Problem 1

Dataset Description:

The blue region (labeled 1) is defined by the following inequalities:

1. $x_1 + x_2 \leq 1$
2. $x_1 + x_2 \geq 4$
3. $x_1 + x_2 \leq 6$
4. $x_1 + x_2 \geq 9$

Objective:

Design a neural network that outputs 1 when a point lies in any of these blue regions and outputs 0 otherwise.

Neural Network Architecture:

- **Input Layer:** 2 neurons (x_1, x_2)
- **Hidden Layer:** 4 neurons
- **Output Layer:** 1 neuron

Activation function:

The summation of inputs to the neuron nodes; x .
Then comparison with threshold.

$$g(x;T) = \begin{cases} 1 & \text{if } x \geq T \\ 0 & \text{otherwise} \end{cases}$$

Weights and Thresholds:

Hidden Layer Neurons:

1. **Neuron N1: Detects** $x_1 + x_2 \leq 1$
   - **Weights:** $w_{N1,x1} = -1$, $w_{N1,x2} = -1$
   - **Threshold:** $T_{N1} = -1$
2. **Neuron N2: Detects** $x_1 + x_2 \geq 4$
   - **Weights:** $w_{N2,x1} = 1$, $w_{N2,x2} = 1$
   - **Threshold:** $T_{N2} = 4$

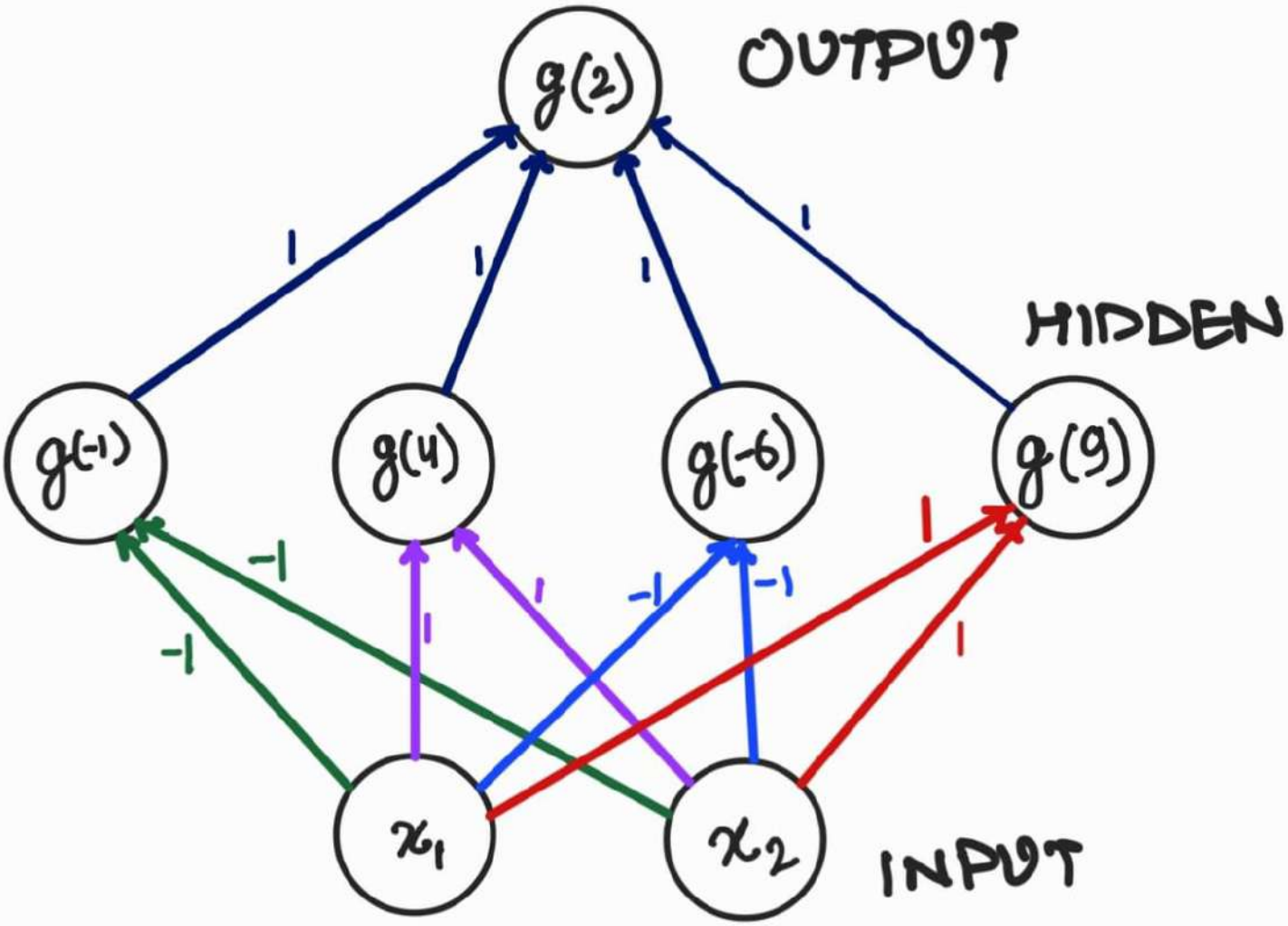
3. **Neuron N3: Detects $x_1 + x_2 \leq 6$)**
   - **Weights:** $w_{N3,x1} = -1$, $w_{N3,x2} = -1$
   - **Threshold:** $T_{N3} = -6$
4. **Neuron N4: Detects $x_1 + x_2 \geq 9$**
   - **Weights:** $w_{N4,x1} = 1$, $w_{N4,x2} = 1$
   - **Threshold:** $T_{N4} = 9$

Output Layer Neuron:

- **Neuron O: Outputs 1 if 2 conditions are met**
  - **Inputs:** Outputs of N1, N2, N3, N4
  - **Weights:** $w_{O,N1} = 1$, $w_{O,N2} = 1$, $w_{O,N3} = 1$, $w_{O,N4} = 1$
  - **Threshold:** $T_O = 2$

Neuron Functions Explanation:

- **N1 to N4:** Each neuron checks one of the inequalities defining the blue region.
- **O:** Outputs 1 if 2 of the conditions are satisfied, at a time only 2 conditions can be true.
- **Neuron activation for different regions:**
  1. For $x_1 + x_2 \leq 1$, N1 and N3 neuron gives 1
  2. For $4 \geq x_1 + x_2 \leq 6$, N2 and N3 neuron gives 1
  3. For $x_1 + x_2 \geq 9$, N2 and N4 neuron gives 1


g(2)
OUTPUT
1
1
1
1
HIDDEN
g(-1)
g(4)
g(-6)
g(9)
-1
-1
1
1
-1
-1
1
1
x_1
x_2
INPUT

Problem 2

Dataset Description:

The blue region (labeled 1) is defined by the following inequalities:

1. $x_2 \leq 6$
2. $-7x_1 + 10x_2 \geq 10$
3. $7x_1 + 10x_2 \geq 80$
4. $10x_1 + 4x_2 \leq 80$
5. $10x_1 - 4x_2 \geq 20$

Objective:

Design a neural network that outputs 1 only when all these conditions are satisfied simultaneously.

Neural Network Architecture:

- **Input Layer:** 2 neurons (x_1, x_2)
- **Hidden Layer:** 5 neurons
- **Output Layer:** 1 neuron

Activation function:

The summation of inputs to the neuron nodes; x .
Then comparison with threshold.

$$g(x;T) = \begin{cases} 1 & \text{if } x \geq T \\ 0 & \text{otherwise} \end{cases}$$

Weights and Thresholds:

Hidden Layer Neurons:

1. **Neuron N1: Detects** $x_2 \leq 6$
   - **Weights:** $w_{N1,x2} = -1$
   - **Threshold:** $T_{N1} = -6$
2. **Neuron N2: Detects** $-7x_1 + 10x_2 \geq 10$
   - **Weights:** $w_{N2,x1} = -7$, $w_{N2,x2} = 10$

- **Threshold:** $T_{N2} = 10$

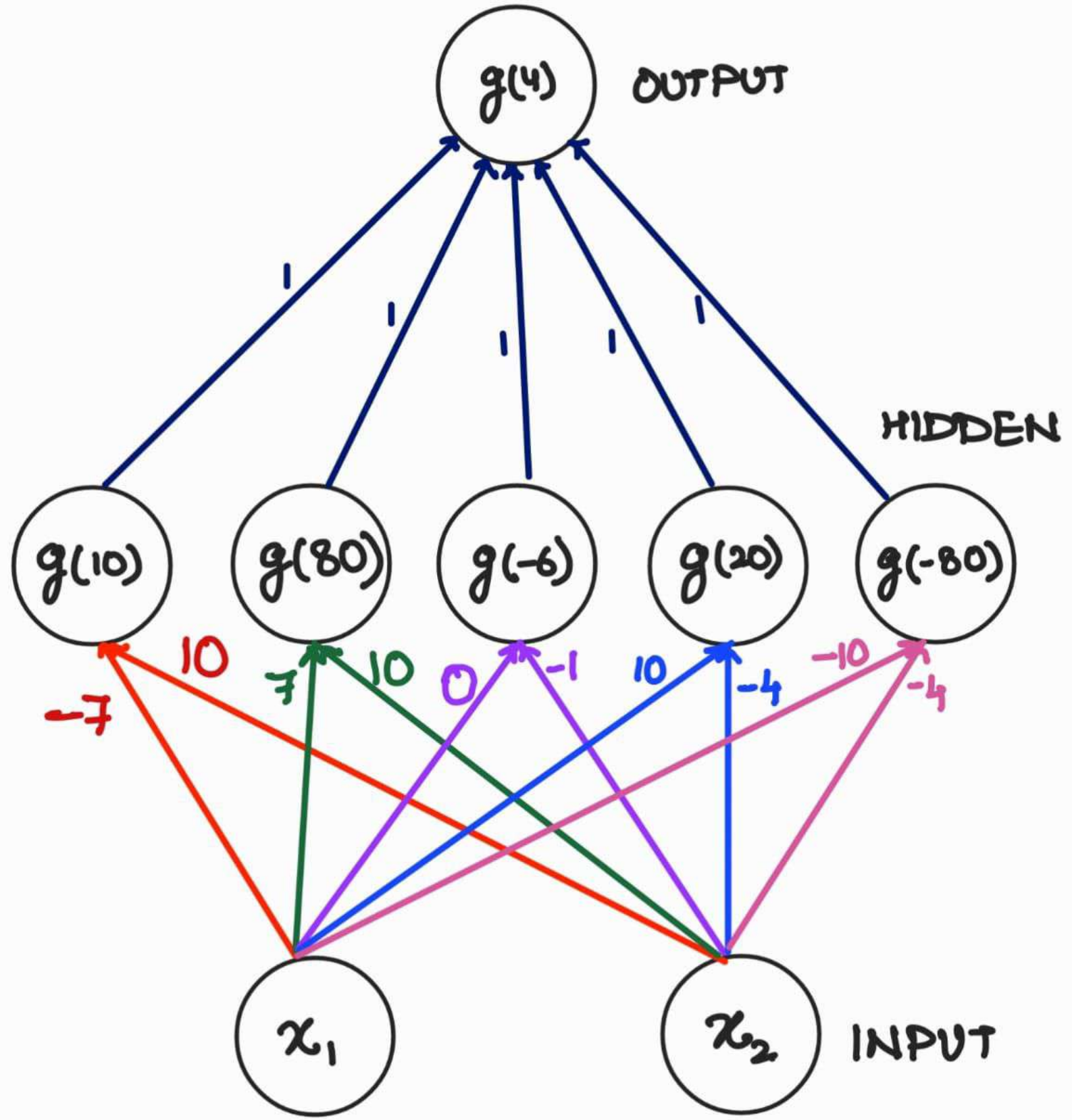
3. **Neuron N3: Detects** $7x_1 + 10x_2 \geq 80$
   - **Weights:** $w_{N3,x1} = 7$, $w_{N3,x2} = 10$
   - **Threshold:** $T_{N3} = 80$
4. **Neuron N4: Detects** $10x_1 + 4x_2 \leq 80$
   - **Weights:** $w_{N4,x1} = -10$, $w_{N4,x2} = -4$
   - **Threshold:** $T_{N4} = -80$
5. **Neuron N5: Detects** $10x_1 - 4x_2 \geq 20$
   - **Weights:** $w_{N5,x1} = 10$, $w_{N5,x2} = -4$
   - **Threshold:** $T_{N5} = 20$

Output Layer Neuron:

- **Neuron O: Outputs 1 if at least 4 conditions are met**
  - **Inputs:** Outputs of N1, N2, N3, N4, N5
  - **Weights:** $w_{O,N1} = 1$, $w_{O,N2} = 1$, $w_{O,N3} = 1$, $w_{O,N4} = 1$, $w_{O,N5} = 1$
  - **Threshold:** $T_O = 4$

Neuron Functions Explanation:

- **N1 to N5:** Each neuron checks one of the inequalities defining the blue region.
- **O:** Outputs 1 if any 4 or all 5 of the conditions are satisfied, ensuring the point is within the blue region.
- **Conditions for pentagon and triangle region:**
  1. For the central pentagon region all equations are satisfied
  2. For triangular arms 4 equations are satified


g(4)
OUTPUT
1
1
1
1
1
HIDDEN
g(10)
g(80)
g(-6)
g(20)
g(-80)
10
-7
7
10
0
-1
10
-4
-10
-4
x_1
x_2
INPUT