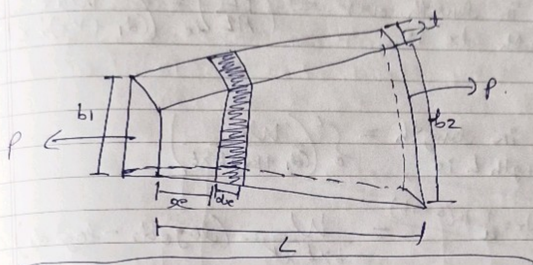
Deformation in Rectangular tapred bar

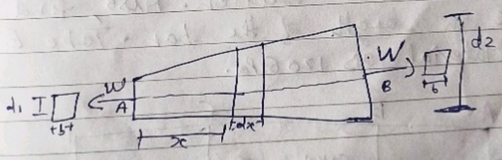
Deformation of tapered bar with rectangular cross-section:-



$$\Rightarrow \boxed{\delta = \frac{2.302 P l}{t E (b_2 - b_1)} (\log b_2 - \log b_1)}$$

where, P = Axial force, E = Modulus of Elasticity, L = Length of the member, b_1 = Smaller depth, b_2 = larger depth, t = Thickness of the member, δ = Change in length of tapered bar.

Consider an element of length, dx at a distance x from A,



Depth at x, $= d_1 + \frac{(d_2 - d_1)}{L} \times x$ ~~c/s area~~

$= d_1 + k \times x$.

c/s area at x, $= (d_1 + kx) b$

Change in length over a length dx is, $= \left(\frac{PL}{AE}\right)_{dx} = \left(\frac{W dx}{(d_1 + kx) b \times E}\right)$

Change in length over a length L is, $= \int_0^L \left(\frac{W dx}{(d_1 + kx) b \times E}\right)$

$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln(ax+b)$ formula

$$= \frac{W}{b \times E \times k} (\log_e d_2 - \log_e d_1)$$

$$= \frac{2.302 \times W \times L}{b \times E \times (d_2 - d_1)} (\log d_2 - \log d_1)$$

$$\boxed{\text{Change in the length} = \frac{2.302 \times W \times L}{b \times E \times (d_2 - d_1)} (\log d_2 - \log d_1)}$$

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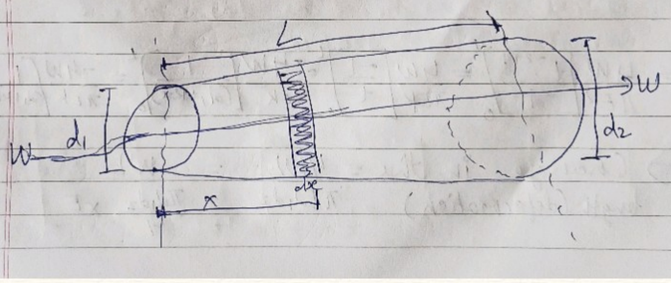
Deformation in Circular tapered bar

Deformation of tapered bar with circular cross-section.

$$\Rightarrow \delta = \frac{4WL}{\pi E d_1 d_2}$$

where, W = Axial force, E = Modulus of elasticity, L = Length of the member, d_1 = smaller diameter, d_2 = Larger diameter, δ = Change in length of tapered bar.



Consider an element of ~~tapered bar~~ length, dx at a distance x for A.

$$\text{Diameter at } x, = d_1 + \frac{(d_2 - d_1)}{L} \times x = d_1 + K \times x$$

$$\text{c/s area at } x, = \frac{\pi d_x^2}{4} = \frac{\pi}{4}(d_1 + Kx)^2.$$

Change in length over a length dx is $= \left(\frac{PL}{AE}\right)_{dx}$

$$= \left(\frac{W dx}{\frac{\pi}{4}(d_1 + Kx)^2 \times E}\right)$$

Change in Length over a length L is $= \int_0^L \left(\frac{W dx}{\frac{\pi}{4}(d_1 + Kx)^2 \times E}\right)$

$$= \int_0^L \left(\frac{W \frac{dt}{K}}{\frac{\pi}{4}(t)^2 \times E}\right)$$

Put $d_1 + Kx = t$, Differentiating w.r.t x Then $K dx = dt$.

$$= \frac{4W}{\pi E K}\left[\frac{t^{-2+1}}{-1}\right]_0^L = \frac{4W}{\pi E K}\left[\frac{-1}{t}\right]_0^L = \frac{-4W}{\pi E K}\left[\frac{1}{(d_1 + Kx)}\right]_0^L = \frac{-4W}{\pi E K}\left[\frac{1}{(d_1 + KL)} - \frac{1}{(d_1)}\right]$$

$$\Rightarrow \text{Change in the length (deformation)} = \frac{4WL}{\pi E d_1 d_2} = \frac{WL}{\frac{\pi d_1 d_2}{4} \times E}$$

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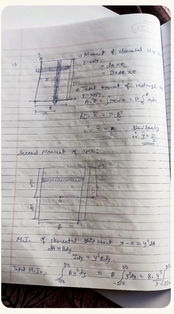
Parallel Axis Theorem(2nd M.I.)



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Principal of TRANSMISSIBILITY

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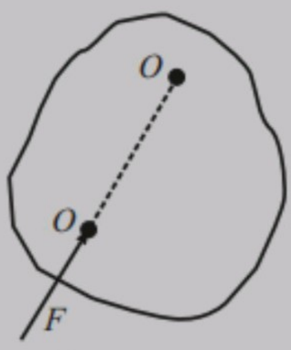
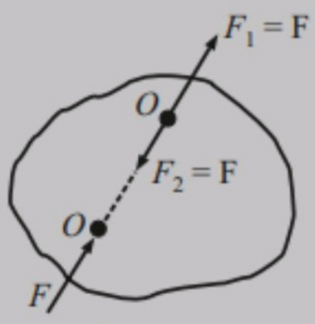
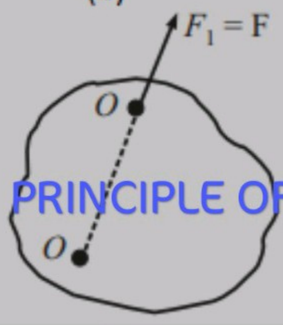
It state that if a force acting at a point on a rigid body, it may be considered to act at any other point on its line of action, provided this point is rigidly connected with the body. The external effect of the force on the body remains unchanged. The problems based on concurrent force system (you study in next article) are solved by application of this principle.



(a)



(b)



PRINCIPLE OF TRANSMISSIBILITY

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Relation bw Angel of Friction and Repose

⇒ Angle of Limiting Friction (ϕ) :-

An angle made by the total rxn (R) with the normal rxn (N) when F has reached F_{max},

$$\tan\phi = \frac{F_{max}}{N}$$

⇒ Angle of Repose :- (α)

The limiting angle up to which the gains (sleep / rest) is called the angle of repose of the granular material.

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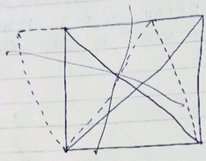
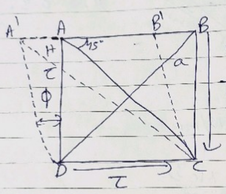
Relationship bw Modulus of Elasticity & Rigidity

A. Relationship b/w modulus of elasticity (E) & modulus of rigidity (G).





→ strain along AC

$= \frac{\tau}{E} - \mu\left(-\frac{\tau}{E}\right) = \frac{\tau}{E}(1+\mu)$ — ①

→ strain along AC

$= \frac{A'H}{AC} = \frac{A'H}{\sqrt{2}a}$ — ②

from Δ' AA'H; $\cos 45° = \frac{A'H}{AA'}$

$\therefore A'H = \frac{AA'}{\sqrt{2}}$

$\Rightarrow \boxed{E = 2G(1+\mu)}$ → from derivation

from Δ^le AA'D; $\tan\phi \simeq \phi = \frac{AA'}{AD} = \frac{AA'}{a}$

$\therefore AA' = a\phi$

eq^n (2) strain along $AC = \frac{AA'}{2a} = \frac{a\phi}{2a} = \frac{\phi}{2}$ — ③

$\rightarrow \frac{\tau}{E}(1+\mu) = \phi/2$

We know that, $G = \frac{\text{Shear strain}}{\text{Shear strain}} = \frac{\tau}{\phi} \Rightarrow \phi = \frac{\tau}{G}$

$\rightarrow \frac{\tau}{E}(1+\mu) = \frac{\tau}{2G} \Rightarrow E = 2G(1+\mu)$.

⇒ Relationship b/w E, G & K.

→ $E = 2G(1+\mu)$ ... ①

→ $E = 3K(1-2\mu)$ ... ②.

& ③ ⇒ $E = \frac{9KG}{3K+G}$

$2G(1+\mu) = 3K(1-2\mu)$

$\cancel{2G(2\mu)}$ $2G + 2\mu G = 3K - 6K\mu$

$2\mu G + 6K\mu = 3K - 2G$

$\mu(2G + 6K) = 3K - 2G$

$\mu = \frac{3K - 2G}{2G + 6K}$ — ③

substitute ③ in ① $E = 2G\left(1 + \frac{3K-2G}{2G+6K}\right)$

$E = 2G\left(\frac{2G + 6K + 3K - 2G}{2G+6K}\right)$

$= \frac{\cancel{2}G(9K)}{\cancel{2}(G+3K)} = \frac{9KG}{3K+G}$

$\Rightarrow E = \frac{9KG}{3K+G}$

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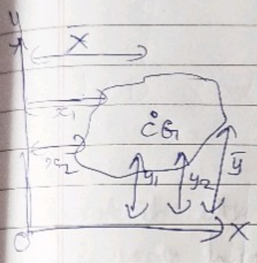
Varignons Theorem(1st moment of area)

A. Using method of moments (First moment of area).

→ Moment of Total area 'A' about y-axis = Algebric sum of moment of elemental ~~dA~~ "dA" about the same axis. (Varignon's Theorem).



$$\bar{x} = \frac{\sum a_i x_i}{A}$$

$$\Rightarrow A\bar{x} = a_1x_1 + a_2x_2 + a_3x_3 + \cdots$$

$$\bar{y} = \frac{\sum a_i y_i}{A}$$

$$\Rightarrow A\bar{y} = a_1y_1 + a_2y_2 + a_3y_3 + \cdots$$

⇒ It is an axis w.r.t which for an elementary area on one side of the axis, there is a corresponding elementary area on the other side of the axis (the first moment of these elementary areas about the axis balance each other.).

⇒ If an area has an axis of symmetry, then the centroid must lie on that axis.

⇒ If an area has 2 axes of symmetry, then the centroid must lie at the point of intersection of these axes.

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