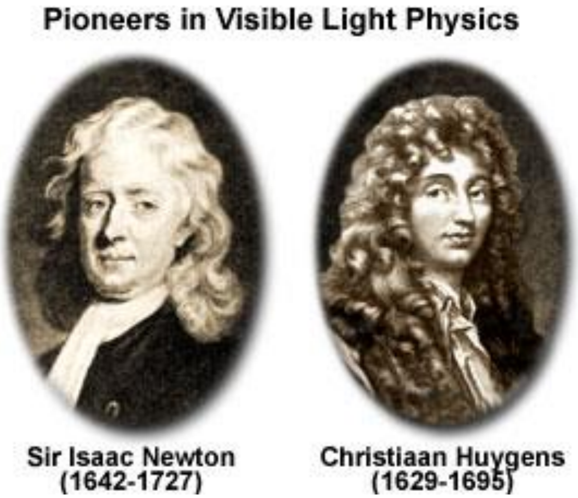
Chapter 1

INTERFERENCE OF LIGHT WAVES

OBJECTIVES

- To understand the principles of interference.
- To explain the intensity distribution in interference under various conditions.
- To explain the interference from thin films.

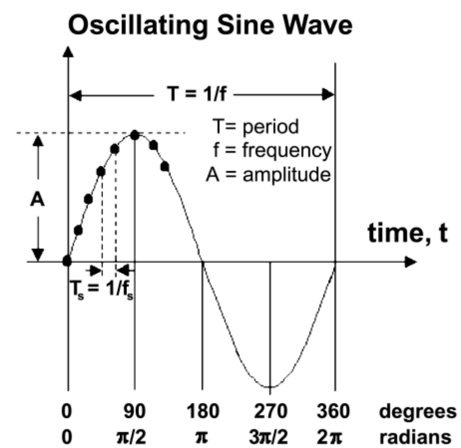


waves

A **wave** is a disturbance or oscillation that travels through space and matter, accompanied by a transfer of energy.

1. Longitudinal waves

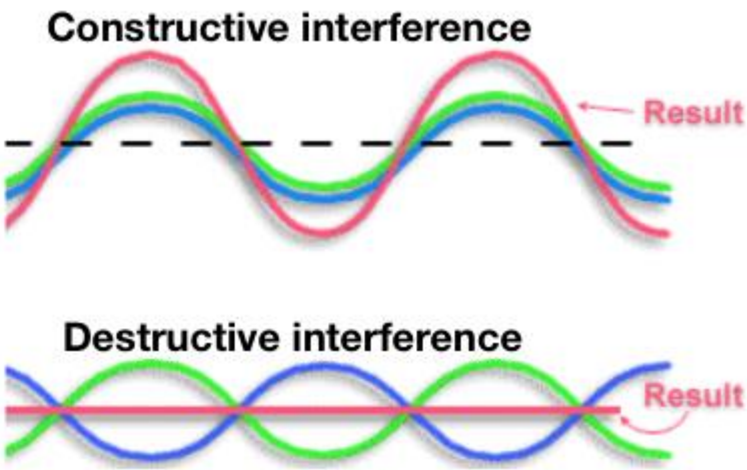
2. Transverse waves

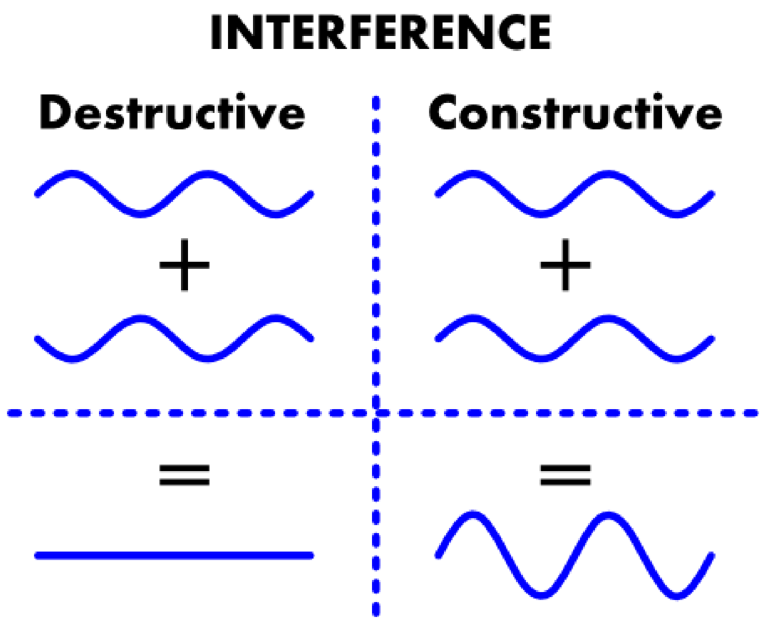


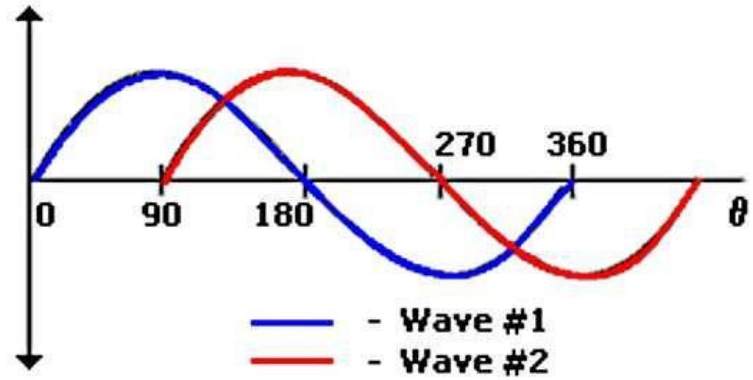
Phase denotes the particular point in the cycle of a waveform, measured as an angle in degrees

One radian of phase equals approximately 57.3°

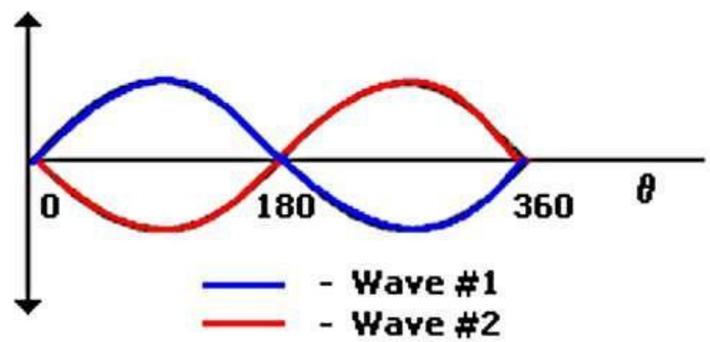
Wave optics (Physical Optics): The study of interference, diffraction, and polarization of light. These phenomena cannot be adequately explained with the ray optics.







Waves #1 and #2 are 90 degrees out of phase.

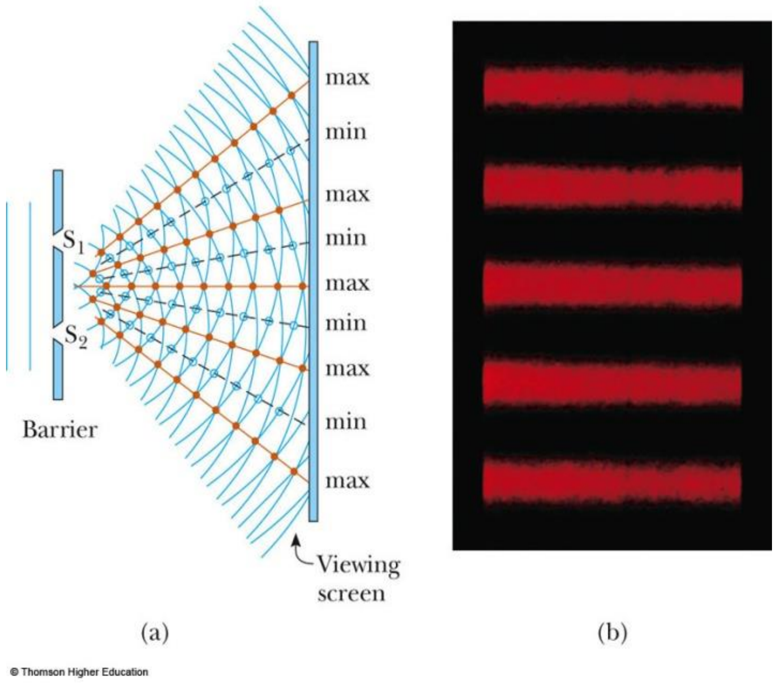


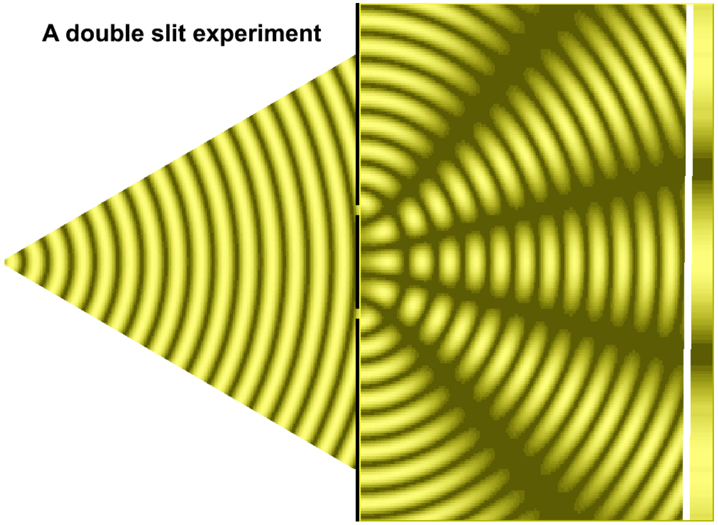
Waves #1 and #2 are 180 degrees out of phase.

When similar waves combine, the outcome can be constructive or destructive interference.

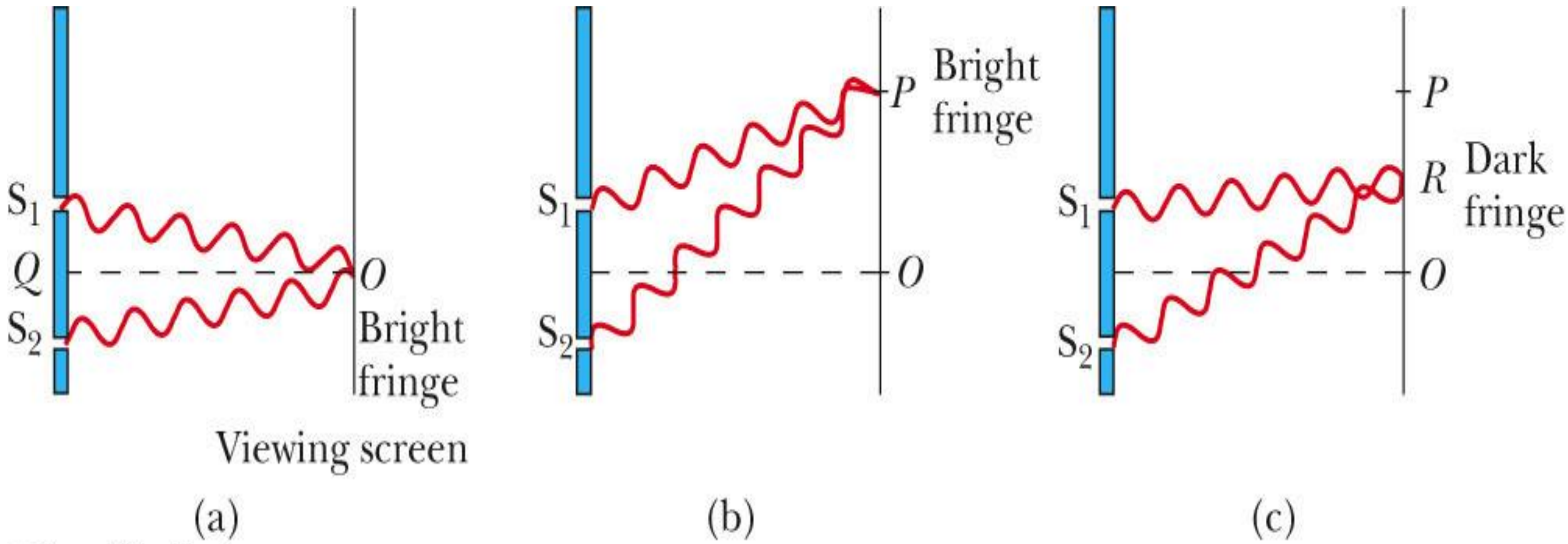
Wave optics (Physical Optics): The study of interference, diffraction, and polarization of light. These phenomena cannot be adequately explained with the ray optics.

Young's Double-Slit Experiment





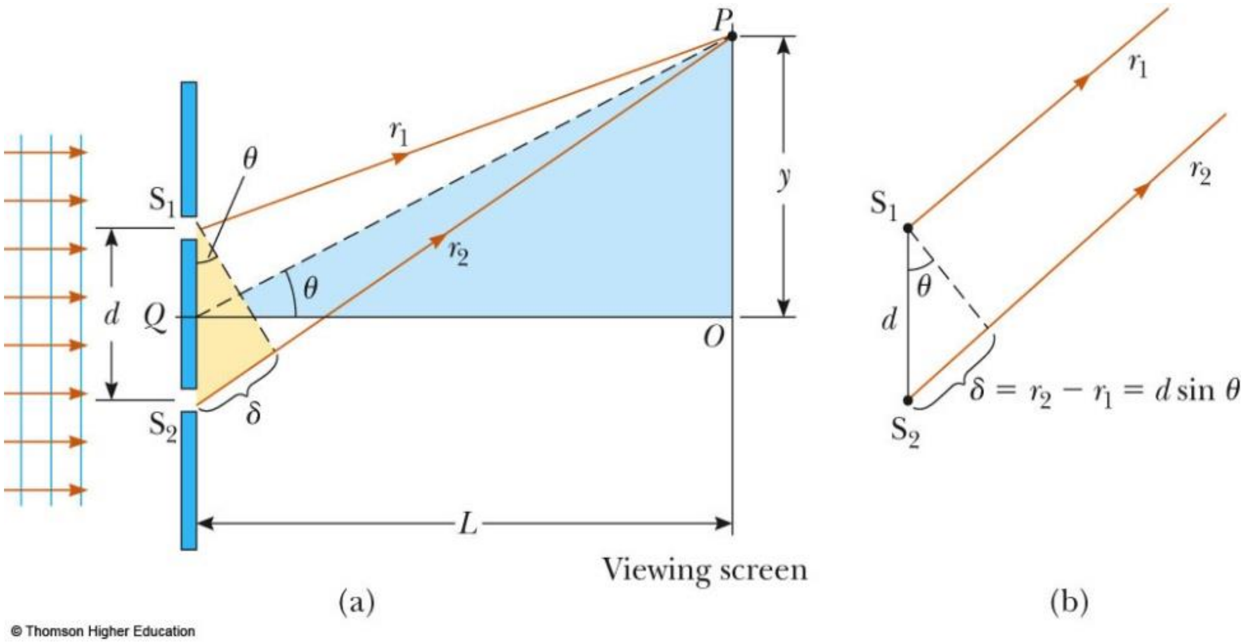
13 June 1773 – 10 May 1829



To observe interference of waves from two sources, the following conditions must be met:

- The sources must be **coherent**; that is, they must maintain a constant phase with respect to each other.
- The sources should be **monochromatic**; that is, they should be of a single wavelength.

Analysis Model: Waves in Interference



Path difference $\delta = r_2 - r_1 = d\sin\theta$

Condition for **constructive interference**, at point *P* is,

$$d\sin\theta_{\text{bright}} = m\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

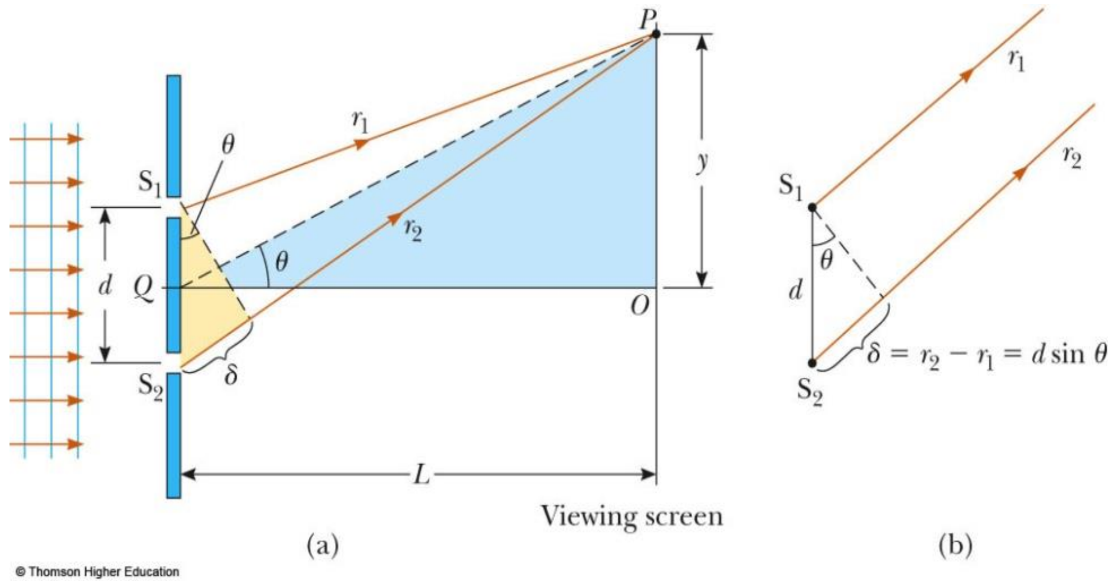
The number *m* is called the **order number**.

Condition for **destructive interference**, at point *P* is,

$$d\sin\theta_{\text{dark}} = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

Linear positions of bright and dark fringes:

From the triangle OPQ,



$$\tan\theta = \frac{y}{L}$$

$$y_{\text{bright}} = L\tan\theta_{\text{bright}}$$

$$y_{\text{bright}} = L\frac{m\lambda}{d} \quad \text{(small angles)}$$

$$y_{\text{dark}} = L\tan\theta_{\text{dark}}$$

$$y_{\text{dark}} = L\frac{\left(m+\frac{1}{2}\right)\lambda}{d}$$

Fringe separation $\Delta y = \dfrac{\lambda L}{d}$

A viewing screen is separated from a double slit by 4.80 m. The distance between the two slits is 0.0300 mm. Monochromatic light is directed toward the double slit and forms an interference pattern on the screen. The first dark fringe is 4.50 cm from the center line on the screen. (A) Determine the wavelength of the light. (B) Calculate the distance between adjacent bright fringes.

Ans : (A) 562 nm   and (B) 8.9 cm

A light source emits visible light of two wavelengths: λ = 430 nm and λ' = 510 nm. The source is used in a double-slit interference experiment in which *L* = 1.50 m and *d* = 0.025 0 mm. Find the separation distance between the third-order bright fringes for the two wavelengths.

Ans: 0.0144 m or 1.44 cm

A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm, and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the center of the pattern. What is the wavelength?

Ans: 515 nm

A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm, and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the center of the pattern. What is the wavelength?

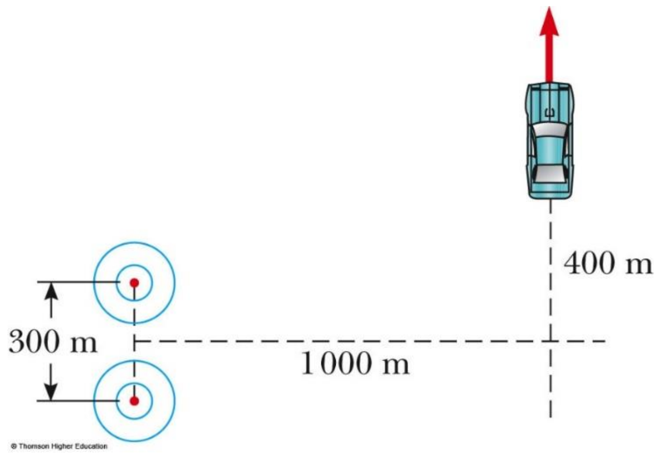
In a Young's interference experiment, the two slits are separated by 0.150 mm and the incident light includes two wavelengths: λ_1 = 540 nm (green) and λ_2= 450 nm (blue). The overlapping interference patterns are observed on a screen 1.40 m from the slits. Calculate the minimum distance from the center of the screen to a point where a bright fringe of the green light coincides with a bright fringe of the blue light.

2.52 cm

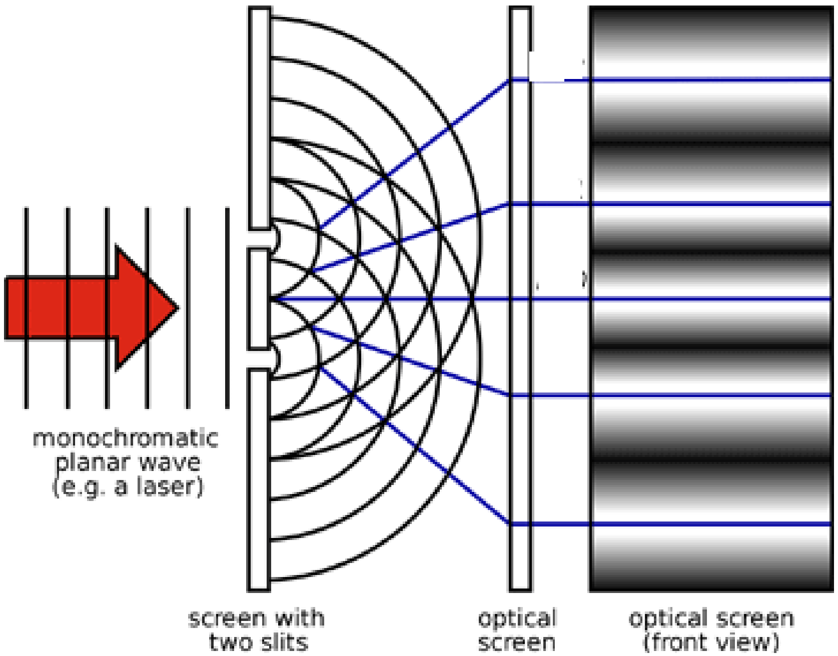
In a Young's interference experiment, the two slits are separated by 0.150 mm and the incident light includes two wavelengths: λ_1 = 540 nm (green) and λ_2= 450 nm (blue). The overlapping interference patterns are observed on a screen 1.40 m from the slits. Calculate the minimum distance from the center of the screen to a point where a bright fringe of the green light coincides with a bright fringe of the blue light.

Two radio antennas separated by *d* = 300 m as shown in figure simultaneously broadcast identical signals at the same wavelength. A car travels due north along a straight line at position *x* = 1000 m from the center point between the antennas, and its radio receives the signals. (a) If the car is at the position of the second maximum after that at point *O* when it has traveled a distance *y* = 400 m northward, what is the wavelength of the signals? (b) How much farther must the car travel from this position to encounter the next minimum in reception? *Note:* Do not use the small-angle approximation in this problem.

Ans: 55.7 m and 124 m



Intensity Distribution of the Double-Slit Interference Pattern



In a pure interference, slit width "a" is negligible compared to the wavelength.

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

Intensity Distribution of the Double-Slit Interference Pattern

Consider two coherent sources of sinusoidal waves such that they have same angular frequency ω and phase difference ϕ.

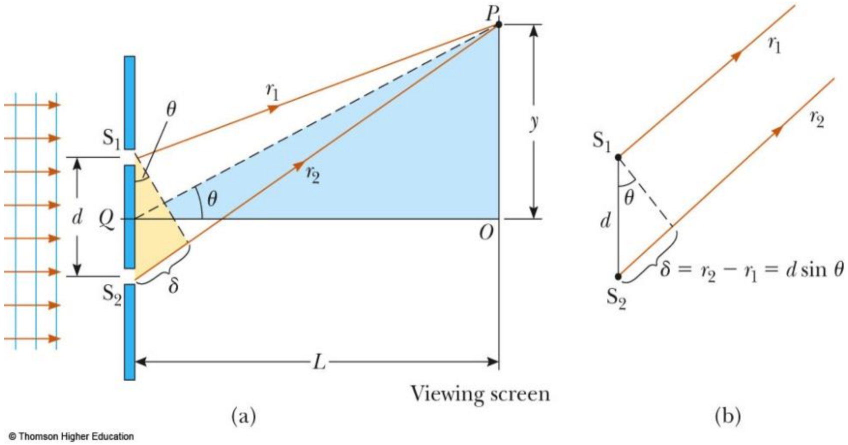
$$E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin(\omega t + \phi)$$

$$\phi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} d \sin\theta$$

Magnitude of resultant electric field at point P is :

$$E_P = E_1 + E_2 = E_0\left[\sin \omega t + \sin(\omega t + \phi)\right]$$

$$E_P = 2E_0 \cos\left(\frac{\phi}{2}\right)\sin\left(\omega t + \frac{\phi}{2}\right)$$



Intensity of a wave is proportional to the square of the resultant electric field magnitude at that point.

$$I \propto E_P^2 = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right)\sin^2\left(\omega t + \frac{\phi}{2}\right) \qquad I_1 \propto E_0^2 \ sin^2 \omega t$$

Most light-detecting instruments measure time-averaged light intensity, and the time averaged value of $\sin^2\left(\omega t + \frac{\phi}{2}\right)$ over one cycle is ½.

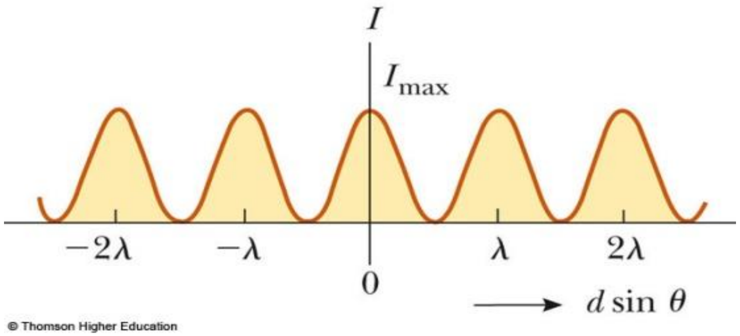
$$I = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$$

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

Alternatively, since $\sin\theta \approx \frac{y}{L}$

for small values of θ , we can write;

$$I = I_{\max} \cos^2\left(\frac{\pi d}{\lambda L} y\right)$$



In a double slit experiment, let *L* = 120 cm and *d* = 0.250 cm. The slits are illuminated with coherent 600-nm light. Calculate the distance y above the central maximum for which the average intensity on the screen is 75.0% of the maximum. **[use the calculator in radian mode]**

$$I = I_{\max} \cos^2\left(\frac{\pi d}{\lambda L} y\right)$$

48 μm

Show that the two waves with wave functions $E_1 = 6.00\sin(100\pi t)$ and $E_2 = 8.00\sin(100\pi t + \pi / 2)$ add to give a wave with the wave function $E_R \sin(100\pi t + \phi)$. Find the required values for E_R and ϕ. Ans: 10 and 53.1°

Phase change on reflection

Denser medium Rarer medium

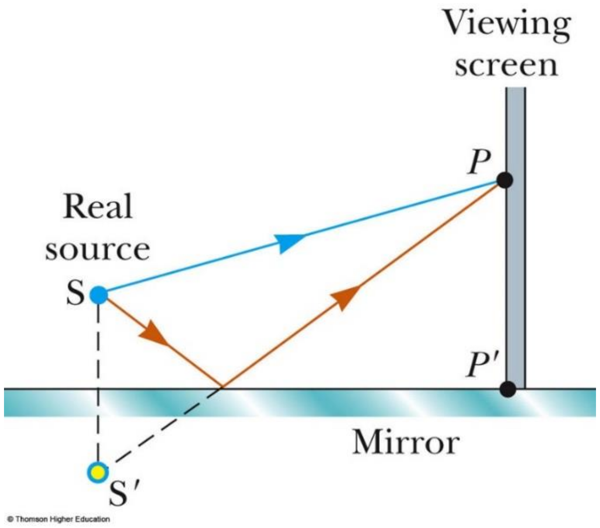
Rarer medium **Denser medium**

no phase change of the reflected wave

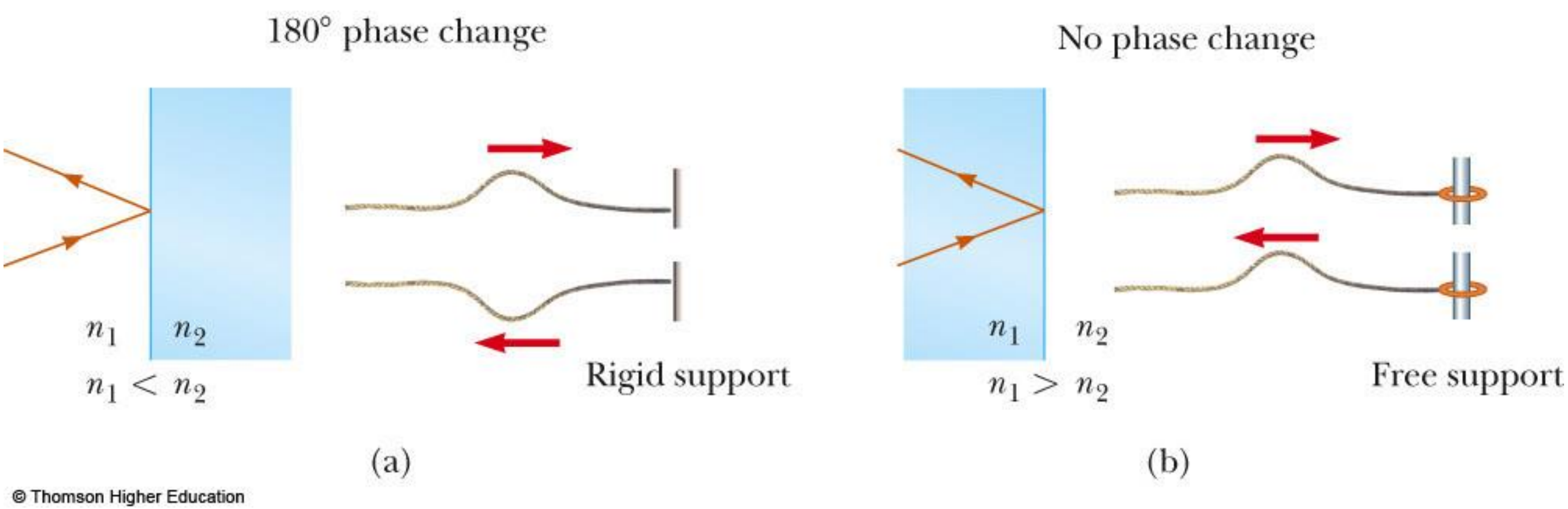
phase change of π (=180°)

Phase change on reflection at a junction between two strings of different linear mass densities.

Change of Phase Due to Reflection



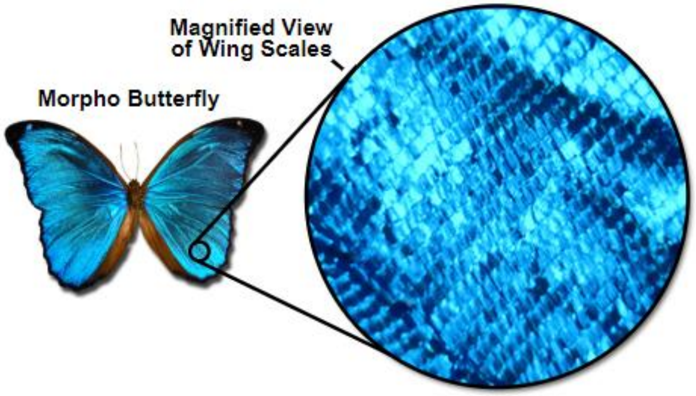
Lloyd's mirror. The reflected ray undergoes a phase change of 180°.

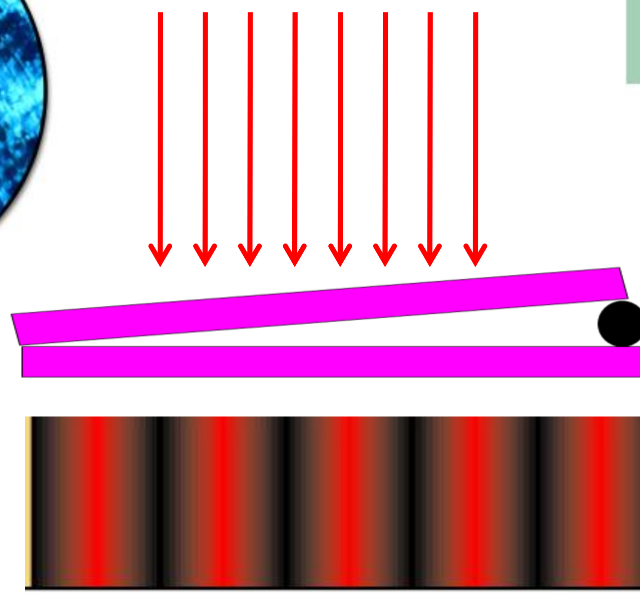


Interference in Thin Films

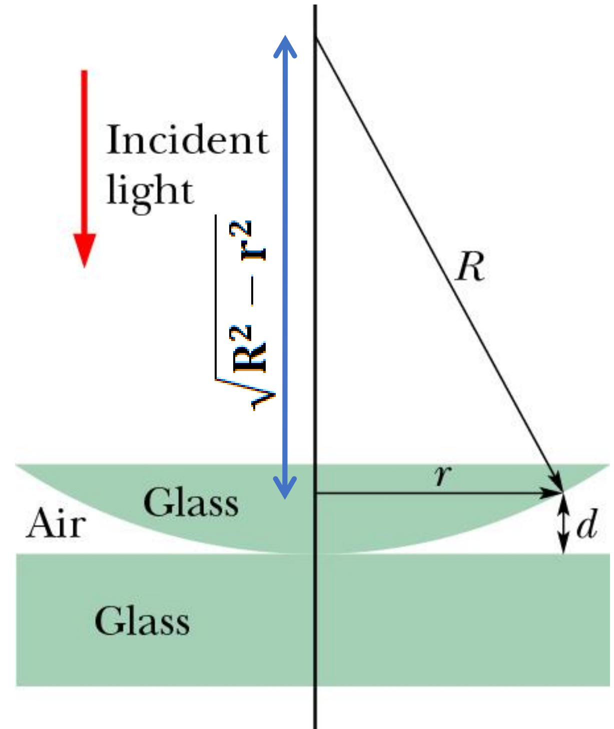
Examples of Interference in Nature

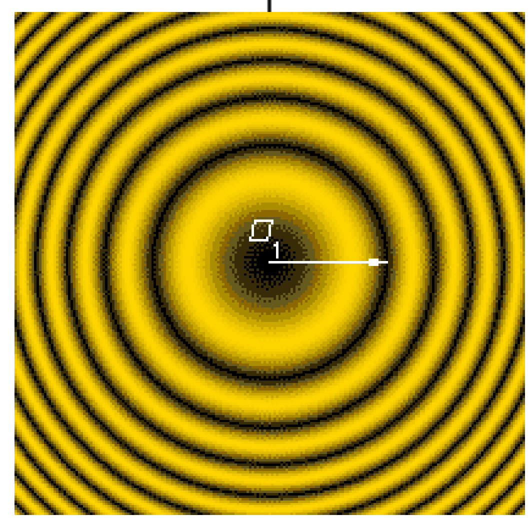
Interference Structures in Butterfly Wings





Air Wedge interference





Newton rings

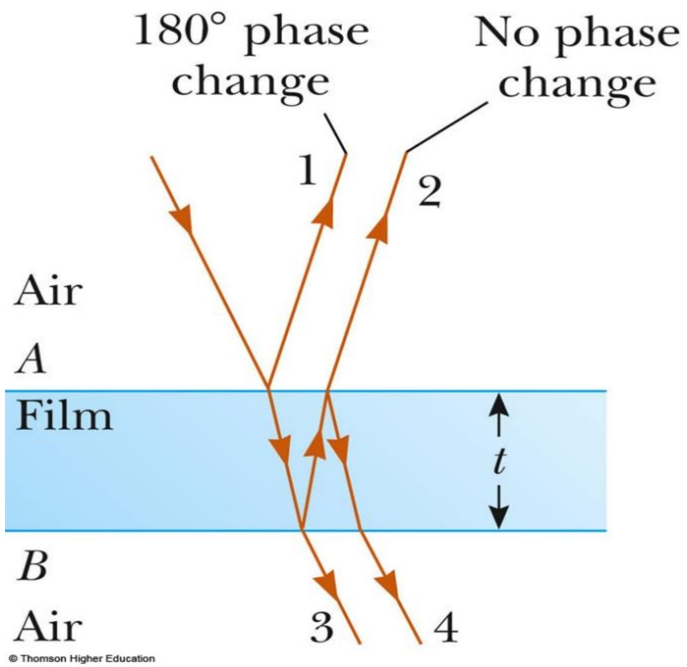
Interference in Thin Films

Consider a film of uniform **thickness t** and **index of refraction n**. Assume light rays traveling in air are nearly **normal** to the two surfaces of the film. If λ is the wavelength of the light in free space and n is the index of refraction of the film material, then the wavelength of light in the film is $\lambda_n = \dfrac{\lambda}{\text{n}}$

The condition for ***constructive*** interference in thin films is,

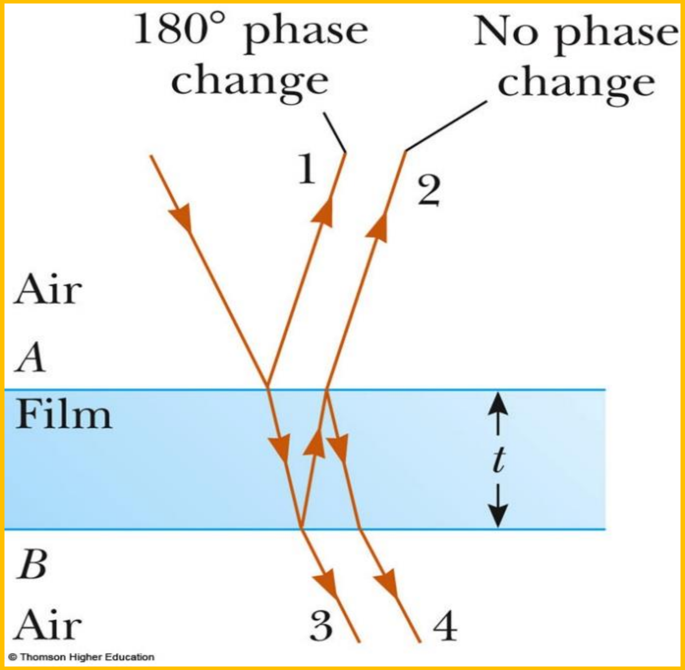
$$2t = \left(m + \frac{1}{2}\right)\lambda_n \quad (m = 0, 1, 2, \ldots)$$

$$2nt = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots)$$



The condition for ***destructive*** interference in thin films is,

$$2nt = m\lambda \quad (m = 0, 1, 2, \ldots)$$



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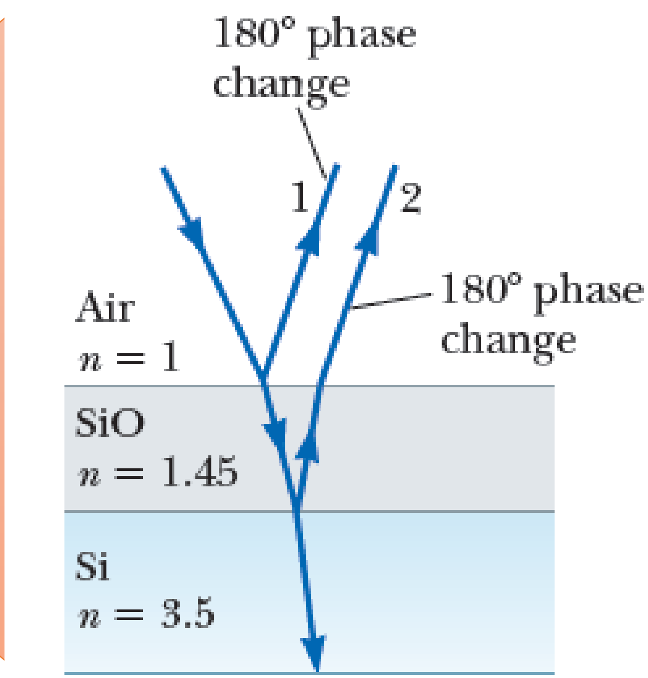

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Calculate the minimum thickness of a soap-bubble film that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is 600 nm. The index of refraction of the soap film is 1.33.

113 nm

Solar cells—devices that generate electricity when exposed to sunlight—are often coated with a transparent, thin film of silicon monoxide (SiO, n = 1.45) to minimize reflective losses from the surface. Suppose a silicon solar cell (n = 3.5) is coated with a thin film of silicon monoxide for this purpose. Determine the **minimum film thickness** that produces the **least reflection** at a wavelength of 550 nm, near the center of the visible spectrum.

94.8 nm

$$t = \frac{\lambda}{4n} = \frac{550x10^{-9}}{4x1.45} = 94.8\ nm$$

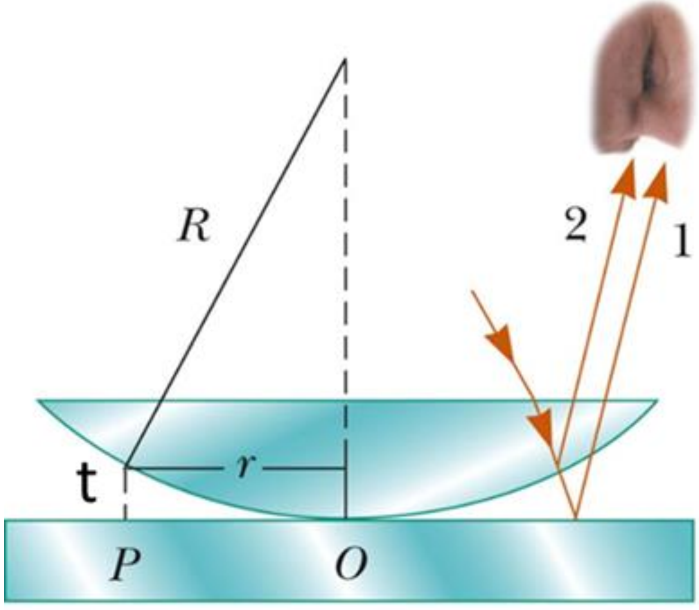
An oil film (n = 1.45) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find (a) the wavelength and color of the light in the visible spectrum **most strongly reflected** and (b) the wavelength and color of the light in the spectrum **most strongly transmitted**. Explain your reasoning.

a) 541 nm and green

b) 406 nm and violet

Newton's Rings

Expressions for radii of the dark rings:



Consider the dark rings (destructive interference)

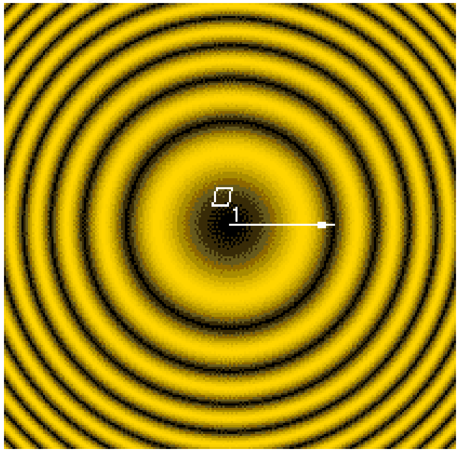
$$2nt = m\lambda, \qquad m = 0, 1, 2, 3\ldots$$

For air film, $n \approx 1$

$$\therefore \quad 2t = m\lambda$$

From the above figure, $t = R - \sqrt{R^2 - r^2}$

$$t = R - R\left[1 - \left(\frac{r}{R}\right)^2\right]^{1/2}$$



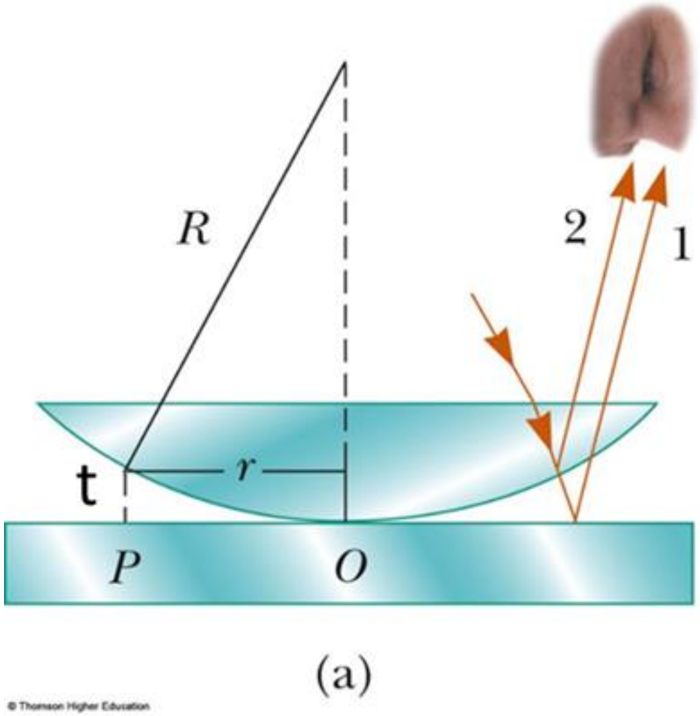
Binomial theorem is, $(1+y)^n = 1 + ny + \dfrac{n(n-1)}{2!}y^2 + \ldots\ldots.$

If $r/R << 1,$

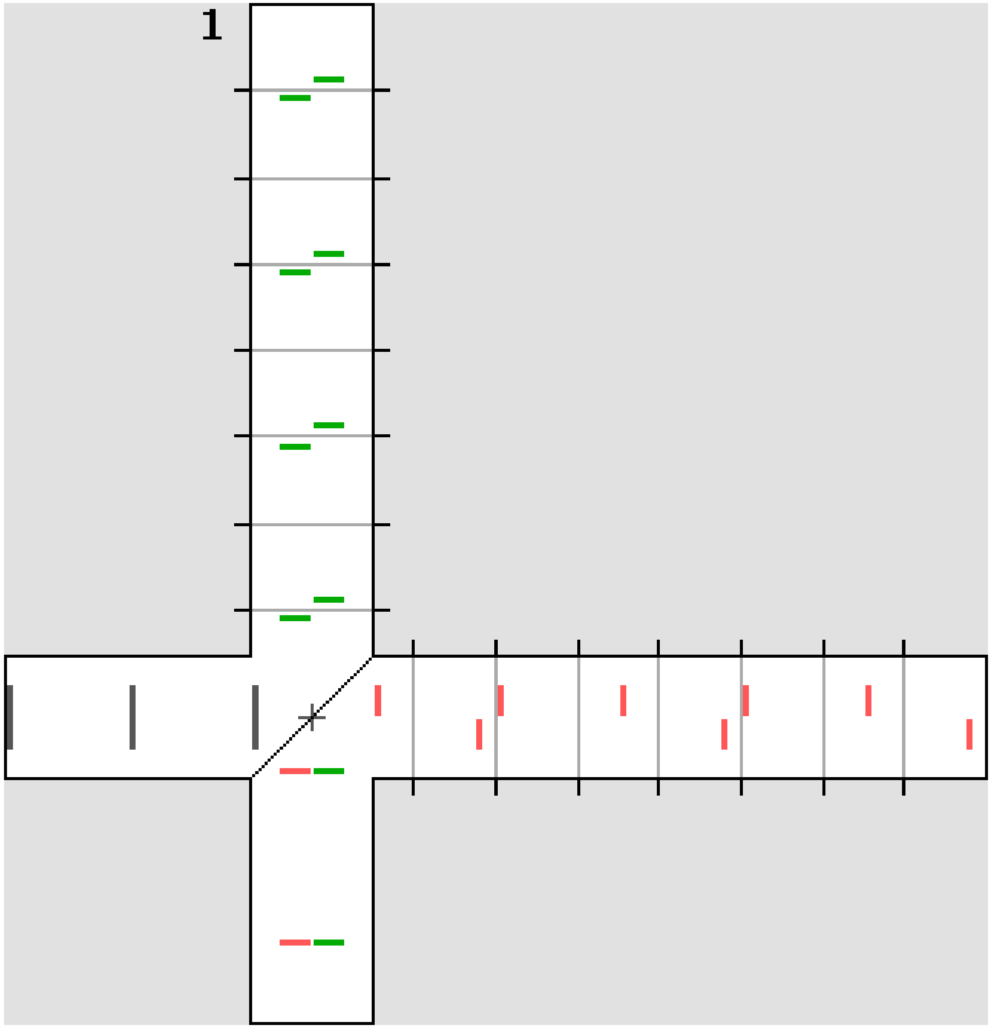
$$t = R - R\left[1 - \frac{1}{2}\left(\frac{r}{R}\right)^2 + \ldots\ldots..\right] \approx \frac{r^2}{2R}$$

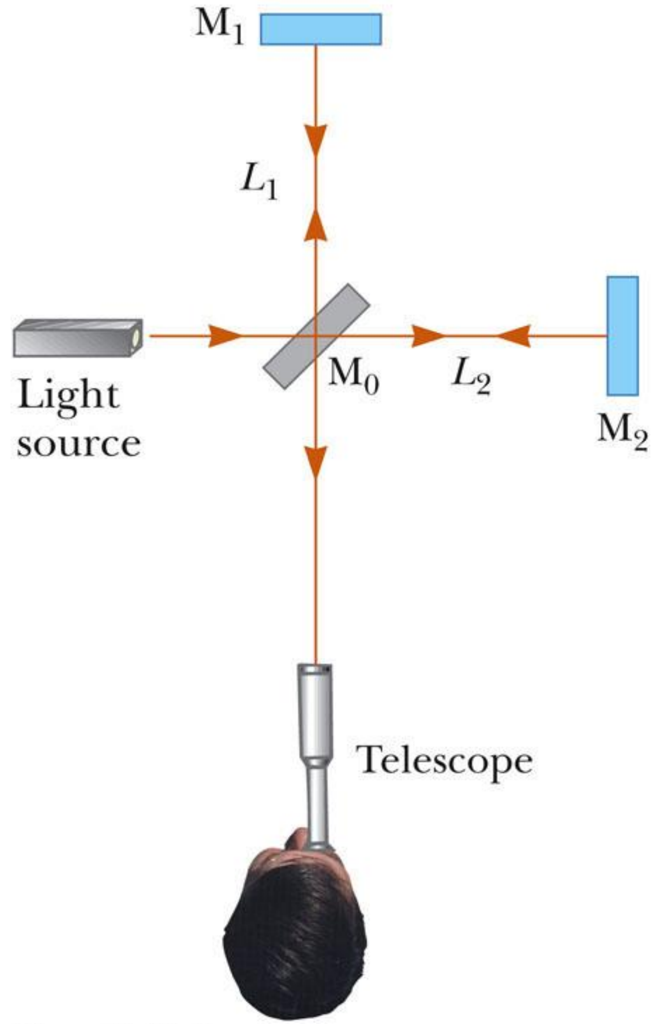
$$r_{dark} \approx \sqrt{mR\lambda} \qquad (m = 0, 1, 2, \ldots)$$

In a Newton's-rings experiment, a plano-convex glass (n = 1.52) lens having radius r = 5.00 cm is placed on a flat plate as shown in Figure. When light of wavelength 650 nm is incident normally, 55 bright rings are observed, with the last one precisely on the edge of the lens. (a) What is the radius R of curvature of the convex surface of the lens? (b) What is the focal length of the lens?



Michelson Interferometer



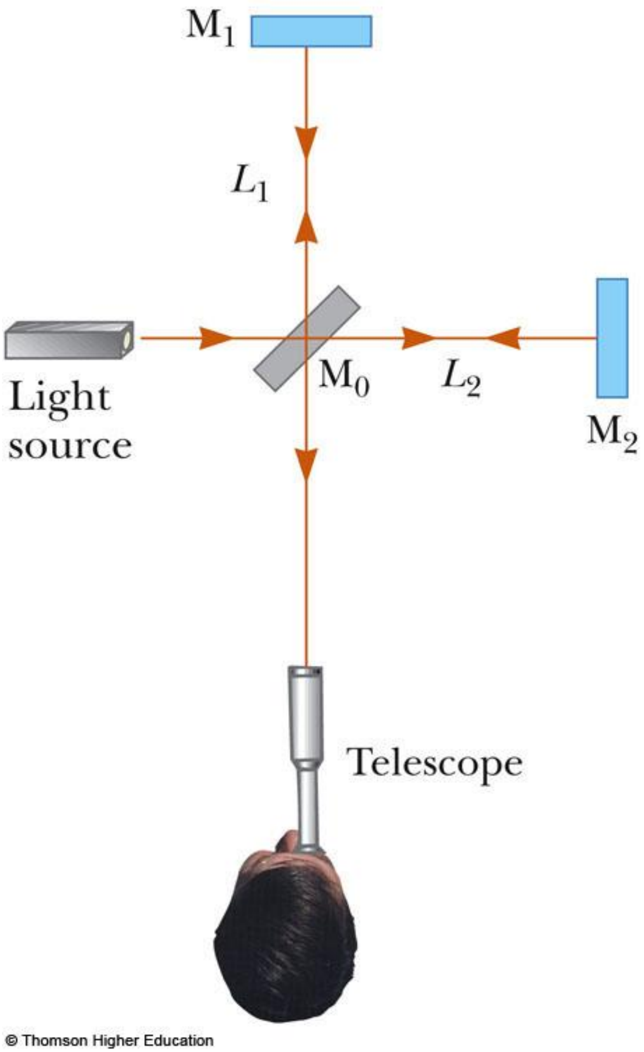


Michelson Interferometer

The **interferometer**, invented by A. A. Michelson, splits a light beam into two parts and then recombines the parts to form an interference pattern.

The interference condition for the two rays is determined by the difference in their path length. When the two mirrors are exactly perpendicular to each other, the interference pattern is a target pattern of bright and dark circular fringes.

If a dark circle appears at the center of the target pattern and M_1 is then moved a distance $\lambda/4$ toward M_0, the path difference changes by $\lambda/2$. This replaces dark circle at center by bright circle. **Therefore, the fringe pattern shifts by one-half fringe each time M_1 is moved a distance $\lambda/4$.**



Mirror M1 in Figure 1.9 is moved through a displacement ΔL. During this displacement, 250 fringe reversals (formation of successive dark or bright bands) are counted. The light being used has a wavelength of 632.8 nm. Calculate the displacement ΔL.

$$2\Delta L = \frac{m\lambda}{2}$$

Ans: 39.6 μm

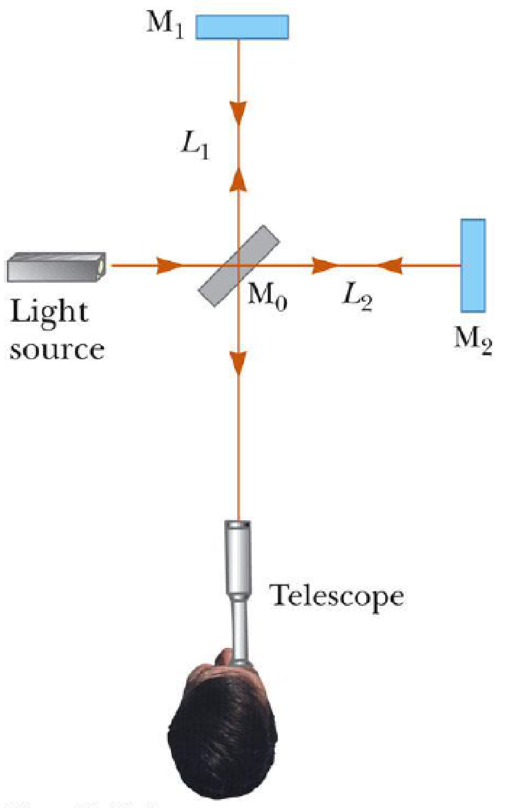


Figure 1.9 Schematic diagram of Michelson Interferometer

Questions

1. What is interference of light waves?
2. What is coherence? Mention its importance.
3. Write the necessary condition for the constructive and destructive interference of two light waves in terms of path/phase difference.
4. Obtain an expression for intensity of light in double-slit interference.
5. Write the conditions for constructive and destructive interference of reflected light from a thin soap film in air, assuming normal incidence.
6. Explain the formation of fringes in Michelson interferometer.