Questions on: Distributions, Function of 1D random variable and Mgf

1. In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would you expected to contain at least 3 defective parts?
   Ans: 323

2. The amount of time required to repair a television is exponentially distributed with mean 2. Find
   (i) The probability that the required time exceeds 2 hours
   (ii) The conditional probability that the required time takes at least 10 hours given that already 9 hours have been spent on repairing the TV.

   Ans: (i) 1/e (ii) $e^{-1/2}$

3. A factory produces 10 glass containers daily. It is assumed that there is a constant probability 0.1 of producing a defective container. Before containers are stored, they are tested and defective ones are set aside. The accuracy of the test is 95%. Let X be the number of containers classified as defective. Find $P(X = k)$ and $P(X \geq 1)$
   Ans: 0.7786

4. If X has pdf $f(x) = \lambda e^{-\lambda(x-a)},\ x \geq a$. Find mgf of X and hence find $E(X)$ and $V(X)$.
   Ans: $E(X) = a + \frac{1}{\lambda}$ and $V(X) = \frac{1}{\lambda^2}$

5. Suppose that life length of two electronic device say D_1 and D_2 have distributions $N(40,36)$ and $N(45,9)$ respectively. If the electronic device is to be used for 45 hours period, which device is to be preferred? If it is to be used for 48 hours period, which device is to be preferred?
   Ans: $P(D_1 \geq 45) = 0.2033 \quad and\ P(D_2 \geq 45) = 0.5 \quad Hene\ D_2\ is\ preferred$
   $P(D_1 \geq 48) = 0.0918 \quad and\ P(D_2 \geq 45) = 0.158 \quad Hene\ D_2\ is\ preferred$

6. The number of telephone lines busy at a particular time is a binomial variable with probability 0.1 that a line is busy. If 10 lines are selected at random, what is the probability that i) no line is busy ii) at least one line is busy iii) at most 2 lines are busy.
   Ans: (i) 0.3487 (ii) 0.6513 (iii) 0.929830.

7. The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3 out of 1000 taxi drivers. Find approximately the number of the drivers with i) no accident in a year ii) more than 3 accidents in a year.
   Ans: (i) 50 (ii) 353

8. A fair coin is tossed 500 times. Find the probabilities that the number of heads will not differ from 250 by (a) more than 10 (b) more than 30.
   [Hint: : Let X represents the number of heads in 500 tosses.
   $\therefore p$ =probability of head turning up= $\frac{1}{2}$ and $n = 500$.
   Mean= $np = 500 \times \frac{1}{2} = 250$ and $\sigma = \sqrt{npq} = \sqrt{500 \times \frac{1}{2} \times \frac{1}{2}} = 11.18033$
   $X \sim N(250, 11.1803)$]
   $Ans: P(240 \leq X \leq 260) = 0.6266 \quad and\ P(220 \leq X \leq 280) = 0.9920$

9. If mean marks is 60 and standard deviation is 10, 70% failed in examination. What is the grace marks given to obtain 70% pass the examination?
Ans: 10.5

10. In an examination, the marks scored by the students follow the normal distribution. It is known that a student passes the examination if he secures 40% or more marks. He is place in first, second and third division, if he secures 60% or more, between 50% and 60%, and between 40% and 50% respectively. He gets a distinction, if he gets 70% or more marks. It is given that 10% of students have failed in the examination and 5% of them obtained distinction. Find the percentage of the students getting second division.(Assume that marks are normally distributed).
Ans: $\mu = 53.1058, \ \sigma = 10.2389 \quad and\ P(50 < X < 60) = 0.3665;$
$\text{percentage of the students getting second division. } 36.65\ \%$

11. An office switch board receives telephone calls at the rate of 3 calls per minute on an average. What is the probability of receiving,
(i) No calls in 1min interval
(ii) At most 3 calls in 5min interval
Ans: $X \sim Poission(\alpha = 3),\ (i)\ e^{-3} \qquad (ii)\ 0.1136$

12. Buses arrive at a specified stop at 15min intervals starting at 7am. i.e., they arrive at 7am, 7:15am, 7:30am, 7:45am and so on. If a passenger arrives at the stop at a time is uniformly distributed between 7am and 7:30am, find the probability that he waits,
(i) Less than 5min for bus
(ii) More than 10 min for bus

Ans: (i) 1/3 (ii) 1/3

13. Suppose X is uniformly distributed over (0,1), find pdf of $Y = \frac{1}{X+1}$.
Ans: $g(y) = \frac{1}{y^2}; \frac{1}{2} < y < 1.$

14. If X is uniformly distributed in (1, 2). Find the pdf of Y if $Y = \frac{1}{X}$.
Ans: Ans: $g(y) = \frac{1}{y^2}; \frac{1}{2} < y < 1.$

15. Suppose X has the pdf $f(x) = \begin{cases} \frac{2}{9}(x+1); & -1 < x < 1 \\ 0; & Otherwise \end{cases}$.
Find pdf of $Y = X^2$.
Ans: $g(y) = \frac{1}{2\sqrt{y}}\left(f(\sqrt{y}) + f(-\sqrt{y})\right) = \frac{1}{2\sqrt{y}}\left(\frac{2(\sqrt{y}+1)}{9} + \frac{2(-\sqrt{y}+1)}{9}\right) = \frac{2}{9\sqrt{y}}; 0 < x < 1$

16. If X is the outcome obtained when a die is tossed, then find the moment generating function. Also find its mean and variance.
Ans: $M_X(t) = 16\ [e^t + e^{2t} + \cdots + e^{6t}] \quad E(X) = \frac{21}{6} \quad and\ E(x^2) = \frac{91}{6}$

17. If X has pdf $f(x) = \lambda e^{-\lambda(x-a)}$ if $x \geq a$. Find its mgf and also find the mean and variance.
Ans: $M_X(t) = \frac{\lambda e^{at}}{\lambda - t},\ \lambda > t \quad E(X) = a + \frac{1}{\lambda}\ , \quad V(X) = \frac{1}{\lambda^2}$