Sol^n is $\int\left(\frac{1}{y}-\frac{2}{x}\right)dx + \int \frac{3}{y}\,dy = c \Rightarrow \frac{1}{y}x - 2\log x + 3\log y = c$

4/11. **Reducible to exact D.E**

Consider $M\,dx + N\,dy = 0$ — ①.

If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ ① is not exact

* $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$, a fn of x alone

  then $I \cdot F = e^{\int f(x)\,dx}$

* $\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = g(y)$, a fn of 'y' alone.

  Then $I \cdot F = e^{-\int g(y)\,dy}$.

Q. Solve $(xy^2 - e^{1/x^3})\,dx - x^2y\,dy = 0$ — ①

Ans. In ① $M = xy^2 - e^{1/x^3}$, $N = -x^2y$

$\frac{\partial M}{\partial y} = 2xy$ $\quad \frac{\partial N}{\partial x} = -2xy$.

ie $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ ① is not exact.

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 4xy$$

$$\therefore \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{-x^2y} \times 4xy = \frac{-4}{x} = f(x)$$

$$I.F = e^{\int -\frac{4}{x}dx} = e^{-4\log x} = \frac{1}{x^4}.$$

$$\therefore ① \Rightarrow \left(\frac{y^2}{x^3} - \frac{e^{-1/x^3}}{x^4}\right)dx - \frac{4}{x^2}\,dy = 0 \quad —②$$

In ②, $M = \frac{y^2}{x^3} - \frac{e^{1/x^3}}{x^4}$, $N = -y/x^2$

$\therefore$ Solⁿ is,

$$y^2\int x^{-3}dx - \int \frac{e^{1/x^3}}{x^4}\,dx = e$$

$$\Rightarrow y^2\left(\frac{-1}{2x^2}\right) + 3\int e^t dt = c$$

put $1/x^3 = t$

$\Rightarrow \frac{dt}{dx} = \frac{-3}{x^4}$

$$\Rightarrow \frac{y^2}{2x^2} + \frac{e^{1/x^3}}{3} = c.$$

$\Rightarrow \frac{dx}{x^4} = \frac{-dt}{3}$

Q. Solve $(xy^3+y)dx + 2(x^2y^2+x+y^4)\,dy = 0$ —①.

Ans. In ① $M = xy^3+y$; $N = 2x^2y^2 + 2x + 2y^4$

$\frac{\partial M}{\partial y} = 3xy^2+1$, $\frac{\partial N}{\partial x} = 4xy^2+2$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ (1) is not exact

$\therefore \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 3xy^2+1 - 4xy^2-2$
$= -1 - xy^2 = -(1+xy^2)$

$\therefore \frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{y(xy^2+1)} - (1+xy^2) = -\frac{1}{y} = g(y)$

$\therefore$ I.F $= e^{-\int g(y)dy} = e^{\int 1/y\,dy} = e^{\log y} = y$.

(1) $\Rightarrow (xy^4+y^2)dx + 2(x^2y^3+xy+y^5)dy = 0$. — (2).

In (2), $M = xy^4+y^2$; $N = 2x^2y^3+2xy+2y^5$.

$\therefore$ Sol^n is $\int (xy^4+y^2)dx + \int 2y^5 dy = C$.

y as const

after integration,

$\frac{x^2y^4}{2} + xy^2 + \frac{y^6}{3} = C$

Q. Solve $(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0$ — (1).

Ans. Here $M = y^4+2y$; $N = xy^3+2y^4-4x$

$\Rightarrow \frac{\partial M}{\partial y} = 4y^3+2$; $\frac{\partial N}{\partial x} = y^3-4$.

ie $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ (1) is not exact

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 3(y^3+2)$.

$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{y(y^3+2)} 3(y^3+2) = 3/y = g(y)$

$\therefore$ I.F $= e^{-\int 3/y\,dy} = \frac{1}{y^3}$.

$\therefore$ ① $\Rightarrow \left(y + \frac{2}{y^2}\right)dx + \left(x + 2y - \frac{4x}{y^3}\right)dy = 0$ — ②.

$$\left(y + \frac{2}{y^2}\right)x + y^2 = c$$

Q. Solve $(6x^2 + 4y^3 + 12y)\,dx + 3x(1+y^2)\,dy = 0$.

Ans. $\frac{\partial M}{\partial y} = 12y^2 + 12$, $\frac{\partial N}{\partial x} = 3 + 3y^2$

$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{3}{x} = f(x)$.

$$I.F = e^{\int f(x)\,dx} = x^3$$

① $\Rightarrow (6x^5 + 4x^3y^3 + 12x^3y)\,dx + 3x^4(1+y^2)\,dy = 0$. — ②

Sol^n

$$\int_{y \text{ as const}} (6x^5 + 4x^3y^3 + 12x^3y)\,dx + \int 0\,dy = c$$

$$x^6 + x^4y^3 + 3x^4y = c$$

Consider $Mdx + Ndy = 0$, is not exact —①.

① can be written as.

$$\underbrace{y\, f(xy)}_{M}dx + \underbrace{x\, g(xy)}_{N}dy = 0 \quad —②.$$

If $Mx - Ny \neq 0$ then $I.F = \dfrac{1}{Mx - Ny}$.

Q. Solve $(xy^2+y)dx - (x^2y - x)dy = 0$ —①.

In ①, $M = xy^2 + y$ $\quad N = -x^2y + x$.

$\dfrac{\partial M}{\partial y} = 2xy + 1 \quad \dfrac{\partial N}{\partial x} = -2xy + 1$

ie $\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x} \Rightarrow$ ① is not exact

①: $y(xy+1)dx - x(xy-1)dy = 0$ —②.

In ②, $M = y(xy+1) \quad N = -x(xy-1)$

$\therefore Mx - Ny = 2x^2y^2 \neq 0$.

$\therefore I.F = \dfrac{1}{Mx - Ny} = \dfrac{1}{2x^2y^2}$.

$$② \Rightarrow \underbrace{\left(\frac{xy+1}{2x^2y}\right)}_{M}dx + \underbrace{\frac{(1-xy)}{2xy^2}}_{N}dy = 0. \quad —③$$

sol^n:

$$\int_{y \text{ as const}} \left(\frac{1}{2x} + \frac{1}{2x^2y}\right)dx + \int -\frac{1}{2y}\,dy = c.$$

$$\int \frac{1}{2x}dx + \frac{1}{2y}\int \frac{1}{x^2} - \frac{1}{2}\int \frac{1}{y}dy = c.$$

$$\log\left(\frac{x}{y}\right) - \frac{1}{xy} = c'$$

Q Solve $(\overset{M}{xy^2\sin(xy) + y\cos(xy)})dx + (\underset{N}{x^2y\sin(xy) - x\cos(xy)})dy = 0$ — ①.

Ans. $\frac{\partial M}{\partial y} = x\frac{\partial}{\partial y}(y^2\sin(xy)) + \frac{\partial}{\partial y}(y\cos(xy))$

$\frac{\partial N}{\partial x} = y\frac{\partial}{\partial x}(x^2\sin(xy)) - \frac{\partial}{\partial x}(x\cos(xy))$.

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ ① is not exact.

① $\Rightarrow y(xy\sin(xy) + \cos(xy))dx + x(xy\sin(xy) - \cos(xy))dy = 0$.

$Mx - Ny = 2xy\cos(xy)$.

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{2xy\cos(xy)}$$

① $\Rightarrow \left(\underset{M}{\frac{y}{2}\tan(xy) + \frac{1}{2x}}\right)dx + \left(\overset{N}{\frac{x}{2}\tan(xy) - \frac{1}{2y}}\right)dy = 0$ — ②

$\therefore$ Sol^n is,

$$\int\left(\frac{y}{2}\tan(xy) + \frac{1}{2x}\right)dx + \int -\frac{1}{2y}\,dy = C$$

y as constant

$$\Rightarrow \frac{y}{2}\int\tan(xy)\,dx + \frac{1}{2}\int\frac{1}{x}\,dx - \frac{1}{2}\int\frac{dy}{y} = C.$$

$$\frac{y}{2}\cdot\frac{1}{y}\log\sec(xy) + \frac{1}{2}\log x - \frac{1}{2}\log y = c.$$

$$\Rightarrow \log(\sec(xy)) + \log(x/y) = \log K.$$

$\rightarrow) \log\left(\sec(xy) \cdot \frac{x}{y} \right) = \log K$

$\left(\frac{x}{y}\right) \sec(xy) = K$

Q. Solve $(xy^2 + 2x^2y^3)\,dx + (x^2y - x^3y^2)\,dy = 0$

Ans. $Mx - Ny = 3x^3y^3$

$I.F = \frac{1}{3x^3y^3}$

Sol^n is, $\log\left(\frac{x^2}{y}\right) - \frac{1}{xy} = K$

Consider $Mdx + Ndy = 0$ — ①

$x^{k_1} y^{k_2} (c_1 y\,dx + c_2 x\,dy) + x^{k_3} y^{k_4} (c_3 y\,dx + c_4 x\,dy) = 0$

then $I.F = x^a y^b$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Q. Solve $x(3y\,dx + 2x\,dy) + 8y^4(y\,dx + 3x\,dy) = 0$ — ①

① becomes, $(3xy + 8y^5)\,dx + (2x^2 + 24xy^4)\,dy = 0$

Multiply ② by $x^a y^b$ we get,

$(3x^{a+1}y^{b+1} + 8x^a y^{b+5})\,dx + (2x^{a+2}y^b + 24x^{a+1}y^{b+4})\,dy = 0$ — ③

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Here, $M = 3x^{a+1}y^{b+1} + 8x^a y^{b+5}$
$N = 2x^{a+2}y^b + 24x^{a+1}y^{b+4}$

Since ③ is exact, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

$\Rightarrow 3x^{a+1}(b+1)y^b + 8x^a(b+5)y^{b+4}$
$= 2(a+2)x^{a+1}y^b + 24(a+1)x^a y^{b+4}$.

Compare,

$3(b+1) = 2a+2$

$8(b+5) = 24(a+1)$

$\therefore a=1, b=1$

$\therefore M = 3x^2y^2 + 8xy^6$
$N = 2x^3y + 24x^2y^5$.

Sol^n is, $\int_{y \text{ as const}} (3x^2y^2 + 8xy^6)dx + \int 0\,dy = C$.

$\Rightarrow x^3y^2 + 4x^2y^6 = C$

Q. Solve. $x(4y\,dx + 2x\,dy) + y^3(3y\,dx + 5x\,dy) = 0$.

Ans. $(4xy + 3y^4)dx + (2x^2 + 5xy^3)dy = 0$. —①

Multiply both sides of ① by $x^a y^b$.
we get.

$(\underbrace{4x^{a+1}y^{b+1} + 3x^a y^{b+4}}_{M})dx + (\underbrace{2x^{a+2}y^b + 5x^{a+1}y^{b+3}}_{N})dy = 0$. —②

Since ② is exact, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

$a=2, b=1$.

$M = 4x^3y^2 + 3x^2y^5$

$N = 2x^4y + 5x^3y^4.$

Soln, $x^4y^2 + x^3y^5 = c$

Matrices

The matrix representation of an n-dimensional vector

$$\vec{X} = a_1 + a_2 \cdots a_n = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = (a_1\ a_2 \cdots a_n)$$

MATRIX SYSTEM OF LINEAR EQUATIONS

→ 'm' equations (linear)

→ 'n' unknowns

$x_1, x_2, \ldots x_n$

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n &= b_3 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}\right\} \circledast$$

The matrix eq^n of $\circledast$ is, $AX = B$.

where $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m\times n}$ $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n\times 1}$

$B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}_{m\times 1}$

→ Consider $AX = B$

Augmented Matrix :- $\left(A \mid B\right)$

$2x_1 + 3x_2 = 6$

$3x_1 + x_2 = 5$.

$AX = B$ where $A = \begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$ $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ $B = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$

∴ Aug matrix $[A:B]$ or $[A/B]$ or $(A|B)$

Here $[A:B] = \begin{pmatrix} 2 & 3 & : & 6 \\ 3 & 1 & : & 5 \end{pmatrix}$

$AX = 0$ $\quad \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{n\times 1}$; ~~trivial~~ sol^n of the homogenous

→ $AX = 0$ $\quad y_1, y_2 \cdots\cdots y_n$

$c_1 y_1 + c_2 y_2 + \cdots + c_n y_n$