Shortest Paths in graphs: Dijkstra's algorithm

This algorithm is used to find the shortest path between the vertices when each edge is associated with a distance.

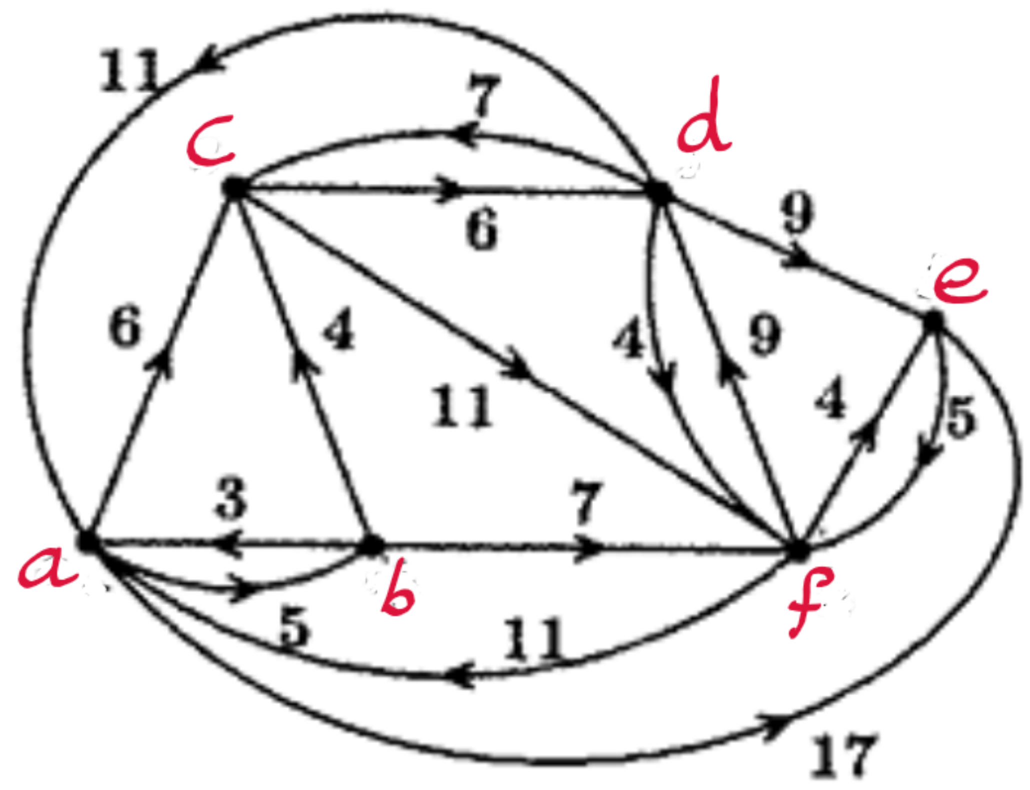
Step 1: We consider **two sets** of vertices K {vertex *r*}, U {all the other vertices except *r*}. For all vertices except *r*, set best $d(i) = d_{ri}$ and tree$(i) = r$.

Step 2: Find the vertex *s* in U which has the minimum value of best *d*. Remove *s* from U and put it in K.

Step 3: For each vertex *u* in U find best $d(s) + d_{su}$ and if it is less than best $d(u)$ replace best $d(u)$ by this new value and let tree$(u) = s$. (In other words a shortest path to *u* has been found by going via vertex *s*.)

Step 4: If U contains only one vertex stop else go back to step 2.

Using Dijkstra's alogorithm, find shortest path from 'c' to all other vertices for the following network.



$$D = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 5 & 6 & \infty & 17 & \infty \\ 3 & 0 & 4 & \infty & \infty & 7 \\ \infty & \infty & 0 & 6 & \infty & 11 \\ 11 & \infty & 7 & 0 & 9 & 4 \\ \infty & \infty & \infty & \infty & 0 & 5 \\ 11 & \infty & \infty & 9 & 4 & 0 \end{bmatrix} \end{array}$$

Step(1): Consider $K = \{c\}$, $U = \{a, b, d, e, f\}$

	a	b	d	e	f
best d	∞	∞	6	∞	11
tree	c	c	c	c	c

Step(2): Minimum best d is 6, remove vertex 'd' from U and put it in K

$\therefore K = \{c, d\}$, $U = \{a, b, e, f\}$

Distance from c to a via 'd' $= 6 + 11 = 17 < \infty$

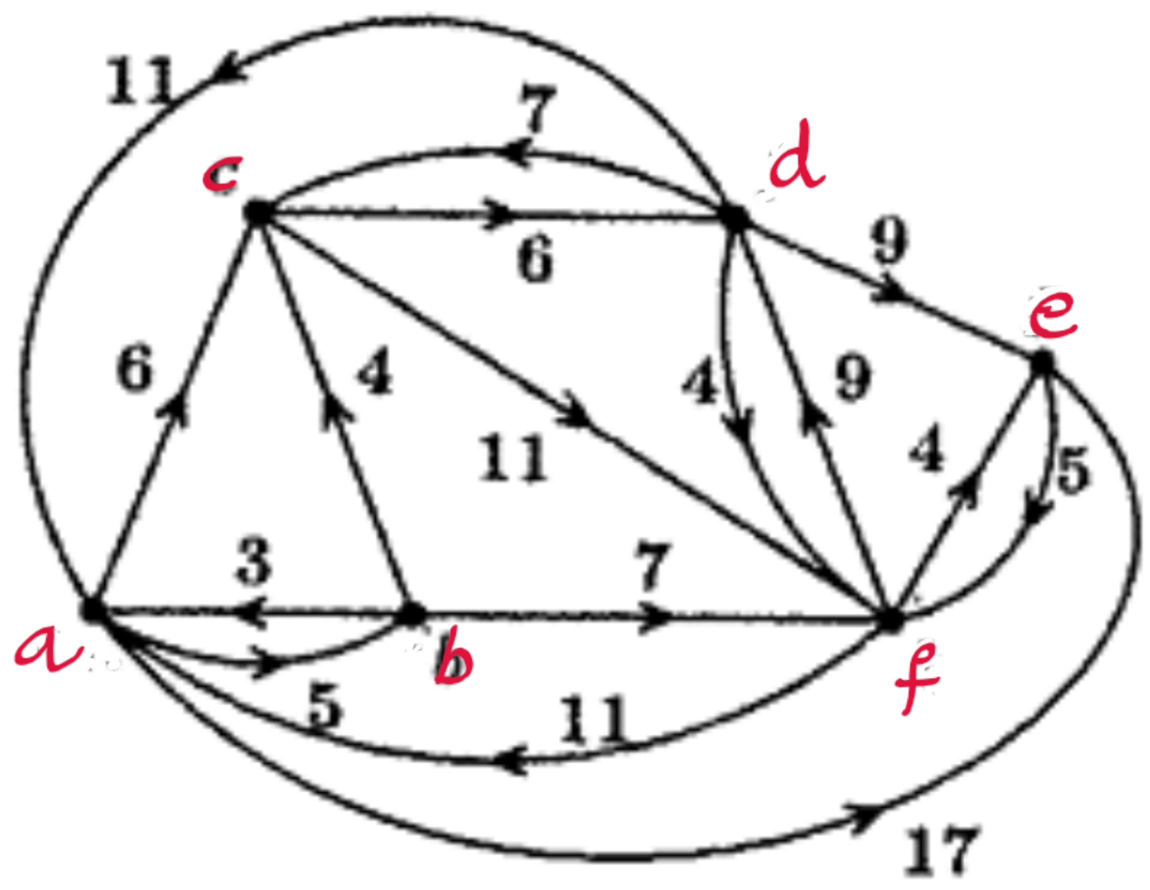
Distance from c to b via 'd' $= 6 + \infty = \infty$

Distance from c to e via d $= 6 + 9 = 15 < \infty$

Distance from c to f via d $= 6 + 4 = 10 < 11$

	a	b	e	f
best d	17	∞	15	10
tree	d	c	d	d

$K = \{c, d, \underline{\underline{f}}\}$, $U = \{a, b, e\}$



Distance from c to a via f $= 10 + 11 = 21 > 17$

Distance from c to b via f $= 10 + \infty = \infty$

Distance from c to e via f $= 10 + 4 = 14 < 15$

	a	b	e
best d	17	∞	14
tree	d	c	$\underline{\underline{f}}$

Minimum best d is 14, remove $\underline{\underline{e}}$ from U & put it in K

$K = \{c, d, f, \underline{\underline{e}}\}$, $U = \{a, b\}$

Distance from c to a via e $= 14 + \infty = \infty > 17$

Distance from c to b via e $= 14 + \infty = \infty$

	a	b
best d	17	∞
tree	$\underline{\underline{d}}$	c

Minimum distance d is 17, remove 'a' from U & put it in K

$K = \{c, d, f, e, \underline{\underline{a}}\}$, $U = \{b\}$

Distance from c to b via 'a' $= 17 + 5 = \underline{\underline{22}} < \infty$

$\therefore$

	a	b	d	e	f
best d	17	22	6	14	10
tree	d	a	c	f	d