

### MAT 2155: PROBLEM SET 3

1. In a class of 76 students, 26 have a brother, 29 have a sister, and 20 have both. How many have no siblings?
2. Number of positive integers between 1 and 100 not divisible by any of 2, 3, and 5.
3. Number of positive integers less than or equal to  $p^a q^b$ , not divisible by either one of  $p$  and  $q$ , where  $p$  and  $q$  are distinct prime numbers and  $a$  and  $b$  are positive integers.
4. Number of derangements of  $n$  objects.
5. Show that the proportion of permutations of  $1, 2, \dots, n$  with no consecutive pair  $i, i + 1$  for any  $i$  is  $\frac{n+1}{ne}$ .
6. Number of distributions of 30 distinct objects into 3 distinct boxes such that no box is empty.
7. Number of subsets of  $\{1, 2, \dots, 2n\}$  such that the sum of all the elements in the subset is odd.