THE PRIME POPULATION

Project by Sanskriti School Team

Research Abstract / Synopsis

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- *Title:* *To predict the optimum population density in a given terrain to maximize the resource distribution, and optimize the welfare expectancy*

- *Research Focus:* *Mathematics and Statistics*

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- *Students in Grade:* *12*

- *Submitted to:* *31st Shri R.S. Niamat Rai Competition for Investigatory Projects in Mathematics, 2022, organized by Springdales School, Pusa Road, New Delhi*

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A BRIEF INTRODUCTION AND THE MOTIVE BEHIND THIS PROJECT

We try to predict the optimum population density in a given terrain to maximize the resource distribution. It's been done before, but our unique approach inculcates the attempt to maximize the welfare expectancy.

From the times of the early man, humans have frequently changed their habitats according to their basic needs. These needs include food, water, shelter from rain etc. During the old ages, kings had empires and they had their people living with them in a certain area with a border. When humans started farming, they moved to areas where they could grow crops. With the commencement of the Age of Industrialization, industries rose up, and people started working in factories. People needed a livelihood which saw a rise in the number of cities and towns. The rise of industries led to a rise in pollution of all sorts.

As time passed by, the factories produced a lot of pollution which resulted in the city being covered in smog. In the current scenario, people are becoming aware about global warming and trying to become sustainable and leave no carbon footprint. People are now becoming more aware of the effects of pollution and are trying to become more sustainable, by reducing their carbon footprint. As a result, they have moved to different places where they can experience a better environment.

Nowadays, people decide where they want to live based on a variety of factors.The primary goal of this research is to provide a brief overview of useful tips and techniques for determining the optimal population density for any given locality, along with some specific examples in various circumstances. Thus we have researched the criteria on which people live where they live and provide you the optimum population density in a given terrain based on the available resources available. The definition of population density is the number of people living per square unit area.

Similar case for animals.

Any wildlife biologist will tell you that the most important factor to consider when establishing a new wildlife habitat is ensuring there's enough food for the animals to survive. The number of animals that can live in a given space (known as "population density") without starving to death or running out of food entirely depends on the resources available. Each species has different nutritional requirements and likely prefers a different type of terrain. For example, foxes are known to thrive in areas with ample tree hollows, while squirrels prefer densely-packed hardwood trees. Reducing the population density of an area—and increasing its carrying capacity--reduces the likelihood of disease spreading among animals and helps keep them from over-exploiting their resources.

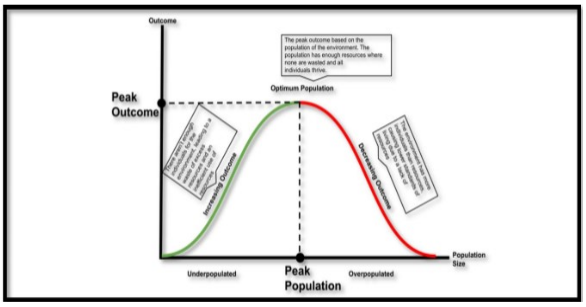
Resource distribution is also very important in the field of medicine.
For example, when the first wave of COVID-19 hit India the distribution of oxygen tanks and vaccines was very biased.
Now if the government had properly planned the resources available based on social-economic factors, the regions and population densities then this wouldn't have been a problem.

Economic disparity is evident in India- the rich get richer and the poor get poorer.

As the resources on Earth deplete too, the rich will have easier access while the poor would have the least access.
Our aim should be to provide basic needful resources to all without discrimination.
Thus the population should spread out in a manner that the government can monitor it and the resource distribution should be optimum.

Some important concepts in play:

The **Optimum Population Density** (OPD) is the population density of a given area that provides the best balance between resource availability and human stress. In other words, it is the optimal number of people living in a particular place, with limited land resources and natural amenities (such as fresh water, clean air, arable land, etc.). The OPD can be calculated based on particular characteristics of a given habitat. The population density of an area is usually measured by its number of inhabitants per unit area – for example, people per km^2. The optimum population density must take into account the natural carrying capacity and available resources in any given region.



Methodologies:

Density as an economic variable for analysis purposes has become common recently in the past few years, owing to the migratory conditions that have arisen. Quite greatly, the population density of any nation can affect the functioning, both internal and external. The case adopted herewith relates to a place with maximum utilization of resources, under the case of complete equilibrium, keeping the numerical utility measured constant. Determining an accurate relation between population and economic density is essential for the working of the model on its economic factors. With this in mind, the approach adopted by the team in relation to the mathematical model is on the basis of 4 criteria, wherein all the conditions have been adjusted to a suitable/sustainable point to maintain the accuracy of our mathematical model:

- ➔ We solve this problem by formulating an equation using linear regression, and work to find the weightages of each of the factors like resource availability, GNP ,etc. by using matrices.
- ➔ We then use bayesian statistics and differential equations to get the least deviation error of population density that can be predicted, to ensure best predicted results.
- ➔ Moreover, we use convolutional neural networking and linear regression programming to verify our final equation and minimize the error.

Data Collection:

In order to work with accurate data, we dug deep into country wide census reports, shortlisting places keeping in mind to keep a balance of places with one unique type of occupation, relief features etc. as well as places with a blend of multiple features. In this way, we can not only study category-specific variations of the population density but also find out what combination of all these factors works better for maximizing population for a limited amount of resources.

The data consists of 20 categories (5 binary, 14 float and 1 subjective) for 40 major cities, arranged in 4 groups of 10 each on the basis of their geographical location, i.e from Japan, Russia, the USA or Europe. This would help us form a case study comparing countries with a (potentially) negative population growth rate (Japan and Russia) with respect to thriving locations in the USA and Europe.

Our Criterias:

We take the following conditions into account:

1) *Terrain (mountain/hills/plains/plateau with sed/met rock/ …)*
2) *Gross National Product(GNP) per capita*
3) *Mean years of schooling*
4) *Health index - (also including the effect of diseases and outbreaks)*
5) *Water availability (and availability of water processing units)*
6) *Food sufficient (staple diets)*
7) *Minerals and other resources index*
8) *Happiness index*
9) *Birth rate*
10) *Mortality rate*
11) *Migration*
12) *Prevalent Industry*

Density as an economic variable for analysis purposes has become common recently in the past few years owing to the migratory conditions that have arisen. The population density of any nation can affect its functioning, both internally and externally. The case adopted herewith relates to a place with maximum utilization of resources, under the case of complete equilibrium, keeping the numerical utility measured constant. Determining an accurate relation between population and economic density is essential for the working of the model on its economic factors.

With this in mind, the approach adopted by the team concerning the mathematical model is based on 4 criteria, wherein all the conditions have been adjusted to a suitable/sustainable point to maintain the accuracy of our mathematical model:

1. *Population size functions in accordance with the Economic Factors, wherein expenditure is kept constant along with the consumer's utility obtained from consumption of resources. Producers and Consumers function with suitable economic laws that ensure a better and well-defined market with sustainable growth in factors like GNP per capita or GDP density that would in turn result in equitable growth/density in population size. Optimum Market Access+ Market Ratio+ Market Functioning*
   Market access is a term used to describe the govt policies on import and export, plus the willingness of foreign producers to trade with that particular region. Market Ratio is the ratio of producer to consumer. The ratio matters because if it's too high, business suffers, If it's too low, prices hike. Finally Market functioning refers to the efficiency of the entire system. For example, Bangalore is notorious for traffic, so any transport operation through or in Bangalore will face delays and hence will come under improper functioning. All of these market based factors have been significant inputs for the mathematical model, and their functions have been defined in the previous sections.

The core aspects of this section focus on defining the given place with optimum population density and we aim to focus adequately on producers and consumers, who function at the core of any economy. With stable growth on economic terms, it is expected that the present market laws ensure equitable access- in terms of preparation and distribution of finished goods, and also inter-community interaction; both of which ensure better overall market functioning, an aspect we require attention to before elaboration on the mathematical models.

Of the divisions mentioned, the focally significant aspects of the Economic factors are as follows:

- <u>1.1: GNP per capita:</u> An increase in GNP per capita would significantly lead to an increase in population density, limited till a particular point. Given the value of a sustainable economy in any region, consumers would profusely migrate to a tenable area for sustenance.
- <u>1.2: GDP density:</u> It is often referred to as GDP per sq km, calculated by multiplying GDP per capita with the Population Density. Although not a requisite in the mathematical model, it provides an idea of the effects of topographical factors on an economy where population density is being considered.
- <u>1.3: Migration:</u> Migration transition helps analyze the shift in consumer demands due to rise in opportunities while moving from one location to another. Higher values of opportunities are directly proportional to the increase in population density, which can be inferred from Ravenstein's Laws of Migration, which provide a theoretical framework for the research of migration.
- <u>1.4: Transportation:</u> At a region with an optimum population density, lower costs directly increase utility obtained from transport for the consumers and producers who would gain maximum output from the given conditions. Despite the polar conditions between positive and normative macroeconomic scenarios, market access and equilibrium are still important aspects of the economic factors of determination.

  Aside from the 4 aspects mentioned within the Economic Factors, other important macroeconomic variables and instances would include region wide economic growth, interest and exchange rates, general price levels and various other such examples. Within the reach of the economic factors, it is observed that these, directly or indirectly, hold quite a significant role in any analyses centered around the population of a region. Not only do they comprehensively define the functioning on a macroeconomic scale, but also supplement the statistical measures/methods used in past research papers.

2. *Population growth is in direct relation to provision of infrastructural components for the consumers to obtain maximum benefit and to provide satisfactory access to necessary goods and services. All the natural resources in the given terrain must be equally distributed and accessible to uphold the diversity*

without compromising the ecosystem. All institutional facilities require maximum utilization to improve the carrying capacity of aspects including education, healthcare, legalities, etc.

Proper employment of resources with their utilization depends on factors such as availability, extensivity and the legalities involved with the same. For this study, it is taken that all such factors have a suitable utilization as per the given region. It is also expected that consumer attitude towards the mentioned facilities is neutral on average, considering that an effective ecosystem involves aspects of consumer behavior.
The importance of infrastructural factors, being of equal importance, are rather classified deeply into the following significant factors:

- 2.1: Academical Institution Facilities: ADFs, comprising Mean years of schooling (MYS) and Literacy Rate, apart from the Infrastructural Educational Institutions, have been a widely considered factor in the determination of population density. It is hypothesized to be in direct relation to more opportunities, thereby resulting in more opportunities for consumers. Thus, ADFs, with their full utilization can lead to higher population density in a region concerned.
- 2.2: Health Provisions: Health provisions, essentially the core of infrastructural development in any nation, is also a major aspect in the determination of population density. Health provisions expand up to points including Medical Tourism, Health Indices and Institutions. With such provisions being considered essentials, the development of these facilities would be directly proportional to population density, whereas an increase in the quality of provisions would lead to an increase in population density. In this case however, the population should not cross a certain threshold for a given health index, as it might lead to depletion of substantial resources, as well as decrease the efficiency of human resources available. Falling under health provisions, Birth Rates are also a major facet of Infrastructural Factors. Being the number of births per thousand of population per year, a booming birth rate in countries like India have led to higher population density throughout the country. Birth rates also highly depend upon land area per square kilometer, an exception that could explain overpopulation in various third world countries- A nation may have an unusually high birth rate, yet still lower density due to more land area available per square km. The general trend of higher birth rates can explained as a simple proportional relationship with population density, and can be examined by the statistics in the data table below:

Country Name	Birth Rate (As per most recent census)	Population Density
UK	**11.322** births per 1,000 population	**281 people per km^2**
USA*	**12.012** births per 1000 people	**36 people per Km^2**
India	**17.163** births per 1000 people	**431.11 people per km^2**
Vietnam	**15.502** births per 1000 people	**296.43 people per km^2**
Iran*	**16.897** births per 1000 people	**16.897 people per km^2**
Spain	**8.014** births per 1000 people	**94.00 people per km^2**

All data has been collected from national government sites and International UN websites

*Exceptions on basis of land area

- 2.3: Natural Resources and Assets: With resources being the most valuable asset for any region, the given resources must be of proper use. Adequate allocation of these resources plays a significant role in determining the population density due to constant utility. Natural resources of any region to be considered as parameters for further research include the following:
    - Water availability: People in ancient times were more likely to go to a place with a good supply of water, and even today are more likely to live in regions where they can satisfy their need for water.
    - Food sufficient: Similar to the above point, we can find a higher population density in places with a satisfactory amount of food being grown.
    - Minerals and other resources index: The presence of mineral resources in a place leads to a higher population density due to increased employment opportunities. The presence of minerals or raw materials, such as coal or iron ore, can attract more population.

Having elaborated on both the aforementioned criteria, most aspects regarding the core of the mathematical model have been resolved. A commonly used factor in similar research is spatial or physiological factors. Terrain has a lot of importance in deciding which is more habitable, we are more likely to see a higher population density in the plains than mountains. The mathematical model is targeted at determining population density in any given terrain with the resources available. Each terrain has a different level of suitability for the consumers and producers, wherein the given resources at one region may be considered an inferior amount to another- In two regions 'A' and 'B', A might have a higher amount of iron ore stock present- our model results in better comparability between both regions wherein the interpretations drawn, can be applied in comparing the growth with respect to these factors.

The functioning and accuracy would depend on the correct analysis of the criteria, irrespective of whether the terrain is mountainous or plain. However, the land area of the region concerned must be kept in mind, as the population density depends on birth rates that go hand in hand with the size of the area. Singapore, with a population density of over 8000 people per square kilometer could be taken as an example, in contrast to the USA, with a population density of merely 34 people per square kilometer.

3. *Increase in population density is on the basis of welfare and progress implications for the community. Aspects including Welfare expectancy should be at an optimal rate, which essentially gives the total lifetime welfare a person born into the community concerned experiences. Keeping in mind the optimum ratio between resource use and population, the optimum population density defined through the sections of this paper is based on regions with adequate welfare interventions that keep the progressive implications growing at a constant rate (if defined quantitatively).*

   This criteria in particular is what defines the unique aspect presented in our model. Welfare expectancy depends a lot on the physiological factors, and also relies on the community itself, covering aspects including both size and functioning- from the life expectancy, to the working of government bodies. To elaborate on welfare interventions, it revolves around the qualitative aspect of every community, which can be expressed in numerical terms. Suppose two regions with different levels of happiness- a region with higher levels of happiness will have greater population density, as long as it covers adequate growth on economic terms and offers ease of access to institutions, primarily on how negative individuals impact community welfare. In essence, the scenario of population density impacting community welfare depends solely on the experiences of the consumer. With the model, we can determine the density of population with sufficient progress levels, and this could provide for the interventions aimed at improving the lives of the people.

Mathematics involved:

1) Linear Regression

Let y denote population density, and let, y_1 , y_2 , y_3 , …., y_n denotes the population densities of n different test cases.
n is the number of data values we have
m is the number of distinct values were considering

Next, we assume $y = a_1x_1 + a_2x_2 + \ldots + a_mx_m$
And, $y_1 = a_1x_{11} + \ldots + a_mx_{1m}$
And, $y_n = a_1x_{n1} + \ldots + a_mx_{nm}$
Thus, let's say for given m=13 factors we've a population density of 5000
y in this case will be 5000 and we'll have 13 values of the magnitudes of 13 factors x, as well as thirteen corresponding weights to them.

To find the coefficients, we could opt for 2 methods:

a) Matrices

Taking X as a n x m matrix, Y as a n x 1 matrix, and A as a m x 1 matrix,

We use: $Y = XA$
Multiplying by transpose on both sides, $X^TY = X^TXA$
Therefore, $(X^TX)^{-1} X^TY = A$

We will henceforth get values of all the coefficients.

b) Minimizing the error using Bayesian Statistics:

If y_1 denotes the actual value of y and y' denotes the predicted value of y calculated using the values a_1 , a_2 , … , a_n ,

Error, $E_1 = y_1 - y' = y_1 - (a_1x_{11} + \ldots + a_mx_{1m})$

E therefore, denotes a n x 1 matrix of all values of E_1 , E_2 , …. , E_n

The Square Sum Error (SSE), is thus, the sum of the squares of the values of the matrix E, i.e.:

$SSE = (E_1)^2 + (E_2)^2 + \ldots. + (E_n)^2 = (y_1 - (a_1x_{11} + \ldots + a_mx_{1m}))^2 + \ldots.$

2) Partial differentiation

$\frac{\delta(SSE)}{\delta a1} = 0$ (to find an a1 such that SSE would be least)

1. **CNN AI**
https://towardsdatascience.com/gentle-dive-into-math-behind-convolutional-neural-networks-79a07dd44cf9 [Image and explanation]

a. Convolution-
The most common example taken for convolution is to compress digital images to a lower size while preserving all relevant information, such that the drop in details is invisible to the user but drastically helping with storage. However, this application is irrelevant for the discussion at hand.

Mathematically speaking, convolution is an operator on two functions (matrices) that produces a third function (matrix), which is the modified input by the other having different features (values in the matrix).
For example, consider a 5x5 matrix(I), and two 3x3 matrices (K and O) as follows:

I =

$$\begin{bmatrix} 3 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 3 & 1 & 2 & 2 & 3 \\ 2 & 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 0 & 1 \end{bmatrix}$$

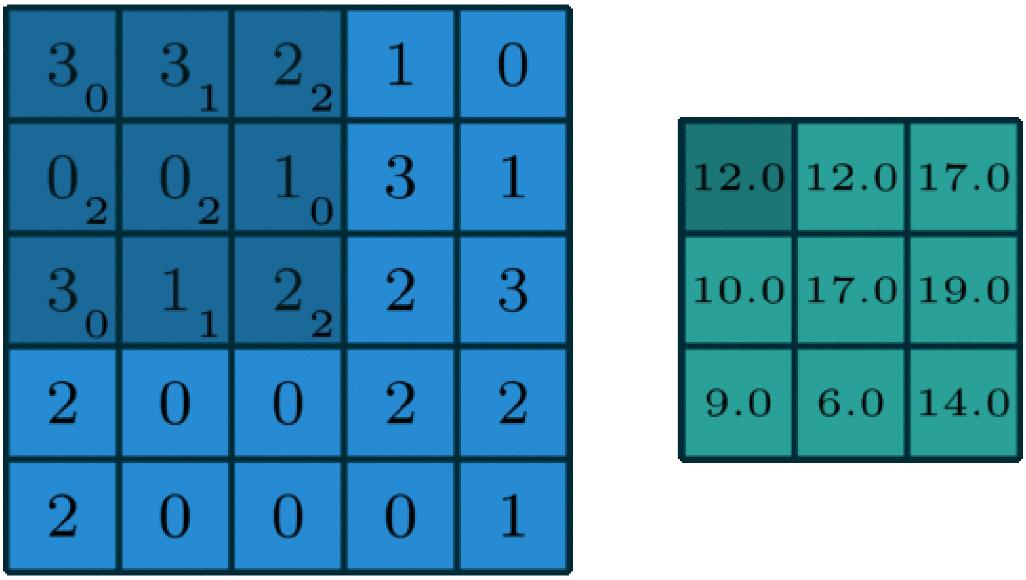
K=

$$\begin{bmatrix} 0 & 1 & 2 \\ 2 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

O=

$$\begin{bmatrix} 12 & 12 & 17 \\ 10 & 17 & 19 \\ 9 & 6 & 14 \end{bmatrix}$$

The following animation depicts the process of convolution-



A few relevant terms:

i. ***Input:*** Matrix I above is the input to the convolution.

ii. ***Output:*** Matrix O in the above example is the output.

iii. ***Kernel:*** A kernel is a small matrix that uses the power of localisation to extract the required features from the given function/matrix.

iv. ***Slide:*** The 3x3 matrix that is a subset of I and whose dot product is taken with the Kernel matrix is referred to as the slide. While the Input, Output and Kernel will stay the same during one convolution cycle, the slide changes in every step.

v. ***Stride:*** The idea of the stride is to skip some of the slide locations of the kernel. A stride of 1 means to pick slides one element apart, so basically every single slide, acting as a standard convolution. A stride of 2 means picking slides 2 elements apart, skipping every other slide in the process, downsizing by roughly a factor of 2.

vi. ***Padding:*** It refers to adding elements at the edges of the Output in order to make it the same size/order as the Input matrix. These elements are usually null or 0 value inputs. It helps in keeping the constant output size, otherwise with the use of kernels, the output is a smaller dimension and could create a bottleneck in some scenarios.

When the padding, stride and kernel in a convolution are symmetric (equal for height and width):

$$o = \left\lfloor \frac{i - k + 2 \cdot p}{s} \right\rfloor + 1$$

Where:
i -> input shape (height = width)
k -> kernel shape
p -> padding along the edges of the image
s -> stride for the convolution (for sliding dot product)

<u>An Alternate Definition</u>

A more technical description of convolution is as follows:

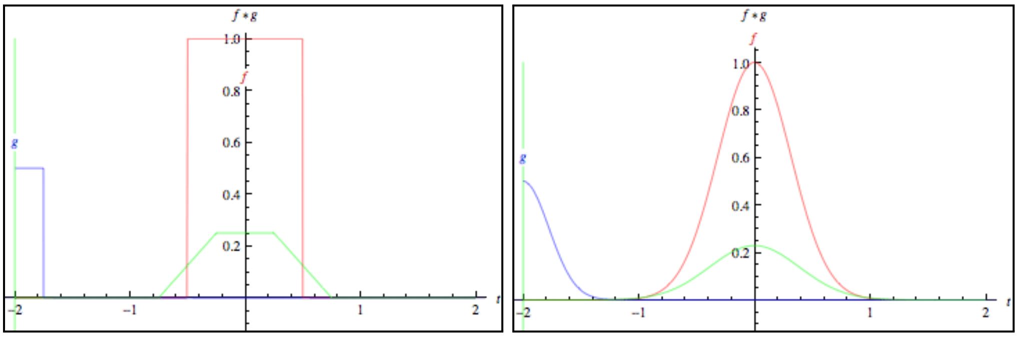
“A convolution is an integral that expresses the amount of overlap of one functioning as it is shifted over another function f. It therefore "blends" one function with another. For example, in synthesis imaging, the measured dirty map is a convolution of the "true" CLEAN map with the dirty beam (the Fourier transform of the sampling distribution).”

Abstractly, a convolution is defined as a product of functions f and g that are objects in the algebra of Schwartz functions in R^n. Convolution of two functions f and g over a finite range [0,t] is given by

$$[f * g](t) \equiv \int_0^t f(\tau)\, g(t-\tau)\, d\tau,$$

where the symbol [f*g](t) denotes convolution of f and g.
Convolution is more often taken over an infinite range,

$$\begin{aligned} f * g &\equiv \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau \\ &= \int_{-\infty}^{\infty} g(\tau)\, f(t-\tau)\, d\tau \end{aligned}$$



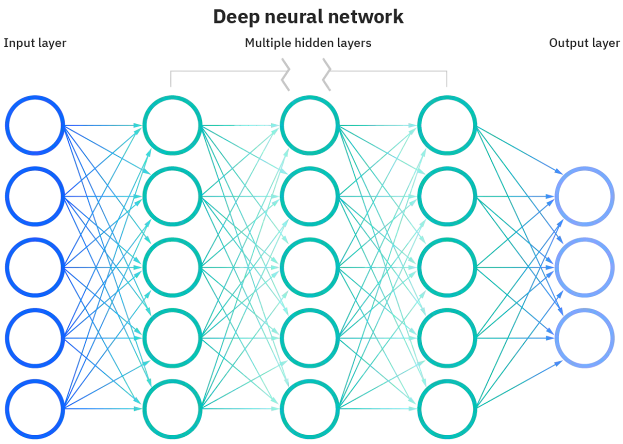
The animations above graphically illustrate the convolution of two boxcar functions (left) and two Gaussians (right). In the plots, the green curve shows the convolution of the blue and red curves as a function of t, the position indicated by the vertical green line. The gray region indicates the product g(τ)f(t-τ) as a function of t, so its area as a function of t is precisely the convolution. One feature to emphasize and which is not conveyed by these illustrations (since they both exclusively involve symmetric functions) is that the function g must be mirrored before lagging it across f and integrating.

b. <u>Neural</u>:

The word Neural refers to the nervous system of a body. Like that of a human, an Artificial Neural Network is based on a collection of connected units called artificial neurons. They are similar to those of human brains. A Neural Network is a system where the neurons are divided into different layers, depicting different tiers of information. Each neuron in a layer is connected to every single neuron in the neighboring layers.

It can be visualized as node layers, containing an input layer, one or more hidden layers and an output layer. Each node has a specific threshold value assigned to it. If the output of any node is more than the threshold, it is activated and sends data to the next layer of the network.

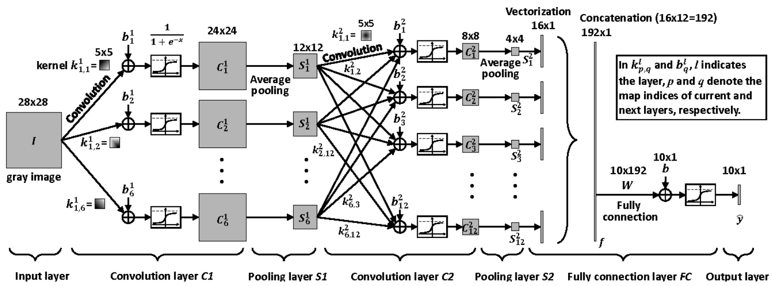
Neural Networks learn by processing examples, consisting of input and results, forming probability weighted associations between the two, which are stored within the data structure of the net itself.They learn by solely getting the information and the result, whether it is true or false. Like the nervous system, the ANN will learn and improve its accuracy over time.



c. <u>Networking:</u>

In each layer of the CNN, each node connects to another. A CNN also has an associated weight; as the layers' filters move across the image, the weights remain fixed -- a condition known as parameter sharing. Convolutional neural networks are composed of multiple layers of artificial neurons. Artificial neurons, a rough imitation of their biological counterparts, are mathematical functions that calculate the weighted sum of multiple inputs and output an activation value. This makes the whole CNN system less computationally intensive than an NN system.

Training a network is a process of finding kernels in convolution layers and weights in fully connected layers which minimize differences between output predictions and given ground truth labels on a training dataset. Backpropagation algorithm is the method commonly used for training neural networks where loss function and gradient descent optimization algorithm play essential roles. A model performance under particular kernels and weights is calculated by a loss function through forward propagation on a training dataset, and learnable parameters, namely kernels and weights, are updated according to the loss value through an optimization algorithm called backpropagation and gradient descent.



1. The mathematical models are targeted at determining population density in any given terrain with the resources available. Each terrain has a different level of suitability, wherein the given resources at one region may be considered an inferior amount to another. What the model results in is better comparability between two regions. Interpretations drawn from the calculations can be applied in comparing the growth in regions with respect to factors apart from population density. The functioning and accuracy would depend on correct analysis of the criteria irrespective of whether the terrain is mountainous or plain.

2. Increase in population density is on the basis of welfare and progress implications of the community. Aspects including Welfare expectancy should be at an optimal rate, which essentially gives the total lifetime welfare a person born into the community concerned experiences. Keeping in mind the optimum ratio between resource use and population, optimum population density defined through the sections of this paper are based on regions with adequate welfare interventions that keep the progressive implications growing at a constant rate (if defined quantitatively).

This criteria in particular is what defines the unique aspect presented in our model. Welfare expectancy depends a lot on the physiological factors, and also relies on the community itself, covering aspects including both size and functioning- from the life expectancy, to the working of government bodies. To elaborate on welfare interventions, it revolves around the qualitative aspect of every community, which can be expressed in numerical terms. Suppose two regions with different levels of happiness- a region with higher levels of happiness will have greater population density, as long as it covers adequate growth on economic terms and offers ease of access to institutions, primarily on how negative individuals impact community welfare. In essence, the scenario of population density impacting community welfare depends solely on the experiences of the consumer. With the model, we can determine the density of population with sufficient progress levels, and this could provide for the interventions aimed at improving the lives of the people.

Concluding remarks, and findings:

Through the given data, one can approximate the optimum places which encourage more population density based on the aforementioned factors. We will also predict a future trend on how the population density will grow, and hence conclude with the best strategy to sustainably help in the development of the region by managing the population in that region and ultimately help in the development of the country.

BIBLIOGRAPHY

- https://www.google.com/url?q=https://worldeconomics.com/Indicator-Data/ESG/Social/Mean-Years-of-Schooling/&sa=D&source=docs&ust=1667410062937214&usg=AOvVaw2TWFW2SrNnOXr-JUrVQuJi
- https://www.google.com/url?q=https://worldpopulationreview.com/country-rankings/immigration-by-country&sa=D&source=docs&ust=1667410062938317&usg=AOvVaw0S3HZSXgGuXz7sDc4qeZ5J
- https://www.google.com/url?q=https://worldpopulationreview.com/country-rankings/immigration-by-country&sa=D&source=docs&ust=1667410062939257&usg=AOvVaw0FbUzltQeWD1abtT2XraFK

- https://www.google.com/url?q=https://medium.com/firm-narrative/want-a-better-pitch-watch-this-328b95c2fd0b&sa=D&source=docs&ust=1667410062922806&usg=AOvVaw3OE2l7aZEOjt3jxK8xtRdz
- https://www.google.com/url?q=https://www.tandfonline.com/doi/abs/10.1080/15230406.2019.1687014&sa=D&source=docs&ust=1667410062919761&usg=AOvVaw33Qh6Bv1dSYf41Q7JOxKqc
- https://www.google.com/url?q=https://overpopulation-project.com/a-new-approach-for-defining-optimum-population-size/amp/&sa=D&source=docs&ust=1667410062920901&usg=AOvVaw1_2J--1CYQ2zd8RpQlhRDT
- https://www.google.com/url?q=https://people.eecs.berkeley.edu/~varaiya/Download/Varaiya-Optimization.pdf&sa=D&source=docs&ust=1667410062921826&usg=AOvVaw2TzwxcVEFgdvAb6V9HTqQR
- https://www.google.com/url?q=https://coral.ise.lehigh.edu/katyas/files/OPTML/Lecture20.pdf&sa=D&source=docs&ust=1667410062923564&usg=AOvVaw1zayiZsFKJTvXQSdt5KC3-
- https://towardsdatascience.com/gentle-dive-into-math-behind-convolutional-neural-networks-79a07dd44cf9
- https://www.google.com/url?q=https://miro.medium.com/max/640/0*TW1CoVdj92yxZF1s.gif&sa=D&source=docs&ust=1667410062926677&usg=AOvVaw0zfNrDp_c61Q5CL0sSXjEW
- https://www.google.com/url?q=https://mathworld.wolfram.com/Convolution.html&sa=D&source=docs&ust=1667410062927928&usg=AOvVaw39PTB-eWZl7wJXodJgdih3
- https://www.google.com/url?q=https://www.techtarget.com/searchenterpriseai/definition/convolutional-neural-network&sa=D&source=docs&ust=1667410062930955&usg=AOvVaw0QIyNppGg9XK1pHCjenCRU
- https://www.google.com/url?q=https://insightsimaging.springeropen.com/articles/10.1007/s13244-018-0639-9&sa=D&source=docs&ust=1667410062931126&usg=AOvVaw22l7G_jUrK4dYUHyZ96gQF