# KARNAUGH MAP (K – MAP)

LECTURE 5 & 6

ABCD

$2^4 = 16$ i/P cmbs

# FOUR VARIABLE K – MAP

00 ↔ 01 ↔ 10 ↔ 11

**A. SOP: -**

| AB \ CD | $\bar C\bar D$ 00 | $\bar C D$ 01 | $C D$ 11 | $C\bar D$ 10 |
|---|---|---|---|---|
| $\bar A\bar B$ 00 | $\bar A\bar B\bar C\bar D$ 0 | $\bar A\bar B\bar C D$ 1 | $\bar A\bar B C D$ 3 | $\bar A\bar B C\bar D$ 2 |
| $\bar A B$ 01 | $\bar A B\bar C\bar D$ 4 | $\bar A B\bar C D$ 5 | $\bar A B C D$ 7 | $\bar A B C\bar D$ 6 |
| $A B$ 11 | $A B\bar C\bar D$ 12 | $A B\bar C D$ 13 | $A B C D$ 15 | $A B C\bar D$ 14 |
| $A\bar B$ 10 | $A\bar B\bar C\bar D$ 8 | $A\bar B\bar C D$ 9 | $A\bar B C D$ 11 | $A\bar B C\bar D$ 10 |

1100 → 12

1101 → 13

1110 → 14

1011 → 11

# FOUR VARIABLE K – MAP

$2^4 = 16$

POS: -

| A+B \ C+D | $C+D$ 0 0 | $C+\overline{D}$ 0 1 | $\overline{C}+\overline{D}$ 1 1 | $\overline{C}+D$ 1 0 |
|---|---|---|---|---|
| $A+B$ 0 0 | $A+B+C+D$ 0 | $A+B+C+\overline{D}$ 1 | $A+B+\overline{C}+\overline{D}$ 3 | $A+B+\overline{C}+D$ 2 |
| $A+\overline{B}$ 0 1 | $A+\overline{B}+C+D$ 4 | $A+\overline{B}+C+\overline{D}$ 5 | $A+\overline{B}+\overline{C}+\overline{D}$ 7 | $A+\overline{B}+\overline{C}+D$ 6 |
| $\overline{A}+\overline{B}$ 1 1 | $\overline{A}+\overline{B}+C+D$ 12 | $\overline{A}+\overline{B}+C+\overline{D}$ 13 | $\overline{A}+\overline{B}+\overline{C}+\overline{D}$ 15 | $\overline{A}+\overline{B}+\overline{C}+D$ 14 |
| $\overline{A}+B$ 1 0 | $\overline{A}+B+C+D$ 8 | $\overline{A}+B+C+\overline{D}$ 9 | $\overline{A}+B+\overline{C}+\overline{D}$ 11 | $\overline{A}+B+\overline{C}+D$ 10 |

# EXAMPLE 1:

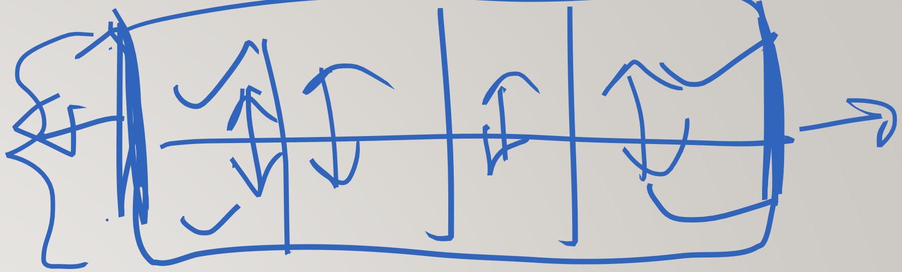
- Simplify the following expression into
  - SOP
  - POS

$$F(A,B,C,D) = \sum(1,5,6,7,8,9,10,14)$$

SOP:

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 (0) | 1 (1) | 0 (3) | 0 (2) |
| 01 | 0 (4) | 1 (5) | 1 (7) | 1 (6) |
| 11 | 0 (12) | 0 (13) | 0 (15) | 1 (14) |
| 10 | 1 (8) | 1 (9) | 0 (11) | 1 (10) |

$\bar{A}\bar{C}D$ $\quad \bar{A}BC \quad BC\bar{D} \quad AC\bar{D} \quad A\bar{B}\bar{C} \quad A\bar{B}\bar{D} \quad \bar{B}\bar{C}D$



$$F = \bar{A}\bar{C}D + \bar{A}BC + AC\bar{D} + A\bar{B}\bar{C}$$

&

$$F = \bar{B}\bar{C}D + BC\bar{D} + A\bar{B}\bar{D} + \bar{A}BD.$$

# CONTINUED…

$$F(A,B,C,D) = \sum(1,5,6,7,8,9,10,14)$$

- POS

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 0 | 0 |
| 01 | 0 | 1 | 1 | 1 |
| 11 | 0 | 0 | 0 | 1 |
| 10 | 1 | 1 | 0 | 1 |

$(A+C+D)$ $(A+B+\bar{C})$ $(A+\bar{B}+C)$ $(\bar{A}+\bar{C}+\bar{D})$

$$F = (A+C+D)(A+B+\bar{C})(\bar{A}+\bar{C}+\bar{D})(\bar{A}+\bar{B}+C)$$

&

$$F = (A+B+D)(\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{D})(B+\bar{C}+\bar{D})$$

# EXAMPLE 2:

$$F(A,B,C,D)=\sum(0,1,2,4,5,6,8,9,10,12,13)$$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 0 | 1 |
| 01 | 1 | 1 | 0 | 1 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 1 | 1 | 0 | 1 |

$\rightarrow \bar{C}$

$\rightarrow \bar{A}\bar{D}$

$\rightarrow \bar{B}\bar{D}$

SOP :- $F = \bar{C} + \bar{A}\bar{D} + \bar{B}\bar{D}$

POS :- $F = (\bar{C} + \bar{D})(\bar{A} + \bar{B} + \bar{C})$

# EXAMPLE 3:

SOP, POS:

$$F(A,B,C,D) = \prod(0,2,3,4,8,9,10,14)$$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 0 | 0 |
| 01 | 0 | 1 | 1 | 1 |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | 0 |

Redundant

SOP:

$$F = \bar{A}\bar{C}D + \bar{A}BC + AB\bar{C} + ACD$$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 0 | 0 |
| 01 | 0 | 1 | 1 | |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | 0 |

Redundant

POS:

$$F = (A+C+D)(A+B+\bar{C})(\bar{A}+\bar{C}+D)(\bar{A}+B+C)$$

# EXAMPLE 4:

$$F(A,B,C,D) = \overline{C}(\overline{A}\,\overline{B}\,\overline{D} + D) + A\overline{B}C + \overline{D}$$

$= \underbrace{\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}} + \underbrace{\overline{C}\,\overline{D}} + A\overline{B}C + \overline{D}$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| $\overline{A}\,\overline{B}$ 00 | 1 | 1 | 0 | 1 |
| $\overline{A}B$ 01 | 1 | 1 | 0 | 1 |
| $AB$ 11 | 1 | 1 | 0 | 1 |
| $A\overline{B}$ 10 | 1 | 1 | 1 | 1 |

$\rightarrow A\overline{B}$

SOP: $F = \overline{C} + \overline{D} + A\overline{B}$

POS: $F = (A + \overline{C} + \overline{D})(\overline{B} + \overline{C} + \overline{D})$

# EXAMPLE 5:

$$F(A,B,C,D) = D(\overline{A}+B) + \overline{B}(C+AD)$$

$$= \overline{A}D + BD + \overline{B}C + A\overline{B}D$$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 1 | 1 |
| 01 | 0 | 1 | 1 | 0 |
| 11 | 0 | 1 | 1 | 0 |
| 10 | 0 | 1 | 1 | 1 |

SOP:

$$F = D + \overline{B}C$$

POS:

$$F = (C+D)(\overline{B}+D)$$

# DON'T CARE CONDITION

- The "Don't Care" conditions indicate the input combinations which are invalid for a particular circuit.
- While forming groups of cells, we can consider a "Don't Care" cell as either 1 or 0 or we can simply ignore that cell.
- Therefore, "Don't Care" condition are used to form a larger group of cells.

$2^4 = 16$ → combination

0 to 9 → 0000 to 1001

1010
1011
1100
1101
1110
1111

BCD no

X

d

# EXAMPLE 1:

$$F(A,B,C)=\sum_m(1,3,5,7)+\sum_d(0,2)$$

→ min

→ don't care

$\Sigma m(1,3,5,7) + \Sigma d(0,2)$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | X | 1 | 1 | X |
| 1 | 0 | 1 | 1 | 0 |

→ C

C

SOP :-

F = C

POS :-

F = C

# EXAMPLE 2:

$$F(W, X, Y, Z) = \sum_m (1,3,7,11,15) + \sum_d (0,2,5)$$

| WX \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | X | 1 | 1 | X |
| 01 | 0 | X | 1 | 0 |
| 11 | 0 | 0 | 1 | 0 |
| 10 | 0 | 0 | 1 | 0 |

$\bar{W}Z$

$\bar{W}\bar{X}$

$YZ$

SOP:

$F = \bar{W}Z + YZ$

$F = \bar{W}\bar{X} + YZ$ ?

POS:

$F = Z(\bar{W} + Y)$

# EXAMPLE 3:

$$F(W,X,Y,Z)=\prod_{M}(0,1,3,5,8,9,14).\prod_{D}(2,6,10)$$

| wx \ yz | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | X |
| 01 | 1 | 0 | 1 | X |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | X |

$w x \bar{y}$

SOP :- $F = x\bar{y}\bar{z} + wxz + xyz + wyz$

| wx \ yz | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | X |
| 01 | 1 | 0 | 1 | X |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | X |

3-Variab

2-Varia

4-Variable

wxyz

POS :- $F = (x+y)\,(w+x)\,(\bar{y}+z)\,(w+y+\bar{z})$

# EXAMPLE 4:

$$F(W, X, Y, Z) = \sum_m (1,4,5,7,9,10,11) + D(14,15)$$

F =

F =

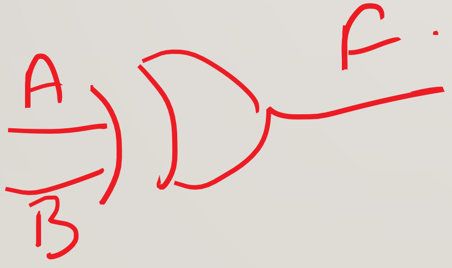
# XOR and XNOR gates:

| A | B | F |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

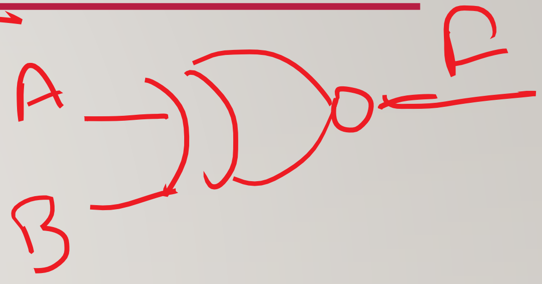
$A \oplus B \oplus C \oplus D$

$(A \oplus B \oplus C \oplus D)$

$$F = A\bar{B} + \bar{A}B$$
$$= A \oplus B$$




| AB | F |
|---|---|
| 00 | 1 |
| 01 | 0 |
| 10 | 0 |
| 11 | 1 |

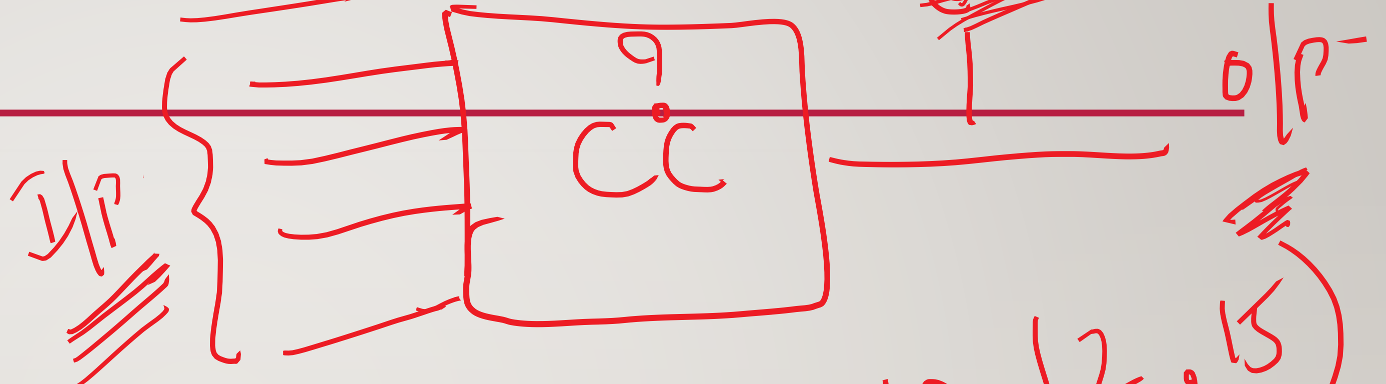



$$F = \bar{A}\bar{B} + AB$$
$$= \overline{(A\bar{B} + \bar{A}B)}$$
$$= A \odot B$$
$$= \overline{A \oplus B}$$

# EXAMPLE 5:

Design a combinational circuit to check for even parity of 4 bits. A logic '1' output is required when the 4 bits constitute an even parity.

even → even no.
odd → odd no.




| | A | B | C | D | F |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 |
| 3 | 0 | 0 | 1 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 1 |
| 7 | 0 | 1 | 1 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 |
| 9 | 1 | 0 | 0 | 1 | 1 |
| 10 | 1 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 |
| 13 | 1 | 1 | 0 | 1 | 0 |
| 14 | 1 | 1 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 1 |

$F = \Sigma(0, 3, 5, 6, 9, 10, 12, 15)$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 1 | 0 |
| 01 | 0 | 1 | 0 | 1 |
| 11 | 1 | 0 | 1 | 0 |
| 10 | 0 | 1 | 0 | 1 |

$F = \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$

$$F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D +$$

$$\bar{A}BC\bar{D} + AB\bar{C}\bar{D} + ABCD$$

$$+ A\bar{B}\bar{C}D + A\bar{B}C\bar{D}$$

$$F = \bar{A}\bar{B}\left[\bar{C}\bar{D} + CD\right] + \bar{A}B\left[\bar{C}D + C\bar{D}\right] + AB\left[\bar{C}\bar{D} + CD\right] + A\bar{B}\left[\bar{C}D + C\bar{D}\right]$$

$$= \bar{A}\bar{B}\left[\overline{C \oplus D}\right] + \bar{A}B\left[C \oplus D\right] + AB\left[\overline{C \oplus D}\right] + A\bar{B}\left[C \oplus D\right]$$

$$F = \overline{[C \oplus D]}\,[\bar{A}\bar{B} + AB] + [C \oplus D][\bar{A}B + A\bar{B}]$$

$$= \overline{[C \oplus D]}\,\overline{[A \oplus B]} + [C \oplus B][A \oplus B]$$

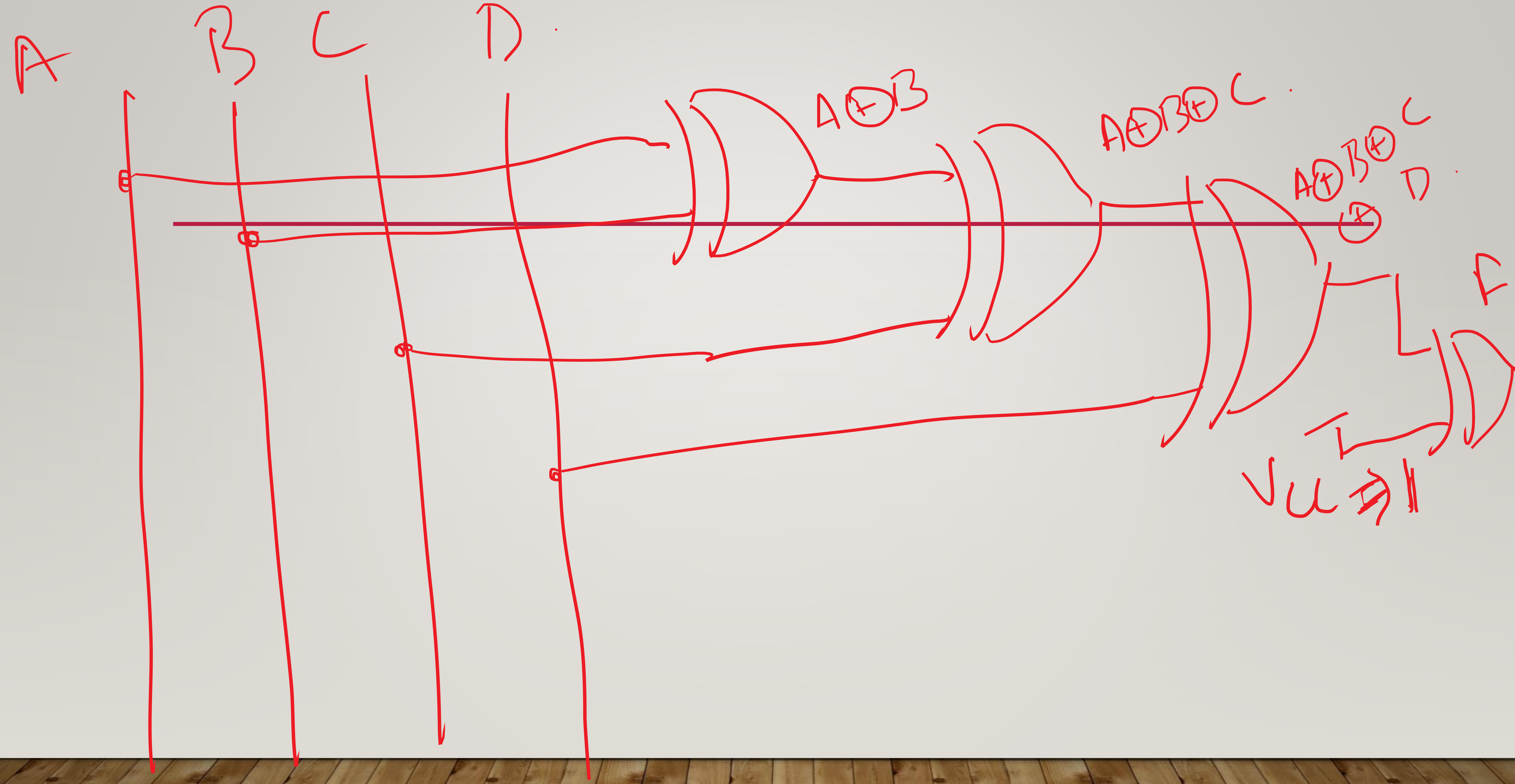
$Y = C \oplus D$

$X = A \oplus B$

$$\Rightarrow F = \bar{X}\bar{Y} + XY$$

$$= \overline{[X \oplus Y]}$$

$$= \overline{[A \oplus B \oplus C \oplus D]}$$


A
B
C
D
A⊕B
A⊕B⊕C
A⊕B⊕C
⊕ D
F

# EXAMPLE 6:

Design a combinational circuit with 4- input lines that represents a decimal digit in BCD and 4- output lines that generates 2’s complement of input digit.

| $B_3$ | $B_2$ | $B_1$ | $B_0$ | $F_3$ | $F_2$ | $F_1$ | $F_0$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | |
| 0 | 0 | 0 | 1 | | | | |
| 0 | 0 | 1 | 0 | | | | |
| 0 | 0 | 1 | 1 | | | | |
| 0 | 1 | 0 | 0 | | | | |
| 0 | 1 | 0 | 1 | | | | |
| 0 | 1 | 1 | 0 | | | | |
| 0 | 1 | 1 | 1 | | | | |
| 1 | 0 | 0 | 0 | | | | |
| 1 | 0 | 0 | 1 | | | | |
| 1 | 0 | 1 | 0 | | | | |
| 1 | 0 | 1 | 1 | | | | |
| 1 | 1 | 0 | 0 | | | | |
| 1 | 1 | 0 | 1 | | | | |
| 1 | 1 | 1 | 0 | | | | |
| 1 | 1 | 1 | 1 | | | | |

# EXAMPLE 7:

Design a combinational circuit that multiplies by '5' an input decimal digit represented in BCD. The output is also in BCD.