Reg. No.										

MANIPAL UNIVERSITY

FIRST SEMESTER B.E. DEGREE EXAMINATION – NOV/DEC 2007

SUBJECT: ENGINEERING MATHEMATICS – I (MAT 101)

(CREDIT SYSTEM)

Friday, December 21, 2007

Time: 3 Hrs. Max. Marks: 100

Answer any FIVE full questions.

1A. Find the n^{th} derivative of i) $\dfrac{x^2}{(x+2)(2x+3)}$ ii) $\cos x \cos 2x \cos 3x$.

1B. A line makes angles $\alpha, \beta, \gamma, \delta$ with the diagonals of a cube, prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta = \dfrac{4}{3}$

1C. Obtain the reduction formula for $\int \sin^n x\,dx$ and hence obtain the value of $\int_0^{\frac{\pi}{2}} \cos^n x\,dx$

(7+6+7 = 20 marks)

2A. If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$. Hence, find the value of y_n when x = 0.

2B. Trace the curve $r^2 = a^2\cos 2\theta$ with explanations.

2C. Find the image of the point (1, 3, 4) in the plane $2x-y+z+3 = 0$.

(8+6+6 = 20 marks)

3A. Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the latus rectum.

3B. Show that the radius of curvature at any point of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ is equal to three times the length of the perpendicular from the origin to the tangent.

3C. Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z = 3$ as a great circle.

(6+8+6 = 20 marks)

4A. Show that the evolute of the tractrix $x = a\left(\cos t+\log \tan\frac{t}{2}\right)$, $y = a \sin t$ is the catenary $y = a\cosh\left(\frac{x}{a}\right)$.

4B. Find the area included between the cycloid $x = a(\theta-\sin\theta)$, $y = a(1-\cos\theta)$ and its base.

4C. State Euler's theorem and verify it for $z = (x^2+xy+y^2)^{-1}$.

(8+6+6 = 20 marks)

5A. State Cauchy's mean value theorem. Show 'C' of Cauchy's mean value theorem is harmonic mean between a and b if $f(x)=\frac{1}{x^2}$ and $F(x)=\frac{1}{x}$.

5B. Find the nature of the series: i) $\sum_{n=1}^{\infty} \frac{n^{1/2}}{n^2+1}$, ii) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.

5C. Show that the lines $\frac{x+4}{3}=\frac{y+6}{5}=\frac{z-1}{-2}$ and 3x-2y+z+5 = 0 = 2x+3y+4z-4 are coplanar. Find their point of intersection and the plane in which they lie.

(6+6+8 = 20 marks)

6A. Evaluate i) $\underset{x\to 0}{lt}\frac{1+\sin x-\cos x+\log(1-x)}{x\tan^2 x}$ ii) $\underset{x\to \frac{\pi}{2}}{lt}(\sin x)^{\tan x}$

6B. Test for convergence of the series

$$\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\ldots\ldots\ldots\ldots\infty$$

6C. If $u=x\sin^{-1}\left(\frac{y}{x}\right)-y\tan^{-1}\left(\frac{x}{y}\right)$, show that:

i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u$ ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0$

(7+6+7 = 20 marks)

7A. Obtain the first three non-zero terms in the Maclaurin's expansion of $\frac{x}{e^x-1}$.

7B. For what values of x the following series is convergent.

$$\frac{x}{1.2}-\frac{x^2}{2.3}+\frac{x^3}{3.4}-\frac{x^4}{4.5}+\ldots\ldots\infty.$$

7C. Show that the tangents drawn at the extremities of any chord of the cardioide r = a(1+cosθ) which passes through the pole are perpendicular to each other.

(8+6+6 = 20 marks)