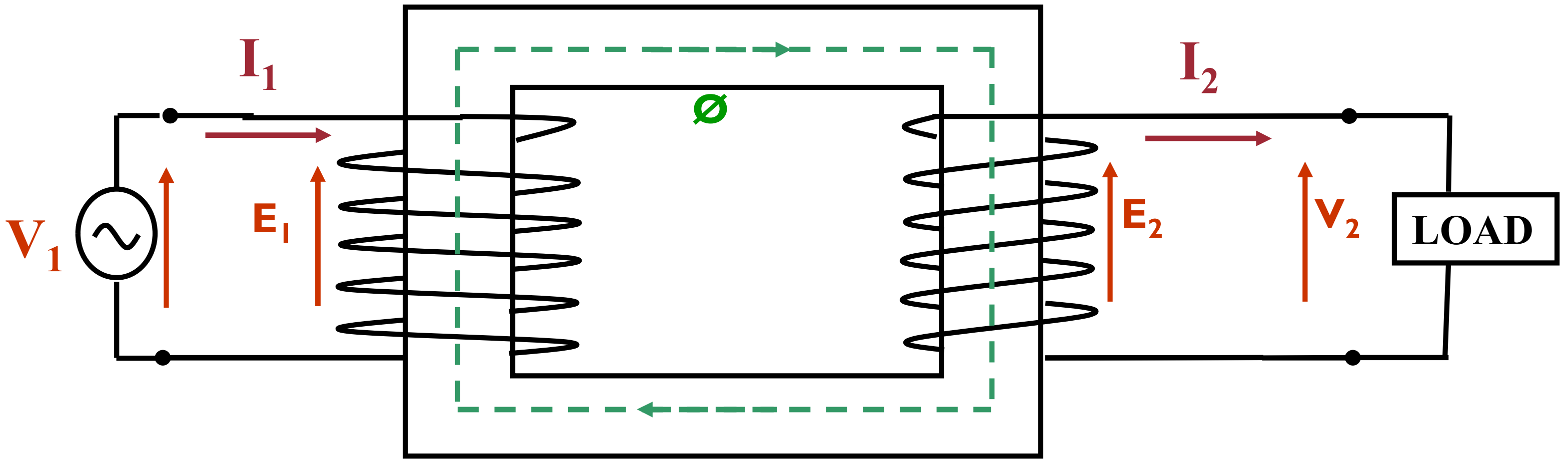


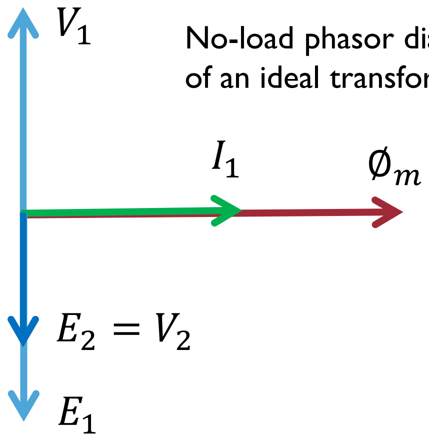
Core type



Shell type

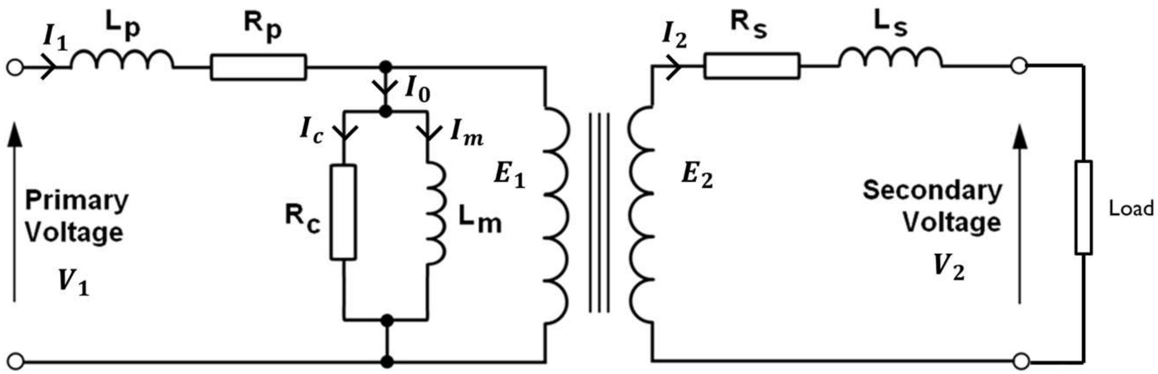
Operation of transformer





No-load phasor diagram of an ideal transformer

- Magnetic core : Flux path
- Flux linkages : Primary & secondary
- Induced Emf :
  - Primary – Self induced emf
  - Secondary – Mutually induced emf



Circuit representation of a practical transformer

EMF equations of transformer

$\boldsymbol{Core\ flux,}$

$$\emptyset = \emptyset_m \sin(\omega t)$$

$\boldsymbol{Induced\ Emf,}$

$$e = -N\frac{d\emptyset}{dt} = N\omega\emptyset_m sin(\omega t - 90°)$$

$$e = E_m sin(\omega t - 90°), \boldsymbol{where}, E_m = N\omega\emptyset_m$$

$\boldsymbol{RMS\ value\ of\ self\ induced\ emf,}$

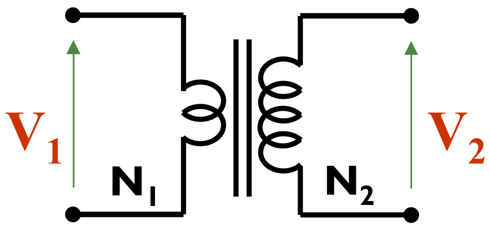
$$E = \frac{E_m}{\sqrt{2}} = \frac{N\omega\emptyset_m}{\sqrt{2}} = \frac{2\pi f N\emptyset_m}{\sqrt{2}}$$

$\boldsymbol{Primary\ Induced\ Emf,} E_1 = 4.44\ N_1 f\emptyset_m$
$\boldsymbol{Secondary\ Induced\ Emf,} E_2 = 4.44\ N_2 f\emptyset_m$

For an ideal transformer,

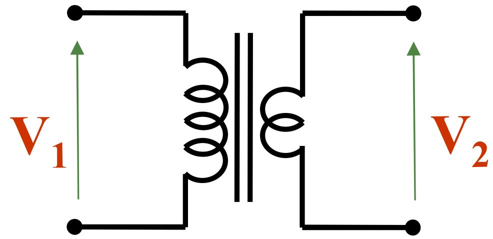
$$\boldsymbol{\frac{V_1}{V_2} \cong \frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a = Turns\ Ratio}$$

$N_2 > N_1$: Step Up



$E_2 > E_1$
$V_2 > V_1$
$I_1 > I_2$

$N_2 < N_1$: Step down



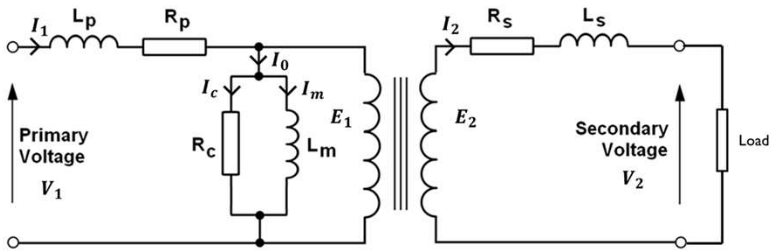
$E_2 < E_1$
$V_2 < V_1$
$I_1 < I_2$

Losses & Efficiency

Total loss = Core loss + copper loss

Core loss (constant)		Copper loss (variable)
Hysteresis loss	Eddy current loss	Winding resistance

- Core loss depends on flux which is constant once core is designed
  - Minimized using high graded core material and lamination
- Copper loss is Current (or load) dependent
- Efficiency: Very high 97% to 99% (since it is a static device)

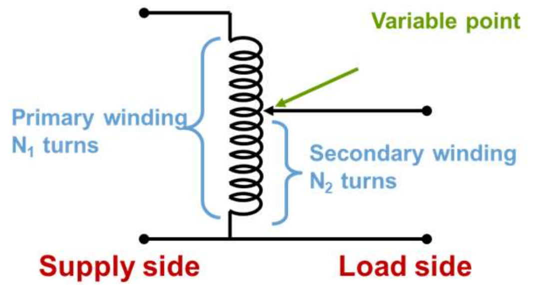


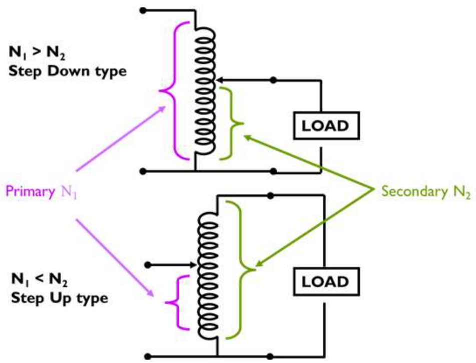
Circuit representation of a practical transformer

Other types

Autotransformer

- One winding transformer
- Secondary winding varied using variable point





Three phase transformer

- Possible connections of primary & secondary windings:
  - star/star
  - star/delta
  - delta/delta
  - delta/star
- 3 single-phase transformers of similar ratings can be connected to form a 3-phase transformer

Power transformer

- Used in electric transmission network

Distribution transformer

- Used in electric distribution networks

Instrument transformers (PT & CT)

- Used for high voltage & current measurement

Isolation transformer

- 1:1 transformers used in circuits to provide electrical isolation.

Constant voltage transformer

- Used as voltage regulators

High frequency transformer

- Transformers designed for operating with high frequency – ferrite core

Problems

Q) An ideal transformer has a turns ratio of 8:1 and the primary current is 3 A when supplied at 240 V. Calculate the secondary voltage and current.

Ans: $\frac{N_1}{N_2} = \frac{8}{1}$

Hence, $V_2 = 30\,\text{V}$

$I_2 = 24\ A$

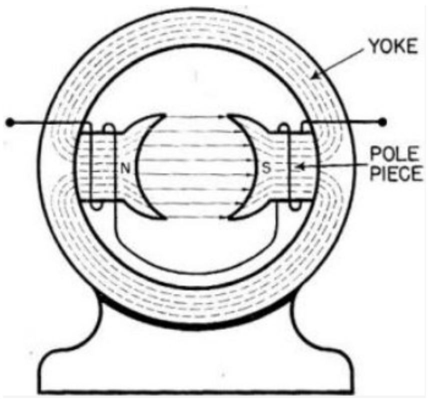
Refer MOOC video for rest of the numerical exercises

DC Motors

Construction

Stator: Houses the field winding *(consists of the yoke, poles, brushes, brush holders, and end covers)*

Rotor: Carries the armature winding *(armature, commutator)*

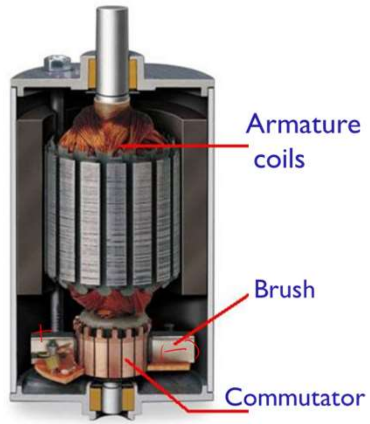


Yoke: Cast steel outer shell housing all the parts

Main poles: Field coils wound when excited with DC produces north and south pole

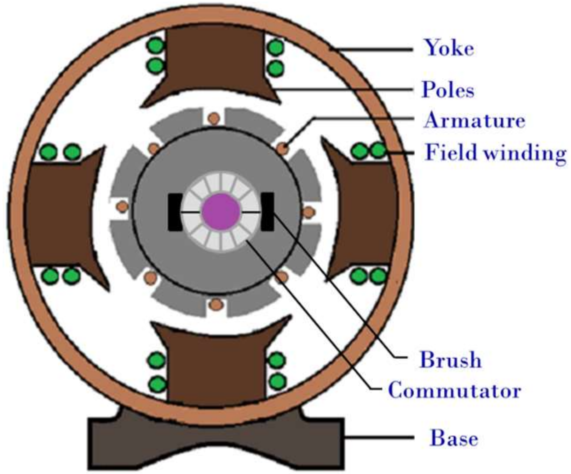
Armature: Rotating part with the armature winding

Commutator: Mechanical rectifier with carbon brushes resting on it



Working principle

- Current carrying armature conductors placed in the magnetic field experience a force that rotates the armature
- Induced emf in armature conductor regulates the current drawn to match with the connected load

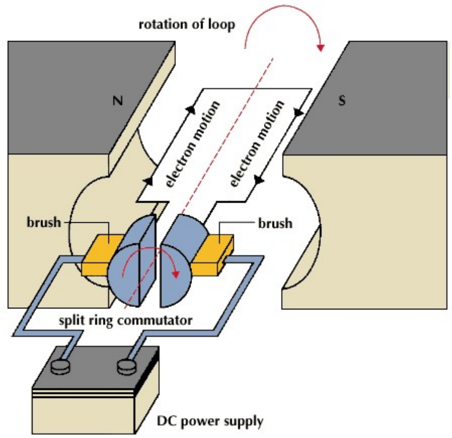


$$V = E_b + I_a R_a$$

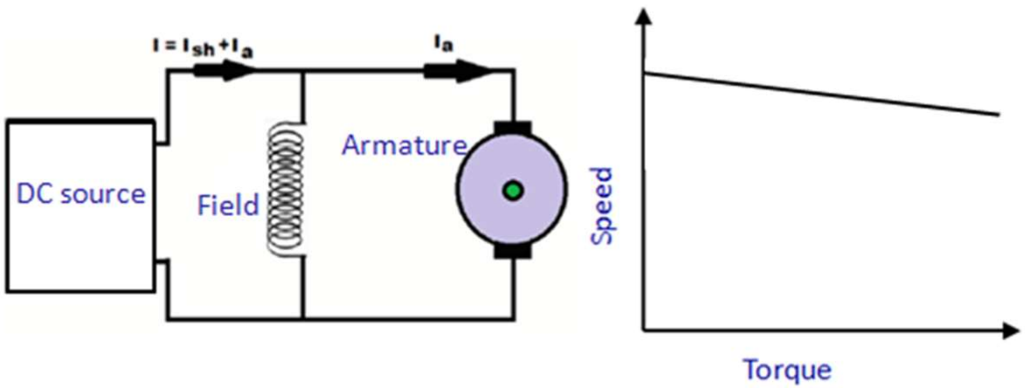
$$N \propto \frac{E_b}{\emptyset}$$

$$T \propto I_a \times \emptyset$$

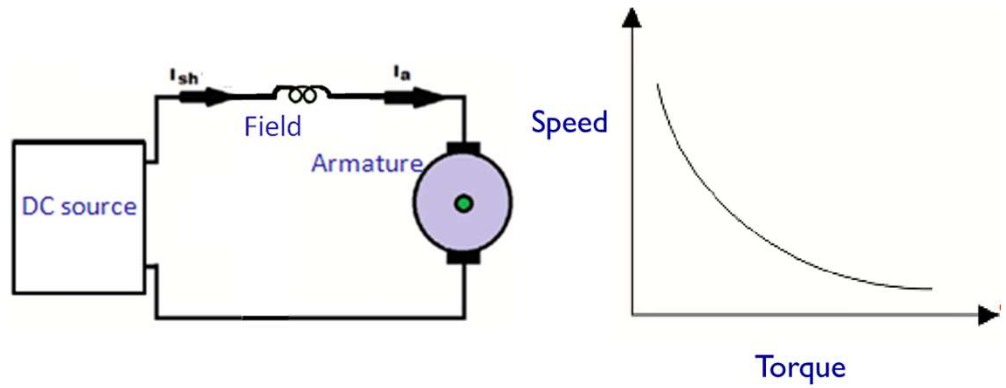
V = Voltage applied(Volts)
Eb = Induced Back e.m.f(Volts)
Ia = Armature current(Amperes)
Ra = Armature resistance(ohms)
N = Speed of the motor(r.p.m)
T = Torque developed(Nm)
Ø = Flux (Webers)



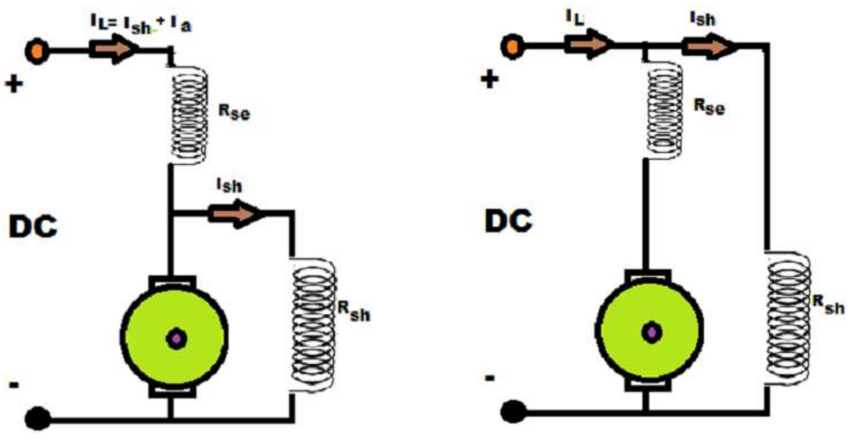
Types of DC motors



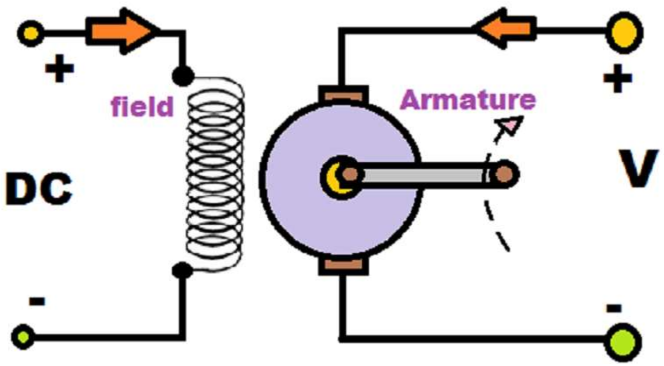
Shunt DC motor



Series DC motor



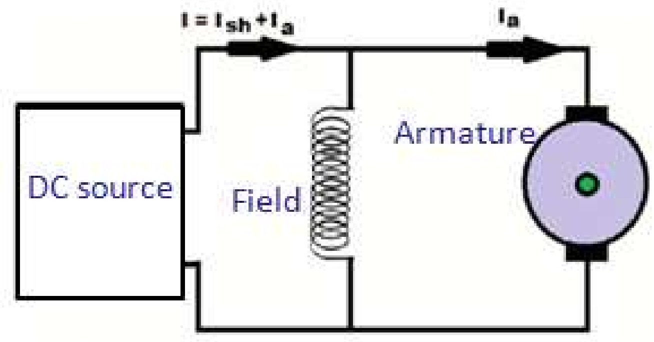
Compound DC motor



Separately excited DC motor

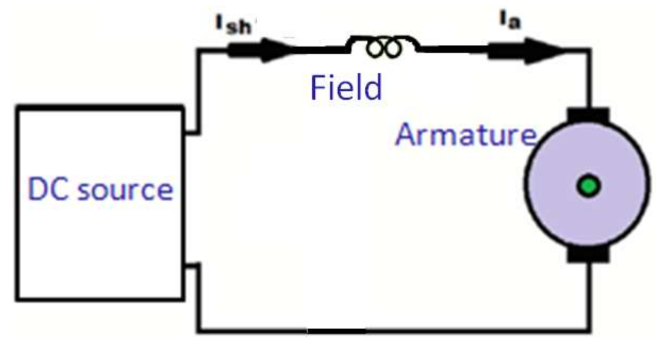
Types of DC motors

DC shunt motor
- Field and armature currents are independent of one another
- Torque proportional to armature current
- Excellent speed control



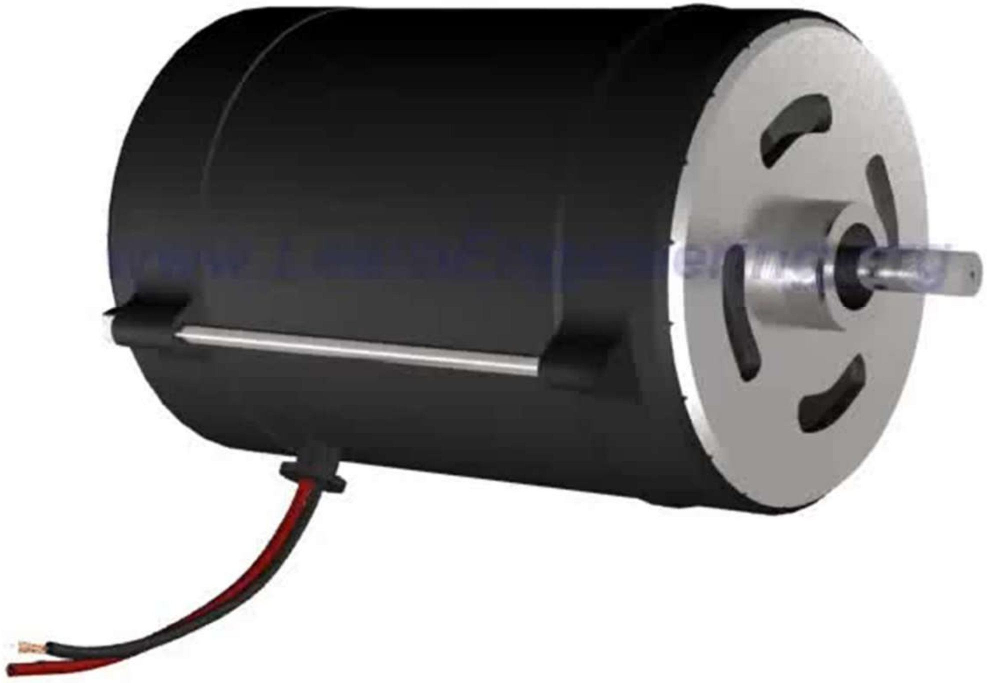
Shunt DC motor

DC series motor
- Field and armature currents are equal
- Torque is proportional to the square of the armature current
- Starting torque is quite high and it gets regulated automatically as speed increases
- Most preferred for traction



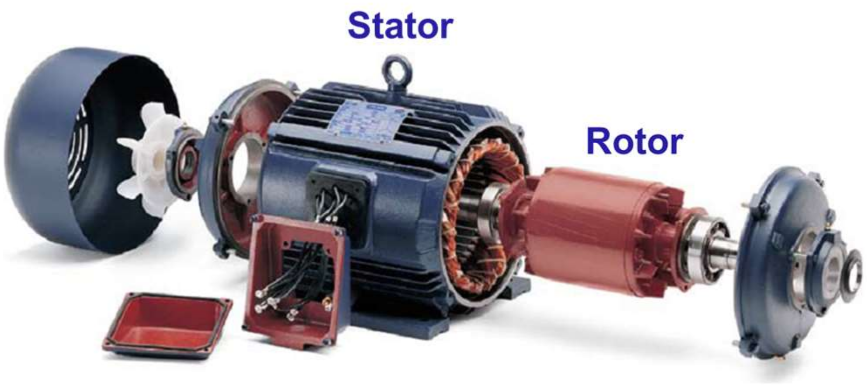
Series DC motor

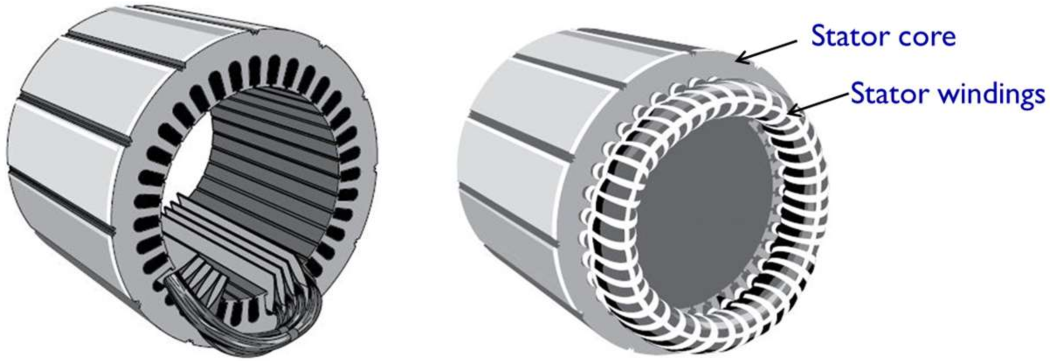
DC compound motor
- Loads that require large momentary torques (e.g. rolling mills)



Three-phase induction motor

Construction

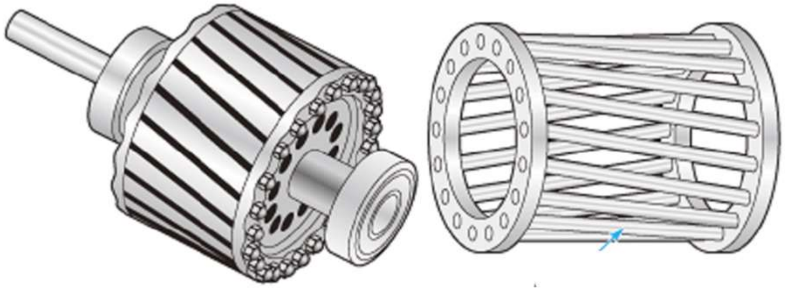




- Stator frame (cast iron) provides mechanical support to the stator core
- Cylindrical stator core laminated and slotted to carry the 3 phase windings
- The balanced windings are displaced in space by 120 degrees electrical
- Slots cut-out on the outer periphery of rotor to place the conductors

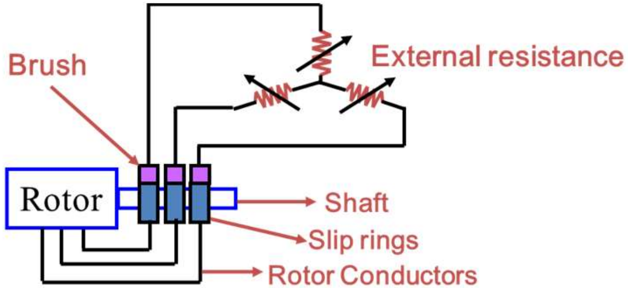
Rotor types

Squirrel cage rotor



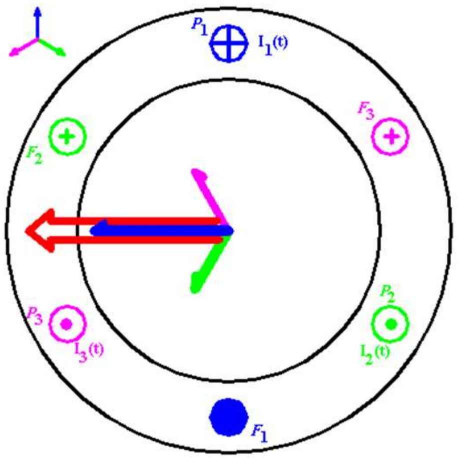
- Skewed arrangement of copper or aluminium bars
- Conductors shorted by end rings – closed rotor circuit
- Cheap, rugged, and needs little or no maintenance

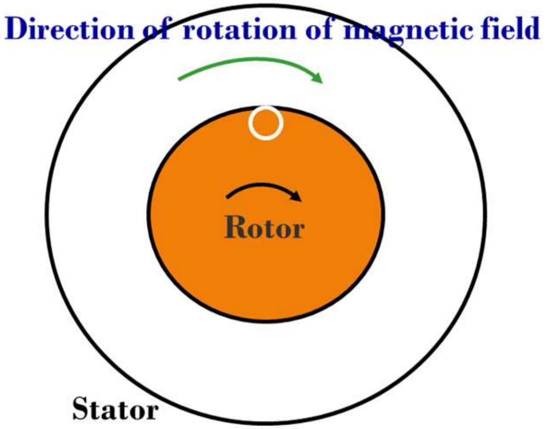
Slip ring rotor



- Rotor winding in star - uniformly distributed
- The terminals of the winding are brought out to three slip rings
- Slip rings in contact with brushes
- Brushes connected to external resistance for higher starting torque

Working principle





- 3-phase currents flowing in the stator winding produce a rotating magnetic field rotating at synchronous speed
- Rotating magnetic field is cut by the rotor conductor
- EMF is induced in rotor conductor
- Current in the rotor conductor sets up a magnetic field which opposes the rotation of the main field
- Main field is independent and hence rotor field tries to catch up the speed of the main field to reduce the relative speed
- Rotor rotates in the same direction as that of rotating magnetic field

Working principle

- Magnetic field rotates at a synchronous speed

$$N_S = \frac{120f}{P}$$

N_S = Speed of Rotating magnetic field, rpm
f = Frequency of ac supply, Hz
P = No. of poles

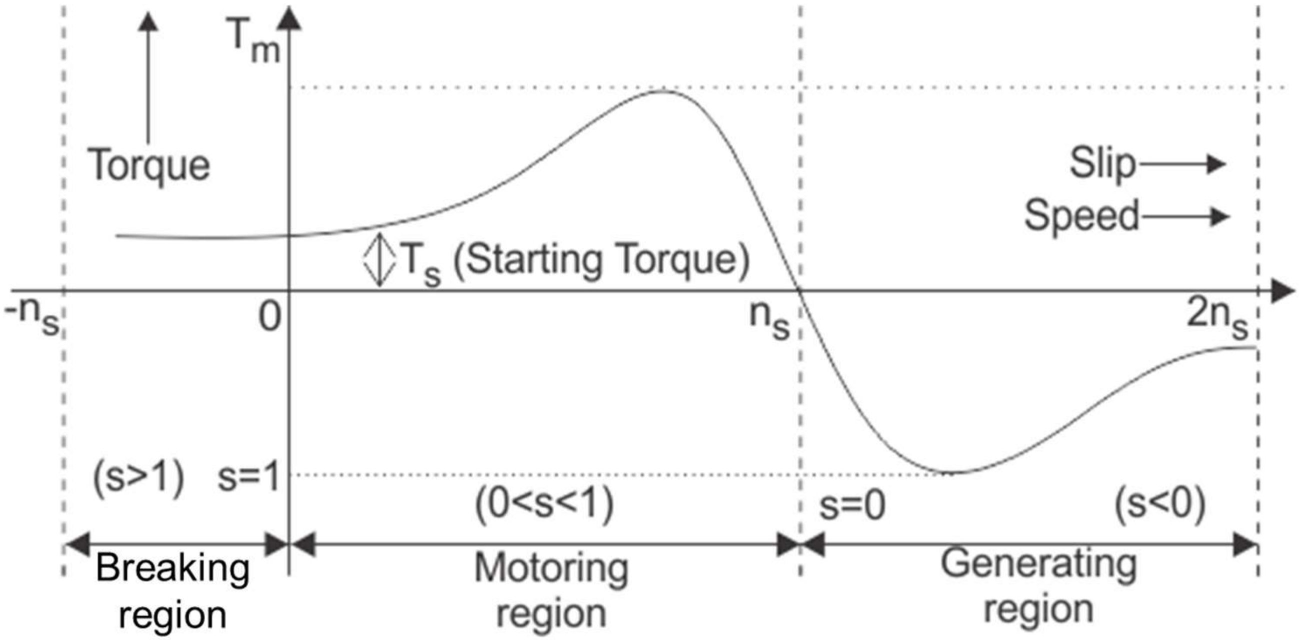
- If rotor speed, $\boldsymbol{N}$ is equal to N_s,
  - No flux cut by rotor conductors
  - No emf induced across rotor conductors
  - No current flow, no torque

- Slip speed, $s = (N_s - N)$, rpm

$$\% \, s = \frac{N_S - N}{N_S} \text{ x } 100 \, \%$$

- Rotor frequency, $f_r = \dfrac{P(N_s - N)}{120} = sf$

Torque – slip characteristics



Applications: Pumping systems, refrigeration systems, compressors, fans & blowers, industrial drives

Special Thanks
Sajith K V

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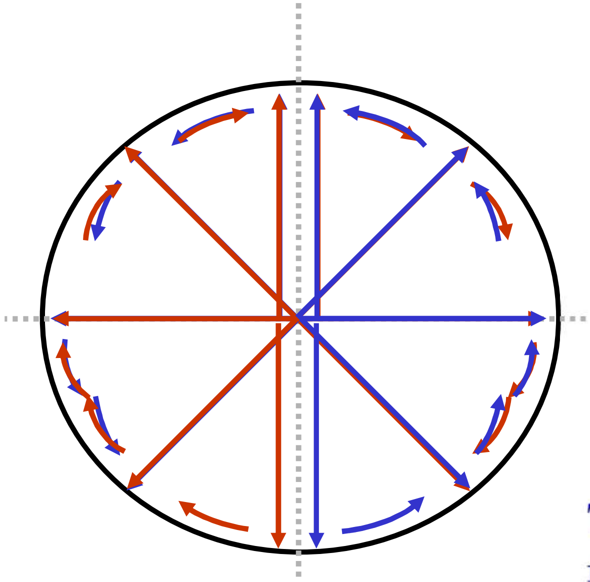
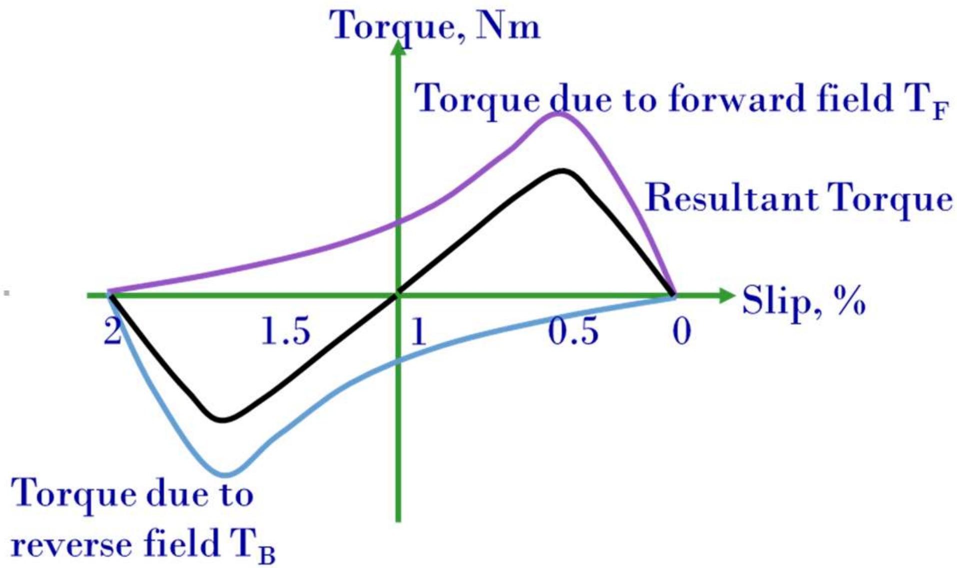
Single-phase induction motor

Double revolving field theory

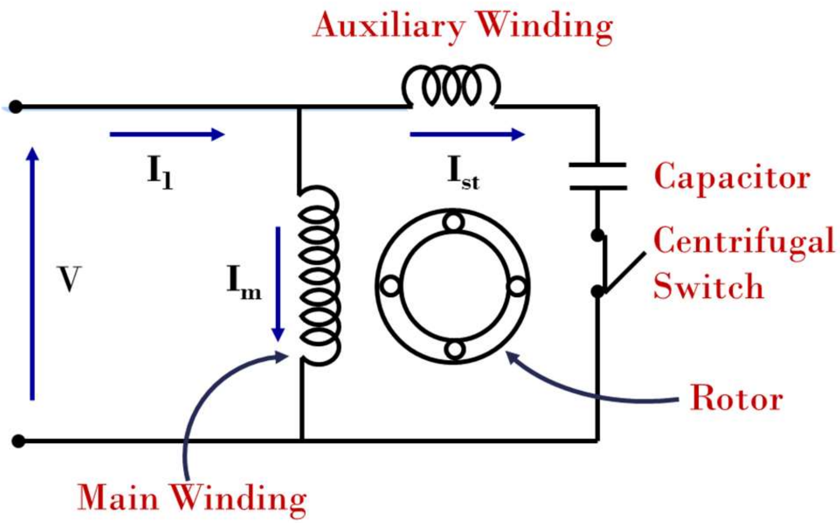


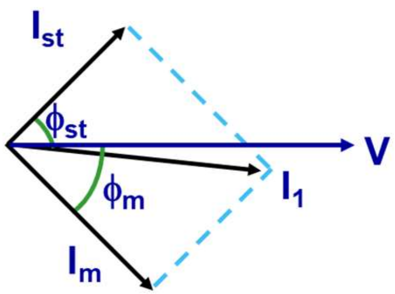


$$\emptyset = \emptyset_m \sin \omega t \cos \alpha = \frac{\emptyset_m}{2} \sin(\omega t + \alpha) + \frac{\emptyset_m}{2} \sin(\omega t - \alpha)$$

Causes T_b Causes T_f

Capacitor start motor





$T_s \propto angle\ between\ I_{st}\ \&\ I_m$

- Auxiliary winding is placed perpendicular to the main winding
- Phase split is achieved by connecting a series capacitor with auxiliary winding
- Centrifugal switch opens the circuit when speed is near about rated speed
- High power factor, high efficiency
- **Application:** High starting torque appliances like compressors, AC, farm tools, lifts, etc.

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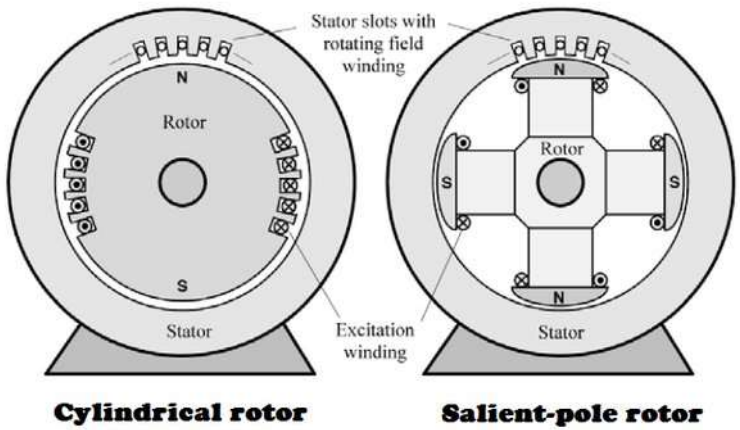
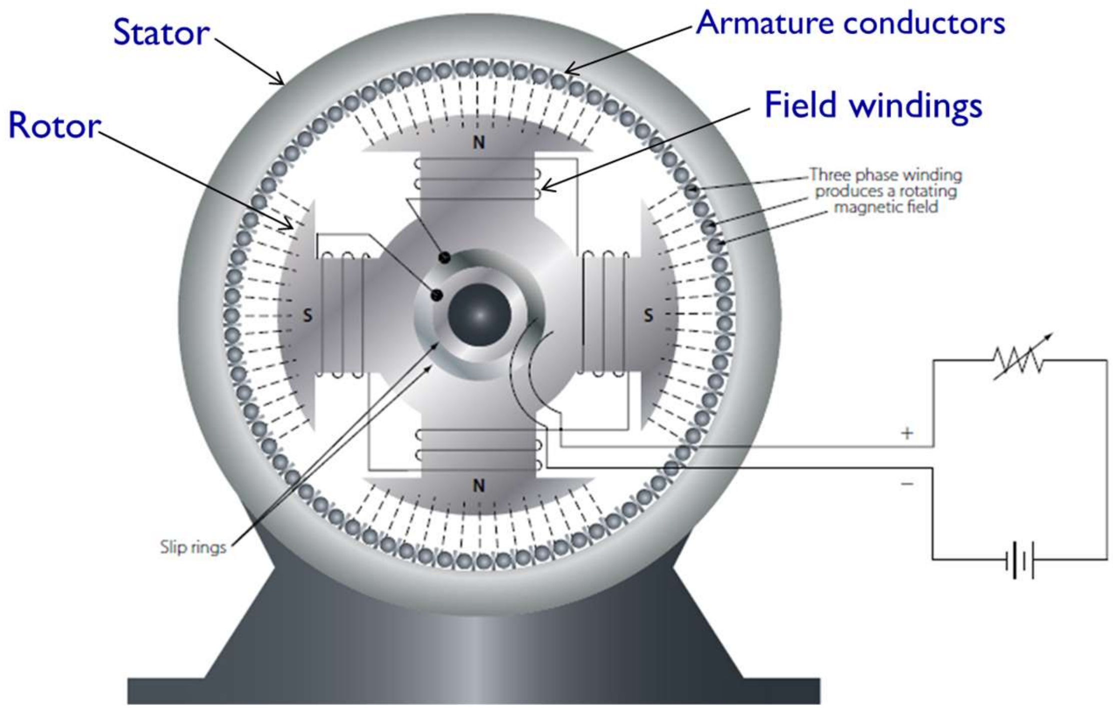
Synchronous motor

Construction

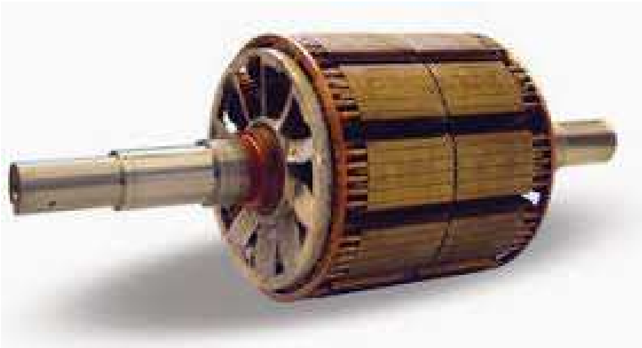
- Stator – accommodates armature windings
- Rotor – carries field windings excited using DC
- Rotor types:
  - Salient-pole: low speed applications
  - Cylindrical: high speed applications



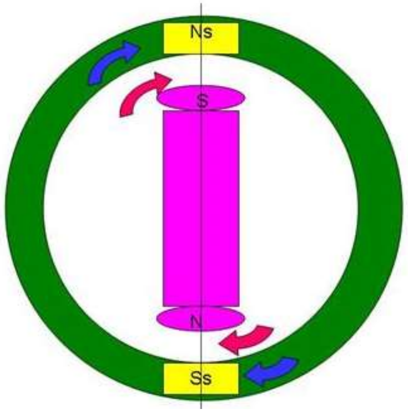


Working principle

- Armature energized from a 3-phase AC source, the machine starts as an induction motor
- After achieving the full speed, the field winding is excited
- Stator and rotor field get magnetically locked



Rotor with damper winding



Magnetic locking near rated speed

Features & applications

- Constant speed AC motor – runs at synchronous speed irrespective of connected load
- Power factor of operation is adjusted by controlling excitation – synchronous condenser
- Used for high power, low-speed applications
  - Lift irrigation
  - Reciprocating pumps
  - Exhaust fans
  - Synchronous condenser

Thank You!